1/1

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{36^2 + 15^2} = 39$$

$$\cos\theta_x = \frac{V_x}{V} = \frac{-36}{39}, \quad \underline{\theta_x = 157.4°}$$

$$\cos\theta_y = \frac{V_y}{V} = \frac{15}{39}, \quad \underline{\theta_y = 67.4°}$$

$$\underline{n} = \frac{\underline{V}}{V} = \frac{-36\underline{i} + 15\underline{j}}{39} = \underline{-0.923\underline{i} + 0.385\underline{j}}$$

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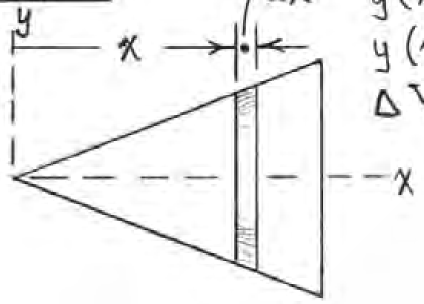
1/10



$y(x) = \frac{r}{h} x$

$y(x+\Delta x) = \frac{r}{h}(x+\Delta x)$

$\Delta V = V(x+\Delta x) - V(x)$

$= \frac{1}{3}\pi[y(x+\Delta x)]^2[x+\Delta x] - \frac{1}{3}\pi[y(x)]^2[x]$

$= \frac{1}{3}\pi\left[\frac{r}{h}(x+\Delta x)\right]^2[x+\Delta x] - \frac{1}{3}\pi\left[\frac{r}{h}x\right]^2[x]$

$= \frac{1}{3}\pi\frac{r^2}{h^2}\left[(x+\Delta x)^3 - x^3\right]$

$= \frac{1}{3}\pi\frac{r^2}{h^2}\left[x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3\right]$

$= \frac{\pi r^2}{h^2}\left[x^2\Delta x + x(\Delta x)^2 + \frac{1}{3}(\Delta x)^3\right]$

In the limit as $\Delta x \Rightarrow dx$, the higher-order terms drop out.

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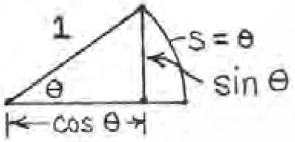
1/11 $20°\left(\frac{\pi \text{ rad}}{180°}\right) = 0.3491 \text{ rad}$

$\sin 20° = 0.3420$

Percent error is $\frac{0.3420 - 0.3491}{0.3420}(100) = \underline{2.06\%}$

$\tan 20° = 0.3640$

Percent error is $\frac{0.3640 - 0.3491}{0.3640}(100) = \underline{4.09\%}$



The approximation $\sin\theta \cong \theta$ involves the approximation that the arclength $s = \theta$ is the vertical side of the triangle. The approximation that $\tan\theta \cong \theta$ involves, in addition, the approximation that 1 is the horizontal side of the triangle.

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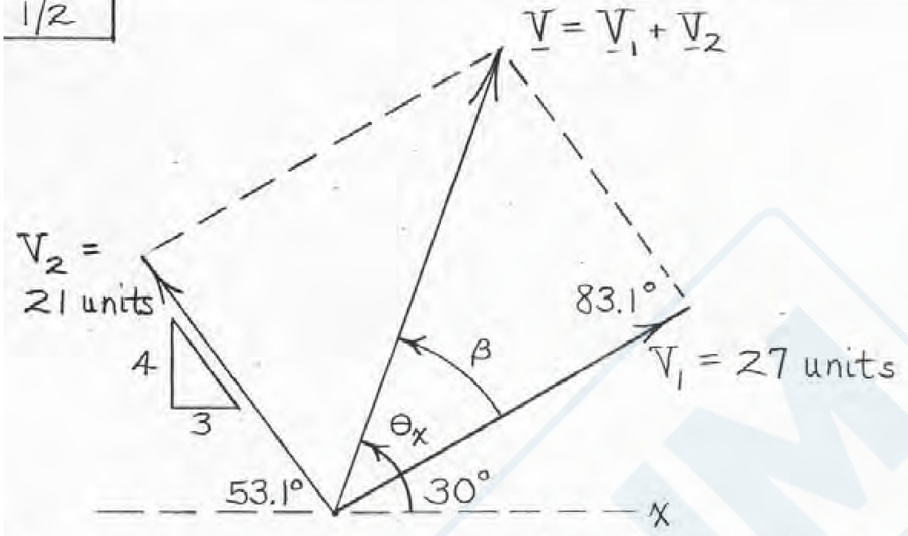
1/2



Graphically, $V = \underline{32 \text{ units}}$, $\underline{\theta_x = 70°}$

Algebraically, $V^2 = 27^2 + 21^2 - 2(27)(21)\cos 83.1°$

$$V = \underline{32.2 \text{ units}}$$

$$\frac{\sin\beta}{21} = \frac{\sin 83.1°}{32.2}, \quad \beta = 40.4°$$

$$\theta_x = \beta + 30° = 40.4° + 30° = \underline{70.4°}$$

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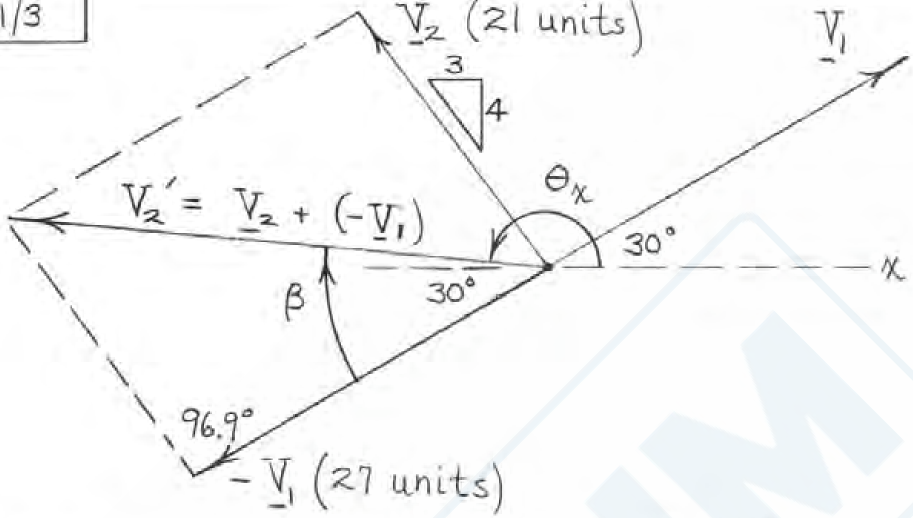
1/3



Graphically, $\underline{V' = 36 \text{ units}}$, $\underline{\theta_x = 175°}$

Algebraically, $V'^2 = 27^2 + 21^2 - 2(27)(21)\cos 96.9°$

$\underline{V' = 36.1 \text{ units}}$

$\frac{\sin\beta}{21} = \frac{\sin 96.9°}{36.1}$, $\beta = 35.2°$

$\theta_x + \beta = 210°$, $\theta_x = 210 - 35.2° = \underline{174.8°}$

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1/4

$$F = \sqrt{160^2 + 80^2 + 120^2} = 215 \text{ lb}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{160}{215} = 0.743, \qquad \underline{\theta_x = 42.0°}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{80}{215} = 0.371, \qquad \underline{\theta_y = 68.2°}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-120}{215} = -0.557, \qquad \underline{\theta_z = 123.9°}$$

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1/5

$$m = \frac{W}{g} = \frac{1000}{32.2} = \underline{31.1 \text{ slugs}}$$

$$m = 31.1 \text{ slugs} \left(\frac{14.594 \text{ kg}}{\text{slug}} \right) = \underline{453 \text{ kg}}$$

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1/6 $F = W = \dfrac{Gm_1m_2}{r^2}$

where $G = 6.673(10^{-11})\ m^3/(kg \cdot s^2)$,

$m_1 = 80\ kg$

$m_2 = 5.976(10^{24})\ kg$

and $r = (6371 + 250)(10^3)\ m$

Substitute these numbers & obtain $\underline{W = 728\ N}$

U.S. units : $W = 728\ N\left(\dfrac{1\ lb}{4.4482\ N}\right) = \underline{163.6\ lb}$

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1/7 $W = (125 \text{ lb})\left(\frac{4.4482 \text{ N}}{\text{lb}}\right) = \underline{556 \text{ N}}$

$$m = \frac{W}{g} = \frac{125}{32.2} = \underline{3.88 \text{ slugs}}$$

$$m = \frac{W}{g} = \frac{556}{9.81} = \underline{56.7 \text{ kg}}$$

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1/8

$A = 8.67, \quad B = 1.429$

$(A+B) = 8.67 + 1.429 = \underline{10.10}$

$(A-B) = 8.67 - 1.429 = \underline{7.24}$

$(AB) = (8.67)(1.429) = \underline{12.39}$

$(A/B) = 8.67/1.429 = \underline{6.07}$

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1/9

$$F = \frac{G m_e m_s}{d^2} = \frac{3.439(10^{-8})(1)(333{,}000)(4.095 \cdot 10^{23})^2}{(92.96 \cdot 10^6 \cdot 5280)^2}$$

$$= \underline{7.97(10^{21}) \text{ lb}}$$

$$F = 7.97(10^{21}) \text{ lb} \left(\frac{4.4482 \text{ N}}{\text{lb}}\right) = \underline{3.55(10^{22}) \text{ N}}$$

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2/1

$$\begin{cases} F_x = -800 \sin 35° = \underline{-459 \text{ N}} \\ F_y = 800 \cos 35° = \underline{655 \text{ N}} \end{cases}$$

$$\underline{\underline{F} = -459\underline{i} + 655\underline{j} \text{ N}}$$

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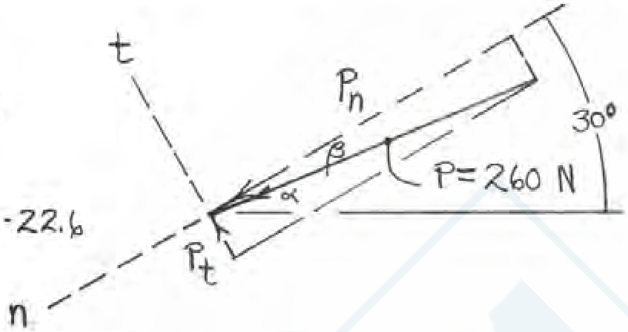
2/10

$\alpha = \tan^{-1}\frac{5}{12}$

$= 22.6°$

$\beta = 30 - \alpha = 30 - 22.6$

$= 7.38°$



$P_n = P\cos\beta = 260 \cos 7.38° = \underline{258 \text{ N}}$

$P_t = P\sin\beta = 260 \sin 7.38° = \underline{33.4 \text{ N}}$

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2/100

$$\underline{T} = T\underline{n}_{BA}$$

$$\underline{T} = 12\left[\frac{-35\underline{i} + 25\underline{j} + 60\underline{k}}{\sqrt{35^2 + 25^2 + 60^2}}\right]$$

$$= -5.69\underline{i} + 4.06\underline{j} + 9.75\underline{k} \text{ kN}$$

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2/101

$$\underline{F} = F\underline{n}_{AB}$$

$$= 750\left[\frac{-40\underline{i} + 70\underline{j} + 65\underline{k}}{\sqrt{40^2 + 70^2 + 65^2}}\right]$$

$$= -290\underline{i} + 507\underline{j} + 471\underline{k} \text{ lb}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{-290}{750}, \quad \theta_x = 112.7°$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{507}{750}, \quad \theta_y = 47.5°$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{471}{750}, \quad \theta_z = 51.1°$$

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2/102 $F_z = 5 \cos 40° = 3.83$ kN

$F_h = 5 \sin 40° = 3.21$ kN

$F_x = -3.21 \sin 35° = -1.843$ kN

$F_y = 3.21 \cos 35° = 2.63$ kN

So $\underline{F} = -1.843\underline{i} + 2.63\underline{j} + 3.21\underline{k}$ kN

$F_{OA} = \underline{F} \cdot \underline{n}_{OA}$

$= (-1.843\underline{i} + 2.63\underline{j} + 3.21\underline{k}) \cdot (\cos 30°\underline{i} + \sin 30°\underline{j})$

$= -0.280$ kN (as a scalar)

$\underline{F}_{OA} = F_{OA}\,\underline{n}_{OA}$

$= -0.280(\cos 30°\underline{i} + \sin 30°\underline{j})$

$= -0.243\underline{i} - 0.1401\underline{j}$ kN

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2/103

$$\underline{F} = F\underline{n} = 300\left[\frac{4\underline{i} - 8\underline{j} - 8\underline{k}}{\sqrt{4^2 + 8^2 + 8^2}}\right]$$

$$= 300\left[\frac{1}{3}\underline{i} - \frac{2}{3}\underline{j} - \frac{2}{3}\underline{k}\right] \text{ lb}$$

$F_x = 100$ lb, $F_y = -200$ lb, $F_z = -200$ lb

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2/104

$$\underline{T} = T\underline{n}_{AB} = 2.4\left(\frac{2\underline{i} + \underline{j} - 5\underline{k}}{\sqrt{2^2 + 1^2 + 5^2}}\right)$$

$$= \underline{0.876\underline{i} + 0.438\underline{j} - 2.19\underline{k} \text{ kN}}$$

Projection $T_{AC} = \underline{T} \cdot \underline{n}_{AC}$

$$= (0.876\underline{i} + 0.438\underline{j} - 2.19\underline{k}) \cdot \left(\frac{2\underline{i} - 2\underline{j} - 5\underline{k}}{\sqrt{2^2 + 2^2 + 5^2}}\right)$$

$$= \underline{2.06 \text{ kN}}$$

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2/105

$$\underline{T} = T\frac{\overline{CD}}{\overline{CD}} = 1.2\,\frac{1.5\underline{i} + 3\underline{j} - 4.5\underline{k}}{\sqrt{1.5^2 + 3^2 + 4.5^2}}$$

$$= \underline{0.321\underline{i} + 0.641\underline{j} - 0.962\,\underline{k}\ \text{kN}}$$

The two indicated coordinate systems are equivalent for the question at hand.

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2/106

$$T_{GF} = \underline{T} \cdot \underline{n}_{GF}$$

$$= (0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}) \cdot \frac{2\underline{i} - 3\underline{k}}{\sqrt{2^2+3^2}}$$

$$= \underline{0.978 \text{ kN}}$$

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2/107

$$\underline{T} = T\underline{n}_{AB} = 10\left[\frac{4\underline{i} - 7.5\underline{j} + 5\underline{k}}{(4^2 + (-7.5)^2 + 5^2)^{1/2}}\right]$$

$$= 10\ (0.406\,\underline{i} - 0.761\,\underline{j} + 0.507\,\underline{k})\ \text{kN}$$

$\cos\theta_x = 0.406$, $\theta_x = 66.1°$

$\cos\theta_y = -0.761$, $\theta_y = 139.5°$

$\cos\theta_z = 0.507$, $\theta_z = 59.5°$

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2/108 The coordinates of point B are

$$(x_B, y_B, z_B) = (1.6, -0.8 \sin 30°, 0.8 \cos 30°)$$
$$= (1.6, -0.4, 0.693) \text{ m}$$

The position vector $\underline{BC}$ is

$$\underline{BC} = (0-1.6)\underline{i} + (0.7-(-0.4))\underline{j} + (1.2-0.693)\underline{k}$$
$$= -1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k} \text{ m}$$

The unit vector which characterizes $\underline{BC}$ is

$$\underline{n}_{BC} = \frac{-1.6\underline{i} + 1.1\underline{j} + 0.507\underline{k}}{\sqrt{1.6^2 + 1.1^2 + 0.507^2}}$$

$$= -0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k}$$

Then $\underline{T} = T\underline{n}_{BC}$

$$= 750(-0.797\underline{i} + 0.548\underline{j} + 0.253\underline{k})$$
$$= -598\underline{i} + 411\underline{j} + 189.5\underline{k} \text{ N}$$

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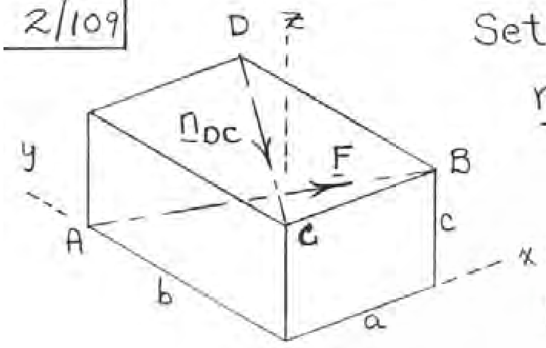
2/109



Set up x-y-z axes shown

$$\mathbf{n}_{DC} = \frac{\overline{DC}}{DC}$$

$$= \frac{-a\mathbf{i} - b\mathbf{j}}{\sqrt{a^2 + b^2}}$$

$$\mathbf{F} = F\mathbf{n}_{AB} = F\frac{\overline{AB}}{AB}$$

$$= F\frac{a\mathbf{i} - b\mathbf{j} + c\mathbf{k}}{\sqrt{a^2 + b^2 + c^2}}$$

$$F_{DC} = \mathbf{F}\cdot\mathbf{n}_{DC} = \frac{(b^2 - a^2)F}{\sqrt{(a^2+b^2)}\sqrt{(a^2+b^2+c^2)}}$$

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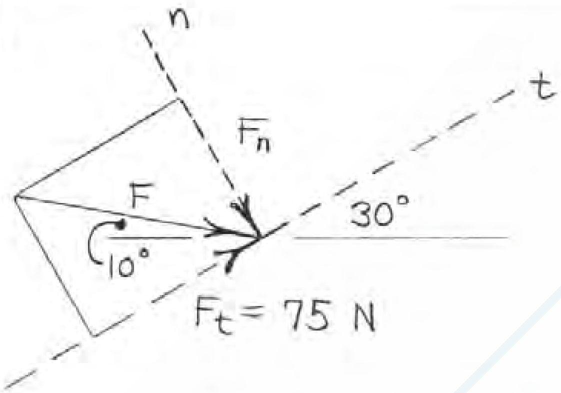
2/11



$$\frac{|F_n|}{F_t} = \tan 40^\circ, \quad |F_n| = F_t \tan 40^\circ = 75 \tan 40^\circ$$

$$F_n = \underline{-62.9\text{ N}}$$

$$\frac{F_t}{F} = \cos 40^\circ, \quad F = \frac{F_t}{\cos 40^\circ} = \frac{75}{\cos 40^\circ}$$

$$= \underline{97.9\text{ N}}$$

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2/110

$$\mathbf{F} = F\mathbf{n}_{OM}$$

$$= F\left[\frac{\frac{a}{2}\mathbf{i} + b\mathbf{j} + \frac{c}{2}\mathbf{k}}{\sqrt{\frac{a^2}{4} + b^2 + \frac{c^2}{4}}}\right]$$

$$= F\left[\frac{a\mathbf{i} + 2b\mathbf{j} + c\mathbf{k}}{\sqrt{a^2 + 4b^2 + c^2}}\right]$$

$$F_{AE} = \mathbf{F}\cdot\mathbf{n}_{AE}$$

$$= F\left[\frac{a\mathbf{i} + 2b\mathbf{j} + c\mathbf{k}}{\sqrt{a^2 + 4b^2 + c^2}}\right]\cdot\left[\frac{-a\mathbf{i} - b\mathbf{j}}{\sqrt{a^2 + b^2}}\right]$$

$$= -\frac{F(a^2 + 2b^2)}{\sqrt{a^2+b^2}\sqrt{a^2+4b^2+c^2}}$$

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2/111

$$\mathbf{F} = F\mathbf{n}_{AB} = 200\left[\frac{-12\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}}{\sqrt{12^2 + 24^2 + 8^2}}\right]$$

$$= -85.7\mathbf{i} + 171.4\mathbf{j} + 57.1\mathbf{k} \text{ lb}$$

$\mathbf{OC} = 12\mathbf{i} + 24\mathbf{j}$ in.

The angle θ between $\mathbf{F}$ and $\mathbf{OC}$ is

$$\theta = \cos^{-1}\frac{\mathbf{F}\cdot\mathbf{OC}}{F(\overline{OC})} = \cos^{-1}\left[\frac{-85.7(12) + 171.4(24)}{200\left(\sqrt{12^2+24^2}\right)}\right]$$

$$= \underline{54.9^\circ}$$

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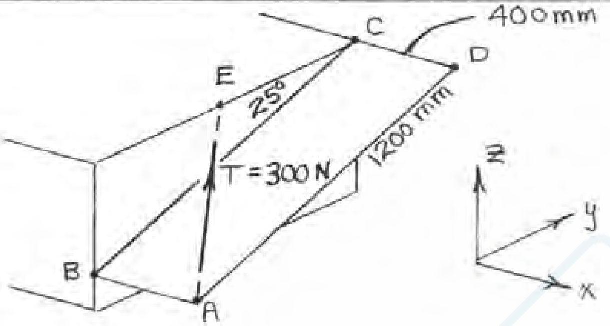
2/112



$$\underline{T} = T\underline{n}_{AE} = 300\left[\frac{-400\underline{i} + 544\underline{j} + 507\underline{k}}{\sqrt{400^2 + 544^2 + 507^2}}\right]$$

$$= 300\left[-0.474\underline{i} + 0.644\underline{j} + 0.601\underline{k}\right] \text{ N}$$

$$\underline{n}_{BC} = \cos 25°\underline{j} + \sin 25°\underline{k}$$

Carry out $T_{BC} = \underline{T}\cdot\underline{n}_{BC}$ to obtain

$$T_{BC} = 251 \text{ N}$$

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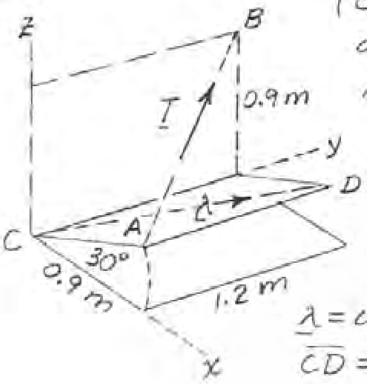
2/113 $T = 150$ N



Coordinates of A are $(0.9\cos 30°, 0, 0.9\sin 30°)$ or $(0.779, 0, 0.45)$ m

$$\overline{AB} = \sqrt{0.779^2 + 1.2^2 + (0.9 - 0.45)^2} = 1.50 \text{ m}$$

$$l = -0.779/1.5,\ m = 1.2/1.5,\ n = \frac{0.45}{1.5}$$

$$\underline{T} = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k}) \text{ N}$$

$\underline{\lambda}$ = unit vector along CD

$$\overline{CD} = \sqrt{0.9^2 + 1.2^2} = 1.5 \text{ m}$$

$$l = 0.779/1.5,\ m = 1.2/1.5,\ n = 0.45/1.5$$

$$\underline{\lambda} = \frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$$

$$T_{CD} = \underline{T} \cdot \underline{\lambda} = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k}) \cdot \frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$$

$$= \frac{100}{2.25}(-0.779^2 + 1.2^2 + 0.45^2) = \frac{100}{2.25}(-0.6075 + 1.44 + 0.2025)$$

$$= \frac{400}{9}(1.035) = \underline{46.0 \text{ N}}$$

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▸2/114 The position of point A is

$\underline{r}_A = 10\cos 15°\underline{i} + L\underline{j} + 10\sin 15°\underline{k}$

$= 9.66\underline{i} + L\underline{j} + 2.59\underline{k}$ in. [L = distance from O to disk center]

$\underline{r}_B = 8\cos 30°\underline{i} + (L+36)\underline{j} - 8\sin 30°\underline{k}$

$= 6.93\underline{i} + (L+36)\underline{j} - 4\underline{k}$ in.

$\underline{r}_{AB} = \underline{r}_B - \underline{r}_A = -2.73\underline{i} + 36\underline{j} - 6.59\underline{k}$ in.

$r_{AB} = \sqrt{2.73^2 + 36^2 + 6.59^2} = 36.7$ in. | unstretched length is $\sqrt{(8-10)^2 + 36^2} = 36.1$ in.

The spring force is $F = k\delta = 15(36.7 - 36.1) = 9.66$ lb

As a vector: $\underline{F} = F\underline{n}_{AB} = F\dfrac{\underline{r}_{AB}}{r_{AB}}$

$\underline{F} = 9.66\left[\dfrac{-2.73\underline{i} + 36\underline{j} - 6.59\underline{k}}{36.7}\right]$

$= -0.719\underline{i} + 9.48\underline{j} - 1.734\underline{k}$ lb

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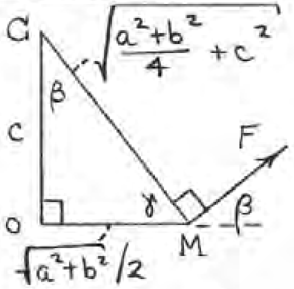
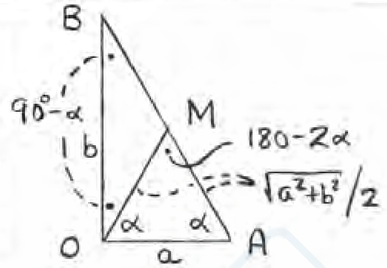




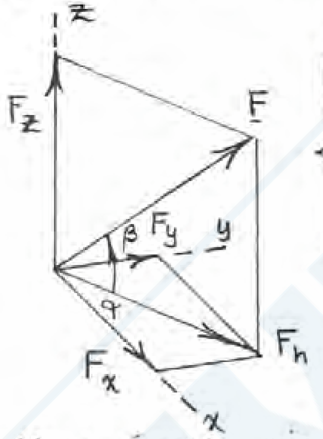
$$\tan\gamma = \frac{c}{\sqrt{a^2+b^2}/2} = \frac{2c}{\sqrt{a^2+b^2}}$$

$$\gamma + 90° + \beta = 180°$$

$$\beta = 90° - \gamma = 90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}$$

$$\tan\alpha = \frac{b}{a}$$

$$\left\{\begin{array}{l} \cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}} \\ \sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}} \end{array}\right.$$



$$\left\{\begin{array}{l} F_z = F\sin\beta \\ F_h = F\cos\beta \\ F_x = F_h\cos\alpha = F\cos\beta\cos\alpha \\ F_y = F_h\sin\alpha = F\cos\beta\sin\alpha \end{array}\right.$$

Now simplify $\sin\beta$ & $\cos\beta$ expressions:

$$\sin\beta = \sin\left[90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$$

$$= \sin 90° \cos\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right] - \cos 90° \sin\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$$

($\cos 90° \to 0$)

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+4c^2}}$$ (Also see that $\sin\beta = \cos\gamma$!)

$$\cos\beta = \sin\gamma = \frac{c}{\sqrt{\frac{a^2+b^2}{4}+c^2}} = \frac{2c}{\sqrt{a^2+b^2+4c^2}}$$

Finally,

$$F_x = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{a}{\sqrt{a^2+b^2}} = \frac{2acF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$$

$$F_y = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{b}{\sqrt{a^2+b^2}} = \frac{2bcF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$$

$$F_z = F\sqrt{\frac{a^2+b^2}{a^2+b^2+4c^2}}$$

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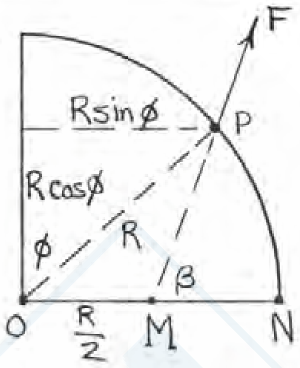
▶2/116

$F_x = F_{xy}\cos\theta,\quad F_y = F_{xy}\sin\theta$

$F_z = F\sin\beta,\quad F_{xy} = F\cos\beta$



$$\tan\beta = \frac{R\cos\phi}{R\sin\phi - \frac{R}{2}} = \frac{2\cos\phi}{2\sin\phi - 1}$$

So $$\sin\beta = \frac{2\cos\phi}{\sqrt{(2\sin\phi - 1)^2 + (2\cos\phi)^2}}$$

$$\cos\beta = \frac{2\sin\phi - 1}{\sqrt{(2\sin\phi - 1)^2 + (2\cos\phi)^2}}$$

Note that $\sqrt{(2\sin\phi-1)^2 + (2\cos\phi)^2} = \sqrt{5 - 4\sin\phi}$

So $\underline{F} = F\left[\cos\theta\cos\beta\,\underline{i} + \sin\theta\cos\beta\,\underline{j} + \sin\beta\,\underline{k}\right]$

$$= \frac{F}{\sqrt{5 - 4\sin\phi}}\left[(2\sin\phi - 1)(\cos\theta\,\underline{i} + \sin\theta\,\underline{j}) + 2\cos\phi\,\underline{k}\right]$$

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2/117 By inspection, $\underline{M}_O = F(c\underline{j} - b\underline{k})$

Or, $\underline{M}_o = \underline{r} \times \underline{F} = (b\underline{j} + c\underline{k}) \times F\underline{i}$

$= F(c\underline{j} - b\underline{k})$ ✓

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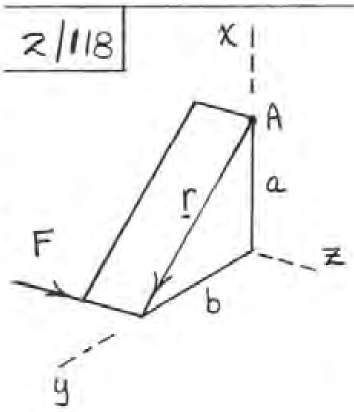

$$\underline{M}_A = \underline{r} \times \underline{F}$$

$$= (-a\underline{i} + b\underline{j}) \times F\underline{k}$$

$$= F(b\underline{i} + a\underline{j})$$

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2/119 (a) $\underline{M}_O = \underline{FL\underline{i}}$

(b) $\underline{M}_O = FL\underline{i} + FD\underline{k} = \underline{F(L\underline{i} + D\underline{k})}$

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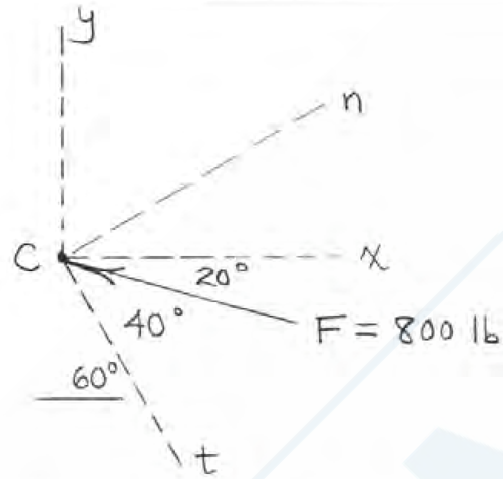
2/12



$$\begin{cases} F_x = -800 \cos 20° = \underline{-752 \text{ lb}} \\ F_y = 800 \sin 20° = \underline{274 \text{ lb}} \end{cases}$$

$$\begin{cases} F_n = -800 \sin 40° = \underline{-514 \text{ lb}} \\ F_t = -800 \cos 40° = \underline{-613 \text{ lb}} \end{cases}$$

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$$\underline{M}_z = (P \cos 30°) d \, \underline{k}$$

$$= (6 \cos 30°)(40) \, \underline{k}$$

$$= \underline{208 \, \underline{k} \text{ lb-in.}}$$

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2/121 $\underline{M} = -150(0.250+0.250)\underline{i} + 150(0.150)\underline{j}$

$= -75\underline{i} + 22.5\underline{j}$ N·m

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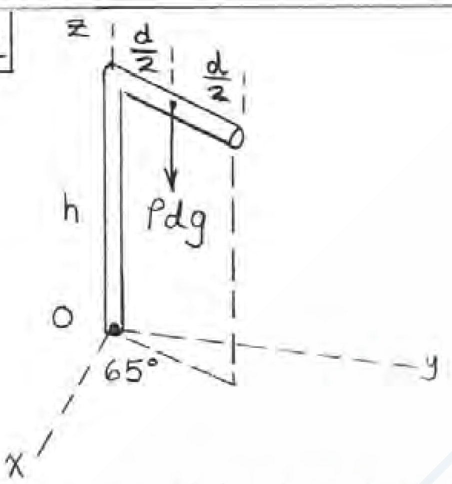

$$\underline{M}_O = -\rho d g\left(\frac{d}{2}\sin 65°\right)\underline{i} + \rho d g\left(\frac{d}{2}\cos 65°\right)\underline{j}$$

$$= \rho g d^2\left(-0.453\,\underline{i} + 0.211\,\underline{j}\right)$$

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$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$= (-6\mathbf{i} + 0.8\mathbf{j} + 1.2\mathbf{k}) \times (-400\mathbf{j})$$

$$= \underline{480\mathbf{i} + 2400\mathbf{k} \ \ \text{N}\cdot\text{m}}$$

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$$\underline{M}_O = \underline{r}_{OA} \times \underline{F}$$

$$= (1.5\underline{j} + 0.75\underline{k}) \times 4(-\cos 30°\underline{i} + \sin 30°\underline{j})$$

$$= -1.5\underline{i} - 2.60\underline{j} + 5.20\underline{k} \text{ lb-in.}$$

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2/125 From the solution to Prob. 2/108, the force is $\underline{R} = \underline{T} = -598\underline{i} + 411\underline{j} + 189.5\underline{k}$ N

The moment associated with the couple is

$\underline{M}_O = \underline{r}_{OC} \times \underline{T}$, where $\underline{r}_{OC} = 0.7\underline{j} + 1.2\underline{k}$ m

Carry out the cross product to obtain

$$\underline{M}_O = -361\underline{i} - 718\underline{j} + 419\underline{k} \text{ N}\cdot\text{m}$$

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$\underline{M} = \underline{r} \times \underline{F}$

$= -0.5\underline{i} \times 400(\cos 15° \underline{j} + \sin 15° \underline{k})$

$= 51.8\underline{j} - 193.2\underline{k}$ N·m

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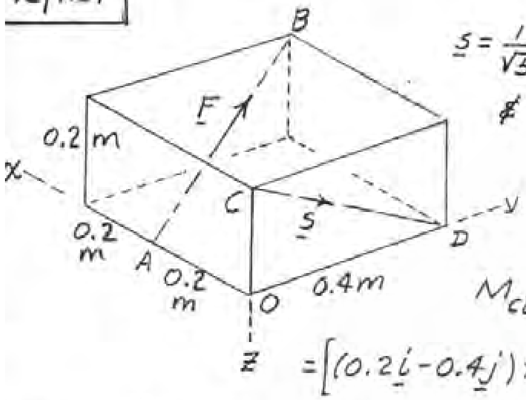

$\underline{s} = \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$

& $\underline{F} = \frac{F}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k})$

Take $\underline{r} = \overrightarrow{DA}$

$= 0.2\underline{i} - 0.4\underline{j}$ m

$M_{CD} = \underline{r} \times \underline{F} \cdot \underline{s}$

$= \left[(0.2\underline{i} - 0.4\underline{j}) \times \frac{F}{\sqrt{6}}(\underline{i} + 2\underline{j} - \underline{k})\right] \cdot \frac{1}{\sqrt{5}}(2\underline{j} + \underline{k})$

So

$$50 = \frac{F}{\sqrt{30}} \begin{vmatrix} 0.2 & -0.4 & 0 \\ 1 & 2 & -1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$= \frac{F}{\sqrt{30}}(0.8 + 0.4) = \frac{1.2F}{\sqrt{30}}, \quad F = \frac{50\sqrt{30}}{1.2} = \underline{228 \text{ N}}$$

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$$\underline{M}_O = 100\,(0.185 \sin 15^\circ)\,\underline{i} - 100\,(0.185 \cos 15^\circ)\,\underline{j}$$

$$= 4.79\,\underline{i} - 17.87\,\underline{j} \quad \text{N·m}$$

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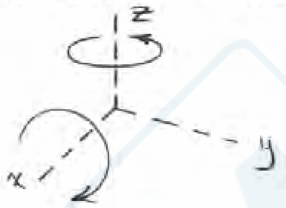
$$\underline{M} = 1.2(40)\underline{k} - 1.2(50)\underline{i}$$

$$= -60\underline{i} + 48\underline{k} \text{ lb-in.}$$

The spacecraft will begin to rotate about its x- and z axes.



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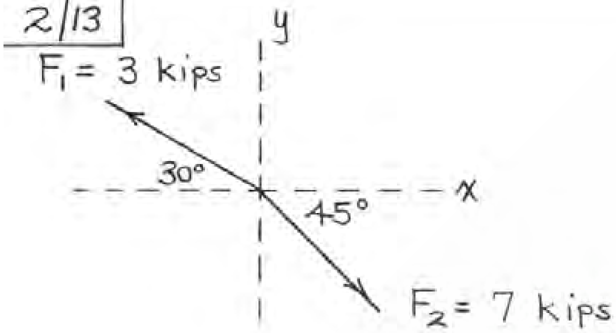

$$R_x = \Sigma F_x = -3\cos 30^\circ + 7\cos 45^\circ = 2.35 \text{ kips}$$

$$R_y = \Sigma F_y = 3\sin 30^\circ - 7\sin 45^\circ = -3.45 \text{ kips}$$

$$\underline{R} = 2.35\,\underline{i} - 3.45\,\underline{j} \text{ kips}$$

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$$M_O = (250 \sin 60^\circ)12 + (250 \cos 60^\circ) \sin 40^\circ (8 - 4.2)$$

$$= \underline{2900 \text{ lb-in. CW}}$$

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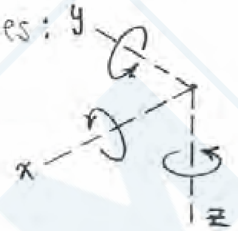
2/131 $\underline{M} = (1700)(2)\underline{i} - (1700)(30)\underline{j} - (1700)(30)\underline{k}$

$= 3400\,\underline{i} - 51\,000\,\underline{j} - 51\,000\,\underline{k}$ N·m

The orbiter would acquire rotational motion about all three axes:



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2/132 $\underline{M}_O = \underline{r}_{OB} \times \underline{T}$, $\underline{r}_{OB} = 6\underline{i} + 13\underline{j}$ m

$$\underline{T} = T\underline{n}_{AB} = 24\left[\frac{6\underline{i} - 5\underline{j} - 30\underline{k}}{\sqrt{6^2 + 5^2 + 30^2}}\right]$$

$$= 4.65\underline{i} - 3.87\underline{j} - 23.2\,\underline{k} \text{ kN}$$

Carry out $\underline{r}_{OB} \times \underline{T}$ to obtain

$$\underline{M}_O = -302\underline{i} + 139.4\underline{j} - 83.6\underline{k} \text{ N}\cdot\text{m}$$

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$$\underline{M}_O = 8(12)\underline{i} - 8(9)\underline{k}$$

$$= \underline{96\underline{i} - 72\underline{k} \text{ lb-in.}}$$

The large moment about the x-axis is an undesirable characteristic of this wrench.

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2/134 $F = 16$ lb (up) will make $(M_o)_x$ zero.

The net moment about O is then

$$\mathbf{M}_o = [16(4) - 8(9)]\mathbf{k} = -8\mathbf{k} \text{ lb-in.}$$

Comment: This wrench should be used only when access considerations make its use absolutely necessary!

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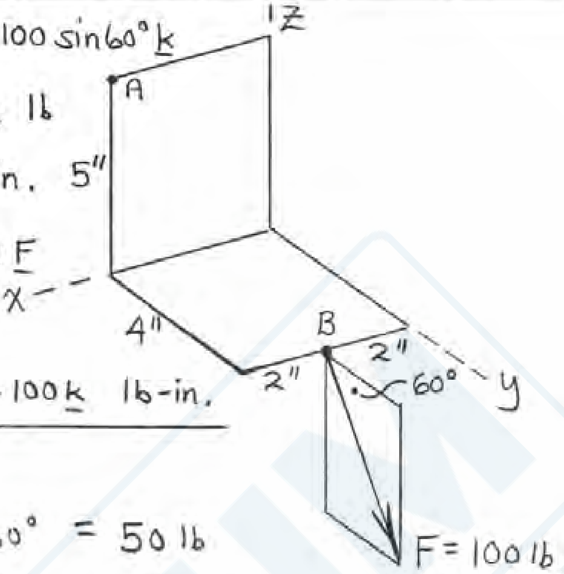
2/135 $\underline{F} = 100\cos 60^\circ \underline{j} - 100\sin 60^\circ \underline{k}$

(a) $= 50\underline{j} - 86.6\underline{k}$ lb

$\underline{r}_{AB} = -2\underline{i} + 4\underline{j} - 5\underline{k}$ in.

Carry out $\underline{M}_A = \underline{r}_{AB} \times \underline{F}$

to obtain



$\underline{M}_A = -96.4\underline{i} - 173.2\underline{j} - 100\underline{k}$ lb-in.

(b)
$$\begin{cases} F_x = 0 \\ F_y = 100\cos 60^\circ = 50 \text{ lb} \\ F_z = -100\sin 60^\circ = -86.6 \text{ lb} \end{cases}$$

$(M_A)_x = 50(5) - 86.6(4) = -96.4$ lb-in.

$(M_A)_y = -86.6(2) = -173.2$ lb-in.

$(M_A)_z = -50(2) = -100$ lb-in.

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$$\underline{M}_O = 0\underline{i} - (200)(0.2 + 0.125 \sin 20°)\underline{j}$$

$$- 200\,(0.125 \cos 20° - 0.070)\,\underline{k}$$

$$= \underline{-48.6\underline{j} - 9.49\underline{k} \quad \text{N·m}}$$

There would be no z-component of $\underline{M}_O$ if

$d \cos 20° - 70 = 0$, $\underline{d = 74.5 \text{ mm}}$

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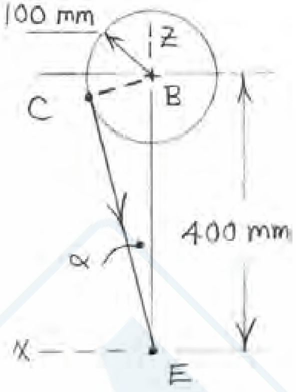
2/137 Wheel detail:



$$\sin\alpha = \frac{\overline{BC}}{\overline{BE}} = \frac{100}{400}$$

$$\alpha = 14.48°$$

So $\underline{T} = T\underline{n}_{CE}$

$$= 80[-\cos\alpha\,\underline{k} - \sin\alpha\,\underline{i}]$$

$$= -20\underline{i} - 77.5\underline{k} \text{ N}$$

$\underline{M}_o = \underline{r}_{OE} \times \underline{T}$, where $\underline{r}_{OE} = 0.2\underline{j}$ m

Carry out to obtain

$$\underline{M}_o = -15.49\underline{i} + 4\underline{k} \text{ N·m}$$

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2/138 $\underline{M}_O = \underline{r}_{OB} \times \underline{F}$, where

$\underline{r}_{OB} = \underline{OA} + \underline{AB}$

$= 4\underline{k} + 16\cos 10°(\cos 20° \underline{i} + \sin 20° \underline{j})$

$+ 16 \sin 10° \underline{k}$

$= 14.81\underline{i} + 5.39\underline{j} + 6.78\underline{k}$ in.

$\underline{F} = 80(-\cos 20° \underline{j} + \sin 20° \underline{i})$

$= 27.4\underline{i} - 75.2\underline{j}$ lb

Carry out $\underline{r}_{OB} \times \underline{F}$ to obtain

$\underline{M}_O = 510\underline{i} + 185.5\underline{j} + 1261\underline{k}$ lb-in.

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$$\underline{T} = T\left[\frac{-0.35\underline{i} - 0.45\cos 20^\circ \underline{j} + (0.4 + 0.45\sin 20^\circ)\underline{k}}{\sqrt{(0.35)^2 + (0.45\cos 20^\circ)^2 + (0.4 + 0.45\sin 20^\circ)^2}}\right]$$

$$= 143.4\left[-0.449\underline{i} - 0.542\underline{j} + 0.710\underline{k}\right] \text{ N}$$

Moment of this force about the x-axis is

$$M_{O_x} = (0.710)(143.4)(0.45\cos 20^\circ) - 0.542(143.4)(0.45\sin 20^\circ) = \underline{31.1 \text{ N}\cdot\text{m}}$$

The moment of the weight W of the 15-kg plate about the x-axis is

$$(M_{O_x})_W = -15(9.81)\frac{0.45\cos 20^\circ}{2} = \underline{-31.1 \text{ N}\cdot\text{m}}$$

The moment of $\underline{T}$ about the line OB is $\underline{\text{zero}}$, because $\underline{T}$ intersects OB.

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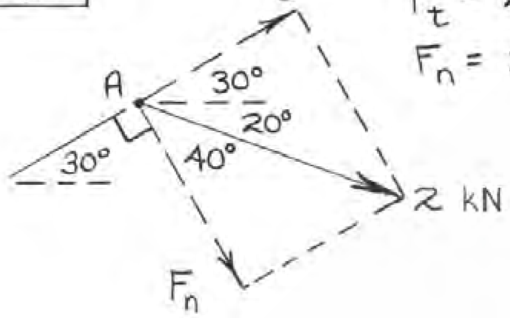

$F_t = 2 \cos 50° = \underline{1.286 \text{ kN}}$

$F_n = 2 \sin 50° = \underline{1.532 \text{ kN}}$

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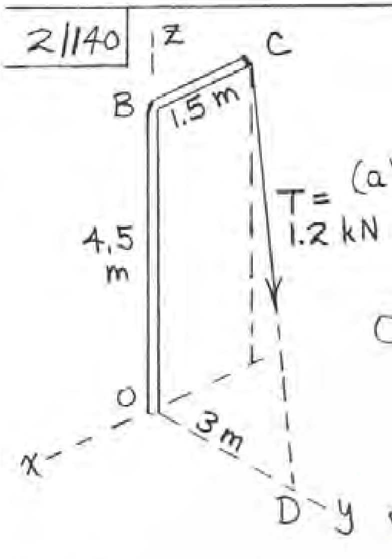

From the solution to Prob. 2/95,

$\underline{T} = 0.321\underline{i} + 0.641\underline{j} - 0.962\underline{k}$ kN

(a) $\underline{M}_O = \underline{r}_{OD} \times \underline{T}$, $\underline{r}_{OD} = 3\underline{j}$ m

$\underline{M}_O = -2.89\underline{i} - 0.962\underline{k}$ kN·m

Could also use

$\underline{M}_O = \underline{r}_{OC} \times \underline{T}$, where

$\underline{r}_{OC} = -1.5\underline{i} + 4.5\underline{k}$ N

Result for $\underline{M}_O$ is the same.

(b) About z-axis:

$\underline{M}_z = -1.5 T_y \underline{k} = -1.5(0.641)\underline{k} = -0.962\underline{k}$ kN·m

Also, $\underline{M}_z = \underline{M}_O \cdot \underline{k}\,\underline{k} = (-2.89\underline{i} - 0.962\underline{k}) \cdot \underline{k}\,\underline{k}$

$= -0.962\underline{k}$ kN·m

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2/141 Using the coordinates of the figure:

$$\underline{M}_A = \underline{r} \times \underline{F} \quad , \quad \underline{F} = -1.8\,\underline{k} \text{ lb}$$

$$\underline{r} = [(2+1)\cos 30°]\,\underline{i} + 3\,\underline{j} + [(2+1)\sin 30°]\,\underline{k} \text{ in.}$$

$$\therefore \quad \underline{M}_A = -5.40\,\underline{i} + 4.68\,\underline{j} \text{ lb-in.}$$

$$\underline{M}_{AB} = (\underline{M}_A \cdot \underline{n}_{AB})\,\underline{n}_{AB} \,, \quad \underline{n}_{AB} = \cos 30°\,\underline{i} + \sin 30°\,\underline{k}$$

$$\therefore \quad \underline{M}_{AB} = -4.05\,\underline{i} - 2.34\,\underline{k} \text{ lb-in.}$$

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2/142 $\underline{M}_O = \underline{r}_{OA} \times \underline{F}$

$$\underline{r}_{OA} = (0.050 + 0.130 \sin 60°)\underline{i} + (-0.140 - 0.130 \cos 60°)\underline{j} + 0.150\underline{k}$$

$$= 0.1626\underline{i} - 0.205\underline{j} + 0.150\underline{k} \text{ m}$$

$$\underline{F} = 600(\cos 45° \sin 60° \underline{i} - \cos 45° \cos 60° \underline{j} + \sin 45° \underline{k})$$

$$= 600(0.612\underline{i} - 0.354\underline{j} + 0.707\underline{k})$$

$$= 367\underline{i} - 212\underline{j} + 424\underline{k} \text{ N}$$

Carry out $\underline{M}_O = \underline{r}_{OA} \times \underline{F}$ to obtain

$$\underline{M}_O = -55.2\underline{i} - 13.86\underline{j} + 40.8\underline{k} \text{ N·m}$$

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$$\begin{cases} R_x = \Sigma F_x = -7 \text{ kN} \\ R_y = \Sigma F_y = 4 - F_3 \cos\theta = -5 \text{ kN} & (1) \\ R_z = \Sigma F_z = F_3 \sin\theta = 6 \text{ kN} & (2) \end{cases}$$

(1): $F_3 \cos\theta = 9$

(2): $F_3 \sin\theta = 6$

Divide Eq. (2) by Eq. 1: $\tan\theta = \frac{2}{3}$

$\underline{\theta = 33.7°}$

Then $\underline{F_3 = 10.82 \text{ kN}}$

$R = \sqrt{7^2 + 5^2 + 6^2} = \underline{10.49 \text{ kN}}$

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$$\begin{cases} \underline{R} = -3F\underline{k} \\ \underline{M}_o = -\frac{\sqrt{3}}{2} bF\underline{i} \end{cases}$$

$\underline{R} \cdot \underline{M}_o = 0$ so $\underline{R} \perp \underline{M}_o$

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2/145 The given loads form two couples, each of which has an associated moment which is in the x-direction. So

$\underline{R} = \Sigma \underline{F} = \underline{\underline{0}}$

$\underline{M}_O = Fb\underline{i} + F\left(b\frac{\sqrt{3}}{2}\right)\underline{i}$

$= \underline{\underline{Fb\left(1+\frac{\sqrt{3}}{2}\right)\underline{i}}}$

The resultant of the system is a couple.

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2/146 $\underline{R} = (1.2 - 1.2 - 1.2)\underline{j} = -1.2\underline{j}$ lb

$\underline{M}_G = 1.2(3)(20)\underline{k} + (1.2 - 1.2 - 1.2)(25)\underline{i}$

$\underline{M}_G = -30\underline{i} + 72\underline{k}$ lb-in.

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2/147 $\underline{R} = \Sigma \underline{F} = -8\underline{i}$ kN

$\underline{M}_G = 50(10)\underline{k} + 8(6)\underline{j} + 8(40)\underline{k}$

$= 48\underline{j} + 820\underline{k}$ kN·m

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$$\underline{R} = (200 + 800)\underline{i} + 1200(\cos 10°\underline{j} - \sin 10°\underline{i})$$

$$= \underline{792\underline{i} + 1182\underline{j}\ \text{N}}$$

$$\underline{M}_O = [(200-800)(0.1) + (1200 \cos 10°)(0.075)]\underline{k}$$

$$+[-(200+800)(0.220+0.330) + 1200 \sin 10°(0.220)]\underline{j}$$

$$+[1200 \cos 10°(0.220)]\underline{i}$$

$$= \underline{260\underline{i} - 504\underline{j} + 28.6\underline{k}\ \text{N}\cdot\text{m}}$$

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2/149 $\mathbf{R} = \Sigma \mathbf{F} = 600(\sin 30° \mathbf{j} + \cos 30° \mathbf{k})$

$$+ 800(-\sin 45° \mathbf{j} + \cos 45° \mathbf{k})$$

$$= -266\mathbf{j} + 1085\mathbf{k} \text{ N}$$

$$\mathbf{M}_O = -0.080\mathbf{i} \times 600(\sin 30° \mathbf{j} + \cos 30° \mathbf{k})$$

$$+ 0.160\mathbf{i} \times 800(-\sin 45° \mathbf{j} + \cos 45° \mathbf{k})$$

$$= -48.9\mathbf{j} - 114.5\mathbf{k} \text{ N·m}$$

$\mathbf{R}$ is not perpendicular to $\mathbf{M}_O$, because

$\mathbf{R} \cdot \mathbf{M}_O \neq 0.$

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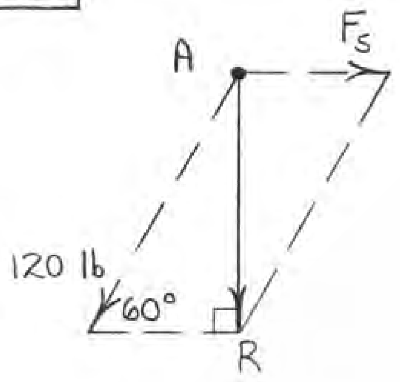

$$\cos 60° = \frac{F_s}{120}$$

$$F_s = 60 \text{ lb}$$

$$\sin 60° = \frac{R}{120}$$

$$R = 103.9 \text{ lb}$$

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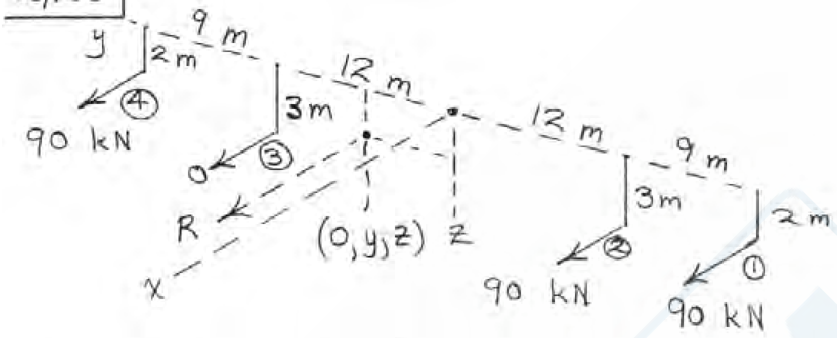

$$R = \Sigma F = 3(90) = 270 \text{ kN}$$

$$\Sigma M_z = -Ry : 90(21) + 90(12) - 90(21) = -270y$$

$$\underline{y = -4 \text{ m}}$$

$$\Sigma M_y = Rz : 2(90)(2) + 90(3) = 270z$$

$$\underline{z = 2.33 \text{ m}}$$

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2/151 The two 160-N forces constitute a couple $160(0.250)\underline{j} = 40\underline{j}$ N·m

$\underline{R} = \Sigma\underline{F} = 120\underline{i} - 180\underline{j} - 100\underline{k}$ N

$\underline{M} = \Sigma\underline{M}_A = [120(0.25) + 100(0.3) + 40]\underline{j} + 50\underline{k}$ N·m

$= 100\underline{j} + 50\underline{k}$ N·m

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2/152 At O: $\underline{R} = \Sigma \underline{F} = (200 + 400)\underline{j} = \underline{600\underline{j}\ \text{lb}}$

$\underline{M}_O = 600(8)\underline{k} + 400(3)\underline{i} = \underline{1200\underline{i} + 4800\underline{k}\ \text{lb-ft}}$

$\underline{R} \cdot \underline{M}_O = 0 \Rightarrow \underline{R} \perp \underline{M}_O$ (Loading system can be represented by single force)

Let P have coordinates $(x, 0, z)$ and let $\underline{R}$ act at P.

$\underline{r}_{OP} \times \underline{R} = \underline{M}_O$: $(x\underline{i} + z\underline{k}) \times 600\underline{j} = 1200\underline{i} + 4800\underline{k}$

$600x\,\underline{k} - 600z\,\underline{i} = 1200\underline{i} + 4800\underline{k}$

$\Rightarrow \underline{x = 8\text{ ft}, \quad z = -2\text{ ft}}$

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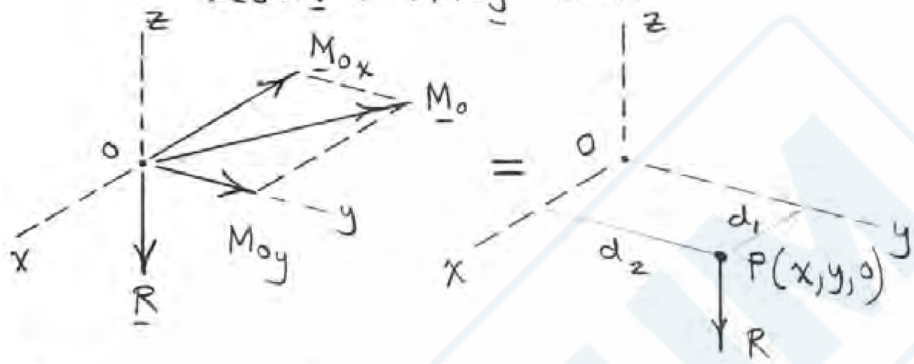
2/153 $\underline{R} = \Sigma \underline{F} = -50\underline{k} - 20\underline{k} = \underline{-70\underline{k}\ \text{lb}}$

$\underline{M}_o = 20(12\underline{j} - 20\underline{i}) + 50(24\underline{j} - 48\underline{i})$

$= -2800\underline{i} + 1440\underline{j}$ lb-ft



Conditions on P :

$R d_1 = |M_{oy}|$, $d_1 = \frac{1440}{70} = 20.6$ in.

$R d_2 = |M_{ox}|$, $d_2 = \frac{2800}{70} = 40$ in.

So the coordinates of P are

$(x, y) = \underline{(20.6, 40)\ \text{in.}}$ (Resultant is a standalone force

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2/154 $R_x = -120$ N, $R_y = 0$, $R_z = -160$ N

$R = \sqrt{120^2 + 160^2} = 200$ N, $\underline{R} = -120\underline{i} - 160\underline{k}$ N

$M_x = 25 - 160(0.2) = -7$ N·m

$M_y = 160(0.075) - 120(0.100 - 0.075) = 9$ N·m

$M_z = 120(0.2) = 24$ N·m

$M = \sqrt{7^2 + 9^2 + 24^2} = 25.5$ N·m

$\underline{M} = -7\underline{i} + 9\underline{j} + 24\underline{k}$ N·m

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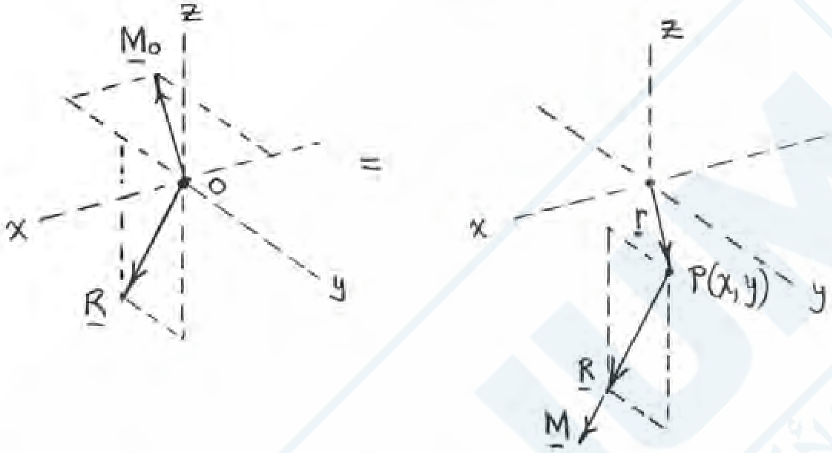
$$\begin{cases} \underline{R} = -20\underline{j} - 40\underline{k} \text{ lb} \quad (= 44.7(-0.447\underline{j} - 0.894\underline{k})) \\ \underline{M}_o = -40(1.4)\underline{i} - 40(8)\underline{j} \\ \quad = -56\underline{i} - 320\underline{j} \text{ lb-in.} \end{cases}$$



$\underline{M}_o: -56\underline{i} - 320\underline{j} = \underline{r} \times \underline{R} + \underline{M} = (x\underline{i} + y\underline{j}) \times (-20\underline{j} - 40\underline{k})$
$+ M(-0.447\underline{j} - 0.894\underline{k})$

Equate coefficients:

$$\begin{cases} \underline{i}: -56 = -40y \\ \underline{j}: -320 = 40x - 0.447M \\ \underline{k}: 0 = -20x - 0.894M \end{cases}$$

Solution:
$x = -6.4$ in.
$y = 1.4$ in.

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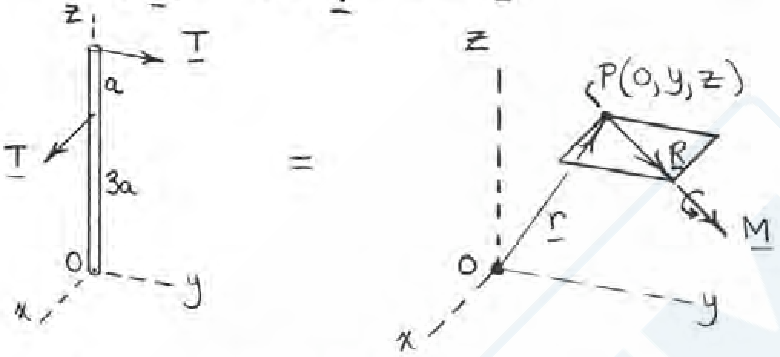
2/156 $\underline{R} = \Sigma \underline{F} = T\underline{i} + T\underline{j} = \sqrt{2}T\left[\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right]$

$\Sigma \underline{M}_o = 3aT\underline{j} - 4aT\underline{i}$



$\Sigma \underline{M}_o = \underline{r} \times \underline{R} + \underline{M}$

$3aT\underline{j} - 4aT\underline{i} = (y\underline{j} + z\underline{k}) \times (T\underline{i} + T\underline{j}) + M\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

$$\Rightarrow \begin{cases} -4aT = -zT + \frac{M}{\sqrt{2}} \\ 3aT = zT + \frac{M}{\sqrt{2}} \\ 0 = -yT \end{cases} \qquad \text{So} \begin{cases} y = 0 \\ z = \frac{7}{2}a \\ M = -\frac{\sqrt{2}}{2}aT \end{cases}$$

So $\underline{M} = -\frac{\sqrt{2}}{2}aT\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

$= \frac{-aT}{2}(\underline{i} + \underline{j})$ (a negative wrench)

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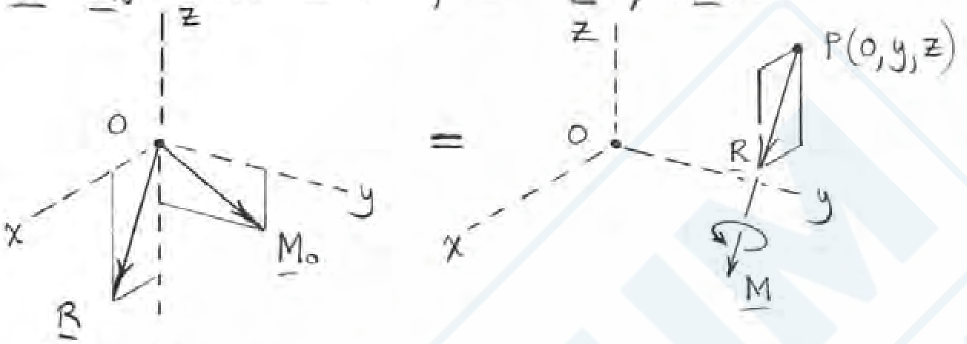
2/157 At O: $\begin{cases} \underline{R} = \Sigma \underline{F} = F\underline{i} - 3F\underline{k} = \sqrt{10}\,F\left(\frac{1}{\sqrt{10}}\underline{i} - \frac{3}{\sqrt{10}}\underline{k}\right) \\ \underline{M}_o = F(2a)\underline{j} - F(a)\underline{k} = Fa(2\underline{j} - \underline{k}) \end{cases}$

$\underline{R} \cdot \underline{M}_o = 3F^2 a \neq 0$, so $\underline{R} \not\perp \underline{M}_o$.



$\underline{M}_o = \underline{M} + \underline{r}_{op} \times \underline{R}$

$Fa(2\underline{j} - \underline{k}) = M\left(\frac{1}{\sqrt{10}}\underline{i} - \frac{3}{\sqrt{10}}\underline{k}\right) + (y\underline{j} + z\underline{k}) \times (F\underline{i} - 3F\underline{k})$

Equate coefficients:

$\underline{i}: \; 0 = -3Fy + \frac{M}{\sqrt{10}}$

$\underline{j}: \; 2Fa = Fz$

$\underline{k}: \; -Fa = -Fy - \frac{3M}{\sqrt{10}}$

Solution:

$y = \frac{a}{10}$

$z = 2a$

$M = 3\sqrt{10}\,aF/10 = 0.949\,Fa$

$\underline{M} = \frac{3\sqrt{10}\,aF}{10}\left(\frac{1}{\sqrt{10}}\underline{i} - \frac{3}{\sqrt{10}}\underline{k}\right) = \frac{3aF}{10}(\underline{i} - 3\underline{k})$

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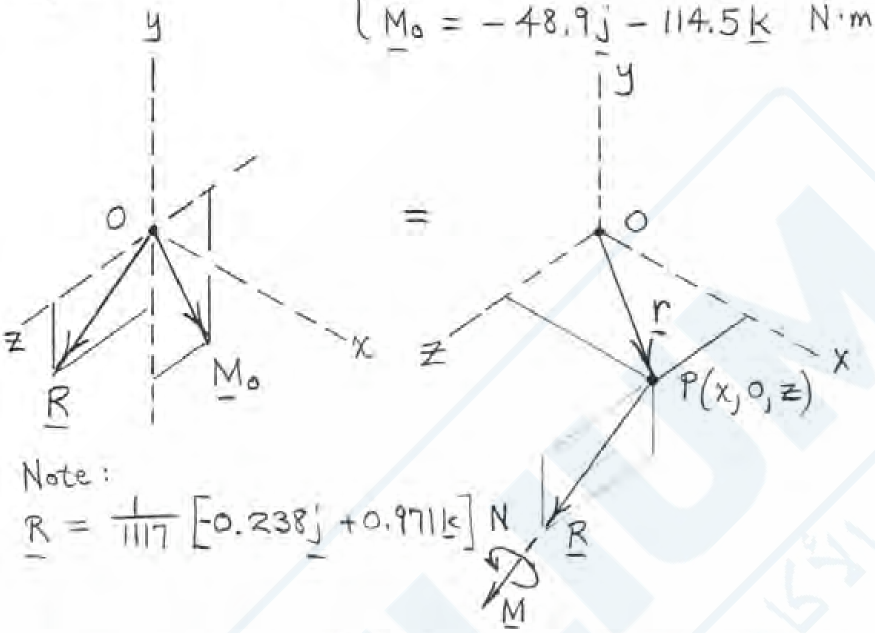
2/158 From Prob. 2/149, the force-couple system at O is

$$\begin{cases} \underline{R} = -266\underline{j} + 1085\underline{k} \text{ N} \\ \underline{M}_O = -48.9\underline{j} - 114.5\underline{k} \text{ N·m} \end{cases}$$



Note:

$\underline{R} = \frac{1}{1117}\left[-0.238\underline{j} + 0.971\underline{k}\right]$ N

$\underline{M}_O = \underline{r} \times \underline{R} + \underline{M} = (x\underline{i} + z\underline{k}) \times (-266\underline{j} + 1085\underline{k}) + M(-0.238\underline{j} + 0.971\underline{k})$

Equate coefficients:

$\underline{i}: \; 0 = 266z$

$\underline{j}: \; -48.9 = -1085x - 0.238M$

$\underline{k}: \; -114.5 = -266x + 0.971M$

Solution:

$x = 0.0669$ m (66.9 mm)

$z = 0$

$M = -99.6$ N·m

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▸2/159

$\underline{R} = \Sigma \underline{F} = 100\underline{i} + 100\underline{j}$ N

Direction cosines $l = 1/\sqrt{2}$, $m = 1/\sqrt{2}$, $n = 0$

Let $P = P(x, 0, z)$

$\underline{M}_P = 100z\underline{i} + 100(0.4 - x)\underline{k} + 100(0.4 - z)\underline{j}$
$\quad - 100(0.3)\underline{k} - 20\underline{j}$
$\quad = 100z\underline{i} + 100(0.2 - z)\underline{j} + 100(0.1 - x)\underline{k}$ N·m

Let $M = |\underline{M}_P|$. Equate direction cosines of $\underline{R}$ & $\underline{M}_P$ to obtain

$$\frac{100z}{M} = \frac{1}{\sqrt{2}}, \quad \frac{100(0.2 - z)}{M} = \frac{1}{\sqrt{2}}, \quad \frac{100(0.1 - x)}{M} = 0$$

Solution : $\begin{cases} x = 0.1 \text{ m} \\ z = 0.1 \text{ m} \\ M = 10\sqrt{2} \text{ N·m} \end{cases}$

$\underline{M} = 10\sqrt{2}\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right) = 10\underline{i} + 10\underline{j}$ N·m

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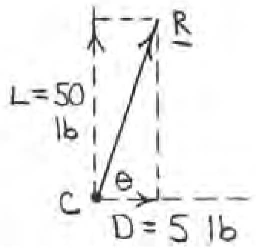
$$\frac{L}{D} = \frac{50}{D} = 10 \; ; \quad D = 5 \text{ lb}$$



$$R = \sqrt{L^2 + D^2} = \sqrt{50^2 + 5^2}$$

$$= \underline{50.2 \text{ lb}}$$

$$\theta = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{50}{5}\right)$$

$$= \underline{84.3°}$$

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▶ 2/160 $\underline{R} = \Sigma \underline{F} = 400(-\cos 24° \underline{j} + \sin 24° \underline{i}) + 800(-\cos 24° \underline{j} - \sin 24° \underline{i})$

$\underline{R} = -162.7\underline{i} - 1096\underline{j}$ N , $R = \sqrt{R_x^2 + R_y^2} = 1108$ N

Dir. cosines: $l = -0.1468$, $m = -0.9892$, $n = 0$

$\underline{M}_A = [(-400(0.2) - 800(0.6))\cos 24°]\underline{i} + [(-400 + 800)(0.2)\cos 24° - 200]\underline{k} + [800(0.6) - 400(0.2)]\sin 24° \underline{j} = -512\underline{i} + 162.7\underline{j} - 126.9\underline{k}$ N·m

$\underline{R} \cdot \underline{M}_A = -95\,100$ N²·m $\neq 0 \Rightarrow \underline{R} \not\parallel \underline{M}_A$

Now move system to $P(x, 0, z)$ & form wrench

$\underline{R} = -162.7\underline{i} - 1096\underline{j}$ N

$\underline{M}_P = \underline{M}_A + \underline{r}_{PA} \times \underline{R}$, where $\underline{r}_{PA} = -x\underline{i} - z\underline{k}$

$= (-512 - 1096z)\underline{i} + (162.7z + 162.7)\underline{j} + (-126.9 + 1096x)\underline{k}$

Let $M = |\underline{M}_P|$ & equate dir. cosines of $\underline{R}$ and $\underline{M}_P$:

$\frac{-512-1096z}{M} = -0.1468$; $\frac{162.7z + 162.7}{M} = -0.9892$; $\frac{-126.9 + 1096x}{M} = 0$

Solve to obtain
$$\begin{cases} M = -85.8 \text{ N·m} \\ x = 0.1158 \text{ m} \\ z = -0.478 \text{ m} \end{cases}$$

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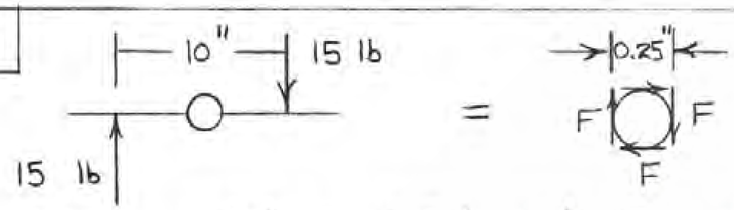

$M = Fd = 15(10) = 2F(0.25)$, $\underline{F = 300 \text{ lb}}$

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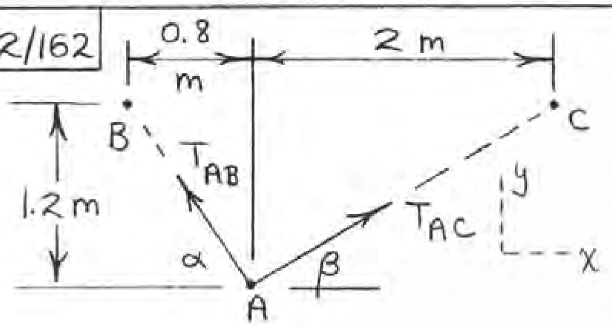

$\alpha = \tan^{-1}\left(\frac{1.2}{0.8}\right)$

$= 56.3°$

$\beta = \tan^{-1}\left(\frac{1.2}{2}\right)$

$= 31.0°$

$$\underline{T}_{AB} = T_{AB}\,\underline{n}_{AB} = 0.858(60)(9.81)\left[-\cos 56.3°\,\underline{i} + \sin 56.3°\,\underline{j}\right]$$

$$= \underline{-280\,\underline{i} + 420\,\underline{j}\ \text{N}}$$

$$\underline{T}_{AC} = T_{AC}\,\underline{n}_{AC} = 0.555(60)(9.81)\left[\cos 31.0°\,\underline{i} + \sin 31.0°\,\underline{j}\right]$$

$$= \underline{280\,\underline{i} + 168.1\,\underline{j}\ \ \text{N}}$$

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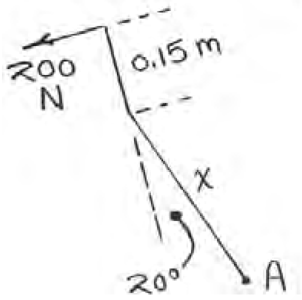
$$M_A = Fd: \quad 80 = 200(0.15 + x\cos 20^\circ)$$

$$x = 0.266 \text{ m or } \underline{266 \text{ mm}}$$



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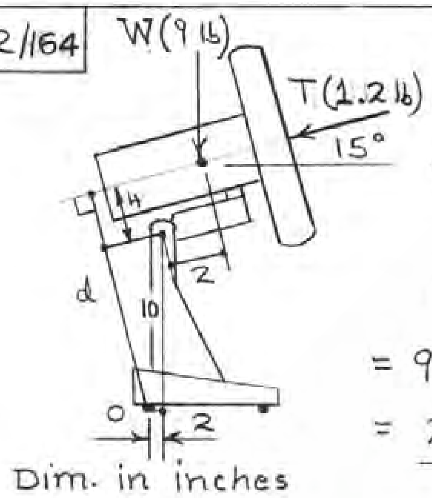

$M_O = Fd$

$= 1.2(4 + 10\cos 15^\circ - 2\sin 15^\circ)$

$= 15.77$ lb-in. (↺)

$M_{O_W} = W d_W$

$= 9(2 + 2\cos 15^\circ - 4\sin 15^\circ)$

$= 26.1$ lb-in. (↻)

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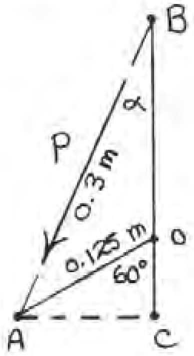

$AC = 0.125 \sin 60° = 0.1083 \text{ m}$

$\alpha = \sin^{-1} \dfrac{0.1083}{0.300} = 21.2°$

$BC = 0.300 \cos \alpha = 0.280 \text{ m}$

$BO = 0.280 - 0.125 \cos 60° = 0.217 \text{ m}$

$\circlearrowleft + \; M_O = 720 = P \sin \alpha \, (BO) = P \sin 21.2° (0.217)$

$\underline{P = 9.18 \text{ kN}}$

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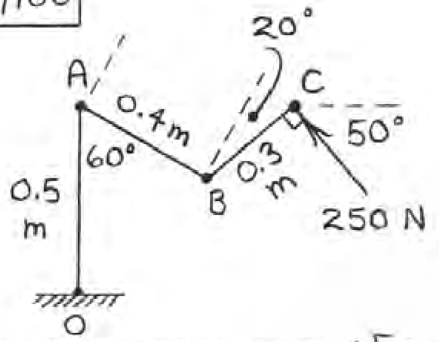

$$+\circlearrowleft\ M_O = 250\cos 50°\left[0.5 - 0.4\cos 60° + 0.3\sin 40°\right]$$

$$+250\sin 50°\left[0.4\sin 60° + 0.3\cos 40°\right]$$

$$= \underline{189.6\ \text{N·m}\ \ \text{CCW}}$$

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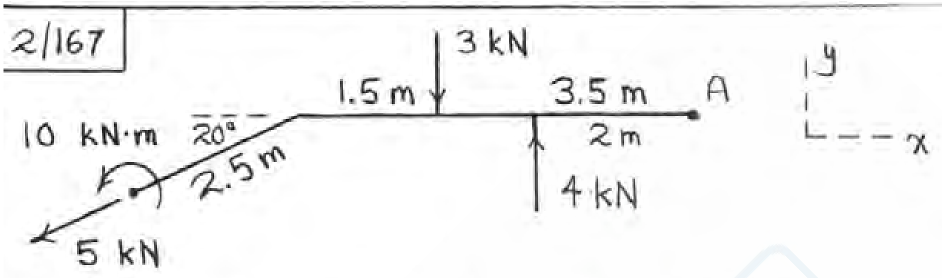

$R_x = \Sigma F_x = -5 \cos 20° = -4.70$ kN

$R_y = \Sigma F_y = -5 \sin 20° + 4 - 3 = -0.710$ kN

$R = \sqrt{R_x^2 + R_y^2} = \underline{4.75 \text{ kN}}$

$\circlearrowleft + \Sigma M_A = 10 + 5 \sin 20° (5) + 3(3.5) - 4(2)$

$= \underline{21.1 \text{ kN·m CCW}}$

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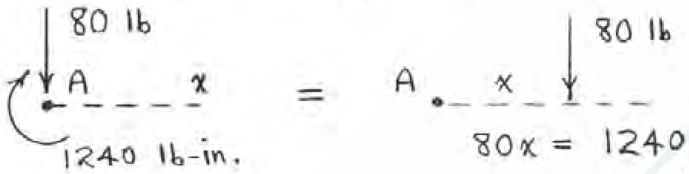
2/168 At A: $R = \Sigma F = 200 + 180 - 300 = \underline{80 \text{ lb} (\downarrow)}$

$\circlearrowleft + M_A = 200(8) + 180(28) - 300(18) = \underline{1240 \text{ lb-in.}}$



$80x = 1240$, $\underline{x = 15.5 \text{ in.}}$

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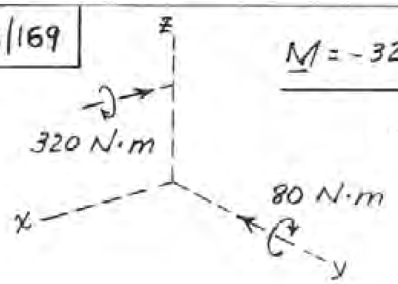

$$\underline{M} = -320\underline{i} - 80\underline{j} \quad N\cdot m$$

$$\cos\theta_x = \frac{M_x}{|\underline{M}|} = \frac{-320}{\sqrt{320^2 + 80^2}} = -0.970$$

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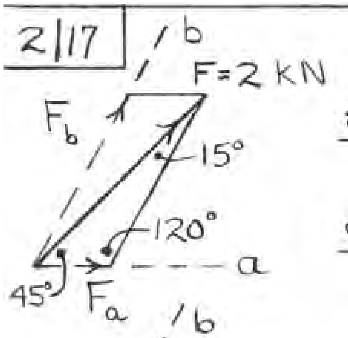
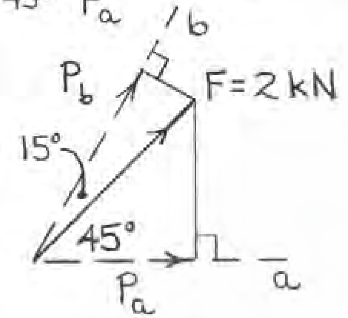


Components

$$\frac{\sin 120^\circ}{2} = \frac{\sin 15^\circ}{F_a}, \quad \underline{F_a = 0.598 \text{ kN}}$$

$$\frac{\sin 120^\circ}{2} = \frac{\sin 45^\circ}{F_b}, \quad \underline{F_b = 1.633 \text{ kN}}$$



Projections

$$P_a = 2 \cos 45^\circ = \underline{1.414 \text{ kN}}$$

$$P_b = 2 \cos 15^\circ = \underline{1.932 \text{ kN}}$$

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2/170 $\underline{P} = P\left(\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}\right)$; $\underline{r}_{AB} = b(-\underline{i} + \underline{j} + \underline{k})$

Carry out $\underline{M}_A = \underline{r}_{AB} \times \underline{P}$ to obtain

$$\underline{M}_A = \frac{Pb}{5}(-3\underline{i} + 4\underline{j} - 7\underline{k})$$

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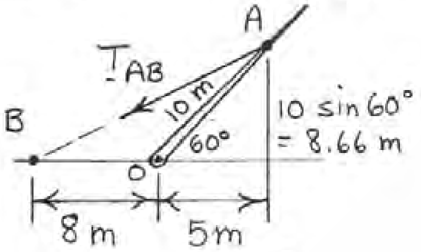
$$\overline{AB}^2 = 8^2 + 10^2 - 2(8)(10)\cos 120°$$

$$\overline{AB} = 15.62 \text{ m}$$



(a) $$\underline{T}_{AB} = 3\left[-\frac{13}{15.62}\cos 35°\underline{i} - \frac{13}{15.62}\sin 35°\underline{j} - \frac{8.66}{15.62}\underline{k}\right]$$

$$= -2.05\underline{i} - 1.432\underline{j} - 1.663\underline{k} \text{ kN}$$

(b) Carry out $\underline{r}_{OB} \times \underline{T}_{AB}$, where $\underline{r}_{OB} = 8(-\cos 35°\underline{i} - \sin 35°\underline{j})$ m to obtain

$$\underline{M}_O = 7.63\underline{i} - 10.90\underline{j} \text{ kN}\cdot\text{m}$$

$\therefore$ $M_{O_x} = 7.63$ kN·m, $M_{O_y} = -10.90$ kN·m, $M_{O_z} = 0$

(c) $T_{AO} = \underline{T}_{AB} \cdot \underline{n}_{AO}$

With $\underline{n}_{AO} = -\cos 60° \cos 35°\underline{i} - \cos 60° \sin 35°\underline{j} - \sin 60°\underline{k}$, we obtain $T_{AO} = 2.69$ kN

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2/172 Coordinates of A: $(x_A, y_A, z_A) = (0, r, 0)$

Coordinates of B: $(x_B, y_B, z_B) = (h, r\cos\theta, r\sin\theta)$

So $\underline{r}_{AB} = h\underline{i} + (r\cos\theta - r)\underline{j} + r\sin\theta\,\underline{k}$

and $\underline{F} = F\underline{n}_{AB} = F\left[\dfrac{h\underline{i} + r(\cos\theta - 1)\underline{j} + r\sin\theta\,\underline{k}}{\sqrt{h^2 + [r(\cos\theta - 1)]^2 + [r\sin\theta]^2}}\right]$

$$= F\left[\frac{h\underline{i} + r(\cos\theta - 1)\underline{j} + r\sin\theta\,\underline{k}}{\sqrt{h^2 + 2r^2(1 - \cos\theta)}}\right]$$

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$$\underline{M}_{100} = -100\,(0.200)\,\underline{i} = -20\underline{i} \text{ N}\cdot\text{m}$$

$$\underline{M}_{80} = 80\,(0.180 \cos 20^\circ)(-\underline{j} \sin 30^\circ - \underline{k} \cos 30^\circ)$$

$$= -6.77\underline{j} - 11.72\underline{k} \text{ N}\cdot\text{m}$$

$$\underline{M}_{120} = -120\,(0.300 \cos 45^\circ)\underline{k} = -25.5\,\underline{k} \text{ N}\cdot\text{m}$$

So $\underline{M} = -20\underline{i} - 6.77\underline{j} - 37.2\underline{k}$ N·m

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$$\underline{R} = 800\left[-\sin 30^\circ \cos 20^\circ \underline{i} + \sin 30^\circ \sin 20^\circ \underline{j} + \cos 30^\circ \underline{k}\right]$$

$$= -376\underline{i} + 136.8\underline{j} + 693\underline{k} \quad \text{N}$$

$$\underline{M}_O = \underline{r}_{OB} \times \underline{F}$$

$$\underline{r}_{OB} = \left[300 \sin 20^\circ \underline{i} + 300 \cos 20^\circ \underline{j} + 250\underline{k}\right] \text{mm}$$

$$\underline{M}_O = 161.1\underline{i} - 165.1\underline{j} + 120\underline{k} \quad \text{N}\cdot\text{m}$$

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2/175 $\underline{R} = \Sigma \underline{F} = 500\cos 45°(\underline{i}) + 400\underline{j} - (600 + 500\sin 45°)\underline{k}$

$= 354\underline{i} + 400\underline{j} - 954\underline{k}$ lb

$R = \sqrt{354^2 + 400^2 + 954^2} = \underline{1093 \text{ lb}}$

$\underline{M} = [500\cos 45°(3) - 600(3) - 400(10)]\underline{i}$

$+ [500\cos 45°(6) + 500\sin 45°(7) + 600(6)]\underline{j}$

$+ [500\sin 45°(3) + 400(3)]\underline{k}$

$= -4739\underline{i} + 8196\underline{j} + 2261\underline{k}$ lb-ft

$M = \sqrt{4739^2 + 8196^2 + 2261^2} = \underline{9730 \text{ lb-ft}}$

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$\Sigma F_x = 0: -360 - 240 \sin\theta + T \sin 30° + 400 \cos 30° = 0$ (1)

$\Sigma F_y = 600: 240 \cos\theta + T \cos 30° + 400 \sin 30° = 600$ (2)

Numerical solution of Eqs. (1) & (2):

$\theta = 21.7°, \quad T = 204 \text{ lb}$

(We could eliminate T between Eqs. (1) & (2), but the resulting equation is still transcendental.)

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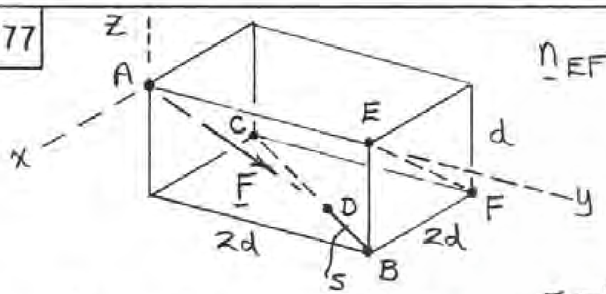

$$\underline{n}_{EF} = \frac{-2d\underline{i} - d\underline{k}}{\sqrt{5}\,d} = \frac{-2\underline{i} - \underline{k}}{\sqrt{5}}$$

$$\underline{AD} = \underline{AB} + \underline{BD} = 2d\underline{j} - d\underline{k} + s\left[\frac{-2d\underline{i} - 2d\underline{j}}{\sqrt{8}\,d}\right]$$

$$= -\frac{s}{\sqrt{2}}\underline{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\underline{j} - d\underline{k}$$

$$|\underline{AD}| = \sqrt{\frac{s^2}{2} + \left(2d - \frac{s}{\sqrt{2}}\right)^2 + d^2} = \sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}$$

$$\underline{F} = F\frac{\underline{AD}}{|\underline{AD}|} = F\,\frac{-\frac{s}{\sqrt{2}}\underline{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\underline{j} - d\underline{k}}{\sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}}$$

Carry out $\underline{F} \cdot \underline{n}_{EF}$ to obtain the projection

$$\underline{F} \cdot \underline{n}_{EF} = \frac{F\left(s\sqrt{2} + d\right)}{\sqrt{5}\sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}}$$

The nondimensionalized fraction n of the magnitude F projected is then

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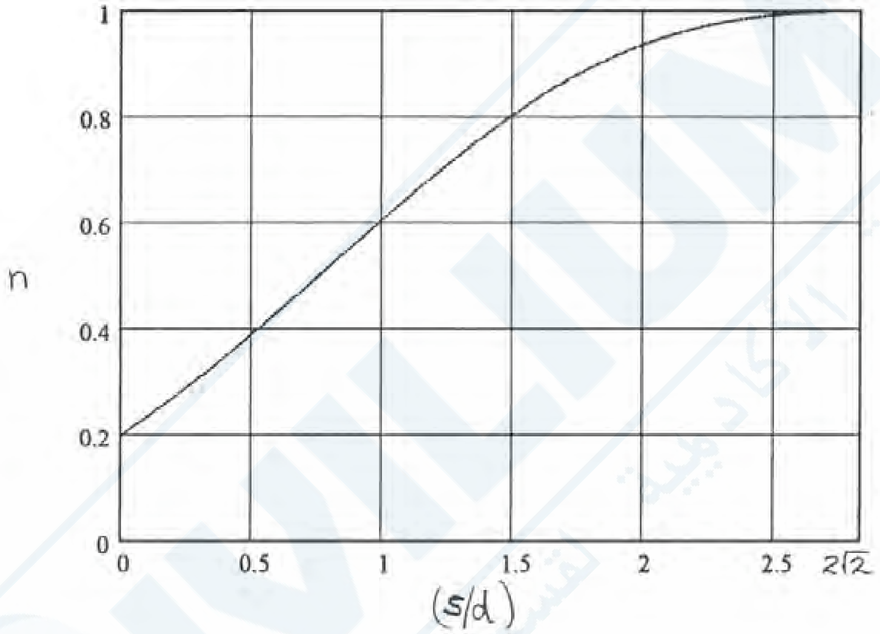
$$n = \frac{F \cdot \underline{n}_{EF}}{F} = \frac{\sqrt{2}\,\frac{s}{d} + 1}{\sqrt{5}\sqrt{\left(\frac{s}{d}\right)^2 + 5 - 2\sqrt{2}\,\frac{s}{d}}}$$

We let $\frac{s}{d}$ vary from 0 to $2\sqrt{2}$ as D moves from B to C. Resulting plot:



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$\underline{T} = T\underline{n}_{AB}$

(axes: y up, x right)

$$\underline{T} = T\left[\frac{(d+40\cos\beta)\underline{i} + 40(1-\sin\beta)\underline{j}}{\sqrt{(d+40\cos\beta)^2 + 40^2(1-\sin\beta)^2}}\right]$$

$$\left(\beta = \theta + \frac{\pi}{4}\right)$$

$\underline{r}_{OB} = (d\underline{i} + 40\underline{j})$

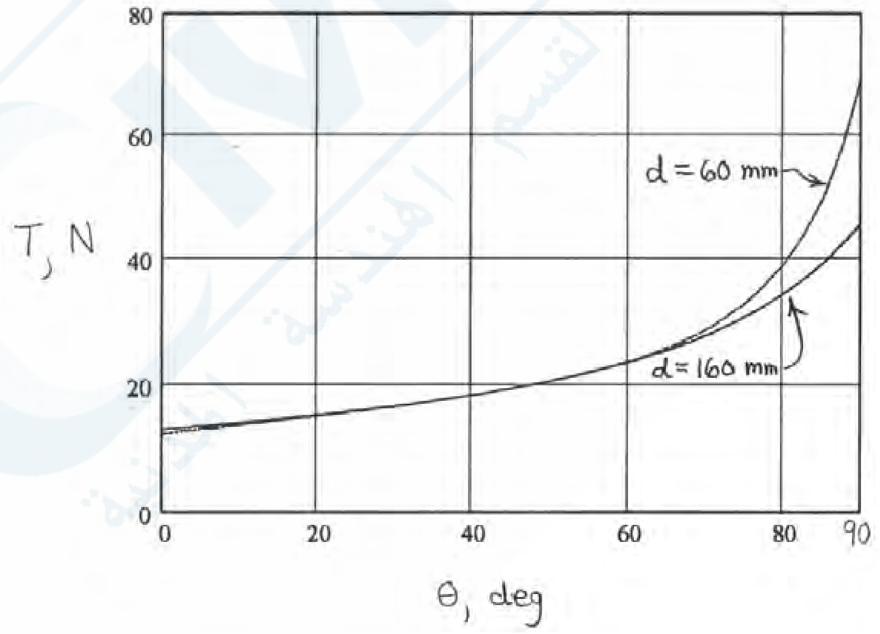
$\Sigma \underline{M}_O = \underline{0}: \quad \underline{r}_{OB} \times \underline{T} + K\beta\underline{k} = \underline{0}$

Carry out the cross product, consider the z-component, and solve for T to obtain

$$T = \frac{12.5\left(\theta+\frac{\pi}{4}\right)\sqrt{d^2 + 80d\cos\left(\theta+\frac{\pi}{4}\right) - 3200\sin\left(\theta+\frac{\pi}{4}\right) + 3200}}{\left[d\sin\left(\theta+\frac{\pi}{4}\right) + 40\cos\left(\theta+\frac{\pi}{4}\right)\right]}$$

(T in N, θ in radians)

Plot of T versus θ for d = 60 mm and for d = 160 mm:



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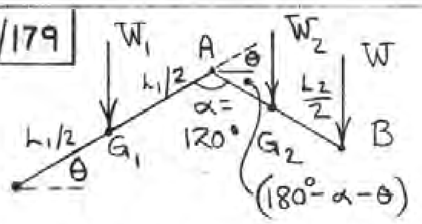


$$\begin{cases} W_1 = 120 \text{ lb} & L_1 = 3 \text{ ft} \\ W_2 = 50 \text{ lb} & L_2 = 2 \text{ ft} \\ W = 300 \text{ lb} & \alpha = 120° \end{cases}$$

$$+\circlearrowleft M_O = W_1 \frac{L_1}{2}\cos\theta + W_2\left(L_1\cos\theta + \frac{L_2}{2}\cos(180° - \alpha - \theta)\right)$$
$$+ W\left(L_1\cos\theta + L_2\cos(180° - \alpha - \theta)\right)$$

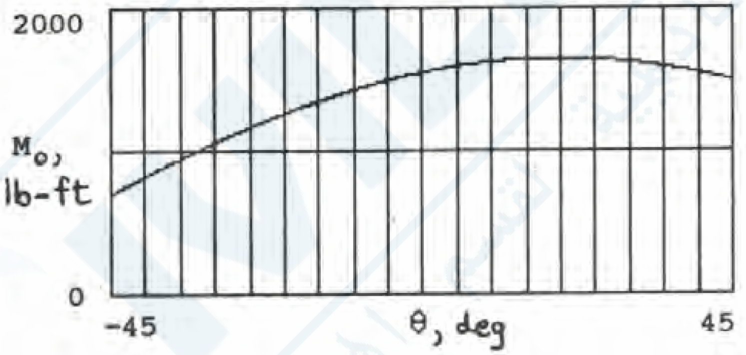
With the above numbers:

$$\underline{M_O = 1230\cos\theta + 650\cos(60° - \theta)} \quad \text{(in lb-ft)}$$

(see plot below)

For $(M_O)_{max}$: $\frac{dM_O}{d\theta} = -1230\sin\theta + 650\sin(60° - \theta) = 0$

Numerical solution : $\underline{\theta = 19.90°}$, $\underline{(M_O)_{max} = 1654 \text{ lb-ft}}$



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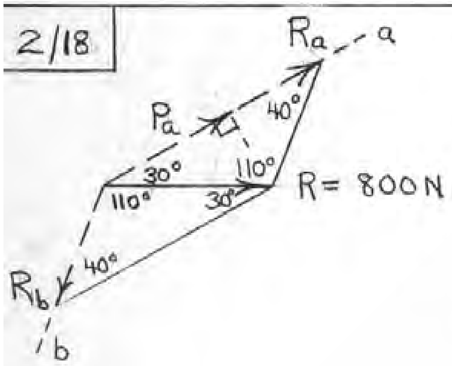

Law of sines:

$$\frac{800}{\sin 40^\circ} = \frac{R_a}{\sin 110^\circ} = \frac{R_b}{\sin 30^\circ}$$

$\underline{R_a = 1170 \text{ N}}$

$\underline{R_b = 622 \text{ N}}$

Projection $P_a = R\cos 30^\circ = 800 \cos 30^\circ = \underline{693 \text{ N}}$

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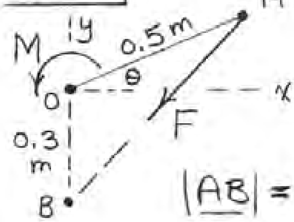

$$\underline{F} = F\underline{n}_{AB} = F\frac{\underline{AB}}{|\underline{AB}|}$$

$$\underline{AB} = -0.5\cos\theta\,\underline{i} - (0.5\sin\theta + 0.3)\underline{j}\ \text{m}$$

$$|\underline{AB}| = \sqrt{(0.5\cos\theta)^2 + (0.5\sin\theta + 0.3)^2}$$

$$= \sqrt{0.34 + 0.3\sin\theta}\ \text{m}$$

$$F = k\delta = 600\left[\sqrt{0.34 + 0.3\sin\theta} - 0.65\right]\ \text{N}$$

So

$$\underline{F} = \frac{600\left[\sqrt{0.34 + 0.3\sin\theta} - 0.65\right]}{\sqrt{0.34 + 0.3\sin\theta}}\left[-0.5\cos\theta\,\underline{i} - (0.5\sin\theta + 0.3)\underline{j}\right]$$

Now form $\underline{r}_{OB} \times \underline{F}$, where $\underline{r}_{OB} = -0.3\underline{j}$ m, to obtain $-\dfrac{90\cos\theta\left(\sqrt{0.34 + 0.3\sin\theta} - 0.65\right)}{\sqrt{0.34 + 0.3\sin\theta}}\underline{k}$

With the above moment plus $M\underline{k}$ summing to zero, we obtain the scalar

$$M = \frac{90\cos\theta\left[\sqrt{0.34 + 0.3\sin\theta} - 0.65\right]}{\sqrt{0.34 + 0.3\sin\theta}}$$

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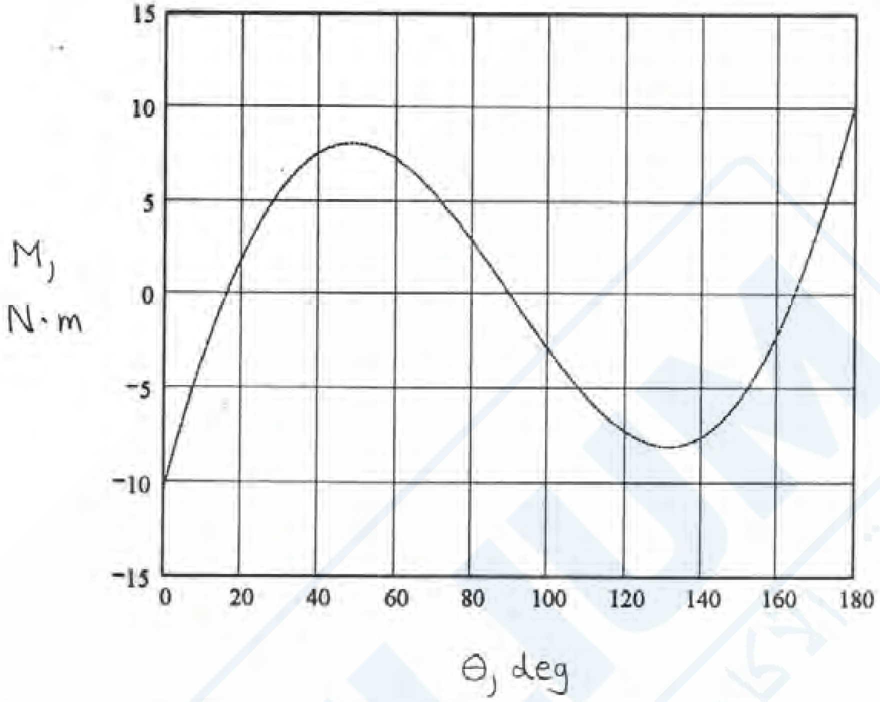
Plot:



(Note: $M(0) = -10.33$ N·m
$M(180°) = 10.33$ N·m)

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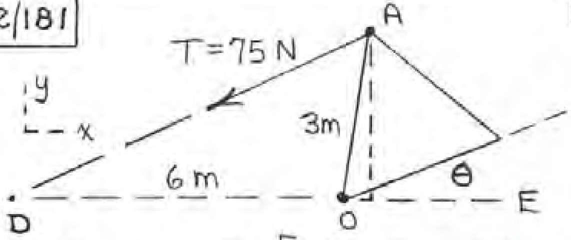
***2/181**



Angle AOE = θ+60°

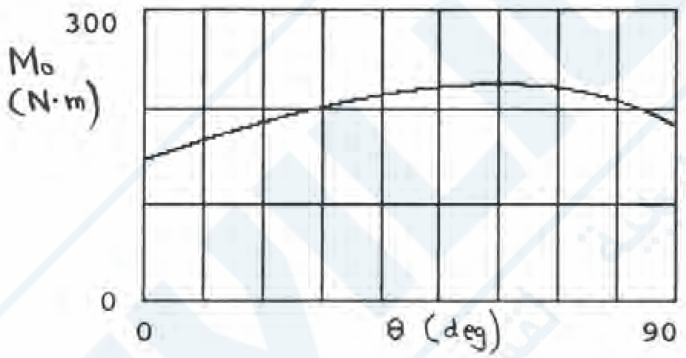
Use $\mathbf{M}_O = \mathbf{r}_{OD} \times \mathbf{T}$

$\mathbf{r}_{OD} = -6\mathbf{i}$ m

$$\mathbf{T} = T\mathbf{n}_{AD} = 75\left[\frac{-(6+3\cos(\theta+60°))\mathbf{i} - 3\sin(\theta+60°)\mathbf{j}}{\sqrt{[6+3\cos(\theta+60°)]^2+[3\sin(\theta+60°)]^2}}\right] \text{N}$$

$$\mathbf{M}_O = \mathbf{r}_{OD}\times\mathbf{T} = \frac{1350\sin(\theta+60°)\,\mathbf{k}}{\sqrt{45+36\cos(\theta+60°)}} \text{ N·m}$$

M_O is a max @ $\underline{\theta = 60°}$: $\underline{M_O = 225 \text{ N·m}}$



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$\underline{T} = T\underline{n}_{AB} = T \dfrac{\underline{AB}}{|\underline{AB}|}$

$= 120\left[\dfrac{0.4\underline{i} + 0.8(1-\sin\theta)\underline{j} - 0.8\cos\theta\,\underline{k}}{\sqrt{0.4^2 + 0.8^2(1-\sin\theta)^2 + 0.8^2\cos^2\theta}}\right]$

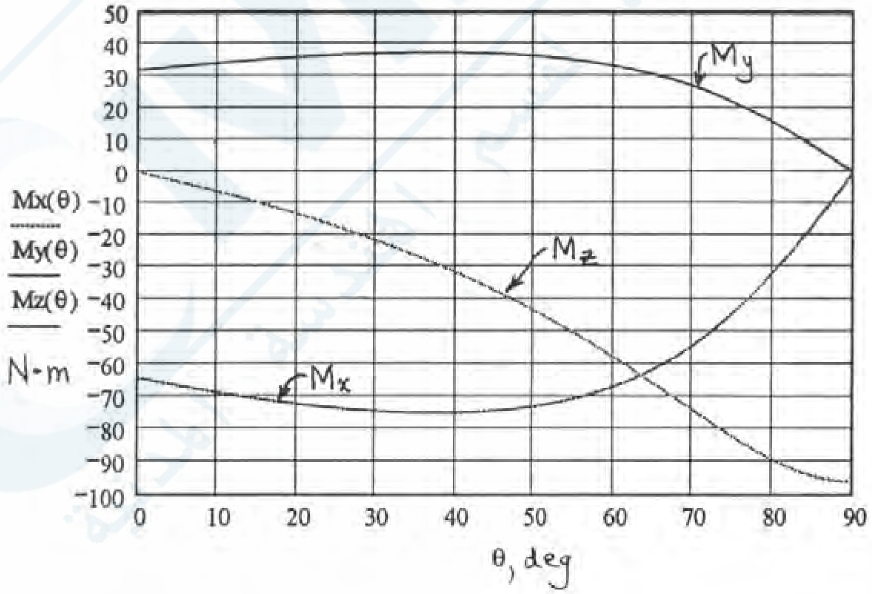
$= \dfrac{48\underline{i} + 96(1-\sin\theta)\underline{j} - 96\cos\theta\,\underline{k}}{\sqrt{1.44 - 1.28\sin\theta}}$ N

$\underline{r}_{OB} = 0.4\underline{i} + 0.8\underline{j}$ m

Carry out $\underline{M}_O = \underline{r}_{OB} \times \underline{T}$ to obtain

$\underline{M}_O = \dfrac{-76.8\cos\theta\,\underline{i} + 38.4\cos\theta\,\underline{j} - 38.4\sin\theta\,\underline{k}}{\sqrt{1.44 - 1.28\sin\theta}}$

The $\underline{i}$, $\underline{j}$, & $\underline{k}$ – components of $\underline{M}_O$ are shown in the following plot:



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$\underline{BC} = -1.2\cos\theta\,\underline{i} + 1.2(1-\sin\theta)\,\underline{j} - 2.4\,\underline{k}$ ft

$$|\underline{BC}| = \sqrt{(1.2\cos\theta)^2 + 1.2^2(1-\sin\theta)^2 + 2.4^2}$$

$$= \sqrt{8.64 - 2.88\sin\theta} \text{ ft}$$

$$F = k\delta = 180\left(\sqrt{8.64 - 2.88\sin\theta} - 2.4\right)$$

$$\underline{F} = F\underline{n}_{BC} = F\frac{\underline{BC}}{|\underline{BC}|}$$

$$= \frac{180\left[\sqrt{8.64-2.88\sin\theta} - 2.4\right]}{\sqrt{8.64-2.88\sin\theta}}\left[-1.2\cos\theta\,\underline{i} + 1.2(1-\sin\theta)\,\underline{j} - 2.4\,\underline{k}\right]$$

lb

Now, carry out $\underline{M}_O = \underline{r}_{OC} \times \underline{F}$, where $\underline{r}_{OC} = 1.2\underline{j}$ ft, to obtain

$$\underline{M}_O = \frac{\sqrt{8.64-2.88\sin\theta} - 2.4}{\sqrt{8.64-2.88\sin\theta}}\left[-518\,\underline{i} + 259\cos\theta\,\underline{k}\right]$$

lb-ft

Plots of the moment components:

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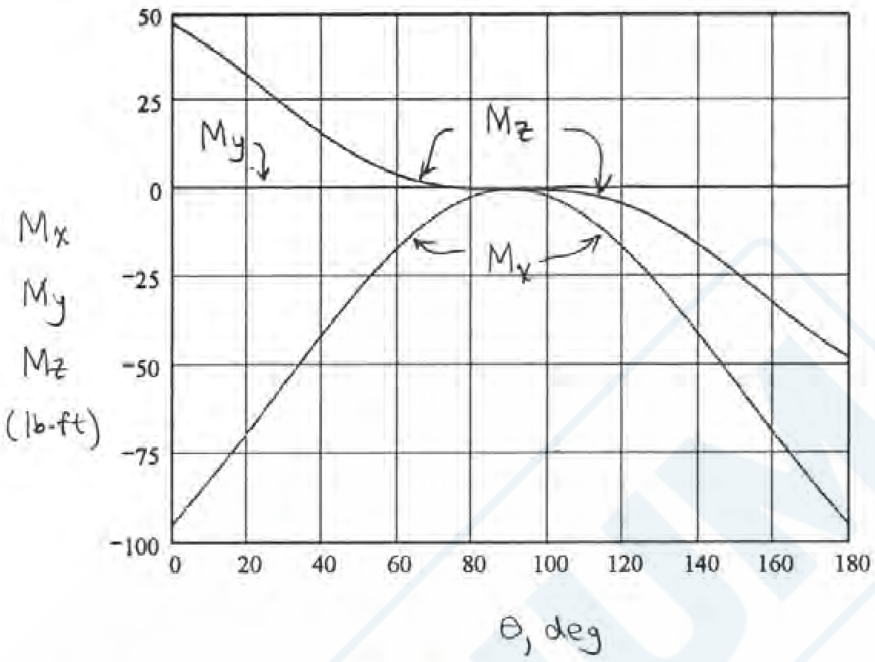

Maximum absolute values:

For M_x : 95.1 lb-ft @ $\theta = 0$ & $\theta = \pi$

For M_z : 47.6 lb-ft @ $\theta = 0$ & $\theta = \pi$

($M_y = 0$ for all θ)

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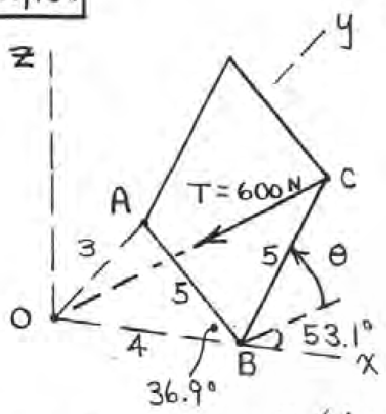


$$C = C(4 + 5\cos 53.1^\circ \cos\theta,\ 5\sin 53.1^\circ \cos\theta,\ 5\sin\theta)\ \text{m}$$

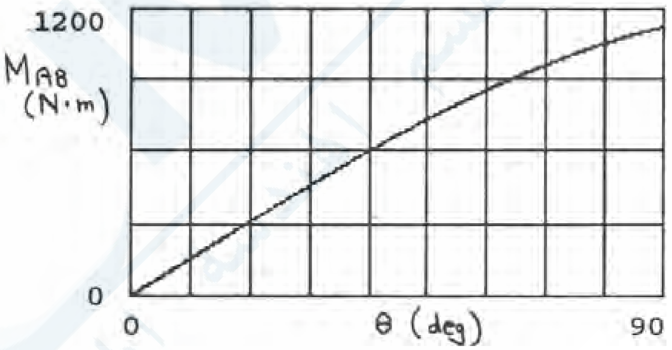
$$\underline{n}_{AB} = 0.8\underline{i} - 0.6\underline{j}$$

$$\underline{T} = T\underline{n}_{CO} = 600\left[\frac{-(4+3\cos\theta)\underline{i} - (4\cos\theta)\underline{j} - (5\sin\theta)\underline{k}}{\sqrt{(4+3\cos\theta)^2 + (4\cos\theta)^2 + (5\sin\theta)^2}}\right]$$

$$= \frac{600\left[-(4+3\cos\theta)\underline{i} - (4\cos\theta)\underline{j} - (5\sin\theta)\underline{k}\right]}{\sqrt{41 + 24\cos\theta}}\ \text{N}$$

$$\underline{M}_B = \underline{r}_{OB} \times \underline{T} = \frac{600}{\sqrt{41+24\cos\theta}}\left(-20\sin\theta\,\underline{j} + 16\cos\theta\,\underline{k}\right)\ \text{N}\cdot\text{m}$$

Now, $M_{AB} = \underline{M}_B \cdot \underline{n}_{AB} = \dfrac{7200\sin\theta}{\sqrt{41 + 24\cos\theta}}\ \text{N}\cdot\text{m}$



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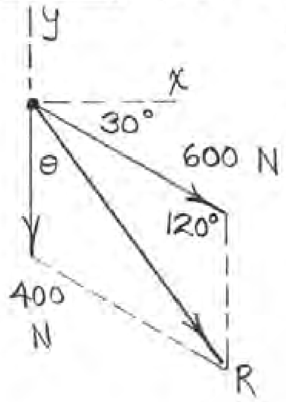
2/19 Law of cosines:



$$R^2 = 400^2 + 600^2 - 2(400)(600)\cos 120°$$

$R = 872\ \text{N}$

Law of sines:

$$\frac{600}{\sin\theta} = \frac{872}{\sin 120°}, \quad \theta = 36.6°$$

$$R_x = \Sigma F_x = 600 \cos 30° = 520\ \text{N}$$

$$R_y = \Sigma F_y = -600 \sin 30° - 400 = -700\ \text{N}$$

So $\mathbf{R} = 520\mathbf{i} - 700\mathbf{j}\ \text{N}$

(Check: $R = \sqrt{520^2 + 700^2} = 872\ \text{N}$ ✓

$\theta = \tan^{-1}\frac{520}{700} = 36.6°$ ✓)

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2/2

$\underline{F} = 600(\cos 30° \underline{i} - \sin 30° \underline{j})$

$= 520\underline{i} - 300\underline{j}$ lb

Scalar components: $\begin{cases} F_x = 520 \text{ lb} \\ F_y = -300 \text{ lb} \end{cases}$

Vector components: $\begin{cases} \underline{F}_x = 520\underline{i} \text{ lb} \\ \underline{F}_y = -300\underline{j} \text{ lb} \end{cases}$

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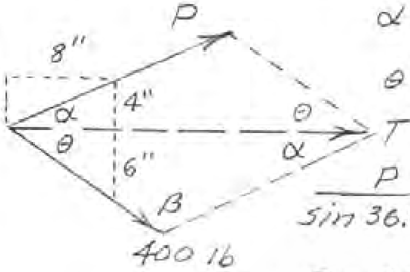
2/20



$\alpha = \tan^{-1}\frac{4}{8} = 26.57°$

$\theta = \tan^{-1}\frac{6}{8} = 36.87°$

$\beta = 180 - (\alpha + \theta) = 116.57°$

$$\frac{P}{\sin 36.87°} = \frac{400}{\sin 26.57°}$$

$$P = 400\,\frac{0.6}{0.4472} = \underline{537 \text{ lb}}$$

$$\frac{T}{\sin 116.56°} = \frac{400}{\sin 26.57°}, \quad T = 400\,\frac{0.8944}{0.4472} = \underline{800 \text{ lb}}$$

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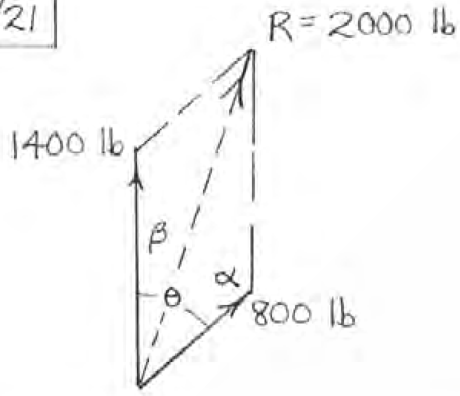

Law of cosines: $2000^2 = 1400^2 + 800^2 - 2(1400)(800)\cos\alpha$

With $\alpha = 180 - \theta$ and $\cos(180-\theta) = -\cos\theta$:

$2000^2 = 1400^2 + 800^2 + 2(1400)(800)\cos\theta$

$\underline{\theta = 51.3°}$

Law of sines: $\dfrac{800}{\sin\beta} = \dfrac{2000}{\sin(180° - 51.3°)}$

$\underline{\beta = 18.19°}$

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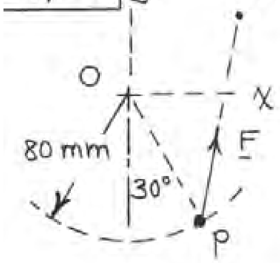
A (60, 40) mm



The coordinates of P are

$(80 \sin 30°, -80 \cos 30°) =$

$(40, -69.3)$ mm

$$\underline{n}_{PA} = \frac{\overline{PA}}{\overline{PA}} = \frac{(60-40)\underline{i} + (40-(-69.3))\underline{j}}{\sqrt{20^2 + 109.3^2}}$$

$$= 0.1800\underline{i} + 0.984\underline{j}$$

The magnitude of $\underline{F}$ is

$$F = kx = 1200\left[\frac{\overline{PA} - 100}{1000}\right] = 1200\left[\frac{111.1 - 100}{1000}\right]$$

$$= 13.32 \text{ N}$$

Then $\underline{F} = F\underline{n}_{PA} = 13.32(0.1800\underline{i} + 0.984\underline{j})$

$$= 2.40\underline{i} + 13.10\underline{j} \text{ N}$$

So $\begin{cases} F_x = 2.40 \text{ N} \\ F_y = 13.10 \text{ N} \end{cases}$

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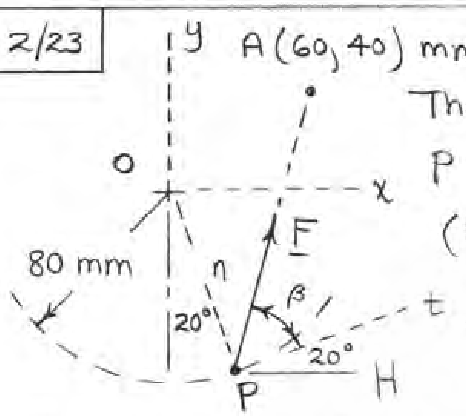

The x-y coordinates of P are

$(80 \sin 20°, -80 \cos 20°)$

or $(27.4, -75.2)$ mm

$$\overline{PA} = \left[(60-27.4)^2 + (40-(-75.2))^2\right]^{1/2} = 119.7 \text{ mm}$$

$$F = kx = 1200\left[\frac{119.7-100}{1000}\right] = 23.7 \text{ N}$$

The angle APH is

$$\alpha = \tan^{-1}\left[\frac{40+75.2}{60-27.4}\right] = 74.2°$$

Then $\beta = \alpha - 20° = 74.2 - 20 = 54.2°$

$$\begin{cases} F_t = F\cos\beta = 23.7 \cos 54.2° = \underline{13.84 \text{ N}} \\ F_n = F\sin\beta = 23.7 \sin 54.2° = \underline{19.18 \text{ N}} \end{cases}$$

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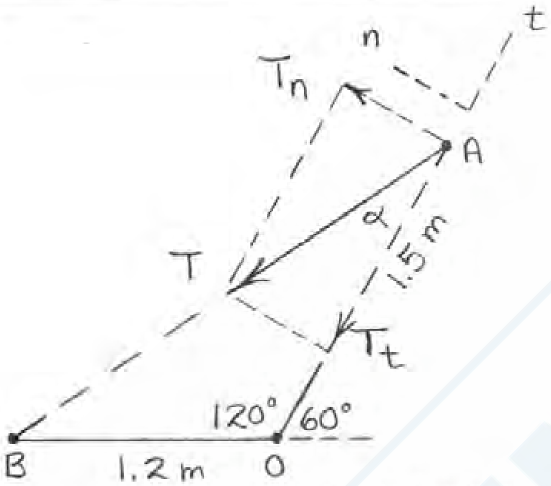

$$\overline{AB}^2 = 1.2^2 + 1.5^2 - 2(1.2)(1.5)\cos 120°$$

$$\overline{AB} = 2.34 \text{ m}$$

$$\frac{\sin\alpha}{1.2} = \frac{\sin 120°}{2.34}, \quad \alpha = 26.3°$$

$$T_n = T\sin\alpha = 750\sin 26.3° = \underline{333 \text{ N}}$$

$$T_t = -T\cos\alpha = -750\cos 26.3° = \underline{-672 \text{ N}}$$

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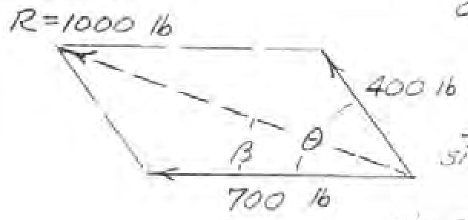
2/25 Law of cosines: $\overline{1000}^2 = \overline{400}^2 + \overline{700}^2 + 2(400)(700)\cos\theta$



$\cos\theta = 0.6250$, $\underline{\theta = 51.3°}$

Law of sines:

$$\frac{1000}{\sin(180-51.3)°} = \frac{400}{\sin\beta}$$

$$\sin\beta = \frac{400}{1000}\,0.7806 = 0.312$$

$$\underline{\beta = 18.19°}$$

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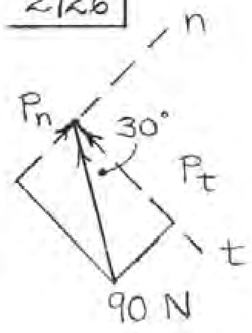
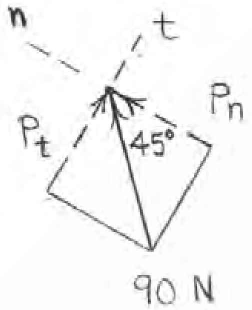


BC

$P_t = -90 \cos 30° = \underline{-77.9 \text{ N}}$

$P_n = 90 \sin 30° = \underline{45.0 \text{ N}}$



AB

$P_t = 90 \sin 45° = \underline{63.6 \text{ N}}$

$P_n = 90 \cos 45° = \underline{63.6 \text{ N}}$

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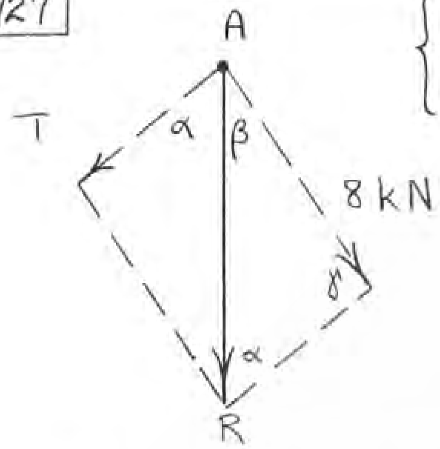

$\alpha = \tan^{-1} \frac{50}{40} = 51.3°$

$\beta = \tan^{-1} \frac{40}{60} = 33.7°$

$\gamma = 180 - \alpha - \beta = 95.0°$

$\frac{\sin\beta}{T} = \frac{\sin\alpha}{8}$

$T = 5.68 \text{ kN}$

$\frac{\sin\gamma}{R} = \frac{\sin\alpha}{8}$, $R = 10.21 \text{ kN}$

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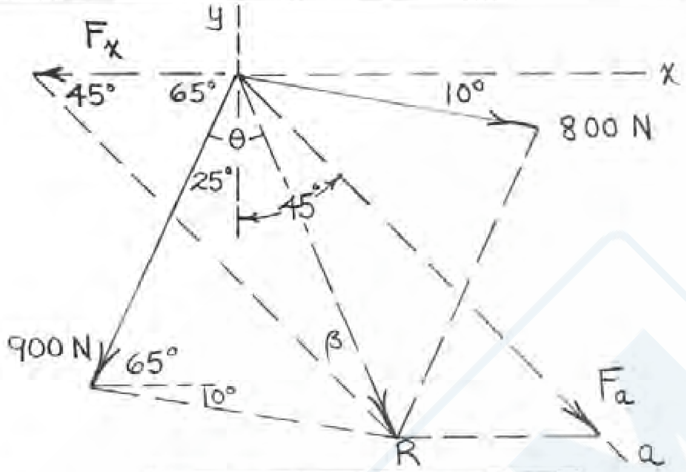

Law of cosines: $R = \sqrt{900^2 + 800^2 - 2(900)(800)\cos 75°} = 1038$ N

Law of sines: $\frac{1038}{\sin 75°} = \frac{800}{\sin\theta}$, $\theta = 48.1°$

$\beta = 180° - 45° - (65° + 48.1°) = 21.9°$

$\frac{1038}{\sin 45°} = \frac{F_x}{\sin 21.9°}$

$\underline{F_x = 547 \text{ N}}$

$\frac{F_a}{\sin(65° + 48.1°)} = \frac{1038}{\sin 45°}$

$\underline{F_a = 1350 \text{ N}}$

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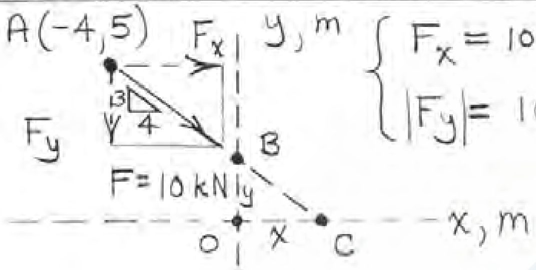

$\left\{ \begin{array}{l} F_x = 10\left(\frac{4}{5}\right) = 8 \text{ kN} \\ |F_y| = 10\left(\frac{3}{5}\right) = 6 \text{ kN} \end{array} \right.$

$+\curvearrowright M_O = 8(5) - 6(4) = \underline{16 \text{ kN}\cdot\text{m CW}}$

By similar triangles, $\frac{4+x}{5} = \frac{4}{3}$, $x = 2.67$ m

$\frac{y}{2.67} = \frac{3}{4}$, $y = 2$ m

So the two intercept points are $(x, y) =$

$\underline{(2.67, 0)}$ m and $\underline{(0, 2) \text{ m}}$.

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2/3

$$\underline{F} = 4.8\left(-\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}\right)$$

$$= -2.88\underline{i} - 3.84\underline{j} \ \text{kN}$$

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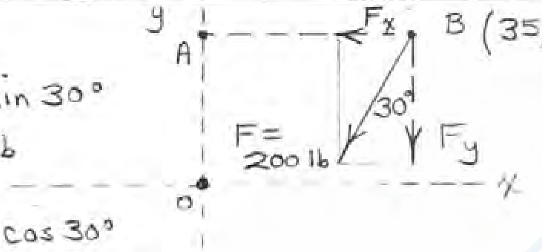

$|F_x| = 200 \sin 30° = 100$ lb

$|F_y| = 200 \cos 30° = 173.2$ lb

$+↻\ M_A = 173.2(35) = 6060$ lb-in. (505 lb-ft) CW

$+↻\ M_O = 173.2(35) - 100(25)$

$= 3560$ lb-in. (297 lb-ft) CW

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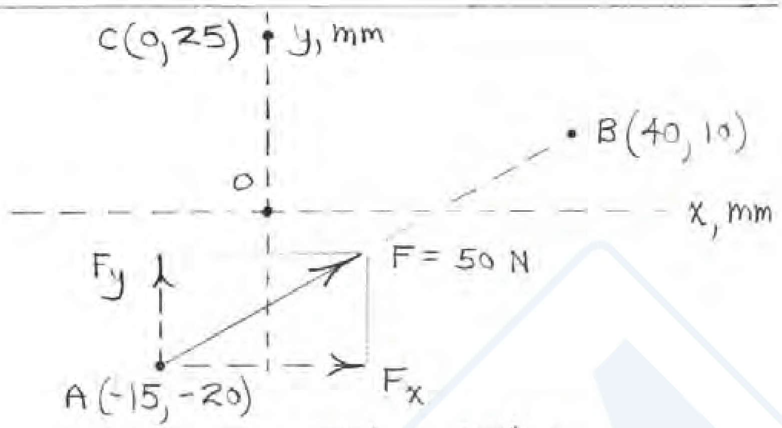

$$\mathbf{F} = F\mathbf{n}_{AB} = 50\left[\frac{55\mathbf{i} + 30\mathbf{j}}{\sqrt{55^2 + 30^2}}\right]$$

$$= 43.9\mathbf{i} + 23.9\mathbf{j} \text{ N}$$

$$\circlearrowleft M_O = 43.9(15) - 23.9(20) = \underline{179.6 \text{ N}\cdot\text{mm CCW}}$$

$$\mathbf{M}_C = \mathbf{r}_{CB} \times \mathbf{F}$$

$$= (40\mathbf{i} - 15\mathbf{j}) \times (43.9\mathbf{i} + 23.9\mathbf{j})$$

$$= \underline{1616\mathbf{k} \text{ N}\cdot\text{mm}} \quad \text{or} \quad \underline{1.616\mathbf{k} \text{ N}\cdot\text{m}}$$

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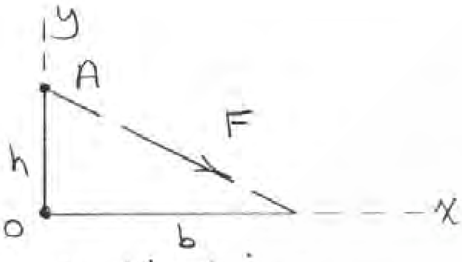

$$\mathbf{F} = F\left[\frac{b\mathbf{i} - h\mathbf{j}}{\sqrt{b^2+h^2}}\right]$$

Acting at A:

$$+\circlearrowright M_O = \frac{Fb}{\sqrt{h^2+b^2}}(h) = \frac{Fbh}{\sqrt{h^2+b^2}} \quad \text{CW}$$

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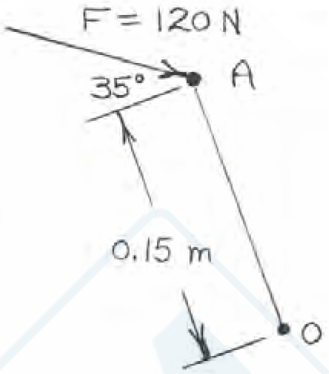
$\overset{+}{\curvearrowright} M_O = 120 \cos 35° (0.15)$

$= \underline{14.74 \text{ N}\cdot\text{m CW}}$



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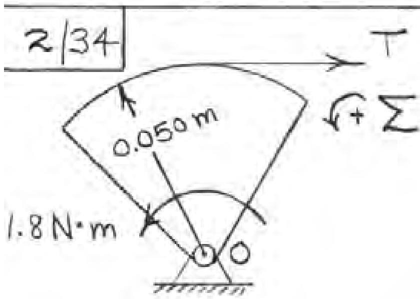

$$\circlearrowleft + \sum M_O = 1.8 - T(0.050) = 0$$

$$\underline{T = 36 \text{ N}}$$

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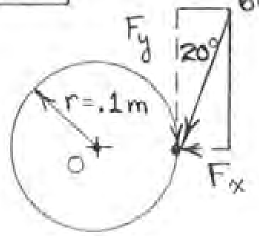

60 N $+\circlearrowleft M_O = r F_y$

$= (0.1)(60 \cos 20°)$

$= \underline{5.64 \text{ N}\cdot\text{m}}$

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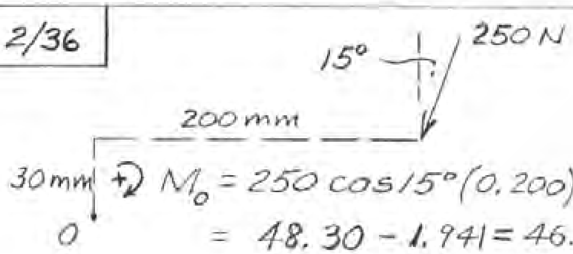

$$+\circlearrowright M_O = 250\cos 15°(0.200) - 250\sin 15°(0.030)$$

$$= 48.30 - 1.941 = \underline{46.4\ N\cdot m}$$

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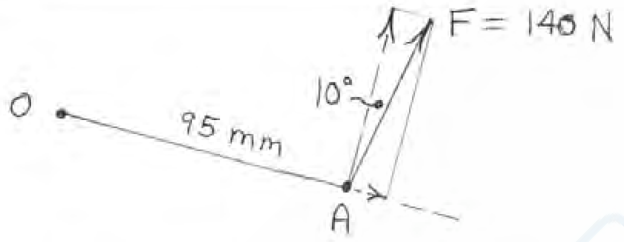

$\circlearrowleft + \; M_O = (140 \cos 10°)(0.095) = \underline{13.10 \text{ N·m CCW}}$

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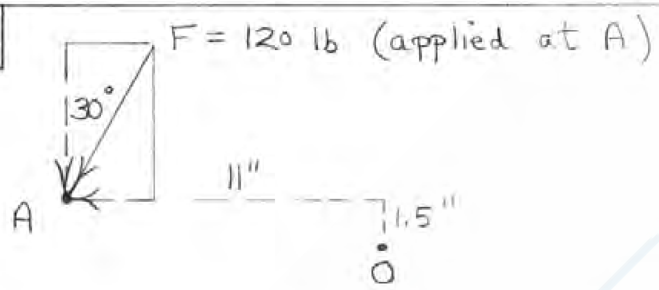

$\curvearrowleft + \; M_O = 120 \cos 30°(11) + 120 \sin 30°(1.5)$

$= 1233$ lb-in. or 102.8 lb-ft CCW

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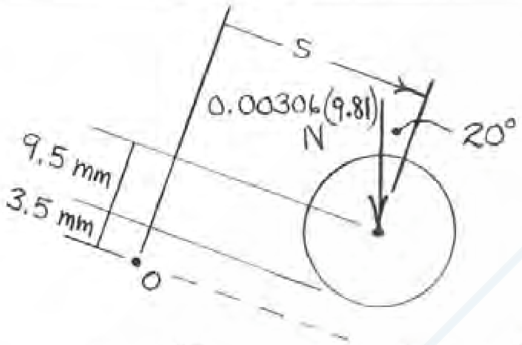

$$\circlearrowleft^{+} M_O = 0.00306(9.81)\left[s\cos 20° + (9.5+3.5)\sin 20°\right]$$

$$= \underline{0.1335 + 0.0282s \text{ N·mm} \quad (s \text{ in mm})}$$

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2/4 $\underline{F} = F\underline{n}_{AB} = 4800\left[\dfrac{45\underline{i} + 30\underline{j}}{\sqrt{45^2 + 30^2}}\right]$

$= 3990\underline{i} + 2660\underline{j}$ lb

Scalar components: $\begin{cases} F_x = 3990 \text{ lb} \\ F_y = 2660 \text{ lb} \end{cases}$

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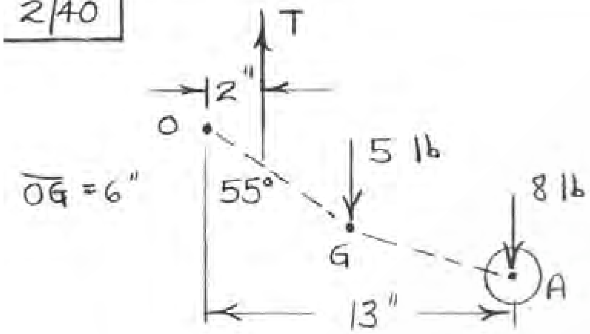

The combined moment about O of the 5-lb and 8-lb weights is

$$\curvearrowright+ \; M_O = 5(6 \sin 55°) + 8(13) = 128.6 \text{ lb-in. (CW)}$$

$$\curvearrowright+ \; \Sigma M_O = 0 : -T(2) + 128.6 = 0$$

$$T = 64.3 \text{ lb}$$

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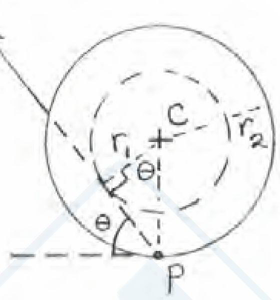
$\circlearrowright + \quad M_C = T r_1 = 32(5)$

$= \underline{160 \text{ lb-in. CW}}$

$\cos\theta = r_1/r_2 = 5/8$

$\underline{\theta = 51.3°}$



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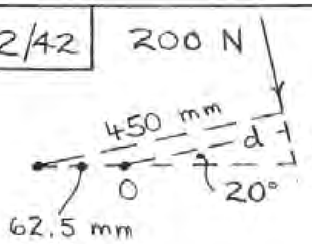

$$d = 450 - 62.5 \cos 20^\circ$$

$$= 391 \text{ mm}$$

$$\circlearrowleft + M = Fd = 200(0.391)$$

$$= \underline{78.3 \text{ N}\cdot\text{m}}$$

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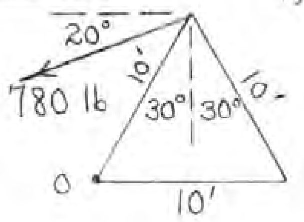

$\circlearrowleft +$ $M_O = 780 \cos 20° (10 \cos 30°)$

$- 780 \sin 20° (5) = \underline{5010 \text{ lb-ft}}$

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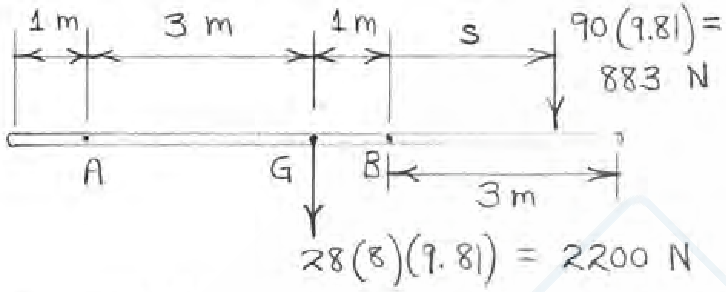

$\curvearrowleft + \; M_B = 2200(1) - 883s = 0, \quad \underline{s = 2.49 \text{ m}}$

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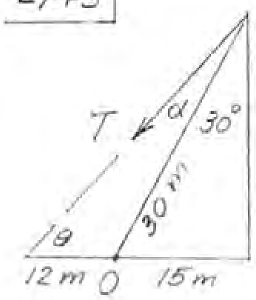

$$\theta = \tan^{-1}\frac{30(0.866)}{12+15} = 43.90°$$

$$\alpha = 90° - (30° + 43.90°) = 16.10°$$

$$M_O = 72 \text{ kN·m}$$

$$= T \sin 16.10° (30) = 8.32T$$

$$T = \frac{72}{8.32} = \underline{8.65 \text{ kN}}$$

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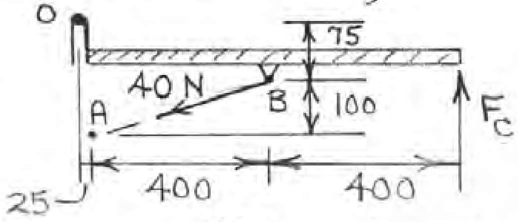
2/46 (Dim. in mm)



$AB = \sqrt{400^2 + 100^2}$

$= 412$ mm

$\circlearrowleft + \; M_O = \left(\frac{400}{412} \cdot 40\right)(75) + \left(\frac{100}{412} \cdot 40\right)(425)$

$= 7030$ N·mm or $\underline{7.03 \text{ N·m}}$

$\circlearrowleft + \; \Sigma M_O = 0 : -F_C(825) + 7030 = 0$

$\underline{F_C = 8.53 \text{ N}}$

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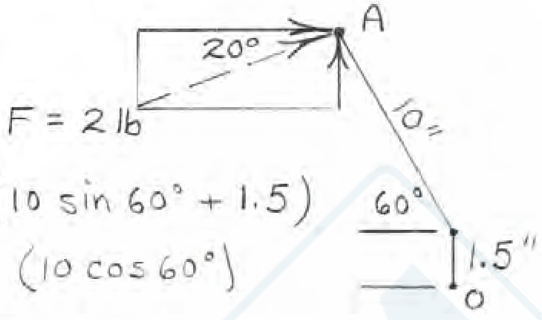

$+\circlearrowright M_O = 2\cos 20°(10\sin 60° + 1.5)$

$+ 2\sin 20°(10\cos 60°)$

$= \underline{22.5 \text{ lb-in. CW}}$

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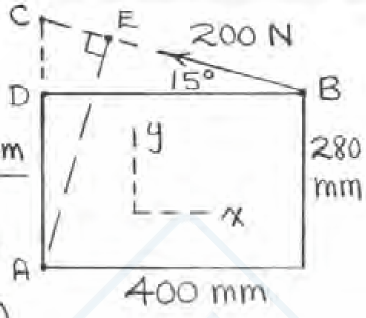

(1) $M_A = 200 \cos 15° (0.280) + 200 \sin 15° (0.400) = \underline{74.8 \text{ N·m}}$

(2) $AE = 0.280 \cos 15° + 0.400 \sin 15°$
$= 0.374 \text{ m}$; $M_A = 200(0.374) = \underline{74.8 \text{ N·m}}$

(3) Apply force at C
$CD = 0.400 \tan 15° = 0.1072 \text{ m}$
$CA = 0.280 + 0.1072 = 0.387 \text{ m}$
$M_A = (200 \cos 15°)(0.387) = \underline{74.8 \text{ N·m}}$

(4) $\underline{M}_A = \underline{r} \times \underline{F} = (0.400\underline{i} + 0.280\underline{j}) \times 200(-\cos 15°\underline{i} + \sin 15°\underline{j}) = \underline{74.8\underline{k} \text{ N·m}}$

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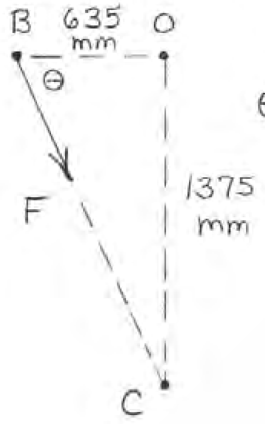

$$F = kx = 60\left(\sqrt{0.635^2 + 1.375^2} - 0.740\right)$$

$$= 46.5 \text{ N}$$

$$\theta = \tan^{-1}\frac{1.375}{0.635} = 65.2°$$

$$\circlearrowleft + \quad M_O = 46.5 \sin 65.2° \,(0.635)$$

$$= \underline{26.8 \text{ N·m CCW}}$$

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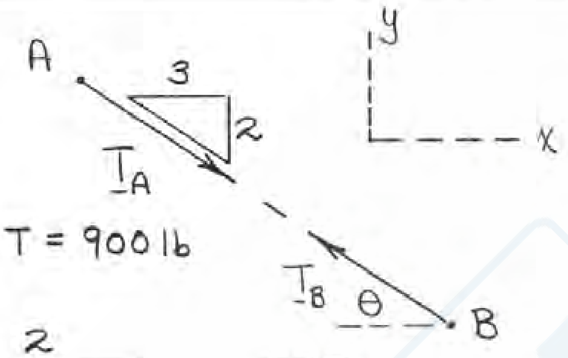

$T_A = T_B = T = 900$ lb

$$\sin\theta = \frac{2}{\sqrt{2^2+3^2}} = 0.555$$

$$\cos\theta = \frac{3}{\sqrt{2^2+3^2}} = 0.832$$

$$\mathbf{T}_A = 900(0.832\mathbf{i} - 0.555\mathbf{j}) = \underline{749\mathbf{i} - 499\mathbf{j} \text{ lb}}$$

$$\mathbf{T}_B = -\mathbf{T}_A = \underline{-749\mathbf{i} + 499\mathbf{j} \text{ lb}}$$

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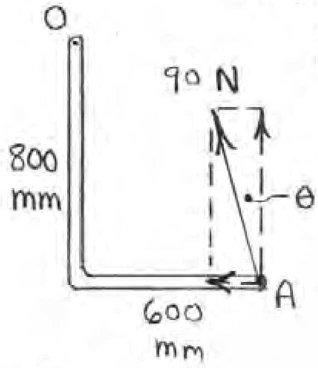
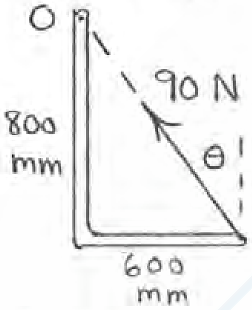
(a) $\theta = 15°$



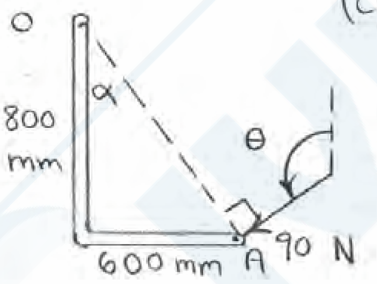
$$\curvearrowleft M_O = 90\cos 15°(0.6) - 90\sin 15°(0.8)$$

$$= \underline{33.5 \text{ N}\cdot\text{m}}$$



(b) $\theta = \tan^{-1}\left(\frac{600}{800}\right) = \underline{36.9°}$

$\underline{(\text{or } 217°)}$



(c) $F \perp OA$, so

$\theta = 90° + \alpha$, $\alpha = \tan^{-1}\frac{600}{800} = 36.9°$

So $\theta = \underline{126.9°}$ (or $\underline{307°}$)

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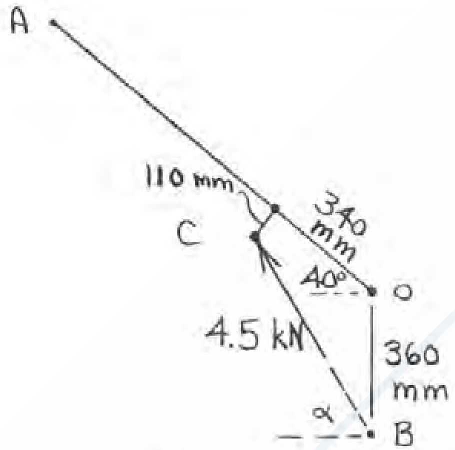

$$\alpha = \tan^{-1}\left[\frac{360 + 340 \sin 40° - 110 \sin 50°}{340 \cos 40° + 110 \cos 50°}\right]$$

$$= 56.2°$$

$$\circlearrowright + \; M_O = 4.5\,(0.360 \cos 56.2°) = \underline{0.902 \text{ kN·m CW}}$$

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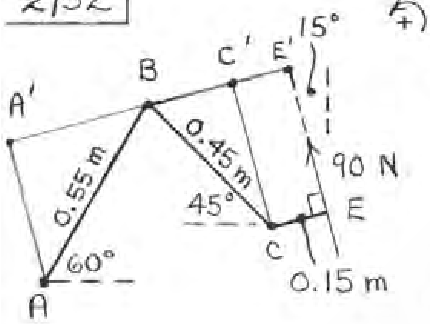

$M_C = F(\overline{CE}) = 90(0.15) = \underline{13.50 \text{ N·m}}$

$M_B = F(\overline{BE'})$

$= 90(0.15 + 0.45 \sin 30°)$

$= \underline{33.8 \text{ N·m}}$

$M_A = F(\overline{A'E'})$

$= 90(0.15 + 0.45 \sin 30° + 0.55 \sin 45°)$

$= \underline{68.8 \text{ N·m}}$

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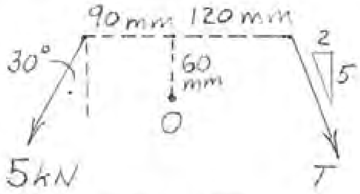
2/53 $M_O = 5[(\cos 30°)90 + (\sin 30°)60]$

$- T\left[\frac{5}{\sqrt{29}}(120) + \frac{2}{\sqrt{29}}(60)\right] = 0$



$\sqrt{2^2+5^2} = \sqrt{29}$

$539.7 - 133.7T = 0$, $\underline{T = 4.04 \text{ kN}}$

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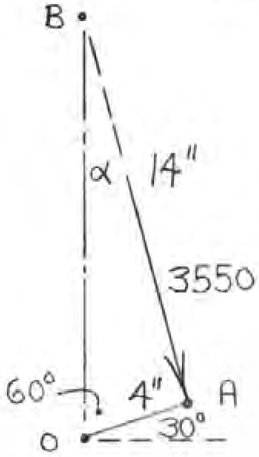

Law of sines: $\frac{4}{\sin\alpha} = \frac{14}{\sin 60°}$

$\alpha = 14.33°$

$\overline{BO} = 14\cos 14.33° + 4\cos 60°$

$= 15.56$ in.

Consider 3550 lb acting at B:

$\circlearrowleft M_O = (3550 \sin 14.33°)(15.56)$

$= \underline{13,670 \text{ lb-in.}}$

(or $M_O = \underline{1139 \text{ lb-ft}}$)

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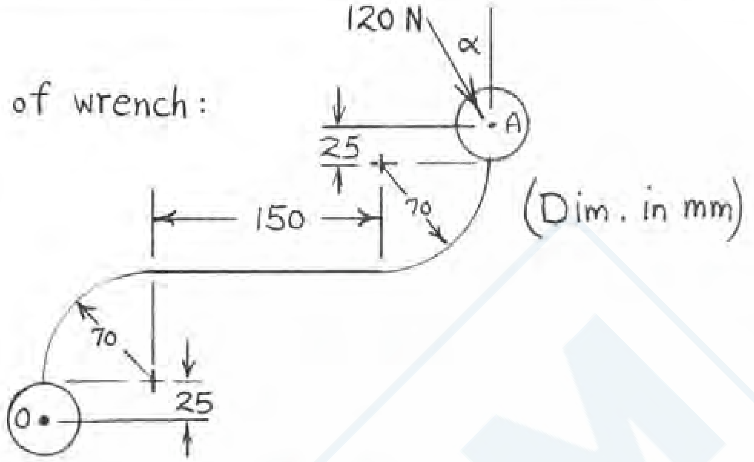
Elements of wrench:



$\alpha = 30°$:

$$\circlearrowleft + M_O = 120 \cos 30° [70 + 150 + 70]$$

$$+ 120 \sin 30° [25 + 70 + 70 + 25] = 41\,500 \text{ N·mm}$$

or $\underline{M_O = 41.5 \text{ N·m CW}}$

For maximum M_O:

$$\alpha = \tan^{-1}\left[\frac{25+70+25+70}{70+150+70}\right] = \underline{33.2°}$$

$$(M_O)_{max} = 120\sqrt{(25+70+25+70)^2 + (70+150+70)^2}$$

$$= 41\,600 \text{ N·mm or } \underline{41.6 \text{ N·m CW}}$$

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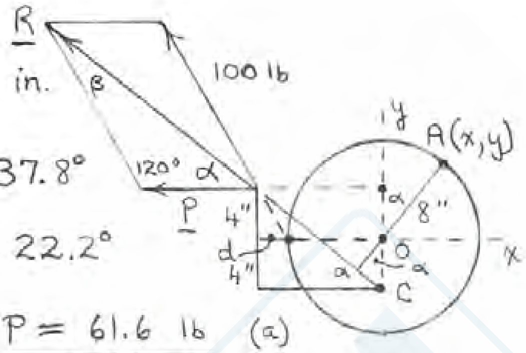

$d = \frac{4}{\tan 60°} = 2.31$ in.

$\alpha = \tan^{-1}\left(\frac{8}{d+8}\right) = 37.8°$

$\beta = 180° - (120° + \alpha) = 22.2°$

$\frac{P}{\sin\beta} = \frac{100}{\sin\alpha}$, $\underline{P = 61.6 \text{ lb}}$ (a)

$\frac{R}{\sin 120°} = \frac{100}{\sin\alpha}$, $\underline{R = 141.3 \text{ lb}}$ 37.8° (b)

$x = 8 \sin\alpha = \underline{4.90}$ in., $y = 8\cos\alpha = \underline{6.32 \text{ in.}}$ (c)

$+\circlearrowleft M_A = Rd = 141.3\,(8 + 4\cos\alpha) = \underline{1577 \text{ lb-in.}}$ (d)

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2/57 $\curvearrowright + \; M = Fd = 80(1.4) = \underline{112 \text{ lb-in. CW}}$

$\underline{(\text{or } 9.33 \text{ lb-ft CW})}$

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2/58 $\underline{F} = 60\left(-\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}\right) = \underline{-36\underline{i} + 48\underline{j}\ \text{N}}$

$\circlearrowleft + \; M_C = 48(50) = 2400\ \text{N}\cdot\text{mm} = \underline{2.4\ \text{N}\cdot\text{m CCW}}$

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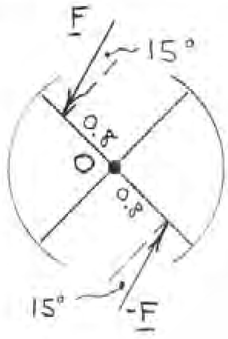

$$\circlearrowleft + \; M_O = \Sigma F d$$

$$25 = 2F(\cos 15^\circ)(0.8)$$

$$F = \underline{16.18 \text{ N}}$$

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2/6

$$\underline{F} = 1800\left(-\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}\right) = -1080\underline{i} - 1440\underline{j} \text{ N}$$

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2/60

$\underline{\mathbf{R} = 6\mathbf{j}\ \text{kN}}$ @ $x = \frac{400}{6000} = 0.0667$ m

or $\underline{x = 66.7 \text{ mm}}$

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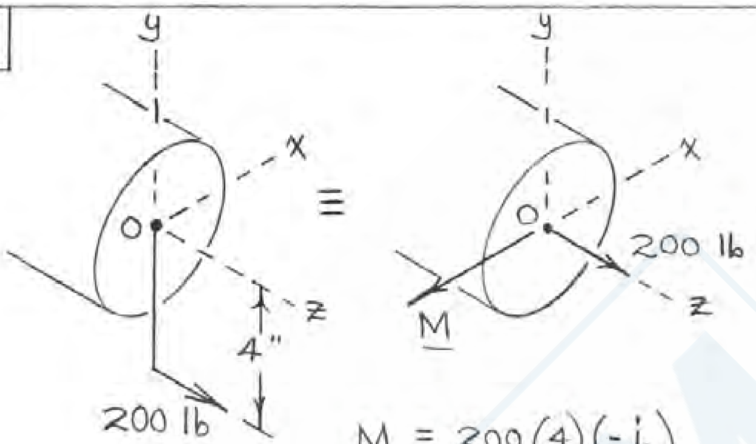

$$\underline{M} = 200(4)(-\underline{i})$$

$$= -800\underline{i} \text{ lb-in.}$$

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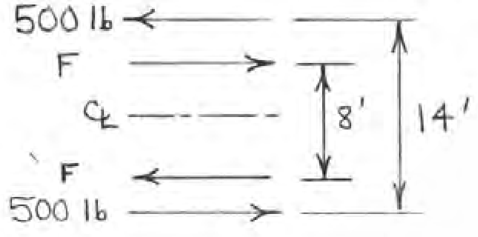

$\circlearrowleft + \; \Sigma M = 500(14) - F(8) = 0$

$F = 875 \text{ lb}$

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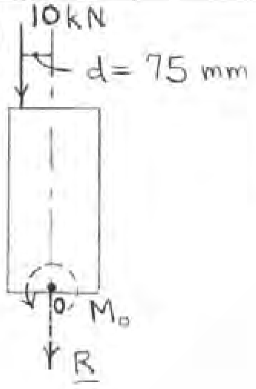

$$R = \underline{10\ \text{kN}}\ (\downarrow)$$

$$M_O = Fd = (10)(0.075)$$

$$= \underline{0.75\ \text{kN}\cdot\text{m}}\ (\text{CCW})$$

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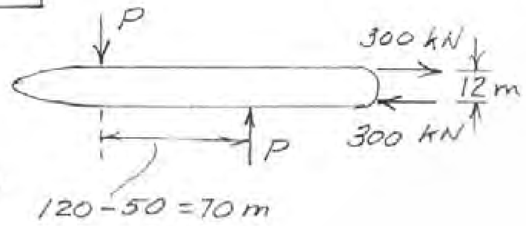

$70P = 300(12)$

$P = 51.4$ kN

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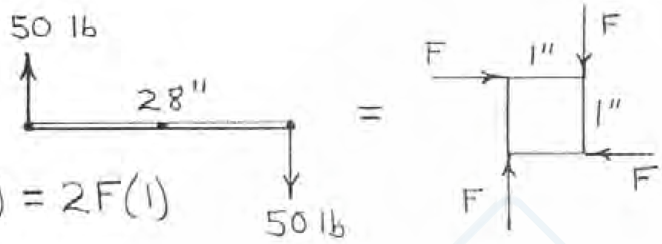

$$M = 50(28) = 2F(1)$$

$$\underline{F = 700 \text{ lb}}$$

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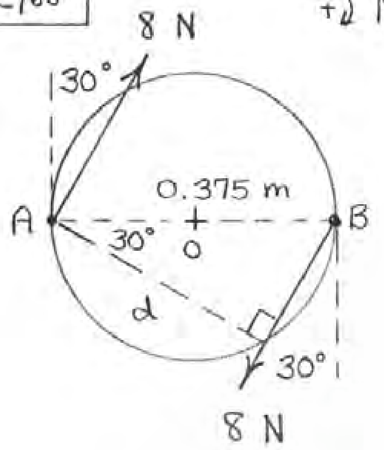

$\curvearrowright^{+}$ $M = Fd = 8\,(0.375 \cos 30°)$

$= \underline{2.60 \text{ N·m CW}}$

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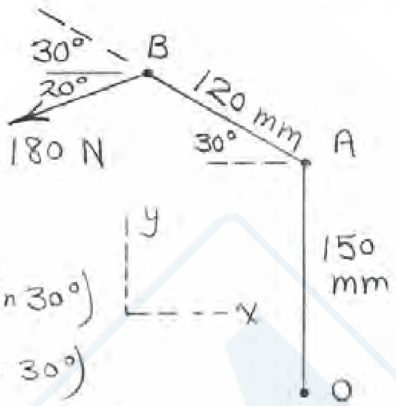
2/67

$\underline{F} = 180(-\cos 20°\underline{i} - \sin 20°\underline{j})$

$= -169.1\underline{i} - 61.6\underline{j}$ N



$\curvearrowleft M_O = 180 \cos 20°(150 + 120 \sin 30°)$

$+ 180 \sin 20°(120 \cos 30°)$

$= 41\,900$ N·mm or 41.9 N·m CCW

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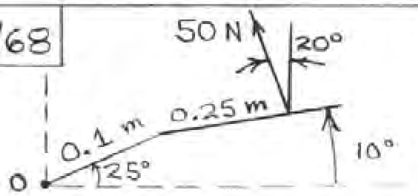
2/68



Use principle of moments.

$$\circlearrowleft_+ \Sigma M_O = 50\cos 20°\left[0.1\cos 25° + 0.25\cos 10°\right]$$

$$+\, 50\sin 20°\left[0.1\sin 25° + 0.25\sin 10°\right]$$

$$= 17.29\ \text{N}\cdot\text{m}$$

Force - Couple System at O:

$$\begin{cases} R = 50\ \text{N} \quad \angle 110° \\ M_O = 17.29\ \text{N}\cdot\text{m} \quad \circlearrowleft \end{cases}$$

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2/69

$\underline{F} = 250(\sin 10°\underline{i} + \cos 10°\underline{j})$

$= \underline{43.4\underline{i} + 246\underline{j} \text{ N}}$

$+\circlearrowright M_O = 250\left[\cos 10°(0.235) + \sin 10°(0.050)\right]$

$= \underline{60.0 \text{ N·m CW}}$

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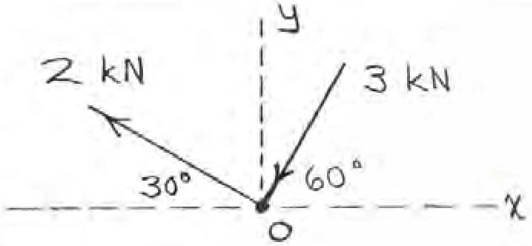

$$R_x = \Sigma F_x = -2\cos 30^\circ - 3\cos 60^\circ = -3.23 \text{ kN}$$

$$R_y = \Sigma F_y = 2\sin 30^\circ - 3\sin 60^\circ = -1.598 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{3.61 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-1.598}{-3.23}\right) = \underline{206^\circ}$$

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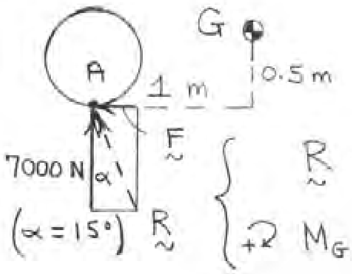

$$\tan 15° = \frac{F}{7000}$$

$$F = 1876 \text{ N}$$

At G:

$$R = \Sigma F = \underline{7250 \text{ N}} \quad \angle 105°$$

$$\curvearrowleft + M_G = 7000(1) + 1876(.5)$$

$$= \underline{7940 \text{ N}\cdot\text{m}}$$

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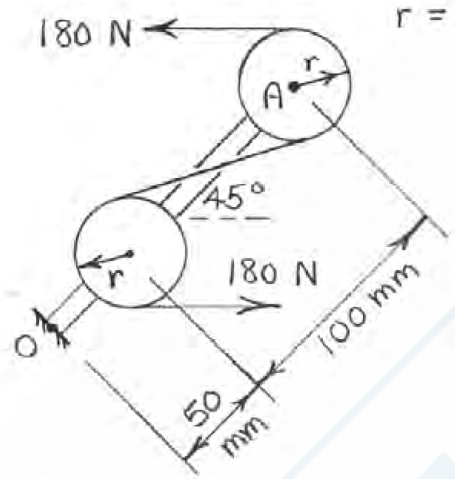

The system at O is a couple.

$\circlearrowleft + M = Fd = 180(100 \sin 45° + 25 + 25)$

$= 21\ 700$ N·mm or $\underline{21.7 \text{ N·m CCW}}$

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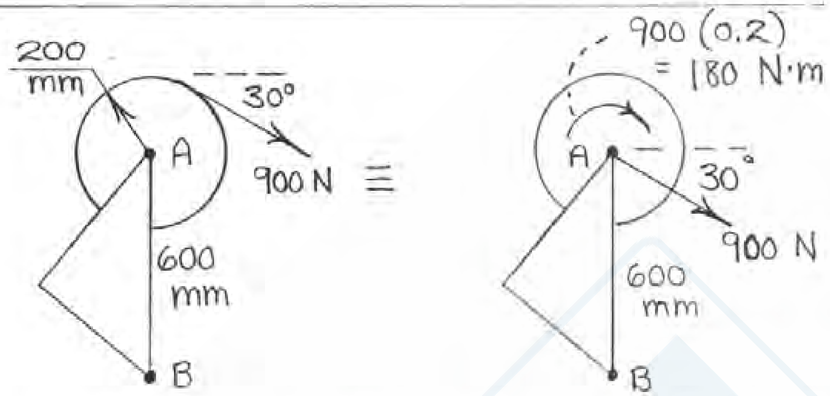

$\circlearrowright M_B = 180 + 900 \cos 30° (0.6) = \underline{648 \text{ N·m CW}}$

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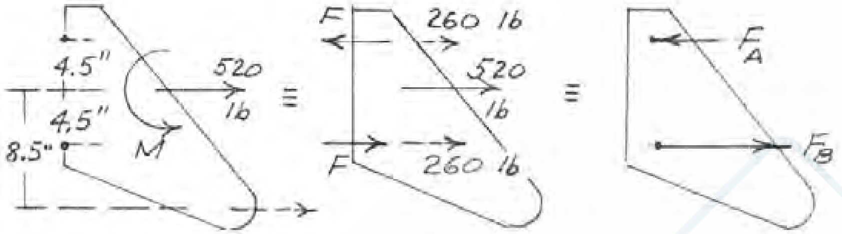

$M = Fd = 520(8.5) = 4420$ lb-in.

$F(9) = 4420, \quad F = 491$ lb

$F_A = 491 - 260 = \underline{231 \text{ lb}}$

$F_B = 491 + 260 = \underline{751 \text{ lb}}$

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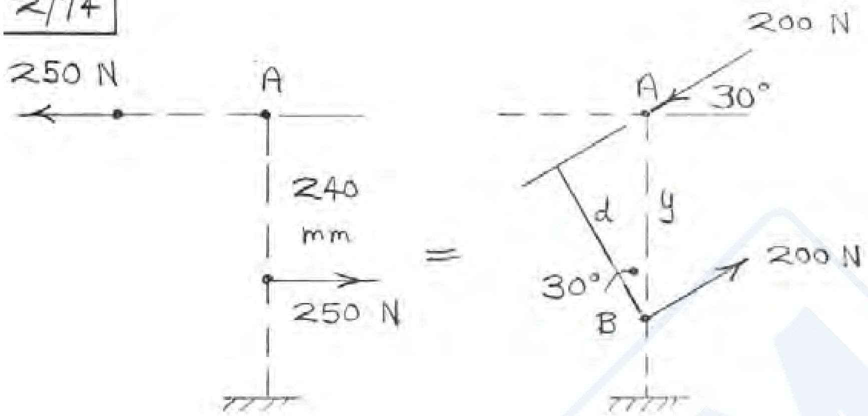

Equal couples :

$$\curvearrowleft_{+} \; 250(240) = 200(y\cos 30^\circ)$$

$$\underline{y = 346 \text{ mm}}$$

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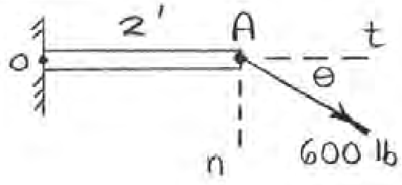

The equivalent force-couple system at O is

$R_t = 600 \cos\theta$ (lb)

$R_n = 600 \sin\theta$ (lb)

$\circlearrowleft + M_O = 2(600 \sin\theta) = 1200 \sin\theta$ (lb-ft)

Constraints:

$$\begin{cases} 600 \cos\theta \leq 550, & \theta \geq 23.6^\circ \\ 600 \sin\theta \leq 550, & \theta \leq 66.4^\circ \\ 1200 \sin\theta < 1000, & \theta \leq 56.4^\circ \end{cases}$$

All considered, $\underline{23.6^\circ \leq \theta \leq 56.4^\circ}$

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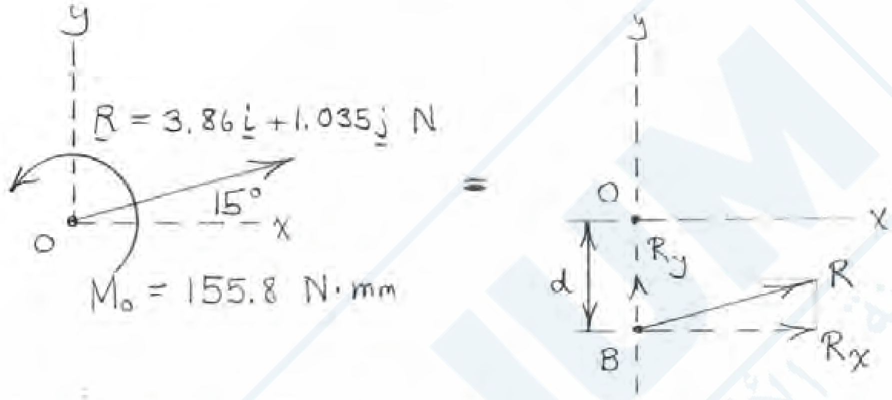
2/76 At O:

$\underline{R} = 4(\cos 15° \underline{i} + \sin 15° \underline{j}) = 3.86\underline{i} + 1.035\underline{j}$ N

$\circlearrowleft + M_O = 300 - 4\cos 15°(40) + 4\sin 15°(10)$

$= 155.8$ N·mm CCW



Condition: $R_x d = M_O$

$3.86 d = 155.8$, $d = 40.3$ mm

So $\underline{y = -40.3 \text{ mm}}$

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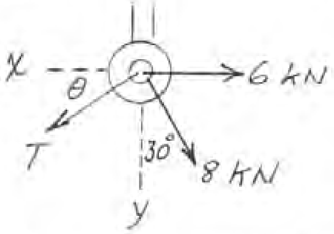

$R = R_y = 15 = T\sin\theta + 8\cos 30°$

$R_x = 0 = T\cos\theta - 6 - 8\sin 30°$

So $T\sin\theta = 8.07$

$T\cos\theta = 10$

Divide & get $\theta = \tan^{-1}\frac{8.07}{10}$

$\underline{\theta = 38.9°}$

$T = \frac{10}{\cos 38.9°} = \underline{12.85 \text{ kN}}$

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2/78 $R_x = \Sigma F_x = 60 \cos 40° + 50 \sin 20° - 30 \cos 20°$

$= 34.87 \text{ kN}$

$R_y = \Sigma F_y = 60 \sin 40° + 40 - 50 \cos 20° + 30 \sin 20°$

$= 41.84 \text{ kN}$

$R = \sqrt{34.87^2 + 41.84^2} = \underline{54.5 \text{ kN}}$

$\theta_x = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{41.84}{34.87} = \underline{50.2°}$

$\underline{\mathbf{R} = 34.9\mathbf{i} + 41.8\mathbf{j} \text{ kN}}$

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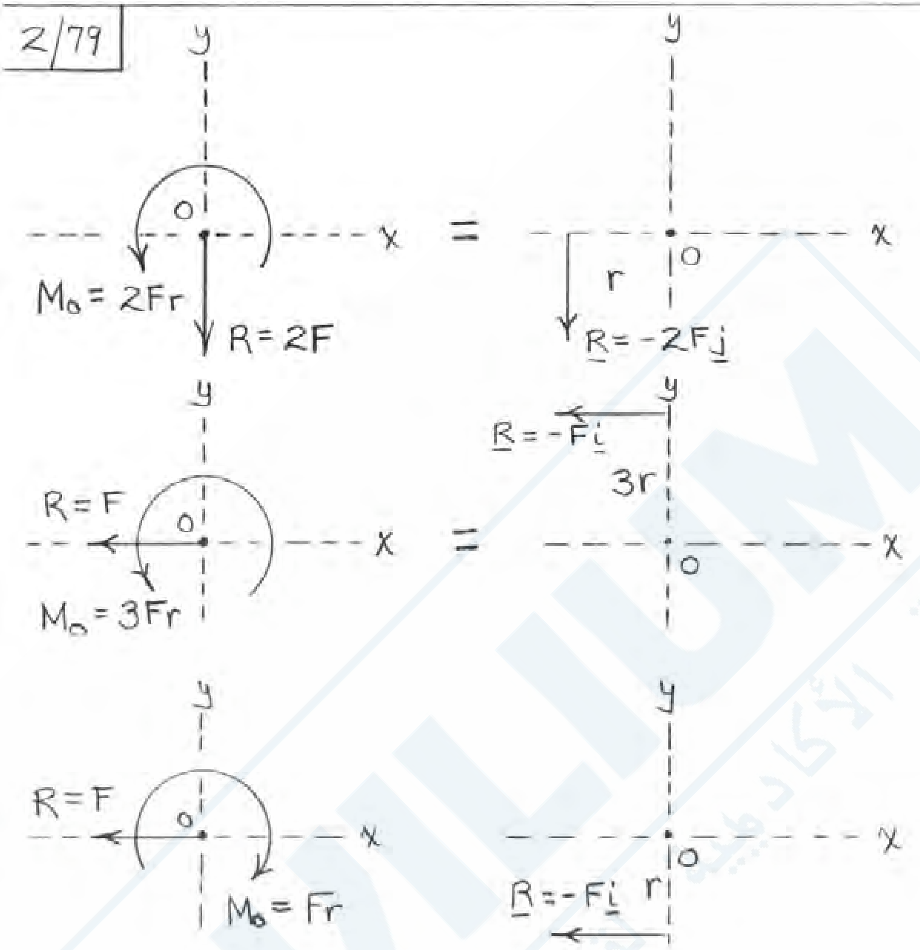

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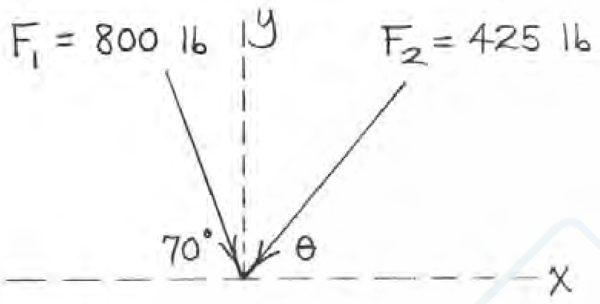

$R_x = \Sigma F_x = 800 \cos 70° - 425 \cos\theta = 0$

$\underline{\theta = 49.9°}$

$R_y = \Sigma F_y = -800 \sin 70° - 425 \sin 49.9°$

$= -1077$ lb

So $\underline{R = 1077 \text{ lb}}$

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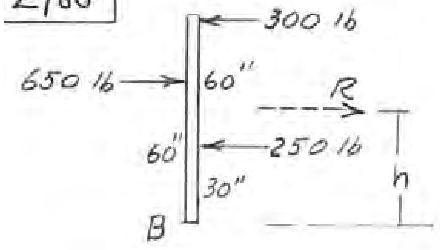

$R = \Sigma F = 650 - 250 - 300$
$= 100 \text{ lb}$

$Rh = \Sigma M_B$;

$100h = 650(60) - 300(90) - 250(30)$

$h = 45 \text{ in.}$

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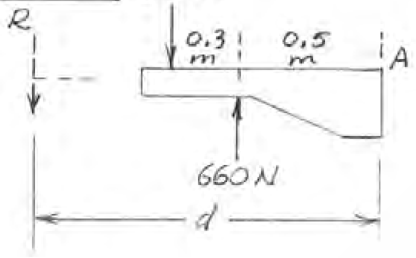

$R = \Sigma F = 680 - 660 = 20\ N$

$Rd = \Sigma M_A$

$20d = 680(0.8) - 660(0.5)$

$d = 10.70\ m$ to the left of A

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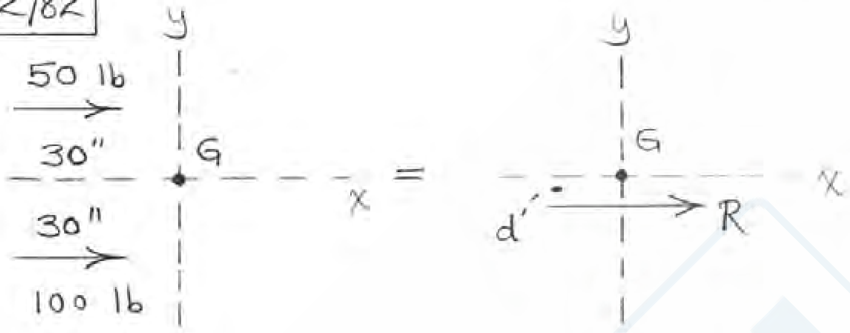

$\overset{+}{\rightarrow} R = \Sigma F = 50 + 100 = 150$ lb (to the right)

$\curvearrowleft M_G = 100(30) - 50(30) = 1500$ lb-ft CCW

$= Rd = 150d, \quad d = 10$ in.

So the y-intercept of the standalone force resultant is $\underline{y = -10 \text{ in.}}$ The effect is to propel the truck forward and rotate it CCW about G.

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2/83

$M_O = 0$, so

$$\circlearrowleft + \quad M - 400(0.150 \cos 30°) - 320(0.300) = 0$$

$$\underline{M = 148.0 \text{ N·m}}$$

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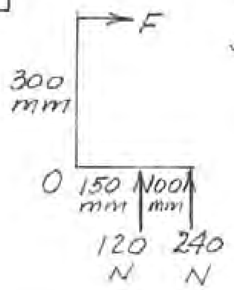

$\Sigma M_O = 0$ for R to pass through O

$300F - 120(150) - 240(250) = 0$

$F = 260$ N

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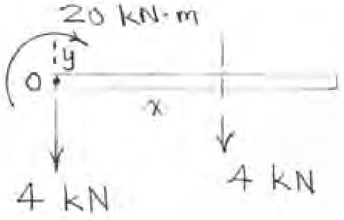
2/85 Force-Couple system at point O:

$\underline{R} = \Sigma \underline{F} = (6-10)\,\underline{j} = -4\underline{j}$ kN

$\circlearrowleft + \; M_O = 6(3) - 10(9) + 52 = -20$ kN·m



$x = \dfrac{M_O}{R} = \dfrac{20}{4}$

$= \underline{5 \text{ m}}$ (on beam!)

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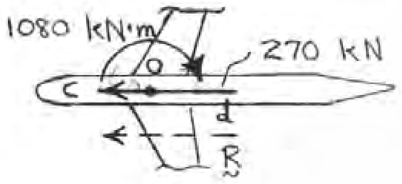
2/86 Force – Couple system at point O:

$$\begin{cases} R = 3(90) = 270 \text{ kN } (\leftarrow) \\ +\circlearrowleft M_O = 12(90) = 1080 \text{ kN}\cdot\text{m} \end{cases}$$



$$d = \frac{M_O}{R} = \frac{1080}{270} = \underline{4 \text{ m}}$$

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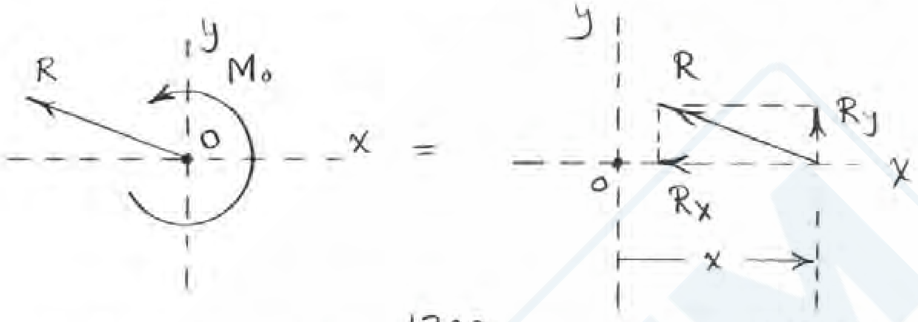
2/87

$\underline{R = -50i + 20j \text{ lb}}$

$\circlearrowleft + M_O = -40(10) + 60(20) + 50(10) = 1300 \text{ lb-in.}$



$R_y x = M_O$, $x = \frac{1300}{20} = \underline{65 \text{ in. (off pipe)}}$

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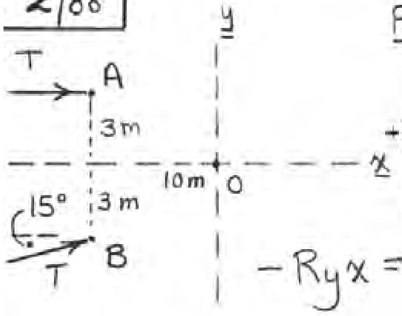
2/88



$$\mathbf{R} = \Sigma \mathbf{F} = T\mathbf{i} + T(\cos 15^\circ \mathbf{i} + \sin 15^\circ \mathbf{j})$$

$$= \underline{1.966T\,\mathbf{i} + 0.259T\,\mathbf{j}}$$

$$+\circlearrowleft M_O = 3T - T\cos 15^\circ (3) + T\sin 15^\circ (10) = \underline{2.69T}$$

$$-R_y x = M_O: \quad -0.259T(x) = 2.69T$$

$$\underline{x = -10.39 \text{ m}}$$

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2/89 Equivalent force - couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (-25 + 20 \sin 30°)\underline{i} + (-30 - 20 \cos 30°)\underline{j} = \underline{-15\underline{i} - 47.3\underline{j} \text{ kN}}$$

$$\circlearrowleft + M_O = 25(5) - 30(9) - (20 \cos 30°)9 - (20 \sin 30°)5 = -351 \text{ kN·m}$$

For final location of $\underline{R}$:

$$\underline{r} \times \underline{R} = \underline{M}_O, \quad (x\underline{i} + y\underline{j}) \times (-15\underline{i} - 47.3\underline{j}) = -351\underline{k}$$

$$-47.3x + 15y = -351$$

Axis intersections: $\underline{x = 7.42 \text{ m}, \ y = -23.4 \text{ m}}$

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2/9

$$P_x = -260\left(\frac{12}{13}\right) = -240 \text{ N}$$

$$P_y = -260\left(\frac{5}{13}\right) = \underline{-100 \text{ N}}$$

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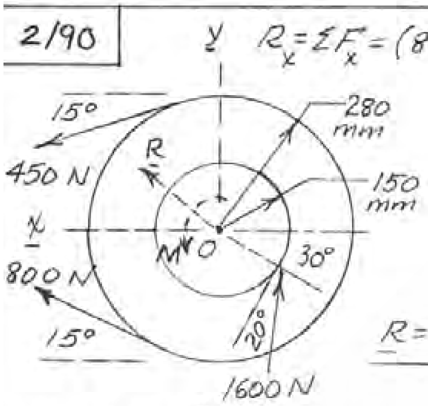
2/90



$R_x = \Sigma F_x = (800 + 450)\cos 15° - 1600 \sin(30° - 20°)$

$= 1207 - 278 = 929.6$ N

$R_y = \Sigma F_y = 1600 \cos 10°$

$+ (800 - 450)\sin 15°$

$= 1576 + 90.6 = 1666$ N

$\underline{\mathbf{R} = 930\mathbf{i} + 1666\mathbf{j} \text{ N}}$

$M = \Sigma M_O$ (+↺); $M = 1600 \cos 20° (0.150) + (450 - 800) 0.280$

$= 225.5 - 98.0 = \underline{127.5 \text{ N·m CCW}}$

so unit is speeding up in CCW dir.

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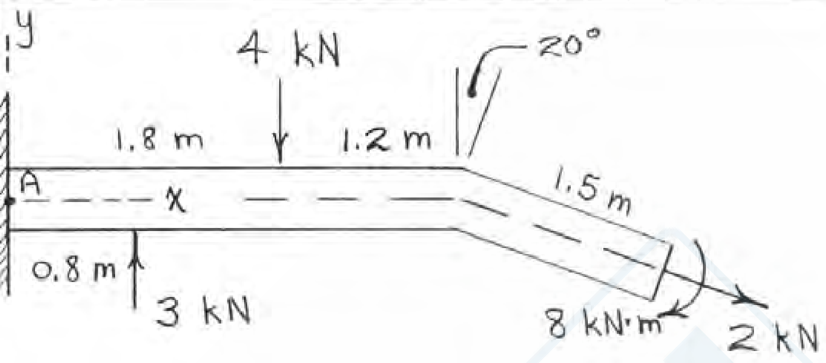

$R_x = \Sigma F_x = 2 \cos 20° = 1.879$ kN

$R_y = \Sigma F_y = 3 - 4 - 2 \sin 20° = -1.684$ kN

$\underline{R} = 1.879\underline{i} - 1.684\underline{j}$ kN, $R = \sqrt{1.879^2 + 1.684^2} = 2.52$ kN

$\circlearrowright + \; M = \Sigma M_A = 4(1.8) - 3(0.8) + 2 \sin 20° (3.0) + 8$

$= 14.85$ kN·m CW

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2/92 $\Sigma M_O = 0$ since $\underline{R}$ passes through O.

$$40(8) + 60(4) - 5P\cos 20° = 0, \quad \underline{P = 119.2 \text{ lb}}$$

Moment of 40-lb & 60-lb forces unaffected by θ
so result for P is not dependent on θ.

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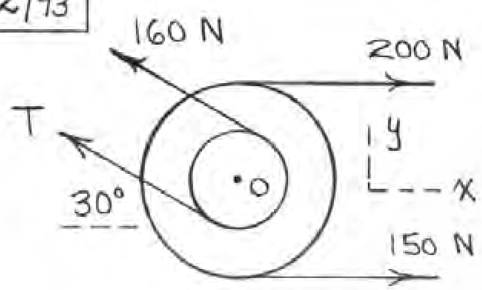

$\begin{cases} r_1 = 0.1 \text{ m} \\ r_2 = 0.2 \text{ m} \end{cases}$

$+\circlearrowleft \Sigma M_O = 0: \quad 200(0.2) - 150(0.2) - 160(0.1) + (0.1)T = 0$

$\underline{T = 60 \text{ N}}$

$R_x = \Sigma F_x = 200 + 150 - (160 + 60)\cos 30° = 159.5 \text{ N}$

$R_y = \Sigma F_y = (160 + 30)\sin 30° = 110 \text{ N}$

$R = \sqrt{R_x^2 + R_y^2} = \underline{193.7 \text{ N}}$

$\theta = \tan^{-1}(R_y/R_x) = \underline{34.6°}$

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2/94 $\underline{R} = 45\underline{i} - 15\underline{j}$ lb

$\circlearrowleft M_A = 25(30) + 15(60) = 1650$ lb-in.

or $\underline{M}_A = -1650\underline{k}$ lb-in.

For final line of action, $\underline{r} \times \underline{R} = \underline{M}_A$

$(x\underline{i} + y\underline{j}) \times (45\underline{i} - 15\underline{j}) = -1650\underline{k}$

$\Rightarrow -15x - 45y = -1650$ or $y = -\frac{1}{3}x + \frac{110}{3}$ in.

(Axis intercepts: $x = 110''$, $y = 110/3''$)

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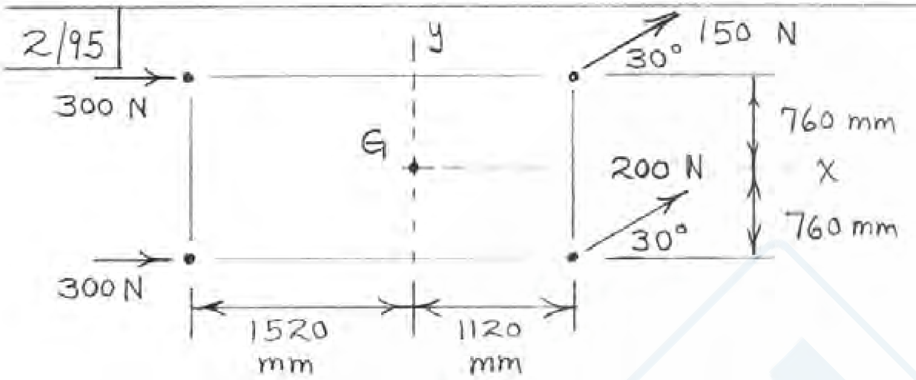
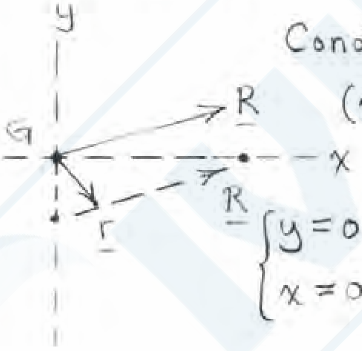
2/95



$$\underline{R} = \Sigma \underline{F} = 2(300\underline{i}) + 150(\cos 30°\underline{i} + \sin 30°\underline{j})$$
$$+ 200(\cos 30°\underline{i} + \sin 30°\underline{j}) = \underline{903\underline{i} + 175\underline{j}\ \text{N}}$$

$$\circlearrowleft M_G = -150\cos 30°(760) + 150\sin 30°(1120)$$
$$+ 200\cos 30°(760) + 200\sin 30°(1120)$$
$$= 229\,000\ \text{N}\cdot\text{mm or } 229\ \text{N}\cdot\text{m CCW}$$



Condition: $\underline{r} \times \underline{R} = \underline{M}_G$

$$(x\underline{i} + y\underline{j}) \times (903\underline{i} + 175\underline{j}) = +229\underline{k}$$
$$175x - 903y = +229$$

$y = 0$: $x = 1.308$ m

$x = 0$: $\underline{y = -0.253\ \text{m}}$

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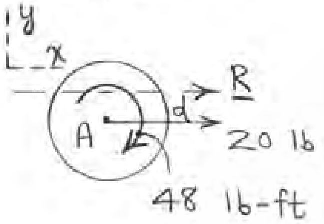
2/96 Force - Couple System at point A:

$$\underline{R} = \Sigma \underline{F} = -500\underline{j} + 60\underline{i} - 100\underline{j} - 40\underline{i} + 600\underline{j}$$

$$= \underline{20\underline{i} \text{ lb}}$$

$$\curvearrowleft + M_A = 2 - 40\left(\frac{15}{12}\right) = -48 \text{ lb-ft}$$



$$Rd = M_A$$

$$d = \frac{M_A}{R} = \frac{48}{20}$$

$$= \underline{2.40 \text{ ft}}$$

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2/97 Use $\overset{y}{\uparrow}\underset{}{\curvearrowleft}{+}\rightarrow x$ system at G:

$$\underline{R} = \Sigma \underline{F} = (80 + 40 + 40 + 50 \sin 30°)\underline{i} + (50 \cos 30° + 70)\underline{j}$$

$$= \underline{185\,\underline{i} + 113.3\,\underline{j} \text{ lb}}$$

$$M_G = 70(66) + 50 \sin 30° (36) = 5520 \text{ lb-in.}$$

$$= \underline{460 \text{ lb-ft}} \ (\curvearrowleft)$$

For line of action of resultant:

$$\underline{r} \times \underline{R} = \underline{M}_G$$

$$(x\underline{i} + y\underline{j}) \times (185\underline{i} + 113.3\underline{j}) = 460\,\underline{k}$$

$$113.3x - 185y = 460$$

$$\underline{x = 4.06 \text{ ft}} \text{ when } y = 0.$$

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2/98 For a zero force-couple system at point O:

$$\underline{R} = \Sigma \underline{F} = (-F_C \sin 30^\circ + F_D \sin 30^\circ)\,\underline{i} + (50 - 10 - 100 - 50 + F_B + F_C \cos 30^\circ + F_D \cos 30^\circ)\,\underline{j} = \underline{0}$$

$$\Rightarrow F_C = F_D = F$$

$$\circlearrowleft M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) - 50(2.5) + 2F\cos 30^\circ (2.9) = 0$$

$$\underline{F = F_C = F_D = 6.42 \text{ N}}, \quad \underline{F_B = 98.9 \text{ N}}$$

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2/99

$\underline{F} = F\underline{n}$

$= 60\left[\dfrac{40\underline{i} - 50\underline{j} + 110\underline{k}}{\sqrt{40^2 + 50^2 + 110^2}}\right]$

$= \underline{18.86\underline{i} - 23.6\underline{j} + 51.9\underline{k}\ \text{N}}$

$\cos\theta_y = \dfrac{F_y}{F} = \dfrac{-23.6}{60}$, $\underline{\theta_y = 113.1^\circ}$

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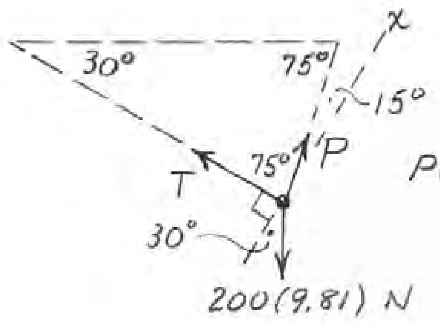

$\Sigma F_x = 0$

$P \cos 15° - 200(9.81) \cos 30° = 0$

$\underline{P = 1759 \text{ N}}$

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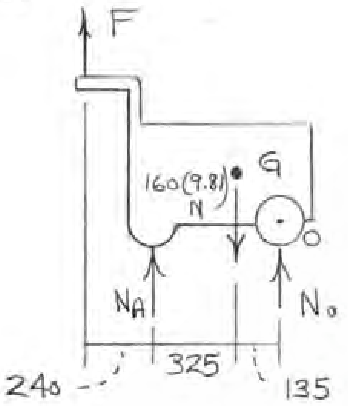

With $F = 0$:

$\Sigma M_O = 0$: $160(9.81)(135) - N_A(460) = 0$

$N_A = 461 \text{ N}$

For $\frac{1}{2} N_A$:

$\Sigma M_O = 0$: $160(9.81)(135) - \frac{461}{2}(460) - F(700) = 0$

$F = 151.4 \text{ N}$

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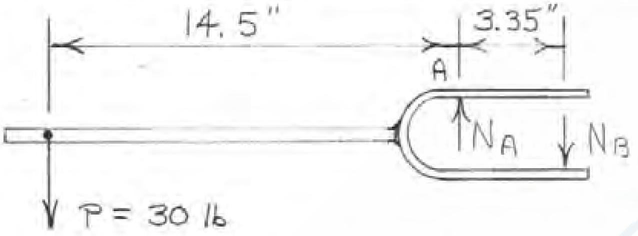

$$\circlearrowleft \Sigma M_A = 0: \quad 30(14.5) - N_B(3.35) = 0$$

$$N_B = 129.9 \text{ lb}$$

$$\uparrow + \Sigma F = 0: \quad -30 + N_A - 129.9 = 0$$

$$N_A = 159.9 \text{ lb}$$

So the forces applied to the stud are

$$\begin{cases} N_A = 159.9 \text{ lb} \downarrow \\ N_B = 129.9 \text{ lb} \uparrow \end{cases}$$

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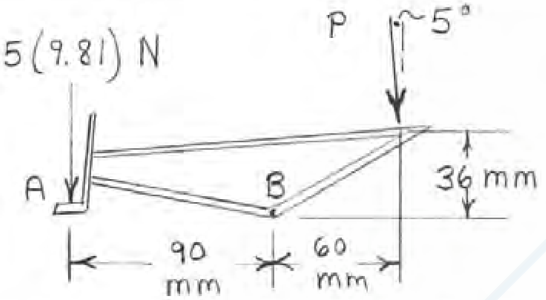

$$\circlearrowleft + \sum M_B = 0: \ 25(9.81)(90) - P\cos 5°(60) - P\sin 5°(36) = 0$$

$$\underline{P = 351 \text{ N}}$$

Assumptions: The weight of the panel acts at the 90-mm dimension as shown above; panel does not slip.

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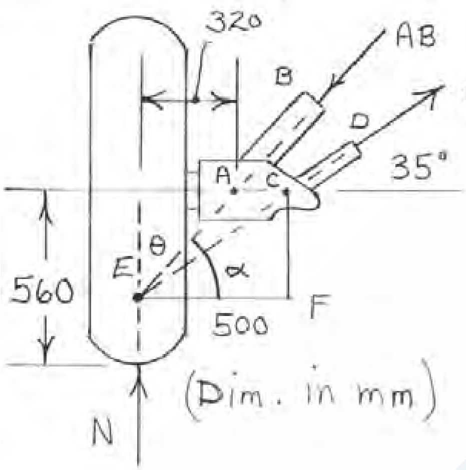
3/102 The three forces must be concurrent at point E.



$\overline{CF} = 500 \tan 35°$

$= 350$ mm

$\tan\theta = \dfrac{320}{350}$

$\theta = 42.4°$

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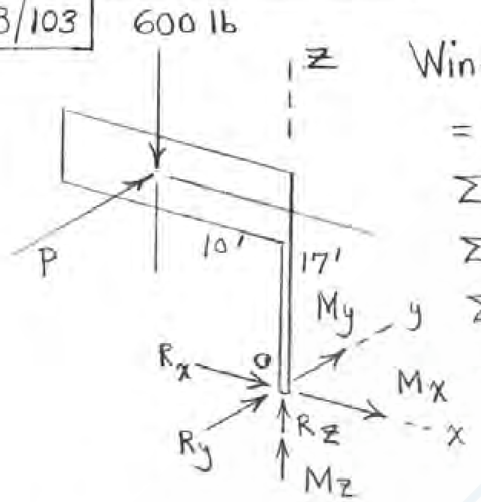

Wind force $P = pA$

$= 17.5(6)(12) = 1260$ lb

$\Sigma F_x = 0 \Rightarrow R_x = 0$

$\Sigma F_y = 0 \Rightarrow R_y = -1260$ lb

$\Sigma F_z = 0 \Rightarrow R_z = 600$ lb

$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$= 1396$ lb

$\Sigma \underline{M}_O = \underline{0}: \quad \underline{M} + (-10\underline{i} + 17\underline{k}) \times (1260\underline{j} - 600\underline{k}) = \underline{0}$

$\Rightarrow \underline{M} = 21,400\,\underline{i} + 6000\,\underline{j} + 12,600\,\underline{k}$ lb-ft

$M = 25,600$ lb-ft

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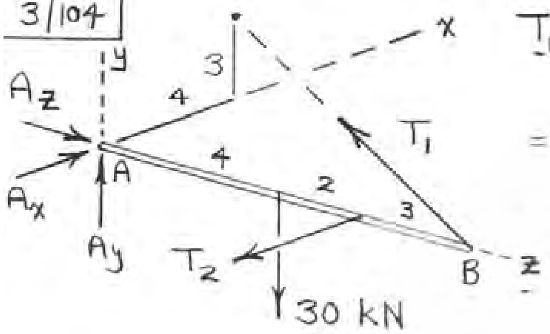

$$\underline{T}_1 = T_1\left[\frac{4\underline{i} + 3\underline{j} - 9\underline{k}}{\sqrt{4^2+3^2+9^2}}\right]$$

$$= T_1(0.389\underline{i} + 0.291\underline{j} - 0.874\underline{k})$$

$$\underline{T}_2 = -T_2\underline{i}$$

$\Sigma M_x = 0: \; 30(4) - 0.291\,T_1(9) = 0, \quad \underline{T_1 = 45.8 \text{ kN}}$

$\Sigma M_y = 0: \; -T_2(6) + 0.389\,T_1(9) = 0, \quad \underline{T_2 = 26.7 \text{ kN}}$

$\Sigma \underline{F} = \underline{0}: \; \underline{A} + 45.8(0.389\underline{i} + 0.291\underline{j} - 0.874\underline{k})$

$\qquad -26.7\underline{i} - 30\underline{j} = 0$

$\underline{A} = 8.89\underline{i} + 16.67\underline{j} + 40.0\underline{k} \quad \text{kN}$

$\underline{A = 44.2 \text{ kN}}$

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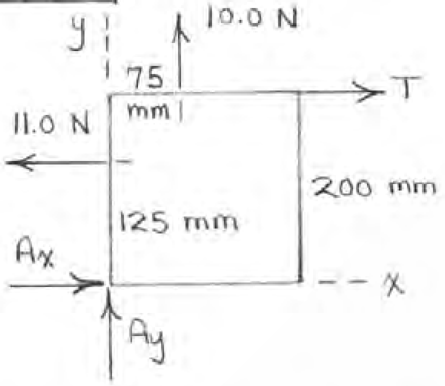

Isolate entire system of plate, pulleys, & tape.

$$\circlearrowleft + \Sigma M_A = 0: \quad T(200) - 10.0(75) - 11.0(125) = 0$$

$$\underline{T = 10.62 \text{ N}}$$

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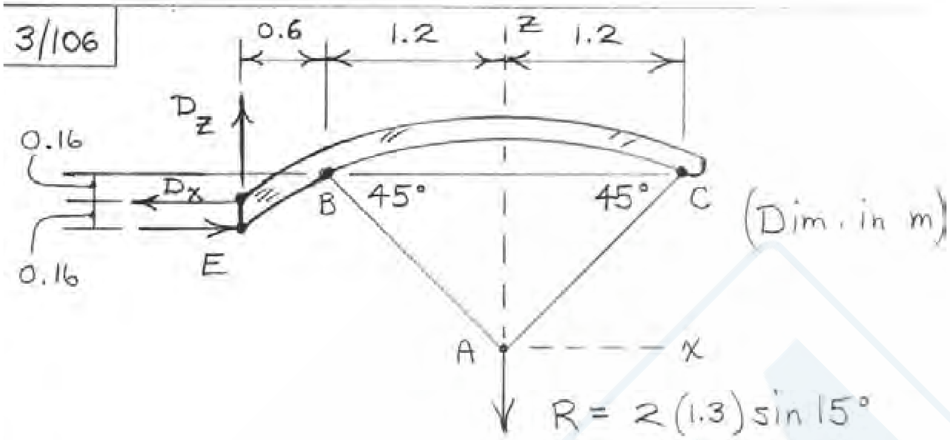

$R = 2(1.3)\sin 15°$
$= 0.673 \text{ kN}$

$\Sigma F_z = 0: \; D_z - 0.673 = 0, \quad D_z = 0.673 \text{ kN}$

$\curvearrowleft + \Sigma M_E = 0: \; 0.16 D_x - 0.673(1.8) = 0$

$D_x = 7.57 \text{ kN}$

$D = \sqrt{0.673^2 + 7.57^2} = \underline{7.60 \text{ kN}}$

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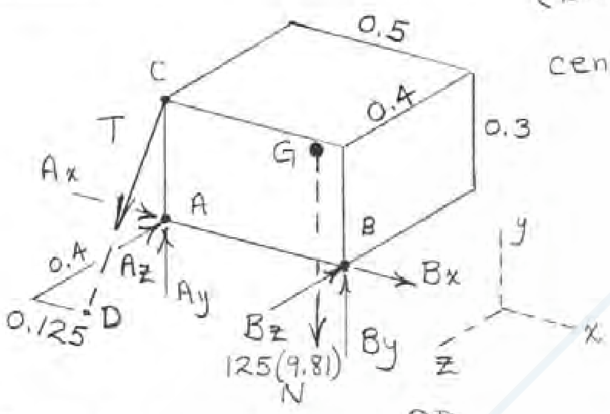
(Dim. in m, G is centered in body)



$$\underline{T} = T\underline{n}_{CD} = T\frac{\overline{CD}}{\overline{CD}}$$

$$= T\left[\frac{0.125\,\underline{i} - 0.3\,\underline{j} + 0.4\,\underline{k}}{\sqrt{0.125^2 + 0.3^2 + 0.4^2}}\right]$$

$$= T\left(0.243\,\underline{i} - 0.582\,\underline{j} + 0.776\,\underline{k}\right)$$

$$\Sigma M_{A_x} = 0: \; 0.776T(0.3) - 125(9.81)(0.2) = 0$$

$$\underline{T = 1053 \text{ N}}$$

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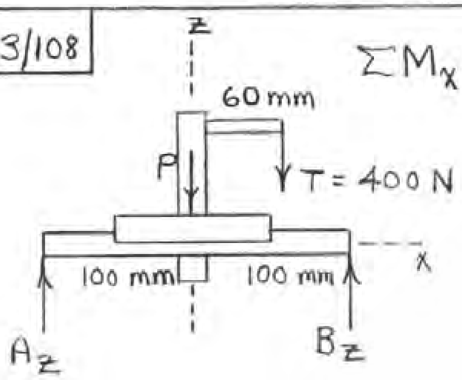

$\Sigma M_x = 0: \ P(200) - 400(120 \cos 30^\circ) = 0$

$P = 208 \text{ N}$

$\Sigma M_{A_y} = 0: \ 208(100) + 400(160) - B_z(200) = 0$

$B_z = 424 \text{ N}$

$\Sigma F_z = 0: \ A_z + 424 - 208 - 400 = 0, \ A_z = 183.9 \text{ N}$

Because $A_y = B_y = 0$, $\underline{A = A_z = 183.9 \text{ N}}$, $\underline{B = B_z = 424 \text{ N}}$

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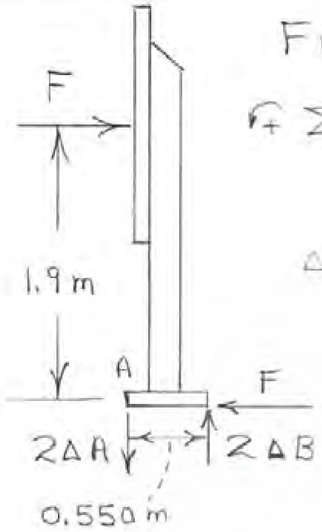

$F = pA = 175(1.6 \cdot 3) = 840$ N

From $\Sigma F = 0$, $\Delta A = \Delta B$

$\Sigma M_A = 0: -840(1.9) + 2\Delta B(0.550) = 0$

$\Delta B = \Delta A = \underline{1451 \text{ N}}$

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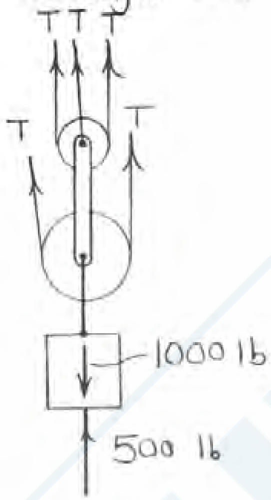
3/11 FBD of 1000-lb weight and lower pair of pulleys:



$\uparrow \Sigma F = 0: \quad 5T + 500 - 1000 = 0, \quad \underline{T = 100 \text{ lb}}$

(We assume that the nonverticality of some of the cables is negligible.)

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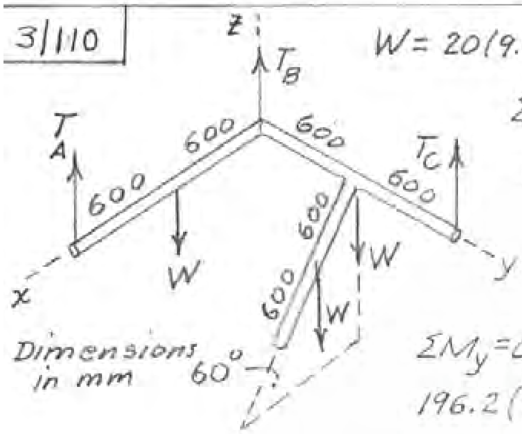

$W = 20(9.81) = 196.2$ N

$\Sigma M_x = 0;$

$1200T_C - 2(196.2)600 = 0$

$\underline{T_C = 196.2 \text{ N}}$

$\Sigma M_y = 0;$

$196.2(600 + 600\cos 60°) - 1200T_A = 0$

$\underline{T_A = 147.2 \text{ N}}$

$\Sigma F_z = 0; \quad T_B + 147.2 + 196.2 - 3(196.2) = 0, \quad \underline{T_B = 245 \text{ N}}$

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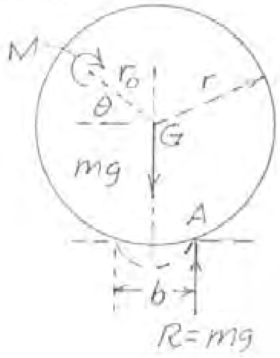

$\Sigma M_A = 0$; $M - mg\frac{b}{2} = 0$

$M = \frac{1}{2} mgb$

Independent of r_o, r, & θ

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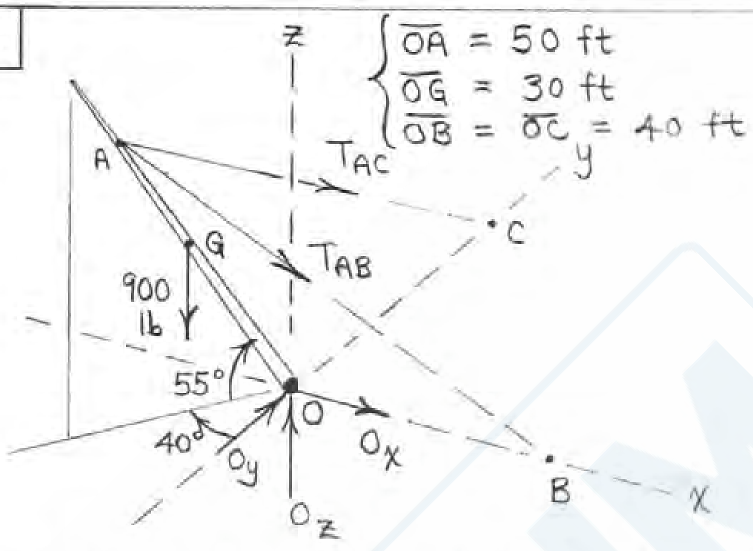

Coordinates of A: $50(-\cos 55° \sin 40°, -\cos 55° \cos 40°, \sin 55°) = (-18.43, -22.0, 41.0)$ ft

Coordinates of G: $30(-\cos 55° \sin 40°, -\cos 55° \cos 40°, \sin 55°) = (-11.06, -13.18, 24.6)$ ft

$$\underline{T}_{AB} = T_{AB}\left[\frac{(18.43+40)\underline{i} + 22.0\underline{j} - 41.0\underline{k}}{\sqrt{(18.43+40)^2 + 22.0^2 + 41.0^2}}\right]$$

$$= T_{AB}\left[0.783\underline{i} + 0.294\underline{j} - 0.549\underline{k}\right]$$

$$T_{AC} = T_{AC}\left[\frac{18.43\underline{i} + (22.0+40)\underline{j} - 41.0\underline{k}}{\sqrt{18.43^2 + (22.0+40)^2 + 41.0^2}}\right]$$

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$$= T_{AC}\left[0.241\underline{i} + 0.810\underline{j} - 0.535\underline{k}\right]$$

$\Sigma F_x = 0: \quad 0.783 T_{AB} + 0.241 T_{AC} + O_x = 0 \qquad (1)$

$\Sigma F_y = 0: \quad 0.294 T_{AB} + 0.810 T_{AC} + O_y = 0 \qquad (2)$

$\Sigma F_z = 0: \quad -0.549 T_{AB} - 0.535 T_{AC} + O_z - 900 = 0 \qquad (3)$

$\Sigma M_{BC} = 0: \quad \Sigma \underline{M}_B \cdot \underline{n}_{BC} = 0:$

$$\left\{-40\underline{i} \times O_z\underline{k} + \left[(-40-11.06)\underline{i} - 13.18\underline{j}\right] \times \left[-900\underline{k}\right]\right\} \cdot \left(-\frac{\sqrt{2}}{2}\underline{i} + \frac{\sqrt{2}}{2}\underline{j}\right) = 0$$

or $\quad (40\,O_z - 46{,}000)\frac{\sqrt{2}}{2} - 11{,}860\frac{\sqrt{2}}{2} = 0 \qquad (4)$

$\Sigma M_{O_y} = 0: \quad 0.549\,T_{AB}(40) - 900(11.06) = 0 \qquad (5)$

Solve Eqs. (1)–(5) in reverse order to obtain

$O_x = -489$ lb $\qquad T_{AB} = 454$ lb

$O_y = -582$ lb $\qquad T_{AC} = 554$ lb

$O_z = 1445$ lb

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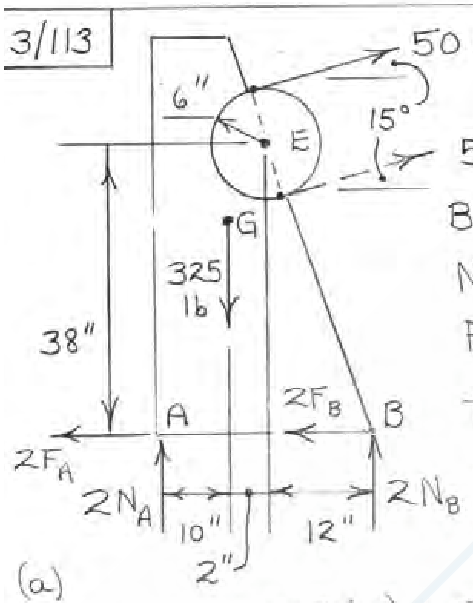

Because of symmetry, $N_A = N_D$ & $N_B = N_C$.
For (a), replace 50-lb force by a 50-lb force & 300 lb-in. CW couple at E (CCW for (b)).

(a)

$\circlearrowleft \Sigma M_A = 0: \; -325(10) + 2N_B(24) - 50\cos 15°(38) + 50\sin 15°(12) - 300 = 0, \; \underline{N_B = N_C = 109.0 \text{ lb}}$

$+\uparrow \Sigma F = 0: \; 2(109.0) + 2N_A - 325 + 50\sin 15° = 0$

$\underline{N_A = N_D = 47.1 \text{ lb}}$

(b) Reverse the sign on the 300 lb-in. couple & obtain

$$\begin{cases} N_B = N_C = 96.5 \text{ lb} \\ N_A = N_D = 59.6 \text{ lb} \end{cases}$$

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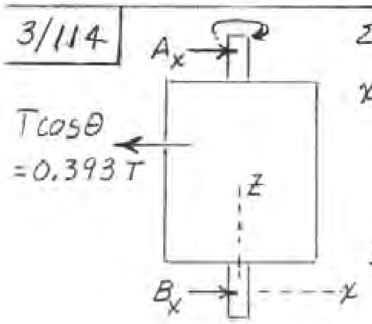


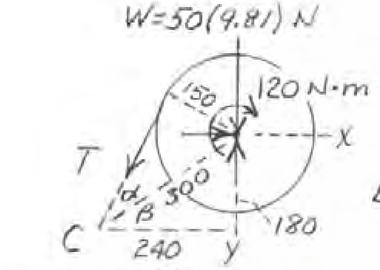
x-z: $\Sigma M_z = 0$; $120 - 0.150T = 0$, $T = 800$ N

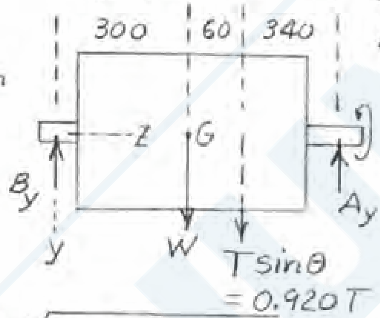
$\Sigma M_B = 0$; $0.393(800)(0.360) - 0.700A_x = 0$; $A_x = 161.6$ N

$\Sigma F_x = 0$; $B_x + 161.6 - 0.393(800) = 0$, $B_x = 152.6$ N

y-z: $\Sigma M_B = 0$; $0.7A_y - 0.920(800)(0.360) - 50(9.81)(0.300) = 0$; $A_y = 588.6$ N

$\Sigma F_y = 0$; $W + T\sin\theta - A_y - B_y = 0$, $B_y = 637.6$ N





$\beta = \tan^{-1}\dfrac{180}{240} = 36.9°$

$\alpha = \sin^{-1}\dfrac{150}{300} = 30°$

$\theta = \alpha + \beta = 66.9°$

$A = \sqrt{(161.6)^2 + (588.6)^2} = \underline{610 \text{ N}}$

$B = \sqrt{(152.6)^2 + (637.6)^2} = \underline{656 \text{ N}}$

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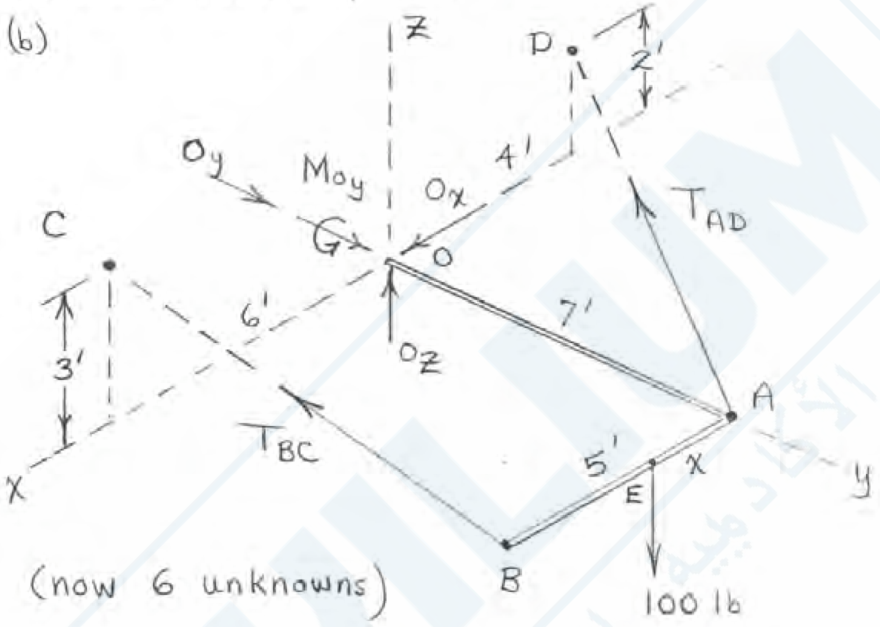
▸3/115 (a) There are 5 unknown constraint forces. The bar is free to rotate about a line which passes through point O and through which the lines of action of both tension forces pass.

(b)



(now 6 unknowns)

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$$\underline{T}_{AD} = T_{AD}\left[\frac{-4\underline{i} - 7\underline{j} + 2\underline{k}}{\sqrt{69}}\right]$$

$$\underline{T}_{BC} = T_{BC}\left[\frac{\underline{i} - 7\underline{j} + 3\underline{k}}{\sqrt{59}}\right]$$

$$\Sigma F_x = 0: \; O_x - \frac{4}{\sqrt{69}} T_{AD} + \frac{1}{\sqrt{59}} T_{BC} = 0 \quad (1)$$

$$\Sigma F_y = 0: \; O_y - \frac{7}{\sqrt{69}} T_{AD} - \frac{7}{\sqrt{59}} T_{BC} = 0 \quad (2)$$

$$\Sigma F_z = 0: \; O_z + \frac{2}{\sqrt{69}} T_{AD} + \frac{3}{\sqrt{59}} T_{BC} - 100 = 0 \quad (3)$$

$$\Sigma M_{O_x} = 0: \; 7\left(\frac{2}{\sqrt{69}} T_{AD}\right) + 7\left(\frac{3}{\sqrt{59}} T_{BC}\right) - 7(100) = 0 \quad (4)$$

$$\Sigma M_{O_y} = 0: \; -5\left(\frac{3}{\sqrt{59}} T_{BC}\right) + M_{O_y} + 100x = 0 \quad (5)$$

$$\Sigma M_{O_z} = 0: \; 7\left(\frac{4}{\sqrt{69}} T_{AD}\right) - 7\left(\frac{1}{\sqrt{59}} T_{BC}\right) - 5\left(\frac{7}{\sqrt{59}} T_{BC}\right) = 0 \quad (6)$$

Solve Eqs. (1)–(6) over $0.5 \le x \le 4.5$ ft and discover that three of the requested quantities are constant:

$$\begin{cases} O_x = 83.3 \text{ lb} \\ O_y = 292 \text{ lb} \\ O_z = 0 \end{cases}$$

$T_{AD} = 208$ lb = constant

$T_{BC} = 128.0$ lb = constant

$O = \sqrt{O_x^2 + O_y^2 + O_z^2} = 303$ lb = constant

and $M_{O_y} = -100x + 250$ (in lb-ft if x in ft)

(Note that O_y could have been obtained from $\Sigma M_{CD} = 0$ & O_z from $\Sigma M_{AB} = 0$)

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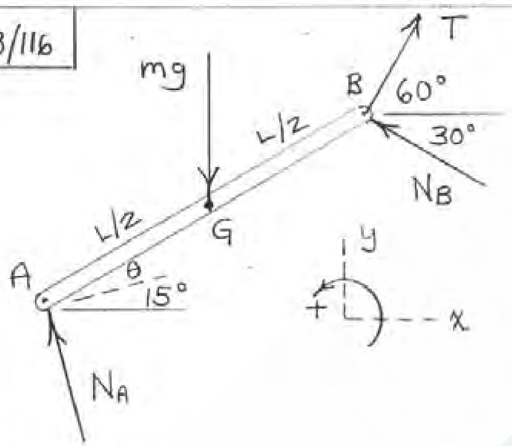
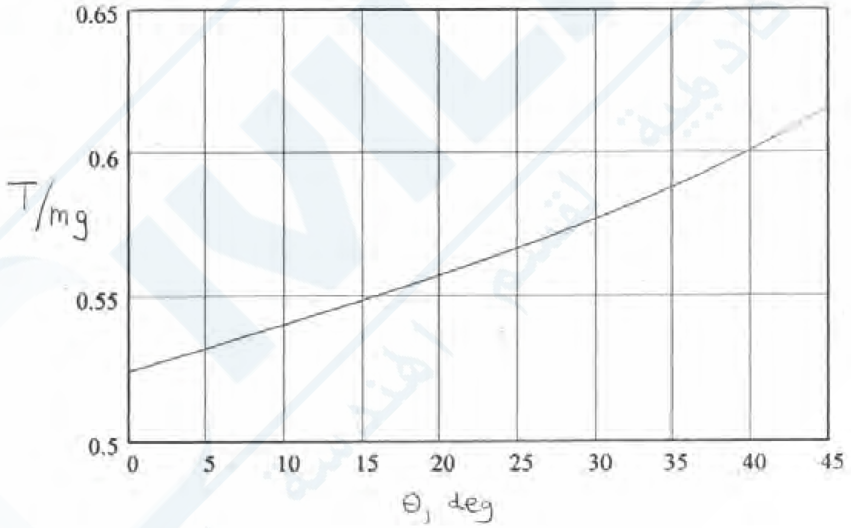


$$\Sigma F_x = 0: \; -N_A \sin 15° - N_B \cos 30° + T \sin 30° = 0$$

$$\Sigma F_y = 0: \; N_A \cos 15° + N_B \sin 30° + T \cos 30° - mg = 0$$

$$\Sigma M_B = 0: \; mg \frac{L}{2} \cos(\theta + 15°) - N_A \cos\theta \,(L) = 0$$

Solving, $$T = mg \, \frac{\frac{\sqrt{3}}{2}\cos\theta - \frac{\sqrt{2}}{4}\cos(\theta + 15°)}{\cos\theta}$$



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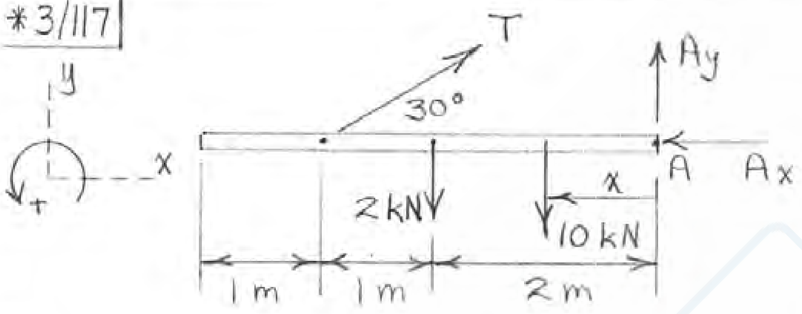

(Weight of beam = 200(10)/1000 = 2 kN)

$\Sigma M_A = 0: \quad 10x + 2(2) - T\sin 30°(3) = 0$

$T = \frac{2}{3}(10x + 4) \quad \text{(in kN)}$

$\Sigma F_x = 0: \quad T\cos 30° - A_x = 0$

$A_x = \frac{1}{\sqrt{3}}(10x + 4)$

$\Sigma F_y = 0: \quad T\sin 30° - 2 - 10 + A_y = 0$

$A_y = \frac{1}{3}(-10x + 32)$

$R = \{A_x^2 + A_y^2\}^{1/2} = \frac{1}{3}\{400x^2 - 400x + 1072\}^{1/2}$

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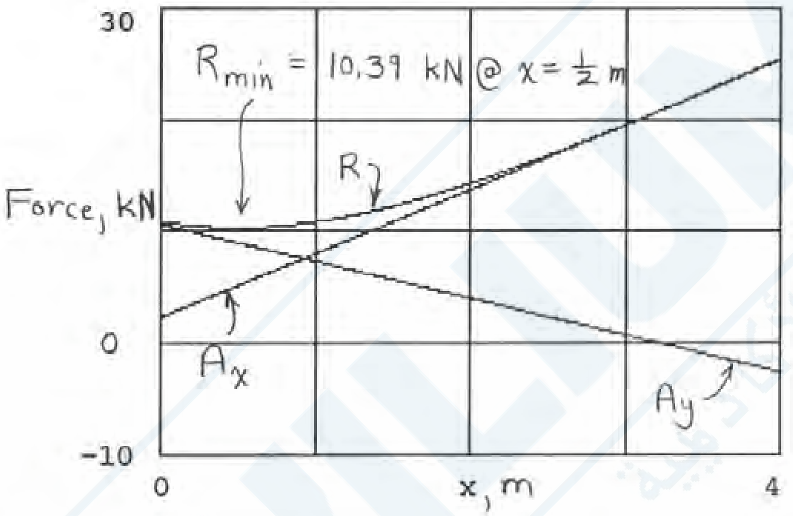
Set $\frac{dR^2}{dx} = 0$: $800x - 400 = 0$

$$x = \tfrac{1}{2}\text{ m}$$

$$R_{min} = \tfrac{1}{3}\left\{400\left(\tfrac{1}{2}\right)^2 - 400\left(\tfrac{1}{2}\right) + 1072\right\}^{1/2}$$

$$= \underline{10.39\text{ kN}} \qquad \text{(b)}$$

Plot of A_x, A_y, and R:



(a)

(c)

$\underline{R_{max} = 24.3\text{ kN @ } x = 3.8\text{ m}}$ is the value of R which must be used for the design of the pin at A.

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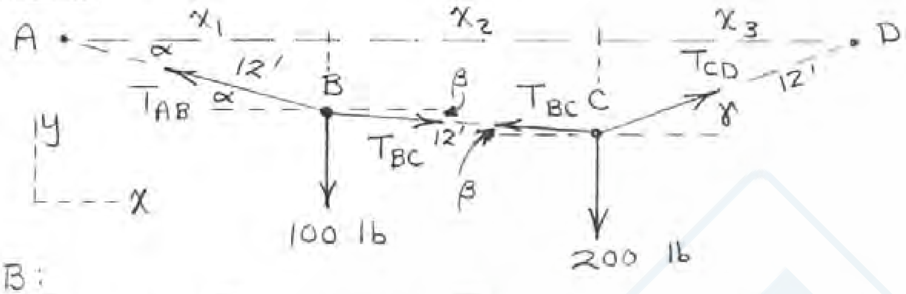

B:

$\Sigma F_x = 0: \quad -T_{AB}\cos\alpha + T_{BC}\cos\beta = 0 \qquad (1)$

$\Sigma F_y = 0: \quad T_{AB}\sin\alpha - T_{BC}\sin\beta - 100 = 0 \qquad (2)$

C:

$\Sigma F_x = 0: \quad -T_{BC}\cos\beta + T_{CD}\cos\gamma = 0 \qquad (3)$

$\Sigma F_y = 0: \quad T_{BC}\sin\beta + T_{CD}\sin\gamma - 200 = 0 \qquad (4)$

$\cos\alpha = \frac{x_1}{12}, \quad \cos\beta = \frac{x_2}{12}, \quad \cos\gamma = \frac{x_3}{12}$

So $\quad 12\cos\alpha + 12\cos\beta + 12\cos\gamma = 35 \qquad (5)$

$\sin\alpha + \sin\beta = \sin\gamma$ (from figure) (6)

Solve numerically:

$\alpha = 14.44°$ $\qquad T_{AB} = 529$ lb

$\beta = 3.57°$ $\qquad T_{BC} = 513$ lb

$\gamma = 18.16°$ $\qquad T_{CD} = 539$ lb

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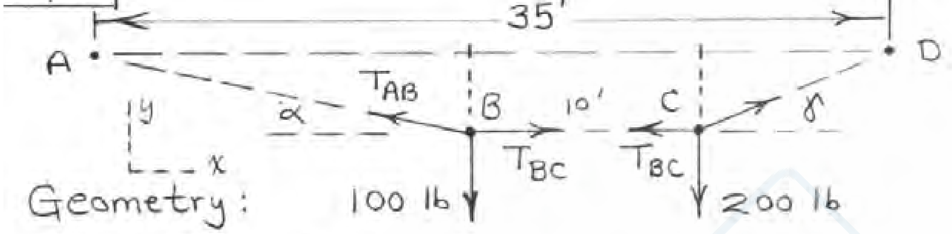

Geometry:

$\overline{AB} + \overline{BC} + \overline{CD} = 36$ ft (1)

$\overline{AB}\cos\alpha + \overline{BC} + \overline{CD}\cos\gamma = 35$ ft (2)

$\overline{AB}\sin\alpha = \overline{CD}\sin\gamma$ (3)

Equilibrium:

(B) $\Sigma F_x = 0: \; -T_{AB}\cos\alpha + T_{BC} = 0$ (4)

$\Sigma F_y = 0: \; T_{AB}\sin\alpha - 100 = 0$ (5)

(C) $\Sigma F_x = 0: \; -T_{BC} + T_{CD}\cos\gamma = 0$ (6)

$\Sigma F_y = 0: \; T_{CD}\sin\gamma - 200 = 0$ (7)

With $\overline{BC}$ set to 10 ft, solve 7 equations in 7 unknowns & obtain

$\overline{AB} = 17.01$ ft	$\alpha = 11.47°$	$T_{AB} = 503$ lb
$\overline{CD} = 8.99$ ft	$\gamma = 22.1°$	$T_{BC} = 493$ lb
		$T_{CD} = 532$ lb

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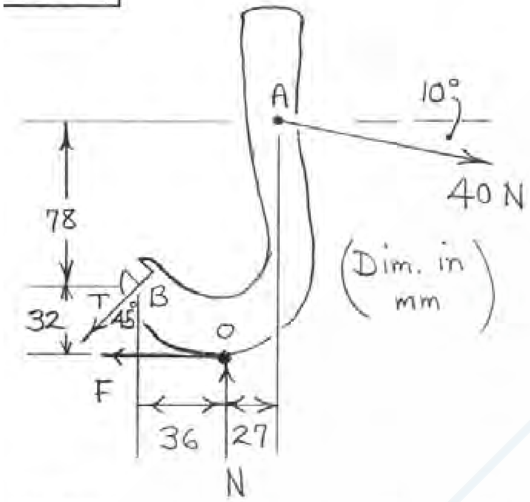

$$\circlearrowleft + \Sigma M_O = 0: \; -40 \cos 10°(110) - 40 \sin 10°(27) + T \cos 45°(32) + T \sin 45°(36) = 0$$

$$\underline{T = 94.0 \text{ N}}$$

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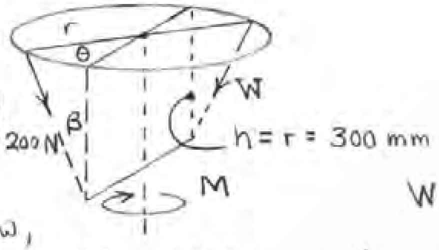
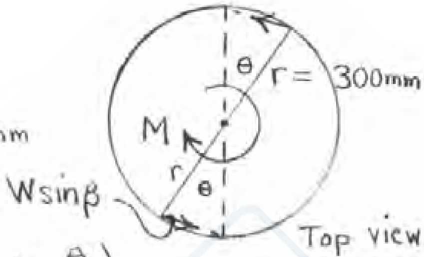


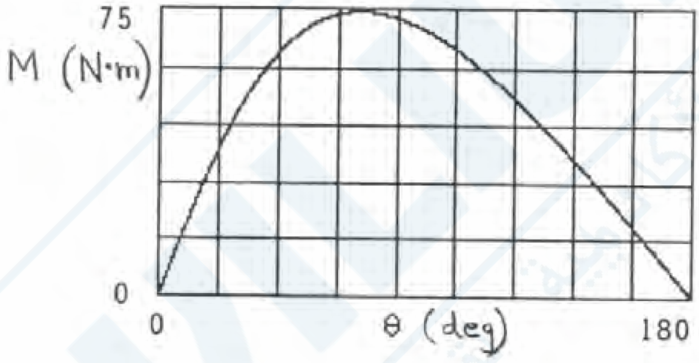


$W = mg = 20(10) = 200$ N

Top view, $\Sigma M = 0$: $M = 2W \sin\beta \left(r \cos\frac{\theta}{2}\right)$

where $\sin\beta = 2r\sin\frac{\theta}{2} \Big/ \sqrt{\left(2r\sin\frac{\theta}{2}\right)^2 + h^2}$

With $h = r$: $M = 2Wr \dfrac{\sin\theta}{\sqrt{4\sin^2\frac{\theta}{2}+1}} = 120 \dfrac{\sin\theta}{\sqrt{4\sin^2\frac{\theta}{2}+1}}$ N·m



$M_{max} = 74.2$ N·m @ $\theta = 67.5°$

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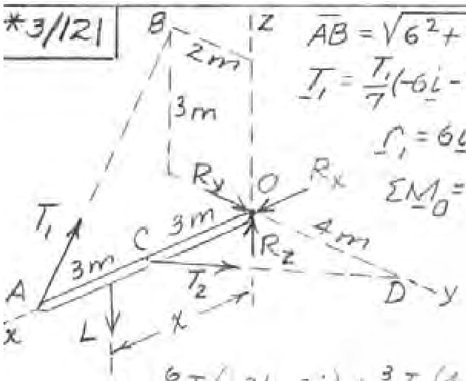


$\overline{AB} = \sqrt{6^2+2^2+3^2} = 7\text{ m}$, $\overline{CD} = \sqrt{3^2+4^2} = 5\text{ m}$

$\underline{T}_1 = \frac{T_1}{7}(-6\underline{i}-2\underline{j}+3\underline{k})$, $\underline{T}_2 = \frac{T_2}{5}(-3\underline{i}+4\underline{j})$, $\underline{L} = -L\underline{k}$

$\underline{r}_1 = 6\underline{i}$, $\underline{r}_2 = 3\underline{i}$, $\underline{r} = x\underline{i}$

$\Sigma \underline{M}_O = \underline{0}: \ \underline{r}_1 \times \underline{T}_1 + \underline{r}_2 \times \underline{T}_2 + \underline{r} \times \underline{L} = \underline{0}$

$6\underline{i} \times \frac{T_1}{7}(-6\underline{i}-2\underline{j}+3\underline{k})$

$+ 3\underline{i} \times \frac{T_2}{5}(-3\underline{i}+4\underline{j}) + x\underline{i} \times (-L\underline{k}) = 0$

Expand: $\frac{6}{7}T_1(-2\underline{k}-3\underline{j}) + \frac{3}{5}T_2(4\underline{k}) + Lx\underline{j} = 0$

$-\frac{12}{7}T_1 + \frac{12}{5}T_2 = 0$, $-\frac{18}{7}T_1 + Lx = 0$, so $T_1 = \frac{7}{18}Lx$, $T_2 = \frac{5}{18}Lx$

$\Sigma F_x = 0$; $R_x - \frac{3}{5}T_2 - \frac{6}{7}T_1 = 0$, $R_x = \frac{3}{5}\frac{5}{18}Lx + \frac{6}{7}\frac{7}{18}Lx = \frac{1}{2}Lx$

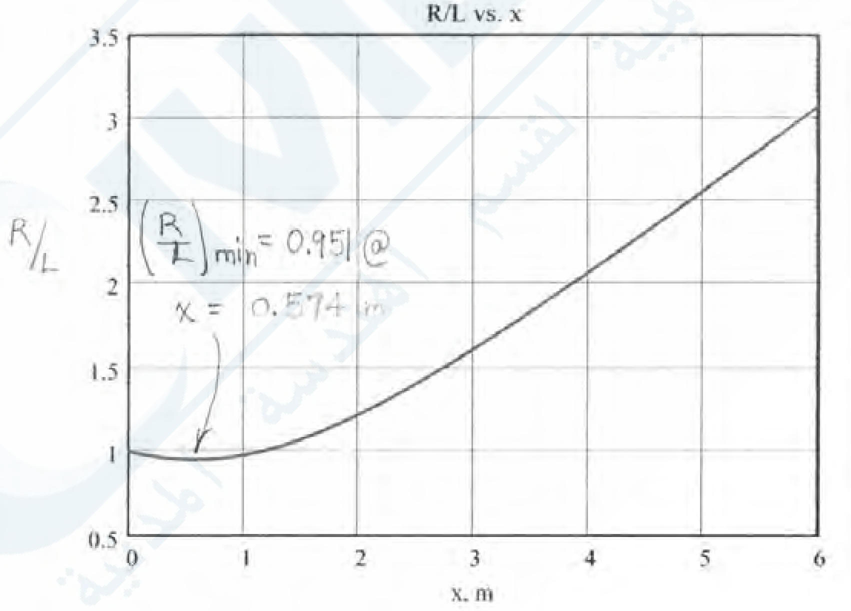
$\Sigma F_y = 0$; $R_y + \frac{4}{5}T_2 - \frac{2}{7}T_1 = 0$, $R_y = -\frac{4}{5}\frac{5}{18}Lx + \frac{2}{7}\frac{7}{18}Lx = -\frac{1}{9}Lx$

$\Sigma F_z = 0$; $R_z - L + \frac{3}{7}T_1 = 0$, $R_z = L - \frac{3}{7}\frac{7}{18}Lx = L(1-\frac{1}{6}x)$

$R^2 = L^2\left(\frac{x^2}{4} + \frac{x^2}{81} + 1 - \frac{x}{3} + \frac{x^2}{36}\right)$

$R/L = \sqrt{(47x^2/162) - x/3 + 1}$

$\frac{dR^2/L^2}{dx} = \frac{47x}{81} - \frac{1}{3} = 0$ for min.; $x = 0.574$ m



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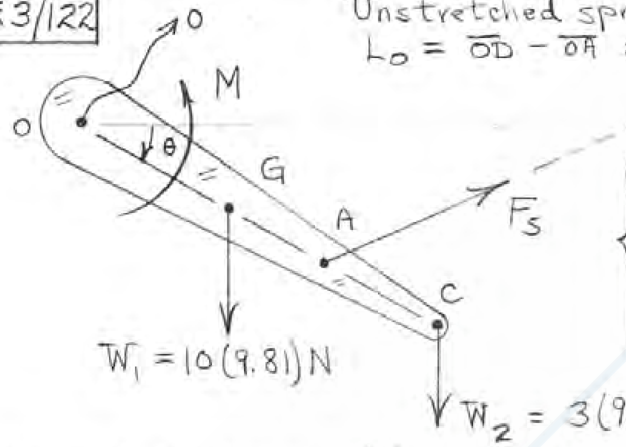
Unstretched spring length is

$L_o = \overline{OD} - \overline{OA} = 480 - 250 = 230$ mm



$\overline{OG} = 150$ mm

$\overline{OA} = 250$ mm

$\overline{OC} = 360$ mm

$\overline{OD} = 480$ mm

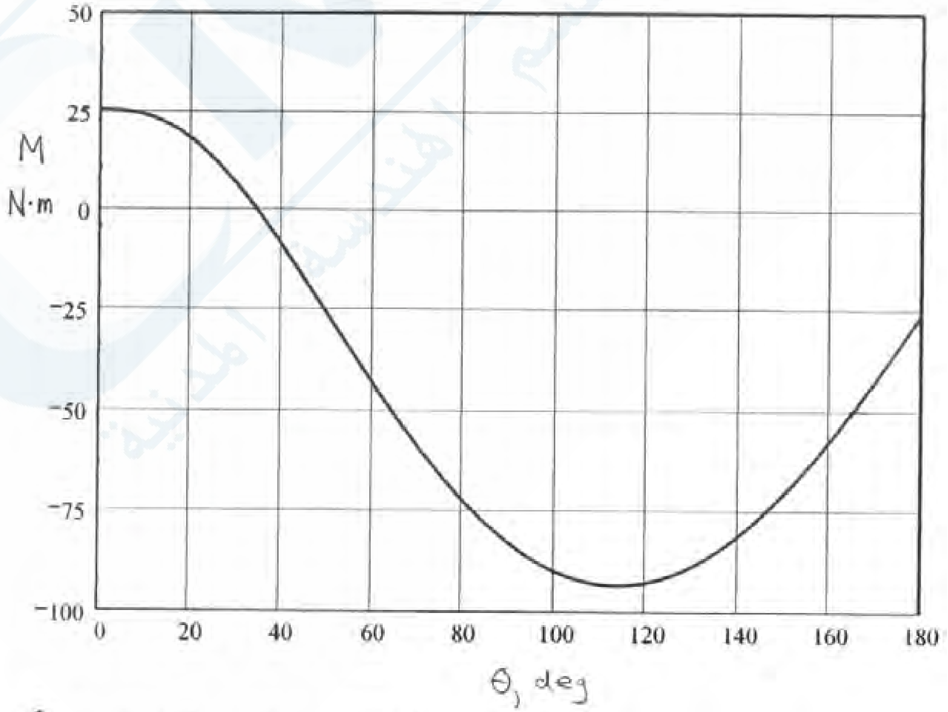
$$\underline{AD} = (\overline{OD} - \overline{OA}\cos\theta)\underline{i} + \overline{OA}\sin\theta\,\underline{j}$$

$$\overline{AD} = \left\{\left[\overline{OD} - \overline{OA}\cos\theta\right]^2 + \left[\overline{OA}\sin\theta\right]^2\right\}^{1/2}$$

$$\underline{F_s} = k\left(\overline{AD} - L_o\right)\frac{\underline{AD}}{\overline{AD}} \quad ; \quad \underline{r}_{OA} = \overline{OA}\left(\cos\theta\,\underline{i} - \sin\theta\,\underline{j}\right)$$

$$\Sigma M_O = 0: \quad M - \left(W_1\overline{OG} + W_2\overline{OC}\right)\cos\theta + \left(\underline{r}_{OA} \times \underline{F_s}\right)_z = 0$$

Carry out algebra, solve for M as a function of θ, & plot to obtain



$M = 0$ @ $\theta = 34.6°$

$M_{min} = -93.4$ N·m @ $\theta = 113.9°$

$M_{max} = 25.3$ N·m @ $\theta = 0$

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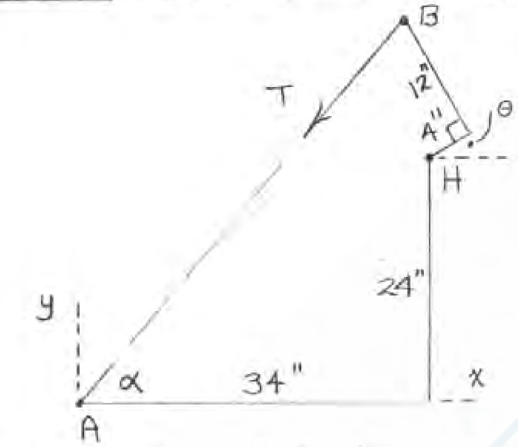
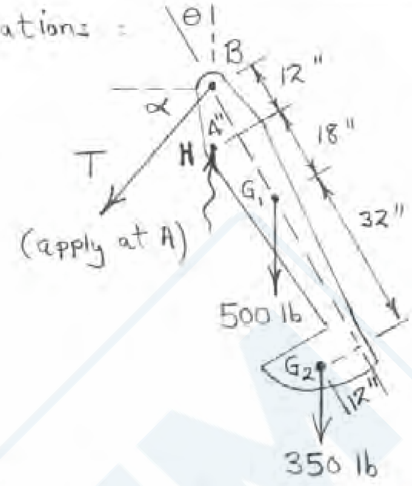
*3/123 Geometrical Considerations:





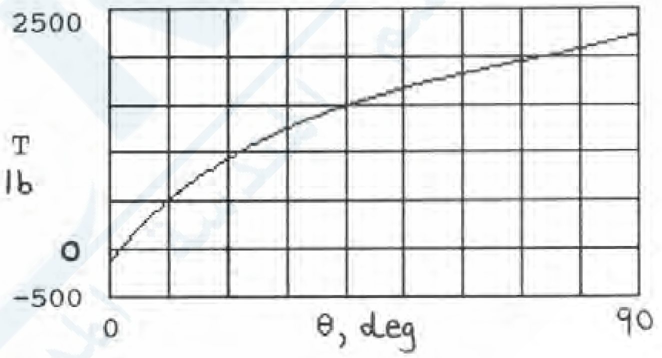
Coordinates of B: $\begin{cases} x = 34 + 4\cos\theta - 12\sin\theta \\ y = 24 + 4\sin\theta + 12\cos\theta \end{cases}$ (in.) (1)

$\therefore\ \alpha = \tan^{-1}\dfrac{y}{x}$ (aims T) (2)

$\circlearrowleft \Sigma M_H = 0:\ -(T\cos\alpha)(24) + (T\sin\alpha)(34) - 500(18\sin\theta + 4\cos\theta) - 350(4\cos\theta + (18+32)\sin\theta - 12\cos\theta) = 0$

$\Rightarrow\ T = \dfrac{26{,}500\sin\theta - 800\cos\theta}{34\sin\alpha - 24\cos\alpha}$ (lb; positive is tension in AB)

Solve (1), (2), & (3) & plot: (Note $T = 0$ @ $\theta = 1.729°$)



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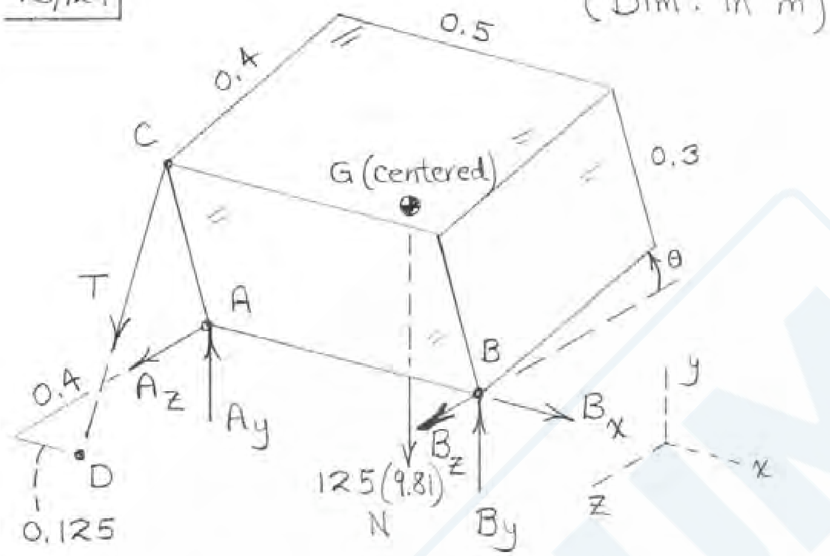
*3/124 (Dim. in m)



$$\underline{CD} = 0.125\underline{i} - 0.3\cos\theta\,\underline{j} + (0.4 - 0.3\sin\theta)\,\underline{k}$$

Then $\underline{T} = T\underline{n}_{CD} = T\,\frac{\underline{CD}}{\overline{CD}}$, or

$$\underline{T} = T\left[\frac{0.125\underline{i} - 0.3\cos\theta\,\underline{j} + (0.4 - 0.3\sin\theta)\,\underline{k}}{\sqrt{0.266 - 0.24\sin\theta}}\right]$$

$$\Sigma F_x = 0: \quad \frac{0.125\,T}{\sqrt{0.266 - 0.24\sin\theta}} + B_x = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad \frac{-0.3\,T\cos\theta}{\sqrt{0.266 - 0.24\sin\theta}} + A_y + B_y - 125(9.81) = 0 \qquad (2)$$

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$$\Sigma F_z = 0: \quad \frac{(0.4 - 0.3\sin\theta)T}{\sqrt{0.266 - 0.24\sin\theta}} + A_z + B_z = 0 \qquad (3)$$

$$\Sigma M_{D_x} = 0: \quad (A_y + B_y)(0.4) - 125(9.81)(0.4 + 0.2\cos\theta - 0.15\sin\theta) = 0 \qquad (4)$$

$$\Sigma M_{D_y} = 0: \quad -B_x(0.4) + A_z(0.125) - B_z(0.375) = 0 \qquad (5)$$

$$\Sigma M_{D_z} = 0: \quad -A_y(0.125) + B_y(0.375) - 125(9.81)(0.125) = 0 \qquad (6)$$

Computer Solution:

$$T = -12.77\,\frac{-4\cos\theta + 3\sin\theta}{\cos\theta}\sqrt{425 - 384\sin\theta}$$

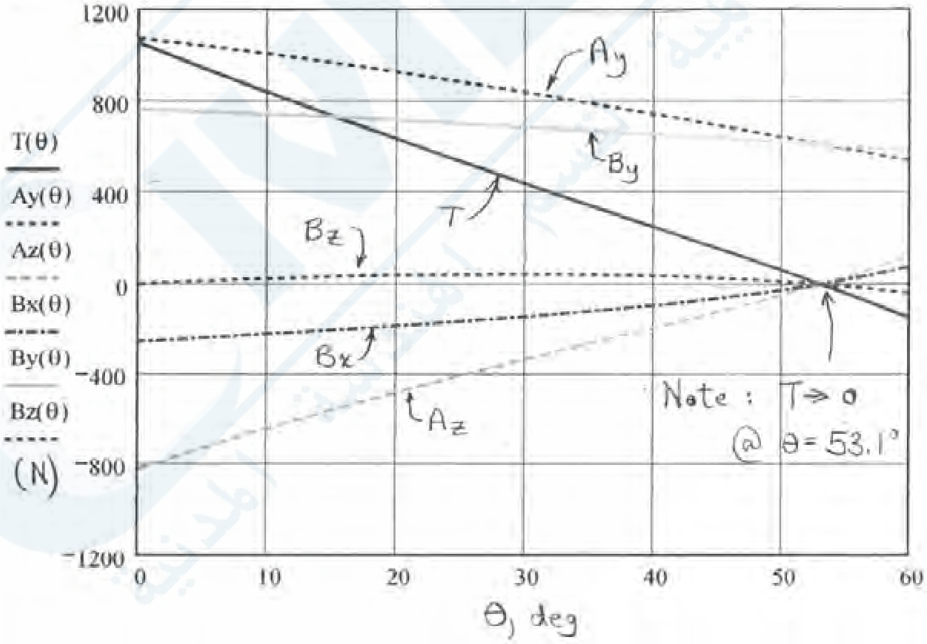
$$A_y = 613 + 460\cos\theta - 345\sin\theta$$

$$A_z = -12.77\,\frac{64\cos\theta - 48\sin\theta - 36\sin\theta\cos\theta + 27\cos^2\theta}{\cos\theta}$$

$$B_x = 63.9\,\frac{-4\cos\theta + 3\sin\theta}{\cos\theta}$$

$$B_y = 613 + 153.3\cos\theta - 115.0\sin\theta$$

$$B_z = -38.3\,\frac{-4\cos\theta + 3\sin\theta}{\cos\theta}\sin\theta$$



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*3/125 With reference to the FBD and solution to Prob. 3/96, the various tension vectors are $\underline{T}_2 = 1000(0\underline{i} - \cos 10° \underline{j} - \sin 10° \underline{k})$ N

$$\underline{T}_1 = T_1(-\sin 30° \cos 10° \underline{i} + \cos 30° \cos 10° \underline{j} - \sin 10° \underline{k})$$

$$\underline{T}_{BE} = \frac{T(10\cos\theta\, \underline{i} + 10\sin\theta\, \underline{j} - 11\underline{k})}{\sqrt{(10\cos\theta)^2 + (10\sin\theta)^2 + 11^2}}$$

$$\underline{T}_{AD} = \frac{T(8\cos\theta\, \underline{i} + 8\sin\theta\, \underline{j} - 9\underline{k})}{\sqrt{(8\cos\theta)^2 + (8\sin\theta)^2 + 9^2}}$$

Needed position vectors are $\underline{r}_{OC} = 13\underline{k}$ m, $\underline{r}_{OB} = 11\underline{k}$ m, & $\underline{r}_{OA} = 9\underline{k}$ m.

$$\Sigma \underline{M}_O = \underline{0}: \quad \underline{r}_{OC} \times (\underline{T}_1 + \underline{T}_2) + \underline{r}_{OB} \times \underline{T}_{BE} + \underline{r}_{OA} \times \underline{T}_{AD} = \underline{0}$$

Carry out cross products, collect terms, & set the $\underline{i}$- & $\underline{j}$-components to zero, solving

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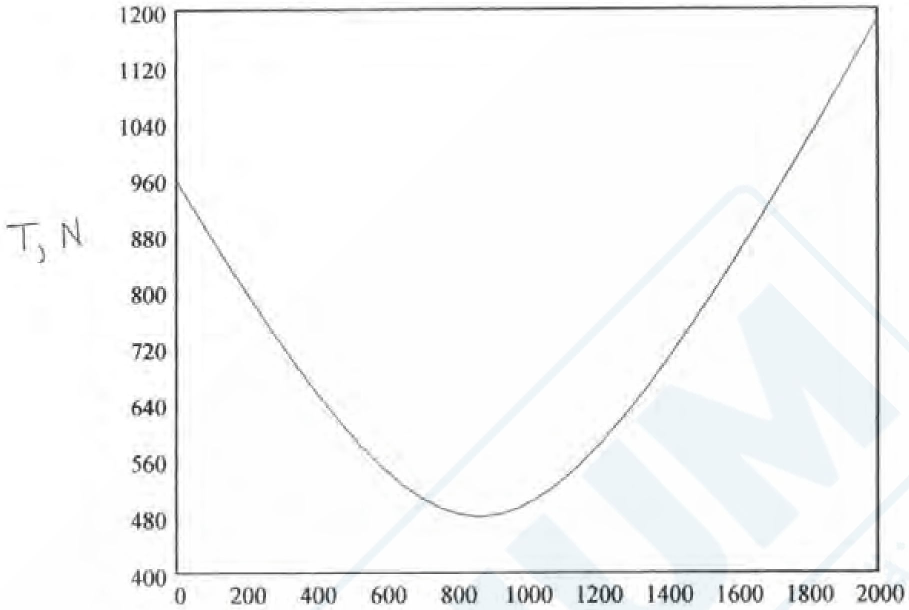
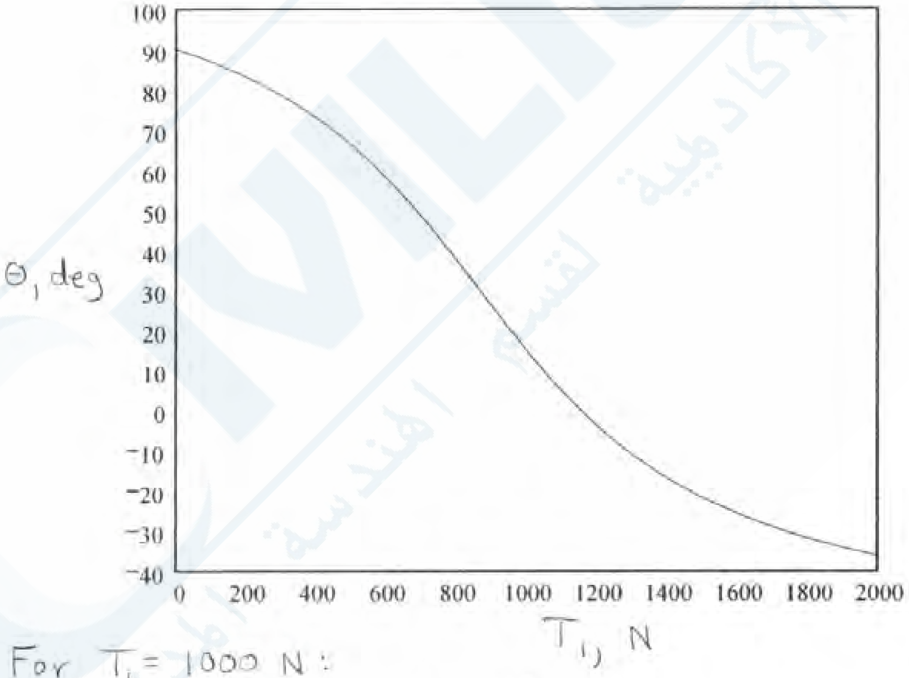
for T and θ for each value of T_1. Resulting plots:





For $T_1 = 1000$ N:

$T = 495$ N, $\theta = 15°$

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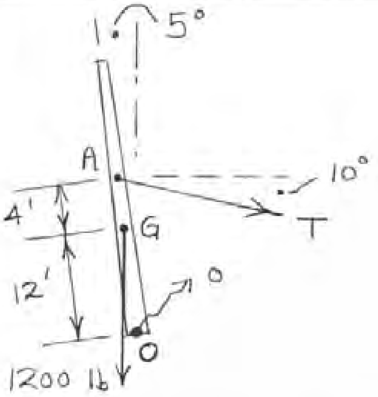

$$\circlearrowleft + \Sigma M_O = 0: \quad 1200(12 \sin 5°) - T \cos 15° (16) = 0$$

$$T = 81.2 \text{ lb}$$

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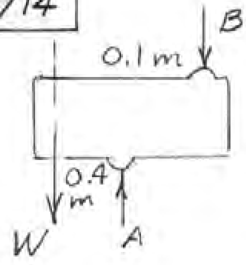

$W = 300(9.81) = 2943$ N

$\Sigma M_A = 0$; $2943(0.4) - B(0.6) = 0$

$\underline{B = 1962 \text{ N}}$ or $\underline{1.962 \text{ kN}}$

$\Sigma F = 0$; $A = 2943 + 1962$

$= \underline{4910 \text{ N}}$ or $\underline{4.91 \text{ kN}}$

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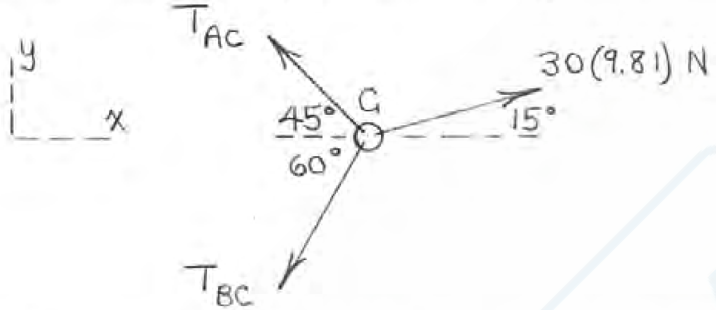
3/15 FBD of junction ring C:



$$\begin{cases} \Sigma F_x = 0: -T_{AC}\cos 45° - T_{BC}\cos 60° + 30(9.81)\cos 15° = 0 \\ \Sigma F_y = 0: T_{AC}\sin 45° - T_{BC}\sin 60° + 30(9.81)\sin 15° = 0 \end{cases}$$

Solve simultaneously to obtain

$$\begin{cases} T_{AC} = 215 \text{ N} \\ T_{BC} = 264 \text{ N} \end{cases}$$

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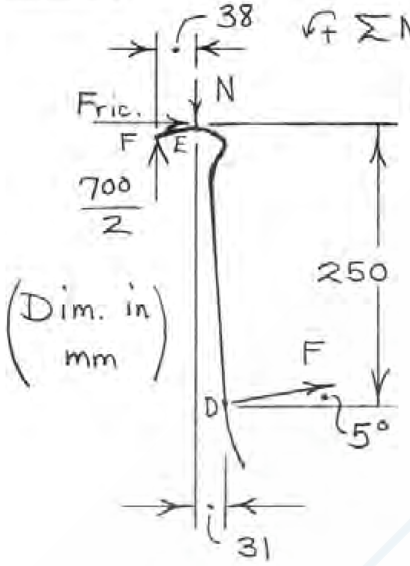
3/16 Consider the right prybar:



$\circlearrowleft + \Sigma M_E = 0: \; -\frac{700}{2}(38) +$

$F\cos 5°(250) + F\sin 5°(31) = 0$

$\underline{F = 52.8 \text{ N}}$

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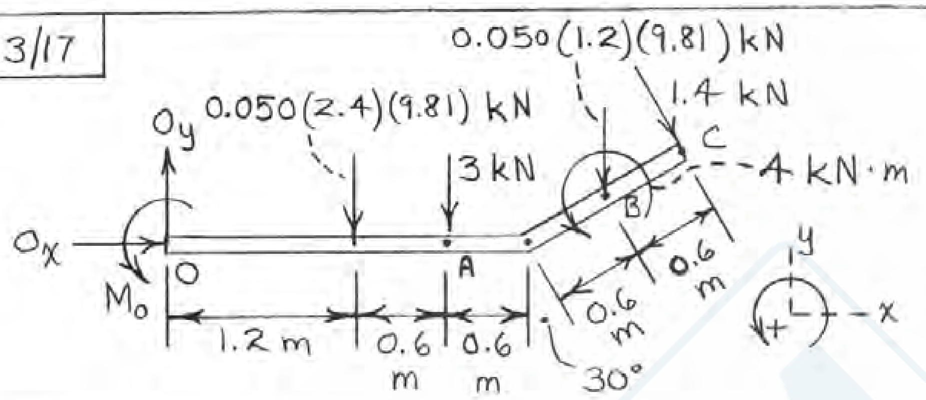

$\Sigma F_x = 0: \quad O_x + 1.4 \sin 30° = 0$

$\underline{O_x = -0.7 \text{ kN}}$

$\Sigma F_y = 0: \quad O_y - 0.050(2.4)(9.81) - 3 - 1.4 \cos 30°$

$-0.050(1.2)(9.81) = 0, \quad \underline{O_y = 5.98 \text{ kN}}$

$\Sigma M_O = 0: \quad M_O - 0.050(2.4)(9.81)(1.2) - 3(1.8)$

$-0.050(1.2)(9.81)(2.4 + 0.6 \cos 30°) + 4$

$-1.4(2.4 \cos 30° + 1.2) = 0, \quad \underline{M_O = 9.12 \text{ kN·m}}$

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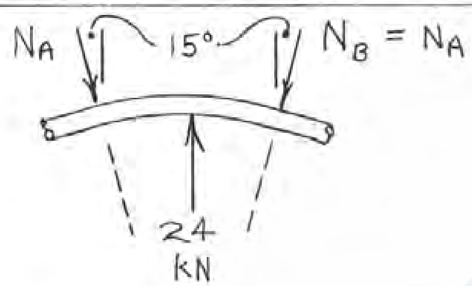

$\uparrow \Sigma F = 0: \quad 24 - 2N_A \cos 15° = 0$

$$N_A = N_B = 12.42 \text{ kN}$$

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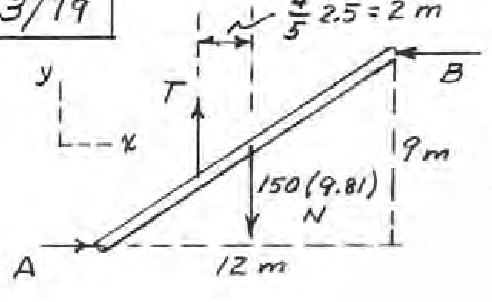

$\Sigma F_y = 0$; $T - 150(9.81) = 0$

$T = 1472$ N

$\Sigma F_x = 0$; $A = B$

$\Sigma M = 0$; $1472(2) - A(9) = 0$

$A = B = 327$ N

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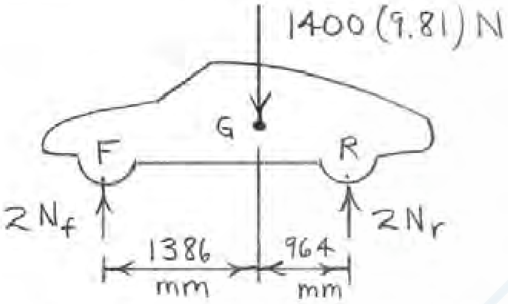

$+\uparrow \Sigma F = 0: \quad 2N_f + 2N_r - 1400(9.81) = 0$

$\circlearrowleft + \Sigma M_F = 0: \quad -1400(9.81)(1386) + 2N_r(1386 + 964) = 0$

Solution : $\begin{cases} N_f = 2820 \text{ N} \\ N_r = 4050 \text{ N} \end{cases}$

Assumes G midway between left and right wheels.

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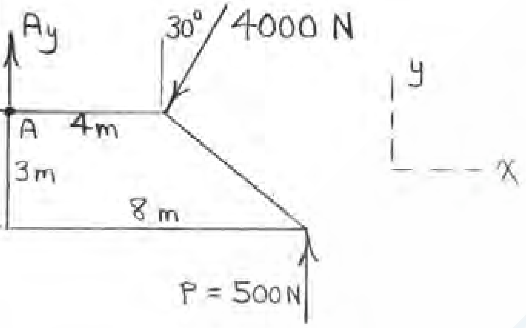

$\Sigma F_x = 0: \; A_x + E_x - 4000 \sin 30° = 0$

$\Sigma F_y = 0: \; A_y - 4000 \cos 30° + 500 = 0$

$\Sigma M_A = 0: \; E_x(3) + 500(8) - 4000 \cos 30° (4) = 0$

$\Rightarrow \; \underline{A_x = -1290 \text{ N}}, \; \underline{A_y = 2960 \text{ N}}, \; \underline{E_x = 3290 \text{ N}}$

For maximum P: $E_x = 0$ and $\Sigma M_A = 0$:

$P(8) - 4000 \cos 30° (4) = 0, \; \underline{P = 1732 \text{ N}}$

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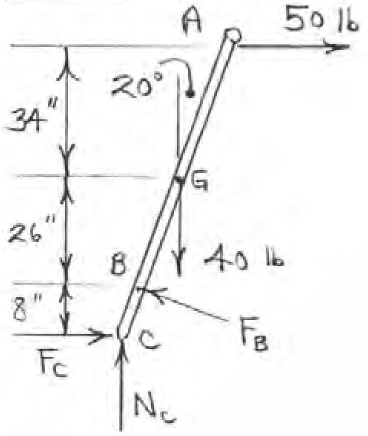

(a) Including 40-lb weight:

$\circlearrowleft \Sigma M_C = 0: \quad 50(68) + 40(34 \tan 20^\circ) - F_B \frac{8}{\cos 20^\circ} = 0$

$F_B = 458 \text{ lb}$

$\rightarrow \Sigma F = 0: \quad F_C - 458 \cos 20^\circ + 50 = 0$

$F_C = 380 \text{ lb}$

(b) Exclude 40-lb weight:

$\circlearrowleft \Sigma M_C = 0: \quad 50(68) - F_B \frac{8}{\cos 20^\circ} = 0$

$F_B = 399 \text{ lb}$

$\rightarrow \Sigma F = 0: \quad F_C - 399 \cos 20^\circ + 50 = 0$

$F_C = 325 \text{ lb}$

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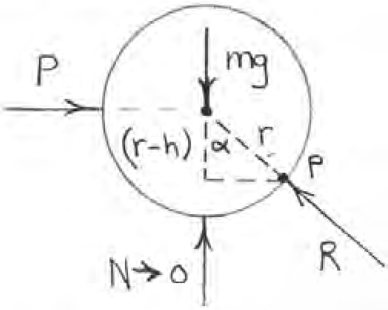
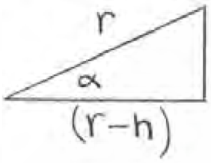
$$\cos\alpha = \frac{r-h}{r}$$





$$\sqrt{r^2-(r-h)^2} = \sqrt{2rh-h^2}$$

$$\sin\alpha = \sqrt{2rh-h^2}\,/\,r$$

$$+\circlearrowright\ \Sigma M_P = 0: \quad P(r-h) - mg\,r\sin\alpha = 0$$

$$\Rightarrow \quad P = \frac{mg\sqrt{2rh-h^2}}{r-h}$$

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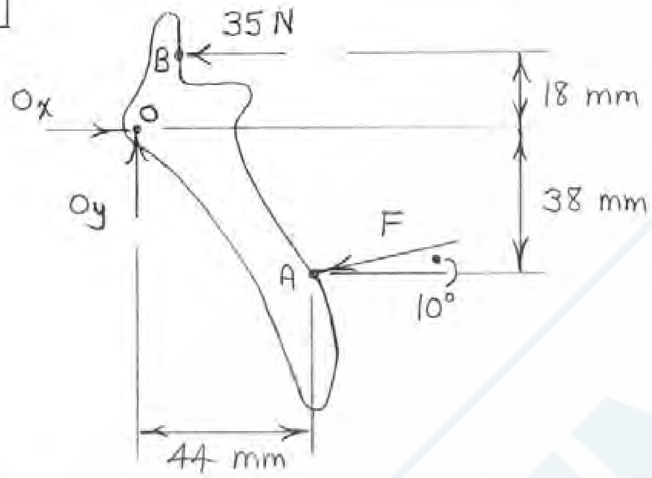

$\circlearrowleft + \Sigma M_O = 0: \quad 35(18) - (F\cos 10°)(38) - (F\sin 10°)(44) = 0$

$\underline{F = 13.98 \text{ N}}$

$\Sigma F_x = 0: \quad O_x - 35 - 13.98\cos 10° = 0$

$O_x = 48.8 \text{ N}$

$\Sigma F_y = 0: \quad O_y - 13.98\sin 10° = 0$

$O_y = 2.43 \text{ N}$

$O = \sqrt{48.8^2 + 2.43^2} = \underline{48.8 \text{ N}}$

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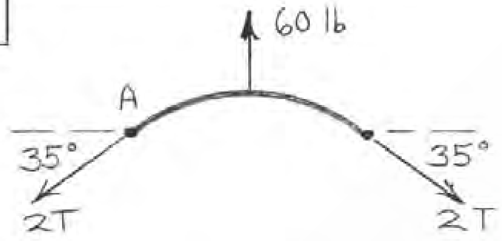

$+\uparrow \Sigma F = 0: \quad 60 - 4T \sin 35° = 0$

$\underline{T = 26.2 \text{ lb}}$

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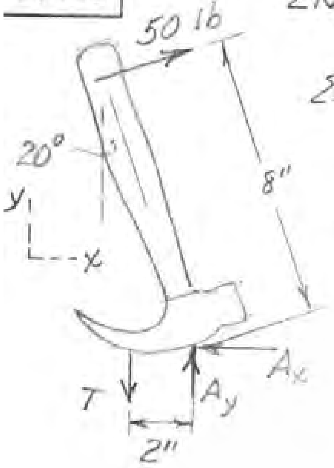

$\Sigma M_A = 0;\ 50(8) - 2T = 0,\ \underline{T = 200 \text{ lb}}$

$\Sigma F_x = 0;\ 50 \cos 20° - A_x = 0$

$A_x = 46.98 \text{ lb}$

$\Sigma F_y = 0;\ A_y + 50 \sin 20° - 200 = 0$

$A_y = 182.9 \text{ lb}$

$A = \sqrt{(46.98)^2 + (182.9)^2} = \underline{188.8 \text{ lb}}$

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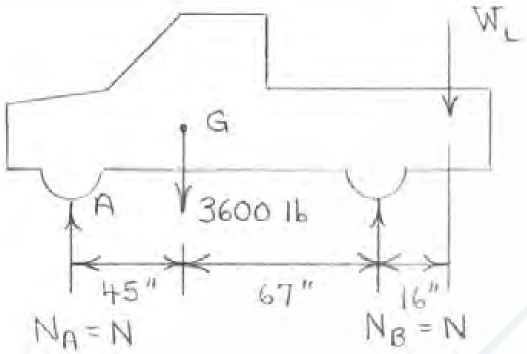

$\circlearrowleft + \sum M_A = 0: \quad 3600(45) - N(112) + W_L(128) = 0$

$+\uparrow \sum F = 0: \quad 2N - 3600 - W_L = 0$

Solve to obtain $N = 2075$ lb

$\underline{W_L = 550 \text{ lb}}$

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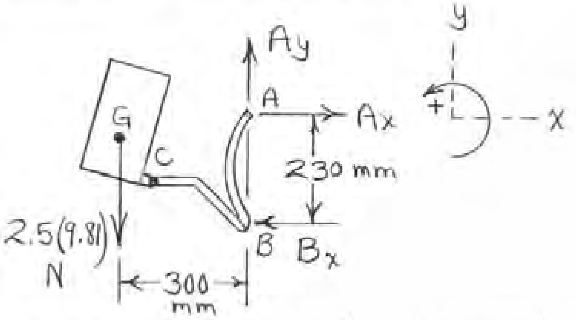
3/27 Entire unit:



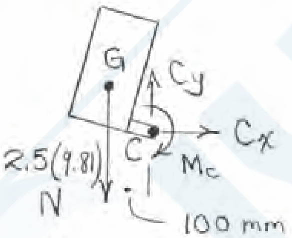
$\Sigma M_A = 0: \; 2.5(9.81)(300) - B_x(230) = 0$

$\underline{B_x = 32.0 \text{ N}}$

$\Sigma F_x = 0: \; A_x - 32.0 = 0, \quad \underline{A_x = 32.0 \text{ N}}$

$\Sigma F_y = 0: \; A_y - 2.5(9.81) = 0, \quad \underline{A_y = 24.5 \text{ N}}$

Fixture only:



$\Sigma M_C = 0: \; 2.5(9.81)(100) - M_C = 0$

$\underline{M_C = 2.45 \text{ N·m CW}}$

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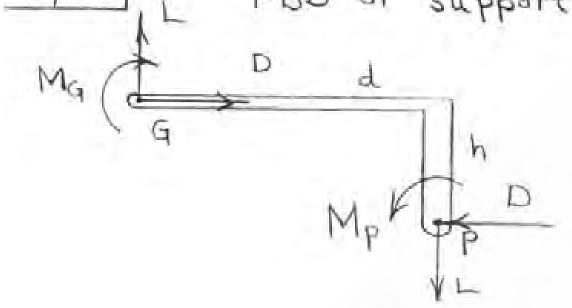
3/28 FBD of support & model:



$\circlearrowleft \Sigma M_P = 0: \quad M_P - M_G - Ld - Dh = 0$

$$M_G = M_P - Ld - Dh$$

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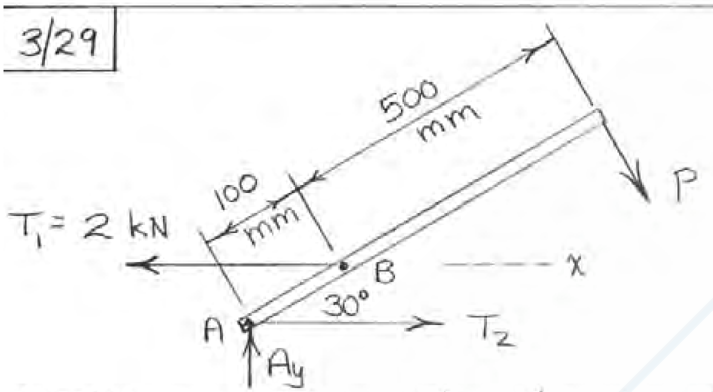

$\circlearrowleft + \; \Sigma M_A = 0: \; P(500+100) - 2(100 \sin 30°) = 0$

$P = 0.1667$ kN or $\underline{P = 166.7 \text{ N}}$

$\Sigma F_x = 0: \; 0.1667 \sin 30° + T_2 - 2 = 0$

$\underline{T_2 = 1.917 \text{ kN}}$

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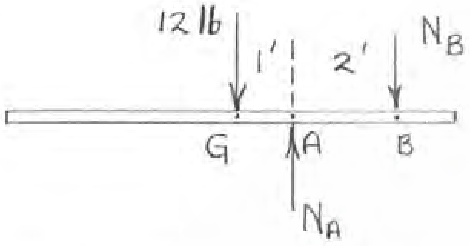

$$\circlearrowleft + \Sigma M_B = 0: \quad 12(3) - N_A(2) = 0$$

$$\underline{N_A = 18 \text{ lb}}$$

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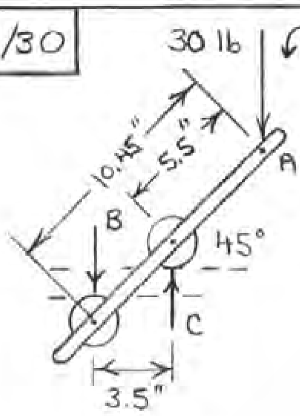

$\curvearrowleft + \Sigma M_C = 0$: $B(3.5) - (30 \cos 45°)(5.5) = 0$, $\underline{B = 33.3 \text{ lb}}$

$\Sigma M_B = 0$: $C(3.5) - (30 \cos 45°)(10.45) = 0$, $\underline{C = 63.3 \text{ lb}}$

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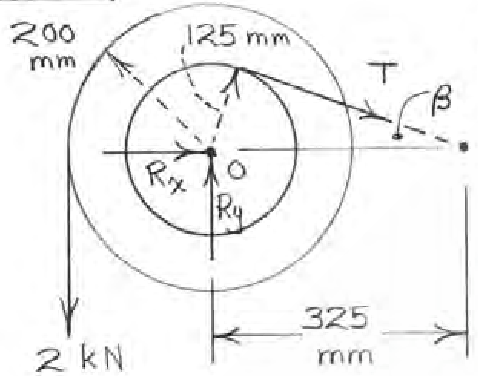

$$\begin{cases} \sin\beta = \frac{125}{325} = \frac{5}{13} \\ \cos\beta = \frac{12}{13} \end{cases}$$

$\circlearrowleft \Sigma M_O = 0: \quad 2(200) - T(125) = 0, \quad T = 3.2 \text{ kN}$

$\Sigma F_x = 0: \quad 3.2\left(\frac{12}{13}\right) + R_x = 0, \quad R_x = -2.95 \text{ kN}$

$\Sigma F_y = 0: \quad R_y - 2 - 3.2\left(\frac{5}{13}\right) = 0, \quad R_y = 3.23 \text{ kN}$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{2.95^2 + 3.23^2} = \underline{4.38 \text{ kN}}$

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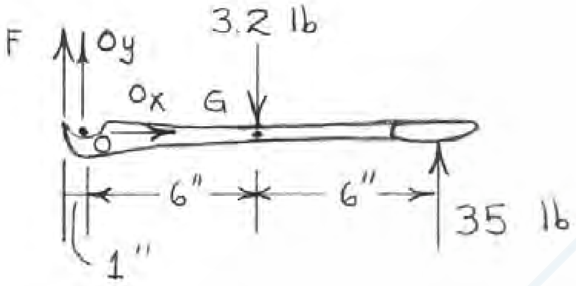

$$\circlearrowleft + \Sigma M_O = 0: \; -F(1) - 3.2(6) + 35(12) = 0$$

$$\underline{F = 401 \text{ lb}}$$

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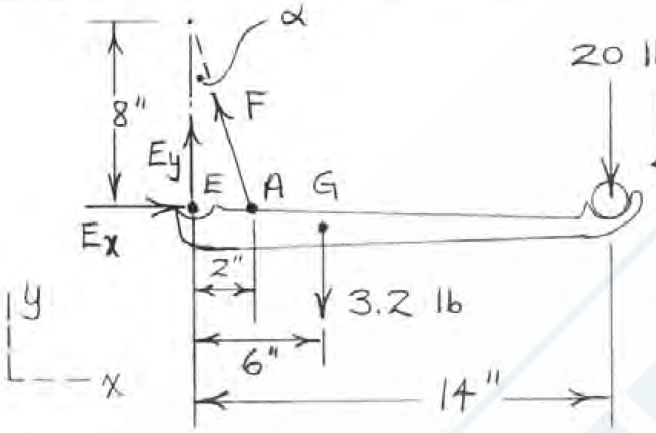
$\alpha = \tan^{-1}\frac{2}{8} = 14.04°$



Assume that A & E are on a horizontal line

$\curvearrowleft + \Sigma M_E = 0: \quad F\cos 14.04°(2) - 3.2(6) - 20(14) = 0$

$F = 154.2 \text{ lb}$

$\Sigma F_x = 0: \quad -154.2(\sin 14.04°) + E_x = 0$

$E_x = 37.4 \text{ lb}$

$\Sigma F_y = 0: \quad 154.2\cos 14.04° - 3.2 - 20 + E_y = 0$

$E_y = -126.4 \text{ lb}$

$E = \sqrt{E_x^2 + E_y^2} = \sqrt{37.4^2 + 126.4^2} = 131.8 \text{ lb}$

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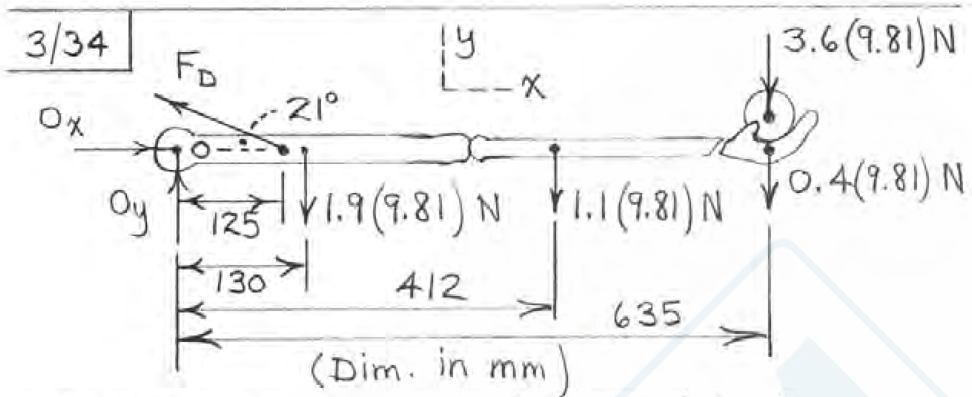

$\circlearrowleft + \sum M_O = 0: \; F_D \sin 21° (125) - 1.9(9.81)(130)$

$- 1.1(9.81)(412) - (3.6 + 0.4)(9.81)(635) = 0$

$\underline{F_D = 710 \text{ N}}$

$\xrightarrow{+} \sum F_x = 0: \; O_x - 710 \cos 21° = 0, \quad \underline{O_x = 662 \text{ N}}$

$+\uparrow \sum F_y = 0: \; O_y + 710 \sin 21° - (1.9 + 1.1 + 3.6 + 0.4)\, 9.81$

$= 0, \quad \underline{O_y = -185.6 \text{ N}}$

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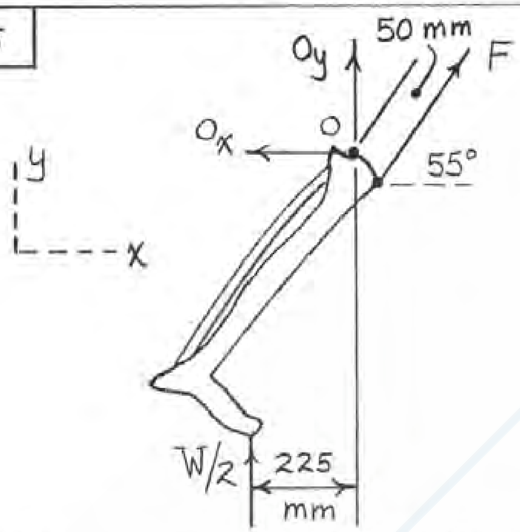

$$\circlearrowleft + \Sigma M_O = 0: \quad F(50) - \frac{W}{2}(225) = 0, \quad \underline{F = 2.25W}$$

$$\Sigma F_x = 0: \quad -O_x + 2.25W\cos 55° = 0, \quad O_x = 1.291W$$

$$\Sigma F_y = 0: \quad \frac{W}{2} + O_y + 2.25W\sin 55° = 0$$

$$O_y = -2.34W$$

$$O = \sqrt{O_x^2 + O_y^2} = \sqrt{(1.291W)^2 + (2.34W)^2} = \underline{2.67W}$$

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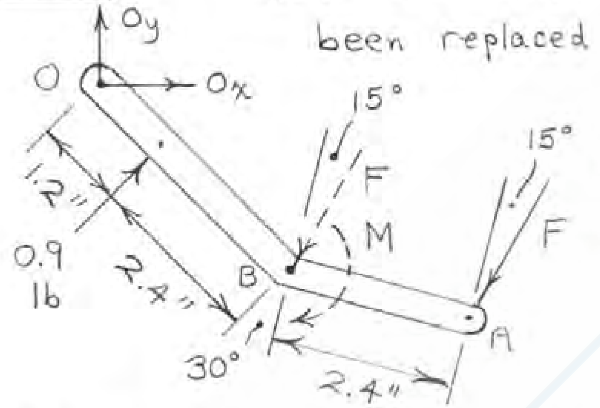
3/36 Note that the force F has been replaced by a force-couple system at B (dashed), where $M = F(2.4 \cos 15°)$



$\curvearrowleft \Sigma M_O = 0: \quad 0.9(1.2) - F(2.4 \cos 15°) - F(3.6 \cos 15°) = 0$

$\underline{F = 0.1863 \text{ lb}}$

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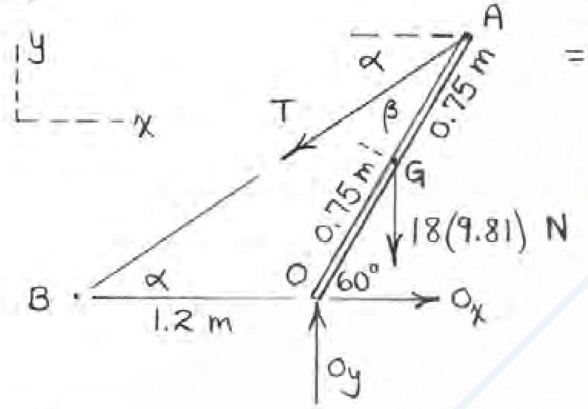
$$\alpha = \tan^{-1}\left[\frac{1.5 \sin 60°}{1.5 \cos 60° + 1.2}\right] = 33.7°$$

$$\beta = 90° - \alpha - 30° = 26.3°$$



$\circlearrowleft + \Sigma M_O = 0: \; T \sin 33.7° (1.2) - 18(9.81)(0.75)\cos 60° = 0$

$\underline{T = 99.5 \text{ N}}$

$\Sigma F_x = 0: \; -99.5 \cos 33.7° + O_x = 0$

$O_x = 82.8 \text{ N}$

$\Sigma F_y = 0: \; -99.5 \sin 33.7° - 18(9.81) + O_y = 0$

$O_y = 232 \text{ N}$

So $\underline{O = 246 \text{ N} \; @ \; 70.3° \text{ CCW from } +x\text{-axis}}$

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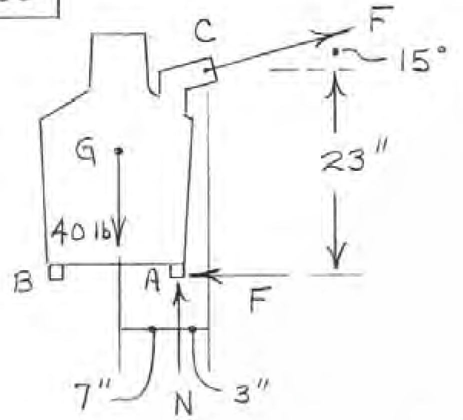

For impending tip, $N_B \to 0$.

$$\circlearrowleft + \Sigma M_A = 0: \; 40(7) - F\cos 15°(23) + F\sin 15°(3) = 0$$

$$\underline{F = 13.06 \text{ lb}}$$

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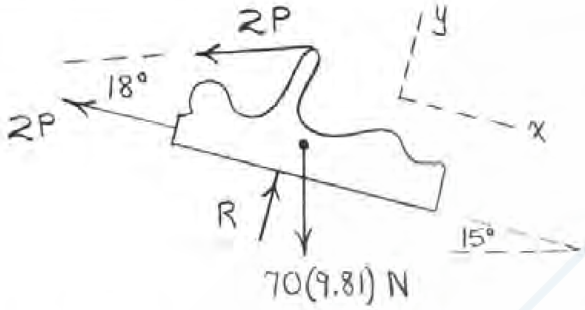

$\Sigma F_x = 0: \; 70(9.81)\sin 15° - 2P - 2P\cos 18° = 0$

$\underline{P = 45.5 \text{ N}}$

$\Sigma F_y = 0: \; R - 70(9.81)\cos 15° - 2(45.5)\sin 18° = 0$

$\underline{R = 691 \text{ N}}$

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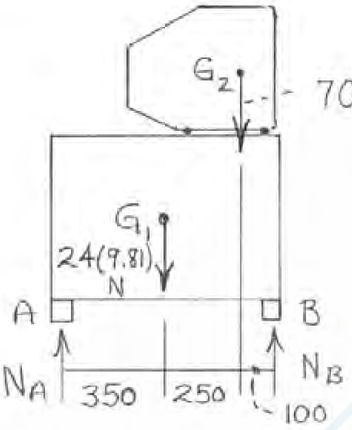

$\circlearrowleft\!+\ \Sigma M_A = 0:\ -24(9.81)350 - 70(9.81)(600) + N_B(700) = 0$

$\underline{N_B = 706\ \text{N}}$

$+\!\uparrow \Sigma F = 0:\ N_A + 706 - (70+24)(9.81) = 0$

$\underline{N_A = 216\ \text{N}}$

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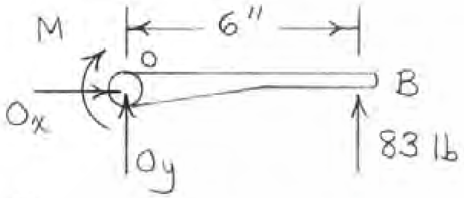
3/40 Arm OB:



$\circlearrowleft + \; \Sigma M_O = 0: \quad 83(6) - M = 0$

$M = 498$ lb-in. or $\underline{41.5 \text{ lb-ft}}$

$F \cos 20° (15) = M = 498$

$\underline{F = 35.3 \text{ lb}}$

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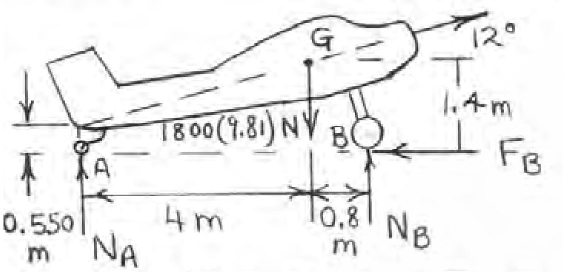

Engine off : $T = 0$, $F_B = 0$

$\Sigma M_A = 0$: $1800(9.81)4 - N_B(4.8) = 0$ $\quad N_B = 14\,720$ N

$\Sigma F_y = 0$: $N_A + 14\,720 - 1800(9.81) = 0$, $\quad N_A = 2940$ N

$\Sigma M_A = 0$: $1800(9.81)4 - N_B'(4.8) + 3000\cos 12° (0.550) = 0$

$N_B' = 15{,}050$ N

$\Sigma F_y = 0$: $N_A' + 15{,}050 - 1800(9.81) + 3000 \sin 12° = 0$, $N_A' = 1983$ N

$n_A = \dfrac{N_A' - N_A}{N_A}(100) = \underline{-32.6\%}$, $\quad n_B = \dfrac{N_B' - N_B}{N_B} = \underline{2.28\%}$

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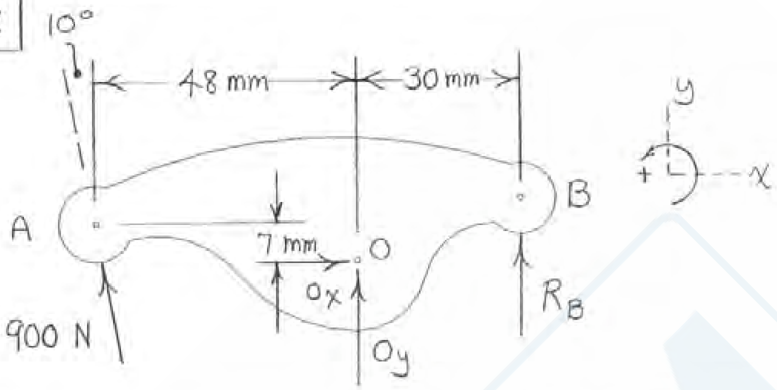

$\Sigma M_O = 0: -900 \cos 10°(48) + 900 \sin 10°(7)$
$+ R_B(30) = 0, \quad \underline{R_B = 1382 \text{ N}}$

$\Sigma F_x = 0: \quad O_x - 900 \sin 10° = 0, \quad O_x = 156.3 \text{ N}$

$\Sigma F_y = 0: \quad 900 \cos 10° + 1382 + O_y = 0$
$O_y = -2270 \text{ N}$

$O = \sqrt{O_x^2 + O_y^2} = \underline{2270 \text{ N}}$

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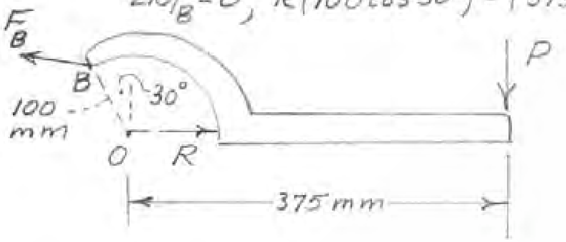
$M = Fd$; $80 = P(0.375)$, $P = 213$ N

$\Sigma M_B = 0$; $R(100\cos 30°) - (375 + 100\sin 30°)(213) = 0$



$R = 1047$ N

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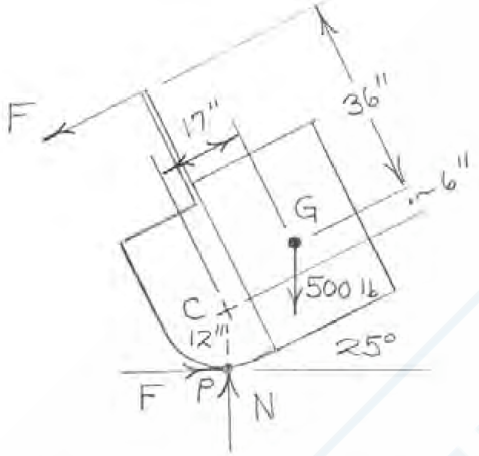

$\circlearrowleft + \Sigma M_P = 0$: $F(36 + 6 + 12\cos 25°) -$

$500[17\cos 25°] + 500[6\sin 25°] = 0$

$\underline{F = 121.7 \text{ lb}}$

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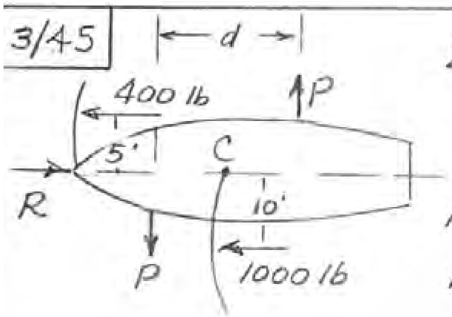

$\Sigma M_C = 0; \quad Pd + 400(5) - 1000(10) = 0$

Resultant of lateral forces is couple

$M = Pd = \underline{8000 \text{ lb-ft}}$

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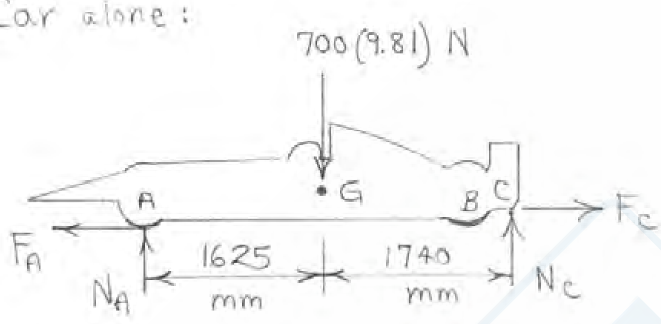
3/46 Car alone:

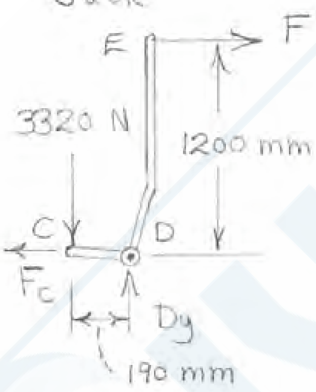


$$\circlearrowleft^{+} \Sigma M_A = 0: \; -700(9.81)(1625) + N_C(1625 + 1740) = 0$$

$$N_C = 3320 \text{ N}$$

(assumes moment of F_C about A is negligible)

Jack



$$\circlearrowleft^{+} \Sigma M_D = 0:$$

$$3320(190) - F(1200) = 0$$

$$\underline{F = 525 \text{ N}}$$

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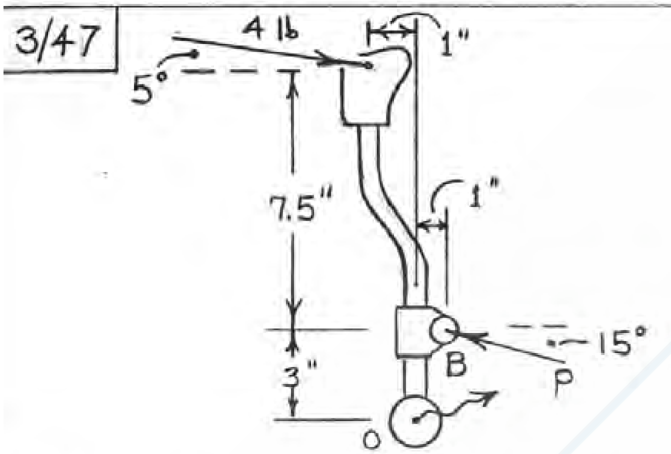

$$\circlearrowleft + \Sigma M_O = 0: -4\cos 5°(10.5) + 4\sin 5°(1)$$

$$+ P\cos 15°(3) + P\sin 15°(1) = 0$$

$$\underline{P = 13.14 \text{ lb}}$$

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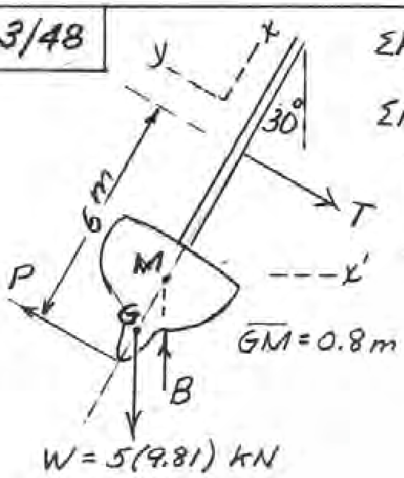

$\Sigma F_x = 0; \; B = 5(9.81) = 49.05$ kN

$\Sigma F_{x'} = 0; \; P = T$

Two couples, so

$\Sigma M = 0; \; 6T - 5(9.81)(0.8)\sin 30^\circ = 0$

$$T = \frac{5(9.81)(0.8)(0.5)}{6}$$

$= \underline{3.27 \text{ kN}}$

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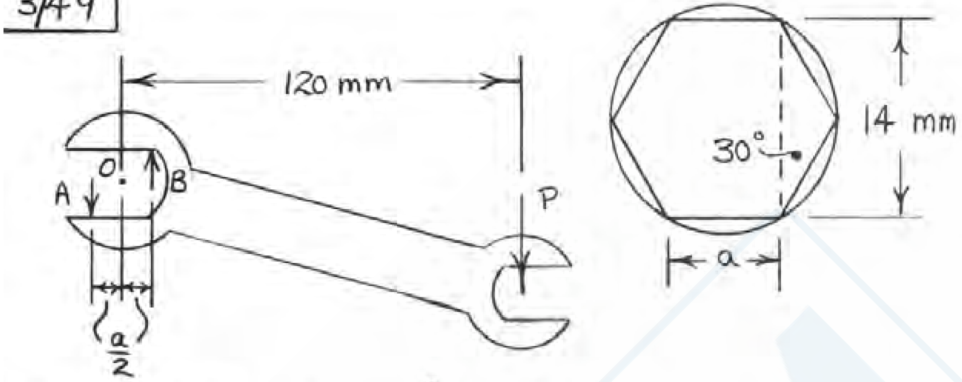

$2a\cos 30° = 14$, $\frac{a}{2} = 4.04$ mm

$\circlearrowleft + \Sigma M_O = 0$: $0.120P - 24 = 0$, $\underline{P = 200\text{ N}}$

(for wrench and bolt)

For wrench alone,

$\circlearrowleft + \Sigma M_A = 0$: $200(0.120 + 0.00404) - B(2 \cdot 0.00404) = 0$, $\underline{B = 3070\text{ N}}$

$+\uparrow \Sigma F = 0$: $-A + 3070 - 200 = 0$, $\underline{A = 2870\text{ N}}$

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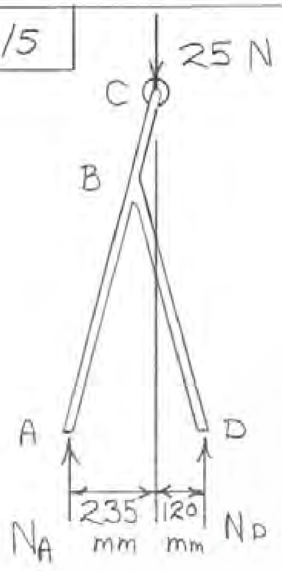

$\circlearrowleft \Sigma M_A = 0:$

$N_D(355) - 25(235) = 0$

$N_D = 16.55$ N

$+\uparrow \Sigma F = 0:$

$N_A + 16.55 - 25 = 0$

$N_A = 8.45$ N

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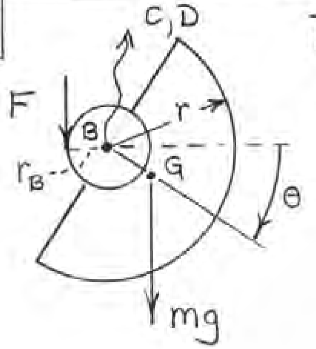

$\overline{BG} = \bar{r} = \frac{3}{8} r$ (from Table D/4)

FBD is of hemisphere, shaft, and gear B.

$$\circlearrowleft + \Sigma M_B = 0: \quad F r_B - mg\left(\frac{3}{8} r \cos\theta\right)$$

$$F = \frac{3}{8} mg \frac{r}{r_B} \cos\theta$$

Gear A:

$$\circlearrowleft + \Sigma M_B = 0: \quad F r_A - M = 0$$

$$M = \left(\frac{3}{8} mg \frac{r}{r_B} \cos\theta\right) r_A$$

$$= \frac{3mgr}{8} \frac{r_A}{r_B} \cos\theta$$

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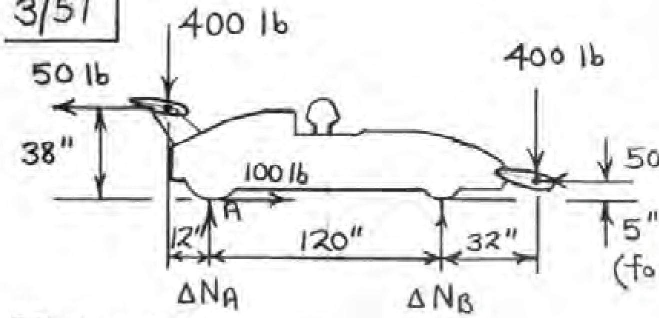

Without airfoils, $N_A = N_B = 850$ lb

With airfoils,

$+\uparrow \Sigma F_y = 0: \Delta N_A + \Delta N_B - 2(400) = 0$

$\circlearrowleft + \Sigma M_A = 0: \; 50(38) + 400(12) + \Delta N_B(120) + 50(5) - 400(152) = 0$

$\left.\begin{array}{l}\Delta N_A = 351 \text{ lb} \\ \Delta N_B = 449 \text{ lb}\end{array}\right\} \Rightarrow \begin{array}{l} N_A = 850 + 351 = 1201 \text{ lb} \; (48.0\%) \\ N_B = 850 + 449 = 1299 \text{ lb} \; (52.0\%)\end{array}$

Note that a 100-lb propulsive force has been added (at A) to maintain equilibrium.

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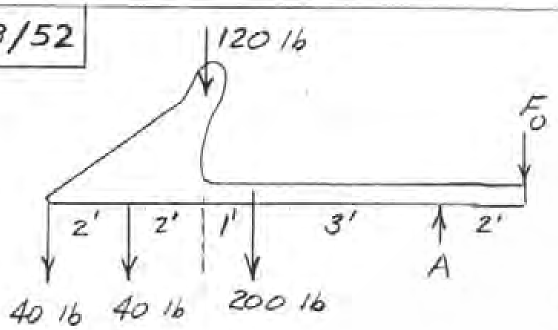

$\Sigma M_A = 0;$

$40(8) + 40(6) + 120(4) + 200(3) - 2F_O = 0$

$F_O = 820\text{ lb}$

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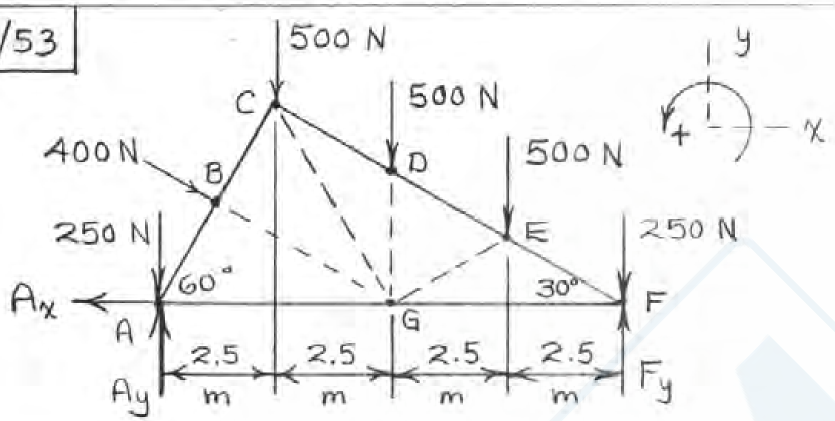

$\Sigma F_x = 0: \; -A_x + 400\cos 30° = 0, \quad \underline{A_x = 346 \text{ N}}$

$\Sigma M_A = 0: \; 400\left(\frac{10}{4}\right) + 500(2.5) + 500(5)$

$+ 500(7.5) + 250(10) - 10F_y = 0$

$\underline{F_y = 1100 \text{ N}}$

$\Sigma F_y = 0: -250 - 400\sin 30° - 500(3) - 250$

$+ 1100 + A_y = 0, \quad \underline{A_y = 1100 \text{ N}}$

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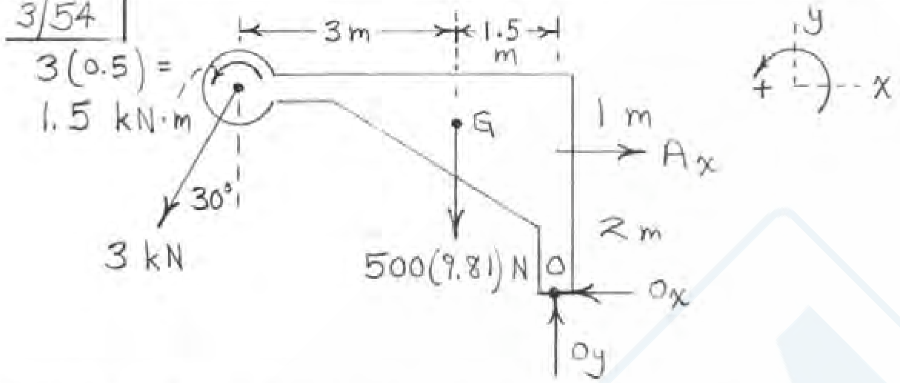

$\Sigma M_O = 0: \; -2A + 500(9.81)(1.5) + 1500$

$+ 3000\cos 30°(4.5) + 3000 \sin 30°(3) = 0$

$A = 12\,520$ N

$\Sigma F_x = 0: \; -3000 \sin 30° + 12\,520 - O_x = 0$

$O_x = 11\,020$ N

$\Sigma F_y = 0: \; -3000 \cos 30° - 500(9.81) + O_y = 0$

$O_y = 7500$ N

$O = \sqrt{11020^2 + 7500^2} = 13\,340$ N or 13.34 kN

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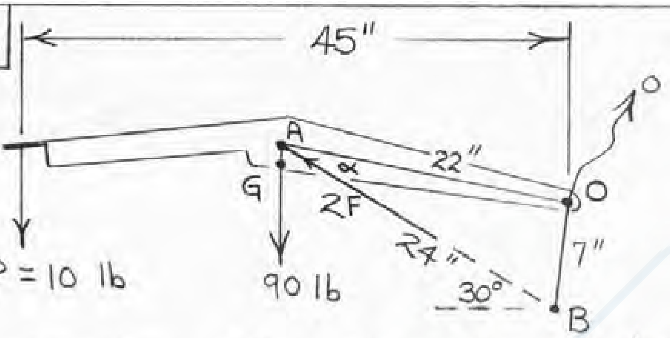

Law of cosines: $7^2 = 22^2 + 24^2 - 2(22)(24)\cos\alpha$

$\alpha = 16.79°$

$\circlearrowleft + \Sigma M_O = 0: \quad 10(45) - 2F(22\sin\alpha) + 90(22\cos(30° - \alpha)) = 0, \quad F = 187.1 \text{ lb}$

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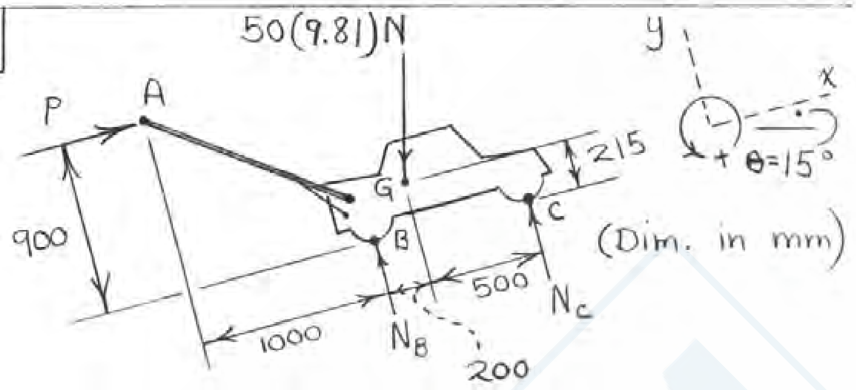

$\Sigma F_x = 0: \quad P - 50(9.81)\sin 15° = 0 \qquad (1)$

$\Sigma F_y = 0: \quad N_B + N_C - 50(9.81)\cos 15° = 0 \qquad (2)$

$\Sigma M_C = 0: \quad -P(900) - N_B(700) + 50(9.81)[500\cos 15° + 215\sin 15°] = 0 \qquad (3)$

Solution to Eqs. (1)-(3):	With $\theta = P = 0$:
$P = 127.0$ N	$P = 0$
$N_B = 214$ N	$N_B = 350$ N
$N_C = 260$ N	$N_C = 140.1$ N

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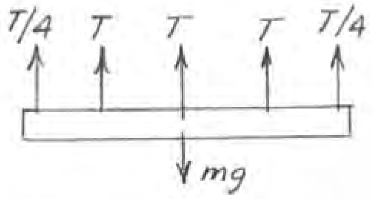

$$\Sigma F = 0;\ \left(3 + \frac{1}{4} + \frac{1}{4}\right)T - mg = 0$$

$$\underline{T = \frac{2}{7}mg}$$

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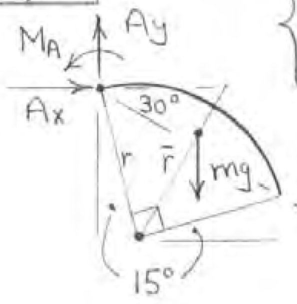

From Table D/3, $\bar{r} = \dfrac{r \sin\alpha}{\alpha}$

For $\alpha = \pi/4$, $\bar{r} = 2\sqrt{2}\, r/\pi$

$\circlearrowleft + \sum M_A = 0$:

$M_A - mg(r \sin 15^\circ + \bar{r} \cos 60^\circ) = 0$

$\underline{M_A = 0.709\ mgr}$

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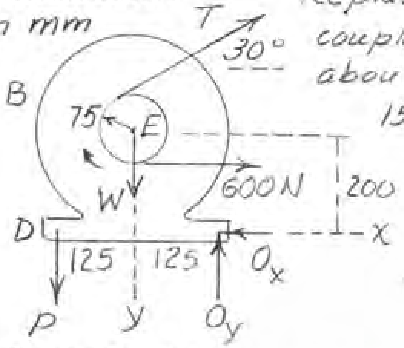
3/59 Torque $M = 100\ N\cdot m = (600 - T)(0.225)$, $T = 155.6\ N$

Dimensions in mm



$W = 100(9.81)\ N$

Replace T by force at E and a couple, so that moment of T about D in CW sense becomes

$155.6(0.2\cos 30° - 0.125\sin 30°) + 155.6(0.075) = 28.89\ N\cdot m$

$\Sigma M_D = 0;\ 28.89 + 981(0.125) + 600(0.2 - 0.075) - 0.250\,O_y = 0$

$O_y = 906\ N$

$\Sigma F_x = 0;\ 155.6\cos 30° + 600 - O_x = 0$

$O_x = 735\ N$

$R = \sqrt{O_x^2 + O_y^2} = \sqrt{(735)^2 + (906)^2} = 1167\ N$ or $1.167\ kN$

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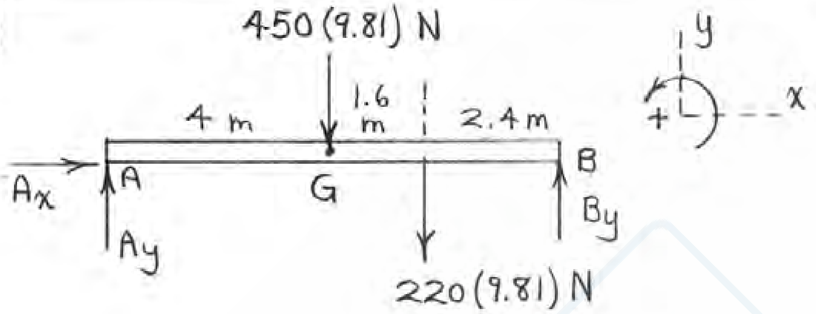

From $\Sigma F_x = 0$, $A_x = 0$

$\Sigma M_A = 0$: $-450(9.81)4 - 220(9.81)(5.6) + B_y(8) = 0$, $\underline{B_y = 3720 \text{ N}}$

$\Sigma F_y = 0$: $A_y - 450(9.81) - 220(9.81) + 3720 = 0$

$\underline{A_y = 2850 \text{ N}}$

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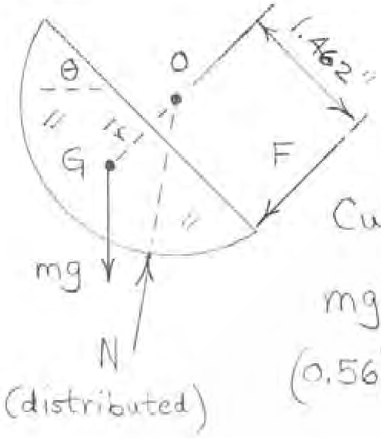
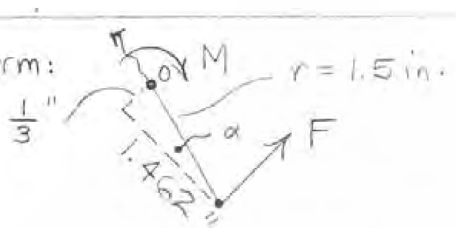
Cube:



Arm:



$$\alpha = \sin^{-1}\frac{1/3}{1.5} = 12.84^\circ$$

Cube: $\circlearrowleft \Sigma M_O = 0$:

$$mg\bar{r}\sin\theta - F(1.462) = 0$$

$$(0.56)(0.55 \cdot 1.5)\sin\theta - F(1.462) = 0$$

$$F = 0.316\sin\theta \quad (\text{in lb})$$

Arm: $\circlearrowleft \Sigma M_O = 0$:

$$-M + 0.316\sin\theta\,(1.462) = 0$$

$$M = 0.462\sin\theta \quad \text{lb-in.}$$

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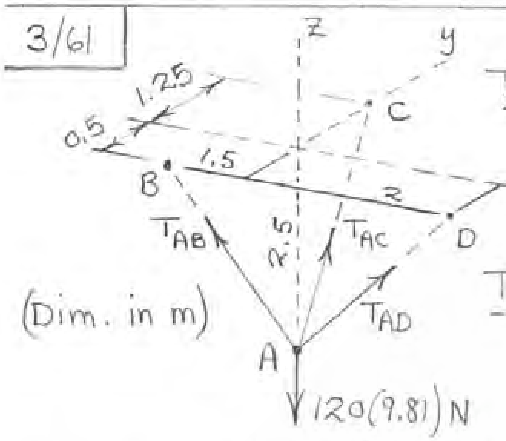

$$\underline{T}_{AB} = T_{AB}\frac{-1.5\underline{i} - 0.5\underline{j} + 2.5\underline{k}}{\sqrt{1.5^2 + 0.5^2 + 2.5^2}},$$

$$= T_{AB}\left(-0.507\underline{i} - 0.1690\underline{j} + 0.845\underline{k}\right)$$

$$\underline{T}_{AC} = T_{AC}\frac{1.25\underline{j} + 2.5\underline{k}}{\sqrt{1.25^2 + 2.5^2}},$$

$$= T_{AC}\left(0.447\underline{j} + 0.894\underline{k}\right)$$

$$\underline{T}_{AD} = T_{AD}\frac{2\underline{i} - 0.5\underline{j} + 2.5\underline{k}}{\sqrt{2^2 + 0.5^2 + 2.5^2}}, = T_{AD}\left(0.617\underline{i} - 0.1543\underline{j} + 0.772\underline{k}\right)$$

$$\begin{cases} \Sigma F_x = 0: -0.507T_{AB} + 0.617T_{AD} = 0 \\ \Sigma F_y = 0: -0.1690T_{AB} + 0.447T_{AC} - 0.1543T_{AD} = 0 \\ \Sigma F_z = 0: 0.845T_{AB} + 0.894T_{AC} + 0.772T_{AD} - 120(9.81) = 0 \end{cases}$$

Solution:

$$\begin{cases} T_{AB} = 569 \text{ N} \\ T_{AC} = 376 \text{ N} \\ T_{AD} = 467 \text{ N} \end{cases}$$

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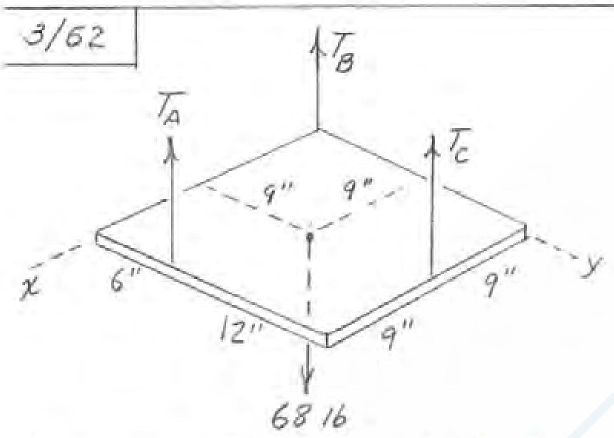

$\Sigma M_x = 0$; $6T_A + 18T_C - 68(9) = 0$, $T_A + 3T_C = 102$

$\Sigma M_y = 0$; $-18T_A - 9T_C + 68(9) = 0$, $2T_A + T_C = 68$

Solve & get $\underline{T_A = 20.4 \text{ lb}}$, $\underline{T_C = 27.2 \text{ lb}}$

$\Sigma F_z = 0$; $20.4 + T_B + 27.2 - 68 = 0$, $\underline{T_B = 20.4 \text{ lb}}$

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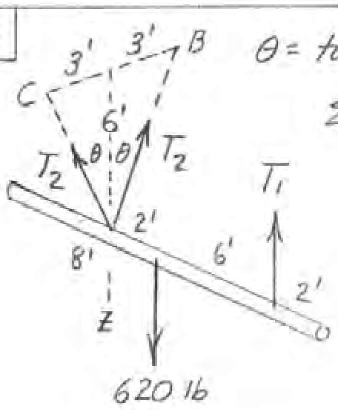

$\theta = \tan^{-1} 3/6 = 26.57°$

$\Sigma M_{CB} = 0;\ 8T_1 - 620(2) = 0$

$\underline{T_1 = 155 \text{ lb}}$

$\Sigma F_z = 0;\ 620 - 155 - 2T_2 \cos\theta = 0$

$T_2 = \dfrac{465}{2 \cos 26.57°} = \underline{260 \text{ lb}}$

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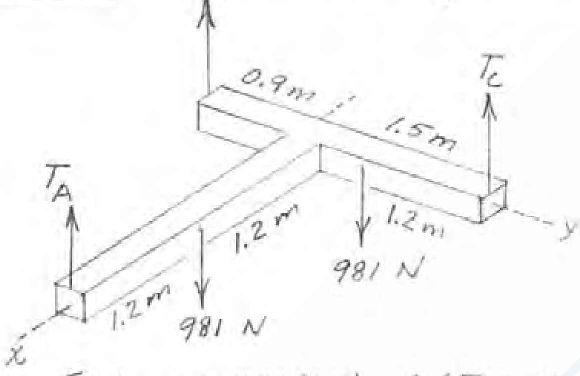
$W = mg = 100(9.81) = 981\ N$



$\Sigma M_y = 0;\ 981(1.2) - 2.4T_A = 0,\quad \underline{T_A = 490\ N}$

$\Sigma M_x = 0;\ 1.5T_C - 0.9T_B - 981(0.3) = 0$

$\Sigma F_z = 0;\ 491 + T_B + T_C - 2(981) = 0,\quad T_B + T_C = 1472\ N$

so $1.5T_C - 0.9(1472 - T_C) = 294.3,\quad \underline{T_C = 674\ N}$

$\underline{T_B = 797\ N}$

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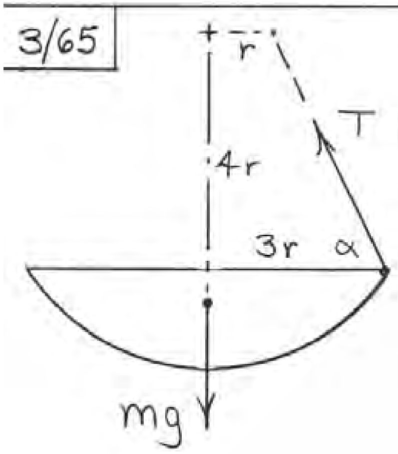

T (one of three)

$$\alpha = \tan^{-1} \frac{4r}{2r} = 63.4°$$

$+\uparrow \Sigma F = 0:$

$$3(T \sin \alpha) - mg = 0$$

$$\underline{T = 0.373mg}$$

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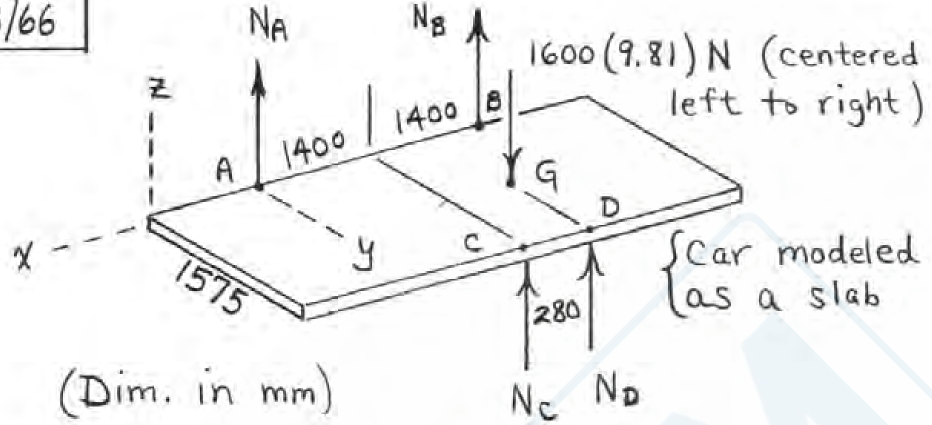

Jacking at C ($N_D = 0$):

$$\Sigma M_x = 0: \; -1600(9.81)\left(\frac{1575}{2}\right) + N_C(1575) = 0$$

$$N_C = 7850 \text{ N}$$

$$\Sigma M_y = 0: \; -1600(9.81)(1680) + N_B(2800) + N_C(1400) = 0$$

$$\Sigma F_z = 0: \; N_A + N_B + N_C - 1600(9.81) = 0$$

$$\Rightarrow N_A = 2350 \text{ N}, \quad N_B = 5490 \text{ N}$$

Jacking at D ($N_C = 0$): $N_D = 7850$ N {Same as for N_C

$$\Sigma M_y = 0: \; -1600(9.81)(1680) + N_B(2800) + N_D(1680) = 0$$

$$\Sigma F_z = 0: \; N_A + N_B + N_D - 1600(9.81) = 0$$

$$\Rightarrow N_A = 3140 \text{ N}, \quad N_B = 4710 \text{ N}$$

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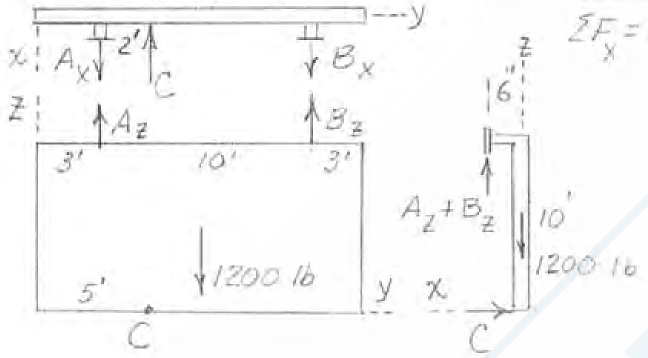
3/67 x-z; $\Sigma M_{AB} = 0$; $10C - 1200(6/12) = 0$, $C = 60$ lb

x-y; $\Sigma M_A = 0$; $2(60) - 10B_x = 0$; $B_x = 12$ lb

$\Sigma F_x = 0$; $A_x + 12 - 60 = 0$

$A_x = 48$ lb



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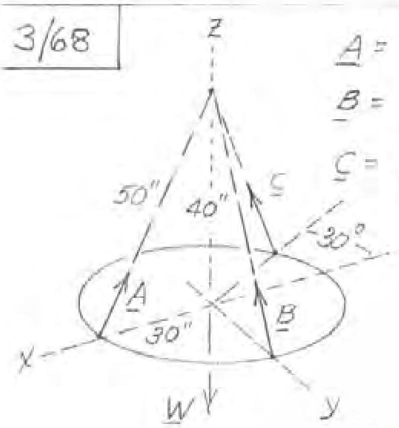

$\underline{A} = \frac{A}{5}(-3\underline{i} + 4\underline{k})$

$\underline{B} = \frac{B}{5}(-3\underline{j} + 4\underline{k})$

$\underline{C} = \frac{C}{5}(3\underline{i}\cos 30° + 3\underline{j}\sin 30° + 4\underline{k})$

$\underline{W} = -600\underline{k}$ lb

$\Sigma \underline{F} = 0;\quad \frac{1}{5}(-3A + \frac{3\sqrt{3}}{2}C)\underline{i} + \frac{1}{5}(-3B + \frac{3}{2}C)\underline{j}$

$+ \frac{1}{5}(4A + 4B + 4C - 5[600])\underline{k} = \underline{0}$

Thus $A = \frac{\sqrt{3}}{2}C$, $B = \frac{C}{2}$, $4(A+B+C) = 3000$

$4(\frac{\sqrt{3}}{2} + \frac{1}{2} + 1)C = 3000$, $C = \frac{3000}{2(3+\sqrt{3})} = \underline{317 \text{ lb}}$

$B = \underline{158 \text{ lb}}$

$A = \underline{275 \text{ lb}}$

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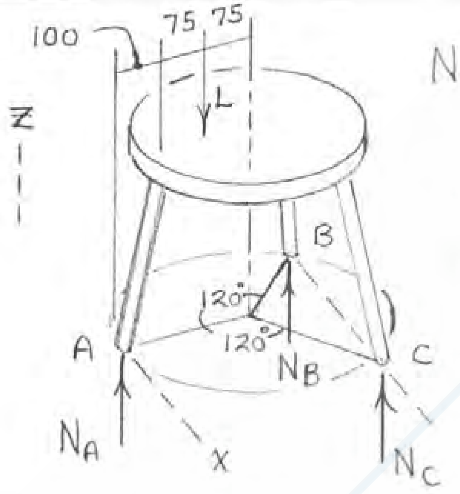

Note: x-axis is ∥ BC.

$N_B = N_C = N$, by symmetry.

$\Sigma M_x = 0: \; -L(175) + 2N(250 + 250\cos 60°) = 0$

$N = 0.233L = N_B = N_C$

$\Sigma F_z = 0: \; N_A + 2(0.233L) - L = 0$

$N_A = 0.533L$

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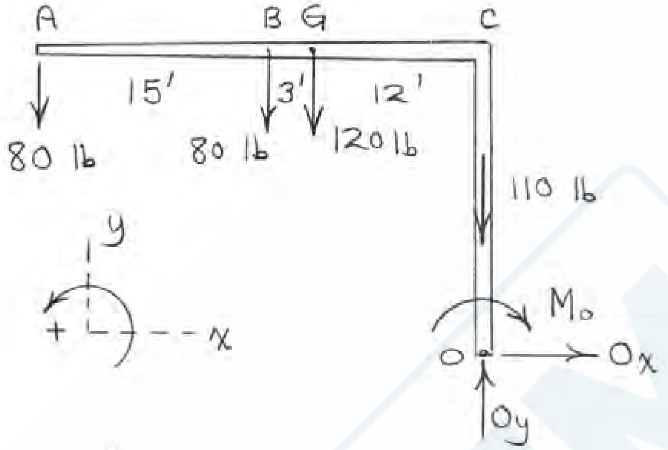

$\Sigma F_x = 0: \quad O_x = 0$

$\Sigma F_y = 0: \quad O_y - 80 - 80 - 120 - 110 = 0$

$O_y = 390 \text{ lb}$

$\Sigma M_O = 0: \quad 80(30) + 80(15) + 120(12) - M_O = 0$

$M_O = 5040 \text{ lb-ft CW}$

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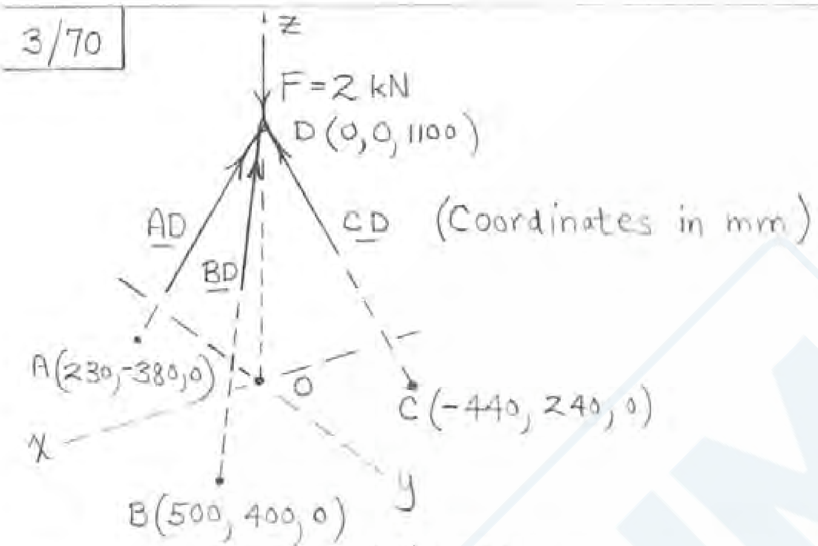

$$\underline{AD} = AD\frac{-230\underline{i} + 380\underline{j} + 1100\underline{k}}{\sqrt{230^2 + 380^2 + 1100^2}} = AD(-0.1939\underline{i} + 0.320\underline{j} + 0.927\underline{k})$$

$$\underline{BD} = BD\frac{-500\underline{i} - 400\underline{j} + 1100\underline{k}}{\sqrt{500^2 + 400^2 + 1100^2}} = BD(-0.393\underline{i} - 0.314\underline{j} + 0.864\underline{k})$$

$$\underline{CD} = CD\frac{440\underline{i} - 240\underline{j} + 1100\underline{k}}{\sqrt{440^2 + 240^2 + 1100^2}} = CD(0.364\underline{i} - 0.1985\underline{j} + 0.910\underline{k})$$

From $\Sigma \underline{F} = \underline{0}$:

$$\begin{cases} -0.1939AD - 0.393BD + 0.364CD = 0 \\ 0.320AD - 0.314BD - 0.1985CD = 0 \\ 0.927AD + 0.864BD + 0.910CD - 2 = 0 \end{cases}$$

Solution: $\underline{AD = 0.925 \text{ kN}}$, $\underline{BD = 0.376 \text{ kN}}$, $\underline{CD = 0.898 \text{ kN}}$

(all compression)

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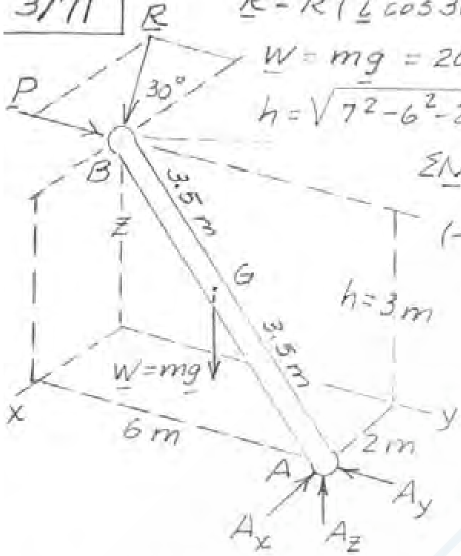

$\underline{R} = R(\underline{i}\cos 30° + \underline{j}\sin 30°) = \frac{R}{2}(\sqrt{3}\,\underline{i} + \underline{j})$

$\underline{W} = mg = 200(9.81)(-\underline{k}) = -1962\,\underline{k}$ N

$h = \sqrt{7^2 - 6^2 - 2^2} = 3$ m

$\Sigma \underline{M}_A = \underline{0};\ \underline{r}_{AG} \times \underline{W} + \underline{r}_{AB} \times (\underline{P} + \underline{R}) = \underline{0}$

$(-1\underline{i} - 3\underline{j} + 1.5\underline{k}) \times (-1962\underline{k})$

$+ (-2\underline{i} - 6\underline{j} + 3\underline{k}) \times (P\underline{j} + \frac{R\sqrt{3}}{2}\underline{i} + \frac{R}{2}\underline{j})$

$= \underline{0}$

Simplify & get

$(5886 - 3P - 3R/2)\underline{i}$

$+ (-1962 + 3\sqrt{3}R/2)\underline{j}$

$+ (-2P + R + 3\sqrt{3}R)\underline{k} = 0$

$R = \frac{2(1962)}{3\sqrt{3}} = \underline{755 \text{ N}}$

$3P = 5886 - \frac{3}{2}755$, $P = \underline{1584 \text{ N}}$

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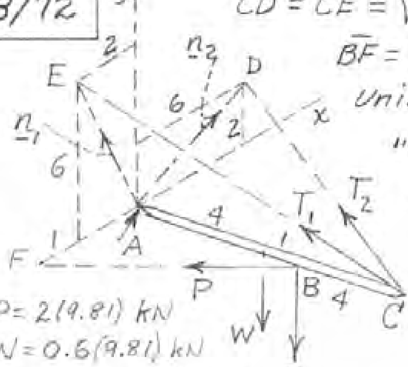

$P = 2(9.81)$ kN

$W = 0.6(9.81)$ kN

Dimensions in (m)

$\overline{CD} = \overline{CE} = \sqrt{6^2+2^2+9^2} = 11$ m

$\overline{BF} = \sqrt{3^2+5^2} = 5.83$ m

unit vector $\underline{n}_1 = (-\underline{i}+3\underline{j})/\sqrt{10}$

" " $\underline{n}_2 = (3\underline{i}+\underline{j})/\sqrt{10}$

$\Sigma M_{AD} = 0;$

$$9\underline{k} \times \frac{T_1}{11}(-2\underline{i}+6\underline{j}-9\underline{k}) \cdot (3\underline{i}+\underline{j})/\sqrt{10}$$

$$+5\underline{k} \times \frac{2(9.81)}{5.83}(-3\underline{i}-5\underline{k}) \cdot (3\underline{i}+\underline{j})/\sqrt{10}$$

$$+5\underline{k} \times 2(9.81)(-\underline{j}) \cdot (3\underline{i}+\underline{j})/\sqrt{10}$$

$$+4.5\underline{k} \times 0.6(9.81)(-\underline{j}) \cdot (3\underline{i}+\underline{j})/\sqrt{10} = 0$$

Simplify & get $\dfrac{180T_1}{11\sqrt{10}} + \dfrac{294.3}{5.83\sqrt{10}} = \dfrac{373.8}{\sqrt{10}}$, $\underline{T_1 = 19.76 \text{ kN}}$

(Similarly, if T_2 is desired, $\Sigma M_{AE} = 0$ will give $T_2 = 16.86$ kN)

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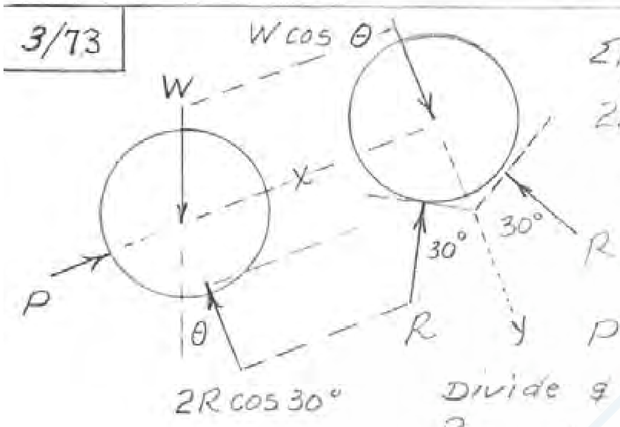

$\Sigma F_y = 0$;

$2R\cos 30° - W\cos\theta = 0$

$R\sqrt{3} = W\cos\theta$

$\Sigma F_x = 0$;

$P - W\sin\theta = 0$

Divide & get

$\frac{P}{R\sqrt{3}} = \tan\theta$

So with $P = R$, $\theta = \tan^{-1} 1/\sqrt{3} = \underline{30°}$

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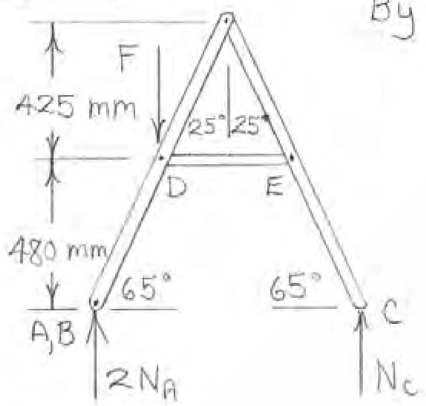

By inspection, $N_A = N_B$.

$\curvearrowleft + \Sigma M_C = 0: \; -2N_A(2 \cdot 905 \tan 25°)$
$+F(905 \tan 25° + 425 \tan 25°) = 0$

$\underline{N_A = 0.367F = N_B}$

$+\uparrow \Sigma F = 0: \; 2(0.367F) + N_C - F = 0$

$\underline{N_C = 0.265F}$

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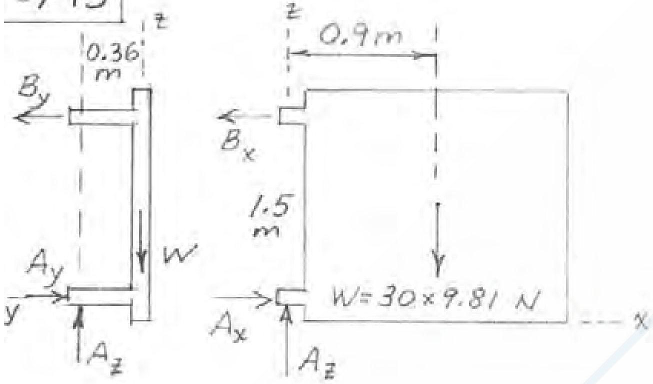

x-z; $\Sigma M_A = 0$; $1.5 B_x - 0.9(30)(9.81) = 0$, $B_x = 176.6$ N

y-z; $\Sigma M_A = 0$; $1.5 B_y - 0.36(30)(9.81) = 0$, $B_y = 70.6$ N

$$B = \sqrt{176.6^2 + 70.6^2} = \underline{190.2 \text{ N}}$$

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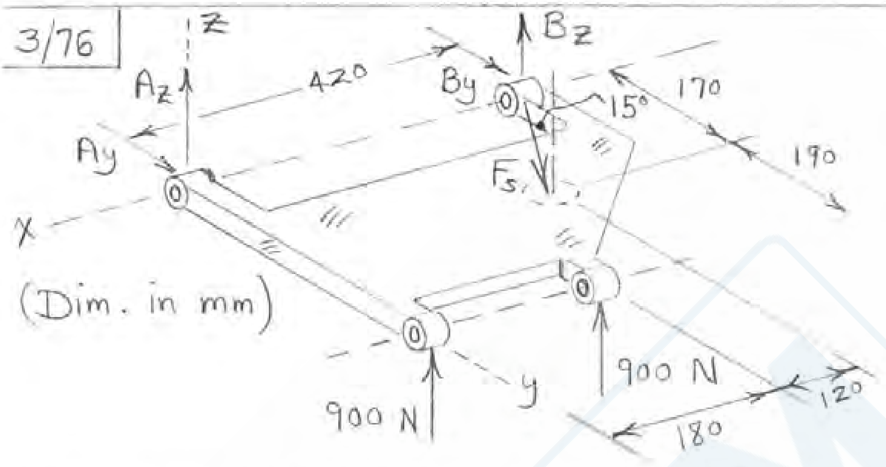

$\Sigma M_x = 0: \ 2(900)(360) - F_s \cos 15° (170) = 0, \ \underline{F_s = 3950 \text{ N}}$

$\Sigma M_y = 0: \ B_z(420) + 900(180) - 3950 \cos 15° (300) = 0$

$B_z = 2340 \text{ N}$

$\Sigma M_z = 0: -B_y(420) - 3950 \sin 15° (300) = 0, \ B_y = -730 \text{ N}$

$\Sigma F_y = 0: \ A_y - 730 + 3950 \sin 15° = 0, \quad A_y = -292 \text{ N}$

$\Sigma F_z = 0: \ A_z + 2340 - 3950 \cos 15° + 1800 = 0, \ A_z = -325 \text{ N}$

$F_A = \sqrt{A_y^2 + A_z^2} = \underline{437 \text{ N}}$

$F_B = \sqrt{B_y^2 + B_z^2} = \underline{2450 \text{ N}}$

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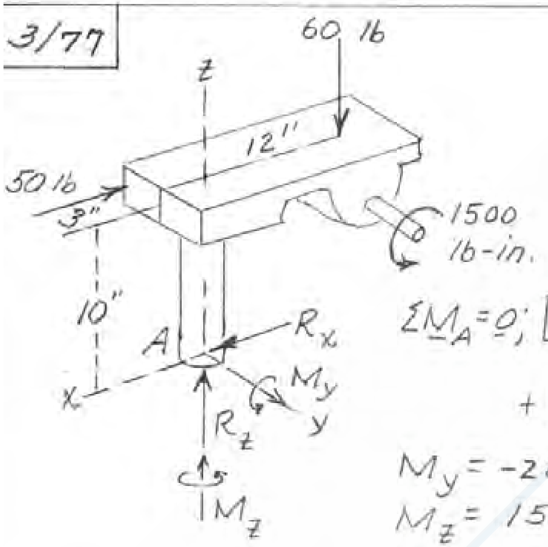

$$\Sigma \underline{M}_A = \underline{0};\ [-60(12) - 50(10) + 1500 + M_y]\underline{j} + [-50(3) + M_z]\underline{k} = \underline{0}$$

$M_y = -280$ lb-in.

$M_z = 150$ lb-in

$\underline{M} = -280\underline{j} + 150\underline{k}$ lb-in.

$$\Sigma \underline{F} = \underline{0};\ (R_x - 50)\underline{i} + (R_z - 60)\underline{k} = \underline{0}$$

$R_x = 50$ lb, $R_z = 60$ lb

$\underline{R} = 50\underline{i} + 60\underline{k}$ lb

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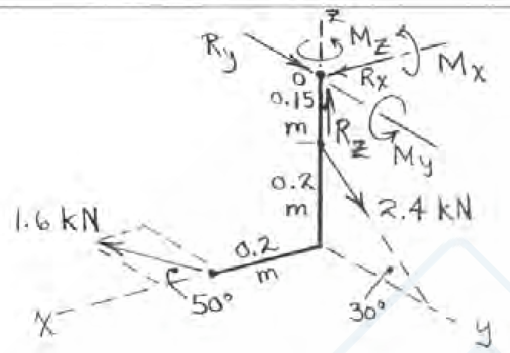

$\Sigma F_x = 0: \; R_x + 1.6 \cos 50°, \qquad R_x = -1.028 \text{ kN}$

$\Sigma F_y = 0: \; R_y + 2.4 \cos 30° - 1.6 \sin 50° = 0, \quad R_y = -0.853 \text{ kN}$

$\Sigma F_z = 0: \; R_z - 2.4 \sin 30° = 0, \qquad R_z = 1.2 \text{ kN}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \underline{1.796 \text{ kN}}$

$\Sigma M_{o_x} = 0: \; M_x + 2.4 \cos 30° (0.15) - 1.6 \sin 50° (0.35) = 0$

$M_x = 0.1172 \text{ kN·m}$

$\Sigma M_{o_y} = 0: \; M_y - 1.6 \cos 50° (0.35) = 0, \quad M_y = 0.360 \text{ kN·m}$

$\Sigma M_{o_z} = 0: \; M_z - 1.6 \sin 50° (0.2) = 0, \quad M_z = 0.245 \text{ kN·m}$

$M = \sqrt{M_x^2 + M_y^2 + M_z^2} = \underline{0.451 \text{ kN·m}}$

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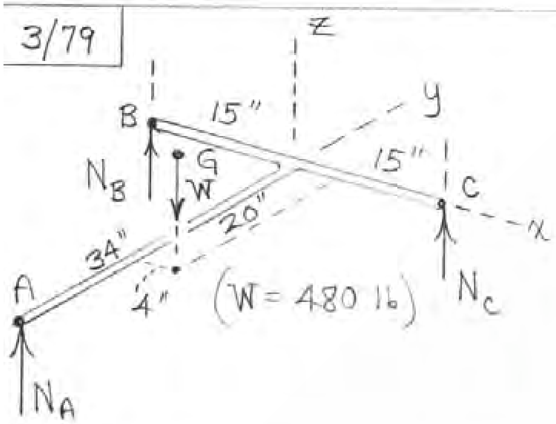

$\Sigma F_z = 0: \quad N_A + N_B + N_C - 480 = 0$

$\Sigma M_x = 0: \quad 480(20) - N_A(34) = 0$

$\Sigma M_y = 0: \quad N_B(15) - N_C(15) + 480(4) = 0$

Solution:
$N_A = 282$ lb
$N_B = 34.8$ lb
$N_C = 162.8$ lb

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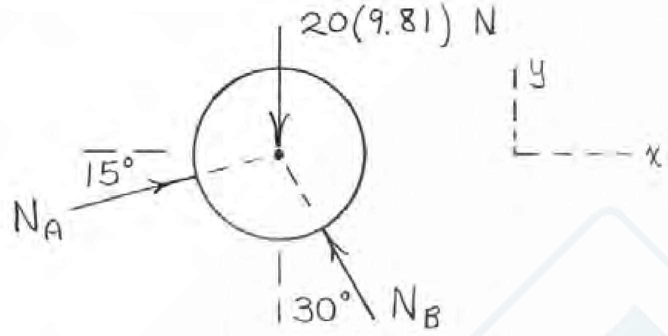

$$\begin{cases} \Sigma F_x = 0: & N_A \cos 15° - N_B \sin 30° = 0 \quad (1) \\ \Sigma F_y = 0 & N_A \sin 15° + N_B \cos 30° - 20(9.81) = 0 \quad (2) \end{cases}$$

Solution: $\begin{cases} N_A = 101.6 \text{ N} \\ N_B = 196.2 \text{ N} \end{cases}$

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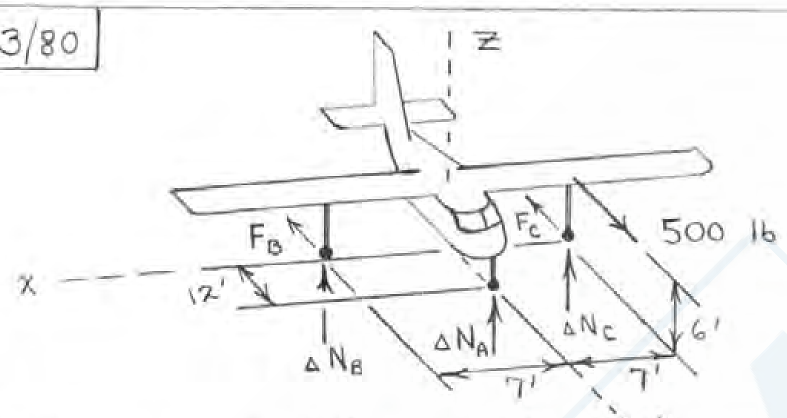

$\Sigma M_x = 0: \quad \Delta N_A(12) - 500(6) = 0, \qquad \underline{\Delta N_A = 250 \text{ lb}}$

$\Sigma F_z = 0: \quad \Delta N_A + \Delta N_B + \Delta N_C = 0$

$\Sigma M_y = 0: \quad \Delta N_C(7) - \Delta N_B(7) = 0$

$\underline{\Delta N_B = \Delta N_C = -125 \text{ lb}}$

More information would be required to determine F_B and F_C. x-components of friction at B and C are possible.

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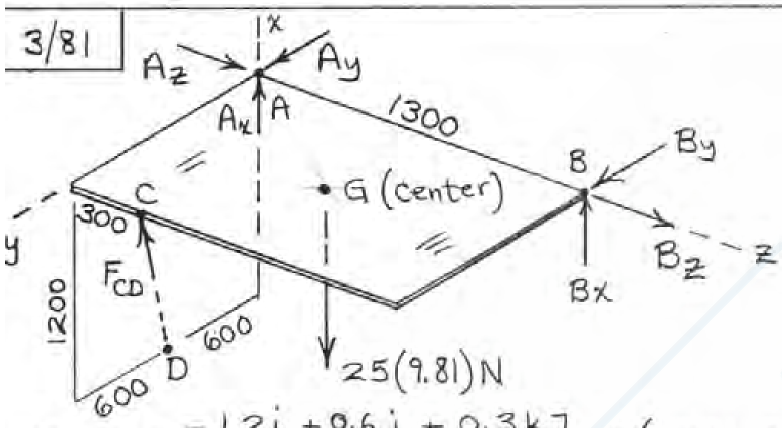

$$\underline{F}_{CD} = F\left[\frac{1.2\underline{i} + 0.6\underline{j} + 0.3\underline{k}}{\sqrt{1.2^2 + 0.6^2 + 0.3^2}}\right] = F(0.873\underline{i} + 0.436\underline{j} + 0.218\underline{k})$$

$\Sigma M_z = 0: \; 25(9.81)(0.6) - 0.873F(1.2) = 0$

$\underline{F = 140.5 \text{ N}}$

$\Sigma M_y = 0: \; B_x(1.3) - 25(9.81)(0.65) + 0.873F(0.3) = 0$

$B_x = 94.3 \text{ N}$

$\Sigma F_x = 0: \; A_x + 94.3 + 0.873F - 25(9.81) = 0$

$A_x = 28.3 \text{ N}$

$\Sigma M_x = 0: \; -B_y(1.3) + 0.218F(0.6) = 0$

$B_y = 14.15 \text{ N}$

$\Sigma F_y = 0: \; A_y + 14.15 + 0.436F = 0, \quad A_y = -75.5 \text{ N}$

$A_n = \sqrt{75.5^2 + 28.3^2} = \underline{80.6 \text{ N}}, \quad B_n = \sqrt{94.3^2 + 14.15^2} = \underline{95.4 \text{ N}}$

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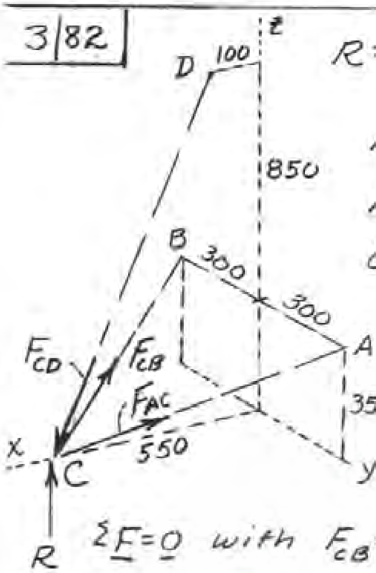

$R = \frac{1}{3} mg$; From Table C2 for Mars $g = 3.73\ m/s^2$

$R = \frac{1}{3} 600(3.73) = 746\ N$

$\overline{AC} = \sqrt{(550)^2 + (300)^2 + (350)^2} = 718\ mm$

$\overline{CD} = \sqrt{(450)^2 + (1200)^2} = 1282\ mm$

$\underline{F}_{AC} = \frac{F_{AC}}{718}(-550\underline{i} + 300\underline{j} + 350\underline{k})$

$\underline{F}_{CB} = \frac{F_{CB}}{718}(-550\underline{i} - 300\underline{j} + 350\underline{k})$

$\underline{F}_{CD} = \frac{F_{CD}}{1282}(450\underline{i} - 1200\underline{k})$

$\Sigma \underline{F} = \underline{0}$ with $F_{CB} = F_{AC}$ gives upon collecting terms

$$\left(-\frac{2F_{AC}}{718}550 + \frac{F_{CD}}{1282}450\right)\underline{i} + \left(746 + \frac{2F_{AC}}{718}350 - \frac{F_{CD}}{1282}1200\right)\underline{k} = \underline{0}$$

Equate coefficients to zero & solve simultaneously to get $F_{CD} = \underline{1046\ N}$ compression

$F_{AC} = F_{CB} = \underline{240\ N\ \text{tension}}$

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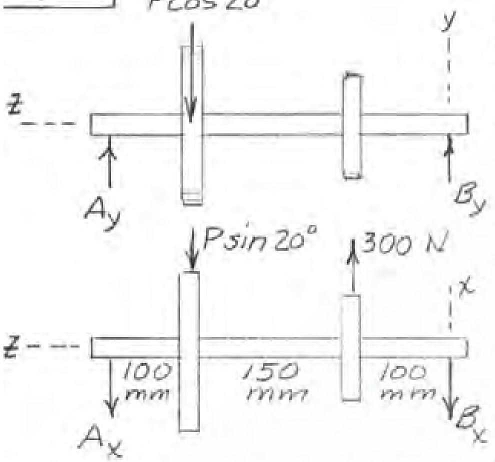
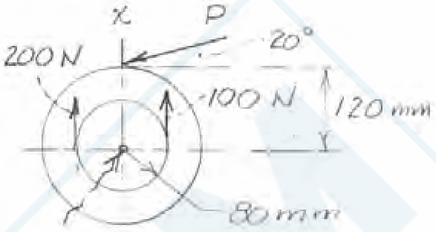


$\Sigma M_z = 0;$

$(200 - 100)0.080 - 0.120\, P \cos 20° = 0$

$\underline{P = 70.9 \text{ N}}$



y-z; $\Sigma M_A = 0;\ 0.35 B_y - 70.9 \cos 20°(0.1) = 0,\quad B_y = 19.05 \text{ N}$

$\Sigma F_y = 0;\ A_y + 19.05 - 70.9 \cos 20° = 0,\quad A_y = 47.6 \text{ N}$

x-z; $\Sigma M_A = 0;\ 300(0.25) - 70.9 \sin 20°(0.1) - 0.35 B_x = 0$

$B_x = 207 \text{ N}$

$\Sigma F_x = 0;\ A_x + 70.9 \sin 20° + 207 - 300 = 0;\quad A_x = 68.4 \text{ N}$

$A = \sqrt{68.4^2 + 47.6^2} = \underline{83.3 \text{ N}}$, $B = \sqrt{207^2 + 19.05^2} = \underline{208 \text{ N}}$

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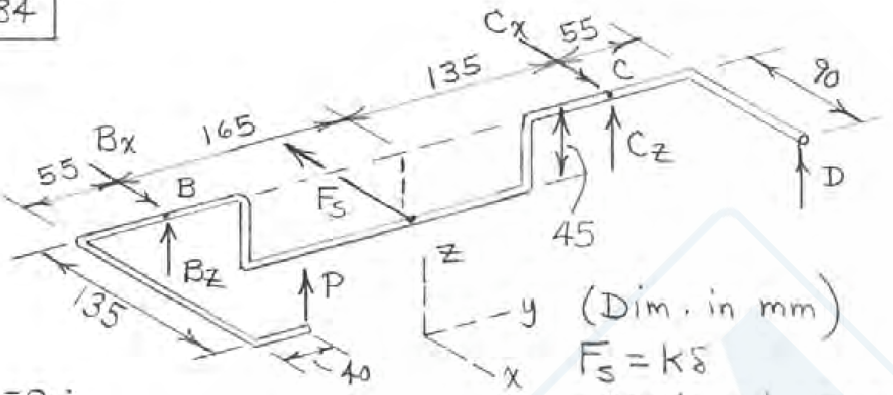

$F_s = k\delta$

$= 900(0.060) = 54\text{ N}$

D=0 :

$\Sigma M_{BC} = 0: \; -P(135) + 54(45) = 0, \quad \underline{P_{min} = 18\text{ N}}$

$\Sigma M_{B_x} = 0: \; -18(15) + C_z(300) = 0, \quad C_z = 0.9\text{ N}$

$\Sigma M_{B_z} = 0: \; 54(165) - C_x(300) = 0, \quad C_x = 29.7\text{ N}$

$\Sigma F_x = 0: \; 29.7 + B_x - 54 = 0, \quad B_x = 24.3\text{ N}$

$\Sigma F_z = 0: \; 0.9 + B_z + 18 = 0, \quad B_z = -18.9\text{ N}$

$B = \sqrt{B_x^2 + B_z^2} = \underline{30.8\text{ N}}, \quad C = \sqrt{C_x^2 + C_z^2} = \underline{29.7\text{ N}}$

If $P = P_{min}/2 = 18/2 = 9\text{ N}, \; (D \neq 0)$:

$\Sigma M_{BC} = 0: \; -9(135) + 54(45) - D(90) = 0, \quad \underline{D = 13.5\text{ N}}$

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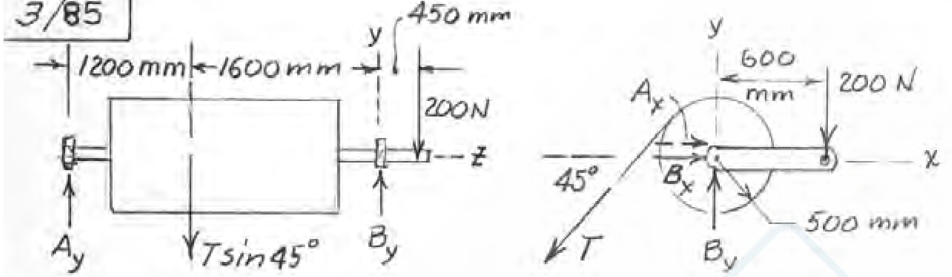

$\Sigma M_z = 0;\ 0.500T - 0.600(200) = 0,\quad T = 240\ N$

$\Sigma M_x = 0;\ 2.800A_y + 0.450(200) - 240(0.707)(1.600) = 0$

$A_y = 64.8\ N$

$\Sigma F_y = 0;\ 64.8 - 240(0.707) + B_y - 200 = 0,\ B_y = 305\ N$

$\Sigma M_y = 0;\ 240(0.707)(1.600) - 2.800A_x = 0,\ A_x = 97.0\ N$

$\Sigma F_x = 0;\ 97.0 + B_x - 240(0.707) = 0,\quad B_x = 72.7\ N$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(97.0)^2 + (64.8)^2} = \underline{116.7\ N}$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(72.7)^2 + (305)^2} = \underline{313\ N}$

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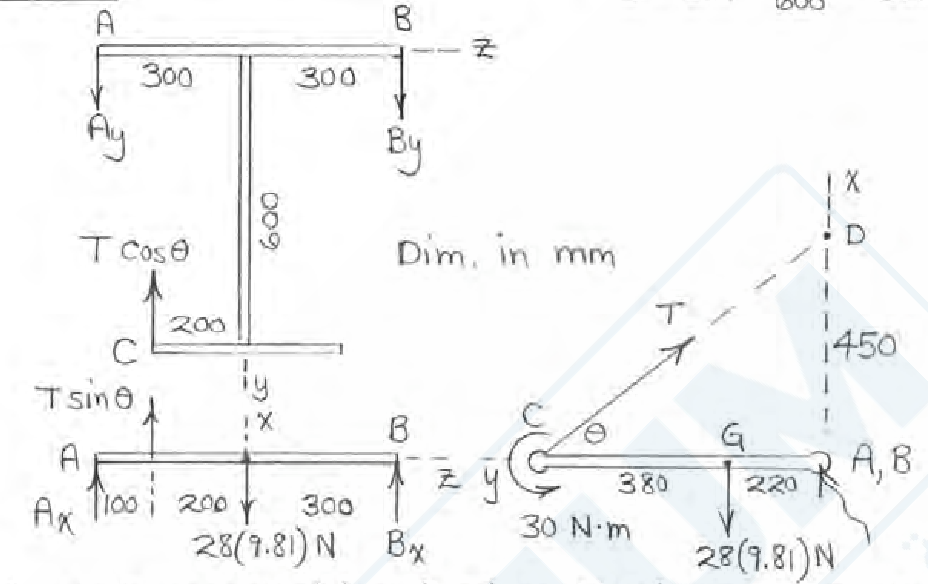
$\theta = \tan^{-1}\frac{450}{600} = 36.9°$



$(x\text{-}y)\ \Sigma M_z = 0:\ 28(9.81)(0.220) - T\sin 36.9°(0.600) + 30 = 0,\quad T = 251\text{ N}$

$(x\text{-}z)\ \Sigma M_B = 0:\ 28(9.81)(0.300) - 251\sin 36.9°(0.500) - 0.600A_x = 0,\ A_x = 11.74\text{ N}$

$\Sigma F_x = 0:\ 11.74 + 251\sin 36.9° - 28(9.81) + B_x = 0,\quad B_x = 112.2\text{ N}$

$(y\text{-}z)\ \Sigma M_B = 0:\ 251\cos 36.9°(0.500) - 0.6A_y = 0,\quad A_y = 167.5\text{ N}$

$\Sigma F_y = 0:\ 167.5 + B_y - 251\cos 36.9° = 0,\quad B_y = 33.5\text{ N}$

$A = \sqrt{11.74^2 + 167.5^2} = \underline{167.9\text{ N}}$

$B = \sqrt{112.2^2 + 33.5^2} = \underline{117.1\text{ N}}$

Couple may be applied at any place on rigid body with the same external effect.

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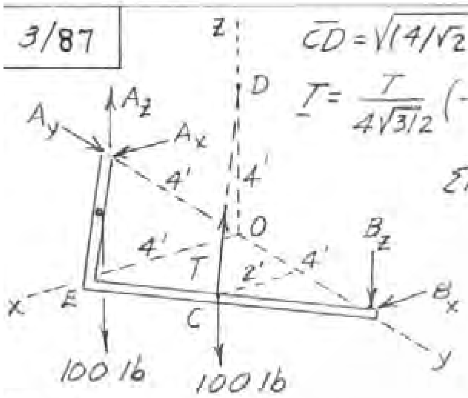

$\overline{CD} = \sqrt{(4/\sqrt{2})^2 + 4^2} = 4\sqrt{3/2}$ ft

$\underline{T} = \frac{T}{4\sqrt{3/2}}(-2\underline{i} - 2\underline{j} + 4\underline{k}) = \frac{T}{\sqrt{6}}(-\underline{i} - \underline{j} + 2\underline{k})$

$\Sigma M_{AB} = 0;\ 200(2) - T_z(2) = 0$

$T_z = \frac{2T}{\sqrt{6}} = 200,\ T = 100\sqrt{6}$ lb

$\underline{T = 245 \text{ lb}}$

$(\Sigma M_B)_x = 0;\ 100(2) + 100(6) - 200(2) - 8A_z = 0,\ A_z = 50$ lb

$\Sigma F_y = 0;\ A_y - \frac{100\sqrt{6}}{\sqrt{6}} = 0,\ A_y = 100$ lb

$(\Sigma M_B)_z = 0;\ 8A_x - \frac{100\sqrt{6}}{\sqrt{6}}(2) - \frac{100\sqrt{6}}{\sqrt{6}}(2) = 0,\ A_x = 50$ lb

$A = \sqrt{50^2 + 100^2 + 50^2} = \underline{122.5 \text{ lb}}$

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3/88 (Dim. in mm)

$pA = 4(10^{-5})(48)(80)$
$= 0.1536$ N

$\Sigma M_{AB} = 0: -P(22 - 16 \sin 15°) + 0.1536(24) = 0$

$P = 0.206$ N

$\Sigma M_{B_x} = 0: \quad A_y(42) - 0.206 \cos 15°(58) = 0$

$A_y = 0.275$ N

$\Sigma M_{A_x} = 0: \quad -B_y(42) - 0.206 \cos 15°(16) = 0$

$B_y = -0.0760$ N

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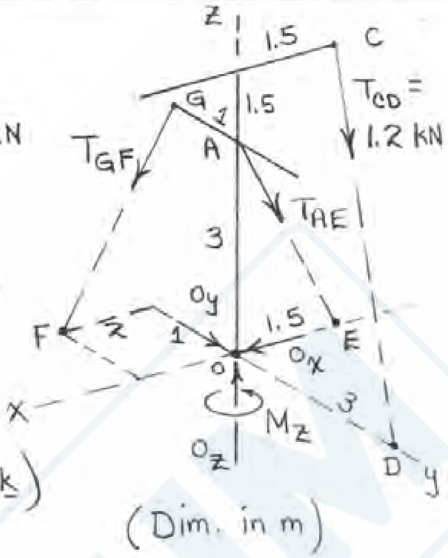
3/89 From Prob. 2/105,



$\mathbf{T}_{CD} = 0.321\mathbf{i} + 0.641\mathbf{j} - 0.962\mathbf{k}$ kN

$$\mathbf{T}_{AE} = T_{AE}\,\frac{-1.5\mathbf{i} - 3\mathbf{k}}{\sqrt{1.5^2 + 3^2}} = T_{AE}(-0.447\mathbf{i} - 0.894\mathbf{k})$$

$$\mathbf{T}_{GF} = T_{GF}\,\frac{2\mathbf{i} - 3\mathbf{k}}{\sqrt{2^2 + 3^2}} = T_{GF}(0.555\mathbf{i} - 0.832\mathbf{k})$$

$\Sigma \mathbf{M}_O = \mathbf{0}$: $\mathbf{OC} \times \mathbf{T}_{CD} + \mathbf{OA} \times \mathbf{T}_{AE} + \mathbf{OG} \times \mathbf{T}_{GF} + M_z\mathbf{k} = \mathbf{0}$

$(-1.5\mathbf{i} + 4.5\mathbf{k}) \times (0.321\mathbf{i} + 0.641\mathbf{j} - 0.962\mathbf{k}) + 3\mathbf{k} \times$
$T_{AE}(-0.447\mathbf{i} - 0.894\mathbf{k}) + (-\mathbf{j} + 3\mathbf{k}) \times T_{GF}(0.555\mathbf{i} - 0.832\mathbf{k})$
$+ M_z\mathbf{k} = \mathbf{0} \Rightarrow \begin{cases} 0.832T_{GF} - 2.89 = 0 \\ 1.664\,T_{GF} - 1.342T_{AE} = 0 \\ 0.555\,T_{GF} - 0.962 + M_z = 0 \end{cases}$

Solve to obtain $T_{GF} = 3.47$ kN, $T_{AE} = 4.30$ kN,
$M_z = -0.962$ kN·m

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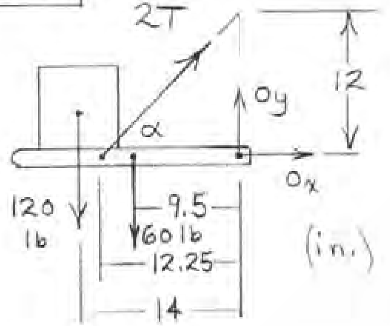
3/9



$\alpha = \tan^{-1}\frac{12}{12.25} = 44.4°$

$\circlearrowleft + \Sigma M_O = 0: \quad 120(14) + 60(9.5) - 2T\sin 44.4°(12.25) = 0$

$T = 131.2 \text{ lb}$

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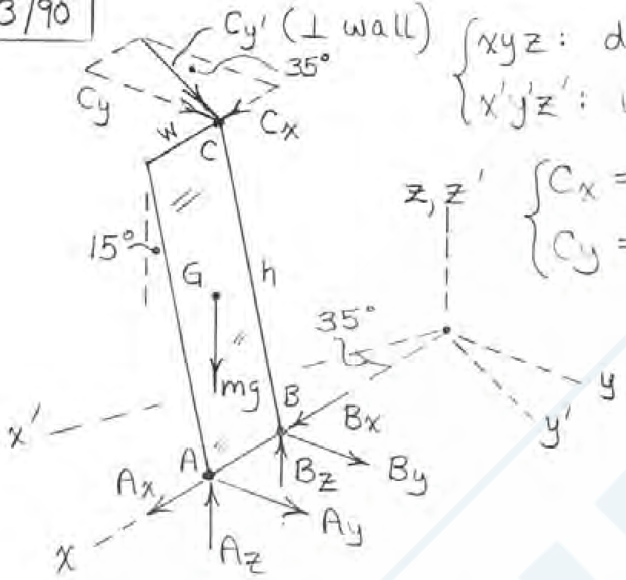
3/90



$\begin{cases} xyz: \text{ door-based} \\ x'y'z': \text{ wall-based} \end{cases}$

$\begin{cases} C_x = C_{y'} \sin 35° \\ C_y = C_{y'} \cos 35° \end{cases}$

$\Sigma M_x = 0: \quad mg\frac{h}{2}\sin 15° - C_y h \cos 15° = 0, \quad C_y = 0.1340\,mg$

$C_{y'} = \dfrac{C_y}{\cos 35°} = \underline{0.1636\,mg}, \quad C_x = C_y \tan 35° = 0.0938\,mg$

$\Sigma M_{B_z} = 0: \quad C_x h \sin 15° + A_y w = 0, \quad \underline{A_y = -0.0243\,mg\frac{h}{w}}$

$\Sigma M_{B_y} = 0: \quad mg\frac{w}{2} + C_x h \cos 15° - A_z w = 0$

$\underline{A_z = mg\left(\frac{1}{2} + 0.0906\frac{h}{w}\right)}$

$\Sigma M_{A_z} = 0: \quad -B_y w - C_y w + C_x h \sin 15° = 0$

$\underline{B_y = mg\left(0.0243\frac{h}{w} - 0.1340\right)}$

$\Sigma M_{A_y} = 0: \quad B_z w - mg\frac{h}{2} + C_x h \cos 15° = 0$

$\underline{B_z = mg\left(\frac{1}{2} - 0.0906\frac{h}{w}\right)}$

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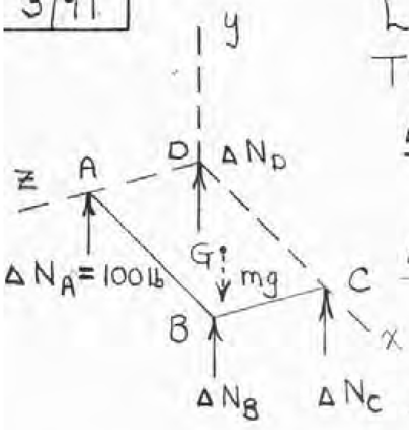
3/91



Location of G does not change. Thus,

$\Delta N_D = -100$ lb (preserves total rear-axle loading)

$\Delta N_B = -100$ lb (preserves total right-side loading)

$\Delta N_C = 100$ lb (preserves total normal force; preserves total front-axle loading)

(Note: The results for ΔN_B & ΔN_C hold only if the track (distance between tire centers) at the front is equal to that at the rear.)

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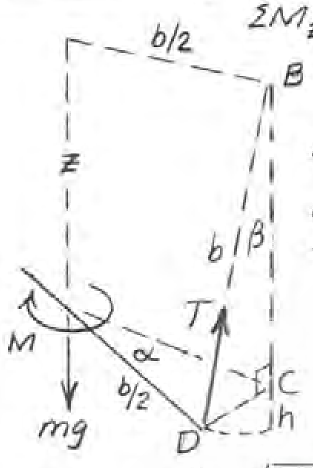
3/92

$\Sigma F_z = 0$; $2T\cos\beta - mg = 0$ - - - - - - - - - - (1)

$\Sigma M_z = 0$; $2T\sin\beta\cos\frac{\alpha}{2}\left(\frac{b}{2}\right) - M = 0$ - - - (2)



$\overline{CD} = 2\frac{b}{2}\sin\frac{\alpha}{2} = b\sin\beta$, $\beta = \frac{\alpha}{2}$ -- (3)

Divide (2) by (1) & substitute (3) & get

$$\frac{2T\frac{b}{2}\sin\beta\cos\beta}{2T\cos\beta} = \frac{M}{mg}, \quad \sin\beta = \frac{2M}{bmg}$$

Thus $\cos\beta = \sqrt{1 - \left(\frac{2M}{bmg}\right)^2}$

& $h = b(1-\cos\beta)$

so $h = b\left(1 - \sqrt{1 - \left(\frac{2M}{bmg}\right)^2}\right)$

For $h \to b$, $\cos\beta \to 0$, $\sin\beta \to \pi/2$ & $M \to \frac{bmg}{2}$

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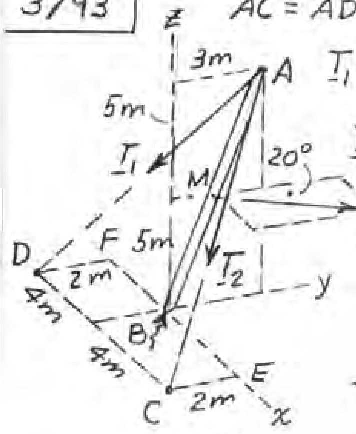

$\overline{AC} = \overline{AD} = \sqrt{4^2+5^2+10^2} = 11.87\text{ m}$; $|\underline{P}| = 20$ kN

$\underline{T}_1 = \dfrac{T_1}{11.87}(-4\underline{i}-5\underline{j}-10\underline{k})$; $\sin 20° = 0.3420$

$\underline{T}_2 = \dfrac{T_2}{11.87}(4\underline{i}-5\underline{j}-10\underline{k})$ $\cos 20° = 0.9397$

$\Sigma M_x = 0$; $\underline{r}_{EC}\times\underline{T}_2\cdot\underline{i} + \underline{r}_{FD}\times\underline{T}_1\cdot\underline{i} + \underline{r}_{BM}\times\underline{P}\cdot\underline{i} = 0$

$-2\underline{j}\times\dfrac{T_2}{11.87}(4\underline{i}-5\underline{j}-10\underline{k})\cdot\underline{i}$

$-2\underline{j}\times\dfrac{T_1}{11.87}(-4\underline{i}-5\underline{j}-10\underline{k})\cdot\underline{i} + (1.5\underline{j}+5\underline{k})\times 20(0.342\underline{i} + 0.9397\underline{j})\cdot\underline{i} = 0$

Simplify & get $T_1 + T_2 = 55.79$ ---- (1)

$\Sigma M_z = 0$; $\underline{r}_{BA}\times(\underline{T}_1+\underline{T}_2)\cdot\underline{k} + \underline{r}_{BM}\times\underline{P}\cdot\underline{k} = 0$

$(3\underline{j}+10\underline{k})\times\dfrac{1}{11.87}([-4T_1+4T_2]\underline{i} + [-5T_1-5T_2]\underline{j} - [10T_1+10T_2]\underline{k})\cdot\underline{k} - 20(0.3420)(1.5) = 0$

Simplify to $T_1 - T_2 = 10.14$ ---- (2)

Solve (1) & (2) & get $\underline{T_1 = 33.0\text{ kN}}$, $T_2 = \underline{22.8\text{ kN}}$

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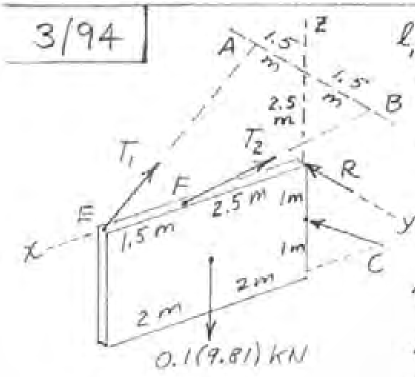

$\ell_1 = \overline{AE} = \sqrt{4^2 + 1.5^2 + 2.5^2} = 4.95$ m

$\ell_2 = \overline{BF} = \sqrt{2.5^2 + 1.5^2 + 2.5^2} = 3.84$ m

$\underline{T}_1 = \frac{T_1}{\ell_1}(-4\underline{i} - 1.5\underline{j} + 2.5\underline{k})$

$\underline{T}_2 = \frac{T_2}{\ell_2}(-2.5\underline{i} + 1.5\underline{j} + 2.5\underline{k})$

Note: $\underline{T}_1$, $\underline{T}_2$, $\underline{R}$, and weight all pass through x-axis, so $\underline{C}$ must also. Thus $C_y = 0$

$\Sigma M_{AB} = 0;\ C_x(3.5) - 9.81(2) = 0,\quad C_x = 0.561$ kN

$\Sigma M_z = 0;\ 4\underline{i} \times \frac{T_1}{\ell_1}(-1.5\underline{j}) + 2.5\underline{i} \times \frac{T_2}{\ell_2}(1.5\underline{j}) = 0,\quad 8T_1/\ell_1 = 5T_2/\ell_2$

$\Sigma F_x = 0;\ -\frac{T_1}{\ell_1}(4) - \frac{T_2}{\ell_2}(2.5) + 0.561 = 0,\quad 8T_1/\ell_1 + 5T_2/\ell_2 = 1.121$ kN

solve & get $T_1 = 1.121\,\ell_1/16 = \underline{0.347 \text{ kN}}$, $T_2 = \frac{1.121}{10}\ell_2 = \underline{0.431 \text{ kN}}$

$\Sigma F_y = 0;\ \frac{1.121}{16}\ell_1 \frac{1.5}{\ell_1} - \frac{1.121}{10}\ell_2 \frac{1.5}{\ell_2} + R = 0,\quad \underline{R = 0.0631 \text{ kN}}$

$\Sigma F_z = 0;\ \frac{1.121}{16}\ell_1 \frac{2.5}{\ell_1} + \frac{1.121}{10}\ell_2 \frac{2.5}{\ell_2} + C_z - 0.981 = 0,\quad C_z = 0.526$ kN

Thus $C = \sqrt{(0.561)^2 + (0.526)^2} = \underline{0.768 \text{ kN}}$

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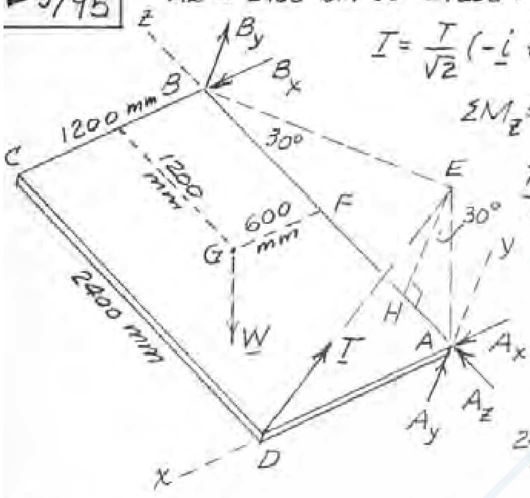
▶ 3/95

$\overline{AE} = 2400 \sin 30° = 1200$ mm, $\overline{AH} = 600$ mm, $\overline{HE} = 1200 \cos 30° = 600\sqrt{3}$ mm



$$\underline{T} = \frac{T}{\sqrt{2}}\left(-\underline{i} + \frac{\sqrt{3}}{2}\underline{j} + \frac{1}{2}\underline{k}\right)$$

$\Sigma M_z = 0$; $T_y \overline{AD} - W_y \overline{GF} = 0$

$$\frac{T\sqrt{3}}{2\sqrt{2}}(1200) = 40(9.81)\frac{\sqrt{3}}{2}(600)$$

$$T = 20(9.81)\sqrt{2} = \underline{277 \text{ N}}$$

$\Sigma M_x = 0$; $-B_y \overline{AB} + W_y \overline{FA} = 0$

$$2400 B_y = 40(9.81)\frac{\sqrt{3}}{2}(1200)$$

$$B_y = 98.1\sqrt{3} = 169.9 \text{ N}$$

$\Sigma M_y = 0$; $B_x \overline{AB} + W_z \overline{GF} - T_z \overline{AD} = 0$

$$2400 B_x + 40(9.81)0.5(600) - \frac{277}{2\sqrt{2}}(1200) = 0$$

$$B_x = 0$$

$B = B_y = \underline{169.9 \text{ N}}$ ($B_x = 0$ can be obtained by inspection by noting $\Sigma M_{AE} = 0$ eliminates all terms except $B_x \overline{BE}$ so $B_x = 0$)

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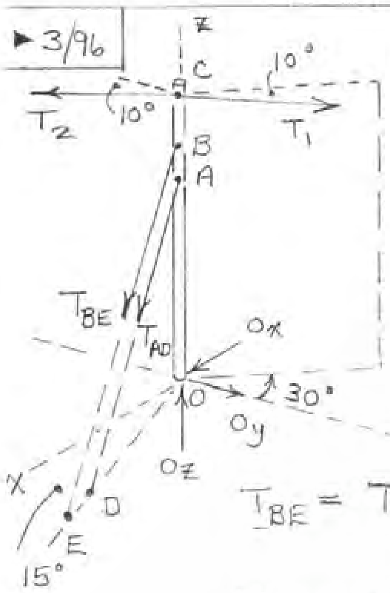
▸3/96



$\overline{OA} = 9\text{ m}$, $\overline{OB} = 11\text{ m}$, $\overline{OC} = 13\text{ m}$

$\overline{OD} = 8\text{ m}$, $\overline{OE} = 10\text{ m}$

$$\underline{T}_1 = 950[-\sin 30° \cos 10° \underline{i} + \cos 30° \cos 10° \underline{j} - \sin 10° \underline{k}]$$

$$\underline{T}_2 = 950[-\cos 10° \underline{j} - \sin 10° \underline{k}]$$

(in N)

$$\underline{T}_{BE} = T\left[\frac{10 \cos 15° \underline{i} + 10 \sin 15° \underline{j} - 11 \underline{k}}{\sqrt{221}}\right]$$

$$\underline{T}_{AD} = T\left[\frac{8 \cos 15° \underline{i} + 8 \sin 15° \underline{j} - 9 \underline{k}}{\sqrt{145}}\right]$$

$\Sigma \underline{M}_o = \underline{0}$: $\underline{r}_{oc} \times (\underline{T}_1 + \underline{T}_2) + \underline{r}_{OB} \times \underline{T}_{BE} + \underline{r}_{OA} \times \underline{T}_{AD} = \underline{0}$

Set $\underline{r}_{oc} = 13\underline{k}$ m, $\underline{r}_{OB} = 11\underline{k}$ m, & $\underline{r}_{OA} = 9\underline{k}$ m, carry out cross products, and set either the $\underline{i}$-component or the $\underline{j}$-component to zero to obtain $\underline{T = 471 \text{ N}}$.

$\Sigma F_x = 0$: $-950 \sin 30° \cos 10° + 471 \dfrac{10 \cos 15°}{\sqrt{221}} + 471 \dfrac{8 \cos 15°}{\sqrt{145}} + O_x = 0$

$O_x = -140.0$ N

$\Sigma F_y = 0$: $950 \cos 30° \cos 10° - 950 \cos 10° + 471 \dfrac{10 \sin 15°}{\sqrt{221}} + 471 \dfrac{8 \sin 15°}{\sqrt{145}} + O_y = 0$

$O_y = -37.5$ N

Then $O = \sqrt{O_x^2 + O_y^2} = \underline{144.9 \text{ N}}$

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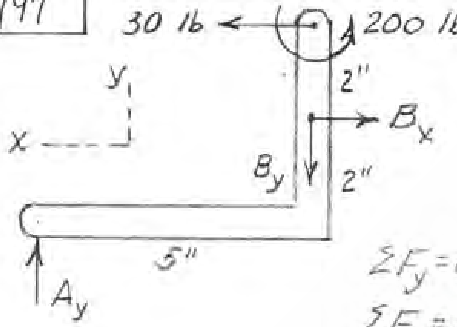

$\Sigma M_B = 0;$

$200 + 30(2) - 5A_y = 0$

$A_y = 52$ lb

$\Sigma F_y = 0; \quad B_y = 52$ lb

$\Sigma F_x = 0; \quad B_x = 30$ lb

$B = \sqrt{(30)^2 + (52)^2} = \underline{60.0 \text{ lb}}$

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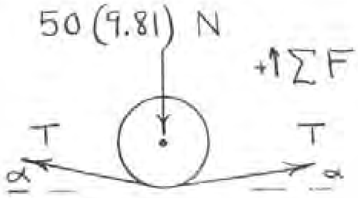
3/98 Isolate wheel of unicycle:



$$\alpha = \tan^{-1}\left(\frac{0.075}{9}\right) = 0.477°$$

$$+\uparrow \Sigma F = 0: \quad 2T\sin\alpha - 50(9.81) = 0$$

$$T = 29\,400 \text{ N}$$

or $\underline{T = 29.4 \text{ kN}}$

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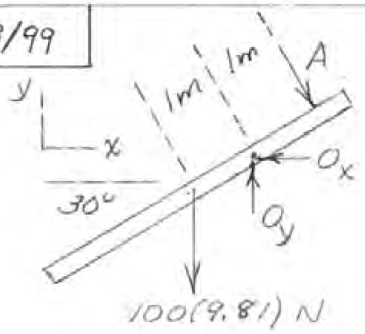

$\Sigma M_O = 0;\ A(1) - (981\cos 30°)1 = 0$

$A = 850\ \text{N}$

$\Sigma F_x = 0;\ O_x - 850\sin 30° = 0$

$O_x = 425\ \text{N}$

$\Sigma F_y = 0;\ O_y - 850\cos 30° - 981 = 0$

$O_y = 1717\ \text{N}$

$O = \sqrt{425^2 + 1717^2} = \underline{1769\ \text{N}}$

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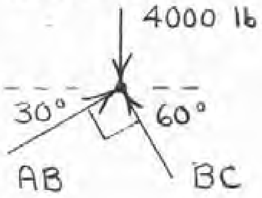
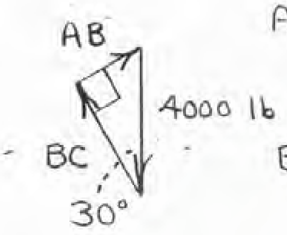
Joint B:





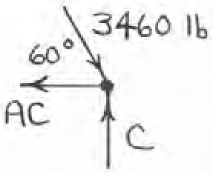
$AB = 4000\left(\frac{1}{2}\right)$

$= \underline{2000 \text{ lb } C}$

$BC = 4000 \sqrt{3}/2$

$= \underline{3460 \text{ lb } C}$

Joint C:



$\overset{+}{\rightarrow}\Sigma F_x = 0: \; -AC + 3460 \cos 60° = 0$

$\underline{AC = 1732 \text{ lb } T}$

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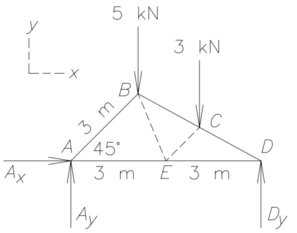


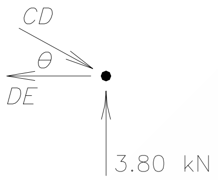
As a whole:

Note: $\overline{CE} = 1.5$ m by similar triangles

$$\Sigma M_A = 0:\ 5(3\cos 45°) + 3(3 + 1.5\cos 45°) - 6D_y = 0$$

$$D_y = 3.80 \text{ kN}$$

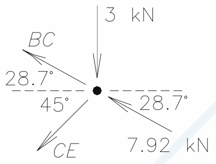
Joint D:



$$\theta = \tan^{-1}\frac{3\sin 45°}{6 - 3\cos 45°} = 28.7°$$

$$\begin{cases} \Sigma F_y = 0:\ 3.80 - CD\sin 28.7° = 0,\ CD = 7.92 \text{ kN } C \\ \Sigma F_x = 0:\ 7.92\cos 28.7° - DE = 0,\ DE = 6.94 \text{ kN } T \end{cases}$$

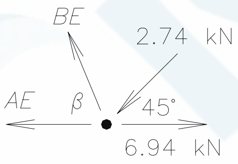
Joint C:



$$\begin{cases} \Sigma F_x = 0:\ -BC\cos 28.7° - CE\cos 45° - 7.92\cos 28.7° = 0 \\ \Sigma F_y = 0:\ BC\sin 28.7° - CE\sin 45° + 7.92\sin 28.7° - 3 = 0 \end{cases}$$

Solve simultaneously to obtain:
$$BC = -5.70 \text{ kN } (C)$$
$$\underline{CE = -2.74 \text{ kN } (C)}$$

Joint E:



$$\beta = \frac{180° - 45°}{2} = 67.5°$$

$$\Sigma F_y = 0:\ BE\sin 67.5° - 2.74\sin 45° = 0$$

$$\underline{BE = 2.10 \text{ kN } T}$$

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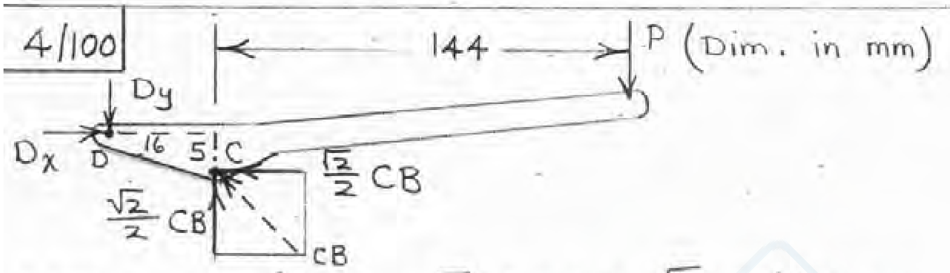
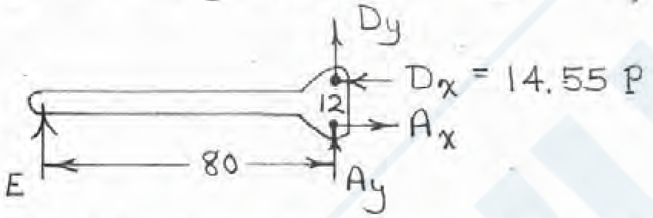


$\circlearrowleft^{+}$ $\Sigma M_D = 0$: $P(160) + \frac{\sqrt{2}}{2} CB(5) - \frac{\sqrt{2}}{2} CB(16) = 0$

$CB = 20.6P$

$\xrightarrow{+}$ $\Sigma F = 0$: $D_x - 20.6P \frac{\sqrt{2}}{2} = 0$, $D_x = 14.55P$



$\circlearrowleft^{+}$ $\Sigma M_A = 0$: $E(80) - 14.55P(12) = 0$

$\underline{E = 2.18P}$

(Note: Mechanical advantage will increase as CB becomes more aligned with CD.)

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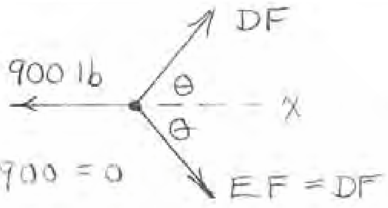
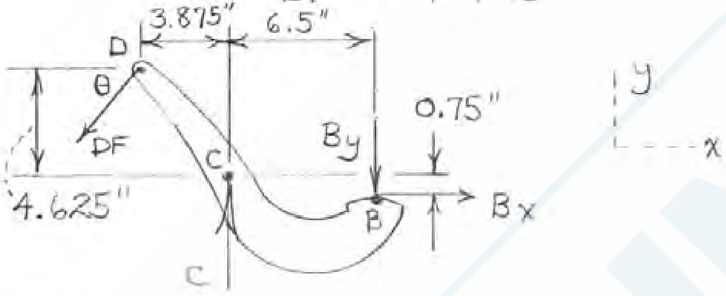
4/101 FBD of joint F:



$\theta = \tan^{-1}\left(\frac{4.625}{3.875}\right) = 50.0°$

$\Sigma F_x = 0: \quad 2DF\cos 50.0° - 900 = 0$

$DF = 701 \text{ lb}$



$\Sigma F_x = 0: \quad -701\cos 50.0° + B_x = 0, \quad B_x = 450 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0: \quad -C(6.5) + 701\cos 50.0°(4.625 + 0.75)$
$+ 701\sin 50.0°(10.375) = 0, \quad \underline{C = 1229 \text{ lb}}$

$\Sigma F_y = 0: \quad -701\sin 50.0° + 1229 - B_y = 0$

$\underline{B_y = 692 \text{ lb} = B_n = A_n}$

(No horizontal force component at C because of symmetry.)

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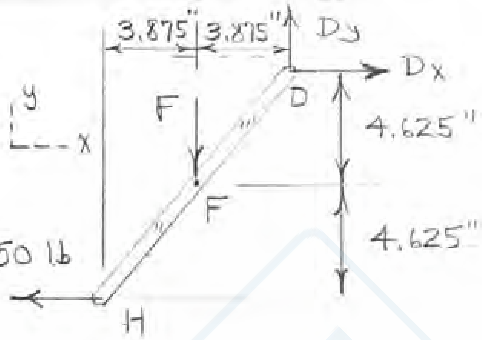
4/102 Member DFH:



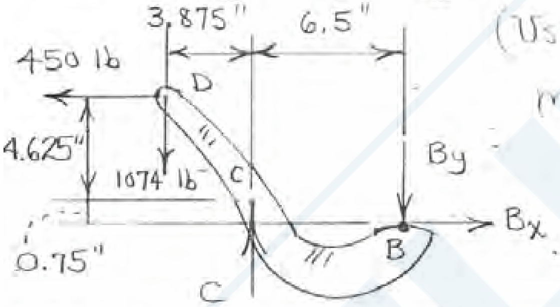
From $\Sigma F_x = 0$,

$D_x = 450$ lb

$\curvearrowleft + \Sigma M_D = 0$:

$F(3.875) - 450(9.25) = 0$, $F = 1074$ lb

From $\Sigma F_y = 0$, $D_y = 1074$ lb



(Use of symmetry made @ C and F.)

$\curvearrowleft + \Sigma M_B = 0$: $450(5.375) + 1074(10.375) - C(6.5) = 0$

$C = 2090$ lb

$\Sigma F_y = 0$: $-1074 + 2090 - B_y = 0$

$B_y = 1012$ lb

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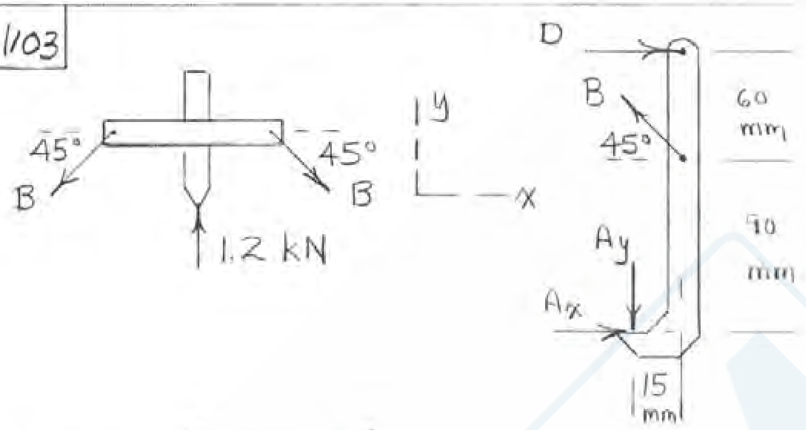

(Upper bar & screw)

$\Sigma F_y = 0: -2B\sin 45° + 1.2 = 0, \quad B = 0.849 \text{ kN}$

(ABD)

$\Sigma M_A = 0: 150D - 0.849\cos 45°(90) - 0.849\sin 45°(15) = 0$

$D = 0.420 \text{ kN}$

$\Sigma F_x = 0: A_x - 0.849\cos 45° + 0.420 = 0, \quad A_x = 0.1800 \text{ kN}$

$\Sigma F_y = 0: -A_y + 0.849\sin 45° = 0, \quad A_y = 0.6 \text{ kN}$

$A = \sqrt{A_x^2 + A_y^2} = \underline{0.626 \text{ kN}}$

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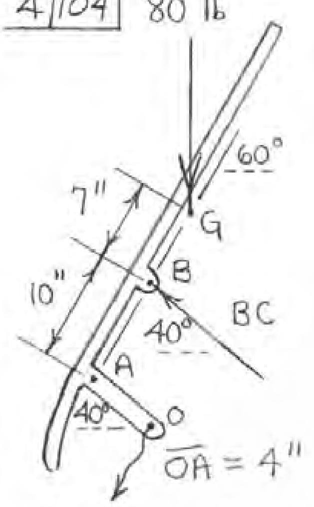
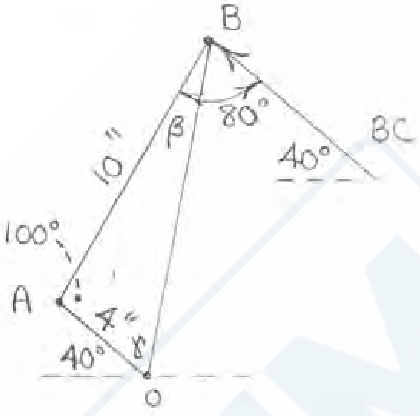
4/104



Blowup:

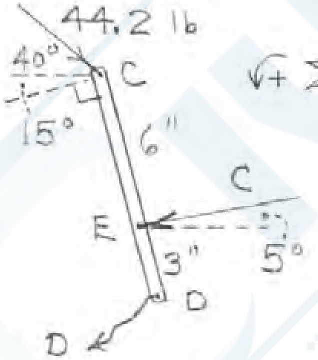


$$\overline{OB}^2 = 4^2 + 10^2 - 2(4)(10)\cos 100°, \quad \overline{OB} = 11.40 \text{ in.}$$

$$\frac{\sin\beta}{4''} = \frac{\sin 100°}{11.40''}, \quad \beta = 20.2°$$

$$\circlearrowleft + \Sigma M_O = 0: \; -80(17\cos 60° - 4\cos 40°)$$

$$+ BC\,(11.40 \sin(80° - 20.2°)) = 0, \quad BC = 44.2 \text{ lb}$$



$$\circlearrowleft + \Sigma M_D = 0: \; -44.2\cos 55°(9)$$

$$+ C\cos 10°(3) = 0$$

$$\underline{C = 77.2 \text{ lb}}$$

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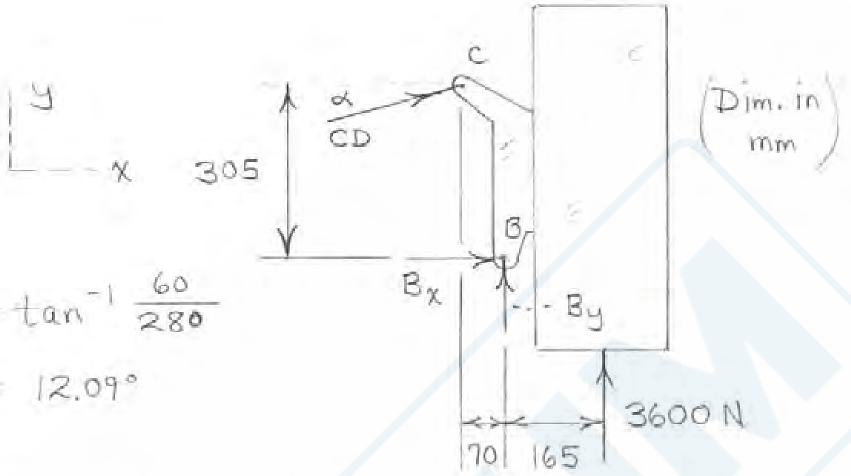
4/105 Wheel unit:



$\alpha = \tan^{-1}\frac{60}{280}$

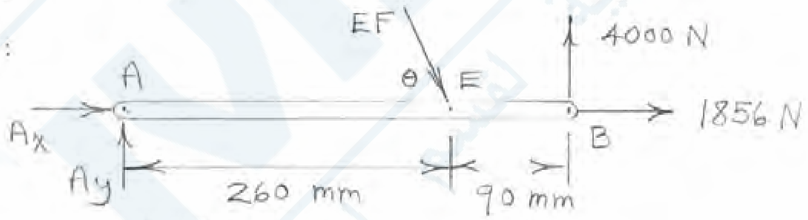
$= 12.09°$

$\circlearrowleft \Sigma M_B = 0: \; 3600(165) - CD\cos\alpha\,(305)$
$- CD\sin\alpha\,(70) = 0, \quad CD = 1898 \text{ N}$

$\Sigma F_x = 0: \; B_x + CD\cos\alpha = 0, \quad B_x = -1856 \text{ N}$

$\Sigma F_y = 0: \; B_y + CD\sin\alpha + 3600 = 0, \quad B_y = -4000 \text{ N}$

AEB:



$\theta = \tan^{-1}\left(\frac{435}{200}\right) = 65.3°$

$\circlearrowleft \Sigma M_A = 0: \; -EF\sin\theta\,(260) + 4000(350) = 0$
$EF = 5920 \text{ N}$

$\Sigma F_x = 0: \; A_x + EF\cos\theta + 1856 = 0$
$A_x = -4330 \text{ N}$

$\Sigma F_y = 0: \; A_y - EF\sin\theta + 4000 = 0$
$A_y = 1384 \text{ N}$

So $A = \sqrt{(-4330)^2 + 1384^2} = \underline{4550 \text{ N}}$

$B = \sqrt{1856^2 + 4000^2} = \underline{4410 \text{ N}}$

$\underline{C = D = 1898 \text{ N}}, \quad \underline{E = F = 5920 \text{ N}}$

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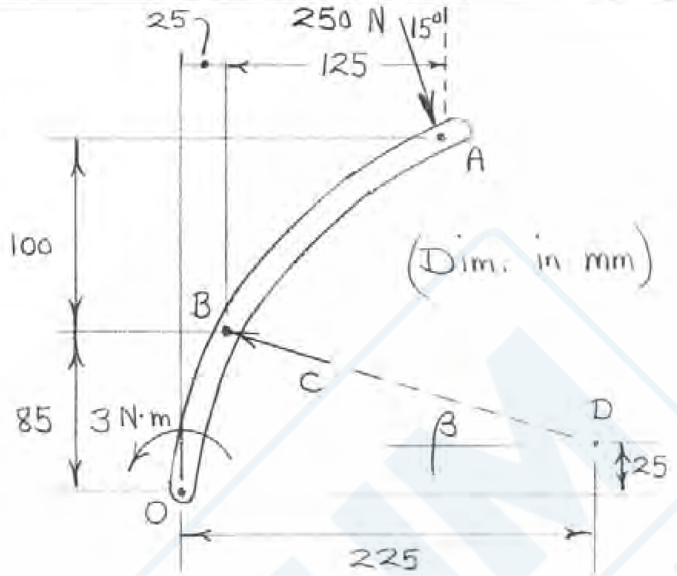
$\beta = \tan^{-1}\frac{60}{200}$

$= 16.70°$



$\circlearrowleft + \Sigma M_O = 0: \ 3000 - 250\cos 15°(150) -$

$250\sin 15°(185) + C\cos\beta(85) + C\sin\beta(25) = 0$

$\underline{C = 510 \text{ N}}$

$C = pA: \quad 510 = p\left(\frac{\pi 45^2}{4}\right)$

$p = 0.321 \frac{\text{N}}{\text{mm}^2}$ or $\underline{321\,000 \text{ Pa}}$

(gauge pressure)

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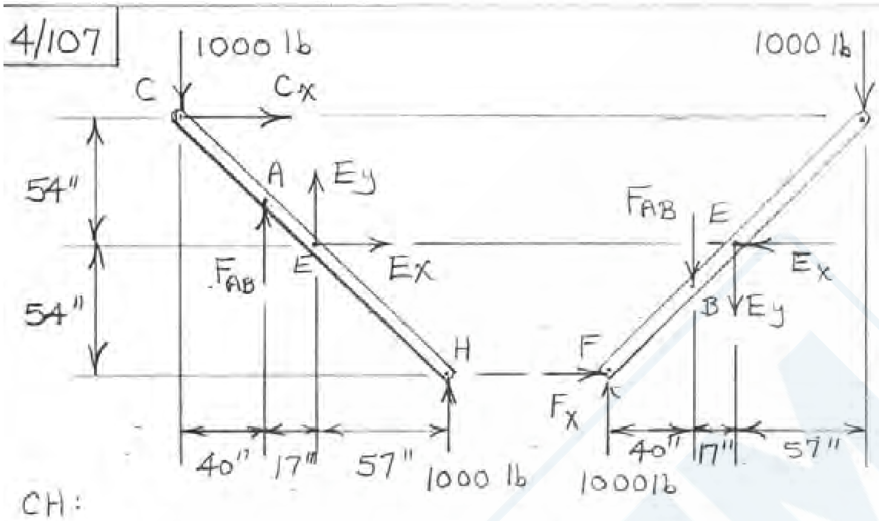
4/107



CH:

$\Sigma F_x = 0: \quad C_x + E_x = 0$ (1)

$\Sigma F_y = 0: \quad F_{AB} + E_y + 1000 - 1000 = 0$ (2)

$\Sigma M_C = 0: \quad F_{AB}(40) + E_y(57) + E_x(54) + 1000(114) = 0$ (3)

DF:

$\Sigma F_x = 0: \quad F_x - E_x = 0$ (4)

$\Sigma F_y = 0: \quad$ (Same as Eq. (2))

$\Sigma M_F = 0: \quad -F_{AB}(40) - E_y(57) + E_x(54) - 1000(114) = 0$ (5)

Solution: $F_{AB} = 6710 \text{ lb}$

$C_x = F_x = E_x = 0, \quad E_y = -6710 \text{ lb}$

(C_x, F_x evident from overall FBDs!)

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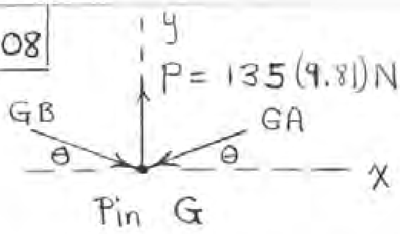
4/108

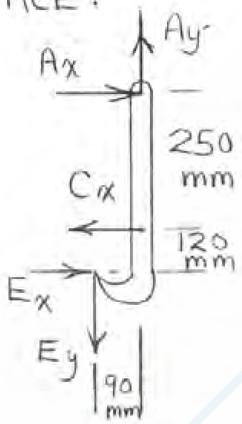


$\theta = \cos^{-1}\dfrac{340}{350} = 13.73°$

$\Sigma F_y = 0: \; 135(9.81) - 2\,GA \sin 13.73° = 0$

$GA = GB = 2790 \text{ N}$

ACE:



$A_x = 2790 \cos 13.73° = 2710 \text{ N}$

$A_y = 2790 \sin 13.73° = 662 \text{ N}$

$\Sigma F_y = 0 \Rightarrow E_y = 662 \text{ N}$

$\Sigma M_C = 0: \; 2710(250) - 662(90)$

$-E_x(120) = 0, \quad E_x = 5150 \text{ N}$

$E = \sqrt{E_x^2 + E_y^2} = 5190 \text{ N}$

or $\underline{5.19 \text{ kN}}$

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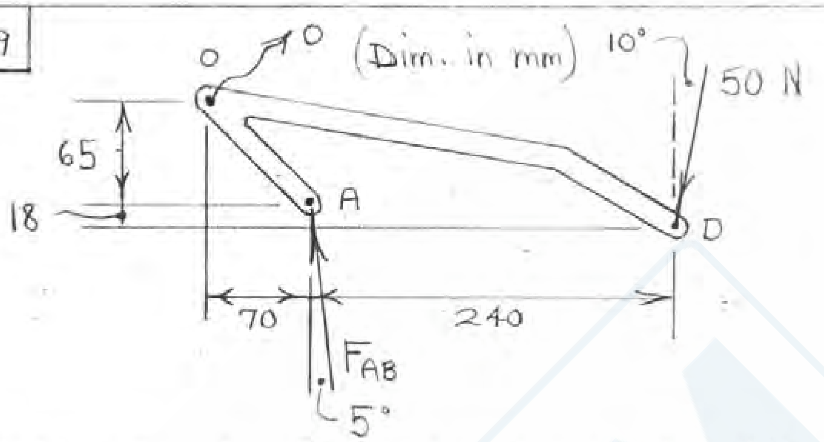
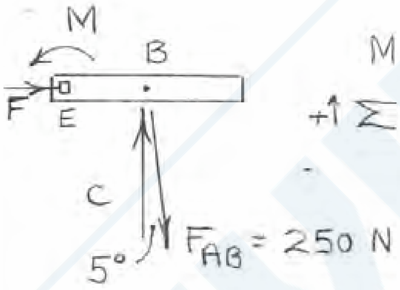


$$\curvearrowleft + \ \Sigma M_O = 0: \quad F_{AB}[\cos 5^\circ(70) - \sin 5^\circ(65)] - 50[\cos 10^\circ(310) + \sin 10^\circ(83)] = 0$$

$$F_{AB} = 250 \text{ N}$$



$M = 0$ (from $\Sigma M_B = 0$)

$$+\uparrow \Sigma F = 0: \quad C - 250 \cos 5^\circ = 0$$

$$\underline{C = 249 \text{ N}}$$

(a factor $\frac{C}{P} = 4.97$)

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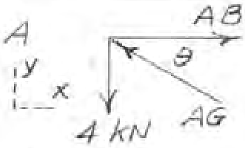
4/11

Joint A



$\theta = \tan^{-1} 1/2 = 26.57°$

$\sin\theta = 1/\sqrt{5}$, $\cos\theta = 2/\sqrt{5}$

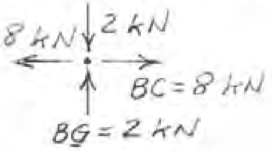
$\Sigma F_y = 0$; $AG/\sqrt{5} - 4 = 0$

$AG = 4\sqrt{5}$ kN C

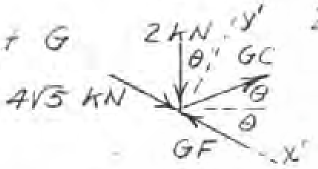
$\Sigma F_x = 0$; $AB - 4\sqrt{5}(2/\sqrt{5}) = 0$

$AB = 8$ kN T

Joint B



Joint G



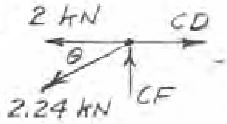
$\Sigma F_{y'} = 0$; $2\left(\frac{2}{\sqrt{5}}\right) - CG \sin 2\theta = 0$

$CG = \frac{4}{\sqrt{5}} \cdot \frac{1}{0.8} = \underline{2.24 \text{ kN T}}$

$\Sigma F_{x'} = 0$; $2.24 \cos 2\theta + 2/\sqrt{5} + 4\sqrt{5} - GF = 0$

$GF = 5\sqrt{5} = 11.18$ kN C

Joint C



$\Sigma F_y = 0$; $CF - 2.24 \sin\theta = 0$

$\underline{CF = 1.00 \text{ kN C}}$

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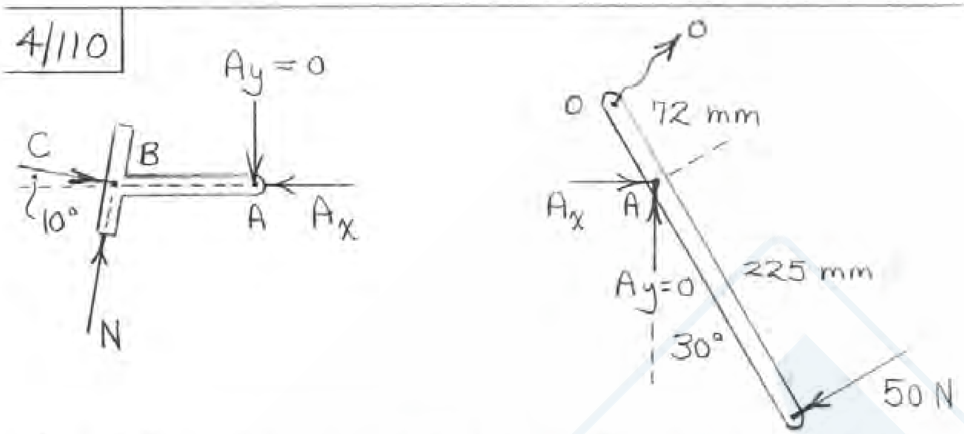

$A_y = 0$ because AB is a three-force body.

Body OA:

$$\circlearrowleft + \Sigma M_O = 0: \quad A_x(72 \cos 30°) - 50(72 + 225) = 0$$

$$A_x = 238 \text{ N}$$

Body AB:

$$\xrightarrow{+} \Sigma F = 0: \quad C - 238 \cos 10° = 0$$

$$C = 235 \text{ N}$$

$$\left(\text{a factor } \frac{C}{P} = 4.69\right)$$

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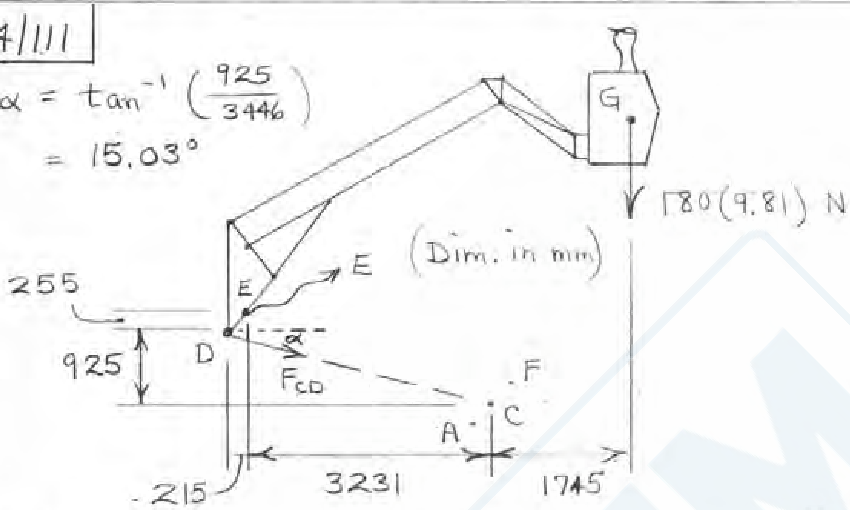
4/111

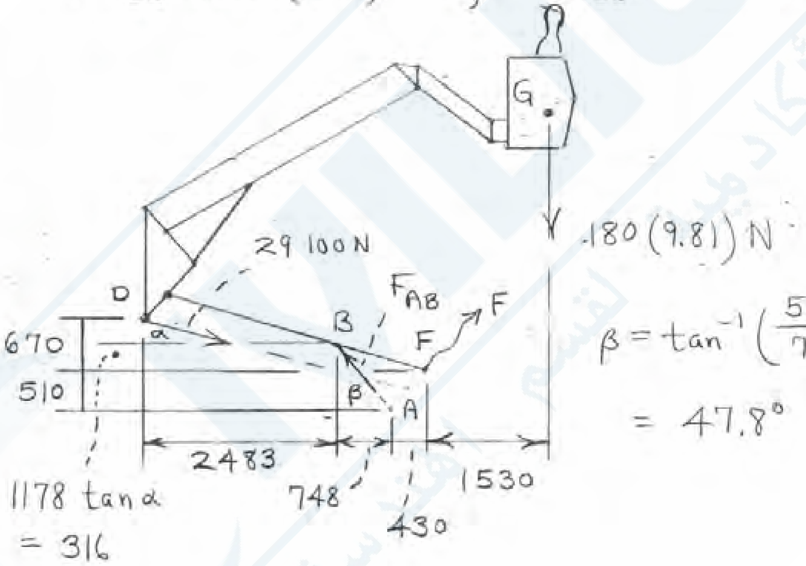
$\alpha = \tan^{-1}\left(\frac{925}{3446}\right)$

$= 15.03°$



$\circlearrowleft + \ \Sigma M_E = 0: -180(9.81)(4976) + F_{CD}\cos\alpha\,(255)$

$+ F_{CD}\sin\alpha\,(215) = 0, \quad F_{CD} = 29\,100 \text{ N}$



$\beta = \tan^{-1}\left(\frac{510+316}{748}\right)$

$= 47.8°$

$\circlearrowleft + \ \Sigma M_F = 0: -29\,100\cos\alpha\,(670) + 29\,100\sin\alpha\,(3661)$

$- F_{AB}\cos\beta\,(510) - F_{AB}\sin\beta\,(430) - 180(9.81)(1530) = 0$

$F_{AB} = 9200 \text{ N}$

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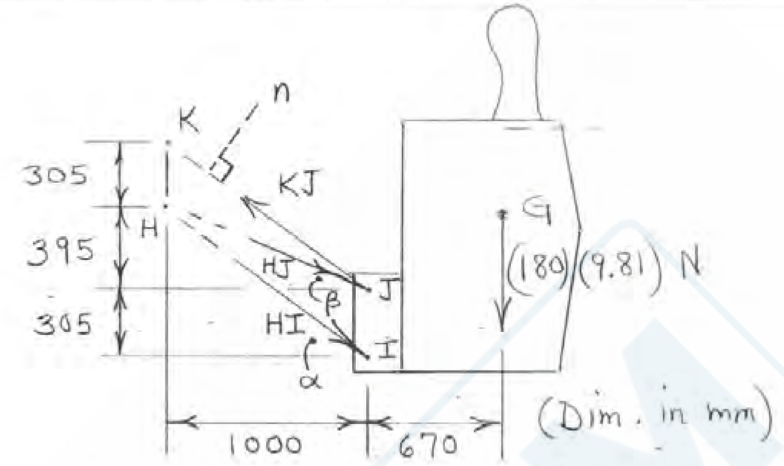

$\alpha = \tan^{-1} \frac{700}{1000} = 35.0°$; $\beta = \tan^{-1} \frac{395}{1000} = 21.6°$

$\Sigma F_n = 0$: $HJ \sin(\alpha - \beta) - 180(9.81)\cos\alpha = 0$

$\underline{HJ = 6220 \text{ N}}$

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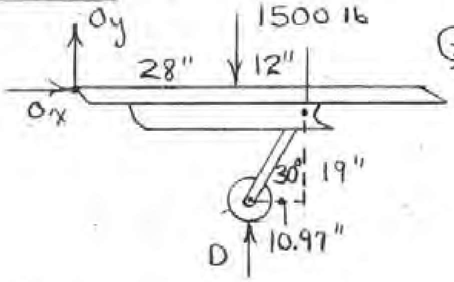
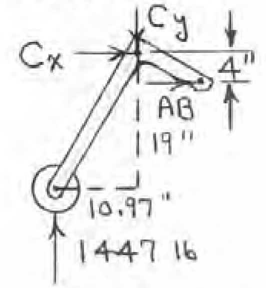
4/113 Entire ramp + mechanism :



$\circlearrowleft^{+} \sum M_O = 0: \; D(28+12-10.97) - 1500(28) = 0$

$D = 1447 \text{ lb}$

Crank BCD :



$\circlearrowleft^{+} \sum M_C = 0: -1447(10.97) + AB(4) = 0, \quad \underline{AB = 3970 \text{ lb}}$

(Cylinder is in compression)

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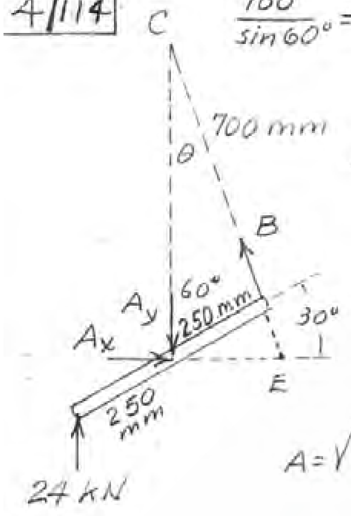

$\dfrac{700}{\sin 60°} = \dfrac{250}{\sin\theta}$, $\theta = \sin^{-1} 0.3093 = 18.02°$

$\cos\theta = 0.9510$, $\tan\theta = 0.3252$

$\overline{AC} = 700(0.9510) + 250(0.5) = 791$ mm

$\Sigma M_C = 0$; $24(250)(0.866) - 791 A_x = 0$

$A_x = 6.57$ kN

$\overline{AE} = \overline{AC}\tan\theta = 791(0.3252) = 257$ mm

$\Sigma M_E = 0$; $A_y(257) - 24(250 \times .866 + 257) = 0$

$A_y = 44.2$ kN

$A = \sqrt{6.57^2 + 44.2^2} = \underline{44.7 \text{ kN}}$

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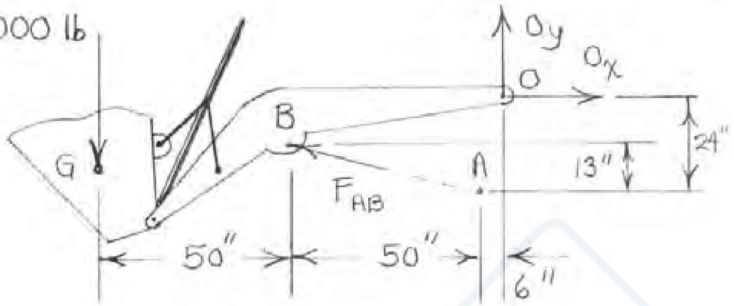

$\angle OBA = \tan^{-1}\left(\frac{11}{56}\right) + \tan^{-1}\left(\frac{13}{50}\right) = 11.11° + 14.57° = 25.7°$

$\overline{OB} = \sqrt{11^2 + 56^2} = 57.1$ in.

$\circlearrowleft + \Sigma M_O = 0: \; 4000(106) - F_{AB} \sin 25.7° (57.1) = 0$

$F_{AB} = 17{,}140$ lb

$\Sigma F_x = 0: \; O_x - 17{,}140 \cos 14.57° = 0$

$O_x = 16{,}590$ lb

$\Sigma F_y = 0: \; O_y + 17{,}140 \sin 14.57° - 4000 = 0$

$O_y = -313$ lb

$O = \sqrt{16{,}590^2 + 313^2} = 16{,}590$ lb

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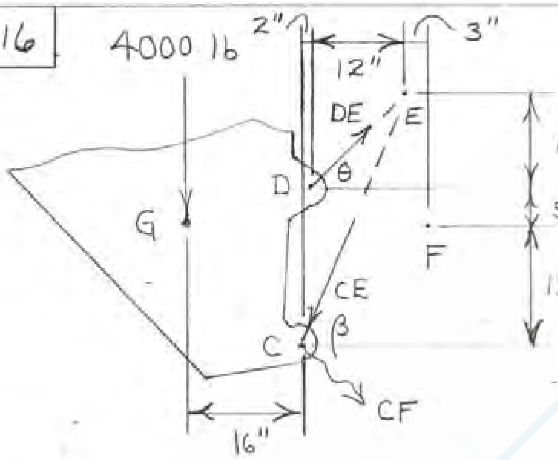

$\theta = 45°$

$\beta = \tan^{-1} \frac{32}{14} = 66.4°$

$\alpha = \tan^{-1} \frac{17}{3} = 80.0°$

$\circlearrowleft + \Sigma M_C = 0: \ 4000(16) - DE \cos 45° (20) + DE \sin 45° (2) = 0$

$DE = 5030 \text{ lb}$

FBD of joint E:

$\Sigma F_x = 0:$

$-DE \cos\theta + CE \cos\beta + EF \cos\alpha = 0$

$\Sigma F_y = 0:$

$-DE \sin\theta + CE \sin\beta - EF \sin\alpha = 0$

Solve simultaneously to obtain

$CE = 7440 \text{ lb}$

$EF = 3310 \text{ lb}$

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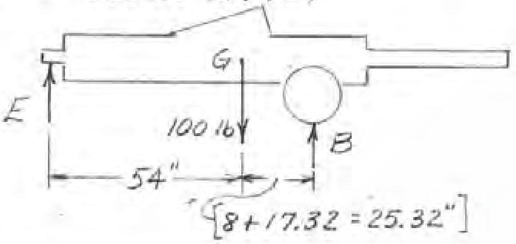
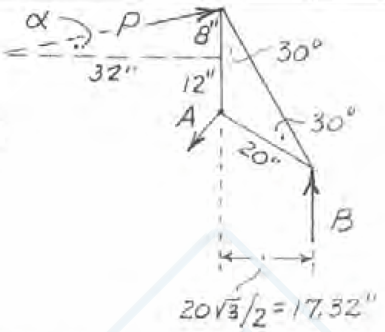
4/117 $\tan\alpha = \frac{8}{32}$, $\alpha = 14.04°$

$\cos\alpha = 0.9701$





$\Sigma M_E = 0;\ (54 + 25.32)B - 54(100) = 0$

$B = 68.1\ \text{lb}$

$\Sigma M_A = 0;\ 0.9701P(20) - 68.1(17.32) = 0$

$\underline{P = 60.8\ \text{lb}}$

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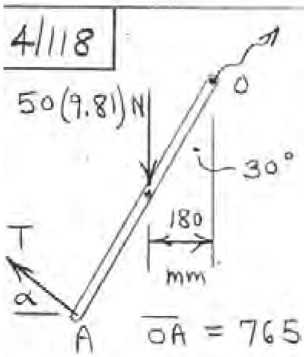


$$\alpha = \tan^{-1}\left[\frac{765\cos 30° - 410}{570 + 180 - 765\sin 30°}\right]$$

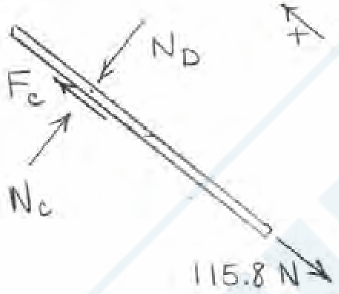
$$= 34.5°$$

$\overline{OA} = 765$ mm

$\circlearrowleft + \ \Sigma M_O = 0: \ 50(9.81)(180) - T\cos\alpha\,(765\cos 30°)$

$- T\sin\alpha\,(765\sin 30°) = 0, \ T = 115.8$ N

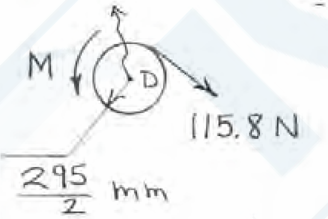
AB:



$\nwarrow + \ \Sigma F = 0: \ F_C - 115.8 = 0$

$F_C = 115.8$ N

Pinion:



$\circlearrowleft + \ \Sigma M_D = 0: \ M - 115.8\left(\frac{295}{2}\right) = 0$

$M = 17\,080$ N·mm

or <u>17.08 N·m</u>

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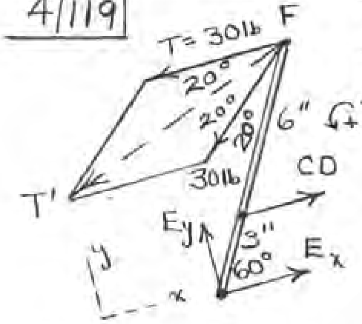


$$T'^2 = 30^2 + 30^2 - 2(30)(30)\cos 140°$$

$$T' = 56.4 \text{ lb}$$

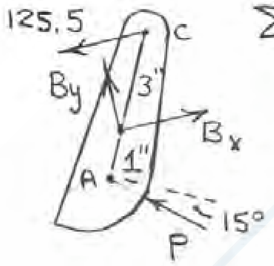
$$\circlearrowleft + \Sigma M_E = 0: \; -CD\,(3 \sin 60°) + 56.4(9 \sin 40°) = 0, \quad CD = 125.5 \text{ lb}$$

$$\Sigma F_x = 0: \; E_x + 125.5 - 56.4 \cos 20° = 0$$

$$E_x = -72.6 \text{ lb}$$

$$\Sigma F_y = 0: \; E_y - 56.4 \sin 20° = 0, \quad E_y = 19.28 \text{ lb}$$

$$E = \sqrt{E_x^2 + E_y^2} = \underline{75.1 \text{ lb}}$$



$$\Sigma M_B = 0: \; 125.5(3 \sin 60°) - P \cos 15°\,(1) = 0, \quad \underline{P = 338 \text{ lb}}$$

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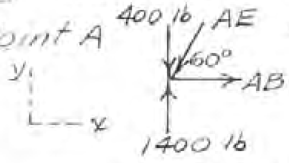
4/12 Total weight of truss $= 7(400) = 2800$ lb

By symmetry, reactions at A & C are 1400 lb

Joint A



$\Sigma F_y = 0;\ AE\cos 30° + 400 - 1400 = 0$

$\underline{AE = 2000/\sqrt{3}\ \text{lb}\ C}$

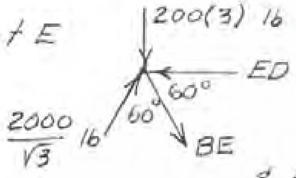
$\Sigma F_x = 0;\ AB - \frac{2000}{\sqrt{3}}\cos 60° = 0$

$\underline{AB = 1000/\sqrt{3}\ \text{lb}\ T}$

By symmetry $\underline{BC = 1000/\sqrt{3}\ \text{lb}\ T}$

$\underline{CD = 2000/\sqrt{3}\ \text{lb}\ C}$

Joint E



$\Sigma F_y = 0;\ BE\sin 60° - \frac{2000}{\sqrt{3}}\sin 60° + 600 = 0$

$\underline{BE = 800/\sqrt{3}\ \text{lb}\ T}$

& by symmetry $\underline{BD = 800/\sqrt{3}\ \text{lb}\ T}$

$\Sigma F_x = 0;\ ED - \frac{2000}{\sqrt{3}}\sin 30° - \frac{800}{\sqrt{3}}\sin 30° = 0$

$\underline{ED = 1400/\sqrt{3}\ \text{lb}\ C}$

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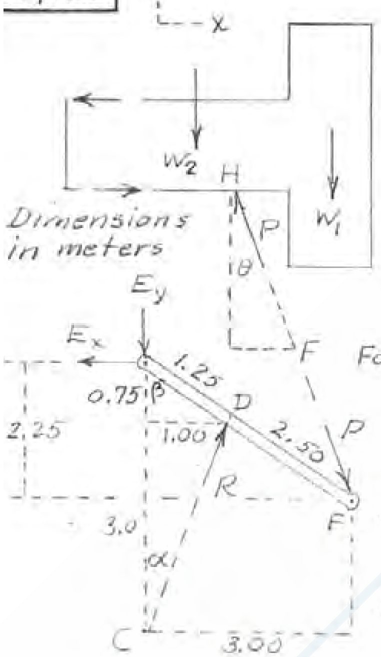

$\sin\theta = \dfrac{(3.00-1.75)}{3.25} = \dfrac{5}{13}$, $\cos\theta = \dfrac{12}{13}$

$\cos\beta = \dfrac{0.75}{1.25} = \dfrac{3}{5}$, $\sin\beta = \dfrac{4}{5}$

$\tan\alpha = 1/3$, $\sin\alpha = 1/\sqrt{10}$, $\cos\alpha = 3/\sqrt{10}$

$W_1 = 1.5(9.81) = 14.72$ kN

$W_2 = 2(9.81) = 19.62$ kN

$\Sigma F_y = 0$; $\frac{12}{13}P - 14.72 - 19.62 = 0$

$P = 37.2$ kN

For EF; $\Sigma M_E = 0$; $37.2\left(\frac{12}{13}\right)(3) - 37.2\left(\frac{5}{13}\right)(2.25) - \frac{R}{\sqrt{10}}(3.0 + 0.75) = 0$

$\underline{R = 59.7 \text{ kN}}$

$\Sigma F_x = 0$; $E_x - 59.7/\sqrt{10} - 37.2(5/13) = 0$

$E_x = 33.2$ kN

$\Sigma F_y = 0$; $E_y + 37.2(12/13) - 59.7(3/\sqrt{10}) = 0$

$E_y = 22.3$ kN

$E = \sqrt{33.2^2 + 22.3^2} = \underline{40.0 \text{ kN}}$

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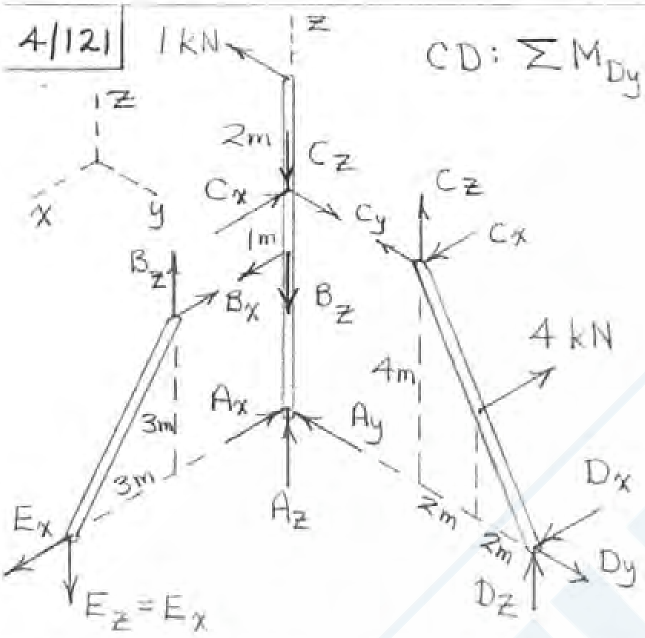

CD: $\Sigma M_{D_y} = 0: \; 4(2) - C_x(4) = 0$

$C_x = 2$ kN

ABC: $\Sigma M_{B_y} = 0: \; 2(1) - A_x(3) = 0, \quad A_x = 0.667$ kN

$\Sigma F_x = 0: \; B_x - 0.667 - 2 = 0, \quad B_x = 2.67$ kN

EB: $B_z = E_z = B_x = E_x = 2.67$ kN

ABC: $\Sigma M_{A_x} = 0: \; 1(6) - C_y(4) = 0, \quad C_y = 1.50$ kN

CD: $\Sigma M_{D_x} = 0: \; C_z = C_y = 1.50$ kN

ABC: $\Sigma F_z = 0: \; A_z - 2.67 - 1.50 = 0, \quad A_z = 4.17$ kN

$\Sigma F_y = 0: \; A_y + 1 - 1.50 = 0, \quad A_y = 0.50$ kN

$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \underline{4.25 \text{ kN}}$

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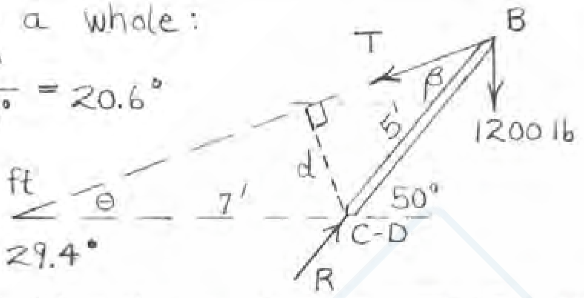
4/122 Frame as a whole:



$\theta = \tan^{-1} \dfrac{5 \sin 50°}{7 + 5 \cos 50°} = 20.6°$

$d = 7 \sin 20.6° = 2.46$ ft

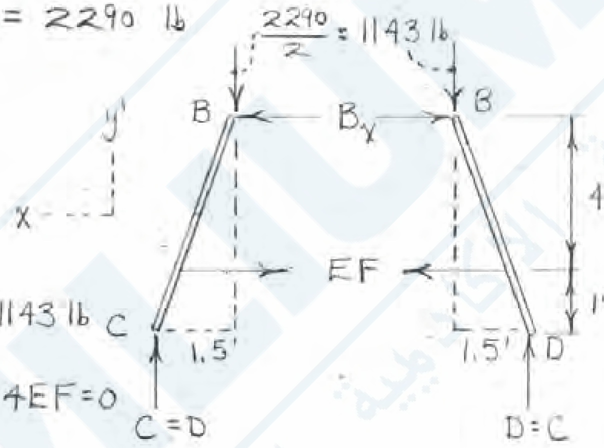
$\beta + 20.6° = 50°, \quad \beta = 29.4°$

$\Sigma M_x = 0: \ (5 \cos 50°)(1200) - 2.46 T = 0, \quad \underline{T = 1569 \text{ lb}}$

$\Sigma F_{(C-D)-B} = 0: \ R - 1200 \cos 40° - 1569 \cos 29.4° = 0$

$R = 2290$ lb

Plane of frame:



$\Sigma F_{y'} = 0: \ C = D = 1143$ lb

$\Sigma M_B = 0: \ 1143(1.5) - 4EF = 0$

$\underline{EF = 429 \text{ lb}}$

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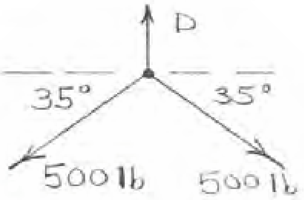
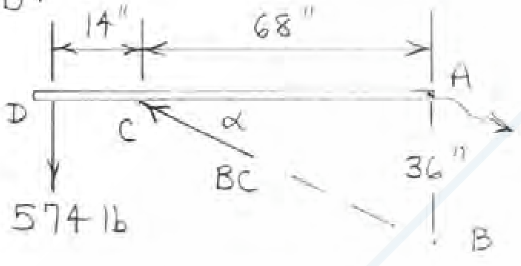
4/123 At D:

$\uparrow \Sigma F = 0: \quad D - 2(500)\sin 35° = 0$

$D = 574 \text{ lb}$



ACD:



$\alpha = \tan^{-1}\dfrac{36}{68}$

$= 27.9°$

$\circlearrowleft \Sigma M_A = 0: \quad 574(82) - BC\sin 27.9°(68) = 0$

$BC = \underline{1478 \text{ lb} = F}$

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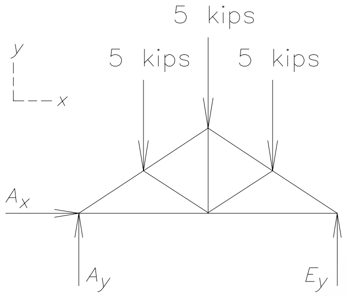
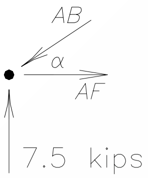


As a whole:

By symmetry, $A_y = E_y = 7.5$ kips

$\Sigma F_x = 0: \ A_x = 0$

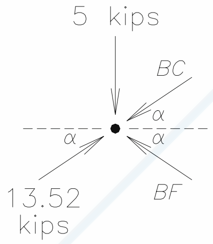
Joint *A*:



$$\alpha = \tan^{-1}\frac{10}{15} = 33.7°$$

$$\begin{cases} \Sigma F_x = 0: \ -13.52\cos 33.7° + AF = 0, \ \underline{AF = 11.25 \text{ kips } T} \\ \Sigma F_y = 0: \ -AB\sin 33.7° + 7.5 = 0, \ \underline{AB = 13.52 \text{ kips } C} \end{cases}$$

Joint *B*:

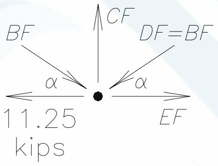


$$\begin{cases} \Sigma F_x = 0: \ (13.52 - BC - BF)\cos\alpha = 0 \\ \Sigma F_y = 0: \ (13.52 - BC = BF)\sin\alpha - 5 = 0 \end{cases}$$

Solve simultaneously to obtain:

<u>*BC* = 9.01 kips *C*</u>, <u>*BF* = 4.51 kips *C*</u>

Joint *F*:



By symmetry, <u>*EF* = *AF* = 11.25 kips *T*</u>
& <u>*DF* = *BF* = 4.51 kips *C*</u>

$\Sigma F_y = 0: \ CF - 2(4.51)\sin 33.7° = 0, \quad \underline{CF = 5 \text{ kips } T}$

By symmetry, <u>*CD* = 9.01 kips *C*</u>, <u>*DE* = 13.52 kips *C*</u>, <u>*DF* = 4.51 kips *C*</u>

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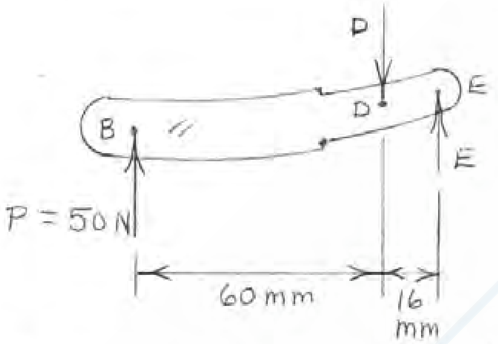
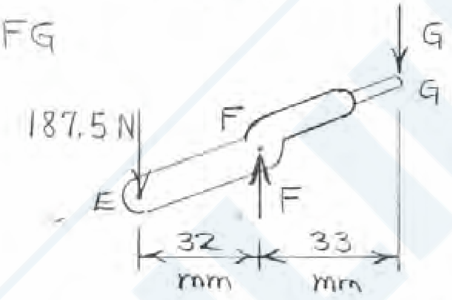
4/125 Handle BDE



$\circlearrowleft + \Sigma M_D = 0: \; -50(60) + E(16) = 0, \quad E = 187.5 \text{ N}$

Jaw EFG



$\circlearrowleft + \Sigma M_F = 0: \; 187.5(32) - G(33) = 0, \quad \underline{G = 181.8 \text{ N}}$

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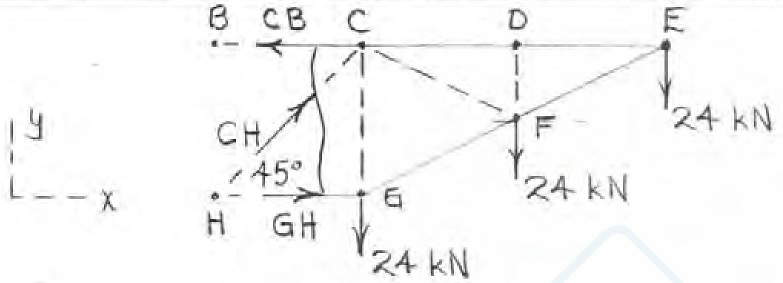
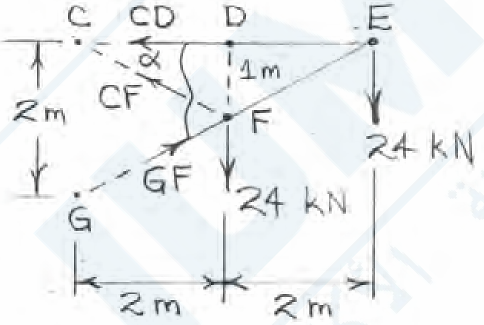


$\Sigma F_y = 0: \; CH \sin 45° - 3(24) = 0$, $CH = 101.8$ kN C

$\alpha = \tan^{-1} \frac{1}{2} = 26.6°$



$\circlearrowleft + \Sigma M_E = 0: \; 24(2) - (CF \sin 26.6°)(4) = 0$

$CF = 26.8$ kN T

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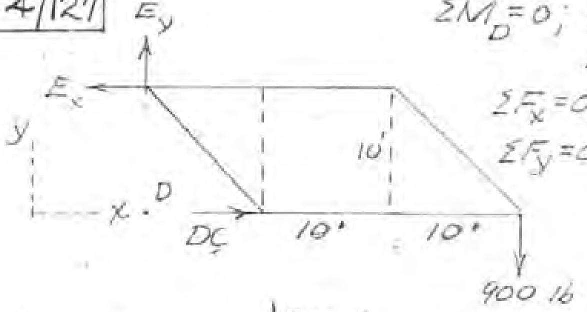


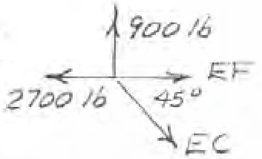
$\Sigma M_D = 0;\ 900(30) - 10E_x = 0$

$E_x = 2700$ lb

$\Sigma F_x = 0;\ DC = 2700$ lb C

$\Sigma F_y = 0;\ E_y = 900$ lb

Joint E;



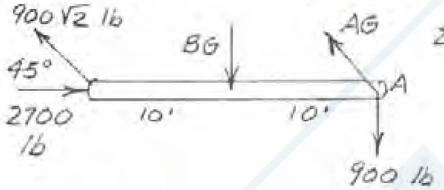
$\Sigma F_y = 0;\ EC/\sqrt{2} - 900 = 0$

$EC = 900\sqrt{2}$ lb

$\Sigma F_x = 0;\ EF + \dfrac{900\sqrt{2}}{\sqrt{2}} - 2700 = 0$

$EF = 1800$ lb T

Joint F; $FG = EF,\ FC = 0$



$\Sigma M_A = 0,\ 10BG - \dfrac{900\sqrt{2}}{\sqrt{2}}20 = 0$

$BG = 1800$ lb C

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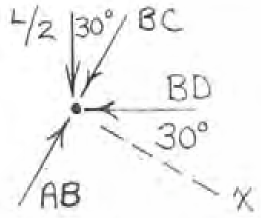
4/128 Joint B:



$\Sigma F_x = 0: \quad \frac{L}{2}\left(\frac{1}{2}\right) - BD\frac{\sqrt{3}}{2} = 0$

$BD = \frac{\sqrt{3}}{6} L$ C, independent of x.

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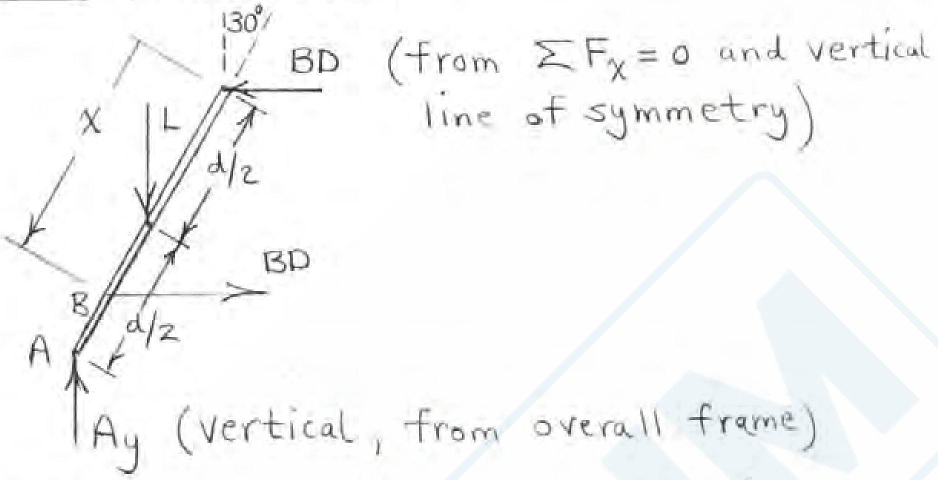
4/129 Member ABC:



$$\circlearrowleft + \Sigma M_A = 0: \; BD(d \cos 30°) - BD(d-x)\cos 30° - L\left(\frac{d}{2}\sin 30°\right) = 0$$

$$BD = \frac{Ld}{2x}\tan 30° = \frac{0.289Ld}{x}$$

(x cannot be zero)

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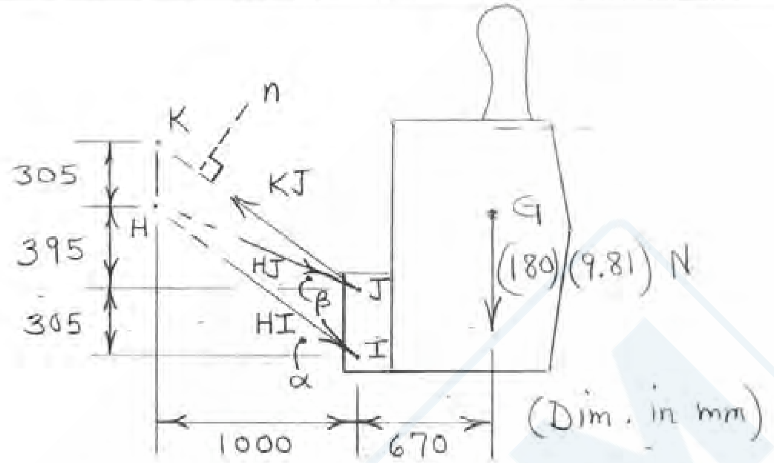
4/112



$\alpha = \tan^{-1}\frac{700}{1000} = 35.0°$; $\beta = \tan^{-1}\frac{395}{1000} = 21.6°$

$\Sigma F_n = 0$: $HJ\sin(\alpha - \beta) - 180(9.81)\cos\alpha = 0$

$\underline{HJ = 6220 \text{ N}}$

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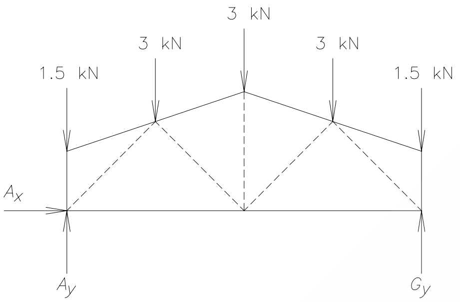
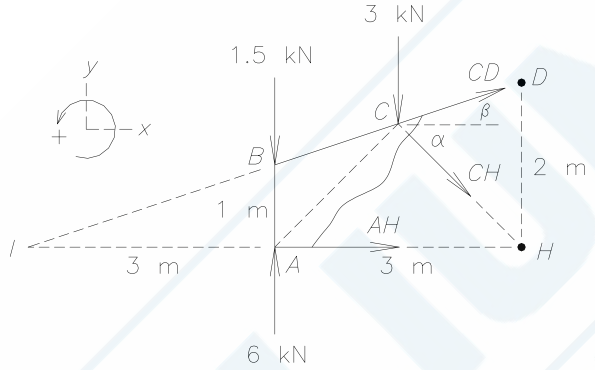


By symmetry, $A_y = G_y = 6$ kN

$\Sigma F_x = 0: \; A_x = 0$



$\alpha = 45°, \; \beta = \tan^{-1}\dfrac{0.5}{1.5} = 18.43°$

$\overline{AC} = \dfrac{3}{\sqrt{2}}$ m

$\Sigma M_I = 0: \; 6(3) - 1.5(3) - 3(4.5) - CH(2\overline{AC}) = 0, \; \underline{CH = 0}$

$\Sigma F_y = 0: \; 6 + CD \sin 18.43° - 1.5 - 3 = 0, \; CD = -4.74$ kN or $\underline{4.74 \text{ kN } C}$

$\Sigma F_x = 0: \; -4.74 \cos 18.43° + AH = 0, \; \underline{AH = 4.5 \text{ kN } T}$

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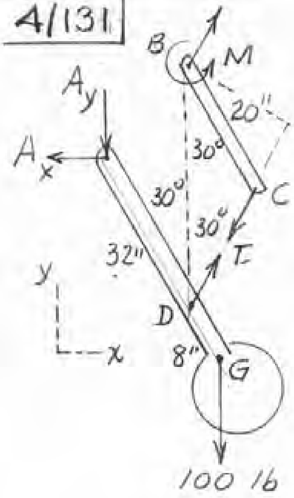

ADG; $\Sigma M_A = 0$; $100(40 \sin 30°) - T(32 \cos 30°) = 0$

$T = 72.2$ lb

BC; $\Sigma M_B = 0$; $72.2(20 \cos 30°) - M = 0$

$M = 1250$ lb-in.

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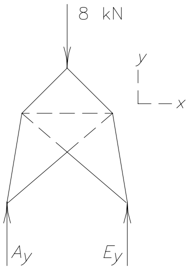
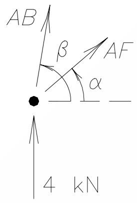


By symmetry, $A_y = E_y = 4$ kN

Joint *A*:

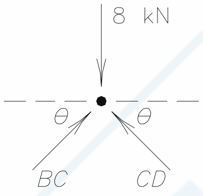


$$\alpha = \tan^{-1}\frac{3}{3.5} = 40.6°,\ \beta = \tan^{-1}\frac{3}{0.5} = 80.5°$$

$$\begin{cases} \Sigma F_x = 0: & AB\cos 80.5° + AF\cos 40.6° + 4 = 0 \\ \Sigma F_y = 0: & AB\sin 80.5° + AF\sin 40.6° = 0 \end{cases}$$

Solve to obtain: $AB = -4.73$ kN or 4.73 kN *C*

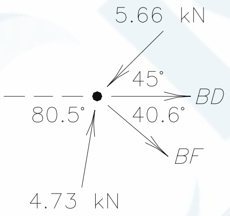
Joint *C*:



By symmetry, *BC* = *CD*

$$\Sigma F_y = 0: \ 2BC\sin 45° - 8 = 0,\ BC = 5.66 \text{ kN } C$$

Joint *B*:



$$\Sigma F_y = 0: \ 4.73\sin 80.5° - BF\sin 40.6° - 5.66\sin 45° = 0$$

$$\underline{BF = 1.024 \text{ kN } T}$$

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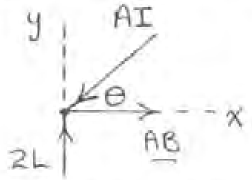
4/133 From whole truss, $A = F = 2L$

A :

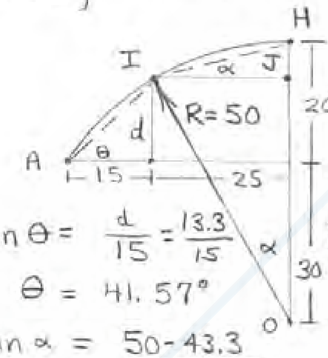


y: $2L - AI \sin\theta$

$AI = 3.01L$ C

x: $-3.01 \cos\theta + AB = 0$

$AB = 2.26L$ T



$\sin\alpha = \frac{25}{50}$

$\alpha = 30°$

$\tan\alpha = \frac{25}{\overline{OJ}}$

$\overline{OJ} = 43.3$

$d = 43.3 - 30$
$= 13.3$ m

$\tan\theta = \frac{d}{15} = \frac{13.3}{15}$

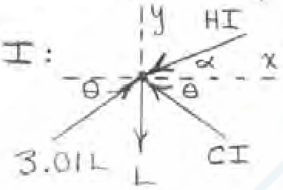
$\theta = 41.57°$

$\tan\alpha = \frac{50 - 43.3}{25}$

$\alpha = 15°$

From joint B, $BI = L$ T

I :



$$\begin{cases} \Sigma F_x = 0 : (3.01L - CI)\cos\theta - HI\cos\alpha = 0 \\ \Sigma F_y = 0 : (3.01L + CI)\sin\theta - HI\sin\alpha - L = 0 \end{cases}$$

Solve to obtain $CI = -0.458L$ T

$HI = 2.69L$ C

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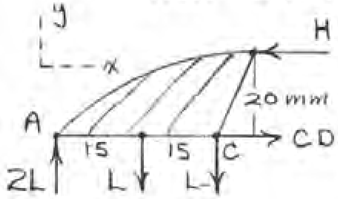
4/134 From whole structure, $A = F = 2L$

Half of structure:

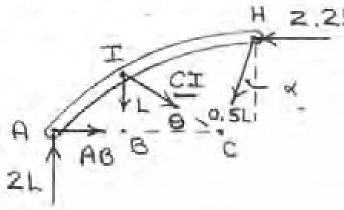


$\curvearrowleft + \sum M_C = 0: 20H + 15L - 30(2L) = 0$; $H = 2.25L$

$\sum F_x = 0 \Rightarrow CD = 2.25L$ T

From joint B, $BI = L$ T

Member AIH:



From previous solution, $\theta = 41.57$, $\alpha = \tan^{-1}\left(\frac{10}{20}\right) = 26.57°$

$\sum F_y = 0: 2L - L - CI \sin\theta - 0.5L\cos\alpha = 0$, $CI = 0.833L$ T

$\sum F_x = 0: AB + 0.833L\cos\theta - 0.5L\sin\alpha - 2.25L = 0$

$AB = 1.850L$ T

Problem not solvable without CH data.

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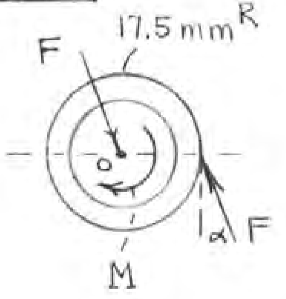

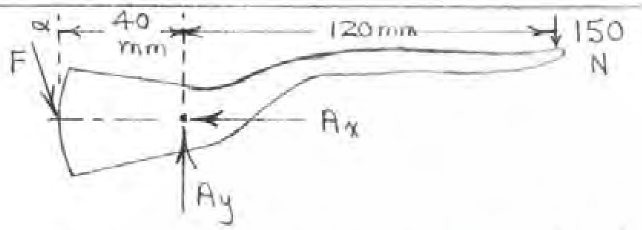

$\Sigma M_A = 0: \; 40 F\cos\alpha = 120(150)$

$F\cos\alpha = 450 \text{ N}$

$\Sigma M_O = 0: \; 450(0.0175) = \underline{M = 7.88 \text{ N}\cdot\text{m}}$

$\Sigma M = 0: \; 57.5(10^{-3})R = 7.88$

$\underline{R = 137.0 \text{ N}}$

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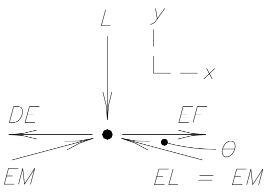
We can begin at joint E without finding the external reactions.

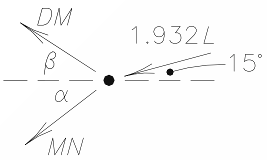
Joint *E*:



$$\theta = \tan^{-1}\frac{24 - 24\sin 60°}{24\sin 30°} = 15°$$

$$\begin{cases} \Sigma F_y = 0: \ 2(EM \sin 15°) - L = 0, \ \underline{EM = 1.932\ L\ C} \\ \Sigma F_x = 0: \ 1.932L\cos 15° - DE = 0, \ \underline{DE = 1.366\ L\ C} \end{cases}$$

Joint *M*:



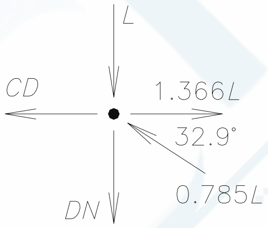
$$\alpha = \tan^{-1}\frac{24\sin 60° - 24\sin 45°}{24\cos 45° - 24\cos 60°} = 37.5°$$

$$\beta = \tan^{-1}\frac{24 - 24\sin 60°}{24\cos 45° - 24\cos 60°} = 32.9°$$

$$\begin{cases} \Sigma F_x = 0: \ -DM\cos 32.9° - MN\cos 37.5° - 1.932L\cos 15° = 0 \\ \Sigma F_y = 0: \ DM\sin 32.9° - MN\sin 37.5° - 1.932\sin 15° = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{DM = 0.785L\ C}$

Joint *D*:



$$\Sigma F_y = 0: \ 0.785L\sin 32.9° - L - DN = 0$$

$$\underline{DN = 0.574L\ C}$$

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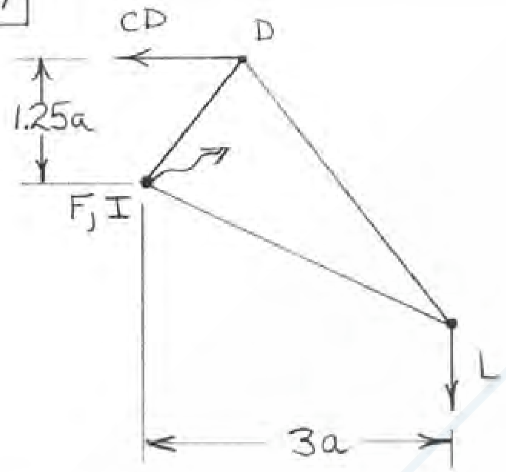

$\circlearrowleft + \sum M_F = 0: \quad CD(1.25a) - L(3a) = 0$

$CD = 2.4L \; T$

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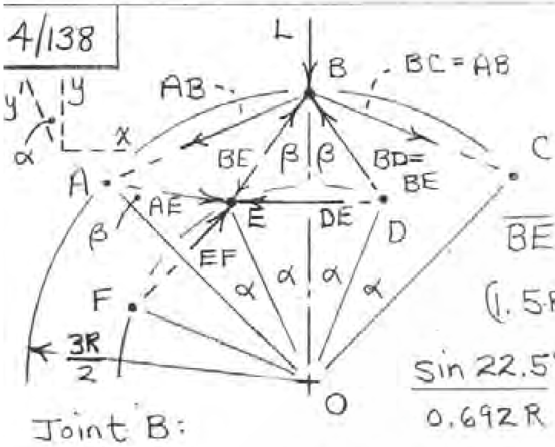

$\begin{cases} \alpha = 22.5^\circ \\ \measuredangle ABO = 67.5^\circ \end{cases}$

$\overline{BE}^2 = R^2 + (1.5R)^2 - 2R \times (1.5R) \cos 22.5^\circ$, $\overline{BE} = 0.692R$

$\dfrac{\sin 22.5^\circ}{0.692R} = \dfrac{\sin\beta}{R}$, $\beta = 33.6^\circ$

Joint B:

$\Sigma F_y = 0: \; 2(0.8L)\cos 33.6^\circ - 2\,AB\cos 67.5^\circ - L = 0$

$\underline{AB = 0.434L \;\; T}$

For joint E, note that EF = DE & AE = BE.

$\Sigma F_{y'} = 0: \; -2(0.8L)\cos 56.1^\circ + 2DE\cos 67.5^\circ = 0$

$\underline{DE = 1.166L \;\; C}$

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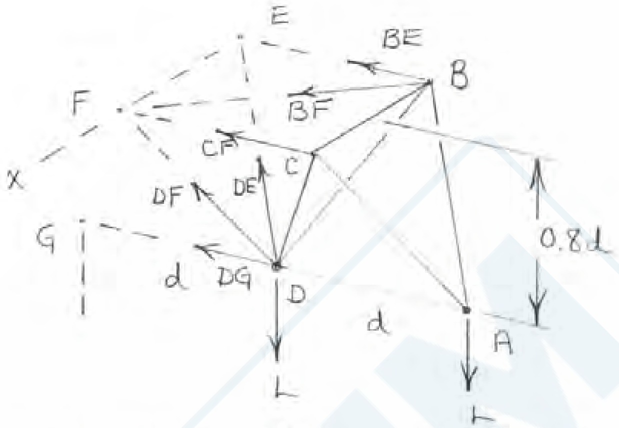
4/139 Section for DG:



$$\Sigma M_x = 0: \quad -DG(0.8d) - L\left(\frac{d}{2}\right) - L\left(\frac{3d}{2}\right) = 0$$

$DG = -2.5L$ (or $2.5L$ C)

Similarly, $AD = 0.625L$ C

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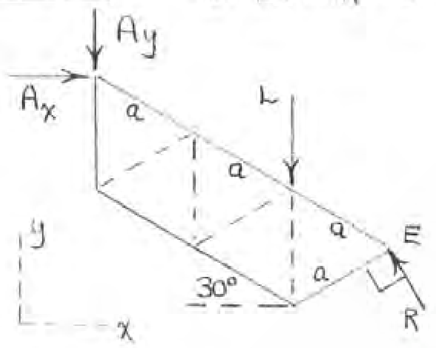
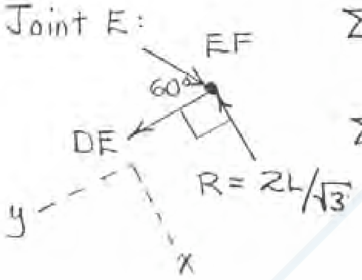
$\circlearrowleft + \; \Sigma M_A = 0: \; L(2a\cos 30°) - R(3a\sin 30°) = 0$

$R = 2L/\sqrt{3}$



Joint E:



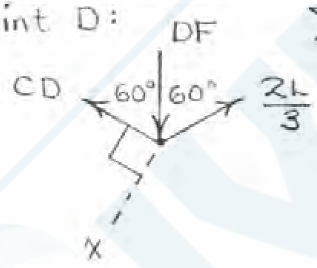
$\Sigma F_x = 0: \; EF\sin 60° - 2L/\sqrt{3} = 0$

$EF = 4L/3 \quad C$

$\Sigma F_y = 0: \; DE - \frac{4L}{3}\cos 60° = 0$

$DE = 2L/3 \quad T$

Joint D:



$\Sigma F_x = 0: \; DF\cos 30° - \frac{2L}{3}\cos 30° = 0$

$DF = 2L/3 \quad C$

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▸4/140 Vector expressions for forces at A (treated as tensions) with $F_{AE} = F_{AF} = F_1$, $F_{BE} = F_{BF} = P$, $F_{BD} = F_{BC} = C$, are

$\mathbf{F}_{AE} = \frac{F_1}{1.552}(-1.2\mathbf{i} - 0.4\mathbf{j} + 0.9\mathbf{k})$, $\mathbf{F}_{AF} = \frac{F_1}{1.552}(-1.2\mathbf{i} + 0.4\mathbf{j} + 0.9\mathbf{k})$

$\mathbf{F}_{AB} = \frac{F_{AB}}{1.432}(-0.3\mathbf{i} + 1.4\mathbf{k})$, $\mathbf{F} = 2.2\mathbf{k}$. For joint A, $\Sigma\mathbf{F} = 0$ gives $\left[\frac{F_{AB}}{1.432}(-0.3) + \frac{2F_1}{1.552}(-1.2)\right]\mathbf{i}$

$+\left[2.2 + \frac{F_{AB}}{1.432}(1.4) + \frac{2F_1}{1.552}(0.9)\mathbf{k}\right] = \mathbf{0}$

Solve to get $\underline{F_{AB} = -2.681 \text{ kN}}$, $\underline{F_1 = 0.363 \text{ kN}}$

On B: $\mathbf{F}_{BE} = \frac{P}{1.105}(-0.9\mathbf{i} - 0.4\mathbf{j} - 0.5\mathbf{k})$

$\mathbf{F}_{BF} = \frac{P}{1.105}(-0.9\mathbf{i} + 0.4\mathbf{j} - 0.5\mathbf{k})$, $\mathbf{F}_{BD} = \frac{C}{1.105}(-0.9\mathbf{i} - 0.4\mathbf{j} + 0.5\mathbf{k})$

$\mathbf{F}_{BC} = \frac{C}{1.105}(-0.9\mathbf{i} + 0.4\mathbf{j} + 0.5\mathbf{k})$

For joint B, $\Sigma\mathbf{F} = \mathbf{0}$ gives

$\left(\frac{-1.8P}{1.105} - \frac{1.8C}{1.105} + 0.3\frac{-2.681}{1.432}\right)\mathbf{i} + \left(\frac{-P}{1.105} + \frac{C}{1.105} - 1.4\frac{-2.681}{1.432}\right)\mathbf{k}$

$+ 0\mathbf{j} = \mathbf{0}$. Solve to get $P = 1.620$ kN,

$C = -1.275$ kN, $\underline{F_{BE} = P = 1.620 \text{ kN}}$

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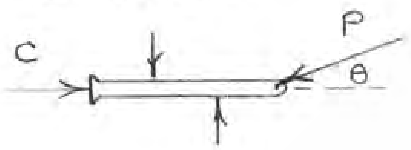
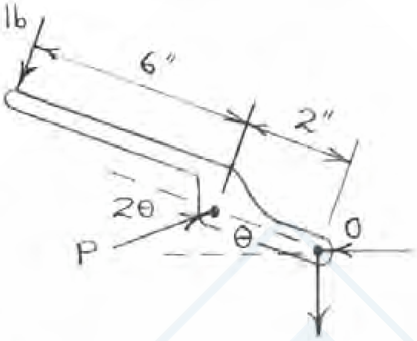
Shaft: $\Sigma F = 0$ gives

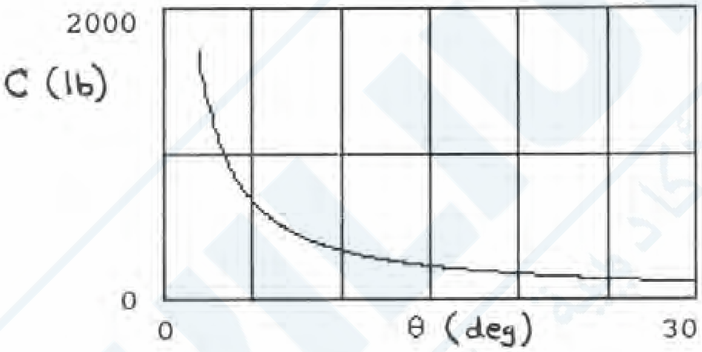
$C = P\cos\theta$





Lever: $\Sigma M_O = 0$: $30(8) - P\sin 2\theta\,(2) = 0$

$P = \dfrac{120}{\sin 2\theta} = \dfrac{60}{\sin\theta\cos\theta}$, $\quad \underline{C = \dfrac{60}{\sin\theta}}$ (lb)



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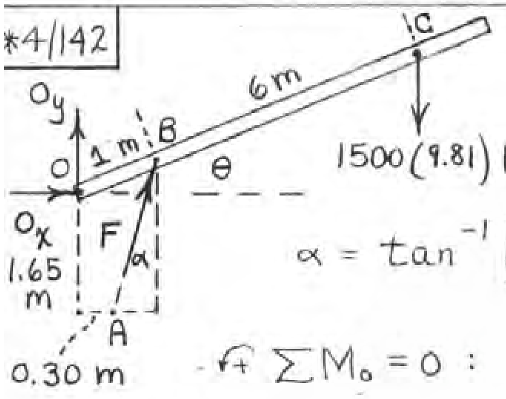
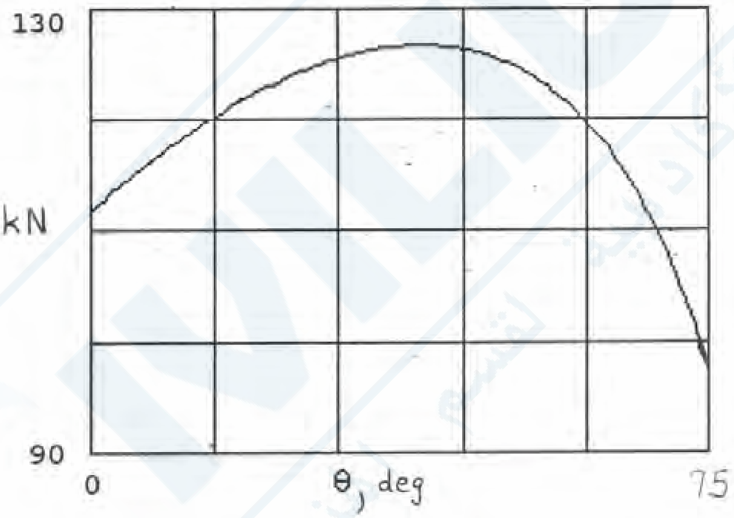


$$\alpha = \tan^{-1}\left[\frac{1\cos\theta - 0.3}{1\sin\theta + 1.65}\right]$$

$\circlearrowleft + \sum M_O = 0$:

$$(F\cos\alpha)(1\cos\theta) - (F\sin\alpha)(1\sin\theta) - 1500(9.81)(7\cos\theta) = 0$$

$$F = \frac{103.0\cos\theta}{(\cos\alpha\cos\theta - \sin\alpha\sin\theta)} \quad \text{(in kN)}$$



$F_{max} = 126.7$ kN @ $\theta = 39.8°$

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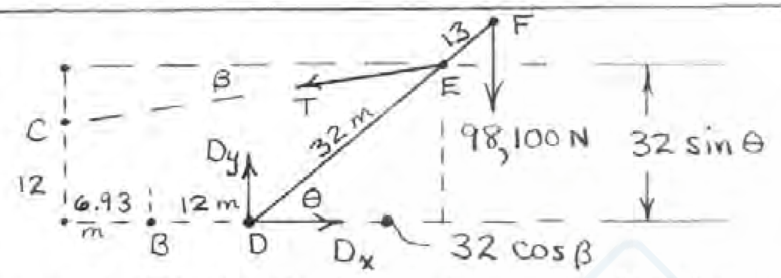
Boom:

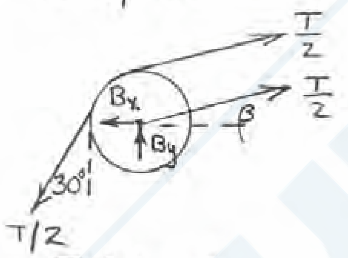


$$\beta = \tan^{-1}\left(\frac{32\sin\theta - 12}{6.93 + 12 + 32\cos\theta}\right) = \tan^{-1}\left(\frac{32\sin\theta - 12}{32\cos\theta + 18.93}\right) \quad (1)$$

$$\circlearrowleft \Sigma M_D = 0: \ (T\cos\beta)(32\sin\theta) - (T\sin\beta)(32\cos\theta) - (98{,}100)(45\cos\theta) = 0$$

So $$T = \frac{137{,}953\cos\theta}{\cos\beta\sin\theta - \sin\beta\cos\theta} \ (\text{in N}) \quad (2)$$

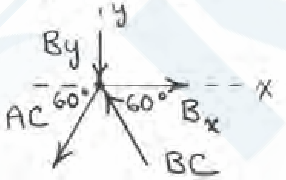
Pulley at C:



Equilibrium yields

$$\begin{cases} B_x = T\cos\beta - \frac{T}{2}\sin 30° \\ B_y = -T\sin\beta + \frac{T}{2}\cos 30° \end{cases} \quad (3)$$

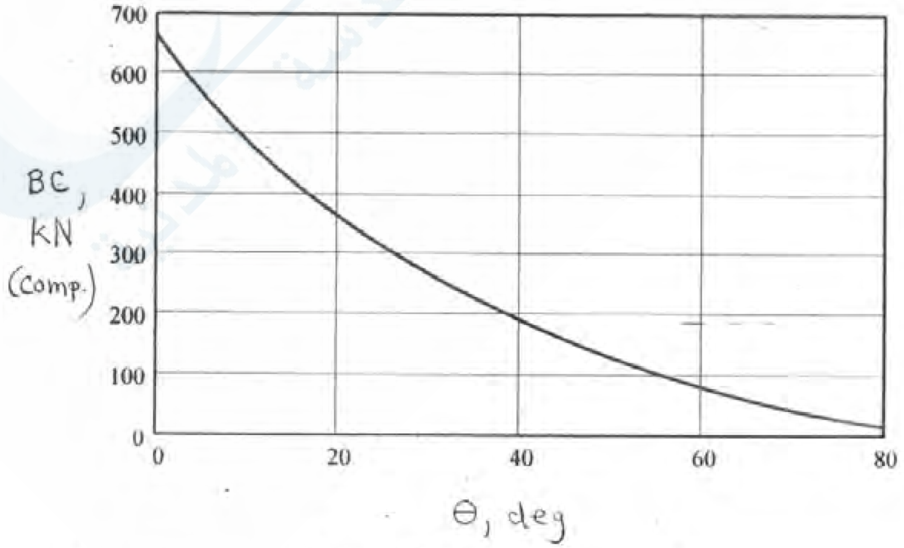
Joint C



Equilibrium yields

$$BC = B_x + 0.5774 B_y \quad (4)$$

$$(AC = B_x - 0.5774 B_y)$$



BC @ θ = 40° = 190.5 kN

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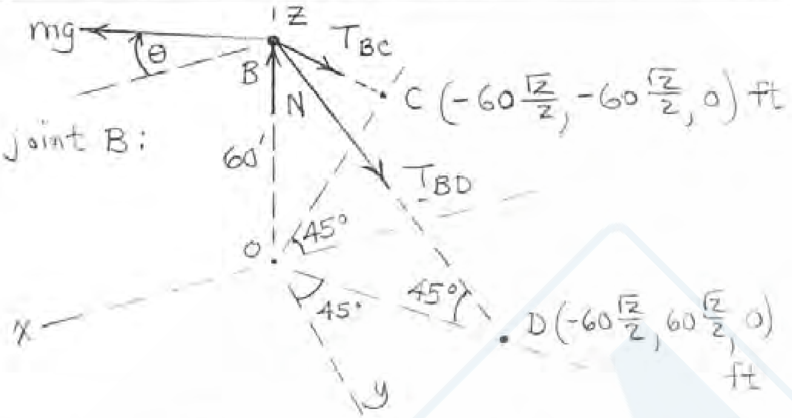
FBD of joint B:



$$\underline{T}_{BC} = T_{BC}\left[\frac{-60\frac{\sqrt{2}}{2}\underline{i} - 60\frac{\sqrt{2}}{2}\underline{j} - 60\underline{k}}{\sqrt{2(60)^2\frac{2}{4} + 60^2}}\right]$$

$$= T_{BC}\left[-0.5\underline{i} - 0.5\underline{j} - 0.707\underline{k}\right]$$

$$\underline{T}_{BD} = T_{BD}\left[-0.5\underline{i} + 0.5\underline{j} - 0.707\underline{k}\right]$$

$$\Sigma F_x = 0: -0.5T_{BC} - 0.5T_{BD} + mg\cos\theta = 0 \quad (1)$$

$$\Sigma F_y = 0: -0.5T_{BC} + 0.5T_{BD} - mg\sin\theta = 0 \quad (2)$$

Solve simultaneously:

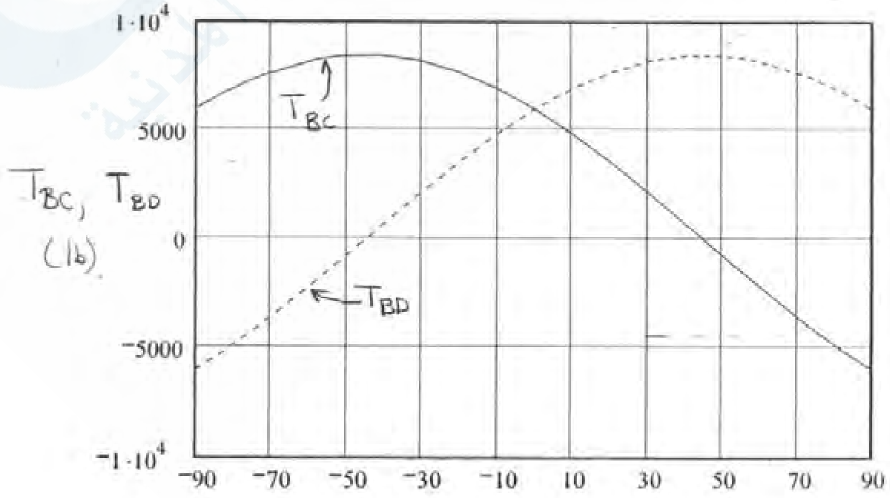
$$T_{BC} = mg(\cos\theta - \sin\theta)$$

$$T_{BD} = mg(\cos\theta + \sin\theta)$$

Maximum values

$T_{BC_{max}} = 8490$ lb @ $\theta = -45°$

$T_{BD\,max} = 8490$ lb @ $\theta = 45°$



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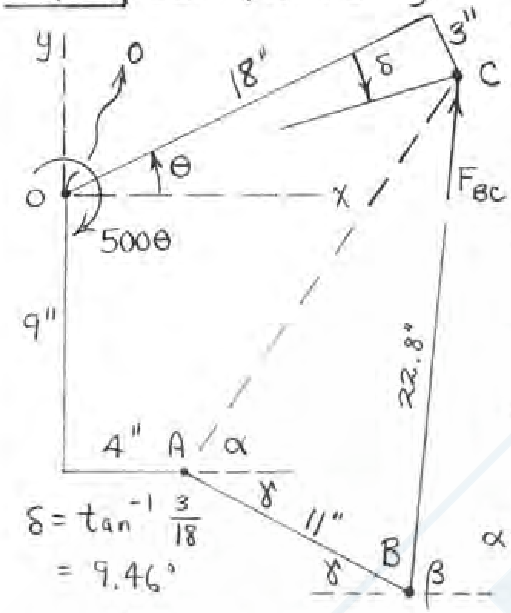
*4/145 $\overline{OC} = \sqrt{18^2 + 3^2}$; $\overline{AB} = 11''$; $\overline{BC} = \sqrt{14^2 + 17^2}$



$$\overline{AC} = \left\{ \left[18\cos\theta + 3\sin\theta - 4 \right]^2 + \left[18\sin\theta - 3\cos\theta + 9 \right]^2 \right\}^{\frac{1}{2}}$$

$$\measuredangle BAC = \cos^{-1}\left[\frac{\overline{AB}^2 + \overline{AC}^2 - \overline{BC}^2}{2\,\overline{AB}\,\overline{AC}} \right]$$

$$\measuredangle ABC = \cos^{-1}\left[\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,\overline{AB}\,\overline{BC}} \right]$$

$$\delta = \tan^{-1}\frac{3}{18} = 9.46^\circ$$

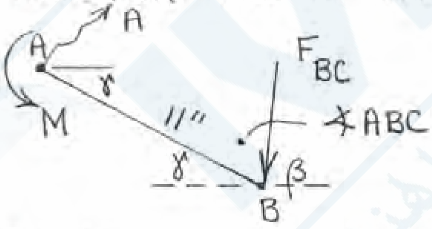
$$\alpha = \tan^{-1}\left[\frac{18\sin\theta - 3\cos\theta + 9}{18\cos\theta + 3\sin\theta - 4} \right]$$

$\gamma = \measuredangle BAC - \alpha$; $\beta = 180^\circ - \gamma - \measuredangle ABC$

From above FBD of door :

$$\circlearrowleft + \Sigma M_O = 0 : \; -500\theta + F_{BC}\left(\overline{OC}\sin(\beta - \theta + \delta)\right) = 0$$

Then from FBD of AB:



$$\circlearrowleft + \Sigma M_A = 0 : \; M - F_{BC}\left(11\sin\measuredangle ABC\right) = 0$$

Solve the above equations (in order) with θ varied from 0° to 90° to obtain the following plots for M and F_{BC}.

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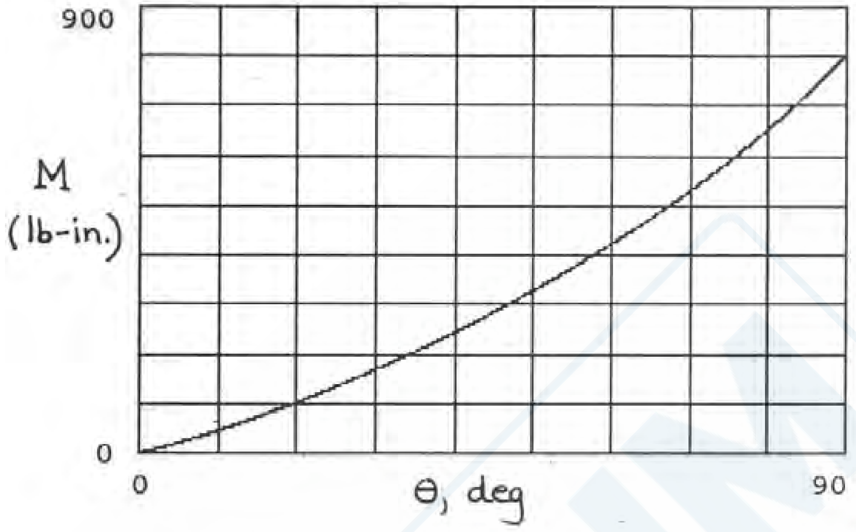

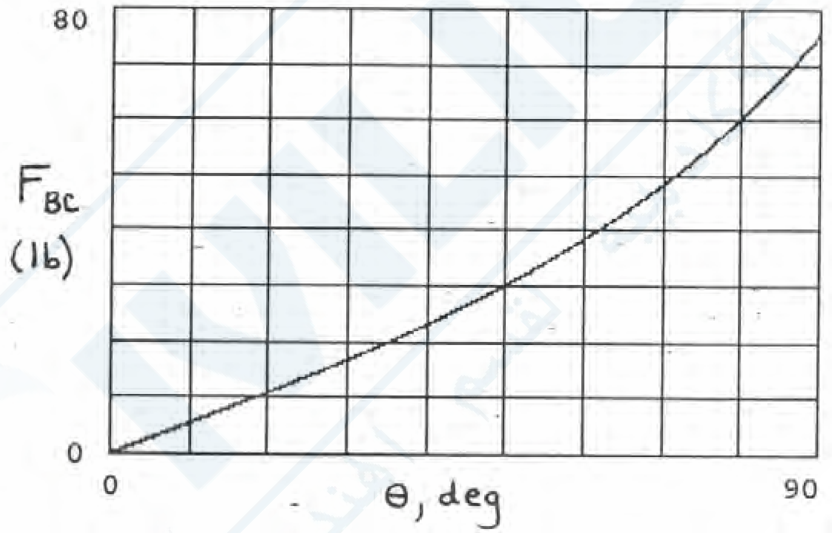

At θ = 45°, M = 285 lb-in.

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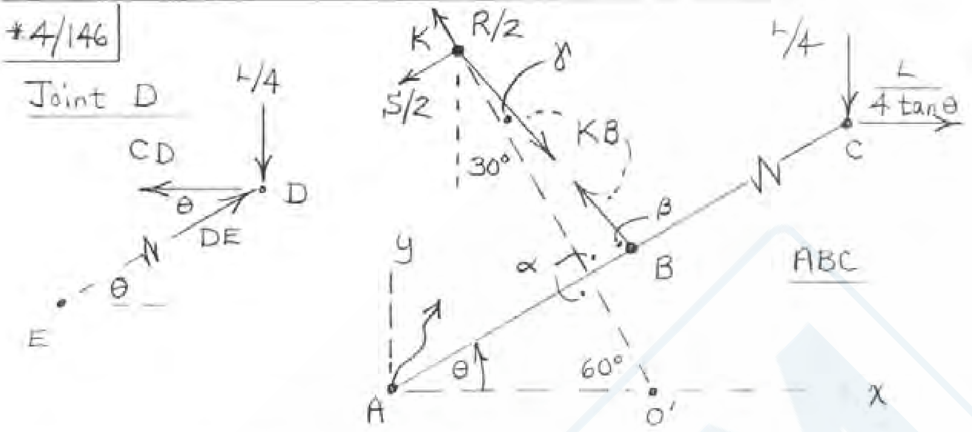

From FBD of joint D,

$$\begin{cases} \Sigma F_y = 0: \quad DE \sin\theta - \frac{L}{4} = 0, & DE = \frac{L}{4 \sin\theta} \quad C \\ \Sigma F_x = 0: \quad \frac{L}{4 \sin\theta} \cos\theta - CD = 0, & CD = \frac{L}{4 \tan\theta} \quad T \end{cases}$$

From above right diagrams:

$$\underline{r}_K = \underline{r}_{B/A} + \underline{r}_{K/B} = \overline{AB}\,(\cos\theta\,\underline{i} + \sin\theta\,\underline{j}) + \overline{BK}\,(-\sin(\gamma + 30^\circ)\,\underline{i} + \cos(\gamma + 30^\circ)\,\underline{j})$$

Also,

$$\underline{r}_K = \underline{r}_{O'/A} + \underline{r}_{K/O'} = \overline{AO'}\,\underline{i} + \overline{O'K}\,(-\cos 60^\circ\,\underline{i} + \sin 60^\circ\,\underline{j})$$

Equate the two $\underline{r}_K$ expressions, then equate corresponding $\underline{i}$- and $\underline{j}$-coefficients, and then numerically solve for γ and $\overline{O'K}$ as θ varies from 5° to 90°.

Then $\alpha = 180^\circ - 60^\circ - \theta$

and $\beta = 180^\circ - \gamma - \alpha$

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From FBD of ABC:

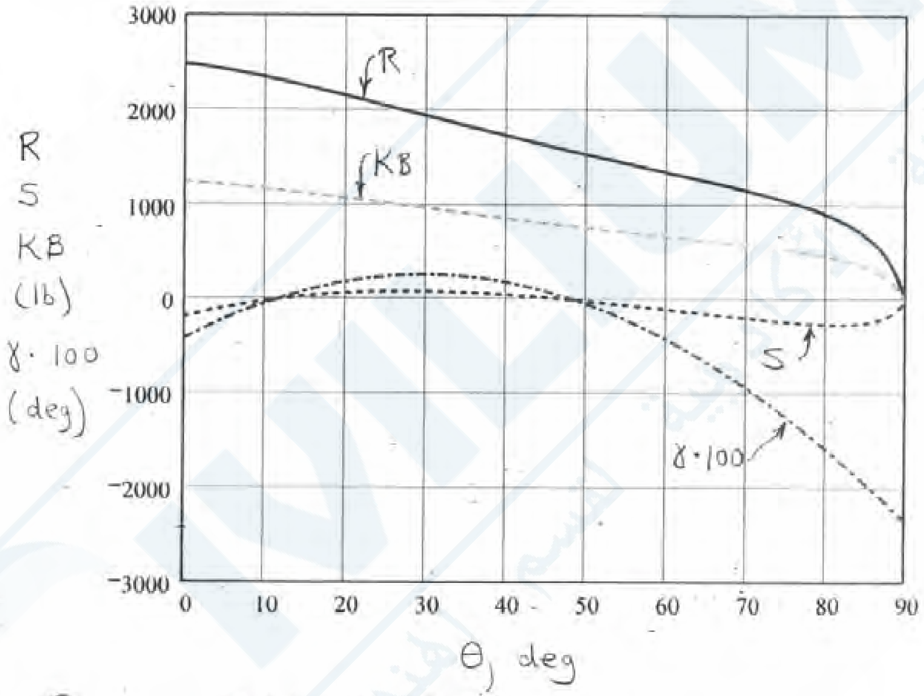
$\stackrel{+}{\curvearrowleft} \Sigma M_A = 0: -\frac{L}{4}(\overline{AC}\cos\theta) - \frac{L}{4\tan\theta}(\overline{AC}\sin\theta)$
$+ KB\cos(\beta-\theta)(\overline{AB}\sin\theta) + KB\sin(\beta-\theta)(\overline{AB}\cos\theta) = 0$

Solve this for force KB. From the final FBD,

$\frac{R}{2} - KB\cos\gamma = 0 \Rightarrow$ Gives R

$\frac{S}{2} - KB\sin\gamma = 0 \Rightarrow$ Gives S

For $L = 800$ lb, $\overline{AO'} = 8.75''$, $\overline{AB} = 8''$, $\overline{BK} = 9''$, and $\overline{AC} = 22.5''$, the following plots are constructed:



$R_{max} = 2490$ lb @ $\theta = 0$

$|S|_{max} = 259$ lb @ $\theta = 81.6°$

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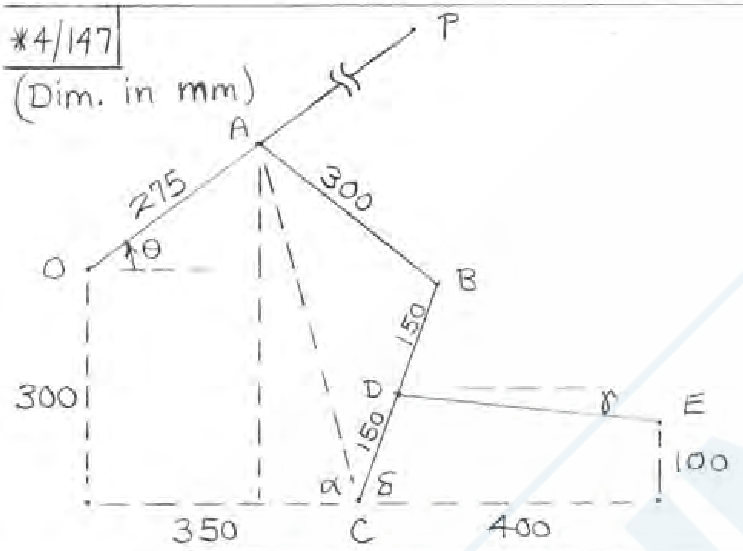
(Dim. in mm)



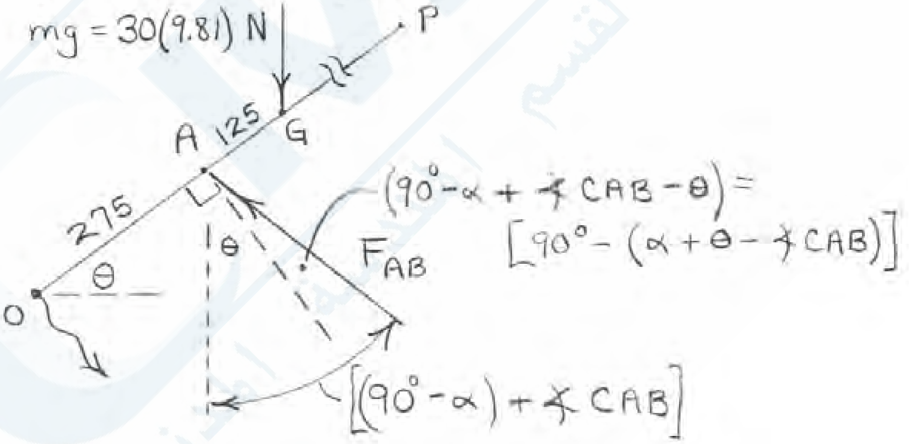
$$\overline{AC} = \sqrt{(350 - 275\cos\theta)^2 + (300 + 275\sin\theta)^2}$$

$$\measuredangle ABC = \cos^{-1}\left[\frac{\overline{AB}^2 + \overline{BC}^2 - \overline{AC}^2}{2\,\overline{AB}\,\overline{BC}}\right]$$

$$\alpha = \tan^{-1}\left[\frac{300 + 275\sin\theta}{350 - 275\cos\theta}\right]$$

$$\measuredangle CAB = \measuredangle ACB = \frac{180^\circ - \measuredangle ABC}{2}$$

$$\delta = 180^\circ - \alpha - \measuredangle ACB$$



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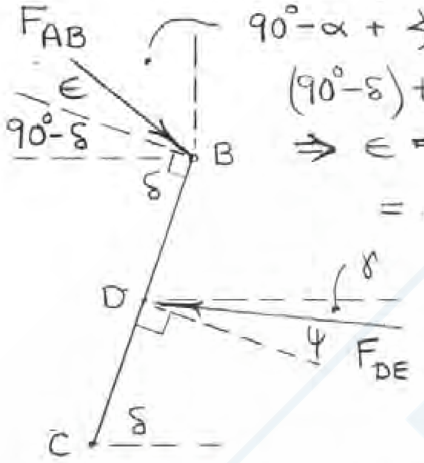
$\circlearrowleft + \sum M_O = 0: \quad F_{AB} \cos\left[90^\circ - (\alpha + \theta - \measuredangle CAB)\right] \overline{OA} - mg\,\overline{OG} \cos\theta = 0$

$$F_{AB} = \frac{mg\,\overline{OG} \cos\theta}{\overline{OA} \sin(\alpha + \theta - \measuredangle CAB)}$$



$$(90^\circ - \delta) + (\epsilon) + (90^\circ - \alpha + \measuredangle CAB) = 90^\circ$$

$$\Rightarrow \epsilon = \alpha + \delta - \measuredangle CAB - 90^\circ$$

$$= \alpha + (180^\circ - \alpha - \measuredangle ACB) - \measuredangle CAB - 90^\circ$$

$$\epsilon = 90^\circ - 2\measuredangle ACB$$

Also, $\gamma + \psi = 90^\circ - \delta$

$$\psi = 90^\circ - \delta - \gamma$$

$$\gamma = \tan^{-1}\left[\frac{150 \sin\delta - 100}{400 - 150\cos\delta}\right]$$

$\circlearrowleft + \sum M_C = 0: \; -F_{AB} \cos\epsilon\,(\overline{CB}) + F_{DE} \cos\psi\,(\overline{CD}) = 0$

$$F_{DE} = \frac{F_{AB} \cos\epsilon\,(\overline{CB})}{\overline{CD} \cos\psi}$$

To find θ_{max}, set the above expression for $\overline{AC}$ to $\overline{AC} = 600$ mm and solve for $\theta_{max} = 65.9^\circ$. Numerically evaluate the above expressions for $0 \le \theta \le \theta_{max}$ to obtain the following plot:

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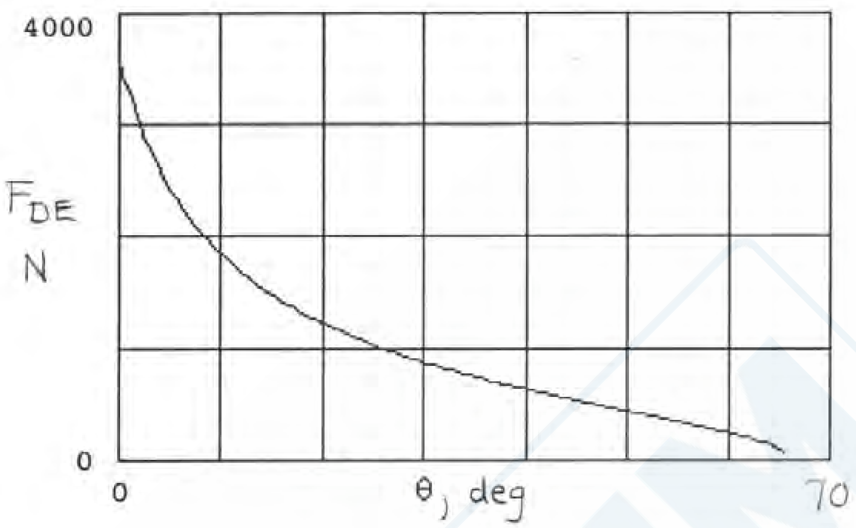

$(F_{DE})_{max} = 3580$ N @ $\theta = 0$

$(F_{DE})_{min} = 0$ @ $\theta_{max} = 65.9°$ (links AB and BC are collinear and serve as (an unstable!) prop for the door)

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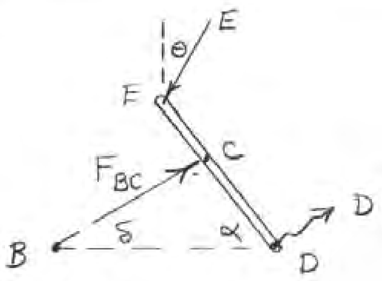

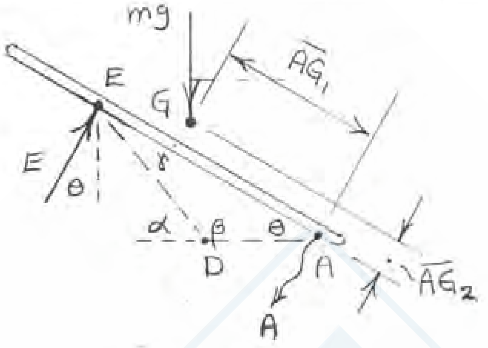

$$\frac{\sin\gamma}{\overline{AD}} = \frac{\sin\theta}{\overline{DE}}, \quad \gamma = \sin^{-1}\left(\frac{\overline{AD}\sin\theta}{\overline{DE}}\right)$$

$$\beta = 180 - (\theta + \gamma), \quad \alpha = 180 - \beta$$

$$\overline{AE} = \sqrt{\overline{DE}^2 + \overline{AD}^2 - 2(\overline{DE})(\overline{AD})\cos\beta}$$

$$\overline{BC} = \sqrt{\overline{BD}^2 + \overline{CD}^2 - 2(\overline{BD})(\overline{CD})\cos\alpha}$$

$$\frac{\sin\alpha}{\overline{BC}} = \frac{\sin\delta}{\overline{CD}}, \quad \delta = \sin^{-1}\left(\frac{\overline{CD}\sin\alpha}{\overline{BC}}\right)$$

Conveyor: $\circlearrowleft \Sigma M_A = 0: \; -E(\overline{AE}) + mg(\overline{AG_1})\cos\theta$

$$- mg(\overline{AG_2})\sin\theta = 0 \quad (1)$$

DE: $\circlearrowleft \Sigma M_D = 0:$

$$E\cos\theta(\overline{DE}\cos\alpha) + E\sin\theta(\overline{DE}\sin\alpha)$$

$$- F_{BC}\sin\delta(\overline{BD}) = 0 \quad (2)$$

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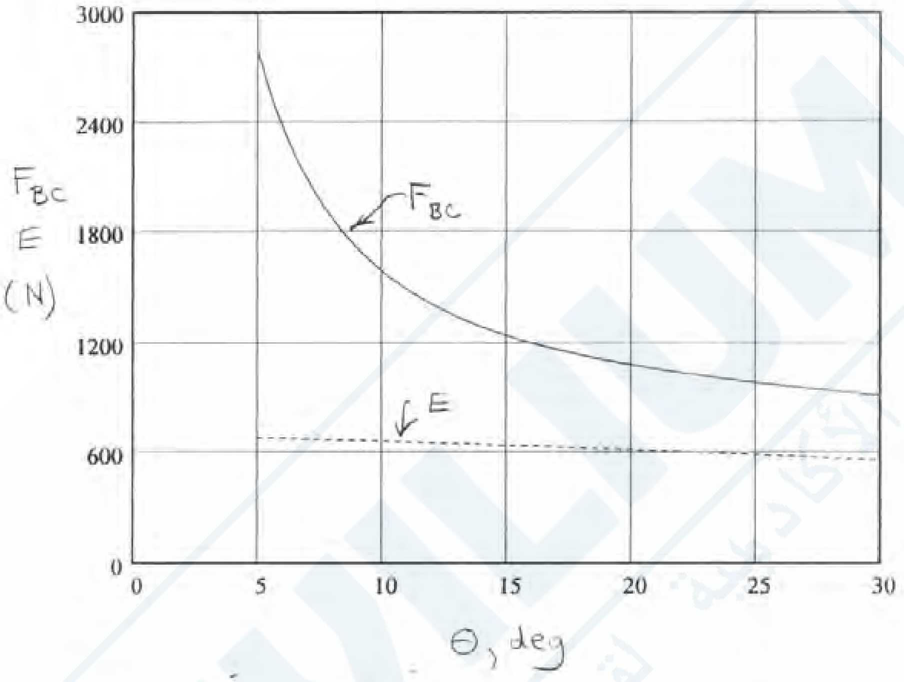
Solve Eqs. (1) & (2) for E and F_{BC} as functions of θ for the values $\overline{AD} = 1060$ mm, $\overline{BD} = 1660$ mm, $\overline{DE} = 1945$ mm, $\overline{CD} = 1150$ mm, $\overline{AG_1} = 2130$ mm, $\overline{AG_2} = 500$ mm, and $mg = 100(9.81)$ N to obtain the following plot:



At $\theta = 5°$, $\underline{(F_{BC})_{max} = 2800 \text{ N}}$

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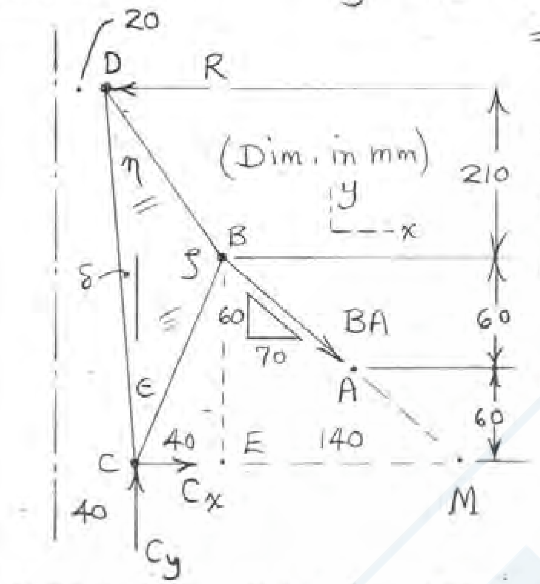
***4/149** $\theta = 0$:

$$C_y = \frac{pA}{2} = \frac{p\pi r^2}{2} = \frac{35(10^6)\pi(0.050)^2}{2} = 137\,400 \text{ N}$$



$$\delta = \tan^{-1}\frac{20}{330} = 3.47°$$

$$\overline{CD} = \sqrt{20^2 + 330^2} = 331 \text{ mm}$$

Similarly, $\overline{BD} = 218$ mm

$\overline{BC} = 126.5$ mm

$$\circlearrowleft \Sigma M_M = 0: \; R(330) - 137\,400(180) = 0, \; \underline{R = 75\,000 \text{ N}}$$

$$\Sigma F_y = 0: \; -BA\sin\left(\tan^{-1}\frac{6}{7}\right) + 137\,400 = 0, \; \underline{BA = 211\,000 \text{ N}}$$

$$\Sigma F_x = 0: \; -75\,000 + 211\,000\cos\left(\tan^{-1}\frac{6}{7}\right) + C_x = 0$$

$$\underline{C_x = -85\,400 \text{ N}}$$

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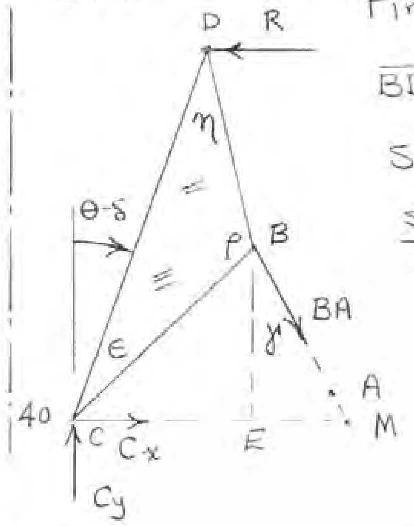
$\theta > 0$



Find ϵ, ρ, and η:

$\overline{BD}^2 = \overline{CD}^2 + \overline{BC}^2 - 2\,\overline{CD}\,\overline{BC}\cos\epsilon$

Solve for ϵ, then use

$\dfrac{\sin\rho}{\overline{CD}} = \dfrac{\sin\epsilon}{\overline{BD}}$ for ρ

$\dfrac{\sin\rho}{\overline{CD}} = \dfrac{\sin\eta}{\overline{BC}}$ for η

$\overline{CE} = \overline{BC}\sin(\theta-\delta+\epsilon)$

$\overline{BE} = \overline{BC}\cos(\theta-\delta+\epsilon)$

$110 = \overline{AB}\sin\gamma + \overline{BC}\sin(\theta-\delta+\epsilon) \Rightarrow$ Solve for γ

Then $\overline{EM} = \overline{BE}\tan\gamma$ and $\overline{CM} = \overline{CE} + \overline{EM}$

$\circlearrowleft + \Sigma M_M = 0: \quad R(\overline{CD}\cos(\theta-\delta)) - C_y(\overline{CM}) = 0$

$$R = \frac{C_y\,\overline{CM}}{\overline{CD}\cos(\theta-\delta)}$$

$\Sigma F_y = 0: \quad C_y - BA\cos\gamma = 0, \quad BA = \dfrac{C_y}{\cos\gamma}$

$\Sigma F_x = 0: \quad -R + BA\sin\gamma + C_x = 0, \quad C_x = R - BA\sin\gamma$

Solve all of the above in sequential order to obtain the plots below. Note that

$R_{min} = 49\,400$ N @ $\theta = 23.2°$

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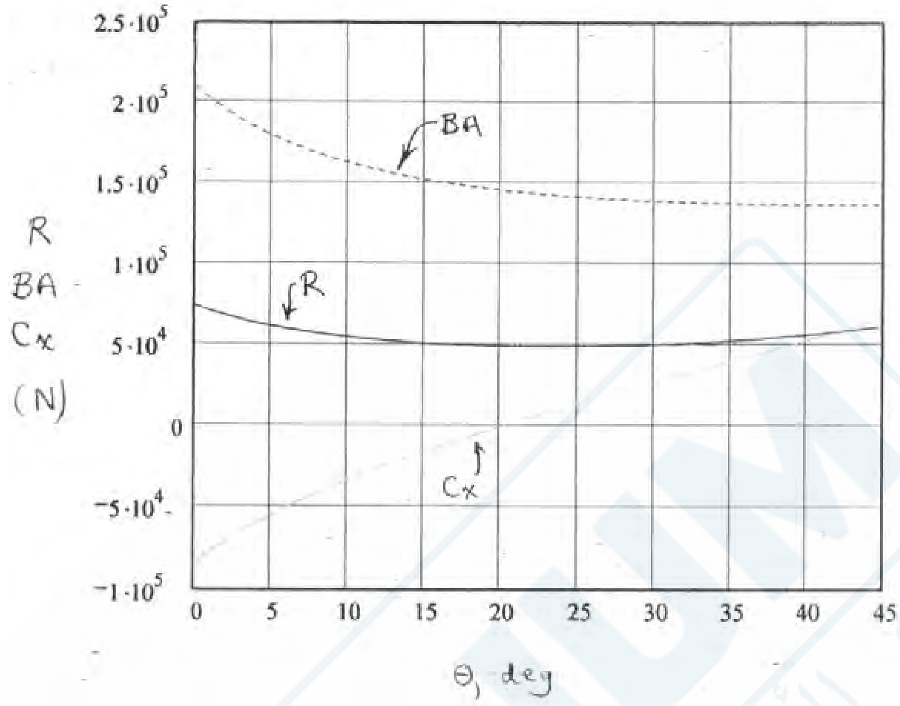

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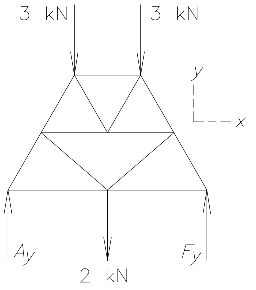
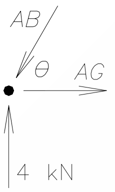


By symmetry, $A_y = F_y = 4$ kN

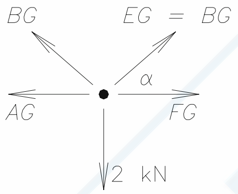
Joint *A*:



$$\theta = \cos^{-1}\frac{1}{2} = 60°$$

$$\begin{cases} \Sigma F_y = 0: \; 4\text{ kN} - AB\sin 60° = 0, \; AB = 4.62\text{ kN } C \\ \Sigma F_x = 0: \; AG - 4.62\cos 60° = 0, \; AG = 2.31\text{ kN } T \end{cases}$$

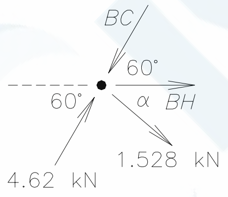
Joint *G*:



$$\alpha = \tan^{-1}\frac{2\sin 60°}{2} = 40.9°$$

$$\Sigma F_y = 0: \; 2BG\sin 40.9° - 2 = 0, \; \underline{BG = 1.528\text{ kN } T}$$

Joint *B*:



$$\Sigma F_y = 0: \; 4.62\sin 60° - BC\sin 60° - 1.528\sin 40.9° = 0$$

$$\underline{BC = 3.46\text{ kN } C}$$

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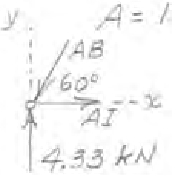
4/16 Truss as a whole

$\Sigma M_F = 0;\ 2(a + \frac{a}{2}) + 4(2a + \frac{a}{2}) - A(3a) = 0$

$A = 13/3 = 4.33\ kN$

Joint A



$\Sigma F_y = 0;\ AB \sin 60° - 4.33 = 0$

$AB = 5.00\ kN\ C$

$\Sigma F_x = 0;\ AI - 5 \cos 60° = 0$

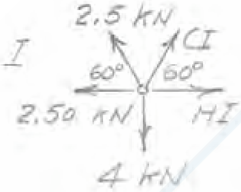
$AI = 2.50\ kN\ T$

Joint B

$\Sigma F_{y'} = 0;\ 5.00 \cos 60° - BI = 0$

$\underline{BI = 2.50\ kN\ T}$

Joint I



$\Sigma F_y = 0;\ (CI + 2.5) \sin 60° - 4 = 0$

$\underline{CI = 2.12\ kN\ T}$

$\Sigma F_x = 0;\ HI + 2.12 \cos 60°$
$- 2.50 - 2.5 \cos 60° = 0$

$\underline{HI = 2.69\ kN\ T}$

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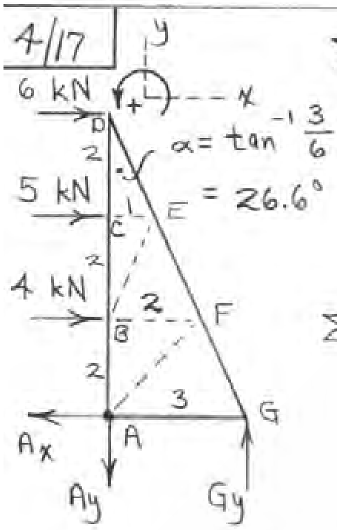


$\Sigma M_A = 0: \; -6(6) - 5(4) - 4(2) + G_y(3) = 0$

$G_y = 21.3 \text{ kN}$

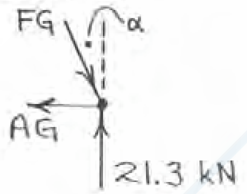
$\Sigma F_y = 0: \; -A_y + G_y = 0$

$A_y = 21.3 \text{ kN}$

$\Sigma F_x = 0: \; 6 + 5 + 4 - A_x = 0$

$A_x = 15 \text{ kN}$

Joint G:



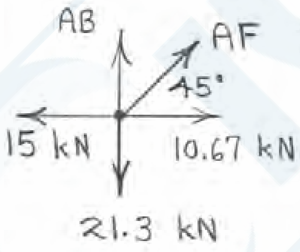
$\Sigma F_y = 0: \; 21.3 - FG \cos\alpha = 0$

$FG = 23.9 \text{ kN } C$

$\Sigma F_x = 0: \; 23.9 \sin\alpha - AG = 0$

$AG = 10.67 \text{ kN } T$

Joint A:



$\Sigma F_x = 0: \; 10.67 - 15 + AF \cos 45^\circ = 0$

$AF = 6.13 \text{ kN } T$

$\Sigma F_y = 0: \; AB + 6.13 \sin 45^\circ - 21.3 = 0$

$AB = 17 \text{ kN } T$

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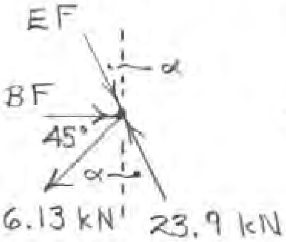
Joint F:



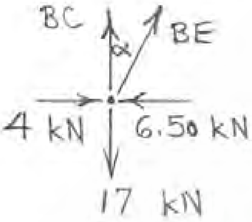
$\Sigma F_y = 0: -EF\cos\alpha + 23.9\cos\alpha$

$-6.13\sin 45° = 0, \quad EF = 19.01 \text{ kN C}$

$\Sigma F_x = 0: 19.01\sin\alpha + BF - 6.13\cos 45°$

$-23.9\sin\alpha = 0, \quad \underline{BF = 6.50 \text{ kN C}}$

Joint B:



$\Sigma F_x = 0: BE\sin\alpha + 4 - 6.50 = 0$

$\underline{BE = 5.59 \text{ kN T}}$

From inspection of joint C, $\underline{CE = 5 \text{ kN C}}$

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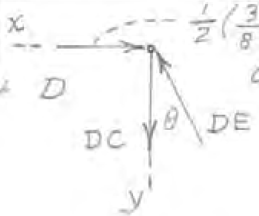
Joint D



$\frac{1}{2}\left(\frac{3}{8}\right)800 = 150\ \text{lb}$; $\theta = \tan^{-1}\frac{1}{2} = 26.57°$

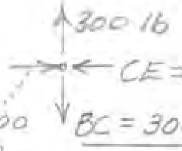
$\cos\theta = 2/\sqrt{5}$, $\sin\theta = 1/\sqrt{5}$

$\Sigma F_x = 0$; $DE\left(\frac{1}{\sqrt{5}}\right) - 150 = 0$

$DE = 335\ \text{lb}\ C$

$\Sigma F_y = 0$; $DC = 335(2/\sqrt{5}) = 300\ \text{lb}\ T$

Joint C



$\frac{5}{8}(800) = 500\ \text{lb}$

$\underline{BC = 300\ \text{lb}}\ T$

Joint E

$\Sigma F_{x'} = 0$; $500\left(\frac{2}{\sqrt{5}}\right) - BE\sin 2\theta = 0$

$\sin 2\theta = 0.8$

$BE = \frac{1000}{\sqrt{5}}\frac{1}{0.8} = \underline{559\ \text{lb}\ T}$

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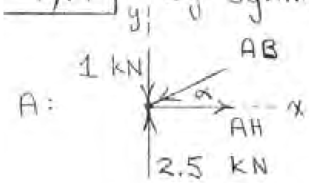
4/19 By symmetry, $A = E = 2.5$ kN; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$

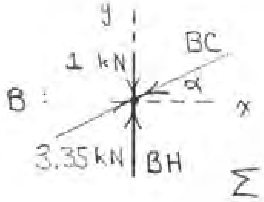


$\Sigma F_y = 0: \; 2.5 - 1 - AB \sin\alpha = 0$

$\underline{AB = 3.35 \text{ kN} \quad C}$

$\Sigma F_x = 0: \; -3.35 \cos\alpha + AH = 0$

$\underline{AH = 3 \text{ kN} \quad T}$

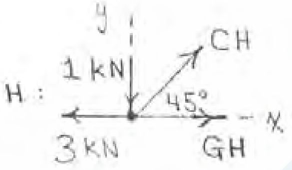


$\Sigma F_x = 0: \; 3.35 \cos\alpha - BC \cos\alpha = 0$

$\underline{BC = 3.35 \text{ kN} \quad C}$

$\Sigma F_y = 0: \; -1 + (3.35 - 3.35)\sin\alpha + BH = 0$

$\underline{BH = 1 \text{ kN} \quad C}$



$\Sigma F_y = 0: \; -1 + CH \sin 45° = 0$

$\underline{CH = 1.414 \text{ kN} \quad T}$

$\Sigma F_x = 0: \; -3 + 1.41 \cos 45° + GH = 0$

$\underline{GH = 2 \text{ kN} \quad T}$

By inspection of joint G and $\Sigma F_y = 0$, $\underline{CG = 0}$.

By symmetry,

$DE = AB = 3.35$ kN C

$CD = BC = 3.35$ kN C

$EF = AH = 3$ kN T

$DF = BH = 1$ kN C

$CF = CH = 1.414$ kN T

$FG = GH = 2$ kN T

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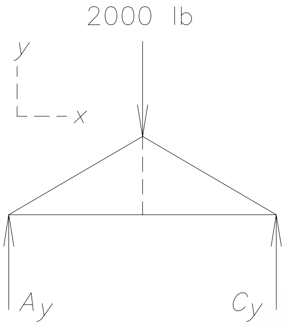
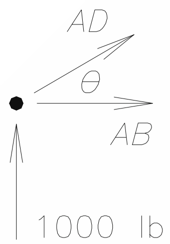


By symmetry, $A_y = C_y = 1000$ lb

Joint *A*:



$$\theta = \tan^{-1}\frac{7}{12} = 30.3°$$

$$\begin{cases} \Sigma F_y = 0: \; AD\sin 30.3° + 1000 = 0, \; \underline{AD = -1985 \text{ lb } (C)} \\ \Sigma F_x = 0: \; AD\cos 30.3° + AB = 0, \; \underline{AB = 1714 \text{ lb } T} \end{cases}$$

By symmetry: *CD* = *AC* = 1985 lb *C*
BC = *AB* = 1714 lb *T*

By inspection of joint *B*, *BD* is a zero-force member.

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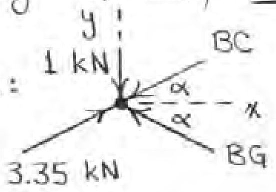
4/20 By symmetry, $A = E = 2.5\text{ kN}$; $\alpha = \tan^{-1}\left(\frac{2}{4}\right) = 26.6°$

Joint A analysis same as Prob. 4/19: $\begin{cases} AB = 3.35\text{ kN } C \\ AH = 3.00\text{ kN } T \end{cases}$

By inspection, $\underline{BH = 0}$ and $\underline{GH = AH}$.

B:

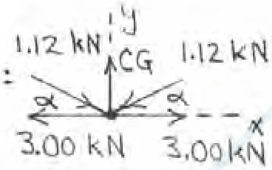


$\Sigma F_y = 0: -1 + 3.35 \sin\alpha + BG \sin\alpha - BC \sin\alpha = 0$

$\Sigma F_x = 0: 3.35 \cos\alpha - BC \cos\alpha - BG \cos\alpha = 0$

$\Rightarrow$ $\underline{BC = 2.24\text{ kN } C, \quad BG = 1.118\text{ kN } C}$

G:



$\Sigma F_y = 0: \; CG - 2(1.12)\sin\alpha = 0$

$\underline{CG = 1.00\text{ kN } T}$

By symmetry,
$$\begin{cases} DE = AB = 3.35\text{ kN } C \\ CD = BC = 2.24\text{ kN } C \\ EF = AH = 3.00\text{ kN } T \\ DF = BH = 0 \\ FG = GH = 3.00\text{ kN } T \\ DG = BG = 1.118\text{ kN } C \end{cases}$$

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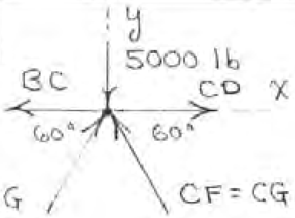
C :



Arguing symmetry,

$\Sigma F_y = 0$: $2CG \sin 60° - 5000 = 0$, $\underline{CG = 2890 \text{ lb } C}$

G :

By inspection,

$\underline{AG = 2890 \text{ lb } C}$

$\underline{BG = 0}$

From joint B : $\underline{AB = 0, \ BC = 0}$.

(Right truss symmetric to left one.)

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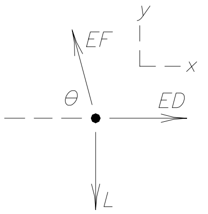
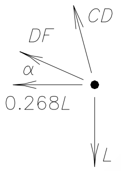
Joint E:



$$\theta = \frac{180° - 30°}{2} = 75°$$

$$\begin{cases} \Sigma F_y = 0: \; EF \sin 75° - L = 0, \; EF = 1.035L \; T \\ \Sigma F_x = 0: \; -1.035L \cos 75° + ED = 0, \; ED = 0.268L \; T \end{cases}$$

Joint D:



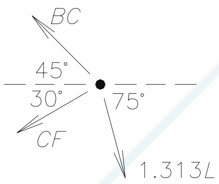
$$\overline{DF}^2 = R^2 + 4R^2 - 2(R)(2R)\cos 30°, \; \overline{DF} = 1.239R$$

$$\frac{\sin 30°}{1.239R} = \frac{\sin \alpha}{R}, \; \alpha = 23.8°$$

$$\begin{cases} \Sigma F_x = 0: \; -0.268L - DF \cos 23.8° - CD \cos 75° = 0 \\ \Sigma F_y = 0: \; -L + DF \sin 23.8° + CD \sin 75° = 0 \end{cases}$$

Solve simultaneously to obtain: $CD = 1.313L \; T$, $DF = 0.664L \; C$

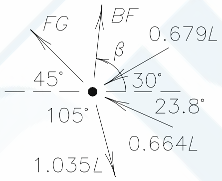
Joint C:



$$\begin{cases} \Sigma F_x = 0: \; -BC \cos 45° - CF \cos 30° + 1.313L \cos 75° = 0 \\ \Sigma F_y = 0: \; BC \sin 45° - CF \sin 30° - 1.313L \sin 75° = 0 \end{cases}$$

Solve simultaneously to obtain: $CF = -0.674L \; (C)$

Joint F:



$$\beta = 30° + 23.8° + 30° = 83.8°$$

$$\begin{cases} \Sigma F_x = 0: \; -FG \cos 45° + BF \cos \beta + EF \cos 75° + CF \cos 30° + DF \cos \alpha = 0 \\ \Sigma F_y = 0: \; FG \sin 45° + BF \sin \beta - EF \sin 75° + CF \sin 30° - DF \sin \alpha = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{BF = 1.814L \; T}$

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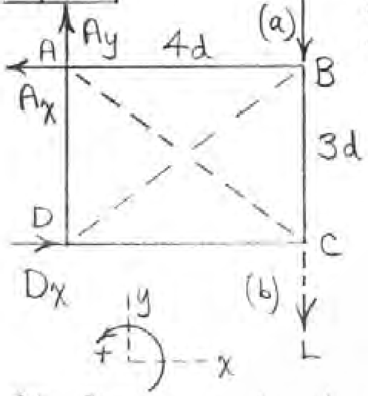


$\Sigma F_y = 0: \ A_y - L = 0, \quad A_y = L$

$\Sigma M_A = 0: \ D_x(3d) - L(4d) = 0$

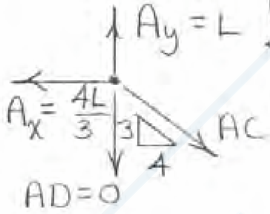
$D_x = \frac{4L}{3}$

$\Sigma F_x = 0: \ A_x = D_x = \frac{4L}{3}$

(Reactions hold for (a) & (b).)

(a) Assume that BD goes slack. From an inspection of joint B, $\underline{AB = 0}$ and $\underline{BC = L \ C}$.
Similarly, from joint D, $\underline{AD = 0}$ and $\underline{CD = \frac{4L}{3} \ C}$.

Joint A:



$\Sigma F_y = 0: \ L - \frac{3}{5} AC = 0, \quad \underline{AC = \frac{5L}{3} \ T}$

$\Sigma F_x = 0: -\frac{4L}{3} + \frac{5L}{3} \cdot \frac{4}{5} = 0$ ✓

Because AC is in tension, assumption is valid.

(b) Assume that BD goes slack. From joint B, $\underline{AB = BC = 0}$. From joint D, $\underline{AD = 0}$ & $\underline{CD = \frac{4L}{3} \ C}$.

Joint C:

$\Sigma F_y = 0: \ AC\left(\frac{3}{5}\right) - L = 0, \quad \underline{AC = \frac{5L}{3} \ T}$

$\Sigma F_x = 0: \ \frac{4L}{3} - \frac{5L}{3}\left(\frac{4}{5}\right) = 0$ ✓

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4/24 m = no. of two-force members
j = no. of joints

(a) [m+3 = 13] > [2j = 12] so redundant members.
Remove one member connecting B, C, D, and E.

(b) [m+3 = 12] = [2j = 12] so sufficient no. of members, but redundancy in external supports. Place A or F on roller.

(c) [m+3 = 9] > [2j = 8] so redundant members. Supports are also redundant. Remove AE or BE. Supports are then OK.

(d) [m+3 = 12] = [2j = 12] so sufficient no. of members, but redundancy in external supports. Place B on roller or remove member CD.

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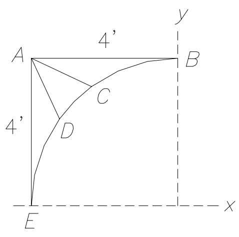


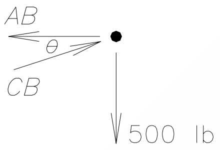
Location of C: $x_c = -\sqrt{4^2 - y_c^{\ 2}}$

$$\overline{AC}^2 = (4 + x_c)^2 + (4 - y_c)^2$$

$$x_c = -(4 - AC\cos 25°),\ y_c = 4 - AC\sin 25°$$

Solve to obtain $\overline{AC} = 1.815$ ft

Joint *B*:



$$\theta = \tan^{-1}\frac{1.815\sin 25°}{4 - 1.815\cos 25°} = 18.04°$$

$$\begin{cases}\Sigma F_y = 0:\ CB\sin 18.04° - 500 = 0,\ CB = 1615 \text{ lb } C\\ \Sigma F_x = 0:\ 1615\cos 18.04° - AB = 0,\ AB = 1535 \text{ lb } T\end{cases}$$

Joint *C*:

$$\delta = \frac{180° - 40°}{2} = 70°,\ \beta = \delta - 25° = 45°$$

$$\begin{cases}\Sigma F_y = 0:\ AC\sin 25° - CD\sin 45° - 1615\sin 18.04° = 0\\ \Sigma F_x = 0:\ -AC\cos 25° - CD\cos 45° - 1615\cos 18.04° = 0\end{cases}$$

Solve simultaneously to obtain: $\underline{AC = -779 \text{ lb or } 779 \text{ lb } C}$

By symmetry across a line from the origin to point *A*, $\underline{AD = AC = 779 \text{ lb } C}$

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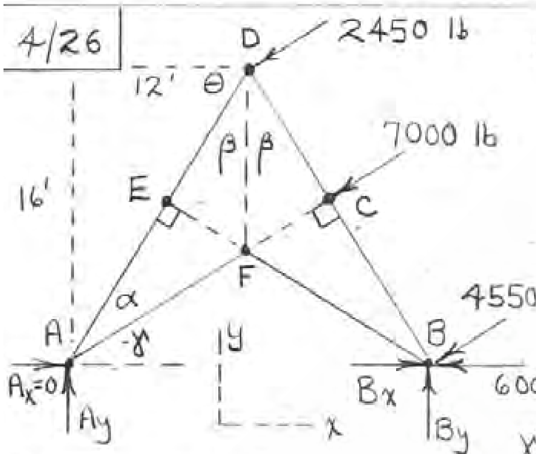

Geometry:

$\theta = \tan^{-1}\frac{16}{12} = 53.1°$

$\beta = 90 - \theta = 36.9°$

$\alpha = 90 - 2\beta = 16.26°$

$\overline{CD} = 20 \sin\alpha = 5.60$ ft

$\overline{CB} = 20 - \overline{CD} = 14.40$ ft

$\gamma = 90 - (90 - \theta) - \alpha = 36.9°$

Truss as a whole:

$\circlearrowleft \Sigma M_B = 0$: $7000(14.4) + 2450(20) - 24A_y = 0$

$A_y = 6240$ lb

$\Sigma F_y = 0$: $B_y + 6240 - (2450 + 7000 + 4550)\sin 36.9°$
$= 0$, $B_y = 2160$ lb

$\Sigma F_x = 0$: $B_x - (2450 + 7000 + 4550)\cos 36.9°$
$- 6000 = 0$, $B_x = 17{,}200$ lb

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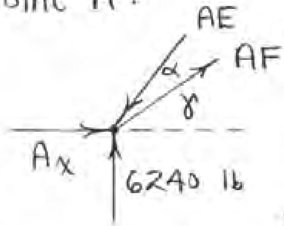
Joint A:



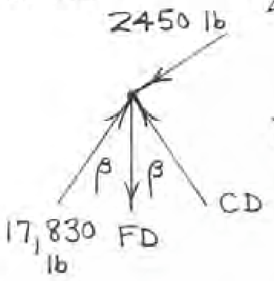
$\Sigma F_x = 0: -AE\cos 53.1° + AF\cos 36.9° = 0$

$\Sigma F_y = 0: 6240 - AE\sin 53.1° + AF\sin 36.9° = 0$

$\Rightarrow AF = 13{,}380$ lb T, $AE = 17{,}830$ lb C

From joint E, $ED = AE = 17{,}830$ lb C

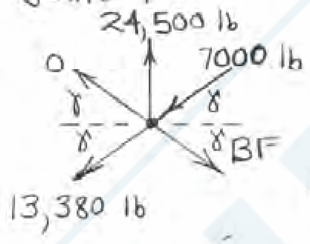
Joint D:



$\Sigma F_x = 0: 17{,}830\sin 36.9° - CD\sin 36.9° - 2450\cos 36.9° = 0$, $CD = 14{,}570$ lb C

$\Sigma F_y = 0: (14{,}570 + 17{,}830)\cos 36.9° - FD - 2450\sin 36.9° = 0$, $FD = 24{,}500$ lb T

Joint F:



$\Sigma F_x = 0: BF\cos 36.9° - (13{,}380 + 7000)\cos 36.9° = 0$, $BF = 20{,}400$ lb T

(Joint B checks.)

The maximum force occurs in member FD:

$FD = 24{,}500$ lb T

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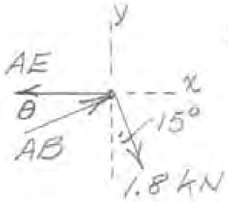
$\theta = \tan^{-1}\frac{1}{2} = 26.6°$

Joint A



$\Sigma F_y = 0;\ AB \sin 26.6° - 1.8 \cos 15° = 0$

$AB = 3.89$ kN C

$\Sigma F_x = 0;\ AE = 1.8 \sin 15° + 3.89 \cos\theta$

$AE = 3.94$ kN T

Joint E gives $EB = 0$

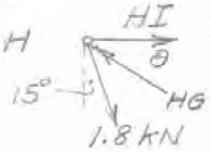
Joint B

$\Sigma F_{y'} = 0$ so $DB = 0$

$CB = 3.89$ kN C

without diagonals, FD would lengthen & CJ shorten, so FD is tension & $CJ = 0$

Joint H



$\Sigma F_y = 0;\ HG \sin 26.6° - 1.8 \cos 15° = 0$

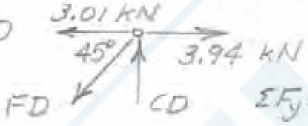
$HG = 3.89$ kN C

$\Sigma F_x = 0;\ HI + 1.8 \sin 15° - 3.89 \cos 26.6° = 0$

$HI = 3.01$ kN T

with $IG = GJ = JC = 0$, $JD = HI$

Joint D



$\Sigma F_x = 0;\ FD \cos 45° + 3.01 = 3.94$

$FD = 1.318$ kN T

$\Sigma F_y = 0;\ CD = 1.318 \sin 45° = 0.932$ kN C

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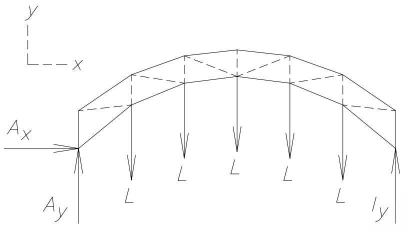


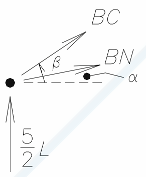
By symmetry, $A_y = I_y = 5/2L$, $A_x = 0$
By inspection of point A , AN = 0

Coordinate origin is at the center of the two concentric arcs.

Location of N: $x^2 + y^2 = 16^2$
For x_N = -8 m: $y_N^2 = -8^2 + 16^2$
y_N = 13.86 m N = (-8, 13.86) m

Location of A: $y_A^2 = -12^2 + 16^2$, y_A = 10.58 m, A = (-12, 10.58) m
Location of B: $y_B^2 = -12^2 + 18^2$ =, y_B = 13.42 m, B = (-12, 13.41) m
Location of C: $y_C^2 = -8^2 + 18^2$ =, y_C = 16.12 m, C = (-8, 16.12) m
Location of D: $y_D^2 = -4^2 + 18^2$ =, y_D = 17.55 m, C = (-4, 17.55) m
Location of M: $y_M^2 = -4^2 + 16^2$ =, y_M = 15.49 m, C = (-4, 15.49) m

Joint B:



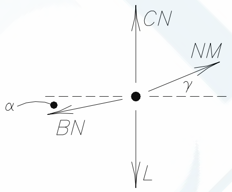
$$\alpha = \tan^{-1}\frac{13.86-13.42}{-8+12} = 6.28°$$

$$\beta = \tan^{-1}\frac{16.12-13.42}{-8+12} = 34.1°$$

$$\begin{cases} \Sigma F_x = 0: \ BC\cos 34.1° + BN\cos 6.28° = 0 \\ \Sigma F_y = 0: \ (5/2)L + BC\sin 34.1° + BN\sin 6.28° = 0 \end{cases}$$

Solve simultaneously to obtain: $BN = 4.44L\ T$, $BC = -5.32L\ (C)$

Joint N:



$$\gamma = \tan^{-1}\frac{15.49-13.86}{-4+8} = 22.2°$$

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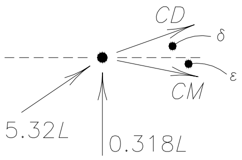
$$\begin{cases}\Sigma F_x = 0: \; -4.44L\cos 6.28° + NM\cos 22.2° = 0, \;\; NM = 4.76L\ T \\ \Sigma F_y = 0: \; -4.44L\sin 6.28° + 4.76\sin 22.2° + CN - L = 0, \;\; CN = 0.318L\ C\end{cases}$$

Joint C:



$$\delta = \tan^{-1}\frac{17.55-16.12}{-4+8} = 19.61°$$

$$\varepsilon = \tan^{-1}\frac{16.12-15.49}{8-4} = 8.99°$$

$$\begin{cases}\Sigma F_x = 0: \; 5.32L\cos 34.1° + CD\cos 19.61° + CM\cos 8.99° = 0 \\ \Sigma F_y = 0: \; 5.32L\sin 34.1° + 0.318L + CD\sin 19.61° - CM\sin 8.99° = 0\end{cases}$$

Solve simultaneously to obtain: $\underline{CM = 3.41L\ T}$

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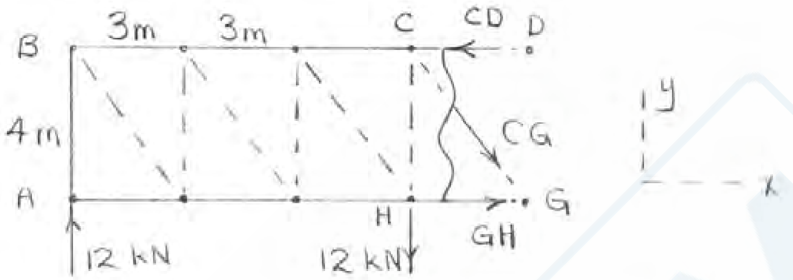
4/29 From truss as a whole, $A_y = F_y = 12$ kN and $A_x = 0$. Left section:



By inspection of $\Sigma F_y = 0$, $\underline{CG = 0}$.

$\circlearrowleft \Sigma M_C = 0: GH(4) - 12(9) = 0$, $\underline{GH = 27 \text{ kN } T}$

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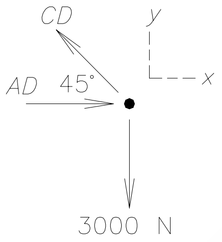
4/3

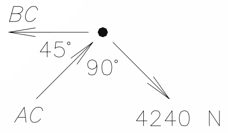
We can begin at joint D without finding the external reactions.

Joint *D*:



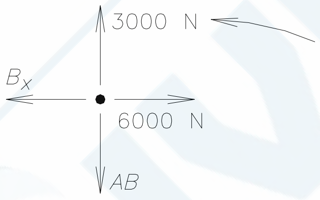
$$\begin{cases}\Sigma F_y = 0: \ CD\sin 45° - 3000 = 0, \ \underline{CD = 4240 \text{ N } T} \\ \Sigma F_x = 0: \ AD - CD\cos 45° = 0, \ \underline{AD = 3000 \text{ N } C}\end{cases}$$

Joint *C*:



$$\begin{cases}\Sigma F_y = 0: \ AC\sin 45° - 4240\sin 45° = 0, \ \underline{AC = 4240 \text{ N } C} \\ \Sigma F_x = 0: \ -BC + 2(4240\cos 45°) = 0, \ \underline{BC = 6000 \text{ N } T}\end{cases}$$

Joint *B*:



From equilibrium of entire truss

From $\Sigma F_y = 0$, $\underline{AB = 3000 \text{ N } T}$

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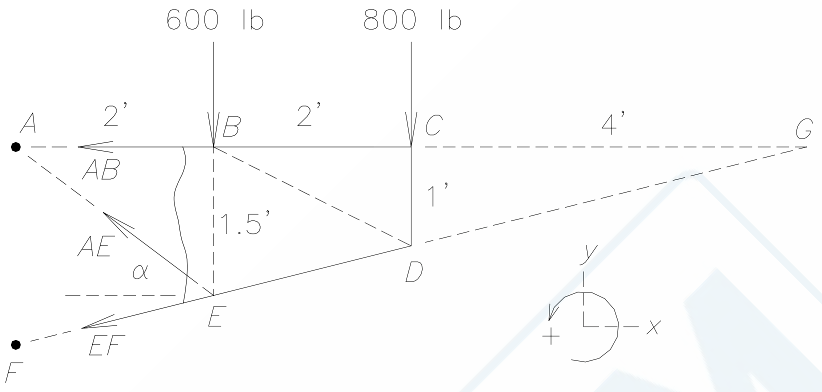

$$\alpha = \tan^{-1}\frac{1.5}{2} = 36.9°$$

$$\Sigma M_G = 0: \; 800(4) + 600(6) - AE\cos 36.9°(1.5) - AE\sin 36.9°(6) = 0$$

$$\underline{AE = 1417 \text{ lb } T}$$

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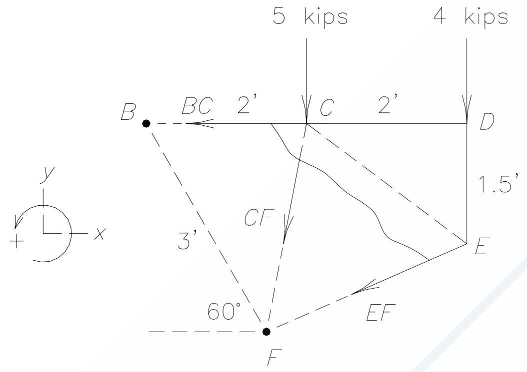

$$\Sigma M_F = 0: \; BC(3\sin 60°) - 5(0.5) - 4(2.5) = 0, \; \underline{BC = 4.81 \text{ kips } T}$$

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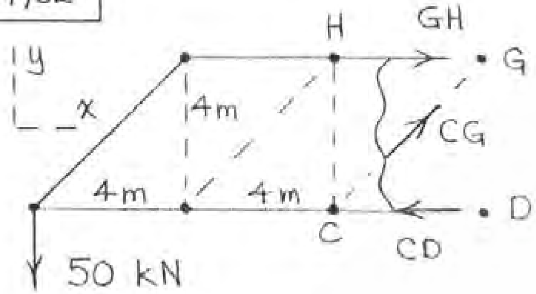

$\Sigma F_y = 0$: $CG \sin 45° - 50 = 0$, $CG = 70.7$ kN T

$\Sigma M_C = 0$: $GH(4) - 50(8) = 0$, $GH = 100$ kN T

All members except EF are statically determinate, so above solution is unaffected by the redundant support.

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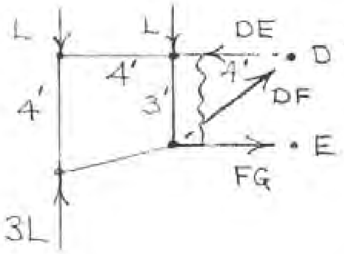
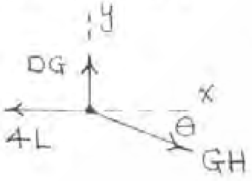
4/33 From entire truss, $A = B = 3L$.



$\circlearrowleft + \Sigma M_D = 0: \; FG(3) + L(4+8) - 3L(8) = 0, \quad FG = 4L \; T$

Joint G:



$\theta = \tan^{-1}\left(\frac{1}{4}\right) = 14.04°$

$\Sigma F_x = 0: \; -4L + GH \cos\theta = 0$

$GH = 4.12L \; T$

$\Sigma F_y = 0: \; DG - 4.12 \sin\theta = 0$

$\underline{DG = L \; T}$

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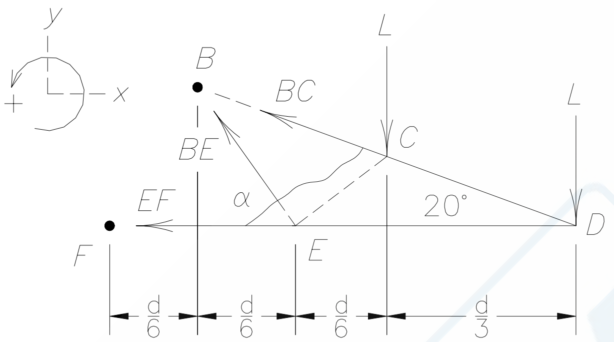

$$\alpha = \tan^{-1}\frac{\frac{4}{6}d\tan 20°}{\frac{d}{6}} = 55.5°$$

$$\Sigma M_D = 0: \; L\left(\frac{d}{3}\right) - BE\left(\frac{d}{2}\right)(\sin 55.5°) = 0$$

$$\underline{BE = 0.809L \; T}$$

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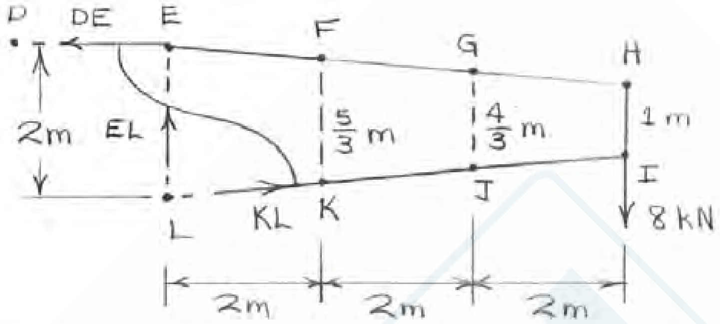
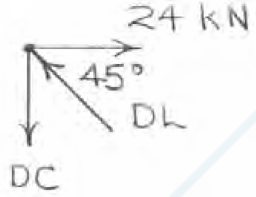


$\Sigma M_L = 0: \ DE(2) - 8(6) = 0,$ $\underline{DE = 24 \text{ kN } T}$

Joint D:



$\xrightarrow{+} \Sigma F_x = 0: \ 24 - DL \cos 45° = 0$

$\underline{DL = 33.9 \text{ kN } C}$

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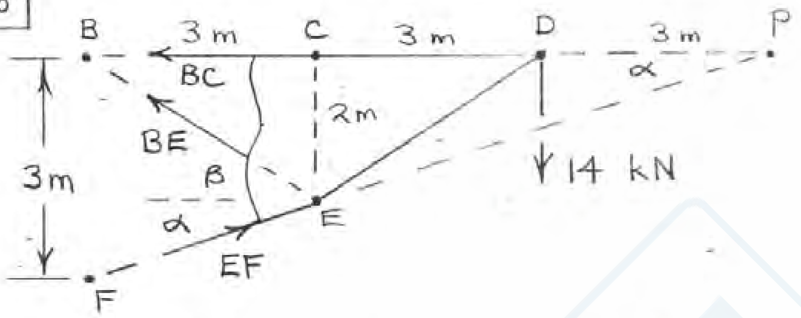

$\alpha = \tan^{-1}\left(\frac{2}{6}\right) = 18.43°, \quad \beta = \tan^{-1}\frac{2}{3} = 33.7°$

$\curvearrowleft + \Sigma M_E = 0: \quad BC(2) - 14(3) = 0, \quad \underline{BC = 21 \text{ kN T}}$

$\curvearrowleft + \Sigma M_P = 0: -BE \sin\beta(9) + 14(3) = 0, \quad \underline{BE = 8.41 \text{ kN T}}$

$\curvearrowleft + \Sigma M_B = 0: \quad EF \cos\alpha(3) - 14(6) = 0, \quad \underline{EF = 29.5 \text{ kN C}}$

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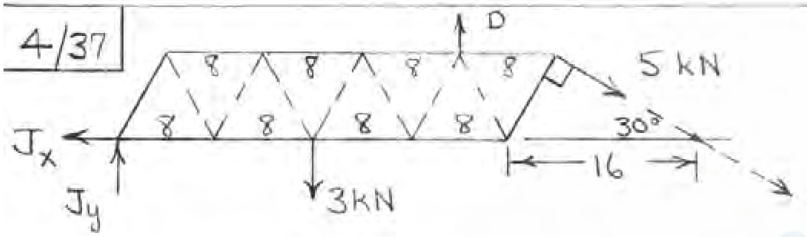
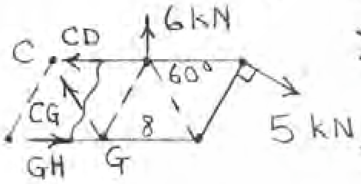


$\Sigma M_J = 0: \quad -3(16) + D(28) - (5 \sin 30°)(48) = 0$

$D = 6 \text{ kN}$



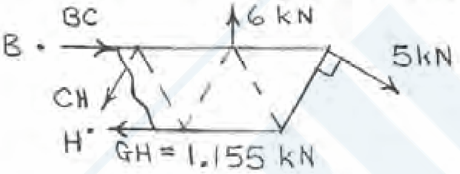
$\Sigma F_y = 0: \quad CG \sin 60° + 6 - 5 \sin 30° = 0$

$\underline{CG = -4.041 \text{ kN} \quad C}$

$\Sigma M_G = 0: \quad CD(8 \sin 60°) + 6(4) - (5 \sin 30°)(24) = 0, \quad \underline{CD = 5.20 \text{ kN} \quad T}$

$\Sigma M_C = 0: \quad GH(8 \sin 60°) + 6(8) - (5 \sin 30°)(16) = 0$

$GH = -1.155 \text{ kN} \quad T$



$\Sigma M_H = 0: \quad 6(12) - (5 \sin 30°)(32) - BC \, 8 \sin 60° = 0$

$\underline{BC = -1.155 \text{ kN} \quad T}$

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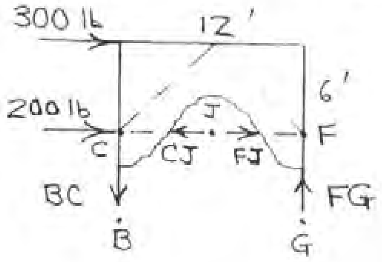

$\circlearrowleft + \Sigma M_C = 0: \; -300(6) + FG(12) = 0$

$\underline{FG = 150 \text{ lb } C}$

$\Sigma M_F = 0: \; -300(6) + BC(12) = 0$

$\underline{BC = 150 \text{ lb } T}$

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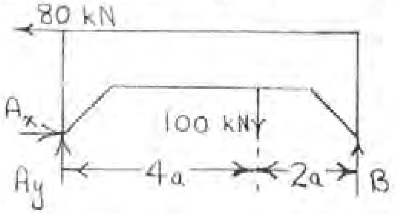
4/39 $\circlearrowleft \Sigma M_A = 0: \quad B(6a) - 100(4a) + 80(2a) = 0$

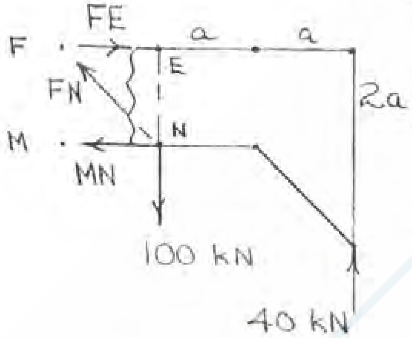
$B = 40 \text{ kN}$



For section,

$\Sigma F_y = 0: \quad \frac{FN}{\sqrt{2}} + 40 - 100 = 0$

$\underline{FN = 84.8 \text{ kN } T}$



$\Sigma M_E = 0: \quad 40(2a) - 84.8 \frac{a}{\sqrt{2}}$

$- MN(a) = 0, \quad \underline{MN = 20 \text{ kN } T}$

For section through GF & LM, $\Sigma F_y = 0$ gives

$\underline{GM = 84.8 \text{ kN } T}$.

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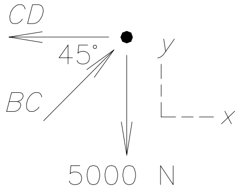
4/4

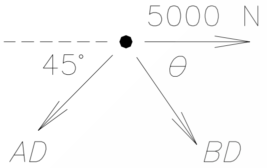
We can begin at joint C without finding the external reactions.

Joint *C*:



$$\begin{cases} \Sigma F_y = 0: \; BC \sin 45° - 5000 = 0, \; \underline{BC = 7070 \text{ N } C} \\ \Sigma F_x = 0: \; 7070 \cos 45° - CD = 0, \; \underline{CD = 5000 \text{ N } T} \end{cases}$$

Joint *D*:



$$\overline{DB}^2 = 5^2 + 6^2 - 2(5)(6)\cos 45°, \; \overline{DB} = 4.31 \text{ m}$$

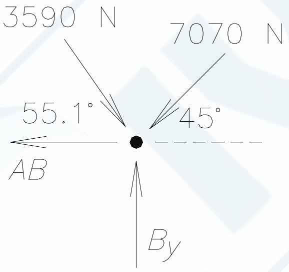
$$\frac{\sin\theta}{5} = \frac{\sin 45°}{4.31} \Rightarrow \theta = 55.1°$$

$$\begin{cases} \Sigma F_x = 0: \; 5000 + BD \cos 55.1° - AD \cos 45° = 0 \\ \Sigma F_y = 0: \; -AD \sin 45° - BD \sin 55.1° = 0 \end{cases}$$

Solve simultaneously to obtain: $\underline{BD = -3590 \text{ N or } 3590 \text{ N } C}$

$\underline{AD = 4170 \text{ N } T}$

Joint *B*:



$$\Sigma F_x = 0: \; 3590 \cos 55.1° - 7070 \cos 45° - AB = 0$$

$$\underline{AB = -2950 \text{ N or } 2950 \text{ N } C}$$

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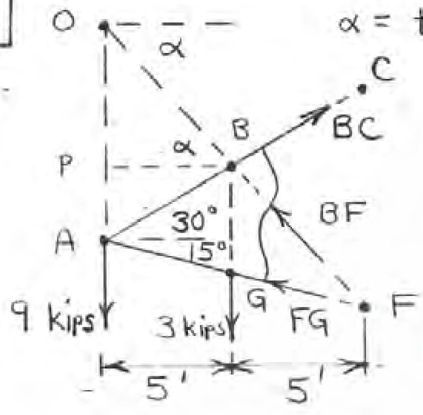

$$\alpha = \tan^{-1}\left[\frac{5\tan 30° + 2(5\tan 15°)}{5}\right] = 48.1°$$

$$\overline{AO} = \overline{AP} + \overline{PO} = \overline{AP} + \overline{PB}\tan\alpha = 5\tan 30° + 5\tan\alpha = 8.45'$$

$$\Sigma M_A = 0: (BF\cos\alpha)\,\overline{AO} - 3(5) = 0$$

$$\underline{BF = 2.66 \text{ kips } C}$$

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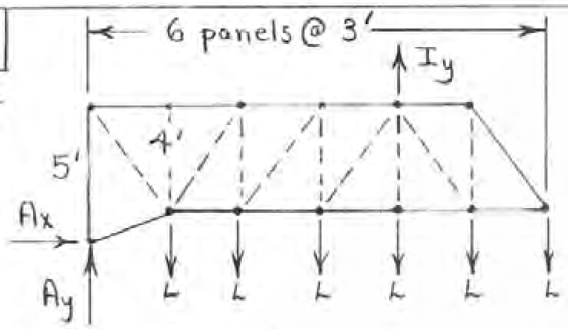
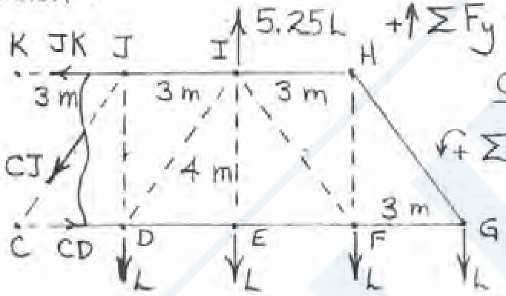
Entire truss:



$\circlearrowleft + \sum M_A = 0: \; I_y(12) - L(3+6+9+12+15+18) = 0$

$I_y = 5.25L$

Section :



$+\uparrow \sum F_y = 0: \; 5.25L - 4L - CJ\left(\frac{4}{5}\right) = 0$

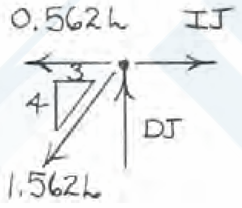
$CJ = 1.562L \;\; T$

$\circlearrowleft + \sum M_J = 0: \; CD(4) + 5.25L(3) - L(3+6+9) = 0$

$CD = 0.562L \;\; C$

From $\xrightarrow{+} \sum F_x = 0$, $JK = 0.562L \;\; T$.

Joint J:



$+\uparrow \sum F_y = 0: \; DJ - 1.562L\left(\frac{4}{5}\right) = 0$

$DJ = 1.250L \;\; C$

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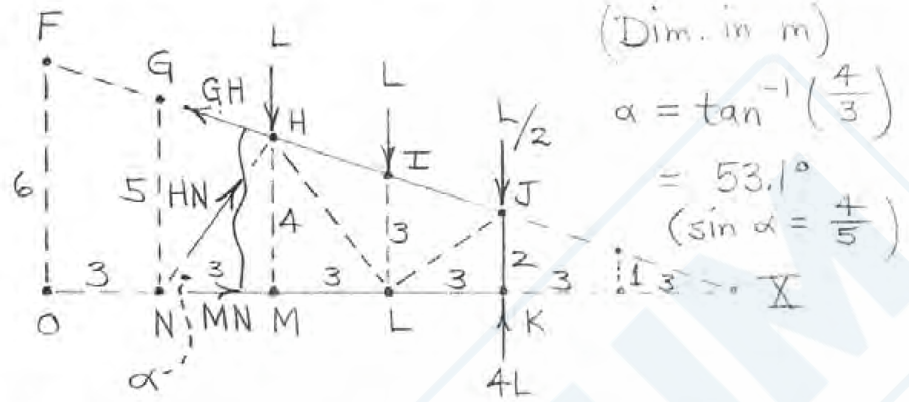
4/42 From the truss as a whole, the external reactions at A and K are 4L (up).



$$\circlearrowleft\!+\ \Sigma M_X = 0: \left(\frac{L}{2} - 4L\right)6 + L(9) + L(12)$$

$$-HN\left(\frac{4}{5}\right)(15) = 0, \quad \underline{HN = 0}$$

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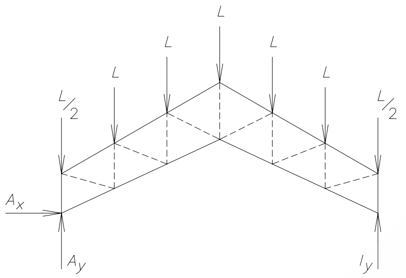
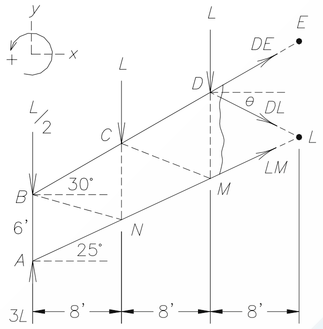


By symmetry, $A_y = I_y = 3L$

$\Sigma F_x = 0: \; A_x = 0$



$$\theta = \tan^{-1}\frac{6 + 16\tan 30° - 24\tan 25°}{8} = 26.8°$$

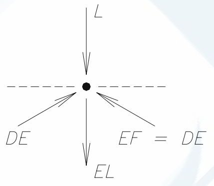
$$\Sigma M_L = 0: \; L(8) + L(16) + \frac{L}{2}(24) - 3L(24) - DE(24\tan 30° - 24\tan 25° + 6)\sin 60° = 0$$

$DE = -4.80L$ or $\underline{4.80L\ C}$

$$\begin{cases} \Sigma F_x = 0: \; -4.80L\cos 30° + DL\cos 26.8° + LM\cos 25° = 0 \\ \Sigma F_y = 0: \; -4.80L\sin 30° - DL\sin 26.8° + LM\sin 25° + 3L - 2L - \dfrac{L}{2} \end{cases}$$

Solve simultaneously to obtain: $\underline{DL = 0.0446L\ T}$, $\underline{LM = 4.54L\ T}$

Joint E:



$\Sigma F_y = 0: \; 2(4.80L\sin 30°) - L - EL = 0$

$\underline{EL = 3.80L\ T}$

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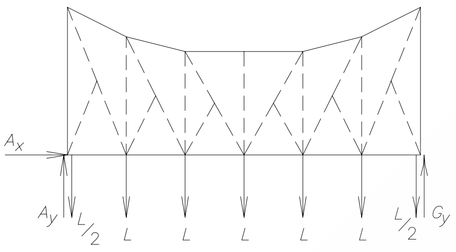
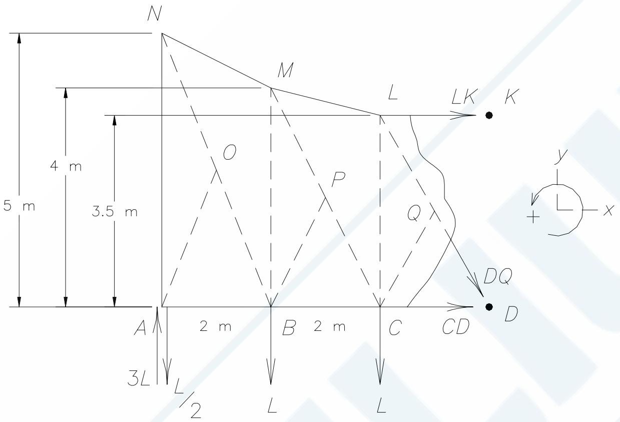


By symmetry, $A_y = G_y = 3L$

$\Sigma F_x = 0: \; A_x = 0$



$$\Sigma F_y = 0: \; 3L - 2L - \frac{L}{2} - DQ \sin\left(\tan^{-1}\frac{3.5}{2}\right) = 0, \; \underline{DQ = 0.576L \; T}$$

By inspection: $\underline{CQ = 0}$

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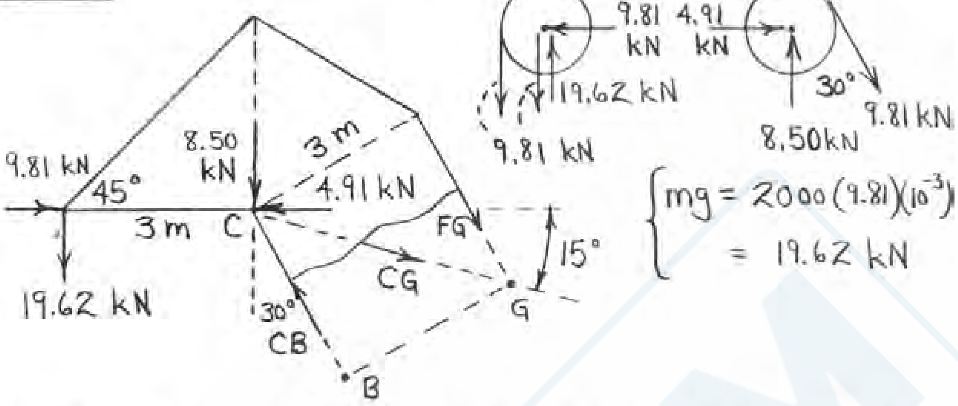

$$\left\{ \begin{aligned} mg &= 2000(9.81)(10^{-3}) \\ &= 19.62 \text{ kN} \end{aligned} \right.$$

$\circlearrowleft + \Sigma M_C = 0: \quad 19.62(3) - FG(3) = 0, \quad \underline{FG = 19.62 \text{ kN } T}$

$\circlearrowleft + \Sigma M_G = 0: \quad 19.62(3 + 3\sqrt{2}\cos 15°) - (9.81 - 4.91)(3\sqrt{2}\sin 15°)$

$+ 8.50(3\sqrt{2}\cos 15°) - CB(3) = 0$

$\underline{CB = 56.2 \text{ kN } C}$

$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad CG\cos 15° + 19.62\sin 30° - 56.2\sin 30°$

$+ 9.81 - 4.91 = 0, \quad \underline{CG = 13.87 \text{ kN } T}$

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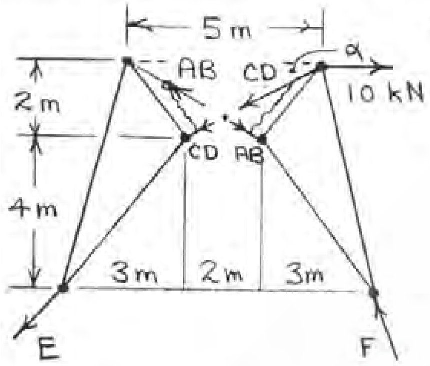
4/46 $\alpha = \tan^{-1}\left(\frac{2}{3.5}\right) = 29.7°$



I. $\Sigma M_E = 0: \; CD(4\cos\alpha) - CD(3\sin\alpha) - AB(6\cos\alpha) - AB(1.5\sin\alpha) = 0$

$CD = 3.00\,AB$

II. $\Sigma M_F = 0: \; 10(6) + AB(4\cos\alpha) - AB(3\sin\alpha) - CD(6\cos\alpha) - CD(1.5\sin\alpha) = 0$

$60 + 1.985\,AB - 5.954\,CD = 0$

Solving simultaneously, $\underline{AB = 3.78 \text{ kN } C}$

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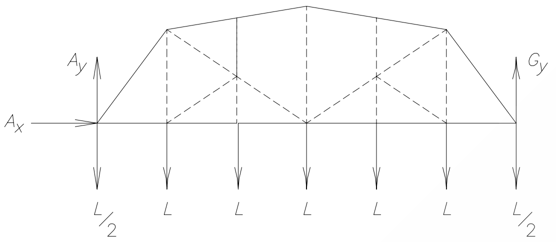
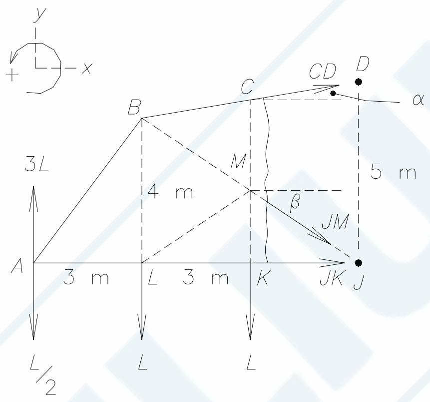


By symmetry, $A_y = G_y = 3L$ $\Sigma F_x = 0: \; A_x = 0$



$$\alpha = \tan^{-1}\frac{1}{6} = 9.46°, \; \beta = \tan^{-1}\frac{4}{6} = 33.7°$$

$$\Sigma M_J = 0: \; \frac{L}{2}(9) + L(6) + L(3) - 3L(9) - CD(5)\left[\sin(90° - 9.46°)\right] = 0$$

$$CD = -2.74L \; (C)$$

$$\Sigma F_y = 0: \; 3L + CD\sin 9.46° - \frac{L}{2} - 2L - JM\sin 33.7° = 0, \; \underline{JM = 0.0901L \; T}$$

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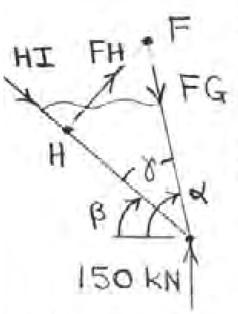
4/48 By symmetry, $A = G = 150$ kN



$\curvearrowleft \Sigma M_F = 0: \; 150(4) + HI\,(7.902) = 0$

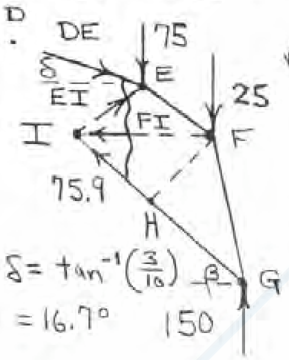
$\underline{HI = -75.9 \text{ kN } T}$

$\left(\text{Note: } \alpha = \tan^{-1}\left(\frac{14}{4}\right) = 74.1°\right.$

$\beta = \tan^{-1}\left(\frac{14}{16}\right) = 41.2°$

$\gamma = \alpha - \beta = 32.9°$

Then $\left. d_\perp = FG \sin\gamma = \sqrt{14^2 + 4^2}\,\sin\gamma = 7.902 \text{ m}\right)$



$\curvearrowleft^{+} \Sigma M_E = 0: \; -25(6) + 150(10)$

$- FI(4) - (75.9 \sin\beta)(6) - (75.9 \cos\beta)(4)$

$= 0, \quad \underline{FI = 205 \text{ kN } T}$

$\Sigma M_I = 0: \; -75(6) - 25(12) + 150(16)$

$-(DE \cos\delta)(4) - (DE \sin\delta)(6) = 0$

$\underline{DE = 297 \text{ kN } C}$

$\Sigma F_y = 0: \; -75 - 25 + 150 - 297 \sin\delta + 75.9 \sin\beta$

$+ EI \frac{4}{\sqrt{52}} = 0, \quad \underline{EI = -26.4 \text{ kN } T}$

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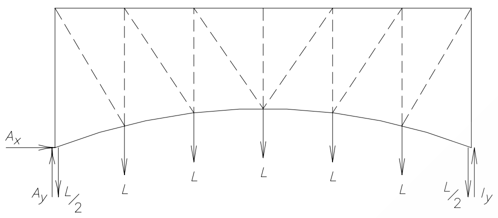
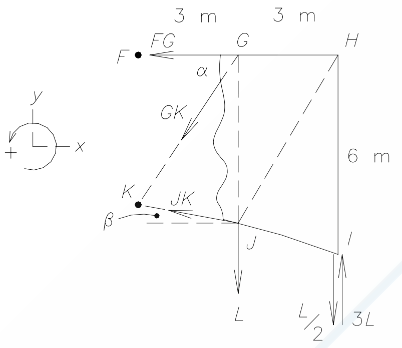


By symmetry, $A_y = H_y = 3L$

$\Sigma F_x = 0: \; A_x = 0$



Origin at center of arc

Location of I: $y_I^2 = 25^2 - 9^2$, $B = (9, 23.3)$ m
Location of J: $y_J^2 = 25^2 - 6^2$, $J = (6, 24.3)$ m
Location of G: $y_G = y_I + 6$, $G = (6, 29.3)$ m
Location of K: $y_K^2 = 25^2 - 3^2$, $K = (3, 24.8)$ m

$$\alpha = \tan^{-1}\frac{29.3 - 24.8}{3} = 56.3°, \; \beta = \tan^{-1}\frac{24.8 - 24.3}{3} = 10.39°$$

$$\Sigma M_K = 0: \; 3L(6) - \frac{L}{2}(6) + FG(y_G - y_K) - L(3) = 0, \; FG = -2.66L \; (C)$$

$$\begin{cases} \Sigma F_x = 0: \; 2.66L - GK\cos 56.3° - JK\cos 10.39° = 0 \\ \Sigma F_y = 0: \; \frac{3}{2}L + JK\sin 10.39° - GK\sin 56.3° \end{cases}$$

Solve simultaneously to obtain: $\underline{GK = 2.13L \; T}$

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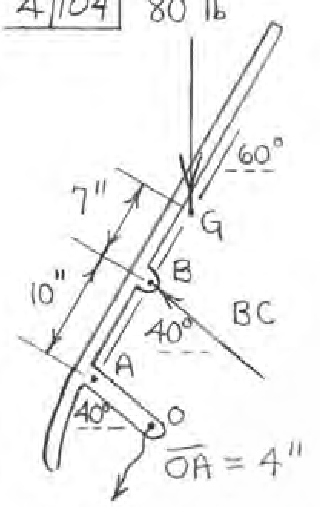
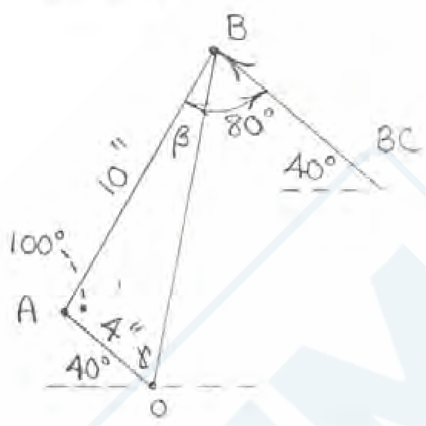


Blowup:

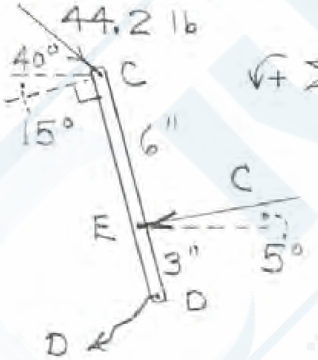


$\overline{OB}^2 = 4^2 + 10^2 - 2(4)(10)\cos 100°$, $\overline{OB} = 11.40$ in.

$\dfrac{\sin\beta}{4''} = \dfrac{\sin 100°}{11.40''}$, $\beta = 20.2°$

$\circlearrowleft + \Sigma M_O = 0: \; -80(17\cos 60° - 4\cos 40°)$

$+ BC(11.40\sin(80° - 20.2°)) = 0$, $BC = 44.2$ lb



$\circlearrowleft + \Sigma M_D = 0: \; -44.2\cos 55°(9)$

$+ C\cos 10°(3) = 0$

$C = 77.2$ lb

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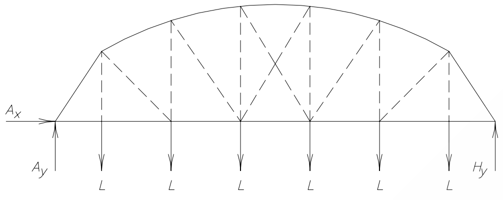
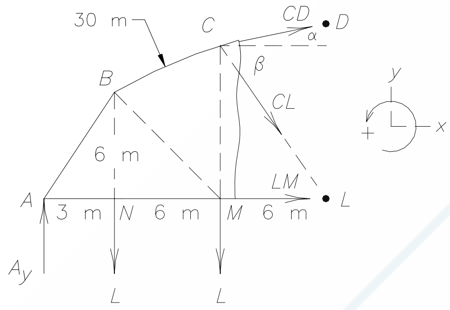


By symmetry, $A_y = H_y = 3L$

$\Sigma F_x = 0: \ A_x = 0$



Origin at center of arc

Location of B: $y_B^2 = 30^2 - (-15)^2$, $B = (-15, 26.0)$ m
Location of A: $y_A = y_B - 6$, $A = (-18, 20.0)$ m
Location of C: $y_C^2 = 30^2 - (-9)^2$, $C = (-9, 28.6)$ m
Location of M: $y_M = y_A$, $M = (-9, 20.0)$ m
Location of D: $y_D^2 = 30^2 - (-3)^2$, $D = (-3, 29.8)$ m
Location of L: $y_L = y_A$, $L = (-3, 20.0)$ m

$$\alpha = \tan^{-1}\frac{29.8 - 28.6}{6} = 11.60°, \ \beta = \tan^{-1}\frac{28.6 - 20.0}{6} = 55.2°$$

$$\Sigma M_L = 0: \ -3L(15) + L(6) + L(12) - CD(29.8 - 20.0)\sin(90° - 11.60°) = 0$$
$$CD = -2.79L \text{ or } CD = 2.79L \ C$$

$$\Sigma F_y = 0: \ 3L - L - L - 2.79L\sin 11.60° - CL\sin 55.2° = 0, \ \underline{CL = 0.534L \ T}$$

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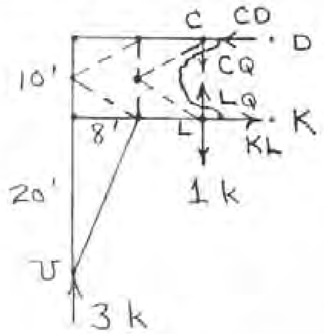
▸4/51 From truss as a whole, $\begin{cases} U = 3 \text{ kips} \\ V = 4 \text{ kips} \end{cases}$



$\circlearrowleft_+ \Sigma M_C = 0: \; KL(10) - 3(16) = 0$

$KL = 4.8 \text{ kips T}$

$\Sigma M_L = 0: \; CD(10) - 3(16) = 0$

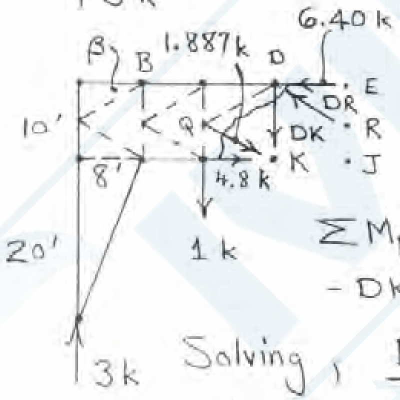
$CD = 4.8 \text{ kips C}$

(From a similar right-hand section, DE = 6.40 kips C.)

$\Sigma M_D = 0: \; -3(24) + 1(8) + KQ\left(\sqrt{8^2+5^2}\,\sin\alpha\right) + 4.8(10) = 0,$

where $\alpha = 180 - 2\tan^{-1}\left(\frac{8}{5}\right) = 64.0°$

Solving, $KQ = 1.887 \text{ kips T}$



$\beta = \tan^{-1}\left(\frac{5}{8}\right) = 32.0°$

$\Sigma F_x = 0: \; -6.40 + 1.887\cos\beta + 4.8 - DR\cos\beta = 0, \; DR = 0$

$\Sigma M_B = 0: \; -3(8) - 1(8) + 4.8(10) - DK(16) = 0$

Solving, $\underline{DK = 1 \text{ kip T}}$

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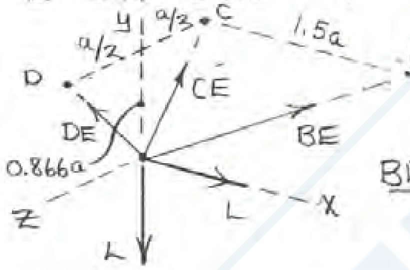
4/60 The truss as a whole is statically determinate with six supporting constraints. $j = 6$ & $m = 12$; $3j = m + 6$, so there are sufficient members for stability. C, B, and D are fixed so E is fixed. A and F are also fixed, so the truss is a rigid unit. From an inspection of joint F, $AF = 0$, $\underline{BF = 0}$, $EF = L\ T$. From an inspection of joint A, $AB = AD = \underline{AE = 0}$. We can now go to joint E and solve for all unknowns there:



$\underline{CE} = CE[0.866\underline{j} - 0.5\underline{k}]$

$\underline{DE} = DE[0.866\underline{j} + 0.5\underline{k}]$

$$\underline{BE} = BE\left[\frac{1.5\underline{i} + 0.866\underline{j} - 0.5\underline{k}}{\sqrt{1.5^2 + 0.866^2 + 0.5^2}}\right]$$

$$= BE[0.832\underline{i} + 0.480\underline{j} - 0.277\underline{k}]$$

$$\left.\begin{aligned} \Sigma F_x = 0:&\ 0.832 BE + L = 0 \\ \Sigma F_y = 0:&\ 0.480 BE + 0.866 CE + 0.866 DE - L = 0 \\ \Sigma F_z = 0:&\ -0.277 BE - 0.5 CE + 0.5 DE = 0 \end{aligned}\right\}$$

Solution: $\underline{BE = -1.202L\ (C)}$, $\underline{CE = 1.244L\ T}$

$DE = 0.577L\ T$

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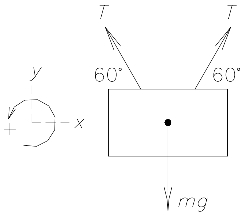
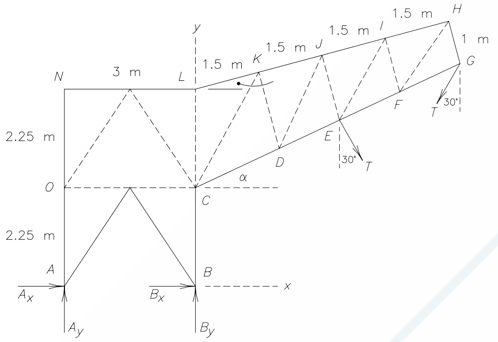


$\Sigma F_y = 0: \ 2T\sin 60° - 1000(9.81) = 0, T = 5660 \text{ N}$



From $\Sigma F_x = 0$, $A_x + B_x = 0$

C = (0, 2.25) m
L = (0, 4.5) m
H = (6cos15°, 4.5 + 6sin15°) = (5.80, 6.05) m
G = (6cos15° + 1cos75°, 4.5 + 6sin15° - 1sin75°) = (6.05, 5.09) m

$$\alpha = \tan^{-1}\frac{5.09 - 2.25}{6.05} = 25.1°$$

$$FG = EF = DE = \frac{1.5}{\cos(25.1° - 15°)} = 1.524 \text{ m}$$

E = (6cos15° + 1cos75°-2(1.524cos25.1°), 4.5 + 6sin15° - 1sin75° - 2(1.524sin25.1°)) = (3.30, 3.79) m

$\Sigma M_B = 0: \ -A_y(3) - T\sin 60°(3.30) - T\cos 60°(3.79) - T\sin 60°(6.05) + T\cos 60°(5.09) = 0$
$A_y = -14\,070 \text{ N}$

$\Sigma F_y = 0: \ -14\,070 + B_y - 1000(9.81) = 0, B_y = 23\,900 \text{ N}$

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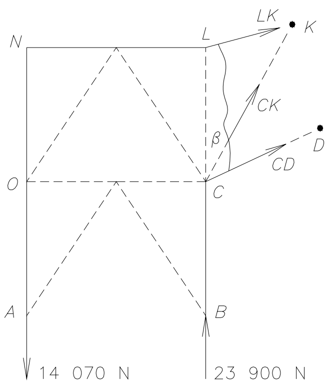

$$\beta = \tan^{-1}\frac{1.5\cos 15°}{2.25 + 1.5\sin 15°} = 28.8°$$

$\Sigma M_C = 0:\ 14\,070(3) - LK(2.25)(\sin 75°) = 0,\ LK = 19\,420\text{ N}$

$$\begin{cases} \Sigma F_x = 0:\ 19\,420\cos 15° + \text{CK}\sin 28.8° + CD\cos 25.1° = 0 \\ \Sigma F_y = 0:\ -14\,070 + 23\,900 + 19\,420\sin 15° + CK\cos 28.8° + CD\sin 25.1° = 0 \end{cases}$$

Solve simultaneously to obtain: $CK = -9290$ N or $\underline{CK = 9290\text{ N } C}$

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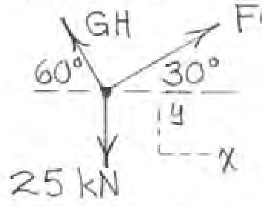
▶ 4/54 $\underline{FI = 0}$, by inspection of joint I.

Joint G:



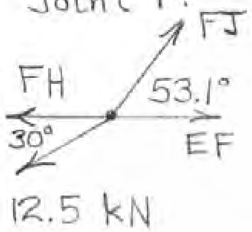
$$\begin{cases} \Sigma F_x = 0: -GH\left(\tfrac{1}{2}\right) + FG\left(\tfrac{\sqrt{3}}{2}\right) = 0 \\ \Sigma F_y = 0: GH\left(\tfrac{\sqrt{3}}{2}\right) + FG\left(\tfrac{1}{2}\right) - 25 = 0 \end{cases}$$

1st eq.: $GH = \sqrt{3}\, FG$

2nd eq.: $\sqrt{3}\, FG\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2} FG = 25$

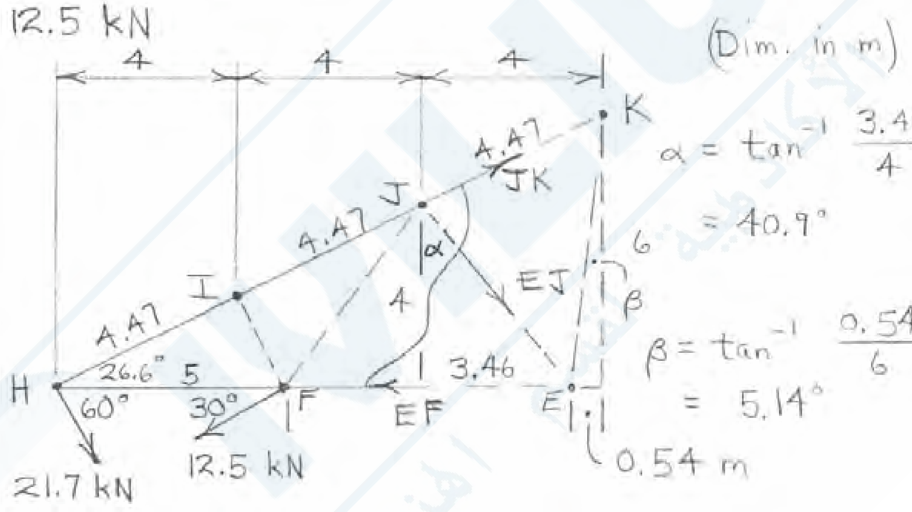
$\Rightarrow FG = 12.5$ kN T, $GH = 21.7$ kN T

Joint F:



$\Sigma F_y = 0: \; FJ(\sin 53.1°) - 12.5 \sin 30° = 0$

$\underline{FJ = 7.81 \text{ kN T}}$



$\alpha = \tan^{-1} \frac{3.46}{4} = 40.9°$

$\beta = \tan^{-1} \frac{0.54}{6} = 5.14°$

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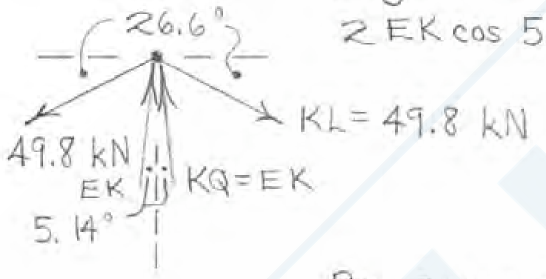
$\curvearrowleft + \Sigma M_H = 0: -12.5\left(\frac{1}{2}\right)(5) - EJ\left[\cos 40.9°(8) + \sin 40.9°(4)\right] = 0, \quad EJ = -3.61 \text{ kN}$

So $\underline{EJ = 3.61 \text{ kN C}}$

$\Sigma F_y = 0: \; JK \sin 26.6° + 3.61 \cos 40.9° - 12.5\left(\frac{1}{2}\right) - 21.7\left(\frac{\sqrt{3}}{2}\right) = 0, \quad JK = 49.8 \text{ kN T}$

Joint K, using symmetry:



$\Sigma F_y = 0: -2(49.8) \sin 26.6° + 2 EK \cos 5.14° = 0,$

$\underline{EK = 22.4 \text{ kN C}}$

By symmetry of loads L and $\Sigma M_K = 0$, $\underline{ER = 0}$

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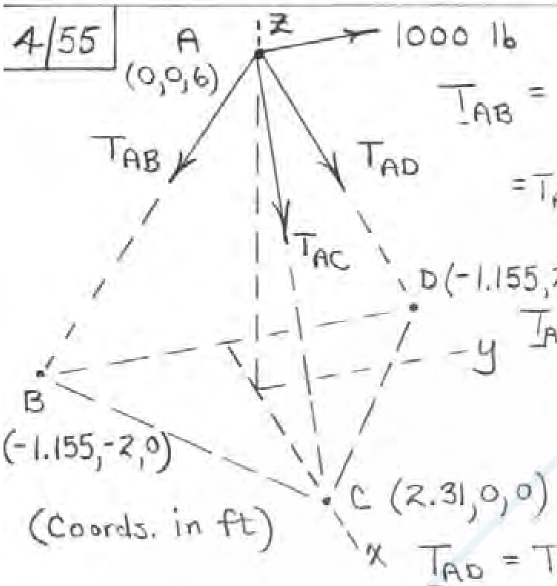

$$\mathbf{T}_{AB} = T_{AB} \frac{-1.155\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}}{\sqrt{1.155^2 + 2^2 + 6^2}} = T_{AB}[-0.1796\mathbf{i} - 0.311\mathbf{j} - 0.933\mathbf{k}]$$

$$\mathbf{T}_{AC} = T_{AC} \frac{2.31\mathbf{i} - 6\mathbf{k}}{\sqrt{2.31^2 + 6^2}} = T_{AC}[0.359\mathbf{i} - 0.933\mathbf{k}]$$

$$\mathbf{T}_{AD} = T_{AD} \frac{-1.155\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}}{\sqrt{1.155^2 + 2^2 + 6^2}} = T_{AD}[-0.1796\mathbf{i} + 0.311\mathbf{j} - 0.933\mathbf{k}]$$

$\Sigma F_x = 0: -0.1796\,T_{AB} + 0.359\,T_{AC} - 0.1796\,T_{AD} = 0$ (1)

$\Sigma F_y = 0: 1000 - 0.311\,T_{AB} + 0.311\,T_{AD} = 0$ (2)

$\Sigma F_z = 0: -0.933\,T_{AB} - 0.933\,T_{AC} - 0.933\,T_{AD} = 0$ (3)

Solve Eqs. (1)-(3):

$$\begin{cases} T_{AB} = 1607 \text{ lb} \quad (T) \\ T_{AC} = 0 \\ T_{AD} = -1607 \text{ lb} \quad (C) \end{cases}$$

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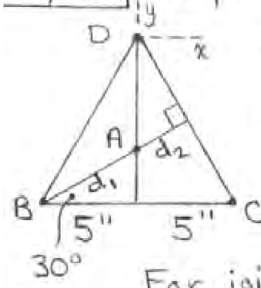
4/56 Top view of base:



$\cos 30° = \frac{d_1+d_2}{10}$, $d_1+d_2 = 8.66"$

$\cos 30° = \frac{5}{d_1}$, $d_1 = 5.77"$

$d_2 = 8.66 - 5.77 = 2.89"$

For joint A, assuming symmetry:

$$\underline{F}_{BA} = P\left[\frac{5\underline{i} + 2.89\underline{j} + 16\underline{k}}{(5^2 + 2.89^2 + 16^2)^{1/2}}\right] = P(0.294\underline{i} + 0.1697\underline{j} + 0.941\underline{k})$$

$\underline{F}_{CA} = P(-0.294\underline{i} + 0.170\underline{j} + 0.941\underline{k})$, $\underline{F}_{DA} = P(-0.339\underline{j} + 0.941\underline{k})$

$\Sigma F_z = 0$ at A: $3(0.941\,P) - 800 = 0$, $P = 283$ lb

For joint C, assuming symmetry:

$\underline{F}_{BC} = -Q\underline{i}$, $\underline{F}_{CD} = Q(-\sin 30°\,\underline{i} + \cos 30°\,\underline{j})$

Normal $\underline{N} = 267\underline{k}$ lb

$\Sigma \underline{F} = \underline{0}$ at C: $\underline{N} + \underline{F}_{BC} + \underline{F}_{CD} + \underline{F}_{AC} = \underline{0}$

$267\underline{k} - Q\underline{i} + 283(0.294\underline{i} - 0.1697\underline{j} - 0.941\underline{k})$

$+ Q(-0.5\underline{i} + 0.866\underline{j}) = \underline{0}$

Solving, $Q = 55.6$ lb $\Rightarrow$ $\underline{BC = BD = CD = 55.6 \text{ lb } T}$

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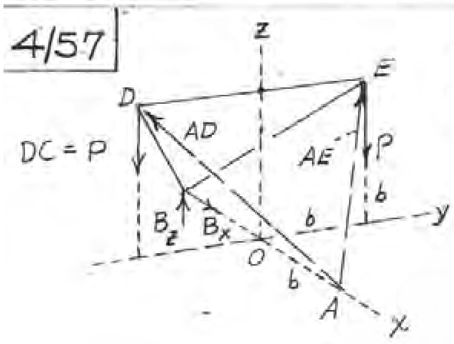

For truss as a whole

$\Sigma M_x = 0$ gives $DC = P$

$\Sigma M_y = 0$ & $\Sigma F_z = 0$ gives $B_z = P$

For unit BDE

$\Sigma M_{DE} = 0$; $B_z b - B_x b = 0$

$B_x = B_z = P$

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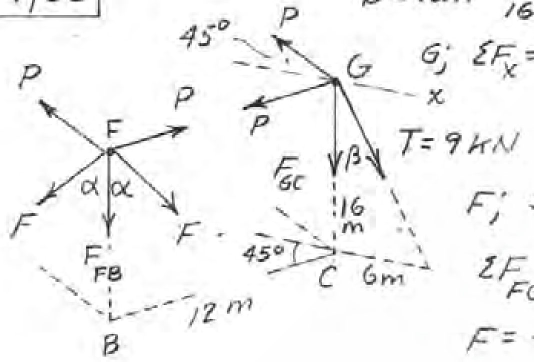

$\beta = \tan^{-1}\frac{6}{16} = 20.6°, \ \sin\beta = 0.351$

$G; \ \Sigma F_x = 0; \ 9(0.351) - 2P(0.707) = 0$

$P = 2.235 \text{ kN}$

$F; \ \sin\alpha = 3/5 = 0.6$

$\Sigma F_{FG} = 0; \ P + F\sin\alpha = 0$

$F = -2.235/0.6 = \underline{-3.72 \text{ kN}}$

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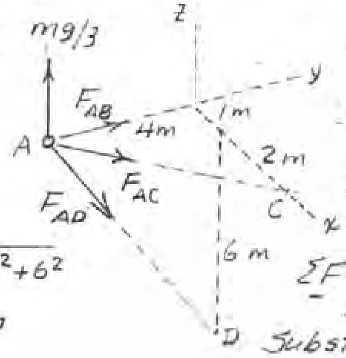
4/59 From truss as a whole $\Sigma M_y = 0$ gives tension in vertical wire at C $T_C = \frac{1}{3} mg$

By symmetry & $\Sigma F_z = 0$; $T_A = T_B = \frac{1}{3} mg$

Joint A;



$\underline{F}_{AB} = F_{AB}\underline{j}$

$\underline{F}_{AC} = \frac{F_{AC}}{5}(3\underline{i} + 4\underline{j})$

$\underline{F}_{AD} = \frac{F_{AD}}{\sqrt{53}}(\underline{i} + 4\underline{j} - 6\underline{k})$

$\overline{AD} = \sqrt{4^2 + 1^2 + 6^2}$
$= \sqrt{53}$ m

$\Sigma \underline{F} = \underline{0}$; $\frac{mg}{3}\underline{k} + \underline{F}_{AB} + \underline{F}_{AC} + \underline{F}_{AD} = \underline{0}$

Substitute & collect terms to get

$$\left(\frac{3F_{AC}}{5} + \frac{F_{AD}}{\sqrt{53}}\right)\underline{i} + \left(F_{AB} + \frac{4F_{AC}}{5} + \frac{4F_{AD}}{\sqrt{53}}\right)\underline{j} + \left(\frac{mg}{3} - \frac{6F_{AD}}{\sqrt{53}}\right)\underline{k} = \underline{0}$$

Equate coefficients of $\underline{i}$-, $\underline{j}$-, & $\underline{k}$-terms to zero & get

$F_{AD} = \frac{\sqrt{53}}{18} mg$, $\underline{F_{AC} = -\frac{5}{54} mg}$, $\underline{F_{AB} = -\frac{4}{27} mg}$

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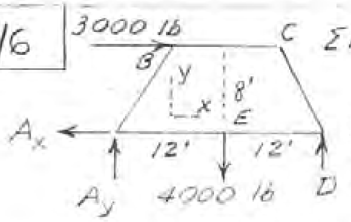


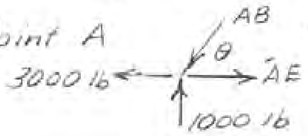
$\Sigma M_A = 0;\ 4000(12) + 3000(8) - 24D = 0$

$D = 3000$ lb

$\Sigma F_x = 0;\ A_x = 3000$ lb

$\Sigma F_y = 0;\ A_y = 4000 - 3000 = 1000$ lb

Joint A

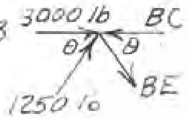


$\Sigma F_y = 0;\ AB\left(\frac{4}{5}\right) - 1000 = 0,\ \underline{AB = 1250 \text{ lb C}}$

$\Sigma F_x = 0;\ AE - 1250\left(\frac{3}{5}\right) - 3000 = 0$

$\underline{AE = 3750 \text{ lb T}}$

Joint B

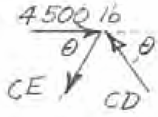


$\Sigma F_y = 0;\ 1250\left(\frac{4}{5}\right) - BE\left(\frac{4}{5}\right) = 0,\ \underline{BE = 1250 \text{ lb T}}$

$\Sigma F_x = 0;\ BC - 3000 - 1250\left(\frac{3}{5}\right) - 1250\left(\frac{3}{5}\right) = 0$

$\underline{BC = 4500 \text{ lb C}}$

Joint C

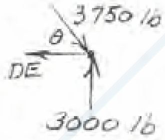


$\Sigma F_y = 0;\ CE \sin\theta - CD \sin\theta = 0,\ CE = CD$

$\Sigma F_x = 0;\ 2CE(3/5) - 4500 = 0,\ \underline{CE = 3750 \text{ lb T}}$

$\underline{CD = 3750 \text{ lb C}}$

Joint D



$\Sigma F_x = 0;\ DE - 3750\left(\frac{3}{5}\right) = 0$

$\underline{DE = 2250 \text{ lb T}}$

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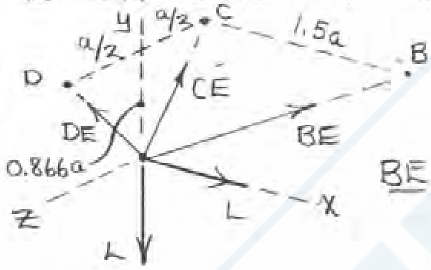
4/60 The truss as a whole is statically determinate with six supporting constraints. $j=6$ & $m=12$; $3j=m+6$, so there are sufficient members for stability. C, B, and D are fixed so E is fixed. A and F are also fixed, so the truss is a rigid unit. From an inspection of joint F, $AF=0$, $BF=0$, $EF = LT$. From an inspection of joint A, $AB = AD = AE = 0$. We can now go to joint E and solve for all unknowns there:



$\underline{CE} = CE[0.866\underline{j} - 0.5\underline{k}]$

$\underline{DE} = DE[0.866\underline{j} + 0.5\underline{k}]$

$$\underline{BE} = BE\left[\frac{1.5\underline{i} + 0.866\underline{j} - 0.5\underline{k}}{\sqrt{1.5^2 + 0.866^2 + 0.5^2}}\right]$$

$$= BE[0.832\underline{i} + 0.480\underline{j} - 0.277\underline{k}]$$

$\Sigma F_x = 0$: $0.832 BE + L = 0$

$\Sigma F_y = 0$: $0.480 BE + 0.866 CE + 0.866 DE - L = 0$

$\Sigma F_z = 0$: $-0.277 BE - 0.5 CE + 0.5 DE = 0$

Solution: $BE = -1.202L$ (C), $CE = 1.244L$ T

$DE = 0.577L$ T

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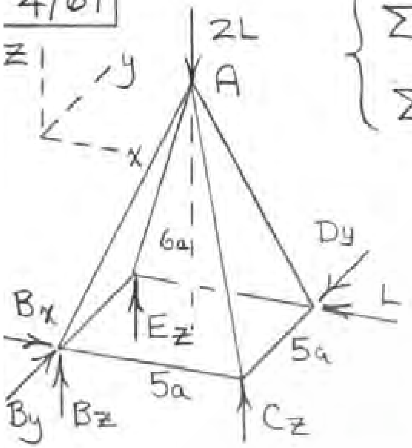
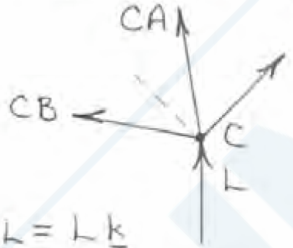


$$\left\{\begin{array}{l}\Sigma M_{BE} = 0 \Rightarrow C_z = L \\ \Sigma M_{B_z} = 0 \Rightarrow D_y = L\end{array}\right.$$

Joint C: (tensions assumed)



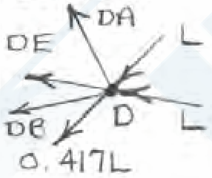
$\underline{CB} = -CB\,\underline{i}$, $\underline{CD} = CD\,\underline{j}$, $\underline{L} = L\,\underline{k}$

$$\underline{CA} = CA\left(\frac{-2.5a\underline{i} + 2.5a\underline{j} + 6a\underline{k}}{\sqrt{(2.5^2 + 2.5^2 + 6^2)a^2}}\right) = CA(-0.359\underline{i} + 0.359\underline{j} + 0.862\underline{k})$$

$\Sigma \underline{F} = \underline{0}$ yields:

$$\left.\begin{array}{ll}\underline{i}: & -CB - 0.359\,CA = 0 \\ \underline{j}: & CD + 0.359\,CA = 0 \\ \underline{k}: & L + 0.862\,CA = 0\end{array}\right\} \begin{array}{l} CA = -0.1667L \\ CD = +0.417L \end{array}$$

Joint D:



$\Sigma \underline{F} = \underline{0}$ yields

$$\left\{\begin{array}{ll}\underline{i}: & -DE - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{j}: & -0.417L - L - 0.359\,DA - \frac{DB}{\sqrt{2}} = 0 \\ \underline{k}: & 0.862\,DA = 0\end{array}\right.$$

$DA = 0$, $\underline{DB = -2.00L}$

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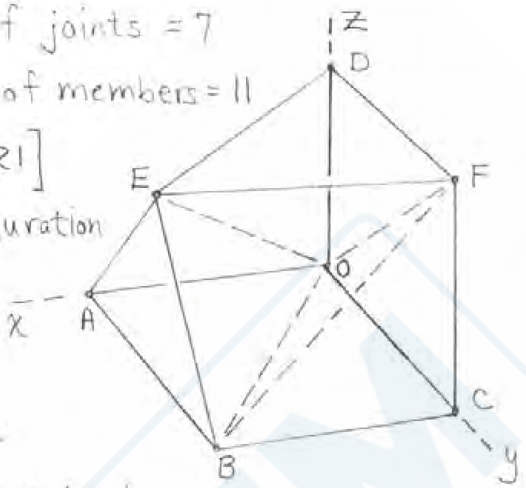
4/62 j = number of joints = 7

m_i = initial number of members = 11

$[m_i + 6 = 17] < [3j = 21]$

So the initial configuration lacks $21 - 17 = 4$ members for internal stability. A stable configuration is achieved by



(1) Adding OB & OE to produce the rigid tetrahedron ABEO.

(2) Adding OF to produce the rigid tetrahedron ODEF.

(3) Adding BF to produce the rigid tetrahedrons OBCF and OBEF.

With 4 new members, $m = 15$ and $m + 6 = 21$. The number of joints remains $j = 7$; $3j = 21$. So $m + 6 = 3j$; sufficient number of members now present.

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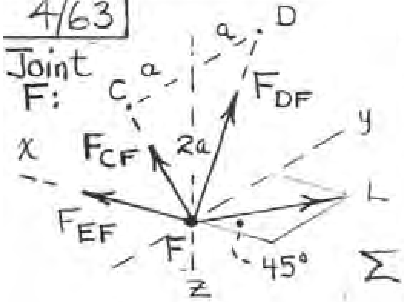
Joint F:



$$\underline{L} = \frac{L}{\sqrt{2}}(-\underline{i}+\underline{j}), \quad \underline{F}_{EF} = F_{EF}\,\underline{i}$$

$$\underline{F}_{CF} = \frac{F_{CF}}{\sqrt{5}}(-\underline{j}-2\underline{k})$$

$$\underline{F}_{DF} = \frac{F_{DF}}{\sqrt{5}}(\underline{j}-2\underline{k})$$

$$\Sigma \underline{F} = \underline{L} + \underline{F}_{EF} + \underline{F}_{CF} + \underline{F}_{DF} = \underline{0}$$

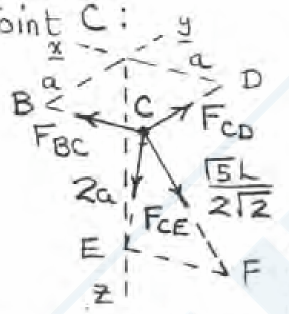
$\underline{i}$: $-\frac{L}{\sqrt{2}} + F_{EF} = 0$, $\quad F_{EF} = \frac{L}{\sqrt{2}}$

$\underline{j}$: $\frac{L}{\sqrt{2}} - \frac{F_{CF}}{\sqrt{5}} + \frac{F_{DF}}{\sqrt{5}} = 0$

$\underline{k}$: $-\frac{2}{\sqrt{5}}F_{CF} - \frac{2}{\sqrt{5}}F_{DF} = 0$

$$F_{CF} = \frac{\sqrt{5}\,L}{2\sqrt{2}}, \quad F_{DF} = -\frac{\sqrt{5}\,L}{2\sqrt{2}}$$

Joint C:



$$\underline{F}_{CD} = F_{CD}\,\underline{j}, \quad \underline{F}_{BC} = F_{BC}\,\underline{i}$$

$$\underline{F}_{CE} = F_{CE}(\underline{i}+\underline{j}+2\underline{k})/\sqrt{6}$$

$$\underline{F}_{CF} = F_{CF}(\underline{j}+2\underline{k})/\sqrt{5} = \frac{L}{2\sqrt{2}}(\underline{j}+2\underline{k})$$

$\Sigma F = 0$ yields:

$\underline{i}$: $0 + F_{BC} + \frac{F_{CE}}{\sqrt{6}} = 0$

$\underline{j}$: $F_{CD} + \frac{F_{CE}}{\sqrt{6}} + \frac{L}{2\sqrt{2}} = 0$

$\underline{k}$: $\frac{2F_{CE}}{\sqrt{6}} + \frac{L}{\sqrt{2}} = 0$

$$F_{BC} = \frac{L\sqrt{2}}{4}$$

$$F_{CD} = 0$$

$$F_{CE} = -\frac{L\sqrt{3}}{2}$$

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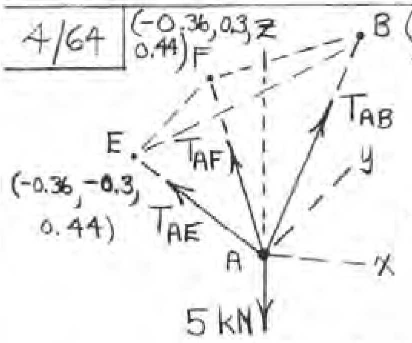


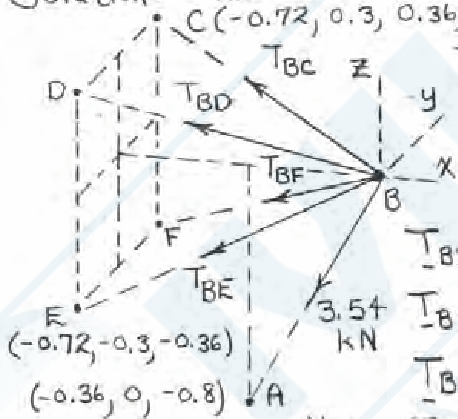
$$\mathbf{T}_{AB} = T_{AB}\frac{0.36\mathbf{i} + 0.8\mathbf{k}}{\sqrt{0.36^2 + 0.8^2}} = T_{AB}[0.410\mathbf{i} + 0.912\mathbf{k}]$$

$$\mathbf{T}_{AE} = T_{AE}\frac{-0.36\mathbf{i} - 0.3\mathbf{j} + 0.44\mathbf{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}} = T_{AE}[-0.560\mathbf{i} - 0.467\mathbf{j} + 0.684\mathbf{k}]$$

$$\mathbf{T}_{AF} = T_{AF}\frac{-0.36\mathbf{i} + 0.3\mathbf{j} + 0.44\mathbf{k}}{\sqrt{0.36^2 + 0.3^2 + 0.44^2}} = T_{AF}[-0.560\mathbf{i} + 0.467\mathbf{j} + 0.684\mathbf{k}]$$

$$\left.\begin{aligned}
\Sigma F_x = 0:&\ 0.410T_{AB} - 0.560T_{AE} - 0.560T_{AF} = 0\\
\Sigma F_y = 0:&\ -0.467T_{AE} + 0.467T_{AF} = 0\\
\Sigma F_z = 0:&\ 0.912T_{AB} + 0.684T_{AE} + 0.684T_{AF} - 5 = 0
\end{aligned}\right\}$$

Solution: $T_{AB} = 3.54$ kN, $T_{AE} = T_{AF} = 1.296$ kN



$$\mathbf{T}_{BC} = T_{BC}\frac{-0.72\mathbf{i} + 0.3\mathbf{j} + 0.36\mathbf{k}}{\sqrt{0.72^2 + 0.3^2 + 0.36^2}} = T_{BC}[-0.838\mathbf{i} + 0.349\mathbf{j} + 0.419\mathbf{k}]$$

$$\mathbf{T}_{BD} = T_{BD}[-0.838\mathbf{i} - 0.349\mathbf{j} + 0.419\mathbf{k}]$$

$$\mathbf{T}_{BE} = T_{BE}[-0.838\mathbf{i} - 0.349\mathbf{j} - 0.419\mathbf{k}]$$

$$\mathbf{T}_{BF} = T_{BF}[-0.838\mathbf{i} + 0.349\mathbf{j} - 0.419\mathbf{k}]$$

Note: $T_{BD} = T_{BC}$; $T_{BE} = T_{BF}$ (symmetry)

Set $\Sigma \mathbf{F} = \mathbf{0}$ to obtain $T_{BD} = T_{BC} = 1.491$ kN

$$T_{BE} = T_{BF} = -2.36 \text{ kN (C)}$$

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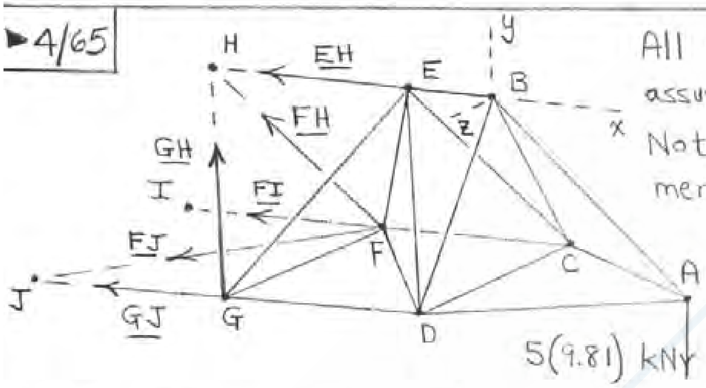

All members assumed in tension. Note that six members are cut!

$\underline{GJ} = -GJ\underline{i}$, $\underline{FI} = -FI\underline{i}$, $\underline{FJ} = \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k})$

$\Sigma \underline{M}_H = \underline{0}$: $-49.05(5)\underline{k} + (-2\cos 30°\underline{j} + 2\sin 30°\underline{k})$
$\times(-GJ)\underline{i} + (-2\cos 30°\underline{j} - 2\sin 30°\underline{k})\times(-FI)\underline{i}$
$+ (\underline{i} - 2\cos 30°\underline{j} - \underline{k}) \times \frac{FJ}{\sqrt{2}}(-\underline{i}+\underline{k}) = \underline{0}$.

Equating unit vector coefficients to zero:

$-1.225\ FJ = 0 \Rightarrow \underline{FJ = 0}$

$\left.\begin{array}{l} -GJ + FI = 0 \\ -1.732\ GJ - 1.732\ FI = 245 \end{array}\right\}$ $FI = GJ = -70.8$ kN

$\therefore$ $\underline{\text{Force in } GJ = 70.8 \text{ kN } C}$

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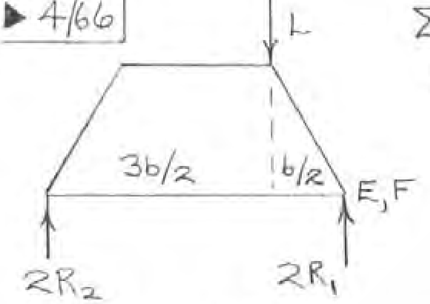
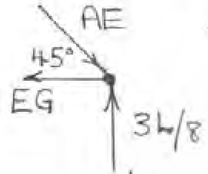


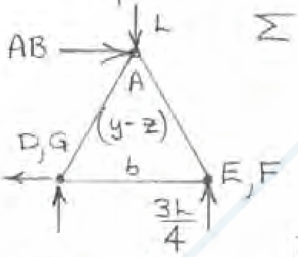
$\Sigma M_{EF} = 0: \; 2R_2(2b) - L\frac{b}{2} = 0$

$R_2 = L/8, \quad R_1 = 3L/8$

Joint E (Vertical plane through AE):



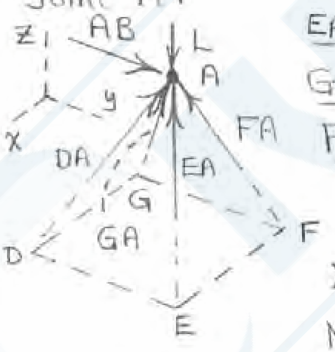
$+\uparrow \Sigma F = 0: \; \frac{3L}{8} - \frac{AE}{\sqrt{2}} = 0, \quad AE = \frac{3L}{4\sqrt{2}}\; C$



$\Sigma M_{DG} = 0: \; AB\frac{b}{\sqrt{2}} + L\frac{b}{2} - \frac{3L}{4}b = 0$

$AB = L\sqrt{2}/4 \; \text{(Comp)}$

Joint A:



$\underline{DA} = DA\left(-\frac{1}{2}\underline{i} + \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k}\right)$

$\underline{EA} = \frac{3L}{4\sqrt{2}}\left(-\frac{1}{2}\underline{i} - \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k}\right)$

$\underline{GA} = GA\left(\frac{1}{2}\underline{i} + \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k}\right)$

$\underline{FA} = \frac{3L}{4\sqrt{2}}\left(\frac{1}{2}\underline{i} - \frac{1}{2}\underline{j} + \frac{1}{\sqrt{2}}\underline{k}\right)$

$\underline{L} = -L\underline{k}, \quad \underline{AB} = \frac{L\sqrt{2}}{4}\underline{j}$

$\Sigma \underline{F} = \underline{0}: \; \underline{L} + \underline{AB} + \underline{DA} + \underline{EA} + \underline{GA} + \underline{FA} = 0$

Note that DA = GA & EA = FA by symmetry & obtain

$\underline{i}: \; -\frac{1}{2}DA - \frac{3L}{8\sqrt{2}} + \frac{1}{2}DA + \frac{3L}{8\sqrt{2}} = 0, \quad 0 = 0 \; ✓$

$\underline{j}: \; \frac{L\sqrt{2}}{4} + \frac{1}{2}DA - \frac{3L}{8\sqrt{2}} + \frac{1}{2}DA - \frac{3L}{8\sqrt{2}} = 0, \quad DA = GA = \frac{L\sqrt{2}}{8} \; \text{(Comp.)}$

$\underline{k}: \; -L + \frac{L}{8} + \frac{3L}{8} + \frac{L}{8} + \frac{3L}{8} = 0, \quad 0 = 0 \; ✓$

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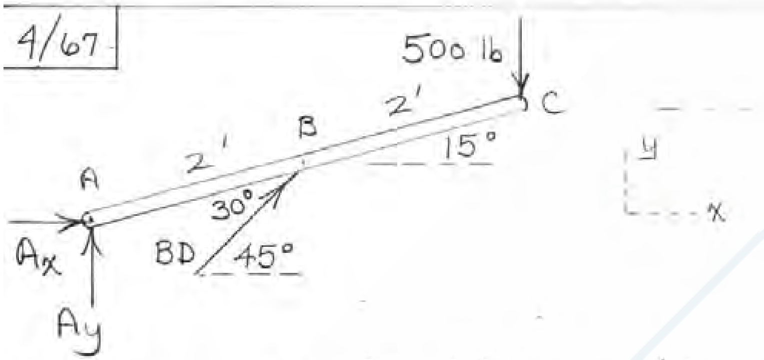

$\circlearrowleft + \Sigma M_A = 0: \; BD \sin 30° (2) - 500 (4 \cos 15°) = 0$

$BD = 1932 \text{ lb}$

Forces at B and D = 1932 lb

$\Sigma F_x = 0: \; A_x + 1932 \cos 45° = 0, \quad A_x = -1366 \text{ lb}$

$\Sigma F_y = 0: \; A_y + 1932 \sin 45° - 500 = 0, \quad A_y = -866 \text{ lb}$

$A = \sqrt{1366^2 + 866^2} = 1617 \text{ lb}$

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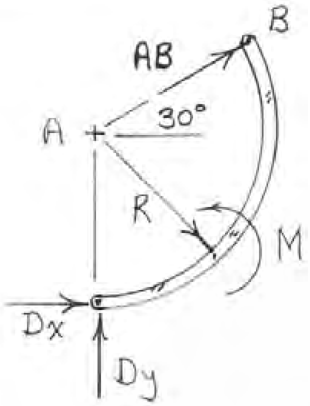

$\curvearrowleft + \Sigma M_D = 0: \quad -AB\cos 30°(R) + M = 0$

$$AB = \frac{M}{R\cos 30°} = \frac{M}{R\,\sqrt{3}/2} = \frac{2\sqrt{3}\,M}{3R}$$

Load is a couple, so reactions form a couple:

$$A = D = \frac{2\sqrt{3}\,M}{3R}$$

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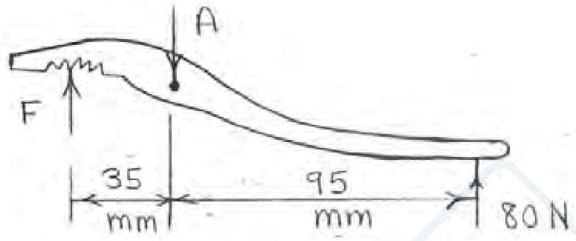

$\circlearrowleft \Sigma M_A = 0: \; 80(95) - F(35) = 0, \quad \underline{F = 217 \text{ N}}$

$+\uparrow \Sigma F = 0: \; 217 - A + 80 = 0, \quad \underline{A = 297 \text{ N}}$

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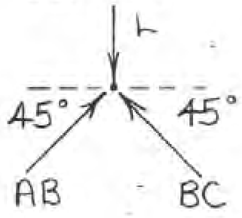
From $\Sigma F_x = 0$, $AB = BC$

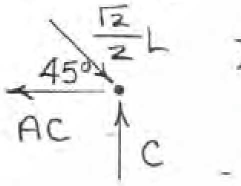
Joint B:



$\Sigma F_y = 0$: $2AB \frac{\sqrt{2}}{2} - L = 0$

$AB = \frac{\sqrt{2}}{2} L = BC$

Joint C:



$\Sigma F_x = 0$: $\frac{\sqrt{2}}{2} L \left(\frac{\sqrt{2}}{2}\right) - AC = 0$

$AC = \frac{L}{2}\ T$

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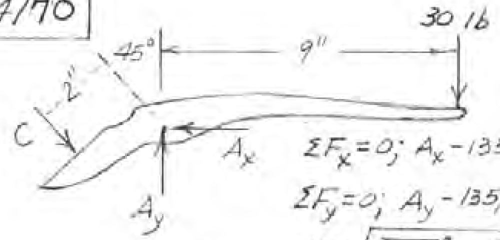

$\Sigma M_A = 0;$

$30(9) - 2C = 0, \quad C = 135 \text{ lb}$

$\Sigma F_x = 0; \; A_x - 135/\sqrt{2} = 0, \quad A_x = 95.4 \text{ lb}$

$\Sigma F_y = 0; \; A_y - 135/\sqrt{2} - 30 = 0, \quad A_y = 125.4 \text{ lb}$

$A = \sqrt{95.4^2 + 125.4^2} = \underline{157.6 \text{ lb}}$

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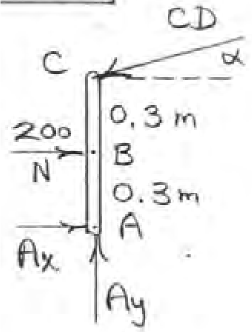
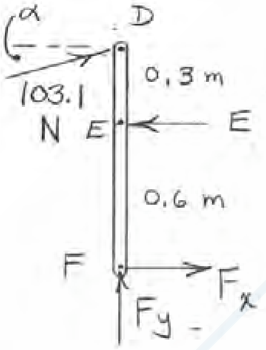
$\alpha = \tan^{-1} \frac{0.3}{1.2} = 14.04°$



$\curvearrowleft + \Sigma M_A = 0: \; CD \cos\alpha \, (0.6) - 200(0.3) = 0, \quad \underline{CD = 103.1 \text{ N}}$



$\curvearrowleft + \Sigma M_F = 0: -103.1 \cos\alpha \, (0.9) + E(0.6) = 0, \quad \underline{E = 150 \text{ N}}$

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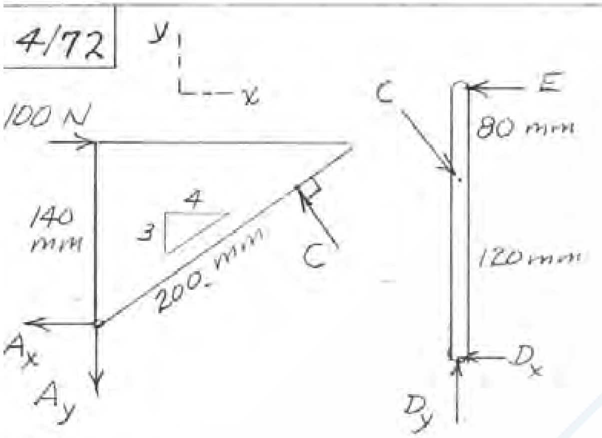

Triangle:

$\Sigma M_A = 0;\ 200C - 140(100) = 0,\quad C = 70\ N$

Bar:

$\Sigma M_D = 0;\ 200E - 70\left(\frac{3}{5}\right)120 = 0;\quad E = 25.2\ N$

$\Sigma F_x = 0;\ D_x + 25.2 - 70(3/5) = 0,\quad D_x = 16.8\ N$

$\Sigma F_y = 0;\ D_y - 70(4/5) = 0,\quad D_y = 56\ N$

$D = \sqrt{D_x^2 + D_y^2} = \sqrt{16.8^2 + 56^2} = \underline{58.5\ N}$

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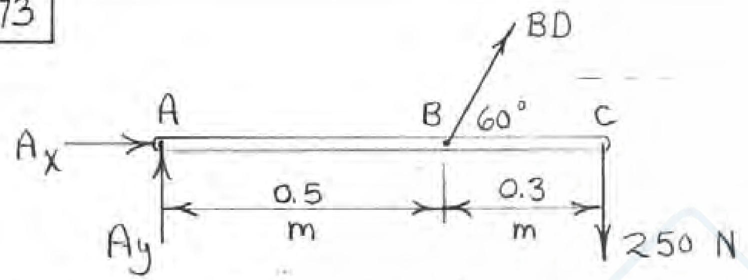
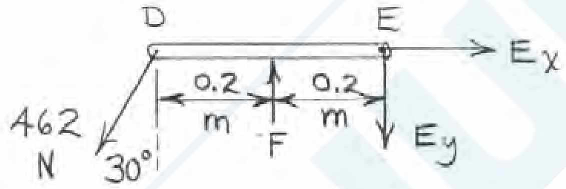


$$\circlearrowleft + \Sigma M_A = 0 : \ (BD \sin 60°)(0.5) - 250(0.8) = 0$$

$$BD = 462 \text{ N}$$



$$\circlearrowleft + \Sigma M_E = 0 : \ 462 \cos 30° (0.4) + F(0.2) = 0$$

$$\underline{F = 800 \text{ N}}$$

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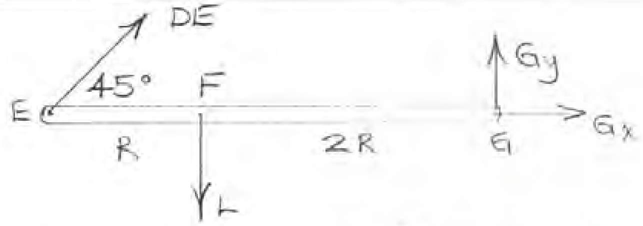
4/74

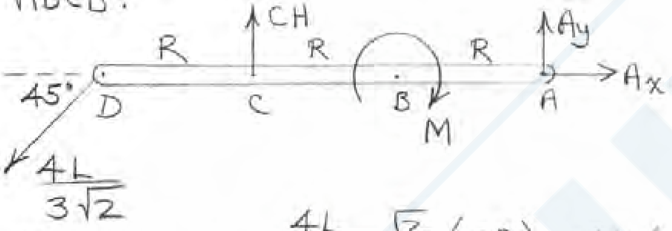
EFG



$$\curvearrowleft\!+\ \Sigma M_G = 0: \quad L(2R) - DE\,\frac{\sqrt{2}}{2}(3R) = 0$$

$$DE = \frac{4L}{3\sqrt{2}}$$

ABCD:



$$\curvearrowleft\!+\ \Sigma M_A = 0: \quad \frac{4L}{3\sqrt{2}}\,\frac{\sqrt{2}}{2}(3R) - CH(2R) - M = 0$$

$$CH = L - \frac{M}{2R} \text{ (tension if positive)}$$

$CH = 0$ if $\underline{M = 2RL}$

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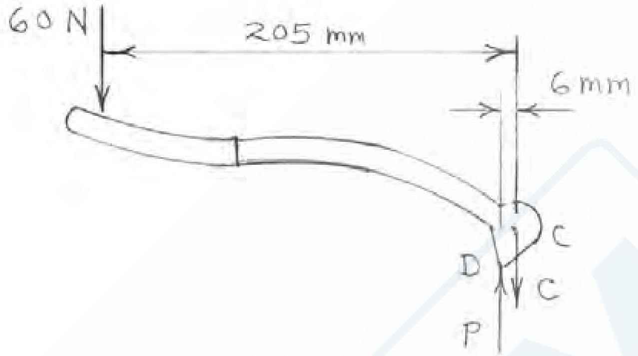
4/75 Upper handle/cam unit:



$$\circlearrowleft + \Sigma M_C = 0: \quad 60(205) - P(6) = 0$$

$$P = 2050\text{ N}$$

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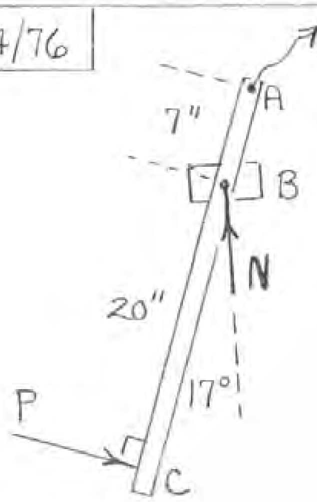
4/76



$\circlearrowleft + \Sigma M_A = 0:$

$$+P(27) - N \sin 17^\circ (7) = 0$$

$$\underline{N = 13.19P}$$

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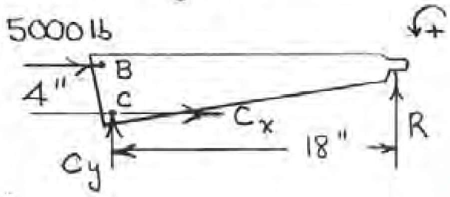
4/77 Piston force = $(500)(20) = 10{,}000$ lb

Force in link $AB = 10{,}000/2 = 5000$ lb

Lower jaw:



$\circlearrowleft + \Sigma M_C = 0: \; R(18) - 5000(4) = 0, \; R = 1111$ lb

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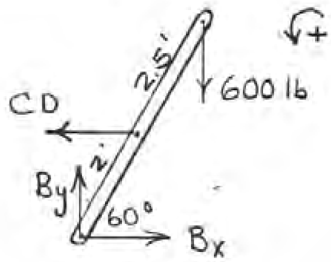
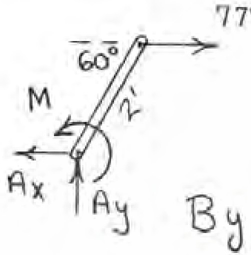
4/78 Member BE:



$\circlearrowleft + \Sigma M_B = 0: \; -600(4.5 \cos 60°) + CD(2 \sin 60°) = 0, \; CD = 779 \text{ lb}$

Member AC:



$\Sigma M_A = 0: \; -779(2 \sin 60°) + M = 0$

$\underline{M = 1350 \text{ lb-ft} \circlearrowleft}$

By inspection, $A_x = 779 \text{ lb}$, $A_y = 0$

$\therefore \; A = \sqrt{A_x^2 + A_y^2} = \underline{779 \text{ lb}}$

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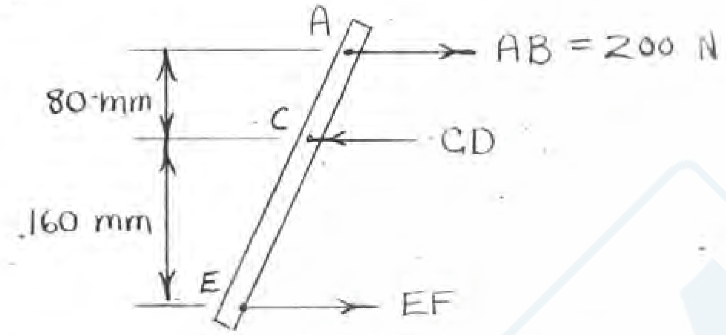

$\circlearrowleft + \Sigma M_C = 0: -200(80) + EF(160) = 0$

$EF = 100 \text{ N } T$

$\xrightarrow{+} \Sigma F = 0: 200 - CD + 100 = 0$

$CD = 300 \text{ N}$

So force supported by pin C is $F = 300 \text{ N}$

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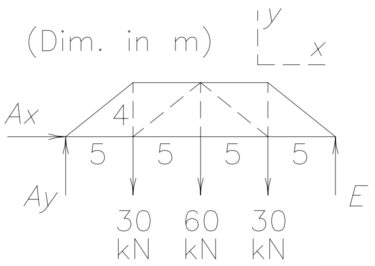
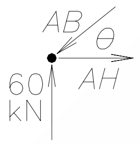


As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

$A_y = E = 60 \text{ kN}$ by

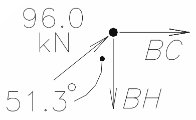
$\Sigma F_y = 0$ and symmetry.

Joint A: $\left(\theta = \tan^{-1}(4/5) = 38.7°\right)$



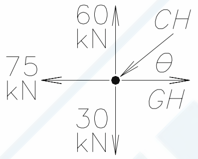
$$\begin{cases} \Sigma F_y = 0: 60 - AB\sin\theta = 0, \; \underline{AB = 96.0 \text{ kN } C} \\ \Sigma F_x = 0: AH - 96.0\cos\theta, \; \underline{AH = 75 \text{ kN } T} \end{cases}$$

Joint B:



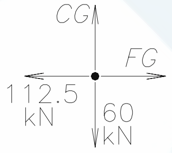
$$\begin{cases} \Sigma F_x = 0: BC + 96.0\sin 51.3° = 0, \; \underline{BC = -75 \text{ kN } (C)} \\ \Sigma F_y = 0: -BH + 96.0\cos 51.3° = 0, \; \underline{BH = 60 \text{ kN } T} \end{cases}$$

Joint H:



$$\begin{cases} \Sigma F_y = 0: -CH\sin\theta + 30 = 0, \; \underline{CH = 48.0 \text{ kN } C} \\ \Sigma F_x = 0: 48.0\cos\theta + GH - 75 = 0, \; \underline{GH = 112.5 \text{ kN } T} \end{cases}$$

Joint G:



$\Sigma F_y = 0 \Rightarrow CG = 60 \text{ kN } T$

By symmetry:
$FG = 112.5$ kN T, $CF = 48.0$ kN C
$CD = 75$ kN C, $DF = 60$ kN T
$\underline{EF = 75 \text{ kN } T, \; DE = 96.0 \text{ kN } C}$

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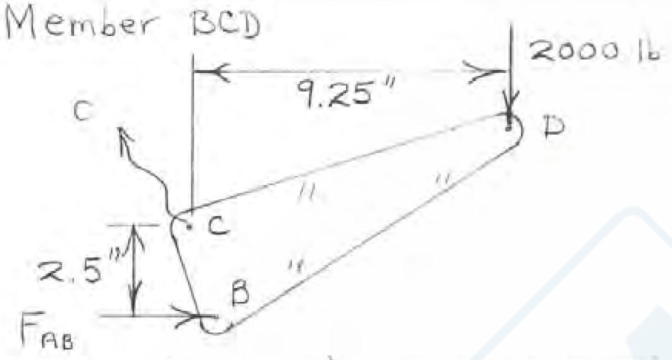
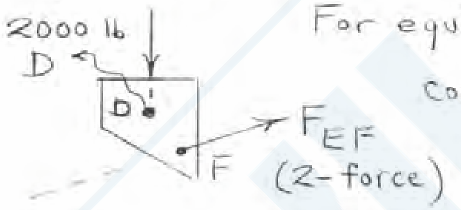
4/80 Member BCD



$\circlearrowleft \Sigma M_C = 0$: $F_{AB}(2.5) - 2000(9.25) = 0$

$F_{AB} = 7400$ lb

The force in member EF is zero, as is disclosed by a FBD of the lifting pad:



For equilibrium, F_{EF} must be concurrent at D, but it cannot be, so it is zero!

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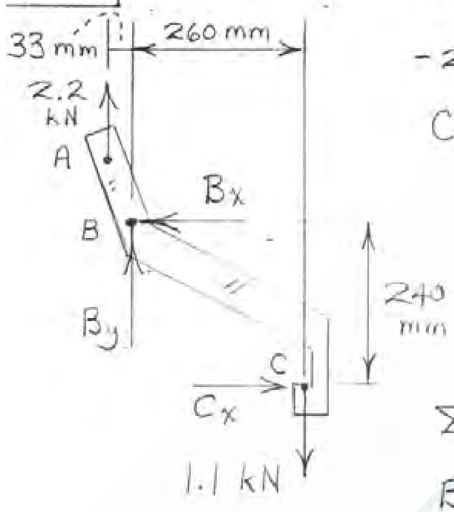

$\circlearrowleft + \ \Sigma M_B = 0:$

$-2.2(33) + C_x(240) - 1.1(260) = 0$

$C_x = 1.494 \text{ kN}$

$\Sigma F_x = 0:$

$1.494 - B_x = 0$

$B_x = 1.494 \text{ kN}$

$\Sigma F_y = 0:$

$B_y + 2.2 - 1.1 = 0$

$B_y = -1.1 \text{ kN}$

$B = \sqrt{1.494^2 + 1.1^2} = \underline{1.855 \text{ kN}}$

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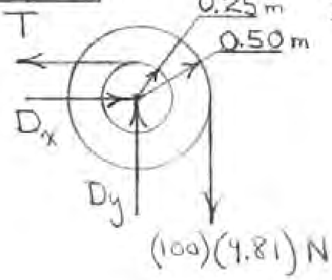
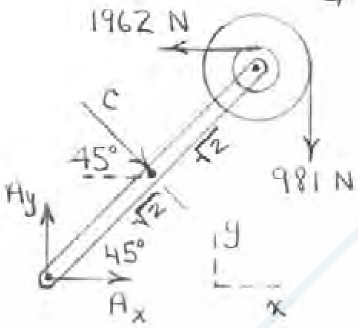
4/82



$\circlearrowleft + \quad \Sigma M_D = 0: \quad T(0.25) - 981(0.5) = 0$

$T = 1962 \text{ N}$



$\circlearrowleft + \quad \Sigma M_A = 0: \quad -C\sqrt{2}$

$+ 1962(2 + 0.25) - 981(2 + 0.50) = 0$

$C = 1390 \text{ N}$

$\therefore \quad C_x = 981 \text{ N}$

$C_y = -981 \text{ N}$

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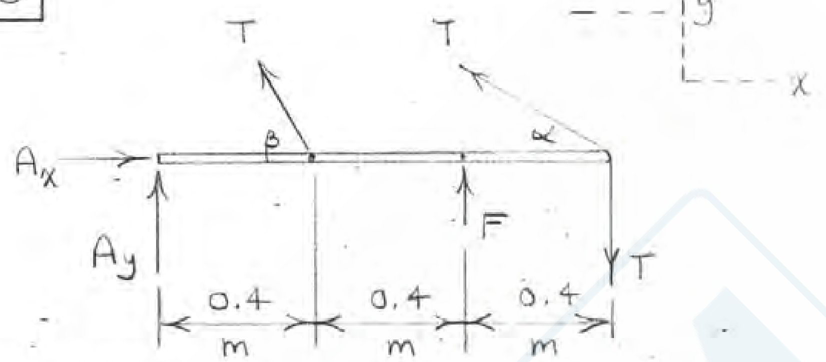

$$\begin{cases} T = 60(9.81) = 589 \text{ N} \\ \alpha = \tan^{-1}\left(\frac{0.5}{1.2}\right) = 22.6° \\ \beta = \tan^{-1}\left(\frac{0.5}{0.4}\right) = 51.3° \end{cases}$$

$\circlearrowleft + \; \Sigma M_A = 0: \; T\sin\beta(0.4) + T\sin\alpha\,(1.2)$

$\quad - T(1.2) + F(0.8) = 0, \quad \underline{F = 314 \text{ N}}$

(Contact at bottom roller)

$\Sigma F_x = 0: \; A_x - T\cos\beta - T\cos\alpha = 0, \; A_x = 911 \text{ N}$

$\Sigma F_y = 0: \; A_y + T\sin\beta + T\sin\alpha - T + F = 0$

$A_y = -411 \text{ N}$

$A = \sqrt{A_x^2 + A_y^2} = \underline{999 \text{ N}}$

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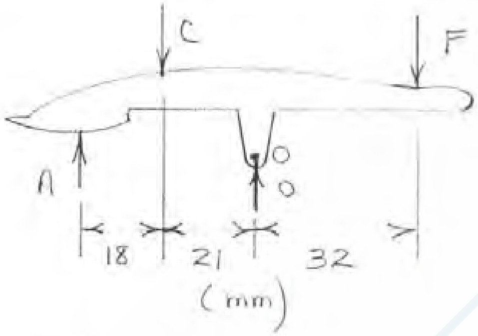

F = 15 N when A = 0:

$\circlearrowleft + \Sigma M_O = 0: \ C(21) - 15(32) = 0, \ C = 22.9 \text{ N}$

When F = 0:

$\circlearrowleft + \Sigma M_O = 0: \ -A(39) + 22.9(21) = 0, \ \underline{A = 12.31 \text{ N}}$

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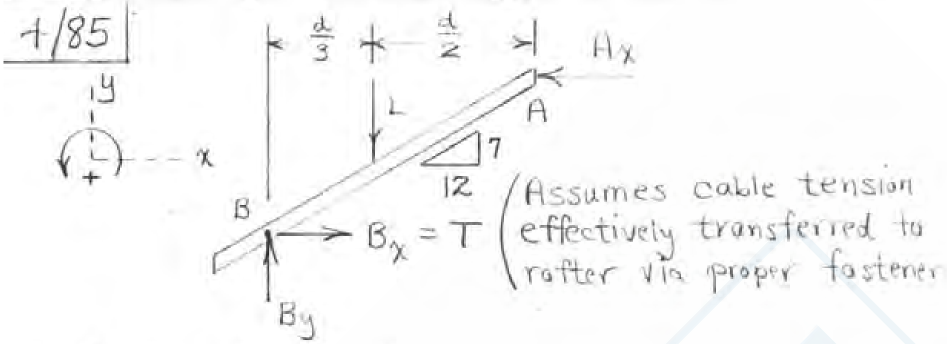

$\Sigma F_x = 0 \Rightarrow T = A_x$

$\Sigma M_B = 0 : \quad -L\left(\frac{d}{3}\right) + T\left(\frac{5d}{6} \cdot \frac{7}{12}\right) = 0$

$T = \frac{24}{35} L$

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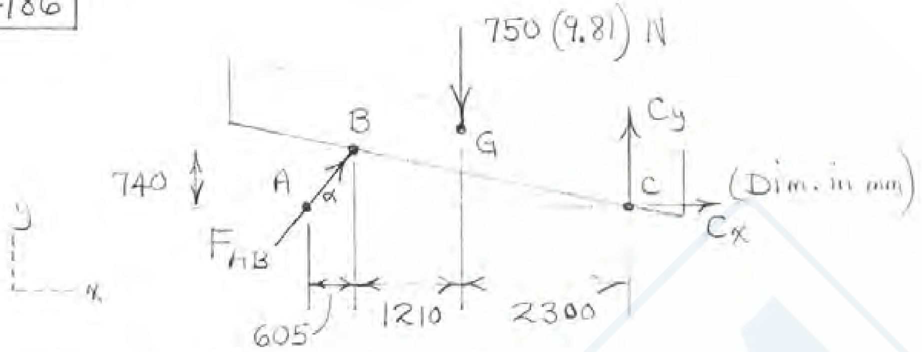

$\alpha = \tan^{-1}\left(\frac{740}{605}\right) = 50.7°$

$\circlearrowleft + \Sigma M_C = 0: \quad 750(9.81)(2300) - F_{AB} \sin\alpha \, (4115) = 0$

$\underline{F_{AB} = 5310 \text{ N}}$

$\Sigma F_x = 0: \quad 5310 \cos 50.7° + C_x = 0, \quad C_x = -3360 \text{ N}$

$\Sigma F_y = 0: \quad 5310 \sin 50.7° - 750(9.81) + C_y = 0$

$C_y = 3250 \text{ N}$

$C = \sqrt{3360^2 + 3250^2} = \underline{4670 \text{ N}}$

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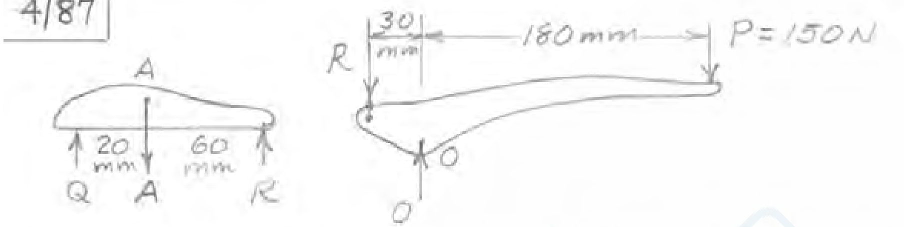

Handle: $\Sigma M_O = 0$; $30R - 180(150) = 0$, $R = 900\text{ N}$

Jaw: $\Sigma M_A = 0$; $20Q - 60(900) = 0$, $Q = 2700\text{ N}$

or $\underline{Q = 2.7\text{ kN}}$

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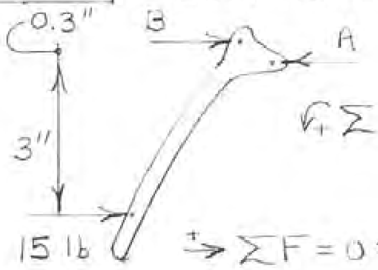
4/88 Left handle (with use of symmetry):

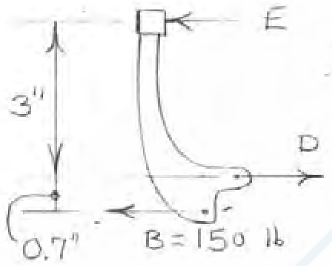


$\circlearrowleft+ \Sigma M_A = 0: \; 15(3) - B(0.3) = 0$

$\underline{B = 150 \text{ lb}}$

$\overset{+}{\rightarrow} \Sigma F = 0: \; 15 + 150 - A = 0, \quad \underline{A = 165 \text{ lb}}$

Left jaw:



$\circlearrowleft+ \Sigma M_D = 0: \; E(3) - 150(0.7) = 0$

$\underline{E = 35 \text{ lb}}$

$\overset{+}{\rightarrow} \Sigma F = 0: \; -35 - 150 + D = 0$

$\underline{D = 185 \text{ lb}}$

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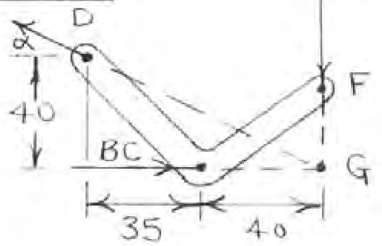
4/89



DCF is a three-force body; forces intersect at G.

$\alpha = \tan^{-1}\left(\frac{40}{75}\right) = 28.1°$

$\Sigma F_y = 0: -200 + D\sin\alpha = 0$

$\underline{D = 425\text{ N}}$

$\Sigma F_x = 0: -D\cos\alpha + BC = 0$, $\underline{BC = 375\text{ N}}$

(BC in compression)

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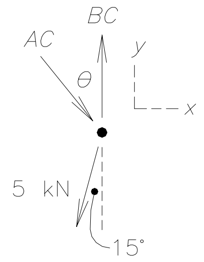
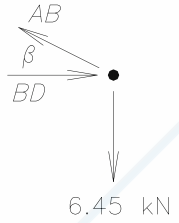
Joint *C*:



$$\theta = \tan^{-1}\frac{4}{5} = 38.7°$$

$$\begin{cases} \Sigma F_x = 0: \ -5\sin 15° + AC\sin 38.7° = 0, \ \underline{AC = 2.07 \text{ kN } C} \\ \Sigma F_y = 0: \ BC - 5\cos 15° - 2.07\cos 38.7° = 0, \ \underline{BC = 6.45 \text{ kN } T} \end{cases}$$

Joint *B*:



$$\beta = \tan^{-1}\frac{2}{4} = 26.6°$$

$$\begin{cases} \Sigma F_y = 0: \ AB\sin 26.6° - 6.45 = 0, \ \underline{AB = 14.42 \text{ kN } T} \\ \Sigma F_x = 0: \ -14.42\cos 26.6° + BD = 0, \ \underline{BD = 12.89 \text{ kN } C} \end{cases}$$

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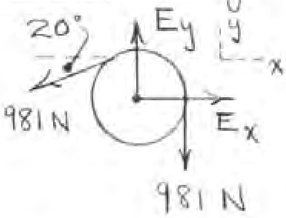
4/90 Pulley:



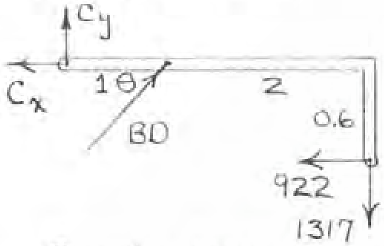
$\Sigma F_x = 0: \; E_x - 981 \cos 20° = 0$

$E_x = 922 \text{ N}$

$\Sigma F_y = 0: \; E_y - 981 - 981 \sin 20° = 0$

$E_y = 1317 \text{ N}$

Member CDE:

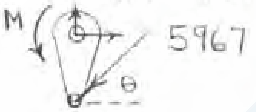


$\theta = \tan^{-1}\left(\frac{1.15}{1}\right) = 48.99°$

$\Sigma M_C = 0: \; (BD \sin\theta)(1) - 922(0.6) - 1317(3) = 0$

$BD = 5967 \text{ N}$

Crank AB:



$\Sigma M_A = 0: \; M - (5967 \cos\theta)(0.5) = 0$

$M = 1958 \text{ N·m}$ ↺

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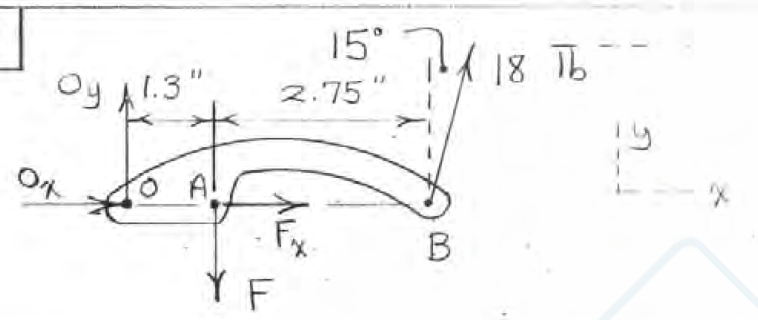

$$\circlearrowleft + \Sigma M_O = 0: -F(1.3) + 18\cos 15°(2.75 + 1.3) = 0$$

$$\underline{F = 54.2 \text{ lb}}$$

(Note: Treatment of member OC as a three-force body would yield a constraint relationship between O_x and O_y.)

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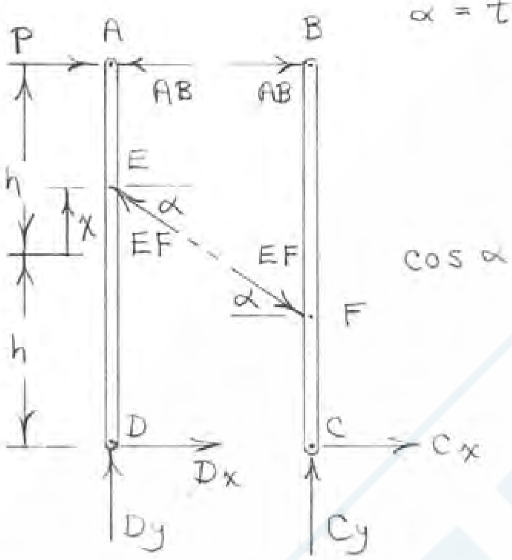
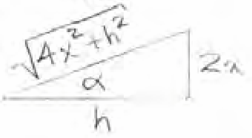


$\alpha = \tan^{-1}\frac{x}{h/2} = \tan^{-1}\frac{2x}{h}$



$\cos\alpha = \frac{h}{\sqrt{4x^2+h^2}}$

Member AED, $\circlearrowleft\!+\ \Sigma M_D = 0$:

$AB(2h) + EF\cos\alpha\,(h+x) - P(2h) = 0 \qquad (1)$

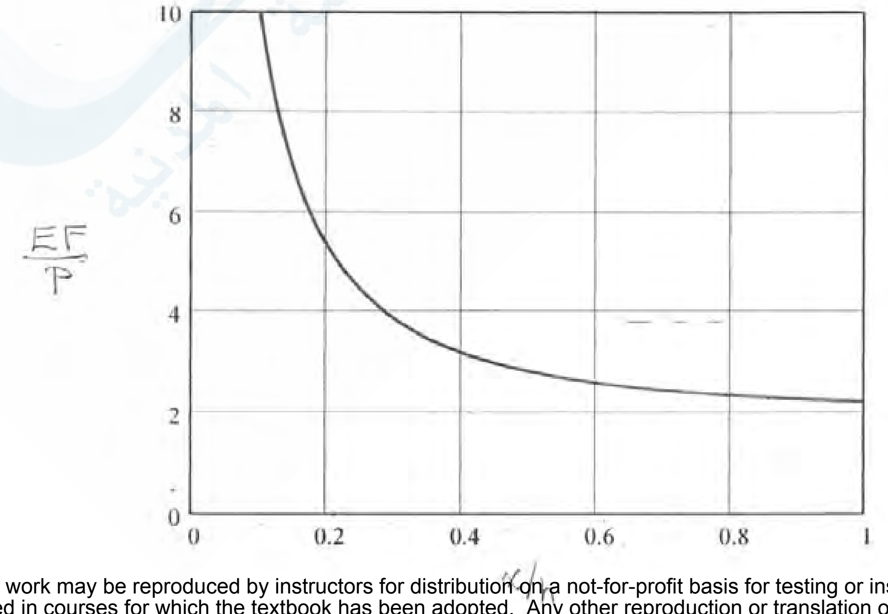
Member BFC, $\circlearrowleft\!+\ \Sigma M_C = 0$:

$-AB(2h) - EF\cos\alpha\,(h-x) = 0 \qquad (2)$

Add (1) & (2), use the above expression for $\cos\alpha$, and solve for EF:

$EF = P\sqrt{4 + \frac{1}{(x/h)^2}}$

Undefined @ $x=0$, but structure collapses for $x=0$



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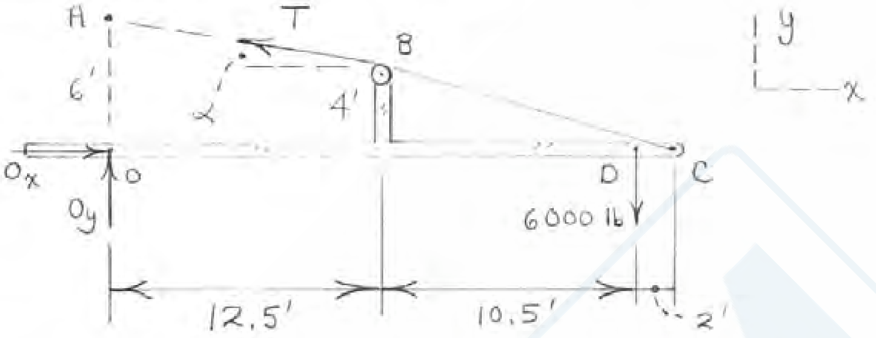
4/93 FBD of OBC and portion of cable:



$\alpha = \tan^{-1}\left(\frac{2}{12.5}\right) = 9.09°$

$\circlearrowleft+ \Sigma M_O = 0: \; T\cos\alpha\,(4) + T\sin\alpha\,(12.5) - 6000\,(23) = 0$

$T = 23{,}300$ lb

$\Sigma F_x = 0: \; O_x - 23{,}300\cos\alpha = 0$

$O_x = 23{,}000$ lb

$\Sigma F_y = 0: \; O_y + 23{,}300\sin\alpha - 6000 = 0$

$O_y = 2320$ lb

$O = \sqrt{23{,}000^2 + 2320^2} = 23{,}100$ lb

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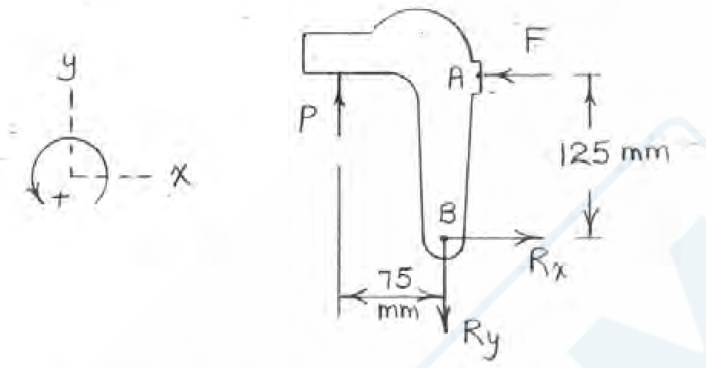

For $P = 3$ kN :

$\Sigma M_B = 0$: $125F - 3(75) = 0$, $F = 1.8$ kN

For $F = 2(1.8) = 3.6$ kN, $P = 3(2) = 6$ kN

$\Sigma F_x = 0$: $R_x - 3.6 = 0$, $R_x = 3.6$ kN

$\Sigma F_y = 0$: $-R_y + 6 = 0$, $R_y = 6$ kN

$R = \sqrt{3.6^2 + 6^2} = \underline{7.00 \text{ kN}}$

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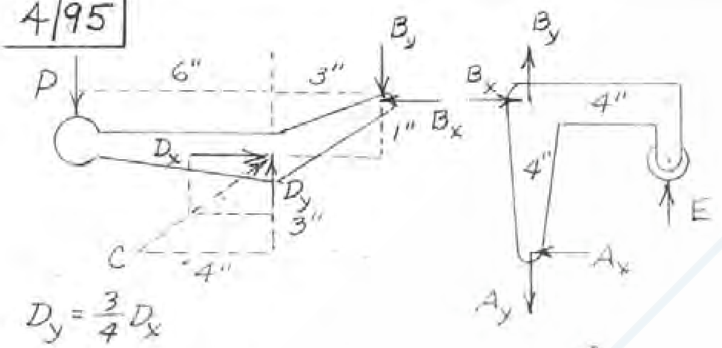

$D_y = \frac{3}{4} D_x$

Lever BD; $\Sigma M_B = 0$; $9P + 1 D_x - 3(\frac{3}{4} D_x) = 0$, $D_x = \frac{36}{5} P$

$D_y = \frac{3}{4} D_x = \frac{27}{5} P$

$\Sigma F_x = 0$; $B_x = D_x = \frac{36}{5} P$

$\Sigma F_y = 0$; $B_y + P - \frac{27}{5} P = 0$, $B_y = \frac{22}{5} P$

ABE; $\Sigma M_A = 0$; $4E - 4(\frac{36}{5} P) = 0$, $\underline{E = \frac{36}{5} P = 7.2 P}$

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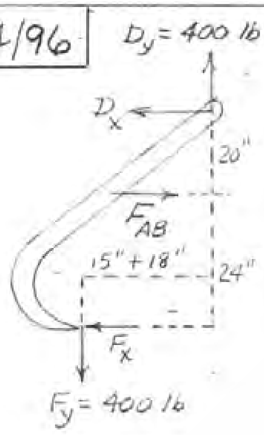

From tongs as a whole

$D_y = \frac{1}{2}(800) = 400 \text{ lb} = F_y$

From ED, $D_x = \frac{18}{12} D_y = \frac{18}{12}(400) = 600 \text{ lb}$

From DF, $\Sigma M_F = 0$;

$F_{AB}(24) - 600(44) - 400(18 + 15) = 0$

$F_{AB} = 1650 \text{ lb}$ tension

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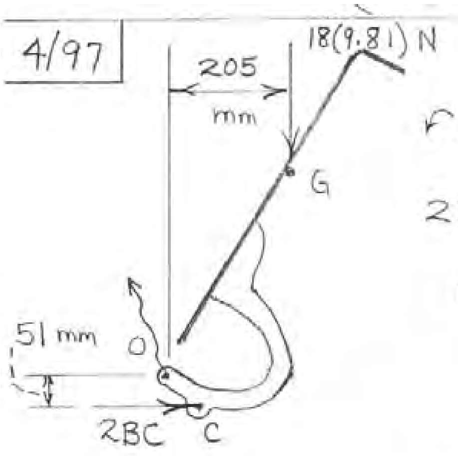
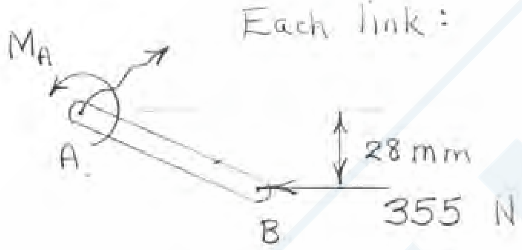


$\curvearrowleft \Sigma M_O = 0:$

$2BC(51) - 18(9.81)(205) = 0$

$BC = 355 \text{ N}$

Each link:



$\curvearrowleft_+ \Sigma M_A = 0: \quad M_A - 355(28) = 0$

$M_A = 9940 \text{ N}\cdot\text{mm}$ or $\underline{9.94 \text{ N}\cdot\text{m CCW}}$

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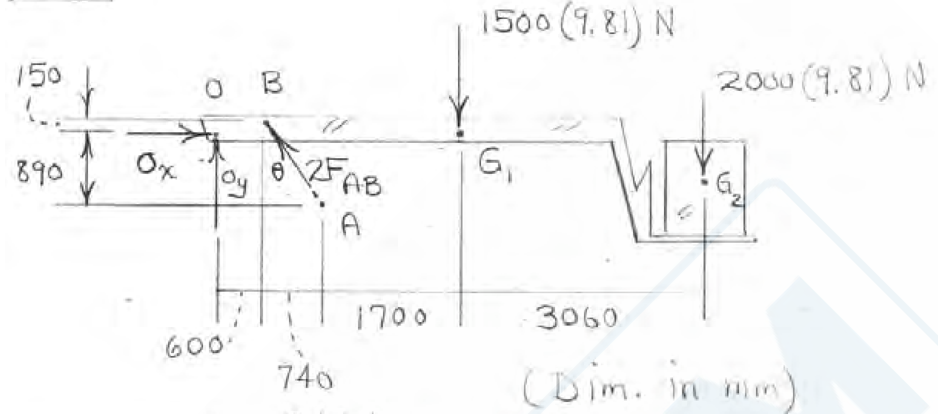

$\theta = \tan^{-1}\left(\frac{740}{1040}\right) = 35.4°$

$\circlearrowleft \Sigma M_O = 0: 2F_{AB}\cos\theta\,(600) + 2F_{AB}\sin\theta\,(150)$

$- 1500(9.81)(3040) - 2000(9.81)(6100) = 0$

$F_{AB} = 142\ 800$ N or $\underline{142.8 \text{ kN}}$

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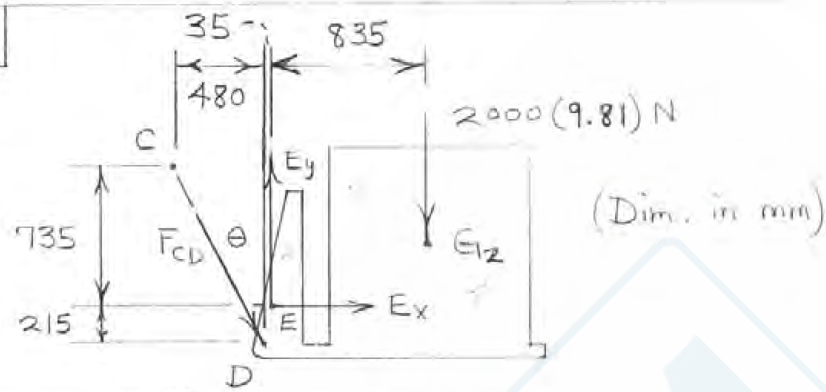

$$\theta = \tan^{-1}\left(\frac{480}{950}\right) = 26.8°$$

$$\circlearrowleft + \Sigma M_E = 0: \; -2000(9.81)(835) + F_{CD}\cos\theta\,(35) + F_{CD}\sin\theta\,(215) = 0$$

$$F_{CD} = 127\,800 \text{ N}$$

or $\underline{127.8 \text{ kN}}$

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5/1 The horizontal coordinate to the centroid is $14 - \frac{1}{3}(14-2) = \underline{10}$

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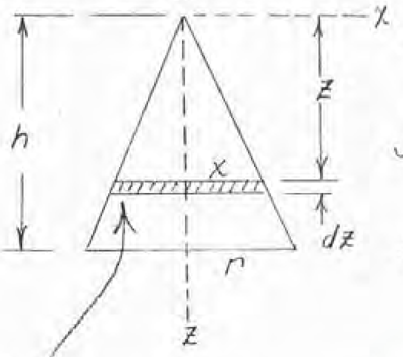
5/10 $x = \frac{r}{h} z$, $dV = \pi x^2 dz = \pi \frac{r^2}{h^2} z^2 dz$



$V = \pi \frac{r^2}{h^2} \int_0^h z^2 dz = \frac{\pi r^2 h}{3}$

$\int z \, dV = \pi \frac{r^2}{h^2} \int_0^h z^3 dz = \frac{\pi r^2 h^2}{4}$

$\bar{z} = \int z \, dV / V = \underline{3h/4}$

(Disk - shaped element viewed edge-on.)

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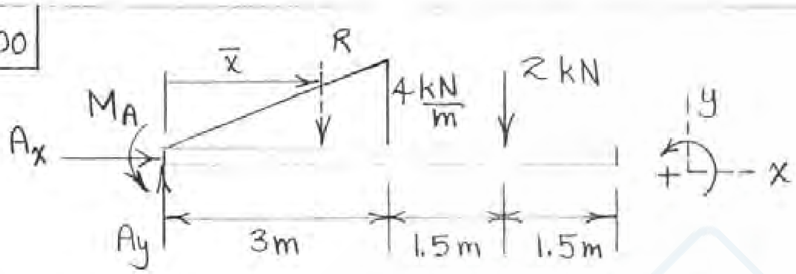
5/100



$R = \frac{1}{2}(3)(4) = 6 \text{ kN}$ @ $\bar{x} = \frac{2}{3}(3) = 2\text{m}$

$\Sigma M_A = 0: \; M_A - 6(2) - 2(4.5) = 0, \quad \underline{M_A = 21 \text{ kN}\cdot\text{m}}$

$\Sigma F_y = 0: \; A_y - 6 - 2 = 0, \quad \underline{A_y = 8 \text{ kN}}$

$\Sigma F_x = 0: \; \underline{A_x = 0}$

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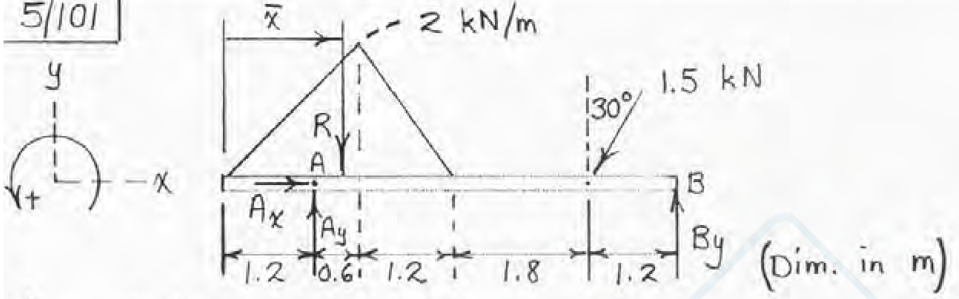

$$R = \tfrac{1}{2}(1.2+0.6+1.2)(2) = 3 \text{ kN}$$

$$\bar{x} = \frac{(1.2+0.6)+(1.2+0.6+1.2)}{3} = 1.6 \text{ m}$$

$\Sigma F_x = 0: \quad A_x - 1.5\sin 30° = 0, \quad \underline{A_x = 750 \text{ N}}$

$\Sigma M_A = 0: \quad -3(1.6-1.2) - 1.5\cos 30°(3.6) - B_y(4.8) = 0$

$\underline{B_y = 1.224 \text{ kN}}$

$\Sigma F_y = 0: \quad A_y - 3 - 1.5\cos 30° + 1.224 = 0$

$\underline{A_y = 3.07 \text{ kN}}$

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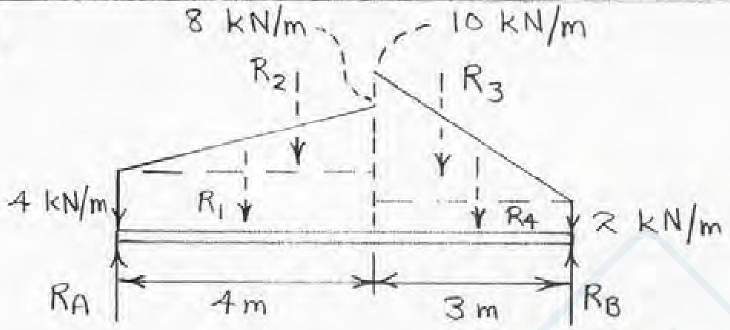

$R_1 = 4(4) = 16$ kN, $R_2 = \frac{1}{2}(4)(4) = 8$ kN

$R_3 = \frac{1}{2}(8)(3) = 12$ kN, $R_4 = 2(3) = 6$ kN

$\circlearrowleft + \Sigma M_A = 0: \; 7R_B - 16(2) - 8(\frac{2}{3}4) - 12(4+1)$

$-6(4+1.5) = 0$, $\underline{R_B = 20.9 \text{ kN}}$

$+\uparrow \Sigma F = 0: \; R_A + 20.9 - (16+8+12+6) = 0$

$\underline{R_A = 21.1 \text{ kN}}$

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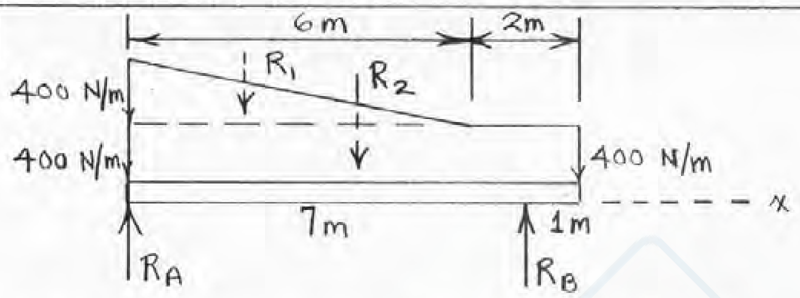

$R_1 = \frac{1}{2}(400)(6) = 1200 \text{ N} \quad @ \quad \bar{x}_1 = \frac{1}{3}(6) = 2\text{m}$

$R_2 = 400(8) = 3200 \text{ N} \quad @ \quad \bar{x}_2 = \frac{1}{2}(8) = 4\text{m}$

$\circlearrowleft + \Sigma M_A = 0: \; R_B(7) - 1200(2) - 3200(4) = 0, \quad \underline{R_B = 2170 \text{ N}}$

$+\uparrow \Sigma F = 0: \; R_A - 1200 - 3200 + 2170 = 0, \quad \underline{R_A = 2230 \text{ N}}$

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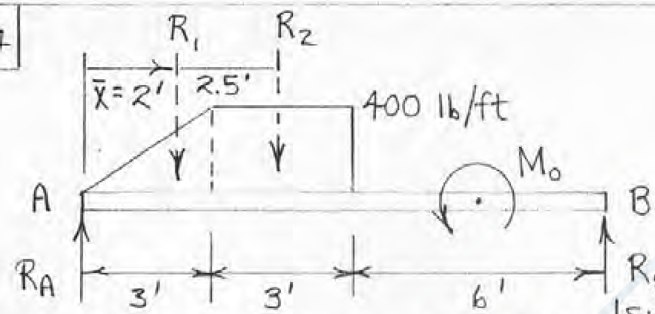

$R_B = 0$ at switch from lower to upper surface

$R_1 = \frac{1}{2}(400)(3) = 600 \text{ lb} \ @ \ \bar{x} = \frac{2}{3}(3) = 2'$

$R_2 = 400(3) = 1200 \text{ lb}$

$\circlearrowleft + \ \Sigma M_A = 0: \ -600(2) - 1200(4.5) + M_0 = 0$

$\underline{M_0 = 6600 \text{ lb-ft}}$

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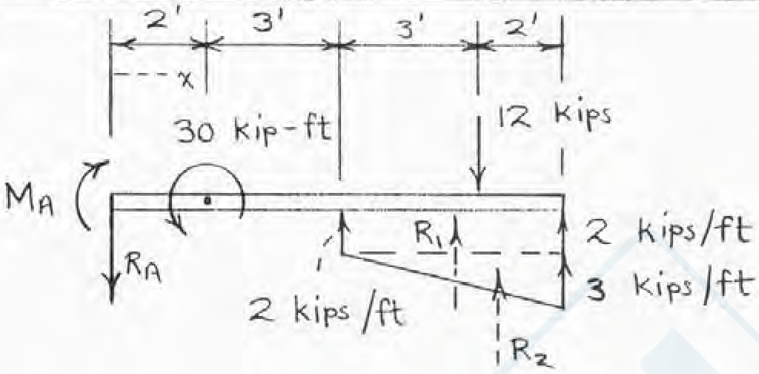

$R_1 = 2(5) = 10$ kips @ $\bar{x}_1 = 7.5'$

$R_2 = \frac{1}{2}(3)(5) = 7.5$ kips @ $\bar{x}_2 = 8.33'$

$\circlearrowleft\!+ \Sigma M_A = 0: \quad M_A - 30 + 12(8) - 10(7.5) - 7.5(8.33) = 0$

$\underline{M_A = 71.5 \text{ kip-ft}}$

$+\uparrow \Sigma F = 0: \; -R_A + 10 + 7.5 - 12 = 0, \quad \underline{R_A = 5.5 \text{ kips}}$

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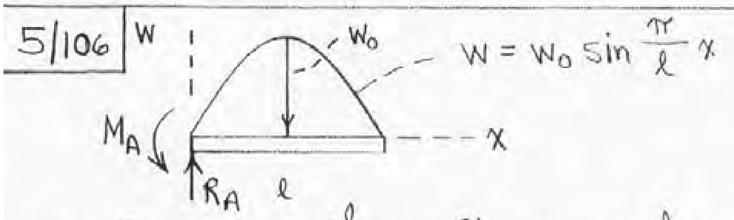

$$R = \int w\,dx = \int_0^l w_0 \sin\frac{\pi}{l}x = -w_0\frac{l}{\pi}\cos\frac{\pi}{l}x\Big|_0^l = \frac{2w_0 l}{\pi}$$

$\bar{x} = \frac{l}{2}$, by inspection

$\circlearrowleft+\ \Sigma M_A = 0: \quad M_A - \frac{2w_0 l}{\pi}\left(\frac{l}{2}\right) = 0, \qquad \underline{M_A = \frac{w_0 l^2}{\pi}}$

$+\uparrow \Sigma F = 0: \quad R_A - \frac{2w_0 l}{\pi} = 0, \qquad \underline{R_A = \frac{2w_0 l}{\pi}}$

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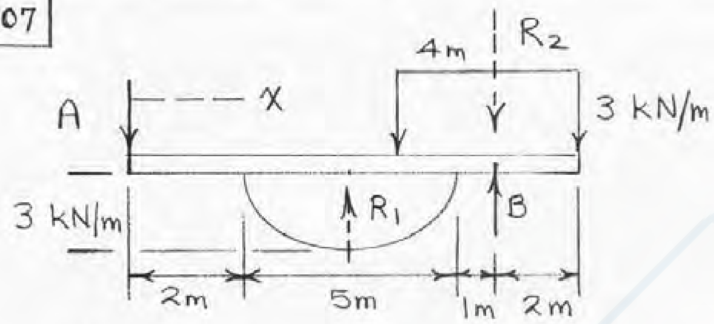

$R_1 = \frac{\pi(5)(3)}{4} = 11.78$ kN @ $\bar{x}_1 = 4.5$ m

$R_2 = 4(3) = 12$ kN @ $\bar{x}_2 = 8$ m

$\circlearrowleft + \Sigma M_B = 0: \quad A(8) - 11.78(3.5) = 0, \quad \underline{A = 5.15 \text{ kN}}$

$+\uparrow \Sigma F = 0 \quad -5.15 + 11.78 - 12 + B = 0, \quad \underline{B = 5.37 \text{ kN}}$

Contact at A is at the upper roller.

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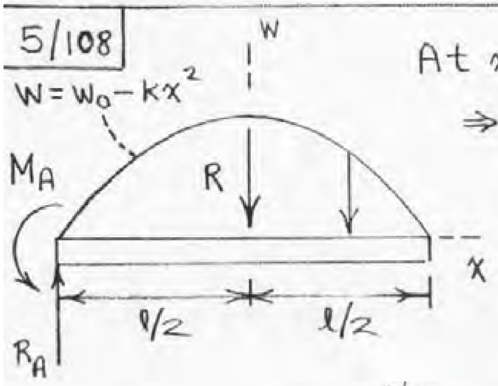
5/108



At $x = \frac{l}{2}$, $w = 0 = w_0 - k\left(\frac{l}{2}\right)^2$

$\Rightarrow k = \frac{4w_0}{l^2}$

$w = w_0\left(1 - \frac{4}{l^2}x^2\right)$

$\bar{x} = 0$, by inspection

$$R = \int w\,dx = 2\int_0^{l/2} w_0\left(1 - \frac{4}{l^2}x^2\right)dx$$

$$= 2w_0\left[x - \frac{4}{3l^2}x^3\right]_0^{l/2} = \frac{2}{3}w_0 l$$

$+\uparrow \Sigma F = 0: \quad R_A - \frac{2}{3}w_0 l = 0, \quad \underline{R_A = \frac{2}{3}w_0 l}$

$\circlearrowleft + \Sigma M_A = 0: \quad M_A - \frac{2}{3}w_0 l\left(\frac{l}{2}\right) = 0, \quad \underline{M_A = \frac{1}{3}w_0 l^2}$

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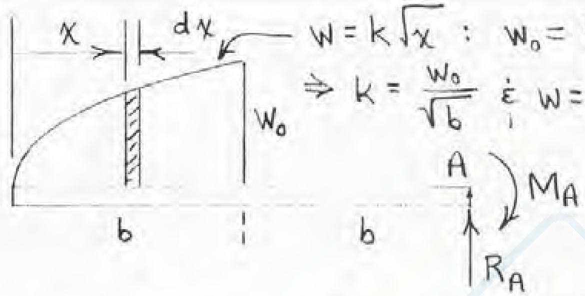

$w = k\sqrt{x}$: $w_0 = k\sqrt{b}$

$\Rightarrow k = \frac{w_0}{\sqrt{b}}$ & $w = w_0\sqrt{\frac{x}{b}}$

The force on dx is $dF = w\,dx = w_0\sqrt{\frac{x}{b}}\,dx$

Total load $F = \int dF = \int_0^b w_0\sqrt{\frac{x}{b}}\,dx = \frac{2}{3}w_0 b$

So $\underline{R_A = \frac{2}{3}w_0 b}$

The moment of dF about A is $w\,dx(2b-x)$

$M_A = \int_0^b w\,(2b-x)\,dx = \int_0^b w_0\sqrt{\frac{x}{b}}\,(2b-x)\,dx$

$= \frac{w_0}{\sqrt{b}}\int_0^b (2b\sqrt{x} - x^{3/2})\,dx = \underline{\underline{\frac{14}{15}w_0 b^2}}$

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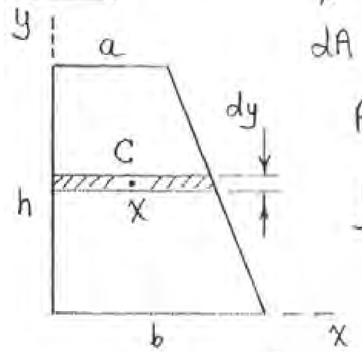

$$x = \left(\frac{a-b}{h}\right)y + b$$

$$dA = x\,dy = \left[\left(\frac{a-b}{h}\right)y + b\right]dy$$

$$A = \left(\frac{a+b}{2}\right)h = \frac{h}{2}(a+b)$$

$$\int x_c\,dA = \int_0^h \frac{x}{2}\left[\left(\frac{a-b}{h}\right)y + b\right]dy$$

$$= \frac{1}{2}\int_0^h \left[\left(\frac{a-b}{h}\right)y + b\right]^2 dy$$

$$\int x_c\,dA = \frac{1}{2}\int_0^h \left(\frac{a-b}{h}\right)^2 y^2 + 2b\left(\frac{a-b}{h}\right)y + b^2\Big]\,dy$$

$$= \frac{1}{2}\left[\left(\frac{a-b}{h}\right)^2 \frac{y^3}{3} + 2b\left(\frac{a-b}{h}\right)\frac{y^2}{2} + b^2 y\right]_0^h$$

$$= \frac{h}{6}\left(a^2 + b^2 + ab\right)$$

$$\int y_c\,dA = \int_0^h y\left[\left(\frac{a-b}{h}\right)y + b\right]dy = \int_0^h \left[\left(\frac{a-b}{h}\right)y^2 + by\right]dy$$

$$= \left[\left(\frac{a-b}{h}\right)\frac{y^3}{3} + b\frac{y^2}{2}\right]_0^h = \frac{h^2}{3}\left(a + \frac{b}{2}\right)$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{h}{6}\left(a^2+b^2+ab\right)}{\frac{h}{2}(a+b)} = \underline{\frac{a^2+b^2+ab}{3(a+b)}}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{h^2}{3}\left(a+\frac{b}{2}\right)}{\frac{h}{2}(a+b)} = \underline{\frac{h(2a+b)}{3(a+b)}}$$

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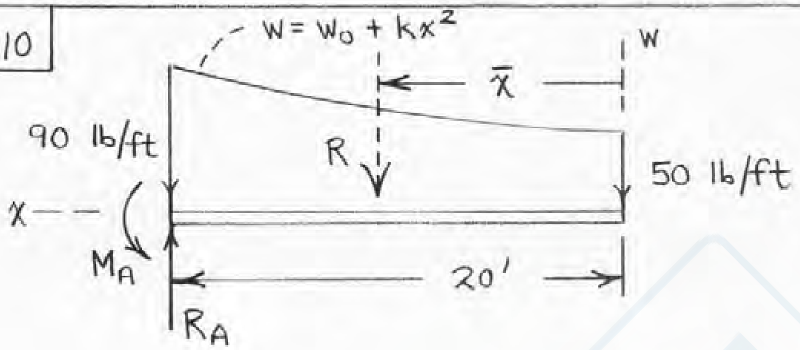

At $x = 0$, $w = 50 = w_0$

At $x = 20'$, $w = 90 = 50 + k(20)^2$, $k = \frac{1}{10}$ lb/ft^3

So $w = 50 + \frac{x^2}{10}$ (lb/ft)

$$R = \int w\,dx = \int_0^{20} \left(50 + \frac{x^2}{10}\right) dx = \left[50x + \frac{x^3}{30}\right]_0^{20} = 1267 \text{ lb}$$

$$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int x\left(50 + \frac{x^2}{10}\right) dx}{1267} = \frac{\left[\frac{50}{2}x^2 + \frac{x^4}{40}\right]_0^{20}}{1267}$$

$= 11.05$ ft

$+\uparrow \Sigma F = 0: \; R_A - 1267 = 0,$ $\underline{R_A = 1267 \text{ lb}}$

$\circlearrowleft \Sigma M_A = 0: \; M_A - 1267(20 - 11.05) = 0,$ $\underline{M_A = 11.33(10^3) \text{ lb-ft}}$

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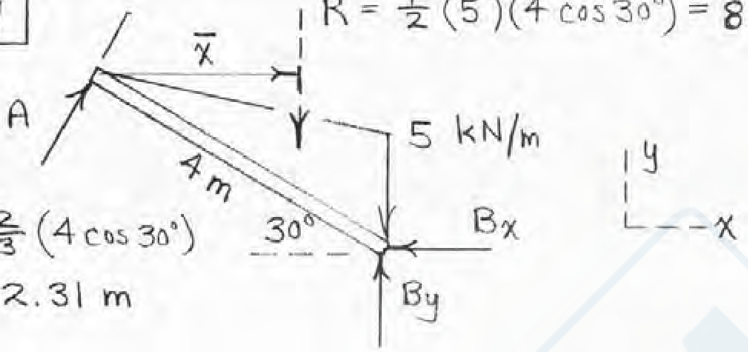
$R = \frac{1}{2}(5)(4\cos 30^\circ) = 8.66$ kN



$\bar{x} = \frac{2}{3}(4\cos 30^\circ)$

$= 2.31$ m

$\Sigma F_x = 0: \quad A\sin 30^\circ - B_x = 0$

$\Sigma F_y = 0: \quad A\cos 30^\circ + B_y - 8.66 = 0$

$\circlearrowleft^{+} \Sigma M_B = 0: \quad -A(4) + 8.66\left(\frac{1}{3}\,4\cos 30^\circ\right) = 0$

$\underline{A = 2.5 \text{ kN}}$, $B_x = 1.25$ kN, $B_y = 6.50$ kN

$B = \sqrt{B_x^2 + B_y^2} = \underline{6.61 \text{ kN}}$

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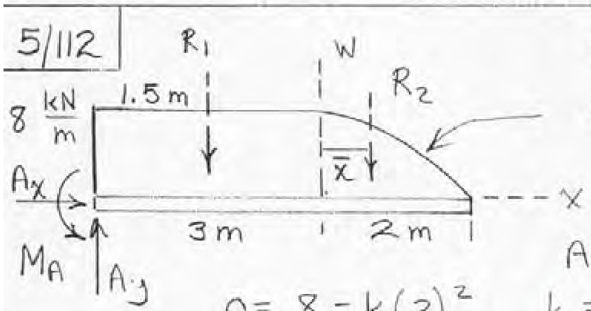

$R_1 = 8(3) = 24 \text{ kN}$

$w = w_0 - kx^2$

$w_0 = 8 \text{ kN/m}$

At $x = 2\text{m}$:

$0 = 8 - k(2)^2, \quad k = 2 \text{ kN/m}^3$

$$R_2 = \int w\,dx = \int_0^2 (8 - 2x^2)\,dx = \left(8x - \frac{2}{3}x^3\right)_0^2$$

$$= 10.67 \text{ kN}$$

$$\bar{x} = \frac{\int x\,w\,dx}{R} = \frac{\int_0^2 (8x - 2x^3)\,dx}{10.67} = \frac{\left(4x^2 - \frac{1}{2}x^4\right)_0^2}{10.67}$$

$$= 0.75 \text{ m}$$

Equilibrium considerations (+ counterclockwise; x, y axes)

$\Sigma F_x = 0 \Rightarrow A_x = 0$

$\Sigma F_y = 0: \quad A_y - 24 - 10.67 = 0, \quad \underline{A_y = 34.7 \text{ kN}}$

$\Sigma M_A = 0: \quad M_A - 24(1.5) - 10.67(3.75) = 0$

$\underline{M_A = 76 \text{ kN}\cdot\text{m}}$

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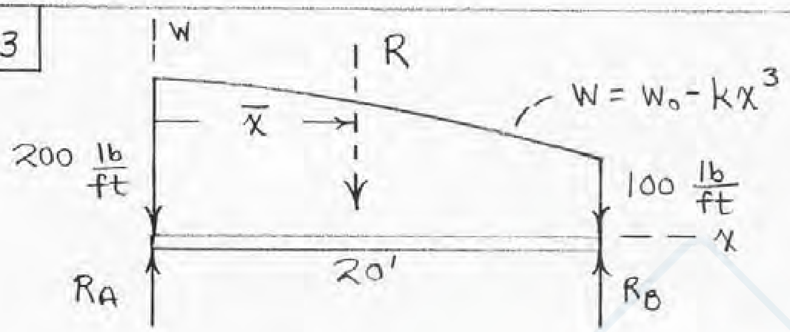

At $x = 0$, $w = w_0 = 200$ lb/ft

At $x = 20'$, $w = 200 - k(20)^3 = 100$, $k = \frac{1}{80}\ \frac{\text{lb}}{\text{ft}^4}$

So $w = 200 - x^3/80$ (lb/ft)

$$R = \int w\,dx = \int_0^{20} \left(200 - \frac{x^3}{80}\right) dx = \left[200x - \frac{x^4}{320}\right]_0^{20} = 3500 \text{ lb}$$

$$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int_0^{20} \left(200 - \frac{x^3}{80}\right) x\,dx}{3500} = \frac{\left[100x^2 - \frac{x^5}{400}\right]_0^{20}}{3500}$$

$$= 9.14'$$

$\circlearrowleft + \Sigma M_A = 0: \ 20R_B - 3500(9.14) = 0$, $\underline{R_B = 1600 \text{ lb}}$

$+\uparrow \Sigma F = 0: \ R_A + 1600 - 3500 = 0$, $\underline{R_A = 1900 \text{ lb}}$

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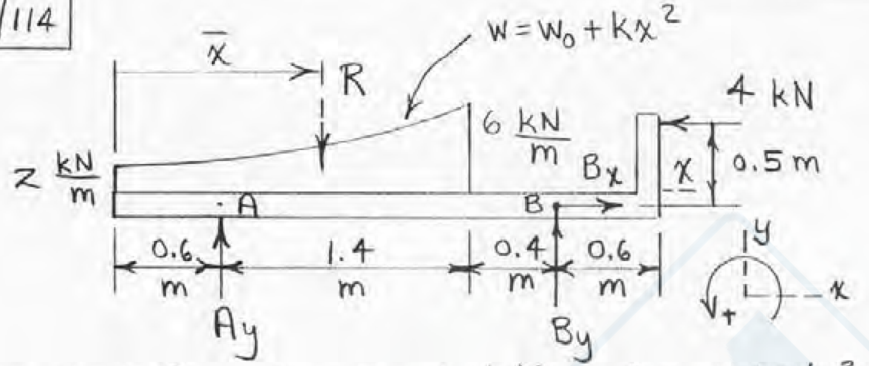

$w = 2 + kx^2 : \quad 6 = 2 + k(2)^2, \quad k = 1 \text{ kN/m}^3,$

$w = 2 + x^2$

$R = \int w\,dx = \int_0^2 (2 + x^2)\,dx = 2x + \frac{x^3}{3}\Big|_0^2 = 6.67 \text{ kN}$

$\bar{x} = \frac{\int x w\,dx}{R} = \frac{\int_0^2 (2x + x^3)\,dx}{6.67} = \frac{x^2 + \frac{x^4}{4}}{6.67}\Big|_0^2 = 1.2 \text{ m}$

$\Sigma F_x = 0: \quad B_x - 4 = 0, \quad \underline{B_x = 4 \text{ kN}}$

$\Sigma M_B = 0: \quad 4(0.5) + 6.67(2.4 - 1.2) - A_y(1.8) = 0$

$\underline{A_y = 5.56 \text{ kN}}$

$\Sigma F_y = 0: \quad 5.56 + B_y - 6.67 = 0, \quad \underline{B_y = 1.111 \text{ kN}}$

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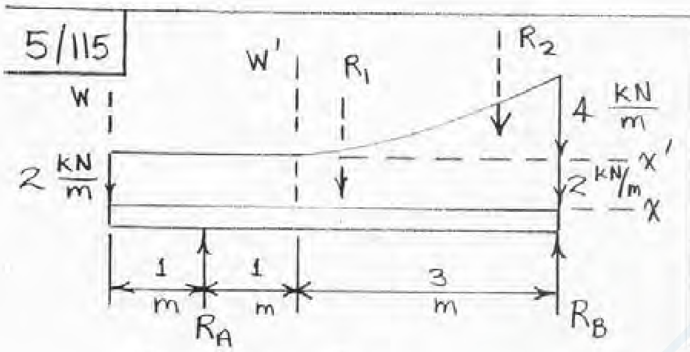

$w' = kx'^2$

At $x' = 3$ m:

$4 = k(3)^2$

$k = \frac{4}{9}$ N/m^3

$w' = \frac{4}{9}x'^2$

$$R_2 = \int_0^3 w'dx' = \int_0^3 \frac{4}{9}x'^2 = \frac{4}{9}\frac{x'^3}{3}\Big|_0^3 = 4 \text{ kN}$$

$$\bar{x}_2' = \int x'w'dx'/R_2 = \frac{1}{4}\int_0^3 \frac{4}{9}x'^3 = \frac{1}{4}\frac{4}{9}\frac{x'^4}{4}\Big|_0^3 = 2.25 \text{ m}$$

$$R_1 = 2(5) = 10 \text{ kN @ } \bar{x}_1 = 2.5 \text{ m} \quad \bar{x}_2 = 4.25 \text{ m}$$

$$\circlearrowleft + \Sigma M_A = 0: -10(1.5) - 4(3.25) + 4R_B = 0$$

$$\underline{R_B = 7 \text{ kN}}$$

$$+\uparrow \Sigma F = 0: \; 7 + R_A - 4 - 10 = 0, \quad \underline{R_A = 7 \text{ kN}}$$

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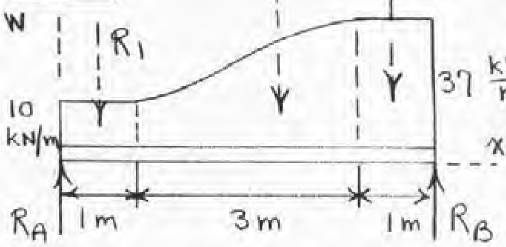

$w = w_0 + k_1 x + k_2 x^2 + k_3 x^3$

$w(1) = 10 = w_0 + k_1 + k_2 + k_3$

$w(4) = 37 = w_0 + k_1(4) + k_2(4)^2 + k_3(4)^3$

$\frac{dw}{dx} = k_1 + 2k_2 x + 3k_3 x^2 : \begin{cases} 0 = k_1 + 2k_2(1) + 3k_3(1)^2 \\ 0 = k_1 + 2k_2(4) + 3k_3(4)^2 \end{cases}$

Solve simultaneously to get $w = 21 - 24x + 15x^2 - 2x^3$

$R_2 = \int w\,dx = \int_1^4 (21 - 24x + 15x^2 - 2x^3)\,dx$

$= \left[21x - 12x^2 + 5x^3 - \frac{1}{2}x^4\right]_1^4 = 70.5 \text{ kN}$

$\bar{x}_2 = \frac{1}{R_2}\int x w\,dx = \frac{1}{70.5}\int_1^4 (21 - 24x + 15x^2 - 2x^3)\,x\,dx$

$= \frac{1}{70.5}\left[\frac{21}{2}x^2 - 8x^3 + \frac{15}{4}x^4 - \frac{2}{5}x^5\right]_1^4 = 2.84 \text{ m}$

$R_1 = 10(1) = 10 \text{ kN} \;@\; \bar{x}_1 = 0.5 \text{ m}$

$R_3 = 37(1) = 37 \text{ kN} \;@\; \bar{x}_3 = 4.5 \text{ m}$

$\Sigma M_A = 0: \; 5R_B - 10(0.5) - 70.5(2.84) - 37(4.5) = 0$

$\underline{R_B = 74.4 \text{ kN}}$

$\Sigma F = 0: \; 74.4 - 10 - 70.5 - 37 + R_A = 0, \quad \underline{R_A = 43.1 \text{ kN}}$

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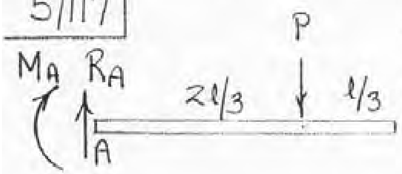
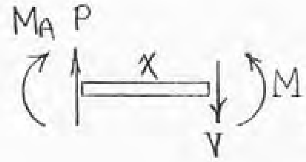


$+\uparrow \Sigma F = 0 \Rightarrow R_A = P$

$\circlearrowleft + \Sigma M_A = 0: \; -M_A - P\left(\frac{2l}{3}\right) = 0$

$M_A = -\frac{2Pl}{3}$

$0 < x < \frac{2l}{3}$:

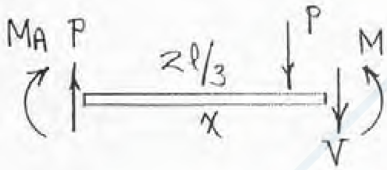


$+\uparrow \Sigma F = 0 \Rightarrow V = P$

$\circlearrowleft + \Sigma M = 0: \; \frac{2Pl}{3} - Px + M = 0$

$M = P\left(x - \frac{2l}{3}\right)$

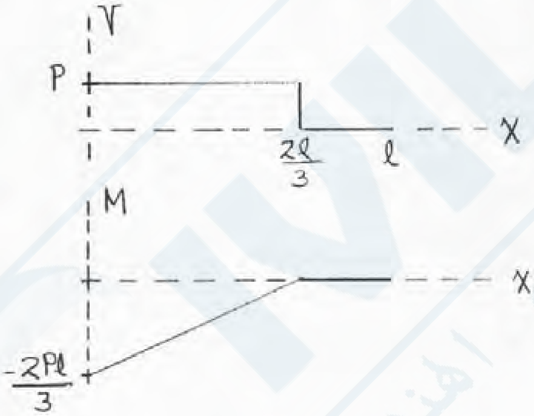
$\frac{2l}{3} < x < l$:



$+\uparrow \Sigma F = 0: \; P - P - V = 0, \; V = 0$

$\circlearrowleft + \Sigma M = 0: \; \frac{2Pl}{3} - P\left(\frac{2l}{3}\right) + M = 0$

$M = 0$



At $x = l/2$:

$V = P$

$M = P\left(\frac{l}{2} - \frac{2l}{3}\right)$

$= -\frac{Pl}{6}$

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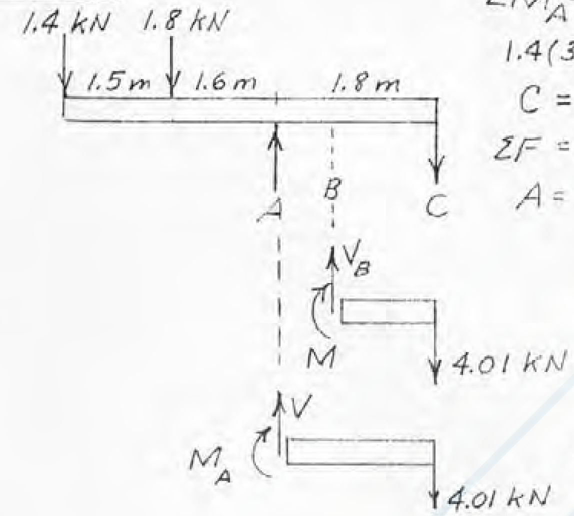

$\Sigma M_A = 0;$

$1.4(3.1) + 1.8(1.6) - 1.8C = 0$

$C = 4.01 \text{ kN}$

$\Sigma F = 0; \; A - 4.01 - 1.4 - 1.8 = 0$

$A = 7.21 \text{ kN}$

$\Sigma F = 0; \; \underline{V_B = 4.01 \text{ kN}}$

$\Sigma M_A = 0; \; M_A + 4.01(1.8) = 0$

$\underline{M_A = -7.22 \text{ kN}\cdot\text{m}}$

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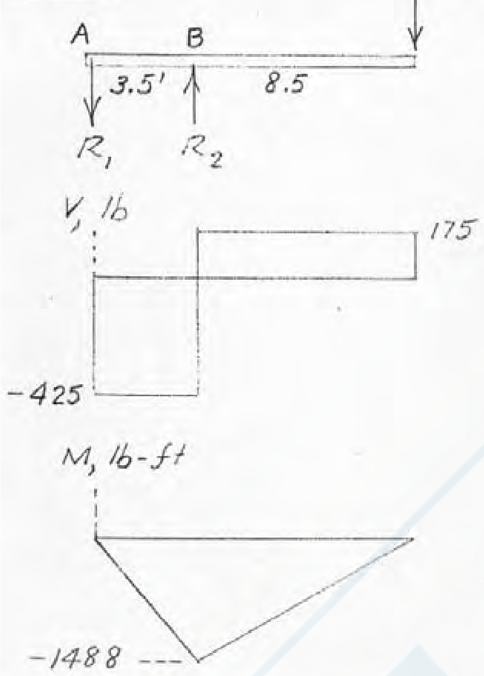

$\Sigma M_{R_1} = 0;\ 12(175) = 3.5 R_2$

$R_2 = 600$ lb

$R_1 = 425$ lb

$M_B = -1488$ lb-ft

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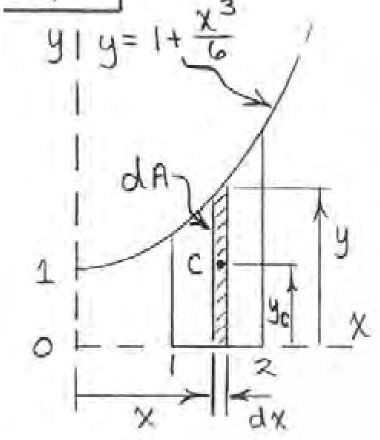

$dA = y\,dx = \left(1 + \frac{x^3}{6}\right)dx$

$A = \int dA = \int_1^2 \left(1 + \frac{x^3}{6}\right)dx$

$= x + \frac{x^4}{24}\Big|_1^2 = \frac{39}{24}$

$\int x_c\,dA = \int_1^2 x\left(1 + \frac{x^3}{6}\right)dx$

$= \frac{x^2}{2} + \frac{x^5}{30}\Big|_1^2 = \frac{38}{15}$

$\int y_c\,dA = \int \frac{y}{2}\,y\,dx = \int \frac{y^2}{2}\,dx = \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{6}\right)^2 dx$

$= \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{3} + \frac{x^6}{36}\right)dx = \frac{1}{2}\left(x + \frac{x^4}{12} + \frac{x^7}{252}\right)\Big|_1^2$

$= \frac{347}{252}$

So $\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{38/15}{39/24} = \underline{1.559}$

$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{347/252}{39/24} = \underline{0.847}$

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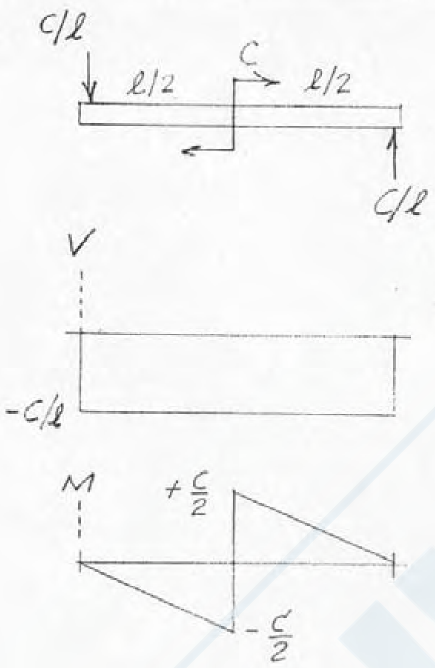

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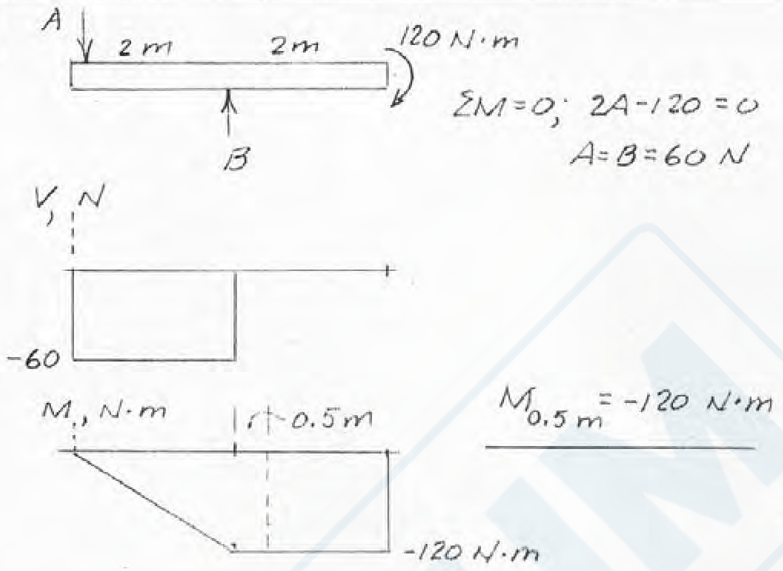

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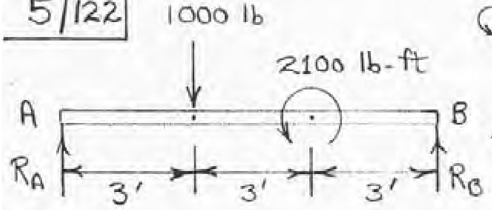
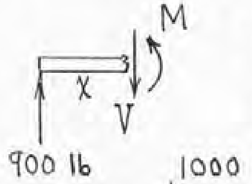


$\circlearrowleft + \Sigma M_A = 0: \; -1000(3) + 2100 + R_B(9) = 0, \quad R_B = 100 \text{ lb}$

$+\uparrow \Sigma F_y = 0: \; R_A - 1000 + 100 = 0$

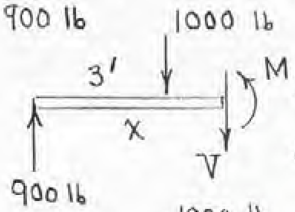
$R_A = 900 \text{ lb}$



$0 < x < 3 \text{ ft}:$

$V = 900 \text{ lb}$

$M = 900x$

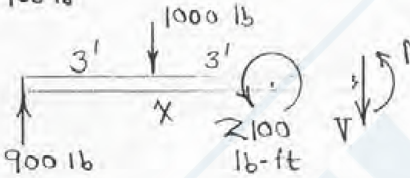


$3 < x < 6 \text{ ft}:$

$+\uparrow \Sigma F = 0: \; 900 - 1000 - V = 0, \; V = -100 \text{ lb}$

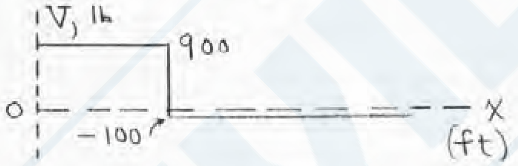
$\circlearrowleft + \Sigma M = 0: \; -900x + 1000(x-3) + M = 0$

$M = 3000 - 100x$



$\Sigma F = 0: \; 900 - 1000 - V = 0, \; V = -100 \text{ lb}$

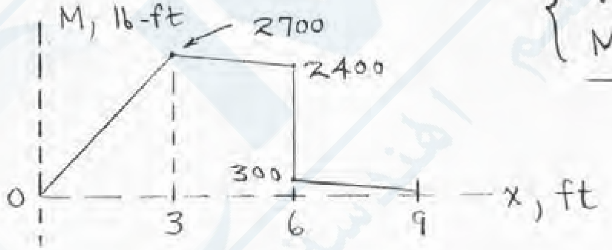
$\Sigma M = 0: \; -900x + 1000(x-3) + 2100 + M = 0, \; M = 900 - 100x$



Values at middle:

$V = -100 \text{ lb}$

$M = 2550 \text{ lb-ft}$



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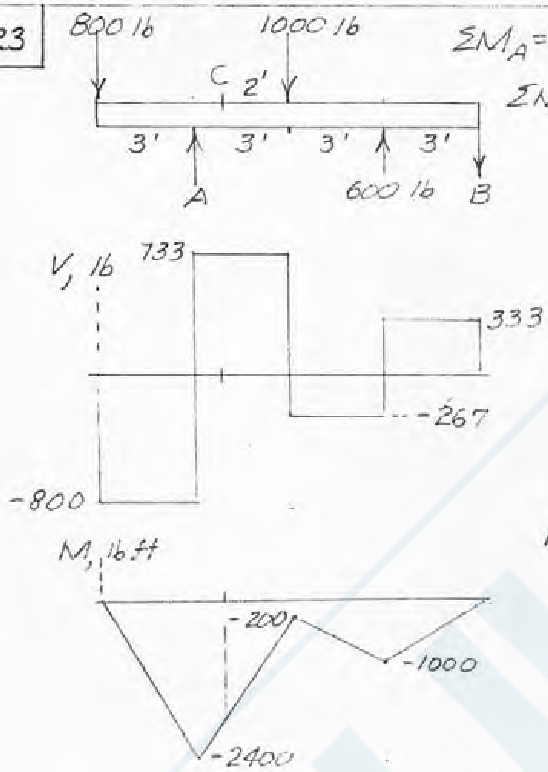

$\Sigma M_A = 0$ gives $B = 333$ lb

$\Sigma M_B = 0$ " $A = 1533$ lb

$M_C = -\left[2400 - \frac{2400-200}{3}\right]$

$= -1667$ lb-ft

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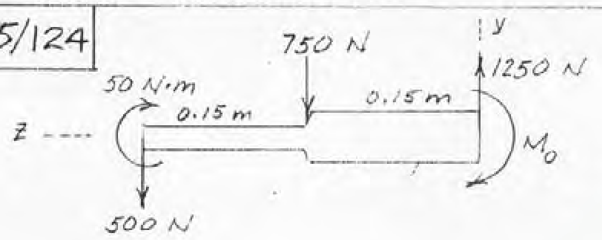
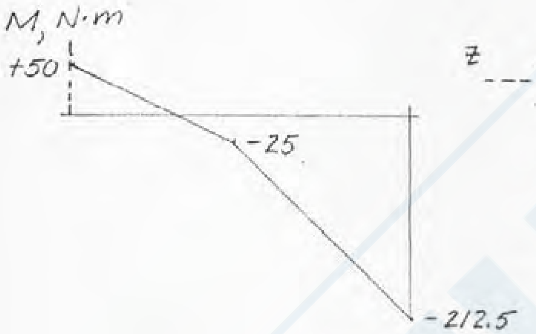
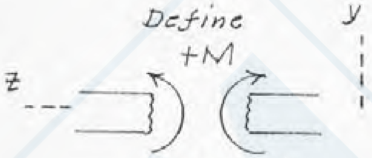


$\Sigma M = 0;\ 50 + M_0 - 500(0.30) - 750(0.15) = 0$

$M_0 = 212.5\ N\cdot m$





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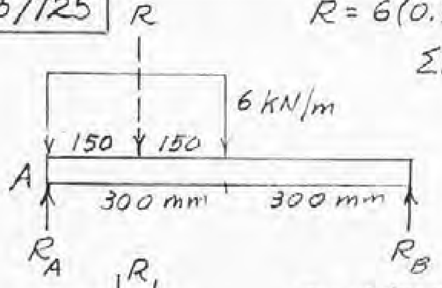
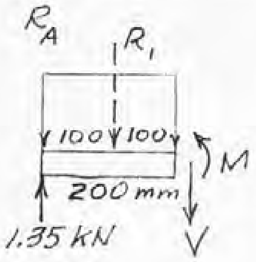


$R = 6(0.3) = 1.8$ kN

$\Sigma M_B = 0;\ 1.8(0.3 + 0.15) - 0.6 R_A = 0$

$R_A = 1.35$ kN



$R_1 = 6(0.2) = 1.2$ kN

$\Sigma F = 0;\ 1.35 - 1.2 - V = 0;$ $\underline{V = 0.15 \text{ kN}}$

$\Sigma M_A = 0;\ M - 1.2(0.1) - 0.15(0.2) = 0$

$\underline{M = 0.15 \text{ kN·m}}$

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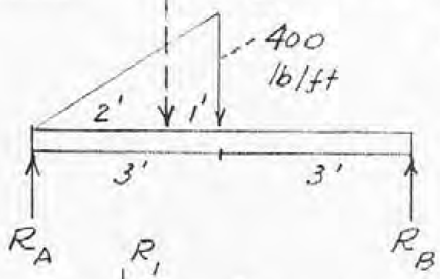
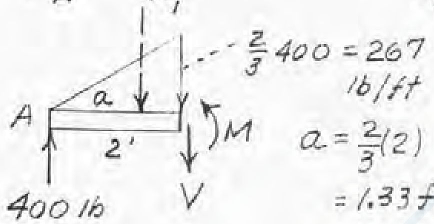
$R = \frac{400}{2}(3) = 600 \text{ lb}$



$\Sigma M_B = 0;\ 600(4) - 6R_A = 0$

$R_A = 400 \text{ lb}$



$a = \frac{2}{3}(2) = 1.33 \text{ ft}$

$R_1 = \frac{267}{2}2 = 267 \text{ lb}$

$\Sigma F = 0;\ 400 - V - 267 = 0$

$\underline{V = 133.3 \text{ lb}}$

$\Sigma M_A = 0;$

$-133.3(2) + M - 267(1.33) = 0$

$\underline{M = 622 \text{ lb-ft}}$

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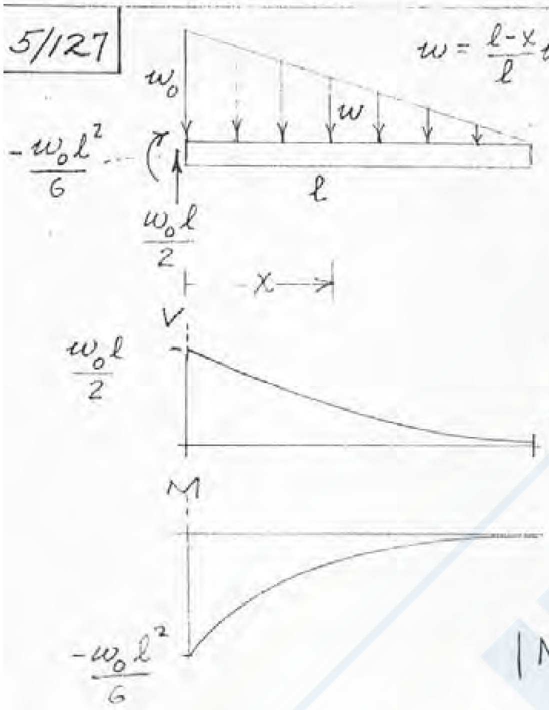
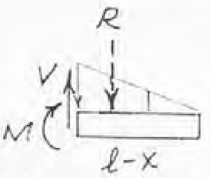


$w = \frac{l-x}{l} w_0$



$R = \frac{w_0}{2}\frac{(l-x)^2}{l}$

$\Sigma F = 0;\ V = \frac{w_0}{2l}(l-x)^2$

$\Sigma M_V = 0;\ R\,\frac{l-x}{3} + M = 0$

$M = -\frac{w_0}{2}\frac{(l-x)^2}{l}\frac{l-x}{3}$

$= -\frac{w_0}{6l}(l-x)^3$

$|M|_{max} = \frac{w_0 l^2}{6}$

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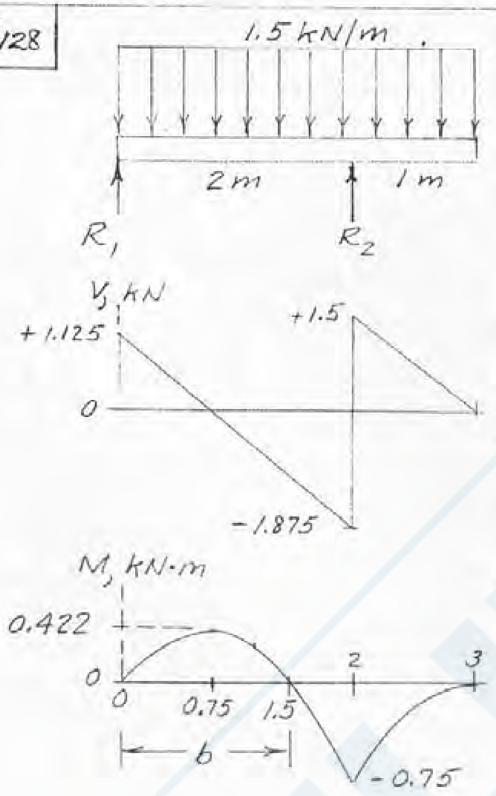

$\Sigma M_{R_1} = 0;$

$1.5(3)(1.5) - 2R_2 = 0$

$R_2 = 3.375$ kN

$\Sigma F = 0;\ R_1 + 3.375 - 1.5(3) = 0$

$R_1 = 1.125$ kN

$\Sigma F = 0;\ V + 1.5x - 1.125 = 0$

$V = 1.125 - 1.5x$

$\Sigma M_V = 0;\ M + 1.5\frac{x^2}{2} - 1.125x = 0$

$M = 1.125x - 0.75x^2$

$b = 1.5$ m

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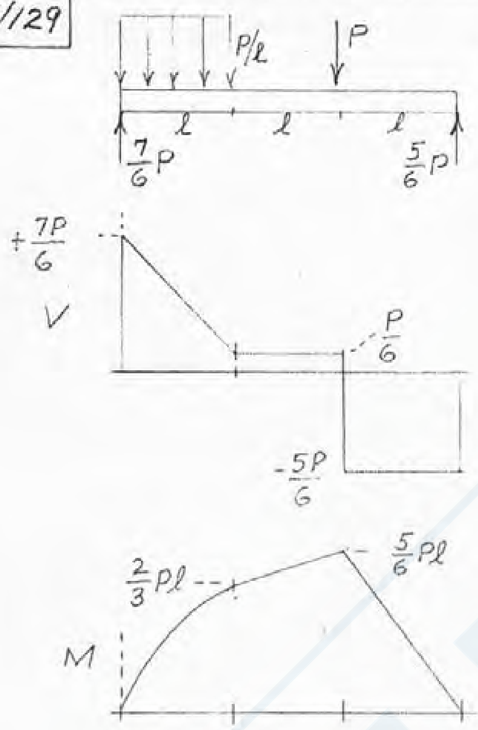

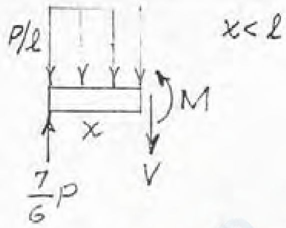

$\Sigma F = 0;\ -V - \frac{P}{l}x + \frac{7}{6}P = 0$

$V = P\left(\frac{7}{6} - \frac{x}{l}\right)$

$\Sigma M_V = 0;\ M + \frac{P}{l}x\frac{x}{2} - \frac{7}{6}Px = 0$

$M = \frac{Px}{2}\left(\frac{7}{3} - \frac{x}{l}\right)$

$M_{max} = \frac{5}{6}Pl$

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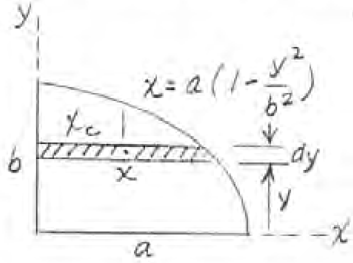
$dA = x\,dy, \quad A = \int_0^b a\left(1-\frac{y^2}{b^2}\right)dy = a\left[y - \frac{y^3}{3b^2}\right]_0^b = \frac{2}{3}ab$



$\bar{x}A = \int x_c\,dA = \int_0^b \frac{x}{2}\,x\,dy$

$= \frac{1}{2}\int_0^b a^2\left(1 - \frac{2y^2}{b^2} + \frac{y^4}{b^4}\right)dy$

$= \frac{a^2}{2}\left[y - \frac{2y^3}{3b^2} + \frac{y^5}{5b^4}\right]_0^b = \frac{4}{15}a^2b$

$\bar{x} = \frac{4a^2b/15}{2ab/3} = \underline{\frac{2}{5}a}$

$\bar{y}A = \int y\,dA = \int_0^b a\left(y - \frac{y^3}{b^2}\right)dy = a\left[\frac{y^2}{2} - \frac{y^4}{4b^2}\right]_0^b = \frac{1}{4}ab^2$

$\bar{y} = \frac{ab^2/4}{2ab/3} = \underline{\frac{3}{8}b}$

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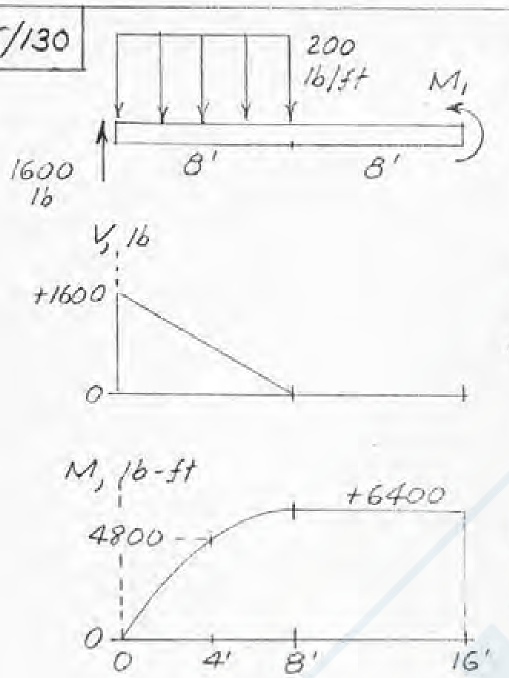
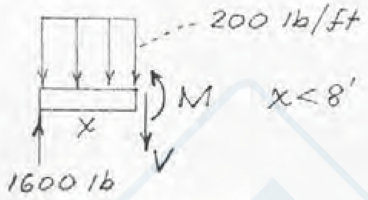


For zero moment on left end,

$M_1 = 200(8)4 = 6400$ lb-ft



$\Sigma F = 0;\ V + 200x - 1600 = 0$

$V = 1600 - 200x$

$\Sigma M_V = 0;\ M + 200\frac{x^2}{2} - 1600x = 0$

$M = 1600x - 100x^2$

$= 100x(16 - x)$

At $x = 4$; $M = 400(12)$

$= \underline{4800 \text{ lb-ft}}$

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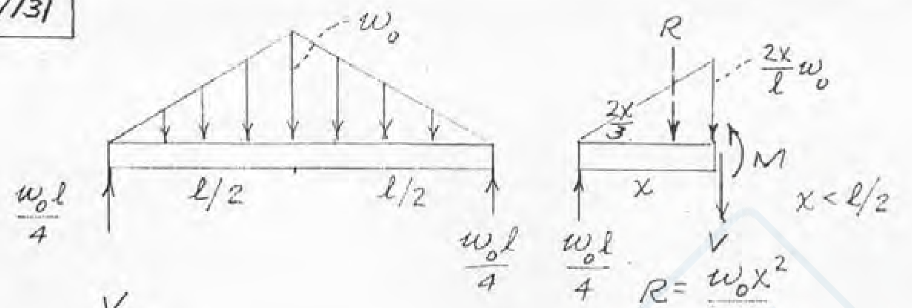
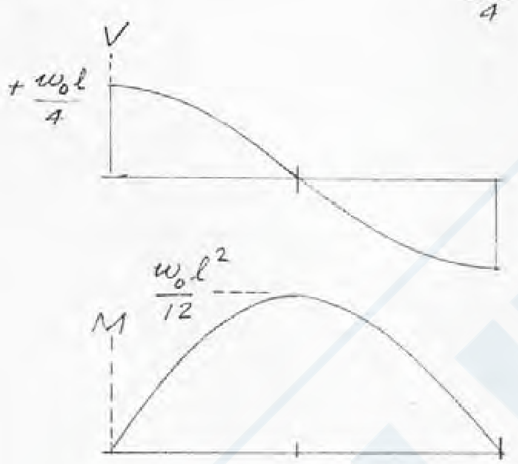


$R = \frac{w_o x^2}{l}$



$\Sigma F = 0; \; V + \frac{w_o x^2}{l} - \frac{w_o l}{4} = 0$

$V = w_o l \left(\frac{1}{4} - \frac{x^2}{l^2}\right)$

$\Sigma M_V = 0; \; M + R\frac{x}{3} - \frac{w_o l}{4} x = 0$

$M = w_o x l \left(\frac{1}{4} - \frac{x^2}{3l^2}\right)$

$M_{max} = \frac{w_o l^2}{12}$

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5/132 $w = -\frac{dV}{dx} = -\frac{d}{dx}(2200x - 40x^3) = \underline{-2200 + 120x^2}$

$dM = V\,dx;\quad \int_{-1600}^{M} dM = \int_{1}^{3} (2200x - 40x^3)\,dx$

$M - (-1600) = 2200\,\frac{3^2 - 1^2}{2} - 10(3^4 - 1^4) = 8000$

$M = 8000 - 1600 = \underline{6400 \text{ lb-ft}}$

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5/133 $M = ky^2$ & $M = Lx$, so $Lx = ky^2$

$L\ell = kh^2$

Thus $k = \frac{L\ell}{h^2}$ so $y^2 = \frac{Lx}{L\ell/h^2}$ so $y = h\sqrt{x/\ell}$

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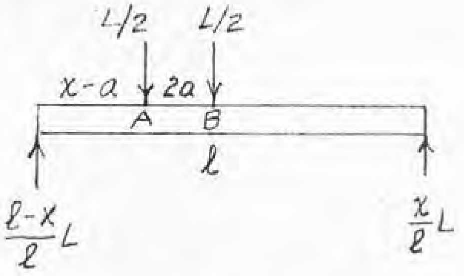

$M_A = \frac{\ell - x}{\ell} L (x-a) = \frac{L}{\ell}(-x^2 + [a+\ell]x - a\ell)$

$\frac{dM_A}{dx} = \frac{L}{\ell}(-2x + a + \ell) = 0$ for max. M_A ; $\underline{x = \frac{a+\ell}{2}}$

& $\underline{M_{A\,max} = \frac{L}{4\ell}(\ell - a)^2}$; $M_{B_{x=\frac{a+\ell}{2}}} = \frac{L}{4\ell}(\ell^2 - 2a\ell - 3a^2) < M_A$

(By symmetry, a second and equal maximum bending moment occurs at $x = \frac{\ell - a}{2}$.)

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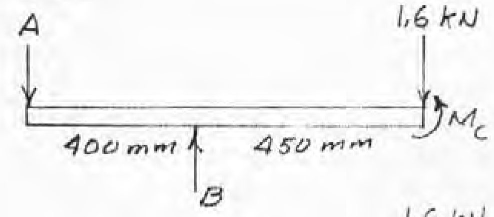


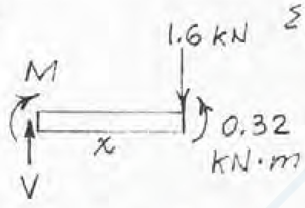
$M_C = 1.6(0.200) = 0.32$ kN·m

$\Sigma M_A = 0;\ 0.4B + 0.32 - 0.85(1.6) = 0$

$B = 2.6$ kN

$\Sigma F = 0;\ A + 1.6 - 2.6 = 0$

$A = 1.0$ kN



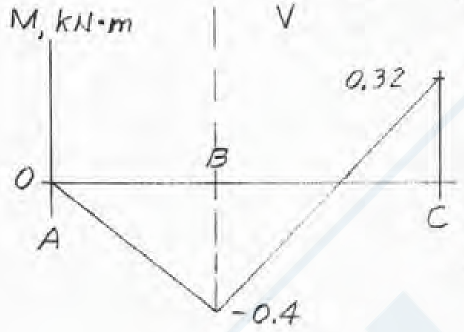
$\Sigma M = 0$

$0.32 - 1.6x - M = 0$

$M = 0.32 - 1.6x$

at B, $x = 0.45$ &

$M_B = -0.40$ kN·m



$M = 0$ when

$x = 0.32/1.6 = 0.2$ m

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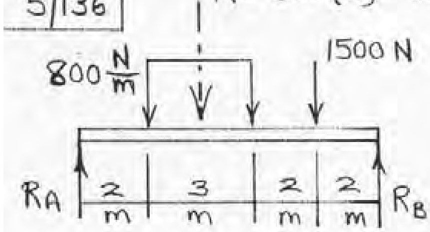
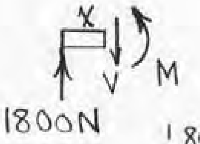
$R = 800(3) = 2400 \text{ N}$



$\Sigma M_A = 0: \; R_B(9) - 1500(7) - 2400(3.5) = 0, \; R_B = 2100 \text{ N}$

$\Sigma F = 0: \; R_A - 2400 - 1500 + 2100 = 0$

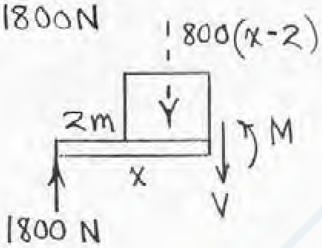
$R_A = 1800 \text{ N}$



$0 < x < 2 \text{ m}:$

$\Sigma F = 0 \Rightarrow V = 1800 \text{ N}$

$\Sigma M = 0 \Rightarrow M = 1800x$



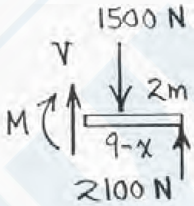
$2 < x < 5 \text{ m}:$

$\Sigma F = 0: \; 1800 - 800(x-2) - V = 0$

$V = 3400 - 800x$

$\Sigma M = 0: \; M + 800(x-2)\dfrac{x-2}{2} - 1800x = 0$

$M = -400x^2 + 3400x - 1600$

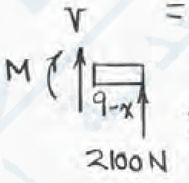


$5 < x < 7:$

$\Sigma F = 0: \; 2100 - 1500 + V = 0$

$V = -600 \text{ N}$

$\Sigma M = 0: \; -M - 1500(7-x) + 2100(9-x) = 0, \; M = 8400 - 600x$



$7 < x < 9 \text{ m}:$

$\Sigma F = 0 \Rightarrow V = -2100 \text{ N}$

$\Sigma M = 0 \Rightarrow M = 18\,900 - 2100x$

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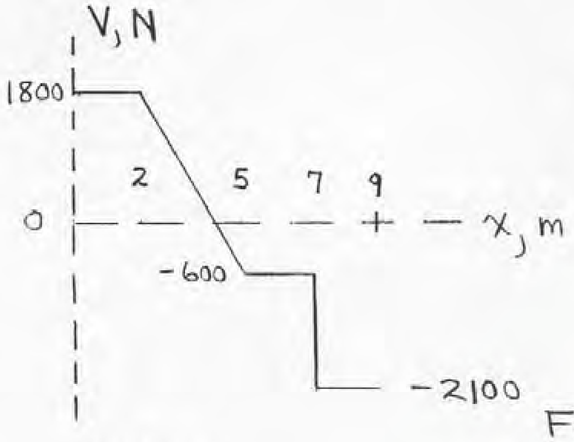


At $x = 6$ m:

$V = -600$ N

$M = 8400 - 600(6)$

$= \underline{4800 \text{ N}\cdot\text{m}}$

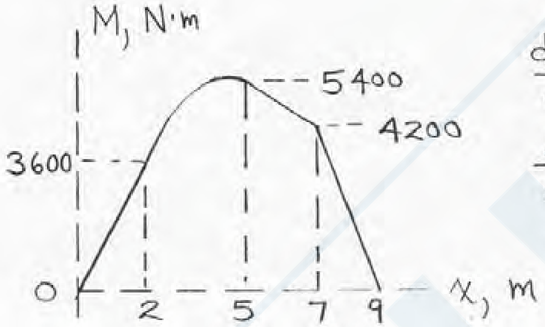
For M_{max},

$$\frac{dM}{dx} = 0$$

$$\frac{d}{dx}\left(-400x^2 + 3400x - 1600\right)$$

$$= -800x + 3400 = 0$$

$$\underline{x = 4.25 \text{ m}}$$



$$M_{max} = -400(4.25)^2 + 3400(4.25) - 1600 = \underline{5620 \text{ N}\cdot\text{m}}$$

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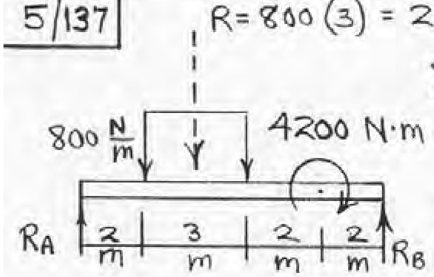
$R = 800(3) = 2400\ \text{N}$



$\Sigma M_A = 0:\ R_B(9) - 2400(3.5) - 4200 = 0,\ R_B = 1400\ \text{N}$

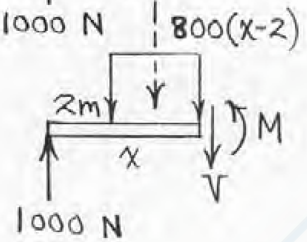
$\Sigma F = 0 \Rightarrow R_A = 1000\ \text{N}$

$0 < x < 2\ \text{m}:$

$\Sigma F = 0 \Rightarrow V = 1000\ \text{N}$

$\Sigma M = 0 \Rightarrow M = 1000x$

$2 < x < 5\ \text{m}:$

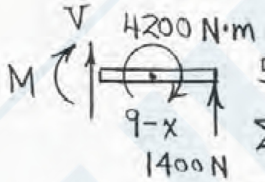


$\Sigma F = 0:\ 1000 - 800(x-2) - V = 0$

$V = 2600 - 800x$

$\Sigma M = 0:\ M + 800(x-2)\frac{x-2}{2} - 1000x = 0$

$M = -400x^2 + 2600x - 1600$

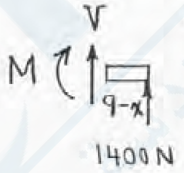


$5 < x < 7\ \text{m}:$

$\Sigma F = 0:\ V + 1400 = 0,\ V = -1400\ \text{N}$

$\Sigma M = 0:\ -M - 4200 + 1400(9-x) = 0$

$M = 8400 - 1400x$



$7 < x < 9\ \text{m}:$

$\Sigma F = 0 \Rightarrow V = -1400\ \text{N}$

$\Sigma M = 0 \Rightarrow M = 12600 - 1400x$

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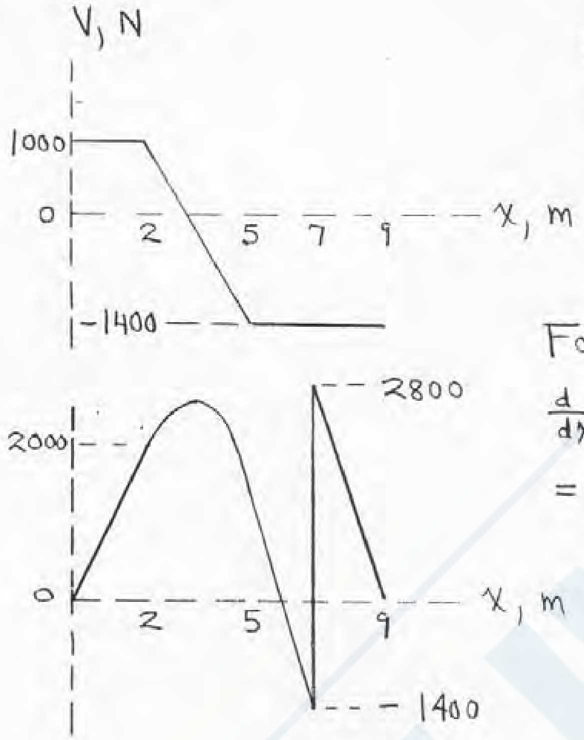

At $x = 6$ m,

$V = -1400$ N

$M = 8400 - 1400(6)$

$= 0$

For M_{max}, $\frac{dM}{dx} = 0$

$\frac{d}{dx}\left(-400x^2 + 2600x - 1600\right)$

$= -800x + 2600 = 0$

$x = 3.25$ m

$M_{x=3.25} = -400(3.25)^2 + 2600(3.25) - 1600 = 2625$ N·m

So $M_{max} = 2800$ N·m

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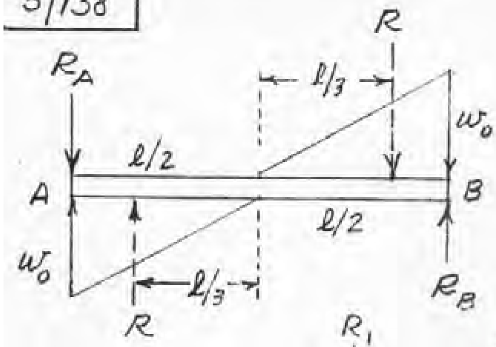


$R = \frac{w_o}{2}\frac{\ell}{2} = w_o \ell/4$

By inspection of the two couples, $R_A = R_B$

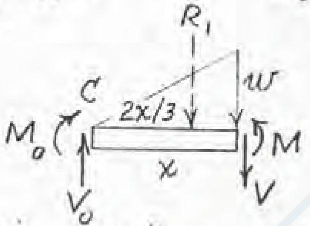
$\Sigma M = 0;\ R_A \ell - \frac{w_o \ell}{4}\frac{2\ell}{3} = 0$

$R_A = R_B = \frac{w_o \ell}{6}$

For complete right half of beam $\Sigma F = 0$ gives

$V_o = R - R_B = w_o \ell/12$

$M_o = 0$



For section of length x, $w = \frac{x}{\ell/2} w_o$ & $R_1 = \frac{w}{2} x = w_o x^2/\ell$

$\Sigma F = 0;\ V + w_o x^2/\ell - w_o \ell/12 = 0,\quad \underline{V = w_o \ell\left(\frac{1}{12} - \frac{x^2}{\ell^2}\right)}$

$\Sigma M_C = 0;\ M - \frac{w_o x^2}{\ell}\frac{2x}{3} - w_o \ell\left(\frac{1}{12} - \frac{x^2}{\ell^2}\right)x = 0,\quad \underline{M = \frac{w_o \ell x}{3}\left(\frac{1}{4} - \frac{x^2}{\ell^2}\right)}$

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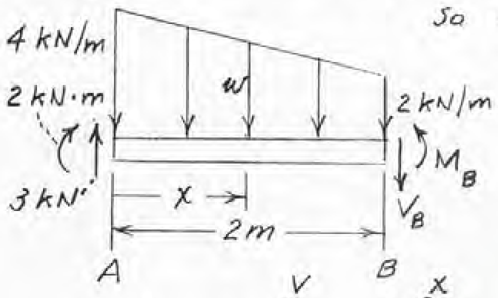
$w = 4 - kx$ where $k = \frac{4-2}{2} = 1\ \text{kN/m}^2$

So $w = 4 - x$ kN/m



$w = -dV/dx$; $\int_3^V dV = -\int_0^x (4-x)\,dx$, $V = 3 - \left[4x - \frac{x^2}{2}\right]_0^x$

$= 3 - 4x + x^2/2$

when $x = 2$ m, $V = V_B = -3$ kN

$V = \frac{dM}{dx}$; $\int_2^{M_B} dM = \int_0^2 \left(3 - 4x + \frac{x^2}{2}\right) dx$

$M_B - 2 = \left[3x - 2x^2 + \frac{x^3}{6}\right]_0^2 = -\frac{2}{3}$

$M_B = 2 - \frac{2}{3} = 4/3$ kN·m

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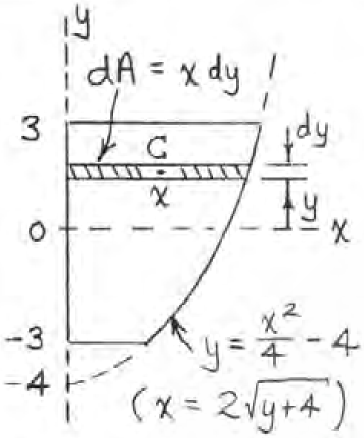

$$A = \int dA = \int x\,dy = \int_{-3}^{3} 2\sqrt{y+4}\,dy$$

$$= 2\,\frac{2}{3}(y+4)^{3/2}\Big|_{-3}^{3} = 23.4$$

$$\int x_c\,dA = \int \frac{x}{2}\,x\,dy = \frac{1}{2}\int x^2 dy$$

$$= \frac{1}{2}\int_{-3}^{3} 4(y+4)\,dy = 2\left(\frac{y^2}{2} + 4y\right)_{-3}^{3}$$

$$= 48$$

$$\int y_c\,dA = \int y\,x\,dy = \int_{-3}^{3} y\,2\sqrt{y+4}\,dy$$

$$= 2\,\frac{2}{15}(3y-8)(y+4)^{3/2}\Big|_{-3}^{3} = 9.47$$

$$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{48}{23.4} = \underline{2.05}$$

$$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{9.47}{23.4} = \underline{0.405}$$

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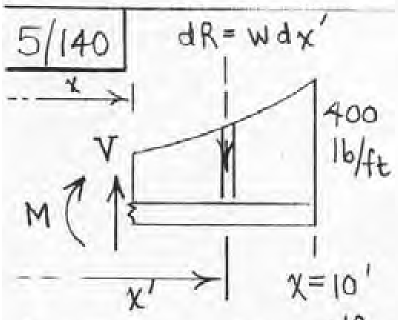

From $w = w_0 + kx^2 = 100 + kx^2$:

$$400 = 100 + k(10)^2$$

$$k = 3 \text{ lb/ft}^3, \quad w = 100 + 3x^2$$

$+\uparrow \Sigma F = 0: \quad V - \int_x^{10} w\,dx' = 0$

$$V = \int_x^{10} (100 + 3x'^2)\,dx' = 100x' + x'^3 \Big|_x^{10}$$

$$\underline{V = 2000 - 100x - x^3} \quad \text{(in lb if } x \text{ is in ft)}$$

$\circlearrowleft \Sigma M = 0: \quad -M - \int_x^{10} (x' - x)\,w\,dx' = 0$

$$M = -\int_x^{10} (-100x + 100x' + 3x'^3 - 3xx'^2)\,dx'$$

$$= -\left[-100xx' + 50x'^2 - xx'^3 + \frac{3}{4}x'^4\right]\Big|_x^{10}$$

$$= \underline{-12{,}500 + 2000x - 50x^2 - \frac{1}{4}x^4}$$

(in lb-ft if x is in ft)

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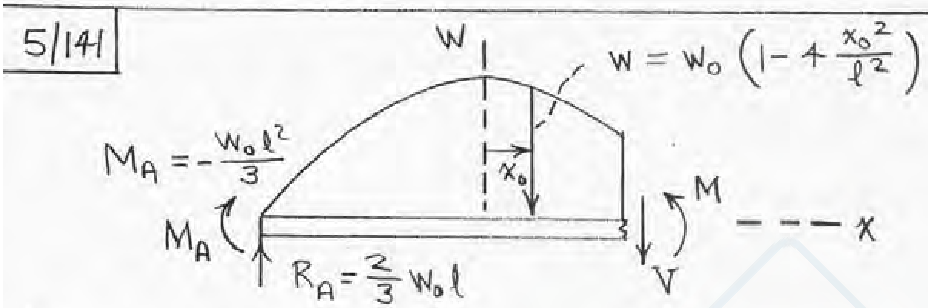

(R_A and M_A from Prob. 5/104)

$$+\uparrow\Sigma F = 0: \quad \frac{2}{3}w_0 l - \int_{-l/2}^{x} w_0\left(1 - 4\frac{x_0^2}{l^2}\right)dx_0 - V = 0$$

$$\underline{V = w_0\left(\frac{l}{3} - x + \frac{4x^3}{3l^2}\right)}$$

$$\circlearrowleft + \Sigma M = 0: \quad M + \int_{-l/2}^{x} w_0\left(1 - 4\frac{x_0^2}{l^2}\right)(x - x_0)\,dx_0$$

$$+ \frac{w_0 l^2}{3} - \frac{2}{3}w_0 l\left(\frac{l}{2} + x\right) = 0, \quad \underline{M = w_0\left(-\frac{l^2}{16} + \frac{xl}{3} - \frac{x^2}{2} + \frac{x^4}{3l^2}\right)}$$

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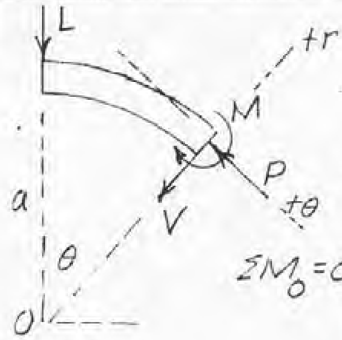

$\Sigma F_\theta = 0;\ -P + L\sin\theta = 0$

$P = L\sin\theta$

$\Sigma F_r = 0;\ -V - L\cos\theta = 0$

$\underline{V = -L\cos\theta}$

$\Sigma M_O = 0;\ Pa - M = 0$

$M = Pa = \underline{La\sin\theta}$

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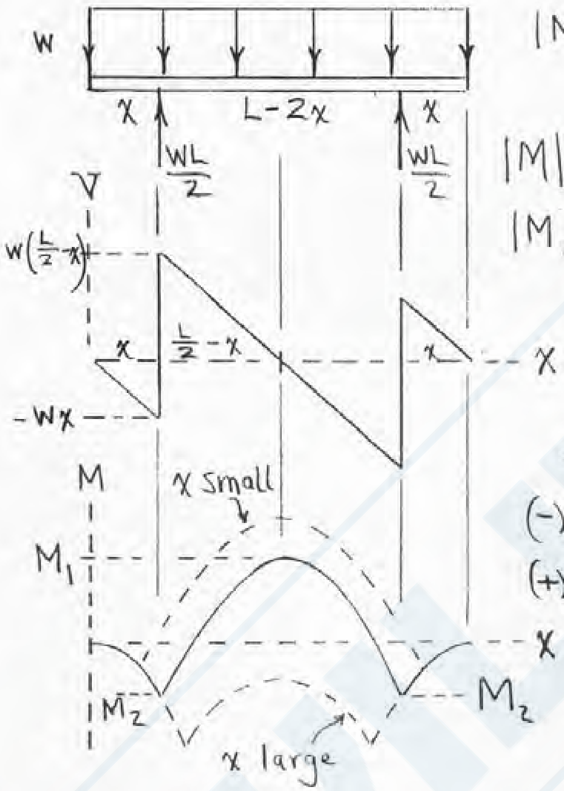
▶5/143 From areas under shear diagram:



$$|M_2| = \frac{1}{2} w x^2$$

$$|M_1| = \frac{w}{2}\left(\frac{L}{2} - x\right)^2 - \frac{1}{2} w x^2$$

$$= \frac{wL}{2}\left(\frac{L}{4} - x\right)$$

$|M|_{min}$ occurs when $|M_1| = |M_2|$. So

$$\frac{wL}{2}\left(\frac{L}{4} - x\right) = \frac{1}{2} w x^2$$

or $x^2 + Lx - \frac{L^2}{4} = 0$

$$\Rightarrow x = \frac{L}{2}\left(-1 \pm \sqrt{2}\right)$$

(−) sign : irrelevant

(+) sign : $x = 0.207L$

$$M_{max} = |M_1|$$

$$= \frac{1}{2} w \left(0.207L\right)^2$$

$$= 0.0214 wL^2$$

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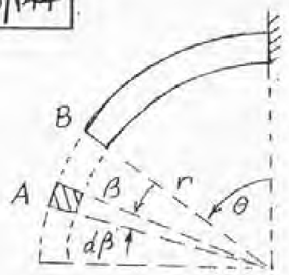

On element A, $dF = wr\,d\beta$

Torsion about B due to dF is

$$dT = dF(r - r\cos\beta) = wr^2(1-\cos\beta)\,d\beta$$

$$T = wr^2\int_0^{\pi/2-\theta}(1-\cos\beta)\,d\beta$$

$$= wr^2\left[\beta - \sin\beta\right]_0^{\frac{\pi}{2}-\theta}$$

$$= wr^2\left[\frac{\pi}{2} - \theta - \sin\left(\frac{\pi}{2}-\theta\right)\right]$$

$$= wr^2\left[\frac{\pi}{2} - \theta - \cos\theta\right]$$

Bending moment about B due to dF is

$$dM = dF\,r\sin\beta = wr^2\sin\beta\,d\beta$$

$$M = wr^2\int_0^{\pi/2-\theta}\sin\beta\,d\beta = -wr^2\cos\beta\Big]_0^{\pi/2-\theta}$$

$$= -wr^2\left[\sin\theta - 1\right] = wr^2(1-\sin\theta)$$

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5/145 Given: $\begin{cases} 2s = 100 \text{ ft}, \quad s = 50 \text{ ft} \\ \mu = 0.00624 \text{ lb/ft} \\ T = 10 \text{ lb} \end{cases}$

$T^2 = \mu^2 s^2 + T_0^2$: $10^2 = (0.00624 \cdot 50)^2 + T_0^2$

$T_0 = 9.995 \text{ lb}$

(Eq. 5/22) $T = T_0 + \mu y$: $10 = 9.995 + 0.00624\, h$

$h = 0.780 \text{ ft}$ or $\underline{h = 9.36 \text{ in.}}$

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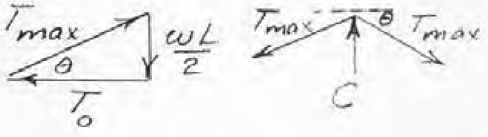
5/146 $L = 4200$ ft, $h = 470$ ft, $w = \frac{21{,}300}{2} = 10650$ lb/ft for each cable

$$T_0 = \frac{wL^2}{8h} = \frac{10650(4200)^2}{8(470)} = \underline{50.0(10^6)\text{ lb}}$$



$$C = 2T_{max}\sin\theta$$

$$= 2\left(\frac{wL}{2}\right) = wL$$

$$= 10650(4200)$$

$$= \underline{44.7(10^6)\text{ lb}}$$

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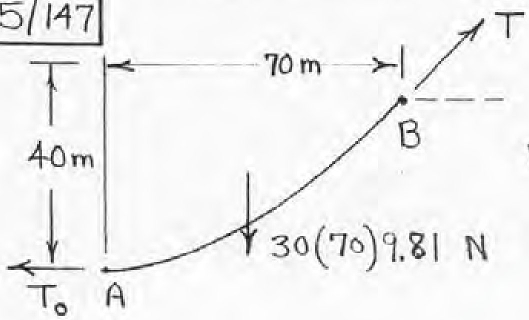

$W = 30(9.81) = 294 \frac{N}{m}$

Eq. 5/14: $y = \frac{wx^2}{2T_0}$

$T_0 = \frac{wx^2}{2y} = \frac{294\,(70)^2}{2(40)} = 18.03(10^3)$ N or 18.03 kN

$\tan\theta = \frac{dy}{dx} = \frac{wx}{T_0}$.

At B, $\theta = \tan^{-1}\left(\frac{294(70)}{18\,030}\right) = 48.8°$

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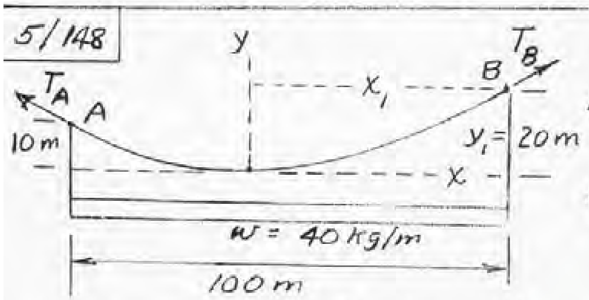

Eq. 5/14 $y = \frac{wx^2}{2T_o}$

A: $10 = \frac{(40)(9.81)(100-x_1)^2}{2T_o}$

B: $20 = \frac{(40)(9.81)x_1^2}{2T_o}$

Combine & get $x_1^2 = 2(100-x_1)^2$, $x_1^2 - 400x_1 + 20000 = 0$

$x_1 = \frac{400}{2} \pm \frac{1}{2}\sqrt{(16-8)10^4} = 100(2 \pm \sqrt{2}) = 58.6\text{ m (or 341 m)}$

At B; $T_o = \frac{40(9.81)(58.6)^2}{2(20)} = 33660\text{ N}$ or $\underline{T_o = 33.7\text{ kN}}$

Also, $T^2 = T_o^2 + w^2x^2$ so

for B, $T_B = \sqrt{(33.7)^2 + [(40)(9.81)(10^{-3})]^2(58.6)^2} = \underline{40.8\text{ kN}}$

for A, $T_A = \sqrt{(33.7)^2 + [(40)(9.81)(10^{-3})]^2(100-58.6)^2} = \underline{37.4\text{ kN}}$

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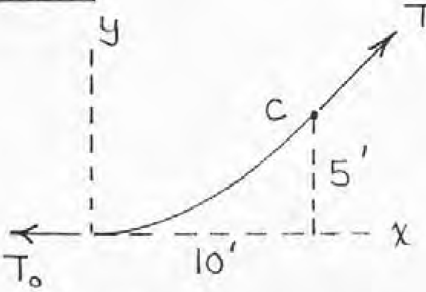

$\mu = 15$ lb/ft

Eq. 5/19 evaluated at point C:

$$5 = \frac{T_0}{\mu}\left[\cosh \frac{10}{T_0/\mu} - 1\right]$$

Numerical solution: $\frac{T_0}{\mu} = 10.74$ ft

Then $T_0 = 10.74(15) = 161.1$ lb

Eq. 5/22 evaluated at C:

$T_C = 161.1 + 15(5) = \underline{236 \text{ lb}}$

Eq. 5/20 : $s = 10.74 \sinh \frac{10}{10.74} = 11.51$ ft

$L = 2s = \underline{23.0 \text{ ft}}$

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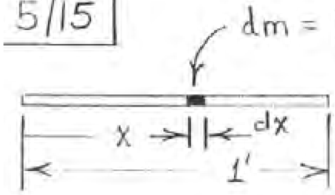

$dm = \rho\, dx$ (ρ = mass per unit length)

$$m = \int dm = \int \rho\, dx = \int_0^{1'} \rho_0 \left(1 - \frac{x}{2}\right) dx$$

$$= \rho_0 \left[x - \frac{x^2}{4}\right]_0^{1'} = \frac{3}{4}\rho_0$$

$$\int x\, dm = \int_0^{1'} x \rho_0 \left(1 - \frac{x}{2}\right) dx = \rho_0 \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_0^{1'} = \frac{\rho_0}{3}$$

$$\bar{x} = \frac{\int x\, dm}{\int dm} = \frac{\rho_0/3}{3\rho_0/4} = \frac{4}{9} \text{ ft}$$

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5/150 Eq. 5/22, $T_B = T_o + \mu y_B$, $T_A = T_o + \mu y_A$

so $T_B - T_A = \mu(y_B - y_A)$ or $T_B - T_A = \mu h$

Thus $h = \frac{1}{\mu}(T_B - T_A) = \frac{1}{0.12(9.81)}(230-110) = \underline{101.9 \text{ m}}$

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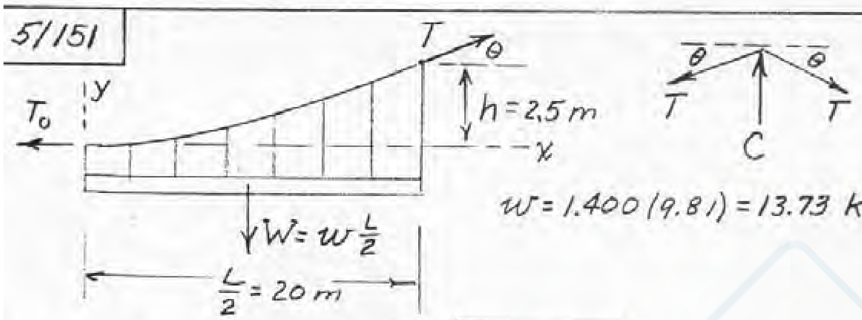

$w = 1.400(9.81) = 13.73$ kN/m

From Eq. 5/15b, $T = \frac{wL}{2}\sqrt{1 + (L/4h)^2}$

$$= \frac{13.73(40)}{2}\sqrt{1 + \left[\frac{40}{4(2.5)}\right]^2} = 1133 \text{ kN}$$

$T^2 = W^2 + T_o^2$, $T_o = \sqrt{(1132)^2 - [(13.73)(20)]^2} = 1099$ kN

$\frac{dy}{dx} = \tan\theta = \frac{w(L/2)}{T_o} = \frac{13.73(20)}{1099} = 0.250$, $\theta = 14.04°$

$\Sigma F_y = 0$ at support; $2T\sin\theta - C = 0$

$$C = 2(1133)\sin 14.04° = \underline{549 \text{ kN}}$$

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5/152 Eq. 5/15 $T = w\sqrt{x^2 + (L^2/8h)^2}$

so $\Delta T = \Delta w\sqrt{x^2 + (L^2/8h)^2}$

For each cable $480{,}000 = \Delta w\sqrt{(600)^2 + \left(\frac{[2500]^2}{8[500]}\right)^2}$

$= 1674\,\Delta w$

$\Delta w = 286.8$ lb/ft per cable

$w' = 2\Delta w = 2(286.8) = \underline{574 \text{ lb/ft}}$ for both cables

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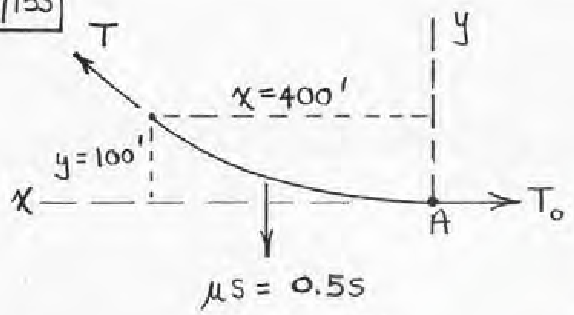

Catenary Eq. 5/19 : $y = \frac{T_o}{\mu}\left(\cosh\frac{\mu x}{T_o} - 1\right)$

$$100 = \frac{T_o}{0.5}\left(\cosh\frac{0.5(400)}{T_o} - 1\right)$$

Solve numerically to obtain $\underline{T_o = 408 \text{ lb}}$

Parabolic Eq. 5/14 : $y = \frac{wx^2}{2T_o} \cong \frac{\mu x^2}{2T_o}$

So $T_o \cong \frac{\mu x^2}{2y} = \frac{0.5(400)^2}{2(100)} = \underline{400 \text{ lb}}$

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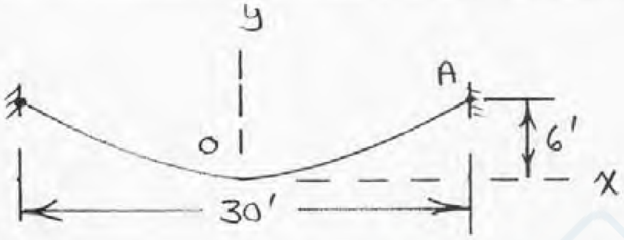

Eq. 5/19, from O to A : $6 = \frac{T_0}{\mu}\left[\cosh \frac{\mu}{T_0}(15) - 1\right]$

By numerical or graphical means, $\frac{T_0}{\mu} = 19.68$ m

Eq. 5/20: $s = \frac{L}{2} = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$\therefore L = 2\left(\frac{T_0}{\mu}\right) \sinh \frac{\mu}{T_0} 15$

$= 2(19.68) \sinh \frac{15}{19.68} = \underline{33.0 \text{ ft}}$

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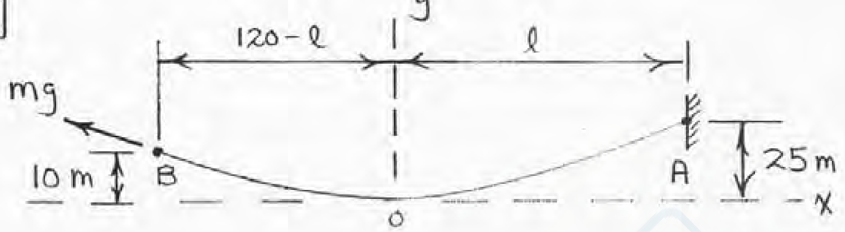

Eq. 5/14 : $y = \frac{wx^2}{2T_0}$

At A: $25 = \frac{40\ell^2}{2T_0}$, At B: $10 = \frac{40(120-\ell)^2}{2T_0}$

Eliminate T_0 : $0.6\ell^2 - 240\ell + 14\,400 = 0$

$\ell = 73.5$ m (or $\ell = 326$ m)

$T_0 = \frac{4}{5}\ell^2 = \frac{4}{5}(73.5)^2 = 4320$ N

Section O-B : $(mg)^2 = T_0^2 + (wx)^2$

$(9.81m)^2 = (4320)^2 + [40(120-73.5)]^2$

$m = 480$ kg

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*5/156 Please refer to the diagram in the solution to Prob. 5/155. Eq. 5/19:

$$y = \frac{T_0}{\mu}\left[\cosh\frac{\mu x}{T_0} - 1\right]$$

At B: $10 = \frac{T_0}{40}\left[\cosh\frac{40(120-\ell)}{T_0} - 1\right]$

At A: $25 = \frac{T_0}{40}\left[\cosh\frac{40\ell}{T_0} - 1\right]$

Numerical solution: $\begin{cases} T_0 = 4440 \text{ N} \\ \ell = 73.2 \text{ m} \end{cases}$

Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

At B: $s_B = \frac{4440}{40}\sinh\frac{40(120-73.2)}{4440} = 48.2 \text{ m}$

Equilibrium of section OB:

$(mg)^2 = T_0^2 + (\mu s_B)^2$: $m^2(9.81)^2 = 4440^2 + (40 \cdot 48.2)^2$

$\underline{m = 494 \text{ kg}}$

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5/157 Eq. 5/19 is $y = \frac{T_0}{\mu}\left(\cosh \frac{\mu x}{T_0} - 1\right)$

So $H - 2 = \frac{300(10^3)}{22(9.81)}\left(\cosh \frac{22(9.81)(250)}{300(10^3)} - 1\right)$

$= 1390(\cosh 0.1798 - 1) = 1390(1.0162 - 1)$

$= 22.5$ m

Thus $\underline{H = 24.5 \text{ m}}$

Eq. 5/20, $s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0} = \frac{300(10^3)}{22(9.81)} \sinh \frac{22(9.81)250}{300(10^3)}$

$= 1390(0.1808) = \underline{251 \text{ m}}$

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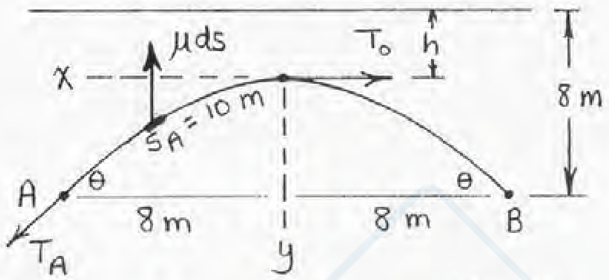

$\mu = 560 - 100$
$= 460$ N/m

Eq. 5/20: $s = \frac{T_o}{\mu} \sinh \frac{\mu x}{T_o}$: $10 = \frac{T_o}{460} \sinh \frac{460(8)}{T_o}$

Numerical or graphical solution: $T_o = 3110$ N

Eq. 5/21 : $T = T_o \cosh \frac{\mu x}{T_o} = 3110 \cosh \frac{460(8)}{3110}$

$T_A = 5550$ N

Eq. 5/22 : $T = T_o + \mu y$ at A: $5550 = 3110 + 460y$

$y = 5.31$ m

Then $h = 8 - 5.31 = \underline{2.69 \text{ m}}$

From Eq. 5/19, $\frac{dy}{dx} = \tan\theta = \sinh \frac{\mu x}{T_o}$

$\tan\theta = \sinh \frac{460(8)}{3110} = \underline{55.9°}$

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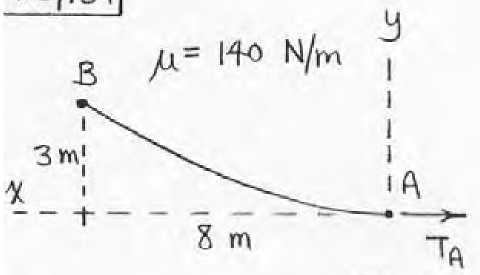

Eq. 5/19: $y = \frac{T_0}{\mu}\left[\cosh\frac{\mu x}{T_0} - 1\right]$

At B: $3 = \frac{T_A}{140}\left[\cosh\frac{140(8)}{T_A} - 1\right]$

Numerical solution: $\underline{T_A = 1559 \text{ N}}$

Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

At B: $L = \frac{1559}{140}\sinh\frac{140(8)}{1559} = \underline{8.71 \text{ m}}$

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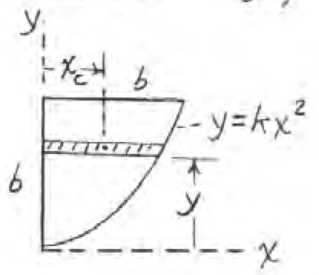
5/16 $dA = x\,dy$; $k = \frac{b}{b^2} = \frac{1}{b}$ so $x^2 = by$, $x_c = x/2$



$$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{\int_0^b \frac{x}{2}\,x\,dy}{\int_0^b x\,dy}$$

$$= \frac{1}{2}\int_0^b by\,dy \Big/ \int_0^b \sqrt{by}\,dy$$

$$= \frac{1}{4}b^3 \Big/ \frac{2}{3}b^2 = \underline{\frac{3}{8}b}$$

$$\bar{y} = \frac{\int y\,dA}{\int dA} = \int_0^b y\sqrt{by}\,dy \Big/ \frac{2}{3}b^2 = \frac{2}{5}b^3 \Big/ \frac{2}{3}b^2 = \underline{\frac{3}{5}b}$$

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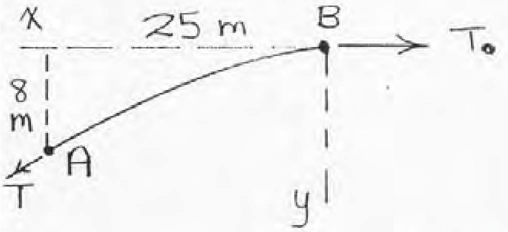
$\mu = 30$ N/m



Eq. 5/19: $y = \frac{T_0}{\mu}\left[\cosh\frac{\mu x}{T_0} - 1\right]$

At A: $8 = \frac{T_0}{\mu}\left[\cosh\frac{25\mu}{T_0} - 1\right]$

Numerical solution: $\frac{T_0}{\mu} = 40.3$ m

$T_0 = 40.3(30) = \underline{1210 \text{ N}}$

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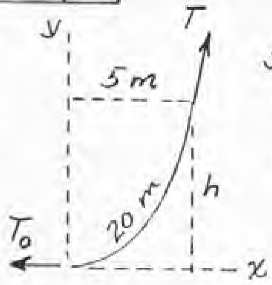

Eq. 5/20, $20 = \frac{T_0}{\mu} \sinh \frac{5\mu}{T_0}$

Solve by computer or graphically & get $T_0/\mu = 1.532$ m

Eq. 5/19, $y = 1.532 \left(\cosh \frac{5}{1.532} - 1\right)$

$h = y = 1.532 (13.09 - 1) = \underline{18.53 \text{ m}}$

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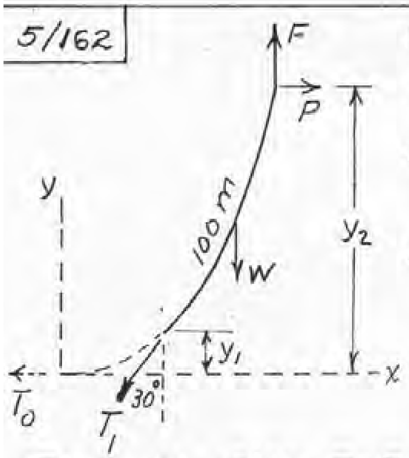

$W = 0.51(9.81)(100) = 500.3\ N$

$T_1 = M/r = 400/\frac{0.5}{2} = 1600\ N$

$\Sigma F_x = 0;\ P - 1600 \sin 30° = 0$

$P = 800\ N\ \ (= T_0)$

$\Sigma F_y = 0;\ F - 500.3 - 1600 \cos 30° = 0$

$F = 1886\ N$

$T = \sqrt{F^2 + P^2} = \sqrt{(1886)^2 + (800)^2}$

$= 2049\ N$

From Eq. 5/22, $T = T_0 + \mu y$, $2049 = 800 + 5.003\, y_2$

$y_2 = 249.6\ m$

Also, $1600 = 800 + 5.003\, y_1$

$y_1 = 159.9\ m$

$H = y_2 - y_1 = \underline{89.7\ m}$

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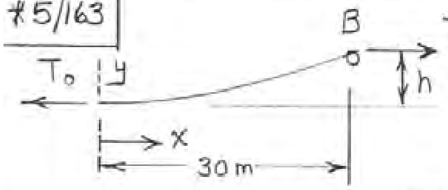


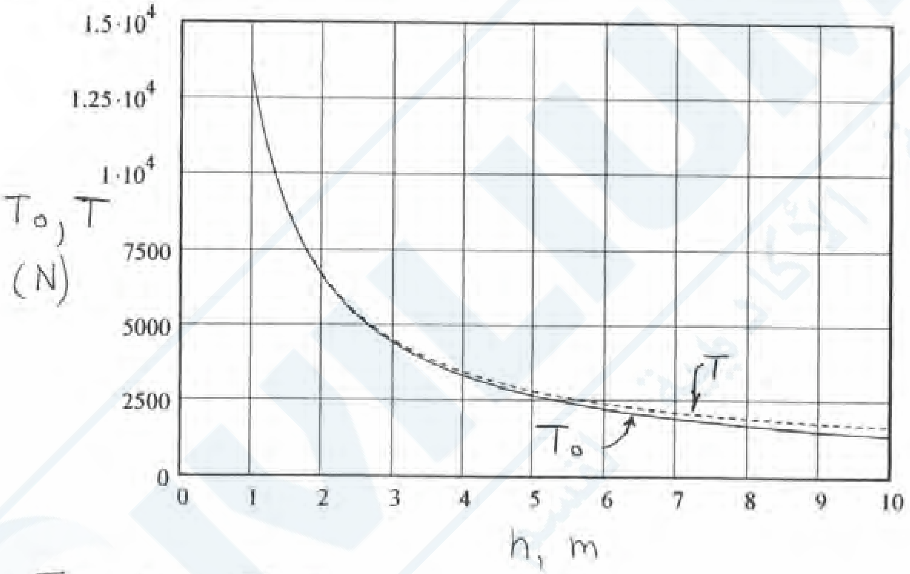
$\mu = \rho g = 3(9.81)$

$= 29.4 \text{ N/m}$

Eq. 5/19 @ B : $h = \frac{T_o}{\mu}\left[\cosh\frac{\mu x}{T_o} - 1\right]$ (1)

Eq. 5/22 @ B : $T = T_o + \mu h$ (2)

Numerically solve (1), then (2) over $1 \leq h \leq 10$ m to obtain



For h = 2 m,

$T_o = 6630$ N, $T = 6690$ N

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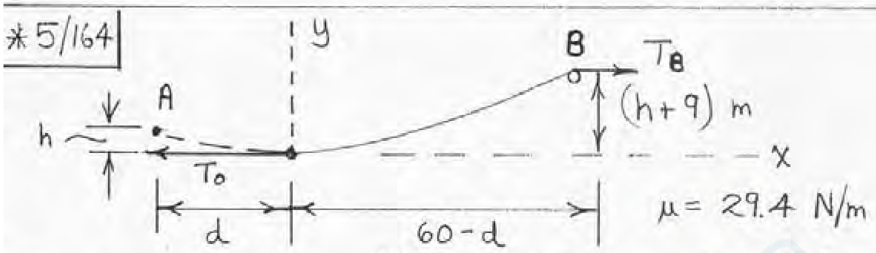
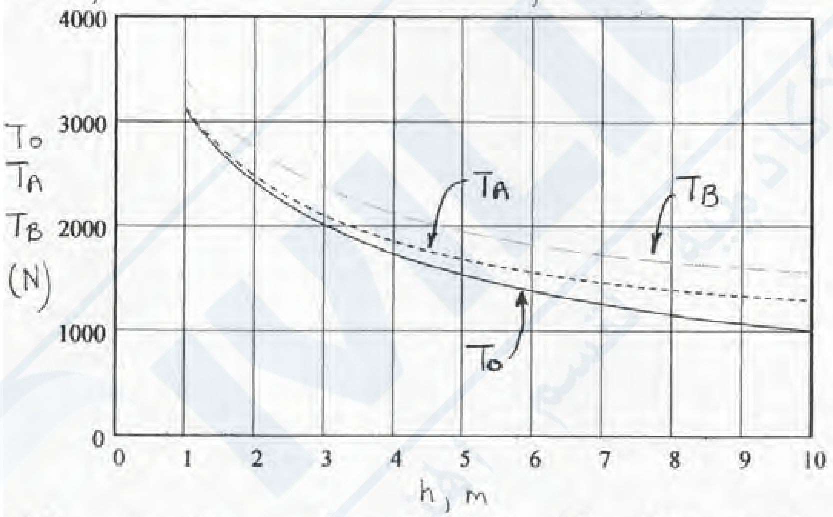


Eq. 5/19 @ B: $h+9 = \frac{T_0}{\mu}\left[\cosh\left(\frac{\mu}{T_0}(60-d)\right) - 1\right]$ (1)

Eq. 5/19 @ A: $h = \frac{T_0}{\mu}\left[\cosh\left(\frac{\mu}{T_0}d\right) - 1\right]$ (2)

Then Eq. 5/22: $\begin{cases} T_A = T_0 + \mu h & (3) \\ T_B = T_0 + \mu(h+9) & (4) \end{cases}$

Numerically solve (1) and (2) (for T_0 & d), then (3) & (4) over $1 \le h \le 10$ m, to obtain



When h = 2 m, $T_0 = 2410$ N, $T_A = 2470$ N, $T_B = 2730$ N

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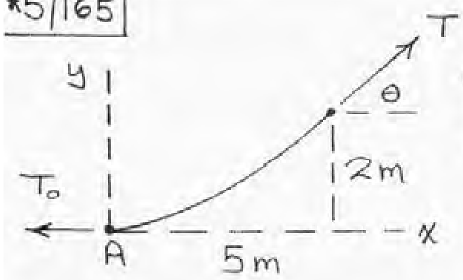

$\mu_e = 0.75(9.81) = 7.36$ N/m

$\mu_f = 1.25(9.81) = 12.26$ N/m

Eq. 5/19: $2 = \frac{T_o}{\mu}\left[\cosh \frac{\mu}{T_o} 5 - 1\right]$

Numerical solution: $\frac{T_o}{\mu} = 6.56$ m

Slope $\frac{dy}{dx} = \sinh \frac{\mu x}{T_o} = \sinh \frac{5}{6.56} = 0.838$

$\theta = \tan^{-1} 0.838 = \underline{40.0°}$ (empty or full)

$T_o = 6.56\mu = 6.56(7.36) = 48.3$ N (empty)

$= 6.56(12.26) = 80.4$ N (full)

$T = T_o + \mu y = 48.3 + 7.36(2) = \underline{63.0 \text{ N}}$ (empty)

$= 80.4 + 12.26(2) = \underline{105.0 \text{ N}}$ (full)

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*5/166 (a) Use $w = \mu = 1.2(9.81) = 11.77$ N/m

Eq. 5/14 : $y = \frac{wx^2}{2T_0}$ @ A : $2.4 = \frac{11.77(5)^2}{2T_0}$

$T_0 = 61.3$ N

So $y_P = \frac{11.77x^2}{2(61.3)} = 0.096x^2$ (see plots below)

Eq. 5/16 : $s_A = l_A\left[1 + \frac{2}{3}\left(\frac{h_A}{l_A}\right)^2 - \frac{2}{5}\left(\frac{h_A}{l_A}\right)^4 + \cdots\right]$

$= 5\left[1 + \frac{2}{3}\left(\frac{2.4}{5}\right)^2 - \frac{2}{5}\left(\frac{2.4}{5}\right)^4 + \cdots\right] = 5.66$ m

So the required length is $L_P = 2s_A = \underline{11.32 \text{ m}}$

(b) Eq. 5/9 : $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

At A : $2.4 = \frac{T_0}{11.77}\left[\cosh\frac{11.77(5)}{T_0} - 1\right]$

Numerical solution : $T_0 = 65.5$ N

So $y_C = 5.57\left[\cosh(0.1796x) - 1\right]$ (see plots)

Eq. 5/20 : $s_A = \frac{65.5}{11.77}\sinh\frac{11.77(5)}{65.6} = 5.70$ m

The required length is $L_C = 2s_A = \underline{11.40 \text{ m}}$

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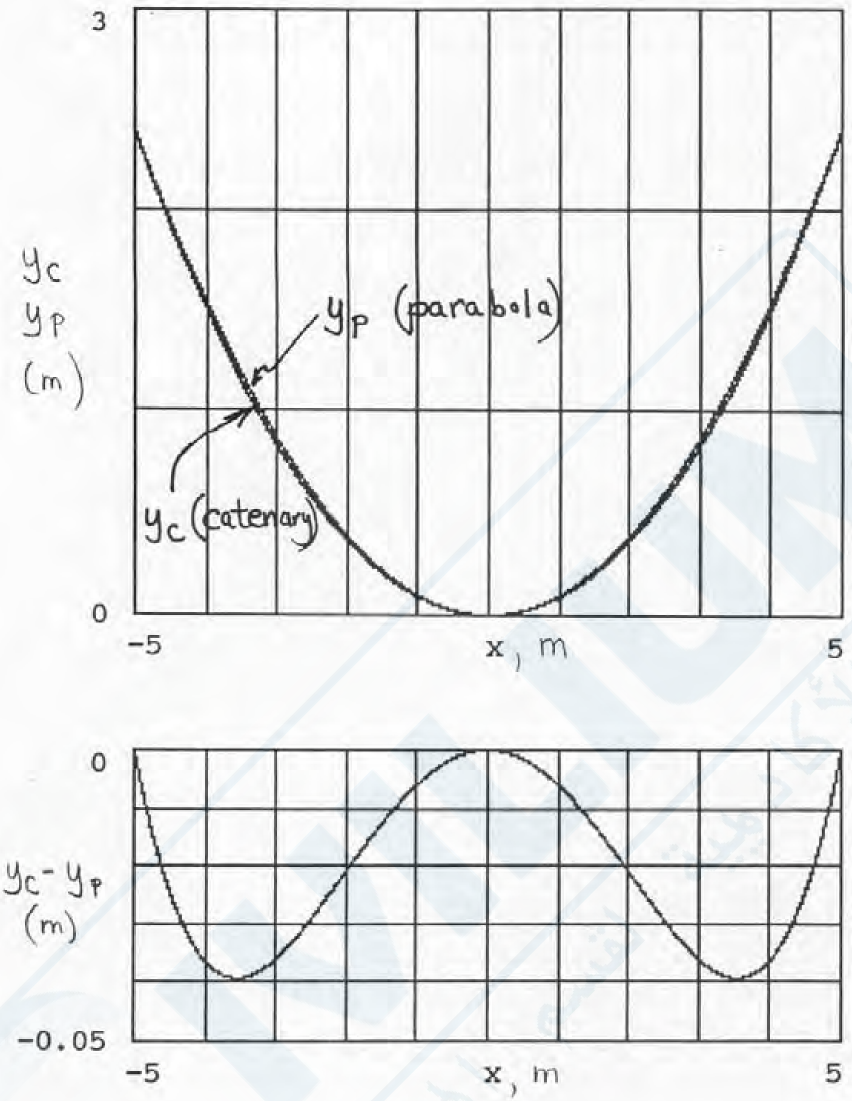

Even with an expanded vertical scale, y_p and y_c are nearly indistinguishable from each other. Note that y_p is above y_c except at the end points (A and B) and the center. So the fact that $L_c > L_p$ makes sense!

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5/167 From Eq. 5/19 with $x = 100$ m, $y = 32$ m

$$32 = \left(\frac{T_o}{\mu}\right)\left[\cosh\left(\frac{100\mu}{T_o}\right) - 1\right]$$

Solve by computer or graphically & get $\frac{T_o}{\mu} = 161.3$ m

From Eq. 5/22, $60(10^3) = T_o + 32\mu$

Solve simultaneously & get $\mu = 310$ N/m

Thus $\rho_{ice} = \frac{\mu}{g} - \rho_{cable} = \frac{310}{9.81} - 18.2 = \underline{13.44 \text{ kg/m of ice}}$

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*5/168 $\mu = 0.5(9.81) = 4.90$ N/m

Eq. 5/19 @ A : $y_A = \frac{T_0}{\mu}\left[\cosh \frac{\mu x_A}{T_0} - 1\right]$ (1)

Eq. 5/19 @ B : $y_A + 9 = \frac{T_0}{\mu}\left[\cosh\left(\frac{\mu}{T_0}(x_A + 12)\right) - 1\right]$ (2)

$\frac{dy}{dx} = \sinh \frac{\mu x}{T_0}$ @ A :

$\tan 15° = \sinh\left[\frac{\mu}{T_0}(x_A)\right]$ (3)

Solve Eqs. (1) – (3) numerically :

$T_0 = 71.5$ N, $x_A = 3.86$ m, $y_A = 0.514$ m

Then

$T_B = T_0 + \mu y_B = 71.5 + 4.90(0.514 + 9)$

$= \underline{118.2 \text{ N}}$

$\tan \theta_B = \sinh\left[\frac{4.90}{71.5}(3.86 + 12)\right]$

$\underline{\theta_B = 52.8°}$

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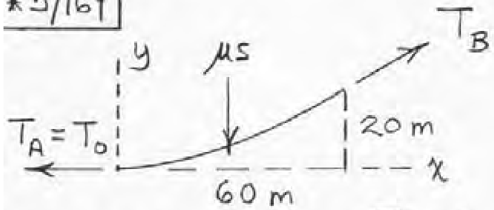

$\mu = (10+20)9.81$
$= 294\ \text{N/m}$

Eq. 5/19: $y = \frac{T_0}{\mu}\left(\cosh\frac{\mu x}{T_0} - 1\right)$

Numbers: $20 = \frac{T_A}{294}\left(\cosh\frac{294(60)}{T_A} - 1\right)$

Numerical solution: $T_A = 27\,400\ \text{N}$
or $\underline{27.4\ \text{kN}}$

Eq. 5/22: $T = T_0 + \mu y$

$T_B = 27\,400 + 294(20) = 33\,300\ \text{N}$
or $\underline{33.3\ \text{kN}}$

Eq. 5/20: $s = \frac{T_0}{\mu}\sinh\frac{\mu x}{T_0}$

$= \frac{27\,400}{294}\sinh\frac{294(60)}{27\,400} = \underline{64.2\ \text{m}}$

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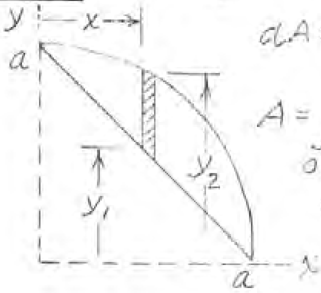
5/17 $x^2 + y_2^2 = a^2$; $x + y_1 = a$



$dA = (y_2 - y_1)dx = (\sqrt{a^2 - x^2} - a + x)dx$

$A = \int_0^a (\sqrt{a^2 - x^2} - a + x)\,dx$

$= \frac{1}{2}\left(x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a}\right) - ax + \frac{x^2}{2}\Big]_0^a$

$= \frac{a^2}{4}(\pi - 2)$

$\int x_c\,dA = \int_0^a (x\sqrt{a^2 - x^2} - ax + x^2)\,dx$

$= -\frac{1}{3}\sqrt{(a^2 - x^2)^3} - \frac{ax^2}{2} + \frac{x^3}{3}\Big]_0^a = \frac{a^3}{6}$

$\bar{x} = \int x_c\,dA / A = \frac{a^3}{6} \Big/ \frac{a^2}{4}(\pi - 2) = \underline{\frac{2a}{3(\pi - 2)}} = \bar{y}$ by symm.

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*5/170 Eq. 5/19: $y = \frac{T_0}{\mu}\left(\cosh \frac{\mu x}{T_0} - 1\right)$

Numbers and rearranging:

$600 = \frac{T_0}{3.10}\left(\cosh \frac{3.10(1600)}{T_0} - 1\right)$ or $\frac{1860}{T_0} = \cosh \frac{4960}{T_0} - 1$

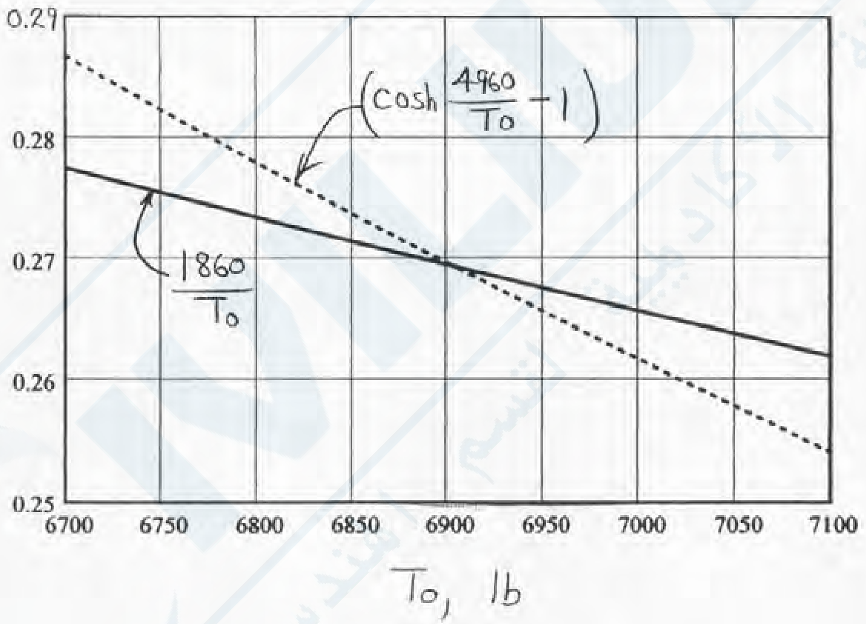
Numerical or graphical (see below) solution:

$\underline{T_0 = 6900 \text{ lb}}$. Then from Eq. 5/22:

$T = T_0 + \mu y = 6900 + 3.10(600) = \underline{8760 \text{ lb}}$

Eq. 5/20: $s = \frac{T_0}{\mu} \sinh \frac{\mu x}{T_0}$

$= \frac{6900}{3.10} \sinh \frac{3.10(1600)}{6900} = \underline{1741 \text{ ft}}$



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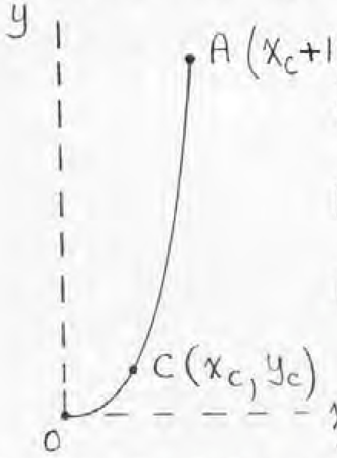
***5/171** Architect's plan : $(T_A)_{arch} = 6(100) = 600$ N

Builder's arrangement :



Eq. 5/19 : $y = \dfrac{T_0}{\mu}\left[\cosh \dfrac{\mu x}{T_0} - 1\right]$

At C : $y_c = \dfrac{T_0}{100}\left[\cosh \dfrac{100\,x_c}{T_0} - 1\right]$

At B : $y_c + 6 = \dfrac{T_0}{100}\left[\cosh \dfrac{100(x_c+1)}{T_0} - 1\right]$

Eq. 5/20 : $s = \dfrac{T_0}{\mu} \sinh \dfrac{\mu x}{T_0}$

So $s_A - s_c = 6.1 = \dfrac{T_0}{100}\left[\sinh \dfrac{100\,(x_c+1)}{T_0} - \sinh \dfrac{100\,x_c}{T_0}\right]$

Numerical solution of three equations :

$x_c = 1.071$ m, $y_c = 1.088$ m, $T_0 = 65.5$ N

Eq. 5/22 : $T = T_0 + \mu y$, so $T_A = 65.5 + 100(1.088 + 6) = 774$ N

Percent increase $n = \dfrac{774 - 600}{600}(100) = \underline{29.0\%}$ (!)

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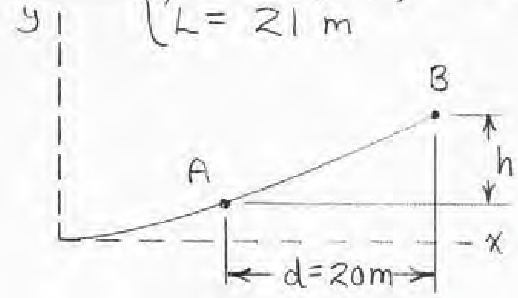
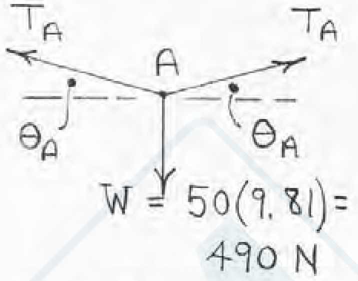
*5/172 $\left\{\begin{array}{l}\mu = 1.2(9.81) = 11.77\ \text{N/m} \\ L = 21\ \text{m}\end{array}\right.$





From FBD of junction ring at A,

$+\uparrow \Sigma F = 0: \quad 2T_A \sin\theta_A - W = 0$

or $\left[T_0 + \mu y_A\right] \sin\left[\tan^{-1}\left(\sinh \frac{\mu x_A}{T_0}\right)\right] - \frac{W}{2} = 0 \quad (1)$

Eq. 5/19 @ A: $y_A = \frac{T_0}{\mu}\left[\cosh \frac{\mu x_A}{T_0} - 1\right] \quad (2)$

Eq. 5/19 @ B: $y_A + h = \frac{T_0}{\mu}\left[\cosh \frac{\mu(x_A + d)}{T_0} - 1\right] \quad (3)$

Eq. 5/20: $s_B - s_A = \frac{T_0}{\mu}\left[\sinh \frac{\mu(x_A + d)}{T_0} - \sinh \frac{\mu x_A}{T_0}\right] = L \quad (4)$

Solution of (1) - (4) with $W = 0$: $h = 5.57$ m

With $W \neq 0$: $h = 6.30$ m

So $\delta = 6.30 - 5.57 = \underline{0.724\ \text{m}}$

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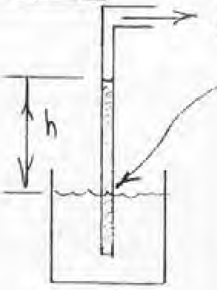

(Assume vacuum pump reduces pressure to zero.)

Pressure here must be atmospheric:

$\rho g h = p_{at}$

$(1000)(9.81)(h) = 1.0133(10^5)$

$\underline{h = 10.33 \text{ m}}$

For mercury: $13\,570(9.81)(h) = 1.0133(10^5)$

$\underline{h = 0.761 \text{ m}}$ (29.97 in.)

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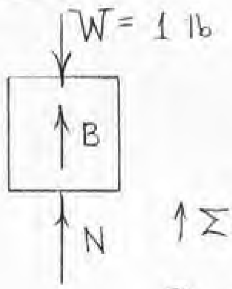

The buoyancy force B is

$$B = \mu_w V = \mu_w \frac{W}{\mu_{ss}}$$

$$= \frac{62.4}{490}(1) = 0.1273 \text{ lb}$$

$\uparrow \Sigma F = 0: \quad N + 0.1273 - 1 = 0, \quad N = 0.873 \text{ lb}$

So the force which the weight exerts on the beaker bottom is 0.873 lb (down). The scale reading, however, increases by 1 lb as the weight is added. The difference is due to the increased water pressure on the beaker bottom as the water level rises.

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5/175 Force on bottom = weight of water

$= \rho g V = \left(1000 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(0.3 m)(0.7 m)(0.4 m)$

$= \underline{824 \text{ N}}$ (down, at center of bottom)

Force on front & back $= p_{av} A_f = \frac{\rho g h}{2} A_f$

$= \frac{1000 (9.81)(0.4)}{2} (0.7)(0.4) = \underline{549 \text{ N}}$ (outward, at $\frac{2}{3}$ depth)

Force on each end glass $= p_{av} A_e = \frac{\rho g h}{2} A_e$

$= \frac{1000 (9.81)(0.4)}{2} (0.3)(0.4) = \underline{235 \text{ N}}$ (outward, at $\frac{2}{3}$ depth)

(All side forces centered horizontally)

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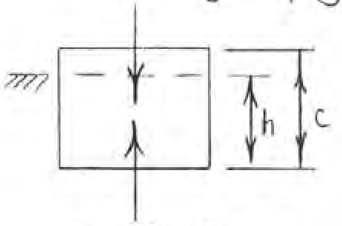
$W = mg = \rho_1 V g = \rho_1 abcg$



$B = \rho_2 V_{sub} g = \rho_2 abhg$

$+\uparrow \Sigma F = 0: \rho_2 abhg - \rho_1 abcg = 0, \quad h = \frac{\rho_1}{\rho_2} c$

$r = \frac{h}{c} = \frac{\rho_1}{\rho_2}$

Oak in water: $r = \frac{800}{1000} = 0.8$

Steel in mercury: $r = \frac{7830}{13\,570} = 0.577$

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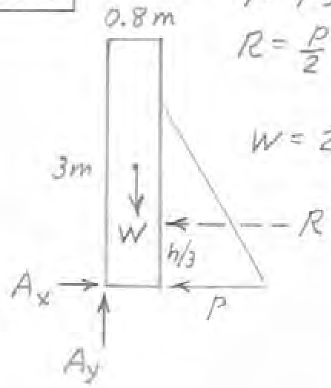

$p = \rho g h = 1760(9.81)h$

$R = \frac{p}{2}h(1) = 1760(9.81)\frac{h^2}{2} = 8633h^2$ N

per meter of horizontal width

$W = 2400(9.81)(3)(0.8) = 56.5(10^3)$ N

$\Sigma M_A = 0;\ R\frac{h}{3} - 0.4W = 0$

$8633h^2\frac{h}{3} = 0.4(56.5)(10^3)$

$h^3 = 7.85$, $\underline{h = 1.988 \text{ m}}$

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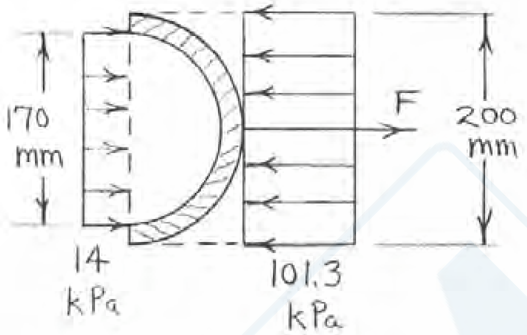

$\overset{+}{\rightarrow} \Sigma F = 0: \quad F + 14(10^3)\dfrac{\pi(0.170)^2}{4} - 101.3(10^3)\dfrac{\pi(0.200)^2}{4} = 0$

$F = 2860$ N or $\underline{F = 2.86 \text{ kN}}$

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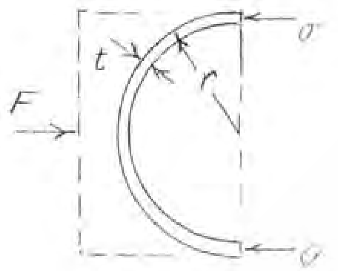
$$F = pA = \rho g h A = \rho g h \pi r^2$$

$$\Sigma F = 0; \quad \rho g h \pi r^2 - \sigma(\pi r^2 - \pi[r-t]^2)$$



$$\sigma = \frac{\rho g h r}{2t} \frac{1}{1 - t/2r}$$

$$= \frac{1.03(10^3)(9.81)\frac{1.500}{2}3(10^3)}{2(0.025)} \frac{1}{1 - \frac{25}{1500}}$$

$$= 454.7(10^6)(1.0169)$$

or $\underline{\sigma = 463 \text{ MPa}}$

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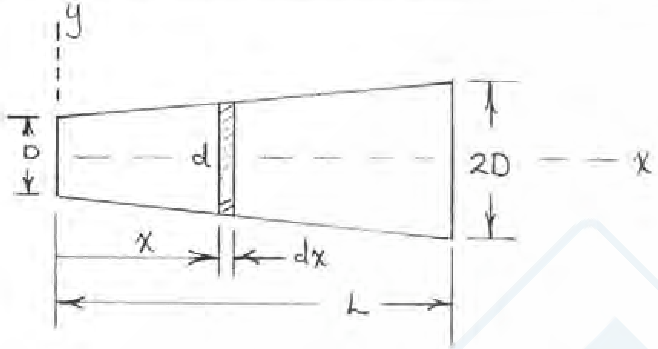

For constant density, $\bar{x}V = \int x\,dV$

Diameter $d = D(1 + x/L)$

So $dV = \dfrac{\pi d^2}{4}dx = \dfrac{\pi D^2}{4}\left(1+\dfrac{x}{L}\right)^2 dx$

$$V = \frac{\pi D^2}{4}\int_0^L \left(1+\frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4}\left[x + \frac{x^2}{L} + \frac{x^3}{3L^2}\right]_0^L$$

$$= \frac{7}{12}\pi D^2 L$$

$$\int x\,dV = \frac{\pi D^2}{4}\int_0^L x\left(1+\frac{x}{L}\right)^2 dx = \frac{\pi D^2}{4}\left[\frac{x^2}{2} + \frac{2x^3}{3L} + \frac{x^4}{4L^2}\right]_0^L$$

$$= \frac{17}{48}\pi D^2 L^2$$

$$\bar{x} = \frac{\frac{17}{48}\pi D^2 L^2}{\frac{7}{12}\pi D^2 L} = \underline{\frac{17}{28}L}$$

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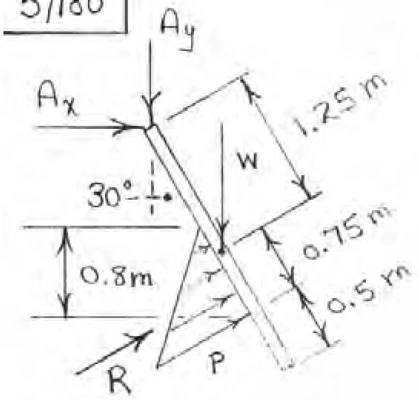

$p = \rho g h = 1000(9.81)(0.8)$

$= 7850 \text{ Pa}$

$R = \frac{1}{2}(7850)\left(\frac{0.8}{\cos 30^\circ}\right)$

$= 3620 \text{ N/m}$

$\circlearrowleft \Sigma M_A = 0: \quad W(1.25 \sin 30^\circ) - 3620\left(2.5 - 0.5 - \frac{1}{3}\frac{0.8}{\cos 30^\circ}\right)$

$= 0; \quad \underline{W = 9810 \text{ N/m}}$

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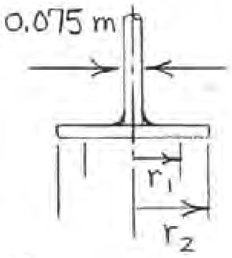

$\begin{cases} r_1 = 0.2 \text{ m} \\ r_2 = 0.3 \text{ m} \end{cases}$

$p = \rho g h = 1030(9.81)(0.6) = 6060 \text{ Pa}$

$R = pA = 6060\pi\left(0.3^2 - \frac{0.075^2}{4}\right) = 1687 \text{ N}$

Pressure supported by seal

$\sigma = R/\pi(r_2^2 - r_1^2) = \frac{1687}{\pi(0.3^2 - 0.2^2)}$

$= 10\,740 \text{ Pa}$ or $\underline{10.74 \text{ kPa}}$

Force to lift plunger

$P = R = 1687 \text{ N}$ or $\underline{1.687 \text{ kN}}$

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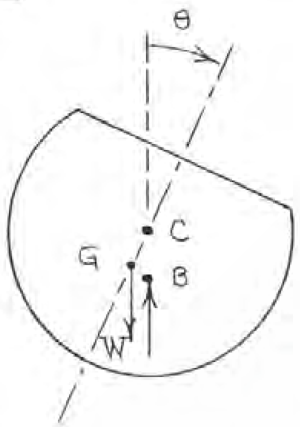
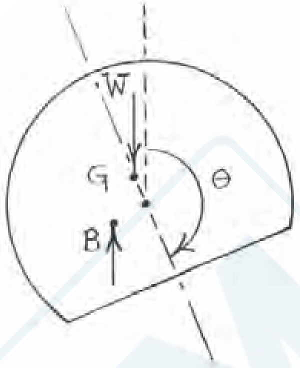


CCW couple tends to make θ = 0



CW couple tends to make θ = 180°

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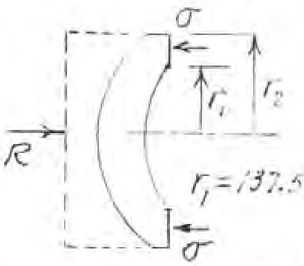
$p = \rho g h = 1030(9.81)(1000) = 10.104(10^6)$ Pa



$r_1 = 137.5$ mm

$r_2 = 175$ mm

$R = pA = 10.104(10^6)\,\pi\,(0.175)^2$

$= 972(10^3)$ N

$\sigma = R/A_0$

$= \dfrac{972(10^3)}{\pi([0.175]^2 - [0.1375]^2)}$

$= 26.4(10^6)$ Pa

or $\sigma = 26.4$ MPa

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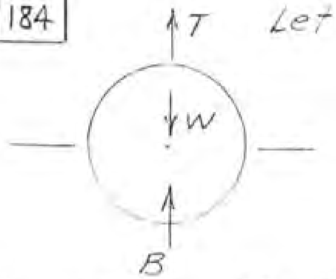

Let γ_c = wt. density of concrete $= 150\ \text{lb/ft}^3$

γ_w = wt. density of fresh water $= 62.4\ \text{lb/ft}^3$

L = length of cylinder = 6 ft

r = radius of cylinder = 2 ft

For equil., $T = W - B$

$$= \gamma_c \pi r^2 L - \gamma_w \frac{\pi r^2}{2} L$$

$$= \pi r^2 L \left(\gamma_c - \frac{1}{2}\gamma_w\right)$$

$$= \pi(2^2)(6)\left(150 - \frac{62.4}{2}\right) = \underline{8960\ \text{lb}}$$

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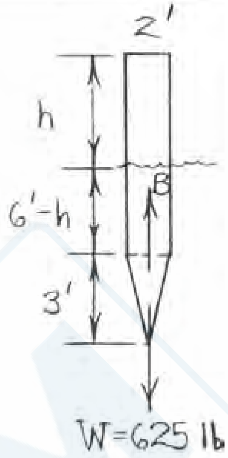
$$B = \mu V = 64\left[\pi \frac{2^2}{4}(6-h) + \frac{1}{3}\pi\frac{2^2}{4}(3)\right]$$

$$= 64\pi\frac{2^2}{4}\left[6-h+1\right]$$

$$= 64\pi\left[7-h\right]$$

$W = B$: $\quad 625 = 64\pi\left[7-h\right]$

$$\underline{h = 3.89 \text{ ft}}$$



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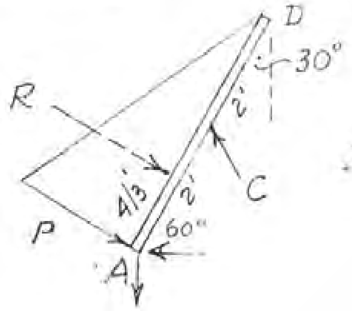

$p = \rho g h = 62.4\,(4\cos 30°) = 216\ \text{lb/ft}^2$

$R = \frac{p}{2}$ Area per panel

$= \frac{216}{2}(4)(2) = 865\ \text{lb}$

$\Sigma M_A = 0;\ 2C\cos 30° - 865\,(4/3) = 0$

$C = \frac{865(2)}{3\cos 30°} = \underline{666\ \text{lb}}$

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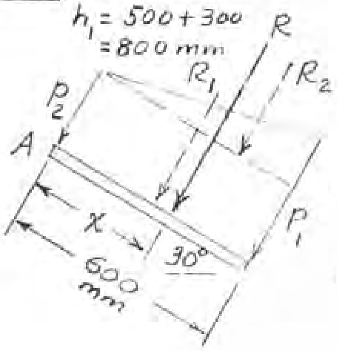
$h_2 = 500$ mm

$h_1 = 500 + 300 = 800$ mm



$p_2 = \rho g h_2 = 900(9.81)(0.500)$
$= 4.415(10^3)$ Pa

$p_1 = \rho g h_1 = 900(9.81)(0.800)$
$= 7.063(10^3)$ Pa

$R_1 = p_2 A = 4.415(10^3)(0.600)(0.400)$
$= 1059$ N

$R_2 = \frac{p_1 - p_2}{2} A$
$= \frac{(7.063 - 4.415)}{2} 10^3 (0.600)(0.400)$
$= 318$ N

$R = R_1 + R_2 = 1059 + 318 = \underline{1377 \text{ N}}$

$Rx = \Sigma M_A$; $1377x = 1060(300) + 318(400)$

$x = \frac{445\,000}{1377} = \underline{323 \text{ mm}}$

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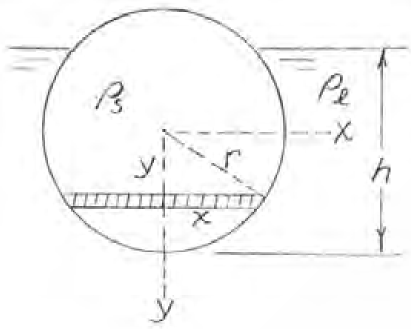
5/188 Buoyancy $B = \rho_\ell g V$, V = submerged volume



$$W = mg = \rho_s \frac{4}{3}\pi r^3 g$$

$$dV = \pi x^2 dy = \pi(r^2 - y^2)\,dy$$

$$B = \rho_\ell g \pi \int_{-(h-r)}^{r} (r^2 - y^2)\,dy$$

$$= \rho_\ell g \pi \left[r^2 y - \frac{y^3}{3} \right]_{-(h-r)}^{r}$$

$$= \frac{1}{3}\rho_\ell g \pi h^2 (3r - h)$$

Thus with $B = W$, $\rho_s g \frac{4}{3}\pi r^3 = \frac{1}{3}\rho_\ell g \pi h^2 (3r - h)$

Rearrange & get $\rho_s = \rho_\ell \left(\frac{h}{2r}\right)^2 \left(3 - \frac{h}{r}\right)$

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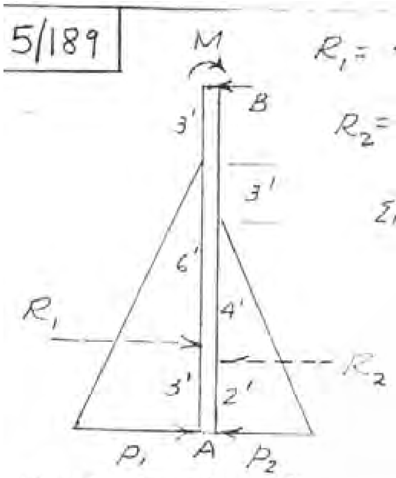

Horiz length of gate is 10 ft

$R_1 = \frac{p_1 A_1}{2} = \frac{62.4(9)}{2}(9)(10) = 25,270$ lb

$R_2 = \frac{p_2 A_2}{2} = \frac{64(6)}{2}(6)(10) = 11,520$ lb

$\Sigma M_B = 0;\ M + 11,520(10) - 25,270(9) = 0$

$M = 11.22(10^4)$ lb-ft

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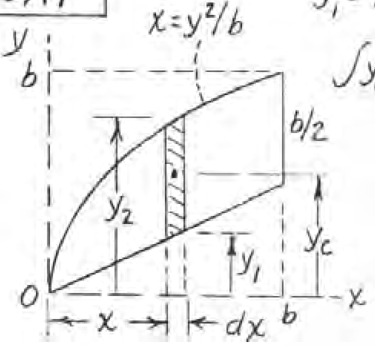

$y_1 = x/2\ ,\quad y_2 = \sqrt{bx}$

$$\int y_c\, dA = \int_0^b \frac{y_1 + y_2}{2}(y_2 - y_1)\, dx$$

$$= \frac{1}{2}\int_0^b (y_2^2 - y_1^2)\, dx$$

$$= \frac{1}{2}\int_0^b \left(bx - \frac{x^2}{4}\right) dx$$

$$= \frac{1}{2}\left[\frac{bx^2}{2} - \frac{x^3}{12}\right]_0^b = \frac{5}{24}b^3$$

$$\int dA = \int_0^b (y_2 - y_1)\, dx \quad = \int_0^b (\sqrt{bx} - x/2)\, dx = \left[\frac{2\sqrt{b}}{3}x^{3/2} - \frac{x^2}{4}\right]_0^b$$

$$= \frac{5}{12}b^2$$

$$\bar{y} = \frac{\int y_c\, dA}{\int dA} = \frac{\frac{5}{24}b^3}{\frac{5}{12}b^2} = \underline{\frac{b}{2}}$$

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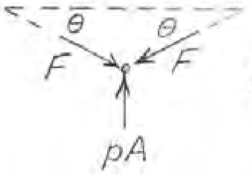
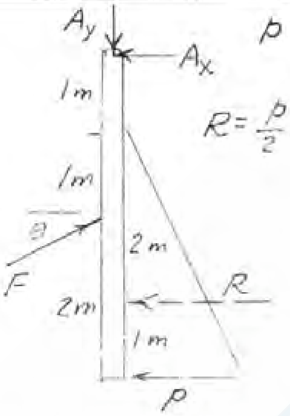
$\theta = \tan^{-1}\frac{0.5}{1}$
$= 26.6°$

$p = \rho g h = 1.0(9.81)(3)$
$= 29.4 \text{ kPa}$

$R = \frac{p}{2}\text{Area} = \frac{29.4}{2}(3)(2)$
$= 88.3 \text{ kN}$





Gate:

$\Sigma M_A = 0;\quad (F\cos 26.6°)2 - 88.3(3) = 0,\quad F = 148.1 \text{ kN}$

Toggle:

$\Sigma F = 0;\quad pA - 2F\sin\theta = 0;\quad \frac{\pi(0.150)^2}{4}p = 2(148.1)(10^3)\sin 26.6°$

$p = 7.49(10^6) \text{ Pa}$ or $\underline{p = 7.49 \text{ MPa}}$

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5/191 Submerged volume V is

$V = 2(350)(40)(25) + 6\pi \frac{(30)^2}{4}(h-25)$

$= 70(10^4) + 0.4241(10^4)(h-25)$

$B = \gamma V = 64(70)10^4 + 64(0.4241)(10^4)(h-25)$

$= 4480(10^4) + 27.14(10^4)(h-25)$

$W = 26{,}000(2240) = 5824(10^4)$ lb

$B = W,\quad 27.14(10^4)(h-25) = (5824 - 4480)10^4$

$h - 25 = 49.5,\quad \underline{h = 74.5 \text{ ft}}$

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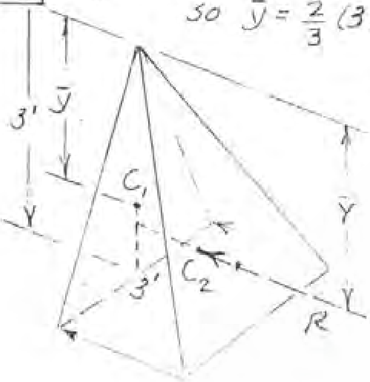
5/192 C_1 = centroid of triangular area

so $\bar{y} = \frac{2}{3}(3) = 2$ ft



C_2 = centroid of tetrahedron

so $H = \bar{Y} = \frac{3}{4}(3) = \underline{2.25 \text{ ft}}$

$R = \rho g \bar{y} A$

$= (62.4)(2)\frac{1}{2}(3)(3)$

$= \underline{562 \text{ lb}}$

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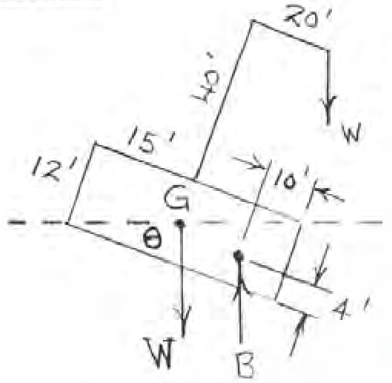

$\theta = \tan^{-1}\frac{12}{30} = 21.8°$

Moment arm of B about G

$(15-10)\cos\theta - (6-4)\sin\theta =$

3.90 ft

Moment arm of w about G

$(40+6)\sin\theta + 20\cos\theta$

$= 35.7$ ft

$B = \rho g V = 64(12)(15)(80) = 922{,}000$ lb

$\Sigma M_G = 0: \; 35.7\,w - 3.90\,(922{,}000) = 0$

$w = 100{,}800$ lb

$W = B - w = 922{,}000 - 100{,}800 = 821{,}000$ lb

or $W = \frac{821{,}000}{2240} = 366$ long tons

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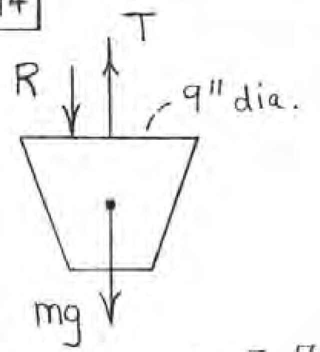

$p = \rho g h = 62.4(20) = 1248 \ \frac{lb}{ft^2}$

$R = pA = 1248\left[\pi \left(\frac{4.5}{12}\right)^2\right]$

$= 551 \text{ lb}$

$mg = \rho V g = \rho g \left[\frac{7\pi r^2 h}{24}\right]$ (from Prob. 5/35)

$= 450\left[\frac{7\pi \ (4.5/12)^2 (12/12)}{24}\right] = 58.0 \text{ lb}$

$+\uparrow \Sigma F = 0: \quad T - 551 - 58.0 = 0, \quad \underline{T = 609 \text{ lb}}$

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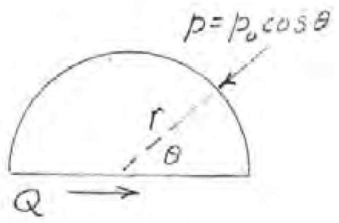
$$Q = \int_0^{\pi} (p_o \cos\theta)(\cos\theta)\, r\, d\theta$$



$$= p_o r \int_0^{\pi} \cos^2\theta\, d\theta$$

$$= p_o r \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi}$$

$$= p_o r \left[\frac{\pi}{2} \right]$$

$$\underline{Q = \tfrac{1}{2}\pi r p_o}$$

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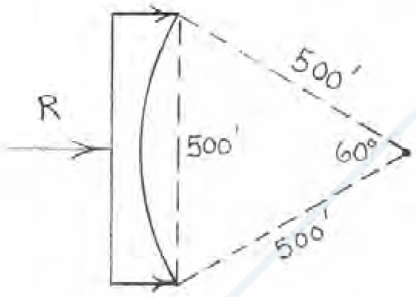

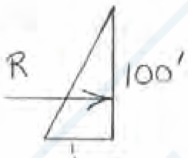

$P_{100} = \mu h = 62.4(100) = 6240 \ \frac{lb}{ft^2}$

$R = P_{av}\,(\text{Area}) = \frac{6240}{2}(500)(100) = \underline{156.0(10^6)\ lb}$

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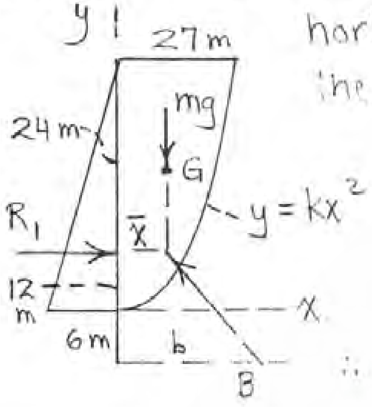
5/197 Take a vertical section of water of unit horizontal length. Let ρ be the water density in t/m^3.



$y = kx^2$: $36 = k(27)^2$, $k = \frac{4}{81}\ m^{-1}$

$\bar{x} = \frac{\int x\,dA}{\int dA}$, $dA = x\,dy = 2\frac{4}{81}x^2 dx$

$\therefore\ \bar{x} = \frac{\int_0^{27} \frac{x}{2}\frac{8}{81}x^2 dx}{\int_0^{27}\frac{8}{81}x^2 dx} = 10.12\ m$

$A = \int dA = 648\ m^2$, $mg = 648\rho g$

$R_1 = \frac{1}{2}\,36\rho g\,(36)(1) = 648\rho g$

Resultant of mg & R_1 passes through B, so $\Sigma M_B = 0$. Thus $648\rho g(18) = 648\rho g\,(b - 10.12)$

$b = 28.1\ m$

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5/198 The gage pressure 12 m below the surface is $p = \rho g h = (1000)(9.81)(12) = 117\ 700\ \text{N/m}^2$.

(a) Cover area $A_{cov} = \pi\left(\frac{0.75}{2}\right)\left(\frac{0.5}{2}\right) = 0.295\ \text{m}^2$

Force on cover $= pA_{cov} = 34\ 700\ \text{N}$

Seal area $A_s = A_{cov} - \pi\left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right) = 0.1325\ \text{m}^2$

$\sigma A_s = pA_{cov}$, $\sigma = \frac{34\ 700}{0.1325} = 262\ 000\ \frac{\text{N}}{\text{m}^2}$

or $\underline{\sigma = 262\ \text{kPa}}$

(b) $16\Delta T = pA_{hole} = 117\ 700\left[\pi\left(\frac{0.55}{2}\right)\left(\frac{0.375}{2}\right)\right]$

$\underline{\Delta T = 1192\ \text{N}}$

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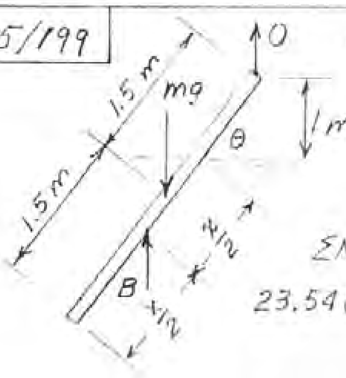

Let A = cross-sectional area of plank

$mg = 800(3)A(9.81) = 23.54(10^3)A$ N

$B = \rho_w g A x$

$= 1000(9.81)A\left(3 - \frac{1}{\sin\theta}\right)$

$\Sigma M_O = 0;$

$23.54(10^3)A(1.5\cos\theta) - 9.81(10^3)A\left(3 - \frac{1}{\sin\theta}\right) \cdot$

$\cdot\left[\frac{1}{2}\left(3 - \frac{1}{\sin\theta}\right) + \frac{1}{\sin\theta}\right]\cos\theta = 0$

Simplify & get $23.54(1.5) = \frac{9.81}{2}\left(9 - \frac{1}{\sin^2\theta}\right)$

& $\sin^2\theta = 0.5556$, $\sin\theta = 0.7454$, $\underline{\theta = 48.2°}$

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5/2 From Sample Problem 5/3 with $r = 8$ and $\alpha = 120° = \frac{2}{3}\pi$:

$$\bar{r} = \frac{2}{3}(8)\frac{\sin 120°}{2\pi/3} = \underline{2.21}$$

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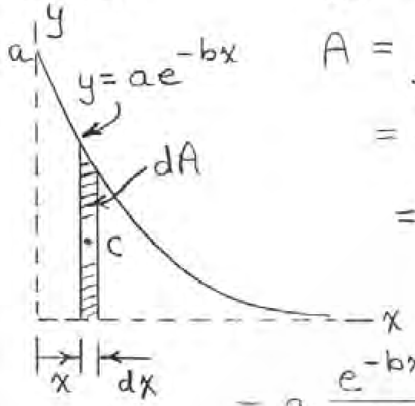

$$dA = y\,dx = ae^{-bx}\,dx$$

$$A = \int dA = \int_0^\infty ae^{-bx}\,dx$$

$$= -\frac{a}{b}e^{-bx}\Big|_0^\infty = -\frac{a}{b}[0-1]$$

$$= a/b$$

$$\int x_c\,dA = a\int_0^\infty xe^{-bx}\,dx$$

$$= a\frac{e^{-bx}}{b^2}\left[-bx-1\right]_0^\infty$$

$$= -\frac{a}{b^2}\left[bxe^{-bx}+e^{-bx}\right]_0^\infty = -\frac{a}{b^2}\left[0+0-(0+1)\right]$$

$$= a/b^2$$

$$\int y_c\,dA = \int \frac{y}{2}y\,dx = \int \frac{y^2}{2}\,dx$$

$$= \frac{1}{2}\int_0^\infty a^2e^{-2bx}\,dx = \frac{a^2}{2}\,\frac{e^{-2bx}}{-2b}\Big|_0^\infty$$

$$= -\frac{a^2}{4b}[0-1] = \frac{a^2}{4b}$$

$$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{a/b^2}{a/b} = \underline{\frac{1}{b}}$$

$$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{a^2/4b}{a/b} = \underline{\frac{a}{4}}$$

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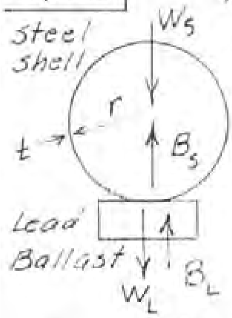

For equilibrium $W_S + W_L = B_S + B_L$

ρ_S = density of steel = 7.83 Mg/m^3

ρ_W = " " salt water = 1.03 Mg/m^3

ρ_L = " " lead = 11.37 Mg/m^3

$r = 1.00$ m, $t = 0.035$ m

V_L = volume of lead, m^3

m = mass of lead = $\rho_L V_L$

so $\rho_S g\, 4\pi r^2 t + \rho_L g V_L = \rho_W g \frac{4}{3}\pi(r+\frac{t}{2})^3 + \rho_W g V_L$

$V_L g(\rho_L - \rho_W) = 4\pi r^2 g\left[\frac{r}{3}(1+\frac{t}{2r})^3\rho_W - \rho_S t\right]$

$m(1-\frac{\rho_W}{\rho_L}) = 4\pi r^2\left[\frac{r}{3}(1+\frac{t}{2r})^3\rho_W - \rho_S t\right]$

$m(1-\frac{1.03}{11.37}) = 4\pi(1)^2\left[\frac{1}{3}(1+\frac{0.035}{2})^3 1.03 - 7.83(0.035)\right]$

$0.9094m = 1.1008$ $m = \underline{1.210 \text{ Mg}}$ (metric tons)

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5/201 The pressure at the bottom of the 3-m wall is $p = \rho g h = 2400(9.81)(3) = 70\,600 \text{ N/m}^2$

Each tie controls an area A given by

$$pA = T, \quad A = \frac{T}{p} = \frac{6\,500}{70\,600} = 0.0920 \text{ m}^2$$

This square area has a side d given by

$$d^2 = A, \quad d = 0.303 \text{ m}$$

Using the pressure at the very bottom of the wall gives us a conservative design; a good figure for d would be $\underline{d = 0.300 \text{ m}}$.

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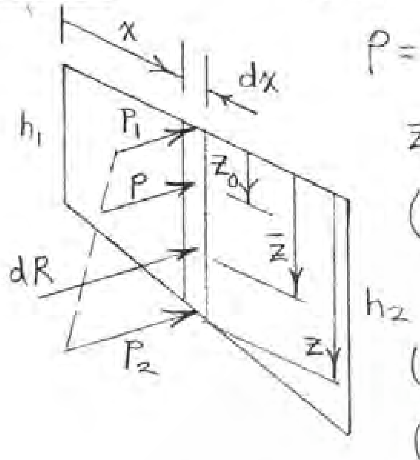


Method I: Direct integration

ρ = water density, $p = \rho g (b + z_0)$

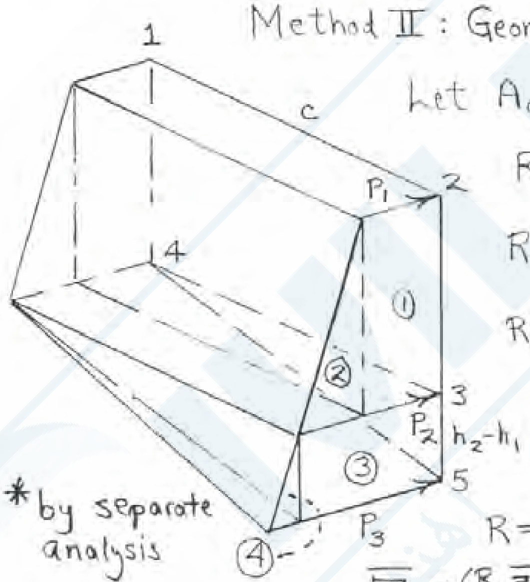
$z = h_1 + kx = f(x)$

(1) Calculate dR for elemental area $z\,dx$ & find $\bar{z}$

(2) Integrate & get $R = \int dR$

(3) $R\bar{z} = \int \bar{z}\, dR$

Method II: Geometry of pressure-area volumes



Let A_a = 1-2-3-4, A_b = 3-4-5

$R_1 = p_1 A_a$, $\bar{z}_1 = h_1/2$

$R_2 = \frac{1}{2}(p_2 - p_1) A_a$, $\bar{z}_2 = \frac{2h_1}{3}$

$R_3 = p_2 A_b$, $\bar{z}_3 = h_1 + \frac{2}{3}(h_2 - h_1)$

$R_4 = \frac{1}{2}(p_3 - p_2)(h_2 - h_1)\frac{c}{3}$

$\bar{z}_4 = \frac{1}{2}(h_2 + h_1)$ *

$R = R_1 + R_2 + R_3 + R_4$

$\bar{z} = (R_1\bar{z}_1 + R_2\bar{z}_2 + R_3\bar{z}_3 + R_4\bar{z}_4)/R$

*by separate analysis

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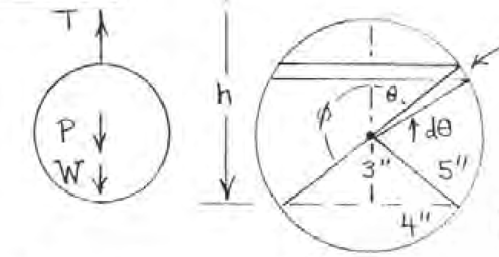

$\Sigma F = 0: \; T - P - W = 0$

Pressure force $P = \int p \, dA \cos\theta$

$\rho g = 62.4/1728 = 0.0361 \text{ lb/in.}^3$

$\phi = 180° - \cos^{-1}(0.6) = 126.9°$

$V = \frac{126.9}{180} \frac{4}{3}\pi(5^3) = 369 \text{ in.}^3, \quad B = 369 \rho g \text{ (lb)}$

$p = \rho g(h - 3 - r\cos\theta) = \rho g(h' - r\cos\theta), \quad h' = h - 3$

$P = \int_0^{\phi} \rho g (h' - r\cos\theta)(2\pi r \sin\theta)\, r\, d\theta \cos\theta$

$= 2\pi r^2 \rho g \int_0^{\phi} (h' \sin\theta \cos\theta - r \sin\theta \cos^2\theta)\, d\theta$

$= 2\pi r^2 \rho g \left[\int_{\theta=0}^{\theta=\phi} \frac{h'}{4} \sin 2\theta \, d(2\theta) - \int_0^{\phi} r(\sin\theta - \sin^3\theta)\, d\theta \right.$

$= 2\pi r^2 \rho g \left[-\frac{h'}{4}\cos 2\theta - r\left(-\cos\theta + \frac{\cos\theta}{3}(2 + \sin^2\theta)\right)\right]_0^{\phi}$

$= 2\pi r^2 \rho g \left[0.32h' - 2.03\right] \text{ (lb)}$

When $T = W$, $P = 0$: $h' = \frac{2.03}{0.32} = h - 3$

$h = 9.33 \text{ in.}$

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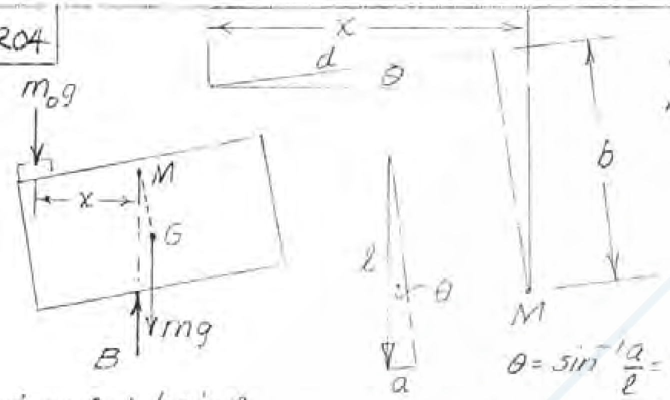

$m_o = 27$ Mg

$m = 12(10^3)$ Mg

$\theta = \sin^{-1}\frac{a}{\ell} = \sin^{-1}\frac{0.2}{6} = 1.91^\circ$

$\sin\theta = 0.0333$, $\cos\theta = 0.9994$

$x = d\cos\theta + b\sin\theta$

$= 7.8(0.9994) + 1.8(0.0333)$

$= 7.86$ m

$\Sigma M_M = 0$; $mg\,\overline{GM}\sin\theta - m_o g x = 0$

$$\overline{GM} = \frac{m_o x}{m\sin\theta} = \frac{27(7.86)}{12(10^3)(0.0333)} = \underline{0.530\text{ m}}$$

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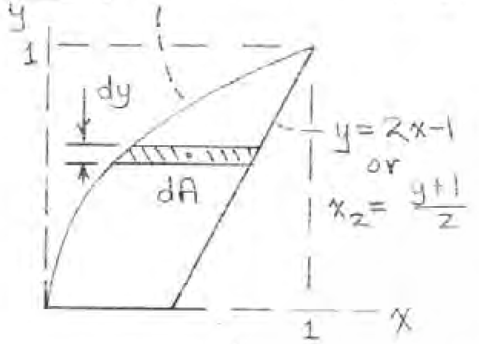
5/205 $y = x^{1/3}$ or $x_1 = y^3$



$dA = (x_2 - x_1)\,dy$
$= \left(\frac{y}{2} + \frac{1}{2} - y^3\right) dy$

$A = \int dA = \int_0^1 \left(\frac{y}{2} + \frac{1}{2} - y^3\right) dy = \left(\frac{y^2}{4} + \frac{y}{2} - \frac{y^4}{4}\right)\Big|_0^1 = \frac{1}{2}$

$\int y_c\, dA = \int_0^1 \left(\frac{y^2}{2} + \frac{y}{2} - y^4\right) dy = \left(\frac{y^3}{6} + \frac{y^2}{4} - \frac{y^5}{5}\right)\Big|_0^1 = \frac{13}{60}$

$\int x_c\, dA = \int_0^1 \left(\frac{x_1 + x_2}{2}\right)(x_2 - x_1)\, dy = \frac{1}{2}\int_0^1 (x_2^2 - x_1^2)\, dy$

$= \frac{1}{2}\int_0^1 \left(\frac{y^2}{4} + \frac{y}{2} + \frac{1}{4} - y^6\right) dy = \frac{1}{2}\left(\frac{y^3}{12} + \frac{y^2}{4} + \frac{y}{4} - \frac{y^7}{7}\right)\Big|_0^1$

$= \frac{37}{168}$

$\bar{y} = \int x_c\, dA \Big/ \int dA = \frac{13/60}{1/2} = 13/30$

$\bar{x} = \int y_c\, dA \Big/ \int dA = \frac{37/168}{1/2} = 37/84$

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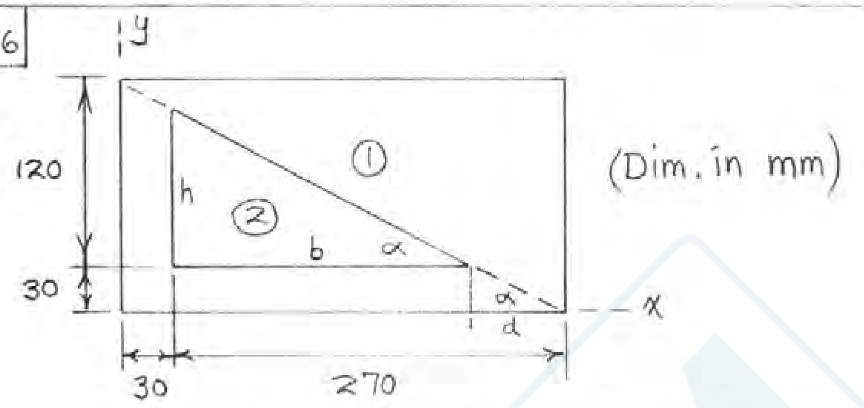

$\tan\alpha = \frac{150}{300} = \frac{1}{2}$; $\frac{30}{d} = \tan\alpha = \frac{1}{2}$, so $d = 60$ mm

Then $b = 300 - (30 + 60) = 210$ mm

$\frac{h}{b} = \tan\alpha = \frac{1}{2}$, $h = \frac{b}{2} = 105$ mm

$A_1 = 150(300) = 45\,000 \text{ mm}^2$

$\bar{x}_1 = 150$ mm, $\bar{y}_1 = 75$ mm

$A_2 = -\frac{1}{2}bh = -\frac{1}{2}(210)(105) = -11\,025 \text{ mm}^2$

$\bar{x}_2 = 30 + \frac{1}{3}(210) = 100$ mm, $\bar{y}_2 = 30 + \frac{1}{3}(105) = 65$ mm

$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{45\,000(150) - 11\,025(100)}{45\,000 - 11\,025} = \underline{166.2 \text{ mm}}$

$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{45\,000(75) - 11\,025(65)}{45\,000 - 11\,025} = \underline{78.2 \text{ mm}}$

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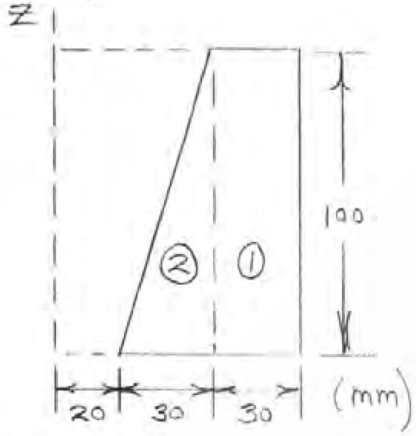

$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi (65)(30 \cdot 100) = 1.225(10^6)\ mm^3$$

$$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi (40) \frac{30 \cdot 100}{2} = 0.377(10^6)\ mm^3$$

$$V = V_1 + V_2 = 1.602(10^6)\ mm^3$$

$$m = \rho V = 7210 \frac{kg}{m^3}\left[1.602(10^6)\ mm^3 \cdot \left(\frac{1\ m}{1000\ mm}\right)^3\right]$$

$$= \underline{11.55\ kg}$$

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For circular arc, $\bar{y} = \dfrac{b \sin 30^\circ}{\pi/6} = 3b/\pi$

$\bar{z} = b$ (See Samp. Prob. 5/1)

$$\bar{Y} = \frac{\Sigma \bar{y} L}{\Sigma L} = \frac{b(0) + 2b\left(\frac{b}{2}\cos 30^\circ\right) + \frac{\pi}{3}b\left(\frac{3b}{\pi}\right)}{b + 2b + \frac{\pi}{3}b}$$

$$= \underline{0.461\,b}$$

$$\bar{Z} = \frac{\Sigma \bar{z} L}{\Sigma L} = \frac{b\left(\frac{b}{2}\right) + 2b(b) + \frac{\pi}{3}b(b)}{b + 2b + \frac{\pi}{3}b}$$

$$= \underline{0.876\,b}$$

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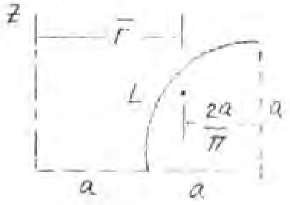
$L = \pi a/2$; $\quad \bar{r} = 2a - \frac{2a}{\pi} = 2a\left(1 - \frac{1}{\pi}\right)$



$A = \frac{\pi}{2}\bar{r}L = \frac{\pi}{2}2a\left(1-\frac{1}{\pi}\right)\frac{\pi a}{2}$

$= \frac{\pi a^2}{2}(\pi - 1)$

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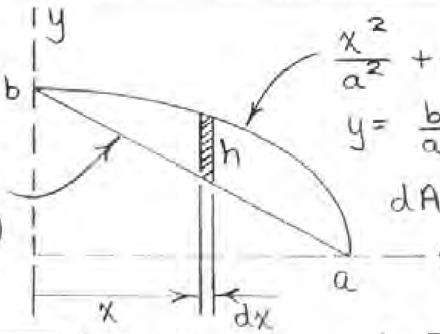

$\frac{x}{a} + \frac{y}{b} = 1$

$y = \frac{b}{a}(a-x)$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$y = \frac{b}{a}\sqrt{a^2 - x^2}$

$dA = h\,dx$

$h = \frac{b}{a}\sqrt{a^2-x^2} - \frac{b}{a}(a-x) = \frac{b}{a}\left[\sqrt{a^2-x^2} - (a-x)\right]$

$A\bar{x} = \int x\,dA$

$A = \int_0^a \frac{b}{a}\left[\sqrt{a^2-x^2} - (a-x)\right]dx$

$= \frac{b}{a}\left[\frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} - \left(ax - \frac{x^2}{2}\right)\right]_0^a = \frac{ab}{2}\left[\frac{\pi}{2} - 1\right]$

$\int x\,dA = \int_0^a x\frac{b}{a}\left[\sqrt{a^2-x^2} - (a-x)\right]dx$

$= \frac{b}{a}\left[-\frac{1}{3}\sqrt{(a^2-x^2)^3} - \left(\frac{ax^2}{2} - \frac{x^3}{3}\right)\right]_0^a = \frac{ba^2}{6}$

$\bar{x} = \frac{\int x\,dA}{A} = \frac{ba^2/6}{\frac{ab}{2}\left(\frac{\pi}{2}-1\right)} = \frac{a}{3\left(\frac{\pi}{2}-1\right)}$

Similarly, $\bar{y} = \frac{b}{3\left(\frac{\pi}{2}-1\right)}$

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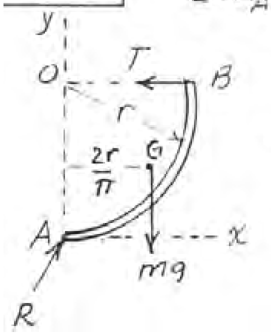

$\Sigma M_A = 0;\ mg\frac{2r}{\pi} - Tr = 0,\ T = \frac{2mg}{\pi}$

$\Sigma F_x = 0;\ R_x = \frac{2mg}{\pi}$

$\Sigma F_y = 0;\ R_y = mg$

$R = \sqrt{R_x^2 + R_y^2} = mg\sqrt{\frac{4}{\pi^2} + 1} = \underline{1.185\,mg}$

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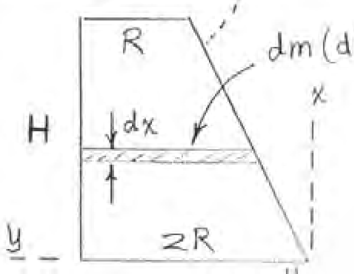

$$dm = \rho\, dV = \rho \pi \left(\frac{2R-y}{2}\right)^2 dx$$

$$= \frac{\pi\rho}{4}\left(4R^2 - 4Ry + y^2\right) dx$$

$$= \frac{\pi\rho}{4}\left[4R^2 - 4R\frac{R}{H}x + \left(\frac{R}{H}x\right)^2\right] dx$$

$$= \frac{\pi\rho R^2}{4}\left[4 - 4\frac{x}{H} + \frac{x^2}{H^2}\right] dx$$

$$m = \int dm = \int_0^H \frac{\pi\rho R^2}{4}\left[4 - 4\frac{x}{H} + \frac{x^2}{H^2}\right] dx$$

$$= \frac{7}{12}\pi\rho R^2 H$$

$$\int x_c\, dm = \int_0^H \frac{\pi\rho R^2}{4}\left[4x - 4\frac{x^2}{H} + \frac{x^3}{H^2}\right] dx$$

$$= \frac{11}{48}\pi\rho R^2 H^2$$

Then $\bar{x} = \dfrac{\int x_c\, dm}{\int dm} = \dfrac{\frac{11}{48}\pi\rho R^2 H^2}{\frac{7}{12}\pi\rho R^2 H}$

$$= \underline{\frac{11}{28} H = \bar{h}}$$

(ρ = mass per unit volume)

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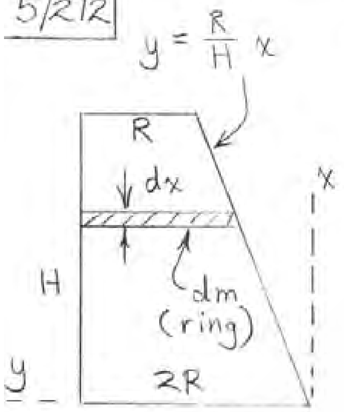

$dm = \rho\, dA = \rho\left[\frac{2R-y}{2} \cdot 2\pi k\right]$

$= \pi\rho\left[2R - \frac{R}{H}x\right]k\,dx$

$= \pi\rho R\left[2 - \frac{x}{H}\right]k\,dx$

$m = \int dm = \int_0^H \pi\rho R\left(2 - \frac{x}{H}\right)k\,dx$

$= \frac{3}{2}k\pi\rho RH$

$$\int x_c\,dm = \int_0^H \pi\rho Rk\left(2x - \frac{x^2}{H}\right)dx = \frac{2}{3}k\pi\rho RH^2$$

$$\bar{x} = \frac{\int x_c\,dm}{\int dm} = \frac{\frac{2}{3}k\pi\rho RH^2}{\frac{3}{2}k\pi\rho RH} = \underline{\frac{4}{9}H = \bar{h}}$$

(ρ = mass per unit area, k is a parameter which accounts for the variation of the slant height of the differential elemental ring around the circumference of the body (the same for all elements.))

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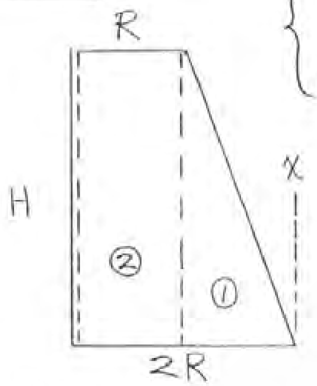

{ ① Solid body (Prob. 5/211)
{ ② Cylindrical hole

(Refer to the solution for Prob. 5/211)

Body	m	$\bar{x}$	$\bar{x}m$
①	$\frac{7}{12}\pi\rho R^2 H$	$\frac{11}{28}H$	$\frac{77}{336}\pi\rho R^2 H^2$
②	$-\pi\rho\left(\frac{R}{2}\right)^2 H$	$\frac{H}{2}$	$-\frac{1}{8}\pi\rho R^2 H^2$

$$\Sigma m = \frac{1}{3}\pi\rho R^2 H \qquad \Sigma \bar{x}m = \frac{35}{336}\pi\rho R^2 H^2$$

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{\frac{35}{336}\pi\rho R^2 H^2}{\frac{1}{3}\pi\rho R^2 H} = \underline{\frac{5}{16}H = \bar{h}}$$

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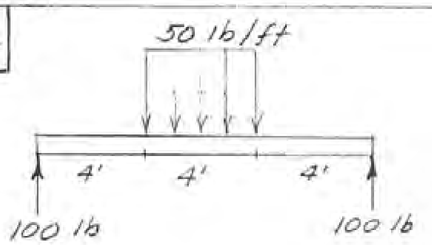

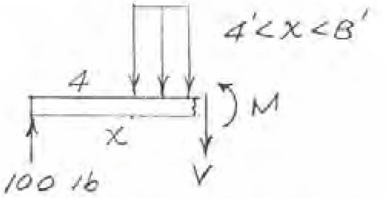

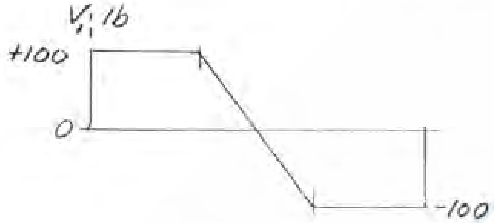

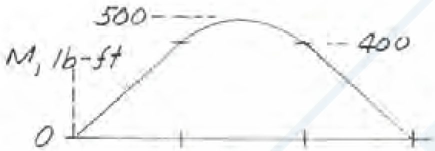

$\Sigma F = 0;\ V + 50(x-4) - 100 = 0$

$V = 50(6-x)$

$\Sigma M_V = 0;\ M + 50\dfrac{(x-4)^2}{2} - 100x = 0$

$M = 25(-x^2 + 12x - 16)$

Set $\dfrac{dM}{dx} = 0$ to get

$M_{max} = 500$ lb-ft

@ $x = 6$ ft

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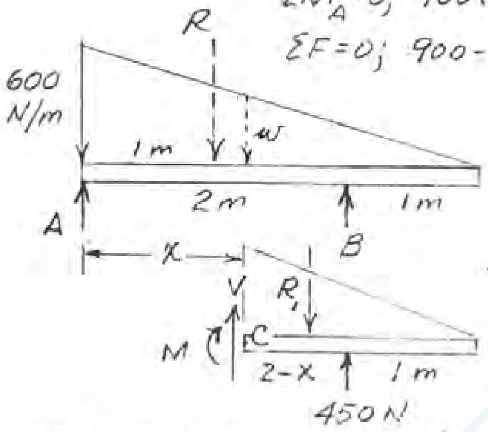
5/215 $R = \frac{600}{2}(3) = 900 \text{ N}$;

$\Sigma M_A = 0$; $900(1) - 2B = 0$, $B = 450 \text{ N}$

$\Sigma F = 0$; $900 - 450 - A = 0$, $A = 450 \text{ N}$



$R_1 = \frac{w}{2}(2 - x + 1) = \frac{3-x}{2} w$

where $w = \frac{3-x}{3} 600$ N/m

So $R_1 = 100(3-x)^2$ N

$\Sigma M_C = 0$;

$450(2-x) = 100(3-x)^2 \frac{3-x}{3} + M$

$M = 450(2-x) - \frac{100}{3}(3-x)^3 \qquad 0 \le x \le 2 \text{ m}$

$\frac{dM}{dx} = -450 + 100(3-x)^2 = 0$ for max or min

$3 - x = \pm\sqrt{4.5}$, $\quad x = 3(1 \pm 1/\sqrt{2}) = \underline{0.879 \text{ m}}$ (or 5.12)

$M_{max} = 450(2 - 0.879) - \frac{100}{3}(3 - 0.879)^3 = \underline{186.4 \text{ N}\cdot\text{m}}$

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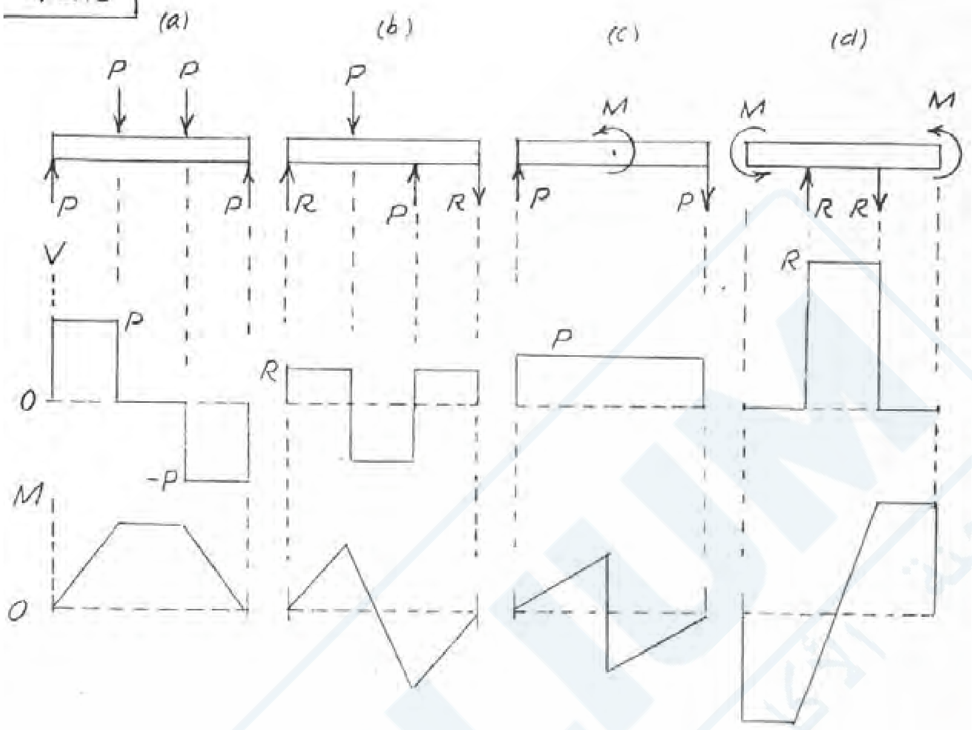

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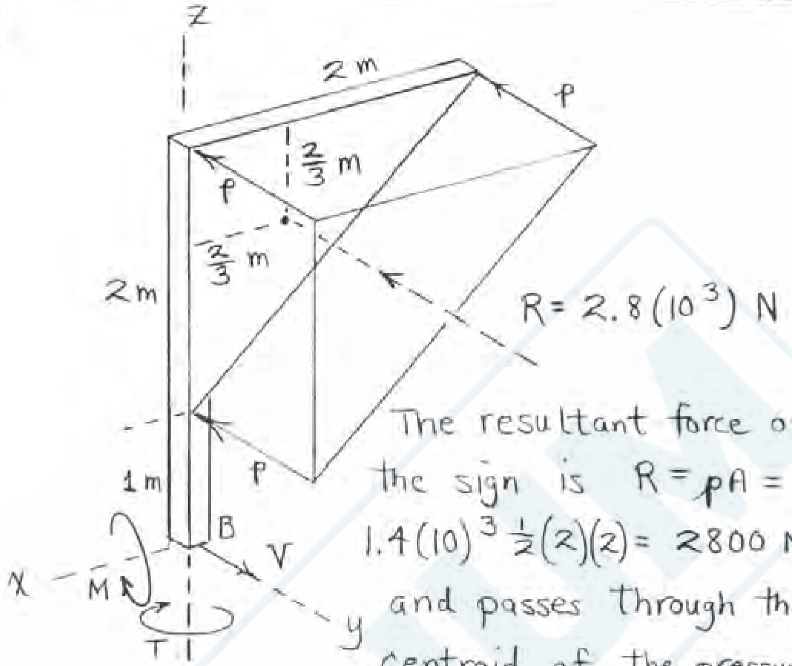

The resultant force on the sign is $R = pA = 1.4(10)^3 \frac{1}{2}(2)(2) = 2800$ N and passes through the centroid of the pressure-area prism.

By inspection,

$$\begin{cases} V = \underline{2.8 \text{ kN}} \\ M = 2.8\left(3 - \frac{2}{3}\right) = \underline{6.53 \text{ kN}\cdot\text{m}} \\ T = 2.8\left(\frac{2}{3}\right) = \underline{1.867 \text{ kN}\cdot\text{m}} \end{cases}$$

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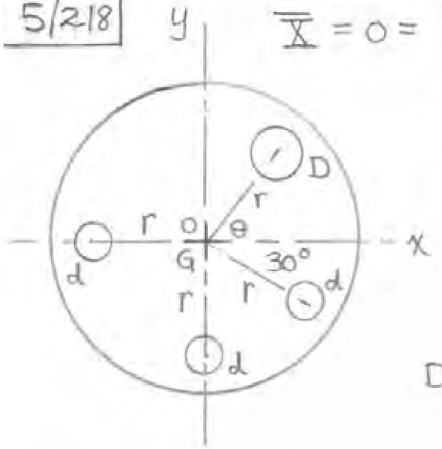

$$\bar{X} = 0 = \frac{\pi D^2}{4} r\cos\theta + \frac{\pi d^2}{4} r\cos 30^\circ - \frac{\pi d^2}{4} r, \quad D^2\cos\theta = d^2\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$\bar{Y} = 0 = \frac{\pi D^2}{4} r\sin\theta - \frac{\pi d^2}{4} r - \frac{\pi d^2}{4} r\sin 30^\circ$$

$$D^2\sin\theta = d^2\left(1 + \frac{1}{2}\right)$$

Divide: $\dfrac{\sin\theta}{\cos\theta} = \dfrac{3/2}{1 - \sqrt{3}/2}$, $\underline{\theta = 84.9^\circ}$

$D^2 = \dfrac{3d^2/2}{\sin 84.9^\circ}$, $\underline{D = 1.227\,d}$

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5/219 Hole: $V = -9h$ (in.^{3}), $\bar{z} = h/2$

Cylinder: $V = \pi 6^2(10) = 1131$ in.^{3}, $\bar{z} = 5$ in.

$$\bar{z} = \frac{\Sigma \bar{z} V}{\Sigma V} = \frac{-9h\left(\frac{h}{2}\right) + 1131(5)}{-9h + 1131}$$

For max. $\bar{z}$, $\frac{d\bar{z}}{dh} = 0$

$$\frac{d\bar{z}}{dh} = \frac{(-9h + 1131)(-9h) - \left(-\frac{9h^2}{2} + 5655\right)(-9)}{(-9h + 1131)^2} = 0$$

$$\Rightarrow 9h^2 - 1131h - 4.5h^2 + 5655 = 0$$

$$4.5h^2 - 1131h + 5655 = 0$$

$$h = \frac{1131 \pm \sqrt{1131^2 - 4(4.5)(5655)}}{9} = \underline{5.10 \text{ in.}} \text{ or } 246 \text{ in.}$$

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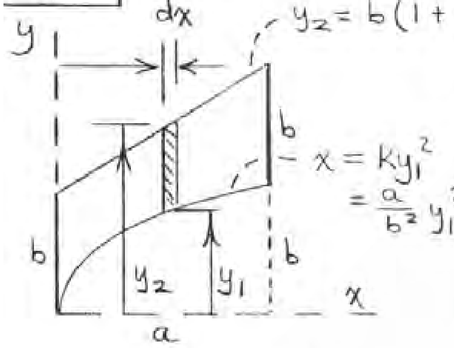

$$A = \int_0^a (y_2 - y_1)\,dx$$

$$= \int_0^a \left[b\left(1+\frac{x}{a}\right) - b\left(\frac{x}{a}\right)^{\frac{1}{2}}\right]dx$$

$$= b\left(x + \frac{x^2}{2a}\right) - \frac{b}{\sqrt{a}}\,\frac{2}{3}x^{3/2}\Big|_0^a$$

$$= \frac{5}{6}ab$$

$$\int y_c\,dA = \int_0^a \left(\frac{y_1+y_2}{2}\right)(y_2-y_1)\,dx = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\,dx$$

$$= \frac{1}{2}\int_0^a \left[b^2\left(1+\frac{x}{a}\right)^2 - \frac{b^2}{a}x\right]dx$$

$$= \frac{1}{2}\left[b^2\left(x + \frac{x^2}{a} + \frac{x^3}{3a^2}\right) - \frac{b^2x^2}{2a}\right]\Big|_0^a = \frac{11}{12}ab^2$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{11ab^2/12}{5ab/6} = \frac{11}{10}b$$

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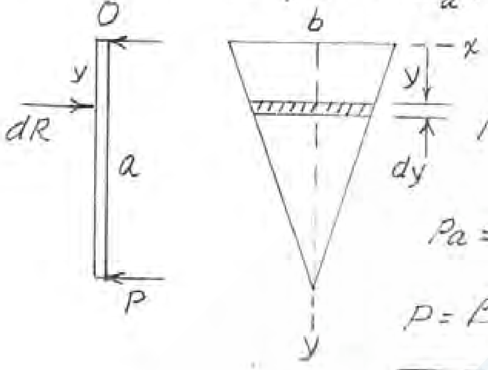
$dA = 2x\,dy = \frac{b}{a}(a-y)\,dy, \quad p = \rho g(h+y)$

$dR = \rho g(h+y)\frac{b}{a}(a-y)\,dy$



$\Sigma M_O = 0; \quad Pa - \int y\,dR = 0$

$Pa = \rho g \frac{b}{a}\int_0^a y(h+y)(a-y)\,dy$

$Pa = \rho g \frac{b}{a}\frac{a^3}{6}\left(h + \frac{a}{2}\right)$

$P = \frac{\rho g a b}{6}\left(h + \frac{a}{2}\right)$

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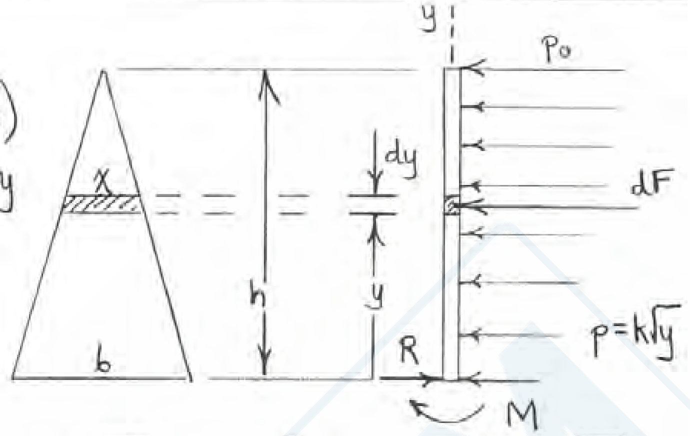
$x = \frac{b}{h}(h-y)$

$dA = \frac{b}{h}(h-y)\,dy$



$p = k\sqrt{y}$: $p_0 = k\sqrt{h}$, $k = \frac{p_0}{\sqrt{h}}$, so $p = p_0\sqrt{\frac{y}{h}}$

$dF = p\,dA = p_0\sqrt{\frac{y}{h}}\,\frac{b}{h}(h-y)\,dy$

$dM = y\,dF = p_0\frac{b}{h\sqrt{h}}\left(y^{3/2}h - y^{5/2}\right)dy$

$M = \int_0^h dM = \frac{p_0 b}{h\sqrt{h}}\left[\frac{y^{5/2}}{5/2}h - \frac{y^{7/2}}{7/2}\right]_0^h = \frac{4}{35}p_0 b h^2$

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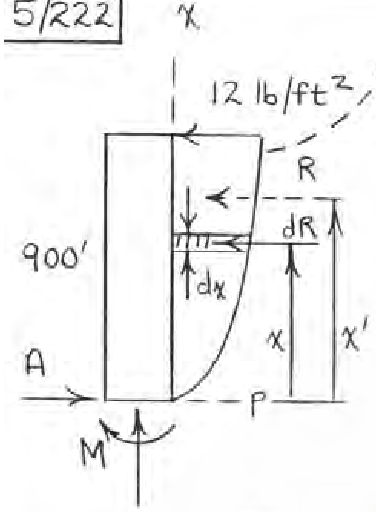

$p = k\sqrt{x}$: $12 = k\sqrt{900}$

$k = 0.4 \frac{\text{lb}}{\text{ft}^{5/2}}$, $p = 0.4\sqrt{x}$

$dR = 200\, p\, dx = 200(0.4\sqrt{x})$

$= 80\sqrt{x}\ dx$

$R = \int dR = \int_0^{900'} 80\sqrt{x}\ dx$

$= 80 \frac{2}{3} x^{3/2} \Big|_0^{900'} = 1.440(10^6)$ lb

$\int x\, dR = \int_0^{900} 80 x^{3/2} dx = 80 \frac{2}{5} x^{5/2} \Big|_0^{900'}$

$= 7.78(10^8)$ lb-ft

Thus, $A = 1.440(10^6)$ lb

$M = 7.78(10^8)$ lb-ft

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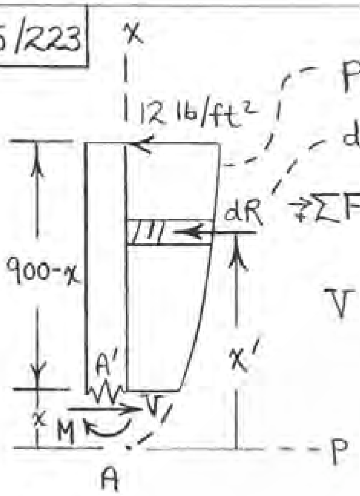

$p = 0.4\sqrt{x}$ (lb/ft²)

$dR = 80\sqrt{x'}\,dx'$

$\rightarrow \Sigma F = 0: \quad V - \int_x^{900'} 80\sqrt{x'}\,dx' = 0$

$$V = 80\,\frac{2}{3}\,x'^{3/2}\Big|_x^{900} =$$

$$\underline{1.440(10^6) - \frac{160}{3}x^{3/2} \text{ (lb)}}$$

$$\underline{V\big|_{x=450'} = 0.931(10^6)\text{ lb}}$$

$$\Sigma M_{A'} = 0: \quad M - \int_x^{900} 80\sqrt{x'}\,(x'-x)\,dx' = 0$$

$$M = 80\left\{\frac{2}{5}x'^{5/2} - x\,\frac{2}{3}x'^{3/2}\right\}\Big|_x^{900}$$

$$= \underline{7.78(10^8) - 1.440(10^6)x + \frac{64}{3}x^{5/2}}$$

$$M\big|_{x=450'} = \underline{2.21(10^8)\text{ lb-ft}}$$

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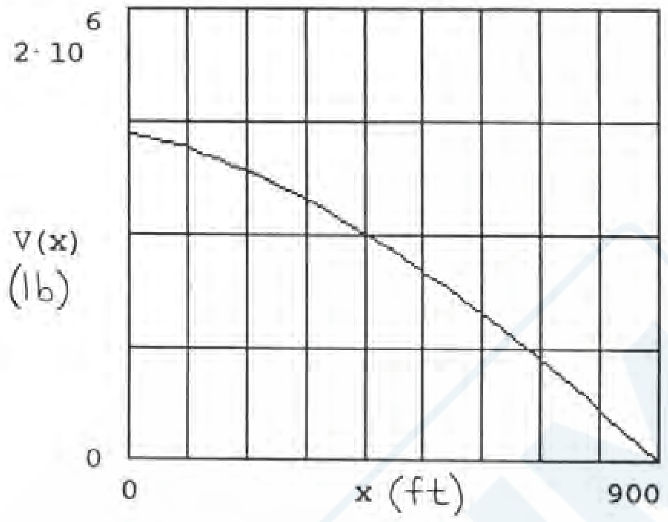

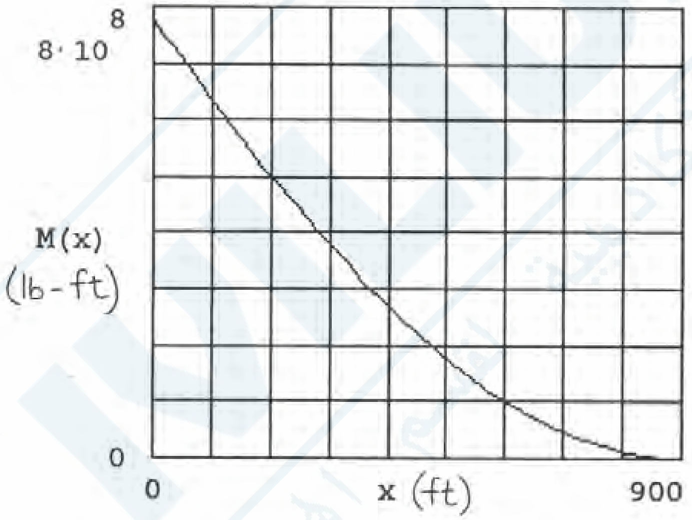

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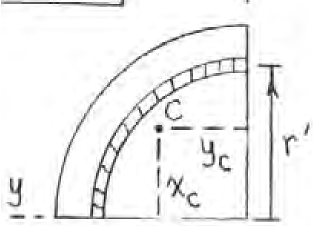


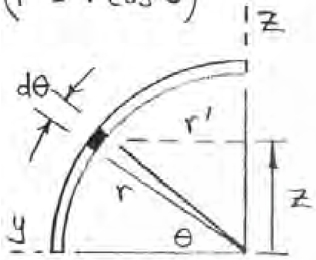
Element is quarter-circular ring parallel to x-y plane with area

$dA = \frac{\pi r'}{2} r\,d\theta = \frac{\pi}{2} r^2 \cos\theta\, d\theta$

$(r' = r\cos\theta)$

Centroid of element is at

$x_c = y_c = \frac{2r'}{\pi} = \frac{2}{\pi} r\cos\theta$ (S.P. 5/1)



$\int x_c\, dA = \int \left(\frac{2}{\pi} r\cos\theta\right)\frac{\pi}{2} r^2 \cos\theta\, d\theta$

$= r^3 \int_0^{\pi/2} \cos^2\theta\, d\theta = \frac{\pi r^3}{4}$

$\int y_c\, dA = \int x_c\, dA = \frac{\pi r^3}{4}$

$\int z_c\, dA = \int (r\sin\theta)\left(\frac{\pi}{2} r^2 \cos\theta\, d\theta\right) = \frac{\pi r^3}{2}\int_0^{\pi/2} \sin\theta\cos\theta\, d\theta$

$= \pi r^3/4$

$A = \int \frac{\pi}{2} r^2 \cos\theta\, d\theta = \frac{\pi r^2}{2}\int_0^{\pi/2} \cos\theta\, d\theta = \pi r^2/2$

From $\bar{x} = \frac{\int x_c\, dA}{A}$, etc., we have

$\bar{x} = \bar{y} = \bar{z} = \frac{\pi r^3/4}{\pi r^2/2} = \underline{r/2}$

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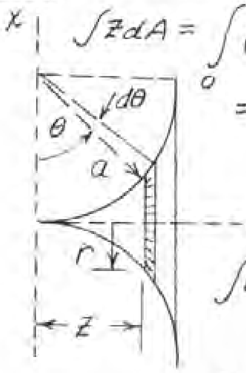
▶ 5/225 $dA = 2\pi r\, a\, d\theta = 2\pi a^2(1-\cos\theta)\, d\theta$



$$\int z\, dA = \int_0^{\pi/2} (a\sin\theta)(2\pi a^2)(1-\cos\theta)\, d\theta$$

$$= 2\pi a^3 \int_0^{\pi/2} (\sin\theta - \sin\theta\cos\theta)\, d\theta$$

$$= 2\pi a^3\left(1-\frac{1}{2}\right) = \pi a^3$$

$$\int dA = 2\pi a^2 \int_0^{\pi/2} (1-\cos\theta)\, d\theta = 2\pi a^2\left(\frac{\pi}{2}-1\right)$$

$$\bar{z} = \frac{\int z\, dA}{A} = \frac{\pi a^3}{2\pi a^2\left(\frac{\pi}{2}-1\right)} = \frac{a}{\pi - 2}$$

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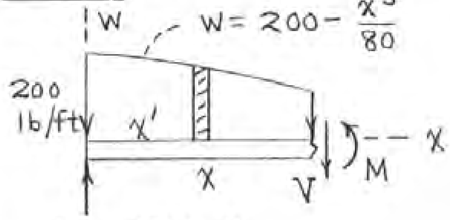
*5/226 (w & R_A from Prob. 5/113)



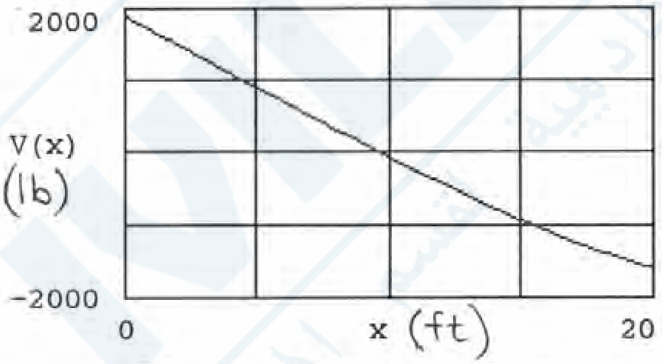
$R_A = 1900$ lb

$+\uparrow \Sigma F = 0: \ 1900 - V - \int_0^x \left(200 - \frac{x'^3}{80}\right) dx'$

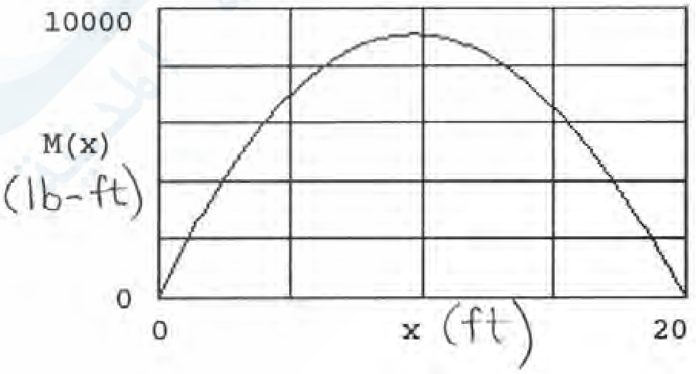
$$\underline{V = 1900 - 200x + \frac{x^4}{320} \quad \text{(lb)}}$$

$\curvearrowleft + \Sigma M = 0: \ M + \int_0^x \left(200 - \frac{x'^3}{80}\right)(x - x')\, dx' - 1900x = 0$

$$\underline{M = 1900x - 100x^2 + \frac{x^5}{1600} \quad \text{(lb-ft)}}$$



$\underline{V_{max} = 1900 \text{ lb @ } x = 0}$



$\underline{M_{max} = 9080 \text{ lb-ft @ } x = 9.63'}$

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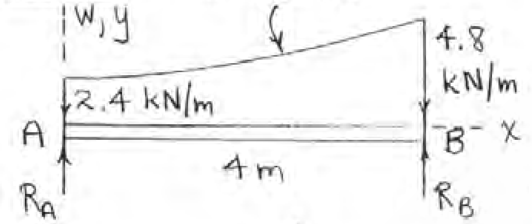


$w_0 = 2.4$ kN/m

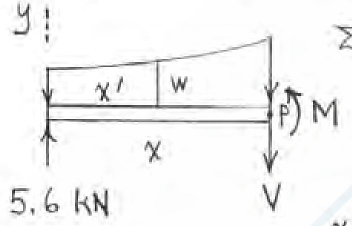
$4.8 = 2.4 + k(4)^2$

$k = 0.15$ kN/m^3

So $w = 2.4 + 0.15x^2$

$\circlearrowleft \Sigma M_A = 0: \quad \int_0^4 (2.4 + 0.15x^2)x\,dx - 4R_B = 0, \quad R_B = 7.2$ kN

$\Sigma F_y = 0: \quad R_A + 7.2 - \int_0^4 (2.4 + 0.15x^2)\,dx = 0, \quad R_A = 5.6$ kN



$\Sigma F_y = 0: \quad 5.6 - \int_0^x (2.4 + 0.15x^2)\,dx - V = 0$

$V = 5.6 - 2.4x - 0.05x^3$

$\circlearrowleft \Sigma M_P = 0: \quad M + \int_0^x (2.4 + 0.15x'^2)(x - x')\,dx' - 5.6x = 0$

$M = 5.6x - \left[2.4xx' + \frac{0.15}{3}x'^3 x - \frac{2.4}{2}x'^2 - \frac{0.15}{4}x'^4\right]_0^x$

$= 5.6x - 1.2x^2 - 0.0125x^4$

For a maximum, $\frac{dM}{dx} = 5.6 - 2.4x - 0.05x^3 = 0$

Solve numerically to obtain

$M_{max} = 6.23$ kN·m @ $x = 2.13$ m

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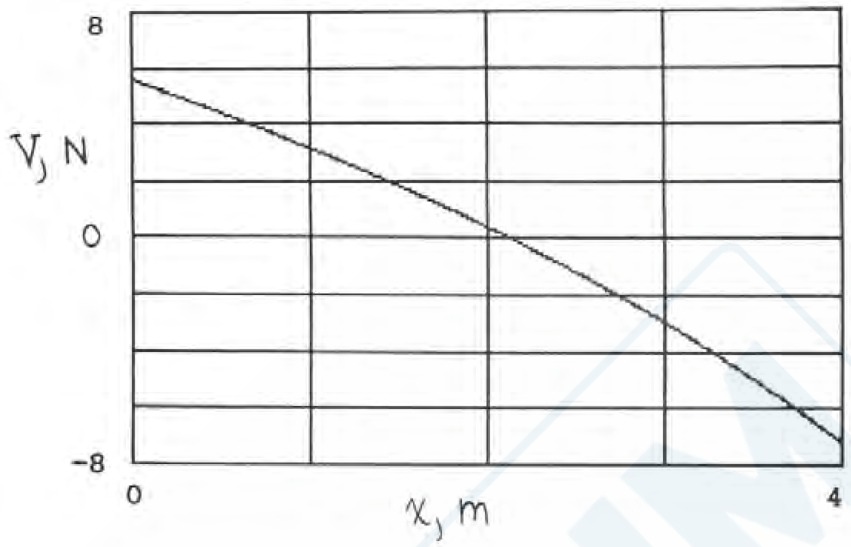

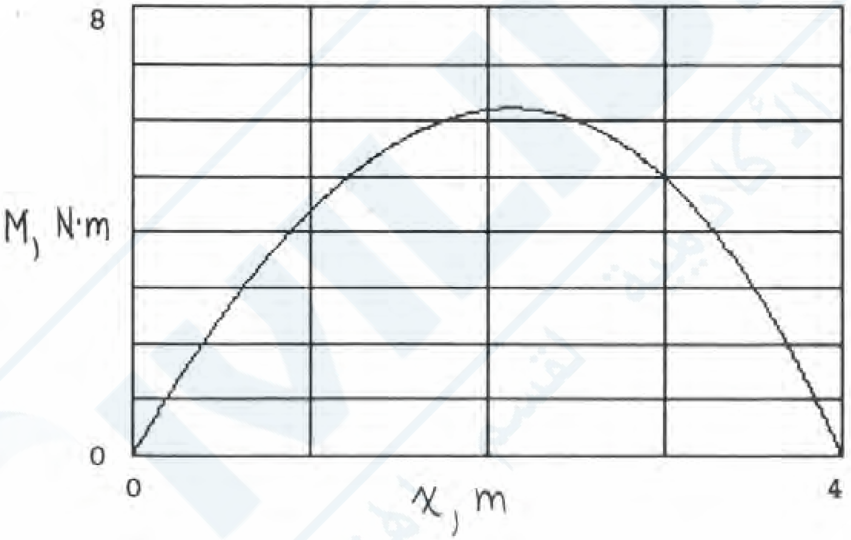

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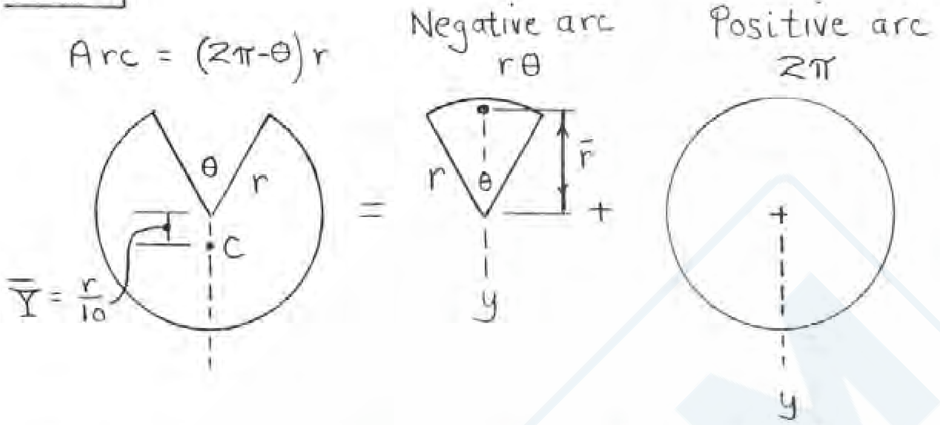
Arc = $(2\pi-\theta)r$



From Sample Problem 5/1, $\bar{r} = \dfrac{r\sin\frac{\theta}{2}}{\theta/2}$

$$\bar{Y} = \frac{\Sigma L\bar{y}}{\Sigma L} : \quad \frac{r}{10} = \frac{(-r\theta)\left(-r\,\frac{\sin\frac{\theta}{2}}{\theta/2}\right) + 2\pi r(0)}{(2\pi-\theta)\,r}$$

Simplify to $10\sin\frac{\theta}{2} = \pi - \frac{\theta}{2}$ and solve numerically to obtain $\underline{\theta = 33.1^\circ}$

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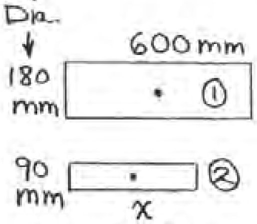


$$V_1 = \frac{\pi\, 180^2}{4}(600) = 15\,270(10^3)\ \text{mm}^3$$

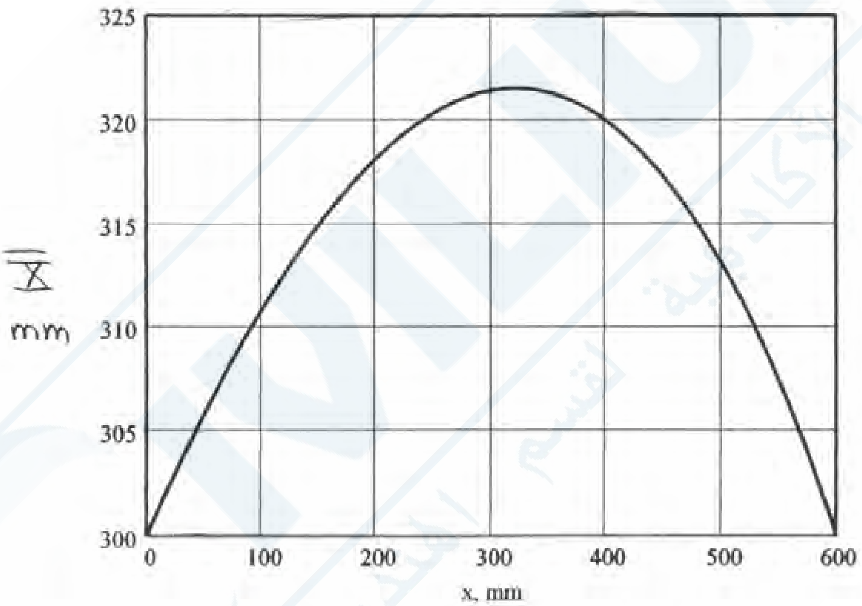
$$\bar{x}_1 = 300\ \text{mm}$$

$$V_2 = -\frac{\pi\, 90^2}{4}x = -6.36(10^3)x$$

$$\bar{x}_2 = -\frac{x}{2}$$

$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{15\,270(10^3)(300) - 6.36(10^3)x\left(\frac{x}{2}\right)}{15\,270(10^3) - 6.36(10^3)x}$$

$$= \frac{4580(10^3) - 3.18x^2}{15\,270 - 6.36x}$$



$$\bar{X}_{max} = 322\ \text{mm} \ @ \ x = 322\ \text{mm}$$

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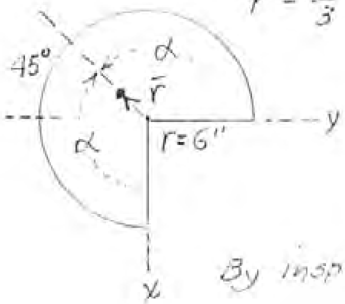
5/23 From Sample Problem 5/3,



$\bar{r} = \frac{2}{3}\frac{r\sin\alpha}{\alpha}$ where $\alpha = 135°$ or $\frac{3}{4}\pi$ rad

$\bar{r} = \frac{2}{3}(6)\frac{\sin 135°}{3\pi/4}$

$= 4\frac{4}{3\pi}\frac{1}{\sqrt{2}} = \frac{16}{3\pi\sqrt{2}}$ in.

$\bar{x} = \bar{y} = -\bar{r}/\sqrt{2} = -\frac{8}{3\pi}$ in.

By inspection $\bar{z} = 5$ in.

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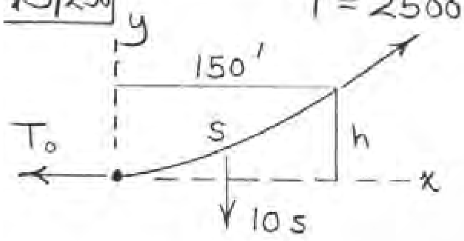


From Eq. 5/21,

$$\frac{T}{T_o} = \cosh \frac{\mu x}{T_o}$$

$$\frac{2500}{T_o} = \cosh \frac{10(150)}{T_o}$$

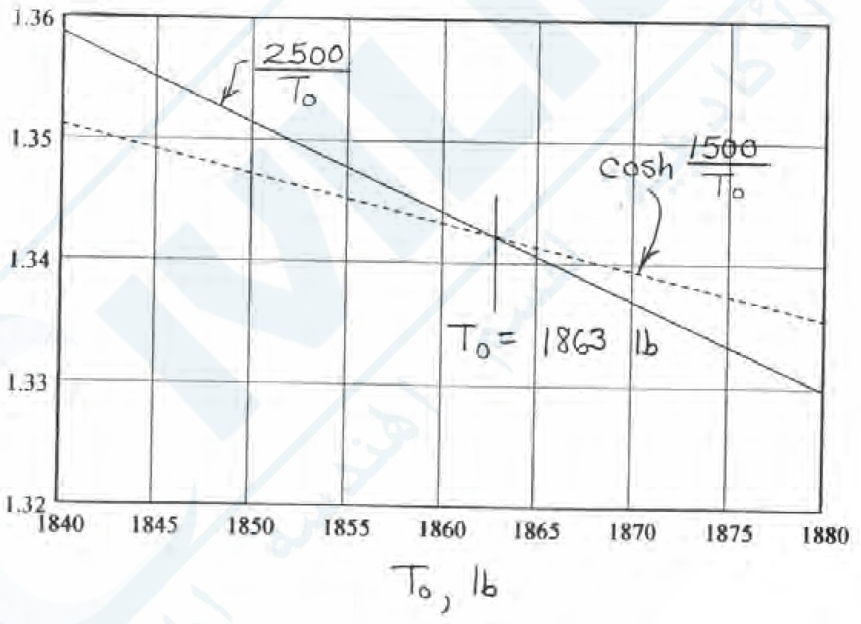
Numerical or graphical (see below) solution:

$T_o = 1863$ lb. Then Eq. 5/22:

$T = T_o + \mu h$: $2500 = 1863 + 10h$, $\underline{h = 63.7 \text{ ft}}$

5/22: $S = 2s = 2\frac{T_o}{\mu} \sinh \frac{\mu x}{T_o}$

$= 2 \frac{1863}{10} \sinh \frac{10(150)}{1863} = \underline{333 \text{ ft}}$



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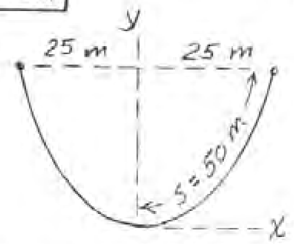

Eq. 5/20 $s = \frac{T_o}{\mu} \sinh \frac{\mu x}{T_o}$

$50 = \frac{T_o}{\mu} \sinh \frac{25\mu}{T_o}$

$\frac{50\mu}{T_o} - \sinh \frac{25\mu}{T_o} = R = 0$

Write and run program for $R = f(\mu/T_o)$ & find μ/T_o for $R = 0$. Result is $\mu/T_o = 0.0871$

From Eq. 5/19, $y = \frac{T_o}{\mu}\left(\cosh \frac{\mu x}{T_o} - 1\right)$

$= \frac{1}{0.0871}\left(\cosh 0.0871[25] - 1\right)$

$h = y = 3.468/0.0871 = \underline{39.8 \text{ m}}$

Result depends only on the geometry of the catenary.

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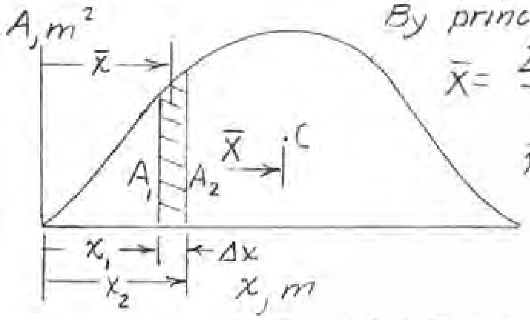
*5/232 $V =$ submerged volume



By principle of moments

$$\bar{X} = \frac{\Sigma \bar{x} \Delta V}{\Delta V}$$

$$\bar{x} = \frac{1}{2}(x_1 + x_2)$$

$$\Delta V = \frac{1}{2}(A_1 + A_2)(x_2 - x_1)$$

Thus $$\bar{X} = \frac{\Sigma \frac{1}{2}(x_1 + x_2)\frac{1}{2}(A_1 + A_2)(x_2 - x_1)}{\Sigma \frac{1}{2}(A_1 + A_2)(x_2 - x_1)}$$

$$= \frac{\Sigma \frac{1}{2}(A_1 + A_2)(x_2^2 - x_1^2)}{\Sigma (A_1 + A_2)(x_2 - x_1)}$$

Write & carry out program for quotient of sums & get $\bar{X} = 24$ m

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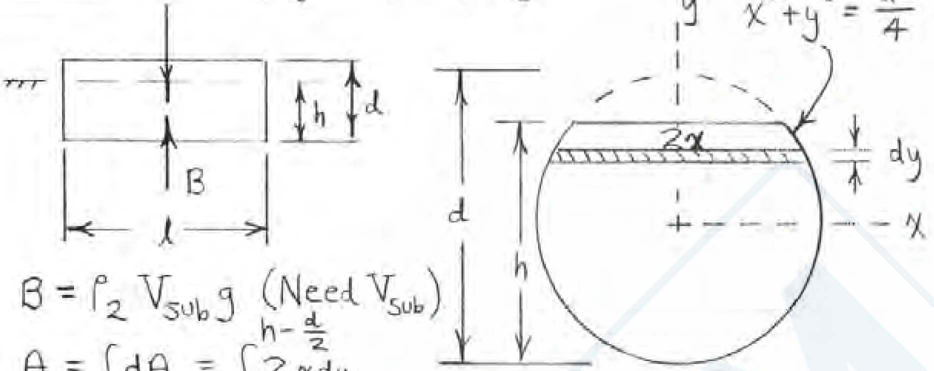
*5/233 $W = \rho_1 V g = \rho_1 \pi \frac{d^2}{4} l g$



$B = \rho_2 V_{sub} g$ (Need V_{sub})

$$A = \int dA = \int_{-d/2}^{h-\frac{d}{2}} 2x\,dy$$

$$= 2\int_{-d/2}^{h-d/2} \sqrt{\frac{d^2}{4}-y^2}\,dy = 2\cdot\frac{1}{2}\left[y\sqrt{\frac{d^2}{4}-y^2} + \frac{d^2}{4}\sin^{-1}\frac{2y}{d}\right]_{-\frac{d}{2}}^{h-\frac{d}{2}}$$

$$= \left(h-\frac{d}{2}\right)\sqrt{hd-h^2} + \frac{d^2}{4}\sin^{-1}\left(\frac{2h-d}{d}\right) + \frac{\pi d^2}{8}$$

So $B = \rho_2 g A l$, A given just above

$\uparrow \Sigma F = 0: \quad \rho_2 g A l - \rho_1 \pi \frac{d^2}{4} l g = 0$

or $\left(h-\frac{d}{2}\right)\sqrt{hd-h^2} + \frac{d^2}{4}\sin^{-1}\left(\frac{2h-d}{d}\right) + \frac{\pi d^2}{8} = \frac{\rho_1}{\rho_2}\pi\frac{d^2}{4}$

Strategy: Set $d=1$ and numerically solve the above equation for h for values of $\frac{\rho_1}{\rho_2}$ between 0 and 1:

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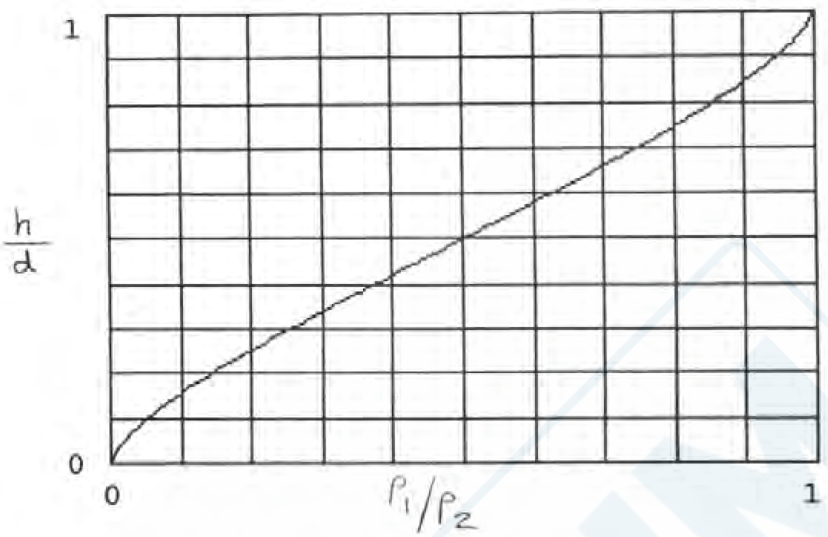

For pine wood and salt water, $\rho_1 = 480\ \frac{kg}{m^3}$ and $\rho_2 = 1030\ \frac{kg}{m^3}$. So $\frac{\rho_1}{\rho_2} = \frac{480}{1030} = 0.466$

Numerical solution : $\frac{h}{d} = \underline{0.473} = r$

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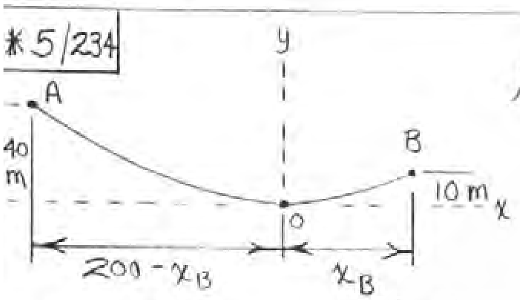

$\mu_{cable} = 20(9.81) = 196.2 \frac{N}{m}$

Eq. 5/17: $y = \frac{T_o}{\mu}\left[\cosh \frac{x}{T_o/\mu} - 1\right]$

At B: $10 = \frac{T_o}{\mu}\left[\cosh \frac{x_B}{T_o/\mu} - 1\right]$

At A: $40 = \frac{T_o}{\mu}\left[\cosh \frac{200 - x_B}{T_o/\mu} - 1\right]$

Simultaneous numerical solution: $x_B = 67.1$ m, $T_o/\mu = 227$ m

$T_A = T_o + \mu y_A$: $75\,000 = 227\mu + \mu(40)$

$\mu = 281$ N/m

$\mu = \mu_{cable} + \mu_{ice}$: $281 = 196.2 + \mu_{ice}$

$\mu_{ice} = 84.7$ N/m

$\rho = \frac{\mu}{g} = \frac{84.7}{9.81} = \underline{8.63 \text{ kg/m}}$

The configuration does not depend on μ.

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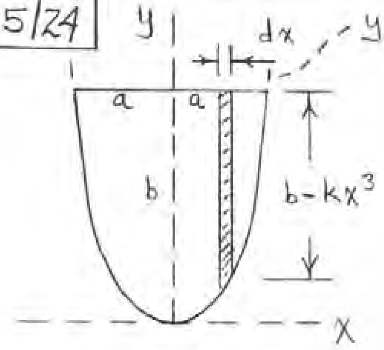

$y = k|x|^3$: $b = ka^3$, $k = b/a^3$

$\therefore \; y = \frac{b}{a^3}|x|^3$

Will treat right half so that absolute value sign may be disregarded.

$$A = 2\int_0^a \left(b - \frac{b}{a^3}x^3\right)dx = 2\left[bx - \frac{b}{4a^3}x^4\right]_0^a = \frac{3}{2}ba$$

$$\int y_c \, dA = 2\int_0^a \frac{b + \frac{b}{a^3}x^3}{2}\left(b - \frac{b}{a^3}x^3\right)dx$$

$$= \int_0^a \left(b^2 - \frac{b^2}{a^6}x^6\right)dx = b^2\left[x - \frac{x^7}{7a^6}\right]_0^a = \frac{6ab^2}{7}$$

$$\bar{y} = \frac{\int y_c \, dA}{A} = \frac{6ab^2/7}{3ab/2} = \underline{\frac{4}{7}b}$$

$\underline{\bar{x} = 0}$, by inspection.

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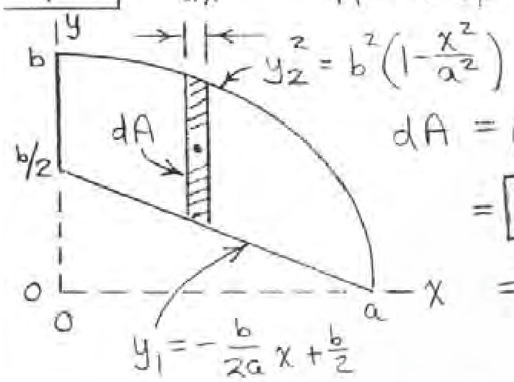
$$A = \pi \frac{ab}{4} - \frac{1}{2} a \frac{b}{2} = \frac{ab}{4}(\pi - 1)$$



$$dA = (y_2 - y_1)\,dx$$

$$= \left[b\sqrt{1 - \frac{x^2}{a^2}} - \left(-\frac{b}{2a}x + \frac{b}{2}\right)\right] dx$$

$$= \frac{b}{a}\left[\sqrt{a^2 - x^2} + \frac{x}{2} - \frac{a}{2}\right] dx$$

$$\int x_c\, dA = \int_0^a x \frac{b}{a}\left[\sqrt{a^2 - x^2} + \frac{x}{2} - \frac{a}{2}\right] dx$$

$$= \frac{b}{a}\left[-\frac{1}{3}\sqrt{(a^2 - x^2)^3} + \frac{x^3}{6} - \frac{ax^2}{4}\right]_0^a = \frac{1}{4} ba^2$$

$$\int y_c\, dA = \int_0^a \left(\frac{y_2 + y_1}{2}\right)(y_2 - y_1)\, dx = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\, dx$$

$$= \frac{1}{2}\int_0^a \left[b^2\left(1 - \frac{x^2}{a^2}\right) - \left(-\frac{b}{2a}x + \frac{b}{2}\right)^2\right] dx$$

$$= \frac{1}{2}\int_0^a \left[b^2\left(1 - \frac{x^2}{a^2}\right) - \frac{b^2}{4a^2}x^2 + \frac{b^2}{2a}x - \frac{b^2}{4}\right] dx$$

$$= \frac{1}{2}\left[b^2\left(x - \frac{x^3}{3a^2}\right) - \frac{b^2}{4a^2}\frac{x^3}{3} + \frac{b^2}{2a}\frac{x^2}{2} - \frac{b^2}{4}x\right]_0^a$$

$$= \frac{7}{24} ab^2$$

$$\bar{x} = \frac{\int x_c\, dA}{A} = \frac{\frac{1}{4}ba^2}{\frac{ab}{4}(\pi - 1)} = \frac{a}{\pi - 1}$$

$$\bar{y} = \frac{\int y_c\, dA}{A} = \frac{\frac{7}{24}ab^2}{\frac{ab}{4}(\pi - 1)} = \frac{7b}{6(\pi - 1)}$$

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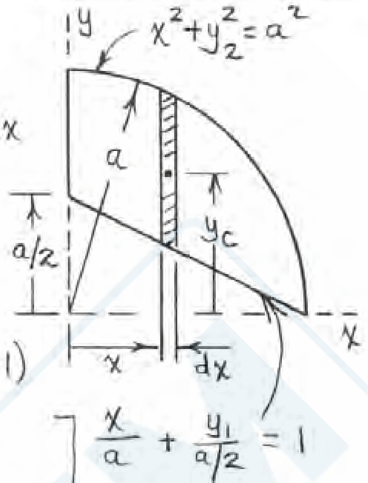
$\bar{x} = \dfrac{\int x\,dA}{\int dA}$

$dA = (y_2 - y_1)\,dx = \left(\sqrt{a^2 - x^2} - \dfrac{a-x}{2}\right)dx$

$A = \int_0^a \left(\sqrt{a^2-x^2} - \dfrac{a-x}{2}\right)dx$

$= \left[\frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2 \sin^{-1}\frac{x}{a}\right) - \frac{1}{2}\left(ax - \frac{x^2}{2}\right)\right]_0^a = \dfrac{a^2}{4}(\pi - 1)$



[Or, by inspection,
$A = \frac{1}{4}\pi a^2 - \frac{1}{2} a \frac{a}{2} = \frac{a^2}{4}(\pi - 1)$]

$\int x\,dA = \int_0^a \left[x\sqrt{a^2-x^2} - \frac{1}{2}(ax - x^2)\right]dx$

$= \left[-\frac{1}{3}\sqrt{(a^2-x^2)^3} - \frac{1}{2}\left(\frac{ax^2}{2} - \frac{x^3}{3}\right)\right]_0^a = \dfrac{a^3}{4}$

So $\bar{x} = \dfrac{a^3/4}{\frac{a^2}{4}(\pi - 1)} = \underline{\dfrac{a}{\pi - 1}}$

Now, $\bar{y} = \dfrac{\int y_c\,dA}{A}$, where $y_c = \dfrac{y_1 + y_2}{2}$

$\int y_c\,dA = \int \frac{y_1+y_2}{2}(y_2 - y_1)\,dx = \frac{1}{2}\int_0^a (y_2^2 - y_1^2)\,dx$

$= \frac{1}{2}\int_0^a \left(a^2 - x^2 - \frac{a^2 - 2ax + x^2}{4}\right)dx = \frac{1}{8}\int_0^a (3a^2 + 2ax - 5x^2)\,dx$

$= \frac{7}{24}a^3$. Then $\bar{y} = \dfrac{7a^3/24}{\frac{a^2}{4}(\pi - 1)} = \underline{\dfrac{7a}{6(\pi - 1)}}$

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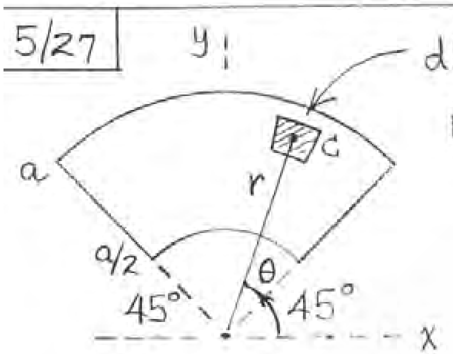

$dA = r\,dr\,d\theta$

$A = \frac{\pi a^2}{4} - \frac{\pi}{4}\left(\frac{a}{2}\right)^2 = \frac{3}{16}\pi a^2$

$$\int y_c\,dA = \iint (r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_{\pi/4}^{3\pi/4}\int_{a/2}^{a} \sin\theta\, r^2\,dr\,d\theta = \int_{\pi/4}^{3\pi/4} \sin\theta\,\frac{7}{24}a^3\,d\theta$$

$$= \frac{7}{24}a^3\left(-\cos\theta\right)\Big|_{\pi/4}^{3\pi/4} = \frac{7\sqrt{2}}{24}a^3$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{7\sqrt{2}}{24}a^3}{\frac{3}{16}\pi a^2} = \frac{14\sqrt{2}}{9\pi}a$$

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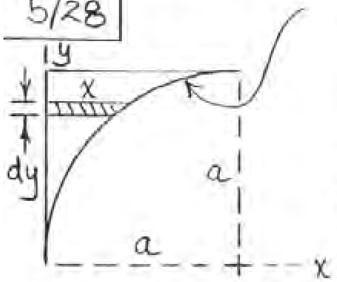

$$(x-a)^2 + y^2 = a^2$$

$$dA = x\,dy = \left(a - \sqrt{a^2-y^2}\right)dy$$

$$\int x_c\,dA = \int \frac{x}{2}\,x\,dy$$

$$= \frac{1}{2}\int_0^a \left(a - \sqrt{a^2-y^2}\right)^2 dy$$

$$\int x_c\,dA = \int_0^a \left(a^2 - a\sqrt{a^2-y^2} - \frac{y^2}{2}\right)dy$$

$$= \left[a^2 y - \frac{a}{2}\left(y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a}\right) - \frac{y^3}{6}\right]_0^a$$

$$= \left(\frac{5}{6} - \frac{\pi}{4}\right)a^3$$

$$\int y_c\,dA = \int y x\,dy = \int_0^a \left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$= \left[\frac{ay^2}{2} + \frac{1}{3}\sqrt{(a^2-y^2)^3}\right]_0^a = \frac{a^3}{6}$$

$$A = a^2 - \frac{1}{4}\pi a^2 = a^2\left(1 - \frac{\pi}{4}\right)$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\left(\frac{5}{6} - \frac{\pi}{4}\right)a^3}{\left(1 - \frac{\pi}{4}\right)a^2} = \frac{10-3\pi}{3(4-\pi)}\,a = \underline{0.223a}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{a^3/6}{\left(1 - \frac{\pi}{4}\right)a^2} = \frac{2a}{3(4-\pi)} = \underline{0.777a}$$

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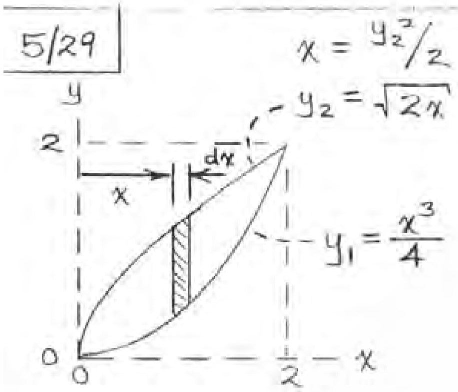

$$A = \int dA = \int_0^2 (y_2 - y_1)\,dx = \int_0^2 \left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$

$$= \left(\frac{2\sqrt{2}}{3} x^{3/2} - \frac{x^4}{16}\right)_0^2 = 5/3$$

$$\int x_c\,dA = \int_0^2 x\left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$

$$= \left(\frac{2\sqrt{2}}{5} x^{5/2} - \frac{x^5}{20}\right)\Big|_0^2 = 8/5$$

$$\int y_c\,dA = \int_0^2 \left(\frac{y_1+y_2}{2}\right)(y_2 - y_1)\,dx = \int_0^2 \frac{1}{2}(y_2^2 - y_1^2)\,dx$$

$$= \frac{1}{2}\int_0^2 \left(2x - \frac{x^6}{16}\right) dx = \frac{1}{2}\left[x^2 - \frac{x^7}{7(16)}\right]_0^2 = 10/7$$

$$\bar{x} = \int x_c\,dA / A = \frac{8/5}{5/3} = \underline{\frac{24}{25}}$$

$$\bar{y} = \int y_c\,dA / A = \frac{10/7}{5/3} = \underline{\frac{6}{7}}$$

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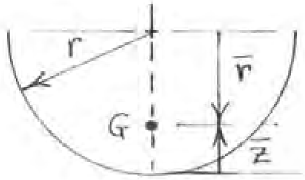

$\bar{r} = \frac{2r}{\pi} = \frac{2(120)}{\pi} = 76.4$ mm

$\bar{z} = r - \bar{r} = 120 - 76.4 = \underline{43.6 \text{ mm}}$

$\underline{\bar{x} = -120 \text{ mm}}$, by inspection

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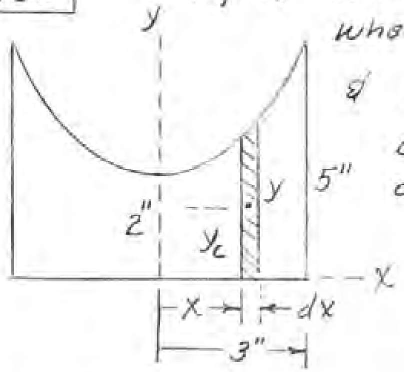

Eq. of curve: $y = 2 + kx^2$

when $x = 3''$, $y = 5''$ so $k = 1/3$

& $y = 2 + x^2/3$

By symmetry use only half of area.

$$A = \int_0^3 y\,dx = \int_0^3 \left(2 + \frac{x^2}{3}\right) dx$$

$$= \left[2x + \frac{x^3}{9}\right]_0^3 = 9 \text{ in.}^2$$

$$\bar{y}A = \int y_c\,dA = \int \frac{y}{2}\,dA = \frac{1}{2}\int_0^3 y^2\,dx = \frac{1}{2}\int_0^3 \left(4 + \frac{4}{3}x^2 + \frac{x^4}{9}\right) dx$$

$$= \frac{1}{2}\left[4x + \frac{4x^3}{9} + \frac{x^5}{45}\right]_0^3 = \frac{1}{2}\left[12 + 12 + \frac{27}{5}\right] = 14.70 \text{ in.}^3$$

Thus $\bar{h} = \bar{y} = \dfrac{14.70}{9} = \underline{1.633 \text{ in.}}$

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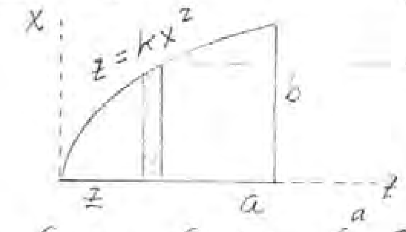
5/31 $a = kb^2$ so $z = \frac{a}{b^2}x^2$ $\quad dV = \frac{\pi x^2}{2}dz = \frac{\pi}{2}\frac{b^2 z}{a}dz$



$$V = \frac{\pi b^2}{2a}\int_0^a z\,dz = \frac{\pi b^2}{2a}\frac{a^2}{2} = \frac{\pi}{4}ab^2$$

$$\int z_c\,dV = \int z\,dV = \int_0^a z\,\frac{\pi}{2}\frac{b^2 z}{a}dz$$

$$= \frac{\pi b^2}{2a}\int_0^a z^2\,dz = \frac{\pi b^2}{2a}\frac{a^3}{3} = \frac{\pi b^2 a^2}{6}$$

$$\bar{z} = \int z_c\,dV / V = \frac{\pi b^2 a^2}{6} \Big/ \frac{\pi}{4}ab^2 = \underline{2a/3}$$

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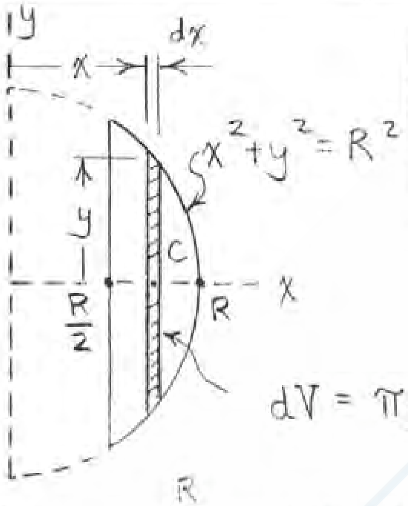

(Note: Shaded element is a circular slice viewed edge-on.

$$dV = \pi y^2 dx = \pi(R^2 - x^2)\,dx$$

$$V = \int dV = \int_{R/2}^{R} \pi(R^2 - x^2)\,dx = \pi\left[R^2 x - \frac{x^3}{3}\right]_{R/2}^{R}$$

$$= \pi\left[R^3 - \frac{R^3}{3} - \left(\frac{R^3}{2} - \frac{R^3}{24}\right)\right] = \frac{5}{24}\pi R^3$$

$$\int x_c\,dV = \int_{R/2}^{R} x\,\pi(R^2 - x^2)\,dx = \pi\left[\frac{R^2 x^2}{2} - \frac{x^4}{4}\right]_{R/2}^{R}$$

$$= \pi\left[\frac{R^4}{2} - \frac{R^4}{4} - \left(\frac{R^4}{8} - \frac{R^4}{64}\right)\right] = \frac{9}{64}\pi R^4$$

$$\bar{x} = \frac{\int x_c\,dV}{\int dV} = \frac{\frac{9}{64}\pi R^4}{\frac{5}{24}\pi R^3} = \underline{\frac{27}{40}R}$$

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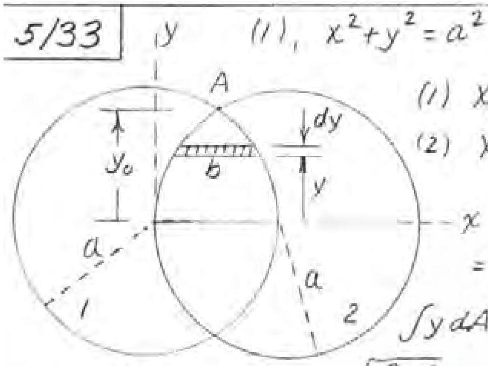

(1), $x^2+y^2=a^2$; (2), $(x-a)^2+y^2=a^2$

(1) $x_1 = +\sqrt{a^2-y^2}$

(2) $x_2 = a - \sqrt{a^2-y^2}$ (Note sign)

$dA = b\,dy = (x_1 - x_2)\,dy$

$= (2\sqrt{a^2-y^2} - a)\,dy$

$\int y\,dA = \int_0^{y_0} (2y\sqrt{a^2-y^2} - ay)\,dy$

For pt A, $x_1 = x_2$ or $\sqrt{a^2-y^2} = a - \sqrt{a^2-y^2}$, $2\sqrt{a^2-y^2} = a$

$y = y_0 = \sqrt{3}a/2$

so $\int y\,dA = -\frac{2}{3}\sqrt{(a^2-y^2)^3} - \frac{ay^2}{2}\Big]_0^{y_0} = 5a^3/24 = 0.208a^3$

$\int dA = \int_0^{y_0} (2\sqrt{a^2-y^2} - a)\,dy = \left(y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a} - ay\right)_0^{y_0}$

$= a^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) = 0.614a^2$

$\bar{y} = \int y\,dA \Big/ \int dA = \frac{0.208a^3}{0.614a^2} = \underline{0.339a}$

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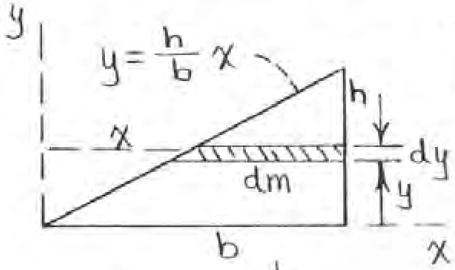
$dm = \rho dV = \rho dA\, t = t\rho (b-x)dy$

$= [t_o(\frac{y}{h}+1)]\rho(b-x)dy$

$= t_o\rho(\frac{y}{h}+1)(b-\frac{b}{h}y)dy$

$= t_o\rho b(1-\frac{y^2}{h^2})dy$



$m = \int dm = \int_0^h t_o\rho b(1-\frac{y^2}{h^2})dy = t_o\rho b\left[y-\frac{y^3}{3h^2}\right]_0^h$

$= \frac{2}{3}\rho t_o bh$

$\int y_c\, dm = \int t_o\rho b(1-\frac{y^2}{h^2})y\,dy = t_o\rho b\left[\frac{y^2}{2}-\frac{y^4}{4h^2}\right]_0^h$

$= \frac{1}{4}\rho t_o bh^2$

$\bar{y} = \frac{\int y_c\, dm}{m} = \frac{\frac{1}{4}\rho t_o bh^2}{\frac{2}{3}\rho t_o bh} = \underline{\frac{3}{8}h}$

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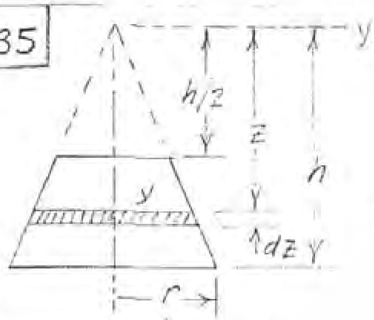

$dV = \pi y^2 dz$ where $y = \frac{r}{h} z$

$= \pi \frac{r^2}{h^2} z^2 dz$

$V = \pi \frac{r^2}{h^2} \int_{h/2}^{h} z^2 dz = \frac{7\pi r^2 h}{24}$

$\int z_c dV = \int_{h/2}^{h} z \pi \frac{r^2}{h^2} z^2 dz = \frac{15}{64} \pi r^2 h^2$

$\bar{z} = \int z_c dV / V = \frac{15}{64} \pi r^2 h^2 \Big/ \frac{7}{24} \pi r^2 h = \frac{45}{56} h$

$\bar{h} = h - \bar{z} = \underline{\frac{11}{56} h}$

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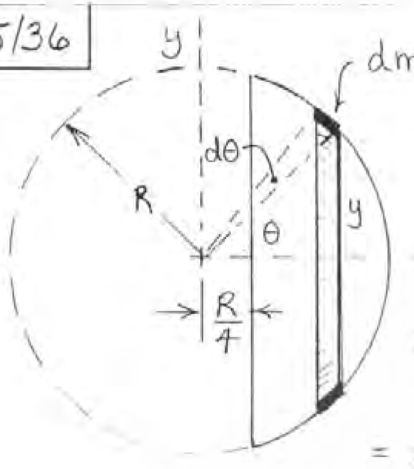

$dm = \rho\, dA = \rho (R\, d\theta)(2\pi y)$

$= 2\pi \rho R (R \sin\theta)\, d\theta$

$= 2\pi \rho R^2 \sin\theta\, d\theta$

$m = \int dm$

$m = \int_0^{\cos^{-1}\frac{1}{4}} 2\pi \rho R^2 \sin\theta\, d\theta$

$= 2\pi \rho R^2 (-\cos\theta)\Big|_0^{\cos^{-1}\frac{1}{4}}$

$= 1.5\pi \rho R^2$

$\int x_c\, dm = \int (R\cos\theta)(2\pi \rho R^2 \sin\theta\, d\theta)$

$= 2\pi \rho R^3 \int \cos\theta \sin\theta\, d\theta = \pi \rho R^3 \int \sin 2\theta\, d\theta$

$= \pi \rho R^3 \left(-\frac{1}{2}\cos 2\theta\right)\Big|_0^{\cos^{-1}\frac{1}{4}} = \frac{15}{16}\pi \rho R^3$

$\bar{x} = \dfrac{\int x_c\, dm}{\int dm} = \dfrac{\frac{15}{16}\pi \rho R^3}{1.5\pi \rho R^2} = \underline{\frac{5}{8} R}$

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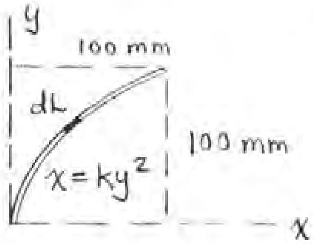
$x = ky^2 = y^2/100, \quad \frac{dx}{dy} = \frac{y}{50}$



$$L = \int dL = \int_0^{100} \sqrt{1 + (dx/dy)^2}\, dy = \int_0^{100} \sqrt{1 + \frac{y^2}{50^2}}\, dy$$

$$= \frac{1}{50} \int_0^{100} \sqrt{50^2 + y^2}\, dy = \frac{1}{50 \cdot 2}\left[y\sqrt{50^2 + y^2} + 50^2 \ln\left(y + \sqrt{50^2 + y^2}\right)\right]_0^{100}$$

$$= 147.9 \text{ mm}$$

$$\int y_c\, dL = \frac{1}{50}\int_0^{100} y\sqrt{50^2 + y^2}\, dy = \frac{1}{50}\frac{1}{3}\left(50^2 + y^2\right)^{3/2}\Big|_0^{100}$$

$$= 8480 \text{ mm}^2$$

$$\bar{y} = \frac{\int y_c\, dL}{L} = \frac{8480}{147.9} = \underline{57.4 \text{ mm}}$$

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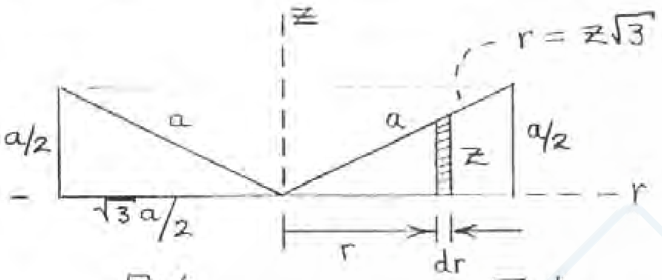

$$V = \int dV = \int_0^{\sqrt{3}a/2} 2\pi r z\, dr = \frac{2\pi}{\sqrt{3}}\int_0^{\sqrt{3}a/2} r^2\, dr$$

$$= \frac{2\pi}{\sqrt{3}}\left(\sqrt{3}a/2\right)^3/3 = \frac{\pi a^3}{4}$$

$$\int z_c\, dV = \int_0^{\sqrt{3}a/2} \frac{r}{2\sqrt{3}}\,\frac{2\pi}{\sqrt{3}}\, r^2\, dr = \frac{\pi}{3}\left[\frac{r^4}{4}\right]_0^{\sqrt{3}a/2} = \frac{3\pi a^4}{64}$$

$$\bar{z} = \frac{\int z_c\, dV}{V} = \frac{3\pi a^4/64}{\pi a^3/4} = \underline{\frac{3a}{16}}$$

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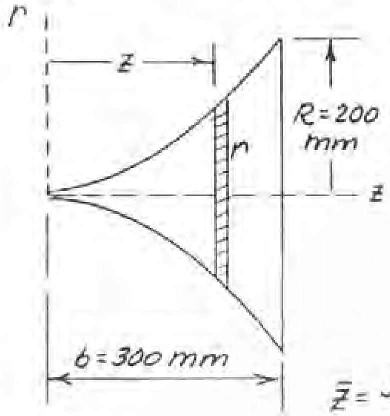
5/39 $r = kz^3$, $R = kb^3$, so $r = \frac{R}{b^3} z^3$



$dV = \pi r^2 dz = \pi \frac{R^2}{b^6} z^6 dz$

$V = \frac{\pi R^2}{b^6} \int_0^b z^6 dz = \frac{\pi}{7} R^2 b$

$\int z_c dV = \int z dV = \frac{\pi R^2}{b^6} \int_0^b z^7 dz$

$= \frac{\pi R^2}{8} b^2$

$\bar{z} = \int z_c dV / V = \frac{\pi R^2 b^2}{8} \Big/ \frac{\pi}{7} R^2 b$

$= 7b/8 = \frac{7(300)}{8} = \underline{263 \text{ mm}}$

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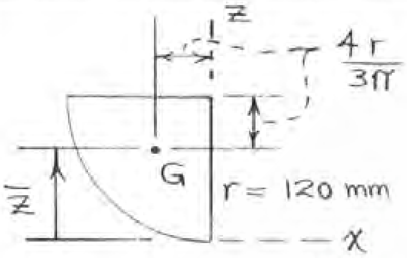

$$\bar{x} = -\frac{4r}{3\pi} = -\frac{4(120)}{3\pi} = \underline{-50.9 \text{ mm}}$$

$$\bar{y} = \underline{120 \text{ mm}}, \text{ by inspection}$$

$$\bar{z} = r - \frac{4r}{3\pi} = 120 - 50.9 = \underline{69.1 \text{ mm}}$$

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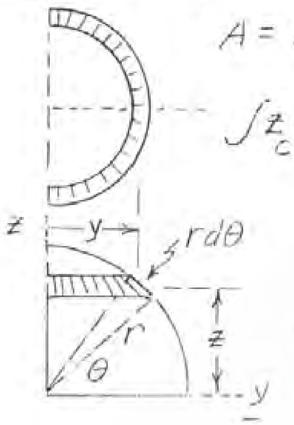

$dA = \pi y \, r d\theta = \pi r^2 \cos\theta \, d\theta$

$A = \pi r^2 \int_0^{\pi/2} \cos\theta \, d\theta = \pi r^2 \sin\theta \Big]_0^{\pi/2} = \pi r^2$

$\int z_c \, dA = \int z \, dA = \int_0^{\pi/2} r\sin\theta \, \pi r^2 \cos\theta \, d\theta$

$= \pi r^3 \int_0^{\pi/2} \frac{\sin 2\theta}{2} d\theta = \frac{\pi r^3}{2}$

$\bar{z} = \int z_c \, dA / A = \frac{\pi r^3}{2} \Big/ \pi r^2 = \underline{\frac{r}{2}}$

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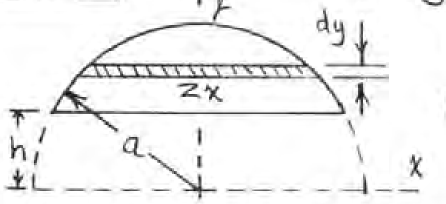

$x^2+y^2=a^2$, $x=+\sqrt{a^2-y^2}$

$dA = 2x\,dy = 2\sqrt{a^2-y^2}\,dy$

$A=\int dA = \int_h^a 2\sqrt{a^2-y^2}\,dy$

$= 2\left(\frac{1}{2}\right)\left[y\sqrt{a^2-y^2}+a^2\sin^{-1}\frac{y}{a}\right]_h^a$

$= a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{h}{a}\right)-h\sqrt{a^2-h^2}$

$\int y\,dA = \int_h^a y\,2\sqrt{a^2-y^2}\,dy = 2\left(-\frac{1}{3}\right)\left(a^2-y^2\right)^{3/2}\Big|_h^a$

$= \frac{2}{3}\left(a^2-h^2\right)^{3/2}$

$\bar{y} = \frac{\int y\,dA}{\int dA} = \frac{\frac{2}{3}\left(a^2-h^2\right)^{3/2}}{a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{h}{a}\right)-h\sqrt{a^2-h^2}}$

Special cases

$h=0$: $\bar{y} = \frac{\frac{2}{3}a^3}{a^2\frac{\pi}{2}} = \frac{4a}{3\pi}$ (the correct result)

$h=\frac{a}{4}$: $\bar{y} = \frac{\frac{2}{3}\left(a^2-\left(\frac{a}{4}\right)^2\right)^{3/2}}{a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{4}\right)-\frac{a}{4}\sqrt{a^2-\left(\frac{a}{4}\right)^2}} = \underline{0.562a}$

$h=\frac{a}{2}$: $\bar{y} = \frac{\frac{2}{3}\left(a^2-\left(\frac{a}{2}\right)^2\right)^{3/2}}{a^2\left(\frac{\pi}{2}-\sin^{-1}\frac{1}{2}\right)-\frac{a}{2}\sqrt{a^2-\left(\frac{a}{2}\right)^2}} = \underline{0.705a}$

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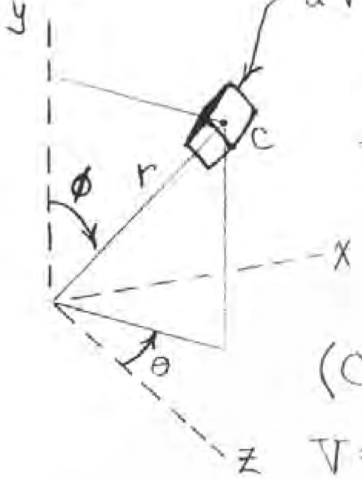

$dV = dr\,(r\,d\phi)(r\sin\phi\,d\theta)$

$= r^2 \sin\phi\, d\phi\, d\theta\, dr$

$V = \int dV = \int_{R/2}^{R}\int_0^{\pi}\int_0^{\pi} r^2 \sin\phi\, d\phi\, d\theta\, dr$

$= \frac{7}{12}\pi R^3$

(Check V by

$V = \frac{\frac{4}{3}\pi R^3}{2} - \frac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{2} = \frac{7}{12}\pi R^3$)

$\int x_c\, dV = \int (r\sin\phi\sin\theta)(r^2\sin\phi\, d\phi\, d\theta\, dr)$

$= \int_{R/2}^{R}\int_0^{\pi}\int_0^{\pi} r^3 \sin^2\phi \sin\theta\, d\phi\, d\theta\, dr = \frac{15}{64}\pi R^4$

$\bar{x} = \frac{\int x_c\, dV}{V} = \frac{\frac{15}{64}\pi R^4}{\frac{7}{12}\pi R^3} = \underline{\frac{45}{112} R}$

(Compare to $\bar{x} = \frac{3}{8} R$ for no hole.)

Note: A hemispherical shell of radius r and thickness dr would be a better element.

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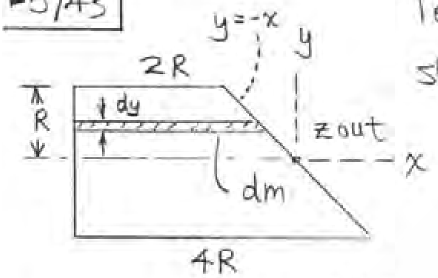

Temporarily use coordinates shown. Element is a thin rectangular plate.

$$dm = \rho\, dV = \rho(3R+x)(2z)\,dy$$
$$= 2\rho[3R-y][\sqrt{R^2-y^2}]\,dy$$
$$= 6\rho R\sqrt{R^2-y^2}\,dy - 2\rho y\sqrt{R^2-y^2}\,dy$$

$$m = \int dm = \int_{-R}^{R} 6\rho R\sqrt{R^2-y^2}\,dy - \int_{-R}^{R} 2\rho y\sqrt{R^2-y^2}\,dy$$

$$m = 6\rho R\,\tfrac{1}{2}\left(y\sqrt{R^2-y^2} + R^2\sin^{-1}\frac{y}{R}\right)\Big|_{-R}^{R} - 2\rho\left(-\tfrac{1}{3}\sqrt{(R^2-y^2)^3}\right)\Big|_{-R}^{R}$$
$$= 3\pi\rho R^3$$

$$\int x_c\, dm = \int_{-R}^{R}\left(\frac{x-3R}{2}\right)\left(6\rho R\sqrt{R^2-y^2}\,dy - 2\rho y\sqrt{R^2-y^2}\,dy\right)$$

$$I_1 = -3\rho R\int_{-R}^{R} y\sqrt{R^2-y^2}\,dy = -3\rho R\left(-\tfrac{1}{3}\sqrt{(R^2-y^2)^3}\right)\Big|_{-R}^{R}$$
$$= 0$$

$$I_2 = -9\rho R^2\int_{-R}^{R}\sqrt{R^2-y^2}\,dy$$
$$= -9\rho R^2\cdot\tfrac{1}{2}\left(y\sqrt{R^2-y^2} + R^2\sin^{-1}\left(\frac{y}{R}\right)\right)\Big|_{-R}^{R}$$
$$= -\tfrac{9}{2}\rho R^2\left(R^2\tfrac{\pi}{2} - \left(-R^2\tfrac{\pi}{2}\right)\right) = -\tfrac{9}{2}\pi\rho R^4$$

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$$I_3 = \rho\int_{-R}^{R} y^2\sqrt{R^2-y^2}\,dy$$

$$= \rho\left(-\frac{y}{4}\sqrt{(R^2-y^2)^3} + \frac{R^2}{8}\left(y\sqrt{R^2-y^2} + R^2\sin^{-1}\frac{y}{R}\right)\right)\Bigg|_{-R}^{R}$$

$$= \rho\left(\frac{R^2}{8}\cdot\left(R^2\left(\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right)\right)\right) = \frac{1}{8}\pi\rho R^4$$

$$I_4 = -3\rho R\int_{-R}^{R} y\sqrt{(R^2-y^2)}\,dy = 0$$

So total is $\int x_c\,dm = \left(-\frac{9}{2}+\frac{1}{8}\right)\pi\rho R^4$

$$= -\frac{35}{8}\pi\rho R^4$$

Then $\bar{x} = \dfrac{\int x_c\,dm}{\int dm}$

$$= \frac{-\frac{35}{8}\pi\rho R^4}{3\pi\rho R^3} = -\frac{35}{24}R$$

Relative to left (flat) end, then,

$$\bar{x} = 3R - \frac{35}{24}R = \frac{37}{24}R \quad (1.542R)$$

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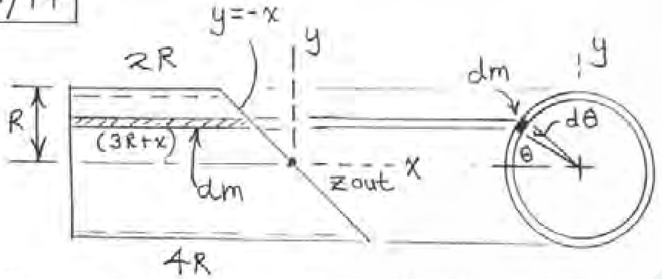

Temporarily use coordinates shown. Element is a "stick". ρ is mass per unit area here.

$$dm = \rho(3R+x)R\,d\theta = \rho R(3R-y)\,d\theta$$
$$= \rho R(3R - R\sin\theta)\,d\theta = \rho R^2(3-\sin\theta)\,d\theta$$

$$m = \int dm = \int_0^{2\pi} \rho R^2(3-\sin\theta)\,d\theta$$
$$= \rho R^2\left[3\theta + \cos\theta\right]_0^{2\pi} = 6\pi\rho R^2$$

$$x_c = -3R + \frac{3R+x}{2} = \frac{x-3R}{2} = -\frac{y+3R}{2}$$
$$= -R\,\frac{\sin\theta+3}{2}$$

$$\int x_c\,dm = \int_0^{2\pi}\left[-\frac{R}{2}(\sin\theta+3)\right]\rho R^2(3-\sin\theta)\,d\theta$$
$$= -\frac{\rho R^3}{2}\int_0^{2\pi}(9-\sin^2\theta)\,d\theta$$
$$= -\frac{\rho R^3}{2}\left[9\theta - \left(\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right)\right]_0^{2\pi} = -\rho R^3\left(\frac{17\pi}{2}\right)$$

So $\bar{x} = \dfrac{\int x_c\,dm}{\int dm} = \dfrac{-\rho R^3(17\pi/2)}{6\pi\rho R^2} = 1.417R$

Relative to left (flat) end,

$$\bar{x} = 3R - 1.417R = \underline{1.583R}$$

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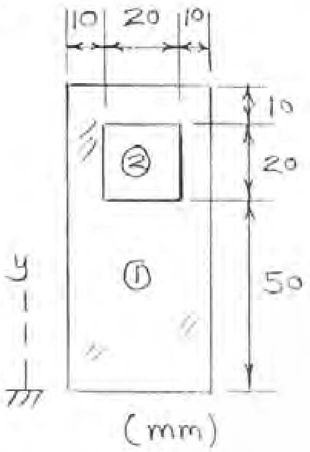

Component	A (mm²)	$\bar{y}$ (mm)	$\bar{y}A$ (mm³)
1	80(40) = 3200	40	128 000
2	− 20(20) = −400	60	−24 000
	ΣA = 2800		$\Sigma \bar{y}A$ = 104 000

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{104\,000}{2800} = \underline{37.1 \text{ mm}}$$

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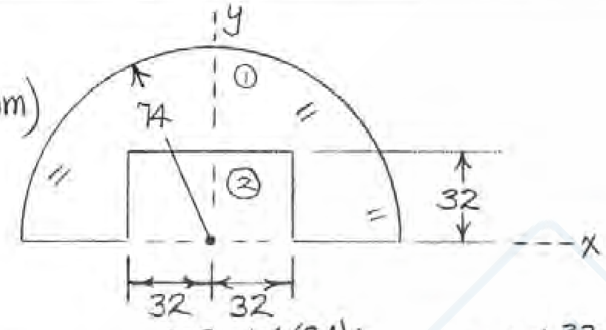
(Dim. in mm)



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{\pi \frac{74^2}{2}\left(\frac{4(74)}{3\pi}\right) - 64(32)\left(\frac{32}{2}\right)}{\pi \frac{74^2}{2} - 64(32)}$$

$= \underline{36.2 \text{ mm}}$

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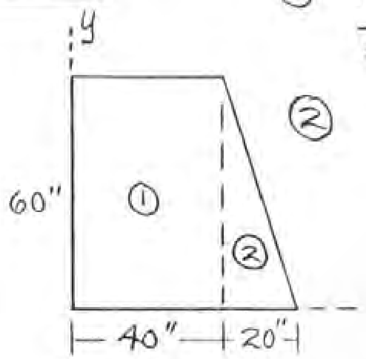

① $A_1 = 40(60) = 2400 \text{ in.}^2$

$\bar{x}_1 = 20 \text{ in.}, \quad \bar{y}_1 = 30 \text{ in.}$

② $A_2 = \frac{1}{2}(20)(60) = 600 \text{ in.}^2$

$\bar{x}_2 = 40 + \frac{20}{3} = 46.7 \text{ in.}$

$\bar{y}_2 = \frac{60}{3} = 20 \text{ in.}$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{2400(20) + 600(46.7)}{2400 + 600} = \underline{25.3 \text{ in.}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2400(30) + 600(20)}{2400 + 600} = \underline{28.0 \text{ in.}}$$

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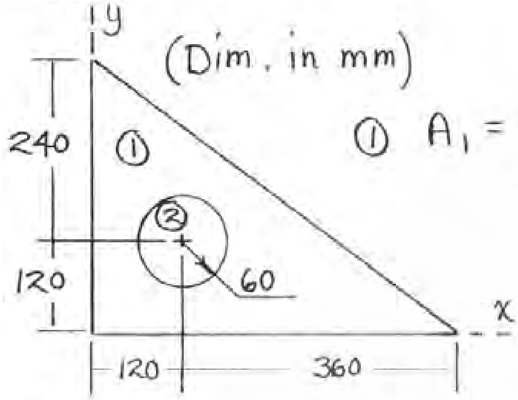

① $A_1 = \frac{1}{2}(480)(360)$
$= 86\,400 \text{ mm}^2$

$\bar{x}_1 = \frac{1}{3}(480) = 160 \text{ mm}$

$\bar{y}_1 = \frac{1}{3}(360) = 120 \text{ mm}$

② $A_2 = -\pi(60)^2 = -11\,310 \text{ mm}^2$

$\bar{x}_2 = \bar{y}_2 = 120 \text{ mm}$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{86\,400(160) - 11\,310(120)}{86\,400 - 11\,310} = \underline{166.0 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{86\,400(120) - 11\,310(120)}{86\,400 - 11\,310} = \underline{120 \text{ mm}}$$

(Because the circular hole is located at the $\bar{y}$ for the triangle, its presence does not alter the $\bar{y}$ result.)

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5/49 Triangle: $A = \frac{1}{2}(2h)h = h^2$

$\bar{y} = h/3$

Semi-circular hole: $A = -\frac{1}{2}\pi(h/2)^2 = -\pi h^2/8$

$\bar{y} = 4(h/2)/3\pi = \frac{2h}{3\pi}$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{h^2(h/3) - (\pi h^2/8)(2h/3\pi)}{h^2 - \pi h^2/8} = \frac{h}{4(1-\pi/8)} = \underline{0.412h}$$

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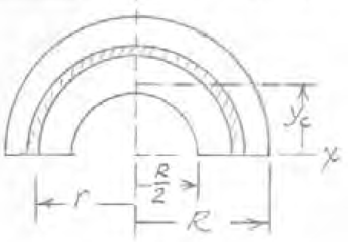
$y_c = \frac{2r}{\pi}$; $dA = \pi r\,dr$, $A = \pi\int_{R/2}^{R} r\,dr = 3\pi R^2/8$



$$\int y_c\,dA = \int_{R/2}^{R} \frac{2r}{\pi}\pi r\,dr$$

$$= \frac{2r^3}{3}\Big]_{R/2}^{R} = \frac{7}{12}R^3$$

$\bar{y}A = \int y_c\,dA$; $\bar{y} = \frac{7}{12}R^3 \Big/ \frac{3\pi R^2}{8} = \underline{\frac{14R}{9\pi}}$

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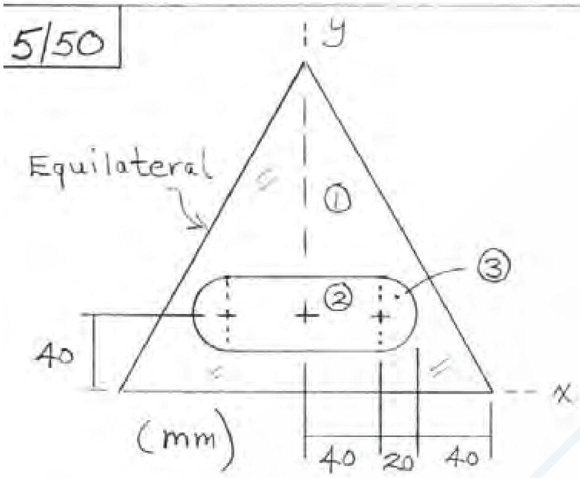

Component	A (mm²)	$\bar{y}$ (mm)	$\bar{y}A$ (mm³)
Triangle 1	17 320	57.7	10^6
Rectangle 2	−3200	40	−128 000
2 semicircles 3	−1257	40	−50,300
	$\Sigma A =$ 12 860		$\Sigma\bar{y}A =$ 822 000

$$\bar{Y} = \frac{\Sigma\bar{y}A}{\Sigma A} \quad \frac{822\,000}{12\,860} = 63.9 \text{ mm}$$

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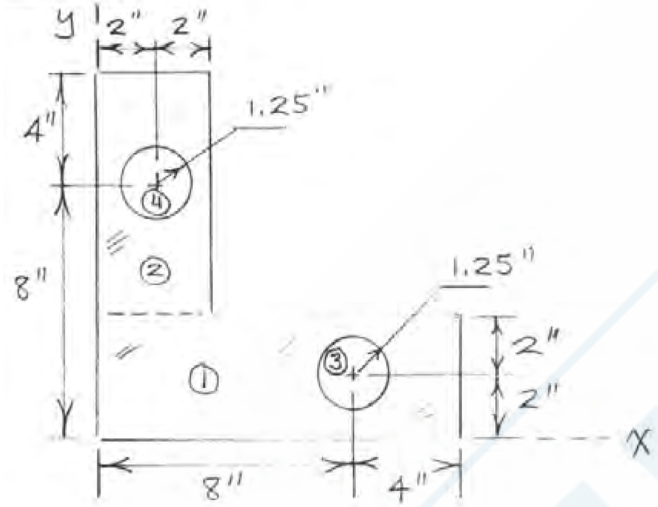

Comp.	A (in.^{2})	$\bar{x}$ (in.)	$\bar{x}A$ (in.^{3})
①	48	6	288
②	32	2	64
③	− 4.91	8	− 39.3
④	− 4.91	2	− 9.82

$\Sigma A = 70.2$ $\qquad \Sigma \bar{x} A = 303$

$$\bar{X} = \bar{Y} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{303}{70.2} = 4.32 \text{ in.}$$

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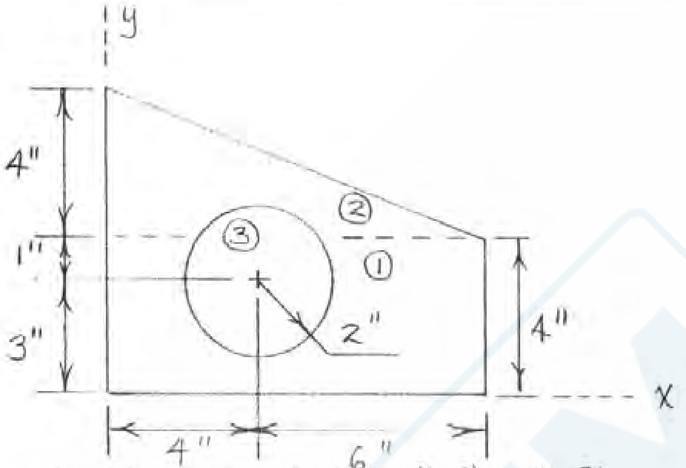

Part	(in.²) A	(in.) $\bar{x}$	(in.) $\bar{y}$	(in.³) $A\bar{x}$	(in.³) $A\bar{y}$
1	40	5	2	200	80
2	20	10/3	4 + 4/3	66.7	106.7
3	$-\pi 2^2$	4	3	-50.3	-37.7
Totals	47.4			216	149.0

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{216}{47.4} = \underline{4.56 \text{ in.}}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{149.0}{47.4} = \underline{3.14 \text{ in.}}$$

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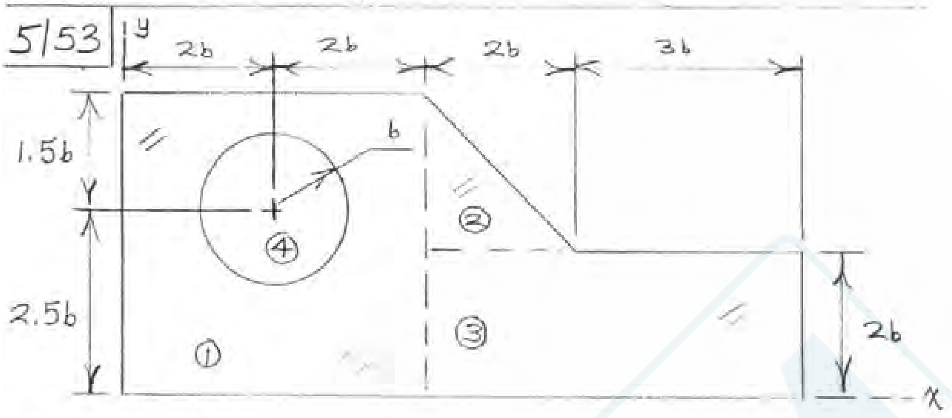

Comp.	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$16b^2$	$2b$	$2b$	$32b^3$	$32b^3$
2	$2b^2$	$\left(4b+\frac{2b}{3}\right)$	$\left(2.5b+\frac{2b}{3}\right)$	$9.33b^3$	$6.33b^3$
3	$10b^2$	$\left(4b+\frac{5b}{2}\right)$	b	$65b^3$	$10b^3$
4	$-\pi b^2$	$2b$	$2.5b$	$-2\pi b^3$	$-2.5\pi b^3$

$\Sigma A = 24.9b^2$ $\qquad \Sigma \bar{x}A = 100.1b^3$ $\qquad \Sigma \bar{y}A = 40.5b^3$

$$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{100.1b^3}{24.9b^2} = 4.02b$$

$$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{40.5b^3}{24.9b^2} = 1.628b$$

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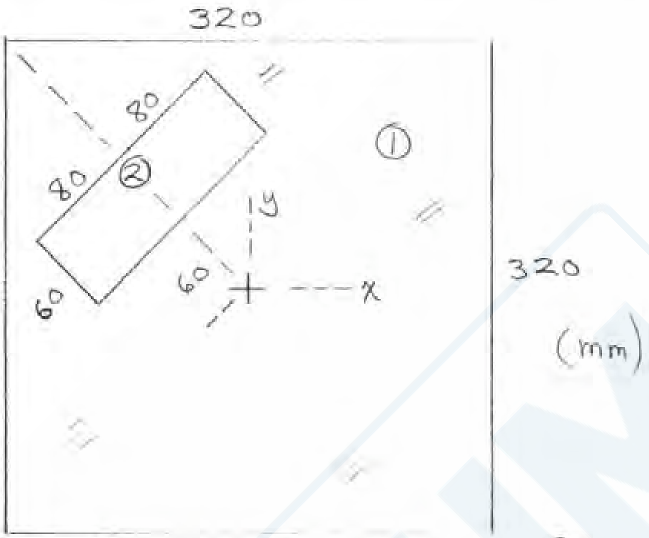

Comp.	A (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}A$ (mm^3)	$\bar{y}A$ (mm^3)
1	$(320)^2$	0	0	0	0
2	$-160(60)$	$-90\frac{\sqrt{2}}{2}$	$90\frac{\sqrt{2}}{2}$	611 000	$-611\,000$

$\Sigma A = 92\,800$

$\Sigma \bar{x}A = 611\,000$

$\Sigma \bar{y}A = -611\,000$

$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{611\,000}{92\,800} = \underline{6.58 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{-611\,000}{92\,800} = \underline{-6.58 \text{ mm}}$$

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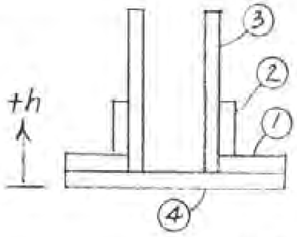

Part	Size mm	A mm^2	$\bar{h}$ mm	$A\bar{h}$ mm^3
1	10×40	800	15	12000
2	10×40	800	40	32000
3	10×120	2400	70	168 000
4	10×160	1600	5	8000
Σ's		5600		220 000

$$\bar{H} = \frac{\Sigma A\bar{h}}{\Sigma A} = \frac{220\,000}{5600} = \underline{39.3 \text{ mm}}$$

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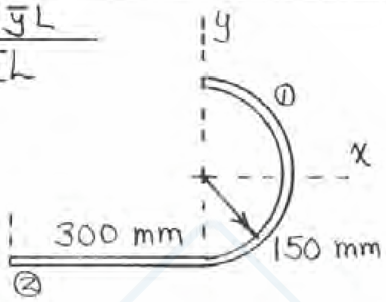
5/56 $\bar{X} = \frac{\Sigma \bar{x} L}{\Sigma L}$, $\bar{Y} = \frac{\Sigma \bar{y} L}{\Sigma L}$



$L_1 = \pi r = 150\pi$ mm

$\bar{x}_1 = \frac{2r}{\pi} = \frac{300}{\pi}$ mm

(from Sample Prob. 5/1)

$\bar{y}_1 = 0$

$L_2 = 300$ mm, $\bar{x}_2 = -150$ mm, $\bar{y}_2 = -150$ mm

So

$$\bar{X} = \frac{\frac{300}{\pi}(150\pi) + 300(-150)}{150\pi + 300} = \underline{0}$$

$$\bar{Y} = \frac{\frac{300}{\pi}(0) + 300(-150)}{150\pi + 300} = \underline{-58.3 \text{ mm}}$$

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$$\bar{z} = \frac{\Sigma m \bar{z}}{\Sigma m} = \frac{2(0) + 1.5(90) + 1(180)}{2 + 1.5 + 1} = \underline{70 \text{ mm}}$$

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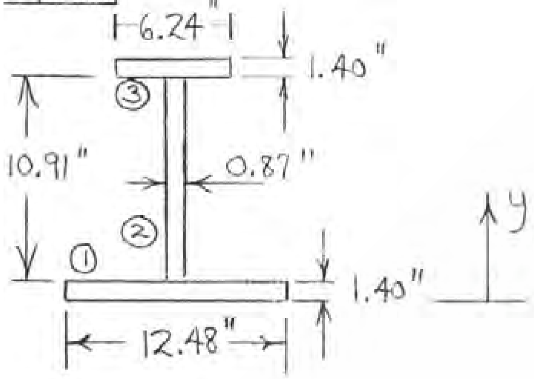

Comp.	A (in.²)	$\bar{y}$ (in.)	$A\bar{y}$ (in.³)
①	(12.48)(1.40)	$\frac{1.40}{2}$	12.23
②	(10.91)(0.87)	$1.4+\frac{10.91}{2}$	65.1
③	(6.24)(1.40)	$1.4+10.91+\frac{1.4}{2}$	113.7

$\Sigma A = 35.7$ $\qquad$ $\Sigma A\bar{y} = 191.0$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \underline{5.35 \text{ in.}}$$

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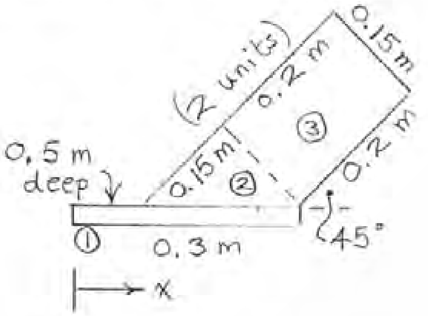
5/59 ρ = mass per unit area



Comp.	m	$\bar{x}$	$\bar{x}m$
①	$(0.5)(0.3)\rho$	0.15	0.0225ρ
②	$2\frac{(0.15)(0.15)}{2}\rho$	$0.3-\left[\frac{2}{3}(0.15)+\frac{1}{3}(0.15)\right]\frac{\sqrt{2}}{2}$	0.00436ρ
③	$2(0.2)(0.15)\rho$	$0.3+\left(\frac{0.2-0.15}{2}\right)\frac{\sqrt{2}}{2}$	0.01906ρ
	$\Sigma m = 0.232\rho$		$\Sigma \bar{x} m = 0.0459\rho$

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{0.0459\rho}{0.232\rho} = \underline{0.1975\text{ m}}$$

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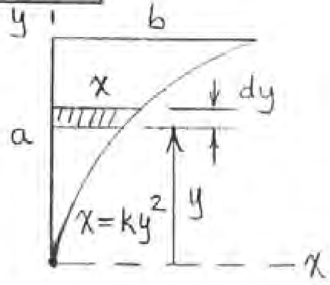

$b = ka^2, \quad k = \frac{b}{a^2}, \quad x = \frac{b}{a^2} y^2$

$A = \int x\,dy = \int_0^a \frac{b}{a^2} y^2\,dy$

$= \frac{b}{a^2}\frac{a^3}{3} = \frac{1}{3}ab$

$\int x_c\,dA = \int \frac{x}{2} x\,dy = \int_0^a \frac{b^2 y^4}{2a^4}\,dy = \frac{ab^2}{10}$

$\bar{x} = \int x_c\,dA / A = \frac{ab^2/10}{ab/3} = \underline{\frac{3}{10} b}$

$\int y_c\,dA = \int y\,x\,dy = \int_0^a y \frac{b}{a^2} y^2\,dy = \frac{ba^2}{4}$

$\bar{y} = \int y_c\,dA / A = \frac{ba^2/4}{ba/3} = \underline{\frac{3}{4} a}$

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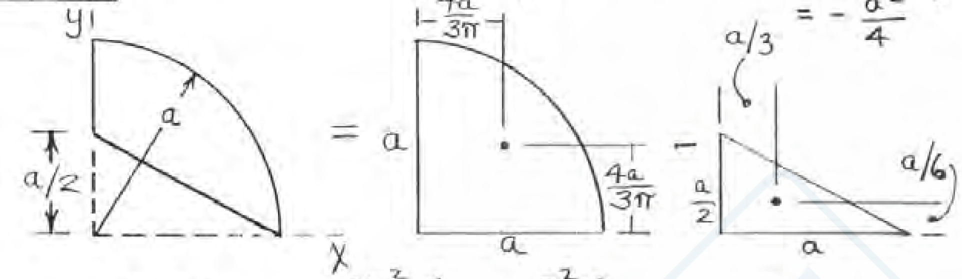
$A = \pi a^2/4$ $\qquad$ $A = -\frac{1}{2}a\left(\frac{a}{2}\right) = -\frac{a^2}{4}$



$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{\frac{\pi a^2}{4}\frac{4a}{3\pi} - \frac{a^2}{4}\frac{a}{3}}{\frac{\pi a^2}{4} - \frac{a^2}{4}} = \underline{\frac{a}{\pi - 1}}$$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{\frac{\pi a^2}{4}\frac{4a}{3\pi} - \frac{a^2}{4}\frac{a}{6}}{\frac{\pi a^2}{4} - \frac{a^2}{4}} = \underline{\frac{7}{6}\frac{a}{\pi - 1}}$$

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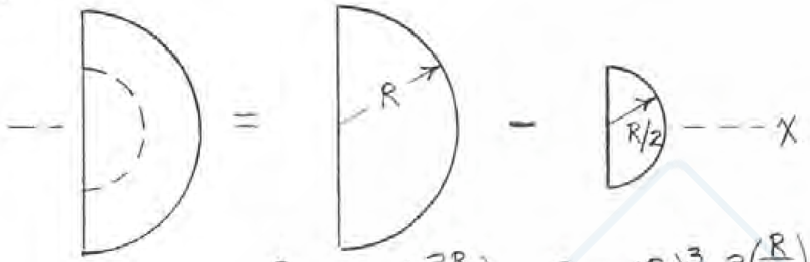

$$\bar{X} = \frac{\Sigma V \bar{x}}{\Sigma V} = \frac{\frac{2}{3}\pi R^3\left(\frac{3R}{8}\right) - \frac{2}{3}\pi\left(\frac{R}{2}\right)^3 \frac{3\left(\frac{R}{2}\right)}{8}}{\frac{2}{3}\pi R^3 - \frac{2}{3}\pi\left(\frac{R}{2}\right)^3}$$

$$= \frac{45}{112} R$$

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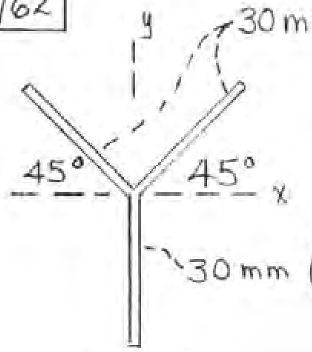

$30\text{ mm}\left(0.3\ \frac{\text{kg}}{\text{m}}\right)\left(\frac{1\text{ m}}{1000\text{ mm}}\right) = 0.009\text{ kg}$

$30\text{ mm}\left(0.5\ \frac{\text{kg}}{\text{m}}\right)\left(\frac{1\text{ m}}{1000\text{ mm}}\right) = 0.0150\text{ kg}$

$$\bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{2(0.009)(15\sin 45°) - 0.0150(15)}{2(0.009) + 0.0150}$$

$$= \underline{-1.033\text{ mm}}$$

$\underline{\bar{X} = 0}$, by inspection.

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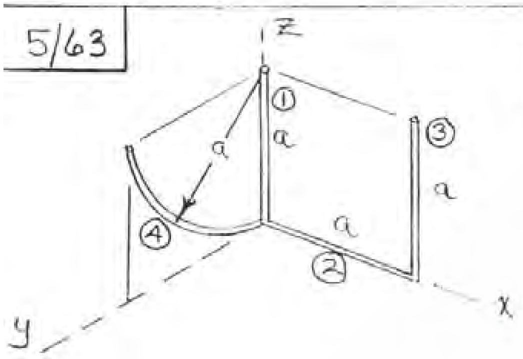

(ρ = mass/length)

Comp.	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}$	$m\bar{y}$	$m\bar{z}$
1	ρa	0	0	$a/2$	0	0	$\rho a^2/2$
2	ρa	$a/2$	0	0	$\rho a^2/2$	0	0
3	ρa	a	0	$a/2$	ρa^2	0	$\rho a^2/2$
4	$\rho \pi a/2$	0	$-2a/\pi$	$a(1-\frac{2}{\pi})$	0	$-\rho a^2$	$\rho a^2(\frac{\pi}{2}-1)$
Totals	$\rho a(3+\frac{\pi}{2})$				$\frac{3}{2}\rho a^2$	$-\rho a^2$	$\frac{\pi}{2}\rho a^2$

$$\bar{X} = \frac{\Sigma m\bar{x}}{\Sigma m} = \frac{\frac{3}{2}\rho a^2}{\rho a(3+\frac{\pi}{2})} = \underline{\frac{3a}{6+\pi}}$$

$$\bar{Y} = \frac{\Sigma m\bar{y}}{\Sigma m} = \frac{-\rho a^2}{\rho a(3+\pi/2)} = \underline{-\frac{2a}{6+\pi}}$$

$$\bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{\frac{\pi}{2}\rho a^2}{\rho a(3+\pi/2)} = \underline{\frac{\pi a}{6+\pi}}$$

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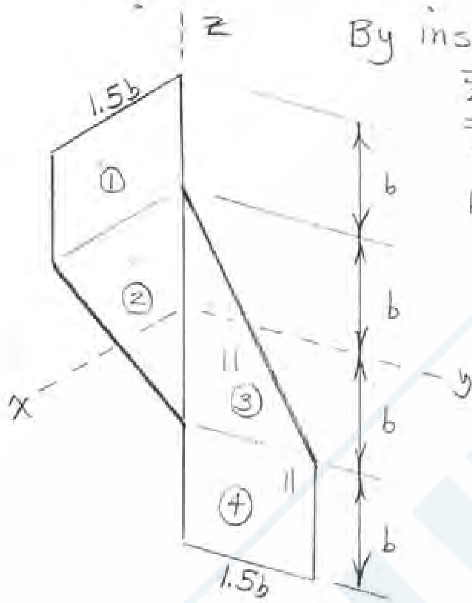

By inspection,
$\bar{Z} = 0$
$\bar{X} = \bar{Y}$
Let ρ = area density

Comp.	m	$\bar{x}$	$m\bar{x}$
1	$1.5b^2\rho$	$1.5b/2$	$1.125b^3\rho$
2	$1.5b^2\rho$	$0.5b$	$0.75b^3\rho$
3	$1.5b^2\rho$	0	0
4	$1.5b^2\rho$	0	0
	$\Sigma A = 6b^2$		$\Sigma \bar{x}A = 1.875b^3$

$$\bar{X} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{1.875b^3\rho}{6b^2\rho} = \underline{0.312b} = \bar{Y}$$

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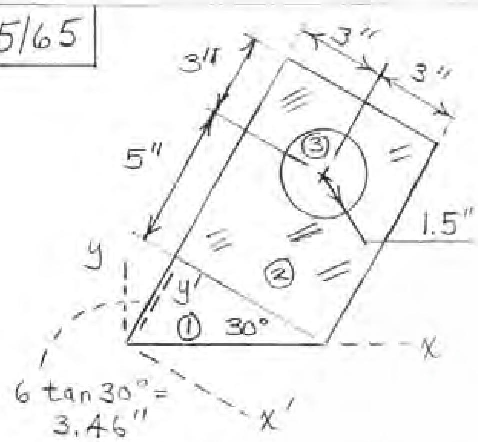

Perform calculations in x'-y' coordinates, then convert to x-y.

Comp.	A	$\bar{x}'$	$\bar{y}'$	$\bar{x}'A$	$\bar{y}'A$
1	$\frac{3.46(6)}{2}$	6/3	$3.46\left(\frac{2}{3}\right)$	20.8	24
2	48	3	7.46	144	358
3	$-\pi(1.5)^2$	3	8.46	−21.2	−59.8
	$\Sigma A = 51.3$			143.6	322

$$\bar{X}' = \frac{\Sigma \bar{x}'A}{\Sigma A} = \frac{143.6}{51.3} = 2.80 \text{ in.}$$

$$\bar{Y}' = \frac{\Sigma \bar{y}'A}{\Sigma A} = \frac{322}{51.3} = 6.28 \text{ in.}$$

Then

$$\bar{X} = 2.80 \cos 30° + 6.28 \sin 30° = \underline{5.56 \text{ in.}}$$

$$\bar{Y} = -2.80 \sin 30° + 6.28 \cos 30° = \underline{4.04 \text{ in.}}$$

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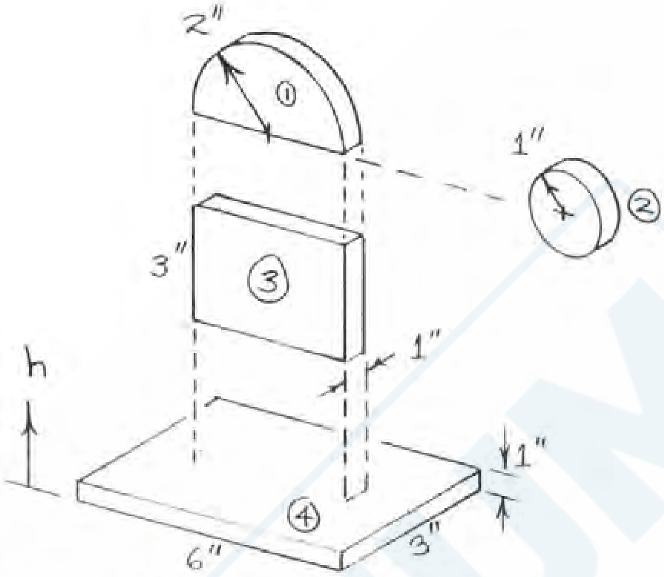

Part	V, in.^{3}	$\bar{h}$, in.	$V\bar{h}$, in.^{4}
①	6.28	4.85	30.5
②	-3.14	4	-12.57
③	12	2.5	30
④	18	0.5	9
Totals	33.1		56.9

$$\bar{H} = \frac{\Sigma V\bar{h}}{\Sigma V} = \frac{56.9}{33.1} = \underline{1.717 \text{ in.}}$$

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Comp.	V	$\bar{x}$	$\bar{y}$	$\bar{z}$	$V\bar{x}$	$V\bar{y}$	$V\bar{z}$
Rect. Sol.	$8.08(10^6)$	0	0	0	0	0	0
Cyl.	$2.26(10^6)$	185	0	0	$418(10^6)$	0	0
Rod	$17.67(10^3)$	0	175	0	0	$3.09(10^6)$	0
Sphere	$524(10^3)$	0	275	0	0	$144.0(10^6)$	0
Totals	$10.88(10^6)$				$418(10^6)$	$147.1(10^6)$	0

$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{418(10^6)}{10.88(10^6)} = \underline{38.5 \text{ mm}}$$

$$\bar{Y} = \frac{\Sigma V\bar{y}}{\Sigma V} = \frac{147.1(10^6)}{10.88(10^6)} = \underline{13.52 \text{ mm}}$$

$$\underline{\bar{Z} = 0}$$

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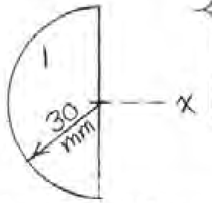

$$\begin{cases} V_1 = \frac{\pi}{2}(30)^2(35) = 49\,500 \text{ mm}^3 \\ \bar{x}_1 = -\frac{4(30)}{3\pi} = -12.73 \text{ mm} \\ \bar{z}_1 = 17.5 \text{ mm} \end{cases}$$

Length = 35 mm

$$V_2 = -\frac{\pi}{2}(20)^2(25) = -15\,710 \text{ mm}^2$$

$$\bar{x}_2 = -\frac{4(20)}{3\pi} = -8.49 \text{ mm}$$

$$\bar{z}_2 = \frac{1}{2}(10 + 35) = 22.5 \text{ mm}$$

Length = 25 mm

$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{49\,500(-12.73) - 15\,710(-8.49)}{49\,500 - 15\,710}$$

$$= \underline{-14.71 \text{ mm}}$$

$$\bar{Z} = \frac{\Sigma V\bar{z}}{\Sigma V} = \frac{49\,500\,(17.5) - 15\,710\,(22.5)}{49\,500 - 15\,710}$$

$$= \underline{15.17 \text{ mm}}$$

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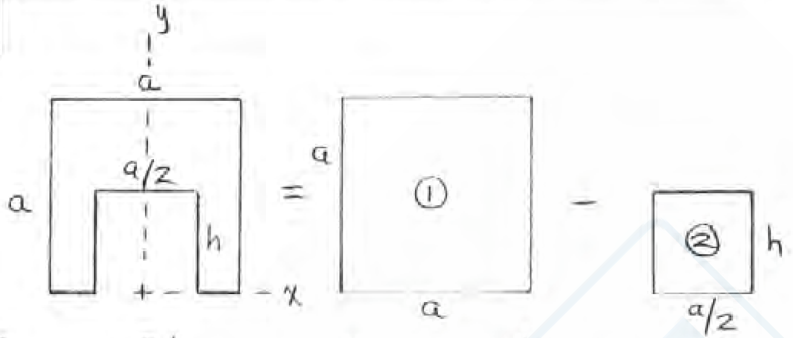

$A_1 = a^2$, $\bar{y}_1 = a/2$; $A_2 = -ah/2$, $\bar{y}_2 = h/2$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{a^2\left(\frac{a}{2}\right) - \frac{ah}{2}\left(\frac{h}{2}\right)}{a^2 - \frac{ah}{2}} = \frac{1}{2}\,\frac{a^2 - h^2/2}{a - h/2}$$

$$\frac{d\bar{Y}}{dh} = \frac{1}{2}\,\frac{(a - h/2)(-h) - (a^2 - h^2/2)(-\frac{1}{2})}{(a - h/2)^2}$$

$$= \frac{1}{2}\,\frac{\frac{h^2}{4} - ah + \frac{a^2}{2}}{(a - \frac{h}{2})^2} = 0 \text{ for max } \bar{Y}.$$

So $\frac{1}{4}(h^2 - 4ah + 2a^2) = 0$, $h = a(2 \pm \sqrt{2})$

The plus sign is rejected because h must be less than a.

Hence $h = a(2 - \sqrt{2}) = \underline{0.586a}$

(Note that $\bar{Y} = 0.586a$ for $h = 0.586a$)

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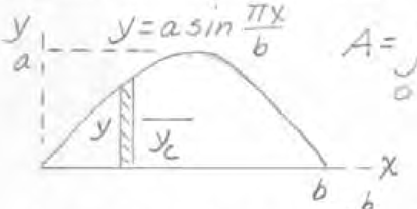

$$dA = y\,dx = a \sin\frac{\pi x}{b}\,dx$$

$$A = \int_0^b a \sin\frac{\pi x}{b}\,dx = -\frac{ab}{\pi}\cos\frac{\pi x}{b}\Big]_0^b$$

$$= \frac{2ab}{\pi}$$

$$\int y_c\,dA = \int \frac{y}{2}\,dA = \int_0^b \frac{1}{2}a^2 \sin^2\frac{\pi x}{b}\,dx = \frac{a^2}{2}\frac{b}{\pi}\left(\frac{\pi x}{2b} - \frac{\sin\frac{2\pi x}{b}}{4}\right)_0^b$$

$$= a^2 b/4$$

$$\bar{y} = \int y_c\,dA / A = \frac{a^2 b}{4} \Big/ \frac{2ab}{\pi} = \underline{\frac{\pi a}{8}}$$

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5/70 Cube: $\begin{cases} V_1 = 350^3 = 42\,875\,000 \text{ mm}^3 \\ \bar{z}_1 = 175 \text{ mm} \end{cases}$

Hole: $\begin{cases} V_2 = -\pi(100)^2 h \\ \bar{z}_2 = \frac{h}{2} \end{cases}$

$$\bar{Z} = \frac{\Sigma V\bar{z}}{\Sigma V} = \frac{42\,875\,000(175) - \pi(100)^2 h \frac{h}{2}}{42\,875\,000 - \pi(100)^2 h}$$

For the maximum $\bar{Z}$, set $\frac{d\bar{Z}}{dh} = 0$:

$$\frac{(42\,875\,000 - \pi\,100^2 h)(-\pi\,100^2 h) - \left(42\,875\,000(175) - \pi\,100^2 \frac{h^2}{2}\right) \times (-\pi\,100^2)}{(\text{denominator})} = 0$$

Set numerator equal to zero to obtain

$$\underline{h = 187.9 \text{ mm}}$$

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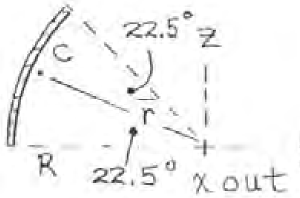
5/71 Detail of "hole" (2): [1 = cyl. shell]

$$\bar{r} = \frac{R \sin 22.5^\circ}{\pi/8} = 0.974R$$



$$y\begin{cases} \bar{x} = -\frac{L}{8} - \frac{1}{2}\frac{3L}{8} = -\frac{5}{16}L \\ \bar{y} = -\bar{r}\cos 22.5^\circ = -0.900R \\ \bar{z} = \bar{r}\sin 22.5^\circ = 0.373R \end{cases}$$

Comp	m	$\bar{x}$	$\bar{y}$	$\bar{z}$
1	$2\rho\pi RL$	$-\frac{L}{2}$	0	0
2	$-\frac{3}{32}\rho\pi RL$	$-\frac{5L}{16}$	$-0.900R$	$0.373R$

$\Sigma m = 1.906\rho\pi RL$

	$\bar{x}m$	$\bar{y}m$	$\bar{z}m$
1	$-\rho\pi RL^2$	0	0
2	$+0.0293\rho\pi RL^2$	$+0.0844\rho\pi R^2L$	$-0.0350\rho\pi R^2L$

$\Sigma\bar{x}m = -0.971\rho\pi RL^2$ $\quad \Sigma\bar{y}m = 0.0844\rho\pi R^2L$ $\quad \Sigma\bar{z}m = -0.0350\rho\pi R^2L$

$$\bar{X} = \frac{\Sigma\bar{x}m}{\Sigma m} = \frac{-0.971\rho\pi RL^2}{1.906\rho\pi RL} = \underline{-0.509L}$$

$$\bar{Y} = \frac{\Sigma\bar{y}m}{\Sigma m} = \frac{0.0844\rho\pi R^2L}{1.906\rho\pi RL} = \underline{0.0443R}$$

$$\bar{Z} = \frac{\Sigma\bar{z}m}{\Sigma m} = \frac{-0.0350\rho\pi R^2L}{1.906\rho\pi RL} = \underline{-0.01834R}$$

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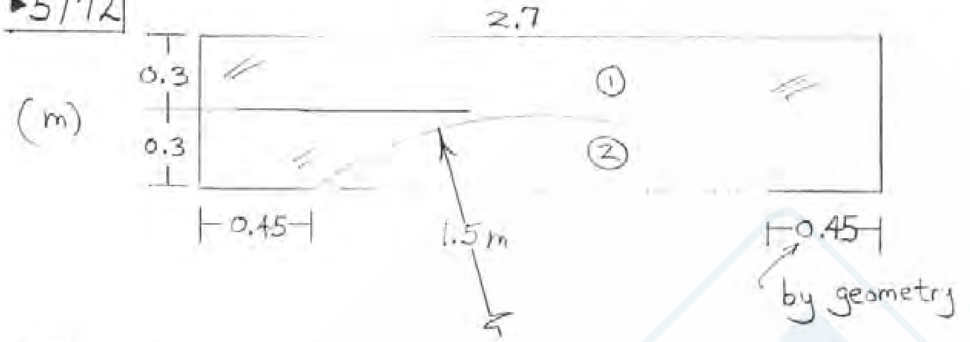

From Prob. 5/41, relative to the base of the current area, we have for the circular portion with $a = 1.5$ m & $h = 1.2$ m:

$$\bar{y} = \frac{\frac{2}{3}\left(1.5^2 - 1.2^2\right)^{3/2}}{1.5^2\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{1.2}{1.5}\right)\right] - 1.2\sqrt{1.5^2 - 1.2^2}} - 1.2$$

$= 0.1211$ m

Comp.	A	$\bar{y}$	$\bar{y}A$
①	(2.7)(0.6)	0.3	0.486
②	− 0.368 *	0.1211	− 0.0445
	$\Sigma A = 1.252$		$\Sigma \bar{y} A = 0.441$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{0.441}{1.252} = \underline{0.353 \text{ m}}$$

* From Prob. 5/41:

$$A = 1.5^2\left(\frac{\pi}{2} - \sin^{-1}\frac{1.2}{1.5}\right) - 1.2\sqrt{1.5^2 - 1.2^2}$$

$= 0.368$ m^2

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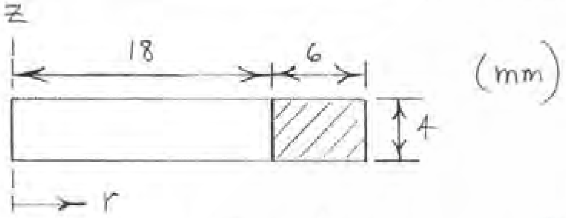

$$A = 2\pi \bar{r} L = 2\pi\left(18+\frac{6}{2}\right)(6+6+4+4)$$

$$= \underline{2640\ \text{mm}^2}$$

$$V = 2\pi \bar{r} A = 2\pi\left(18+\frac{6}{2}\right)(6\cdot 4)$$

$$= \underline{3170\ \text{mm}^3}$$

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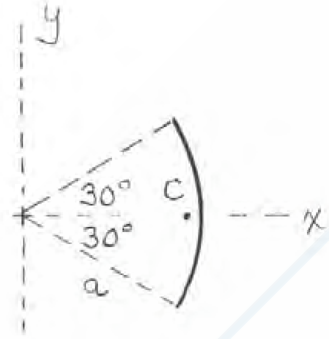

From Table D/3,

$$\bar{x} = \frac{a \sin \pi/6}{\pi/6} = \frac{3a}{\pi}$$

$$A = 2\pi \bar{x} L = 2\pi \left(\frac{3a}{\pi}\right)\left(\frac{\pi}{3} a\right)$$

$$= \underline{2\pi a^2}$$

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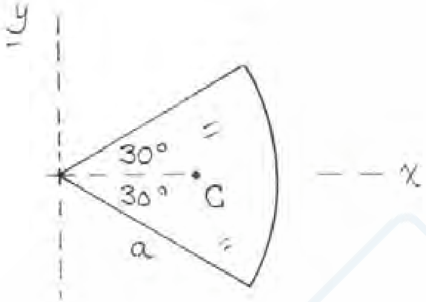

From Table D/3:

$$\bar{x} = \frac{2}{3} a \frac{\sin \pi/6}{\pi/6} = \frac{2a}{3} \frac{1}{2} \frac{6}{\pi} = \frac{2a}{\pi}$$

$$V = \pi \bar{x} A = \pi \left(\frac{2a}{\pi}\right)\left(\frac{\pi a^2}{6}\right)$$

$$= \underline{\frac{\pi a^3}{3}}$$

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5/76 $V = \theta \bar{r} A = \pi\left(8 + \frac{2}{3}(12)\right)\frac{1}{2}(12)(12) = \underline{3620\ \text{mm}^3}$

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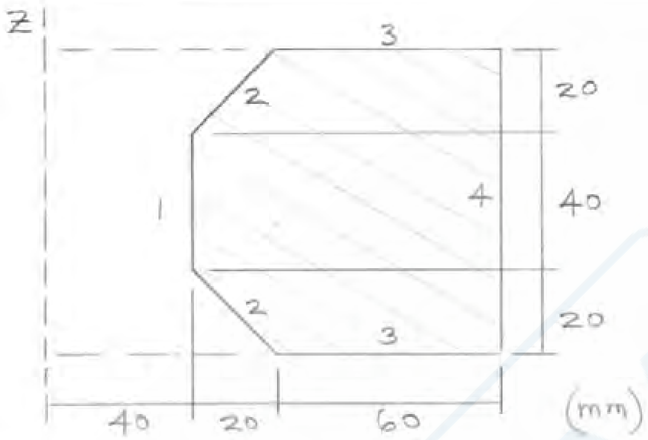

$A_1 = \pi \bar{r}_1 L_1 = \pi(40)(40) = 1600\pi \text{ mm}^2$

$A_2 = 2(\pi \bar{r}_2 L_2) = 2\pi(50)(20\sqrt{2}) = 2830\pi \text{ mm}^2$

$A_3 = 2(\pi r_3 L_3) = 2\pi(90)(60) = 10\,800\pi \text{ mm}^2$

$A_4 = \pi r_4 L_4 = \pi(120)(80) = 9600\pi \text{ mm}^2$

End faces: $2\{80(60) + 40(20) + 20(20)\} = 12\,000 \text{ mm}^2$

Total: $\underline{90\,000 \text{ mm}^2}$

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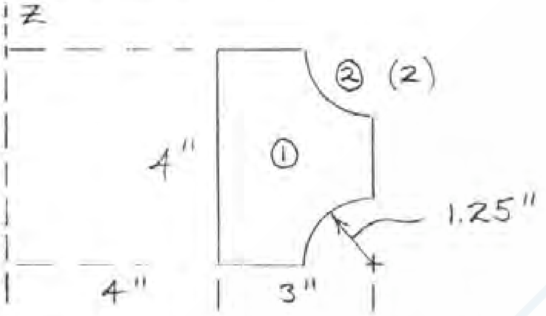

$$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi(4 + 3/2)(4 \cdot 3) = 415 \text{ in.}^3$$

$$V_2 = -2(2\pi \bar{r}_2 A_2) = -4\pi\left(7 - \frac{4(1.25)}{3\pi}\right)\left(\frac{\pi(1.25)^2}{4}\right)$$

$$= -99.8 \text{ in.}^3$$

$$V = V_1 + V_2 = \underline{315 \text{ in.}^3}$$

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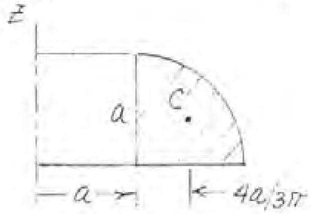

$$V = \bar{r}\theta A = \left(a + \frac{4a}{3\pi}\right)\frac{\pi}{2}\frac{\pi a^2}{4}$$

$$= \frac{\pi a^3}{24}(3\pi + 4)$$

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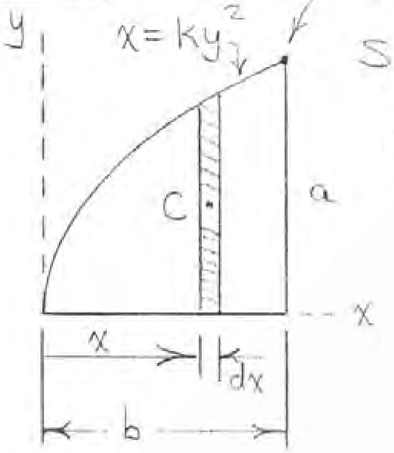

$x = ky^2$: $b = ka^2$, $k = \frac{b}{a^2}$

So $x = \frac{b}{a^2} y^2$

$$A = \int dA = \int y\,dx$$

$$= \int_0^b a\sqrt{\frac{x}{b}}\,dx = \frac{a}{\sqrt{b}} \frac{x^{3/2}}{3/2}\Big|_0^b$$

$$= \frac{2}{3}ab$$

$$\int x_c\,dA = \int_0^b \frac{a}{\sqrt{b}} x^{3/2}\,dx = \frac{a}{\sqrt{b}} \frac{x^{5/2}}{5/2}\Big|_0^b = \frac{2}{5}ab^2$$

$$\int y_c\,dA = \int \frac{y}{2} y\,dx = \frac{1}{2}\int y^2\,dx$$

$$= \frac{1}{2}\int_0^b \frac{a^2}{b} x\,dx = \frac{1}{2}\frac{a^2}{b}\frac{x^2}{2}\Big|_0^b = \frac{1}{4}a^2b$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{\frac{2}{5}ab^2}{\frac{2}{3}ab} = \underline{\frac{3}{5}b}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{\frac{1}{4}a^2b}{\frac{2}{3}ab} = \underline{\frac{3}{8}a}$$

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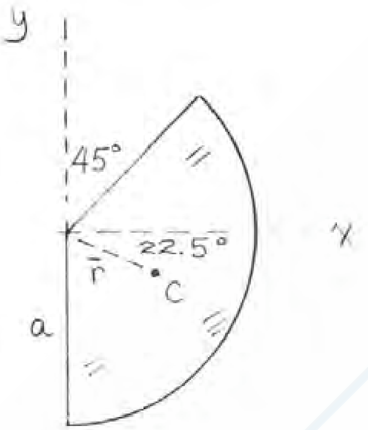

From Table D/3, with $\alpha = 67.5° \; (0.375\pi)$

$$\bar{r} = \frac{2}{3}\frac{a \sin(0.375\pi)}{0.375\pi} = 0.523a$$

$$V = 2\pi\bar{x}A = 2\pi(0.523a\cos 22.5°)\left(\pi a^2 \frac{0.75\pi}{2\pi}\right)$$

$$= \underline{3.58a^3}$$

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$A = 2\pi \bar{r} L + \pi d h$

$= 2\pi(8.2)34 + \pi(8)(18) = 2204 \text{ ft}^2$

No. of gal. for 2 coats $= \frac{2204}{500} \times 2 = \underline{8.82 \text{ gal}}$

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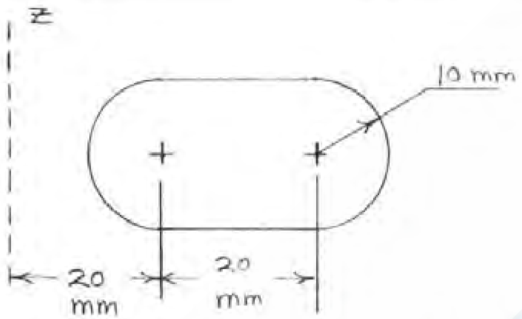

$$A = 2\pi \bar{r} L = 2\pi (30)\left[20 + 20 + \pi 10 + \pi 10\right]$$

$$= \underline{19\ 380\ \text{mm}^2}$$

$$V = 2\pi \bar{r} A = 2\pi (30)\left[20(20) + \pi(10^2)\right]$$

$$= \underline{134\ 600\ \text{mm}^3}$$

For steel,

$$m = \rho V = 7830\ \frac{\text{kg}}{\text{m}^3}\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)^3 (134\ 600\ \text{mm}^3)$$

$$= \underline{1.054\ \text{kg}}$$

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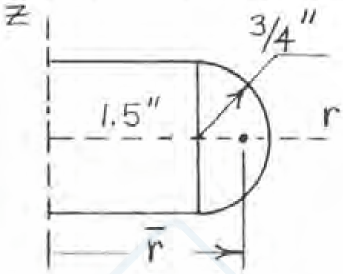
$$V = 2\pi \bar{r}_{area} A$$

$$= 2\pi\left(1.5 + \frac{4(0.75)}{3\pi}\right) \frac{\pi(0.75)^2}{2}$$

$$= \underline{10.09 \text{ in.}^3}$$



$$A = 2\pi \bar{r}_{arc} L$$

$$= 2\pi\left(1.5 + \frac{2(0.75)}{\pi}\right)(\pi \times 0.75)$$

$$= \underline{29.3 \text{ in.}^2}$$

$$\begin{cases} \bar{r}_{area} = \dfrac{4r}{3\pi} + 1.5 \\ \bar{r}_{arc} = \dfrac{2r}{\pi} + 1.5 \end{cases}$$

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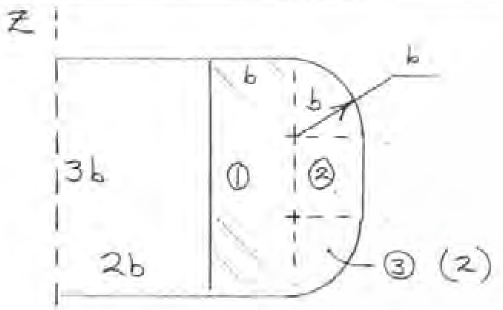

$V_1 = 2\pi \bar{r}_1 A_1 = 2\pi\left(2b+\frac{b}{2}\right)(3b^2) = 15\pi b^3$

$V_2 = 2\pi \bar{r}_2 A_2 = 2\pi\left(3b+\frac{b}{2}\right)(b^2) = 7\pi b^3$

$V_3 = 2(2\pi \bar{r}_3 A_3) = 4\pi\left(3b+\frac{4b}{3\pi}\right)\left(\frac{\pi b^2}{4}\right) = \pi^2 b^3\left(3+\frac{4}{3\pi}\right)$

$V = V_1 + V_2 + V_3 = \underline{\pi b^3\left(\frac{70}{3}+3\pi\right)} = \underline{102.9\, b^3}$

$A_{1_{inner}} = 2\pi \bar{r}_{1i} L_{1i} = 2\pi(2b)(3b) = 12\pi b^2$

$A_{1_{top\,\&\,bott.}} = 2\cdot 2\pi \bar{r}_{1t} L_{1t} = 4\pi\,\frac{5b}{2}(b) = 10\pi b^2$

$A_2 = 2\pi \bar{r}_2 L_2 = 2\pi(4b)(b) = 8\pi b^2$

$A_3 = 2\cdot 2\pi \bar{r}_3 L_3 = 4\pi\left(3b+\frac{2b}{\pi}\right)\left(\frac{\pi b}{2}\right) = 2\pi^2 b^2\left(3+\frac{2}{\pi}\right)$

$A = A_1 + A_2 + A_3 = \underline{\pi b^2(34+6\pi)} = \underline{166.0\, b^2}$

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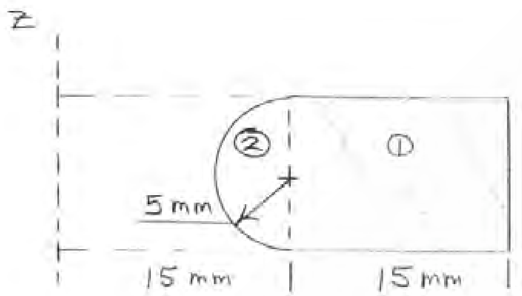

$$\bar{V}_1 = 2\pi \bar{r}_1 A_1 = 2\pi\left(15 + \frac{15}{2}\right)(15 \cdot 10) = 21\ 200\ \text{mm}^3$$

$$\bar{V}_2 = 2\pi \bar{r}_2 A_2 = 2\pi\left(15 - \frac{4 \cdot 5}{3\pi}\right)\left(\frac{\pi 5^2}{2}\right) = 3\ 180\ \text{mm}^3$$

$$\bar{V} = \bar{V}_1 + \bar{V}_2 = \underline{24\ 400\ \text{mm}^3}$$

$$A_{1\,outer} = 2\pi \bar{r}_{1o} L_{1o} = 2\pi(30)(10) = 1885\ \text{mm}^2$$

$$A_{1\,top\,\&\,bottom} = 2 \cdot 2\pi r_{1t} L_{1t} = 4\pi\left(15 + \frac{15}{2}\right)(15) = 4240\ \text{mm}^2$$

$$A_2 = 2\pi \bar{r}_2 L_2 = 2\pi\left(15 - \frac{2 \cdot 5}{\pi}\right)(\pi 5) = 1166\ \text{mm}^2$$

$$A = A_1 + A_2 = \underline{7290\ \text{mm}^2}$$

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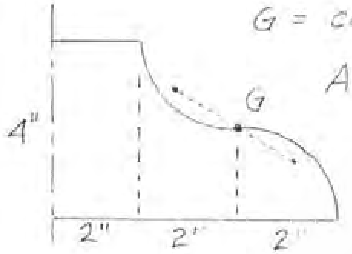

G = centroid by symmetry

$A = 2\pi \bar{r} L = 2\pi(4)\pi(2)$

$= 16\pi^2 = \underline{157.9 \text{ in.}^2}$

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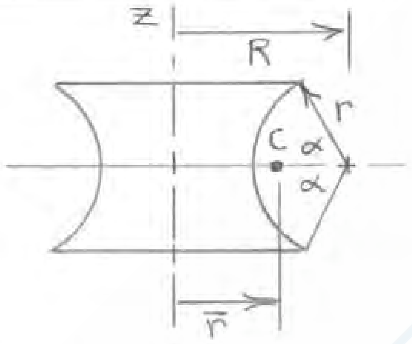

$\bar{r} = R - r\dfrac{\sin\alpha}{\alpha}$ from Sample Problem 5/1

$L = 2r\alpha$

$A = 2\pi\bar{r}L = 2\pi\left(R - r\dfrac{\sin\alpha}{\alpha}\right)(2r\alpha)$

$\quad = \underline{4\pi r\,(R\alpha - r\sin\alpha)}$

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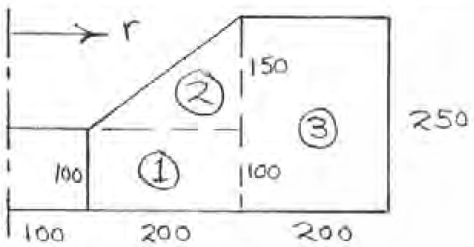

$$V = 2\pi \bar{r} A = 2\pi \sum_{i=1}^{3} \bar{r}_i A_i$$

$A_1 = 200(100) = 20(10^3)\ mm^2$, $r_1 = 100 + \frac{200}{2} = 200\ mm$

$A_2 = \frac{200(150)}{2} = 15(10^3)\ mm^2$, $r_2 = 100 + \frac{2}{3}200 = 233\ mm$

$A_3 = 200(250) = 50(10^3)\ mm^2$, $r_3 = 300 + \frac{200}{2} = 400\ mm$

$$V = 2\pi\left[200(20\times10^3) + 233(15\times10^3) + 400(50\times10^3)\right]$$
$$= 172.8(10^6)\ mm^3 \quad \text{or} \quad \underline{V = 0.1728\ m^3}$$

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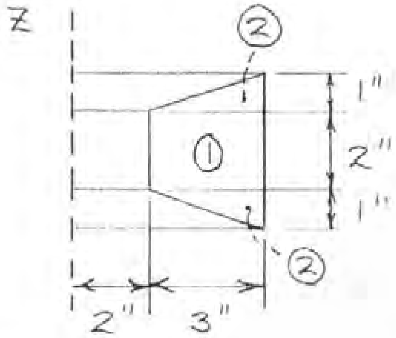

$$V_1 = \pi \bar{r}_1 A_1 = \pi\left(2+\frac{3}{2}\right)(3\cdot 2) = 66.0 \text{ in.}^3$$

$$V_2 = 2(\pi \bar{r}_2 A_2) = 2\pi\left(5-\frac{3}{3}\right)\left(\frac{1}{2}(3)(1)\right) = 37.7 \text{ in.}^3$$

$$V = V_1 + V_2 = 103.7 \text{ in.}^3$$

$$W = \mu V = \left(168 \frac{\text{lb}}{\text{ft}^3}\right)\left(103.7 \text{ in.}^3\right)\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= \underline{10.08 \text{ lb}}$$

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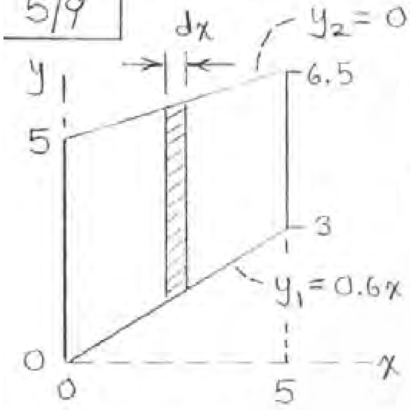

$$A = \int_0^5 (y_2 - y_1)\,dx$$

$$= \int_0^5 (0.3x + 5 - 0.6x)\,dx$$

$$= \int_0^5 (5 - 0.3x)\,dx$$

$$= \left(5x - \frac{0.3x^2}{2}\right)_0^5 = 21.25$$

(Note: Trapezoidal area formula could be used)

$$\int x_c\,dA = \int_0^5 x(5 - 0.3x)\,dx = \left(\frac{5}{2}x^2 - 0.1x^3\right)_0^5 = 50$$

$$\int y_c\,dA = \int_0^5 \left(\frac{y_1 + y_2}{2}\right)(y_2 - y_1)\,dx = \frac{1}{2}\int_0^5 (y_2^2 - y_1^2)\,dx$$

$$= \frac{1}{2}\int_0^5 \left[(0.3x + 5)^2 - (0.6x)^2\right]dx = \frac{1}{2}\int_0^5 (25 + 3x - 0.27x^2)\,dx$$

$$= \frac{1}{2}\left[25x + \frac{3x^2}{2} - \frac{0.27x^3}{3}\right]_0^5 = 75.6$$

$$\bar{x} = \int x_c\,dA / A = 50/21.25 = \underline{2.35}$$

$$\bar{y} = \int y_c\,dA / A = 75.6/21.25 = \underline{3.56}$$

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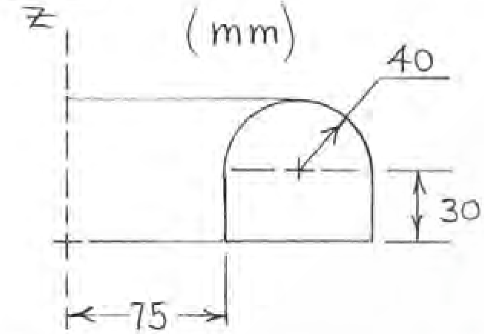

$A_1 = \pi \bar{r} L = \pi(75+40)(2 \cdot 30 + 80 + \pi 40)$

$= 96\,000 \text{ mm}^2$

End areas $A_2 = 2\left(\frac{\pi}{2} \cdot 40^2 + 80(30)\right)$

$= 9830 \text{ mm}^2$

Total area $A = A_1 + A_2 = \underline{105\,800 \text{ mm}^2}$

$V = \pi \bar{r} A = \pi(75+40)(30 \cdot 80 + \pi 40^2/2)$

$= \underline{1.775(10^6) \text{ mm}^3}$

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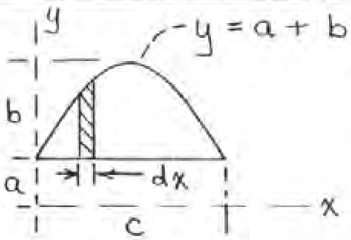

First, find $\bar{y}$ by

$$\bar{y} = \int y_c\,dA \Big/ \int dA$$

$$A = \int dA = \int_0^c b\sin\frac{\pi x}{c}\,dx = -\frac{bc}{\pi}\cos\frac{\pi x}{c}\Big|_0^c = \frac{2bc}{\pi}$$

$$\int y_c\,dA = \int_0^c \left[a + \frac{b}{2}\sin\frac{\pi x}{c}\right]\left[b\sin\frac{\pi x}{c}\,dx\right]$$

$$= \int_0^c ab\sin\frac{\pi x}{c}\,dx + \int_0^c \frac{b^2}{2}\sin^2\frac{\pi x}{c}\,dx$$

$$= -ab\frac{c}{\pi}\cos\frac{\pi x}{c}\Big|_0^c + \frac{b^2}{2}\left[\frac{x}{2} - \frac{1}{4}\sin\frac{2\pi x}{c}\right]_0^c$$

$$= \frac{2abc}{\pi} + \frac{b^2c}{4} = bc\left[\frac{2a}{\pi} + \frac{b}{4}\right]$$

$$\therefore\ \bar{y} = \frac{bc\left[2a/\pi + b/4\right]}{2bc/\pi} = a + \frac{b}{8}\pi$$

$$V = 2\pi\bar{y}A = 2\pi\left(a + \frac{b}{8}\pi\right)\left(2bc/\pi\right)$$

$$= 4bc\left(a + \frac{b\pi}{8}\right)$$

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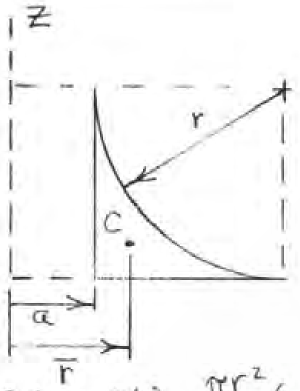

(Refer to Sample Problem 5/3)

$$\bar{r} = \frac{r^2(a + r/2) - \frac{\pi r^2}{4}\left(a + r - \frac{4r}{3\pi}\right)}{r^2 - \pi r^2/4} = a + \frac{10 - 3\pi}{3(4-\pi)} r$$

$$V = \theta \bar{r} A = \frac{\pi}{2}\left[a + \frac{10-3\pi}{3(4-\pi)} r\right]\left(1 - \frac{\pi}{4}\right) r^2$$

$$= \frac{\pi r^2}{8}\left[(4-\pi)a + \frac{10-3\pi}{3} r\right]$$

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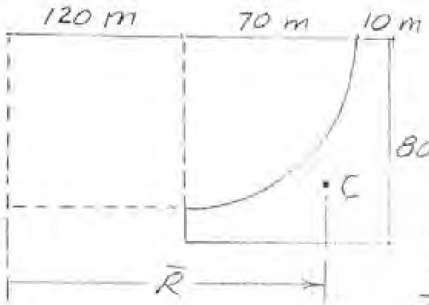

Square: $A = 80^2 = 6400 \ m^2$

$\frac{1}{4}$ circle: $A = \frac{1}{4}\pi(70^2)$

$= 3848 \ m^2$

Net area $= 2552 \ m^2$

$\bar{r}_{square} = 120 + 40 = 160 \ m$

$\bar{r}_{\frac{1}{4}circle} = 120 + \frac{4(70)}{3\pi} = 149.7 \ m$

$\bar{R} = \frac{\Sigma A\bar{r}}{\Sigma A} = \frac{6400(160) - 3848(149.7)}{2552}$

$= 175.5 \ m$

$V = \theta \bar{R} A = \frac{\pi}{3}(175.5)(2552) = 469\,000 \ m^3$

$m = \rho V = 2.40(469\,000) = \underline{1.126 \times 10^6 \ Mg}$

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5/94 From the solution to Prob. 5/11,

$$\bar{r} = 8 - \frac{2}{3}\,\frac{2(1.5)+2}{1.5+2} = 7.05 \text{ m}$$

$$A = \frac{2+1.5}{2}(2) = 3.5 \text{ m}^2$$

$$\theta = \pi/3$$

So $V = \theta \bar{r} A = \frac{\pi}{3}(7.05)(3.5) = 25.8 \text{ m}^3$

$$W = \rho g V = 2400(9.81)(25.8) = 608(10^3) \text{ N}$$

or $\underline{W = 608 \text{ kN}}$

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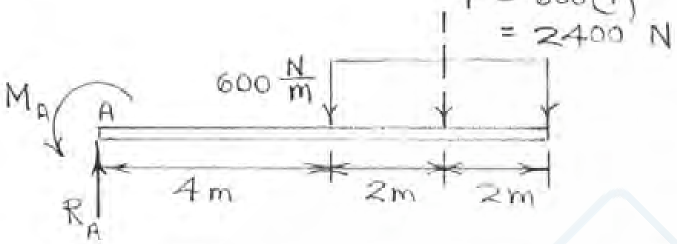

$+\uparrow \Sigma F = 0: \quad R_A - 2400 = 0, \qquad \underline{R_A = 2.4 \text{ kN}}$

$\circlearrowleft + \Sigma M_A = 0: \quad M_A - 2400(6) = 0, \qquad \underline{M_A = 14.4 \text{ kN}\cdot\text{m}}$

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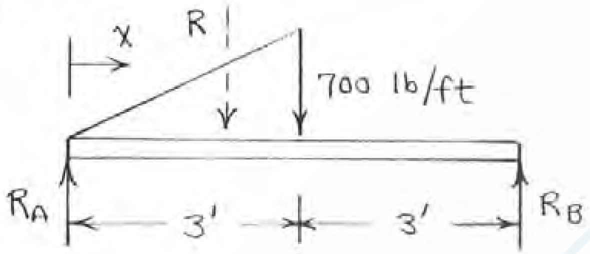

$R = \frac{1}{2}(700)(3) = 1050 \text{ lb}$ @ $\bar{x} = \frac{2}{3}(3) = 2'$

$\circlearrowleft + \Sigma M_A = 0: \quad R_B(6) - 1050(2) = 0, \quad \underline{R_B = 350 \text{ lb}}$

$+\uparrow \Sigma F = 0: \quad R_A + 350 - 1050 = 0, \quad \underline{R_A = 700 \text{ lb}}$

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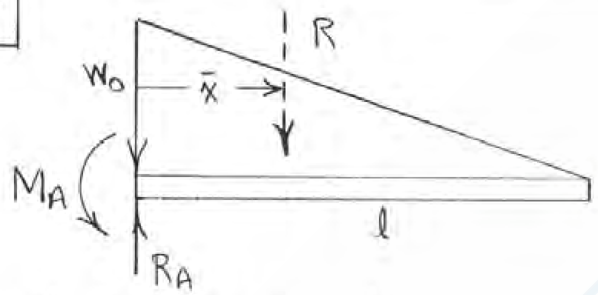

$R = \frac{1}{2} w_0 l$ @ $\bar{x} = \frac{1}{3} l$

$\circlearrowleft + \Sigma M_A = 0$: $M_A - \frac{1}{2} w_0 l \left(l/3 \right) = 0$, $\underline{M_A = \frac{1}{6} w_0 l^2}$

$+\uparrow \Sigma F = 0$: $R_A - \frac{1}{2} w_0 l = 0$, $\underline{R_A = \frac{1}{2} w_0 l}$

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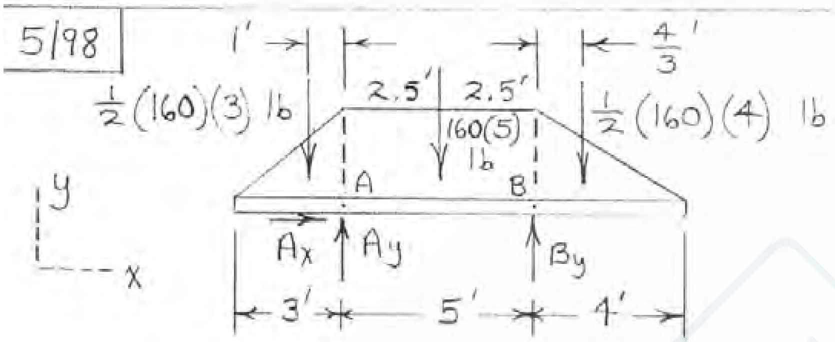

$$\curvearrowleft + \Sigma M_A = 0: \; 240(1) - 800(2.5) - 320(6.33) + B_y(5) = 0$$

$$\underline{B_y = 757 \text{ lb}}$$

$$\Sigma F_y = 0: \; A_y + 757 - 240 - 800 - 320 = 0$$

$$\underline{A_y = 603 \text{ lb}}$$

$$\Sigma F_x = 0 \Rightarrow \underline{A_x = 0}$$

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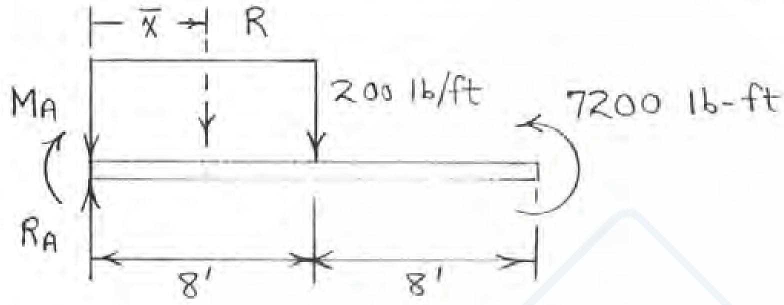

$R = 200(8) = 1600$ lb @ $\bar{x} = 4'$

$\circlearrowleft + \Sigma M_A = 0: \; -M_A - 1600(4) + 7200 = 0$

$\underline{M_A = 800 \text{ lb-ft}}$

$+\uparrow \Sigma F = 0: \; R_A - 1600 = 0, \quad \underline{R_A = 1600 \text{ lb}}$

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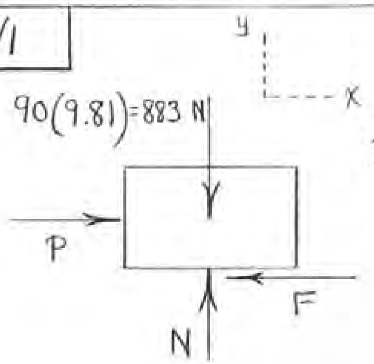

$\Sigma F_y = 0: \quad N - 883 = 0, \quad N = 883\text{ N}$

$F_{max} = \mu_s N = 0.5(883) = 441\text{ N}$

$\Sigma F_x = 0$ yields $F = P$ for equilibrium

(a) $P = 300\text{ N}$, $\underline{F = 300\text{ N}} < F_{max}$, OK

(b) $P = 400\text{ N}$, $\underline{F = 400\text{ N}} < F_{max}$, OK

(c) $P = 500\text{ N}$, $F = 500\text{ N} > F_{max}$, motion

So $F = \mu_k N = 0.4(883) = \underline{353\text{ N}}$

(all to the left)

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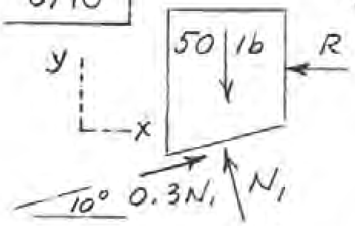
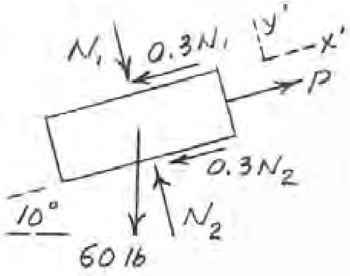


50-lb block: $\Sigma F_y = 0;$

$$N_1 \cos 10° + 0.3 N_1 \sin 10° - 50 = 0$$

$$N_1 (0.985 + 0.0521) = 50$$

$$N_1 = 50/1.0369 = 48.2 \text{ lb}$$



60-lb block;

$\Sigma F_{y'} = 0;\ N_2 - 48.2 - 60 \cos 10° = 0$

$$N_2 = 107.3 \text{ lb}$$

$\Sigma F_{x'} = 0;\ P - 0.3(48.2 + 107.3) - 60 \sin 10° = 0$

$$P = 46.7 + 10.4$$

$$\underline{P = 57.1 \text{ lb}}$$

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6/100

$T_2 = T_1 e^{\mu\beta}$, $\beta = 2$ turns $+ 60°$

$= 2(360°) + 60° = 780°$

or 13.61 rad

$T = \left(\frac{2}{16}\right) e^{0.7(13.61)} = \underline{1720 \text{ lb}}$

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6/101 $\frac{4}{mg} = e^{\mu\beta}$, $\frac{mg}{1.6} = e^{\mu\beta}$

Thus $\frac{4}{mg} = \frac{mg}{1.6}$, $m^2 g^2 = 4(1.6)$

$m = \frac{\sqrt{4(1.6)}}{9.81} = 0.258$ Mg or $m = 258$ kg

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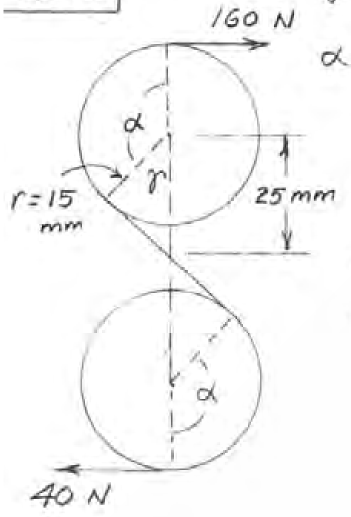

$$\gamma = \cos^{-1}\frac{15}{25} = 53.13°$$

$$\alpha = 180 - 53.13 = 126.87°$$

$$\beta = 2\alpha = \frac{2(126.87)}{180}\pi = 4.429 \text{ rad}$$

$$\frac{T_2}{T_1} = e^{\mu\beta}, \quad \frac{160}{40} = e^{4.429\mu}$$

$$4.429\mu = 1.3863, \quad \underline{\mu = 0.313}$$

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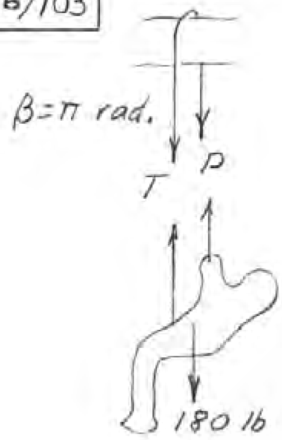

$T = Pe^{\mu\beta}$

$= Pe^{0.6\pi} = 6.59\,P$

Man; $\Sigma F = 0$; $T + P = 180$

Combine & get

$P(1 + 6.59) = 180$ lb

$P = \frac{180}{7.59} = \underline{23.7\text{ lb}}$

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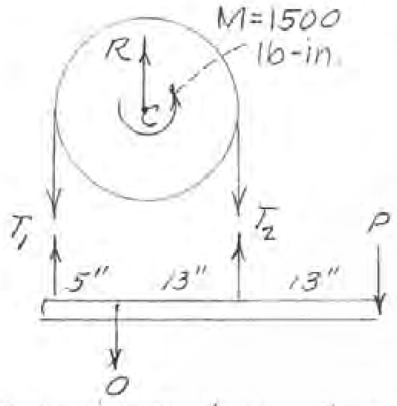
6/104 $T_2/T_1 = e^{\mu\beta} = e^{0.20\pi} = 1.874$ ----(a)



$\Sigma M_C = 0;\ (T_2 - T_1)\,9 = 1500$

$T_2 - T_1 = 166.7$ --- (b)

Lever: $\Sigma M_O = 0;\ 26P + 5T_1 - 13T_2 = 0$ ---(c)

Combine (a) & (b) & get

$T_2 = 1.874(T_2 - 166.7),\ T_2 = 357$ lb.

$T_1 = 357 - 166.7 = 190.7$ lb

Substitute in (c) & get

$26P + 5(190.7) - 13(357) = 0$, $\underline{P = 142.0 \text{ lb}}$

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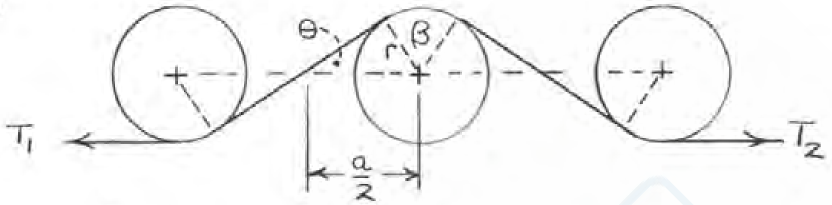

$\theta = \sin^{-1} \dfrac{r}{a/2}$, $\beta = 2\theta = 2\sin^{-1} \dfrac{2r}{a}$

$\dfrac{T_2}{T_1} = e^{\mu\beta}$: $1.15 = e^{0.1\beta}$, $\beta = 1.398$ rad or $80.1°$

Thus $80.1° = 2\sin^{-1} \dfrac{2(20)}{a}$, $\underline{a = 62.2 \text{ mm}}$

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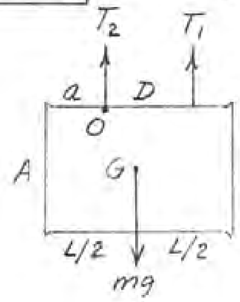
6/106 Slipping impends for rope when $T_2 = T_1 e^{\mu\beta}$ (1)



Equil. of drum

$\Sigma F = 0;\ T_1 + T_2 = mg$ ----------- (2)

$\Sigma M_O = 0;\ mg\left(\frac{L}{2} - a\right) = T_1 D$ --------- (3)

(2) & (3) $T_1\left(\frac{L}{2} - a - D\right) = T_2\left(a - \frac{L}{2}\right)$

Combine with (1) & get

$\dfrac{\frac{L}{2} - a - D}{a - \frac{L}{2}} = e^{\mu\beta}$, $\underline{a = \frac{L}{2} - \frac{D}{1 + e^{\mu\pi}}}$ where $\beta = \pi$ rad.

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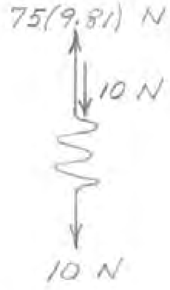
6/107 $L = 75(9.81)\ N, \quad T_2 = T_1 e^{\mu\beta},$



$$75(9.81) - 10 = 10e^{\mu(3+\frac{1}{2})2\pi}$$

$$72.6 = e^{21.99\mu}$$

$$21.99\mu = 4.285, \quad \underline{\mu = 0.1948}$$

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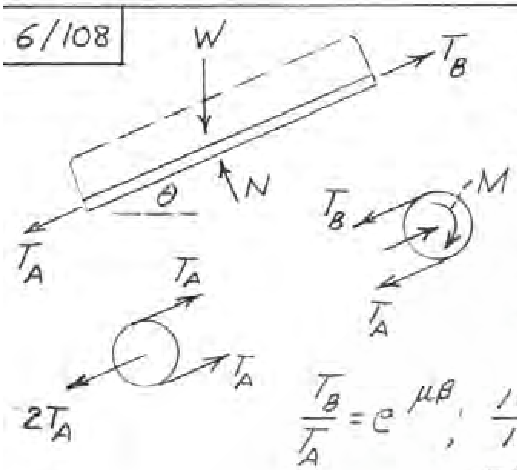

$\theta = \tan^{-1}\frac{20}{50} = 21.8°$

$\sin\theta = 0.3714$

$W\sin\theta = 30(150)(0.3714)$
$= 1671$ lb

$T_B = 1000 + \frac{1671}{2} = 1836$ lb

$T_A = 1000 - \frac{1671}{2} = 164$ lb

$\frac{T_B}{T_A} = e^{\mu\beta}$, $\frac{1836}{164} = e^{\pi\mu} = 11.17$

$\pi\mu = \ln 11.17 = 2.413$

$\mu = \frac{2.413}{\pi} = \underline{0.768}$

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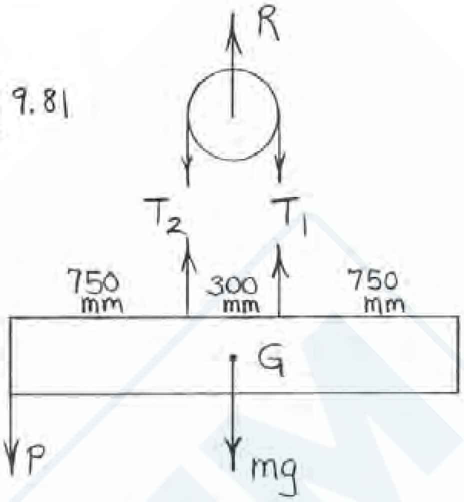
6/109

$mg = 74(0.75 + 0.30 + 0.75)\,9.81$

$= 1307\text{ N}$



Beam $+\uparrow \Sigma F = 0: \; T_1 + T_2 - P - 1307 = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_G = 0: \; P\left(0.75 + \frac{0.30}{2}\right) + \frac{0.30}{2} T_1 - \frac{0.30}{2} T_2 = 0 \qquad (2)$

Drum: $T_2 = T_1 e^{\mu\beta} = T_1 e^{0.5\pi} \qquad (3)$

Solve Eqs. (1), (2), and (3): $\begin{cases} T_1 = 252\text{ N} \\ T_2 = 1215\text{ N} \\ P = 160.3\text{ N} \end{cases}$

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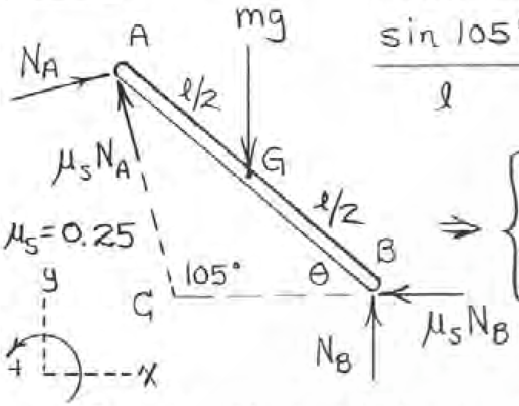

From law of sines:

$$\frac{\sin 105^\circ}{l} = \frac{\sin\theta}{\overline{AC}} = \frac{\sin(75^\circ-\theta)}{\overline{BC}}$$

$$\Rightarrow \begin{cases} \overline{AC} = \dfrac{l\sin\theta}{\sin 105^\circ} \\ \overline{BC} = \dfrac{l\sin(75^\circ-\theta)}{\sin 105^\circ} \end{cases}$$

$\Sigma F_x = 0: \; N_A \cos 15^\circ - 0.25 N_A \sin 15^\circ - 0.25 N_B = 0$

$\Sigma F_y = 0: \; N_A \sin 15^\circ + 0.25 N_A \cos 15^\circ + N_B - mg = 0$

$$\Sigma M_C = 0: \; -N_A\left(\frac{l\sin\theta}{\sin 105^\circ}\right) + N_B\left(\frac{l\sin(75^\circ-\theta)}{\sin 105^\circ}\right) - mg\left[\frac{l\sin(75^\circ-\theta)}{\sin 105^\circ} - \frac{l}{2}\cos\theta\right] = 0$$

Solution:
$$\begin{cases} N_A = 0.244mg \\ N_B = 0.878mg \\ \theta = 59.9^\circ \end{cases}$$

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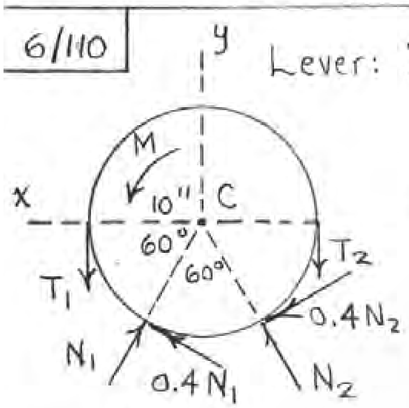


Lever: $\Sigma M_O = 0: \quad \dfrac{4T_2}{\sqrt{2}} = 25\left(15 + \dfrac{4}{\sqrt{2}}\right)$

$T_2 = 157.6 \text{ lb}$

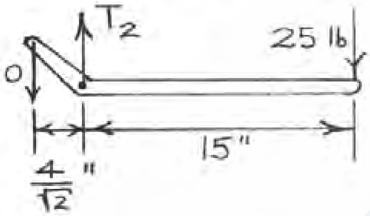
Band: $\dfrac{T_2}{T_1} = e^{\mu\beta}$

$\dfrac{157.6}{T_1} = e^{0.3\pi}, \quad T_1 = 61.4 \text{ lb}$

Pipe: $\Sigma F_y = 0:$

$N_1\left(\dfrac{\sqrt{3}}{2} + 0.4\left(\dfrac{1}{2}\right)\right) + N_2\left(\dfrac{\sqrt{3}}{2} - 0.4\left(\dfrac{1}{2}\right)\right)$

$- 157.6 - 61.4 = 0 \qquad (a)$



$\Sigma F_x = 0: \quad N_1\left(\dfrac{1}{2} - 0.4\dfrac{\sqrt{3}}{2}\right)$

$- N_2\left(\dfrac{1}{2} + 0.4\dfrac{\sqrt{3}}{2}\right) = 0 \qquad (b)$

Solve (a) & (b): $N_1 = 184.5 \text{ lb}, \quad N_2 = 33.5 \text{ lb}$

$\circlearrowleft + \Sigma M_C = 0: \quad M - 157.6(10) + 61.4(10)$

$- 0.4(184.5 + 33.5)(10) = 0, \qquad \underline{M = 1834 \text{ lb-in.}}$

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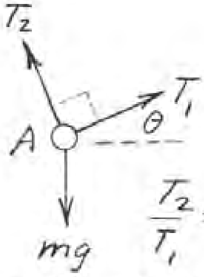

Equilibrium of A gives

$T_1 = mg \sin\theta$

$T_2 = mg \cos\theta$

$\frac{T_2}{T_1} = e^{\mu\beta}, \quad \frac{mg \cos\theta}{mg \sin\theta} = e^{3\pi\mu/2}$

For $\theta = 20°$, $\text{ctn } 20° = e^{3\pi\mu/2} = 2.747$

or $\mu = \frac{2}{3\pi} \ln 2.747 = \underline{0.214}$

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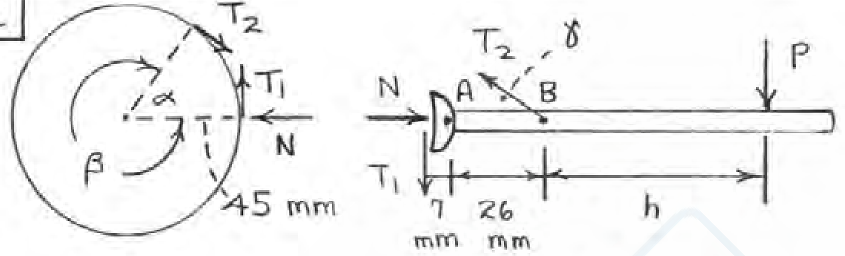

$\cos\alpha = \dfrac{45}{45+7+26}$, $\alpha = 54.8°$, $\gamma = 180-90-\alpha = 35.2°$

$\beta = 360-\alpha = 305°$ or 5.33 rad

$\dfrac{T_2}{T_1} = e^{\mu\beta} = e^{0.25(5.33)} = 3.79 \qquad (a)$

Bar: $\Sigma M_P = 0$: $T_1(h+7+26) - T_2 \sin\gamma\,(h) = 0$

or $\dfrac{T_2}{T_1} = \dfrac{h+33}{0.577h} \qquad (b)$

From (a) & (b), $\underline{h = 27.8 \text{ mm}}$

(For actual wrench, $h \cong 100$ mm)

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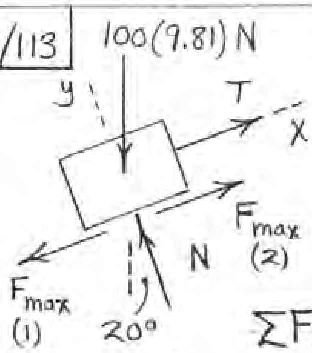

$\Sigma F_y = 0: \ N - 100(9.81)\cos 20° = 0$

$N = 922 \text{ N}$ (throughout)

(a) No cord friction: $T = mg$

(1) Motion impends up incline:

$\Sigma F_x = 0: -0.3(922) - 981\sin 20° + T = 0$

$(F_{max} = 0.3N)$ $\quad T = 612 \text{ N}, \quad m = \frac{612}{9.81} = 62.4 \text{ kg}$

(2) Motion impends down the incline:

$\Sigma F_x = 0: \ 0.3(922) - 981\sin 20° + T = 0, \quad T = 59.0 \text{ N}$

$m = \frac{59.0}{9.81} = 6.01 \text{ kg}$; so $\underline{6.01 \text{ kg} \le m \le 62.4 \text{ kg}}$

(b) Cord friction: $T \neq mg$, $\beta = 110°\left(\frac{\pi}{180°}\right) = 1.920$ rad

(1) Motion impends up: $\frac{mg}{612} = e^{0.3(1.920)}$, $m = 111.0 \text{ kg}$

(2) Motion impends down: $\frac{59.0}{mg} = e^{0.3(1.920)}$, $m = 3.38 \text{ kg}$

So $\underline{3.38 \text{ kg} \le m \le 111.0 \text{ kg}}$

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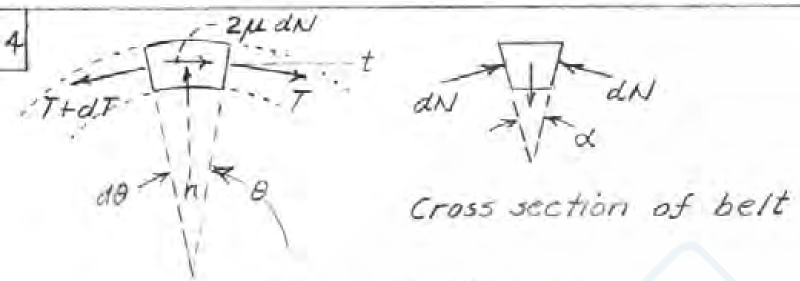

$\Sigma F_n = 0;\quad T\sin\frac{d\theta}{2} + (T+dT)\sin\frac{d\theta}{2} = 2\,dN\,\sin\frac{\alpha}{2}$

or $\quad T\,d\theta = 2\,dN\,\sin\frac{\alpha}{2}$

$\Sigma F_t = 0;\quad T\cos\frac{d\theta}{2} + 2\mu\,dN = (T+dT)\cos\frac{d\theta}{2}$

or $\quad 2\mu\,dN = dT$

combine & get $\quad \frac{dT}{T} = \frac{\mu}{\sin\frac{\alpha}{2}}\,d\theta$

$\int_{T_1}^{T_2}\frac{dT}{T} = \frac{\mu}{\sin\frac{\alpha}{2}}\int_0^\beta d\theta,\quad \ln\frac{T_2}{T_1} = \frac{\mu\beta}{\sin\frac{\alpha}{2}},\quad \underline{T_2 = T_1 e^{\mu\beta/\sin\frac{\alpha}{2}}}$

$n = 1/\sin 17.5^\circ = \underline{3.33}$

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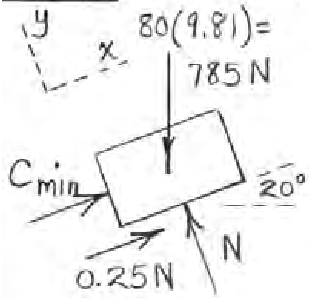
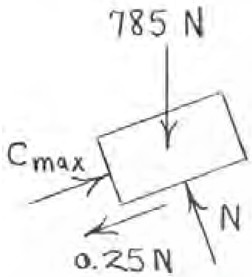


$\Sigma F_y = 0: \quad N - 785 \cos 20° = 0$

$N = 737 \text{ N}$ Throughout

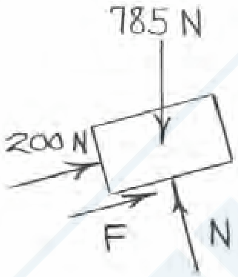
(a) C_{min}

$\Sigma F_x = 0: \; C_{min} + 0.25(737) - 785 \sin 20° = 0, \quad \underline{C_{min} = 84.0 \text{ N}}$



(b) C_{max}

$\Sigma F_x = 0: \; C_{max} - 0.25(737) - 785 \sin 20° = 0, \quad C_{max} = \underline{453 \text{ N}}$



(c) $C = 200$ N

$\Sigma F_x = 0: \; 200 + F - 785 \sin 20° = 0$

$\underline{F = 68.4 \text{ N up the incline}}$

($F < \mu_s N = 0.25(737) = 184.4$ N, so no motion occurs)

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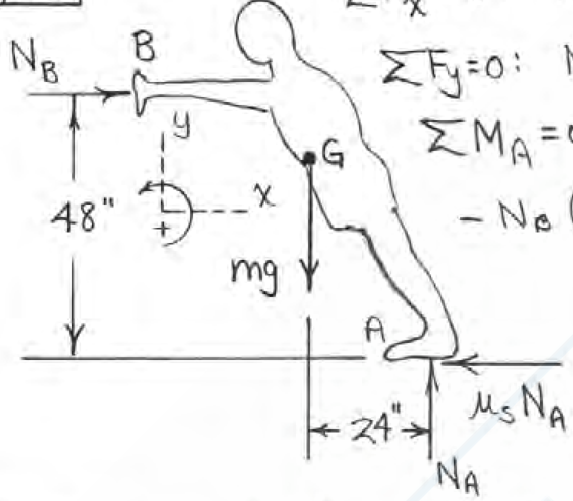
$\Sigma F_x = 0: \quad N_B - \mu_s N_A = 0 \quad (1)$

$\Sigma F_y = 0: \quad N_A - mg = 0 \quad (2)$

$\Sigma M_A = 0:$

$-N_B(48) + mg(24) = 0 \quad (3)$



Solve Eqs. (1)-(3) to obtain

$N_A = mg$

$N_B = \frac{1}{2} mg$

$\mu_s = \frac{1}{2}$

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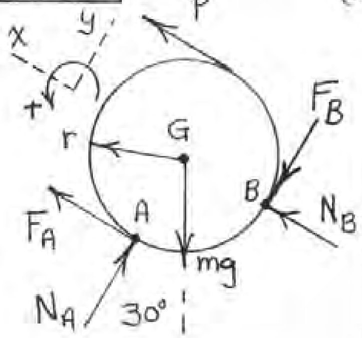

(1) Assume no slippage until contact at B is lost:

$F_B = N_B = 0$

$\Sigma F_x = 0: \quad P + F_A - mg \sin 30° = 0$

$\Sigma F_y = 0: \quad N_A - mg \cos 30° = 0$

$\Sigma M_G = 0: \quad Pr - F_A r = 0$

Solution: $P = F_A = \frac{mg}{4}, \quad N_A = 0.866 mg$

$(F_A)_{max} = \mu_s N_A = 0.25(0.866 mg) = 0.217 mg < F_A$; Assumption invalid

(2) Assume rotational slippage impends:

$F_A = \mu_s N_A = 0.25 N_A, \quad F_B = \mu_s N_B = 0.25 N_B$

$\Sigma F_x = 0: \quad P + 0.25 N_A - mg \sin 30° + N_B = 0$

$\Sigma F_y = 0: \quad N_A - mg \cos 30° - 0.25 N_B = 0$

$\Sigma M_G = 0: \quad Pr - 0.25 N_A r - 0.25 N_B r = 0$

Solution: $P = 0.232 mg, \quad N_A = 0.878 mg, \quad N_B = 0.0487 mg$

So rotational slippage occurs first, at $P = 0.232 mg$

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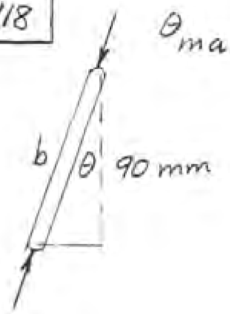

$\theta_{max} = \phi = \tan^{-1}\mu = \tan^{-1} 0.40 = 21.80°$

$b = 90/\cos 21.80° = \underline{96.9 \text{ mm}}$

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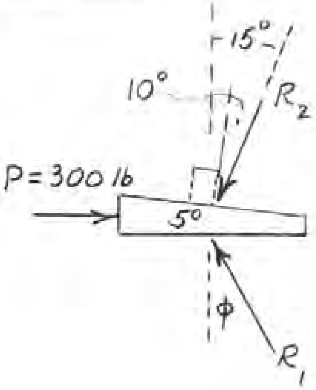
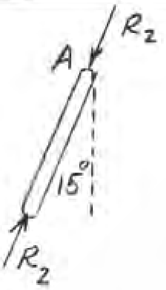
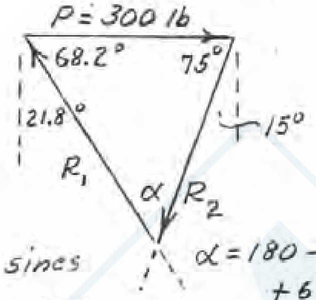




Wedge $\phi = \tan^{-1} 0.40 = 21.80°$



$\alpha = 180 - (75 + 68.2) = 36.8°$

Law of sines

$$\frac{R_2}{\sin 68.2} = \frac{300}{\sin 36.8°}$$

$$R_2 = 300 \, \frac{0.9285}{0.5990} = 465 \text{ lb}$$

Friction force at A is $F = R_2 \sin 15° = 465(0.2588)$

$= \underline{120.3 \text{ lb}}$

$10° < (\tan^{-1} 0.4 = 21.8°)$ so toggle does not slip & $F \neq \mu N$.

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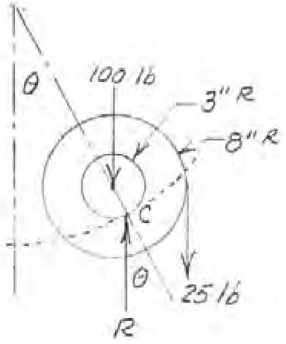

$\Sigma M_C = 0$; $100(3 \sin\theta) - 25(8 - 3\sin\theta) = 0$

$\sin\theta = 8/15$, $\underline{\theta = 32.2°}$

$\mu_{min} = \tan\theta = \tan 32.2°$

$= \underline{0.630}$

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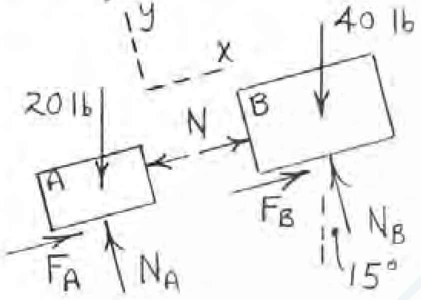
6/120 From $\theta_{max} = \tan^{-1}\mu_s$, we have

$(\theta_{max})_A = \tan^{-1} 0.30 = 16.70°$

$(\theta_{max})_B = \tan^{-1} 0.20 = 11.31°$

$(\theta_{max})_C = \tan^{-1} 0.35 = 19.29°$

So C remains stationary. By themselves, B would slide, A would not.



From $\Sigma F_y = 0$:

$N_A = 19.32$ lb

$N_B = 38.6$ lb

Assume that slipping impends for B:

$\Sigma F_x = 0$: $N + 0.2(38.6) - 40 \sin 15° = 0$, $N = 2.63$ lb

$\Sigma F_x = 0$ for A: $F_A - 20 \sin 15° - 2.63 = 0$

$F_A = 7.80$ lb; $(F_A)_{max} = 0.30(19.32) = 5.80$ lb

Because $(F_A)_{max} < F$, both A and B slip.

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6/121 Friction angle $\phi = \tan^{-1}\mu = \tan^{-1} 0.15 = 8.53°$

$\tan\alpha = \frac{L}{2\pi r}$; Critical when $\alpha = \phi$

$\therefore$ Lead $L = 2\pi r \tan\phi = 2\pi \frac{3/8}{2} \tan 8.53°$

$= 0.1767$ in. per revolution

$N = 1/L = 1/0.1767 = $ 5.66 threads per inch

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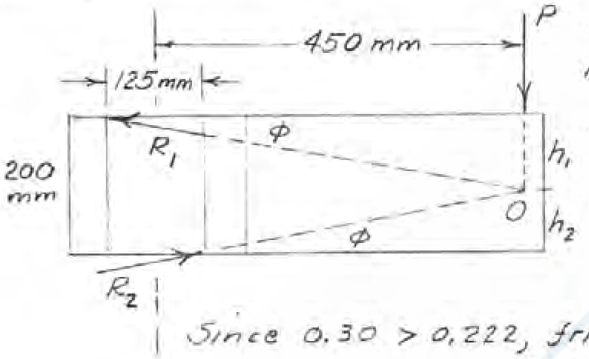
6/122 Forces concurrent at O gives minimum ϕ



$h_1 = (450 + 62.5) \tan\phi$

$h_2 = (450 - 62.5) \tan\phi$

$h_1 + h_2 = 200$ mm

$(512.5 + 387.5) \tan\phi = 200$

$\tan\phi = \dfrac{200}{900} = 0.222$

Since $0.30 > 0.222$, friction will prevent slipping

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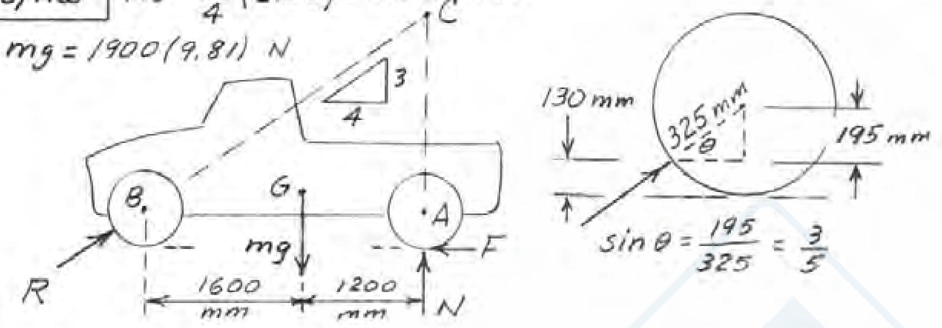
6/123 $\overline{AC} = \frac{3}{4}(2800) = 2100$ mm

$mg = 1900(9.81)$ N



$\sin\theta = \frac{195}{325} = \frac{3}{5}$

$\Sigma M_C = 0;\ F(2100 + 325) - 1200\,mg = 0,\ F = 0.495\,mg$

$\Sigma M_B = 0;\ 1600\,mg + 0.495\,mg\,(325) - 2800\,N = 0,\ N = 0.629\,mg$

Thus $\mu_{min} = F/N = 0.495/0.629 = \underline{0.787}$

$M = Fr = 0.495(1900)(9.81)(0.325) = 2998$ N·m

or $\underline{M = 3.00 \text{ kN·m}}$

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$$\phi = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.31°$$

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{2.5}{2\pi(5)} = 4.55°$$

(a) Tighten: $M = Pr\tan(\phi + \alpha)$

$$F(100) = 600\left(\frac{10}{2}\right)\tan(11.31° + 4.55°)$$

$$\underline{F = 8.52\ N}$$

(b) Loosen: $M = Pr\tan(\phi - \alpha)$

$$F(100) = 600\left(\frac{10}{2}\right)\tan(11.31° - 4.55°)$$

$$\underline{F = 3.56\ N}$$

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6/125 Helix angle $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{8}{2\pi\frac{25}{2}} = 5.82°$

$\phi = \tan^{-1}0.25 = 14.04°$

Screw: (a) $M_s = Wr\tan(\alpha+\phi) = 4(\frac{25}{2})\tan 19.86°$

$= 18.05$ N·m

(b) $M_s = Wr\tan(\phi-\alpha) = 4(\frac{25}{2})\tan 8.22°$

$= 7.22$ N·m

Bearing (worn) $M_B = \frac{1}{2}\mu P(R_o+R_i) = \frac{1}{2}(0.25)(4)\frac{20+4}{2}$

$= 6.00$ N·m

Total moment (a) $M = 18.05 + 6.00 = \underline{24.1 \text{ N·m}}$

(b) $M = 7.22 + 6.00 = \underline{13.22 \text{ N·m}}$

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6/126 Helix angle $\alpha = \tan^{-1}\frac{24}{40\pi} = 10.81°$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1} 0.15 = 8.53°$

$\alpha > \phi$ so screw is not self-locking.

$\alpha + \phi = 19.34°$; $\alpha - \phi = 2.28°$

(a) $M = Pr\tan(\alpha - \phi)$: $60 = P(0.020)\tan 2.28°$

$P = 75\,300$ N or $\underline{75.3 \text{ kN}}$

(b) $M = Pr\tan(\alpha + \phi)$: $60 = P(0.020)\tan 19.34°$

$P = 8550$ N or $\underline{8.55 \text{ kN}}$

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$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{13}{2\pi(78)} = 3.04°$$

$$\phi = \tan^{-1}\mu = \tan^{-1}0.25 = 14.04°$$

(a) To raise, $M = Wr\tan(\alpha + \phi)$

$$= \frac{2.2(10^3)9.81}{2}\frac{(0.078)}{2}\tan(3.04° + 14.04°)$$

$$= \underline{129.3\ N\cdot m}$$

(b) To lower, $M = Wr\tan(\phi - \alpha)$

$$= \frac{2.2(10^3)9.81}{2}\frac{(0.078)}{2}\tan(14.04 - 3.04)$$

$$= \underline{81.8\ N\cdot m}$$

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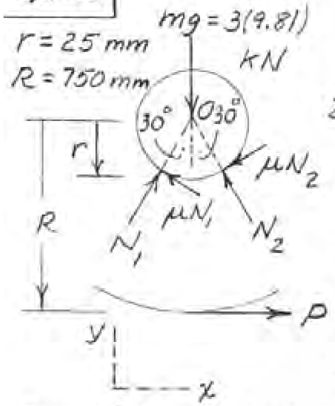

$\Sigma M_O = 0$; $PR = \mu r(N_1 + N_2)$ (a)

$\Sigma F_x = 0$; $P + (N_1 - N_2)\frac{1}{2} = \mu(N_1 + N_2)\frac{\sqrt{3}}{2}$ (b)

$\Sigma F_y = 0$; $mg = (N_1 + N_2)\frac{\sqrt{3}}{2} + \mu(N_1 - N_2)\frac{1}{2}$ (c)

Combine (a) & (c) to give

$$\frac{1}{2}(N_1 - N_2) = \frac{mg}{\mu} - \frac{\sqrt{3}}{2}\frac{PR}{\mu^2 r} \quad (d)$$

Combine (a) & (b) to give

$$\frac{1}{2}(N_1 - N_2) = -P + \frac{PR}{r}\frac{\sqrt{3}}{2} \quad (e)$$

Combine (d) & (e) & get

$$P = \frac{mg/\mu}{\frac{R}{r}\frac{\sqrt{3}}{2}\left(1 + \frac{1}{\mu^2}\right) - 1} = \frac{3(9.81)/0.30}{\frac{750}{25}\frac{\sqrt{3}}{2}\left(1 + \frac{1}{0.3^2}\right) - 1} = 0.313 \text{ kN}$$

or 313 N

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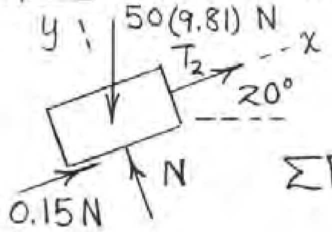
6/129 Motion impending down plane:



$\Sigma F_y = 0: N = 50(9.81)\cos 20°$

$N = 461$ N (throughout)

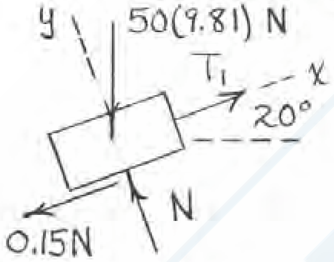
$\Sigma F_x = 0: T_2 - 50(9.81)\sin 20° + 0.15(461) = 0$

$T_2 = 98.6$ N

$T_2 = T_1 e^{\mu\beta}: \quad 98.6 = \frac{mg}{2} e^{0.25(110 \cdot \frac{\pi}{180})}$

$m = 12.44$ kg

Motion impending up plane:



$\Sigma F_x = 0: T_1 - 50(9.81)\sin 20° - 0.15(461) = 0$

$T_1 = 237$ N

$T_2 = T_1 e^{\mu\beta}: \quad \frac{mg}{2} = 237 e^{0.25(110 \cdot \frac{\pi}{180})}$

$m = 78.0$ kg

So range is $12.44 \leq m \leq 78.0$ kg

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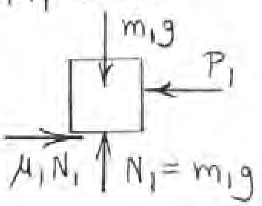
m_1 alone:

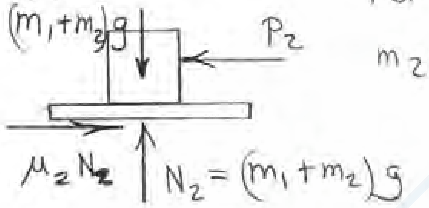


For impending slip between person and board,

$\xrightarrow{+} \Sigma F = 0: \quad \mu_1 m_1 g - P_1 = 0$

$P_1 = \mu_1 m_1 g$

m_1 plus m_2:



For impending slip beneath m_2, $\xrightarrow{+} \Sigma F = 0$:

$-P_2 + \mu_2 (m_1 + m_2) g = 0$

$P_2 = \mu_2 (m_1 + m_2) g$

For first slipping to occur beneath m_2:

$P_2 < P_1$

$\mu_2 (m_1 + m_2) g < \mu_1 m_1 g$

$$\mu_2 < \frac{m_1}{m_1 + m_2} \mu_1$$

Numbers: $\mu_2 < \frac{80}{80+10}(0.60) = 0.533$

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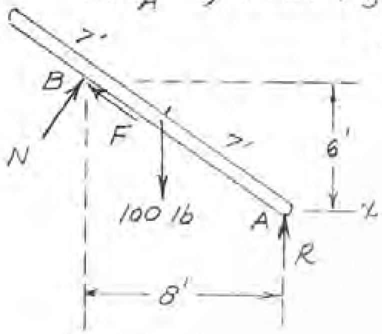
Assume equilibrium

$\Sigma M_A = 0;\ 100(7)\frac{4}{5} - N(10) = 0,\ N = 56\ lb$



$\Sigma F_x = 0;\ \frac{4}{5}F - \frac{3}{5}(56) = 0$

$F = 42\ lb$

$\mu_{s_{min}} = \frac{F}{N} = \frac{42}{56} = 0.75 < 0.80$

so assumption is OK.

& $F = 42\ lb$

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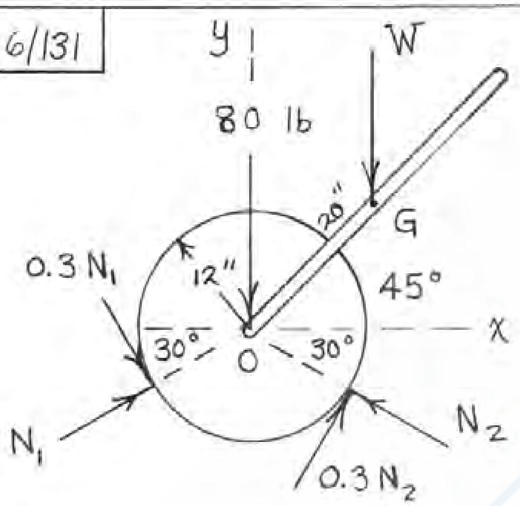

($c = \cos$, $s = \sin$)

$\Sigma F_x = 0$: $N_1 c30° + 0.3N_1 s30° - N_2 c30° + 0.3N_2 s30° = 0$

$\Sigma F_y = 0$: $N_1 s30° - 0.3N_1 c30° + N_2 s30° + 0.3N_2 c30° - 80 - W = 0$

$\Sigma M_O = 0$: $(0.3N_1 + 0.3N_2)(12) - W(20c45°) = 0$

Solve to obtain
$N_1 = 113.9$ lb
$N_2 = 161.6$ lb
$W = 70.1$ lb

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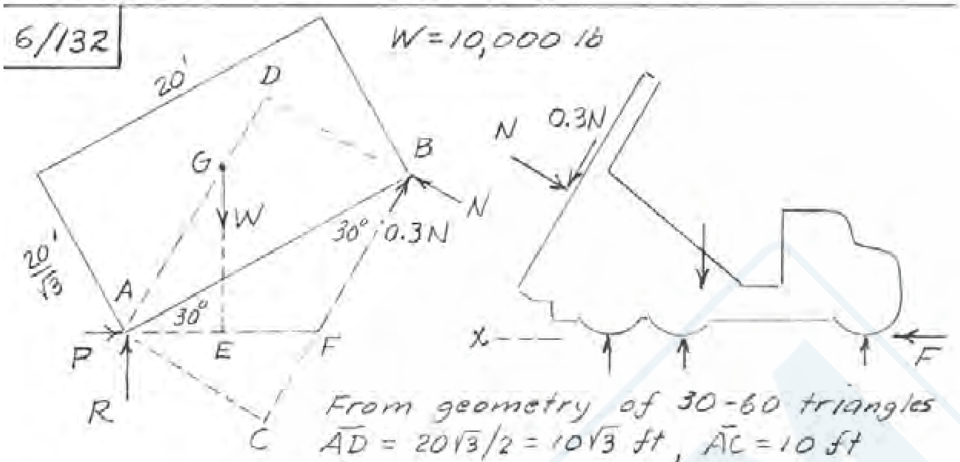

From geometry of 30-60 triangles

$\overline{AD} = 20\sqrt{3}/2 = 10\sqrt{3}$ ft, $\overline{AC} = 10$ ft

$\overline{AE} = \frac{1}{2}\overline{AF} = \frac{1}{2}\,10/\frac{\sqrt{3}}{2} = 10/\sqrt{3}$ ft.

Box:

$\Sigma M_A = 0;\quad 10{,}000(10/\sqrt{3}) - 10\sqrt{3}\,N - 10(0.3N) = 0$

$N = 2840$ lb

Truck:

$\Sigma F_x = 0;\quad F = N\left(\frac{\sqrt{3}}{2}\right) - 0.3N\left(\frac{1}{2}\right) = 0.7160(2840) = \underline{2030 \text{ lb}}$

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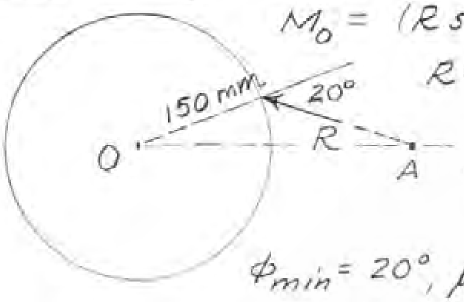
$M_O = PL = 150(0.600) = 90$ N·m



$M_O = (R \sin 20°)(0.150) = 90$

$R = \dfrac{90}{0.3420(0.150)} = 1754$ N

Force on pin at A is

$\underline{R = 1.754 \text{ kN}}$

$\phi_{min} = 20°$, $\mu_{s(min)} = \tan\phi_{min} = \tan 20°$

$= \underline{0.364}$

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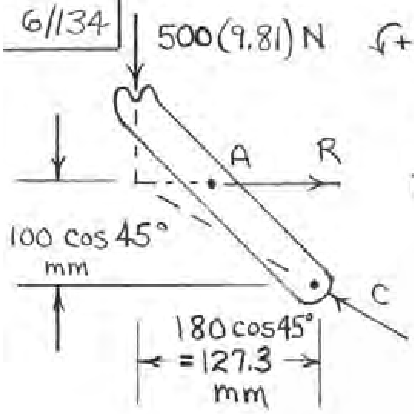

$\curvearrowleft + \sum M_C = 0: \; 500(9.81)(127.3) - R(100\cos 45°) = 0, \; R = 8830\text{ N}$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1}(0.20) = 11.31°$

$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{0.002}{2\pi(0.005)} = 3.64°$

Raise load: $M = Rr\tan(\phi + \alpha)$

$P(0.150) = 8830(0.005)\tan(11.31° + 3.64°)$

$\underline{P = 78.6\text{ N}}$

Lower load: $M = Rr\tan(\phi - \alpha)$

$P(0.150) = 8830(0.005)\tan(11.31° - 3.64°)$

$\underline{P = 39.6\text{ N}}$

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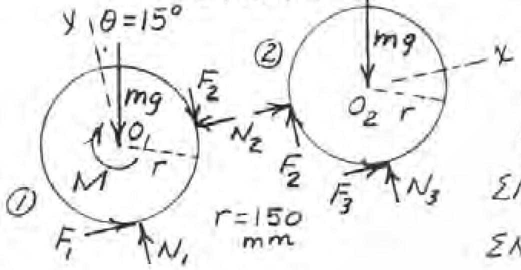
6/135 $mg = 20(9.81) = 196.2\ N$ $\mu_s = 0.60$, $\mu_k = 0.50$



Assume slipping occurs only at contact between cylinders

so $F_2 = \mu_k N_2 = \mu N_2$

$\Sigma M_{O_1} = 0$; $M = (F_1 - F_2) r$

$\Sigma M_{O_2} = 0$; $F_2 = F_3$

① $\Sigma F_x = 0$; $F_1 - N_2 - mg \sin\theta = 0$

$\Sigma F_y = 0$; $N_1 - F_2 - mg \cos\theta = 0$

② $\Sigma F_x = 0$; $F_3 + N_2 - mg \sin\theta = 0$

$\Sigma F_y = 0$; $N_3 + F_2 - mg \cos\theta = 0$

7 eqs. in 7 unknowns

Solve, substitute values & get

$F_2 = \frac{\mu}{1+\mu} mg \sin\theta = F_3 = 16.93\ N$

$F_1 = \frac{2+\mu}{1+\mu} mg \sin\theta = 84.6\ N$

$N_1 = 206.4\ N$

$N_2 = 33.9\ N$

$N_3 = 172.6\ N$

$F_1 < \mu_s N_1$ & $F_3 < \mu_s N_3$ so assumption valid ✓

$M = (84.6 - 16.93)(0.150) = \underline{10.16\ N \cdot m}$

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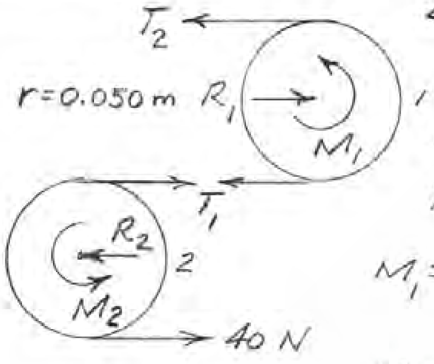

$$\frac{T_1}{40} = e^{0.2(\pi)} = 1.874 \quad ; \quad \frac{T_1}{T_2} = e^{0.3(\pi)} = 2.566$$

$$T_1 = 40(1.874) = 74.98 \text{ N}$$

$$T_2 = 74.98/2.566 = 29.22 \text{ N}$$

$$M_1 = (T_1 - T_2) r = (74.98 - 29.22)(0.050) = \underline{2.29 \text{ N·m}}$$

$$M_2 = (T_1 - 40) r = (74.98 - 40)(0.050) = \underline{1.75 \text{ N·m}}$$

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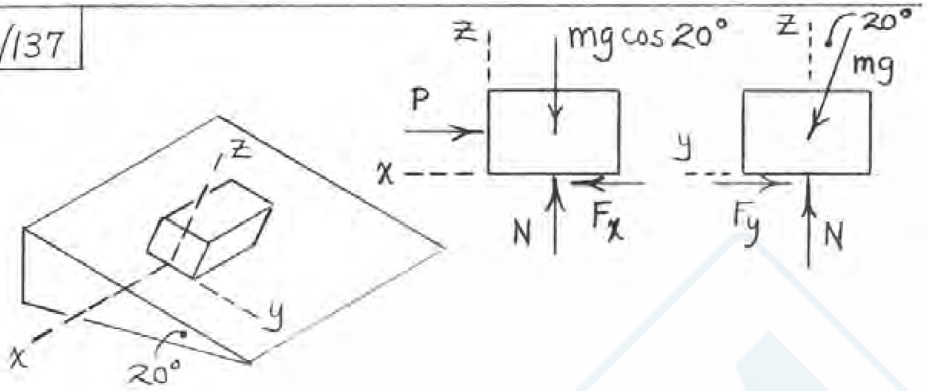

(x - z) $\begin{cases} \Sigma F_z = 0: \; N - 8(9.81)\cos 20° = 0, & N = 73.7 \text{ N} \\ \Sigma F_x = 0: \; F_x - P = 0, & F_x = P \end{cases}$

(y - z) $\{ \Sigma F_y = 0: -F_y + 8(9.81)\sin 20° = 0, \; F_y = 26.8 \text{ N}$

$F = \sqrt{F_x^2 + F_y^2} = \mu_s N: \sqrt{P^2 + 26.8^2} = 0.5(73.7)$

$\underline{P = 25.3 \text{ N}}$

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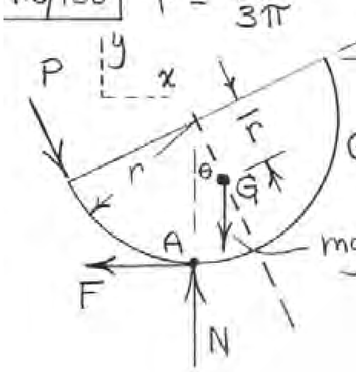
*6/138 $\bar{r} = \frac{4r}{3\pi}$

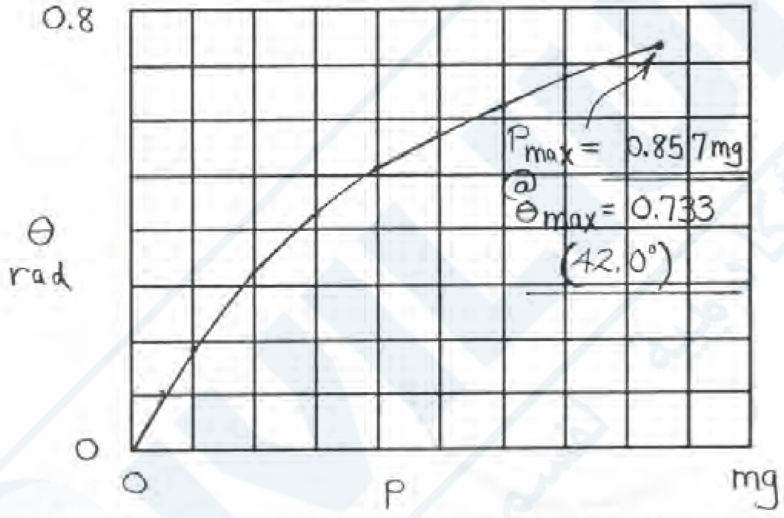


$\Sigma F_x = 0: \; P\sin\theta - F = 0 \quad (1)$

$\Sigma F_y = 0: \; N - mg - P\cos\theta = 0 \quad (2)$

$\circlearrowleft \Sigma M_G = 0: \; Pr - N\bar{r}\sin\theta - F(r - \bar{r}\cos\theta) = 0 \quad (3)$

Vary P and numerically solve Eqs. (1), (2), and (3) for θ, F, and N. Plot for θ:



(Note: Could vary θ & solve for P; then plot!)

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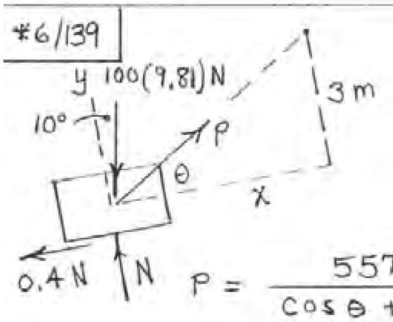


$\Sigma F_x = 0:$

$$P\cos\theta - 0.4N - 981\sin 10° = 0$$

$\Sigma F_y = 0:$

$$N + P\sin\theta - 981\cos 10° = 0$$

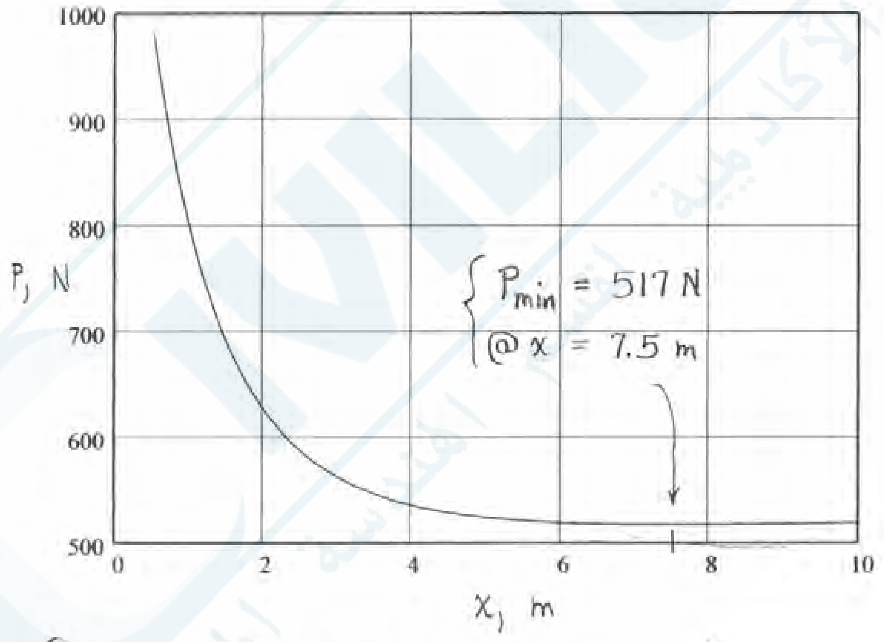
$$P = \frac{557}{\cos\theta + 0.4\sin\theta}, \quad N = \frac{-170.3\sin\theta + 966\cos\theta}{\cos\theta + 0.4\sin\theta}$$

$$\frac{dP}{d\theta} = \frac{-557(-\sin\theta + 0.4\cos\theta)}{(\cos\theta + 0.4\sin\theta)^2} = 0$$

$$\Rightarrow -\sin\theta + 0.4\cos\theta = 0 \text{ or } \tan\theta = 0.4, \quad \theta = 21.8°$$

From $\tan\theta = \frac{3}{x}$, $x = \frac{3}{\tan\theta} = \frac{3}{0.4} = \underline{7.5 \text{ m}}$

At $\theta = 21.8°$, $\underline{P = 517 \text{ N}}$



(Note: $N \to 0$ @ $x = 0.529$ m)

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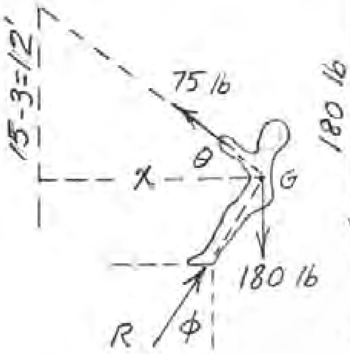

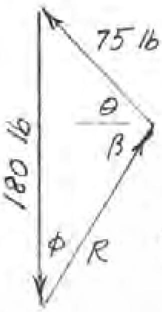

$\phi = \tan^{-1} 0.40 = 21.8^\circ$

$\beta = 90 - 21.8 = 68.2^\circ$

Law of sines

$$\frac{180}{\sin(\theta+\beta)} = \frac{75}{\sin 21.8^\circ}$$

$$\theta + \beta = \sin^{-1} \frac{180 \sin 21.8}{75}$$

$$= \sin^{-1} 0.891 = 63.0^\circ \text{ or } 117.0^\circ$$

63.0° sol. not possible,

so $\theta = 117.0 - 68.2 = 48.8^\circ$

$\frac{12}{x} = \tan 48.8^\circ$, $x = 12/1.14 = \underline{10.52 \text{ ft}}$

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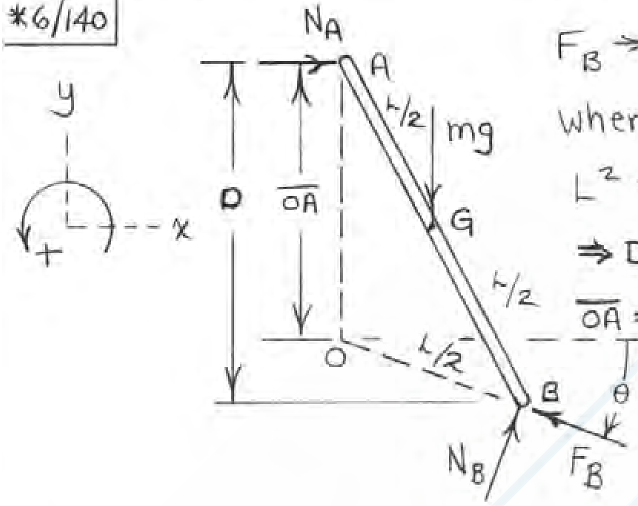

$F_B \rightarrow \mu_s N_B = 0.4 N_B$ when slipping impends.

$L^2 = D^2 + \left(\frac{L}{2}\cos\theta\right)^2$

$\Rightarrow D = L\sqrt{1 - \frac{\cos^2\theta}{4}}$

$\overline{OA} = L\sqrt{1 - \frac{\cos^2\theta}{4}} - \frac{L}{2}\sin\theta$

$= \frac{L}{2}\left[\sqrt{4 - \cos^2\theta} - \sin\theta\right]$

$\Sigma F_x = 0: \; N_A + N_B \sin\theta - 0.4 N_B \cos\theta = 0 \qquad (1)$

$\Sigma F_y = 0: \; N_B \cos\theta + 0.4 N_B \sin\theta - mg = 0 \qquad (2)$

$\Sigma M_O = 0: \; N_B\left(\frac{L}{2}\right) - N_A \frac{L}{2}\left[\sqrt{4 - \cos^2\theta} - \sin\theta\right]$

$- mg\frac{L}{4}\cos\theta = 0 \qquad (3)$

Numerical solution:

$N_A = 0.287 mg$

$N_B = 0.966 mg$

$\theta = 5.80°$

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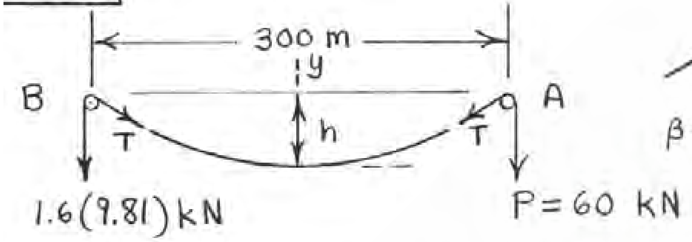

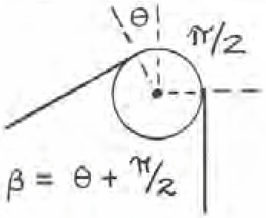

During slipping, Eq. 6/7 @ A: $60 = Te^{\mu_k \beta}$

Eq. 6/7 @ B: $T = 1.6(9.81)e^{\mu_k \beta}$

Eliminate $\mu_k \beta$ & get $\underline{T = 30.7 \text{ kN}}$

Then $e^{\mu_k \beta} = 60/30.7$, $\mu_k \beta = 0.670$

Eq. 5/21: $T = T_0 \cosh \frac{\rho g x}{T_0}$: $30.7 = T_0 \cosh \frac{12(9.81)10^{-3} \cdot 150}{T_0}$

or $30.7 = T_0 \cosh \frac{17.66}{T_0}$. Solve numerically: $T_0 = 23.8$ kN

$\frac{dy}{dx} = \sinh \frac{\rho g x}{T_0}$: $\tan\theta = \sinh \frac{17.66}{23.8} = 0.810$

$\theta = 0.681$ rad, $\beta = \theta + \pi/2 = 2.25$ rad

$\mu_k = \frac{0.670}{2.25} = \underline{0.298}$

Eq. 5/22: $T = T_0 + \rho g h$, $h = \frac{T - T_0}{\rho g}$

$= \frac{30.7 - 23.8}{12(9.81)(10^{-3})} = \underline{58.1 \text{ m}}$

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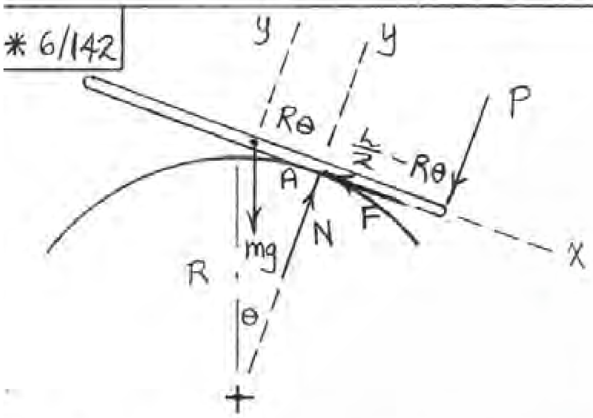

$\Sigma F_x = 0: \quad mg \sin\theta - \mu_s N = 0 \qquad (1)$

$\Sigma F_y = 0: \quad N - mg\cos\theta - P = 0 \qquad (2)$

$\circlearrowleft\!+\ \Sigma M_A = 0: \quad P\left(\frac{L}{2} - R\theta\right) - mg\,R\theta\cos\theta = 0 \qquad (3)$

$(1): \quad N = \dfrac{mg\sin\theta}{\mu_s}$

$(2): \quad \dfrac{mg\sin\theta}{\mu_s} - mg\cos\theta = P$

$(3): \quad mg\left(\dfrac{\sin\theta}{\mu_s} - \cos\theta\right)\left(\dfrac{L}{2} - R\theta\right) - mg\,R\theta\cos\theta = 0$

Simplify: $\tan\theta = \mu_s \dfrac{1}{1 - \frac{2R\theta}{L}} = 0.15\dfrac{1}{1 - 2(0.6)\theta}$

or $\tan\theta - \dfrac{0.15}{1 - 1.2\theta} = 0$. Numerical solution: $\underline{\theta = 11.04°}$

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*6/143 From Prob. 6/142, the equilibrium equations are

$$mg\sin\theta - \mu_s N = 0 \qquad (1)$$

$$N - mg\cos\theta - P = 0 \qquad (2)$$

$$P\left(\frac{L}{2} - R\theta\right) - mgR\theta\cos\theta = 0 \qquad (3)$$

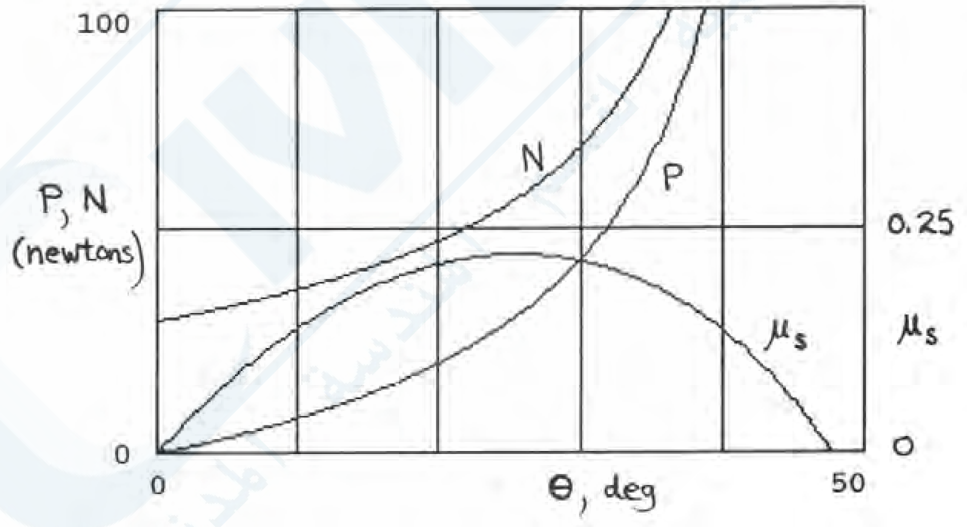
(2) & (3): $(N - mg\cos\theta)\left(\frac{L}{2} - R\theta\right) - mgR\theta\cos\theta = 0$

$$N = mg\cos\theta\,\frac{1}{1-\frac{2R\theta}{L}} = 3(9.81)\,\frac{\cos\theta}{1-\frac{2(1.2)\theta}{2}} = \underline{\frac{29.4\cos\theta}{1-1.2\theta}}$$

(3): $P = mg\,\frac{2R\theta}{L-2R\theta}\cos\theta = \underline{\frac{35.3\theta\cos\theta}{1-1.2\theta}}$

(1) & (2): $\mu_s = \left(1 - \frac{2R\theta}{L}\right)\tan\theta = \underline{(1-1.2\theta)\tan\theta}$

$\underline{\mu_{s_{max}} = 0.222 \text{ @ } \theta = 25.5°}$



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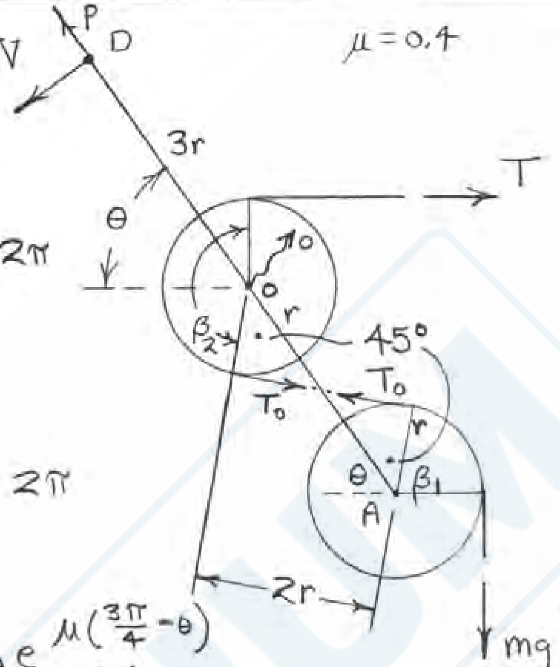
$\mu = 0.4$



From geometry,

upper wheel:

$\theta + \frac{\pi}{4} + \frac{\pi}{2} + \beta_2 = 2\pi$

$\beta_2 = \frac{5\pi}{4} - \theta$

lower wheel:

$\theta + \frac{\pi}{4} + \beta_1 + \pi = 2\pi$

$\beta_1 = \frac{3\pi}{4} - \theta$

Forces: $\begin{cases} mg = T_0 e^{\mu(\frac{3\pi}{4} - \theta)} \\ T_0 = T e^{\mu(\frac{5\pi}{4} - \theta)} \end{cases}$

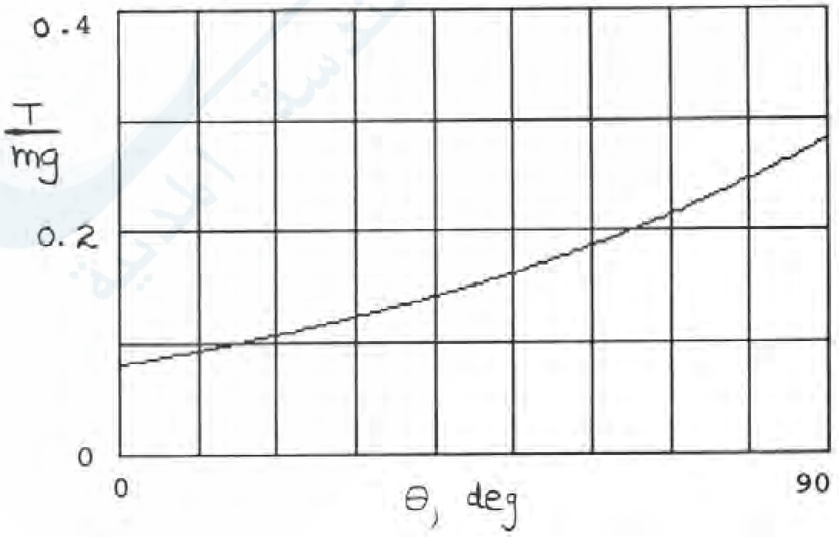
So $mg = Te^{2\mu(\pi - \theta)}$; $\frac{T}{mg} = e^{-0.8(\pi - \theta)}$

Device as a whole:

$\circlearrowleft + \Sigma M_O = 0: \; 3rV - Tr - mg(r + 2r\sqrt{2}\cos\theta) = 0$

$V = \frac{1}{3}\left[T + mg(1 + 2\sqrt{2}\cos\theta)\right]$

For $\theta = 60°$, $\frac{T}{mg} = 0.1872$, $\underline{V = 0.867mg}$



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*6/145 Refer to Article 6/8. $dN = T\,d\theta$

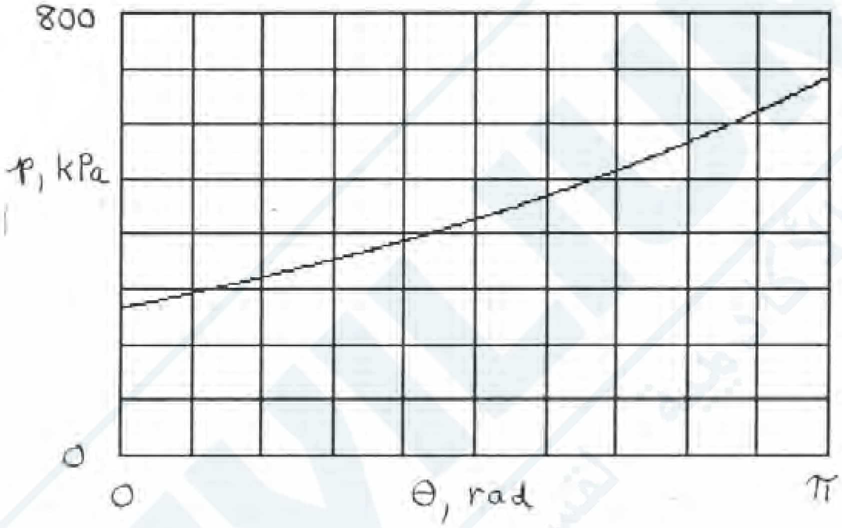
But $dN = pbr\,d\theta$ and $T = T_1 e^{\mu\theta}$ so

$pbr\,d\theta = T_1 e^{\mu\theta} d\theta$. Hence $p = \frac{T_1}{br} e^{\mu\theta}$

Numbers: $p = \frac{1000 e^{0.3\theta}}{(0.050)(0.075)}$ Pa or $p = 267 e^{0.3\theta}$ kPa

$T_2 = T_1 e^{\mu\pi} = 1000 e^{0.3\pi} = \underline{2570 \text{ N}}$

$M = (T_2 - T_1) r = (2570 - 1000)(0.075) = \underline{117.5 \text{ N·m}}$



$\underline{p_1 = 267 \text{ kPa}}$ $\underline{p_2 = 684 \text{ kPa}}$

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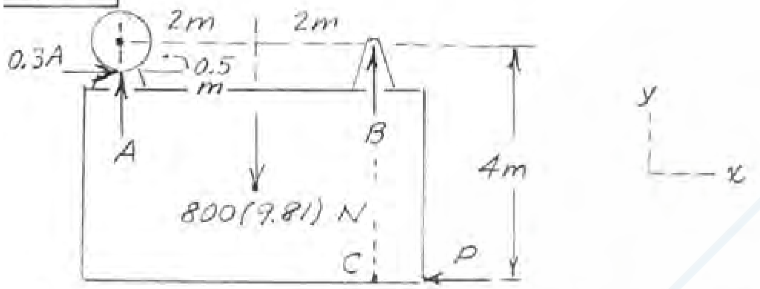

$\Sigma M_C = 0;\ 800(9.81)2 - 4A - 0.3A(4-0.5) = 0,\ A = 3108\text{ N}$

$\Sigma F_x = 0;\ P - 0.3(3108) = 0,\ \underline{P = 932\text{ N}}$

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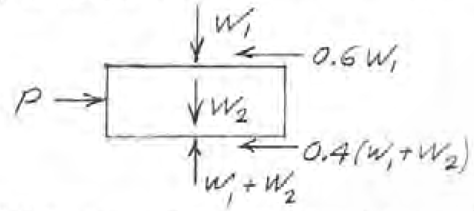
6/16 There are two possibilities

(a) Middle block moves; bottom one does not

$W_1 = 100(9.81) = 981$ N

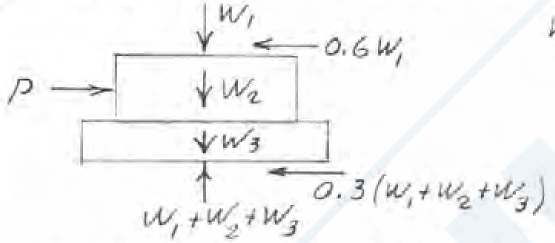
$W_2 = 50(9.81) = 490.5$ N



$\Sigma F = 0;\quad P = 0.6(981) + 0.4(981 + 490.5) = 1177$ N

(b) Bottom block moves with middle block

$W_3 = 20(9.81) = 196.2$ N



$\Sigma F = 0;\quad P = 0.6(981) + 0.3(981 + 490.5 + 196.2) = 1089$ N

1088 < 1177 so case (b) occurs & $\underline{P = 1089 \text{ N}}$

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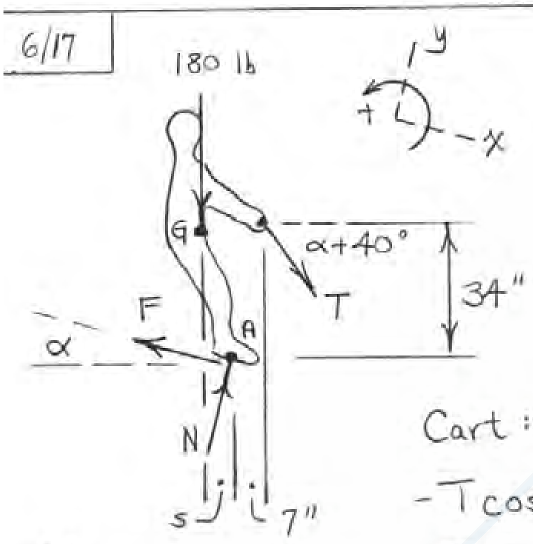

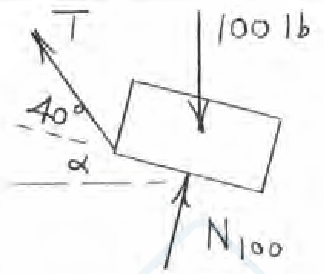

$\alpha = \tan^{-1}\frac{3}{10} = 16.70°$

Cart: $\Sigma F_x = 0$:

$-T\cos 40° + 100 \sin\alpha = 0$

$T = 37.5$ lb

Man:

$\Sigma F_x = 0: \; -F + 180\sin\alpha + 37.5\cos 40° = 0$

$F = 80.5$ lb

$\Sigma F_y = 0: \; N - 180\cos\alpha - 37.5\sin 40° = 0$

$N = 196.5$ lb

$\mu_s = \frac{F}{N} = \frac{80.5}{196.5} = \underline{0.409}$

$\Sigma M_A = 0: \; 180s - 37.5\cos(\alpha + 40°)(34)$

$- 37.5\sin(\alpha + 40°)(7) = 0$

$\underline{s = 5.11 \text{ in.}}$

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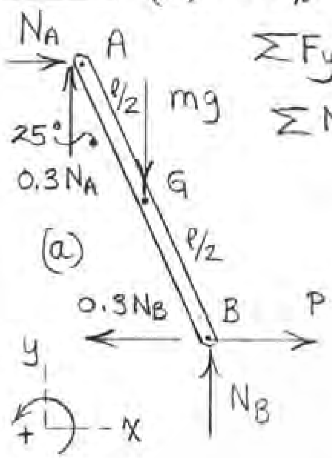
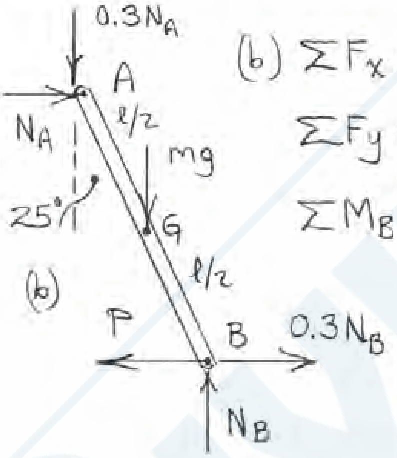


(a) $\Sigma F_x = 0: \; P - 0.3N_B + N_A = 0$

$\Sigma F_y = 0: \; N_B - mg + 0.3N_A = 0$

$\Sigma M_B = 0: \; mg\left(\frac{\ell}{2}\sin 25^\circ\right) - N_A(\ell\cos 25^\circ) - 0.3N_A(\ell \sin 25^\circ) = 0$

Solution: $\begin{cases} N_A = 0.205\,mg \\ N_B = 0.939\,mg \\ P = 0.0771\,mg \end{cases}$



(b) $\Sigma F_x = 0: \; -P + 0.3N_B + N_A = 0$

$\Sigma F_y = 0: \; N_B - mg - 0.3N_A = 0$

$\Sigma M_B = 0: \; mg\left(\frac{\ell}{2}\sin 25^\circ\right) - N_A(\ell\cos 25^\circ) + 0.3N_A(\ell\sin 25^\circ) = 0$

Solution: $\begin{cases} N_A = 0.271\,mg \\ N_B = 1.081\,mg \\ P = 0.595\,mg \end{cases}$

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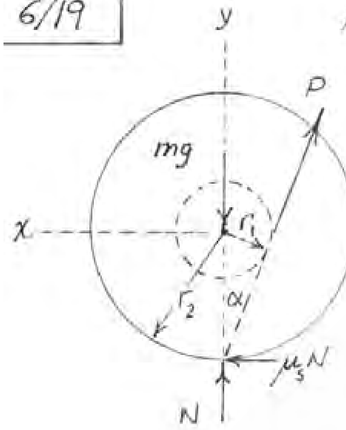

Forces must be concurrent for equilibrium so $\alpha = \sin^{-1}\frac{r_1}{r_2}$

$\Sigma F_y = 0$; $P\cos\alpha + N - mg = 0$

$\Sigma F_x = 0$; $P\sin\alpha - \mu_s N = 0$

Eliminate N & get

$$P = \frac{\mu_s mg}{\sin\alpha + \mu_s\cos\alpha} = \frac{\mu_s mg r_2}{r_1 + \mu_s\sqrt{r_2^2 - r_1^2}}$$

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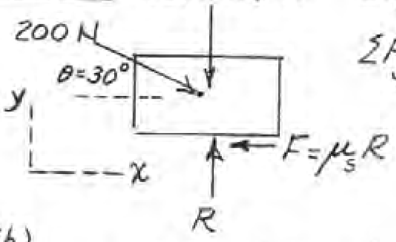
6/2

(a)



$\Sigma F_x = 0;\ 200 \cos 30° - \mu_s R = 0$

$\Sigma F_y = 0;\ R - 200 \sin 30° - 50(9.81) = 0$

$R = 590.5 \text{ N}$

So $\mu_s = \dfrac{200 \cos 30°}{590.5} = \underline{0.29}$

(b)

For $\theta = 45°$, $\Sigma F_x = 0$ gives $F = 200 \cos 45° = \underline{141.4 \text{ N}}$

which is $< \mu_s R_b$

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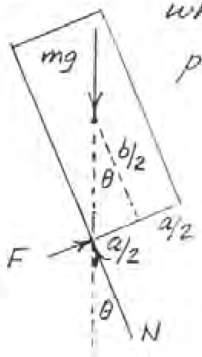
6/20 Block slips if $F = \mu N$ or $mg \sin\theta = \mu mg \cos\theta$

when angle reaches $\theta = \tan^{-1}\mu$

provided $\frac{a}{2} > \frac{b}{2}\tan\theta$ or $a > \mu b$

Tips first if $a < \mu b$



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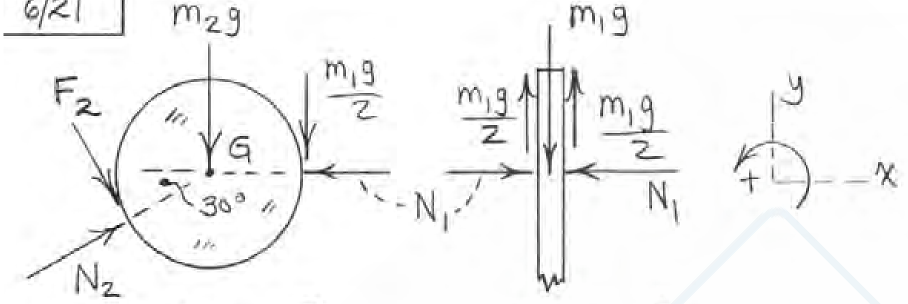

Cylinder:

$$\Sigma F_x = 0: \quad N_2 \cos 30° + F_2 \sin 30° - N_1 = 0 \qquad (1)$$

$$\Sigma F_y = 0: \quad N_2 \sin 30° - F_2 \cos 30° - m_2 g - \frac{m_1 g}{2} = 0 \qquad (2)$$

$$\Sigma M_G = 0: \quad F_2 r - \frac{m_1 g}{2} r = 0 \qquad (3)$$

Solve simultaneously to obtain

$$\mu = \frac{m_1 g/2}{N_1} = \frac{m_1}{3.73 m_1 + 3.46 m_2} > \frac{F_2}{N_2}$$

where we have assumed and then verified that slipping occurs first on right side of the cylinder.

For $m_1 \gg m_2$, $\underline{\mu = 0.268}$

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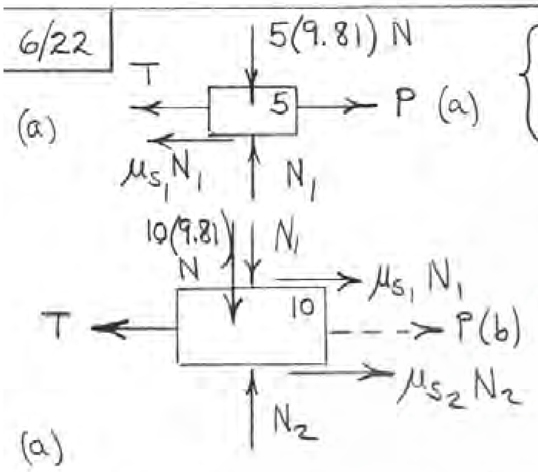

$$\begin{cases} \mu_{s_1} = 0.50 \\ \mu_{s_2} = 0.60 \end{cases}$$

From $\Sigma F_y = 0$,

$$\begin{cases} N_1 = 49.0 \text{ N} \\ N_2 = 147.2 \text{ N} \end{cases}$$

$$\Sigma F_x = 0: \begin{cases} P - T - 0.50(49.0) = 0 \\ -T + 0.50(49.0) + 0.60(147.2) = 0 \end{cases}$$

$\underline{T = 112.8 \text{ N}}$, $\underline{P = 137.3 \text{ N}}$

(b) Now P is applied to 10-kg block & we reverse all friction forces above:

$$\Sigma F_x = 0: \begin{cases} -T + 0.50(49.0) = 0 \\ -T - 0.50(49.0) - 0.60(147.2) + P = 0 \end{cases}$$

$\underline{T = 24.5 \text{ N}}$, $\underline{P = 137.3 \text{ N}}$

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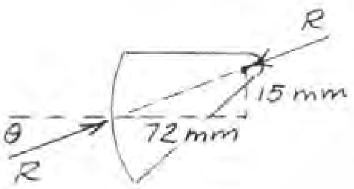

Req'd. minimum coeff. of friction is

$\mu_s = \tan\phi = \tan\theta$

$= \frac{15}{72} = 0.208$

$\Sigma F = 0$ for rope: $2R\sin\theta = 600$

$R = \frac{300}{15/\sqrt{15^2 + 72^2}} = 1471\text{ N}$

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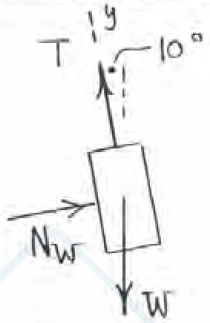
6/24 $\Sigma F_y = 0: \; T - W\cos 10° = 0$

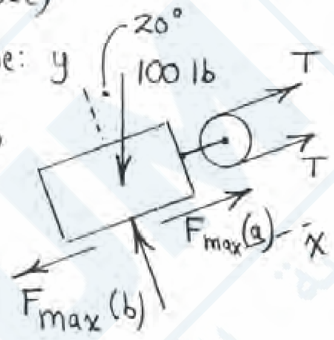
$$T = W\cos 10°$$



100-lb block:

$\Sigma F_y = 0: \quad N - 100\cos 20° = 0$

$$N = 94.0 \text{ lb} \quad \text{(throughout)}$$



(a) Motion impends down incline:

$\Sigma F_x = 0: \quad 2T - 100\sin 20° + F_{max} = 0$

With $F_{max} = \mu_s N = 0.3(94.0)$

$= 28.2$ lb and $T = W\cos 10°$,

$W = 3.05$ lb

(b) Motion impends up incline:

$\Sigma F_x = 0: \quad 2T - 100\sin 20° - F_{max} = 0$

Similarly, $\quad W = 31.7$ lb

Hence the allowable range is $\underline{3.05 \le W \le 31.7 \text{ lb}}$

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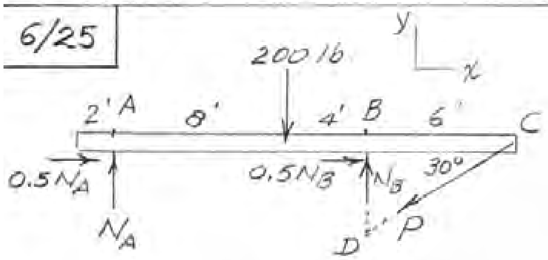

$\overline{BD} = 6 \tan 30° = 6/\sqrt{3}$

$\Sigma M_C = 0;\quad 10(200) - 18N_A - 6N_B = 0$

$\Sigma M_D = 0;\quad 4(200) - 0.5(N_A + N_B)\frac{6}{\sqrt{3}} - 12N_A = 0$

Solve simultaneously & get $N_A = 26.08$ lb

$N_B = 255.1$ lb

$\Sigma F_x = 0;\quad 0.866P - 0.5(26.08 + 255.1) = 0$

$P = \underline{162.3 \text{ lb}}$

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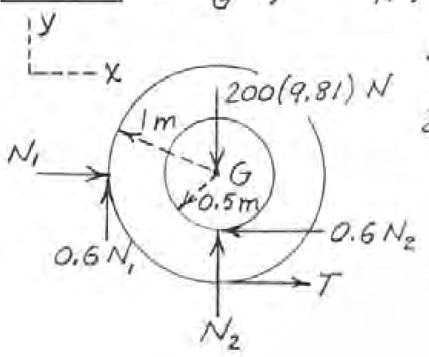

$\Sigma M_G = 0;\ 0.6N_1(1) + 0.6N_2(0.5) - T(1) = 0$

$\Sigma F_x = 0;\ T - 0.6N_2 + N_1 = 0$

$\Sigma F_y = 0;\ N_2 + 0.6N_1 - 1962 = 0$

Combine & get

$N_1 = 331\ N,\ N_2 = 1764\ N,\ \underline{T = 727\ N}$

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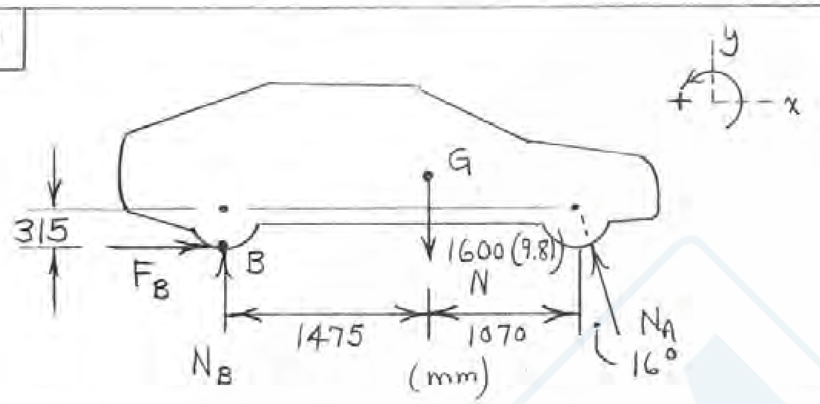

$$\begin{cases} \Sigma F_x = 0: & F_B - N_A \sin 16° = 0 \quad (1) \\ \Sigma F_y = 0: & N_B + N_A \cos 16° - 1600(9.81) = 0 \quad (2) \\ \Sigma M_B = 0: & -1600(9.81)(1475) + N_A \cos 16° (2545) \\ & + N_A \sin 16° (315) = 0 \quad (3) \end{cases}$$

Solution: $\begin{cases} N_A = 9140 \text{ N} \\ N_B = 6910 \text{ N} \\ F_B = 2520 \text{ N} \end{cases}$

$$\mu_s = \frac{F_B}{N_B} = \frac{2520}{6910} = \underline{0.365}$$

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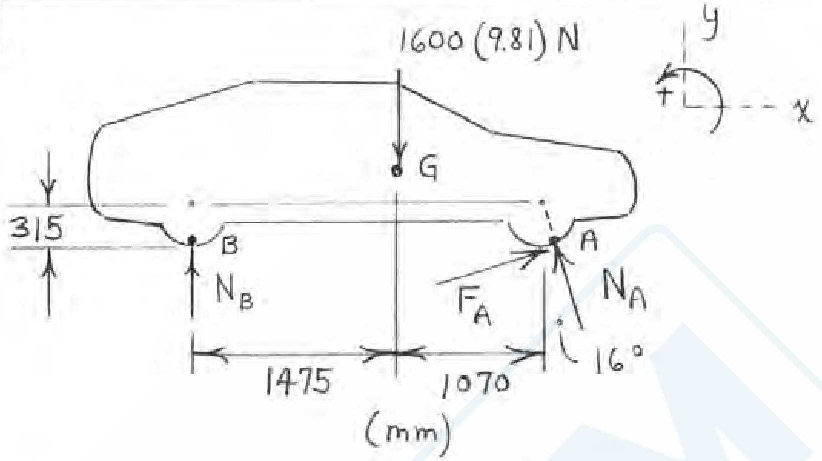

$$\begin{cases} \Sigma F_x = 0: \quad F_A \cos 16^\circ - N_A \sin 16^\circ = 0 & (1) \\ \Sigma F_y = 0: \quad N_B + F_A \sin 16^\circ + N_A \cos 16^\circ - 1600(9.81) = 0 & (2) \\ \Sigma M_A = 0: -N_B\,(2545 + 315 \sin 16^\circ) \\ \qquad + 1600\,(9.81)\,(1070 + 315 \sin 16^\circ) = 0 & (3) \end{cases}$$

Solution:
$$\begin{cases} N_A = 8460 \text{ N} \\ N_B = 6900 \text{ N} \\ F_A = 2420 \text{ N} \end{cases}$$

$$\mu_s = \frac{F_A}{N_A} = \frac{2420}{8460} = \underline{0.287}$$

Note that the value of μ_s can be determined from Eq. (1) alone.

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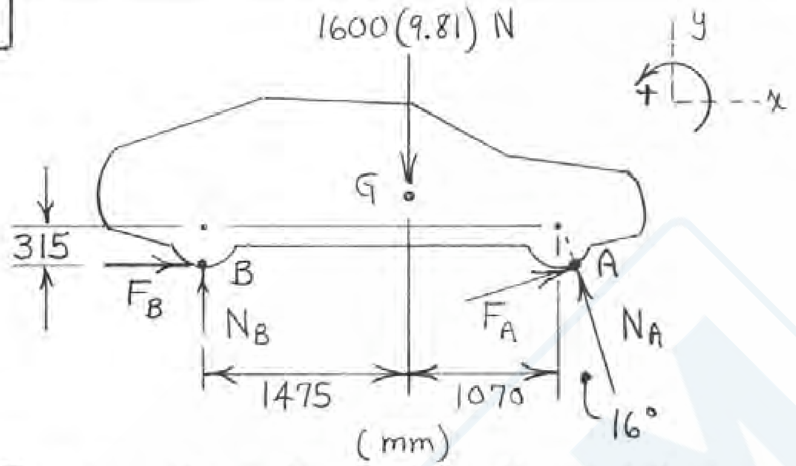

$$\begin{cases} \Sigma F_x = 0: \quad F_B + F_A \cos 16° - N_A \sin 16° = 0 & (1) \\ \Sigma F_y = 0: \quad N_B + F_A \sin 16° + N_A \cos 16° - 1600(9.81) = 0 & (2) \\ \Sigma M_A = 0: \quad -N_B(2545 + 315 \sin 16°) & \end{cases}$$

$$+ F_B(315 - 315\cos 16°) + 1600(9.81)(1070 + 315 \sin 16°) = 0 \quad (3)$$

Also, set $\begin{cases} F_A = \mu_s N_A & (4) \\ F_B = \mu_s N_B & (5) \end{cases}$

Solution: $F_A = 1378$ N $\qquad F_B = 1087$ N

$N_A = 8750$ N $\qquad N_B = 6900$ N

$\underline{\mu_s = 0.1575}$

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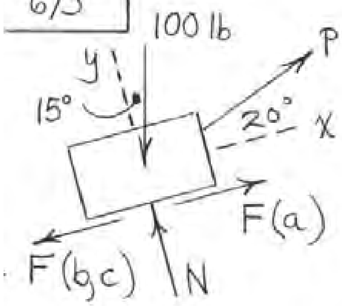

(a) $P = 0$

$\Sigma F_y = 0: \quad N - 100\cos 15° = 0$

$N = 96.6 \text{ lb}$

Assume equilibrium:

$\Sigma F_x = 0: \quad F - 100\sin 15° = 0$

$F = 25.9 \text{ lb}$

$F_{max} = \mu_s N = 0.25(96.6) = 24.1 < F$; assumption invalid and $F = F_k = \mu_k N = 0.2(96.6) = \underline{19.32 \text{ lb}}$ up the incline.

(b) $P = 40$ lb

$\Sigma F_y = 0: \quad N - 100\cos 15° + 40\sin 20° = 0, \quad N = 82.9 \text{ lb}$

$\Sigma F_x = 0: \quad 40\cos 20° - 100\sin 15° - F = 0, \quad \underline{F = 11.71 \text{ lb}}$

$F_{max} = \mu_s N = 0.25(82.9) = 20.7 \text{ lb}$; assumption OK

(c) $P = 60$ lb

$\Sigma F_y = 0: \quad N - 100\cos 15° + 60\sin 20° = 0, \quad N = 76.1 \text{ lb}$

$\Sigma F_x = 0: \quad 60\cos 20° - 100\sin 15° - F = 0, \quad F = 30.5 \text{ lb}$

$F_{max} = \mu_s N = 0.25(76.1) = 19.02 \text{ lb} < F$; assumption invalid

$F = \mu_k N = 0.2(76.1) = \underline{15.21 \text{ lb}}$ down the incline

(d) To initiate motion, set $F = \mu_s N = 0.25N$ down the incline:

$\Sigma F_y = 0: \quad N - 100\cos 15° + P\sin 20° = 0$

$\Sigma F_x = 0: \quad P\cos 20° - 100\sin 15° - 0.25N = 0$

Solve to obtain $\begin{cases} N = 79.9 \text{ lb} \\ \underline{P = 48.8 \text{ lb}} \end{cases}$

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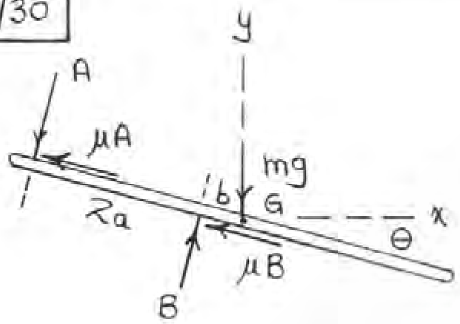

$\circlearrowleft + \Sigma M_G = 0: \quad A(2a+b) - Bb = 0, \quad \frac{B}{A} = \frac{2a+b}{b}$

$\Sigma F_x = 0: \quad (B-A)\sin\theta - \mu(A+B)\cos\theta = 0$

$$\tan\theta = \mu\frac{A+B}{B-A} = \mu\frac{1+B/A}{\frac{B}{A}-1}$$

Substitute $\frac{B}{A}$: $\tan\theta = \mu\frac{1+(2a+b)/b}{(2a+b)/b - 1}$

or $\underline{\theta = \tan^{-1}\left(\mu\frac{a+b}{a}\right)}$

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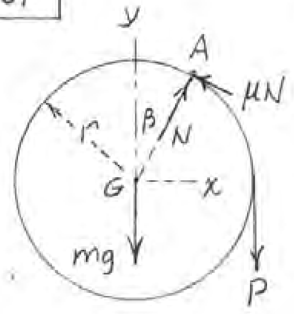

$\Sigma F_x = 0;\ N \sin\beta - \mu N \cos\beta = 0$

$\tan\beta = \mu$

$\beta = \tan^{-1}\mu$

$\Sigma M_A = 0;$

$P(r - r\sin\beta) - mg\, r \sin\beta = 0$

$P = mg \dfrac{\sin\beta}{1-\sin\beta}$

But $\sin\beta = \dfrac{\mu}{\sqrt{1+\mu^2}}$ so $P = mg \dfrac{\mu}{\sqrt{1+\mu^2} - \mu}$

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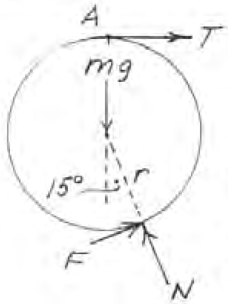

Assume equilibrium, $\Sigma M_A = 0$

$F(r + r\cos 15°) = N r \sin 15°$

$\frac{F}{N} = \frac{0.2588}{1 + 0.9659} = 0.1317$

But $F_{max} = 0.20 N$ $(> 0.1317 N)$

so $F = 0.1317 N$ can be supported

& paper rolls without slipping

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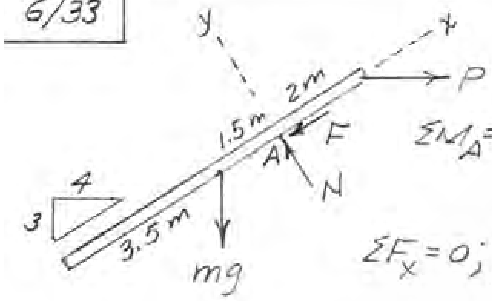

$\Sigma M_A = 0;\ \frac{3}{5}P(2) - \frac{4}{5}mg(1.5) = 0$

$P = mg$

$\Sigma F_x = 0;\ \frac{4}{5}mg - F - \frac{3}{5}mg = 0$

$F = \frac{1}{5}mg$

$\Sigma F_y = 0;\ N - \frac{3}{5}mg - \frac{4}{5}mg = 0,\ N = \frac{7}{5}mg$

$\mu_s(min) = F/N = \frac{1}{5} / \frac{7}{5} = \frac{1}{7} = 0.14 < 0.30$ so friction is sufficient to maintain equil. & ans. is <u>yes</u>.

$F = \frac{1}{5}60(9.81) = \underline{117.7\ N}$

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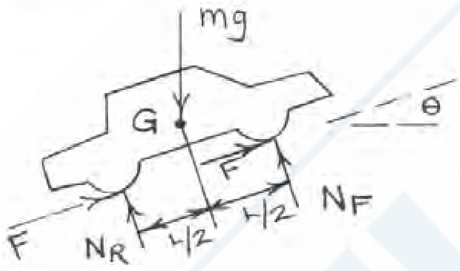
6/34 Consider the FBD below and the equilibrium equation $\Sigma M_G = 0$. The presence of the propulsive friction forces F, whether applied at the front or at the rear, increases the rear normal forces and decreases the front ones. Increased normals mean increased available propulsive friction forces.



Thus the rear-wheel drive car would climb the steeper grade.

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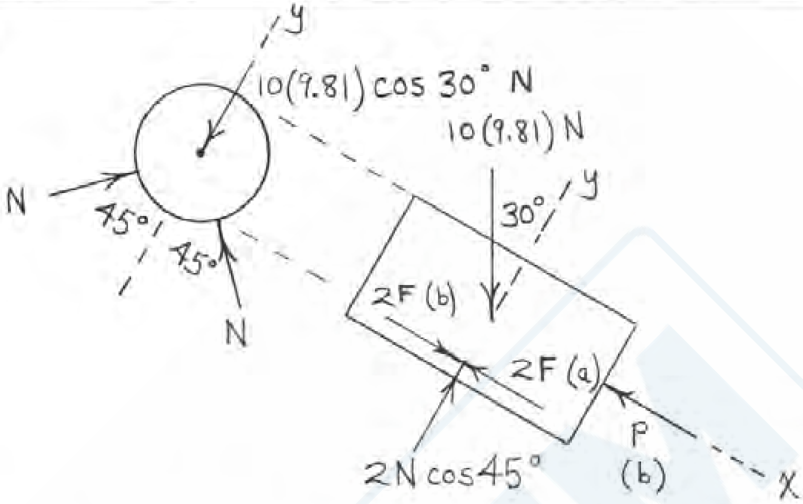

$\Sigma F_y = 0: \quad 2N\cos 45° - 10(9.81)\cos 30° = 0, \quad N = 60.1 \text{ N}$

(a) $P = 0$

$\Sigma F_x = 0: -2F + 10(9.81)\sin 30° = 0, \quad \underline{F = 24.5 \text{ N}}$

Check: $F_{max} = \mu_s N = 0.5(60.1) = 30.0 \text{ N} > F = 24.5 \text{ N}$

So we indeed have static equilibrium.

(b) $P \neq 0$

$\Sigma F_x = 0: -P + 10(9.81)\sin 30° + 2(0.5 \cdot 60.1) = 0$

$\underline{P = 109.1 \text{ N}}$

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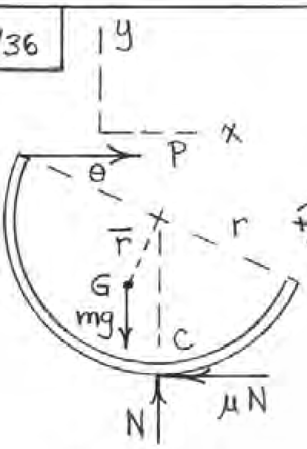

$\Sigma F_y = 0 \Rightarrow N = mg$

$\Sigma F_x = 0 \Rightarrow P = \mu_s N$

$\circlearrowleft^{+} \Sigma M_C = 0: \; P(r + r\sin\theta) - mg\bar{r}\sin\theta = 0, \quad \bar{r} = \frac{2r}{\pi}$

$\therefore \mu_s mg(r + r\sin\theta) = mg\frac{2r}{\pi}\sin\theta$

$\sin\theta\left[\frac{2}{\pi} - \mu_s\right] = \mu_s, \quad \underline{\theta = \sin^{-1}\left(\frac{\pi\mu_s}{2 - \pi\mu_s}\right)}$

$\theta = 90°$ when $\frac{\pi\mu_s}{2 - \pi\mu_s} = 1$ or $\underline{\mu_{90°} = \frac{1}{\pi} = 0.318}$

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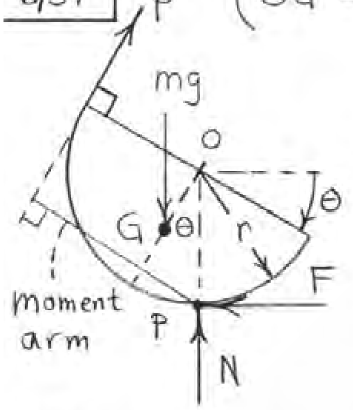
6/37 $\left(\overline{OG} = \bar{r} = \frac{4r}{3\pi}\right)$



$\Sigma F_x = 0: \; P\sin\theta - F = 0 \quad (1)$

$\Sigma F_y = 0: \; N - mg + P\cos\theta = 0 \quad (2)$

$\Sigma M_P = 0: \; mg\,\frac{4r}{3\pi}\sin\theta - P(r + r\sin\theta) = 0 \quad (3)$

When slipping impends, $F = \mu_s N \quad (4)$

$(4) \to (1): \; P\sin\theta - \mu_s N = 0 \quad (5)$

$(2): \; N = mg - P\cos\theta \quad (6)$

$(6) \to (5): \; P\sin\theta - \mu_s(mg - P\cos\theta) = 0$

$$\Rightarrow P = \frac{\mu_s mg}{\sin\theta + \mu_s\cos\theta} \quad (7)$$

$(7) \to (3)$ & simplification yields

$$\mu_s = \frac{4\sin^2\theta}{3\pi(1+\sin\theta) - 4\sin\theta\cos\theta}$$

For $\theta = 40°$, $\underline{\mu_s = 0.1223}$

(7) then gives $\underline{P = 0.1661\,mg}$

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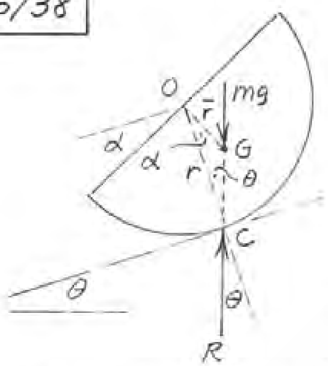

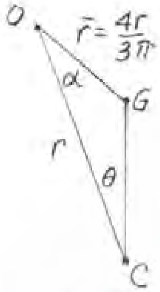

$$\frac{r}{\sin(\pi - \theta - \alpha)} = \frac{\bar{r}}{\sin\theta}$$

$$\frac{r}{\sin(\theta + \alpha)} = \frac{4r/3\pi}{\sin\theta}$$

Max. angle occurs when

$\theta = \phi = \tan^{-1}\mu = \tan^{-1}0.30 = \underline{16.70°}$

Thus $\sin(\theta + \alpha) = \frac{3\pi}{4}\sin\theta = \frac{3\pi}{4}(0.2873) = 0.6770$

so $\theta + \alpha = 42.6°$ & $\alpha = 42.6 - 16.7 = \underline{25.9°}$

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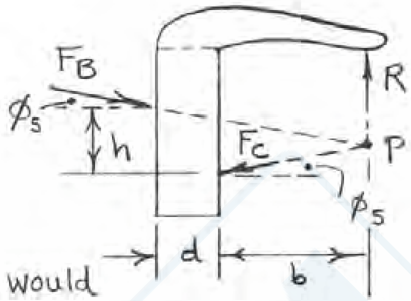
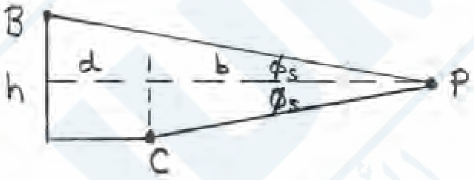
For equilibrium the three forces must be concurrent at P. For b larger than that shown, friction angles ϕ would be less than $\phi_s = \tan^{-1}\mu_s$ and no slippage is possible.



From geometry,



$$\frac{h - b\tan\phi_s}{b+d} = \tan\phi_s$$

$$h - b\mu_s = (b+d)\mu_s\,, \quad \underline{b = \frac{1}{2}\left(\frac{h}{\mu_s} - d\right)}$$

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$\Sigma F_x = 0: \; 0.08N - mg\sin\theta = 0$

$N = \frac{mg\sin\theta}{0.08}$

$\Sigma F_y = 0: \; N - mg\cos\theta = 0$

$N = mg\cos\theta$

So $\frac{mg\sin\theta}{0.08} = mg\cos\theta$, $\tan\theta = 0.08$, $\underline{\theta = 4.57°}$

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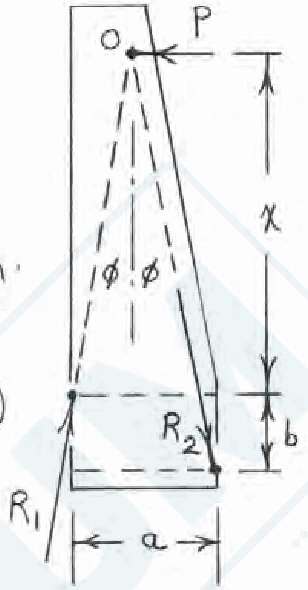
For equilibrium, the forces must be concurrent at O and $\phi = \tan^{-1}\mu_s$ for impending motion with $x = x_{min}$.



$a = x\tan\phi + (x+b)\tan\phi$

$= x\mu_s + (x+b)\mu_s = \mu_s(2x+b)$

$$x = \frac{a - b\mu_s}{2\mu_s}$$

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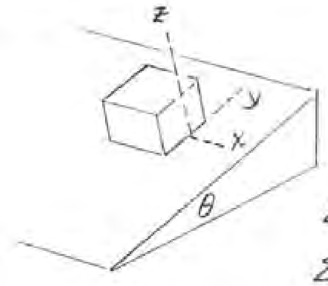
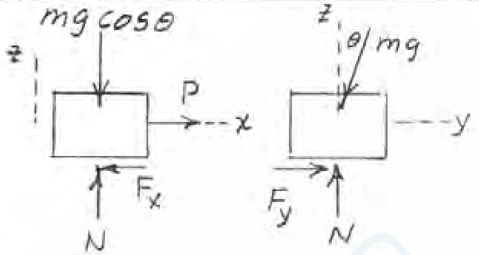




$\Sigma F_y = 0;\quad F_y = mg\sin\theta$

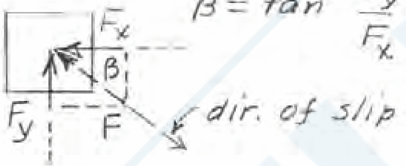
$\Sigma F_x = 0;\quad F_x = P\ ;\ \Sigma F_z = 0;\quad N = mg\cos\theta$

$F = \mu_s N = \sqrt{F_x^2 + F_y^2}$ so $(\mu_s mg\cos\theta)^2 = P^2 + (mg\sin\theta)^2$

so $\underline{P = mg\sqrt{\mu_s^2\cos^2\theta - \sin^2\theta}}$

where $\mu_s > \tan\theta$ otherwise block slips without P

$\beta = \tan^{-1}\dfrac{F_y}{F_x} = \underline{\tan^{-1}\left(\dfrac{mg\sin\theta}{P}\right)}$



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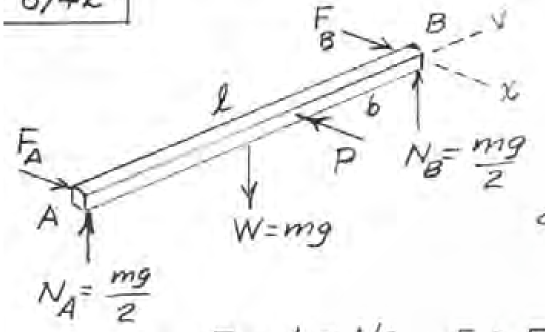

$\Sigma M_{B_z} = 0$

$Pb - F_A \ell = 0$

$F_A = Pb/\ell$

$\Sigma F_x = 0;\ F_B + \frac{Pb}{\ell} - P = 0$

$F_B = P(1 - b/\ell)$

For $b < \ell/2$, $F_B > F_A$ so slips first at B

Hence $F_B = \mu_s N_B = \mu_s mg/2$

Thus $\mu_s mg/2 = P(1 - b/\ell)$ so $P = \dfrac{\mu_s mg}{2(1 - b/\ell)}$

& $F_A = \dfrac{\mu_s mg}{2(1 - b/\ell)} \dfrac{b}{\ell} = \dfrac{\mu_s mg}{2(\frac{\ell}{b} - 1)}$

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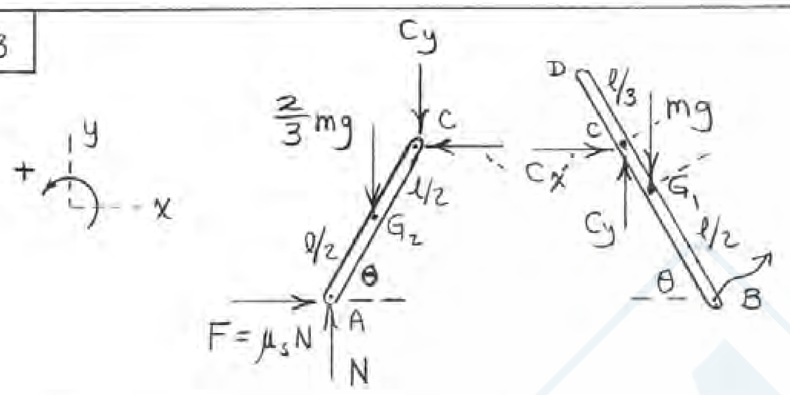


Body BD:

$$\Sigma M_B = 0: -C_x\left(\frac{2\ell}{3}\sin\theta\right) - C_y\left(\frac{2\ell}{3}\cos\theta\right) + mg\frac{\ell}{2}\cos\theta = 0 \quad (1)$$

Body AC:

$$\Sigma F_x = 0: \quad \mu_s N - C_x = 0 \quad (2)$$

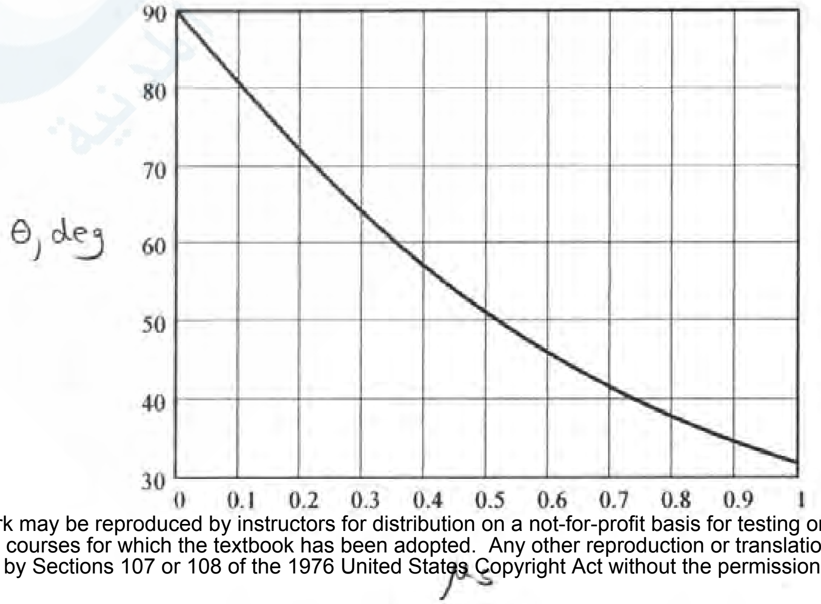
$$\Sigma F_y = 0: \quad N - \frac{2}{3}mg - C_y = 0 \quad (3)$$

$$\Sigma M_C = 0: \mu_s N\left(\frac{2\ell}{3}\sin\theta\right) - N\left(\frac{2\ell}{3}\cos\theta\right) + \frac{2}{3}mg\left(\frac{\ell}{3}\cos\theta\right) = 0 \quad (4)$$

Solve Eqs. (1)-(4) for C_x, C_y, N, and θ as functions of μ_s. Solution for θ is

$$\theta = \tan^{-1}\frac{13}{21\mu_s} \text{ (plotted below)}$$

$\theta \to 90°$ as $\mu_s \to 0$
$\theta = 51.1°$ for $\mu_s = 0.50$
$\theta = 31.8°$ for $\mu_s = 1$



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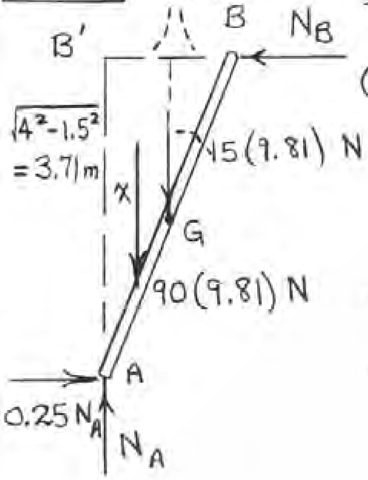

$+\uparrow\Sigma F = 0 \Rightarrow N_A = (90+15)9.81 = 1030\text{ N}$

$\circlearrowleft + \Sigma M_{B'} = 0: 0.25 N_A(3.71) - 90(9.81)x$

$- 15(9.81)(0.75) = 0, \quad x = 0.957\text{ m}$

$\dfrac{S}{x} = \dfrac{4}{1.5}, \quad \underline{S = 2.55\text{ m}}$

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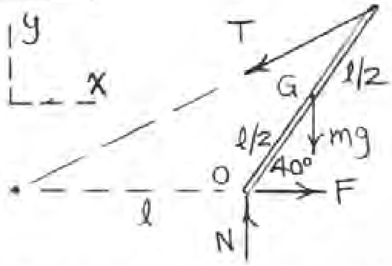

$$\underline{T} = T\left\{\frac{(-\ell-\ell\cos 40°)\underline{i} - (\ell\sin 40°)\underline{j}}{\sqrt{(\ell+\ell\cos 45°)^2+(\ell\sin 45°)^2}}\right\}$$

$$= T\{-0.940\underline{i} - 0.342\underline{j}\}$$

$\circlearrowleft + \Sigma M_O = 0: \quad 0.342T(\ell) - mg\frac{\ell}{2}\cos 40° = 0$

$T = 1.120\, mg$

$\Sigma F_x = 0: \quad F - 1.120\, mg\,(0.940) = 0, \quad F = 1.052\, mg$

$\Sigma F_y = 0: \quad N - 1.120\, mg\,(0.342) - mg = 0$

$N = 1.383\, mg$

$$\mu_s = \frac{F}{N} = \frac{1.052\, mg}{1.383\, mg} = \underline{0.761}$$

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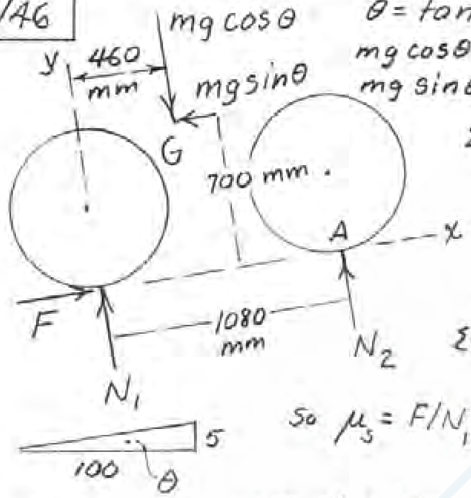

$\theta = \tan^{-1} 5/100 = 2.86°$

$mg\cos\theta = 82(9.81)(0.9998) = 803.4\ N$

$mg\sin\theta = 82(9.81)(0.0499) = 40.2\ N$

$\Sigma M_A = 0;$

$803.4(620) + 40.2(700) - 1080 N_1 = 0$

$N_1 = 487.3\ N$

$\Sigma F_x = 0;\ F = 40.2\ N$

So $\mu_s = F/N_1 = 40.2/487.3 = \underline{0.082}$

If μ_s were doubled, friction force remains $\underline{40.2\ N}$

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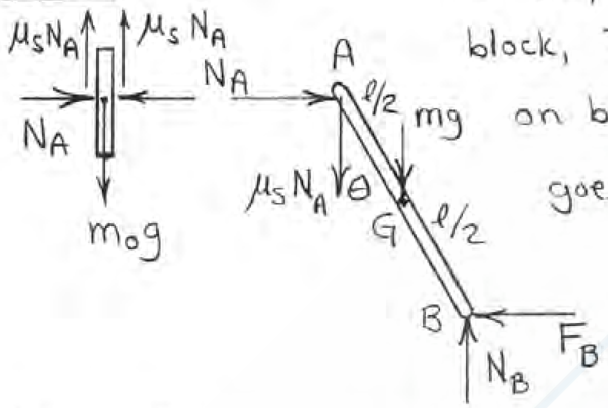
For impending slip of the block, the friction force on both of its faces goes to the maximum.



(Block) $+\uparrow \Sigma F = 0$: $2\mu_s N_A - m_0 g = 0$, $N_A = \dfrac{m_0 g}{2\mu_s}$ (1)

(Bar) $\circlearrowleft + \Sigma M_B = 0$: $mg\left(\frac{l}{2}\sin\theta\right) + \mu_s N_A (l \sin\theta)$
$- N_A (l\cos\theta) = 0$ (2)

With Eq. (1), Eq. (2) becomes

$$mg\left(\frac{l}{2}\sin\theta\right) + \mu_s \frac{m_0 g}{2\mu_s} l\sin\theta - \frac{m_0 g}{2\mu_s} l\cos\theta = 0$$

Solving for θ: $\theta = \tan^{-1}\left[\dfrac{1}{\mu_s\left(1+\frac{m}{m_0}\right)}\right] = \theta_{min}$

(Bar) $\overset{+}{\rightarrow} \Sigma F = 0$: $N_A - F_B = 0$

$$F_B = N_A = \frac{m_0 g}{2\mu_s}$$

$+\uparrow \Sigma F = 0$: $N_B - mg - \mu_s N_A = 0$

$$N_B = mg + \mu_s \frac{m_0 g}{2\mu_s} = \left(m + \frac{m_0}{2}\right) g$$

$$(\mu_s)_B = \frac{F_B}{N_B} = \frac{m_0 g/(2\mu_s)}{\left(m + \frac{m_0}{2}\right) g} = \frac{1}{\mu_s\left(1 + 2\frac{m}{m_0}\right)}$$

Numbers ($\mu_s = 0.5$ throughout)

(a) $\frac{m}{m_0} = 0.1$: $\theta_{min} = 61.2°$, $(\mu_s)_B = 1.667$

($\mu_s > 1$ perhaps unusual, but quite possible)

(b) $\frac{m}{m_0} = 1$: $\theta_{min} = 45°$, $(\mu_s)_B = 0.667$

(c) $\frac{m}{m_0} = 10$: $\theta_{min} = 10.30°$, $(\mu_s)_B = 0.0952$

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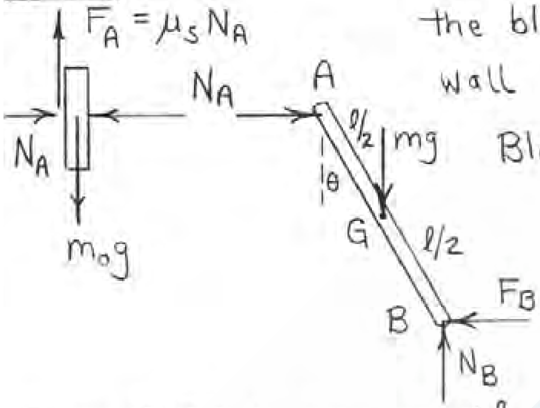

The friction force F_A between the block m_o and the vertical wall is set to the maximum.

Block: $+\uparrow \Sigma F = 0$:

$$\mu_s N_A = m_o g$$

$$N_A = \frac{m_o g}{\mu_s} \quad (1)$$

Bar: $\circlearrowleft + \Sigma M_B = 0$: $mg\left(\frac{l}{2}\sin\theta\right) - N_A(l\cos\theta) = 0$

$$N_A = \frac{mg}{2}\tan\theta \quad (2)$$

Combine (1) & (2): $\frac{m_o g}{\mu_s} = \frac{mg}{2}\tan\theta$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{2}{\mu_s}\frac{m_o}{m}\right) = \theta_{min}$$

Numbers: $\theta_{min} = \tan^{-1}\left(\frac{2}{0.5}\cdot\frac{1}{10}\right) = \underline{21.8°}$

Check to see that no slippage occurs at B.

(Bar) $\xrightarrow{+} \Sigma F = 0$: $N_A - F_B = 0$

So $F_B = N_A = \frac{mg}{2}\tan\theta$ (from Eq. (2))

$+\uparrow \Sigma F = 0$: $N_B - mg = 0$, $N_B = mg$

$$(F_B)_{max} = \mu_s N_B = \mu_s mg$$

Numbers: $\begin{cases} F_B = \frac{mg}{2}\tan 21.8° = 0.2mg \\ (F_B)_{max} = 0.5mg \end{cases}$

$F_B < (F_B)_{max}$, so no slippage at B.

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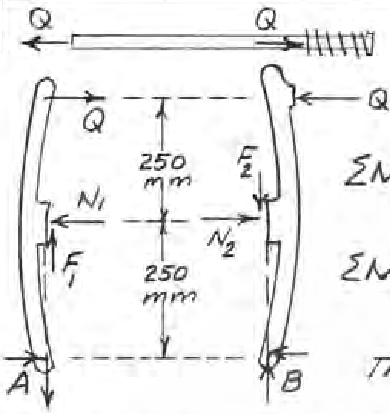

FBD's for $P=0$

$F_1 = 0.2N_1$, $F_2 = 0.2N_2$

$\Sigma M_A = 0;\ 0.250N_1 = 0.500Q$

$N_1 = 2Q$

$\Sigma M_B = 0;\ 0.250N_2 = 0.500Q$

$N_2 = 2Q$

Thus $N_1 = N_2 = N$

Flywheel: $\Sigma M = 0;\ M = F_1 r + F_2 r$

$100 = 0.2N(0.200) + 0.2N(0.200)$

$N = 1250\ \text{N}$

So $Q = N/2 = 625\ \text{N}$; $Q = k\delta$, $k = \dfrac{625}{0.030} = \underline{20.8(10^3)\ \dfrac{\text{N}}{\text{m}}}$

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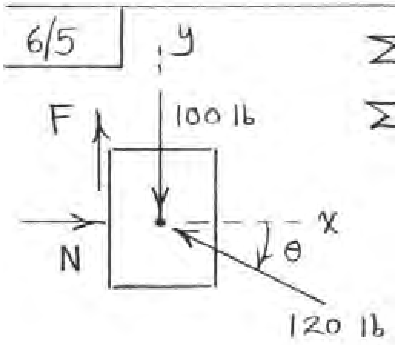

$\Sigma F_x = 0: \quad N - 120\cos\theta = 0$

$\Sigma F_y = 0: \quad F - 100 + 120\sin\theta = 0$

(a) $\theta = 15°$

$N = 115.9 \text{ lb}, \quad F = 68.9 \text{ lb}$

$F_{max} = \mu_s N = 0.50(115.9)$
$= 58.0 \text{ lb}$

$F > F_{max}$, so assumption of equilibrium is invalid and $F = F_k = \mu_k N = 0.40(115.9)$
$= \underline{46.4 \text{ lb (up)}}$

(b) $\theta = 30°$

$N = 103.9 \text{ lb}, \quad F = 40 \text{ lb}$

$F_{max} = \mu_s N = 0.50(103.9) = 52.0 \text{ lb}$

$F < F_{max}$, so equilibrium assumption OK

↑ $\underline{F = 40 \text{ lb (up)}}$

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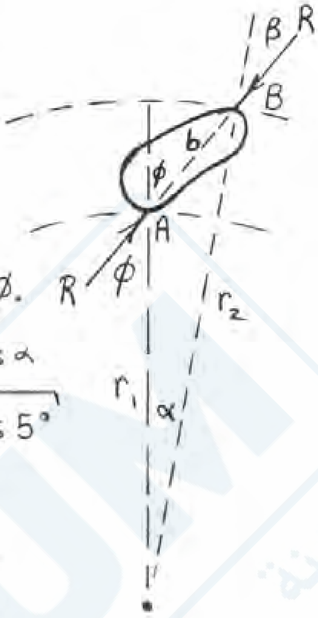
▶6/50 Given: $\begin{cases} r_1 = \overline{OA} = 5/16'' \\ r_2 = \overline{OB} = 7/16'' \\ \alpha = 5° \end{cases}$



$\phi > \beta$ so slipping would occur first at A. Hence $\mu_{min} = \tan\phi$.

Law of cosines: $b^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\alpha$

$$b = \sqrt{(5/16)^2 + (7/16)^2 - 2\left(\frac{5}{16}\right)\left(\frac{7}{16}\right)\cos 5°}$$

$$= 0.1291 \text{ in.}$$

Law of sines: $\dfrac{b}{\sin\alpha} = \dfrac{r_2}{\sin(\pi-\phi)}$

$$\frac{0.1291}{\sin 5°} = \frac{7/16}{\sin(\pi-\phi)} = \frac{7/16}{\sin\phi}$$

$\phi = 17.18°$, $\mu_{min} = \tan\phi = \tan 17.18° = \underline{0.309}$

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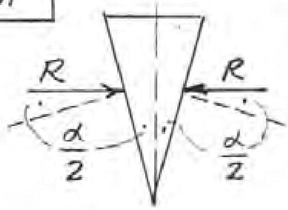

Critical α occurs when

$$\frac{\alpha}{2} = \phi = \tan^{-1}\mu$$

so $\alpha = 2\tan^{-1} 0.20$

$= 2(11.31°) = \underline{22.6°}$

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6/52 Self-locking condition is

$$\alpha = \phi$$

$$\tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \mu_s$$

$$\mu_s = \frac{L}{2\pi r} = \frac{5}{2\pi(3)}$$

$$= \underline{0.265}$$

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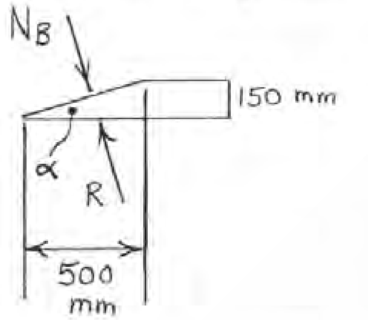
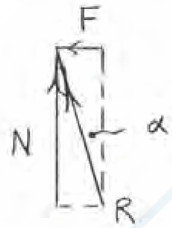
6/53 FBD of portable ramp:



$$\alpha = \tan^{-1}\left(\frac{150}{500}\right) = 16.70°$$

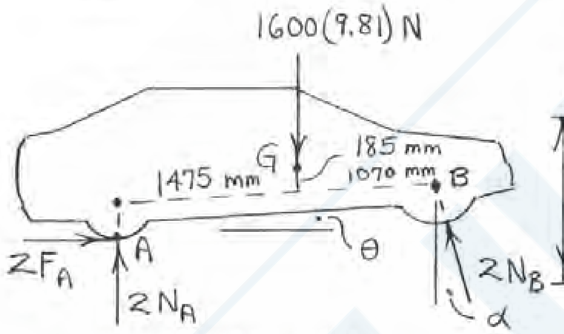


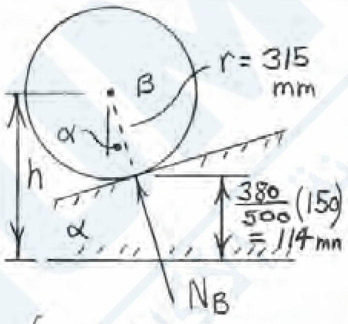
$$\tan\alpha = \frac{F}{N} = \mu_s$$

So $\mu_s = \tan\alpha$

$= \tan 16.70°$

$\underline{\mu_s = 0.3}$





So $\theta = \sin^{-1}\dfrac{416 - 315}{2545} = 2.27°$

$\left\{ \begin{array}{l} h = 114 + 315\cos\alpha \\ \quad = 416 \text{ mm} \end{array} \right.$

$\circlearrowleft^{+} \Sigma M_A = 0: -1600(9.81)(1475\cos\theta - 185\sin\theta)$

$+ 2N_B\cos(\alpha - \theta)(315\sin\theta + 2545) + 2N_B\sin(\alpha - \theta)\, 315\cos\theta = 0$

$$N_B = 4500 \text{ N}$$

$\xrightarrow{+} \Sigma F = 0: \quad 2F_A - 2N_B\sin\alpha = 0$

$$\underline{F_A = 1294 \text{ N}}$$

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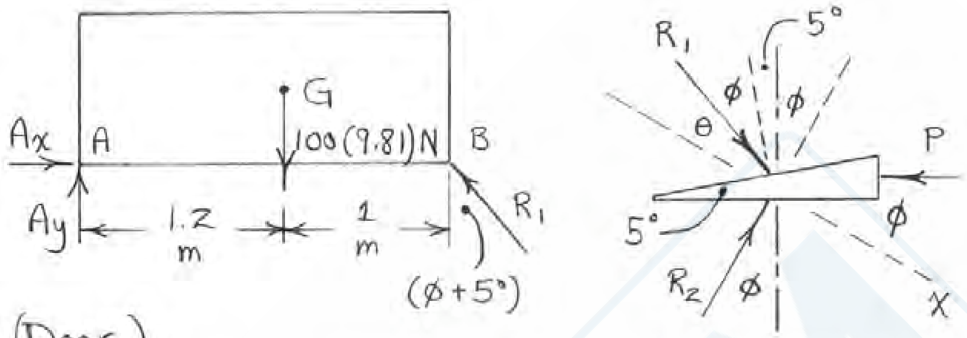
6/54 $\phi = \tan^{-1}(0.60) = 31.0°$



(Door)

$\Sigma M_A = 0: \; R_1 \cos(31.0° + 5°)(2.2) - 100(9.81)(1.2) = 0$

$R_1 = 661 \text{ N}$

(Wedge) (Note $\theta = 90° - 2\phi - 5° = 23.1°$)

$\Sigma F_x = 0: \; 661 \cos 23.1° - P \cos 31.0° = 0$

$\underline{P = 709 \text{ N}}$

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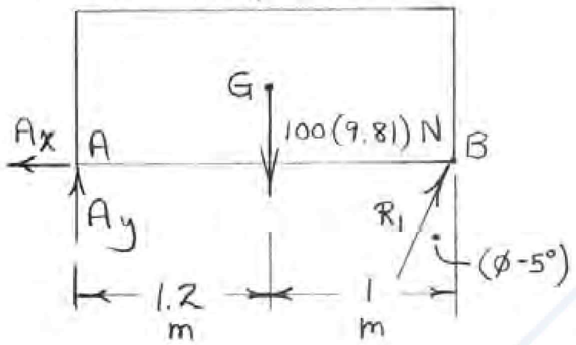
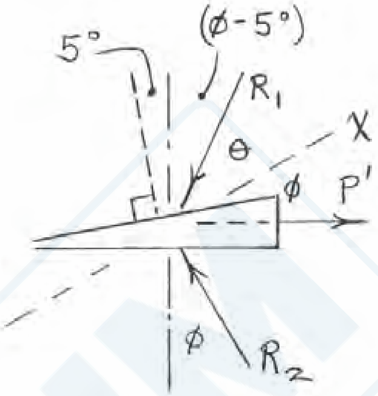
6/55 $\phi = \tan^{-1}(0.60) = 31.0°$

Assume A pinned:





(Door)

$\Sigma M_A = 0: \; R_1 \cos(31.0° - 5°)(2.2) - 100(9.81)(1.2) = 0$

$R_1 = 595 \text{ N}$

(Wedge) (Note $\theta = 90° - 2\phi + 5° = 33.1°$)

$\Sigma F_x = 0: \; P' \cos 31.0° - 595 \cos 33.1° = 0$

$P' = 582 \text{ N}$

(Door) $\Sigma F_x = 0: \; -A_x + 595 \sin(\phi - 5°) = 0, \; A_x = 261 \text{ N}$

$\Sigma F_y = 0: \; A_y - 100(9.81) + 595 \cos(\phi - 5°) = 0, \; A_y = 446 \text{ N}$

$\dfrac{A_x}{A_y} = \dfrac{261}{446} = 0.584 < (\mu_s = 0.6)$; assumption OK, A will not slip.

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6/56

Helix angle $\alpha = \tan^{-1}\frac{L}{\pi d} = \tan^{-1}\frac{1/12}{\pi(3/8)} = 4.05°$

Friction angle $\phi = \tan^{-1}\mu_s = \tan^{-1}(0.20) = 11.31°$

Tighten : $M = Pr\tan(\alpha+\phi) = 80\,\frac{3/8}{2}\tan(4.05°+11.31°)$

$= \underline{4.12 \text{ lb-in.}}$

Loosen : $M' = Pr\tan(\phi-\alpha) = 80\,\frac{3/8}{2}\tan(11.31°-4.05°)$

$= \underline{1.912 \text{ lb-in.}}$ (in direction opposite to that of M)

Note : $\alpha < \phi$, so screw is self-locking (a good feature for a clamp!)

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6/57 $M = Wr \tan(\alpha + \phi)$

where $W = 450$ N, $r = 0.025$ m,

$\alpha = \tan^{-1}\frac{\text{Lead}}{2\pi r} = \tan^{-1}\frac{0.020}{2\pi(0.025/2)} = \tan^{-1} 0.255$

$= 14.29°$

$\phi = \tan^{-1}\mu = \tan^{-1} 0.20 = 11.31°$; $\phi + \alpha = 25.60°$

So $M = 450\left(\frac{0.025}{2}\right)\tan 25.60° = \underline{2.69\ \text{N}\cdot\text{m}}$

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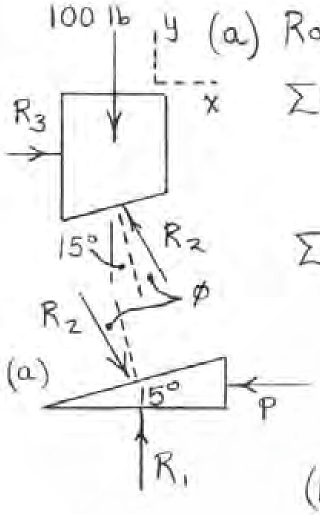
6/58 Friction angle $\phi = \tan^{-1}(0.2) = 11.31°$

(a) Rollers under wedge



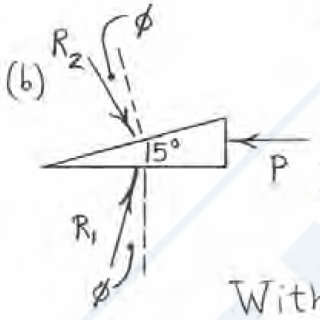
$\Sigma F_y = 0: \; -100 + R_2 \cos(15° + 11.31°) = 0$

$R_2 = 111.6$ lb

$\Sigma F_x = 0: \; R_2 \sin(15° + 11.31°) - P = 0$

$P = 49.4$ lb

(b) Rollers removed



Value of R_2 from 100-lb body is unchanged.

$\Sigma F_x = 0: \; R_2 \sin(15° + 11.31°) - P + R_1 \sin(11.31°) = 0$

With R_1 determined from overall equilibrium as $R_1 = 100/\cos 11.31° = 102.0$ lb, we solve for P as

$P = 69.4$ lb

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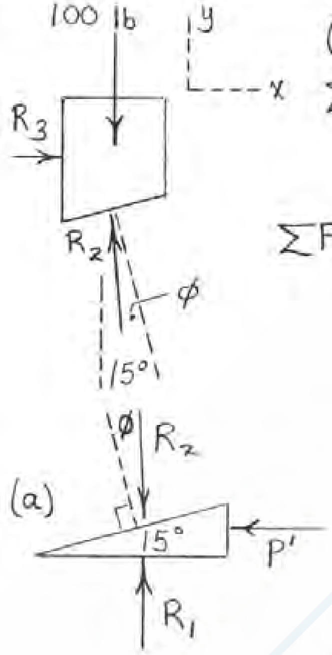
6/59 Friction angle $\phi = \tan^{-1}(0.2) = 11.31°$



(a) Rollers under wedge

$\Sigma F_y = 0: -100 + R_2 \cos(15° - 11.31°) = 0$

$R_2 = 100.2$ lb

$\Sigma F_x = 0: R_2 \sin(15° - 11.31°) - P' = 0$

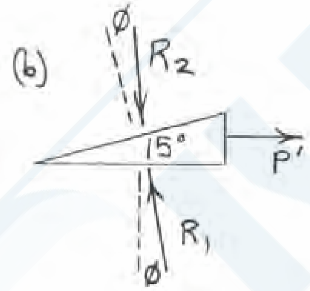
$P' = 6.45$ lb (to the left)

(b) Rollers removed

$R_2 = 100.2$ lb, from (a).

From overall equilibrium,

$$R_1 = \frac{100}{\cos 11.31°} = 102.0 \text{ lb}$$



$\Sigma F_x = 0: R_2 \sin(15° - 11.31°) + P'$

$- R_1 \sin(11.31°) = 0$

$P' = 13.55$ lb (to the right)

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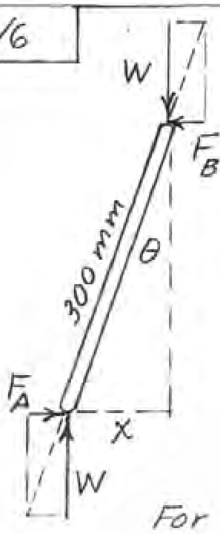
6/6



For $x = 75$ mm, $\theta = \sin^{-1}\frac{75}{300} = 14.5°$

Friction angle $\phi = \tan^{-1}\mu_s$

for A is $\phi_A = \tan^{-1} 0.40 = 21.8°$

" B " $\phi_B = \tan^{-1} 0.30 = 16.7°$

Since $\theta < \phi_A$ & ϕ_B, bar does not slip & $F_A = F_B = W \tan\theta$

$= 50(9.81) \tan 14.5°$

$= \underline{126.6 \text{ N}}$

For increased x bar slips first at B with $\theta = \phi_B = 16.7°$. Thus $x_{max} = 300 \sin 16.7°$

$= \underline{86.2 \text{ mm}}$

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6/60 Helix angle: $\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{1/5}{2\pi\left(\frac{1.150}{2}\right)} = 3.17°$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1}(0.25) = 14.04°$

(a) To tighten, $M_a = 2Tr\tan(\alpha + \phi)$

$$M_a = 2(10{,}000)\frac{1.150}{2}\tan(3.17° + 14.04°) = \underline{3560 \text{ lb-in.}}$$

(b) To loosen, $M_b = 2Tr\tan(\phi - \alpha)$

$$M_b = 2(10{,}000)\frac{1.150}{2}\tan(14.04° - 3.17°) = \underline{2210 \text{ lb-in.}}$$

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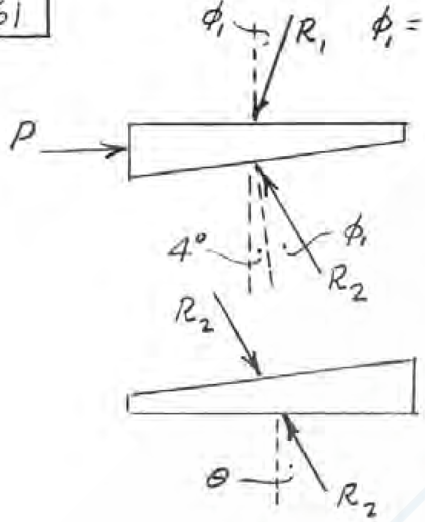

$\phi_1 = \tan^{-1} 0.3 = 16.70°$

In order for column to be raised, bottom wedge must not move. Therefore μ_2 must be greater than $\tan\theta$ or

$$(\mu_2)_{min} = \tan(4 + 16.70) = 0.378$$

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6/62 For the screw,

$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{6}{20\pi} = 5.45°$

$\phi = \tan^{-1}(0.25) = 14.04°$

Eq. 6/3: $M = Wr\tan(\alpha + \phi)$

$24 = P\,\frac{20/2}{1000}\tan(5.45° + 14.04°)$

$P = 6780$ N (to remove collar)

Collar: $\mu p A = P$: $0.30p\,(0.050\pi \cdot 0.060) = 6780$

$p = 2.40(10^6)$ Pa or $\underline{2400 \text{ kPa}}$

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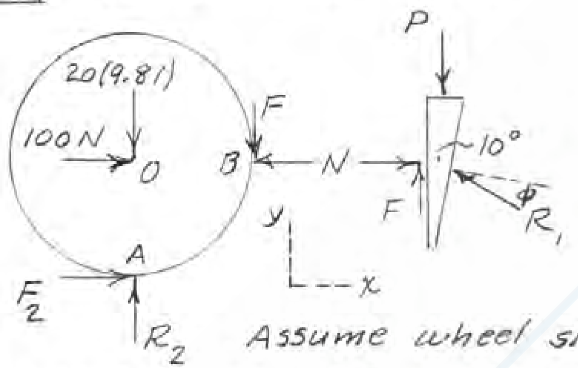
6/63 $\phi = \tan^{-1} 0.30 = 16.70°$



Assume wheel slips at B

wheel: $\Sigma M_O = 0$; $F_2 = F = 0.3N$
$\Sigma F_x = 0$; $N = 100 + F_2$

$0.7N = 100$
$N = 142.9$ N
$F_2 = F = 0.3(142.9) = 42.9$

$\Sigma F_y = 0$; $R_2 = 196.2 + 42.9 = 239.1$ N

$(F_2 = 42.9) < (0.25 \times 239.1 = 59.8)$ so assumption OK

Wedge: $\Sigma F_x = 0$; $R_1 \cos 26.70° = 142.9$, $R_1 = 159.9$ N

$\Sigma F_y = 0$; $P = 42.9 + 159.9 \sin 26.70° = \underline{114.7 \text{ N}}$

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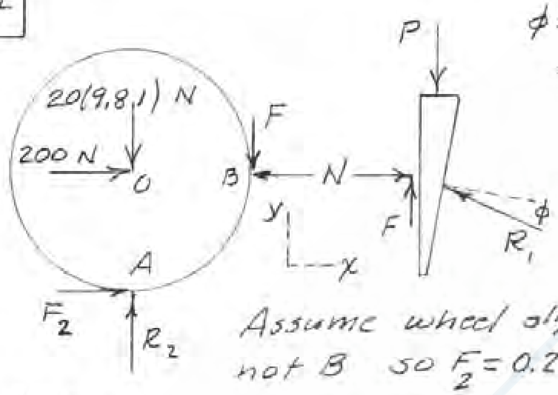

Assume wheel slips at A but not B so $F_2 = 0.25 R_2$

Wheel: $\Sigma M_O = 0$; $F = F_2 = 0.25 R_2$

$\Sigma F_y = 0$; $R_2 = 196.2 + F = 196.2 + 0.25 R_2$

Solve & get $R_2 = 262$ N & $F = 0.25(262) = 65.4$ N

$\Sigma F_x = 0$; $N = 200 + 65.4 = 265.4$ N

$(F = 65.4) < (0.3 \times 265.4 = 79.6)$ so assumption OK

Wedge: $\Sigma F_x = 0$; $R_1 \cos 26.70° = 265.4$, $R_1 = 297$ N

$\Sigma F_y = 0$; $P = 65.4 + 297 \sin 26.70° = \underline{198.8\ N}$

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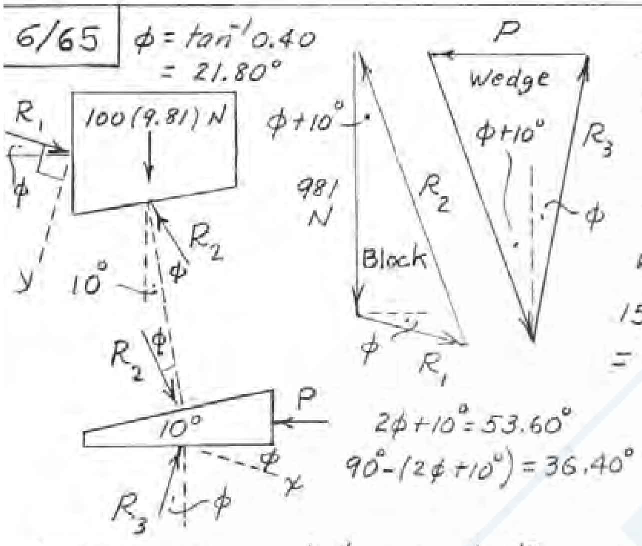
6/65 $\phi = \tan^{-1} 0.40 = 21.80°$



$2\phi + 10° = 53.60°$

$90° - (2\phi + 10°) = 36.40°$

Block: $\Sigma F_y = 0;$

$981 \cos 21.80° = R_2 \cos 53.60°$

$R_2 = 1535$ N

Wedge: $\Sigma F_x = 0;$

$1535 \cos 36.40° = P \cos 21.80°$

$P = 1331$ N

Screw: $\alpha = \tan^{-1} \frac{L}{2\pi r} = \tan^{-1} \frac{10}{2\pi(15)} = 6.06°$; $\phi = \tan^{-1} 0.25 = 14.04°$

$\phi + \alpha = 20.09°$

$M = Pr \tan(\phi + \alpha) = 1331(0.015) \tan 20.09° = \underline{7.30 \text{ N}\cdot\text{m}}$

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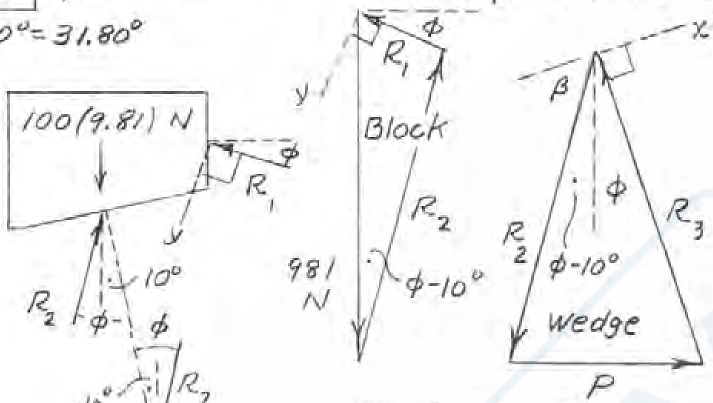
6/66 $\phi = \tan^{-1} 0.40 = 21.80°$ $\beta = 90° - (2\phi - 10°) = 56.40°$

$\phi + 10° = 31.80°$



Block: $\Sigma F_y = 0$;

$981 \cos 21.80 = R_2 \cos 10°$

$R_2 = 925$ N

Wedge: $\Sigma F_{x'} = 0$;

$925 \cos 56.40 = P \cos 21.80$

$P = 551$ N

For screw,

$\alpha = 6.06°$ & $\phi = 14.04°$ (Prob. 6/65)

$M' = Pr \tan(\phi + \alpha) = 551(0.015) \tan 20.09° = \underline{3.02 \text{ N}\cdot\text{m}}$

Note (+) sign, $(\phi + \alpha)$

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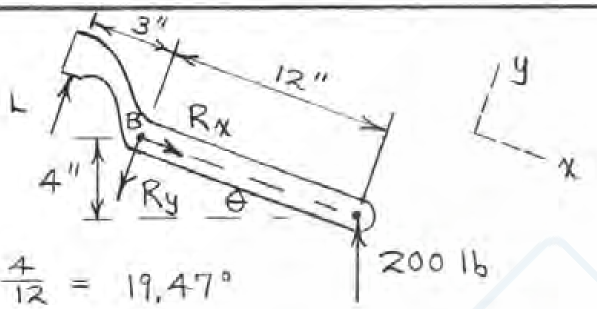

$\theta = \sin^{-1}\frac{4}{12} = 19.47°$

$\Sigma M_B = 0$: $200\cos 19.47°(12) - 3L = 0$, $L = 754$ lb

Screw: $\begin{cases}\text{helix angle } \alpha = \tan^{-1}\frac{1/12}{\pi(1/2)} = 3.04° \\ \text{friction angle } \phi = \tan^{-1}(0.20) = 11.31°\end{cases}$

Tighten screw: $M = Lr\tan(\phi+\alpha)$

$= 754(0.25)\tan(11.31°+3.04°) = 48.2$ lb-in.

Loosen screw: $M' = Lr\tan(\phi-\alpha)$

$= 754(0.25)\tan(11.31°-3.04°) = 27.4$ lb-in.

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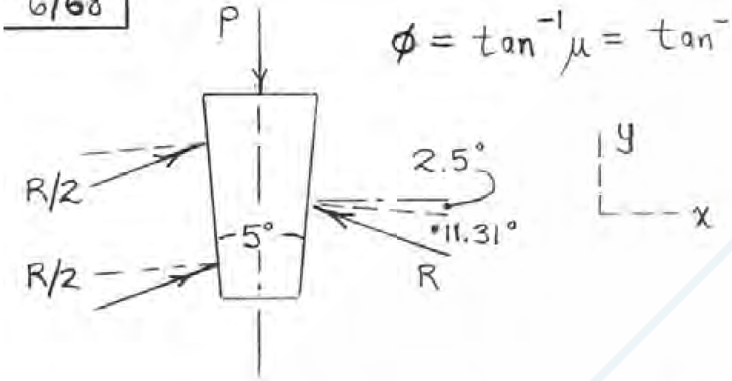

$\phi = \tan^{-1}\mu = \tan^{-1} 0.20 = 11.31°$

$\Sigma F_x = 0$ for shaft: $R\cos(11.31° + 2.5°) = T = 200$

$R = 206$ lb

$\Sigma F_y = 0$ for wedge: $P = 2R\sin(11.31° + 2.5°)$

$\underline{P = 98.3 \text{ lb}}$

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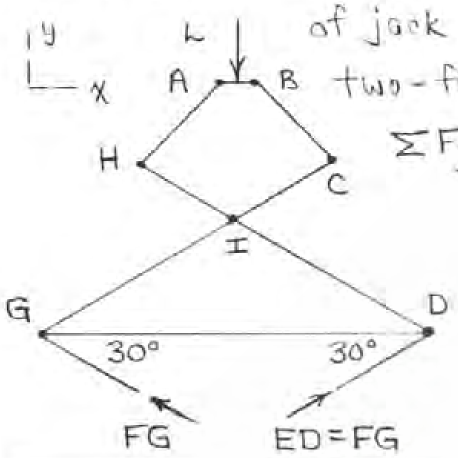
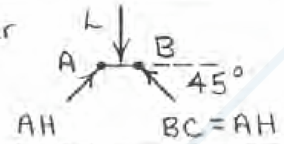
▸6/69 Calculate force in member FG. FBD of jack with FG & DE (both two-force members) cut:



$\Sigma F_y = 0$: $2FG \sin 30° = L$

$FG = L$

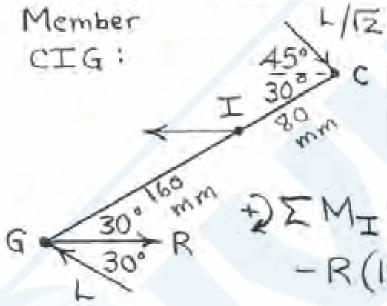
Member AB:



$\Sigma F_y = 0$: $2AH \sin 45° = L$

$AH = L/\sqrt{2}$

Member CIG:



(Due to symmetry, force at I is horizontal; R is force from rod.)

$\circlearrowleft \Sigma M_I = 0$: $\frac{L}{\sqrt{2}}(80 \sin 75°) + L(160 \sin 60°) - R(160 \sin 30°) = 0$, $R = 2.42L$

Friction angle $\phi = \tan^{-1}\mu = \tan^{-1}(0.20) = 11.31°$

$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{0.003}{2\pi(0.010/2)} = 5.45°$

To raise load: $M = Rr\tan(\alpha + \phi)$

$= [2.42(1000)(9.81)]\left[\frac{0.010}{2}\right]\tan(11.31° + 5.45°)$

$= \underline{35.7 \text{ N·m}}$

To lower load: $M' = Rr\tan(\phi - \alpha)$

$= [2.42(1000)(9.81)]\left[\frac{0.010}{2}\right]\tan(11.31° - 5.45°)$

$= \underline{12.15 \text{ N·m}}$

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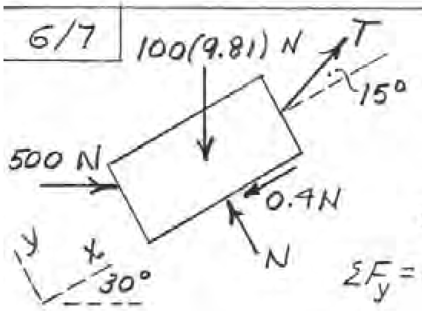
6/7



$\Sigma F_x = 0;$

$T\cos 15° - 981 \sin 30° - 0.4N + 500 \cos 30° = 0$

$0.966T - 0.4N = 57.5$

$\Sigma F_y = 0;\ N - 500 \sin 30° - 981 \cos 30° + T\sin 15° = 0$

$0.259T + N = 1099.6$

Solve simultaneously & get $N = 979$ N

$\underline{T = 465 \text{ N}}$

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▸6/70 For equil. of screw (refer to prob. illust.)

$\Sigma F = 0$; $W = \Sigma R_1 \cos(\alpha+\gamma) = \cos(\alpha+\gamma)\,\Sigma R_1$

$\Sigma M = 0$; $M = \Sigma R_1 r \sin(\alpha+\gamma) = r\sin(\alpha+\gamma)\,\Sigma R_1$

combine & get $M = Wr\tan(\alpha+\gamma) = Wr\dfrac{\tan\alpha + \tan\gamma}{1-\tan\alpha\tan\gamma}$

But $\tan\gamma = \dfrac{R\sin\phi}{R\cos\phi\cos\beta/2} = \mu/\cos\beta/2$

& $\tan\beta/2 = \dfrac{L}{2h}\cos\alpha$, $\tan\dfrac{\theta}{2} = \dfrac{L}{2h}$, so $\tan\dfrac{\beta}{2} = \tan\dfrac{\theta}{2}\cos\alpha$

& $\cos\dfrac{\beta}{2} = 1\Big/\sqrt{1+\tan^2\dfrac{\theta}{2}\cos^2\alpha}$. Thus $\tan\gamma = \mu\sqrt{1+\tan^2\dfrac{\theta}{2}\cos^2\alpha}$

Hence

$$M = Wr\,\frac{\tan\alpha + \mu\sqrt{1+\tan^2\frac{\theta}{2}\cos^2\alpha}}{1-\mu\tan\alpha\sqrt{1+\tan^2\frac{\theta}{2}\cos^2\alpha}}$$

where $\tan\alpha = \dfrac{L}{2\pi r}$

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6/71 $M = Rr\sin\phi$, $\sin\phi = \frac{M}{Rr} = \frac{3}{2(40)(9.81)(0.040/2)}$

$\phi = 11.02°$

$\mu = \tan\phi = \underline{0.1947}$

$r_f = r\sin\phi = \frac{0.040}{2}\sin 11.02° = 0.00382$ m

or $\underline{r_f = 3.82 \text{ mm}}$

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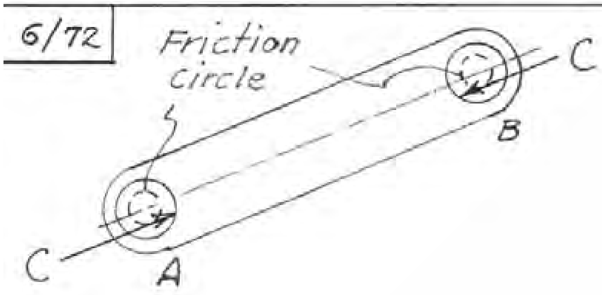
6/72



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6/73

$M = \frac{2}{3}\mu P R$

A on B: $M = \frac{2}{3}(0.40)(80)\left(\frac{9}{2}\right) =$ $\underline{96 \text{ lb-in.}}$

B on C: $96.0 = \frac{2}{3}\mu(80)\left(\frac{12}{2}\right)$, $\underline{\mu = 0.3}$

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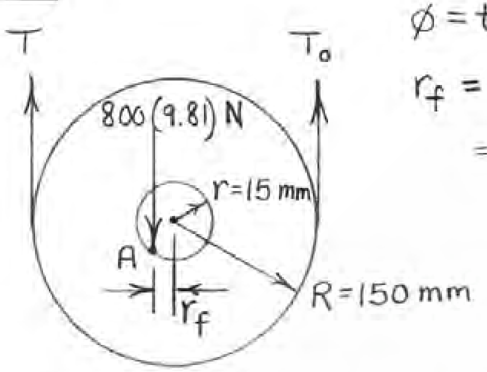
6/74



$\phi = \tan^{-1} 0.25 = 14.04°$

$r_f = r \sin\phi = 15 \sin 14.04$

$= 3.64$ mm

$+\uparrow \Sigma F = 0: \quad T + T_o - 800(9.81) = 0 \qquad (1)$

$\circlearrowleft \Sigma M_A = 0: \quad T(150 - 3.64) - T_o(150 + 3.64) = 0 \qquad (2)$

Solve (1) & (2) : $\begin{cases} T = 4020 \text{ N} \\ T_o = 3830 \text{ N} \end{cases}$

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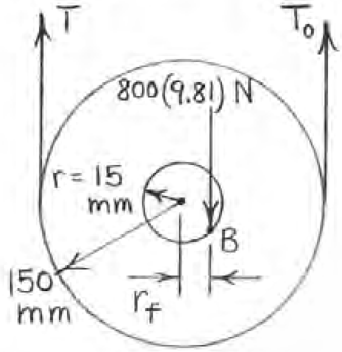
6/75



From the solution to Prob. 6/74, $r_f = 3.64$ mm

$+\uparrow \Sigma F = 0: \; T + T_0 - 800(9.81) = 0 \qquad (1)$

$\circlearrowleft + \Sigma M_B = 0: \; T(150 + 3.64) - T_0(150 - 3.64) = 0 \qquad (2)$

Solve (1) & (2): $\begin{cases} T = 3830 \text{ N} \\ T_0 = 4020 \text{ N} \end{cases}$

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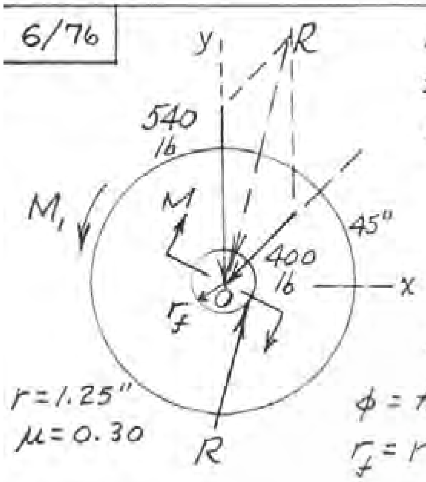
6/76



Replace 400-lb force by force and couple M_1

$M_1 = 400(22) = 8800$ lb-in.

$R_x = 400 \cos 45° = 282.8$ lb

$R_y = 540 + 400 \sin 45°$
$= 822.8$ lb

$R = \sqrt{(282.8)^2 + (822.8)^2} = 870.1$ lb

$\phi = \tan^{-1} 0.30 = 16.70°$

$r_f = r \sin\phi = 1.25 \sin 16.70° = 0.359$ in.

$\Sigma M_O = 0;\quad M - 8800 - 870.1(0.359) = 0$

$M = 8800 + 313 = 9113$ lb-in.

or $M = 9113/12 = \underline{759 \text{ lb-ft}}$

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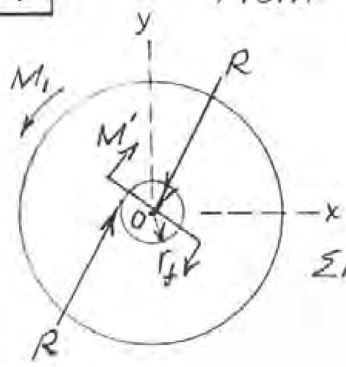
6/77

From Prob. 6/76, $R = 870.0$ lb
$\phi = 16.70°$
$r_f = r\sin\phi = 0.359$ in.
$M_1 = 8800$ lb-in.



$\Sigma M_O = 0;\ M' + 870.0(0.359) - 8800 = 0$

$M' = 8800 - 313$

$= 8487$ lb-in.

or $M' = 707$ lb-ft

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6/78 $\mu = 0.80 - kr$: $0.50 = 0.80 - k(3)$, $k = \frac{1}{10}$ in.$^{-1}$

So $\mu = 0.80 - \frac{r}{10}$ (r in in.)

Downward force $R = pA$, $p = \frac{6+10}{\pi(3)^2} = \frac{16}{9\pi}$ lb/in.^{2}

$$M_z = \int \mu p \, dA \times r = \int_0^{2\pi}\int_0^3 \left(0.80 - \frac{r}{10}\right)\frac{16r}{9\pi}\, r\,dr\,d\theta$$

$$= 2\pi \frac{16}{9\pi}\int_0^3 \left(0.80 - \frac{r}{10}\right) r^2 dr = \frac{32}{9}\left[\frac{0.80r^3}{3} - \frac{r^4}{40}\right]_0^3$$

$$= \underline{18.40 \text{ lb-in.}}$$

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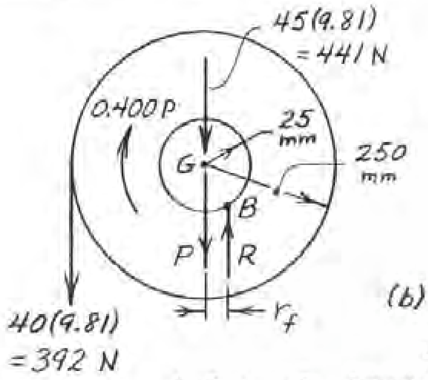
6/79 FBD of shaft and attached drum, with force P replaced by a force-couple system at G:



(a) $r_f = 0$:

$\circlearrowleft + \Sigma M_G = 0$: $392(0.250) - 0.400P = 0$,

$\underline{P = 245 \text{ N}}$

(b) $\varphi = \tan^{-1} 0.2 = 11.31°$

$r_f = r \sin\varphi = 25 \sin 11.31° = 4.90$ mm

$\circlearrowleft + \Sigma M_B = 0$: $392(0.250 + 0.00490) + 441(0.00490) + P(0.00490) - 0.400P = 0$

$\underline{P = 259 \text{ N}}$

(This solution assumes that the bearing reaction can be represented by a single force R as shown above.)

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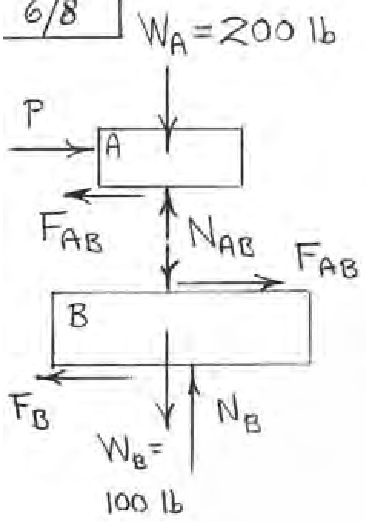
6/8



From $\uparrow \Sigma F_y = 0$,

$N_{AB} = 200$ lb, $N_B = 300$ lb

$$\begin{cases} F_{AB_{max}} = \mu_s N_{AB} = 0.5(200) = 100 \text{ lb} \\ F_{B_{max}} = \mu_s N_B = 0.25(300) = 75 \text{ lb} \end{cases}$$

(a) $P = 60$ lb : Both blocks remain stationary, as $\rightarrow \Sigma F_x = 0$ yields $F_{AB} = 60$ lb and $F_B = 60$ lb, both less than the maximum

(b) $P = 80$ lb : No relative motion between A and B, but sliding between B and the supporting surface; motion is to the right.

(c) $P = 120$ lb : A slides to the right relative to B; $F_{AB} = \mu_k N_{AB} = 0.3(200) = 60$ lb $< F_{B_{max}}$) so B remains stationary.

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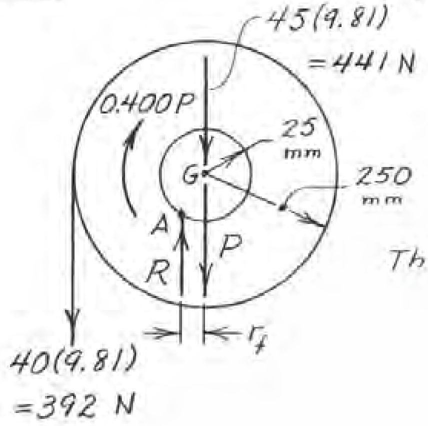
6/80



$r_f = 4.90$ mm (from Prob. 6/79)

$\circlearrowleft \Sigma M_A = 0: 392(0.250 - 0.00490)$

$- 0.400P$

$- (441 + P)(0.00490) = 0$

$\underline{P = 232 \text{ N}}$

The values are NOT symmetric:

$$\begin{cases} P_{down} = 232.18 \text{ N} \\ P_{n.f.} = 245.25 \text{ N} \\ P_{up} = 258.64 \text{ N} \end{cases}$$

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6/81

$dM = (\mu p \, dA) r \quad$ where $p = k/r^2$

$$M = \int_0^{2\pi}\int_{r_i}^{r_o} \mu p r (r \, dr \, d\theta) = 2\pi \mu k \int_{r_i}^{r_o} dr = 2\pi \mu k (r_o - r_i)$$

Also $L = \int p \, dA = \int_0^{2\pi}\int_{r_i}^{r_o} \frac{k}{r^2} r \, dr \, d\theta = 2\pi k \ln r \Big]_{r_i}^{r_o}$

or $L = 2\pi k \ln \frac{r_o}{r_i}$, $2\pi k = \frac{L}{\ln r_o/r_i}$

Thus $M = \mu L \frac{r_o - r_i}{\ln (r_o/r_i)}$

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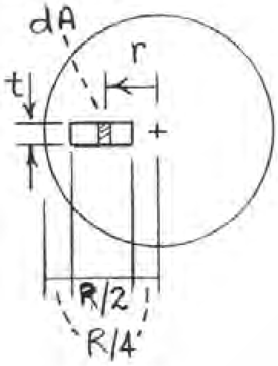
6/82



(t small)

$$P = pA = p\left(t\frac{R}{2}\right)$$

$$M = \int \mu p r\,dA = \mu p \int_{R/4}^{3R/4} r t\,dr$$

$$= \mu p t \left.\frac{r^2}{2}\right|_{R/4}^{3R/4}$$

$$= \frac{\mu p t}{2}\left[\left(\frac{3R}{4}\right)^2 - \left(\frac{R}{4}\right)^2\right]$$

$$= \frac{1}{4}\mu p t R^2$$

$$= \frac{1}{4}\mu\left(\frac{P}{tR/2}\right)t R^2 = \underline{\frac{1}{2}\mu P R}$$

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6/83 For constant pressure Eq. 6/5a gives

$$M = \frac{2}{3}\mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = \frac{2}{3}(0.35)(1)\frac{(150)^3 - (75)^3}{(150)^2 - (75)^2} = 40.8 \text{ N}\cdot\text{m}$$

For wheel $\Sigma M = 0$; $F(0.3) - 40.8 = 0$, $F = 136.1$ N

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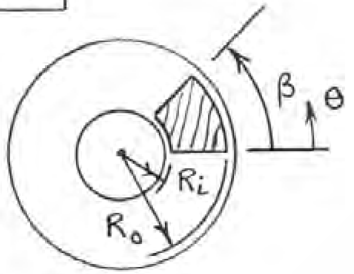
6/84



$$P = pA = p\int_0^\beta \int_{R_i}^{R_o} r\,dr\,d\theta$$

$$= \frac{p}{2}\int_0^\beta (R_o^2 - R_i^2)\,d\theta$$

$$= \frac{p}{2}(R_o^2 - R_i^2)\beta$$

$$M = 2\int \mu p r\,dA = 2\mu p \int_0^\beta \int_{R_i}^{R_o} r^2\,dr\,d\theta$$

$$= \frac{2\mu p}{3}(R_o^3 - R_i^3)\beta$$

$$= \frac{2\mu}{3}\,\frac{2P}{(R_o^2 - R_i^2)\beta}(R_o^3 - R_i^3)\beta$$

$$= \frac{4\mu P}{3}\,\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$$

Same form as Eq. 6/5a except for factor of 2 for 2 pads. No β dependence. Pressure variation with θ would not change the moment M.

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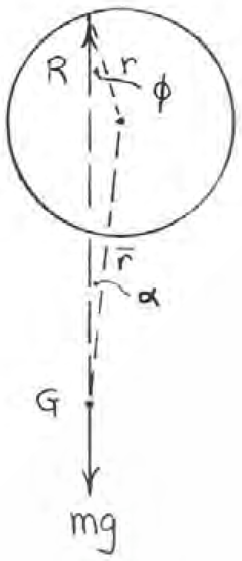
6/85



$$r \sin\phi = \bar{r}\sin\alpha$$

$$\mu = \tan\phi = \frac{\bar{r}\sin\alpha}{\sqrt{r^2 - \bar{r}^2\sin^2\alpha}}$$

or

$$\mu = \frac{1}{\sqrt{\left(\frac{d/2}{\bar{r}\sin\alpha}\right)^2 - 1}}$$

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6/86 $p = p_o\left(1 - \frac{r}{2a}\right)$; $dA = 2\pi r\, dr$

$$L = \int p\, dA = \int_0^a p_o\left(1 - \frac{r}{2a}\right) 2\pi r\, dr = 2\pi p_o\left[\frac{r^2}{2} - \frac{r^3}{6a}\right]_0^a$$

$$= \frac{2}{3}\pi p_o a^2 \quad \text{so} \quad p_o = \frac{3L}{2\pi a^2}$$

$$M = \int \mu p r\, dA = \int_0^a \mu p_o\left(r - \frac{r^2}{2a}\right) 2\pi r\, dr = 2\pi\mu p_o\left[\frac{r^3}{3} - \frac{r^4}{8a}\right]_0^a$$

$$= \frac{5}{12}\pi\mu p_o a^3 = \underline{\frac{5}{8}\mu L a}$$

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6/87 $L = \frac{1}{4}(960 - 4 \times 40) = 200 \text{ lb}$

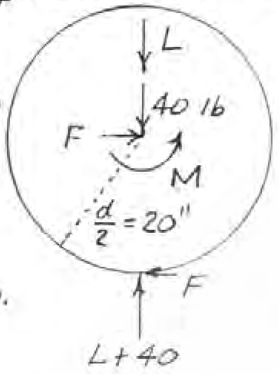
$P = 4F$

$F = \frac{1}{4} 16 = 4 \text{ lb}$

$M = F\frac{d}{2}$

$= 4(20)$

$= 80 \text{ lb-in.}$



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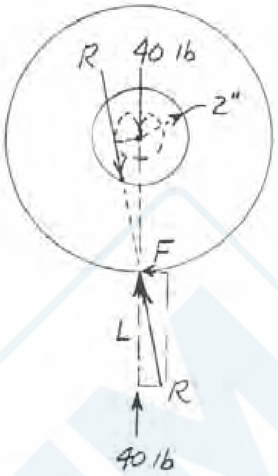

$M = Rr\sin\phi \approx Lr\sin\phi$

$80 = 200(2)\sin\phi$

$\phi = \sin^{-1} 0.2 = 11.54°, \quad \mu = \tan\phi = \underline{0.204}$

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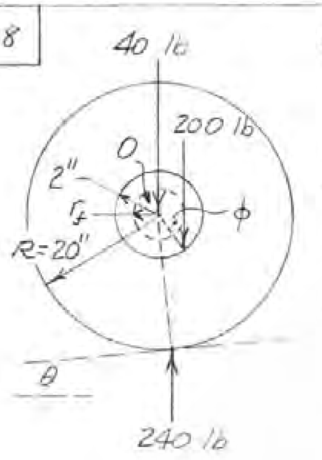
6/88



$r_f = r \sin\phi = 2 \sin(\tan^{-1} 0.2)$

$= 2 \sin^{-1} 11.31° = 2(0.1961)$

$= 0.3922$ in.

$\Sigma M_O = 0;$

$200\, r_f - 240\, R \sin\theta = 0$

$\sin\theta = \dfrac{200 \times 0.3922}{240 \times 20} = 0.01634$

$\underline{\theta = 0.936°}$

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6/89

$$\alpha = \tan^{-1}\frac{L}{2\pi r} = \tan^{-1}\frac{11}{2\pi\,\frac{120}{2}} = 1.671^\circ$$

$$\phi = \tan^{-1} 0.15 = 8.53^\circ$$

Screw: (a) Raise : $M_s = Wr\tan(\alpha+\phi)$

$$= \tfrac{1}{2}(10+3)(9.81)\,\frac{120}{2}\,\tan(1.671^\circ + 8.53^\circ) = 689 \text{ N·m}$$

(b) Lower : $M_s = Wr\tan(\phi-\alpha)$

$$= \tfrac{1}{2}(10+3)(9.81)\,\frac{120}{2}\,\tan(8.53^\circ - 1.671^\circ) = 460 \text{ N·m}$$

Collar bearing : $M_c = \frac{2}{3}\mu P\,\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}$

$$= \frac{2}{3}(0.15)\left(\frac{10+3}{2} + 0.9\right)(9.81)\,\frac{(250/2)^3 - (125/2)^3}{(250/2)^2 - (125/2)^2}$$

$$= 1059 \text{ N·m}$$

Total moment per screw:
(a) $M = 689 + 1059 = \underline{1747 \text{ N·m}}$
(b) $M = 460 + 1059 = \underline{1519 \text{ N·m}}$

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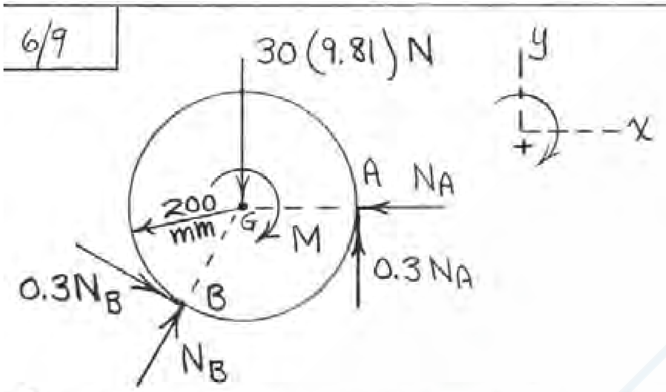
6/9



$$\Sigma M_G = 0: \quad M - 0.3(N_A + N_B)\,0.2 = 0 \qquad (1)$$

$$\Sigma F_x = 0: \quad N_B \sin 30° + 0.3 N_B \cos 30° - N_A = 0 \qquad (2)$$

$$\Sigma F_y = 0: \quad N_B \cos 30° - 0.3 N_B \sin 30° - 30(9.81) + 0.3 N_A = 0 \qquad (3)$$

Solution of Eqs. (1)–(3):

$$N_B = 312 \text{ N}$$
$$N_A = 237 \text{ N}$$
$$M = 32.9 \text{ N·m}$$

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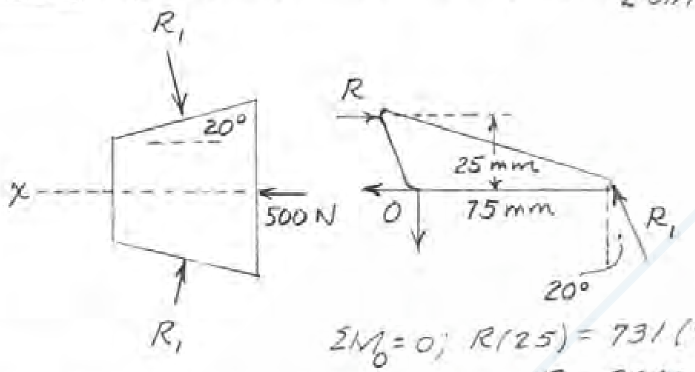
6/90 $\Sigma F_x = 0$; $2R_1 \sin 20° = 500$, $R_1 = \dfrac{500}{2 \sin 20°} = 731\ N$



$\Sigma M_O = 0$; $R(25) = 731\,(75 \cos 20°)$

$R = 2060\ N$

For 5 pairs of surfaces with uniform pressure

$$M = 5\left(\tfrac{2}{3}\right)\mu P \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = \frac{10}{3}(0.15)(4120)\frac{\overline{0.15}^3 - \overline{0.05}^3}{\overline{0.15}^2 - \overline{0.05}^2}$$

$M = 335\ N\cdot m$

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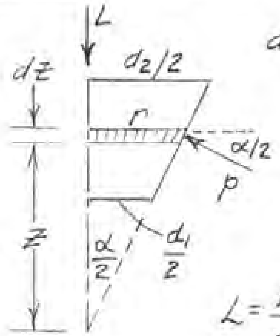
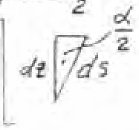
▶6/91

$$dL = p(2\pi r)\,ds\,\sin\frac{\alpha}{2} \quad \text{where} \quad ds = dz/\cos\frac{\alpha}{2}$$



$$dL = 2\pi p\,\sin\frac{\alpha}{2}\left(z\tan\frac{\alpha}{2}\right)\frac{dz}{\cos\alpha/2}$$



$$= 2\pi p\,\tan^2\frac{\alpha}{2}\; z\,dz$$

$$L = 2\pi p\,\tan^2\frac{\alpha}{2}\int_{z_1}^{z_2} z\,dz \quad \text{where} \quad z_1 = \frac{d_1/2}{\tan\alpha/2}$$

$$z_2 = \frac{d_2/2}{\tan\alpha/2}$$

$$L = \frac{\pi p}{4}(d_2^2 - d_1^2)$$

$$M = \int r\mu p\,dA = \mu p\int_{z_1}^{z_2}\left(z\tan\frac{\alpha}{2}\right)\left(2\pi z\tan\frac{\alpha}{2}\right)\frac{dz}{\cos\alpha/2}$$

$$= 2\pi\mu p\,\frac{\tan^2\frac{\alpha}{2}}{\cos\alpha/2}\int_{z_1}^{z_2} z^2\,dz = 2\pi\mu p\,\frac{1}{24\sin\frac{\alpha}{2}}(d_2^3 - d_1^3)$$

$$M = \frac{\mu L}{3\sin\frac{\alpha}{2}}\,\frac{d_2^3 - d_1^3}{d_2^2 - d_1^2}$$

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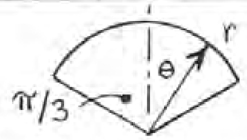
▸6/92



$$dM = \mu p\, dA\, (r \sin\theta) = \mu p\, (r\, d\theta)(2\pi r \sin\theta)\, r \sin\theta$$

$$= 2\pi \mu r^3 p_0 \cos\theta \sin^2\theta \, d\theta$$

$$M = 2\pi\mu r^3 p_0 \int_0^{\pi/3} \cos\theta \sin^2\theta \, d\theta$$

$$= 2\pi\mu r^3 p_0 \left. \frac{\sin^3\theta}{3} \right|_0^{\pi/3} = \frac{\sqrt{3}}{4} \pi\mu r^3 p_0$$

But $L = \int p\cos\theta\, dA = \int_0^{\pi/3} p_0 \cos^2\theta\, (r\, d\theta)(2\pi r \sin\theta)$

$$= 2\pi r^2 p_0 \int_0^{\pi/3} \cos^2\theta \sin\theta\, d\theta = 2\pi r^2 p_0 \left(-\frac{\cos^3\theta}{3} \right)_0^{\pi/3}$$

$$= \frac{7}{12} \pi r^2 p_0$$

Substitute $\pi r^2 p_0 = \frac{12}{7} L$ to obtain

$$M = \frac{\sqrt{3}}{4} \mu r \left(\frac{12L}{7} \right) = \underline{\frac{3\sqrt{3}}{7} \mu r L}$$

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6/93

$$\frac{T_2}{T_1} = e^{\mu\beta}: \quad \frac{mg}{mg/10} = e^{\mu(3\pi)}, \quad \underline{\mu = 0.244}$$

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6/94 Use $\frac{T_2}{T_1} = e^{\mu\beta}$, where $\beta = \frac{\pi}{2}$

(a) $\frac{P}{W} = e^{0.4(\pi/2)}$, $\underline{P = 1.874W}$

(b) $\frac{W}{P} = e^{0.4(\pi/2)}$, $\underline{P = 0.533W}$

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6/95 $\frac{T_2}{T_1} = e^{\mu\beta}$, $\frac{T}{200} = e^{0.30(5\pi/2)}$ where $\beta = \frac{5}{4}(2\pi)$ rad

$\frac{T}{200} = e^{2.356} = 10.55$, $T = 10.55(200) = 2110$ N

or $\underline{T = 2.11 \text{ kN}}$

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6/96

$T_2 = T_1 e^{\mu\beta}$

$18\,000 = 240 e^{0.30\beta}$, $e^{0.30\beta} = 75.0$

$0.30\beta = 4.32$, $\beta = 14.39$ rad

No. of turns $n = \frac{\beta}{2\pi} = \frac{14.39}{2\pi} = \underline{2.29}$ turns

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6/97 $T_2 = T_1 e^{\mu\beta}$, $T_2 = 50$ lb, $T_1 = 20$ lb, $\beta = \pi$

So $\frac{50}{20} = e^{\pi\mu}$, $e^{\pi\mu} = 2.5$, $\pi\mu = \ln 2.5$

$\pi\mu = 0.9163$, $\underline{\mu = 0.292}$

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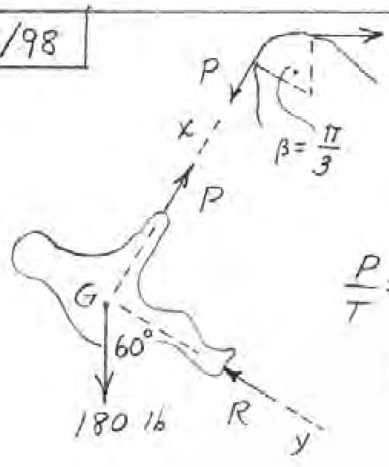

Equil. of climber, $\Sigma F_x = 0$
gives $P = 180 \sin 60°$
$= 155.9$ lb

$\frac{P}{T} = e^{\mu\beta}$; $\frac{155.9}{75} = e^{\pi\mu/3}$

$\frac{\pi}{3}\mu = \ln 2.078 = 0.732$

$\mu = 3(0.732)/\pi = \underline{0.699}$

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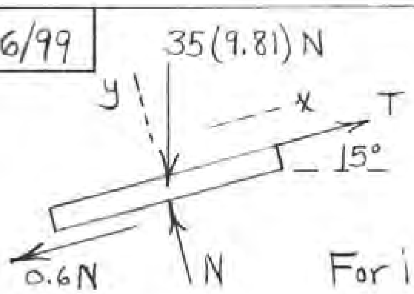

$\Sigma F_y = 0$:

$N - 35(9.81)\cos 15° = 0$

$N = 332$ N

For impending motion up incline:

$-0.6(332) - 35(9.81)\sin 15° + T = 0$, $T = 288$ N

Cord friction: $\dfrac{P}{288} = e^{0.4\left[15 \frac{\pi}{180}\right]}$

$P = 320$ N

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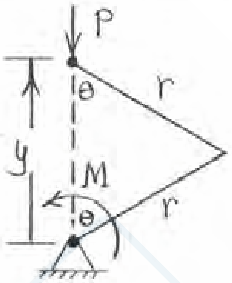
7/1

$\delta U = 0: \; -M\delta\theta - P\delta y = 0$

$y = 2r\cos\theta, \quad \delta y = -2r\sin\theta\,\delta\theta$

So $\quad M\delta\theta = P(2r\sin\theta\,\delta\theta)$

$$\underline{M = 2Pr\sin\theta}$$



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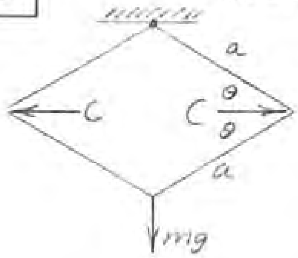

$\delta U = 0;\ 2C\,\delta(a\cos\theta) + mg\,\delta(2a\sin\theta) = 0$

$-2Ca\sin\theta\,\delta\theta + 2mga\cos\theta = 0$

$C = mg\,\text{ctn}\,\theta$

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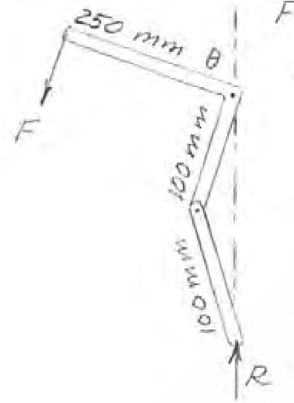

$\delta U = 0$

$F(250\,\delta\theta) - R\,\delta(2[100 \sin\theta]) = 0$

$250F\,\delta\theta = 200R\cos\theta\,\delta\theta$

$\underline{F = 0.8R\cos\theta}$

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7/12 $e = \dfrac{\text{output work}}{\text{input work}}$

To raise, $0.75 = \dfrac{250(1/4)}{P(1)}$, $\underline{P = 83.3 \text{ lb}}$

To lower, $0.75 = \dfrac{P'(1)}{250(1/4)}$, $\underline{P' = 46.9 \text{ lb}}$

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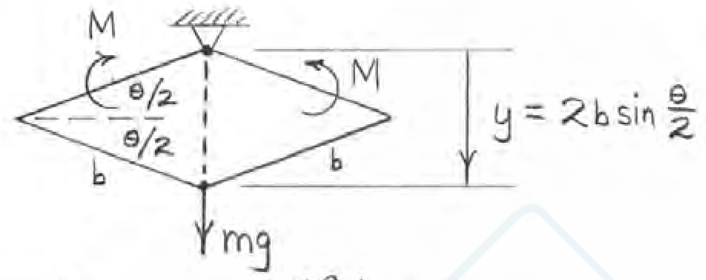

$\delta U = 0: \quad mg\,\delta y - 2M\,\delta\left(\frac{\theta}{2}\right) = 0$

$\delta y = b\cos\frac{\theta}{2}\,\delta\theta$, so $mgb\cos\frac{\theta}{2}\,\delta\theta = 2\left(\frac{1}{2}\right)M\,\delta\theta$

$\underline{M = mgb\cos\frac{\theta}{2}}$

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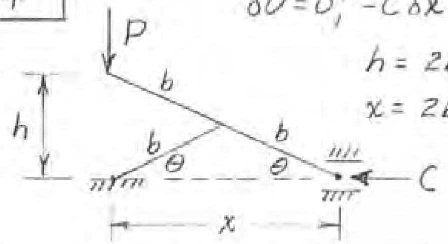
7/14

$\delta U = 0;\ -C\delta x - P\delta h = 0$



$h = 2b\sin\theta,\ \delta h = 2b\cos\theta\,\delta\theta$

$x = 2b\cos\theta,\ \delta x = -2b\sin\theta\,\delta\theta$

Thus $-C(-2b\sin\theta\,\delta\theta) - P(2b\cos\theta\,\delta\theta) = 0$

$C\sin\theta = P\cos\theta\ ,\ C = P\,\text{ctn}\,\theta$

But $\text{ctn}\,\theta = \sqrt{4b^2 - h^2}/h = \sqrt{(2b/h)^2 - 1}$, so $C = P\sqrt{(2b/h)^2 - 1}$

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7/15

$\delta U = 0$: $160 F\delta\theta - 0.4(160 F\delta\theta) - 100(9.81)\left(150\,\delta\frac{\theta}{25}\right) = 0$

$0.6(160)F = 981(6)$, $\underline{F = 61.3 \text{ N}}$

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7/16 Let $\delta\theta$ = virtual angle of input rotation

Then $\delta\theta/40$ = " " " output "

$$e = \frac{\text{output work}}{\text{input work}} = \frac{1180(\delta\theta/40)}{30\delta\theta} = \frac{1180}{30(40)} = \underline{0.983}$$

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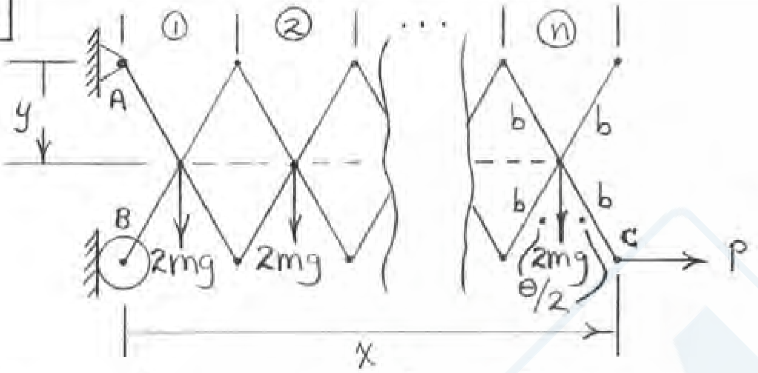

$y = b\cos\frac{\theta}{2}$, $\delta y = -\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta$

$x = n\left(2b\sin\frac{\theta}{2}\right)$, $\delta x = nb\cos\frac{\theta}{2}\,\delta\theta$

$\delta U = 0: \quad P\delta x + n(2mg)\delta y = 0$

$$P\left(nb\cos\frac{\theta}{2}\,\delta\theta\right) = -2nmg\left(-\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta\right)$$

$$\underline{P = mg\tan\frac{\theta}{2}}$$

P does not depend on the number n of sections present.

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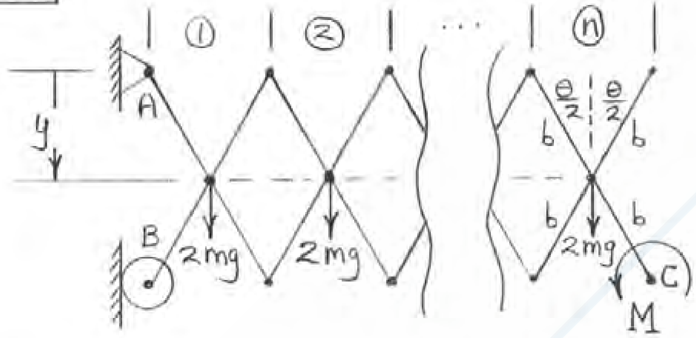

$y = b\cos\frac{\theta}{2}$, $\delta y = -\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta$

$\delta U = 0$: $M\,\delta\left(\frac{\theta}{2}\right) + n(2mg)\,\delta y = 0$

$\frac{M}{2}\,\delta\theta = -2nmg\left(-\frac{b}{2}\sin\frac{\theta}{2}\,\delta\theta\right)$

$M = 2nmgb\sin\frac{\theta}{2}$

M does depend on the number n of sections present.

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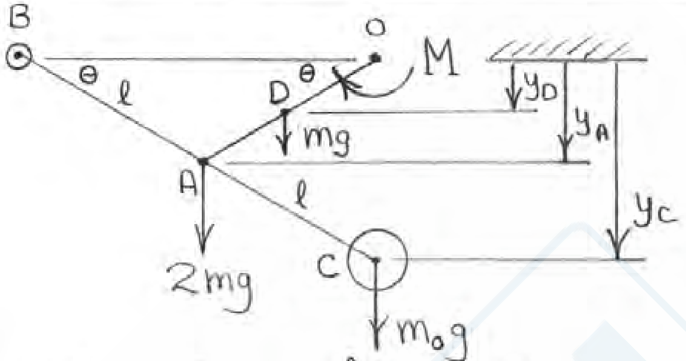

$y_D = \frac{\ell}{2}\sin\theta$, $\delta y_D = \frac{\ell}{2}\cos\theta\,\delta\theta$

$y_A = \ell\sin\theta$, $\delta y_A = \ell\cos\theta\,\delta\theta$

$y_C = 2\ell\sin\theta$, $\delta y_C = 2\ell\cos\theta\,\delta\theta$

$\delta U = 0$: $-M\delta\theta + mg\,\delta y_D + 2mg\,\delta y_A + m_0 g\,\delta y_C = 0$

$$-M\delta\theta + mg\left(\frac{\ell}{2}\cos\theta\,\delta\theta\right) + 2mg\left(\ell\cos\theta\,\delta\theta\right) + m_0 g\left(2\ell\cos\theta\,\delta\theta\right) = 0$$

$$\Rightarrow M = \left(\frac{5}{2}m + 2m_0\right)g\ell\cos\theta$$

For $\theta = 30°$: $M = \left(\frac{5}{4}m + m_0\right)g\ell\sqrt{3}$

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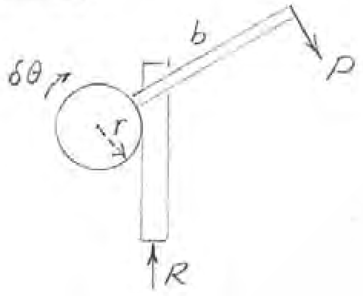

$\delta U = 0;\ Pb\delta\theta - Rr\delta\theta = 0$

$R = P\dfrac{b}{r}$

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7/20 $e = \dfrac{\text{output work}}{\text{input work}}$

Let $\delta\theta$ = virtual crank angle, radians

δh = " movement of lifting pad, inches

where $\dfrac{\delta\theta}{\delta h} = \dfrac{12(2\pi)}{1}$, $\delta\theta = 24\pi\,\delta h$

To raise, $e = \dfrac{L\,\delta h}{Fr\,\delta\theta} = \dfrac{2700\,\delta h}{10(6)\,24\pi\,\delta h} = \underline{0.597}$

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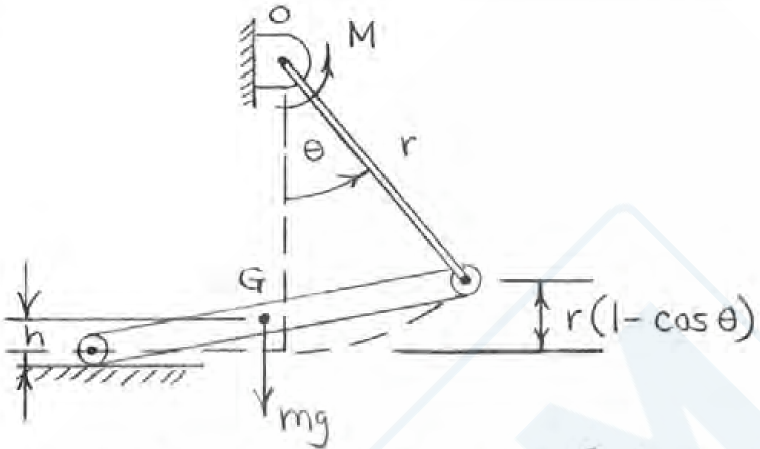

$\delta U = 0: \quad M\,\delta\theta - mg\,\delta h = 0, \quad h = \frac{r}{2}(1-\cos\theta)$

$\delta h = \frac{r}{2}\sin\theta\,\delta\theta$

$\therefore \quad M\,\delta\theta - mg\left(\frac{r}{2}\sin\theta\,\delta\theta\right) = 0$

$\left(M - \frac{mgr}{2}\sin\theta\right)\delta\theta = 0, \quad \underline{\theta = \sin^{-1}\left(\frac{2M}{mgr}\right)}$

$M_{max} = \frac{mgr}{2}$ @ $\theta = 90°$. No equilibrium for $M > \frac{mgr}{2}$

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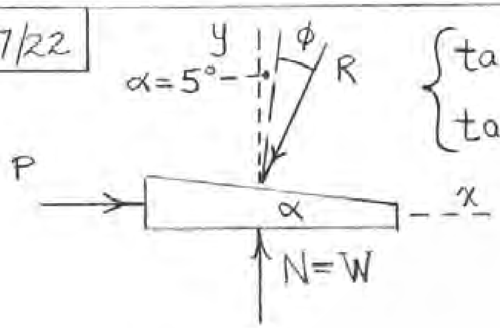

$$\begin{cases} \tan\phi = 0.30, & \phi = 16.70° \\ \tan\alpha = \tan 5° = 0.0875 \end{cases}$$

$\Sigma F_x = 0: \quad P = R\sin(\phi+\alpha)$

$\Sigma F_y = 0: \quad W = R\cos(\phi+\alpha)$

$\tan(\phi+\alpha) = \dfrac{P}{W}$

Work input: $P\,\delta x$

Work output: $W\,\delta y$

where $\delta y = \delta x \tan\alpha$

$$e = \frac{W\,\delta y}{P\,\delta x} = \frac{\tan\alpha}{\tan(\phi+\alpha)} = \frac{\tan 5°}{\tan(16.70°+5°)}$$

or $\underline{e = 0.220}$

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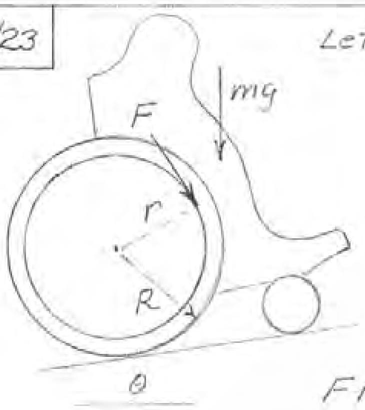

Let β = angle through which wheel turns

s = corresponding displacement along incline.

$s = R\beta$ so $\delta s = R\,\delta\beta$

$\delta U = 0$

$F r\,\delta\beta - mg\,\delta s \sin\theta = 0$

$F r\,\delta\beta = mg R \sin\theta\,\delta\beta$

$F = mg\dfrac{R}{r}\sin\theta$

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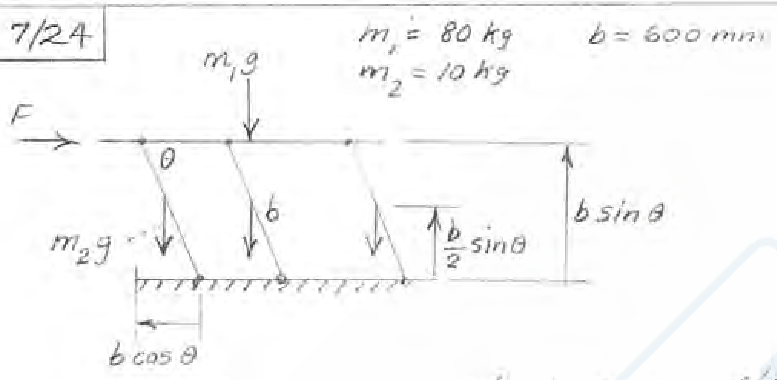

$$\delta U = 0; \quad -F\delta(b\cos\theta) - m_1 g\, \delta(b\sin\theta) - 3m_2 g\, \delta\left(\frac{b}{2}\sin\theta\right) = 0$$

$$Fb\sin\theta\,\delta\theta = m_1 g b\cos\theta\,\delta\theta + \frac{3}{2}m_2 g b\cos\theta\,\delta\theta$$

$$F = g\,\text{ctn}\,\theta\left(m_1 + \frac{3}{2}m_2\right)$$

$$= 9.81\left(80 + \frac{3}{2}10\right)\text{ctn}\,\theta = \underline{932\,\text{ctn}\,\theta \text{ N}}$$

Solution by force and moment equilibrium would require dismemberment with four FBD's and eventual elimination of unwanted forces and dimensions

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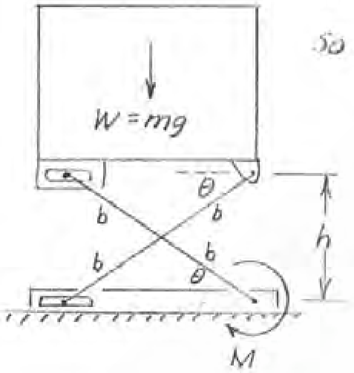
7/25 $\delta U = 0;\quad M\delta\theta - mg\,\delta h = 0$

$h = 2b\sin\theta,\quad \delta h = 2b\cos\theta\,\delta\theta$



So $M\delta\theta = mg(2b\cos\theta)\delta\theta$

$M = 2mgb\cos\theta$

But since $\sin\theta = \frac{h}{2b}$,

$\cos\theta = \sqrt{1 - \left(\frac{h}{2b}\right)^2}$

Thus

$M = 2mgb\sqrt{1 - \left(\frac{h}{2b}\right)^2}$

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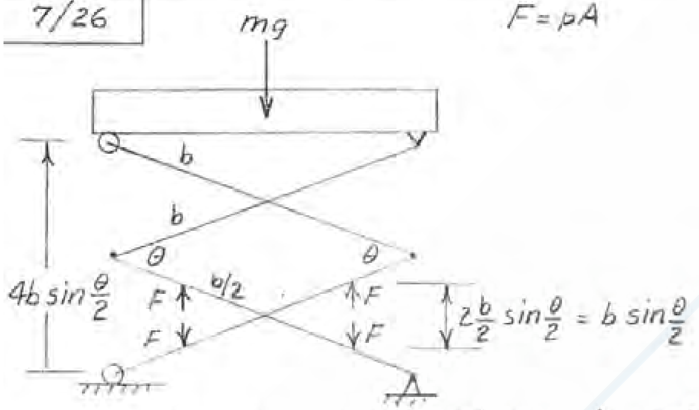

Work done by each cylinder is $F\delta(b\sin\frac{\theta}{2})$

$= \frac{Fb}{2}\cos\frac{\theta}{2}\,\delta\theta$

$\delta U = 0;\ -mg\,\delta(4b\sin\frac{\theta}{2}) + 2\,\frac{Fb}{2}\cos\frac{\theta}{2}\,\delta\theta = 0$

$Fb\cos\frac{\theta}{2} = 2mgb\cos\frac{\theta}{2}$, $F = pA = 2mg$

so $p = 2mg/A$ independent of b & θ

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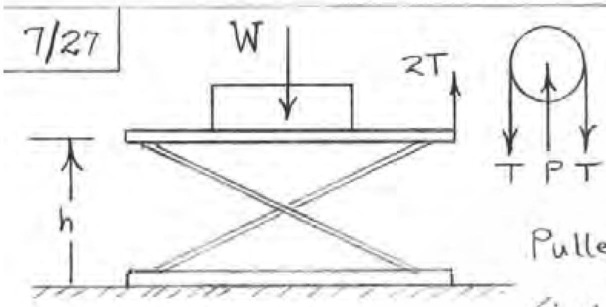
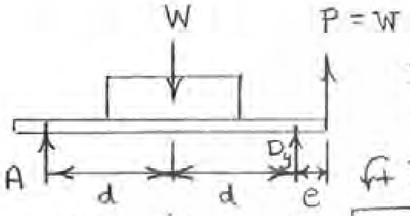


$\delta U = 0: \; 2T\delta h - W\delta h = 0$

$T = W/2$

Pulley: $P = 2T$, so $\underline{P = W}$

(independent of h)



D_y and A form a clockwise couple

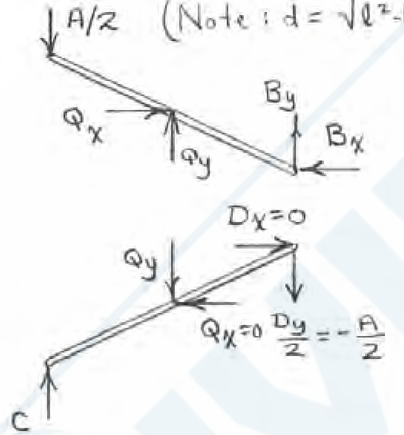
$D_y = -A$ (Note $D_x = 0$)

$\curvearrowleft + \Sigma M = 0: \; W(d+e) - A(2d) = 0$

(Note: $d = \sqrt{\ell^2 - h^2}/2$)

$A = \frac{W}{2}\left(1 + \frac{e}{d}\right)$

$= \frac{W}{2}\left(1 + \frac{2e}{\sqrt{\ell^2 - h^2}}\right)$



CD: $\Sigma M_C = 0$:

$Q_y d - \frac{A}{2}(2d) = 0$

$\underline{Q = Q_y = A = \frac{W}{2}\left(1 + \frac{2e}{\sqrt{\ell^2 - h^2}}\right)}$

(From drawing, $d > e$, so $Q \cong \frac{W}{2}$)

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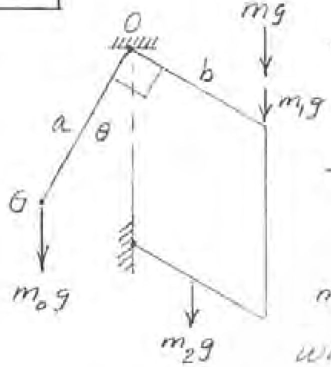

$\delta U = 0;\; m_o g\,\delta(a\cos\theta)$

$+ (m + m_1) g\,\delta(b\sin\theta)$

$+ m_2 g\,\delta\left(\frac{b}{2}\sin\theta\right) = 0$

$-m_o a\sin\theta\,\delta\theta + (m + m_1) b\cos\theta\,\delta\theta$

$+\frac{m_2 b}{2}\cos\theta\,\delta\theta = 0$

$m_o a\tan\theta = mb + \left(m_1 + \frac{m_2}{2}\right)b \quad \text{--- (1)}$

When $\theta = \theta_o$, $mg = 0$ so

$m_o a\tan\theta_o = \left(m_1 + \frac{m_2}{2}\right)b \quad \text{--- (2)}$

Eliminate m_1 & m_2 from Eqs. (1) & (2) & get

$m_o a\tan\theta = mb + m_o a\tan\theta_o$

$m = \frac{a}{b} m_o(\tan\theta - \tan\theta_o)$

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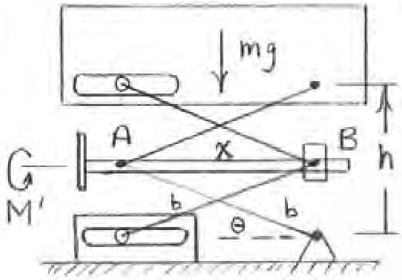
7/29 M' = necessary moment without friction



Let β = angle through which screw turns

$\delta U = 0: \quad M'\delta\beta - mg\,\delta h = 0$

$\frac{L}{2\pi} = \frac{-\delta(x)}{\delta\beta}, \quad \delta\beta = \frac{2\pi}{L}(-\delta x)$

$x = 2b\cos\theta, \quad \delta x = -2b\sin\theta\,\delta\theta$

$\delta\beta = \frac{4\pi b}{L}\sin\theta\,\delta\theta$

$h = 4b\sin\theta, \quad \delta h = 4b\cos\theta\,\delta\theta$

Thus $M'\frac{4\pi b}{L}\sin\theta\,\delta\theta - mg\,(4b\cos\theta\,\delta\theta) = 0$

$M' = \frac{mgL}{\pi}\cot\theta$

$M = M_f + \frac{mgL}{\pi}\cot\theta$

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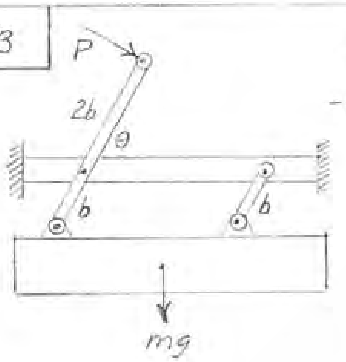

$\delta U = 0;$

$-P(2b\,\delta\theta) + mg\,\delta(b\sin\theta) = 0$

$2Pb\,\delta\theta = mgb\cos\theta\,\delta\theta$

$\cos\theta = \frac{2P}{mg}$

$\theta = \cos^{-1}\frac{2P}{mg}$

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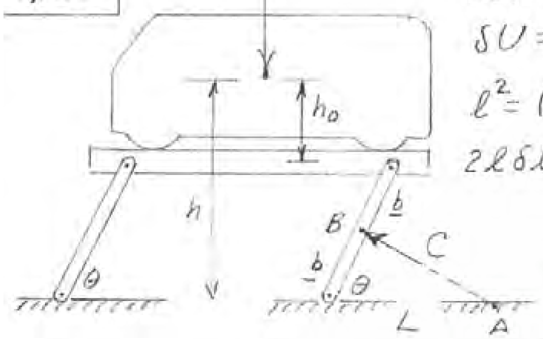

Let $\overline{AB} = \ell$

$\delta U = 0$; $C\delta\ell - mg\delta h = 0$

$\ell^2 = (b\sin\theta)^2 + (L - b\cos\theta)^2$

$2\ell\delta\ell = 2b^2\sin\theta\cos\theta\,\delta\theta$

$\quad + 2(L - b\cos\theta)(b\sin\theta\,\delta\theta)$

$\quad = 2Lb\sin\theta\,\delta\theta$

$\delta\ell = \frac{Lb}{\ell}\sin\theta\,\delta\theta$

$h = 2b\sin\theta + h_0$, $\delta h = 2b\cos\theta\,\delta\theta + 0$

Thus $C\frac{Lb}{\ell}\sin\theta\,\delta\theta - mg(2b\cos\theta\,\delta\theta) = 0$

$$C = 2mg\frac{\ell}{L}\operatorname{ctn}\theta = \frac{2mg}{L}\sqrt{(b\sin\theta)^2 + (L - b\cos\theta)^2}\operatorname{ctn}\theta$$

$$C = 2mg\sqrt{1 + \left(\frac{b}{L}\right)^2 - 2\frac{b}{L}\cos\theta}\ \operatorname{ctn}\theta$$

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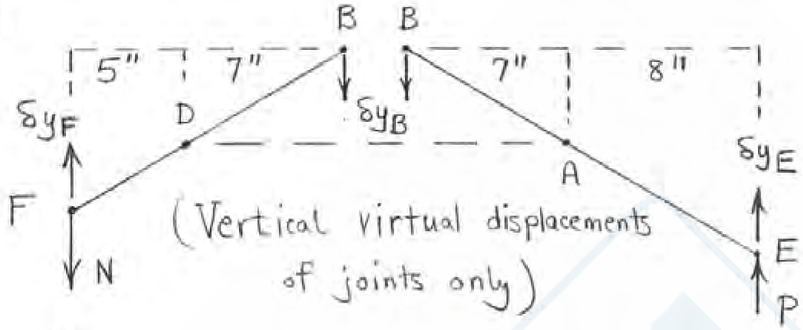

$\delta y_B = \frac{7}{8}\delta y_E$, $\delta y_F = \frac{5}{7}\delta y_B$

So $\delta y_F = \frac{5}{7}\cdot\frac{7}{8}\delta y_E = \frac{5}{8}\delta y_E$

$\delta U = 0$: $P\delta y_E - N\delta y_F = 0$

$$P\delta y_E = N\left(\frac{5}{8}\delta y_E\right)$$

$$N = \frac{8}{5}P = \underline{1.6P}$$

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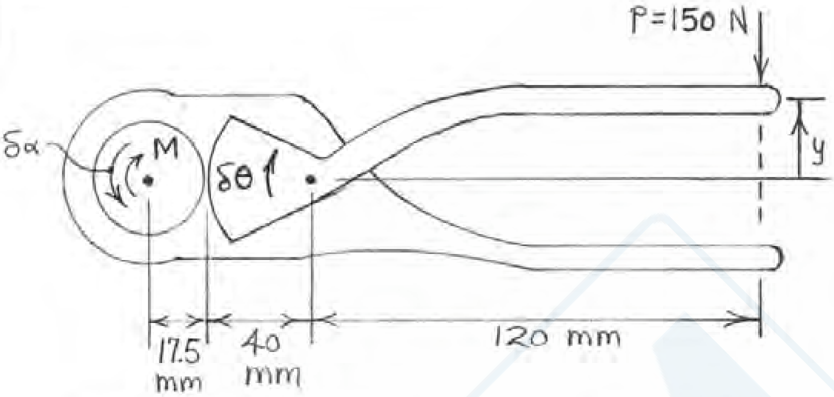

$\delta\alpha$ = rotation of socket on bolt head

$\delta\theta$ = rotation of upper handle (lower handle and frame taken as fixed)

$17.5\,\delta\alpha = 40\,\delta\theta$, $\delta y = -120\,\delta\theta$

$\delta U = 0: \; -M\delta\alpha + P(-\delta y) = 0$

$$M\left(\frac{40}{17.5}\delta\theta\right) = 150\,(120\,\delta\theta)$$

$M = 7880$ N·mm or $\underline{M = 7.88 \text{ N·m}}$

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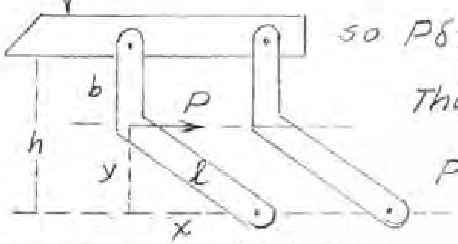
$\delta U = 0; \quad -P\delta x - W\delta h = 0$

$W = 250(9.81)$ N

$x^2 + y^2 = \ell^2, \quad x\delta x = -y\delta y$

$\delta h = \delta(b + y) = \delta y$

so $P\delta x = -W\delta y = -W\left(-\frac{x}{y}\right)\delta x$



Thus $P = W\frac{x}{y}$

$P = 250(9.81)\,\frac{500}{350} = 3500$ N

or $\underline{P = 3.5 \text{ kN}}$

$h = 650$ mm, $b = 300$ mm

$y = 350$ mm, $x = 500$ mm

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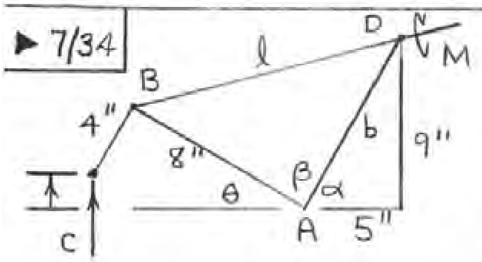

$\delta U = 0: C\delta h + M\delta(2\pi\gamma) = 0$

where $\delta\gamma$ = screw rotation in rev.

With L = lead = $\frac{1}{6}$ in.,

$\delta l = L\,\delta\gamma = \frac{1}{6}\delta\gamma$ in.

$b = \sqrt{5^2 + 9^2} = 10.30$ in.

$h = 8\sin\theta - 4\cos\theta, \quad \delta h = (8\cos\theta + 4\sin\theta)\delta\theta$

$l^2 = 8^2 + b^2 - 2(8)b\cos\beta, \quad 2l\,\delta l = 16b\sin\beta\,\delta\beta$

Because α = constant, $\delta\beta = -\delta\theta$: $l\,\delta l = -8b\sin\beta\,\delta\theta$

Thus $C(8\cos\theta + 4\sin\theta)\delta\theta + M\,2\pi\,6\,\frac{-8b}{l}\sin\beta\,\delta\theta = 0$

$$C = 2\pi M\,\frac{12b}{l}\,\frac{\sin\beta}{2\cos\theta + \sin\theta}$$

For $\theta = 30°$, $\beta = 180° - 30° - \tan^{-1}\frac{9}{5} = 89.1°$

$l = \sqrt{8^2 + 5^2 + 9^2 - 16(10.30)(0.0165)} = 12.93$ in.

$$C = 2\pi\,\frac{12(10.30)\sin 89.1°\;M}{12.93\,(2\cos 30° + \sin 30°)} = \underline{26.9\,M}$$

(M in lb-in., C in lb)

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7/35 $V = 6x^3 - 9x^2 - 7$

$\frac{dV}{dx} = 18x^2 - 18x = 0$ for equil. $x = 0$ or $x = 1$

$\frac{d^2V}{dx^2} = 36x - 18 = -18$ for $x = 0$ so unstable

$= +18$ " $x = 1$ so stable

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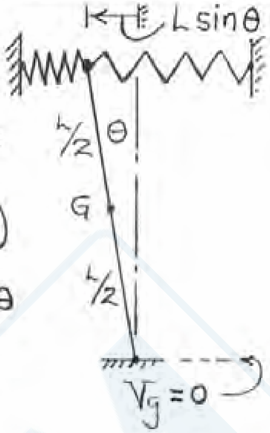
7/36 | δ = initial spring compression



$$V = V_g + V_e = mg\frac{L}{2}\cos\theta + \frac{1}{2}k(\delta + L\sin\theta)^2 + \frac{1}{2}k(\delta - L\sin\theta)^2$$

$$= \frac{1}{2}mgL\cos\theta + k(\delta^2 + L^2\sin^2\theta)$$

$$\frac{dV}{d\theta} = -\frac{1}{2}mgL\sin\theta + 2kL^2\sin\theta\cos\theta$$

$$\frac{d^2V}{d\theta^2} = -\frac{1}{2}mgL\cos\theta + 2kL^2\cos 2\theta$$

For equilibrium, $\frac{dV}{d\theta} = 0$, so

$$\left(-\frac{1}{2}mg + 2kL\cos\theta\right)\sin\theta = 0$$

$$\Rightarrow \theta = 0 \text{ or } \cos\theta = \frac{mg}{4kL}$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = -\frac{1}{2}mgL + 2kL^2$

> 0 (Stable) if $2kL^2 > \frac{1}{2}mgL$

So $k_{min} = \frac{mg}{4L}$

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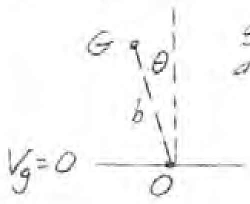
$V = V_g = mgb\cos\theta$



$\frac{dV}{d\theta} = -mgb\sin\theta = 0$ for equil.

$\theta = 0°$ or $\theta = 180°$

$\frac{d^2V}{d\theta^2} = -mgb\cos\theta$

$\theta = 0$, $\frac{d^2V}{d\theta^2} = -mgb$ so unstable

$\theta = 180°$, $\frac{d^2V}{d\theta^2} = -mgb(-1) = +mgb$ stable

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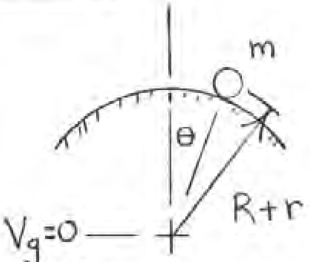
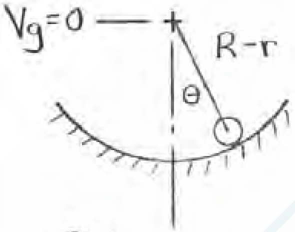


$$V = V_g = (R+r)\cos\theta$$

$$\frac{dV}{d\theta} = -(R+r)\sin\theta = 0 \quad \left(\text{for equil.}\right.$$

$$\theta = 0, \pi \ (\text{reject})$$

$$\frac{d^2V}{d\theta^2} = -(R+r)\cos\theta < 0 \quad @ \quad \theta = 0: \ \underline{\text{unstable}}$$



$$V = V_g = -(R-r)\cos\theta$$

$$\frac{dV}{d\theta} = (R-r)\sin\theta = 0 \quad \left(\text{for equil.}\right.$$

$$\theta = 0, \pi \quad (\text{reject})$$

$$\frac{d^2V}{d\theta^2} = (R-r)\cos\theta > 0 \ @ \ \theta = 0: \ \underline{\text{stable}}$$

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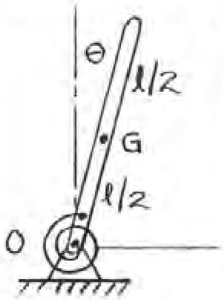

$$V = V_e + V_g = \frac{1}{2}K\theta^2 + mg\frac{l}{2}\cos\theta$$

$$\frac{dV}{d\theta} = K\theta - mg\frac{l}{2}\sin\theta$$

$$\frac{d^2V}{d\theta^2} = K - mg\frac{l}{2}\cos\theta$$

$$\frac{d^2V}{d\theta^2} > 0 \quad @ \quad \theta = 0:$$

$$K - mg\frac{l}{2} > 0$$

or $$l < \frac{2K}{mg}$$

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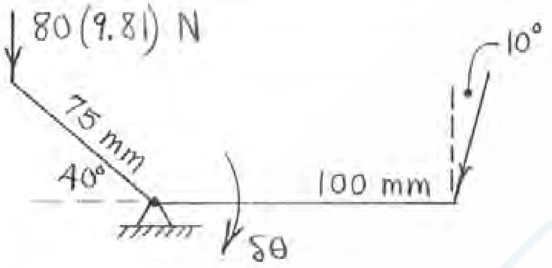

For a virtual displacement δθ of the lever,

$\delta U = 0: \; P\cos 10°(100\,\delta\theta) - 80(9.81)[75\,\delta\theta\,\cos 40°] = 0$

$P = 458$ N

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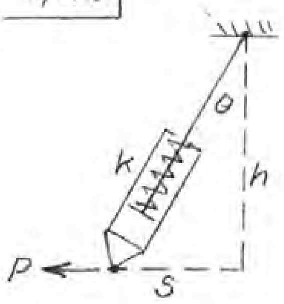

x = spring compression

$= \frac{h}{\cos\theta} - h = h(\sec\theta - 1)$

$\delta U = \delta V_e \; ; \; P\delta s = \delta(\frac{1}{2}kx^2)$

$= kx\,\delta x$

$s = h\tan\theta$ so that

$P\delta(h\tan\theta) = kh^2(\sec\theta - 1)\sec\theta\tan\theta\,\delta\theta$

$Ph\sec^2\theta\,\delta\theta = kh^2(\sec\theta - 1)\sec^2\theta\sin\theta\,\delta\theta$

$P = kh(\sec\theta - 1)\sin\theta$

or $\underline{P = kh\tan\theta(1 - \cos\theta)}$

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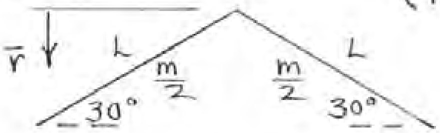
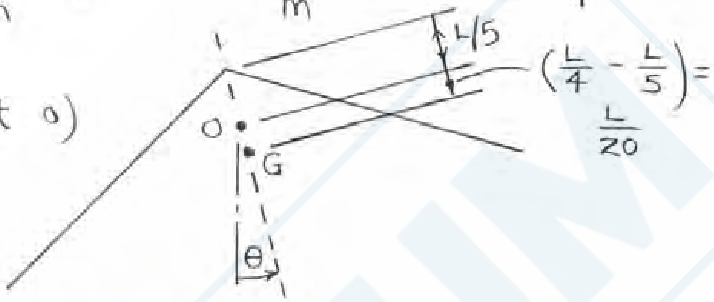
(Total mass m)



$$\bar{r} = \frac{\Sigma m\bar{r}}{\Sigma m} = \frac{2\left[\frac{m}{2}\frac{L}{2}\sin 30^\circ\right]}{m} = \frac{L}{4}$$

(Datum at o)



$$V = V_g = -mg\frac{L}{20}\cos\theta$$

$$\frac{dV}{d\theta} = \frac{mgL}{20}\sin\theta \quad (\text{Zero @ } \theta = 0)$$

$$\frac{d^2V}{d\theta^2} = \frac{mgL}{20}\cos\theta > 0 \text{ @ } \theta = 0$$ <u>so stable</u>.

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7/42 Take $V_g = 0$ through AO & $V_e = 0$ when $\theta = 0$

So $V_g = -mgh = -60(9.81)(0.7\sin\theta) = -412.0\sin\theta$

$$V_e = \frac{1}{2}kx^2 = \frac{1}{2}(160)\left[2(1.4)\sin\frac{\theta}{2}\right]^2 = 627.2\sin^2\frac{\theta}{2}$$

$$V = V_e + V_g = 627.2\sin^2\frac{\theta}{2} - 412.0\sin\theta$$

$$\frac{dV}{d\theta} = \frac{2}{2}(627.2)\sin\frac{\theta}{2}\cos\frac{\theta}{2} - 412.0\cos\theta$$

$$= 313.6\sin\theta - 412.0\cos\theta = 0 \text{ for equil.}$$

$$\text{or } \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{412.0}{313.6} = 1.314$$

$$\underline{\theta = 52.7°}$$

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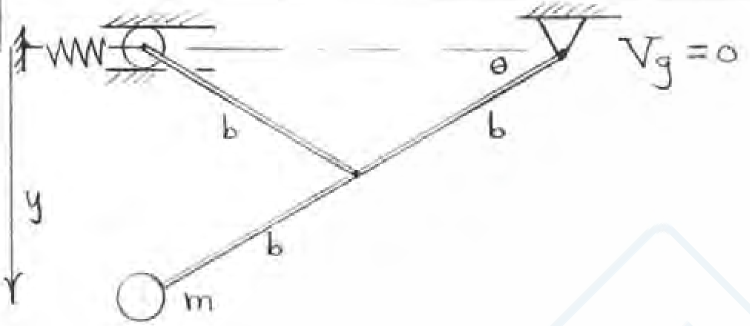

Spring stretch $= 2b - 2b\cos\theta = 2b(1-\cos\theta)$

$V_e = \frac{1}{2}k[2b(1-\cos\theta)]^2 = 2kb^2(1-\cos\theta)^2$

$V_g = -mg(2b\sin\theta) = -2mgb\sin\theta$

$V = 2kb^2(1-\cos\theta)^2 - 2mgb\sin\theta$

$\frac{dV}{d\theta} = 4kb^2(1-\cos\theta)\sin\theta - 2mgb\cos\theta = 0$
(for equilibrium)

$2kb(1-\cos\theta)\sin\theta = mg\cos\theta$, $k = \frac{mg}{2b}\frac{\cot\theta}{1-\cos\theta}$

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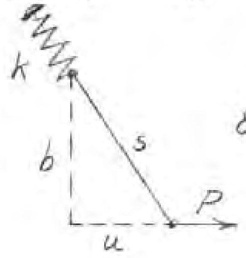

$\delta V_e = kx\,\delta x$ where $x = s - b = \sqrt{b^2+u^2} - b$

$\&\ \delta x = \dfrac{u\,\delta u}{\sqrt{b^2+u^2}}$

$\delta U' = \delta V_e;\ P\delta u = k\left(\sqrt{b^2+u^2} - b\right)\dfrac{u\,\delta u}{\sqrt{b^2+u^2}}$

$P = \left(1 - \dfrac{b}{\sqrt{b^2+u^2}}\right)ku$

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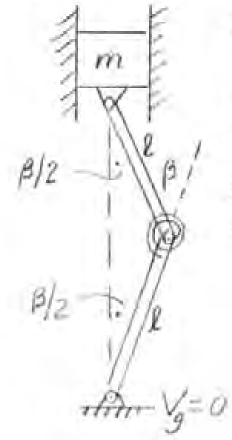

$$V_g = 2mg\ell \cos\frac{\beta}{2}, \quad V_e = \frac{1}{2}K\beta^2$$

$$V = V_g + V_e = 2mg\ell\cos\frac{\beta}{2} + \frac{1}{2}K\beta^2$$

$$\frac{dV}{d\beta} = -mg\ell\sin\frac{\beta}{2} + K\beta, \quad \frac{dV}{d\beta} = 0 \text{ for } \beta = 0$$

$$\frac{d^2V}{d\beta^2} = -\frac{1}{2}mg\ell\cos\frac{\beta}{2} + K$$

$$= -\frac{1}{2}mg\ell + K \text{ for } \beta = 0$$

$$= (+) \text{ stable if } K > \frac{1}{2}mg\ell$$

Thus $K_{min} = \frac{1}{2}mg\ell$

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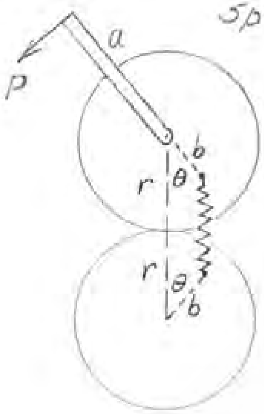
$\delta U' = \delta V_e$; $\delta U' = Pa\,\delta\theta$; $\delta V_e = kx\,\delta x$



Spring stretch $x = (2r - 2b\cos\theta) - (2r - 2b)$

$= 2b(1 - \cos\theta)$

$\delta x = 2b\sin\theta\,\delta\theta$

Thus $Pa\,\delta\theta = 4b^2k(1-\cos\theta)\sin\theta\,\delta\theta$

$$P = \frac{4kb^2}{a}\sin\theta(1-\cos\theta)$$

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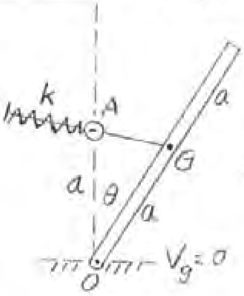
$V_e = \frac{1}{2} k x^2$ where $x = 2a \sin\frac{\theta}{2}$



$V_g = mg\, a \cos\theta$

$V = V_e + V_g = 2ka^2 \sin^2\frac{\theta}{2} + mga \cos\theta$

$\frac{dV}{d\theta} = 2ka^2 \sin\frac{\theta}{2} \cos\frac{\theta}{2} - mga \sin\theta$

$= ka^2 \sin\theta - mga \sin\theta = (ka - mg) a \sin\theta$

$\frac{dV}{d\theta} = 0$ for equil. gives $k = mg/a$ for all θ

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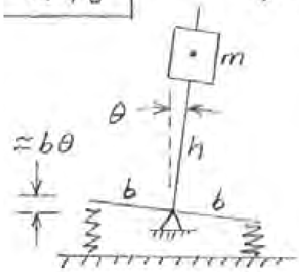

$V = V_e + V_g = \frac{1}{2}k(\Delta + b\theta)^2 + \frac{1}{2}k(\Delta - b\theta)^2 + mgh\cos\theta$

for θ small

$V = k(\Delta^2 + b^2\theta^2) + mgh\cos\theta$

$\frac{dV}{d\theta} = 2kb^2\theta - mgh\sin\theta$

$\frac{d^2V}{d\theta^2} = 2kb^2 - mgh\cos\theta$

Let preset of springs be Δ when $\theta = 0$

For $\theta \to 0$, $\frac{d^2V}{d\theta^2}$ is (+) if $2kb^2 > mgh$

Thus $\theta = 0$ is stable if $h < \frac{2kb^2}{mg}$

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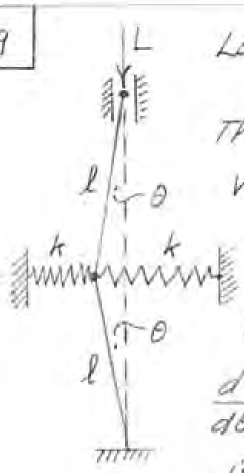

Let Δ = initial compression in each spring

Then for small angles

$$V = V_e + V_g = 2\left[\frac{1}{2}k(\Delta - \ell\sin\theta)^2 + \frac{1}{2}k(\Delta + \ell\sin\theta)^2\right] + 2L\ell\cos\theta$$

$$V = 2k(\Delta^2 + \ell^2\sin^2\theta) + 2L\ell\cos\theta$$

$$\frac{dV}{d\theta} = 4k\ell^2\sin\theta\cos\theta - 2L\ell\sin\theta$$

$$\frac{d^2V}{d\theta^2} = 4k\ell^2\cos 2\theta - 2L\ell\cos\theta$$

$\frac{dV}{d\theta} = 0$ for equil. $\theta = 0$ & $\theta = \cos^{-1}\frac{L}{2k\ell}$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = 4k\ell^2(1) - 2L\ell(1) = (+)$ stable if

$$k > \frac{L}{2\ell}$$

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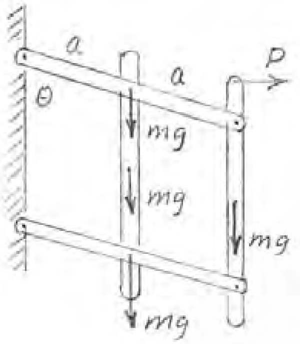

$\delta U = 0; \quad P\delta(2a \sin\theta)$

$+ mg\,\delta(2a\cos\theta) + 3mg\,\delta(a\cos\theta) = 0$

$2Pa\cos\theta\,\delta\theta = 5mga\sin\theta\,\delta\theta$

$P = \frac{5}{2} mg \tan\theta$

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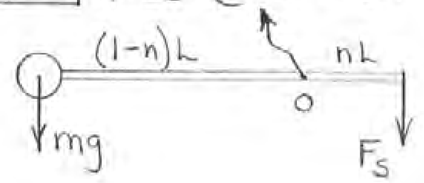
7/50 FBD @ $\theta = 0$:



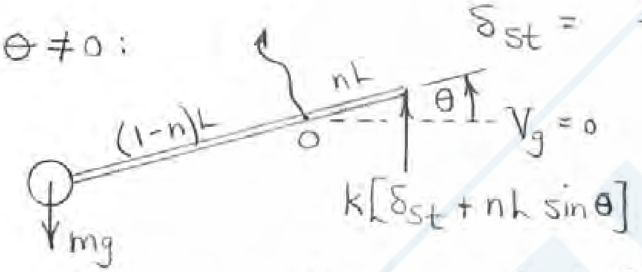
$\circlearrowleft + \sum M_O = 0$:

$$mg(1-n)L - F_s(nL) = 0$$

$$F_s = \frac{mg(1-n)}{n} = k\delta_{st}$$

$$\delta_{st} = \frac{mg(1-n)}{nk}$$

$\theta \neq 0$:



$$V = V_g + V_e = -mg[(1-n)L\sin\theta] + \frac{1}{2}k\left[\frac{mg(1-n)}{kn} + nL\sin\theta\right]^2$$

$$= \frac{m^2g^2(1-n)^2}{2kn^2} + \frac{n^2L^2k\sin^2\theta}{2}$$

$$\frac{dV}{d\theta} = n^2L^2k\sin\theta\cos\theta = 0 \quad \text{at} \quad \theta = 0$$

$$\frac{d^2V}{d\theta^2} = n^2L^2k\left[\cos^2\theta - \sin^2\theta\right] > 0 \quad \text{at} \quad \theta = 0$$

for all $0 < n < 1$

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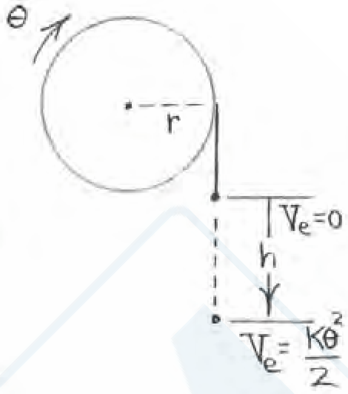
$\delta U' = \delta V = 0 - mgr\,\delta\theta + \delta\left(\frac{1}{2}K\theta^2\right)$

$mgr = K\theta$

With $h = r\theta$: $mgr = K\left(\frac{h}{r}\right)$

$h = \dfrac{mgr^2}{K}$



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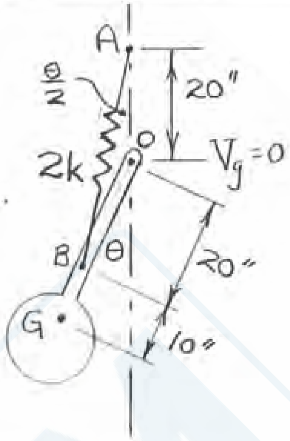

Length $AB = 2(20)\cos\frac{\theta}{2}$ (in.)

Unstretched length $= 40 - 4 = 36$ in.

Spring stretch for arbitrary θ

is $x = 40\cos\frac{\theta}{2} - 36$ in.

$$V_e = 2\tfrac{1}{2}k\left(40\cos\tfrac{\theta}{2} - 36\right)^2$$

$$= k\left[1600\cos^2\tfrac{\theta}{2} - 2880\cos\tfrac{\theta}{2} + 1296\right] \text{ in.-lb}$$

$$V_g = -3(20\cos\theta) = -60\cos\theta \text{ in.-lb}$$

$$V = V_e + V_g = k\left[1600\cos^2\tfrac{\theta}{2} - 2880\cos\tfrac{\theta}{2} + 1296\right] - 60\cos\theta \text{ in.-lb}$$

$$\frac{dV}{d\theta} = k\left[-1600\sin\tfrac{\theta}{2}\cos\tfrac{\theta}{2} + 1440\sin\tfrac{\theta}{2}\right] + 60\sin\theta$$

$$= k\left[-800\sin\theta + 1440\sin\tfrac{\theta}{2}\right] + 60\sin\theta$$

$$\frac{d^2V}{d\theta^2} = k\left[-800\cos\theta + 720\cos\tfrac{\theta}{2}\right] + 60\cos\theta$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{\theta=0} = k[-800 + 720] + 60 > 0 \text{ (Stable) if } k \text{ does}$$

not exceed $\frac{60}{80} = \underline{0.75 \text{ lb/in.} = k_{max}}$

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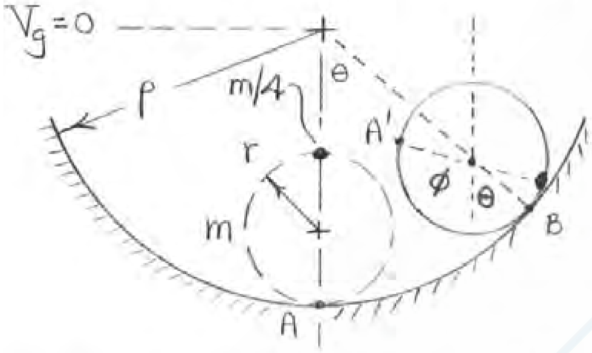
Arc $\widehat{AB}$ = Arc $\widehat{A'B}$: $\rho\theta = r(\theta + \phi)$

$(\rho - r)\theta = r\phi$

$\phi = \frac{\rho - r}{r}\theta$



$$V = V_g = -mg(\rho - r)\cos\theta - \frac{m}{4}g\left[(\rho - r)\cos\theta - r\cos\phi\right]$$

$$= \frac{mg}{4}\left[-5(\rho - r)\cos\theta + r\cos\left(\frac{\rho - r}{r}\theta\right)\right]$$

$$\frac{dV}{d\theta} = \frac{mg}{4}\left[5(\rho - r)\sin\theta - (\rho - r)\sin\left(\frac{\rho - r}{r}\theta\right)\right]$$

= 0 for equilibrium, $\theta = 0$ is desired solution

$$\frac{d^2V}{d\theta} = \frac{mg}{4}\left[5(\rho - r)\cos\theta - \frac{(\rho - r)^2}{r}\cos\left(\frac{\rho - r}{r}\theta\right)\right]$$

At $\theta = 0$: $5(\rho - r) - \frac{(\rho - r)^2}{r} > 0$ for stability

$\rho^2 - 7\rho r + 6r^2 < 0$

$(\rho - 6r)(\rho - r) < 0$ ($\rho = r$ not possible)

$\Rightarrow$ $(\rho - 6r) < 0$ & $(\rho - r) > 0$ (or the reverse which is impossible)

So $\underline{\rho < 6r}$

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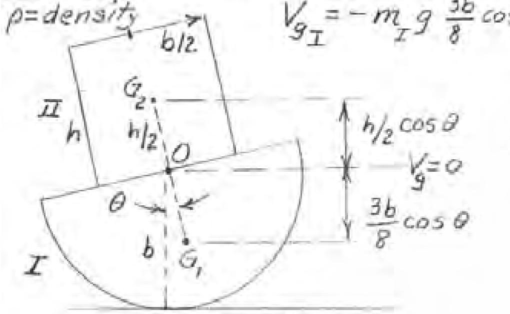
7/54 For solid hemisphere $\overline{OG_1} = 3b/8$



$$V_{g_I} = -m_I g \frac{3b}{8}\cos\theta = -\frac{2}{3}\pi b^3 \rho g \frac{3b}{8}\cos\theta$$

$$= -\pi \rho g \frac{b^4}{4}\cos\theta$$

$$V_{g_{II}} = +m_{II} g \frac{h}{2}\cos\theta$$

$$= \pi\left(\frac{b}{2}\right)^2 h \rho g \frac{h}{2}\cos\theta$$

$$= \pi \rho g \frac{b^2 h^2}{8}\cos\theta$$

$$V = V_{II} + V_I = \pi\rho g \frac{b^2}{4}\left(\frac{h^2}{2} - b^2\right)\cos\theta$$

$$\frac{d^2V}{d\theta^2} = -\pi\rho g \frac{b^2}{4}\left(\frac{h^2}{2} - b^2\right)\cos\theta = +\pi\rho g \frac{b^2}{4}\left(b^2 - \frac{h^2}{2}\right)\cos\theta$$

$= (+)$ stable if $\frac{h^2}{2} < b^2$ or $\underline{h < b\sqrt{2}}$

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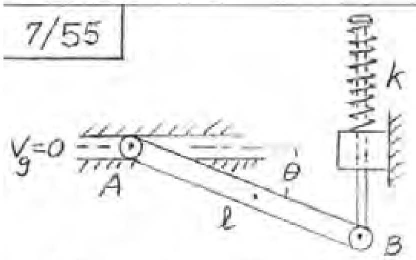

Spring compressed $\ell \sin\theta$

so $V_e = \frac{1}{2} k (\ell \sin\theta)^2$

$V_g = -mg\frac{\ell}{2}\sin\theta$

$V = V_e + V_g = \frac{k}{2}\ell^2 \sin^2\theta - mg\frac{\ell}{2}\sin\theta$

$$\frac{dV}{d\theta} = k\ell^2 \sin\theta\cos\theta - mg\frac{\ell}{2}\cos\theta = \frac{k\ell^2}{2}\sin 2\theta - mg\frac{\ell}{2}\cos\theta$$

$$= \ell\cos\theta\,(k\ell\sin\theta - mg/2) = 0 \text{ for equil.}$$

(1) $\cos\theta = 0$, $\theta = \pi/2$

(2) $\sin\theta = \frac{mg}{2k\ell}$

$$\frac{d^2V}{d\theta^2} = k\ell^2\cos 2\theta + mg\frac{\ell}{2}\sin\theta$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(1)} = k\ell^2\cos\pi + mg\frac{\ell}{2}(1) = k\ell^2\left(-1 + \left[\frac{mg}{2k\ell}\right]\right)$$

$$= (+) \text{ stable if } k < \frac{mg}{2\ell}$$

$$\left(\frac{d^2V}{d\theta^2}\right)_{(2)} = k\ell^2\left(1 - 2\left[\frac{mg}{2k\ell}\right]^2\right) + \frac{mg\ell}{2}\frac{mg}{2k\ell} = k\ell^2\left[1 - \left(\frac{mg}{2k\ell}\right)^2\right]$$

$$= (+) \text{ stable if } k > \frac{mg}{2\ell}$$

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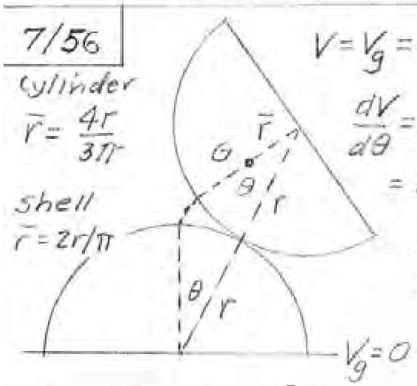
cylinder

$\bar{r} = \frac{4r}{3\pi}$

shell

$\bar{r} = 2r/\pi$



$V = V_g = mg(2r\cos\theta - \bar{r}\cos 2\theta)$

$\frac{dV}{d\theta} = mg(-2r\sin\theta + 2\bar{r}\sin 2\theta)$

$= 2mg(-r\sin\theta + \bar{r}\sin 2\theta)$

$\frac{d^2V}{d\theta^2} = 2mg(-r\cos\theta + 2\bar{r}\cos 2\theta)$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{2\bar{r}}{r}\right)$

For cylinder $\bar{r}/r = \frac{4}{3\pi}$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{8}{3\pi}\right) = (-)$

<u>unstable</u>

For shell $\bar{r}/r = \frac{2}{\pi}$, $\frac{d^2V}{d\theta^2} = 2mrg\left(-1 + \frac{4}{\pi}\right) = (+)$

<u>stable</u>

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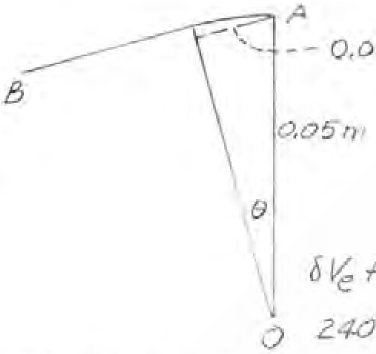
7/57 $\delta V_g = \delta(0.3\, mg \cos\theta) = -0.3(80)(9.81)\sin\theta\,\delta\theta$

$= -235 \sin\theta\,\delta\theta$



$0.05\sin\theta \approx$ spring compression x

$\delta V_e = kx\,\delta x$

$= (96\times10^3)(0.05\sin\theta)\,\delta(0.05\sin\theta)$

$= 240\sin\theta\cos\theta\,\delta\theta$

$\delta V_e + \delta V_g = 0,$

$240\sin\theta\cos\theta\,\delta\theta - 235\sin\theta\,\delta\theta = 0$

$\sin\theta = 0$ or $\cos\theta = \dfrac{235}{240} = 0.9810, \quad \underline{\theta = 11.19°}$

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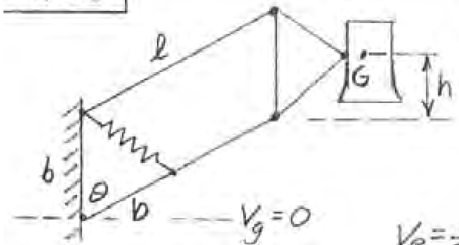

$V_g = mg(l\cos\theta + h)$

Spring length $= 2b\sin\frac{\theta}{2}$

" stretch $x = 2b\sin\frac{\theta}{2} - \frac{b}{2}$

$= \frac{b}{2}(4\sin\frac{\theta}{2} - 1)$

$V_e = \frac{1}{2}kx^2 = \frac{kb^2}{8}(4\sin\frac{\theta}{2} - 1)^2$

$V = V_e + V_g$; $\frac{dV}{d\theta} = -mgl\sin\theta + \frac{kb^2}{4}(4\sin\frac{\theta}{2} - 1)\,2\cos\frac{\theta}{2}$

$= (kb^2 - mgl)\sin\theta - \frac{kb^2}{2}\cos\frac{\theta}{2} = 0$ for equil.

Thus $\left[2(kb^2 - mgl)\sin\frac{\theta}{2} - \frac{kb^2}{2}\right]\cos\frac{\theta}{2} = 0$

& [ ] = 0 gives $k = \frac{mgl}{b^2}\,\frac{1}{1 - \frac{1}{4}\csc\frac{\theta}{2}}$

$\frac{d^2V}{d\theta^2} = (kb^2 - mgl)\cos\theta + \frac{kb^2}{4}\sin\frac{\theta}{2}$; substitute $\sin\frac{\theta}{2} = \frac{kb^2}{4(kb^2 - mgl)}$

& get $\frac{d^2V}{d\theta^2} = +$ (stable) within range of

$\theta = 29°,\ k = \infty$ to $\theta = 180°,\ k = \frac{4mgl}{3b^2}$

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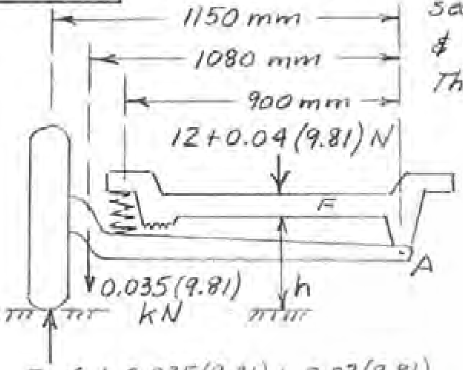
▶7/59 Let x = compression of spring (most easily seen by considering A fixed & wheels moving up)



Thus $x = \frac{900}{1150}(0.35 - h)$ meters

With F & hence A fixed,

$\delta U' = -2(6.54)\delta h + 2(0.035)9.81\frac{1080}{1150}\delta h$

$= (-13.08 + 0.645)\delta h$

$= -12.43\,\delta h$

$P = 6 + 0.035(9.81) + 0.02(9.81)$
$= 6.54$ kN

$\delta V_e = 2(kx\,\delta x) = 2(120)\frac{900}{1150}(0.35-h)\frac{900}{1150}(-\delta h)$

$= -147.0(0.35-h)\delta h$

$\delta U' = \delta V_e$; $-12.43\,\delta h = -147.0(0.35-h)\delta h$

$h = 0.35 - \frac{12.43}{147.0} = 0.265$ m or $\underline{h = 265 \text{ mm}}$

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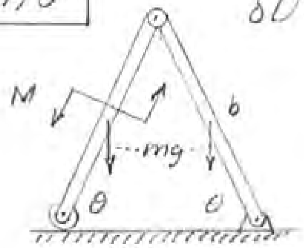
7/6



$\delta U = 0;\quad M\delta\theta - 2mg\,\delta\left(\frac{b}{2}\sin\theta\right) = 0$

$M\,\delta\theta = mgb\cos\theta\,\delta\theta$

$\theta = \cos^{-1}\dfrac{M}{mgb}$

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▶ 7/60 For $\frac{1}{2}$ of door $V_g = \frac{mg}{2}\frac{r}{2}(1-\cos\theta)$

Spring length $\ell = \sqrt{r^2+a^2+2ra\cos\theta}$

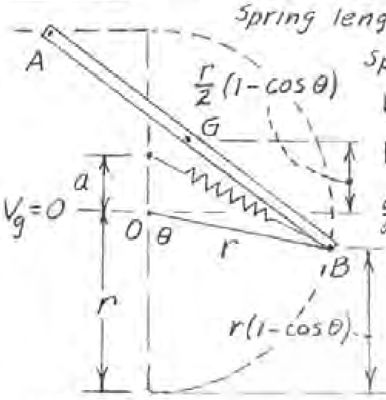
Spring stretch is $\ell - (r-a)$

$V_e = \frac{1}{2}k(\ell - [r-a])^2$

$V = V_g + V_e = \frac{mgr}{4}(1-\cos\theta) + \frac{k}{2}(\ell-[r-a])^2$

$\frac{dV}{d\theta} = \frac{mgr}{4}\sin\theta + k(\ell - [r-a])\frac{-2ra\sin\theta}{2\ell}$

$= \left(\frac{mgr}{4} - kar\right)\sin\theta + \frac{kar(r-a)\sin\theta}{\sqrt{r^2+a^2+2ra\cos\theta}}$



$\frac{d^2V}{d\theta^2} = \left(\frac{mgr}{4} - kar\right)\cos\theta + kar(r-a)\frac{\ell\cos\theta - \sin\theta(-ra\sin\theta)/\ell}{\ell^2}$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = \frac{mgr}{4} - kar + kar(r-a)\left(\frac{1}{r+a} + 0\right)$

$= 0$ for neutral equil. (insensitive response)

Thus $\frac{mgr}{4} = kar\left(1 - \frac{r-a}{r+a}\right)$, $k = \frac{mg(r+a)}{8a^2}$

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7/61 For system $\delta U = 0$

$P_1 \delta x - P_2 \delta x/4 = 0$, $P_2 = 4P_1 = 4(100)$

$= \underline{400\text{ N}}$

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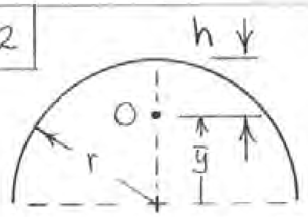
7/62



The pivot O must be at or above the mass center G of the shell, which is located at $\bar{y} = \frac{2r}{\pi}$. So

$$h_{max} = r - \bar{y} = r\left(1 - \frac{2}{\pi}\right) = \underline{0.363r}$$

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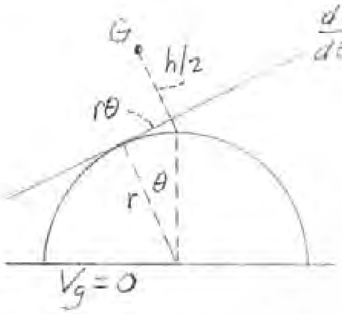
7/63



$$V = V_g = mg(r\cos\theta + r\theta\sin\theta + \frac{h}{2}\cos\theta)$$

$$\frac{dV}{d\theta} = mg(-r\sin\theta + r\sin\theta + r\theta\cos\theta - \frac{h}{2}\sin\theta)$$

$$= mg(r\theta\cos\theta - \frac{h}{2}\sin\theta)$$

$$\frac{d^2V}{d\theta^2} = mg(r\cos\theta - r\theta\sin\theta - \frac{h}{2}\cos\theta)$$

For $\theta = 0$ position, $\frac{d^2V}{d\theta^2} = mg(r - 0 - \frac{h}{2})$

$= (+)$ stable if $\underline{h < 2r}$

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7/64 Force & moment equilibrium (A)

(a), (b), (d)

Virtual work (B)

(c), (e), (f)

(c) $\delta U' = \delta V_g$

(e) $\delta U' = \delta V_g + \delta V_e$

(f) $d^2V/d\theta^2$ must be (+)

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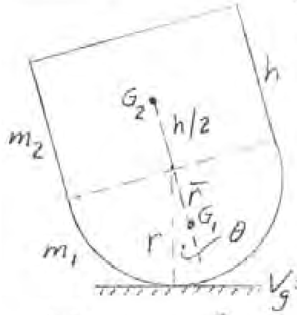
7/65 Let ρ = mass per unit area of shell

$m_1 = 2\pi r^2 \rho$, $m_2 = 2\pi r h \rho$

$\bar{r} = r/2$ for hemispherical shell



$V = V_{g_1} + V_{g_2}$

$= 2\pi r^2 \rho g (r - \bar{r}\cos\theta) + 2\pi r h \rho g (r + \frac{h}{2}\cos\theta)$

$= 2\pi r \rho g [(r^2 + hr) - \frac{1}{2}(r^2 - h^2)\cos\theta]$

$\frac{dV}{d\theta} = 2\pi r \rho g [0 + \frac{1}{2}(r^2 - h^2)\sin\theta]$

$\frac{d^2V}{d\theta^2} = \pi r \rho g (r^2 - h^2)\cos\theta$

For equil. $\frac{dV}{d\theta} = 0$ gives $\theta = 0$ & $h = r$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = (+)$ if $\underline{h < r}$

For $h = r$, neutral equilibrium

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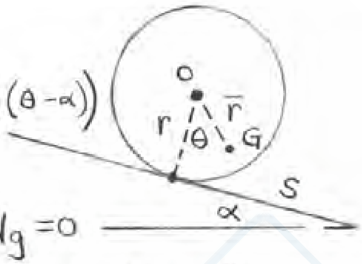

$V_g = W(s\sin\alpha + r\cos\alpha - \bar{r}\cos(\theta-\alpha))$

But $s = s_0 + r\theta$, $\frac{ds}{d\theta} = r$

So $\frac{dV_g}{d\theta} = W(r\sin\alpha + \bar{r}\sin(\theta-\alpha))$

For $\alpha = 10°$, $r = 100$ mm, $\bar{r} = 60$ mm:

$\frac{dV_g}{d\theta} = W[0.1\sin 10° + 0.060\sin(\theta-10°)]$

$= 0$ for equilibrium

$-0.1\sin 10° = 0.060\sin(\theta-10°)$

$\theta - 10° = \sin^{-1}\left(-\frac{0.1}{0.06}\sin 10°\right) = -16.82°$ or $196.8°$

$\Rightarrow$ $\underline{\theta = -6.82°}$ or $\underline{\theta = 207°}$

$\frac{d^2V_g}{d\theta^2} = W[0 + \bar{r}\cos(\theta-\alpha)]$

$\theta = -6.82°$: $d^2V_g/d\theta^2 = W\bar{r}\cos(-16.82°) > 0$ Stable

$\theta = 207°$: $d^2V_g/d\theta^2 = W\bar{r}\cos(196.8°) < 0$ Unstable

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7/67 $V = V_e = \frac{1}{2}k(2a\sin\frac{\theta}{2})^2 = 2ka^2\sin^2\frac{\theta}{2}$

$\delta U = \delta V_e$; $P\delta(2a\sin\theta) = \delta(2ka^2\sin^2\frac{\theta}{2})$

$2Pa\cos\theta\,\delta\theta = 2ka^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\,\delta\theta$

$2Pa\cos\theta = ka^2\sin\theta$

$\tan\theta = \frac{2P}{ka}$, $\underline{\theta = \tan^{-1}\frac{2P}{ka}}$

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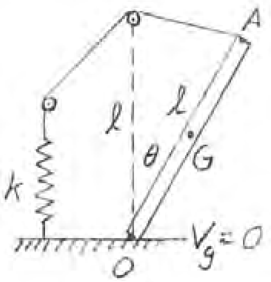
7/68 $V = V_e + V_g = \frac{1}{2}k\left(2\ell\sin\frac{\theta}{2}\right)^2 + mg\frac{\ell}{2}\cos\theta$



$$\frac{dV}{d\theta} = 2k\ell^2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - mg\frac{\ell}{2}\sin\theta$$

$$= k\ell^2\sin\theta - mg\frac{\ell}{2}\sin\theta$$

$$= \ell\left(k\ell - \frac{1}{2}mg\right)\sin\theta = 0 \text{ for equil.}$$

$\theta = 0$ and $\underline{k = \frac{mg}{2\ell}}$ independent of θ

$$\frac{d^2V}{d\theta^2} = \ell\left(k\ell - \frac{1}{2}mg\right)\cos\theta$$

For $\theta = 0$, $\frac{d^2V}{d\theta^2} = \ell\left(k\ell - \frac{1}{2}mg\right)$ $= (+)$ stable if $k > \frac{mg}{2\ell}$

$(-)$ unstable if $k < \frac{mg}{2\ell}$

For $k = \frac{mg}{2\ell}$, $V = \frac{mg\ell}{2}$ const., <u>neutral equil</u>

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7/69 Total length of door is $2.5 + 0.6\pi/2 = 3.44$ m

Unit mass is $135/3.44 = 39.22$ kg/m

Take $V_g = 0$ through A

Let potential energy of cylindrical portion be $-V_0$ which remains constant

so $V_g = 0 - V_0 - 39.22(9.81)x(0.6 + \frac{x}{2})$

$= -V_0 - 384.7x(0.6 + \frac{x}{2})$ J

$V_e = 2(\frac{1}{2}k\theta^2) = \frac{10}{2\pi}\theta^2 = \frac{5}{\pi}(\frac{x}{0.080})^2 = 248.7x^2$ J

$V = V_e + V_g = 248.7x^2 - V_0 - 384.7x(0.6 + \frac{x}{2})$

$\frac{dV}{dx} = 497.4x - 230.8 - 384.7x = 112.7x - 230.8$

$= 0$ for equilibrium, so $x = \frac{230.8}{112.7} = \underline{2.05 \text{ m}}$

$\frac{d^2V}{dx^2} = 112.7$ (+) so $\underline{\text{stable}}$

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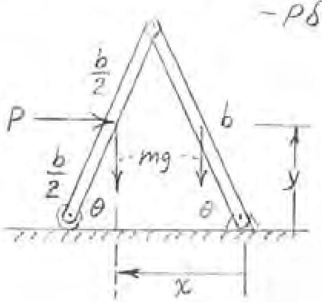
7/7 $\delta U = 0;\quad -P\,\delta x - 2mg\,\delta y = 0$



$$-P\delta\left(\frac{3b}{2}\cos\theta\right) - 2mg\,\delta\left(\frac{b}{2}\sin\theta\right) = 0$$

$$\frac{3Pb}{2}\sin\theta\,\delta\theta - mgb\cos\theta\,\delta\theta = 0$$

$$\tan\theta = \frac{2mg}{3P}$$

$$\theta = \tan^{-1}\frac{2mg}{3P}$$

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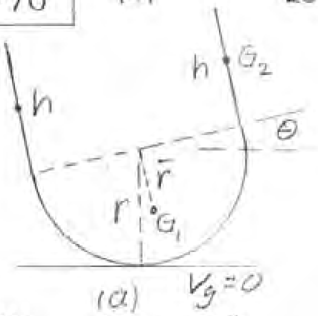
7/70 (a)



Let ρ = mass per unit periphery of shell

$\bar{r} = 2r/\pi$

$$V_g = \rho\left[\pi r\left(r - \frac{2r}{\pi}\cos\theta\right) + h\left(r + r\sin\theta + \frac{h}{2}\cos\theta\right) + h\left(r - r\sin\theta + \frac{h}{2}\cos\theta\right)\right]$$

$$= \rho\left[\pi r^2 - 2r^2\cos\theta + 2hr + h^2\cos\theta\right]$$

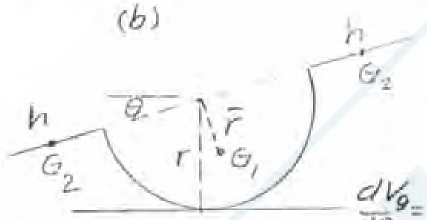
$$\frac{dV_g}{d\theta} = \rho\left[2r^2 - h^2\right]\sin\theta, \quad \frac{d^2V_g}{d\theta^2} = \rho\left[2r^2 - h^2\right]\cos\theta$$

Equil. at $\theta = 0$ stable if $h < r\sqrt{2}$

unstable if $h > r\sqrt{2}$

Neutral equil. if $h = r\sqrt{2}$ for any θ

(b)



$$V_g = \rho\left[\pi r\left(r - \frac{2r}{\pi}\cos\theta\right) + h\left(r + \left[r + \frac{h}{2}\sin\theta\right]\right) + h\left(r - \left[r + h/2\sin\theta\right]\right)\right]$$

$$= \rho\left[\pi r^2 - 2r^2\cos\theta + 2hr\right]$$

$\frac{dV_g}{d\theta} = 2\rho r^2\sin\theta$, $\theta = 0$ for stable equil. independent of θ

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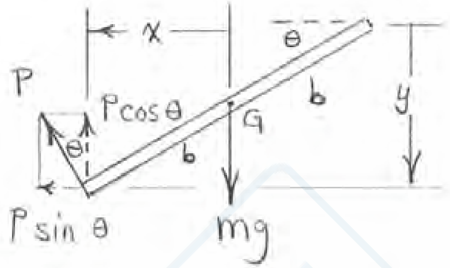
7/71

$y = 2b \sin\theta$

$\delta y = 2b \cos\theta \, \delta\theta$

$x = b \cos\theta$

$\delta x = -b \sin\theta \, \delta\theta$



$\delta U = 0: \; mg\,\delta\left(\frac{y}{2}\right) - P\cos\theta\,(\delta y) + P\sin\theta\,(\delta x) = 0$

$mgb\cos\theta\,\delta\theta - P\cos\theta\,(2b\cos\theta\,\delta\theta) + P\sin\theta\,(-b\sin\theta\,\delta\theta) = 0$

$mg\cos\theta = P(\sin^2\theta + 2\cos^2\theta) = P(1 + \cos^2\theta)$

$$P = \frac{mg\cos\theta}{1 + \cos^2\theta}$$

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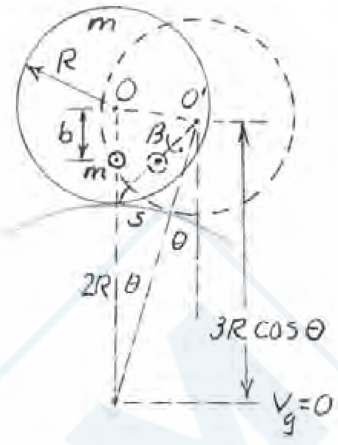
7/72 In displaced position



$V_g = mg(3R\cos\theta)$
$\quad + mg(3R\cos\theta - b\cos(\theta+\beta))$

But $s = 2R\theta = R\beta$

so $\beta = 2\theta$

& $V_g = 6mgR\cos\theta - mgb\cos 3\theta$

$\frac{dV_g}{d\theta} = -6mgR\sin\theta + 3mgb\sin 3\theta$

$(=0 \text{ for } \theta = 0)$

$\frac{d^2V_g}{d\theta^2} = -6mgR\cos\theta + 9mgb\cos 3\theta$

$\left(\frac{d^2V_g}{d\theta^2}\right)_{\theta=0} = 3mg(-2R+3b) = +$ stable if $3b > 2R$

so $\underline{b_{min} = \frac{2}{3}R}$

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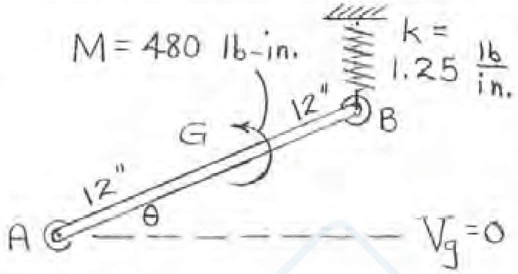
*7/73



$\delta U' = \delta V_g$

$\delta U' = 480\ \delta\theta$

$V = V_g + V_e = 20(12\sin\theta) + \frac{1}{2}(1.25)(24\sin\theta)^2$

$\delta V = 240\cos\theta\ \delta\theta + 360(2\sin\theta\cos\theta)\ \delta\theta$

So $480\ \delta\theta = 240\cos\theta\ \delta\theta + 360\sin 2\theta\ \delta\theta$

or $4 = 2\cos\theta + 3\sin 2\theta$

Numerically solve to obtain $\underline{\theta = 23.0°}$

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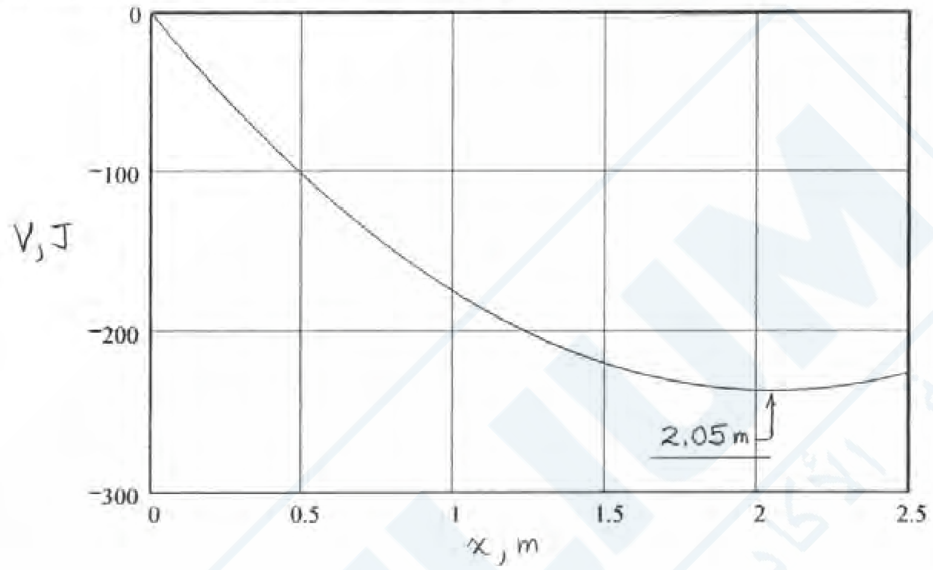
*7/74 From Prob. 7/69,

$V = 248.7x^2 - V_0 - 384.7x(0.6 + \frac{x}{2})$ in Joules

$= 56.33x^2 - 230.8x$ with $V_0 = 0$

where V_0 is the potential energy of the cylindrical section. Compute V and get



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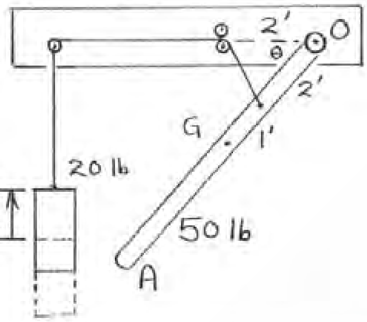
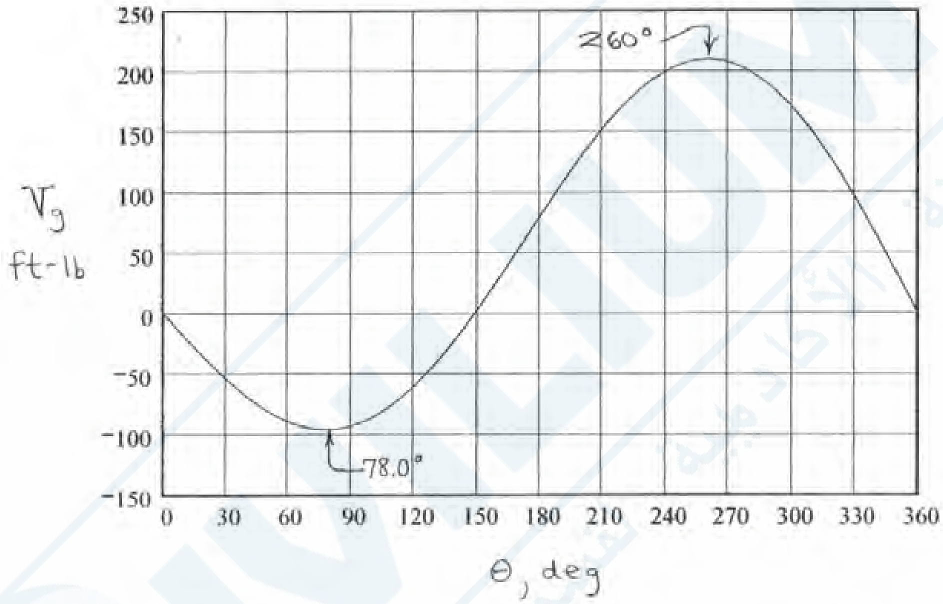
***7/75** $h = 2\left(2\sin\frac{\theta}{2}\right) = 4\sin\frac{\theta}{2}$



$V_g = 20\left(4\sin\frac{\theta}{2}\right) - 50\,(3\sin\theta)$

$= 80\sin\frac{\theta}{2} - 150\sin\theta$ ft-lb



θ = 78.0° stable

θ = 260° unstable

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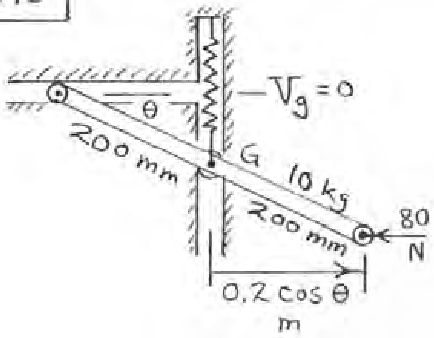
*7/76



$$\delta U = \delta V_g + \delta V_e$$

$$\delta U = -80\delta(0.2\cos\theta) = 16\sin\theta\,\delta\theta$$

$$\delta V_g = \delta(-mgh) = -\delta[10(9.81)(0.2\sin\theta)]$$
$$= -19.62\cos\theta\,\delta\theta$$

$$\delta V_e = \delta(\tfrac{1}{2}kx^2) = kx\,\delta x$$
$$= 1500(0.2\sin\theta)\,\delta(0.2\sin\theta)$$
$$= 60\sin\theta\cos\theta\,\delta\theta = 30\sin 2\theta\,\delta\theta$$

Thus $16\sin\theta\,\delta\theta = -19.62\cos\theta\,\delta\theta + 30\sin 2\theta\,\delta\theta$

or $(16\sin\theta + 19.62\cos\theta - 30\sin 2\theta)\,\delta\theta = 0$

Numerical solution : $\underline{\theta = 27.9^\circ}$

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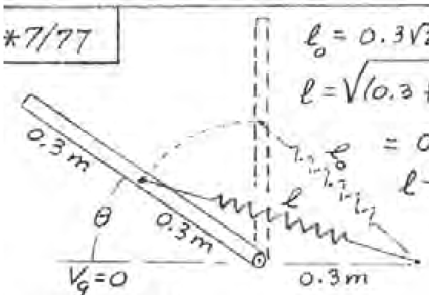
*7/77



$k = 1250$ N/m

$\ell_o = 0.3\sqrt{2} = 0.4243$ m

$\ell = \sqrt{(0.3 + 0.3\cos\theta)^2 + (0.3\sin\theta)^2}$

$= 0.3\sqrt{2}\sqrt{1+\cos\theta} = 0.4243\sqrt{1+\cos\theta}$

$\ell - \ell_o = 0.4243(\sqrt{1+\cos\theta} - 1)$

$V_e = \frac{1}{2}k(\ell - \ell_o)^2$

$= \frac{1250}{2} \times 0.18\left[2 + \cos\theta - 2\sqrt{1+\cos\theta}\right]$

$= 112.5\left[2 + \cos\theta - 2\sqrt{1+\cos\theta}\right]$

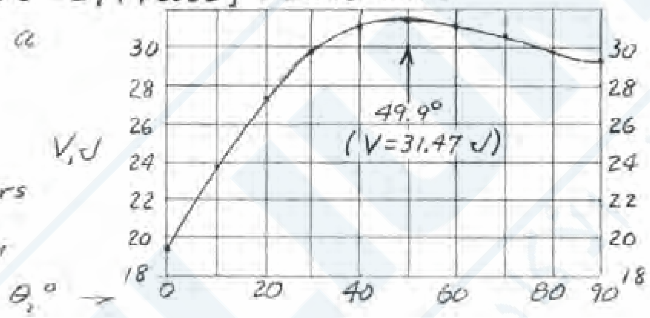
$V_g = mgh = 10(9.81)(0.3\sin\theta)$

$= 29.43\sin\theta$

$V = V_e + V_g$

$V = 112.5\left[2 + \cos\theta - 2\sqrt{1+\cos\theta}\right] + 29.43\sin\theta$

Compute V as a function of θ & get →



unstable equilibrium occurs at $\theta = 49.9°$

Stable equilibrium at $\theta = \pm 90°$

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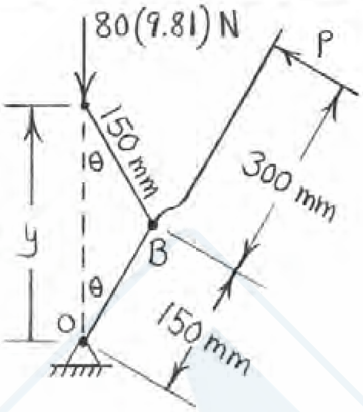
*7/78



$$y = 2(150)\cos\theta$$

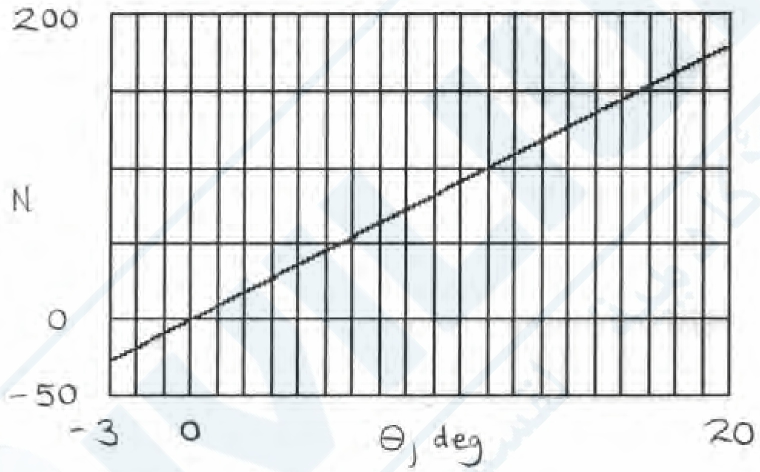
$$\delta y = -300 \sin\theta \, \delta\theta$$

$\delta U = 0$:

$$-P(450\,\delta\theta) - 80(9.81)\delta y = 0$$

$$P = \frac{80(9.81)(300 \sin\theta)}{450}$$

$$= \underline{523 \sin\theta \quad (\text{in newtons})}$$



(At $\theta = -3°$, $P = -27.4$ N
At $\theta = 20°$, $P = 178.9$ N)

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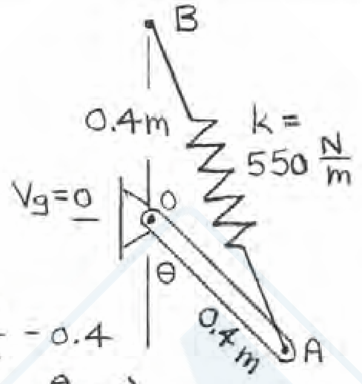
*7/79

$V_g = 20(9.81)(-0.2\cos\theta)$

$= -39.24\cos\theta$ J

$\overline{AB} = 2(0.4)\sin\left(\frac{180-\theta}{2}\right)$

$= 0.8\cos\frac{\theta}{2}$



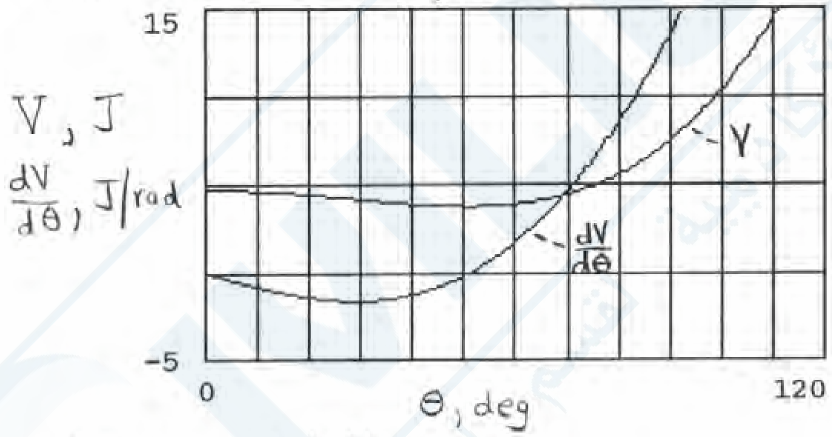
Stretch of spring $= 0.8\cos\frac{\theta}{2} - 0.4$

$= 0.4(2\cos\frac{\theta}{2} - 1)$

$V_e = \frac{1}{2}550[0.4(2\cos\frac{\theta}{2}-1)]^2 = 44(2\cos\frac{\theta}{2}-1)^2$

$V = V_e + V_g = 44(2\cos\frac{\theta}{2}-1)^2 - 39.24\cos\theta$

$dV/d\theta = 88(2\cos\frac{\theta}{2}-1)(-\sin\frac{\theta}{2}) + 39.24\sin\theta$



$\theta = 0$, unstable

$\theta = 51.1°$, stable

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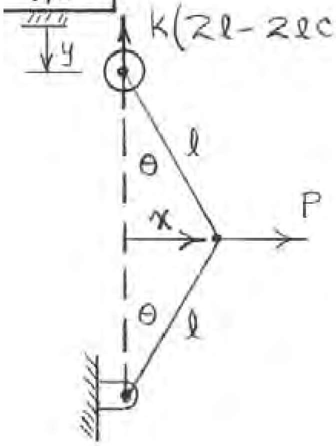
7/8



$\delta U = 0: \; P\delta x - k(2l - 2l\cos\theta)\delta y = 0$

$x = l\sin\theta, \quad \delta x = l\cos\theta\,\delta\theta$

$y = 2l - 2l\cos\theta$ (measured from wheel position when spring is unstretched)

$\delta y = 2l\sin\theta\,\delta\theta$

So $P(l\cos\theta\,\delta\theta) - k(2l - 2l\cos\theta)(2l\sin\theta\,\delta\theta) = 0$

$$\Rightarrow P = \frac{4kl(\sin\theta - \sin\theta\cos\theta)}{\cos\theta}$$

or $P = 4kl(\tan\theta - \sin\theta)$

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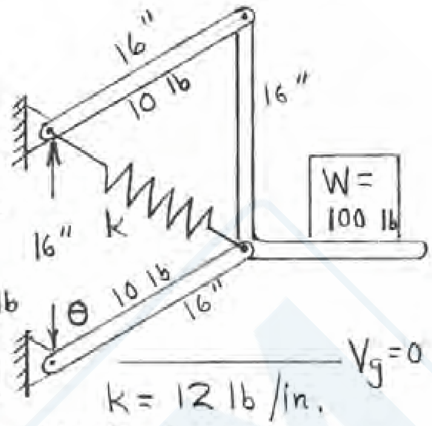

Spring stretch $= 2(16 \sin\frac{\theta}{2}) - 8$

$= 8(4\sin\frac{\theta}{2} - 1)$ in.

$V_e = \frac{1}{2}(12)(8)^2(4\sin\frac{\theta}{2} - 1)^2$

$= 384(4\sin\frac{\theta}{2} - 1)^2$ in.-lb

$V_g = 10(8\cos\theta) + 10(16 + 8\cos\theta)$

$+100(16\cos\theta) = 1760\cos\theta + 160$ in.-lb

$V = V_e + V_g = 384(4\sin\frac{\theta}{2} - 1)^2 + 1760\cos\theta + 160$ in.-lb

$\frac{dV}{d\theta} = 768(4\sin\frac{\theta}{2} - 1)2\cos\frac{\theta}{2} - 1760\sin\theta$

$= 0$ for equilibrium

Numerical solution : $\underline{\theta = 71.7°}$

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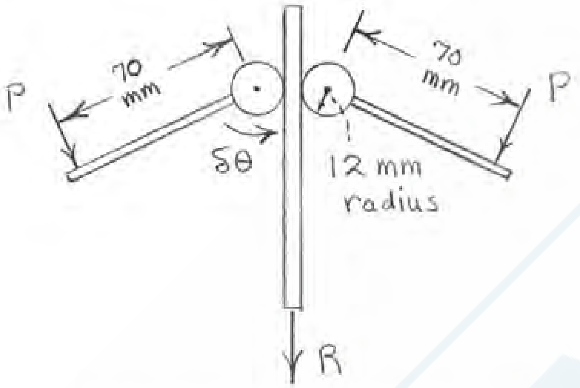
7/9



$$\delta U = 0: \; 2P(70)\delta\theta - R(12)\delta\theta = 0$$

$$\underline{R = 11.67P}$$

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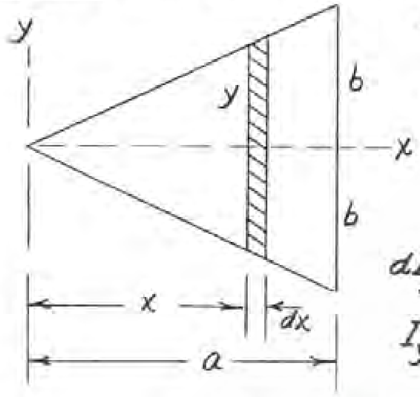
A/1 Using the results of Sample Prob. A/1



$dI_x = \frac{1}{12}(2y)^3 dx = \frac{2}{3} y^3 dx$

But $y = \frac{b}{a} x$ so

$$I_x = \frac{2b^3}{3a^3}\int_0^a x^3 dx = \frac{1}{6} ab^3$$

$dI_y = x^2(2y\,dx) = 2\frac{b}{a} x^3 dx$

$$I_y = \frac{2b}{a}\int_0^a x^3 dx = \frac{2b}{a}\frac{a^4}{4} = \frac{1}{2} ba^3$$

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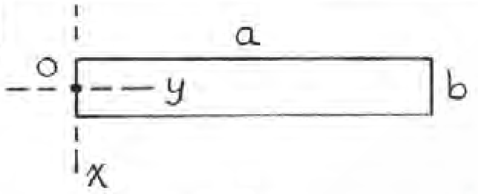
A/10



$I_x = \frac{1}{3} A a^2 = \frac{1}{3} a^3 b$, $I_y = \frac{1}{12} ab^3$

$I_o = I_x + I_y = \frac{1}{3} ab \left(a^2 + \frac{b^2}{4}\right)$

$n = \%\ \text{error} = \frac{I_x - I_o}{I_o} (100) = \frac{-\frac{1}{12} ab^3}{\frac{1}{3} ab\left(a^2 + \frac{b^2}{4}\right)} 100$

$= -\frac{1}{4} \frac{b^2}{a^2 + \frac{b^2}{4}} 100$

For $\frac{b}{a} = \frac{1}{10}$, $n = -25 \frac{1}{10^2 + \frac{1}{4}} = \underline{-0.249\ \%}$

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A/11

$$A \cong \frac{90-30-15}{360} 2\pi (300)(10) = 2360 \text{ mm}^2$$

$$I_o \cong Ar^2 = 2360(300)^2 = \underline{212(10^6) \text{ mm}^4}$$

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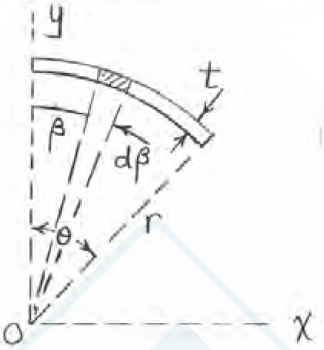
A/12



$I_x = \int y^2 dA$, $I_y = \int x^2 dA$

where $y = r\cos\beta$, $x = r\sin\beta$,

and $dA = rt\,d\beta$

$$I_x = \int_0^\theta r^2\cos^2\beta\; rt\,d\beta = r^3 t\left[\frac{\beta}{2} + \frac{\sin 2\beta}{4}\right]_0^\theta$$

But $A = r\theta t$, so $I_x = \frac{Ar^2}{4}\left(2 + \frac{\sin 2\theta}{\theta}\right)$

$$I_y = \int_0^\theta r^2\sin^2\beta\; rt\,d\beta = r^3 t\left[\frac{\beta}{2} - \frac{\sin 2\beta}{4}\right]_0^\theta = \frac{Ar^2}{4}\left(2 - \frac{\sin 2\theta}{\theta}\right)$$

$I_o = I_x + I_y = Ar^2$

For $\theta = 45°$, $r = 300\text{ mm}^2$, and $t = 10$ mm

from Prob. A/11, we have

$$A = 45\left(\frac{\pi}{180}\right)(300)(10) = 2360\text{ mm}^2$$

So $I_o = 2360(300)^2 = 212(10^6)\text{ mm}^4$

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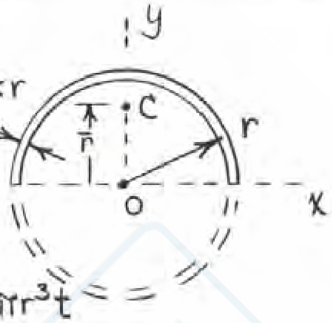
A/13



For complete ring,

$I_o = Ar^2 = 2\pi r t r^2 = 2\pi r^3 t$

and $I_o = I_x + I_y$, $I_x = I_y$

So for complete ring, $I_x = \frac{I_o}{2} = \pi r^3 t$

For half-ring, $\underline{I_x = \frac{1}{2}\pi r^3 t}$ and $I_y = I_x$

by symmetry so $\underline{I_y = \frac{1}{2}\pi r^3 t}$

For half-ring, $I_o = \frac{1}{2}(2\pi r^3 t) = \pi r^3 t$

$I_C = I_o - A\bar{r}^2 = \pi r^3 t - \pi r t\left(\frac{2r}{\pi}\right)^2$

$= \underline{\pi r^3 t\left(1 - \frac{4}{\pi^2}\right)}$

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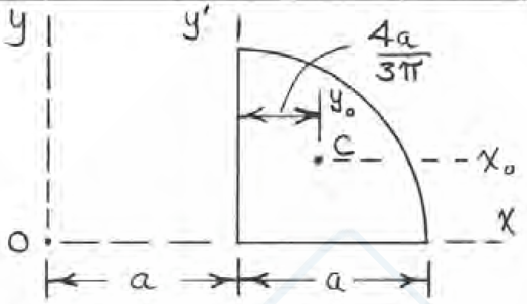
A/14



From Table D/3

$$\underline{I_x} = I_{y'} = \underline{\frac{\pi a^4}{16}}$$

So $I_{x_0} = I_{y_0} =$

$$\frac{\pi a^4}{16} - \frac{\pi a^2}{4}\left(\frac{4a}{3\pi}\right)^2 = a^4\left[\frac{\pi}{16} - \frac{4}{9\pi}\right]$$

Then $I_y = I_{y_0} + Ad^2 = a^4\left[\frac{\pi}{16} - \frac{4}{9\pi}\right] + \frac{\pi a^2}{4}\left[a + \frac{4a}{3\pi}\right]^2$

$$= \underline{a^4\left[\frac{5\pi}{16} + \frac{2}{3}\right]}$$

$$I_o = I_x + I_y = a^4\left[\frac{3\pi}{8} + \frac{2}{3}\right]$$

$$k_o = \sqrt{I_o/A} = \sqrt{a^4\left[\frac{3\pi}{8} + \frac{2}{3}\right]\Big/\frac{\pi a^2}{4}} = \underline{a\sqrt{\frac{8}{3\pi} + \frac{3}{2}}}$$

Decimal forms: $\underline{I_x = 0.1963\, a^4}$, $\underline{I_y = 1.648 a^4}$

$\underline{k_o = 1.533 a}$

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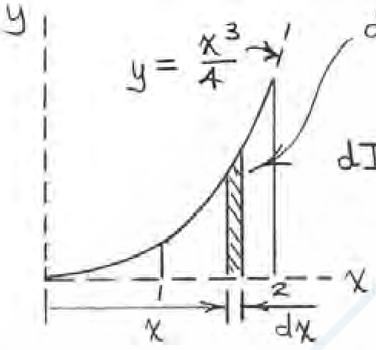
A/15



$dA = y\,dx = \frac{x^3}{4}dx$

$dI_y = x^2 dA = \frac{x^5}{4}dx$

$dI_x = \frac{1}{3}y^3 dx = \frac{x^9}{192}dx$

$$A = \int dA = \int_1^2 \frac{x^3}{4}dx = \frac{1}{4}\frac{x^4}{4}\Big|_1^2 = \frac{15}{16}$$

$$I_y = \int dI_y = \int_1^2 \frac{x^5}{4}dx = \frac{1}{4}\frac{x^6}{6}\Big|_1^2 = \frac{63}{24}$$

$$I_x = \int dI_x = \int_1^2 \frac{x^9}{192}dx = \frac{1}{192}\frac{x^{10}}{10}\Big|_1^2 = \frac{1023}{1920}$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{63/24}{15/16}} = \sqrt{14/5} = \underline{1.673}$$

$$k_x = \sqrt{I_x/A} = \sqrt{\frac{1023/1920}{15/16}} = \underline{0.754}$$

$$k_z = \sqrt{k_x^2 + k_y^2} = \sqrt{1.673^2 + 0.754^2} = \underline{1.835}$$

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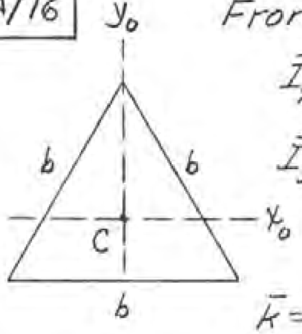
A/16



From Sample Problem A/2

$$\bar{I}_x = \frac{1}{36} b \left(b \frac{\sqrt{3}}{2}\right)^3 = \frac{b^4}{96}\sqrt{3}$$

$$\bar{I}_y = 2\left(\frac{1}{12} b \frac{\sqrt{3}}{2}\left[\frac{b}{2}\right]^3\right) = \frac{b^4}{96}\sqrt{3}$$

$$\bar{J} = \bar{I}_x + \bar{I}_y = \frac{b^4}{48}\sqrt{3}$$

$$\bar{k} = \sqrt{\bar{J}/A} = \sqrt{\frac{b^4\sqrt{3}}{48} \Big/ \left(\frac{b^2}{2}\frac{\sqrt{3}}{2}\right)} = \frac{b}{2\sqrt{3}}$$

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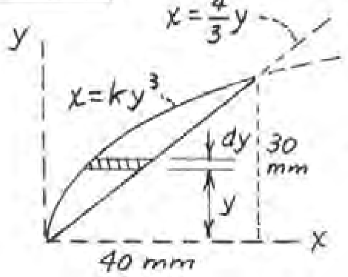
A/17



For $x = 40$ mm & $y = 30$ mm, $k = \frac{40}{27(10^3)}$

$$dI_x = y^2 dA = y^2\left(\frac{4}{3}y - \frac{4}{2700}y^3\right)dy$$

$$I_x = \int_0^{30}\left(\frac{4}{3}y^3 - \frac{4}{2700}y^5\right)dy$$

$$= \left[\frac{y^4}{3} - \frac{y^6}{4050}\right]_0^{30} = \underline{9(10^4)\ \text{mm}^4}$$

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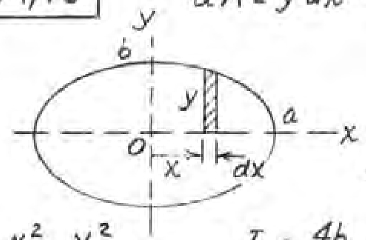
A/18



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$dA = y\,dx = \frac{b}{a}\sqrt{a^2 - x^2}\,dx$$

$$A = \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx = \pi a b$$

$$dI_y = x^2 y\,dx = \frac{b}{a}x^2\sqrt{a^2 - x^2}\,dx$$

$$I_y = \frac{4b}{a}\int_0^a x^2\sqrt{a^2 - x^2}\,dx$$

$$= \frac{4b}{a}\left[-\frac{x}{4}\sqrt{(a^2 - x^2)^3} + \frac{a^2}{8}\left(x\sqrt{a^2 - x^2} + a^2\sin^{-1}\frac{x}{a}\right)\right]_0^a$$

$$= \frac{b}{a}\left[0 + \frac{a^2}{2}\left(0 + a^2\frac{\pi}{2}\right)\right] = \underline{\frac{\pi a^3 b}{4}}$$

Similarly $\quad I_x = \dfrac{\pi a b^3}{4}$

So $\quad I_O = I_x + I_y = \dfrac{\pi a b}{4}(a^2 + b^2)$

& $\quad k_O = \sqrt{I_O/A} = \underline{\dfrac{1}{2}\sqrt{a^2 + b^2}}$

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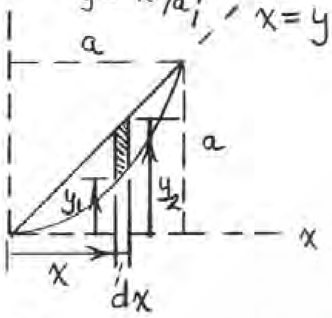
A/19



$$dA = (y_2 - y_1)\,dx = \left(x - \frac{x^2}{a}\right)dx$$

$$I_x = \int \frac{1}{3} y_2^3\,dx - \int \frac{1}{3} y_1^3\,dx = \frac{1}{3}\int_0^a \left(x^3 - \frac{x^6}{a^3}\right)dx$$

$$= \frac{1}{3}\left[\frac{x^4}{4} - \frac{x^7}{7a^3}\right]_0^a = \frac{a^4}{3}\left(\frac{1}{4} - \frac{1}{7}\right) = \underline{a^4/28}$$

$$I_y = \int x^2\,dA = \int x^2\left(x - \frac{x^2}{a}\right)dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^5}{5a}\right]_0^a = \frac{a^4}{4} - \frac{a^4}{5} = \underline{a^4/20}$$

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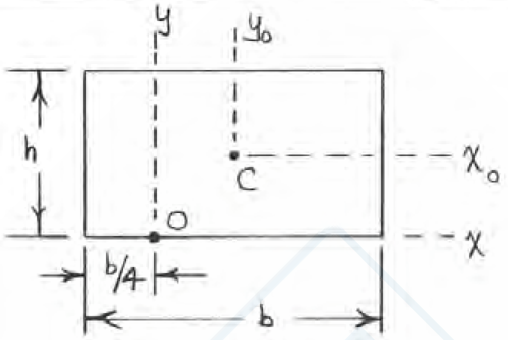
A/2

From Sample Prob. A/1,

$I_{x_o} = \frac{1}{12} bh^3$

$I_{y_o} = \frac{1}{12} hb^3$



So

$I_x = \frac{1}{12} bh^3 + bh\left(\frac{h}{2}\right)^2 = \underline{\frac{1}{3} bh^3}$

$I_y = \frac{1}{12} hb^3 + bh\left(\frac{b}{4}\right)^2 = \underline{\frac{7}{48} hb^3}$

$I_o = I_x + I_y = \underline{\frac{bh}{3}\left(h^2 + \frac{7}{16} b^2\right)}$

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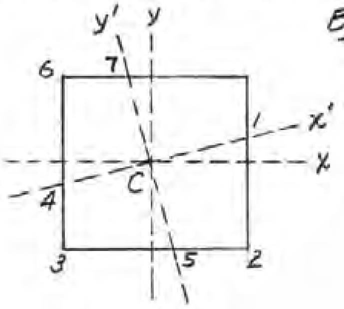
A/20

$I_C = I_x + I_y = I_{x'} + I_{y'}$



By symmetry of the figures 1-2-3-4 and 5-3-6-7

$I_{x'} = I_{y'}$. Thus since $I_x = I_y$,

$2I_x = I_C = 2I_{x'}$ so $\underline{I_{x'} = I_x}$

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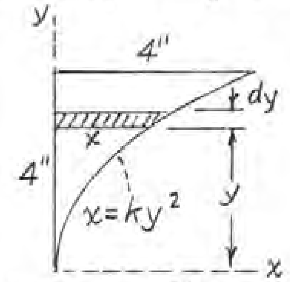
A/21

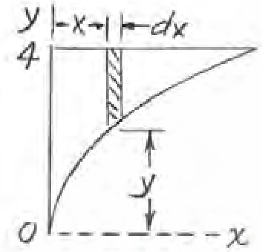
$dI_x = y^2 dA = y^2 x\,dy = y^2(y^2/4)\,dy = (y^4/4)\,dy$



$$I_x = \frac{1}{4}\int_0^4 y^4\,dy = \frac{1}{4}\frac{y^5}{5}\Big]_0^4 = \frac{256}{5}$$

$$= \underline{51.2 \text{ in.}^4}$$

$k = 1/4,\ x = y^2/4$



$$dI_x = \frac{1}{3}(dx)(4^3 - y^3)$$

$$= \frac{1}{3}(4^3 - [2\sqrt{x}]^3)\,dx$$

$$I_x = \frac{1}{3}\int_0^4 (4^3 - 8x^{3/2})\,dx$$

$$= \frac{1}{3}\left[4^3 x - 8\,\frac{2}{5}x^{5/2}\right]_0^4$$

$$= \frac{256}{3}\left(1 - \frac{2}{5}\right) = \frac{256}{5} = \underline{51.2 \text{ in.}^4}$$

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A/22 (a) $k_o^2 = k_c^2 + \overline{OC}^2$ where $k_c^2 = I_c/A = \frac{40(10^4)}{1600}\ mm^2$
$= 250\ mm^2$

$k_o^2 = 250 + (30\sqrt{2})^2 = 2050\ mm^2$

$k_o = \sqrt{2050} = \underline{45.3\ mm}$

(b) $I_{x_o} = k_{x_o}^2 A$ & $I_{x_o} + I_{y_o} = I_c$ & $I_{y_o} = I_{x_o}$ so $I_{x_o} = \frac{1}{2} I_c$

so $k_{x_o}^2 = I_c/2A$, $k_{x_o} = \sqrt{\frac{40(10^4)}{2(1600)}} = \underline{11.18\ mm}$

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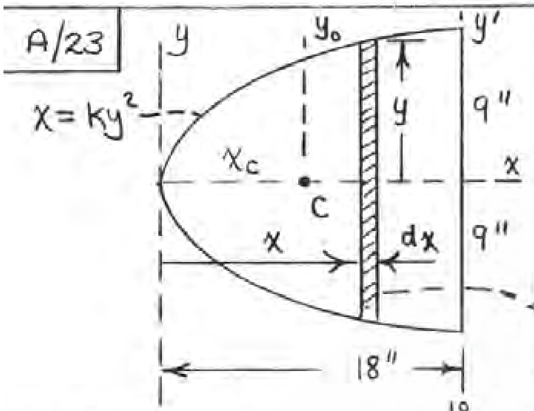
A/23



$x = ky^2$: $18 = k(9^2)$

$k = \frac{2}{9}$ in.$^{-1}$

$x = \frac{2}{9}y^2$, $y = 3\sqrt{x/2}$

$\begin{cases} dA = 2y\,dx \\ dI_y = dA\,x^2 = 2yx^2\,dx \end{cases}$

$A = \int dA = \int 2y\,dx = \int_0^{18} 2(3\sqrt{x/2})\,dx = \frac{6}{\sqrt{2}}\int_0^{18} x^{1/2}\,dx$

$= \frac{6}{\sqrt{2}}\frac{x^{3/2}}{3/2}\Big|_0^{18} = \frac{4}{\sqrt{2}}18^{3/2} = 216 \text{ in.}^2$

$I_y = \int dI_y = \int 2(3\sqrt{x/2})x^2\,dx = \frac{6}{\sqrt{2}}\int_0^{18} x^{5/2}\,dx$

$= \frac{6}{\sqrt{2}}\frac{x^{7/2}}{7/2}\Big|_0^{18} = \underline{30{,}000 \text{ in.}^4}$

$\int x\,dA = \int 2(3\sqrt{x/2})x\,dx = \frac{6}{\sqrt{2}}\int_0^{18} x^{3/2}\,dx = 2330 \text{ in.}^3$

$x_c = \int x\,dA / A = 2330/216 = 10.80$ in.

$I_{y_0} = I_y - Ax_c^2 = 30{,}000 - 216(10.80)^2 = 4800 \text{ in.}^4$

$I_{y'} = I_{y_0} + A(18 - x_c)^2 = 4800 + 216(18-10.8)^2 = \underline{16{,}000 \text{ in.}^4}$

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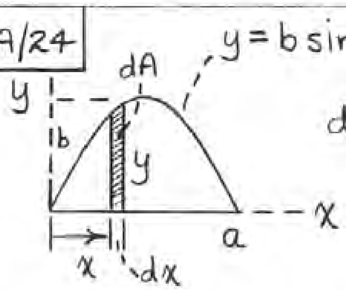
A/24



$$dI_x = \frac{1}{3} dA\, y^2 = \frac{1}{3}(y\,dx)y^2 = \frac{1}{3}y^3 dx$$

$$dI_y = dA\, x^2 = (y\,dx)x^2 = x^2 y\,dx$$

$$I_x = \int dI_x = \int \frac{1}{3} y^3 dx = \frac{1}{3}\int_0^a \left(b \sin \frac{\pi}{a}x\right)^3 dx$$

$$= \frac{b^3}{3}\left[-\frac{a}{\pi}\cos\frac{\pi}{a}x + \frac{a}{3\pi}\cos^3\frac{\pi}{a}x\right]_0^a$$

$$= \underline{\frac{4}{9}\frac{ab^3}{\pi}}$$

$$I_y = \int dI_y = \int_0^a x^2 b \sin\frac{\pi}{a}x\,dx$$

$$= b\left[\frac{2x}{(\pi/a)^2}\sin\frac{\pi}{a}x + \frac{2}{(\pi/a)^3}\cos\frac{\pi}{a}x - \frac{x^2}{\pi/a}\cos\frac{\pi}{a}x\right]_0^a$$

$$= \underline{\frac{ba^3}{\pi}\left(1 - \frac{4}{\pi^2}\right)}$$

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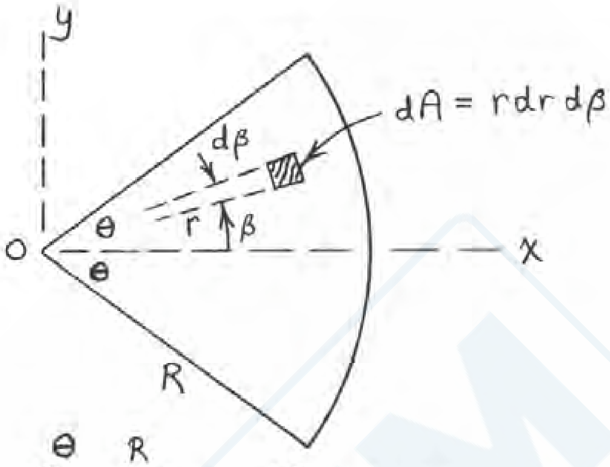
A/25



$$I_x = \int y^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \sin\beta)^2 \, r \, dr \, d\beta$$

$$= \frac{R^4}{4}\left(\frac{\beta}{2} - \frac{\sin 2\beta}{4}\right)_{-\theta}^{\theta} = \underline{\frac{R^4}{4}\left(\theta - \frac{1}{2}\sin 2\theta\right)}$$

$$I_y = \int x^2 dA = \int_{-\theta}^{\theta} \int_0^R (r \cos\beta)^2 \, r \, dr \, d\beta$$

$$= \frac{R^4}{4}\left(\frac{\beta}{2} + \frac{\sin 2\beta}{4}\right)_{-\theta}^{\theta} = \underline{\frac{R^4}{4}\left(\theta + \frac{1}{2}\sin 2\theta\right)}$$

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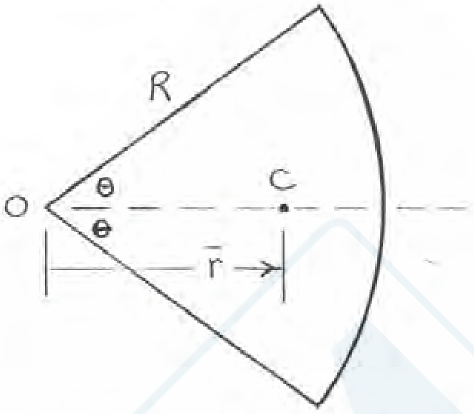
A/26



$$I_C = I_O - A\bar{r}^2$$

$$\bar{r} = \frac{2}{3}\frac{R\sin\theta}{\theta}$$

$$A = \frac{2\theta}{2\pi}\pi R^2 = \theta R^2$$

From Prob. A/25,

$$I_O = I_x + I_y = \frac{R^4}{4}\left(\theta - \frac{\sin\theta}{2} + \theta + \frac{\sin\theta}{2}\right) = \frac{1}{2}R^4\theta$$

So $$I_C = \frac{1}{2}R^4\theta - \theta R^2\left(\frac{2}{3}\frac{R\sin\theta}{\theta}\right)^2$$

$$= R^4\theta\left(\frac{1}{2} - \frac{4}{9}\frac{\sin^2\theta}{\theta^2}\right)$$

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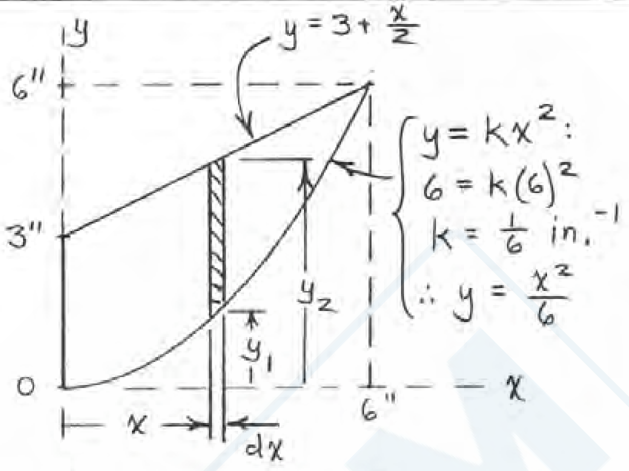
A/27

$y_2 = 3 + \frac{x}{2}$

$y_1 = x^2/6$



From Sample Problem A/1:

$$dI_x = \frac{1}{3}(y_2\,dx)y_2^2 - \frac{1}{3}(y_1\,dx)y_1^2$$

$$= \frac{1}{3}(y_2^3 - y_1^3)\,dx$$

$$I_x = \frac{1}{3}\int_0^6 \left[\left(3+\frac{x}{2}\right)^3 - \left(\frac{x^2}{6}\right)^3\right] dx$$

$$= \frac{1}{3}\left[27x + \frac{27}{4}x^2 + \frac{3}{4}x^3 + \frac{1}{32}x^4 - \frac{1}{1512}x^7\right]_0^6$$

$$= \underline{140.8 \text{ in.}^4}$$

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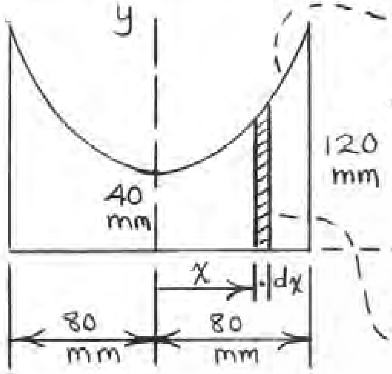
A/28



$$y = a + bx^2$$

$$\begin{cases} 40 = a + b(0)^2 \\ 120 = a + b(80)^2 \end{cases} \qquad \begin{cases} a = 40 \text{ mm} \\ b = \dfrac{1}{80 \text{ mm}} \end{cases}$$

$$\begin{cases} dA = y\,dx \\ dI_y = dA\,x^2 \end{cases}$$

$$A = \int dA = \int y\,dx = 2\int_0^{80}\left(40 + \frac{1}{80}x^2\right)dx$$

$$= 2\left[40x + \frac{x^3}{240}\right]_0^{80} = 10\,670 \text{ mm}^2$$

$$I_y = \int dI_y = \int x^2 y\,dx = 2\int_0^{80}\left[40x^2 + \frac{x^4}{80}\right]dx$$

$$= 2\left[\frac{40}{3}x^3 + \frac{x^5}{400}\right]_0^{80} = 30.0(10^6) \text{ mm}^4$$

$$k_y = \sqrt{I_y/A} = \sqrt{\frac{30.0(10^6)}{10\,670}} = \underline{53.1 \text{ mm}}$$

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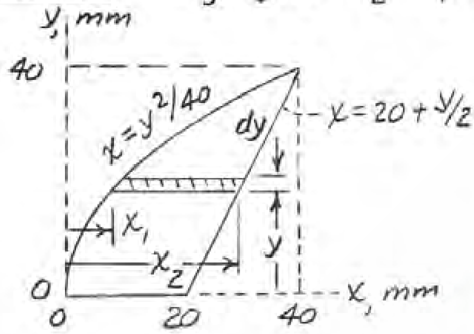
A/29 $dI_y = \frac{1}{3} dy \left(x_2^3 - x_1^3\right) = \frac{1}{3}\left[\left(20 + \frac{y}{2}\right)^3 - \left(\frac{y^2}{40}\right)^3\right] dy$



$$\int_0^{40} \left(20 + \frac{y}{2}\right)^3 dy$$

$$= \frac{1}{2}\left(20 + \frac{y}{2}\right)^4 \Big]_0^{40} = 120(10^4) \text{ mm}^4$$

$$\int_0^{40} \frac{y^6}{40^3} dy = \frac{1}{40^3} \frac{y^7}{7} \Big]_0^{40}$$

$$= 36.57(10^4) \text{ mm}^4$$

$$I_y = \frac{1}{3}(120 - 36.6)10^4 = \underline{27.8(10^4) \text{ mm}^4}$$

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A/3

$$I_x = Ad^2 = 300(15)^2 = 67.5(10^3)\ \text{mm}^4$$

$$J_O = I_x + I_y = 67.5(10^3) + 35(10^3) = 102.5(10^3)\ \text{mm}^4$$

$$k_O = \sqrt{J_O/A} = \sqrt{\frac{102.5(10^3)}{300}} = \underline{18.48\ \text{mm}}$$

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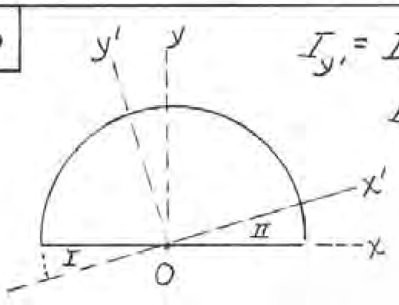
A/30



$I_{y'} = I_{half\ circle} - I_I + I_{II}$

But $I_I = I_{II}$ by symmetry

Similarly for $I_{x'}$

So $I_{x'} = I_{y'} = I_x = I_y$

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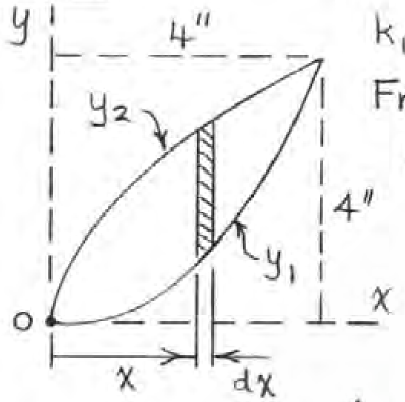
A/31



From $y_1 = k_1 x^3$: $4 = k_1 4^3$

$k_1 = \frac{1}{16}$ in.$^{-2}$ & $y_1 = \frac{x^3}{16}$

From $y_2 = k_2\sqrt{x}$: $4 = k_2\sqrt{4}$

$k_2 = 2$ in.$^{1/2}$ & $y_2 = 2\sqrt{x}$

$$dA = (y_2 - y_1)\,dx$$
$$= \left(2\sqrt{x} - \frac{x^3}{16}\right)dx$$

$$I_y = \int x^2 dA = \int_0^4 x^2\left(2x^{\frac{1}{2}} - \frac{x^3}{16}\right)dx$$

$$= \left[\frac{4}{7}x^{7/2} - \frac{x^6}{96}\right]_0^4 = \underline{30.5 \text{ in.}^4}$$

$$I_x = \int\left[\frac{1}{3}y_2^3\,dx - \frac{1}{3}y_1^3\,dx\right] = \frac{1}{3}\int_0^4\left[(2\sqrt{x})^3 - \left(\frac{x^3}{16}\right)^3\right]dx$$

$$= \frac{1}{3}\left[8\cdot\frac{2}{5}x^{5/2} - \frac{x^{10}}{10\cdot 16^3}\right]_0^4 = \underline{25.6 \text{ in.}^4}$$

$$I_O = I_x + I_y = \underline{56.1 \text{ in.}^4}$$

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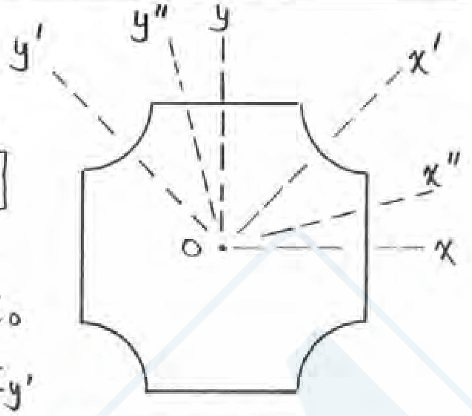
A/32



$I_x + I_y = I_o$, $I_x = I_y$

So $I_x = \frac{1}{2} I_o = \frac{1}{2}[16(10^6)]$

$= \underline{8(10^6)\ mm^4}$

Similarly, $I_{x'} + I_{y'} = I_o$

and $I_{x'} = I_{y'}$

So $I_{x'} = \frac{1}{2} I_o = I_x = \underline{8(10^6)\ mm^4}$

By symmetry it is seen that $I_{x''} = I_{y''}$ and because $I_{x''} + I_{y''} = I_o$, it follows that the moment of inertia about every axis through O has the same value, $8(10^6)\ mm^4$.

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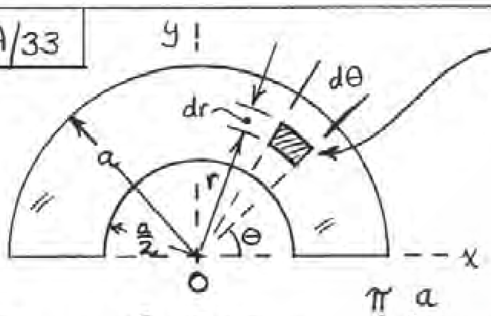
A/33



$$dA = r\,dr\,d\theta$$

$$A = \frac{1}{2}\left[\pi a^2 - \pi\left(\frac{a}{2}\right)^2\right] = \frac{3}{8}\pi a^2$$

$$I_x = \int y^2 dA = \int_0^{\pi}\int_{a/2}^{a} (r\sin\theta)^2\, r\,dr\,d\theta$$

$$= \int_0^{\pi} \frac{15}{64} a^4 \sin^2\theta\, d\theta = \frac{15}{128}\pi a^4$$

$$I_y = \int x^2 dA = 2\int_0^{\pi/2}\int_{a/2}^{a} (r\cos\theta)^2\, r\,dr\,d\theta$$

$$= 2\int_{a/2}^{a} \frac{15}{64} a^4 \cos^2\theta\, d\theta = \frac{15}{128}\pi a^4$$

$$\underline{k_x} = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{15}{128}\pi a^4}{\frac{3}{8}\pi a^2}} = \underline{\frac{\sqrt{5}}{4} a = k_y}$$

$$k_z^2 = k_x^2 + k_y^2 = 2\left(\frac{5}{16} a^2\right)$$

$$\underline{k_z = \frac{\sqrt{10}}{4} a}$$

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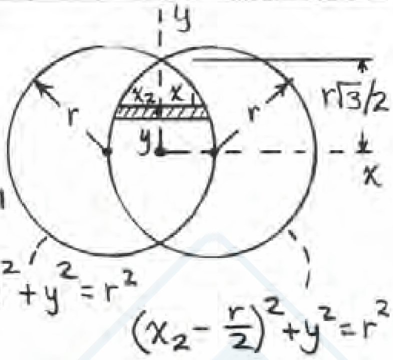
▶ A/34

$x_2 = \frac{r}{2} \pm \sqrt{r^2 - y^2}$ use (−) sign

$x_1 = -\frac{r}{2} \pm \sqrt{r^2 - y^2}$ use (+) sign



$$(x_1 - x_2) = -\frac{r}{2} + \sqrt{r^2-y^2} - \frac{r}{2} + \sqrt{r^2-y^2} = 2\sqrt{r^2-y^2} - r$$

$$dA = \left(2\sqrt{r^2-y^2} - r\right) dy$$

$$I_x = \int y^2 dA = 2\int_0^{r\sqrt{3}/2} y^2 \left(2\sqrt{r^2-y^2} - r\right) dy$$

$$= 4\left\{-\frac{y}{4}\sqrt{(r^2-y^2)^3} + \frac{r^2}{8}\left(y\sqrt{r^2-y^2} + r^2\sin^{-1}\frac{y}{r}\right)\right\} - \frac{2r}{3}y^3 \Bigg|_0^{\frac{r\sqrt{3}}{2}}$$

$$= 4\left\{-\frac{r\sqrt{3}}{8}\frac{r^3}{8} + \frac{r^2}{8}\left(\frac{r\sqrt{3}}{2}\frac{r}{2} + r^2\frac{\pi}{3}\right)\right\} - \frac{2\sqrt{3}}{8}r^4 - 0$$

$$= \frac{r^4}{2}\left\{-\frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4} + \frac{\pi}{3} - \frac{\sqrt{3}}{2}\right\} = \frac{r^4}{2}\left\{\frac{\pi}{3} - \frac{3\sqrt{3}}{8}\right\}$$

$$= 0.1988\, r^4$$

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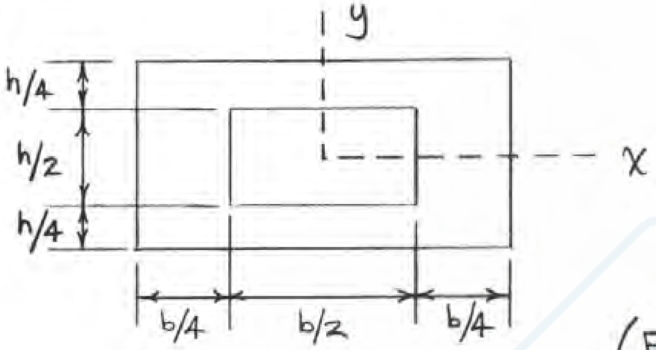
A/35



For solid plate: $I_z = \frac{1}{12} bh(b^2+h^2)$ (From Sample Problem A/1)

For plate with hole:

$$(I_z)_h = \frac{1}{12} bh(b^2+h^2) - \frac{1}{12}\frac{b}{2}\frac{h}{2}\left[\left(\frac{b}{2}\right)^2+\left(\frac{h}{2}\right)^2\right]$$

$$= \frac{15}{16}\left[\frac{1}{12} bh(b^2+h^2)\right]$$

The percent reduction $n = \frac{I_z - (I_z)_h}{I_z}(100)$

$$n = \left(\frac{1-\frac{15}{16}}{1}\right)100 = \underline{6.25\%}$$

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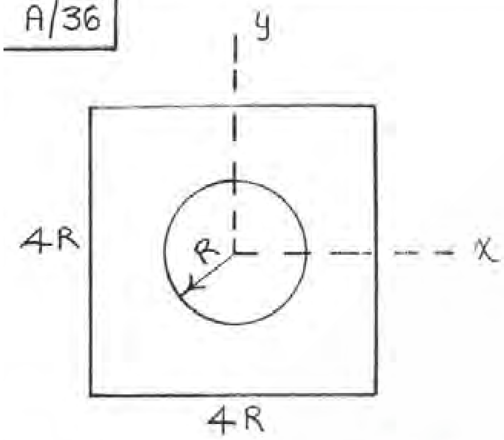

Without hole, $I_x = \frac{1}{12}(4R)(4R)^3 = \frac{64}{3}R^4$
$(21.3R^4)$

With hole, $I_x = \frac{64}{3}R^4 - \frac{1}{4}(\pi R^2)R^2$
$= \underline{20.6R^4}$

(a 3.68% reduction)

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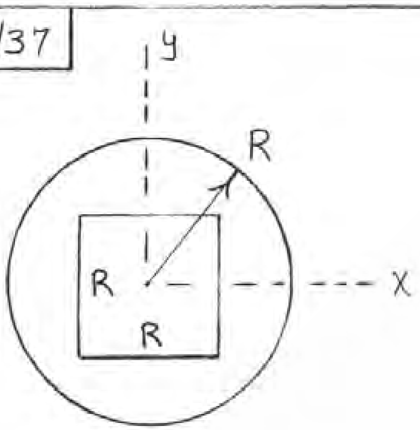

Without square hole:

$I_z = 2I_x = 2\left(\frac{1}{4}\pi R^2 \cdot R^2\right) = \underline{1.571 R^4}$

With hole:

$I_z = 1.571 R^4 - 2\left(\frac{1}{12} R \cdot R^3\right) = \underline{1.404 R^4}$

(a reduction of 10.61%)

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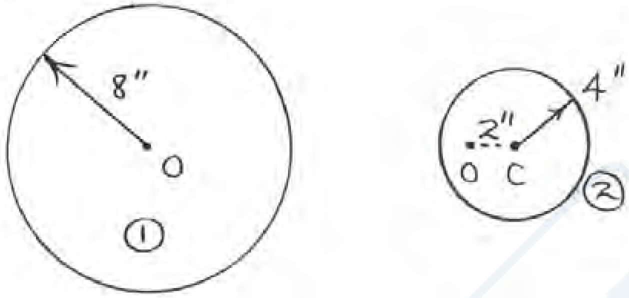
A/38



Area $A = A_1 - A_2 = \pi(8^2 - 4^2) = 48\pi \text{ in.}^2$

① $I_{O_1} = \frac{1}{2}(\pi 8^2)8^2 = 2048\pi \text{ in.}^4$

② $I_{O_2} = +\frac{1}{2}(\pi 4^2)4^2 + \pi(4)^2(2^2) = +192\pi \text{ in.}^4$

So $I_O = I_{O_1} - I_{O_2} = 1856\pi \text{ in.}^4$

$k_O = \sqrt{I_O/A} = \sqrt{\frac{1856\pi}{48\pi}} = \underline{6.22 \text{ in.}}$

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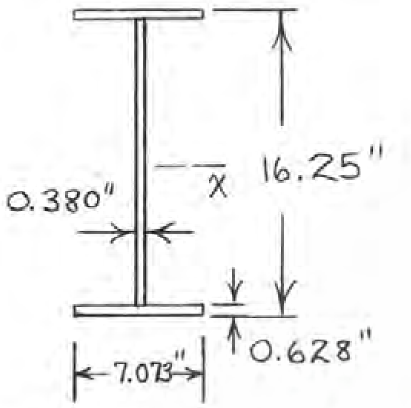

$$\bar{I}_x = \frac{1}{12}(0.380)\left[16.25 - 2(0.628)\right]^3$$

$$+ 2\left\{\frac{1}{12}(7.073)(0.628)^3 + 7.073(0.628)\left[\frac{16.25}{2} - \frac{0.628}{2}\right]^2\right\}$$

$$= \underline{649 \text{ in.}^4}$$

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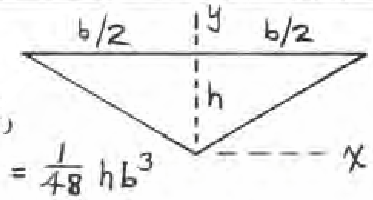
A/4

From Sample Problem A/2,



$I_x = \frac{1}{4} bh^3$, $I_y = 2\left\{\frac{1}{12} h \left(\frac{b}{2}\right)^3\right\} = \frac{1}{48} hb^3$

$I_x = I_y$ if $\frac{1}{4} bh^3 = \frac{1}{48} hb^3$, $\underline{\frac{b}{h} = 2\sqrt{3}}$

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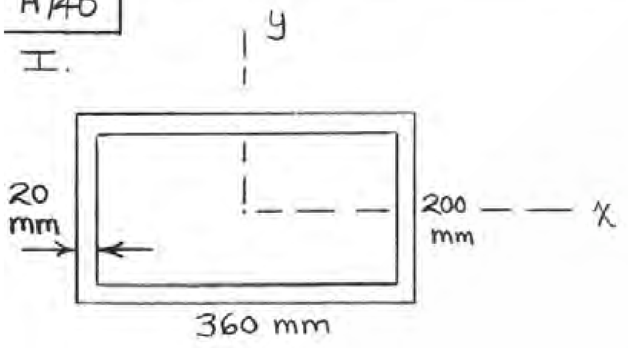
A/40

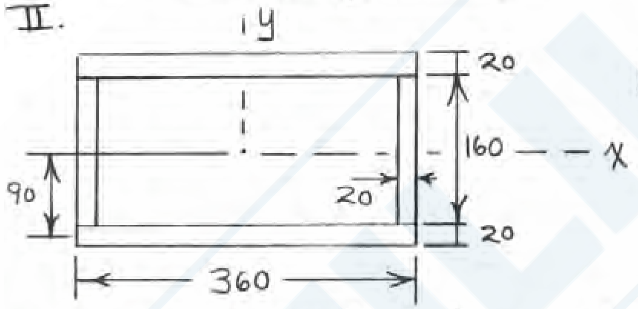
I.



$$I_x = \frac{1}{12}(360)(200)^3 - \frac{1}{12}(320)(160)^3$$

$$= \underline{130.8(10^6)\ \text{mm}^4}$$

II.



$$I_x = 2\left[\frac{1}{12}(360)(20^3) + 360(20)(90)^2\right]$$

$$+ 2\left[\frac{1}{12}(20)(160)^3\right] = \underline{130.8(10^6)\ \text{mm}^4}$$

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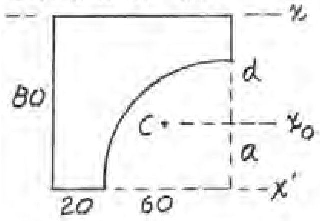
A/41

Square: $I_x = \frac{1}{3} b^4 = \frac{1}{3}(80)^4 = 13.65(10^6)\ \text{mm}^4$

Dimen. in mm



Quarter-circle: $a = \frac{4r}{3\pi} = \frac{4(60)}{3\pi}$

$= 25.46$ mm

$d = 80 - 25.46 = 54.54$ mm

$I_x = \bar{I}_{x_0} + Ad^2 = I_{x'} - Aa^2 + Ad^2$

$= \frac{-1}{4}\frac{\pi r^4}{4} - \frac{\pi r^2}{4}(d^2 - a^2) = -\frac{\pi r^2}{4}\left(\frac{r^2}{4} + d^2 - a^2\right)$

$= -\frac{\pi(60)^2}{4}\left[\frac{60^2}{4} + (54.54)^2 - (25.46)^2\right]$

$= -9.120(10^6)\ \text{mm}^4$

Total $\quad I_x = (13.65 - 9.120)(10^6) = \underline{4.53(10^6)\ \text{mm}^4}$

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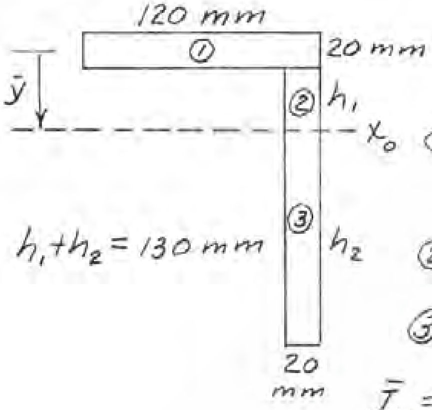
A/42

$$\bar{y} = \Sigma A y / \Sigma A = \frac{0 + (130)(20)(65+10)}{250(20)} = 39.0 \text{ mm}$$



so $h_1 = 29.0$ mm

$h_2 = 101.0$ mm

① $\bar{I}_x = \frac{1}{12}(120)(20^3) + (120)(20)(39.0)^2$

$= 3.73(10^6) \text{ mm}^4$

② $\bar{I}_x = \frac{1}{3}(20)(29.0)^3 = 0.163(10^6) \text{ mm}^4$

③ $\bar{I}_x = \frac{1}{3}(20)(101.0)^3 = 6.87(10^6) \text{ mm}^4$

$\bar{I}_x = (3.73 + 0.163 + 6.87)10^6$

$= \underline{10.76(10^6) \text{ mm}^4}$

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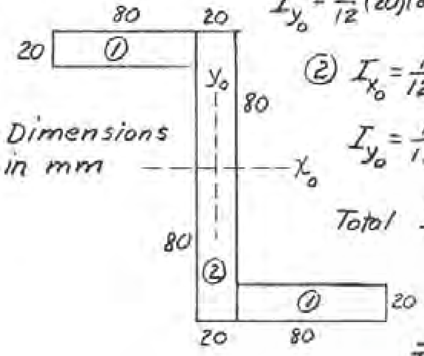
A/43



① $I_{x_o} = \frac{1}{12}(80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6)\ mm^4$

$I_{y_o} = \frac{1}{12}(20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6)\ mm^4$

② $I_{x_o} = \frac{1}{12}(20)(160)^3 = 6.83(10^6)\ mm^4$

$I_{y_o} = \frac{1}{12}(160)(20)^3 = 0.1067(10^6)\ mm^4$

Total $\bar{I}_x = [2(7.89) + 6.83](10^6)$

$= \underline{22.6(10^6)\ mm^4}$

$\bar{I}_y = [2(4.85) + 0.1067](10^6)$

$= \underline{9.81(10^6)\ mm^4}$

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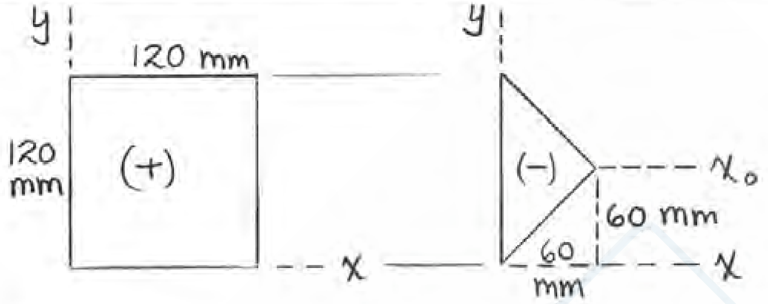
A/44



Positive area:

$$I_x = I_y = \frac{1}{3}Ah^2 = \frac{1}{3}(120)^2(120)^2 = 69.1(10^6)\text{ mm}^4$$

$$I_z = I_x + I_y = 2(69.1)(10^6) = 138.2(10^6)\text{ mm}^4$$

Negative area:

$$I_x = I_{x_0} + A\bar{y}^2 = -2\left(\frac{bh^3}{12}\right) - A(60)^2$$

$$= -2\left(\frac{60 \times 60^3}{12}\right) - \frac{1}{2}(60)(120)(60)^2 = -15.12(10^6)\text{ mm}^4$$

$$I_y = -\frac{bh^3}{12} = -\frac{120(60)^3}{12} = -2.16(10^6)\text{ mm}^4$$

$$I_z = I_x + I_y = -15.12(10^6) - 2.16(10^6) = -17.28(10^6)\text{ mm}^4$$

For composite area:

$$I_x = [69.1 - 15.12]10^6 = \underline{54(10^6)\text{ mm}^4}$$

$$I_y = [69.1 - 2.16]10^6 = \underline{67.0(10^6)\text{ mm}^4}$$

$$I_z = I_x + I_y = [54 + 67.0]10^6 = \underline{121.0(10^6)\text{ mm}^4}$$

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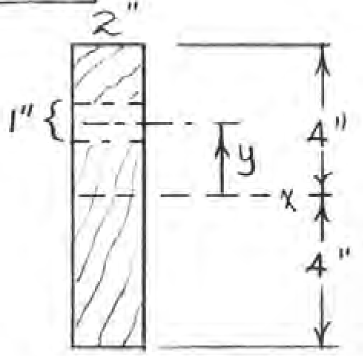
A/45



Without hole,

$$I_x = \frac{1}{12} bh^3 = \frac{1}{12}(2)(8)^3$$
$$= 85.3 \text{ in.}^4$$

With hole, $I_x' = I_x - (\bar{I}_{hole} + Ay^2)$

$$I_x' = 85.3 - \left[\frac{1}{12}(2)(1)^3 + (2)(1)y^2\right]$$
$$= 85.2 - 2y^2 \quad (y \text{ in in., } I_x' \text{ in in.}^4)$$

Percent reduction $n = \frac{I_x - I_x'}{I_x}(100\%)$

$$n = \frac{85.3 - (85.2 - 2y^2)}{85.3}(100\%) = \underline{0.1953 + 2.34y^2}$$
(in percent)

For $y = 2$ in.,

$$n = 0.1953 + 2.34(2)^2 = \underline{9.57\%}$$

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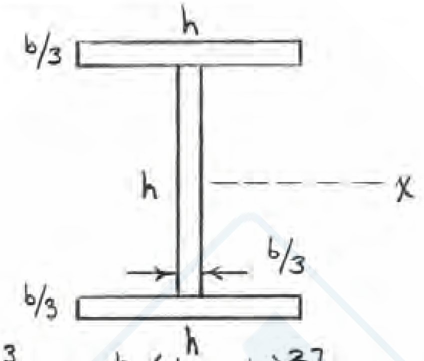
A/46



For area (a),

$$I_x = \frac{1}{12} b h^3$$

For area (b),

$$I_x = \frac{1}{12}\frac{b}{3}h^3 + 2\left[\frac{1}{12}h\left(\frac{b}{3}\right)^3 + h\frac{b}{3}\left(\frac{h}{2}+\frac{b}{6}\right)^2\right]$$

$$= \frac{hb}{9}\left(\frac{7}{4}h^2 + \frac{2}{9}b^2 + hb\right)$$

If $h = 200$ mm and $b = 60$ mm, we have

(a) $I_x = \frac{1}{12}(60)(200)^3 = 40(10^6)\text{ mm}^4$

(b) $I_x = \frac{200(60)}{9}\left(\frac{7}{4}(200)^2 + \frac{2}{9}(60)^2 + 200(60)\right)$

$= 110.4(10^6)\text{ mm}^4$

Percent increase $n = \frac{110.4-40}{40}(100\%) = 176.0\%$

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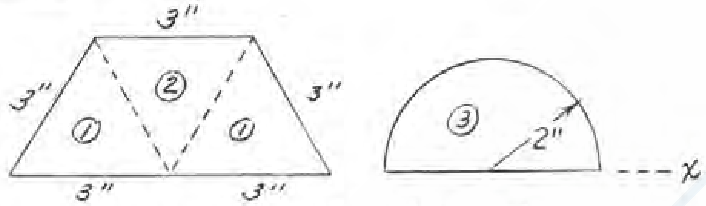
A/47



① $I_x = 2\frac{1}{12}(3)(3\sqrt{3}/2)^3 = \frac{81}{16}\sqrt{3}$ in.^{4}

② $I_x = \frac{1}{4}(3)(3\sqrt{3}/2)^3 = \frac{243}{32}\sqrt{3}$ in.^{4}

③ $I_x = \frac{-1}{2}\left(\frac{1}{4}\pi 2^4\right) = -2\pi = -6.28$ in.^{4}

Total $I_x = \frac{81}{16}\sqrt{3} + \frac{243}{32}\sqrt{3} - 6.28 = \underline{15.64 \text{ in.}^4}$

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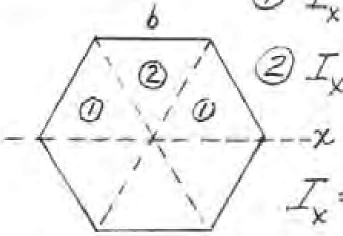
A/48 From Sample Problem A2



① $I_x = \frac{1}{12} b (b\sqrt{3}/2)^3 = \frac{\sqrt{3}}{32} b^4$

② $I_x = \frac{1}{4} b h^3 = \frac{1}{4} b (b\sqrt{3}/2)^3 = \frac{3\sqrt{3}}{32} b^4$

$I_x = 4 I_{①} + 2 I_{②} = \frac{\sqrt{3}}{8} b^4 + \frac{3\sqrt{3}}{16} b^4$

$= \frac{5\sqrt{3}}{16} b^4$

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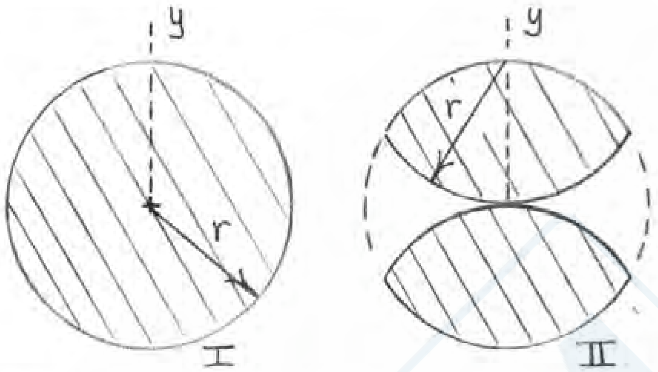
A/49



$I_I = \frac{1}{4}\pi r^4$; $I_{II} = 2(0.1988 r^4) = 0.398 r^4$

$I_y = I_I - I_{II} = 0.785 r^4 - 0.398 r^4 = \underline{0.388 r^4}$

(Refer to Prob. A/34 for I_{II}.)

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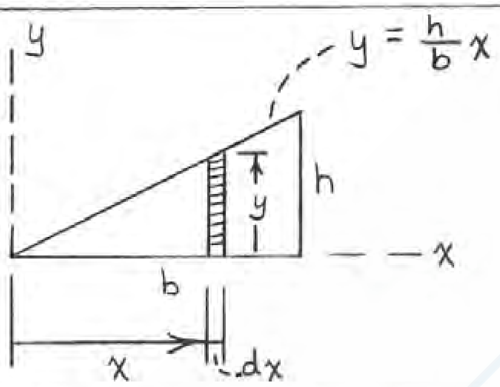
A/5



$$I_y = \int x^2 dA = \int x^2 y\,dx = \int x^2 \left(\frac{h}{b}x\right) dx$$

$$= \frac{h}{b}\int_0^b x^3 dx = \frac{h}{4b} x^4 \Big|_0^b = \frac{hb^3}{4}$$

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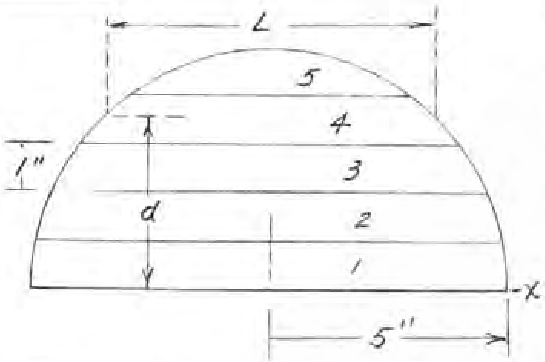
A/50



	L in.	A in.^{2}	d in.	Ad^2 in.^{4}
1	9.9	9.9	0.5	2.5
2	9.5	9.5	1.5	21.5
3	8.7	8.7	2.5	54.1
4	7.1	7.1	3.5	87.5
5	4.4	4.4	4.5	88.3

$I_x \approx \Sigma A d^2 = \underline{253.8 \text{ in.}^4}$

$I_{x\,exact} = \frac{1}{2}\left(\frac{\pi r^4}{4}\right) = \frac{\pi}{8} 5^4 = \underline{245.4 \text{ in.}^4}$; Error $= \frac{253.8 - 245.4}{245.4} 100$

$= \underline{+3\ \%}$

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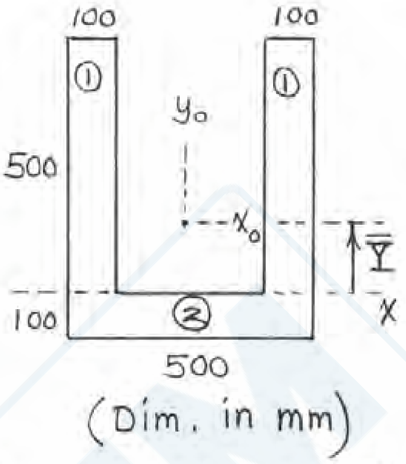
A/51



$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \frac{2[(100)(500)(250)] + 500(100)(-50)}{2(100)(500) + 100(500)}$$

$$= 150 \text{ mm}$$

$$A = 2(100)(500) + 100(500)$$

$$= 15(10^4) \text{ mm}^2$$

① + ① $I_{x_0} = 2\left[\frac{1}{12}(100)(500)^3 + 100(500)(250-150)^2\right]$

$$= 30.8(10^8) \text{ mm}^4$$

$I_{y_0} = 2\left[\frac{1}{12}(500)(100)^3 + 100(500)(150+50)^2\right] = 40.8(10^8) \text{ mm}^4$

② $I_{x_0} = \frac{1}{12}(500)(100)^3 + 100(500)(50+150)^2 = 20.4(10^8) \text{ mm}^4$

$I_{y_0} = \frac{1}{12}(100)(500)^3 = 10.42(10^8) \text{ mm}^4$

Totals ① + ① + ② : $I_{x_0} = 51.2(10^8) \text{ mm}^4$

$I_{y_0} = 51.2(10^8) \text{ mm}^4$

$I_C = I_{x_0} + I_{y_0} = 102.5(10^8) \text{ mm}^4$

$k_C = \sqrt{I_C/A} = \sqrt{\frac{102.5(10^8)}{15(10^4)}} = 261 \text{ mm}$

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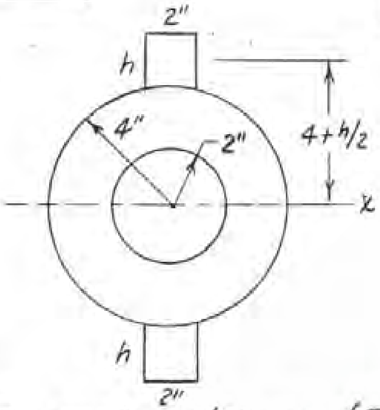
A/52



Circular section

$I_x = \pi r_2^4/4 - \pi r_1^4/4$

$= \frac{\pi}{4}(4^4 - 2^4) = 188.50 \text{ in.}^4$

Rectangular strips

$I_x = \bar{I}_x + Ad^2$

$I_x = 2\left[\frac{1}{12}(2)(h)^3 + 2h\left(4+\frac{h}{2}\right)^2\right]$

$= \frac{h^3}{3} + 64h + 16h^2 + h^3$

$= \frac{4h^3}{3} + 16h^2 + 64h$

To double stiffness $(I_x)_\square = [I_x]_o$

so $\frac{4h^3}{3} + 16h^2 + 64h = 188.50$

solve for h & get $\underline{h = 1.900 \text{ in.}}$

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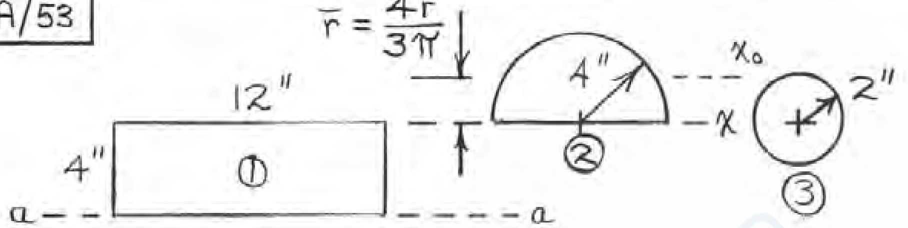
A/53



Part 1: $I_{a\text{-}a} = \frac{1}{3}(12)4^3 = 256 \text{ in.}^4$

Part 2: $I_{a\text{-}a} = I_{x_0} + A\left(4 + \frac{4 \cdot 4}{3\pi}\right)^2$

where $I_{x_0} = I_x - A\bar{r}^2 = \frac{1}{8}\pi 4^4 - \pi \frac{4^2}{2}\left(\frac{4 \cdot 4}{3\pi}\right)^2$

$= 28.1 \text{ in.}^4$

So $I_{a\text{-}a} = 28.1 + \frac{\pi 4^2}{2}(32.5) = 844 \text{ in.}^4$

Part 3: $I_{a\text{-}a} = I_x + A(4)^2 = \frac{1}{4}\pi 2^4 + \pi 2^2 (4)^2$

$= 214 \text{ in.}^4$

Combined: $I_{a\text{-}a} = 256 + 844 - 214 = 886 \text{ in.}^4$

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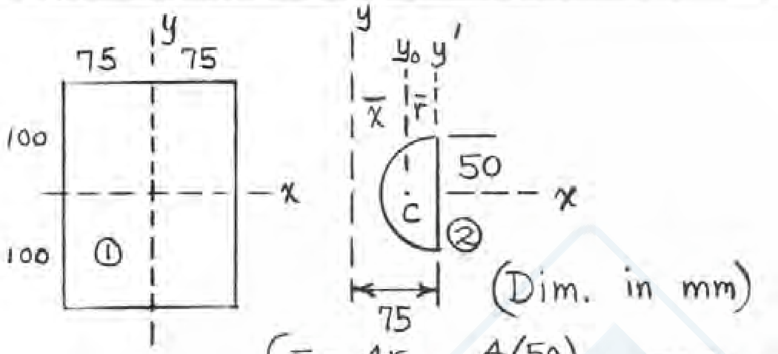
A/54



$$\bar{r} = \frac{4r}{3\pi} = \frac{4(50)}{3\pi} = 21.2 \text{ mm}$$

$$\bar{x} = 75 - 21.2 = 53.8 \text{ mm}$$

Part I: $I_x = \frac{1}{12}(150)(200)^3 = 100(10^6) \text{ mm}^4$

$I_y = \frac{1}{12}(200)(150)^3 = 56.2(10^6) \text{ mm}^4$

Parts II: $I_x = \frac{1}{4}\pi(50)^4 = 4.91(10^6) \text{ mm}^4$ (for both together)

$$I_y = I_{y_0} + A\bar{x}^2 = I_{y'} - A\bar{r}^2 + A\bar{x}^2 = I_{y'} + A(\bar{x}^2 - \bar{r}^2)$$

$$= \frac{1}{2}\left(\frac{1}{4}\pi\, 50^4\right) + \frac{\pi(50)^2}{2}(53.8^2 - 21.2^2)$$

$= 12.04(10^6) \text{ mm}^4$ for each, $24.1(10^6) \text{ mm}^4$ for both

Combined: $I_x = 100(10^6) - 4.91(10^6) = \underline{95.1(10^6) \text{ mm}^4}$

$I_y = 56.2(10^6) - 24.1(10^6) = \underline{32.2(10^6) \text{ mm}^4}$

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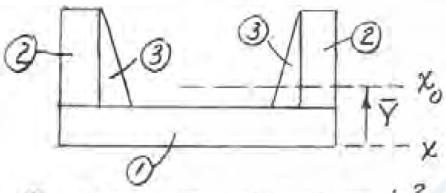
A/55

Part	A in.^{2}	$\bar{y}$ in.	$\bar{y}A$ in.^{3}	$\bar{I}_x$ in.^{4}	d in.	Ad^2 in.^{4}
1	8.40	0.35	2.94	0.343	0.696	4.07
2	4.29	2.35	10.08	3.894	1.304	7.30
3	0.33	1.80	0.59	0.200	0.754	0.19
Totals	13.02		13.61	4.437		11.56



① $A_1 = 0.70 \times 12 = 8.40$ in.^{2}

② $A_2 = 0.65 \times 3.30 \times 2 = 4.29$ in.^{2}

③ $A_3 = 2\left(\frac{1}{2}\right)(0.10)(3.30) = 0.33$ in.^{2}

$$\bar{Y} = \frac{13.61}{13.02} = 1.046 \text{ in.}$$

$$\bar{I}_x = I_{x_0} = \Sigma \bar{I}_x + \Sigma A d^2$$

$$= 4.437 + 11.56$$

$$= \underline{16.00 \text{ in.}^4}$$

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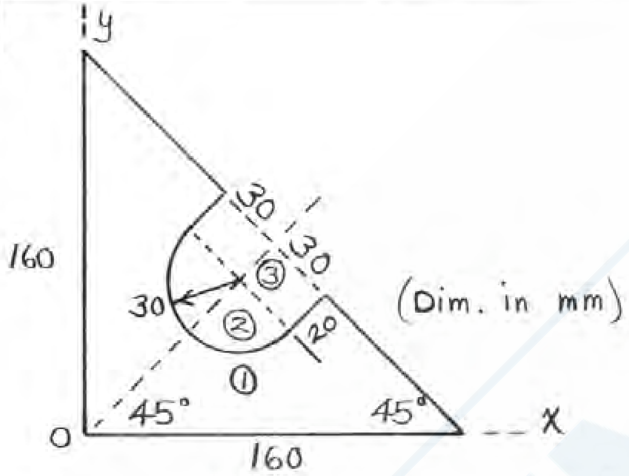
A/56



Triangle ①: $I_o = I_x + I_y = 2I_x = 2\left[\frac{160(160)^3}{12}\right]$

$= 109.2(10^6)\ \text{mm}^4$

Semicircular area ②:

$$I_o = -\frac{\pi(30)^4}{4} + \frac{\pi(30)^2}{2}\left(\frac{4\cdot30}{3\pi}\right)^2 - \frac{\pi(30)^2}{2}\left(\frac{160}{\sqrt{2}} - 20 - \frac{4\cdot30}{3\pi}\right)^2$$

$= -9.55(10^6)\ \text{mm}^4$

Rectangular area ③:

$$I_o = -\frac{60(20)}{12}(60^2 + 20^2) - 60(20)\left(\frac{160}{\sqrt{2}} - 10\right)^2$$

$= -13.16(10^6)\ \text{mm}^4$

Total $I_o = \underline{86.5(10^4)\ \text{mm}^4}$

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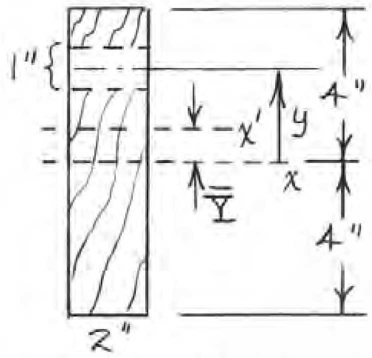
►A/57 Without hole, $x' = x$ and $I_x = I_{x'} = \frac{1}{12}bh^3 = \frac{1}{12}(2)(8)^3 = 85.3 \text{ in.}^4$



Centroid location:

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{8(2)(0) - 2(1)y}{8(2) - 2(1)}$$

$$= -0.1429y \quad (\text{or } -\tfrac{1}{7}y)$$

$$I'_{x'} = \tfrac{1}{3}(2)(4+\bar{Y})(4+\bar{Y})^2 + \tfrac{1}{3}(2)(4-\bar{Y})(4-\bar{Y})^2 - \left[\tfrac{1}{12}(2)(1)^3 + 2(1)(y-\bar{Y})^2\right]$$

$$= 85.2 + 16\bar{Y}^2 - 2(y-\bar{Y})^2$$

With $\bar{Y} = -\frac{1}{7}y$: $I'_{x'} = 85.2 + 16(-\frac{1}{7}y)^2 - 2(y + \frac{1}{7}y)^2$

or $I'_{x'} = 85.2 - \frac{112}{49}y^2 = 85.2 - 2.29y^2$

Percent reduction $n = \dfrac{I_{x'} - I'_{x'}}{I_{x'}}(100\%)$

$$= (100\%)\frac{85.3 - (85.2 - 2.29y^2)}{85.3} = 0.1953 + 2.68y^2$$

<u>(in percent)</u>

For $y = 2$ in., <u>$n = 10.91\%$</u>

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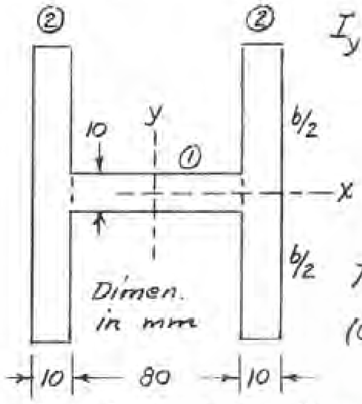
▸A/58



① $I_x = \frac{1}{12}(80)(10)^3 = 0.00667(10^6)\ mm^4$

$I_y = \frac{1}{12}(10)(80)^3 = 0.427(10^6)\ mm^4$

② $I_x = 2\left[\frac{1}{12}(10)\,b^3\right] = 1.667b^3$

$I_y = 2\left[\frac{1}{12}b(10)^3\right] + (10b)(45)^2]$

$= 0.0407(10^6)b$

Total I_x = Total I_y

$(0.00667)(10^6) + 1.667b^3$

$= (0.427 + 0.0407)(10^6)$

or $b^3 - 0.0244(10^6)b - 0.252(10^6) = 0$

Solve by cubic formula; $\left[\frac{0.252(10^6)}{2}\right]^2 < \left[\frac{0.0244(10^6)}{3}\right]^3$ so 3 real roots

$\cos u = \frac{q}{p\sqrt{p}}$ where $q = \frac{252(10^3)}{2} = 126(10^3)$, $p = \frac{24.4(10^3)}{3} = 8.13(10^3)$

$\cos u = 126(10^3)/[8.13(10^3)90.2] = 0.1718$, $u = 80.11°$

$b_1 = 2\sqrt{p}\cos u/3 = 2(90.2)(0.8933) = 161.1\ mm$ or $\underline{b = 161.1\ mm}$

$b_2 = 2\sqrt{p}\cos\left(\frac{u}{3} + 120°\right) = (-)$, $b_3 = 2\sqrt{p}\cos\left(\frac{u}{3} + 240°\right) = -(\ )$

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A/59 $A = (30)(60) = 1800 \text{ mm}^2$ for each area.

$\bar{I}_{xy} = 0$ for each area, so $I_{xy} = 0 + A d_x d_y$.

(a) $I_{xy} = 50(40)(1800) = 360(10^4) \text{ mm}^4$

(b) $I_{xy} = 50(-40)(1800) = -360(10^4) \text{ mm}^4$

(c) $I_{xy} = (-50)(10)(1800) = \underline{-90(10^4) \text{ mm}^4}$

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A/6

$I_p = I_c + A(3)^2, \quad I_{p'} = I_c + A(2)^2$

$I_p - I_{p'} = 50 = A(3^2 - 2^2), \quad \underline{A = 10 \text{ in.}^2}$

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A/60

$$I_{xy} = -3(3)\left[(-3.5)(3.5) + (3.5)(-3.5)\right]$$

$$= \underline{220 \text{ in.}^4}$$

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$$I_x = \frac{1}{12}(400)(200)^3 - 3\left[\frac{1}{4}\pi(30^4) + \pi(30)^2(50)^2\right]$$

$$= \underline{2.44(10^8)\ mm^4}$$

$$I_y = \frac{1}{12}(200)(400)^3 - 3\left[\frac{1}{4}\pi(30^4) + \pi(30)^2(100)^2\right]$$

$$= \underline{9.80(10^8)\ mm^4}$$

$$I_{xy} = -\pi(30)^2\left[(100)(50) + (-100)(50) + (-100)(-50)\right]$$

$$= \underline{-14.14(10^6)\ mm^4}$$

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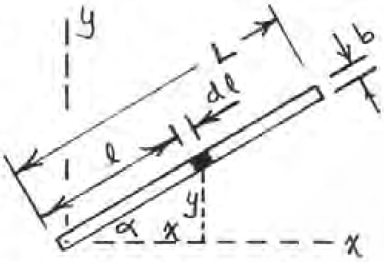

$$I_{xy} = \int xy\,dA = \int_0^L (l\cos\alpha)(l\sin\alpha)\,b\,dl$$

$$= b\sin\alpha\cos\alpha\int_0^L l^2 dl$$

$$= \frac{bL^3}{3}\sin\alpha\cos\alpha \quad \text{or} \quad \underline{\frac{1}{6}bL^3\sin 2\alpha}$$

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A/63

(a) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)(40)(80)(50)$

$= \underline{9.60(10^6)\ mm^4}$

(b) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(40)(\pi \cdot 25^2)$

$= \underline{-4.71\ (10^6)\ mm^4}$

(c) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(-40)(80)(50)$

$= \underline{9.60\ (10^6)\ mm^4}$

(d) $I_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)\left(-40 - \frac{4(25)}{3\pi}\right)$

$\times\ (\pi \cdot 25^2)/2$

$= \underline{-2.98(10^6)\ mm^4}$

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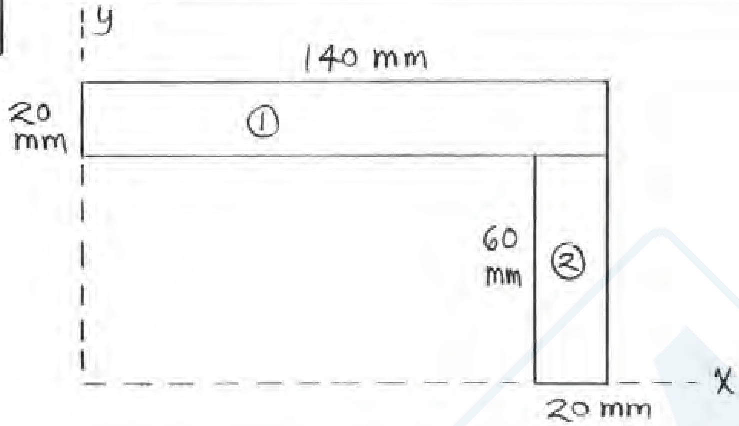
A/64



$$I_{xy_1} = 20(140)(70)(70) = 13.72(10^6)\ \text{mm}^4$$

$$I_{xy_2} = 60(20)(130)(30) = 4.68(10^6)\ \text{mm}^4$$

$$\text{Total}: \underline{I_{xy} = 18.40(10^6)\ \text{mm}^4}$$

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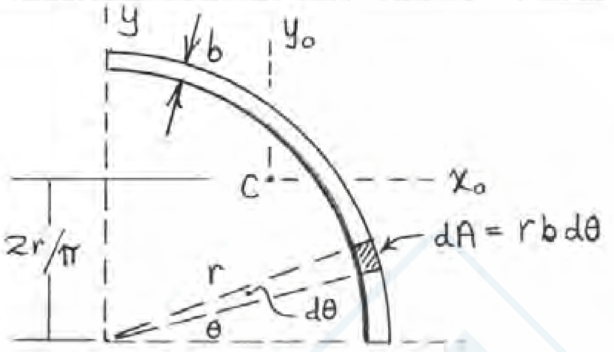
A/65



$$I_{xy} = \int xy\,dA = \int_0^{\pi/2} (r\cos\theta)(r\sin\theta)\,rb\,d\theta$$

$$= r^3 b \left.\frac{\sin^2\theta}{2}\right|_0^{\pi/2} = \underline{\tfrac{1}{2} r^3 b}$$

$$I_{x_0y_0} = I_{xy} - d_x d_y A$$

$$= \tfrac{1}{2} r^3 b - \left(\frac{2r}{\pi}\right)\left(\frac{2r}{\pi}\right) rb\,\frac{\pi}{2} = \underline{\left(\frac{1}{2} - \frac{2}{\pi}\right) r^3 b}$$

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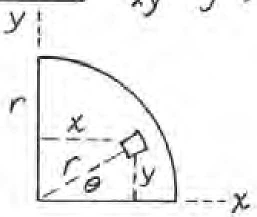
A/66



$$I_{xy} = \int xy\,dA = \int_0^{\pi/2}\int_0^r (r\cos\theta)(r\sin\theta)\,r\,dr\,d\theta$$

$$= \int_0^{\pi/2} \frac{\sin 2\theta}{2}\frac{r^4}{4}\,d\theta = \frac{r^4}{16}\left(-\cos 2\theta\right)\Big|_0^{\pi/2}$$

$$= \frac{r^4}{16}\left(1-[-1]\right) = \underline{r^4/8}$$

$$\bar{I}_{xy} = I_{xy} - d_x d_y A = \frac{r^4}{8} - \frac{4r}{3\pi}\left(\frac{4r}{3\pi}\right)\frac{\pi r^2}{4} = \frac{r^4}{8}\left(1-\frac{32}{9\pi}\right)$$

$$= \underline{-0.01647 r^4}$$

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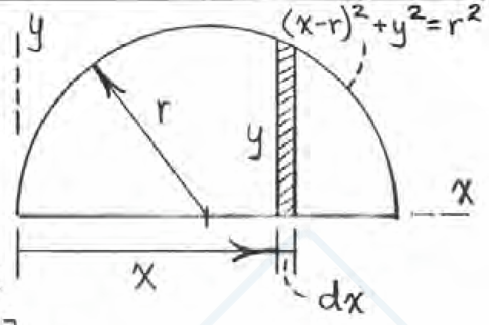
A/67

(1) By direct integration



For elemental strip,

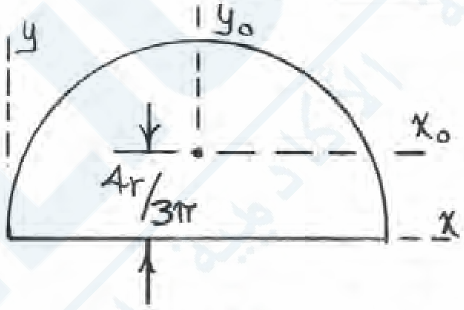
$$dI_{xy} = x\frac{y}{2}dA = \frac{xy}{2}y\,dx$$

$$= \frac{x}{2}\left[r^2 - (x-r)^2\right]dx$$

$$I_{xy} = \frac{1}{2}\int_0^{2r}(xr^2 - x^3 + 2rx^2 - r^2x)\,dx$$

$$= \frac{1}{2}\left[\frac{x^2r^2}{2} - \frac{x^4}{4} + \frac{2rx^3}{3} - \frac{r^2x^2}{2}\right]_0^{2r} = \underline{\frac{2}{3}r^4}$$

(2) By axis transfer



$$I_{xy} = I_{x_0y_0} + Ad_xd_y = 0 + \frac{\pi r^2}{2}(r)\left(\frac{4r}{3\pi}\right) = \underline{\frac{2}{3}r^4}$$

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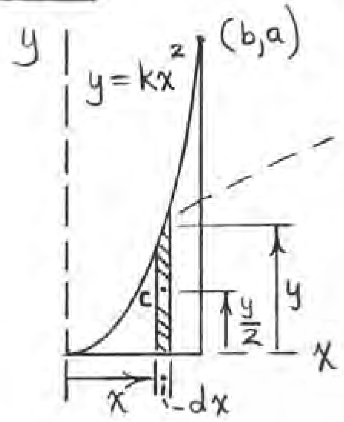
A/68



$y = kx^2$: $a = kb^2$, $k = a/b^2$

$\Rightarrow y = \frac{a}{b^2} x^2$

$dA = y\,dx = \frac{a}{b^2} x^2\,dx$

$d\bar{I}_{xy} = 0$

$dI_{xy} = d\bar{I}_{xy} + dA\,(x)\left(\frac{y}{2}\right)$

$= 0 + y\,dx\,(x)\left(\frac{y}{2}\right)$

$= \frac{1}{2}\frac{a^2}{b^4} x^5\,dx$

$I_{xy} = \int dI_{xy} = \int_0^b \frac{1}{2}\frac{a^2}{b^4} x^5\,dx$

$= \frac{1}{2}\frac{a^2}{b^4}\frac{x^6}{6}\Big|_0^b = \underline{\frac{1}{12}a^2b^2}$

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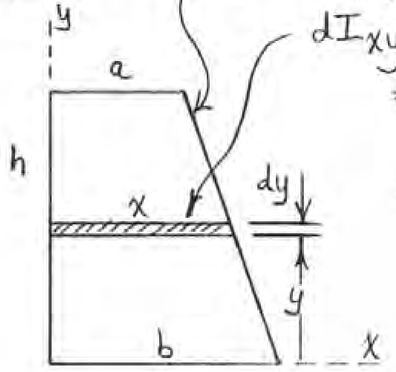
A/69



$$x = \left(\frac{a-b}{h}\right)y + b$$

$$dI_{xy} = x\,dy\left(\frac{x}{2}\right)(y) = x^2\frac{y}{2}\,dy$$

$$= \left[\left(\frac{a-b}{h}\right)y + b\right]^2 \frac{y}{2}\,dy$$

$$= \frac{1}{2}\left[\left(\frac{a-b}{h}\right)^2 y^3 + 2\left(\frac{a-b}{h}\right)by^2 + b^2 y\right]dy$$

$$I_{xy} = \int dI_{xy} = \frac{1}{2}\int_0^h \left[\left(\frac{a-b}{h}\right)^2 y^3 + 2\left(\frac{a-b}{h}\right)by^2 + b^2 y\right]dy$$

$$= \frac{1}{2}\left[\left(\frac{a-b}{h}\right)^2 \frac{y^4}{4} + 2\left(\frac{a-b}{h}\right)b\frac{y^3}{3} + b^2\frac{y^2}{2}\right]_0^h$$

$$= \underline{\frac{h^2}{4}\left(\frac{a^2}{2} + \frac{ab}{3} + \frac{b^2}{6}\right)} \text{ or } \underline{\frac{h^2}{24}\left(3a^2 + 2ab + b^2\right)}$$

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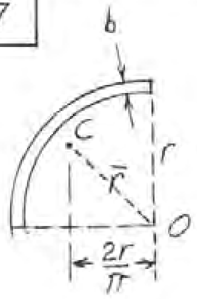
A/7



$$I_C = I_O - A\bar{r}^2$$

$$= Ar^2 - A\left(\frac{2r}{\pi}\sqrt{2}\right)^2$$

$$= Ar^2\left(1 - \frac{8}{\pi^2}\right) = \frac{\pi b r^3}{2}\left(1 - \frac{8}{\pi^2}\right)$$

$$= br^3\left(\frac{\pi}{2} - \frac{4}{\pi}\right)$$

$$= \underline{0.298\, br^3}$$

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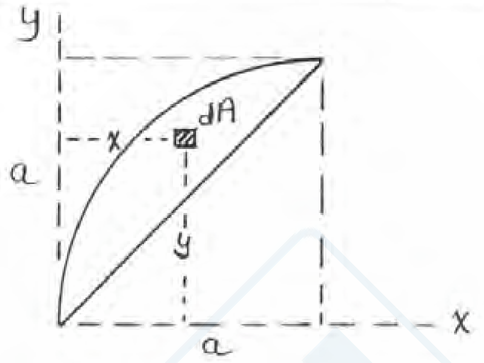
A/70



$$I_{xy} = \int xy\,dA$$
$$= \iint xy\,dx\,dy$$
$$= \int_0^a y\left[\int_{x_1}^{x_2} x\,dx\right]dy$$

where $x_2 = y$ and $(x_1 - a)^2 + y^2 = a^2$

$x_1 = a - \sqrt{a^2 - y^2}$

(note minus sign)

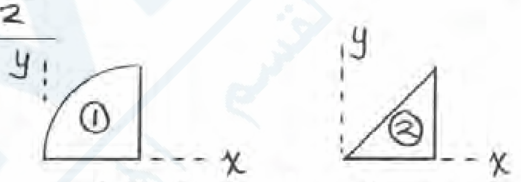
$$\int_{x_1}^{x_2} x\,dx = \left.\frac{x^2}{2}\right|_{a-\sqrt{a^2-y^2}}^{y} = y^2 - a^2 + a\sqrt{a^2-y^2}$$

So $I_{xy} = \int_0^a y\left(y^2 - a^2 + a\sqrt{a^2-y^2}\right)dy$

$$= \left[\frac{y^4}{4} - \frac{a^2y^2}{2} - \frac{a}{3}\sqrt{(a^2-y^2)^3}\right]_0^a$$

$$= \frac{a^4}{12}$$

Alternatively



$$I_{xy} = (I_{xy})_{①} - (I_{xy})_{②}$$

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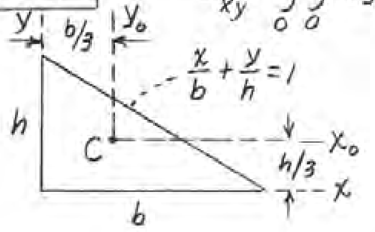
A/71



$$I_{xy} = \int_0^h \int_0^x xy\,dx\,dy = \int_0^h y\left(\frac{x^2}{2}\right)\Big]_0^{b(1-y/h)} dy$$

$$= \int_0^h \frac{b^2}{2}\left(y - \frac{2y^2}{h} + \frac{y^3}{h^2}\right) dy$$

$$= \frac{b^2}{2}\left[\frac{h^2}{2} - \frac{2h^2}{3} + \frac{h^2}{4}\right] = \underline{\frac{b^2h^2}{24}}$$

$$I_{x_0y_0} = \bar{I}_{xy} = I_{xy} - d_x d_y A = \frac{b^2h^2}{24} - \frac{bh}{2}\left(\frac{h}{3}\right)\left(\frac{b}{3}\right) = \underline{-\frac{b^2h^2}{72}}$$

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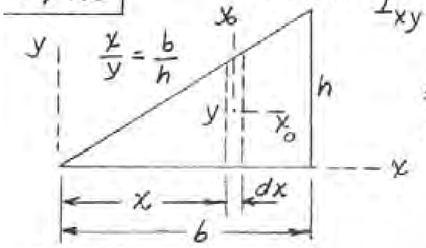
A/72

First:
$$I_{xy} = \int_0^b \int_0^y xy\,dy\,dx$$



$$= \int_0^b \frac{x}{2} y^2 \Big]_0^{\frac{hx}{b}} dx = \int_0^b \frac{h^2}{2b^2} x^3 dx$$

$$= \frac{h^2}{2b^2}\frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$$

Second:
$$dI_{xy} = dI_{x_0y_0} + d_x d_y (dA)$$

$$= 0 + \frac{y}{2} x (y\,dx) = \frac{h^2}{2b^2} x^3 dx$$

$$I_{xy} = \frac{h^2}{2b^2}\int_0^b x^3 dx = \frac{h^2}{2b^2}\frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$$

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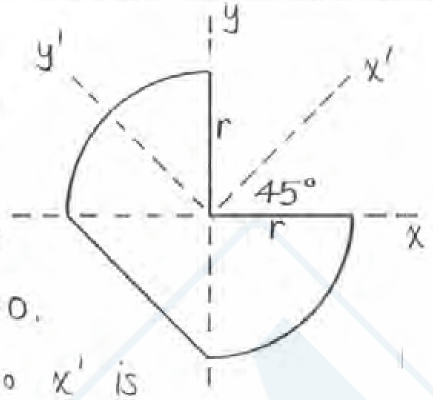
A/73



By inspection, x' is an axis of symmetry, so x' and y' are principal axes of inertia where $I_{x'y'} = 0$.

By inspection $I_{x'} > I_{y'}$ so x' is an axis of maximum inertia.

For each quarter-circular area,

$I_x = I_y = \frac{1}{16}\pi r^4$, $I_{xy} = -\frac{1}{8} r^4$ (from Prob. A/66)

For triangular area

$I_x = I_y = \frac{1}{12} r^4$, $I_{xy} = \frac{1}{24} r^4$ (from Prob. A/71 with $h = b = r$)

For composite area

$$I_x = I_y = \left(\frac{1}{12} + 2\frac{\pi}{16}\right) r^4 = 0.476\, r^4$$

$$I_{xy} = \left(-\frac{1}{8}(2) + \frac{1}{24}\right) r^4 = -0.208\, r^4$$

Eq. A/11:

$$I_{max} = 0.476 r^4 + \frac{1}{2}\sqrt{0 + 4(-0.208)^2 r^8}$$

$$= \underline{0.684\, r^4}$$

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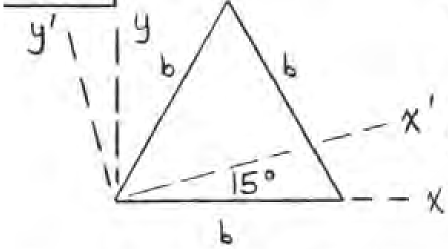
A/74



$$I_x = \frac{1}{12} b \left(\frac{b\sqrt{3}}{2}\right)^3 = \frac{\sqrt{3}}{32} b^4$$

$$\bar{I}_y = \bar{I}_x = \frac{\sqrt{3}}{32} b^4 - \frac{1}{2} b^2 \frac{\sqrt{3}}{2} x \left(\frac{1}{3} b \frac{\sqrt{3}}{2}\right)^2 = \frac{\sqrt{3}}{96} b^4$$

$$I_y = \bar{I}_y + A d_y^2 = \frac{\sqrt{3}}{96} b^4 + b\left(\frac{b\sqrt{3}}{4}\right)\left(\frac{b}{2}\right)^2 = \frac{7\sqrt{3}}{96} b^4$$

$$I_{xy} = \bar{I}_{xy} + A d_x d_y = 0 + b\left(\frac{b\sqrt{3}}{4}\right)\left(\frac{b}{2}\right)\left(\frac{1}{3}\frac{\sqrt{3}}{2} b\right) = \frac{3}{48} b^4$$

Eqs. A/9 & A/9a, with $\theta = 15°$:

$$I_{x'} = \frac{5\sqrt{3}}{96} b^4 - \frac{\sqrt{3}}{48} b^4 \cos 30° - \frac{3}{48} b^4 \sin 30° = \underline{0.0277 b^4}$$

$$I_{y'} = \frac{5\sqrt{3}}{96} b^4 + \frac{\sqrt{3}}{48} b^4 \cos 30° + \frac{3}{48} b^4 \sin 30° = \underline{0.1527 b^4}$$

$$I_{x'y'} = -\frac{\sqrt{3}}{48} b^4 \sin 30° + \frac{3}{48} b^4 \cos 30° = \underline{0.0361 b^4}$$

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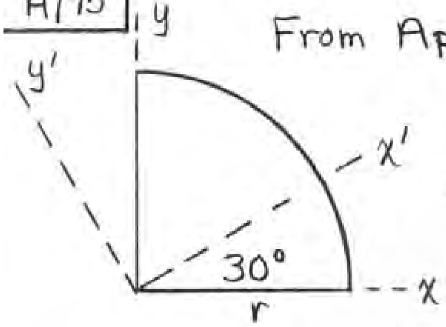
A/75



From Appendix D: $I_x = I_y = \frac{\pi r^4}{16}$

From Prob. A/67:

$I_{xy} = \frac{r^4}{8}$

Eq. A/9: $I_{x'} = \frac{\pi r^4}{16} + 0 - \frac{r^4}{8}\sin 60° = \underline{\frac{r^4}{16}\left[\pi - \sqrt{3}\right]}$

$I_{y'} = \frac{\pi r^4}{16} - 0 + \frac{r^4}{8}\sin 60° = \underline{\frac{r^4}{16}\left[\pi + \sqrt{3}\right]}$

Eq. A/9a :

$I_{x'y'} = 0 + \frac{r^4}{8}\cos 60° = \underline{\frac{r^4}{16}}$

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A/76 $I_x = \frac{1}{3} b(b^3) = \frac{1}{3} b^4$; $I_y = \frac{1}{3} b^4$

$I_{xy} = 0 + \frac{b}{2} \frac{b}{2} b^2 = \frac{1}{4} b^4$

with $\theta = 30°$, Eqs. A/9 & A/9a give

$I_{x'} = \frac{b^4}{3} + 0 - \frac{1}{4} b^4 \sin 60° = \left(\frac{1}{3} - \frac{\sqrt{3}}{8}\right) b^4 = \underline{0.1168\, b^4}$

$I_{y'} = \frac{b^4}{3} + 0 + \frac{1}{4} b^4 \sin 60° = \left(\frac{1}{3} + \frac{\sqrt{3}}{8}\right) b^4 = \underline{0.5498\, b^4}$

$I_{x'y'} = 0 + \frac{b^4}{4} \frac{1}{2} = \frac{b^4}{8} = \underline{0.1250\, b^4}$

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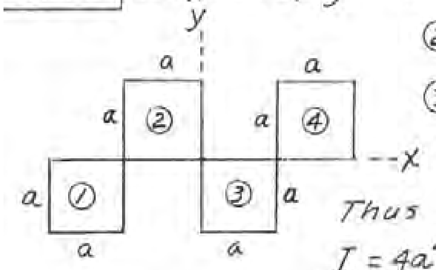
A/77

① $I_x = a^4/3$, $I_y = a^4/12 + a^2(3a/2)^2 = 7a^4/3$; $I_{xy} = +3a^4/4$



② $I_x = a^4/3$, $I_y = a^4/3$, $I_{xy} = -a^4/4$

③ same as ②

④ same as ①

Thus for composite area

$I_x = 4a^4/3$, $I_y = 16a^4/3$, $I_{xy} = +a^4$

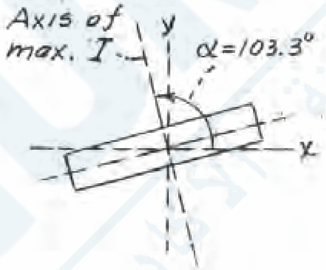
From Eq. A/11,

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^{\,2}} = \left(\frac{10}{3} + \sqrt{5}\right)a^4 = \underline{5.57a^4}$$

$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2} = \left(\frac{10}{3} - \sqrt{5}\right)a^4 = \underline{1.097a^4}$$

From Eq. A/10

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2a^4}{(16/3 - 4/3)a^4} = +\frac{1}{2}$$



$2\alpha = 26.6°$ or $206.6°$

$\alpha = 13.3°$ or $\underline{103.3°}$

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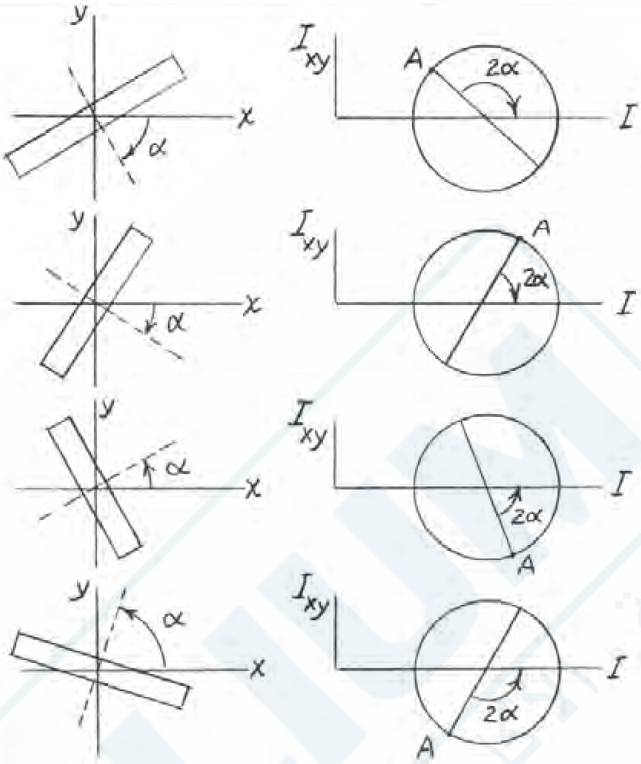
A/78

(a) $I_x < I_y$, I_{xy} (+)

(b) $I_x > I_y$, I_{xy} (+)

(c) $I_x > I_y$, I_{xy} (−)

(d) $I_x < I_y$, I_{xy} (−)



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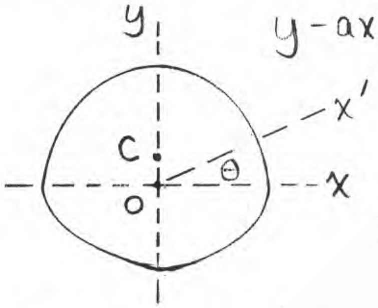
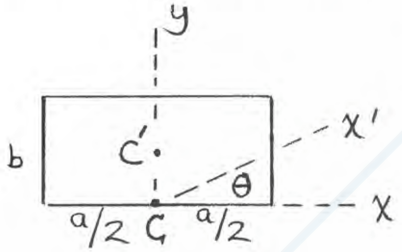
A/79 For an area symmetrical about the y-axis, $I_{xy} = 0$ and if $I_x = I_y$, Eq. A/9 becomes



$$I_{x'} = \frac{I_x + I_y}{2} + 0 - 0 = I_x$$

So $I_{x'}$ is independent of θ.



The rectangular area meets the condition of y-axis symmetry, so $I_{xy} = 0$. $I_x = I_y$ if $\frac{1}{3}ab^3 = \frac{1}{12}ba^3 \Rightarrow \underline{a = 2b}$.

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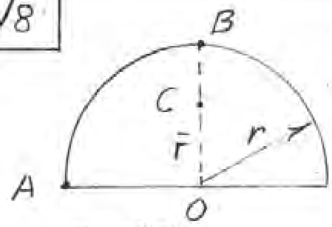
A/8



$\bar{r} = 4r/3\pi$

For complete circle

$I_A = I_0 + Ar^2 = \frac{1}{2}Ar^2 + Ar^2$

$= \frac{3}{2}Ar^2$

For half circle

$I_A = \frac{1}{2}\left(\frac{3}{2}\pi r^4\right) = \frac{3}{4}\pi r^4$

For half circle, $I_0 = \frac{1}{4}\pi r^4$

$I_B = I_C + A(r-\bar{r})^2 = I_0 - A\bar{r}^2 + A(r-\bar{r})^2$

$= I_0 + A(r^2 - 2r\bar{r})$

$= \frac{1}{4}\pi r^4 + \frac{\pi r^4}{2}\left(1 - \frac{8}{3\pi}\right) = r^4\left(\frac{3\pi}{4} - \frac{4}{3}\right)$

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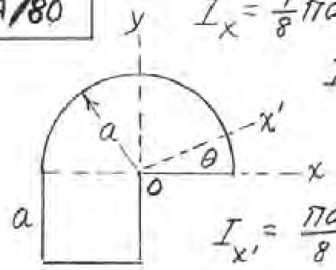
A/80



$I_x = \frac{1}{8}\pi a^4 + \frac{1}{3}a^4, \quad I_y = \frac{1}{8}\pi a^4 + \frac{1}{3}a^4 = I_x$

$I_{xy} = 0 + a^2(-\frac{a}{2})(-\frac{a}{2}) = \frac{a^4}{4}$

From Eq. A/9

$I_{x'} = \frac{\pi a^4}{8} + \frac{a^4}{3} + 0 - \frac{a^4}{4}\sin 2\theta$

$= a^4(\frac{\pi}{8} + \frac{1}{3} - \frac{1}{4}\sin 2\theta)$

$\frac{dI_{x'}}{d\theta} = a^4(-\frac{1}{2}\cos 2\theta) = 0$ for max. or min.

so $2\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ so $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

$\underline{\theta = \pi/4}$, $I_{x'} = a^4(\frac{\pi}{8} + \frac{1}{3} - \frac{1}{4}[1]) = \underline{0.476a^4}$ (min)

$I_{y'} = a^4(\frac{\pi}{8} + \frac{1}{3} + \frac{1}{4}[1]) = \underline{0.976a^4}$ (max)

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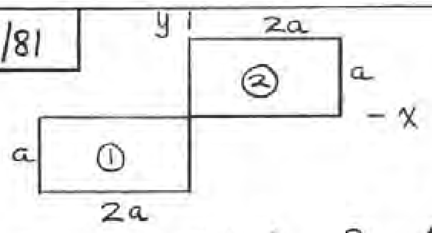
A/81



① $I_x = \frac{1}{3}(2a)(a^3) = \frac{2}{3}a^4,\quad I_y = \frac{1}{3}(a)(2a)^3 = \frac{8}{3}a^4$

$I_{xy} = (2a^2)(a)(a/2) = a^4$

② $I_x = \frac{2}{3}a^4,\quad I_y = \frac{8}{3}a^4,\quad I_{xy} = a^4$

Eq. A/11 : $I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$= \frac{10}{3}a^4 - \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{0.505a^4}$

$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x^2 - I_y^2) + 4I_{xy}^2}$

$= \frac{10}{3}a^4 + \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{6.16a^4}$

Eq. A/10: $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4a^4}{\left(\frac{16}{3} - \frac{4}{3}\right)a^4}$

$2\alpha = 45°$ or $225°$

$\alpha = 22.5°$ for I_{min}

or $\alpha = \underline{112.5°}$ for I_{max}

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A/82 From figure, I_{xy} is (−)

Add Eqs. A/11 & get $I_{max} + I_{min} = I_x + I_y$

so $I_x + I_y = (12 + 2)10^6 = 14(10^6)\ mm^4$

From the 1st of Eqs. A/11,

$$(I_x - I_y)^2 = [2I_{max} - (I_x + I_y)]^2 - 4I_{xy}^2$$

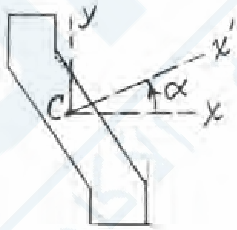
$$= [2(12) - 14]^2 10^{12} - 4(-4)^2 10^{12} = 36(10^{12})\ mm^8$$

$$\left.\begin{aligned} I_x - I_y &= 6(10^6)\ mm^4 \\ I_x + I_y &= 14(10^6)\ mm^4 \end{aligned}\right\}$$ add & get $\underline{I_x = 10(10^6)\ mm^4}$

& $\underline{I_y = 4(10^6)\ mm^4}$

From Eq. A/10,

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(-4)(10^6)}{-6(10^6)} = 4/3$$



$2\alpha = 53.13°$, $\underline{\alpha = 26.6°}$

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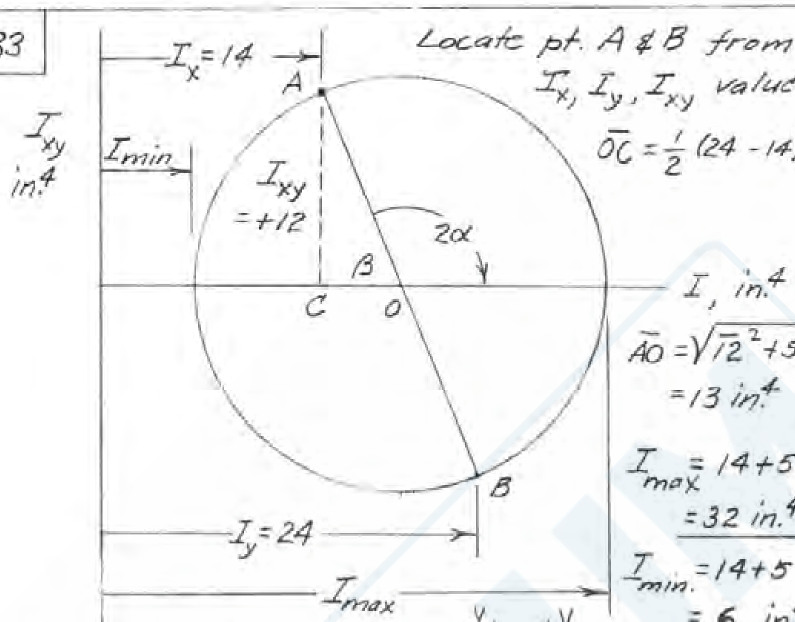
A/83

Locate pt. A & B from I_x, I_y, I_{xy} values

$\overline{OC} = \frac{1}{2}(24 - 14) = 5 \text{ in.}^4$



$\overline{AO} = \sqrt{12^2 + 5^2} = 13 \text{ in.}^4$

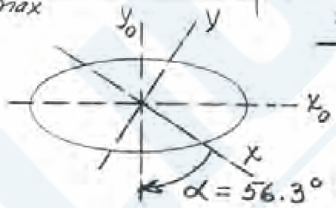
$I_{max} = 14 + 5 + 13 = 32 \text{ in.}^4$

$I_{min.} = 14 + 5 - 13 = 6 \text{ in.}^4$

$\beta = \sin^{-1}\frac{12}{13} = 67.38°$,

$2\alpha = 180 - 67.38 = 112.6°$

$\alpha = 56.3°$ Clockwise



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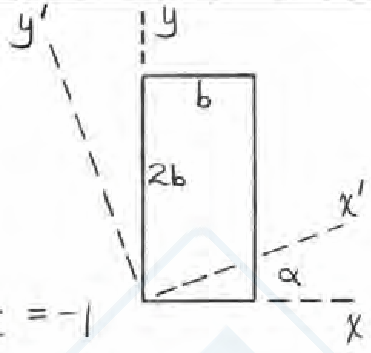
A/84



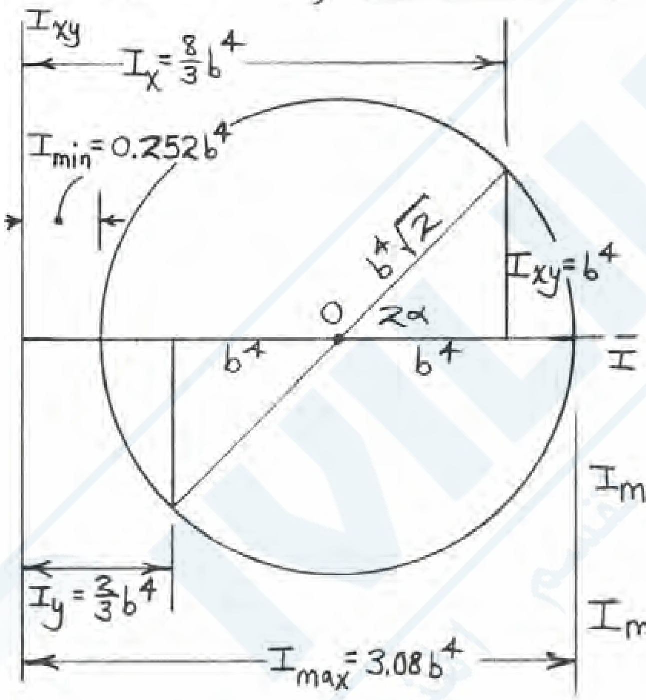
$$I_x = \frac{1}{3} 2b^2 (2b)^2 = \frac{8}{3} b^4$$

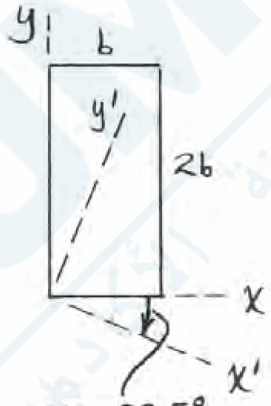
$$I_y = \frac{1}{3} 2b^2 (b)^2 = \frac{2}{3} b^4$$

$$I_{xy} = 2b^2 \left(\frac{b}{2}\right)(b) = b^4$$

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2b^4}{\frac{2}{3}b^4 - \frac{8}{3}b^4} = -1$$

$$\Rightarrow 2\alpha = -45^\circ, \quad \underline{\alpha = -22.5^\circ}$$





$\alpha = -22.5^\circ$

$$I_{max} = \frac{5}{3}b^4 + b^4\sqrt{2} = \underline{3.08\, b^4}$$

$$I_{min} = \frac{5}{3}b^4 - b^4\sqrt{2} = \underline{0.252\, b^4}$$

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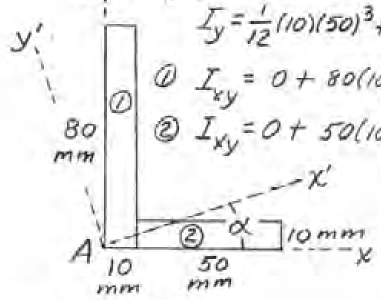
A/85

① $I_x = \frac{1}{3}(10)(80)^3 = 1.707(10^6)\ mm^4$, $I_y = \frac{1}{3}(80)(10)^3 = 0.0267(10^6)\ mm^4$

② $I_x = \frac{1}{3}(50)(10)^3 = 0.0167(10^6)\ mm^4$,

$I_y = \frac{1}{12}(10)(50)^3 + 10(50)(35)^2 = 0.7167(10^6)\ mm^4$



① $I_{xy} = 0 + 80(10)(40)(5) = 0.1600(10^6)\ mm^4$

② $I_{xy} = 0 + 50(10)(35)(5) = 0.0875(10^6)\ mm^4$

Totals: $I_x = 1.723(10^6)\ mm^4$

$I_y = 0.743(10^6)\ mm^4$

$I_{xy} = 0.248(10^6)\ mm^4$

From Eqs. A/11

$$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2},\quad I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$I_{max} = \left[\frac{1.723 + 0.743}{2} + \frac{1}{2}\sqrt{(1.723 - 0.743)^2 + 4(0.248)^2}\right]10^6 = \underline{1.782(10^6)\ mm^4}$$

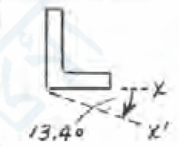
$$I_{min} = \left[\frac{1.723 + 0.743}{2} - \frac{1}{2}\sqrt{(1.723 - 0.743)^2 + 4(0.248)^2}\right]10^6 = \underline{0.684(10^6)\ mm^4}$$

From Eq. A/10

$$\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(0.248)}{0.743 - 1.723} = -0.5051$$

$2\alpha = -26.8°$

$\underline{\alpha = -13.40°}$



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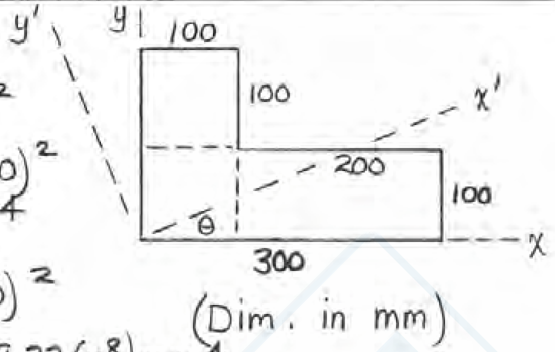
*A/86



$$I_x = \frac{1}{3}(100\cdot 200)(100)^2 + \frac{1}{3}(100\cdot 200)(200)^2 = 3.33(10^8)\ \text{mm}^4$$

$$I_y = \frac{1}{3}(100\cdot 300)(300)^2 + \frac{1}{3}(100\cdot 100)(100)^2 = 9.33(10^8)\ \text{mm}^4$$

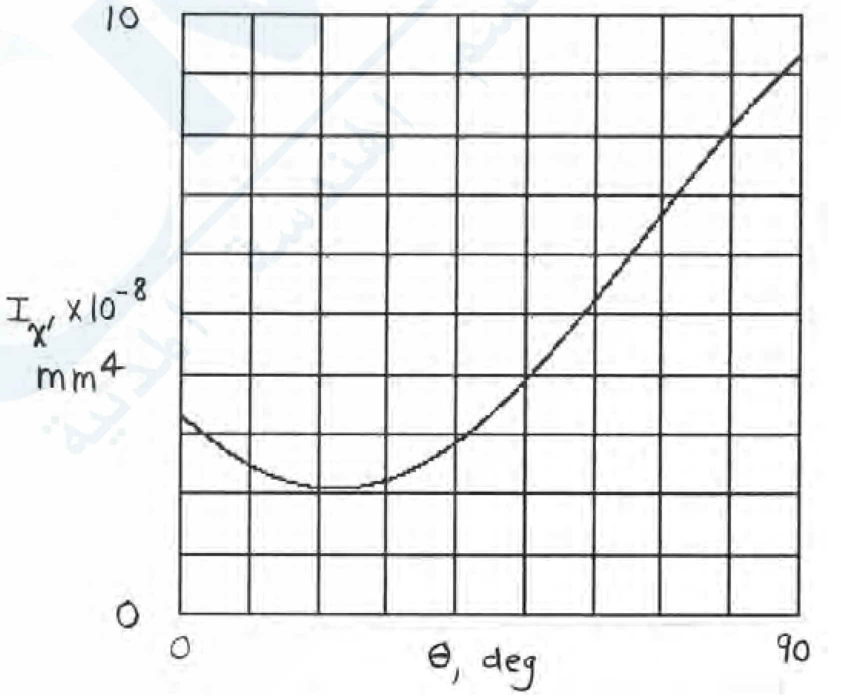
$$I_{xy} = (100\cdot 200)(50)(100) + (100\cdot 200)(200)(50) = 3(10^8)\ \text{mm}^4$$

Eq. A/9: $I_{x'} = \left\{\frac{3.33+9.33}{2} + \frac{3.33-9.33}{2}\cos 2\theta - 3\sin 2\theta\right\}10^8$

$$= \{6.33 - 3\cos 2\theta - 3\sin 2\theta\}\,10^8\ \text{mm}^4$$

$$\frac{dI_{x'}}{d\theta} = 6\{\sin 2\theta - \cos 2\theta\}\,10^8\ \text{mm}^4/\text{rad}$$

Set $\frac{dI_{x'}}{d\theta} = 0$ to find $(I_{x'})_{min} = 2.09(10^8)\ \text{mm}^4$ @ $\theta = 22.5°$



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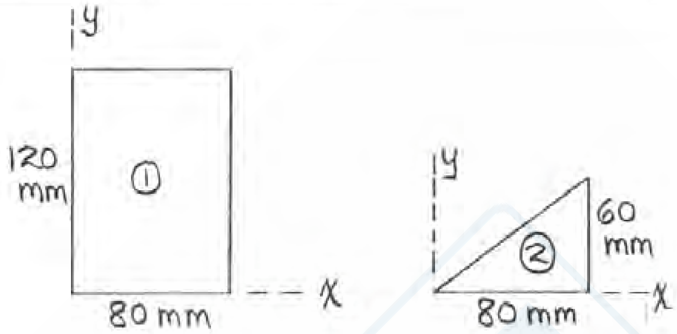
*A/87



① $I_x = \frac{1}{3}(80)(120)^3 = 46.1(10^6)\ mm^4$

$I_y = \frac{1}{3}(120)(80)^3 = 20.5\ (10^6)\ mm^4$

$I_{xy} = 80(120)(40)(60) = 23.0(10^6)\ mm^4$

② $I_x = -\frac{1}{12}(80)(60)^3 = -1.440\ (10^6)\ mm^4$

$I_y = -\frac{1}{4}(60)(80)^3 = -7.68\ (10^6)\ mm^4$

$I_{xy} = -\frac{b^2h^2}{8} = -\frac{80^2 60^2}{8} = -2.88(10^6)\ mm^4$

(See Prob. A/72)

So for the composite body:

$I_x = (46.1 - 1.440)10^6 = 44.6(10^6)\ mm^4$

$I_y = (20.5 - 7.68)10^6 = 12.80\ (10^6)\ mm^4$

$I_{xy} = (23.0 - 2.88)10^6 = 20.2\ (10^6)\ mm^4$

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$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

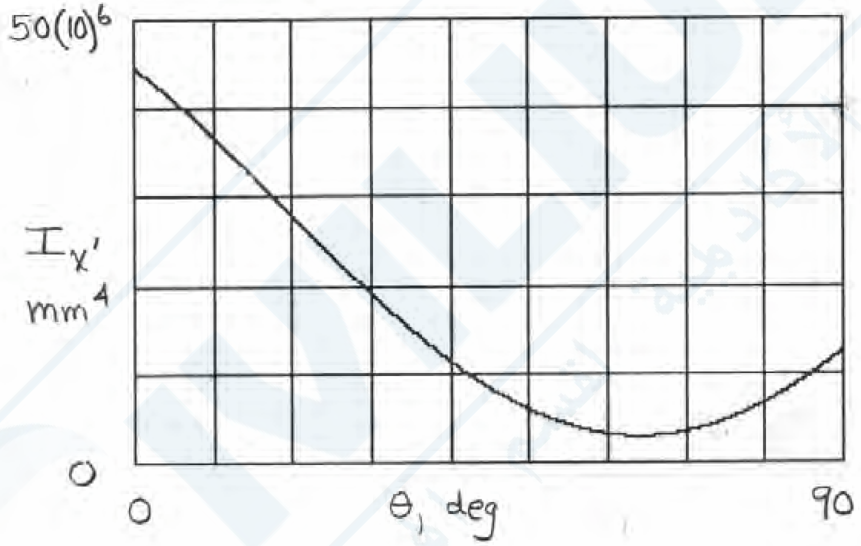
(See plot below)

$$I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= \left\{\frac{44.6 + 12.80}{2} - \frac{1}{2}\sqrt{(44.6 - 12.80)^2 + 4(20.2)^2}\right\} 10^6$$

$$= \underline{3.03(10^6)\text{ mm}^4}$$

$$\tan 2\theta_{cr} = \frac{2I_{xy}}{I_y - I_x} = \frac{2(20.2)}{12.80 - 44.6}, \quad \underline{\theta_{cr} = 64.1^\circ}$$



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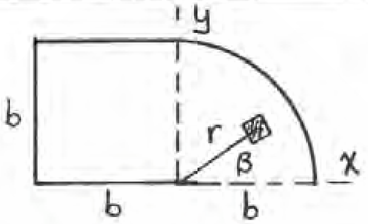
∗ A/88



$I_x = \frac{1}{3}b^4 + \frac{1}{16}\pi b^4 = 0.530\,b^4$

$I_y = \frac{1}{3}b^4 + \frac{1}{16}\pi b^4 = 0.530\,b^4$

Quarter circle : $I_{xy} = \int_0^{\pi/2}\int_0^{b} (r\cos\beta)(r\sin\beta)\, r\,dr\,d\beta$

$= \frac{r^4}{4}\Big|_0^b \times \left(-\frac{1}{4}\cos 2\beta\right)\Big|_0^{\pi/2} = \frac{b^4}{4}\left(\frac{1}{4}+\frac{1}{4}\right) = \frac{b^4}{8}$

Square : $I_{xy} = b^2\left(-\frac{b}{2}\right)\left(\frac{b}{2}\right) = -\frac{b^4}{4} = -0.25\,b^4$

Combined : $I_{xy} = \frac{b^4}{8} - \frac{b^4}{4} = -\frac{b^4}{8} = -0.125\,b^4$

Eq. A/9 : $I_{x'} = \frac{2(0.530\,b^4)}{2} + 0 - (-0.125b^4)\sin 2\theta$

$= (0.530 + 0.125\sin 2\theta)\,b^4$

For critical angle $\theta = \alpha$, Eq. A/10 gives

$\tan 2\alpha = \frac{2(0.530\,b^4)}{0}, \quad 2\alpha = \frac{\pi}{2}, \quad \alpha = \frac{\pi}{4}$

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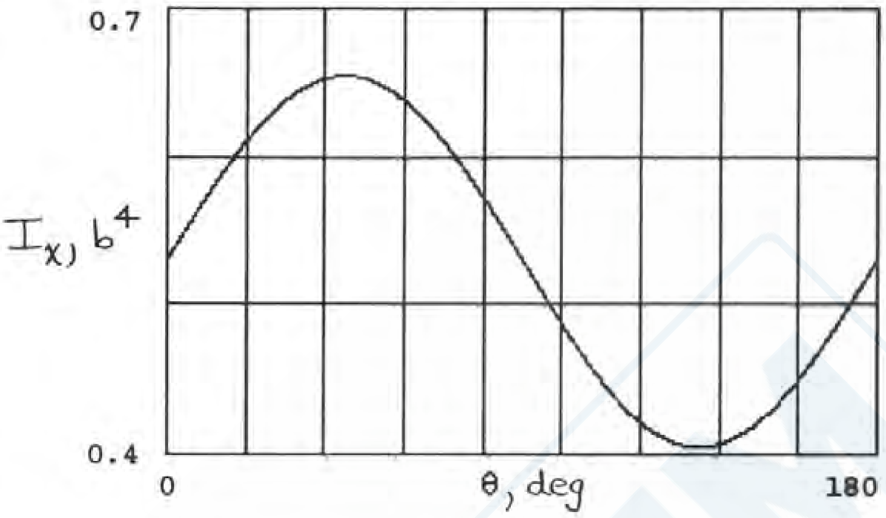

$I_{max} = 0.655\,b^4$ @ $\theta = 45°$

$I_{min} = 0.405\,b^4$ @ $\theta = 135°$

Eqs. A/11:

$$I_{max} = 0.530\,b^4 + \frac{1}{2}\sqrt{0^2 + 4(-0.125b^4)^2}$$
$$= 0.655\,b^4 \checkmark$$

$$I_{min} = 0.530\,b^4 - \frac{1}{2}\sqrt{0^2 + 4(-0.125\,b^4)^2}$$
$$= 0.405\,b^4 \checkmark$$

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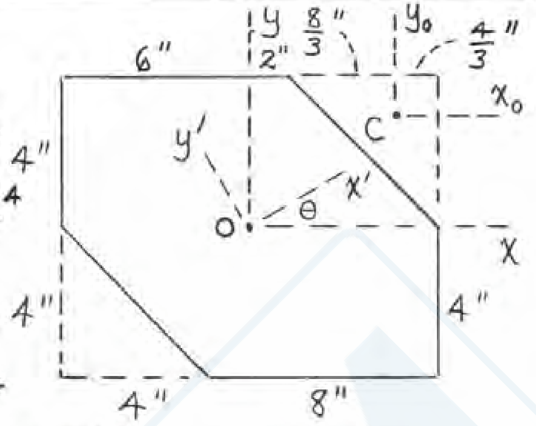
*A/89

Complete rectangle:

$I_x = \frac{1}{12}(8\cdot 12)8^2 = 512 \text{ in.}^4$

$I_y = \frac{1}{12}(8\cdot 12)12^2 = 1152 \text{ in.}^4$

$I_{xy} = 0$



Each dashed triangle:

$I_x = \frac{1}{4}(4)(4^3) = 64 \text{ in.}^4$

(see Sample Problem A/2)

$I_y = \frac{1}{36}4(4)^3 + \frac{1}{2}(4)(4)\left(2+\frac{8}{3}\right)^2 = 181.3 \text{ in.}^4$

(see Sample Problem A/2 and Eq. A/6)

$I_{xy} = -\frac{4^4}{72} + \frac{1}{2}4^2\left(2+\frac{8}{3}\right)\left(\frac{8}{3}\right) = 96 \text{ in.}^4$

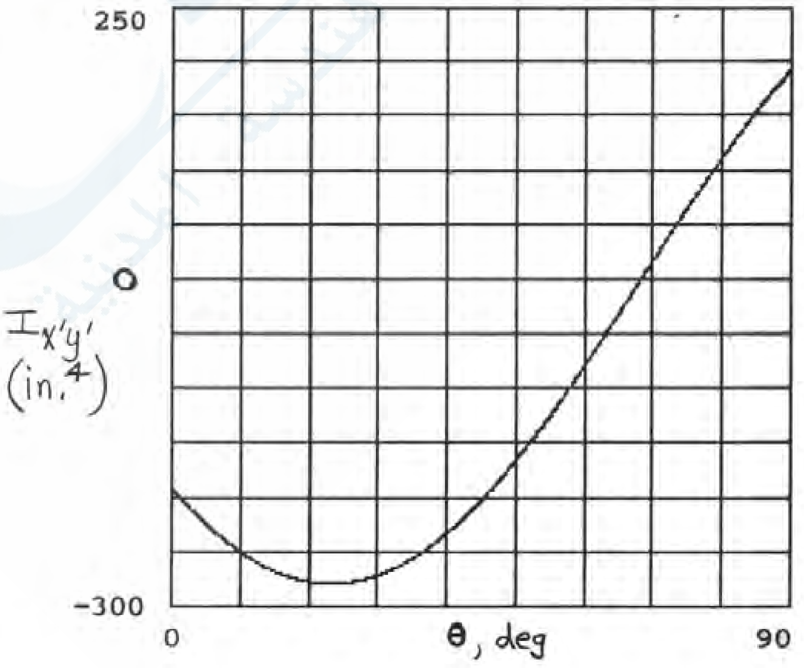
(see Prob. A/69 and Eq. A/8)

Totals: $I_x = 384 \text{ in.}^4$ $\quad I_{xy} = -2(96) = -192 \text{ in.}^4$

$I_y = 789 \text{ in.}^4$

Eq. A/9a: $I_{x'y'} = \frac{384-789}{2}\sin 2\theta - 192\cos 2\theta$

$= -203 \sin 2\theta - 192 \cos 2\theta$



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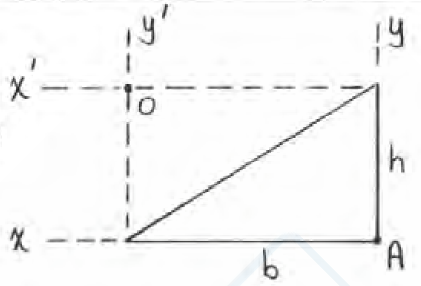
$I_{x'y'} = 0$ @ $\underline{\theta = 68.3°}$

A/9



From Sample Problem A/2,

$I_x = \frac{1}{12}bh^3$, $I_y = \frac{1}{12}b^3h$

$I_A = I_x + I_y$

$= \frac{1}{12}bh(h^2 + b^2)$

$k_A = \sqrt{I_A/A} = \sqrt{\frac{1}{12}bh(h^2+b^2)/\frac{bh}{2}} = \underline{\sqrt{(h^2+b^2)/6}}$

$I_{x'} = \frac{1}{4}bh^3$, $I_{y'} = \frac{1}{4}b^3h$

$I_o = I_{x'} + I_{y'} = \frac{1}{4}bh(h^2+b^2)$

$k_o = \sqrt{I_o/A} = \sqrt{\frac{1}{4}bh(h^2+b^2)/\frac{bh}{2}} = \underline{\sqrt{(h^2+b^2)/2}}$

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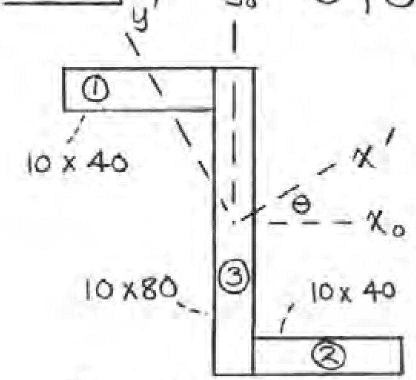
***A/90**



(Dim. in mm)

①&②: $I_{x_o y_o} = \bar{I}_{xy} + d_x d_y A$

$= 0 + (35)(-25)(10 \times 40)$

$= -0.350(10^6)\ \text{mm}^4$

③: $I_{x_o y_o} = 0$

①&②: $I_{x_o} = \frac{1}{12}(40)(10^3)$

$+ (10 \times 40)\,35^2 = 0.493(10^6)\ \text{mm}^4$

$I_{y_o} = \frac{1}{12}(10)(40)^3 + (10 \times 40)\,25^2 = 0.303(10^6)\ \text{mm}^4$

③: $I_{x_o} = \frac{1}{12}(10)(80)^3 = 0.427(10^6)\ \text{mm}^4$

$I_{y_o} = \frac{1}{12}(80)(10)^3 = 0.00667(10^6)\ \text{mm}^4$

Totals:

$$\begin{cases} I_{x_o} = 1.413(10^6)\ \text{mm}^4 \\ I_{y_o} = 0.613(10^6)\ \text{mm}^4 \\ I_{x_o y_o} = -0.700(10^6)\ \text{mm}^4 \end{cases}$$

From Eq. A/9:

$$I_{x'} = \left[\frac{1.413 + 0.613}{2} + \frac{1.413 - 0.613}{2}\cos 2\theta + 0.700 \sin 2\theta\right]10^6$$

$$= \left[1.013 + 0.4\cos 2\theta + 0.7\sin 2\theta\right]10^6$$

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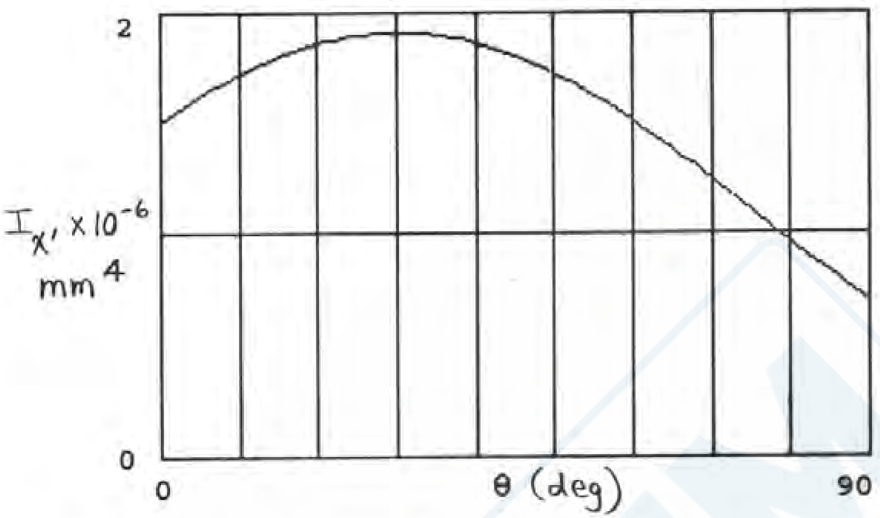

Eq. (A/11): $I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$= \frac{1.413 + 0.613}{2} 10^6 + \frac{1}{2}\sqrt{(1.413 - 0.613)^2(10^6)^2 + 4(-0.7 \times 10^6)^2}$

$= \underline{1.820(10^6)\ mm^4}$

Eq. (A/10): $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$

$= \frac{2(-0.7)}{0.613 - 1.413}$

$\Rightarrow \alpha = \underline{30.1°}, 120.1°$

(Values from Eqs. A/10 & A/11 agree with plot.)

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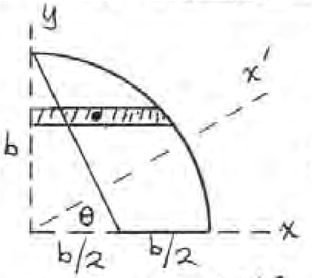
*A/91 Quarter circle $I_x = I_y = \frac{1}{16}\pi r^4$



For horizontal strip:

$dI_{xy} = 0 + (x\,dy)\,y\,\frac{x}{2} = \frac{b^2y - y^3}{2}dy$

$I_{xy} = \frac{1}{2}\int_0^b (b^2y - y^3)\,dy = \frac{b^4}{8}$

Triangle $I_x = \frac{1}{12}\frac{b}{2}b^3 = \frac{b^4}{24}$

$I_y = \frac{1}{12}b\left(\frac{b}{2}\right)^3 = \frac{b^4}{96}$, $I_{xy} = \frac{1}{24}\left(\frac{b}{2}\right)^2 b^2 = \frac{b^4}{96}$

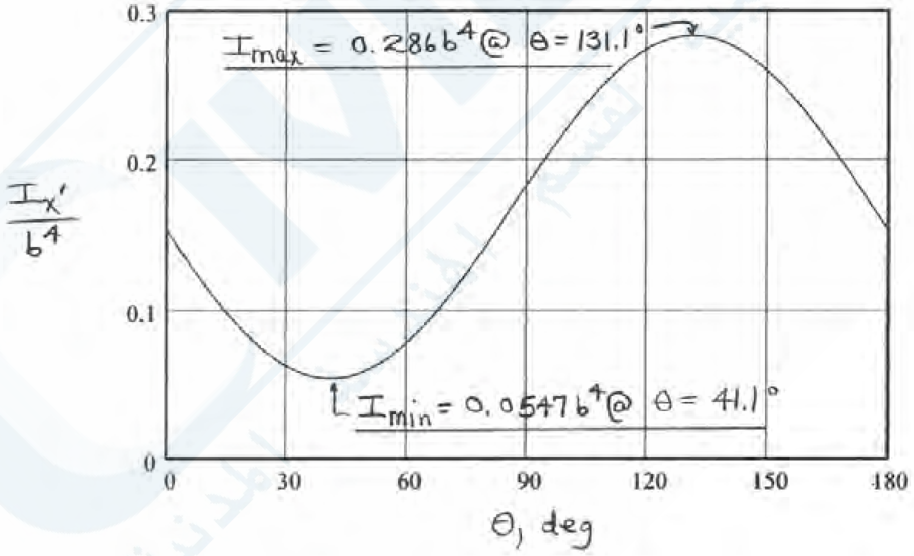
(Prob. A/71)

Composite
$$\begin{cases} I_x = \left(\frac{\pi}{16} - \frac{1}{24}\right)b^4 = 0.1547\,b^4 \\ I_y = \left(\frac{\pi}{16} - \frac{1}{96}\right)b^4 = 0.1859\,b^4 \\ I_{xy} = \left(\frac{1}{8} - \frac{1}{96}\right)b^4 = 0.1146\,b^4 \end{cases}$$

From Eq. A/9:

$$I_{x'} = \left[\frac{0.1547 + 0.1859}{2} + \frac{0.1547 - 0.1859}{2}\cos 2\theta - 0.1146\sin 2\theta\right]b^4$$

$$= \left[0.1703 - 0.01562\cos 2\theta - 0.1146\sin 2\theta\right]b^4$$



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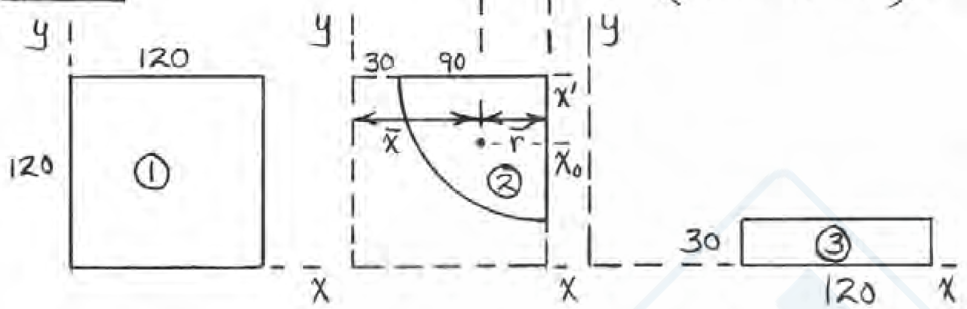
*A/92 (Dim. in mm)



① $I_x = I_y = \frac{1}{3}\,120^4 = 69.1(10^6)\text{ mm}^4$, $A = 120^2 = 1.44(10^4)\text{mm}^2$

$I_{xy} = 120^2(60)(60) = 51.8\text{ mm}^4$

② $\bar{r} = \frac{4r}{3\pi} = \frac{4(90)}{3\pi} = 38.2\text{mm}$, $\bar{x} = 120 - \bar{r} = 81.8\text{ mm}$

$A = \frac{1}{4}\pi(90^2) = 0.636(10^4)\text{ mm}^2$

$I_{x'} = I_{y'} = \frac{1}{4}\left(\frac{1}{4}\pi \times 90^4\right) = 12.88(10^6)\text{ mm}^4$

$I_x = I_y = I_{y_0} + A\bar{x}^2 = I_{y'} - A\bar{r}^2 + A\bar{x}^2$

$= [12.88 + 0.00636(81.8^2 - 38.2^2)]10^6 = 46.2(10^6)\text{mm}^4$

$I_{xy} = I_{x_0y_0} + A\bar{x}\bar{y} = I_{x'y'} - A\bar{r}\bar{r} + A\bar{x}\bar{y}$

where $I_{x'y'} = \frac{r^4}{8} = \frac{90^4}{8} = 8.20(10^6)\text{ mm}^4$ {See Sol. to Prob. A/65

$\therefore I_{xy} = [8.20 + 0.00636(81.8^2 - 38.2^2)]10^6 = 41.5(10^6)\text{ mm}^4$

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③ $I_x = \frac{1}{3}(120)(30)^3 = 1.08(10^6)\ mm^4$

$I_y = \frac{1}{12}(30)(120)^3 + 30(120)(180^2) = 121.0(10^6)\ mm^4$

$I_{xy} = 30(120)(180)(15) = 9.72(10^6)\ mm^4$

Combined:
$$\begin{cases} I_x = (69.1 - 46.2 + 1.08)10^6 = 24.0(10^6)\ mm^4 \\ I_y = (69.1 - 46.2 + 121.0)10^6 = 143.9(10^6)\ mm^4 \\ I_{xy} = (51.8 - 41.5 + 9.72)10^6 = 20.1(10^6)\ mm^4 \end{cases}$$

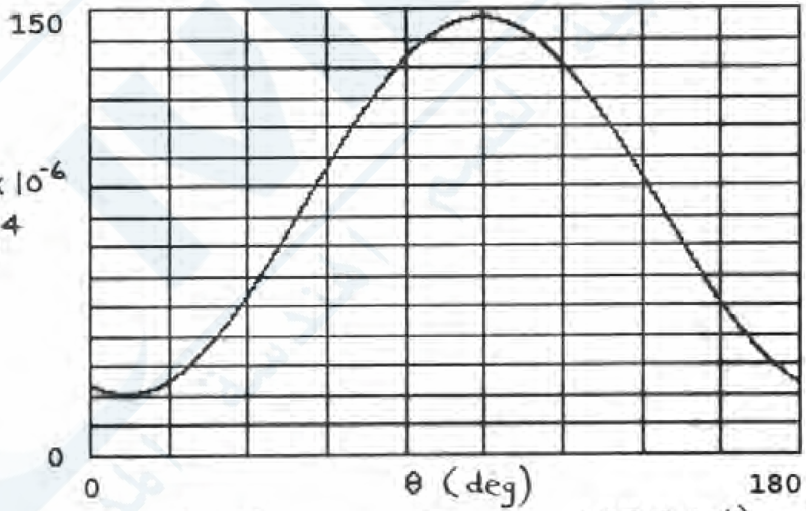
A/9: $I_{x'} = \left\{\frac{24.0 + 143.9}{2} + \frac{24.0 - 143.9}{2}\cos 2\theta - 20.1 \sin 2\theta\right\} \times 10^6$

$I_{x'}(10^{-6}) = 84.0 - 59.9 \cos 2\theta - 20.1 \sin 2\theta \quad mm^4$

A/10: $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{2(20.1)}{143.9 - 24.0} = 0.335$

$2\alpha = 18.51°, \quad \alpha = 9.26°$ (minimum I)

$\alpha = 9.26 + 90 = 99.3°$ (maximum I)



$I_{min} = 20.8(10^6)\ mm^4$ @ $\theta = 9.26°$ $\quad I_{max} = 147.2(10^6)\ mm^4$ @ $\theta = 99.3°$

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Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Appendix A: Area Moments of Inertia.

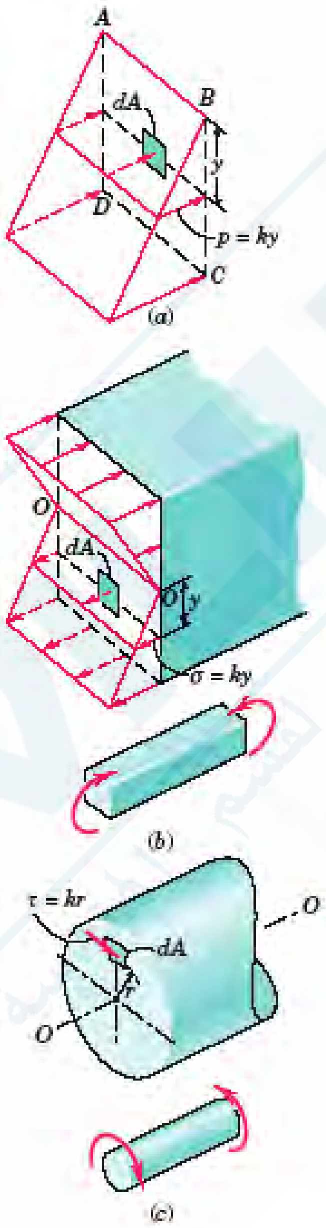


Figure A/1

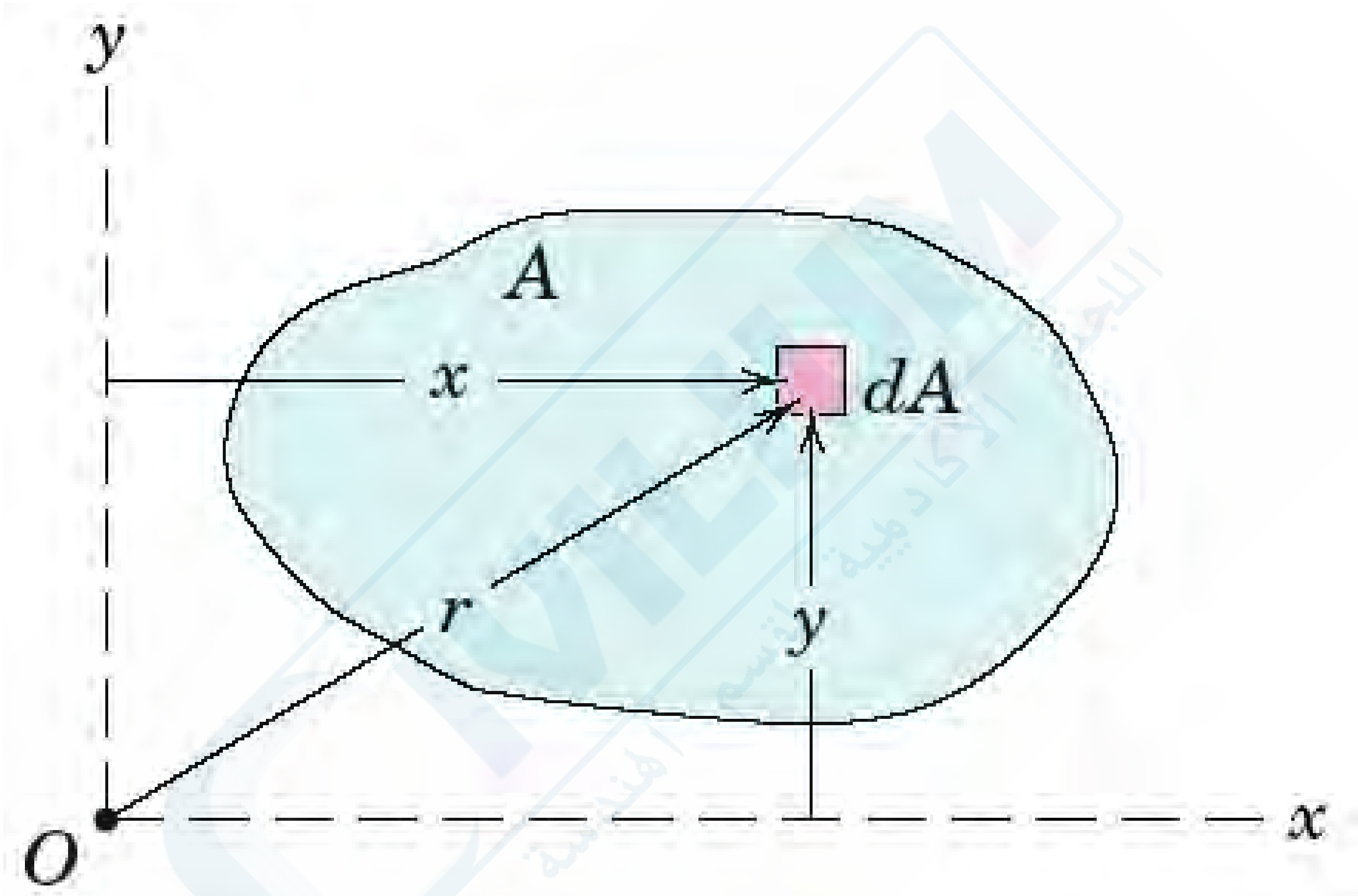


Figure A/2

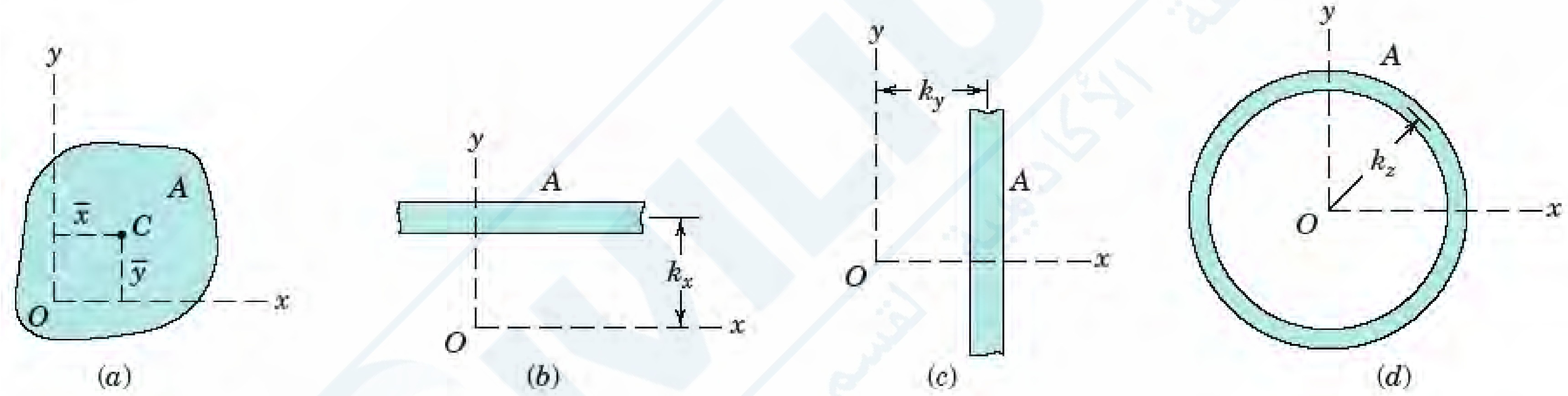


Figure A/3

A_03

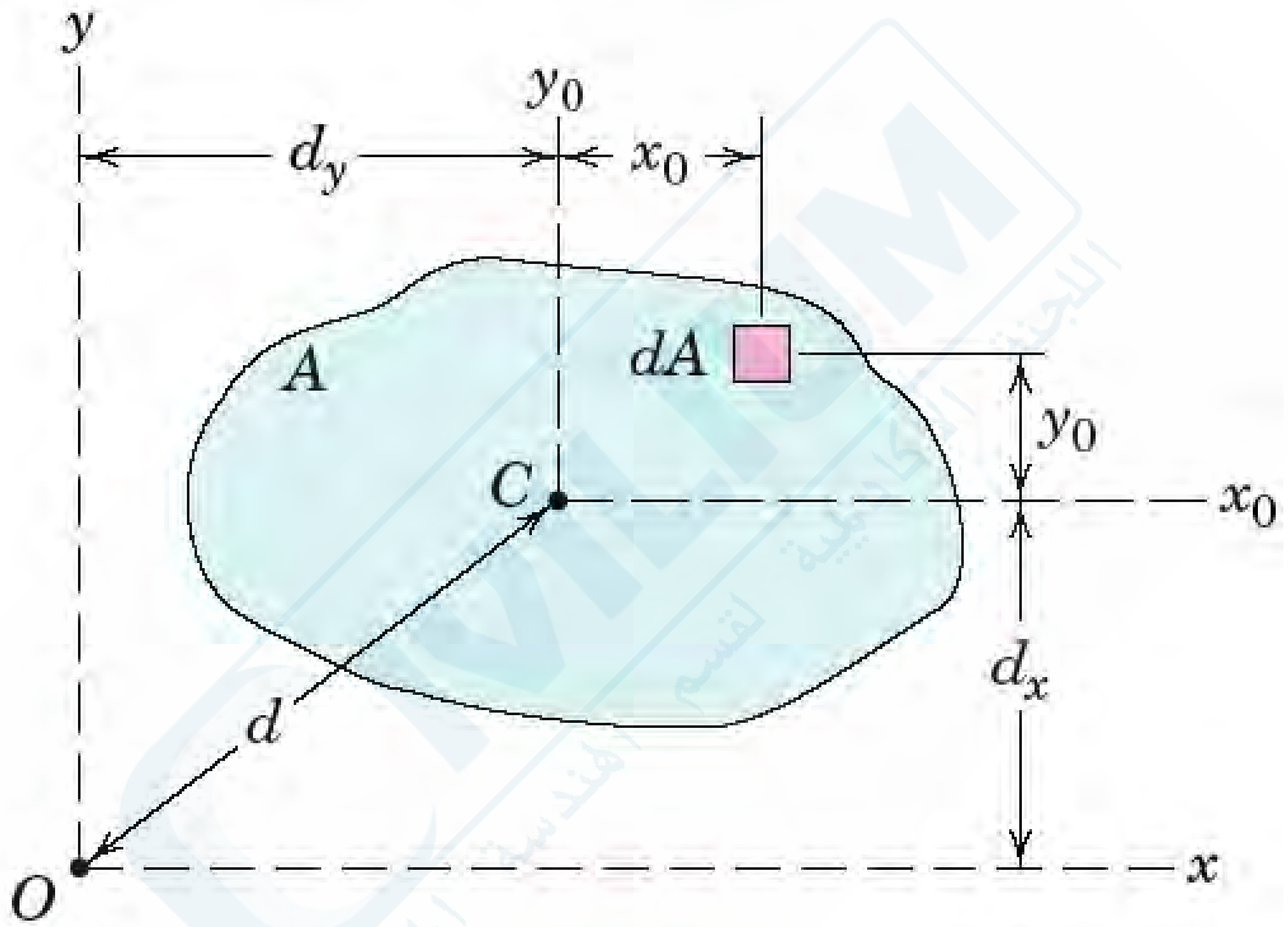


Figure A/4

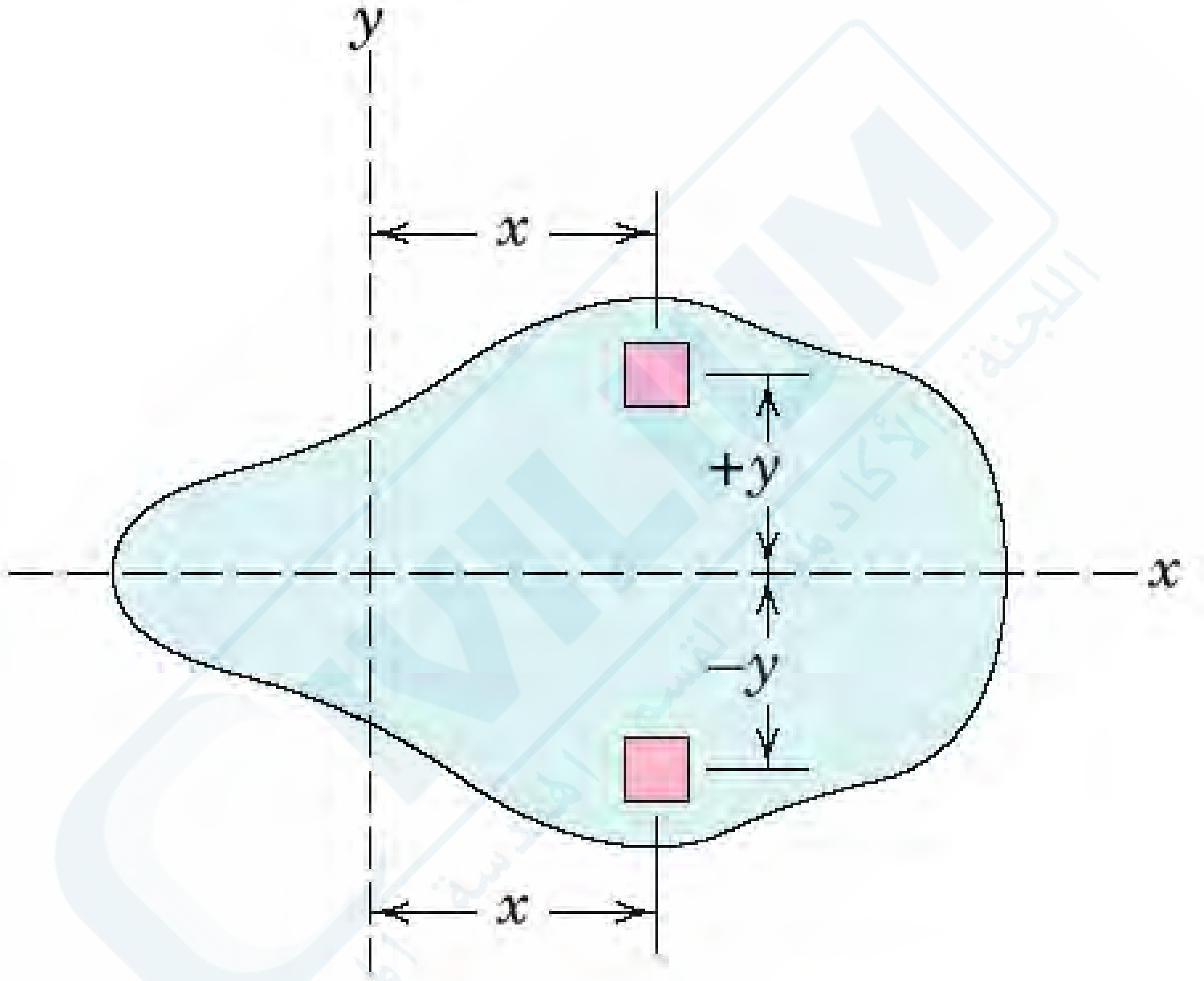


Figure A/5

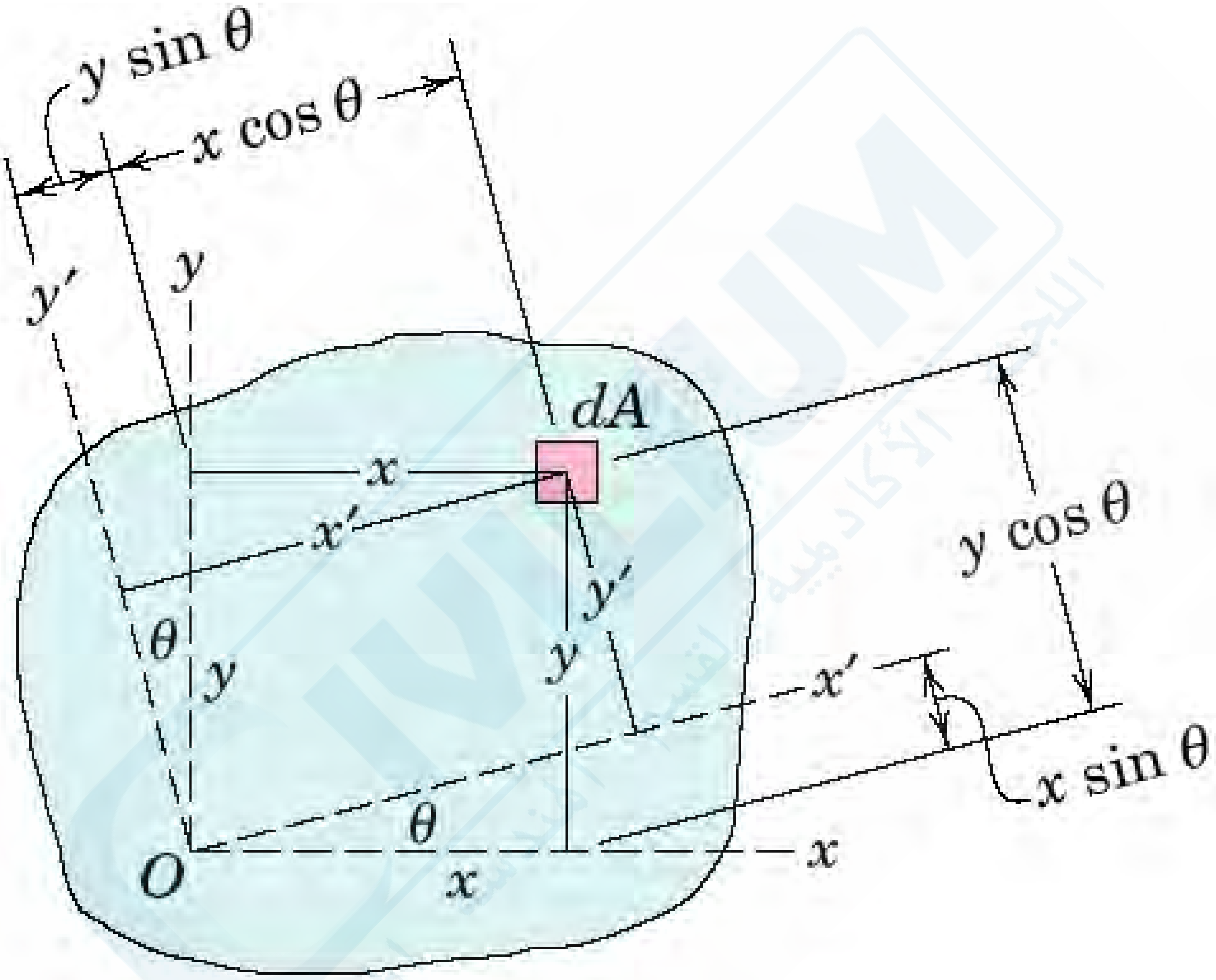
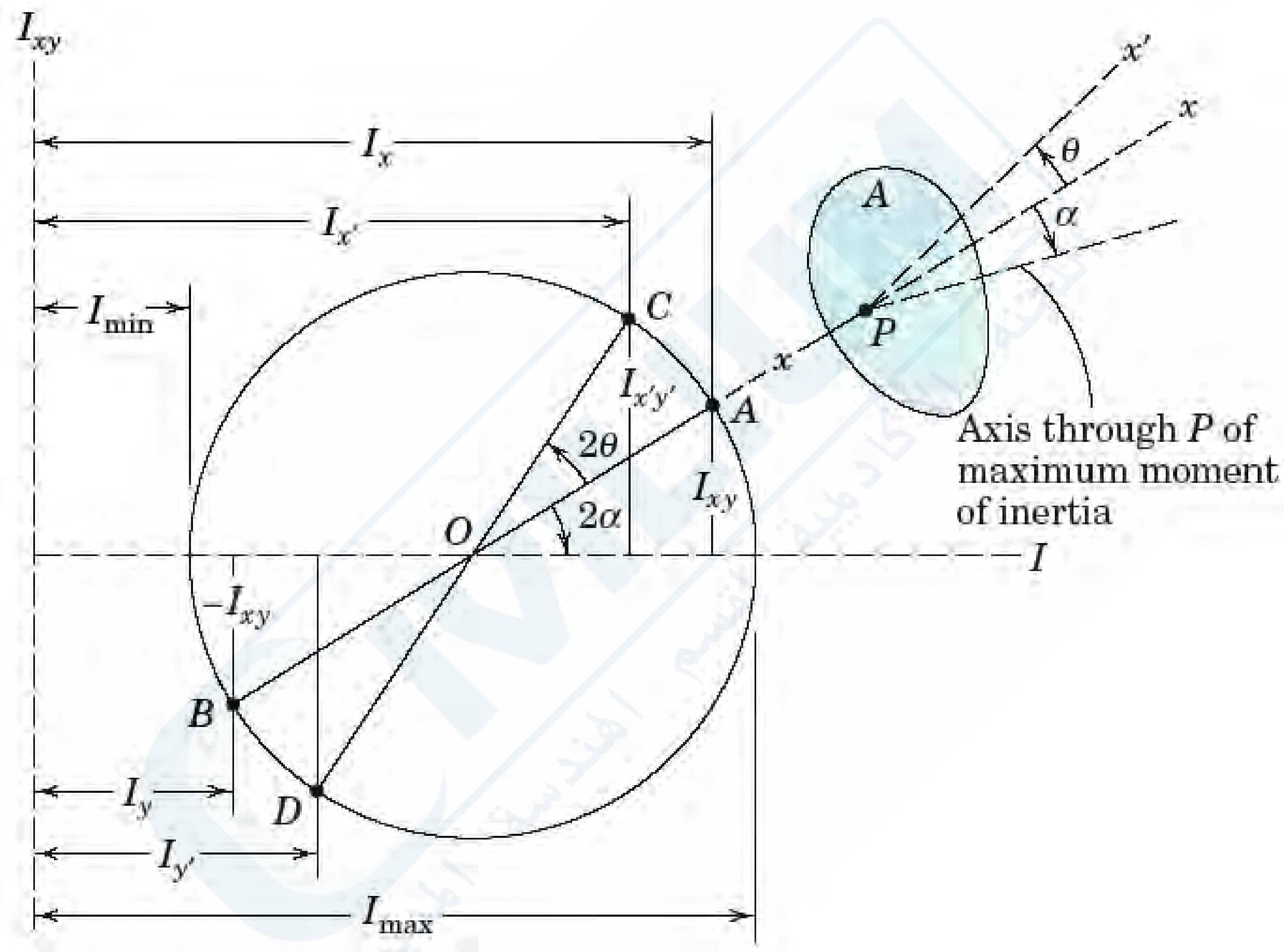
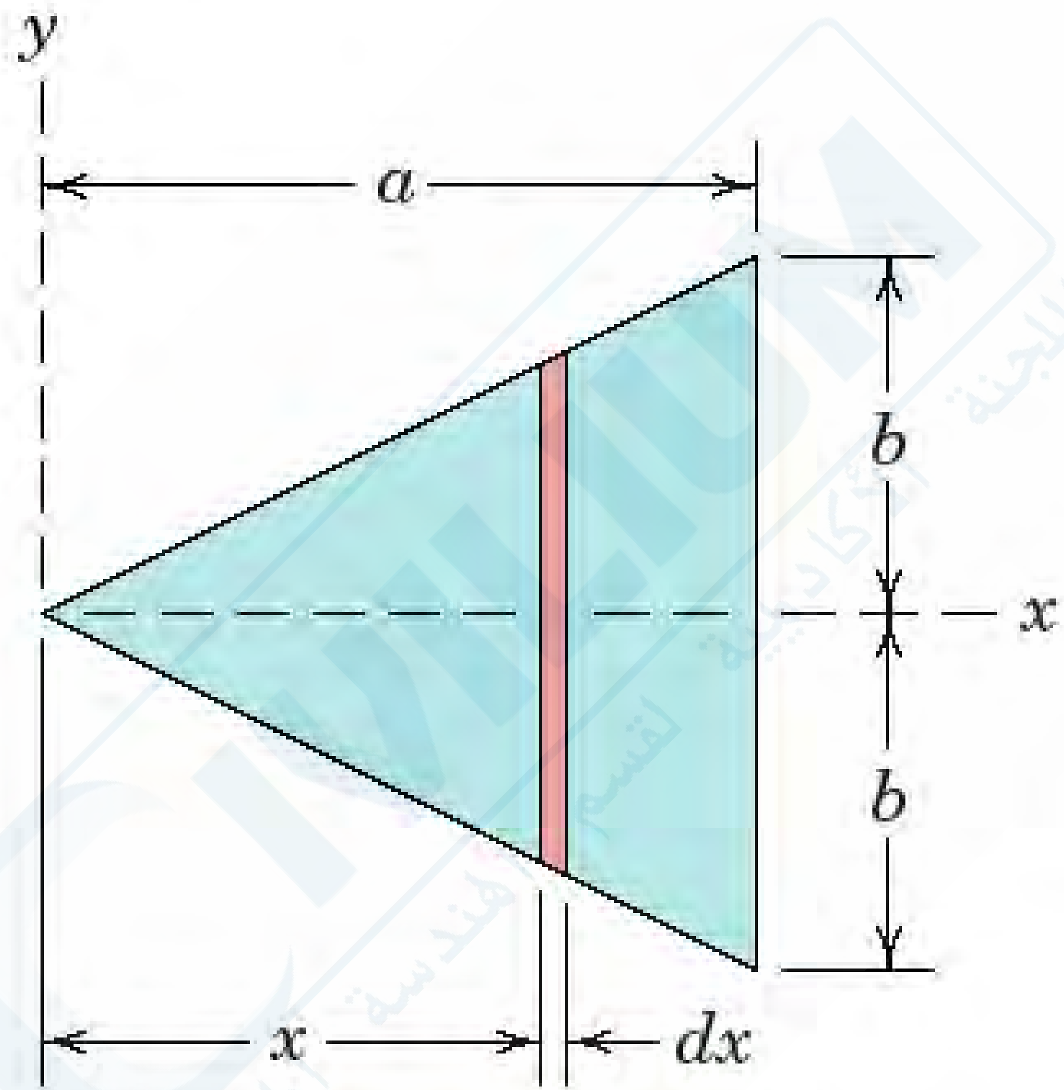


Figure A/6

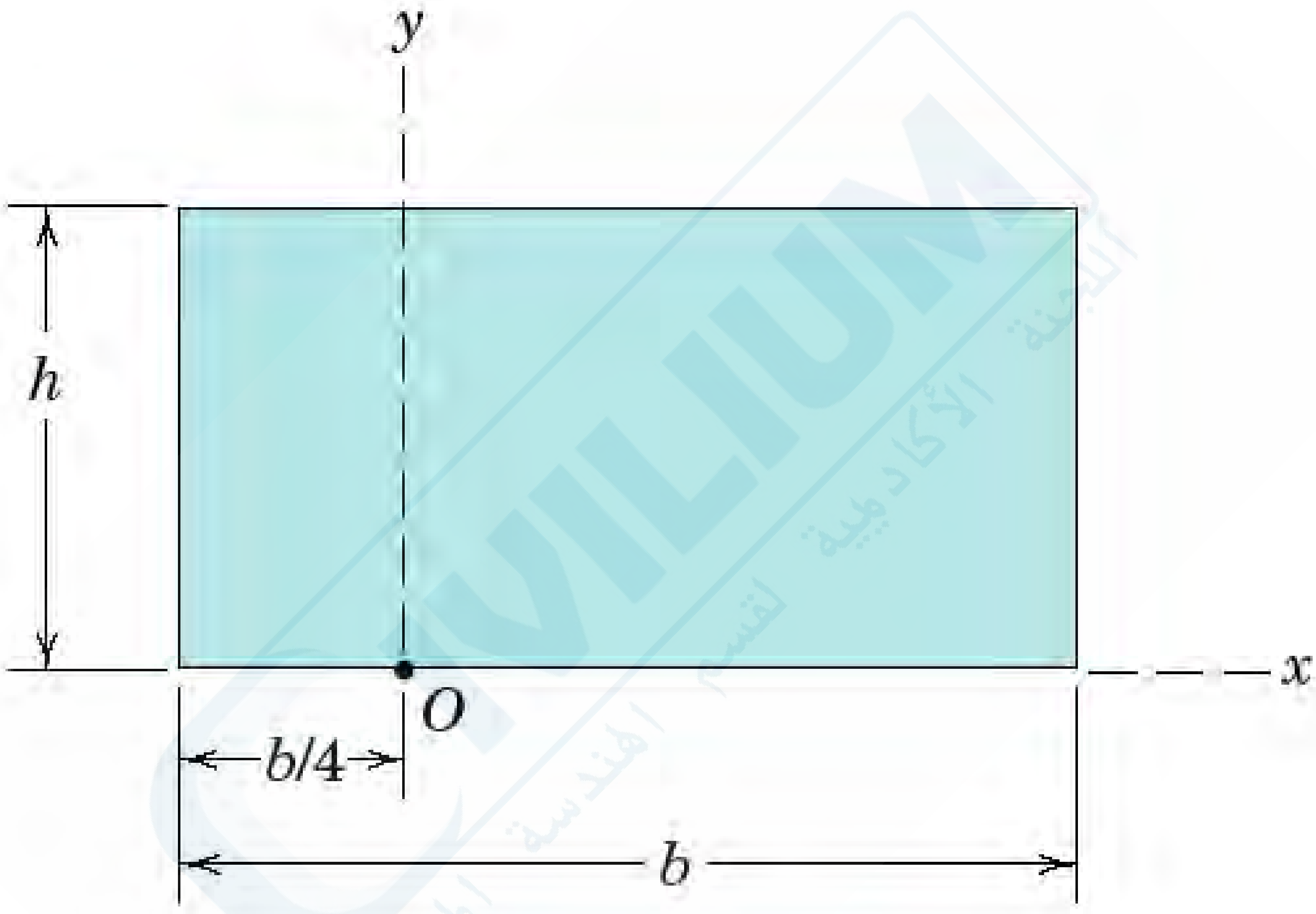


I_{xy}
I_x
$I_{x'}$
I_{min}
C
$I_{x'y'}$
A
2θ
2α
I_{xy}
O
I
$-I_{xy}$
B
D
I_y
$I_{y'}$
I_{max}
x'
x
θ
α
A
P
x
Axis through P of maximum moment of inertia

Figure A/7

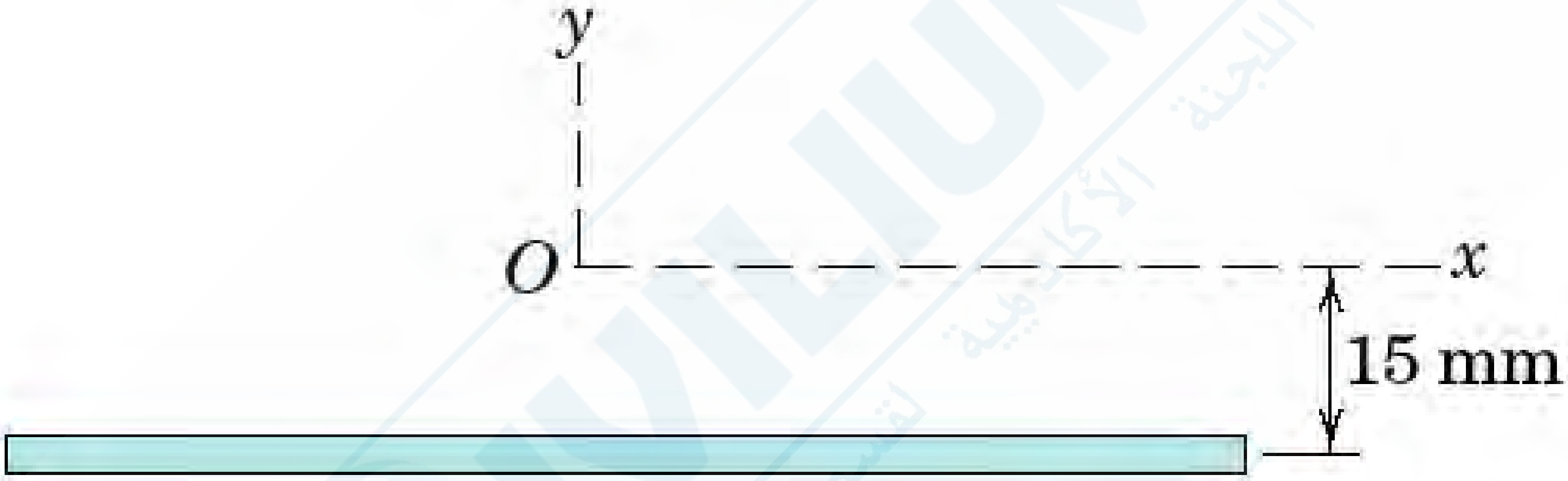


Problem A/1



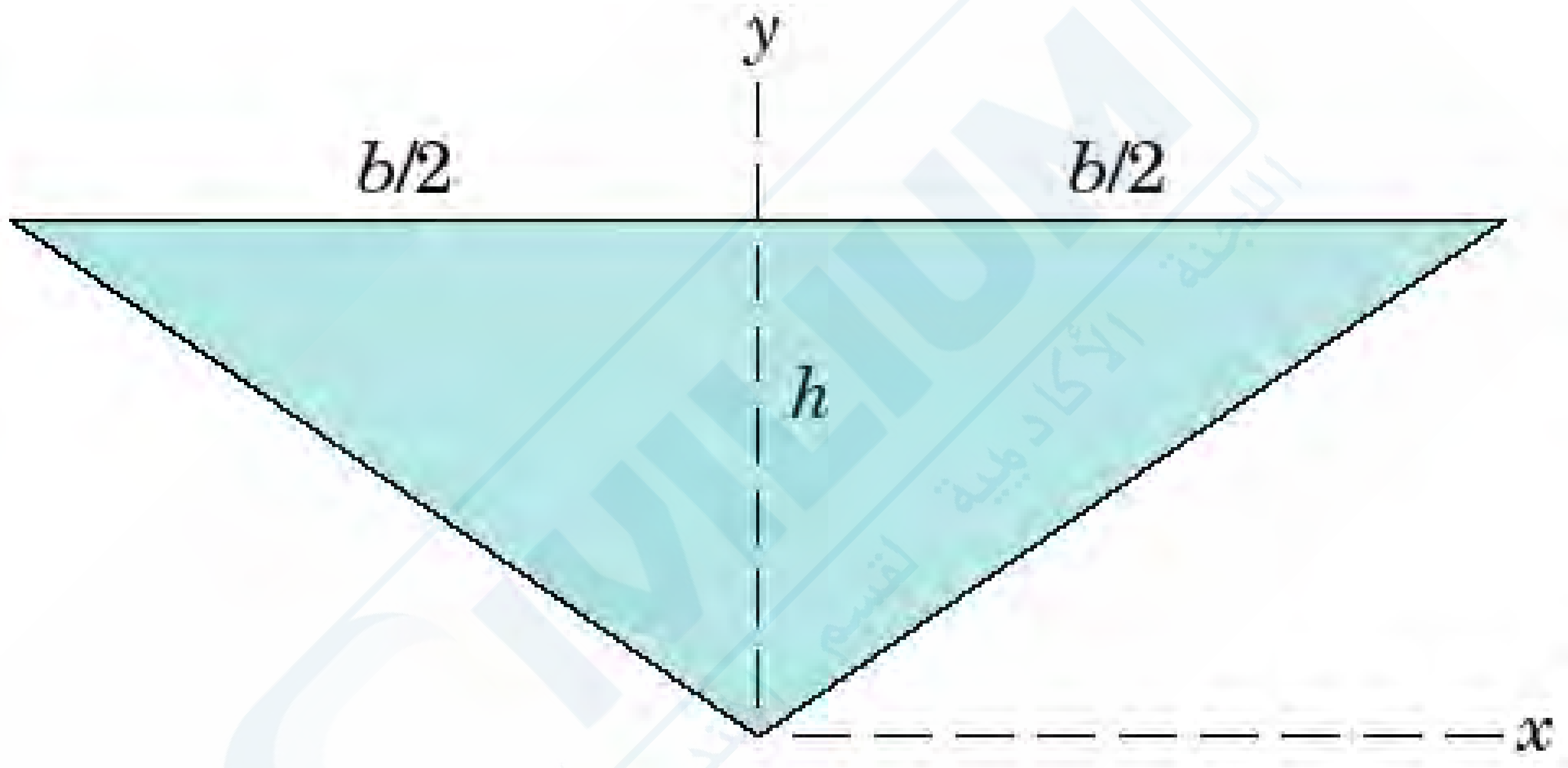
Problem A/2

P A 02



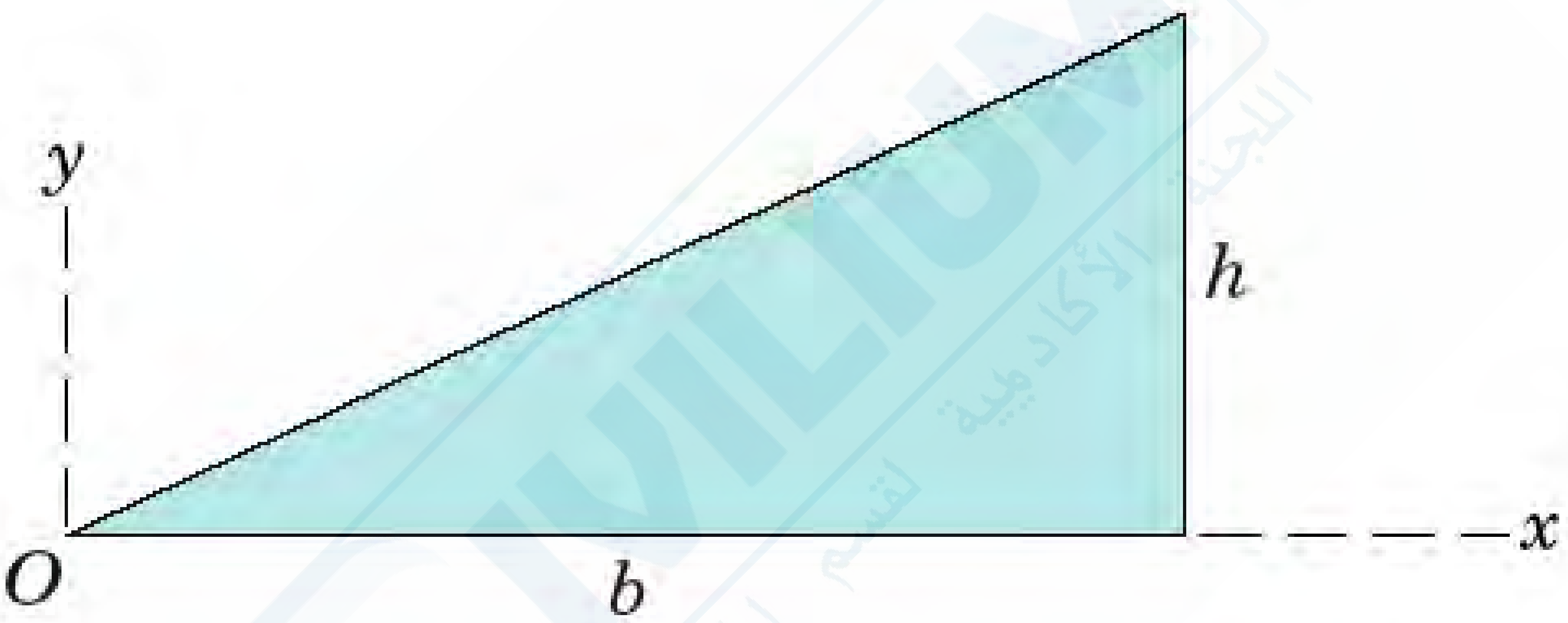
Problem A/3

P_A_03



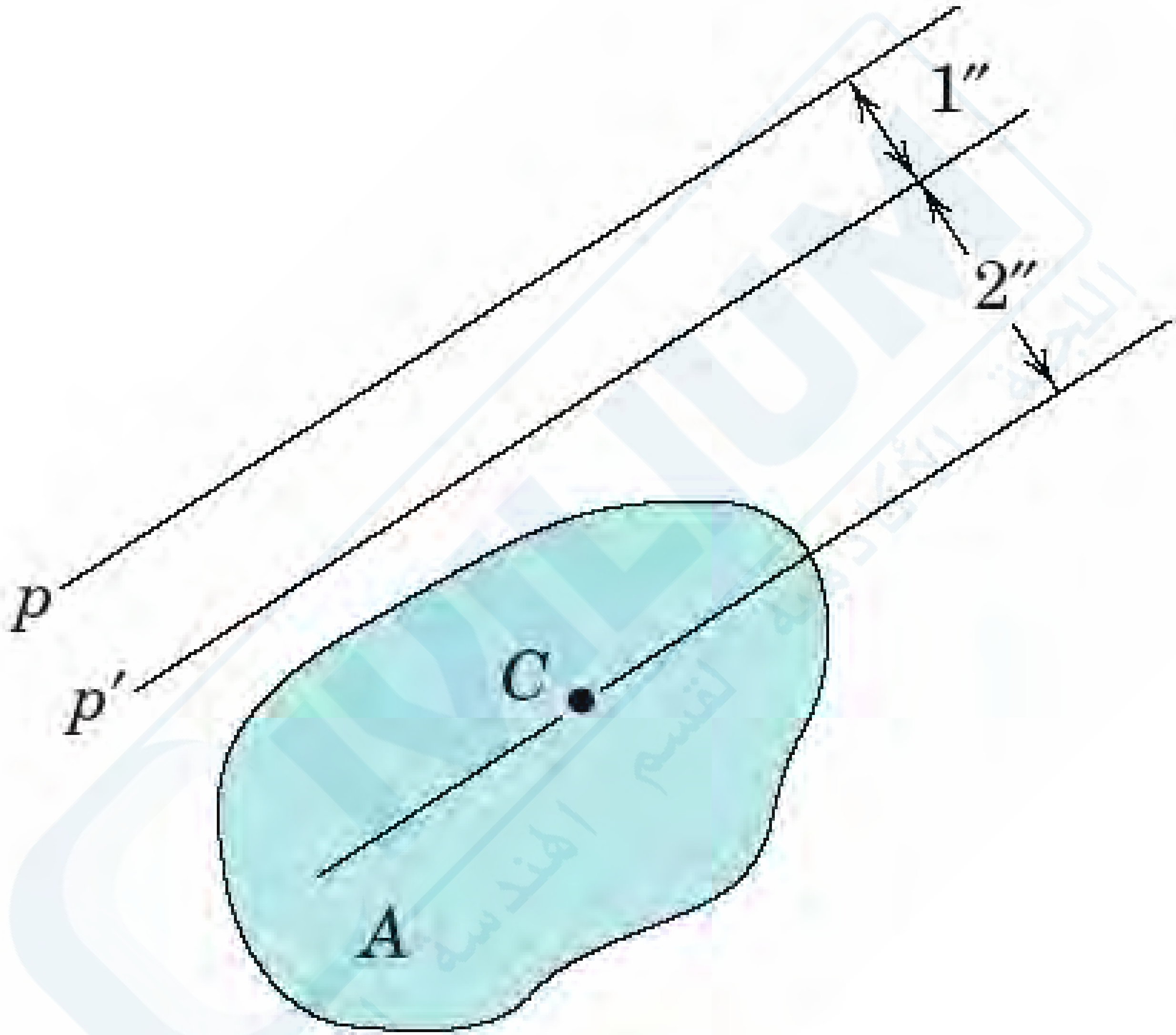
Problem A/4

P_A_04

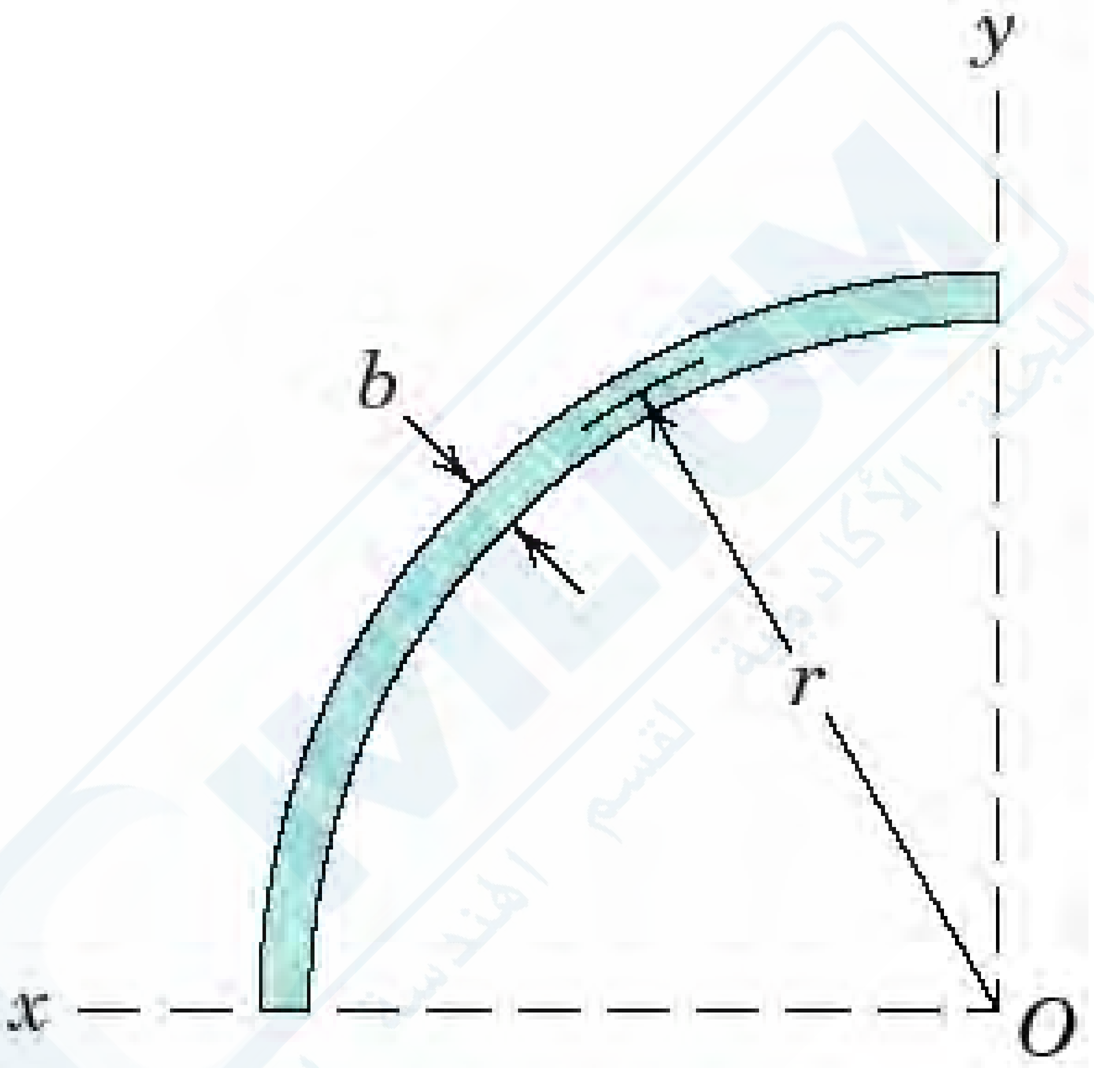


Problem A/5

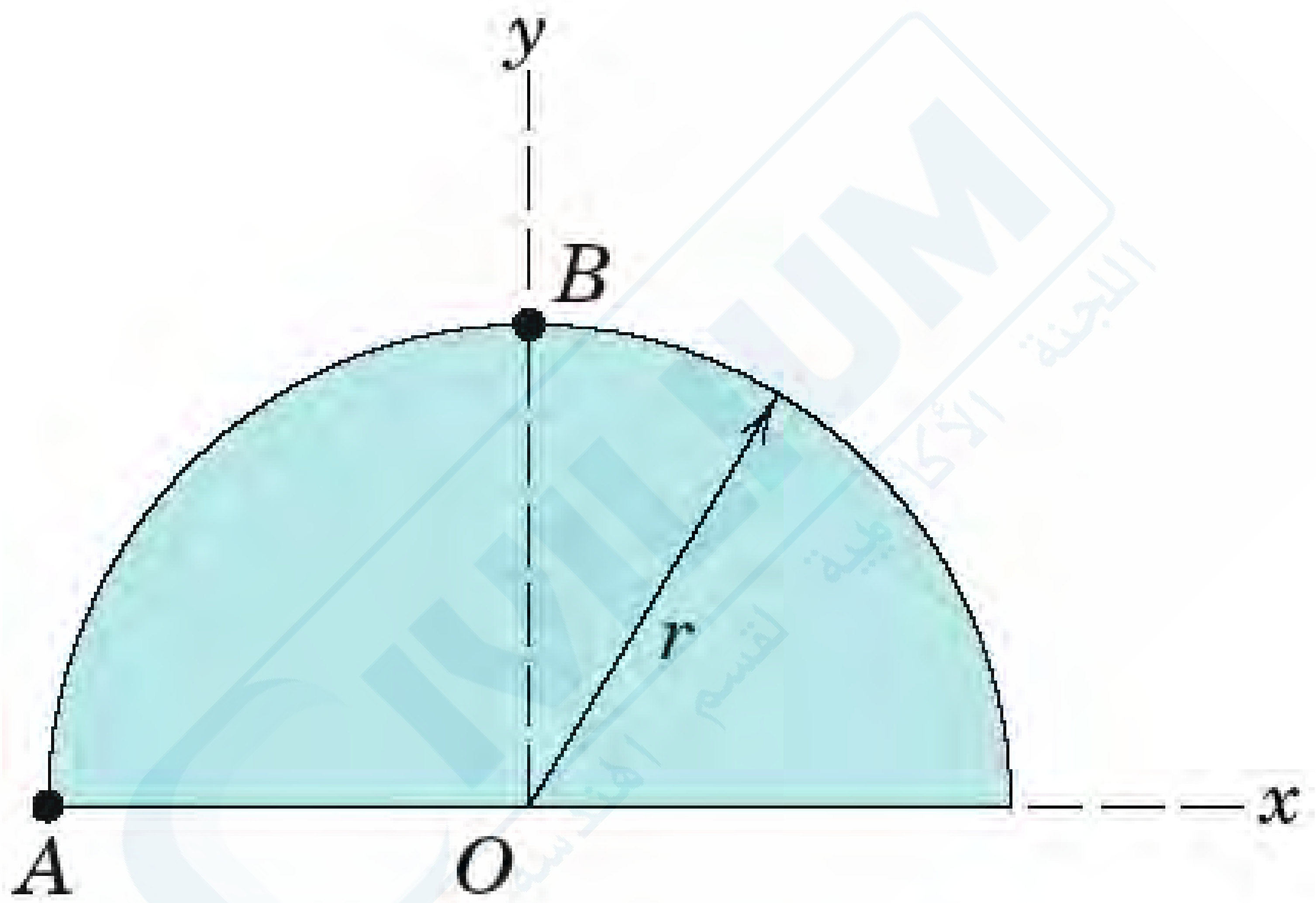
P_A_05



Problem A/6

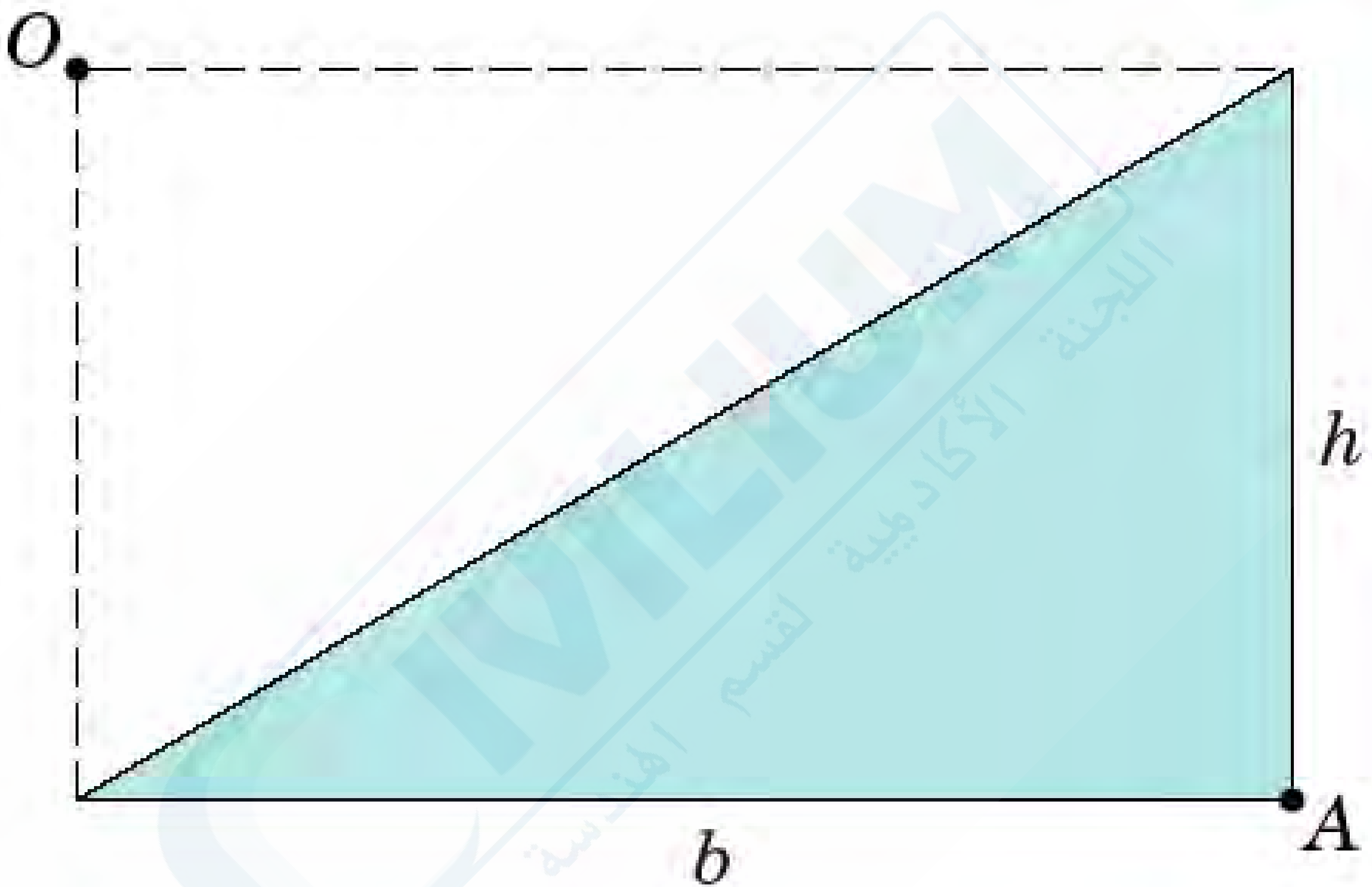


Problem A/7



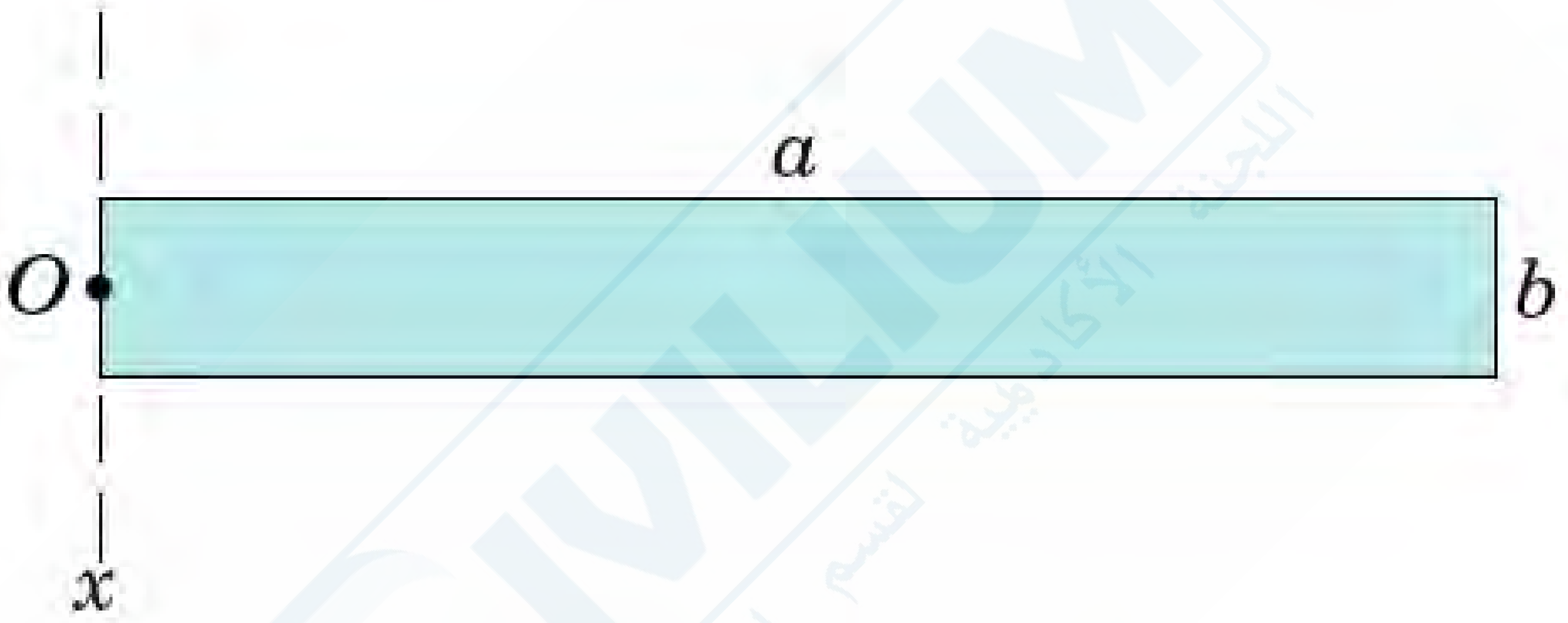
Problem A/8

P A 08



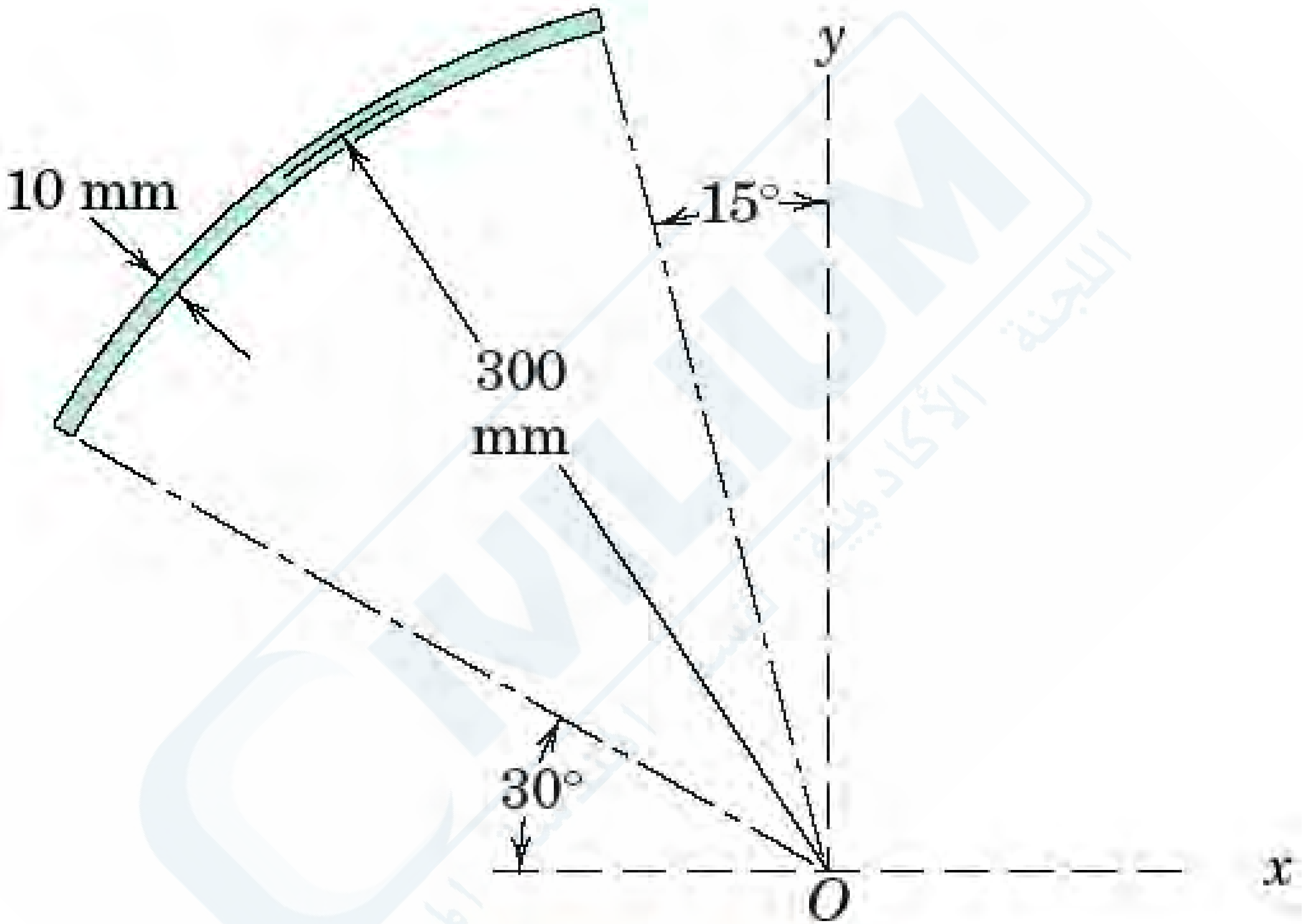
Problem A/9

P A 09



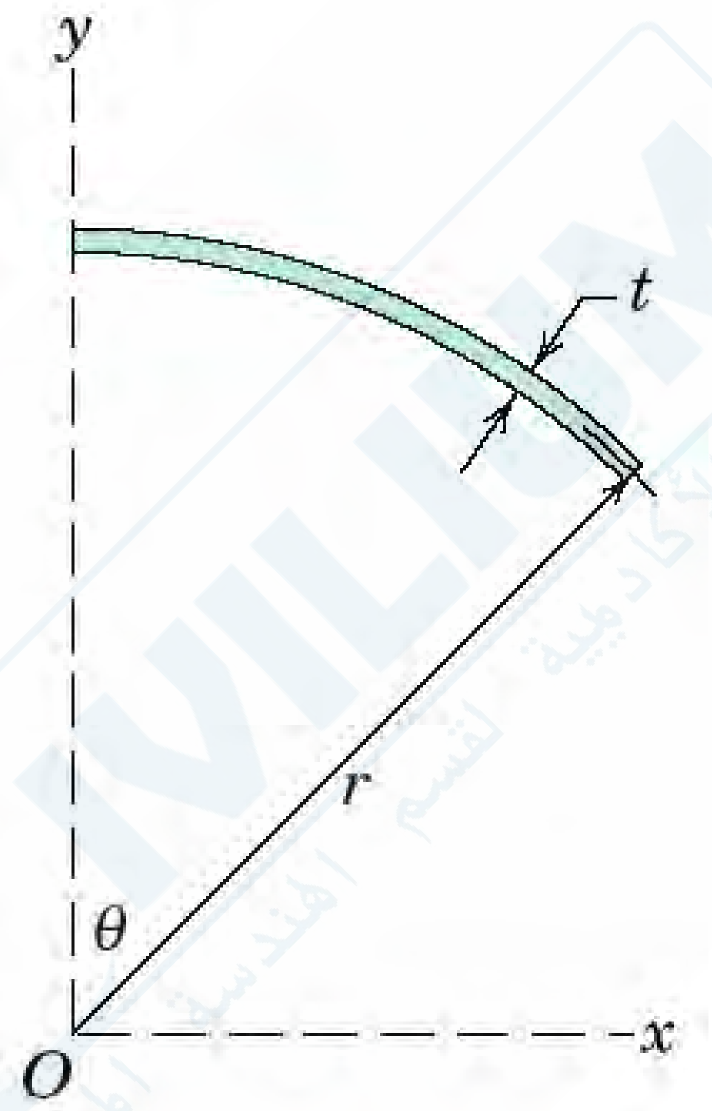
Problem A/10

P_A_10



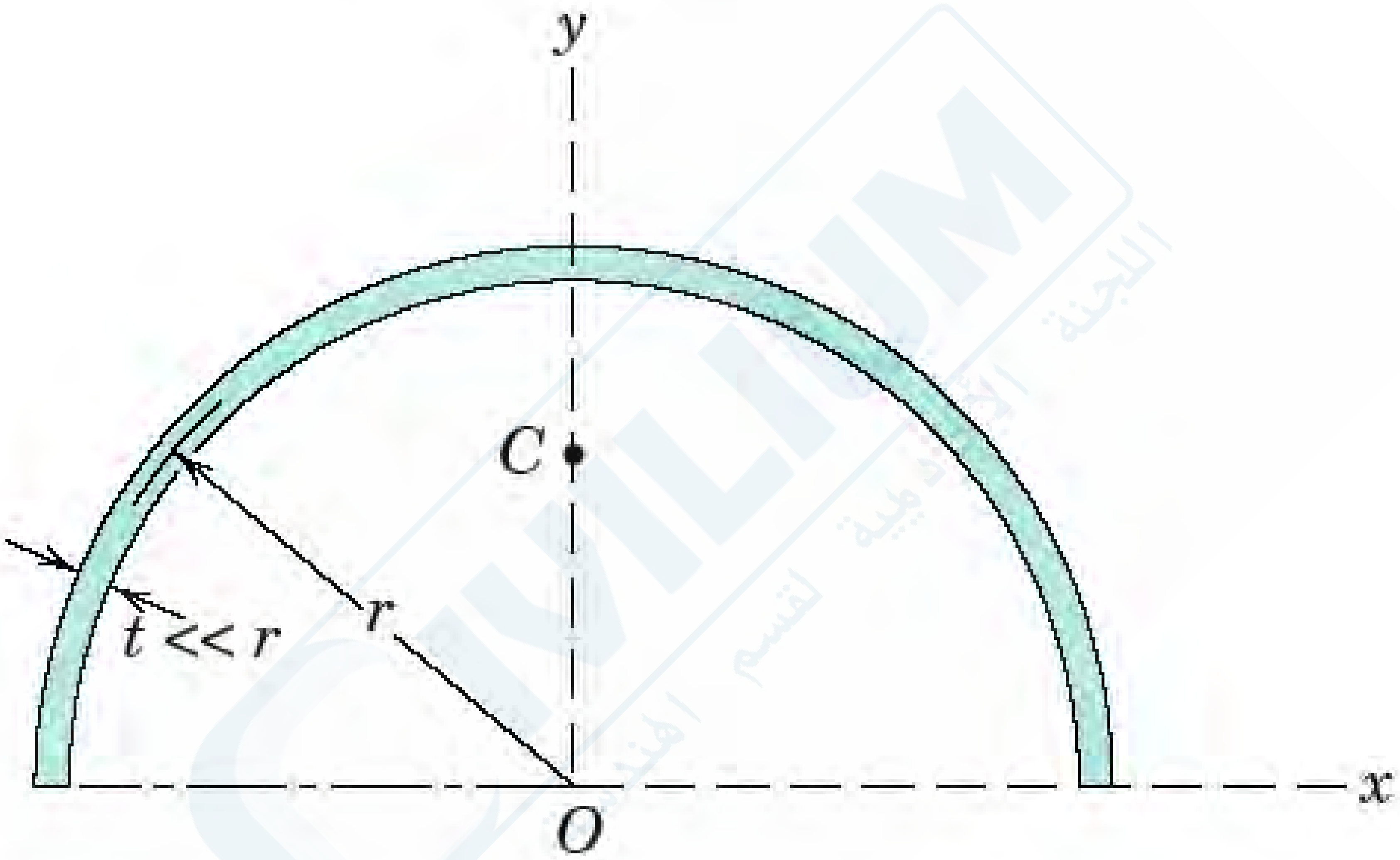
Problem A/11

P A 11



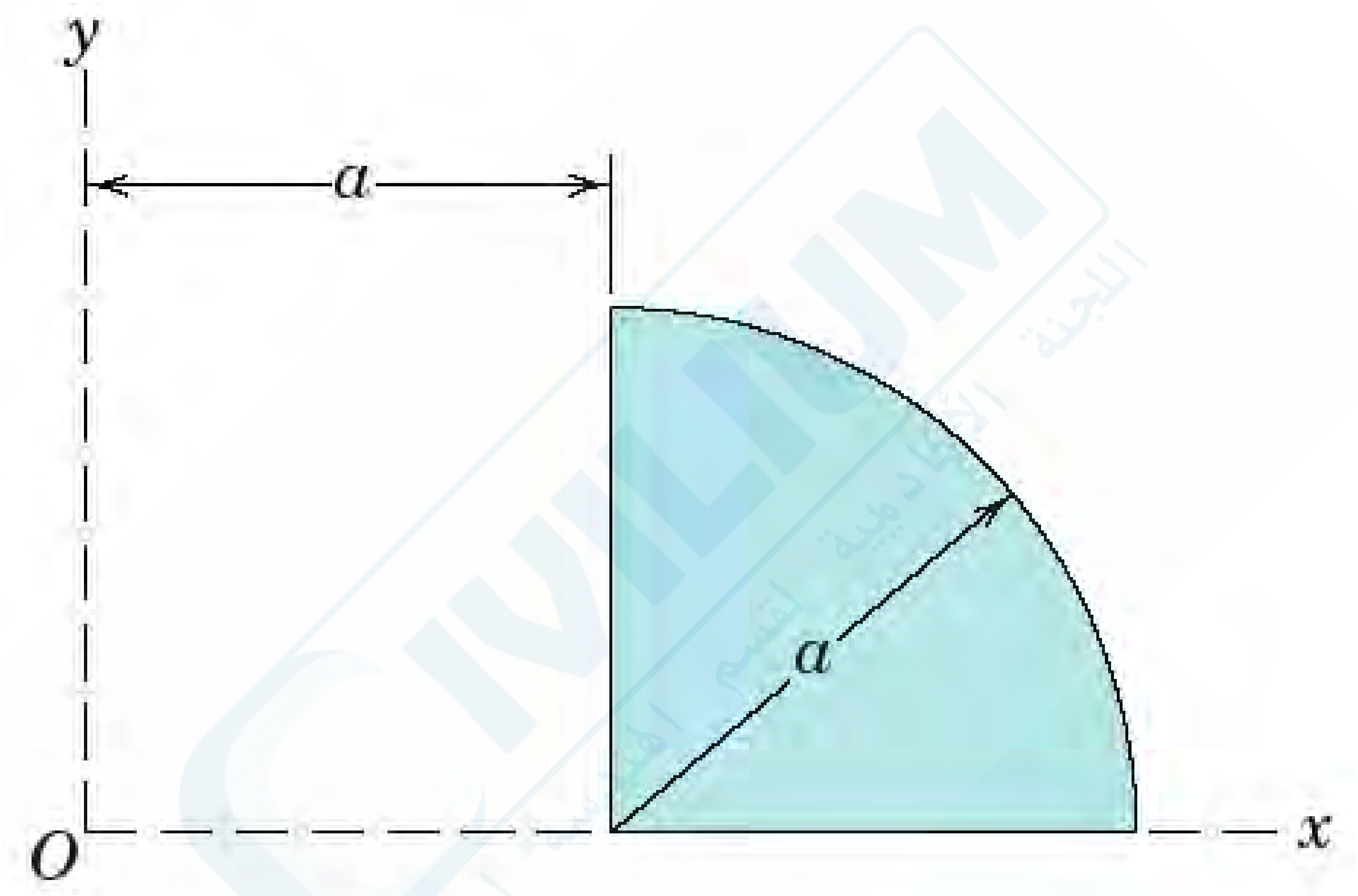
Problem A/12

P A 12



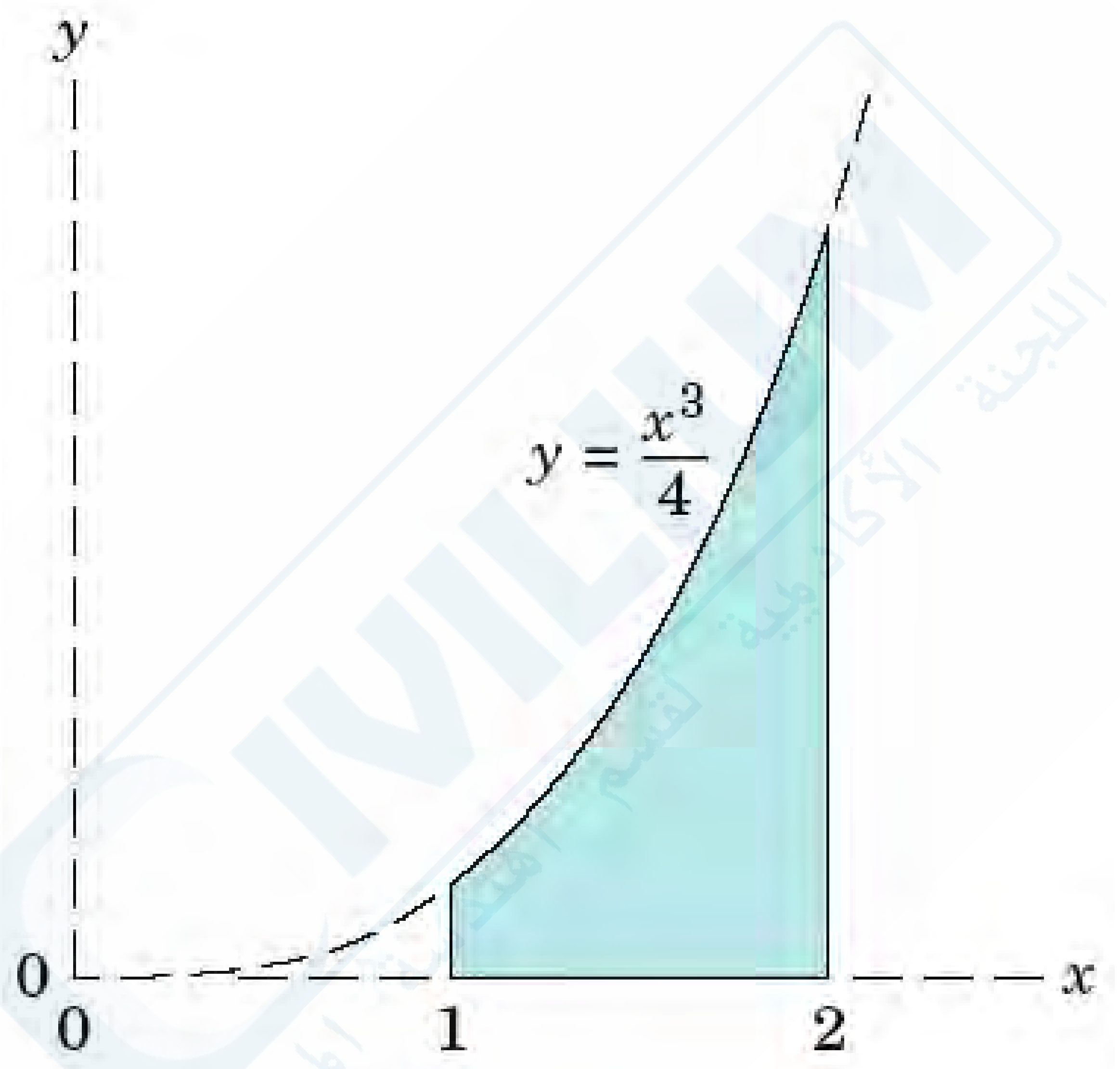
Problem A/13

P_A_13



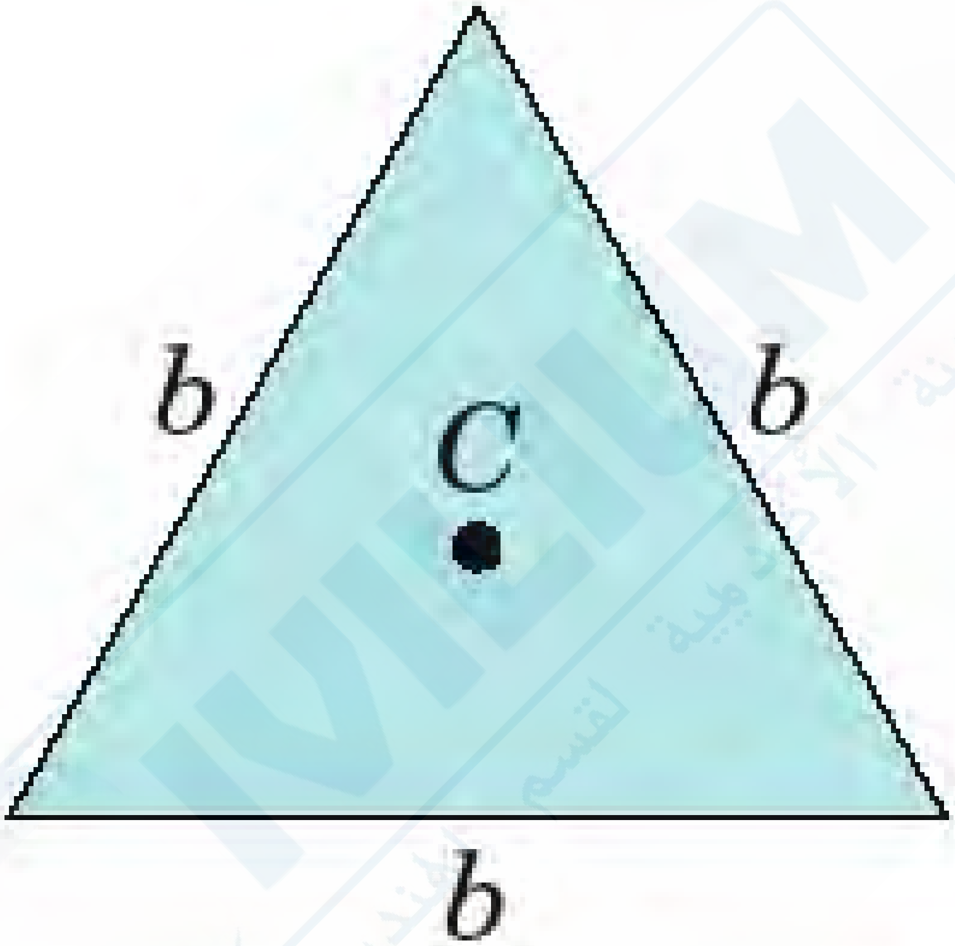
Problem A/14

P A 14



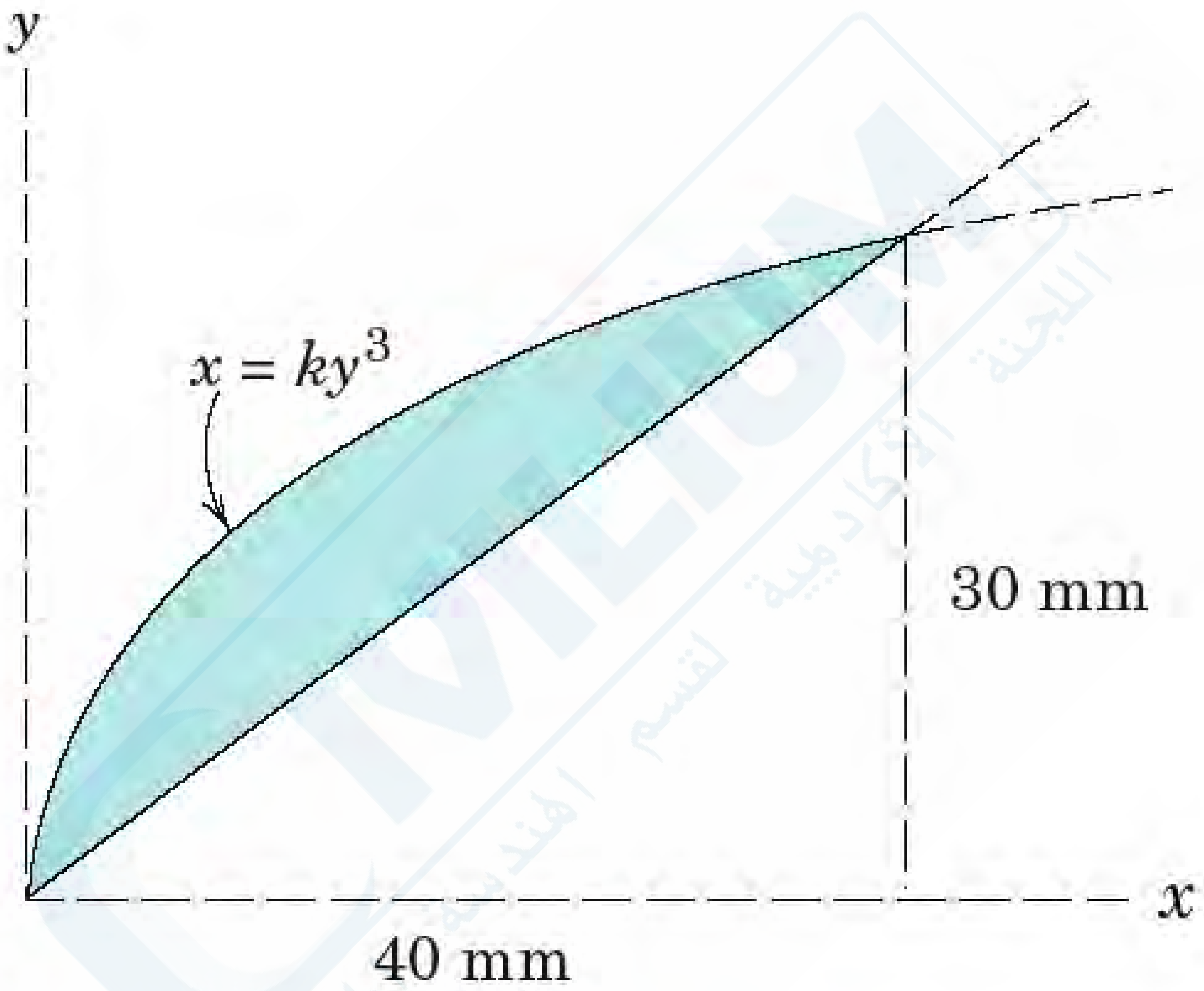
Problem A/15

P A 15



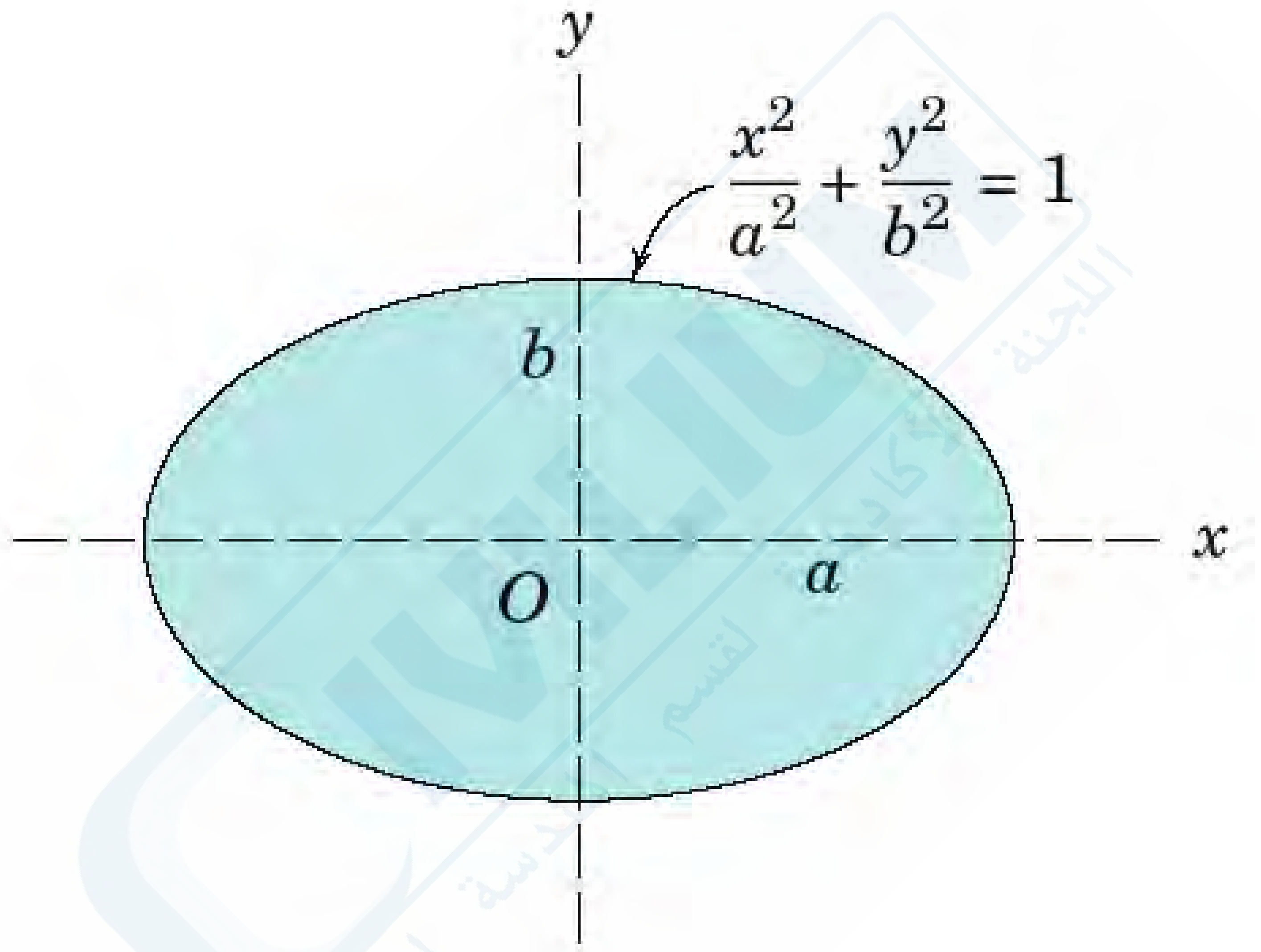
Problem A/16

P A 16

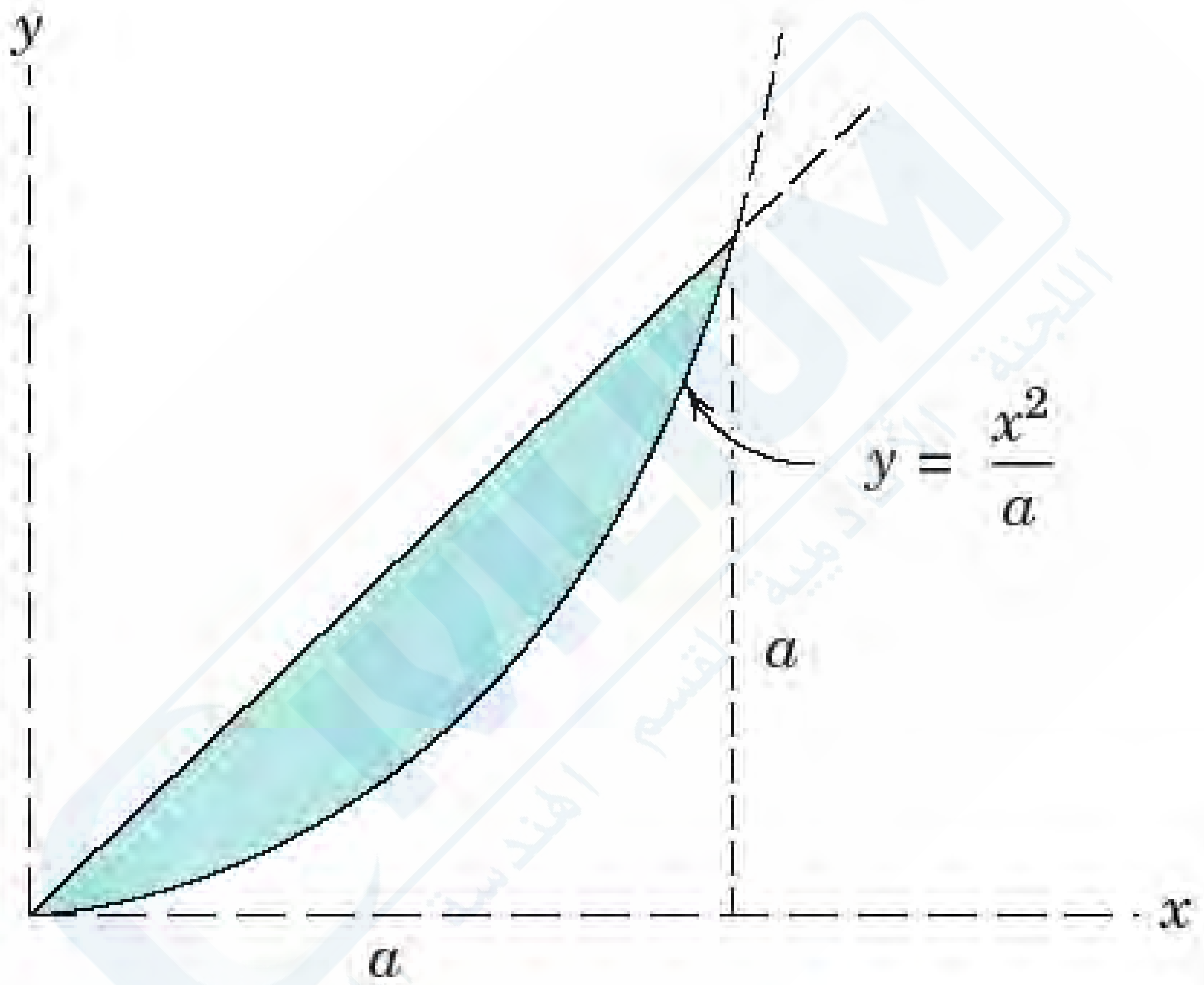


Problem A/17

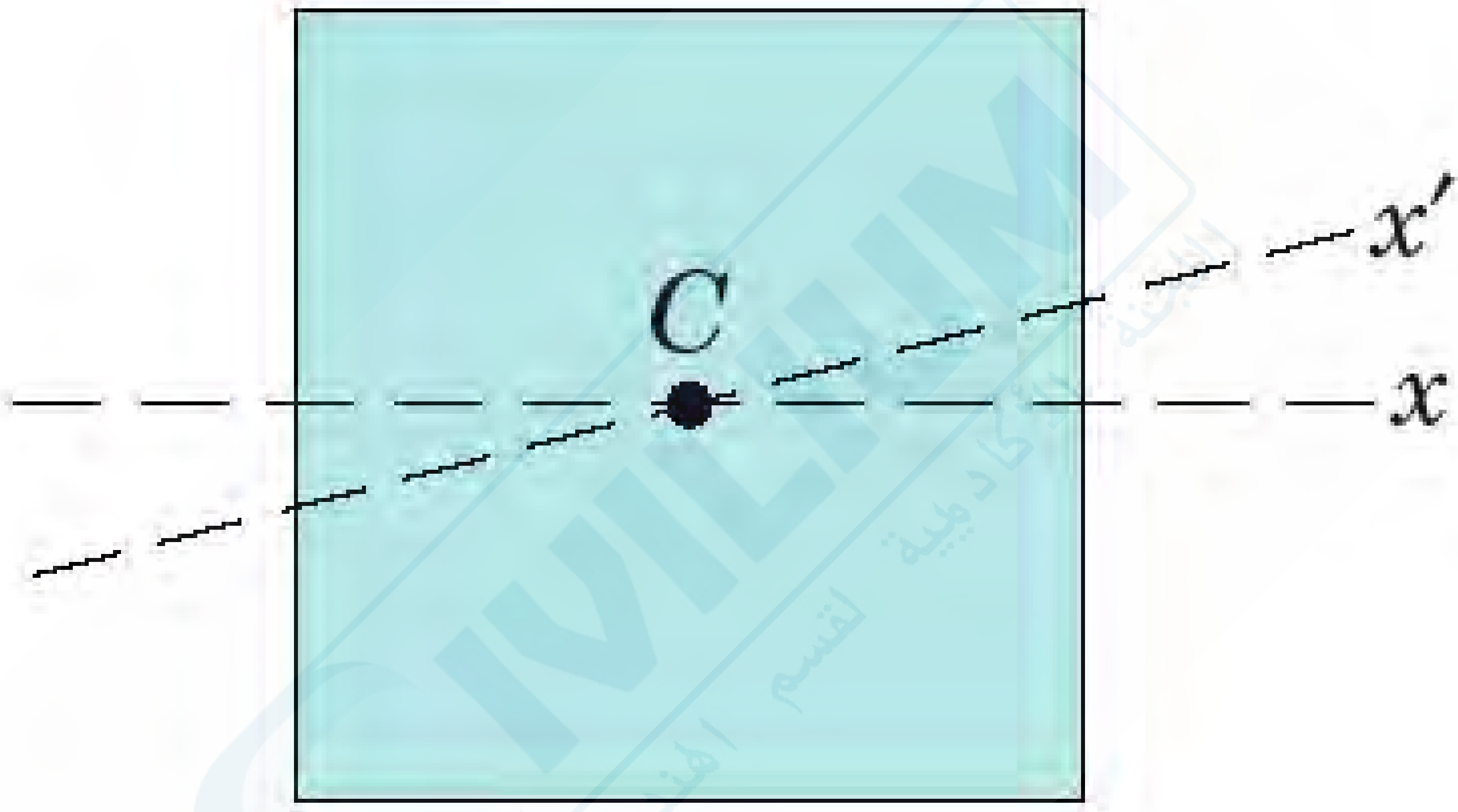
P A 17



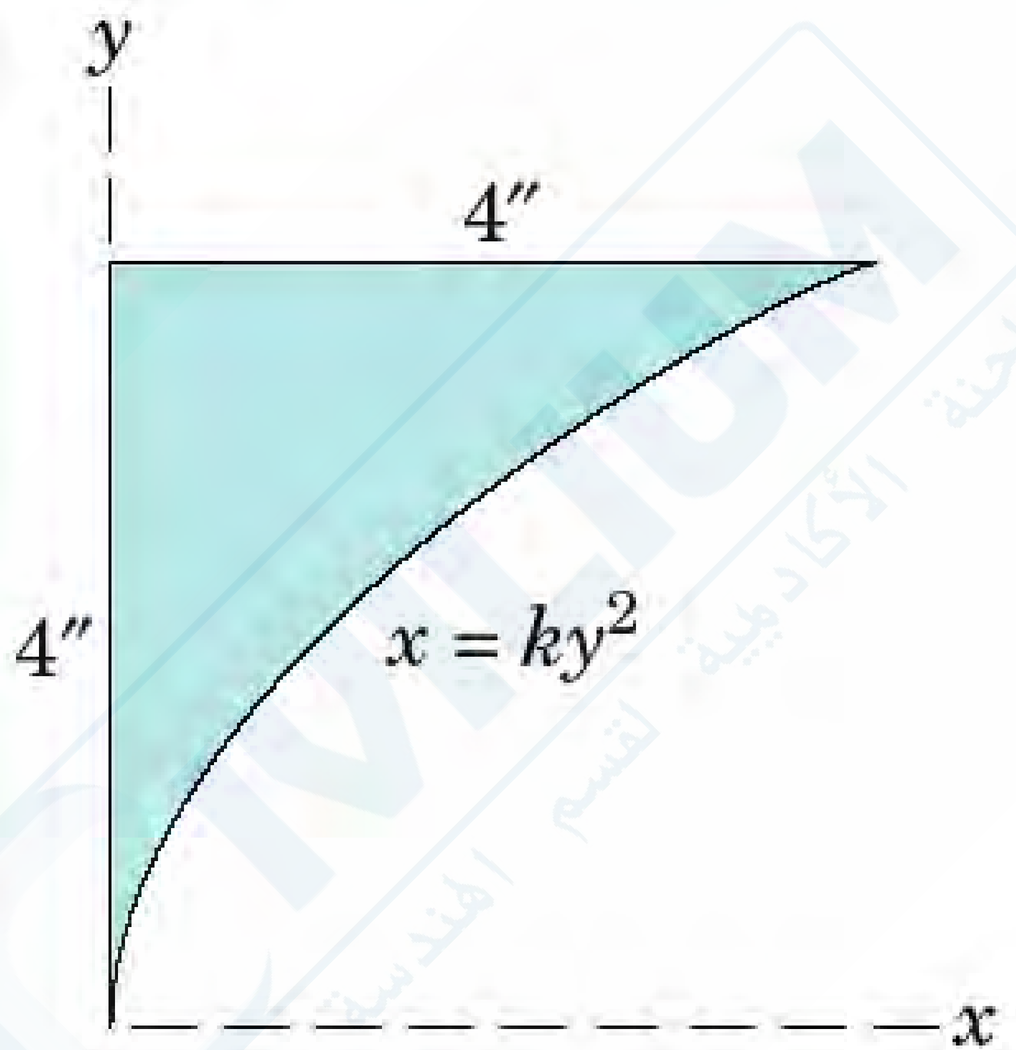
Problem A/18



Problem A/19

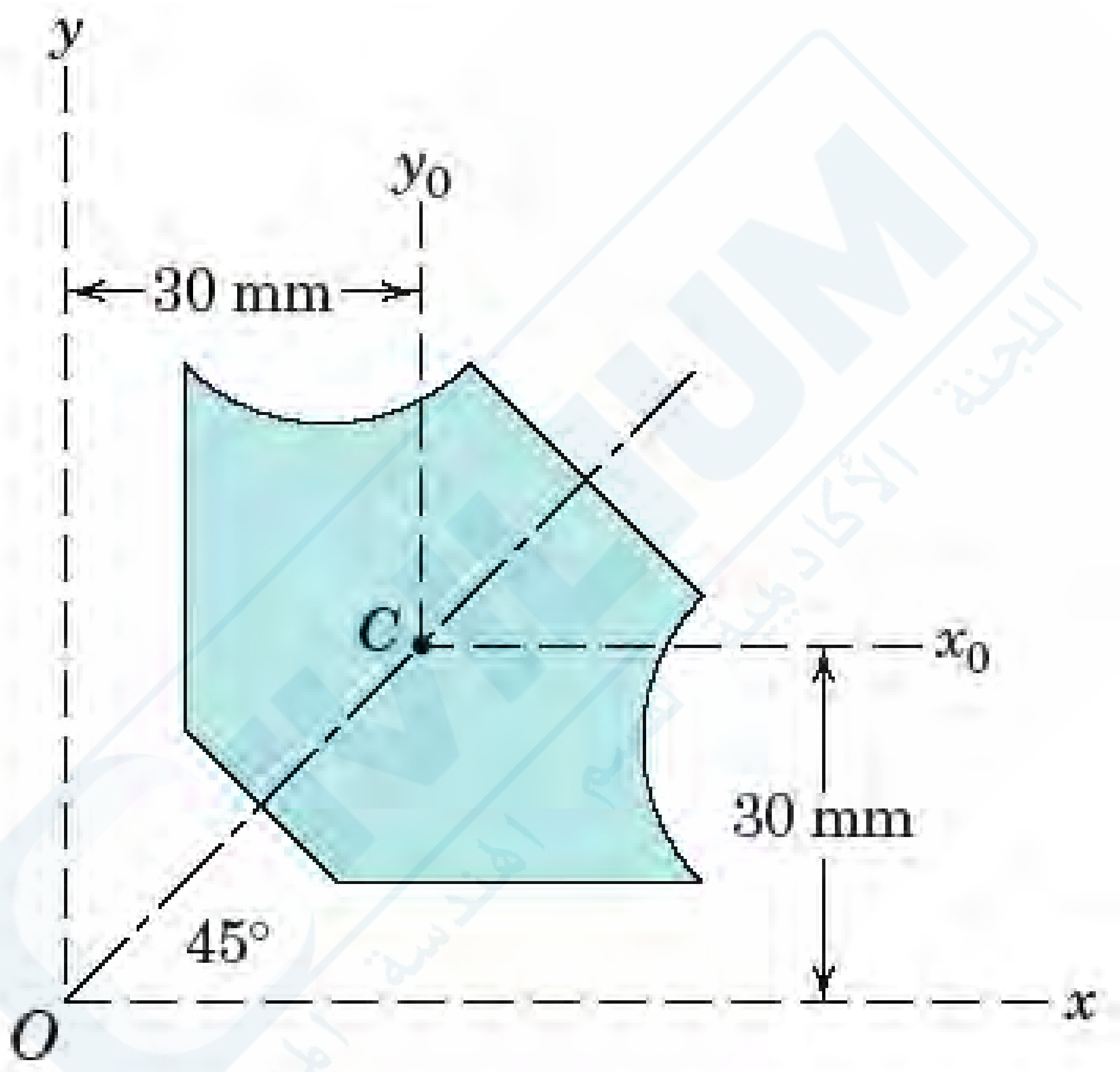


Problem A/20



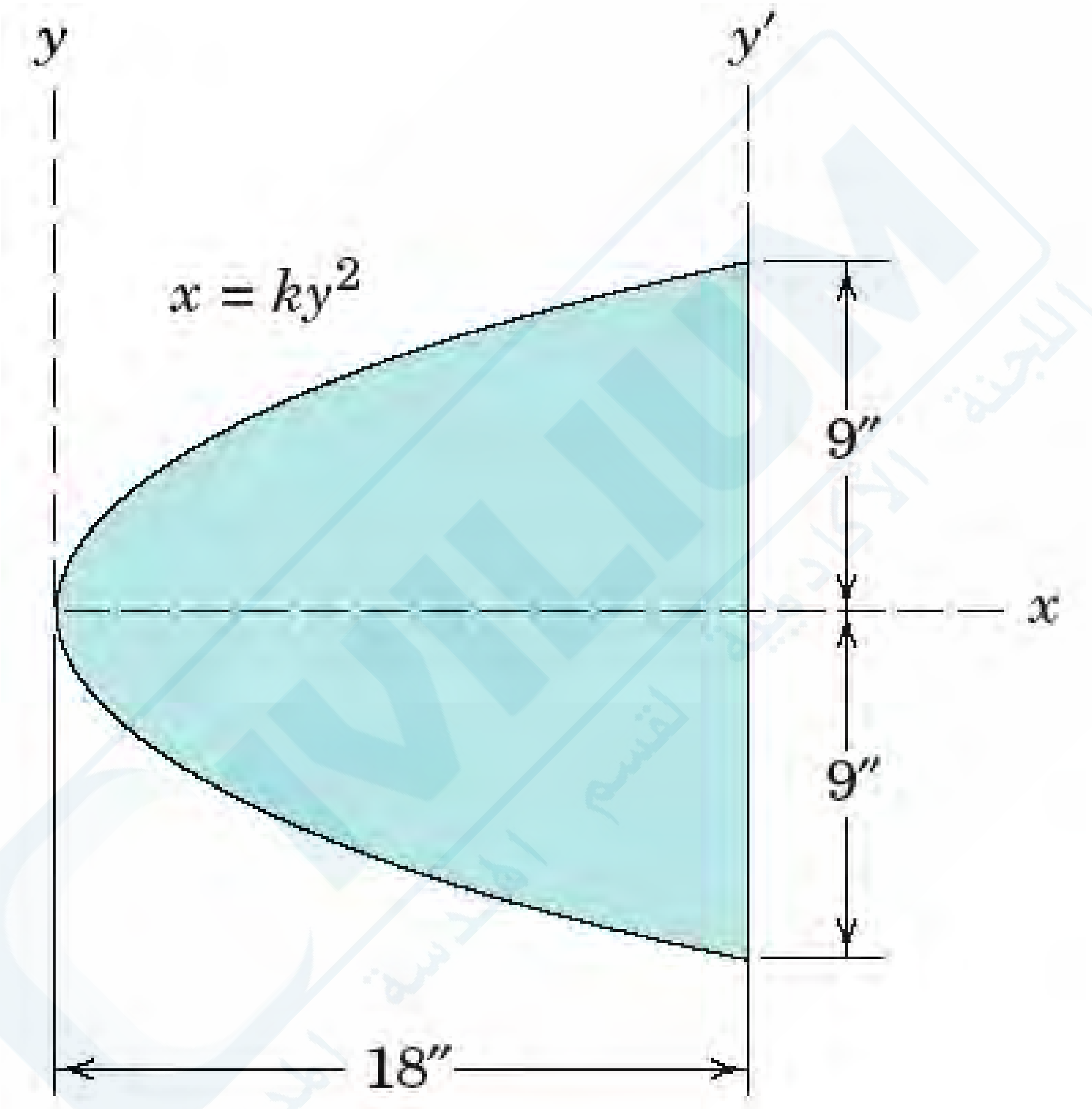
Problem A/21

P A 21



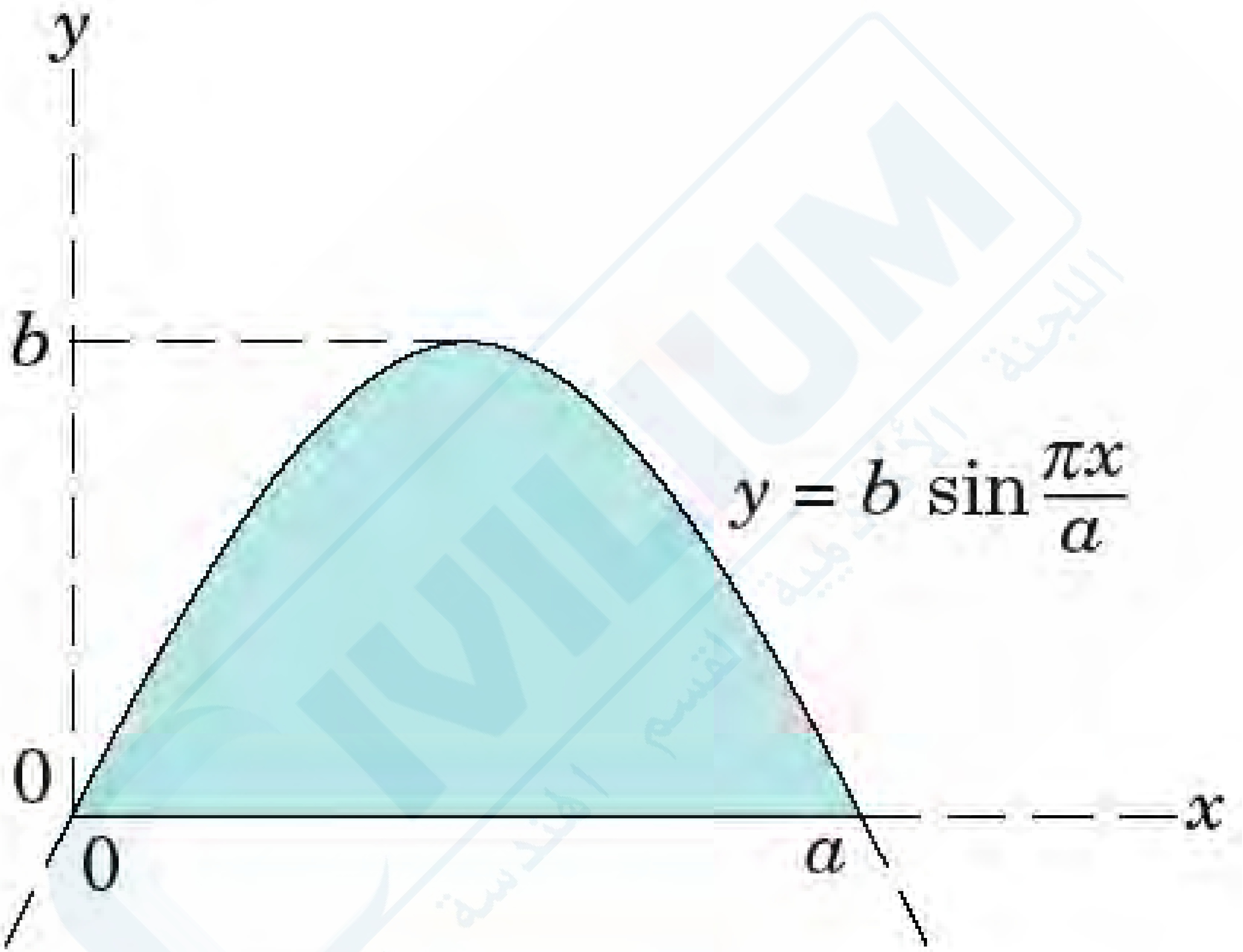
Problem A/22

P A 22

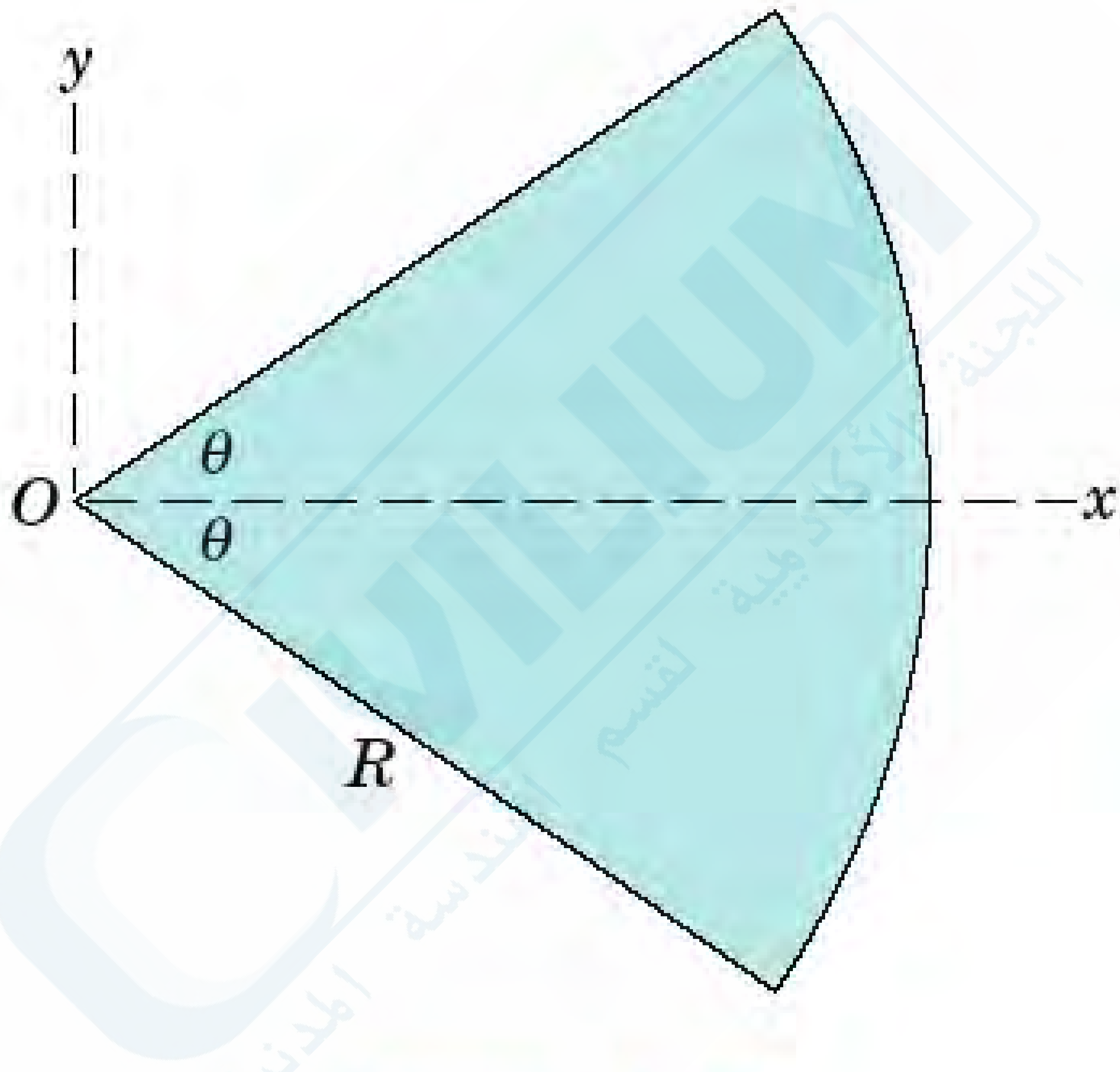


Problem A/23

P A 23

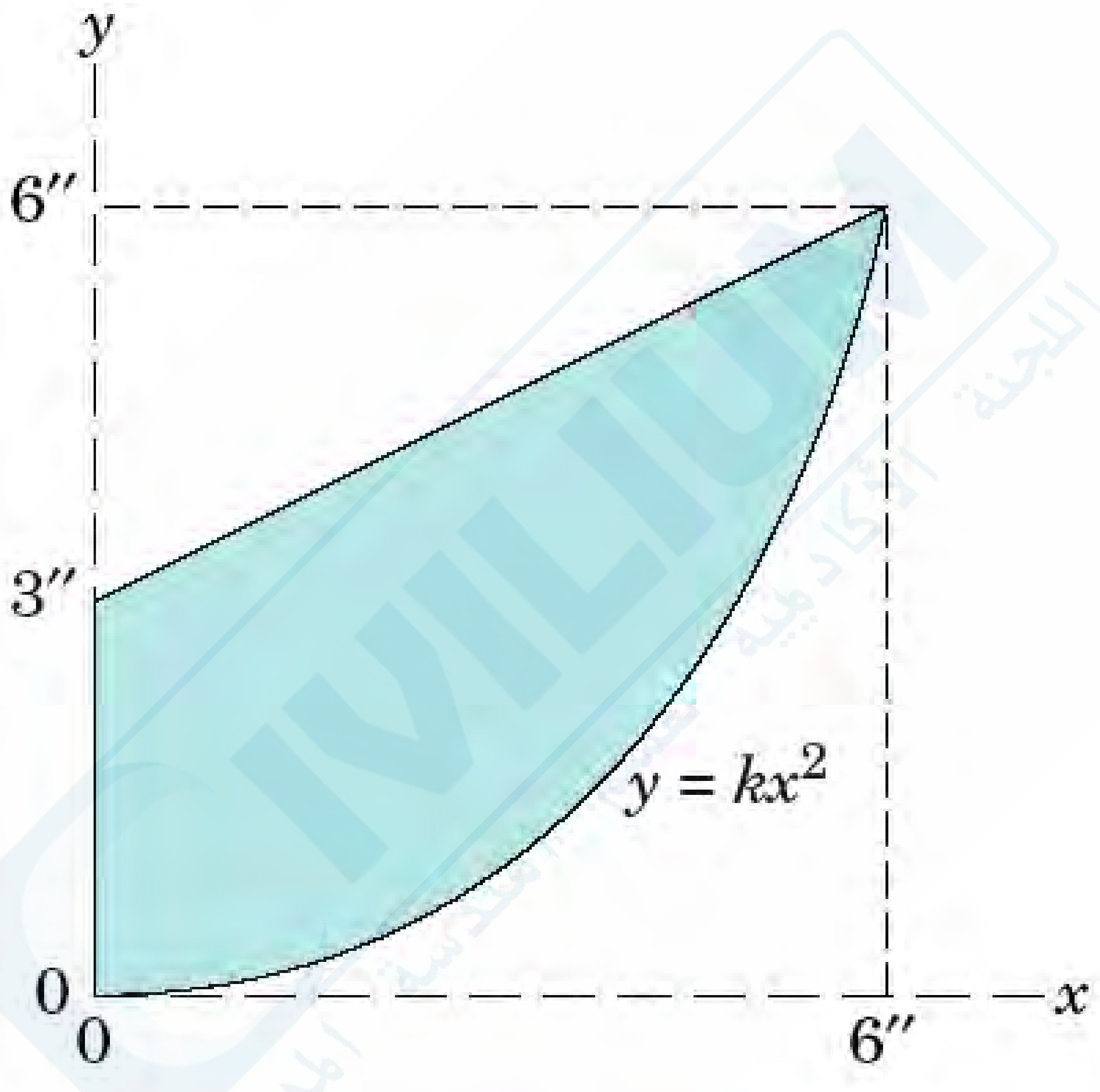


Problem A/24



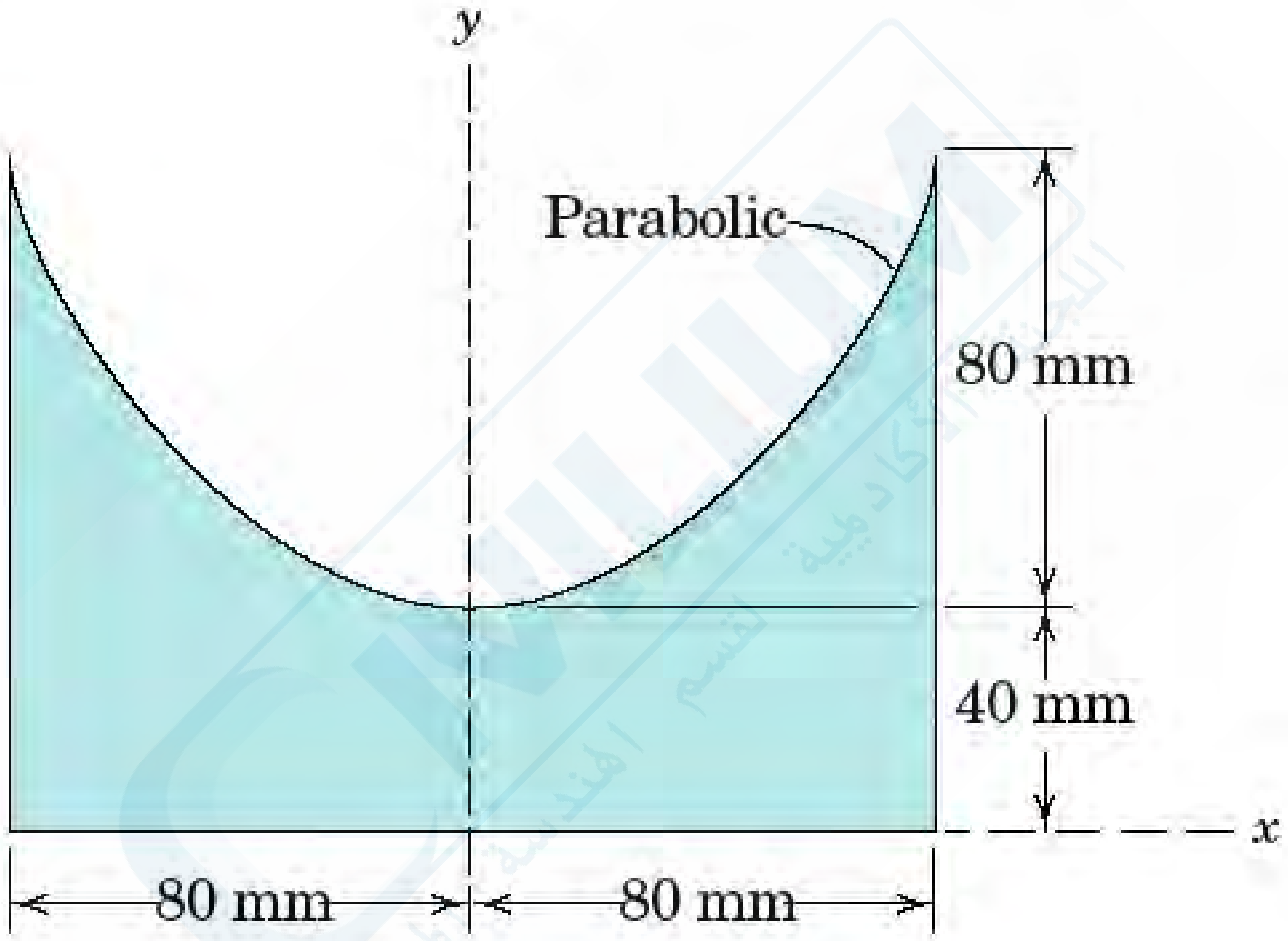
Problem A/25

P A 25



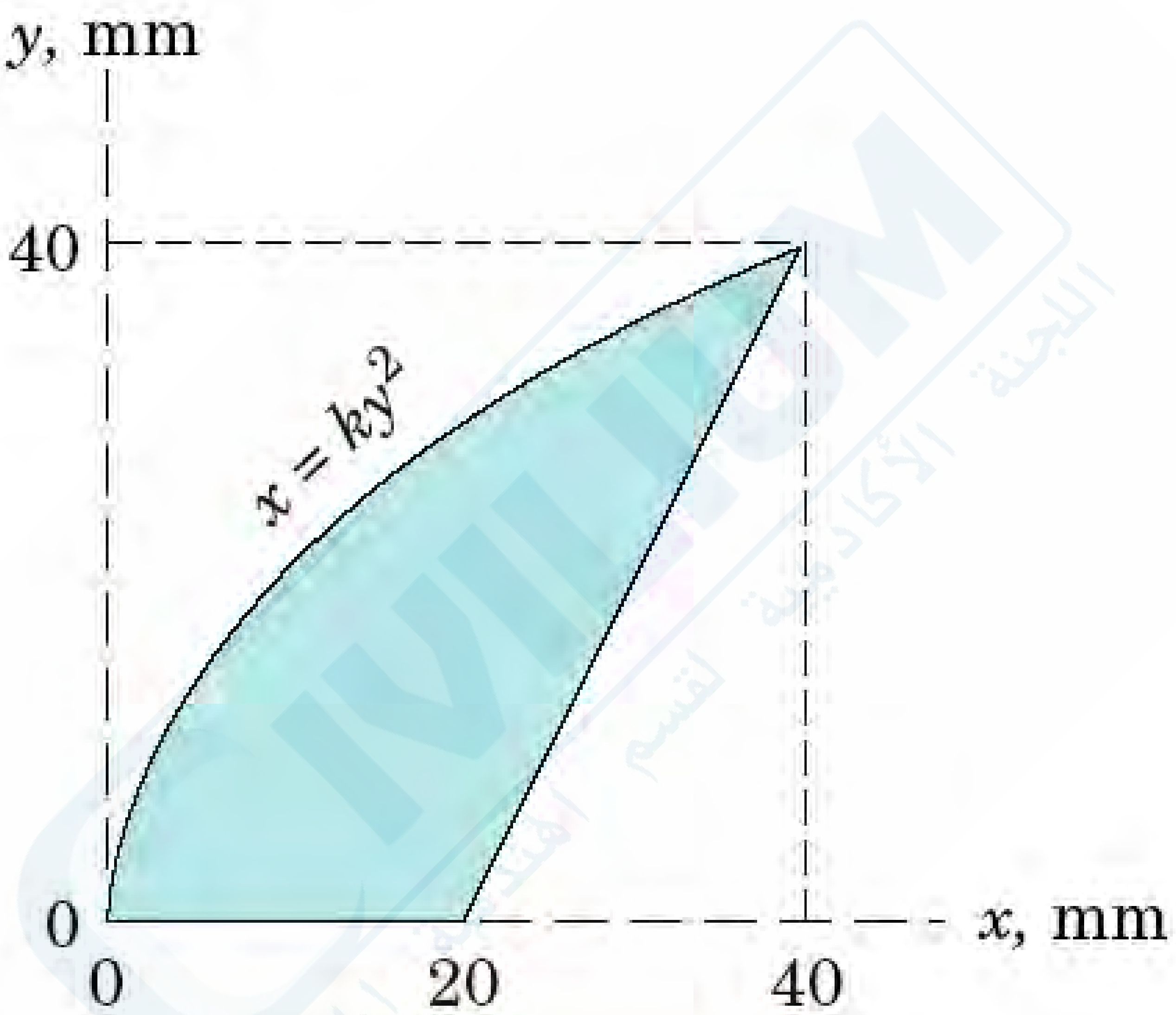
Problem A/27

P A 27



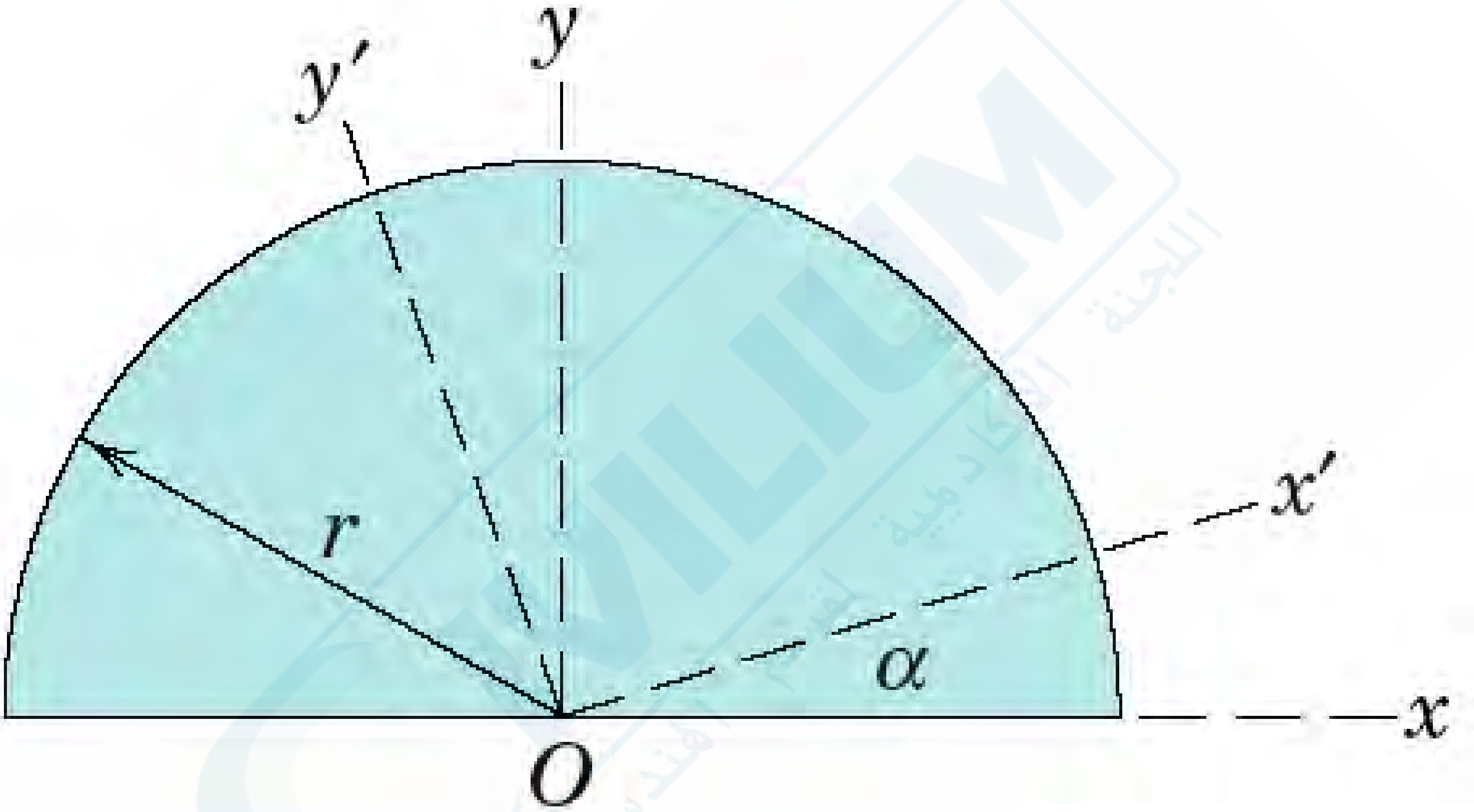
Problem A/28

P A 28



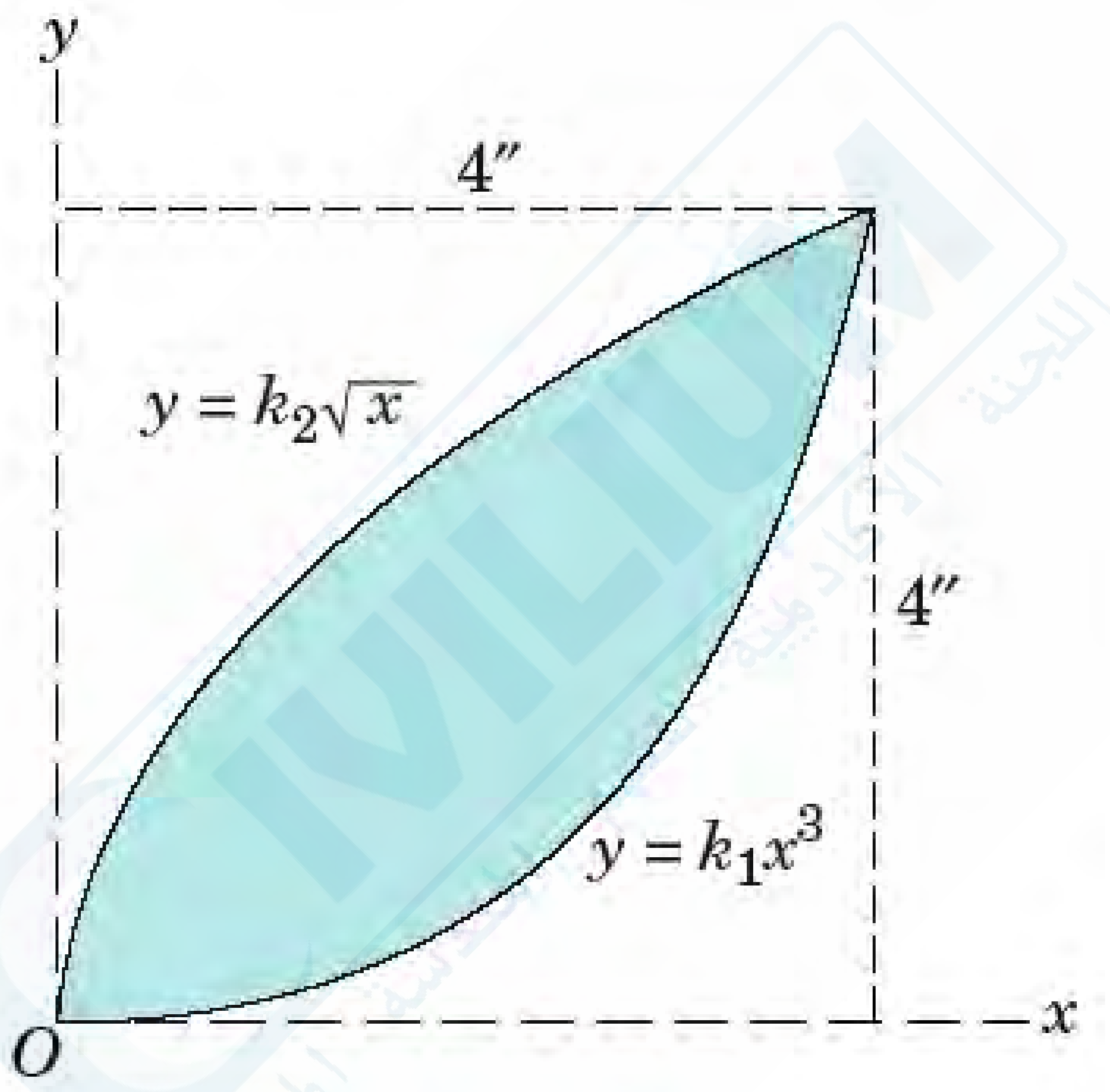
Problem A/29

P A 29



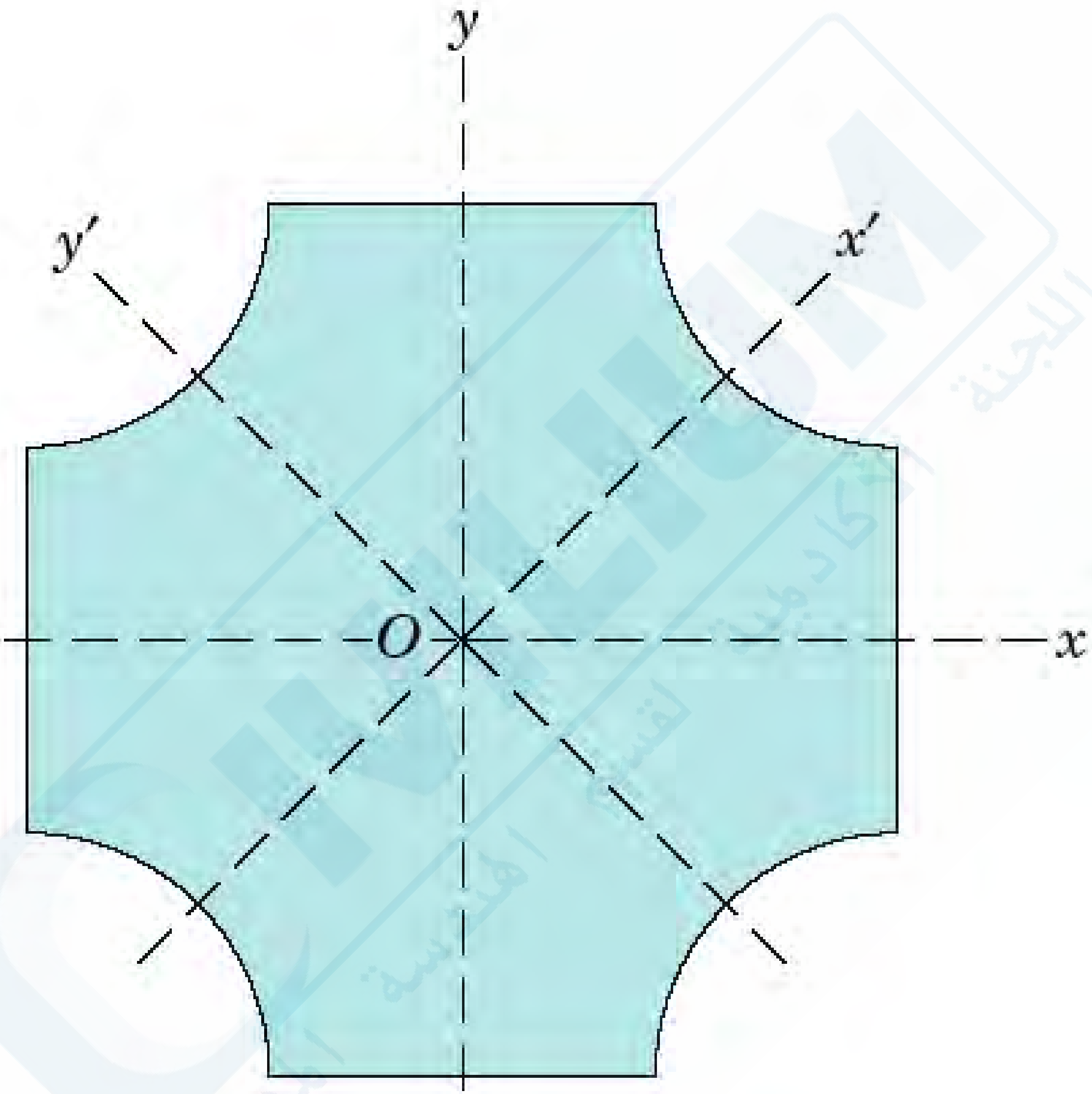
Problem A/30

P_A_30



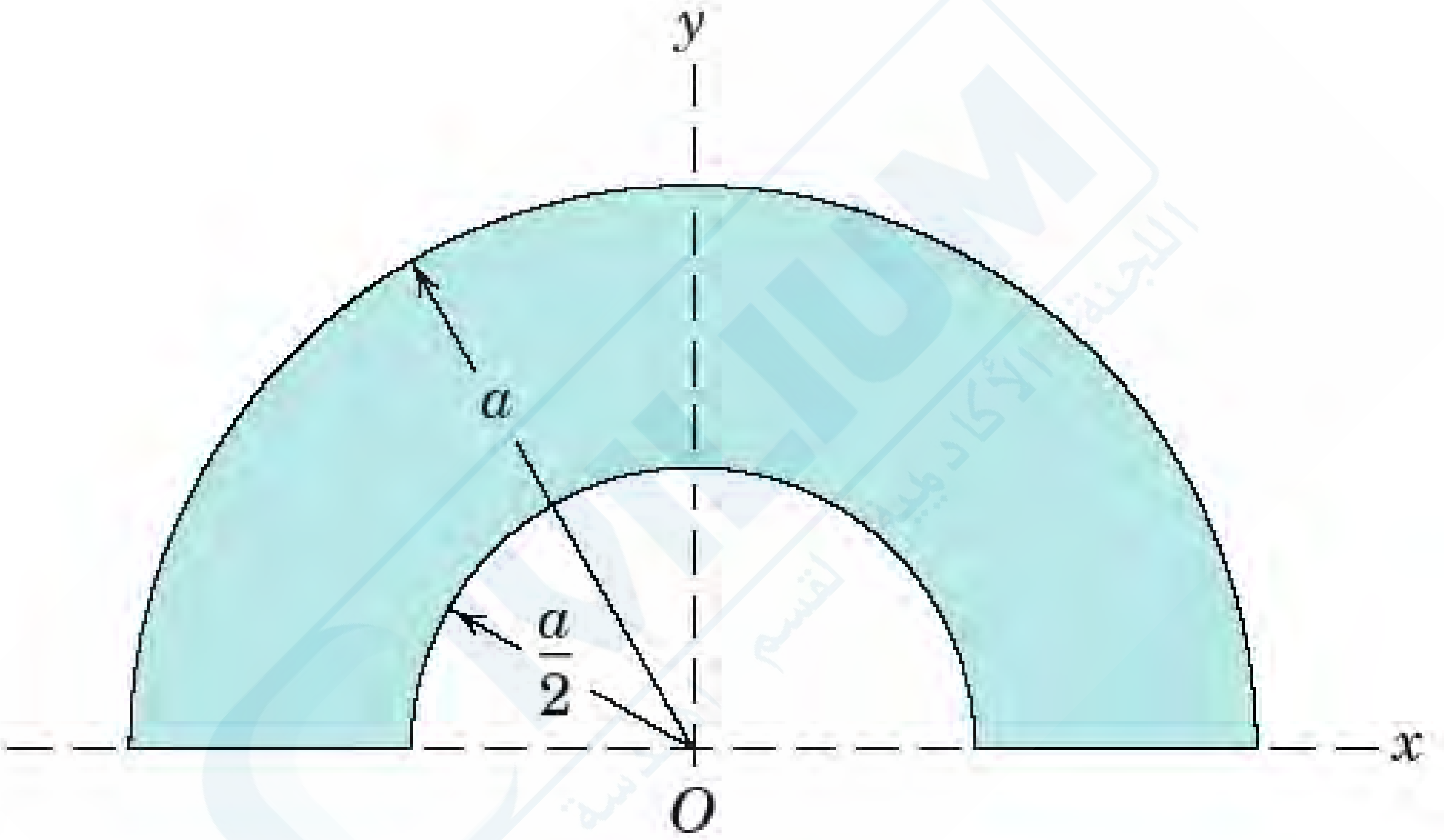
Problem A/31

P A 31



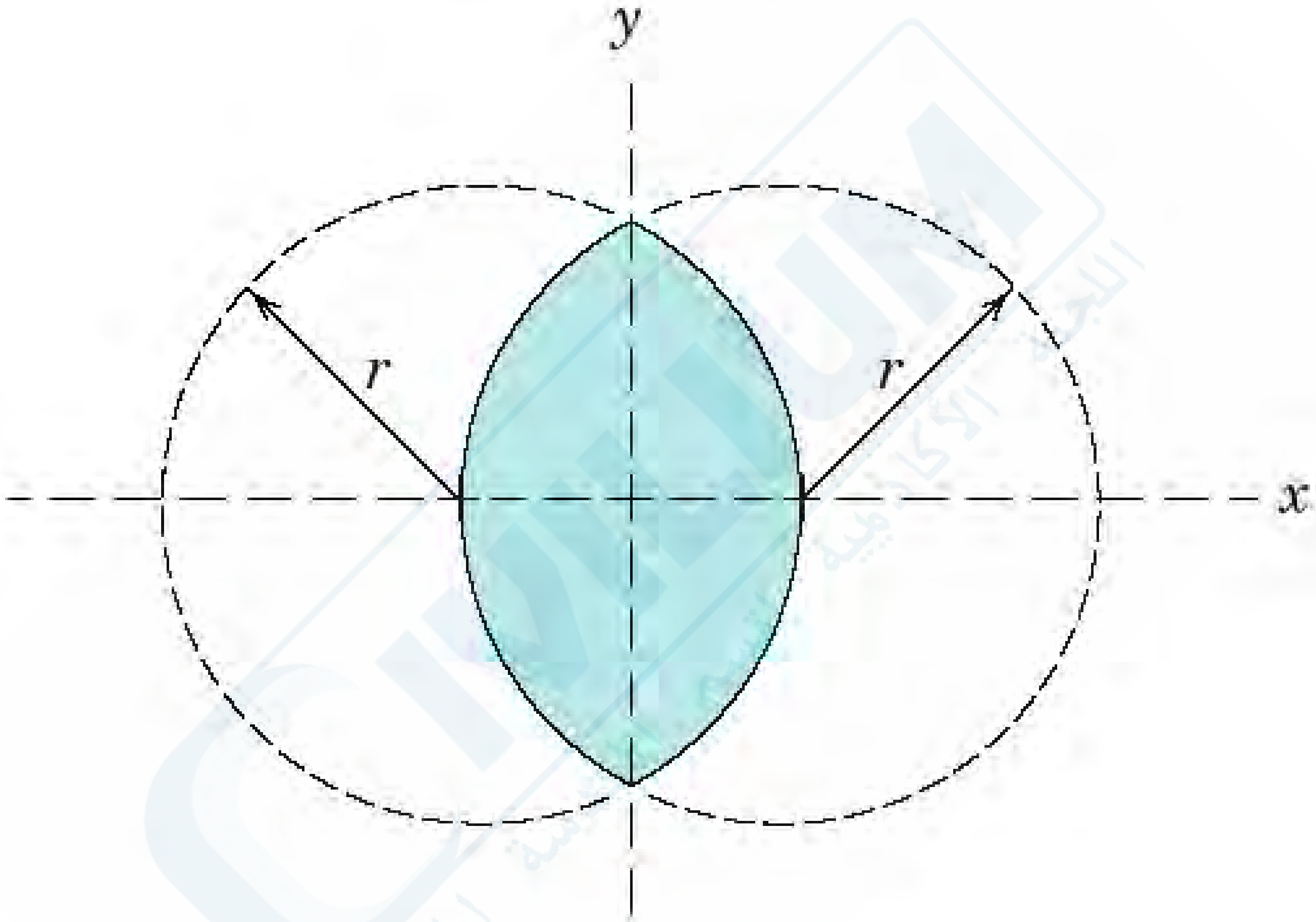
Problem A/32

P A 32



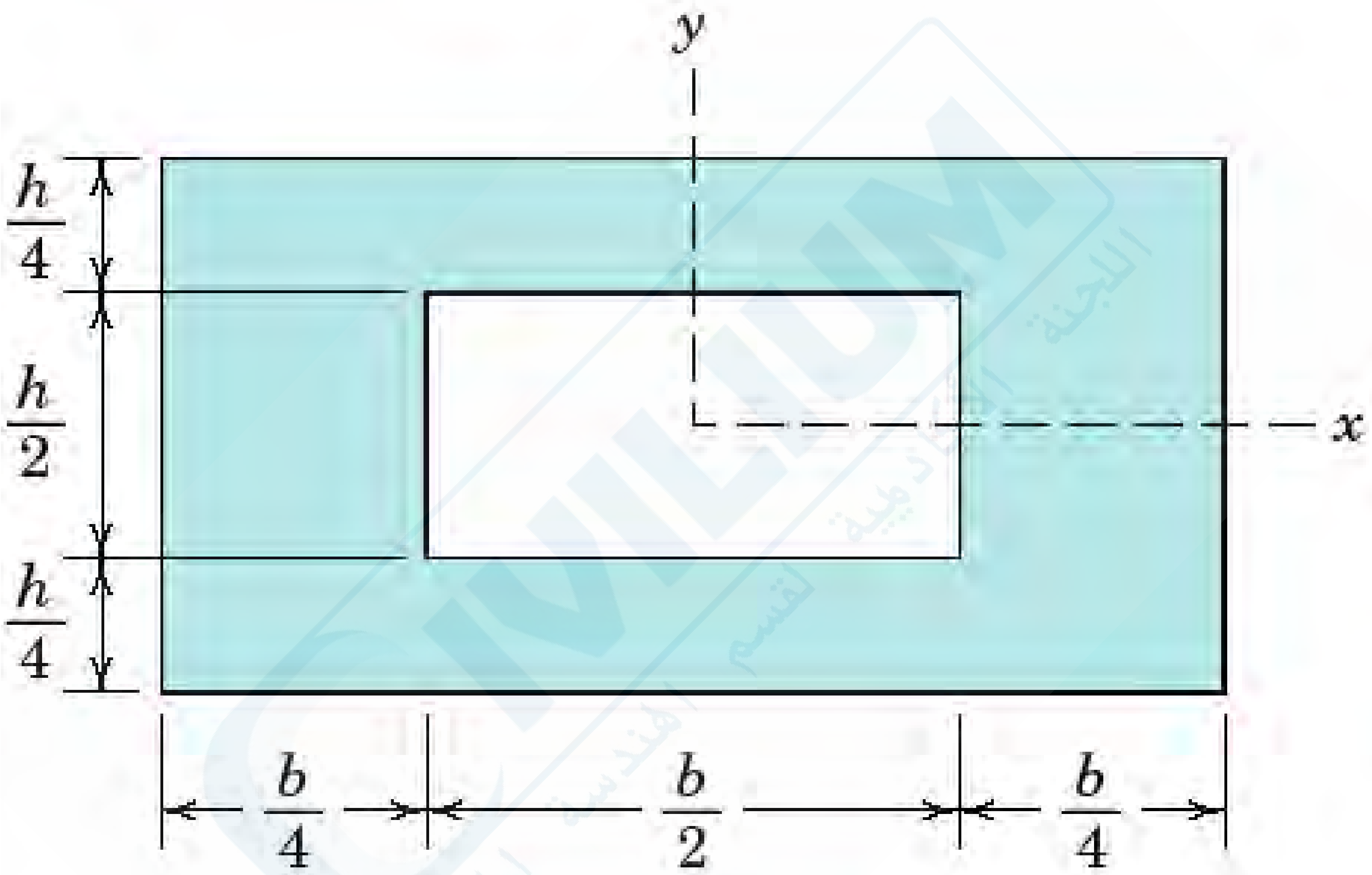
Problem A/33

P_A_33



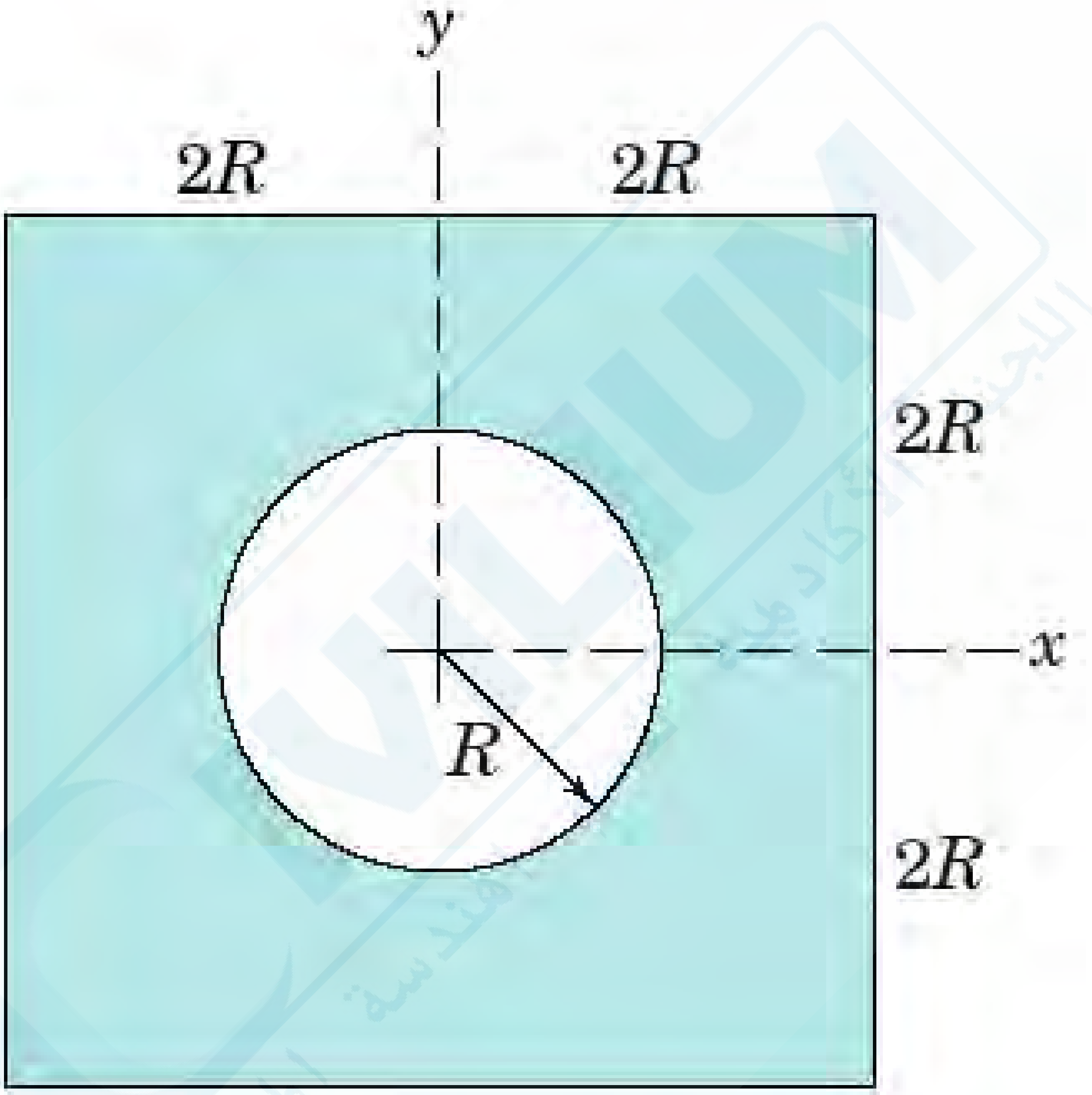
Problem A/34

P A 34



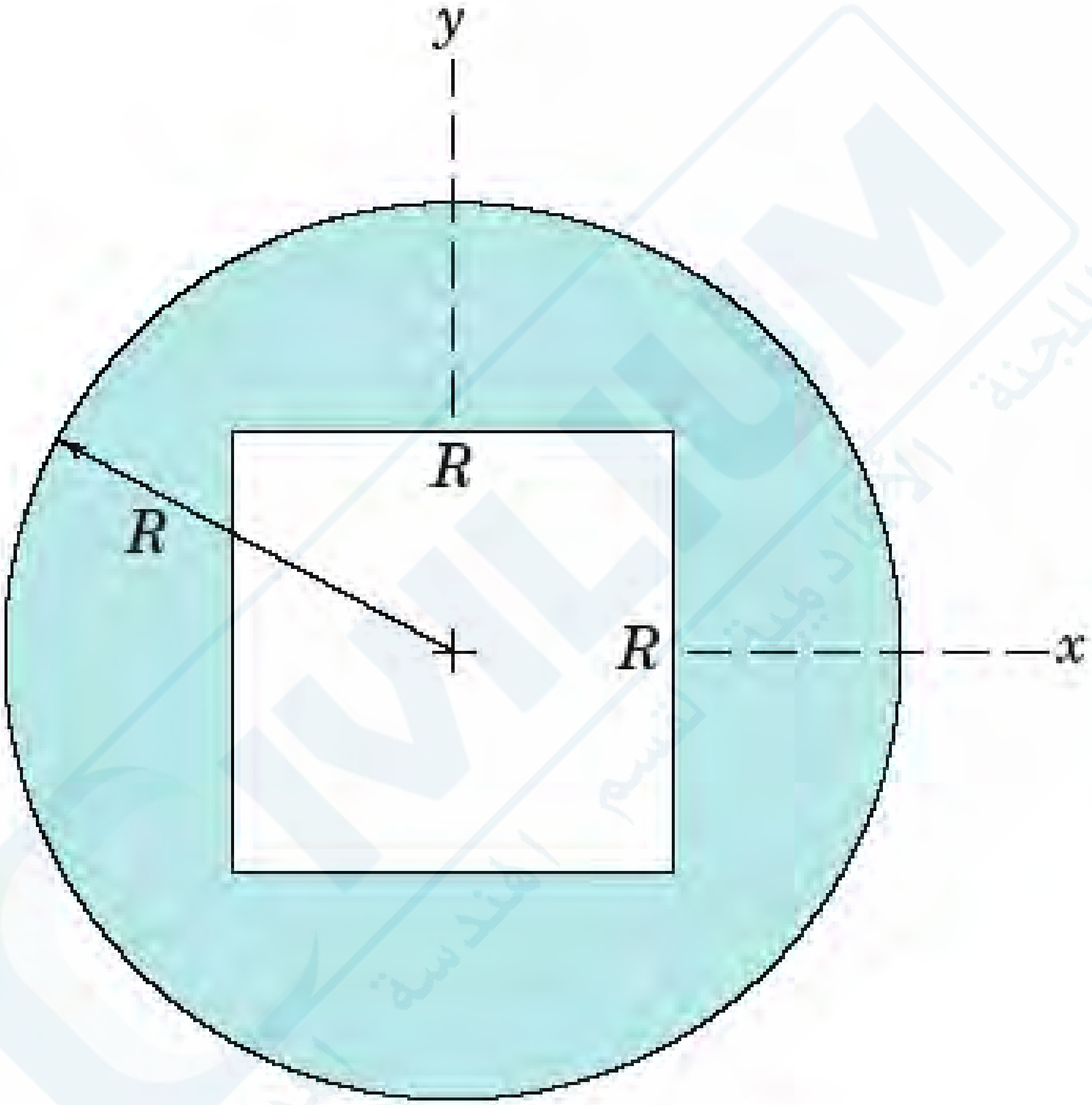
Problem A/35

P A 35



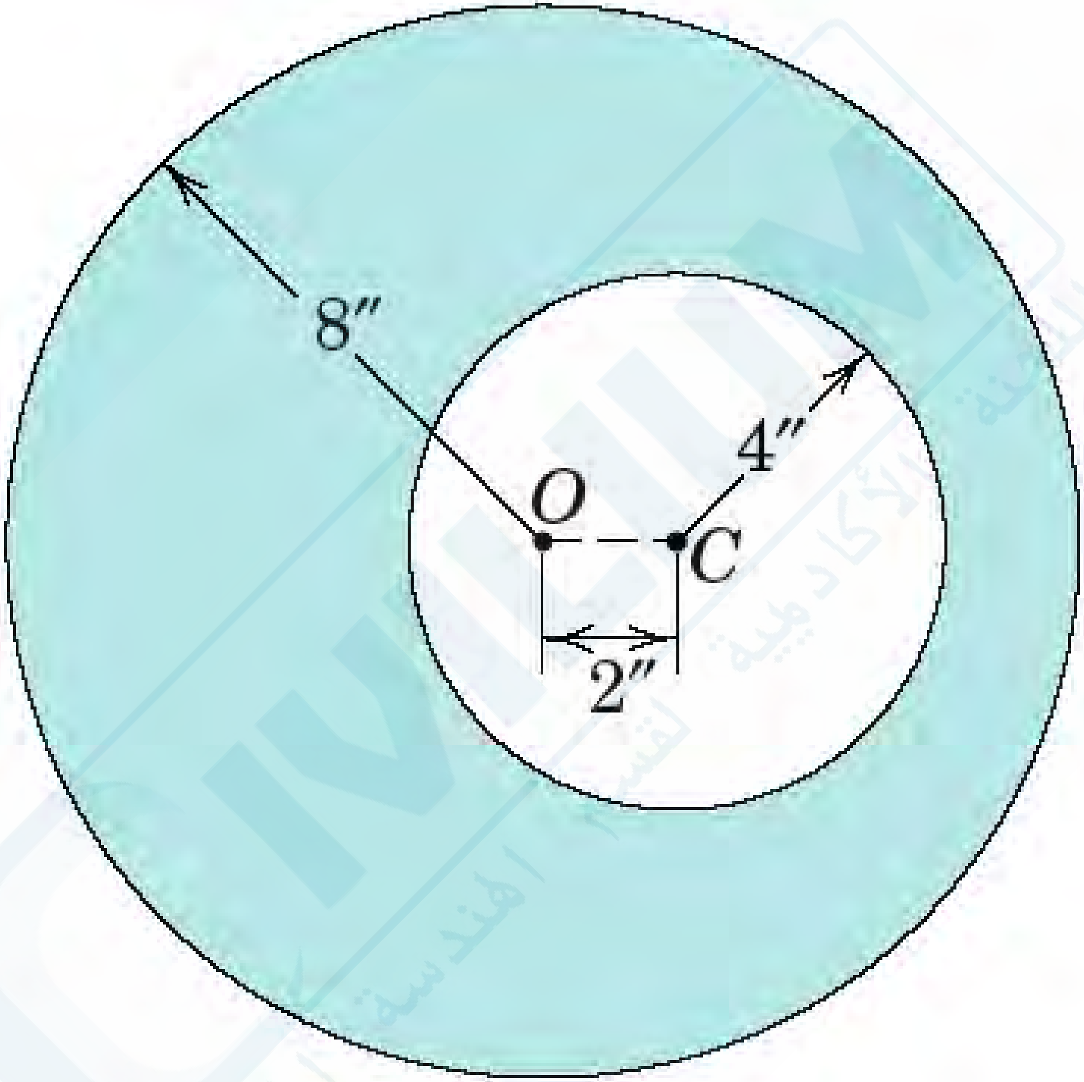
Problem A/36

P A 36



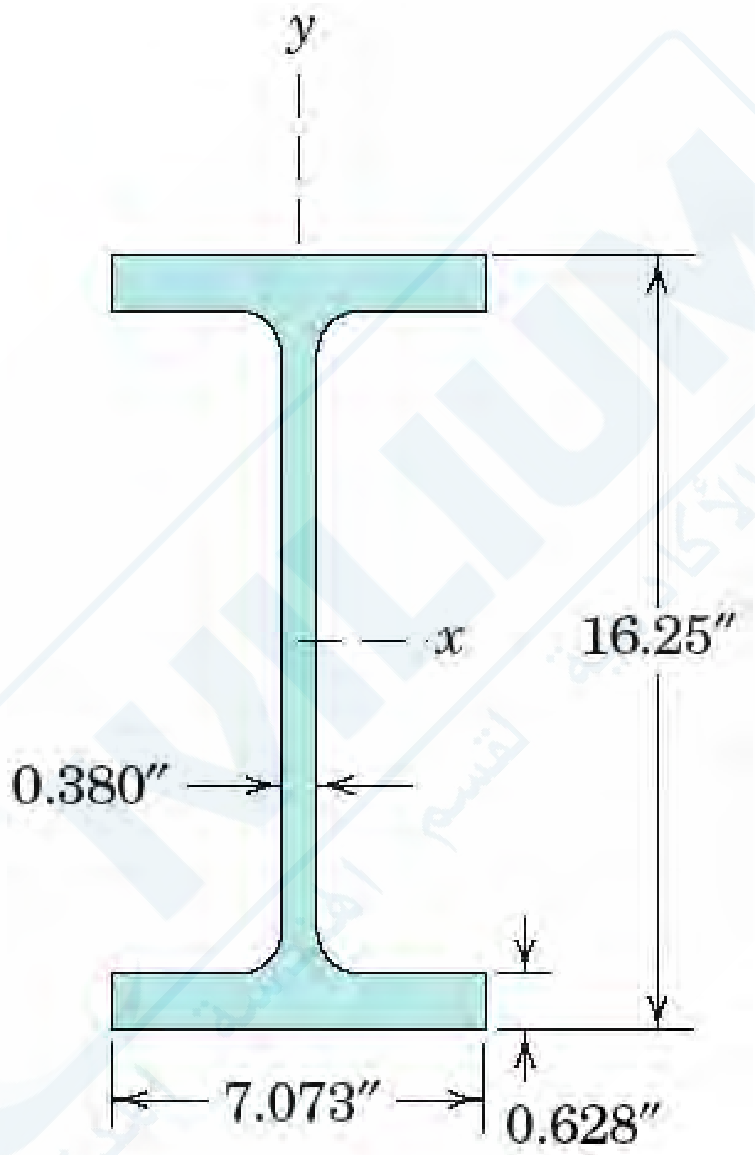
Problem A/37

P A 37



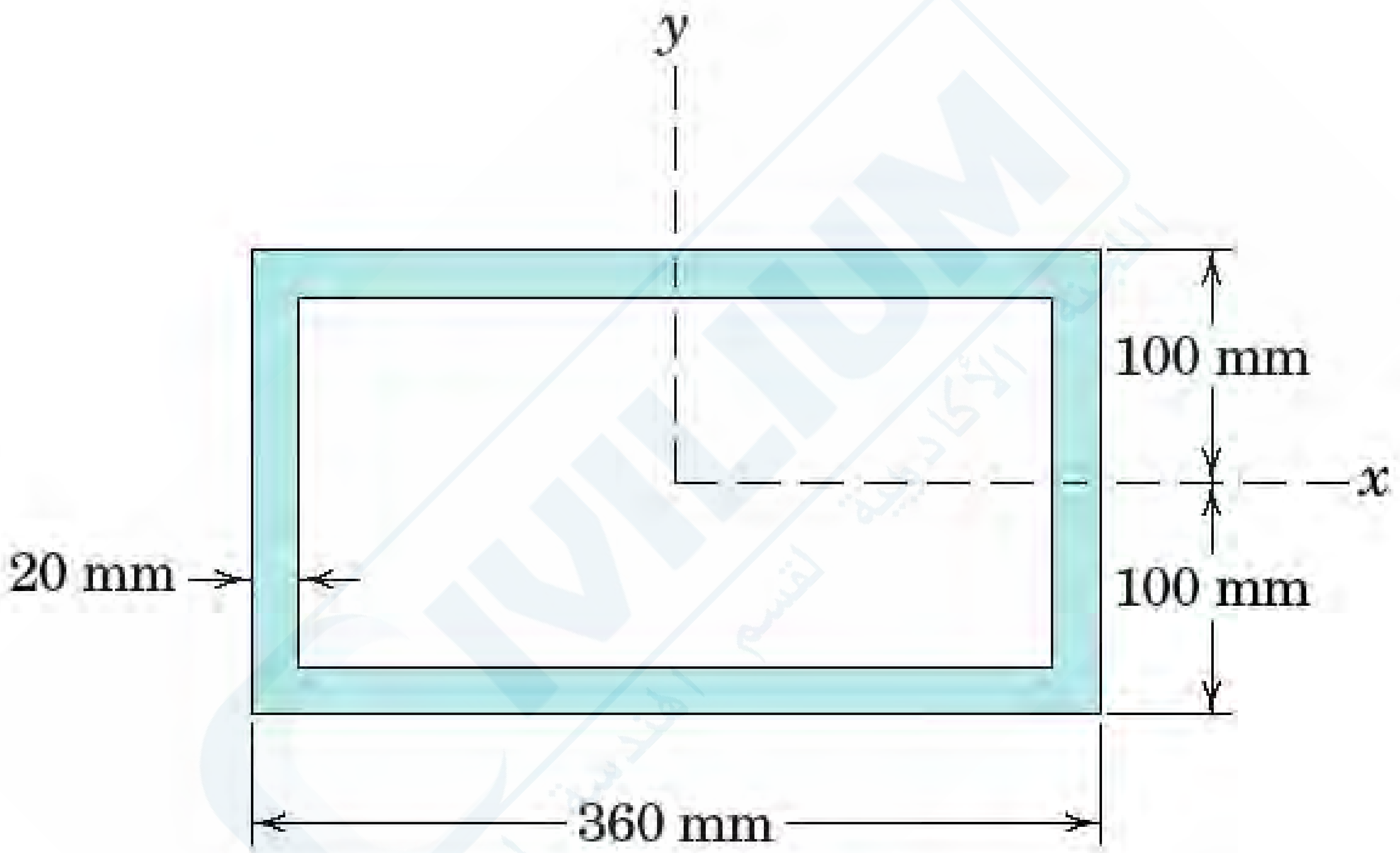
Problem A/38

P A 38



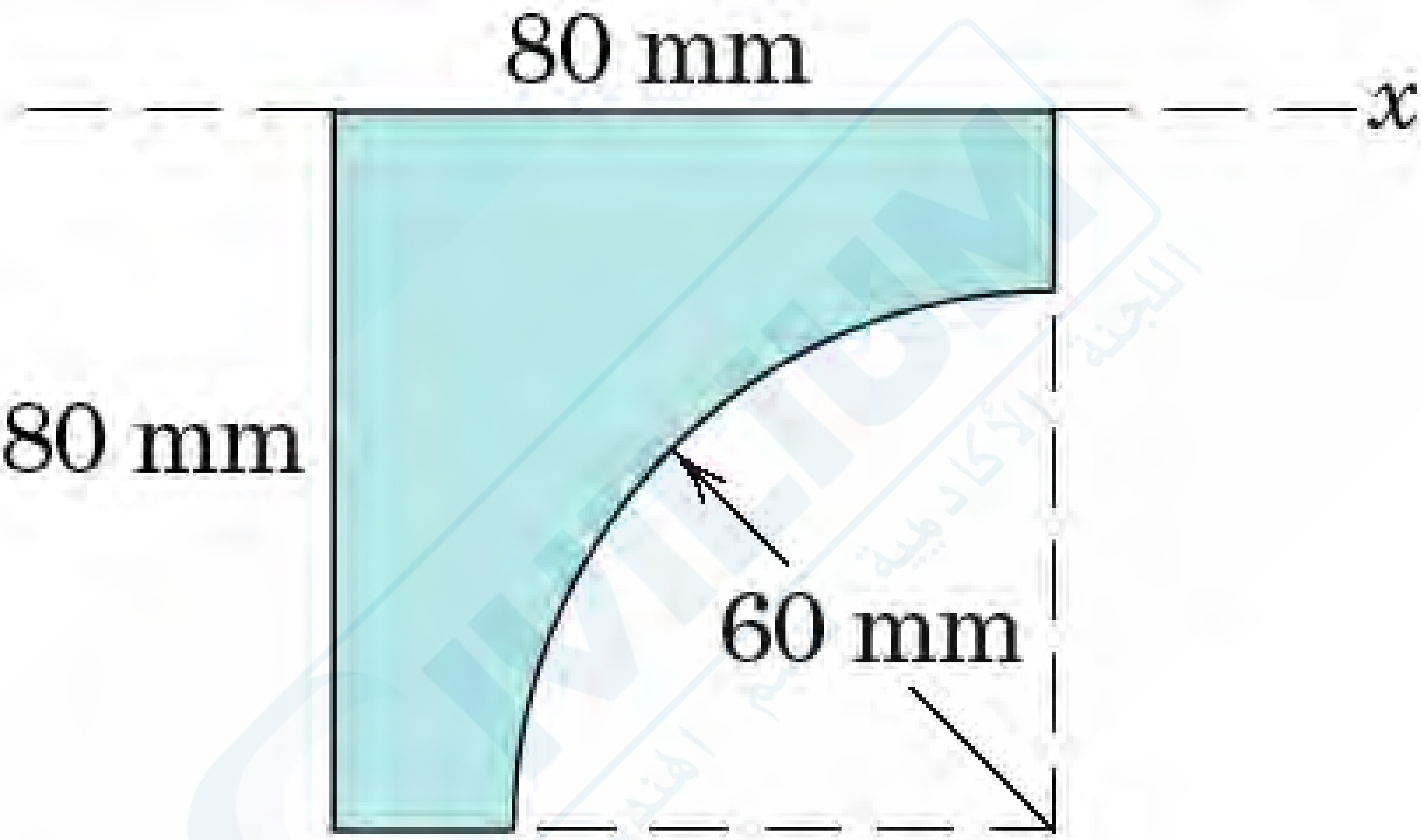
Problem A/39

P A 39



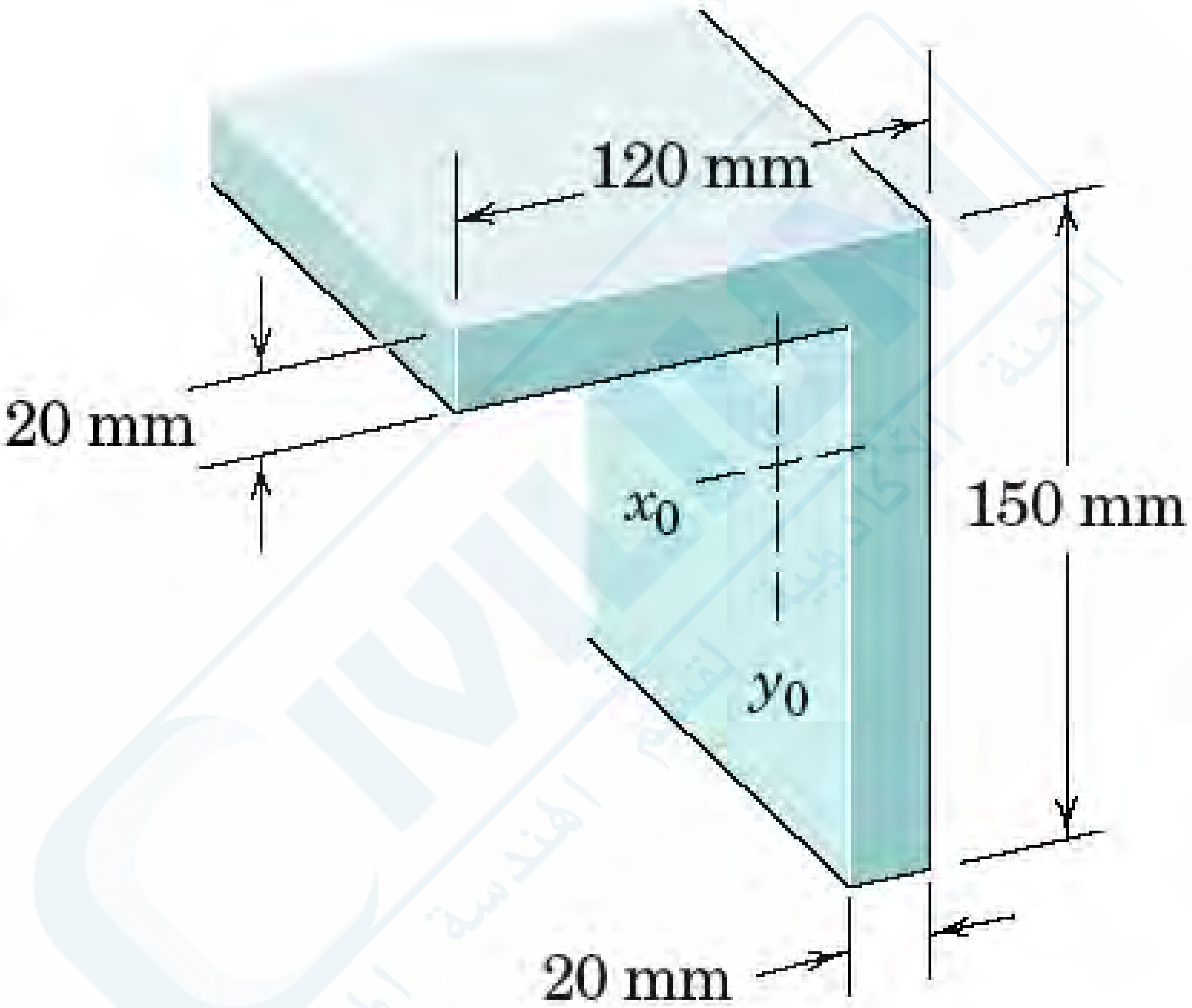
Problem A/40

P_A_40



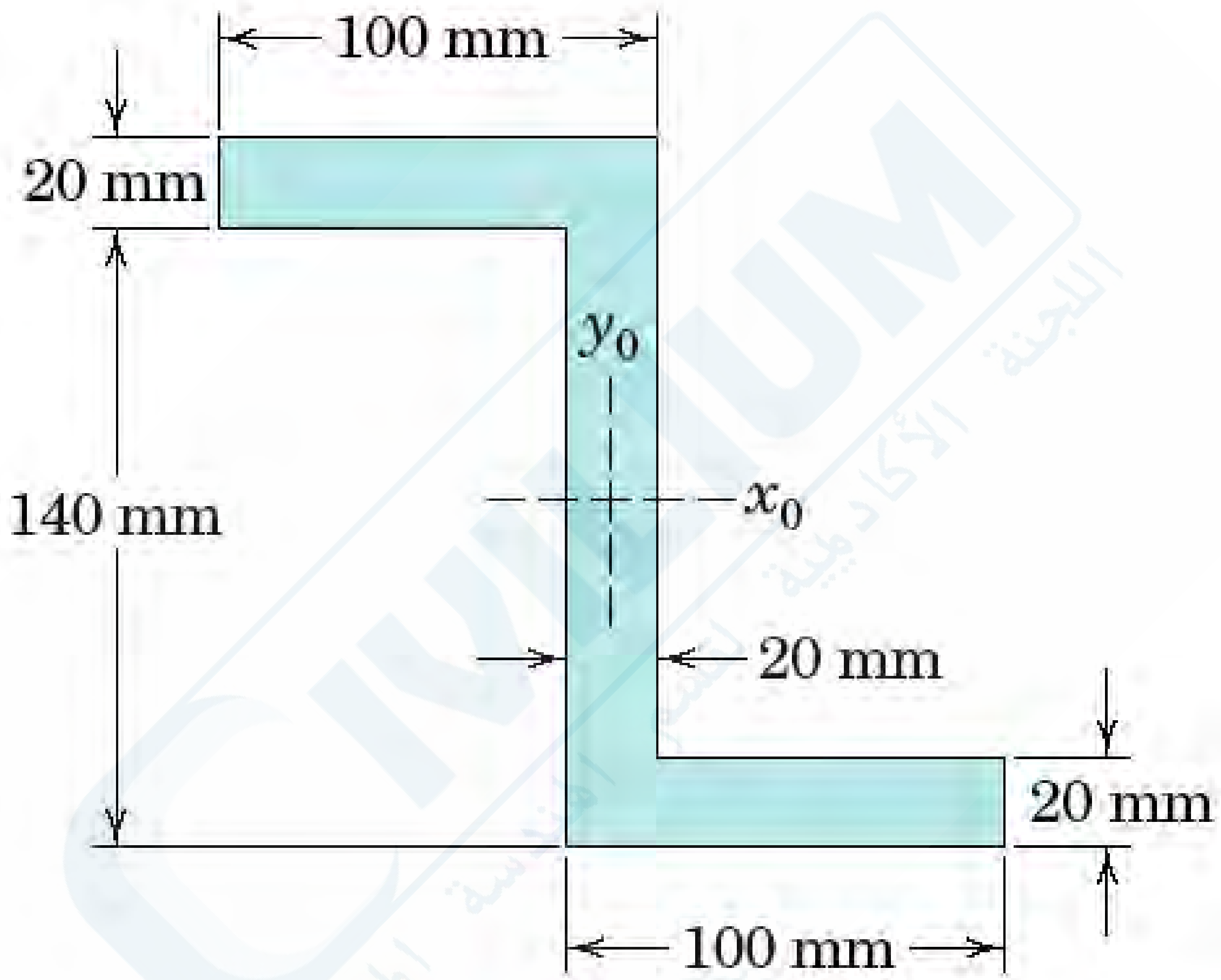
Problem A/41

P_A_41



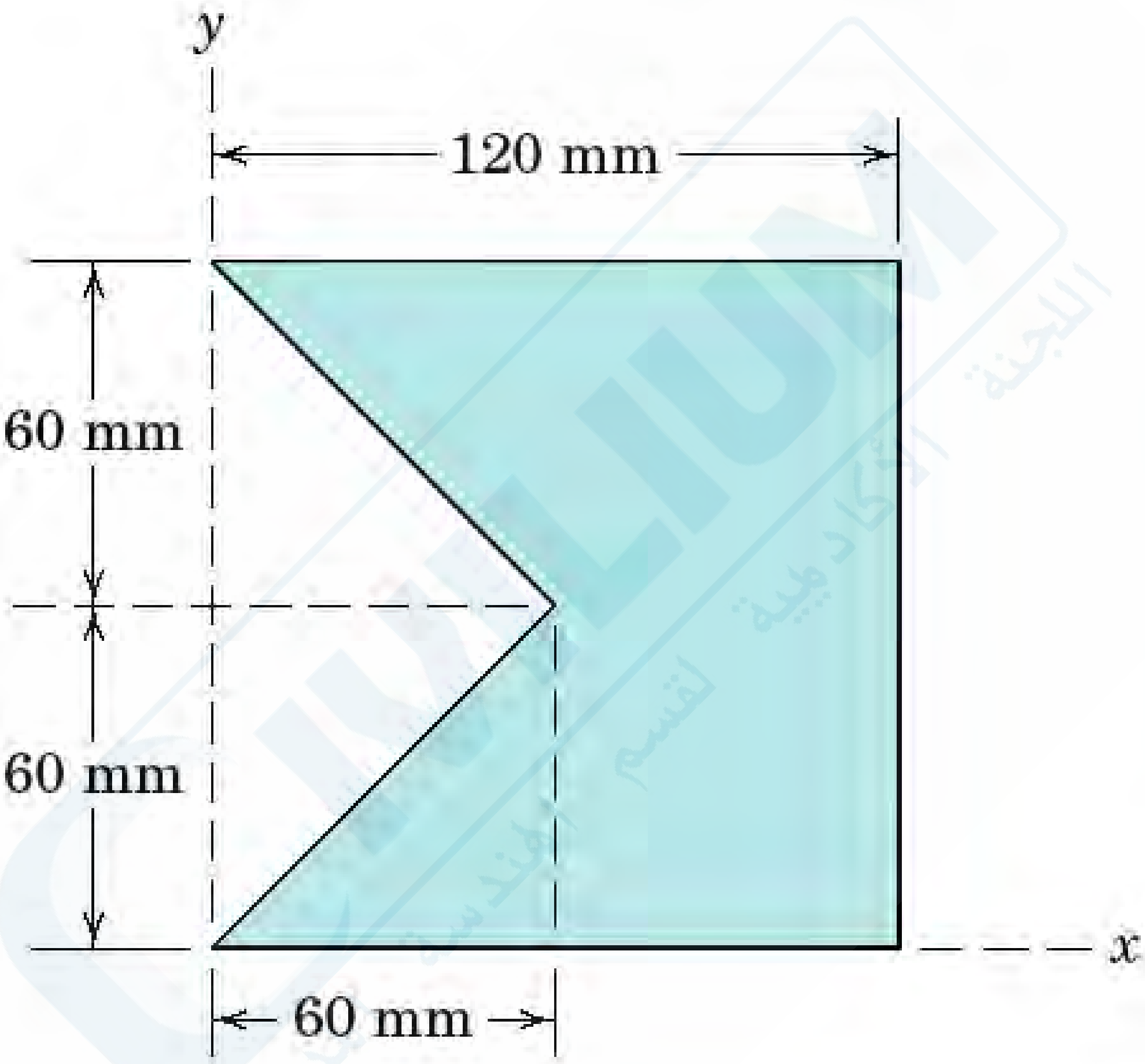
Problem A/42

P A 42



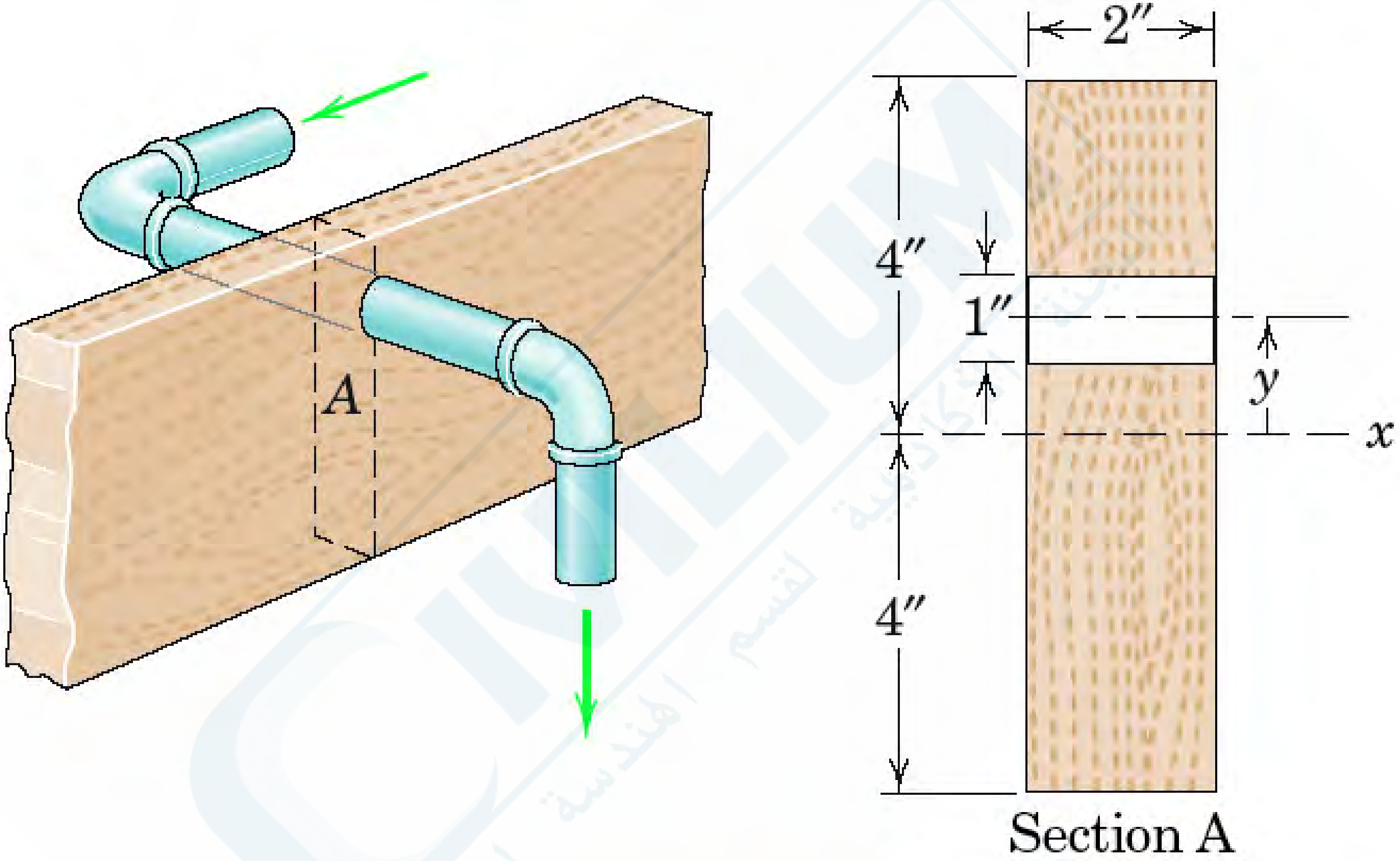
Problem A/43

P A 43



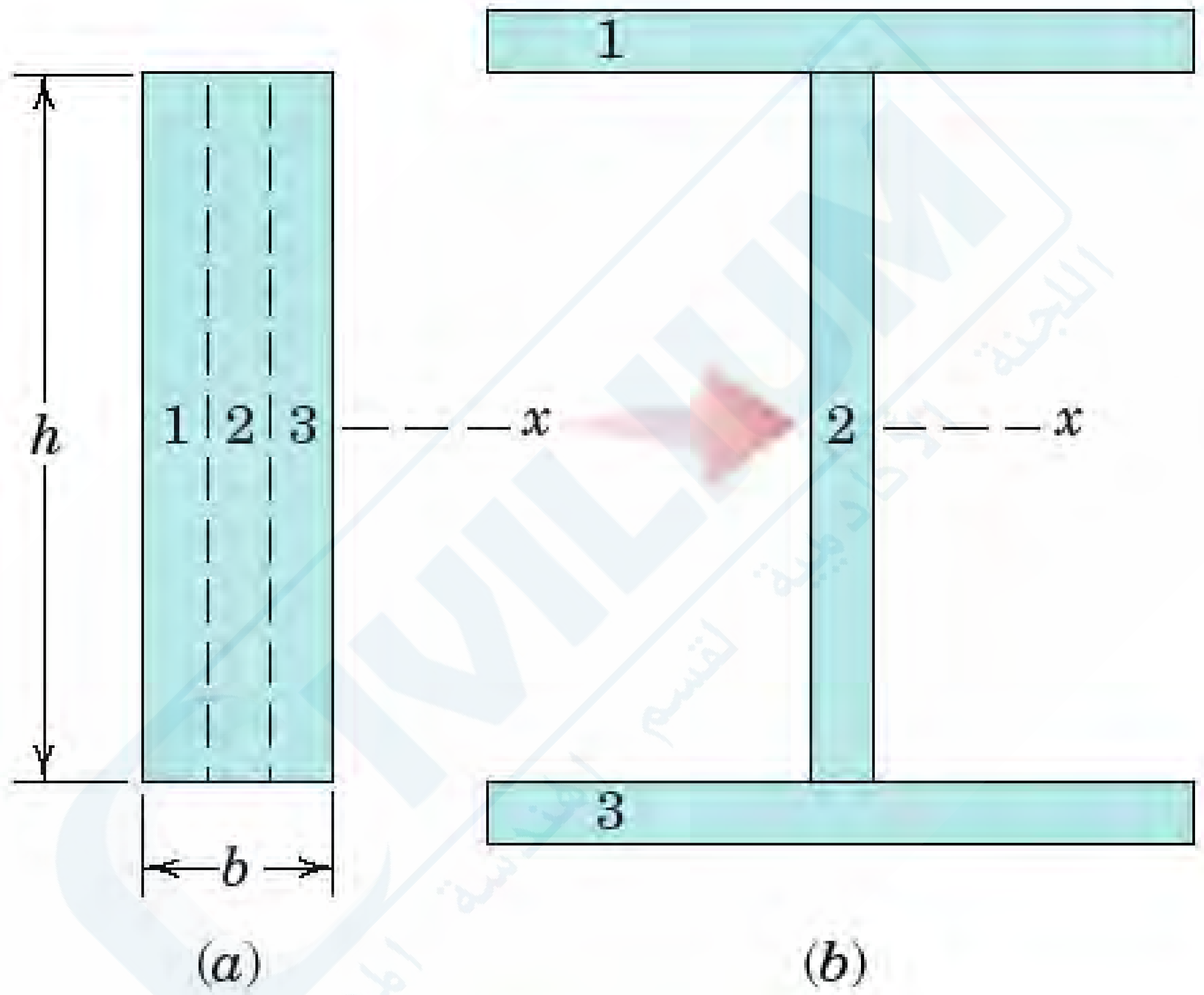
Problem A/44

P A 44



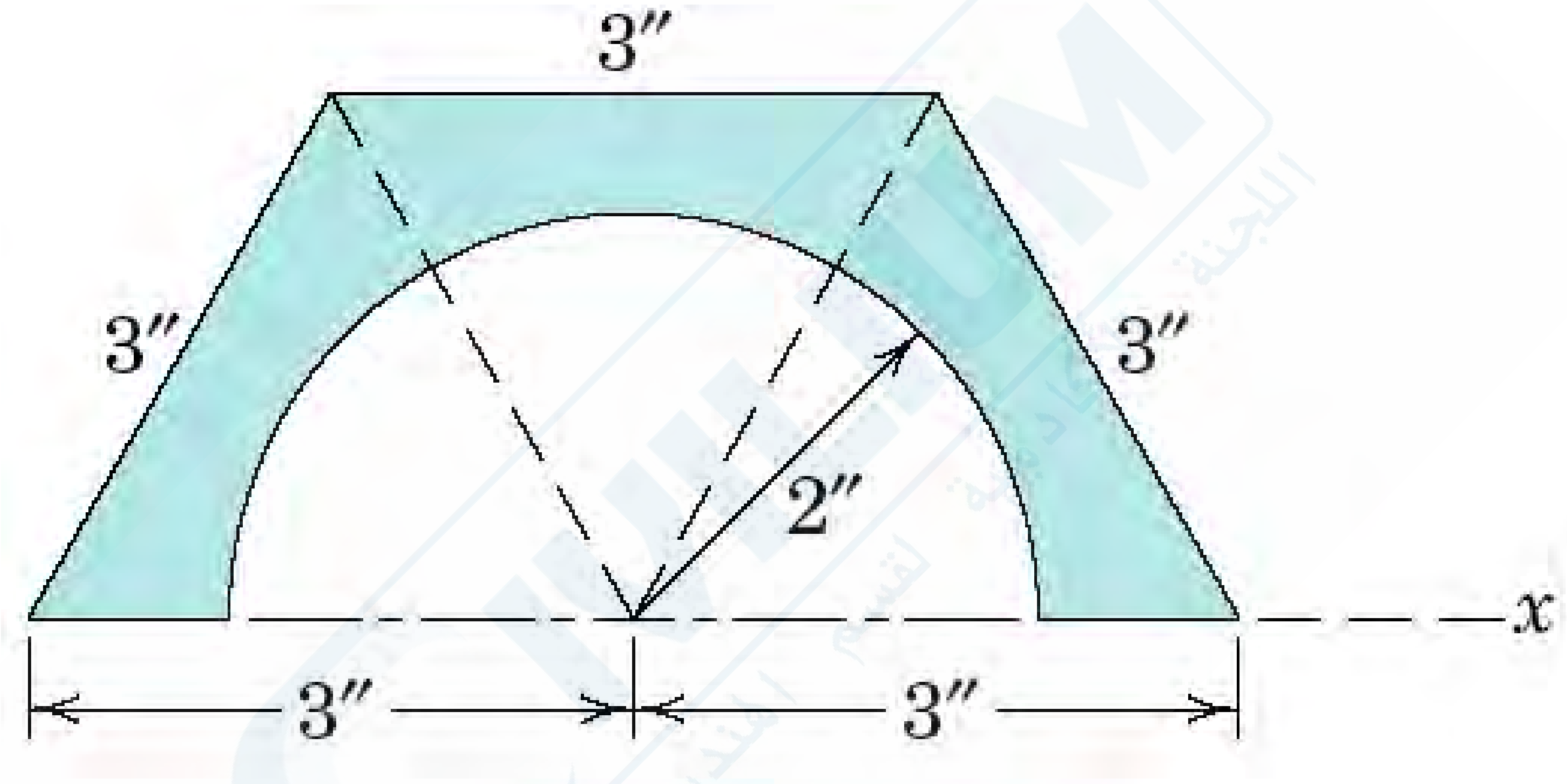
Problem A/45

P_A_45



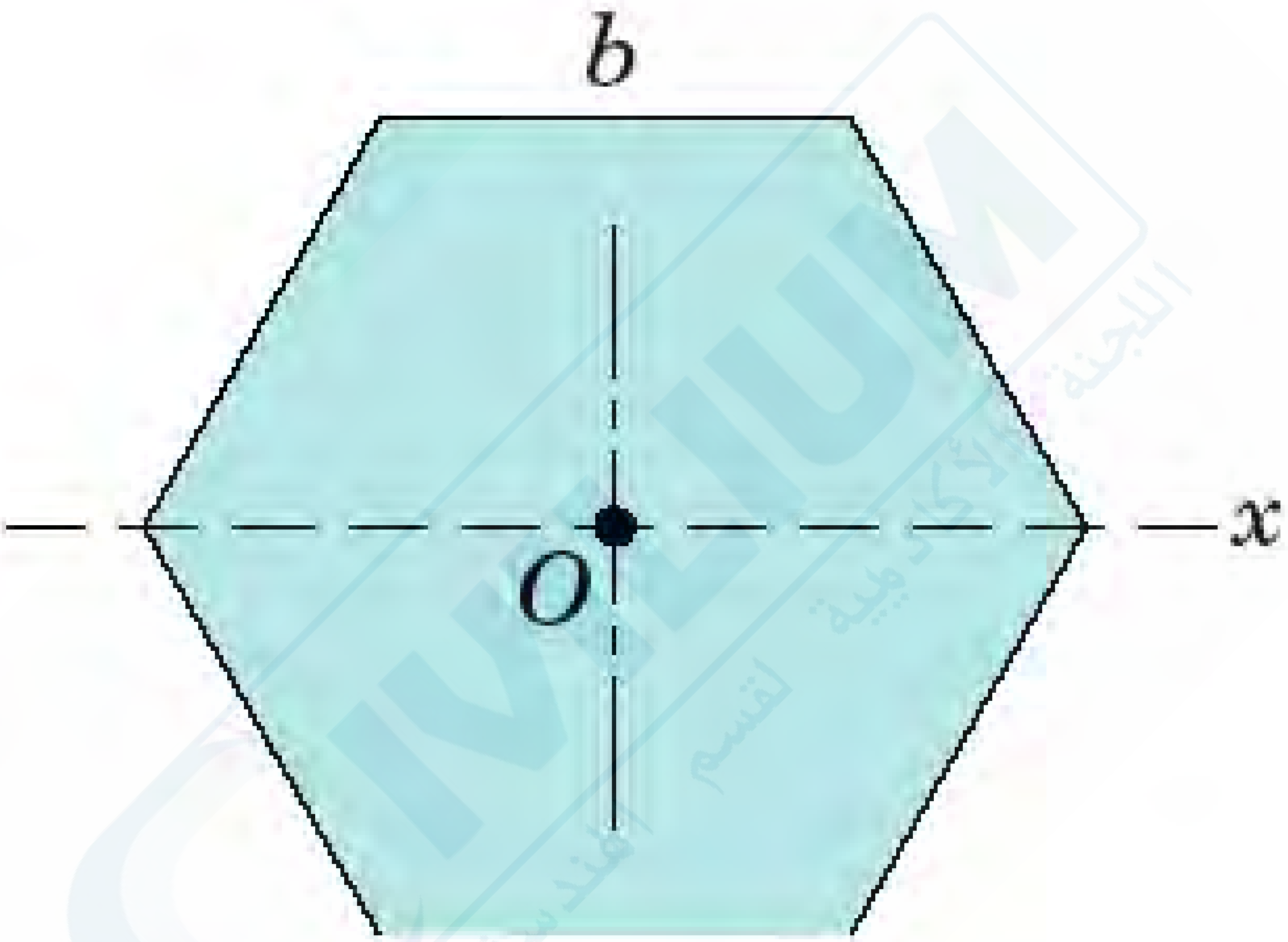
Problem A/46

P A 46



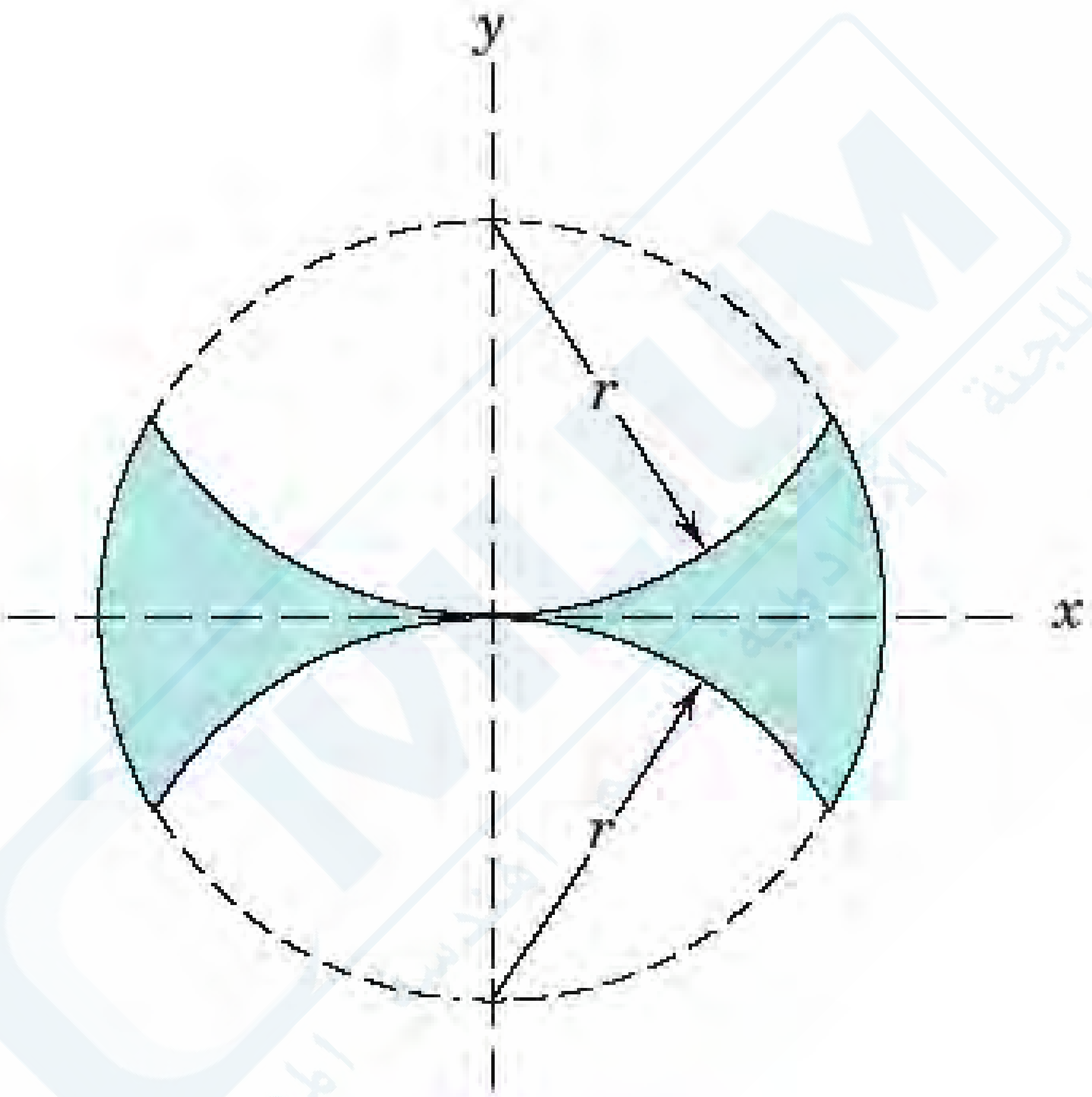
Problem A/47

P_A_47

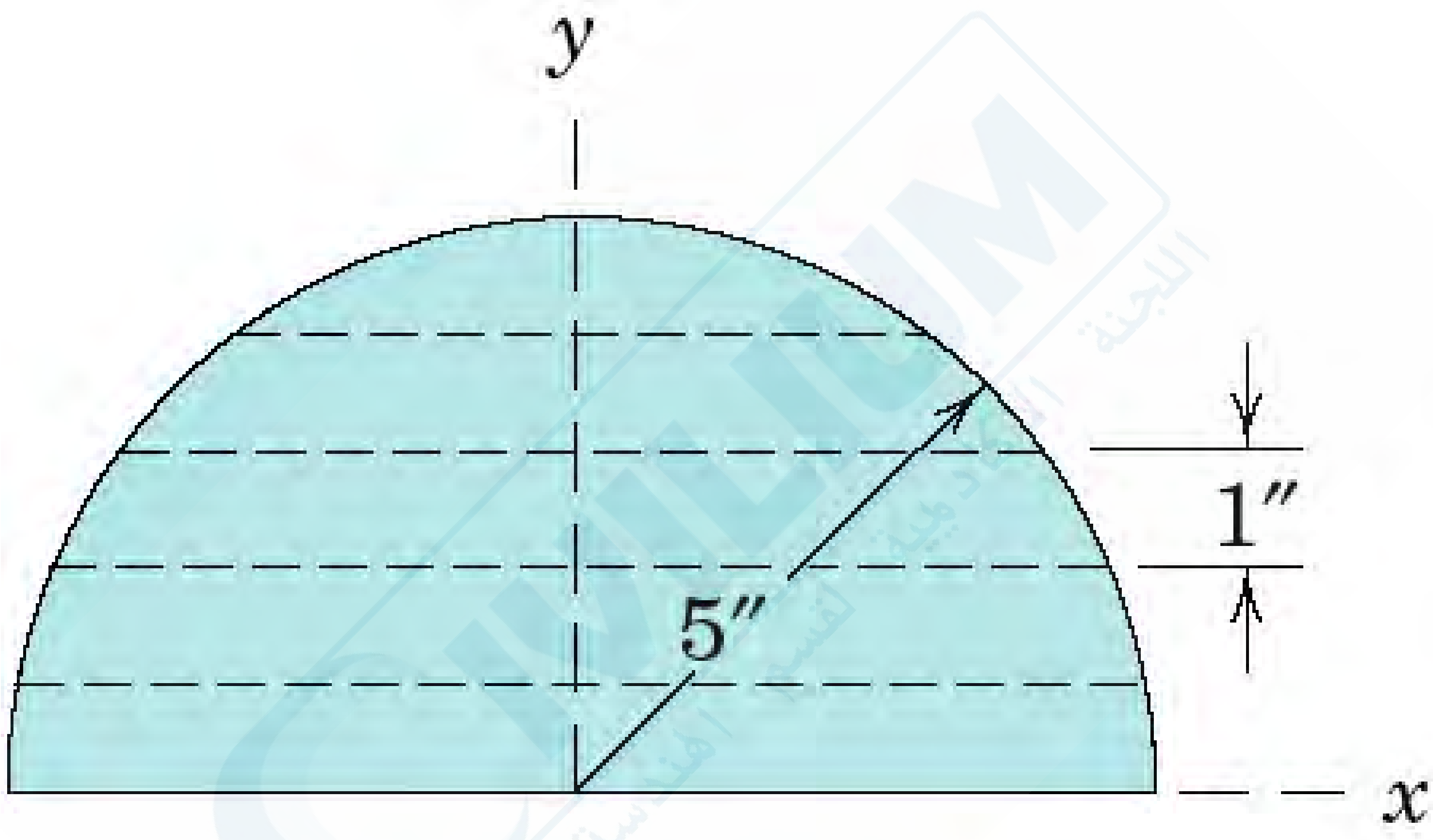


Problem A/48

P A 48

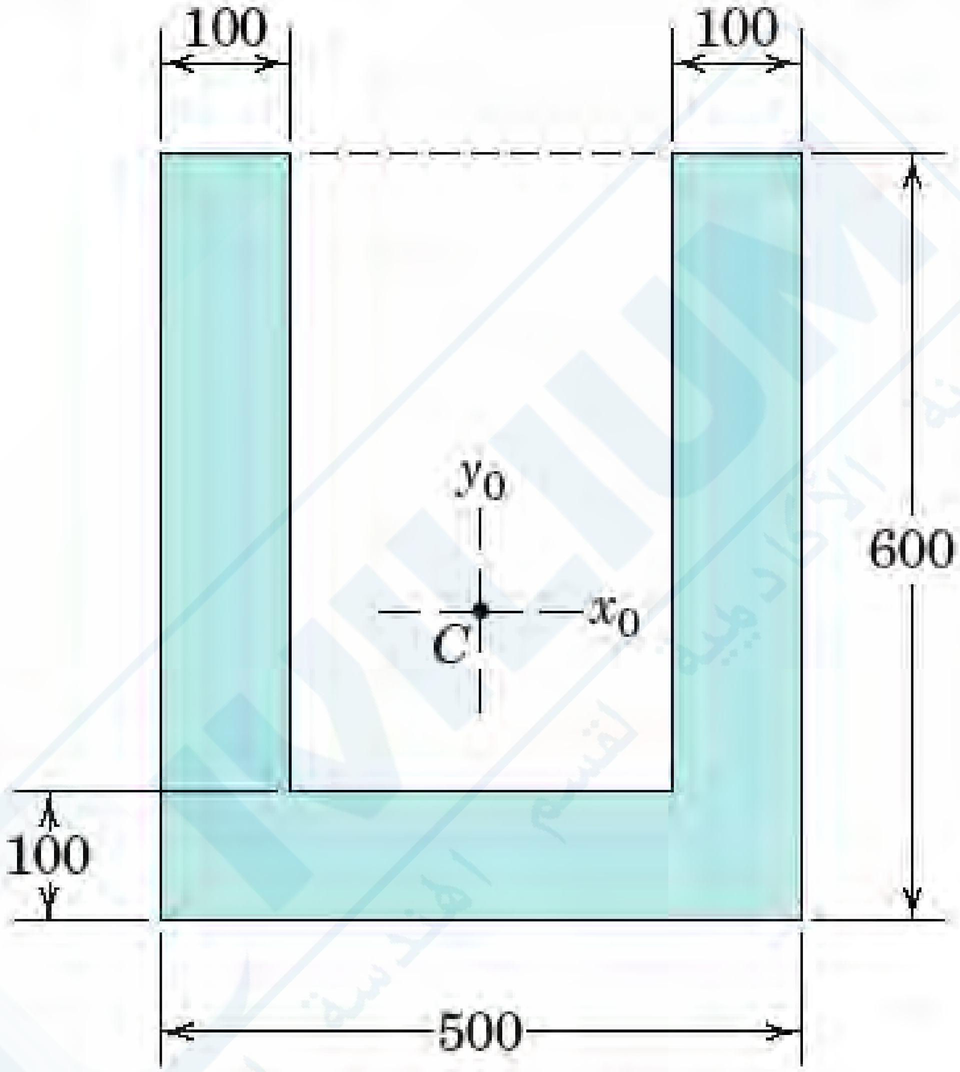


Problem A/49



Problem A/50

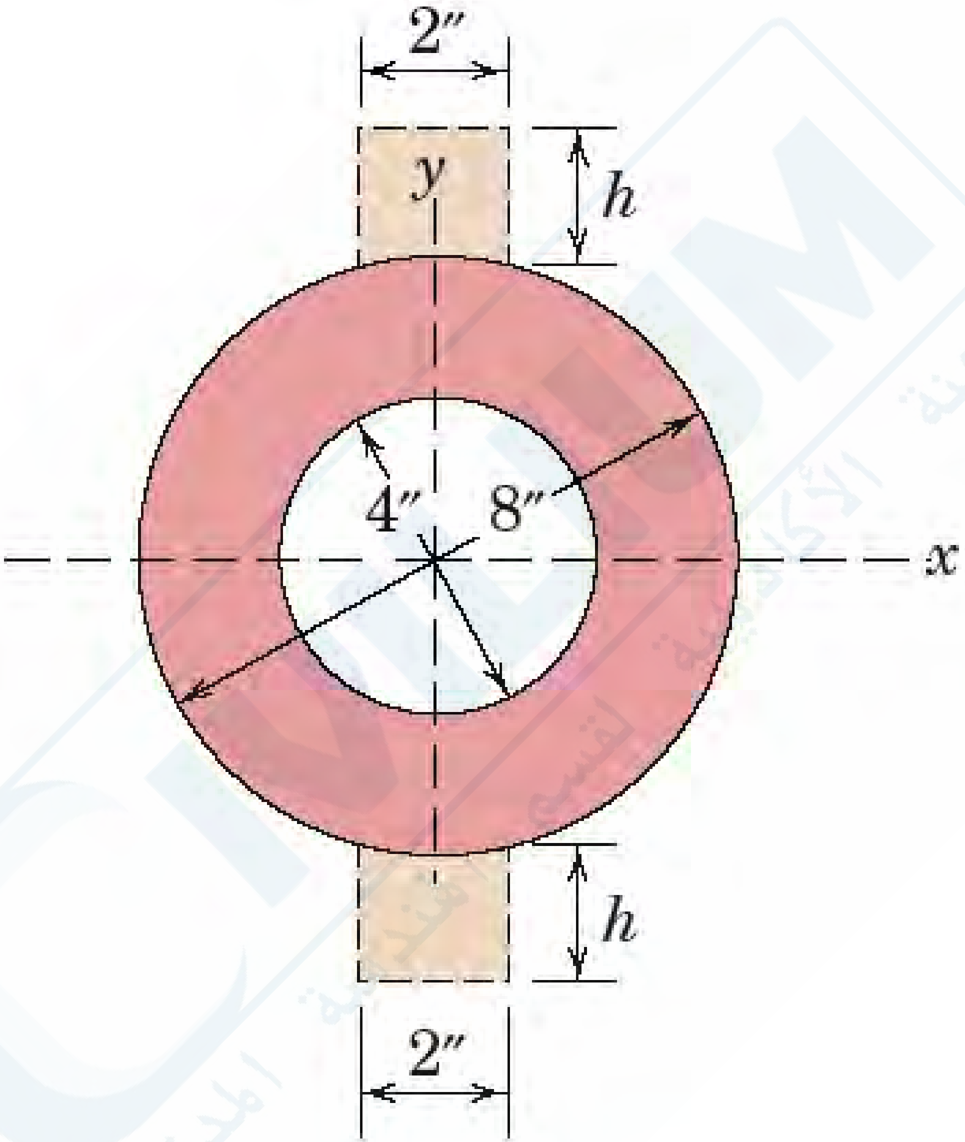
P_A_50



Dimensions in millimeters

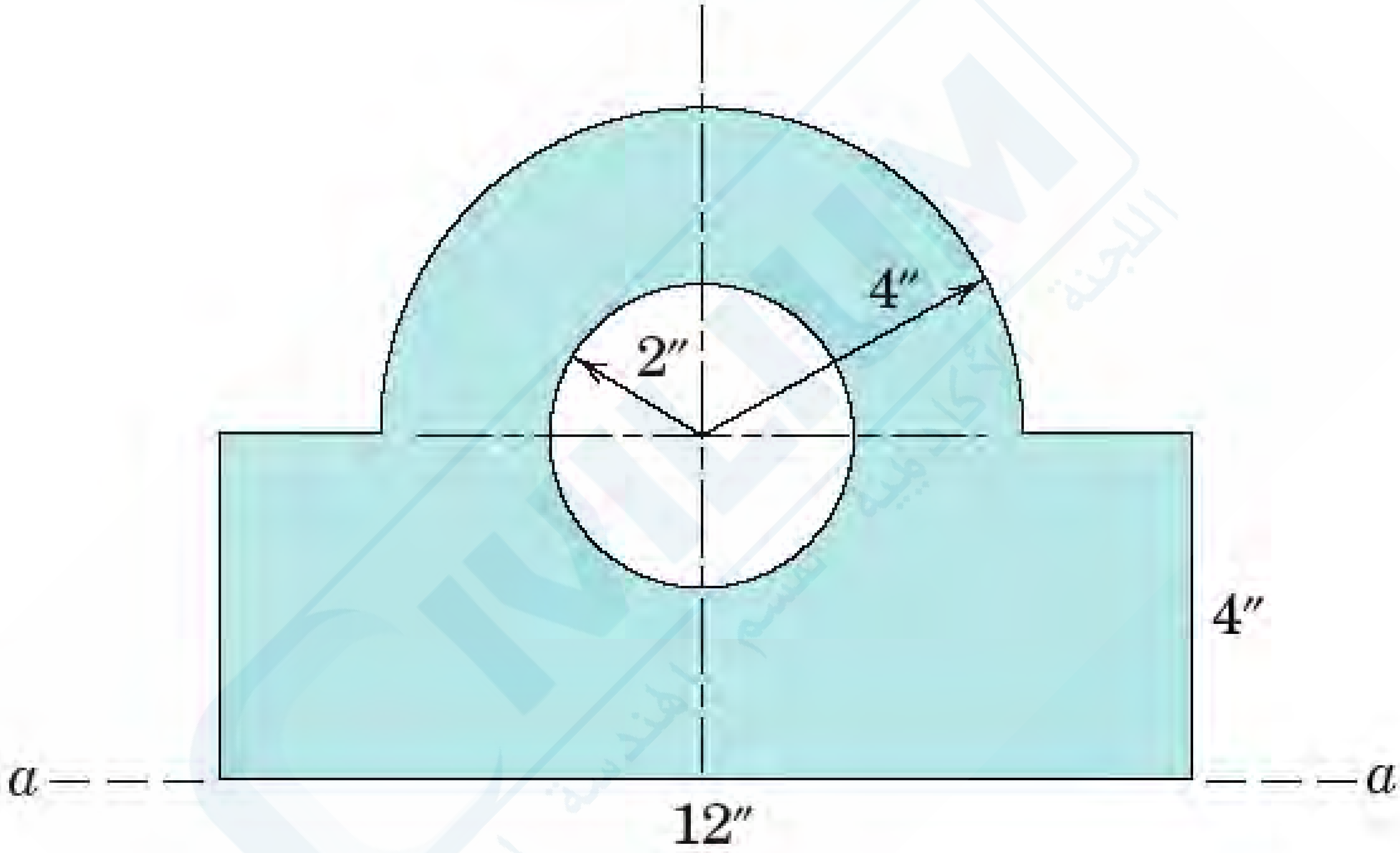
Problem A/51

P A 51



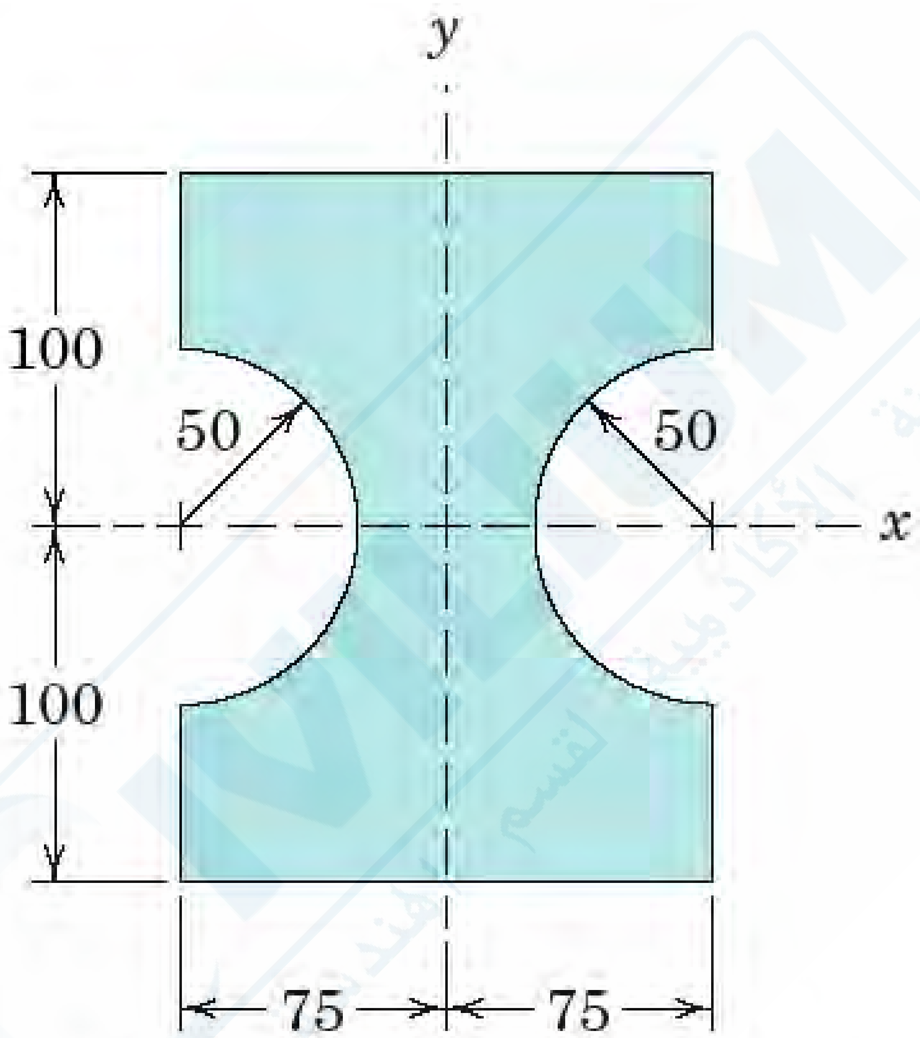
Problem A/52

P A 52



Problem A/53

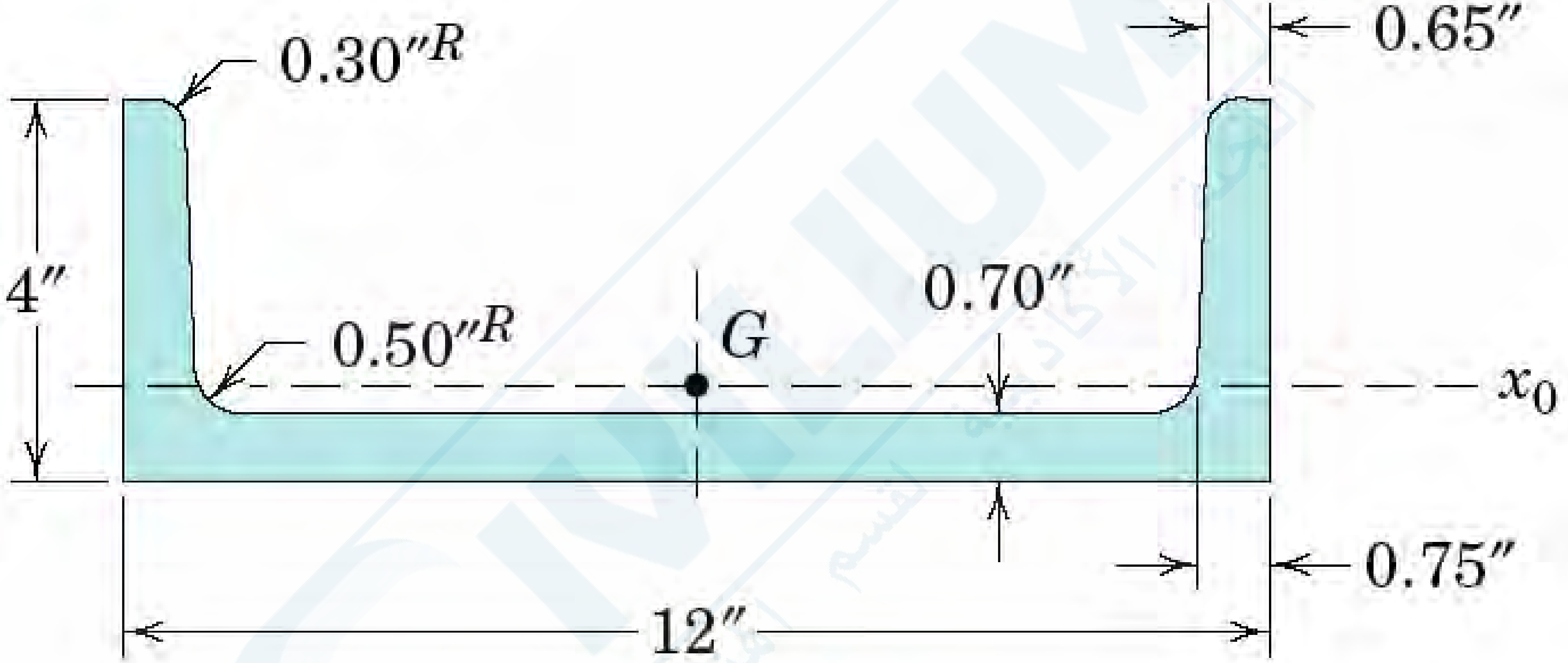
P_A_53



Dimensions in millimeters

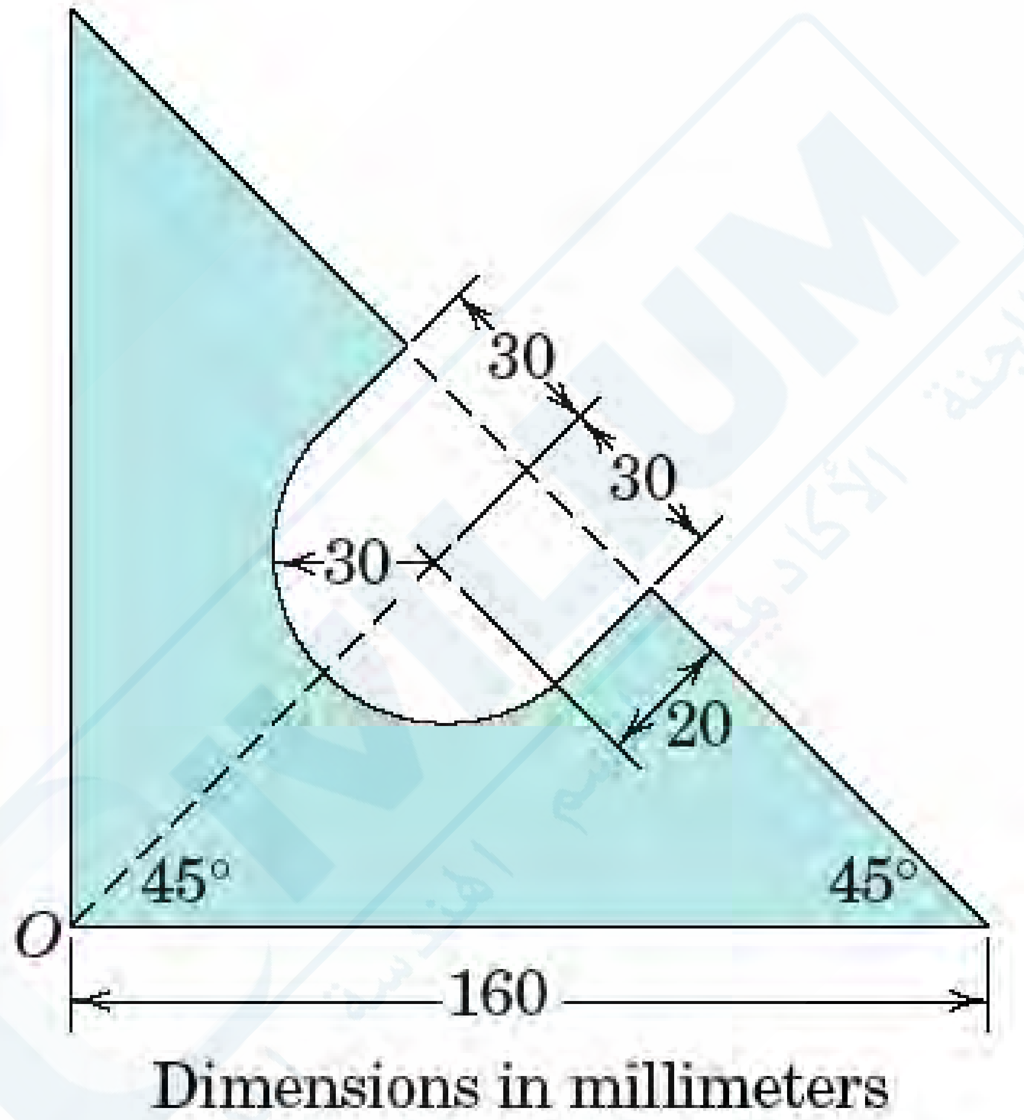
Problem A/54

P A 54

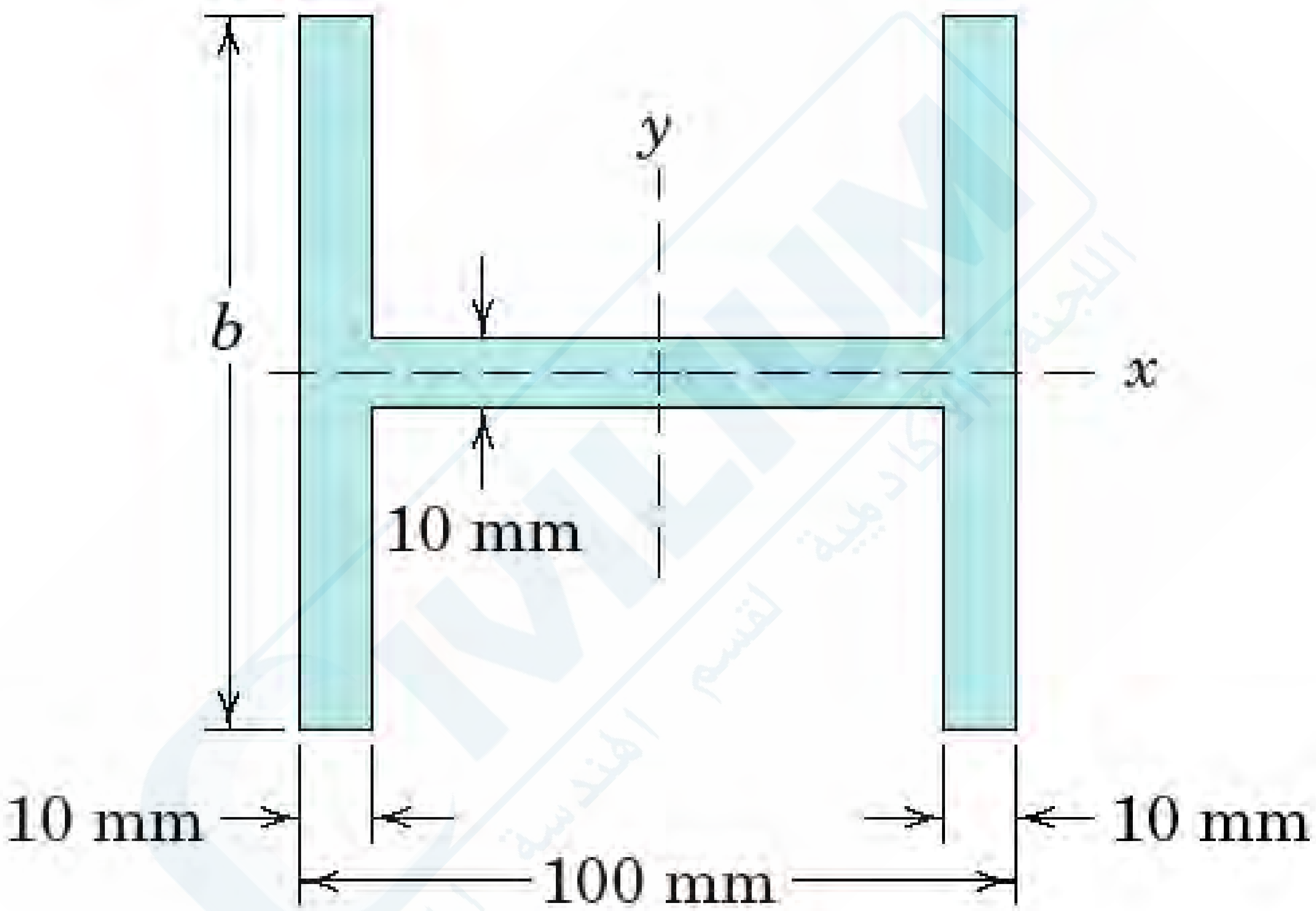


Problem A/55

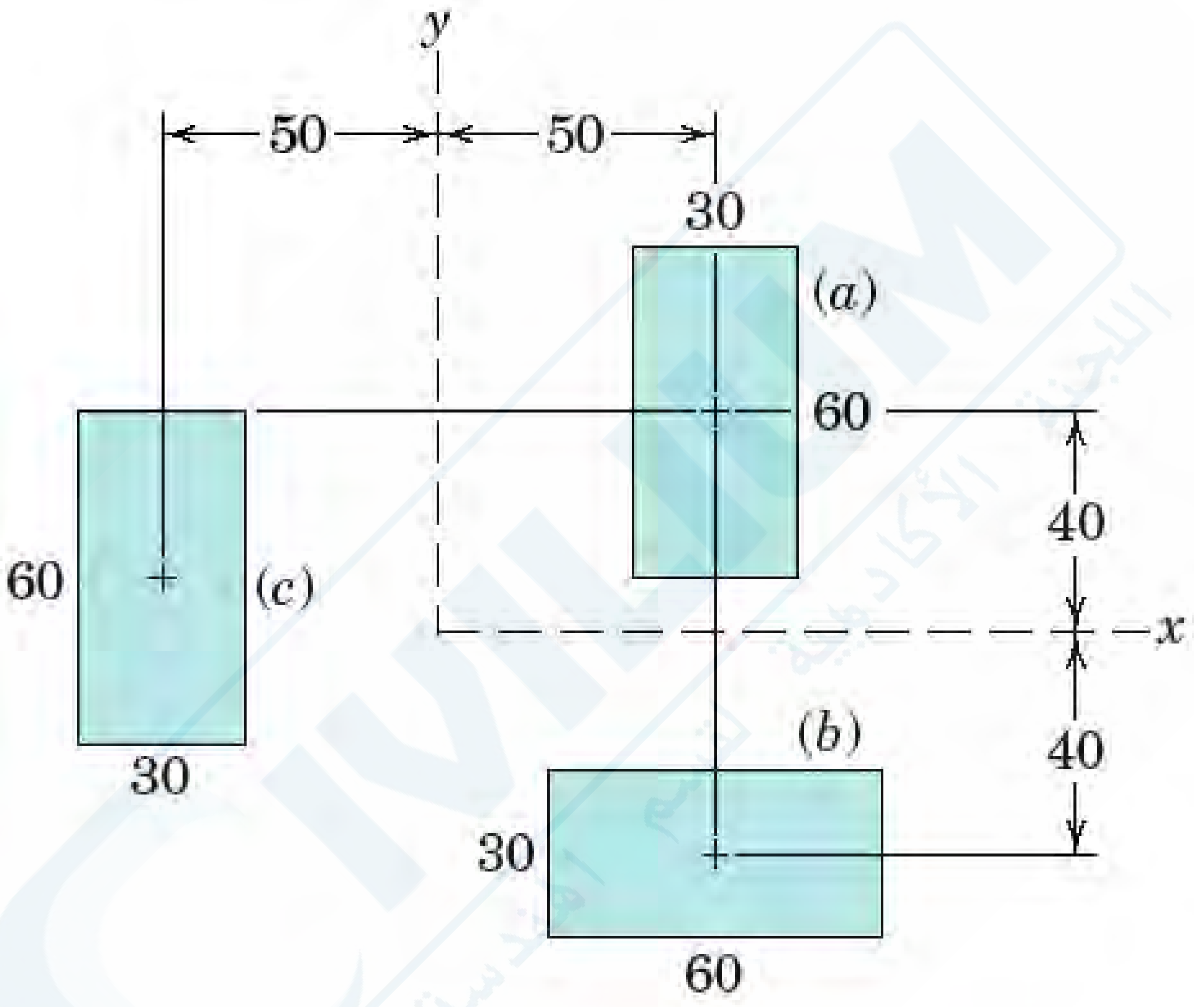
P_A_55



Problem A/56

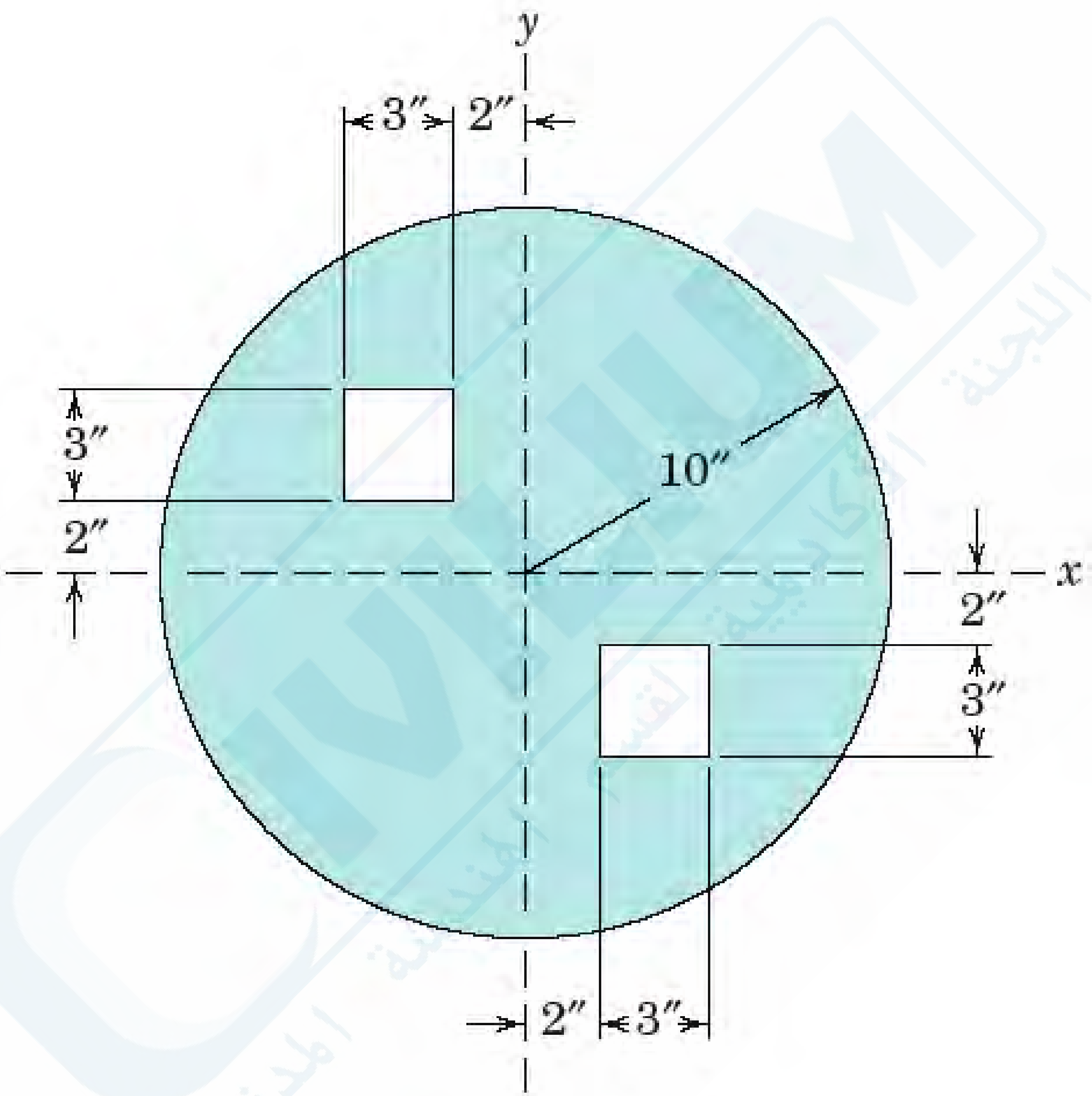


Problem A/58



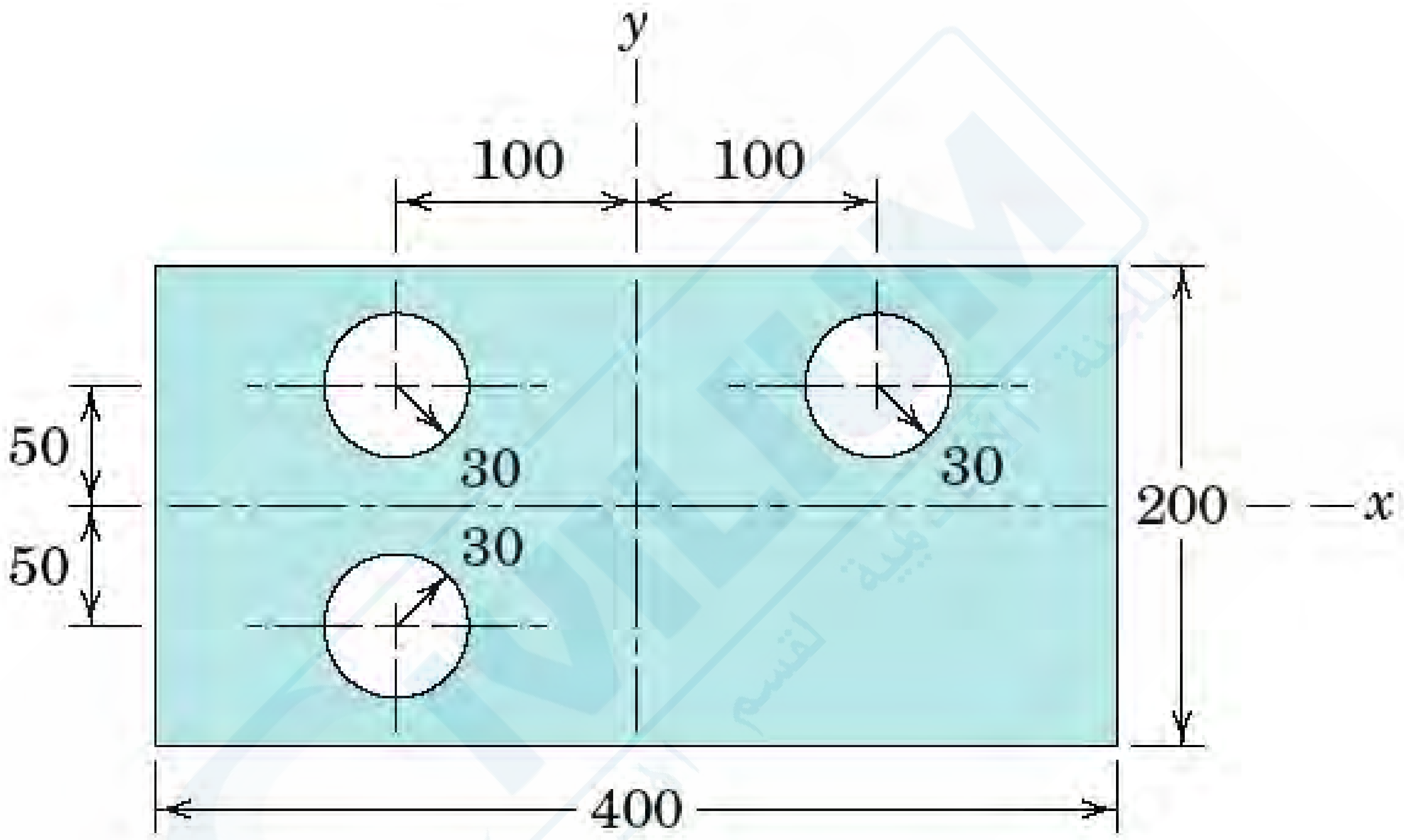
Dimensions in millimeters

Problem A/59



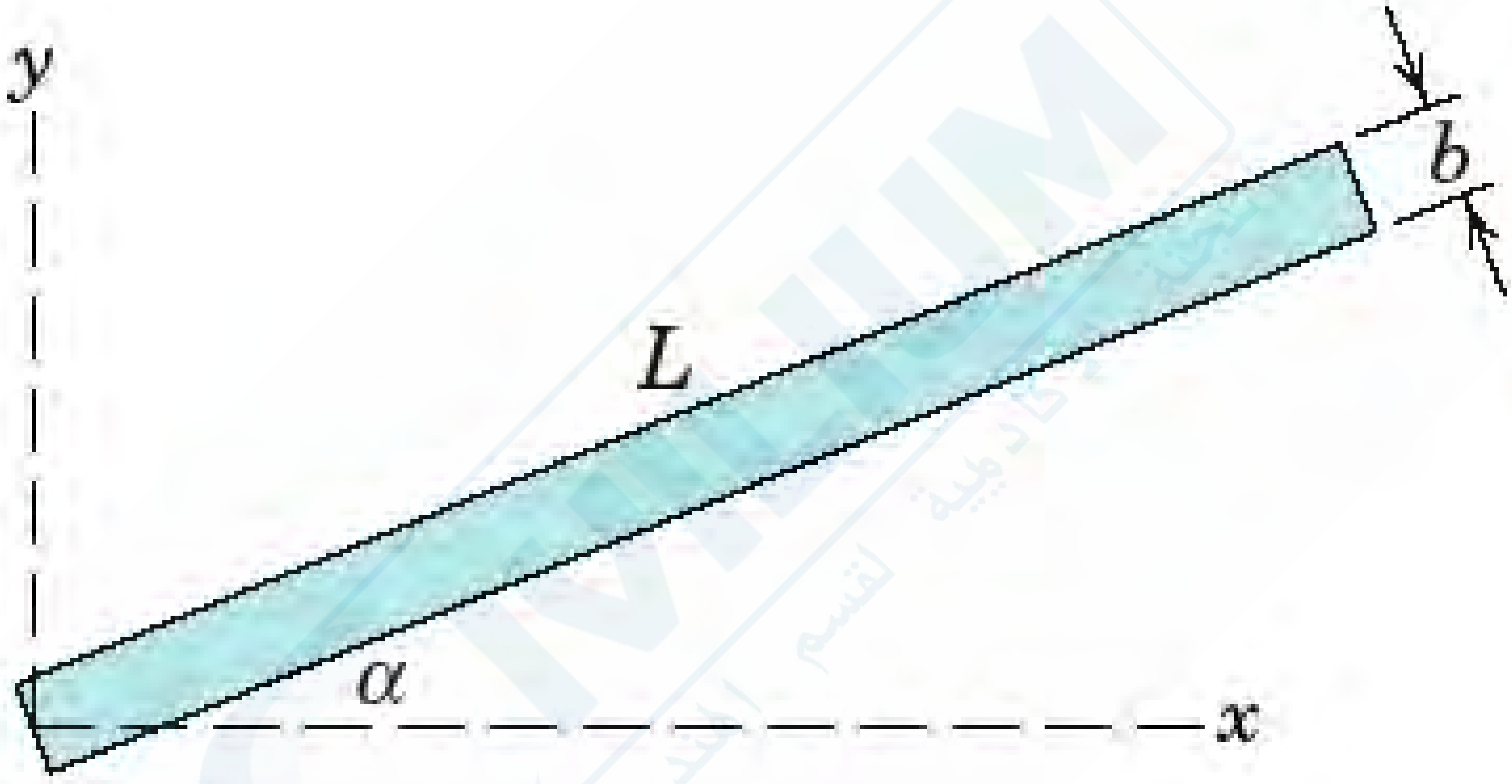
Problem A/60

P A 60



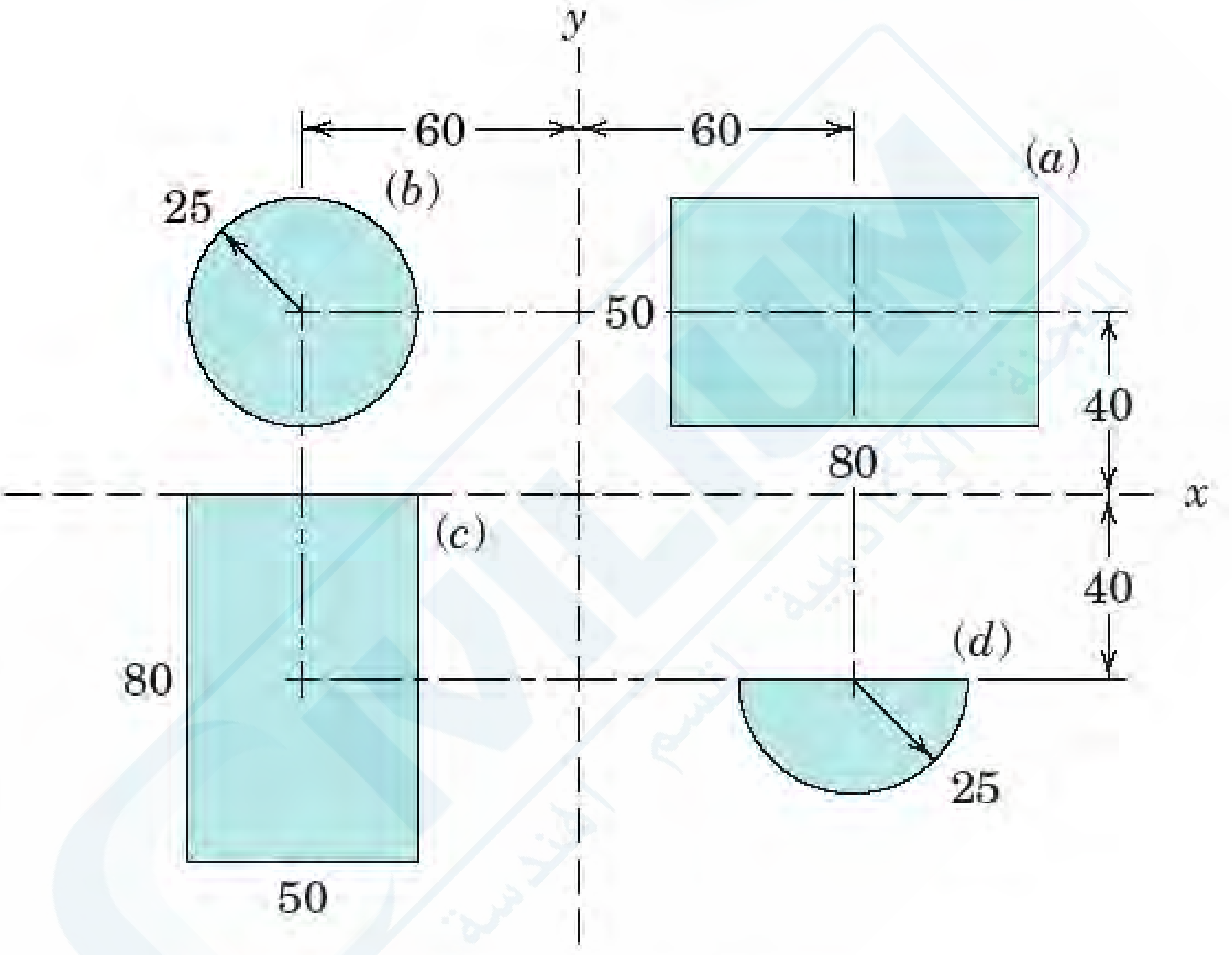
Problem A/61

P A 61



Problem A/62

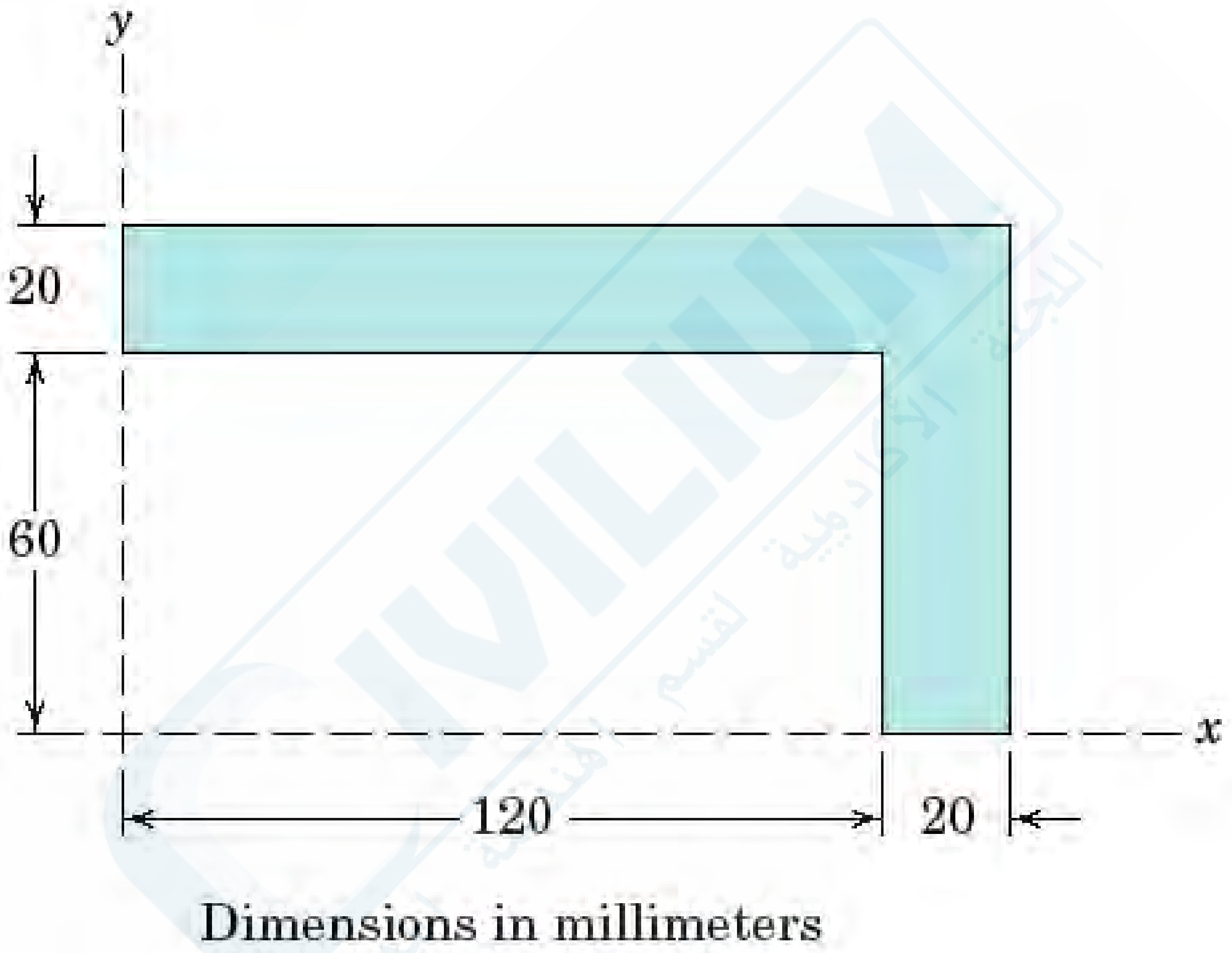
P_A_62



Dimensions in millimeters

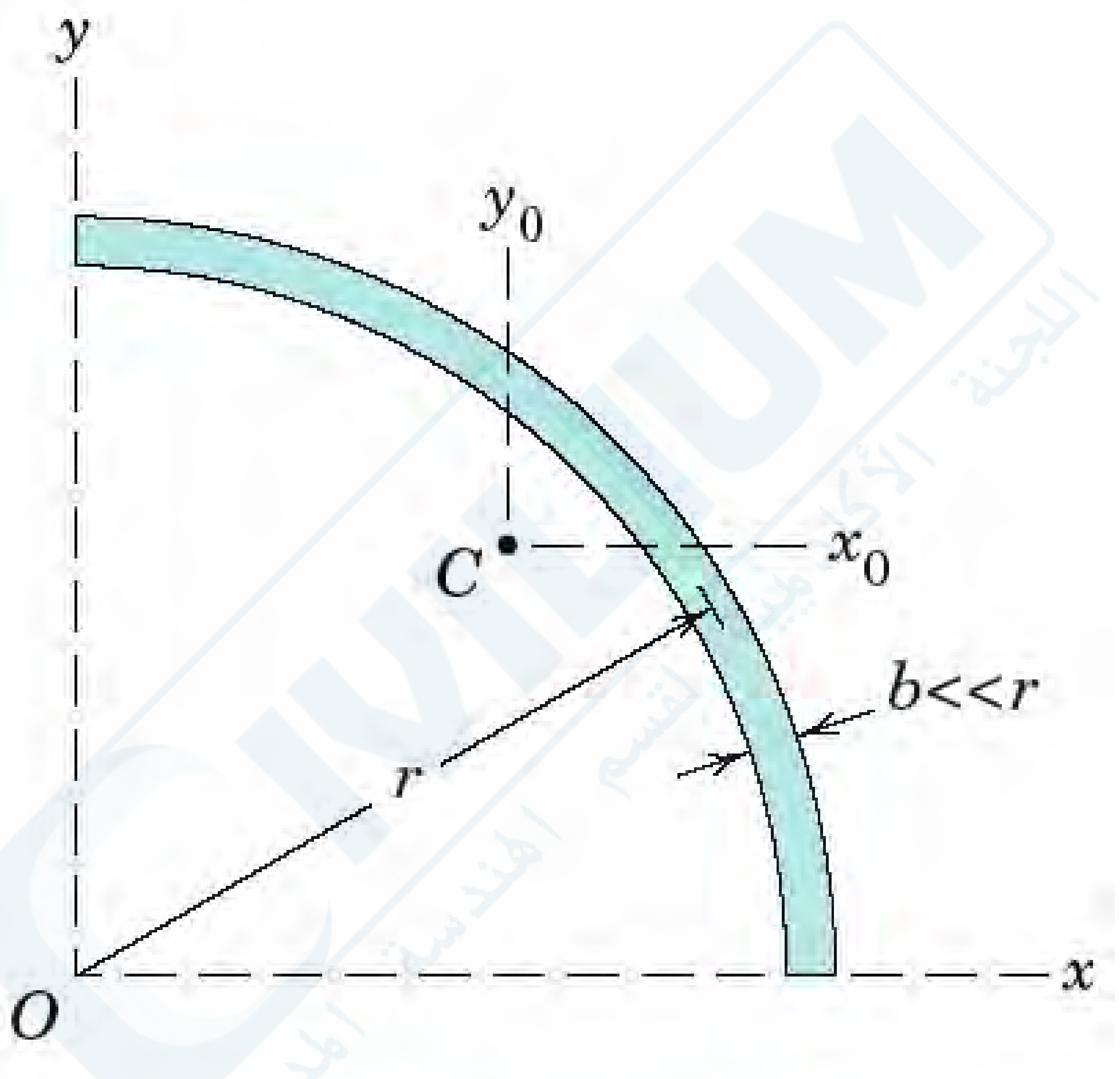
Problem A/63

P A 63



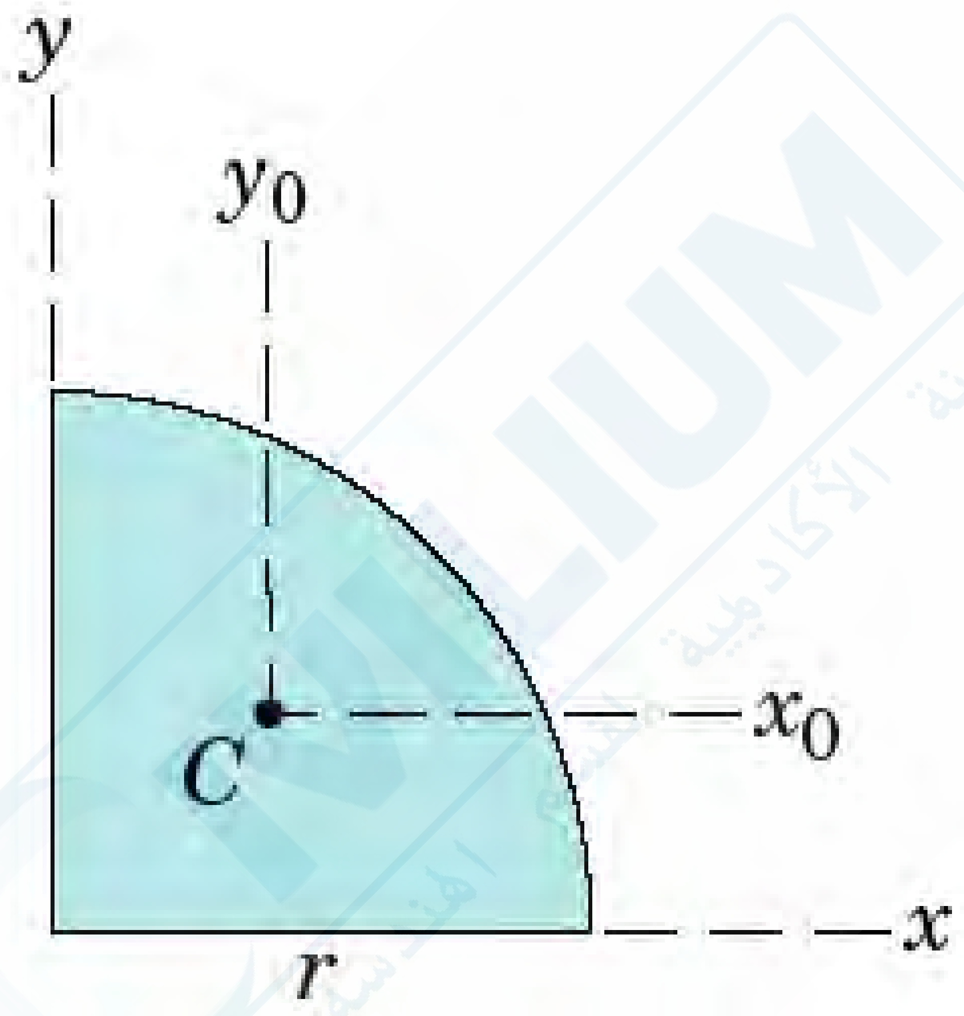
Problem A/64

P A 64



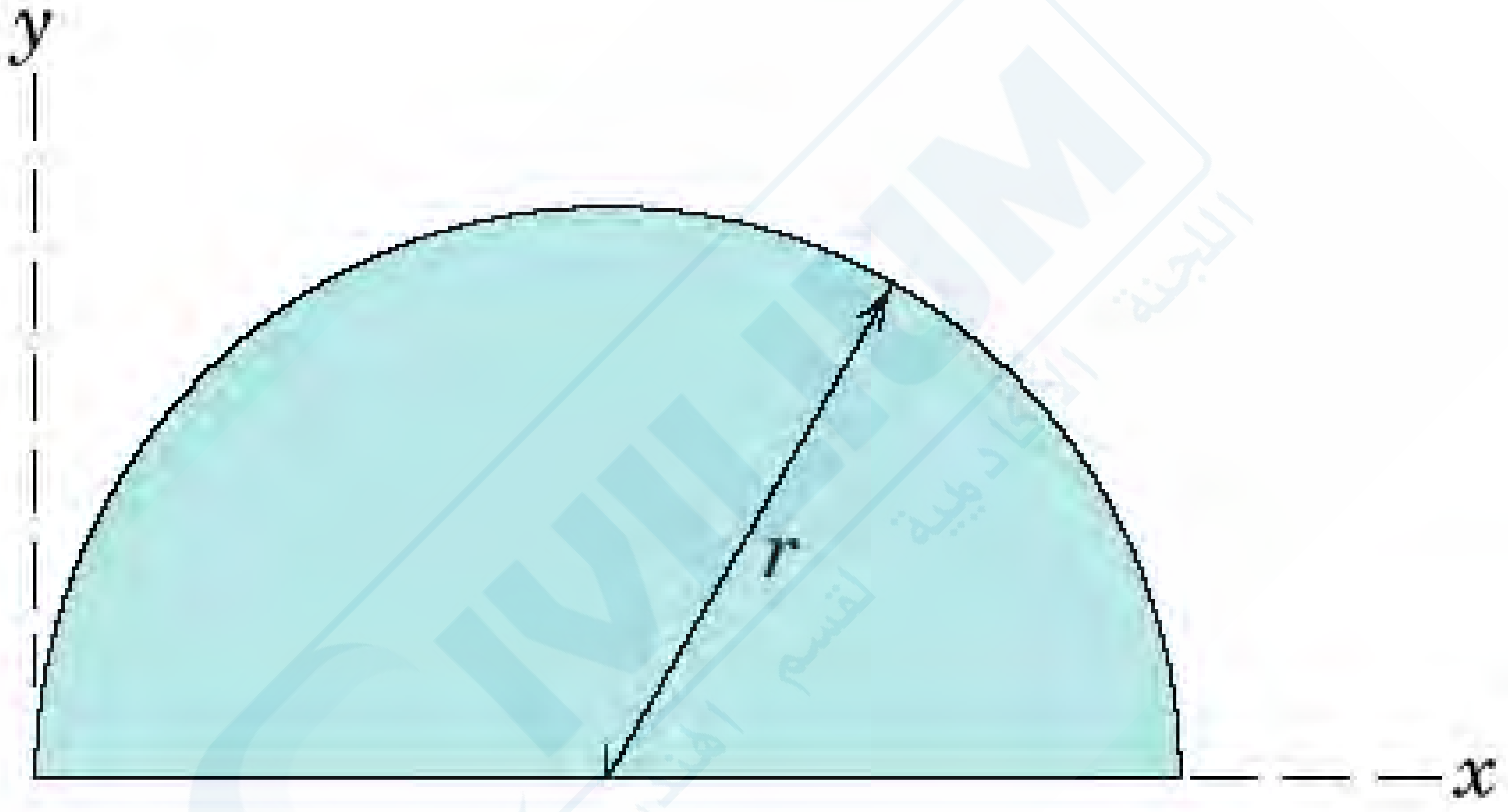
Problem A/65

P A 65



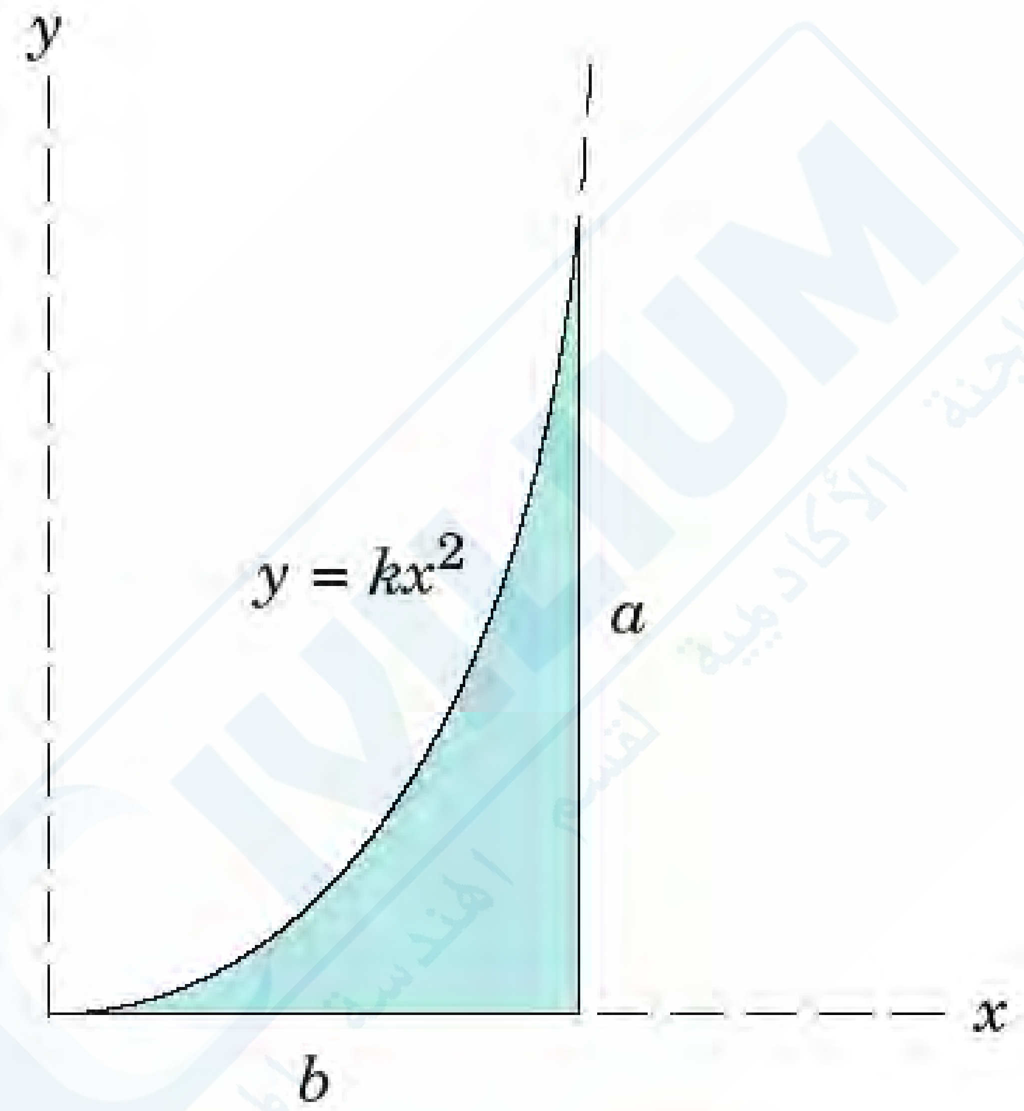
Problem A/66

P A 66



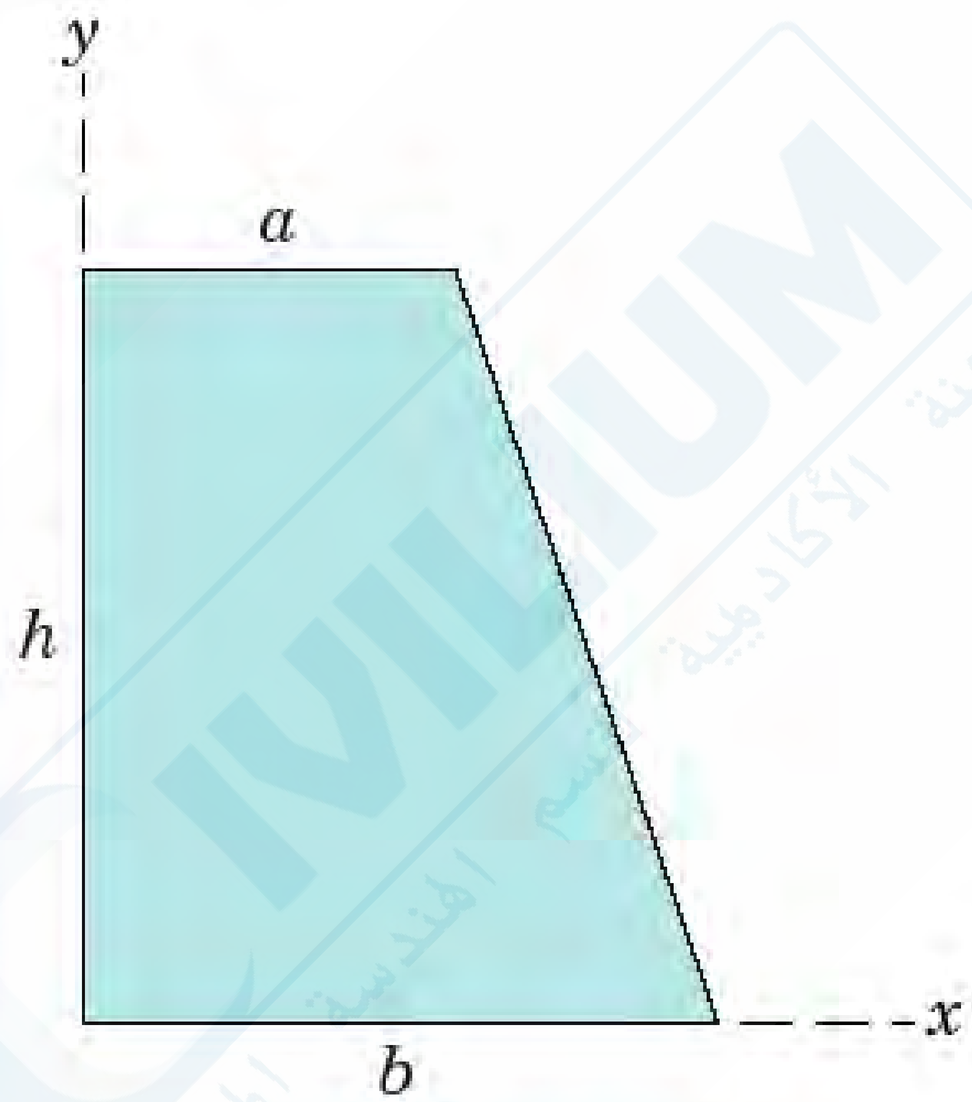
Problem A/67

P_A_67



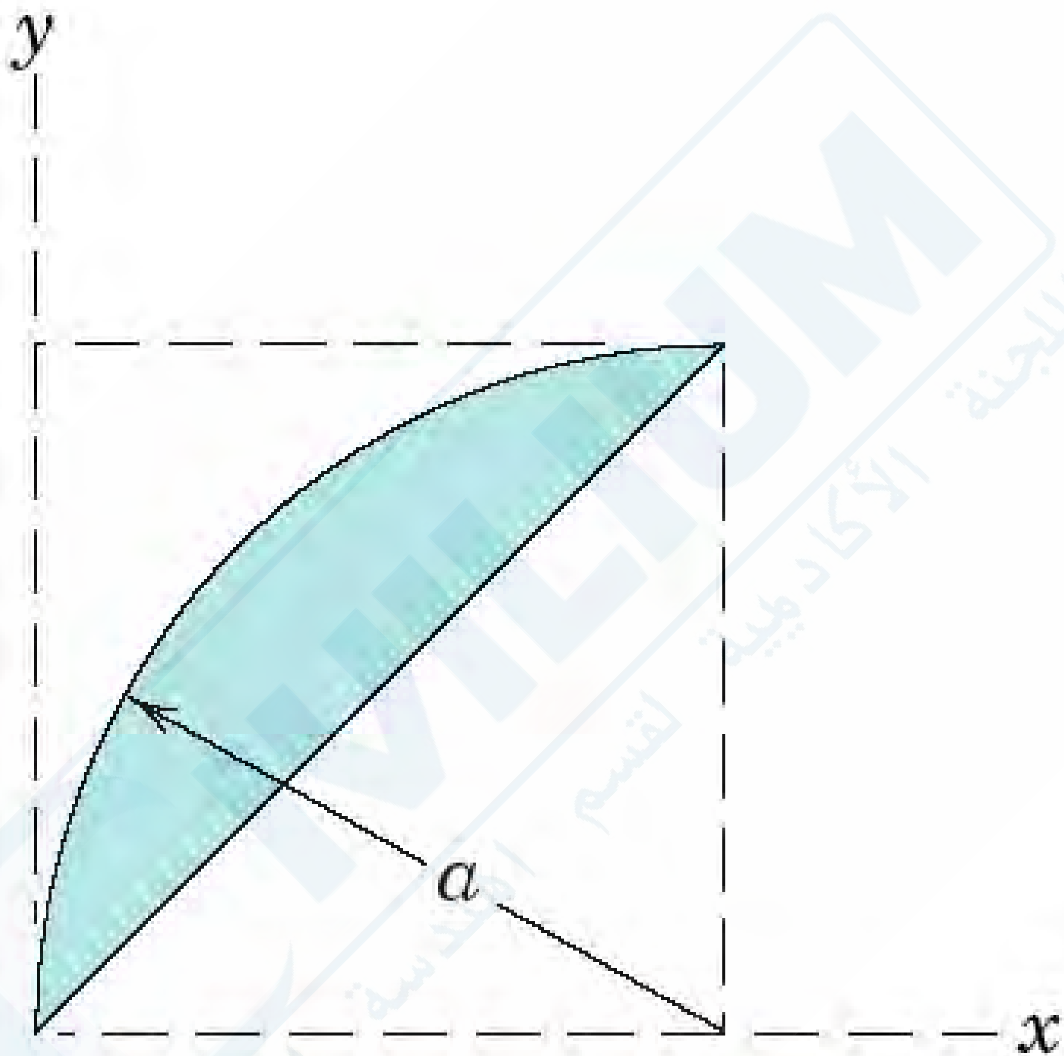
Problem A/68

P A 68



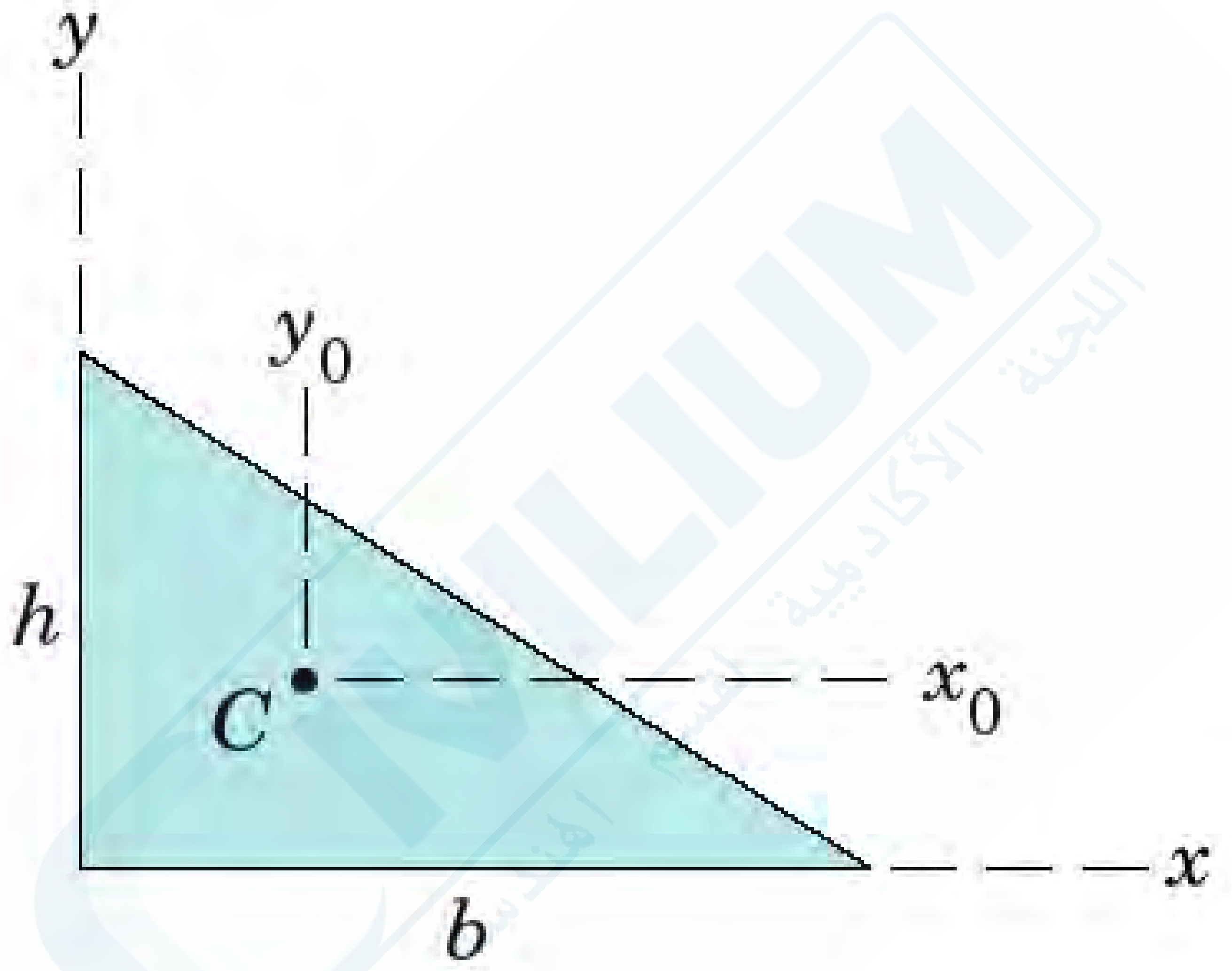
Problem A/69

P A 69

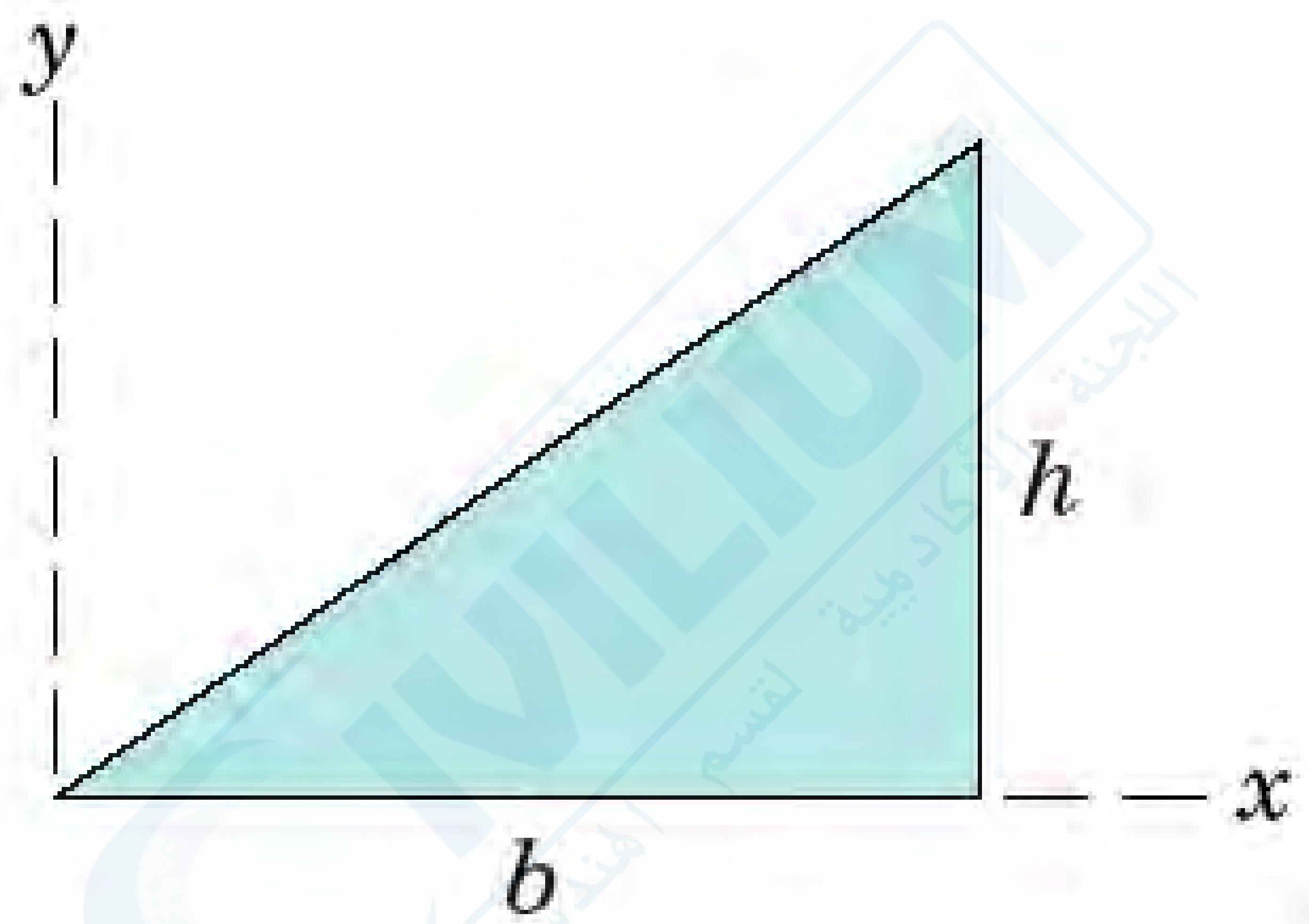


Problem A/70

P A 70

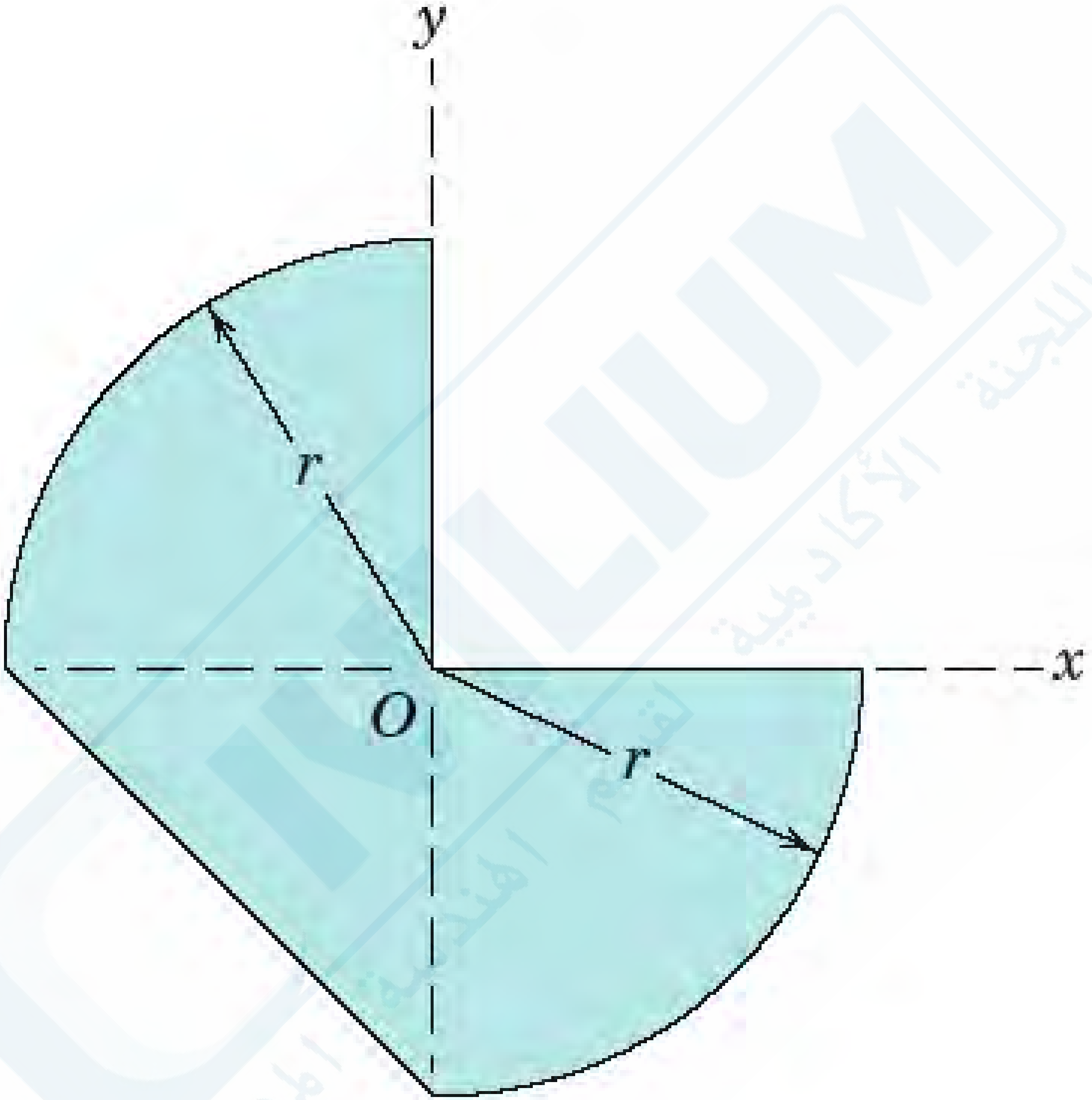


Problem A/71



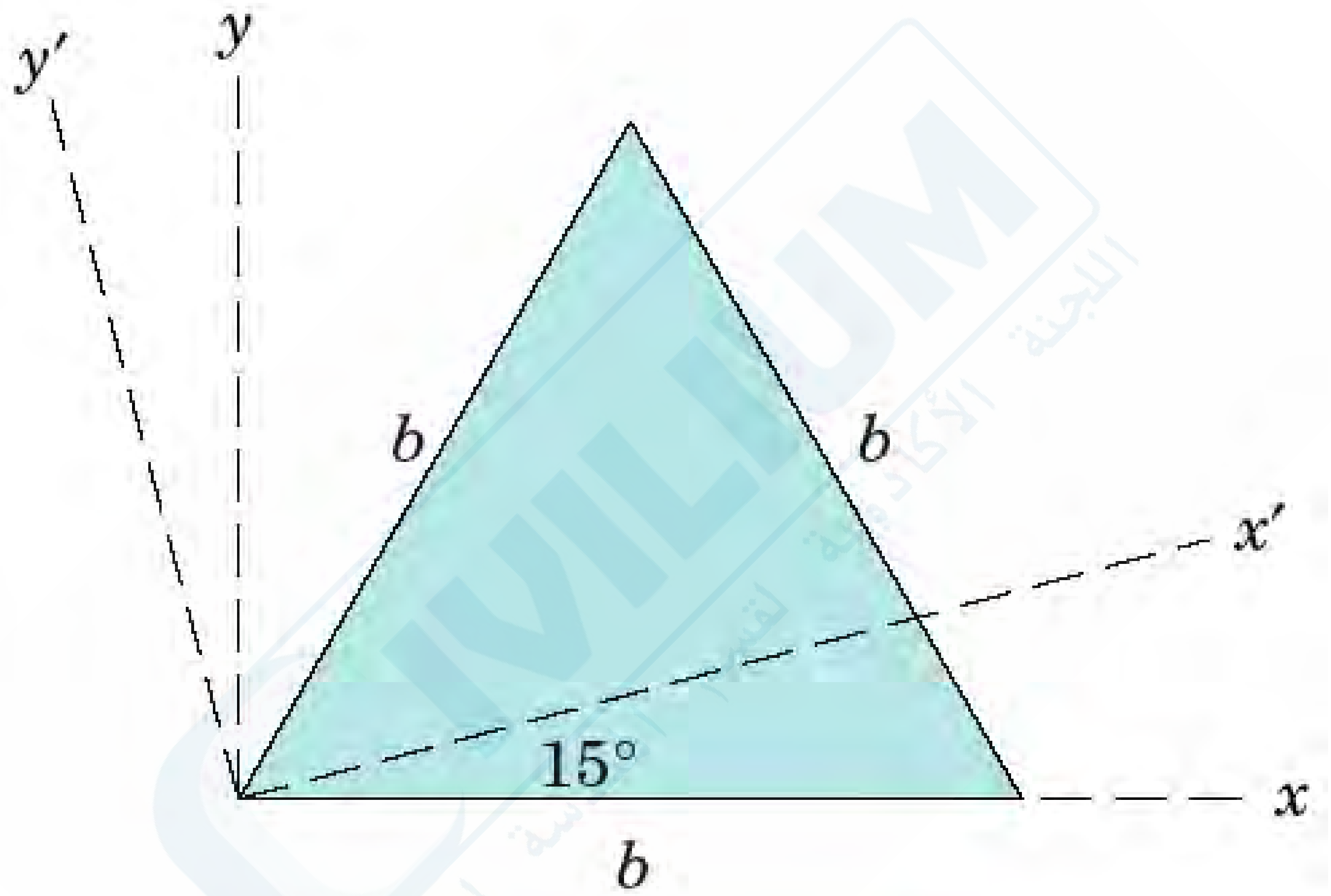
Problem A/72

P A 72



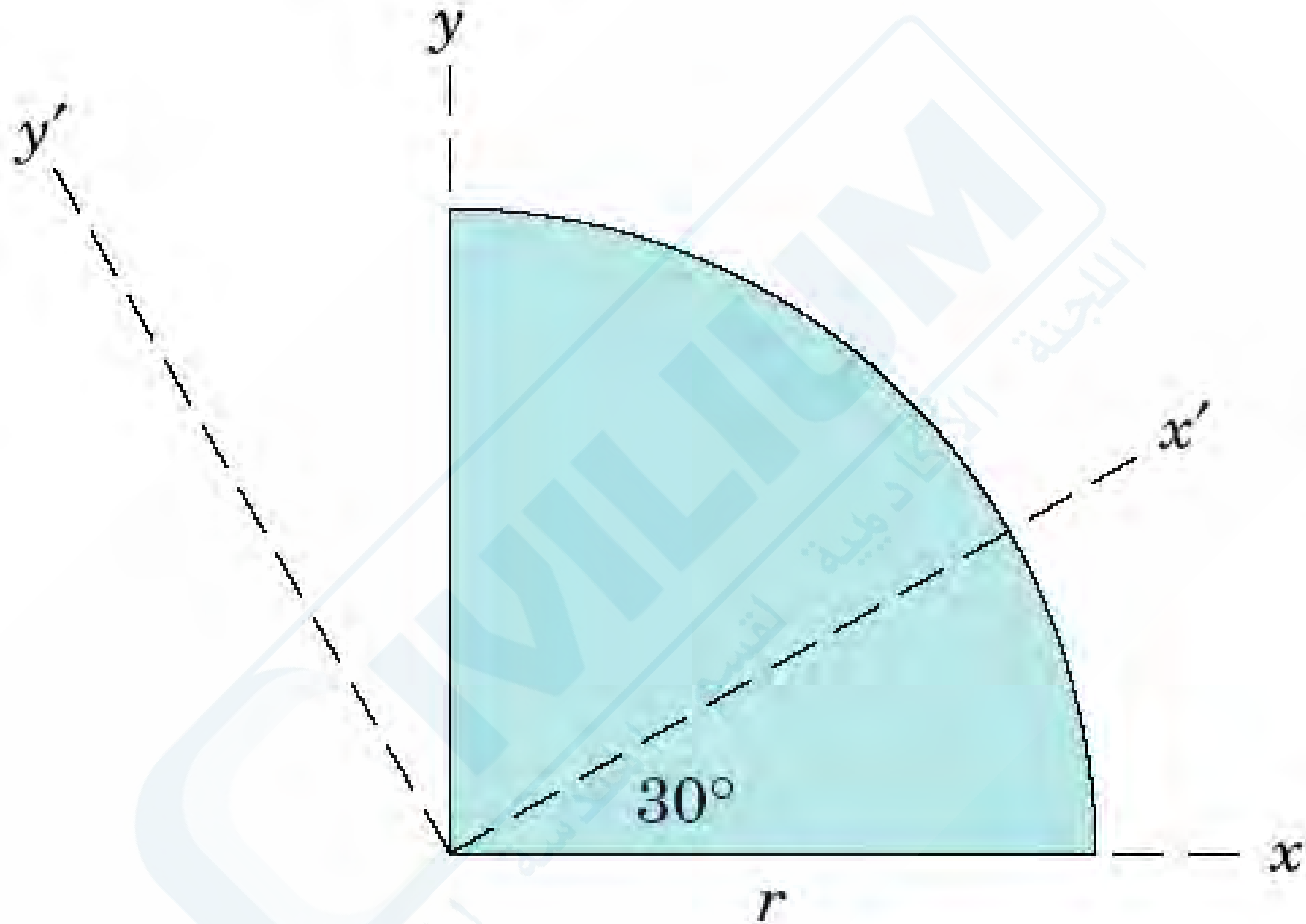
Problem A/73

P A 73



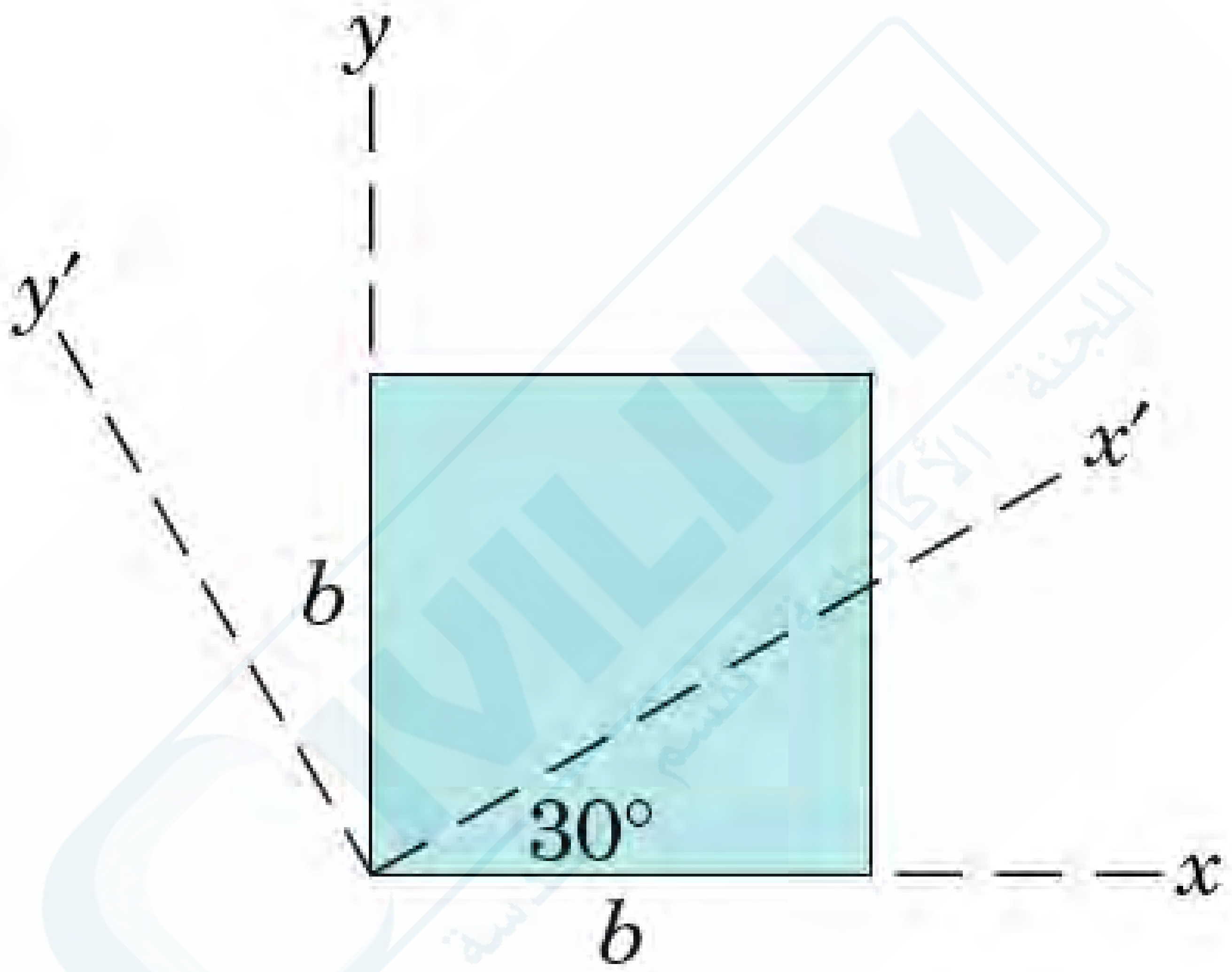
Problem A/74

P A 74



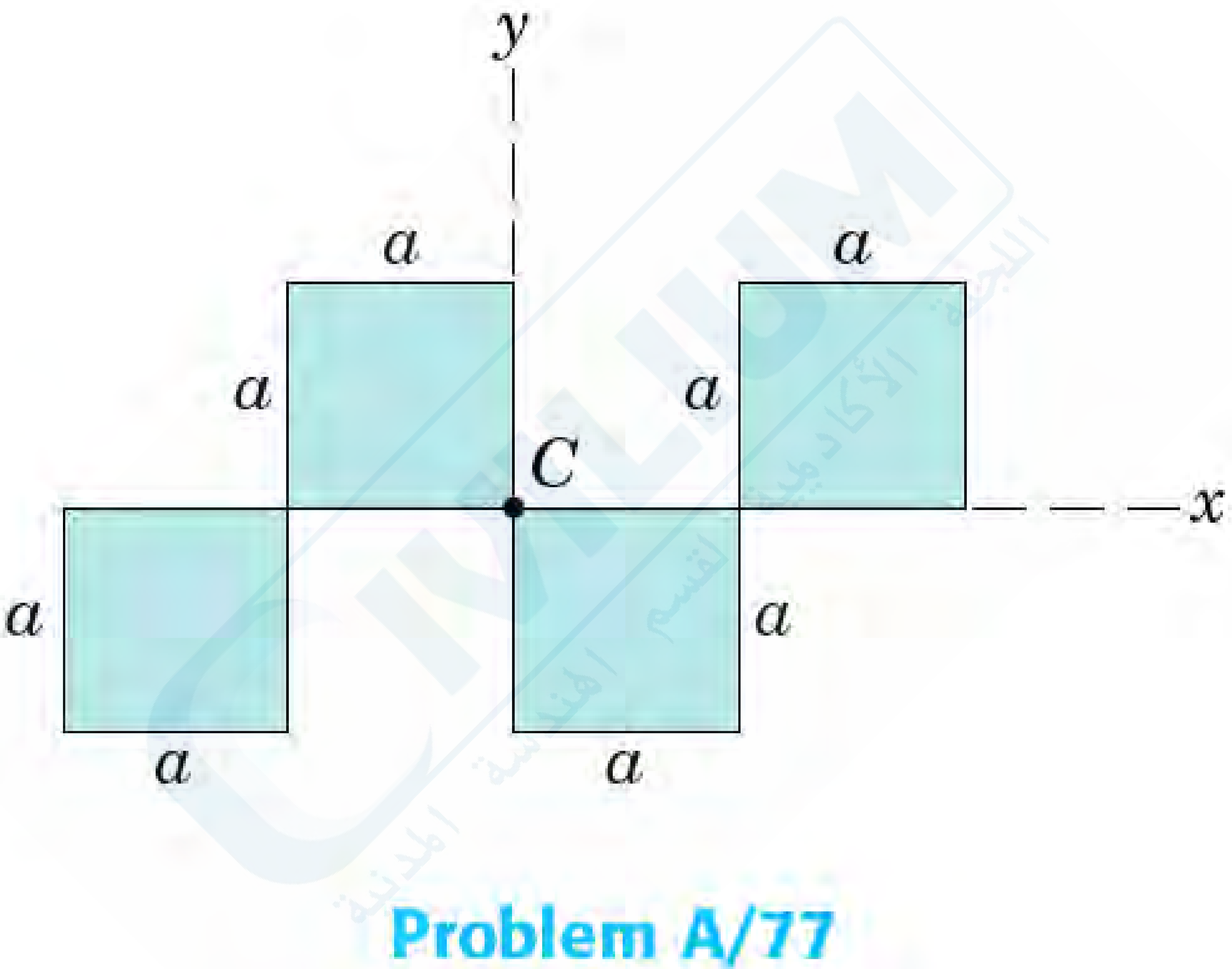
Problem A/75

P A 75

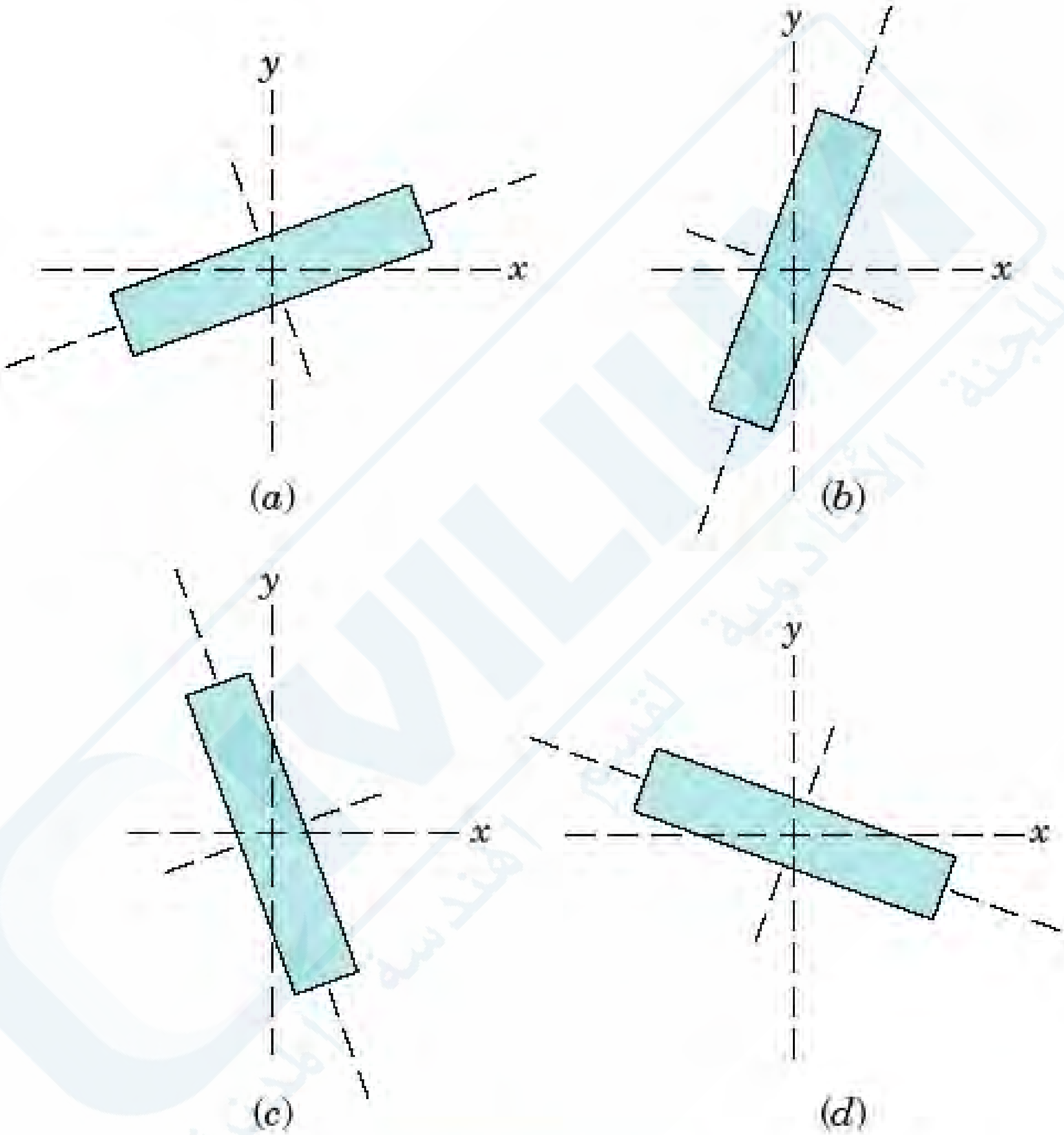


Problem A/76

P A 76

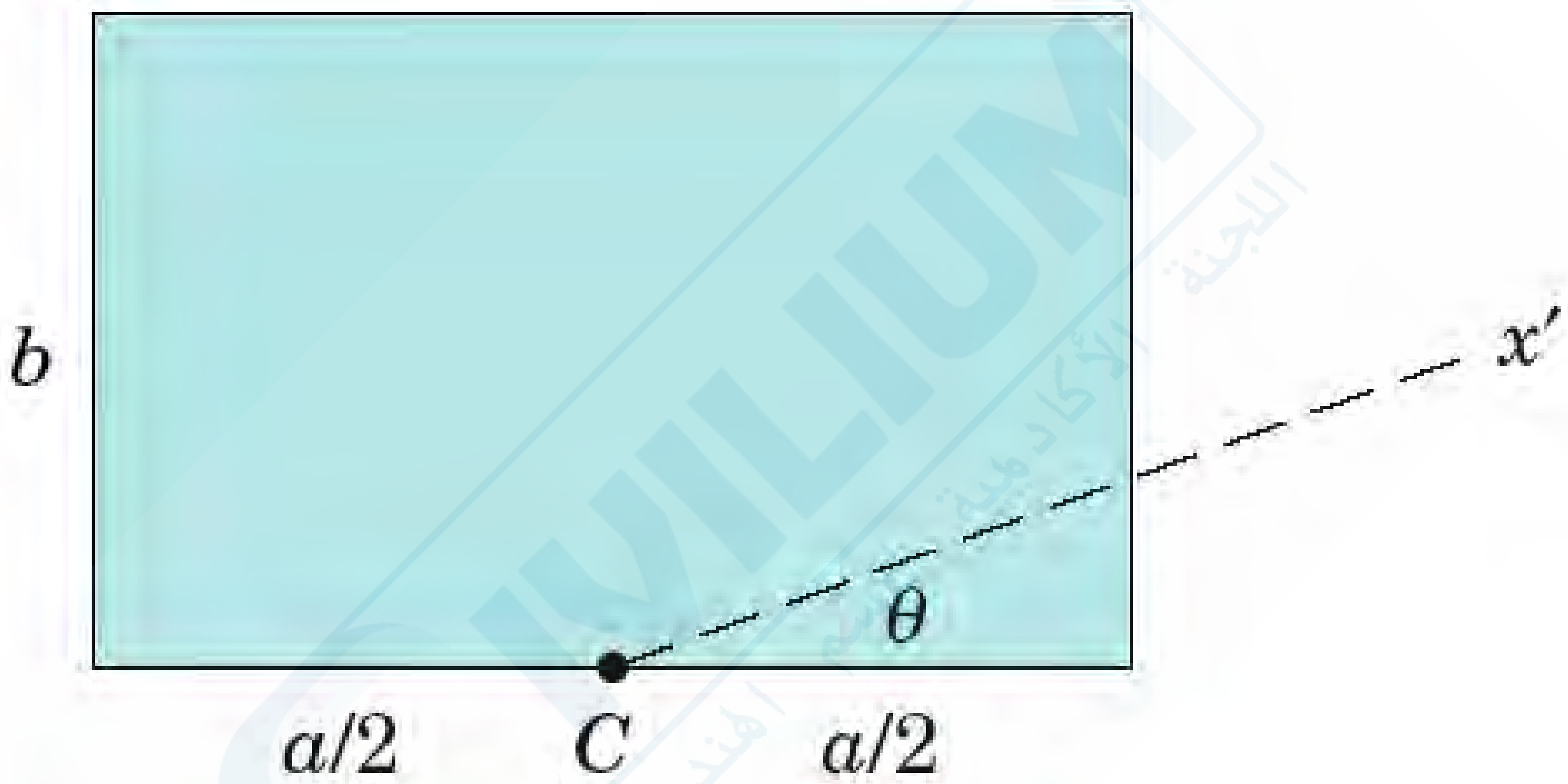


P_A_77



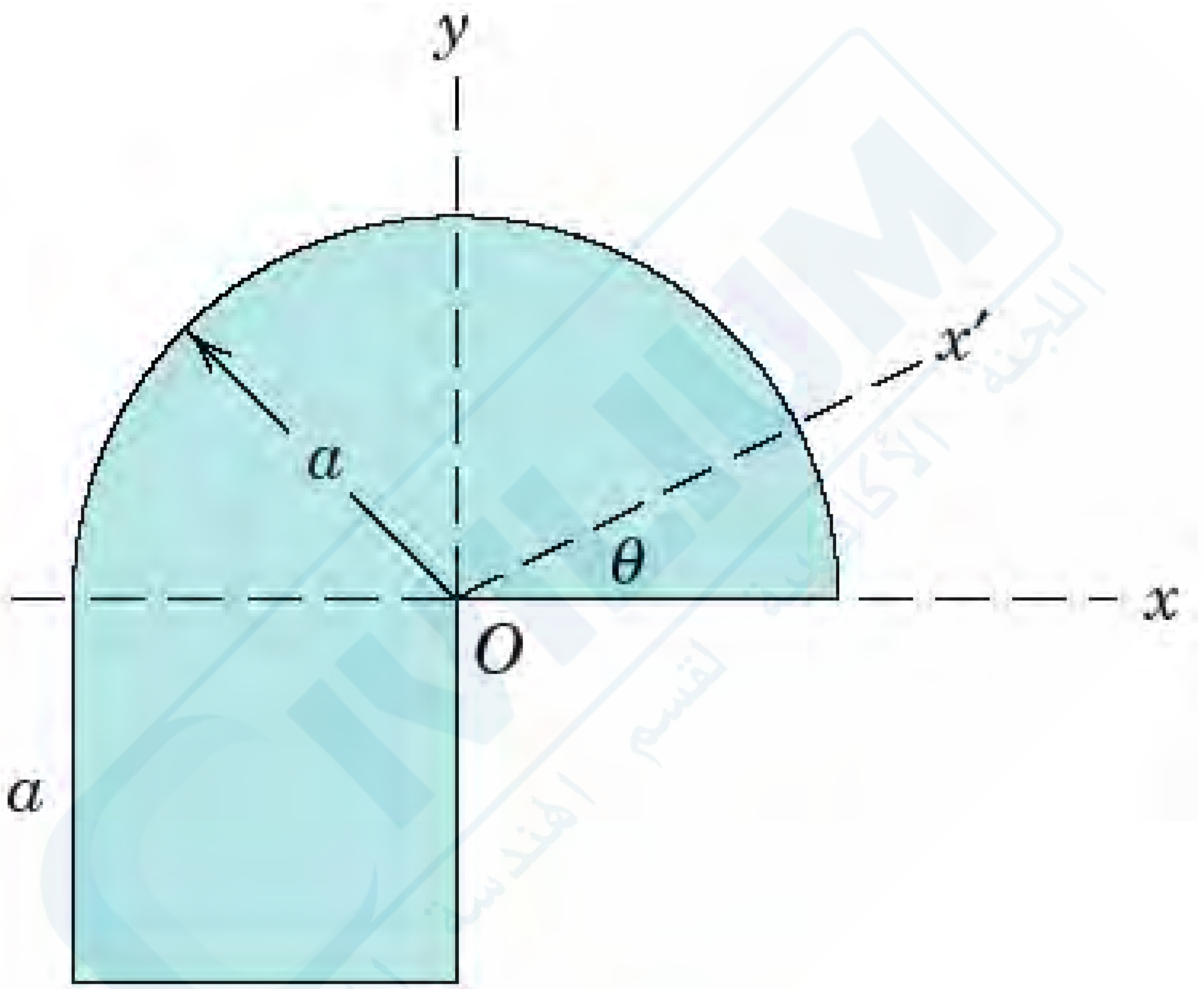
Problem A/78

P A 78

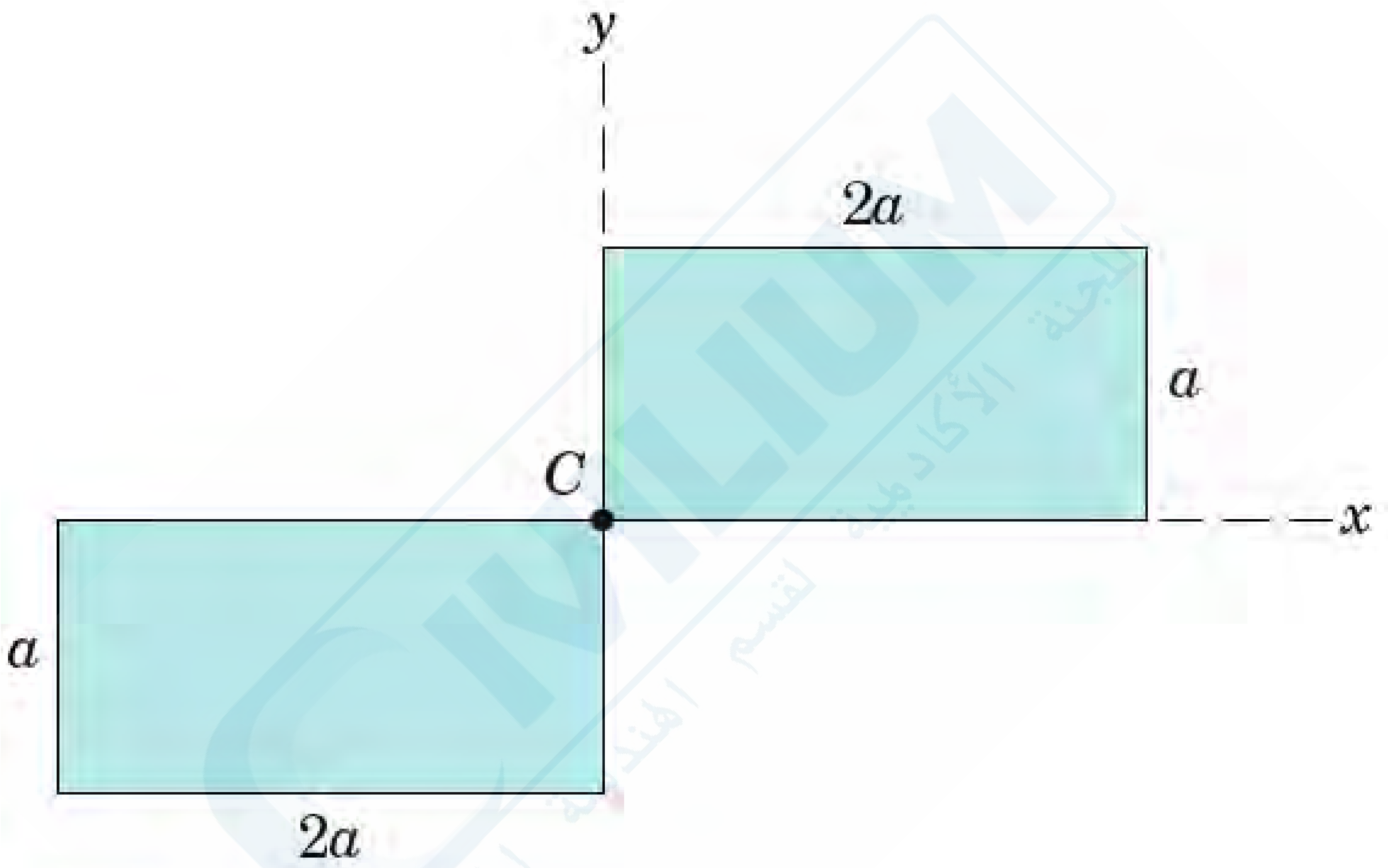


Problem A/79

P_A_79

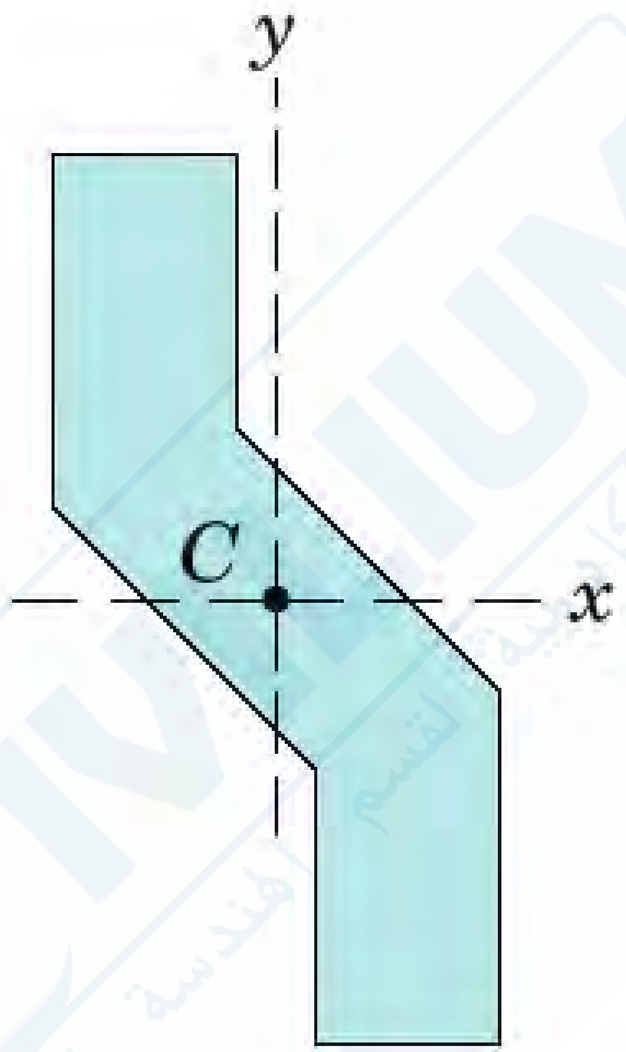


Problem A/80



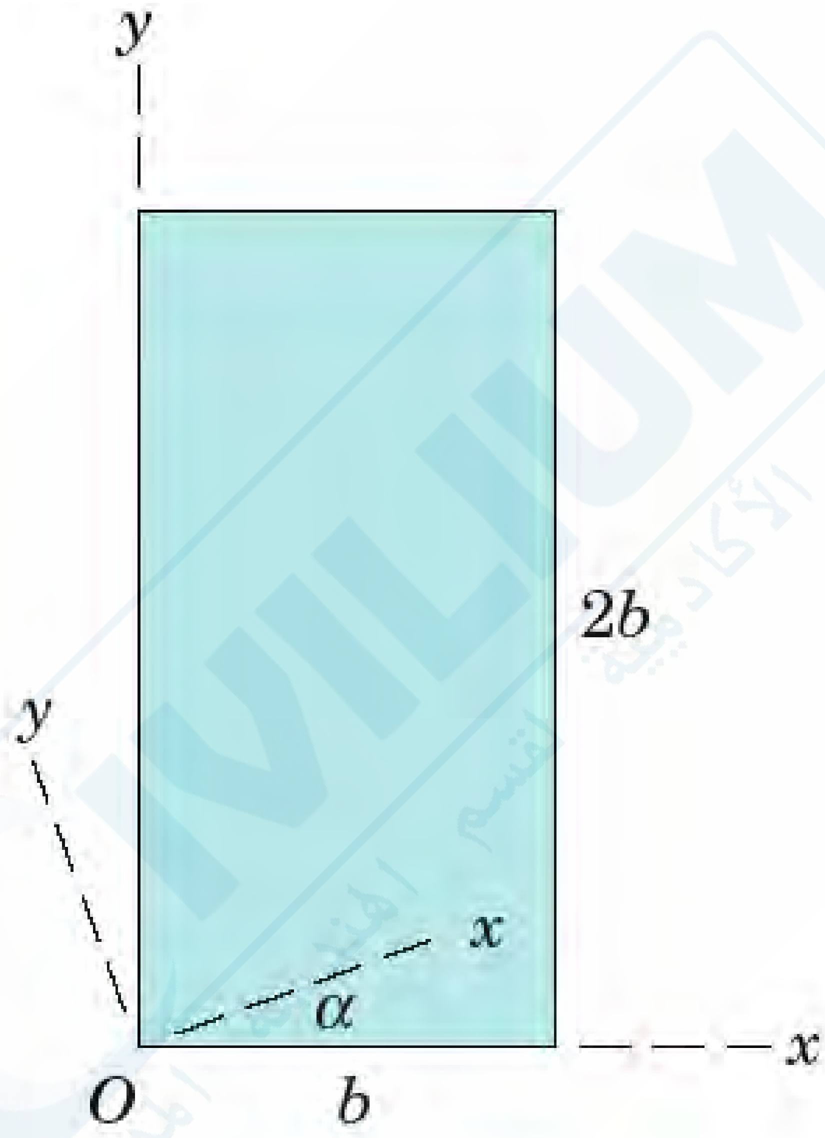
Problem A/81

P_A_81



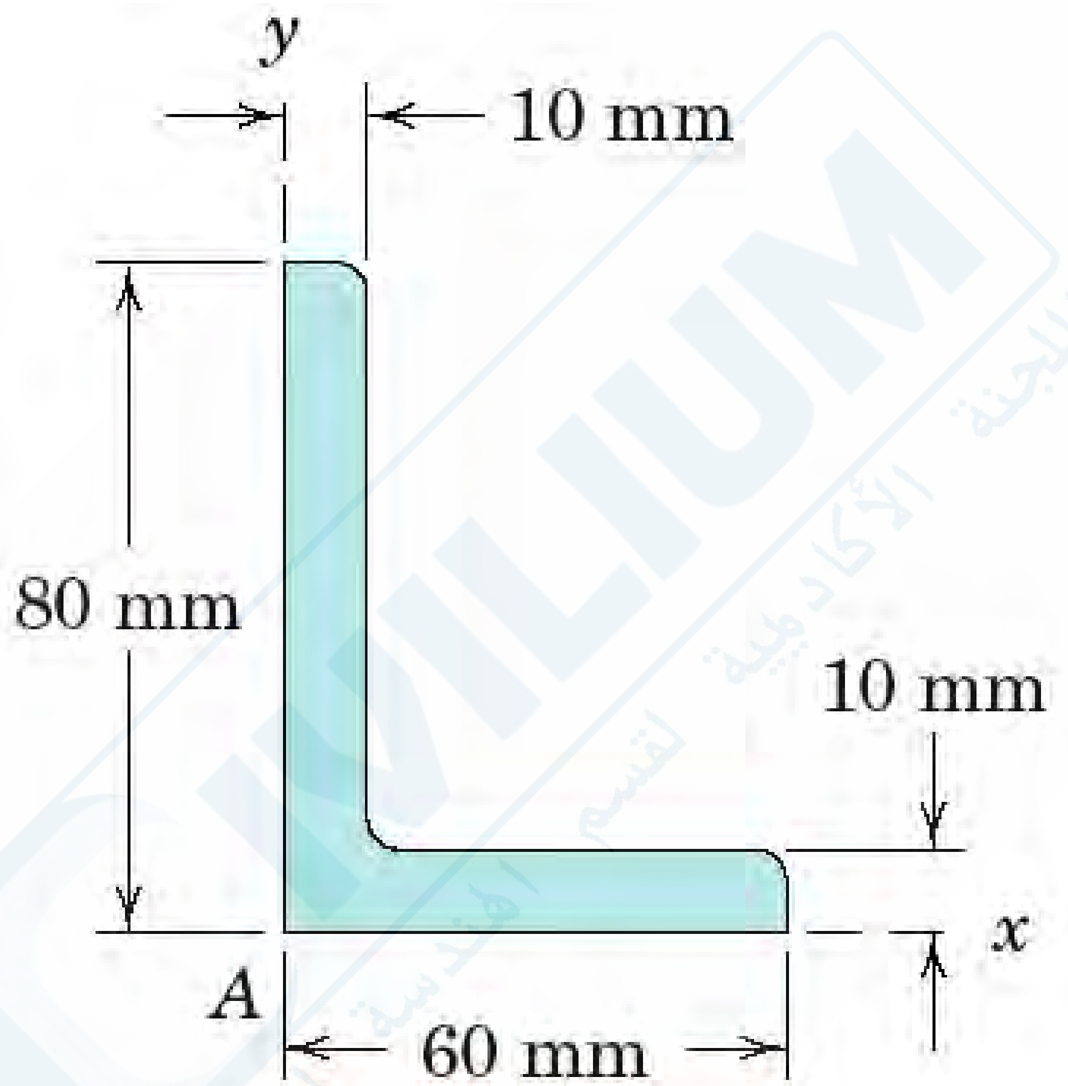
Problem A/82

P A 82

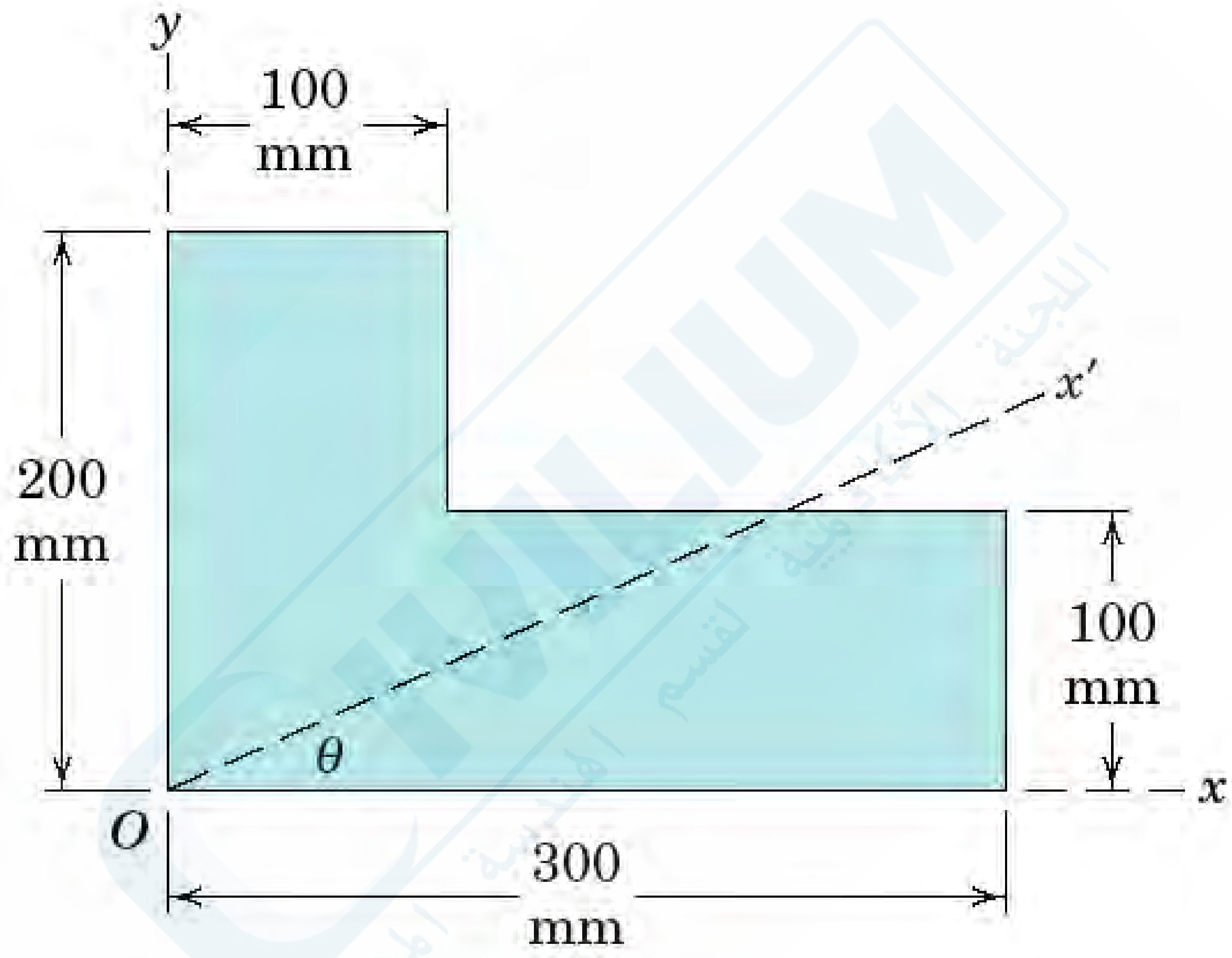


Problem A/84

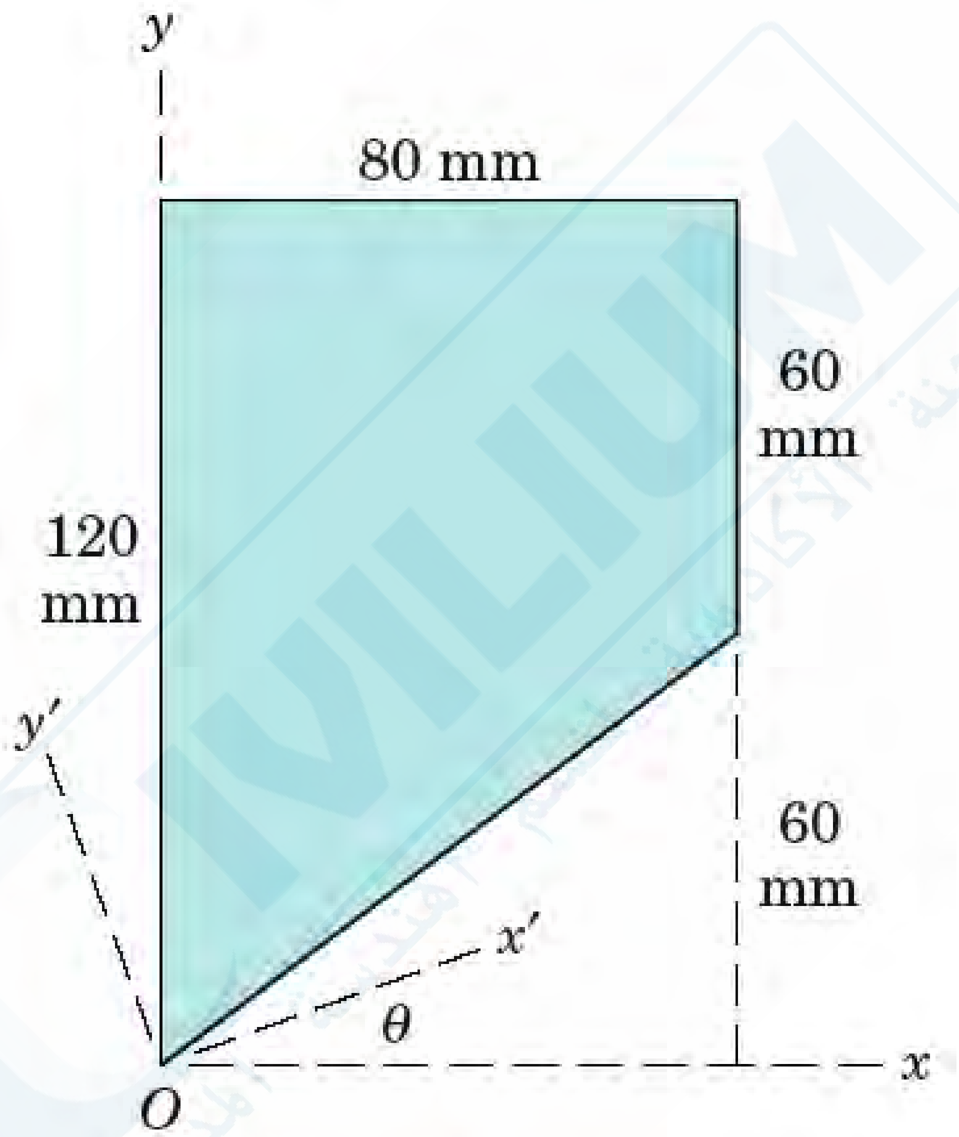
P A 84



Problem A/85

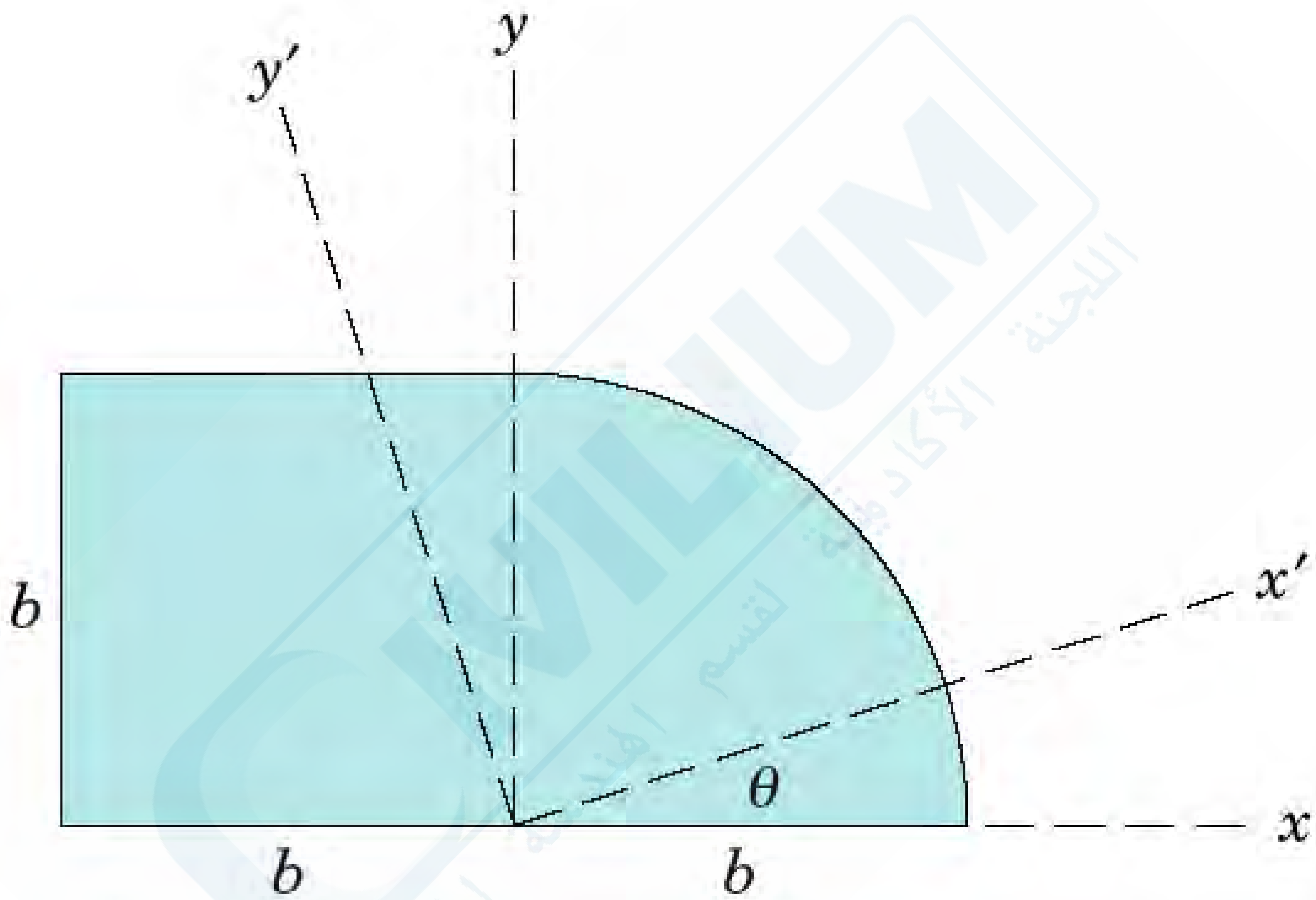


Problem A/86



Problem A/87

P A 87



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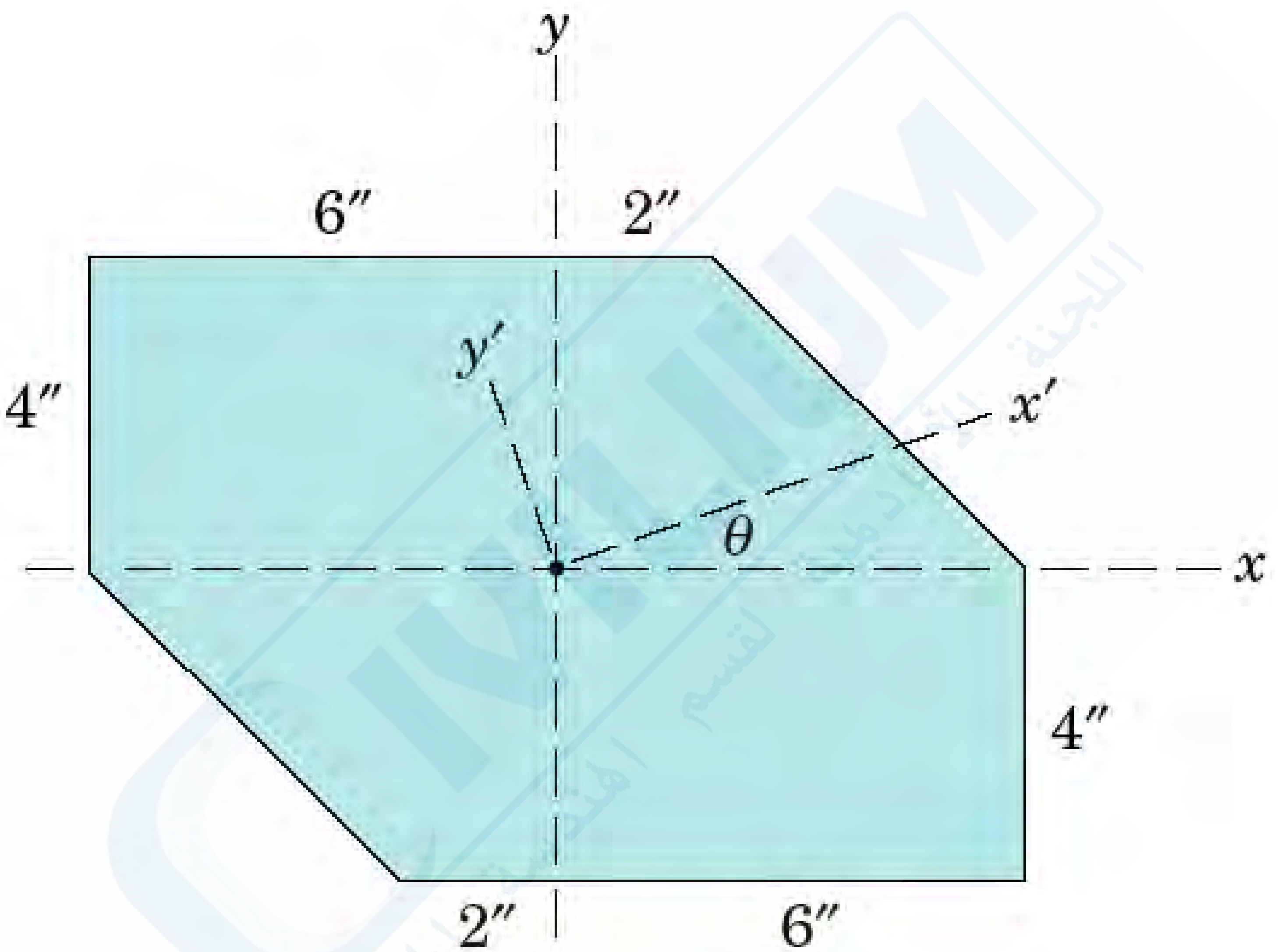
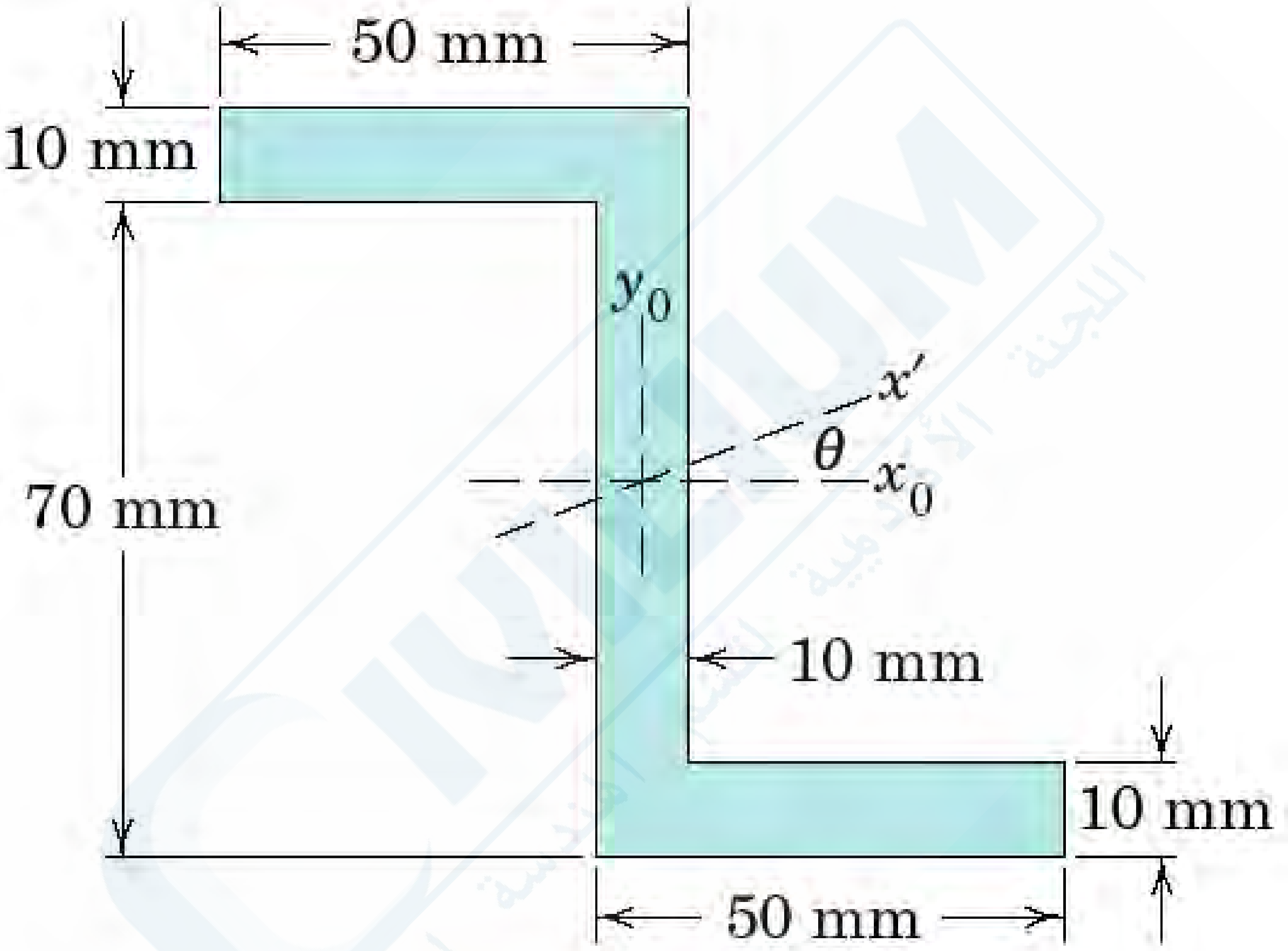
Problem A/88

P A 88



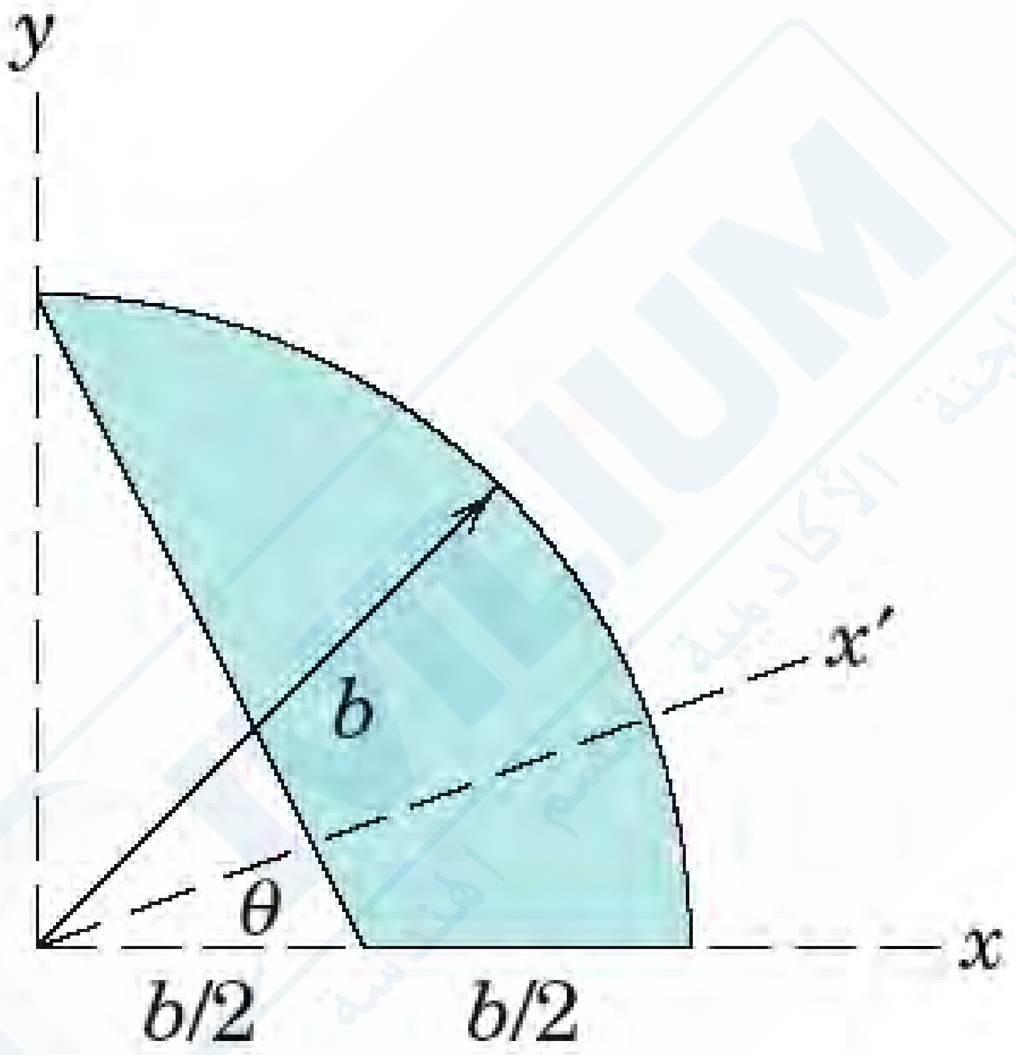
Problem A/89

P A 89



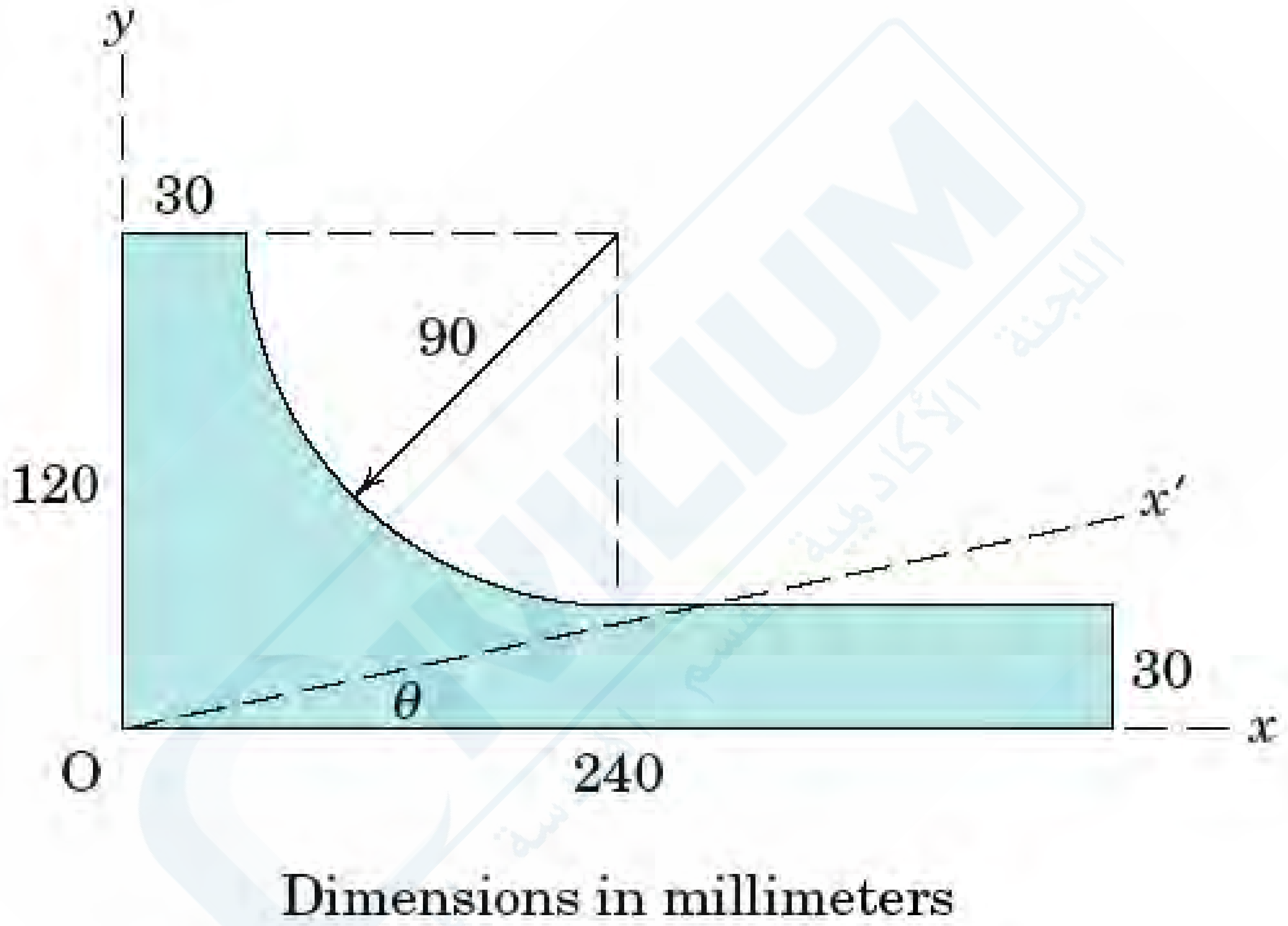
Problem A/90

P A 90



Problem A/91

P A 91



Problem A/92

P A 92

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Appendix B: Summary of Mass Moments of Inertia.

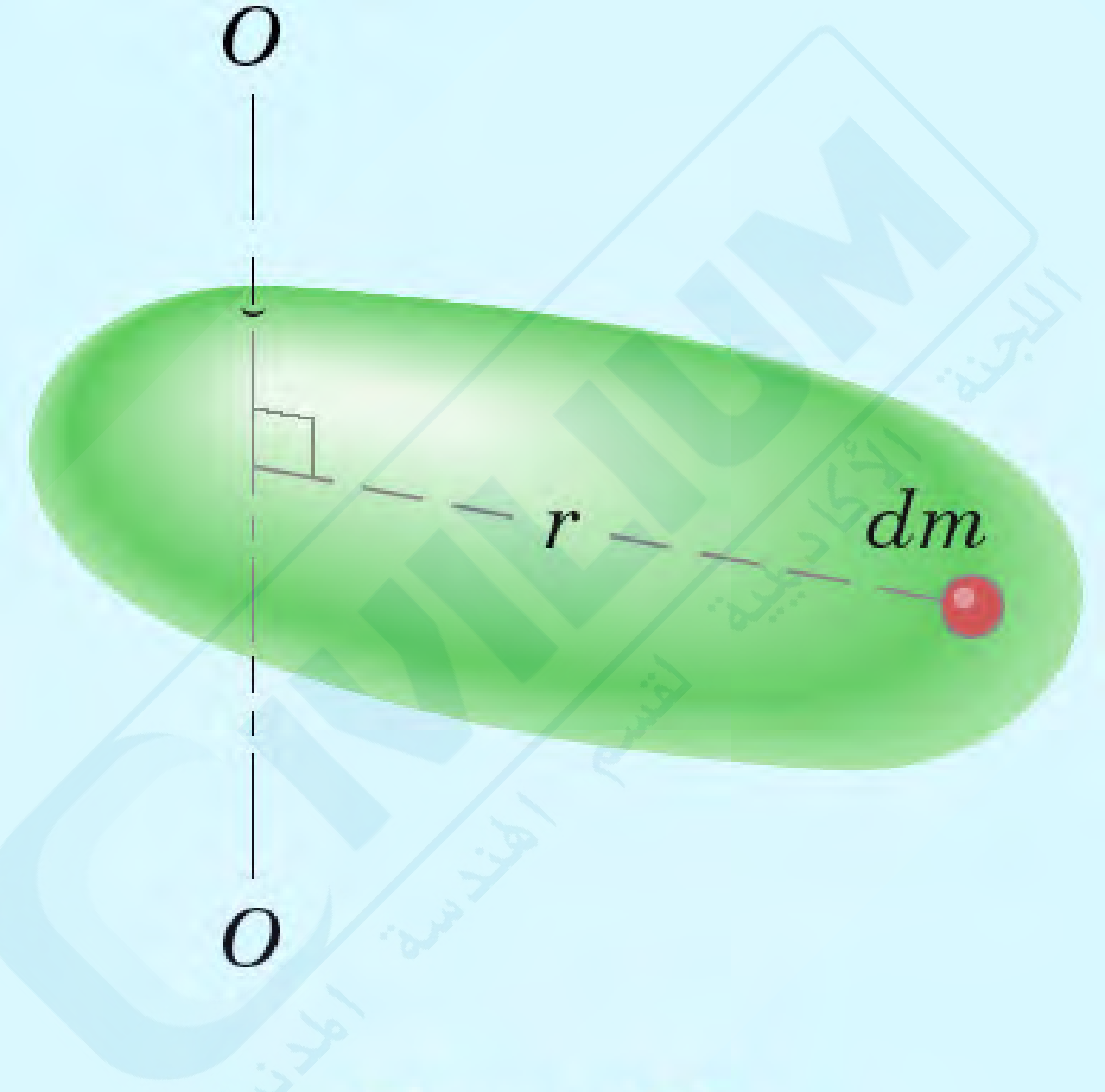
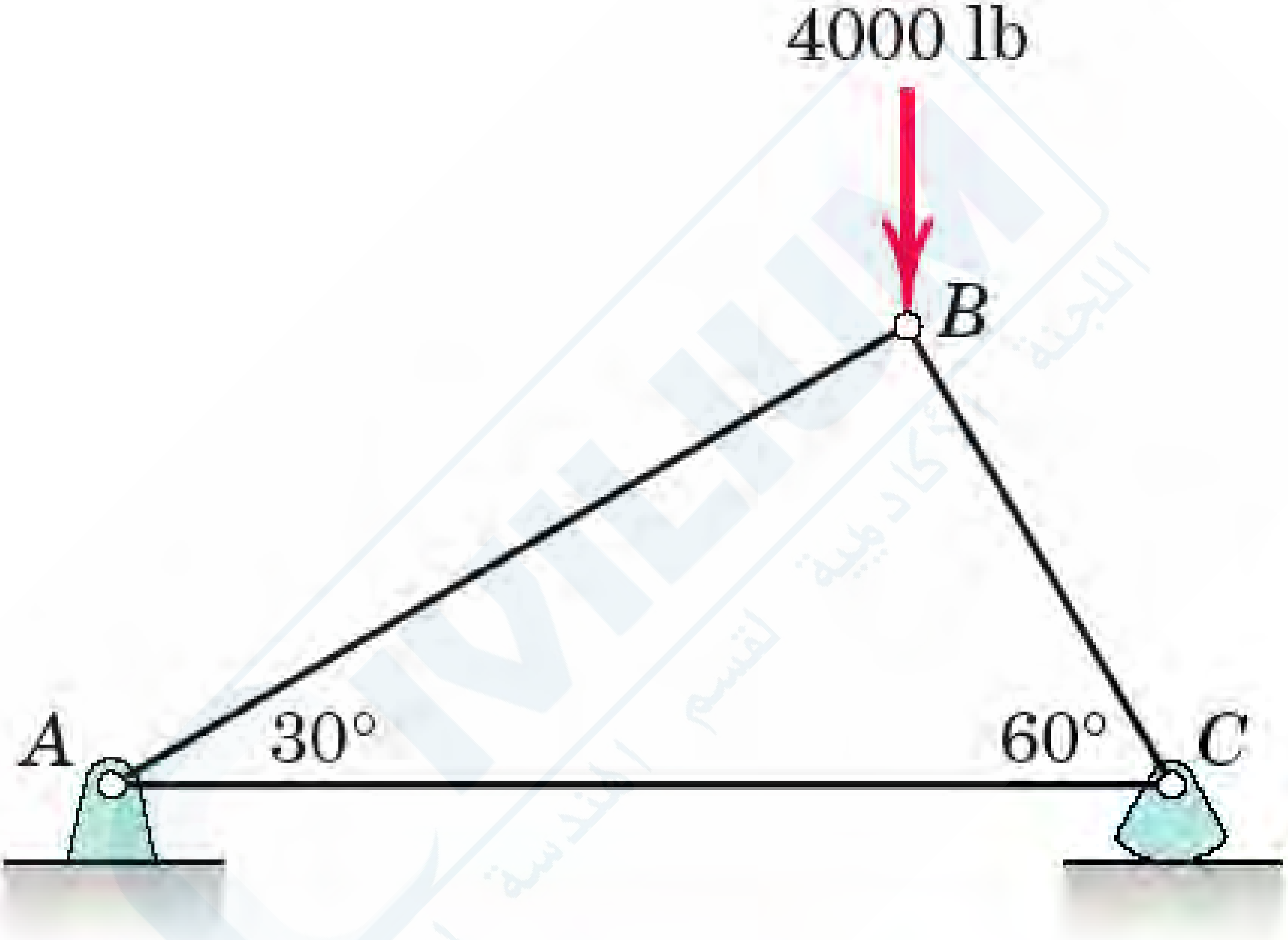
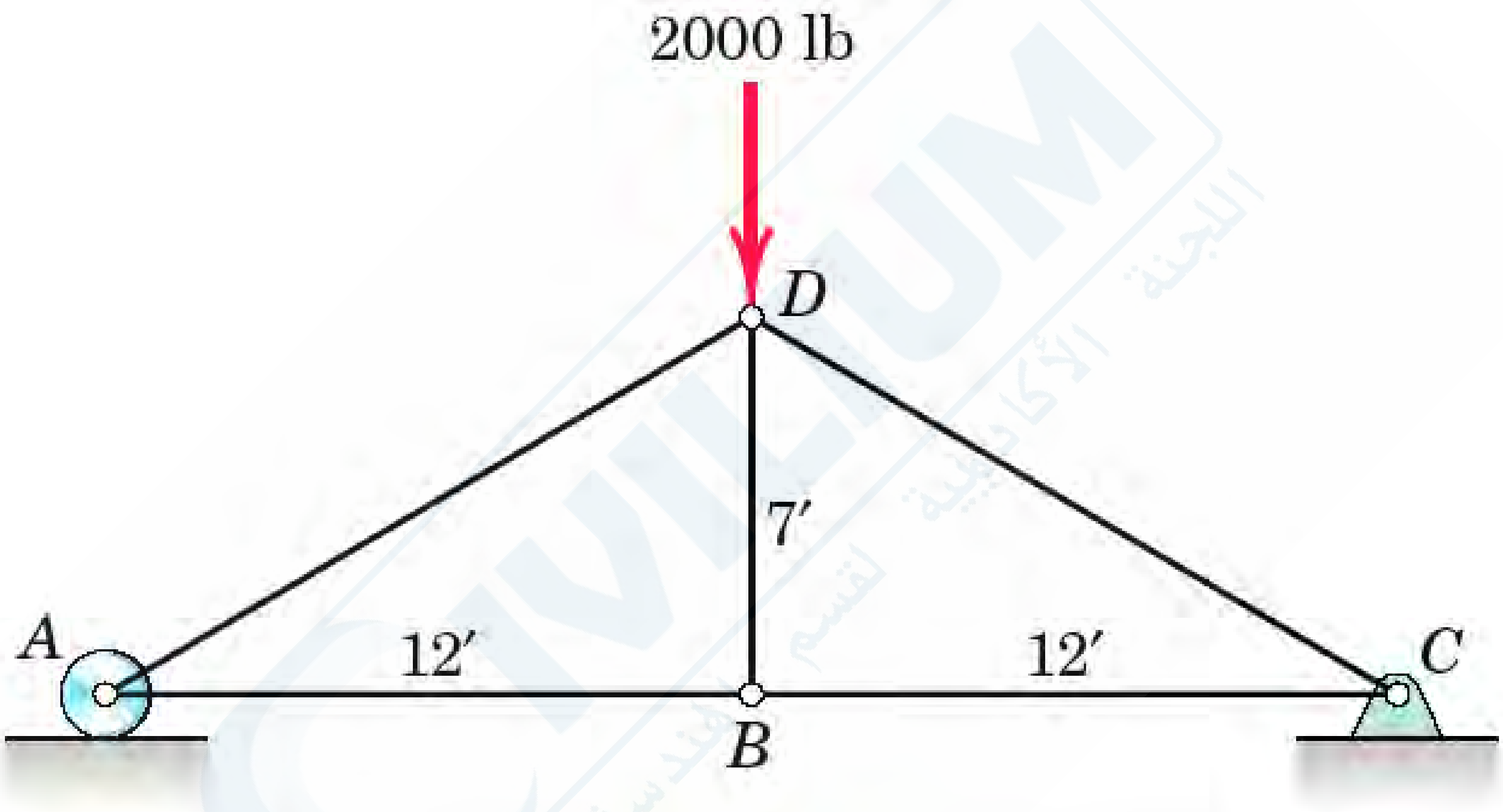


Figure B/1



Problem 4/1

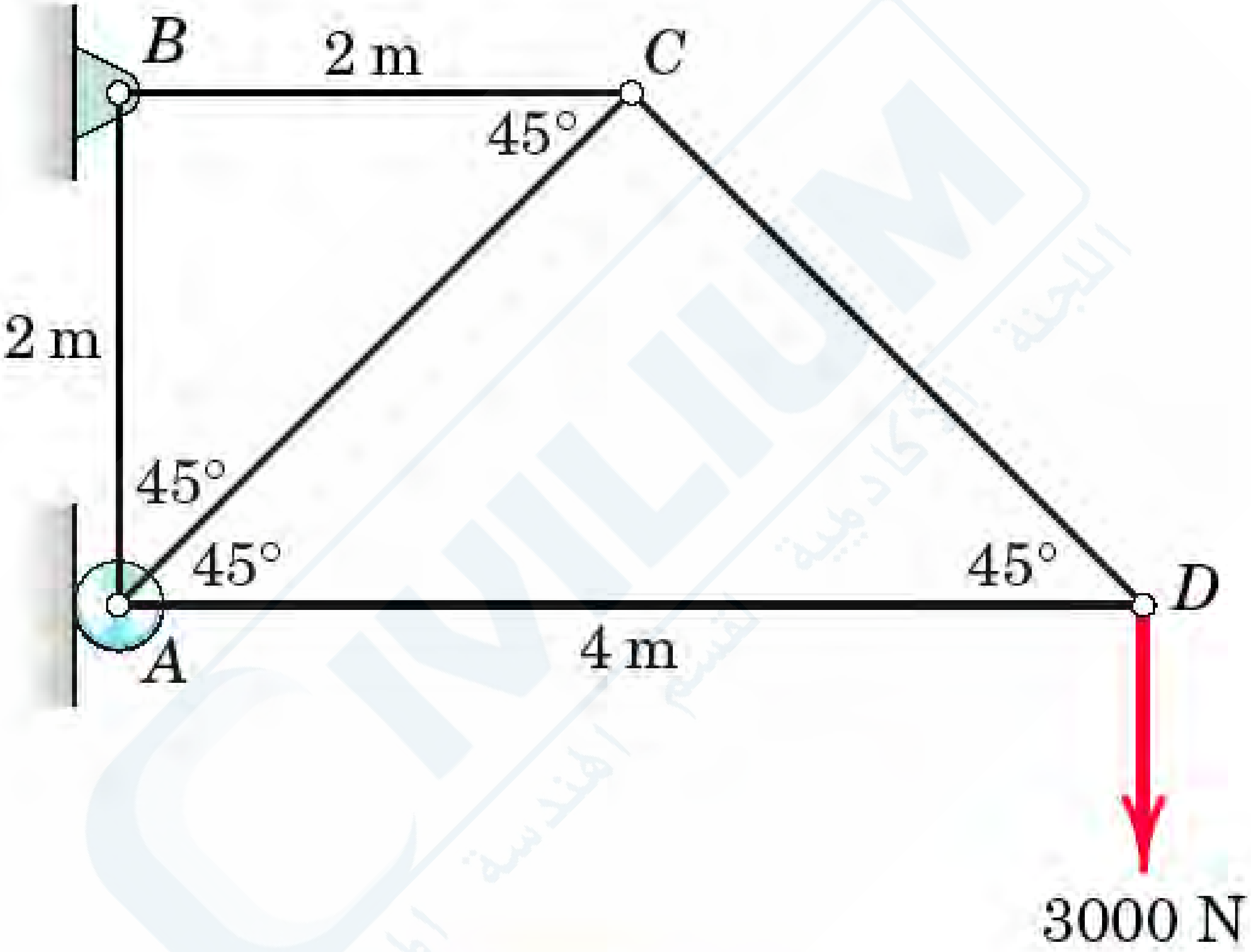
P_04_01



Problem 4/2

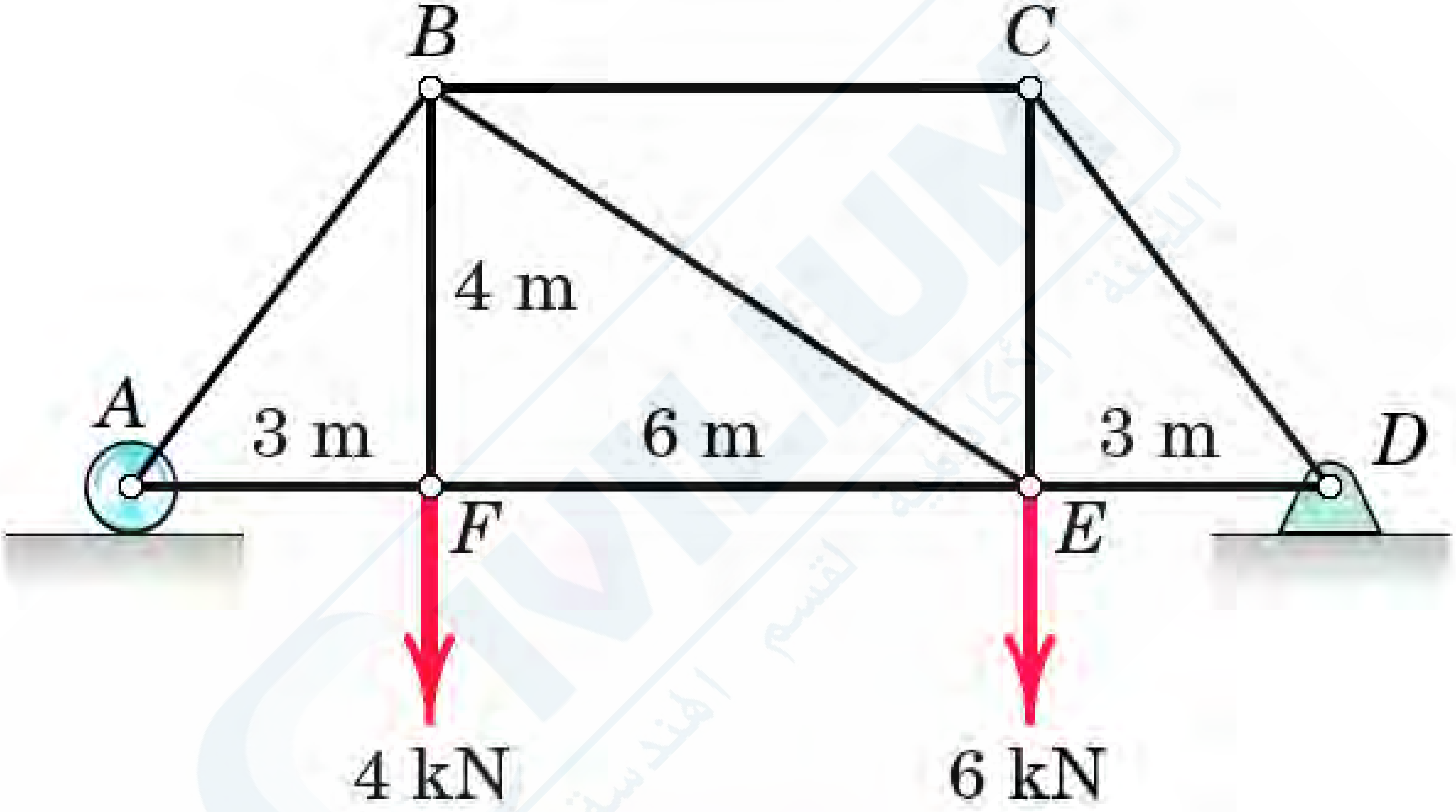
P_04_02

CIVILIUM



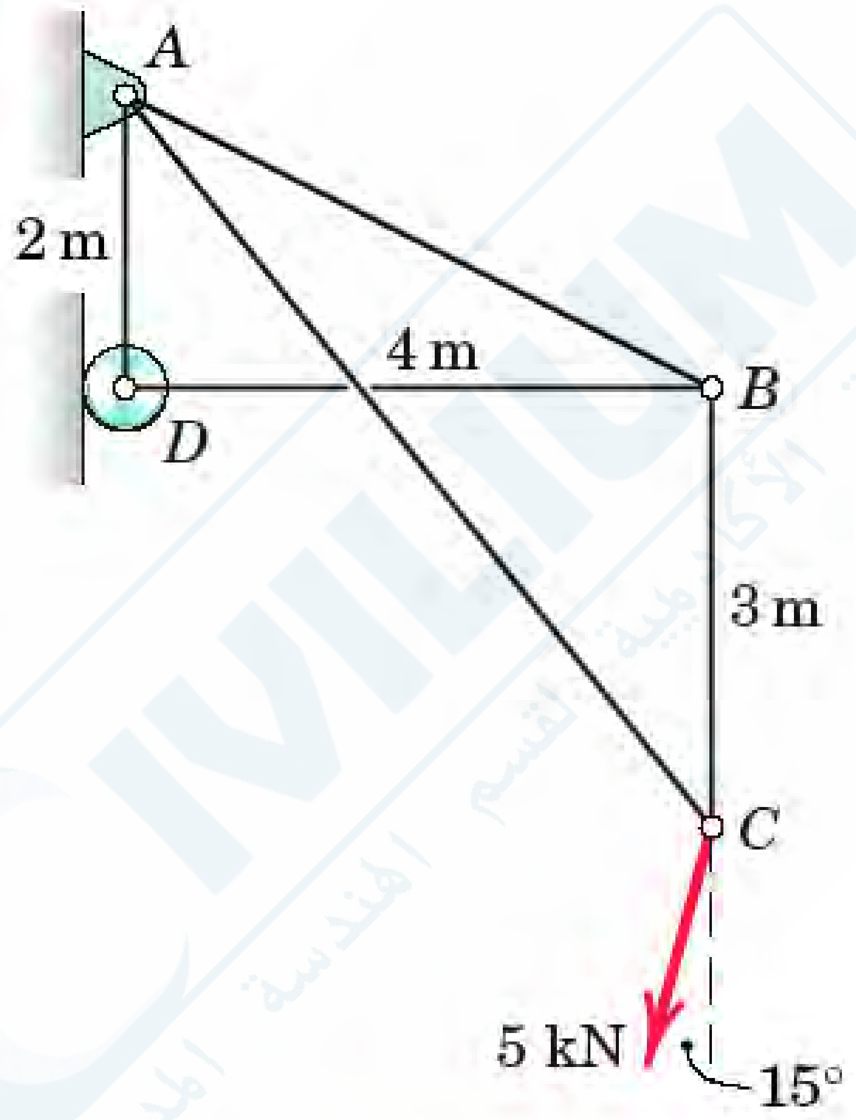
Problem 4/3

P 04 03

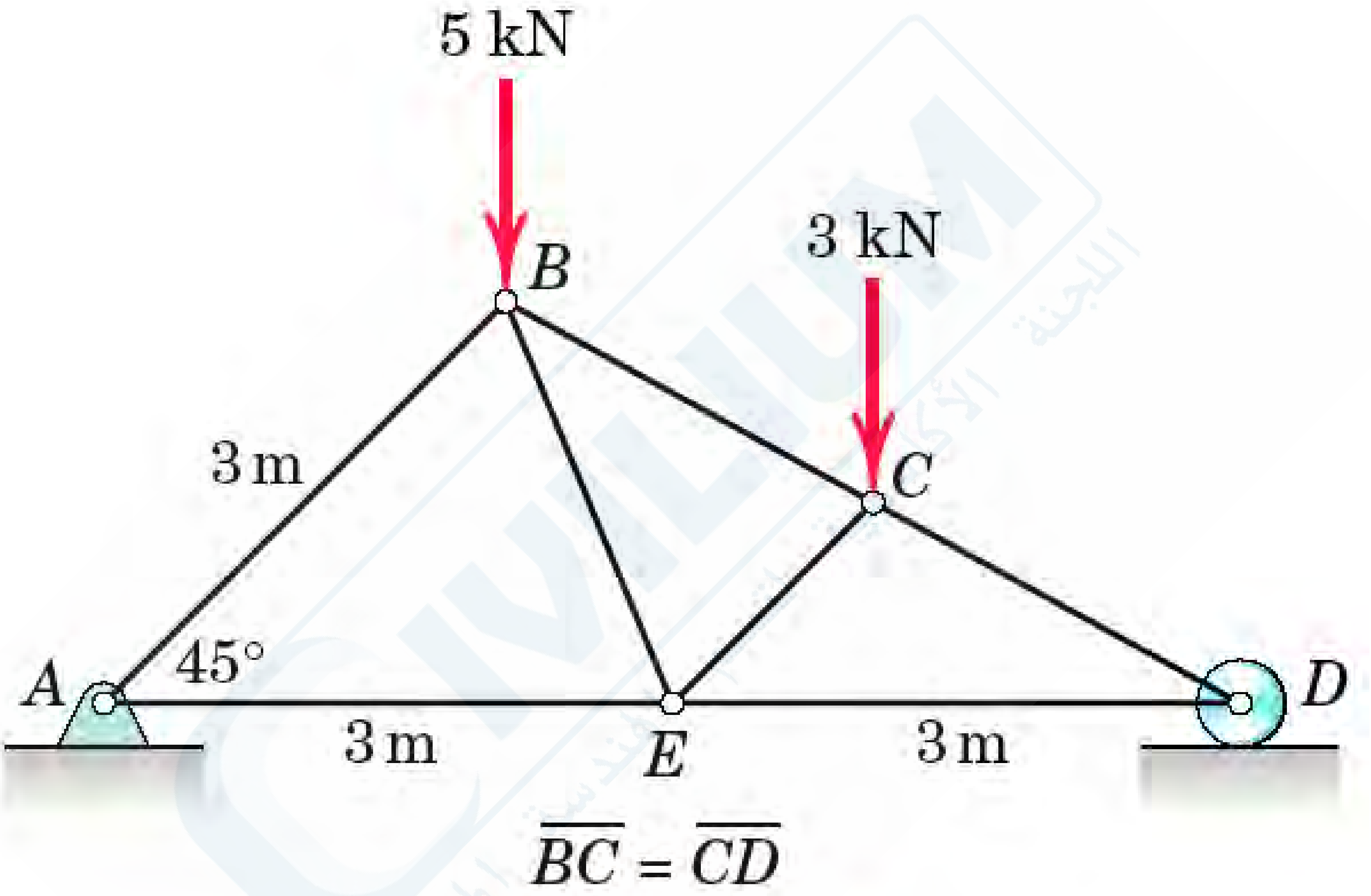


Problem 4/5

P_04_05

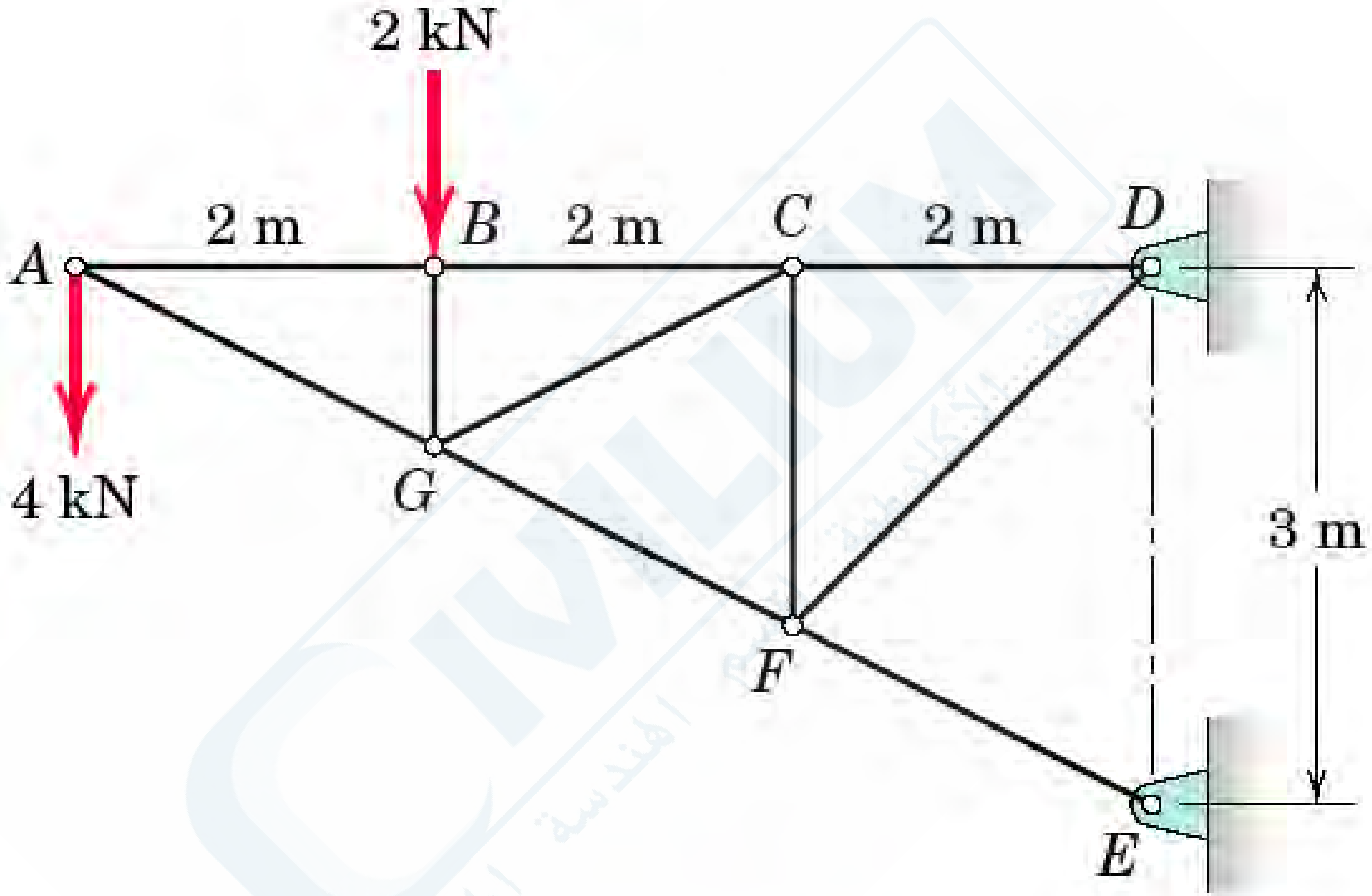


Problem 4/9

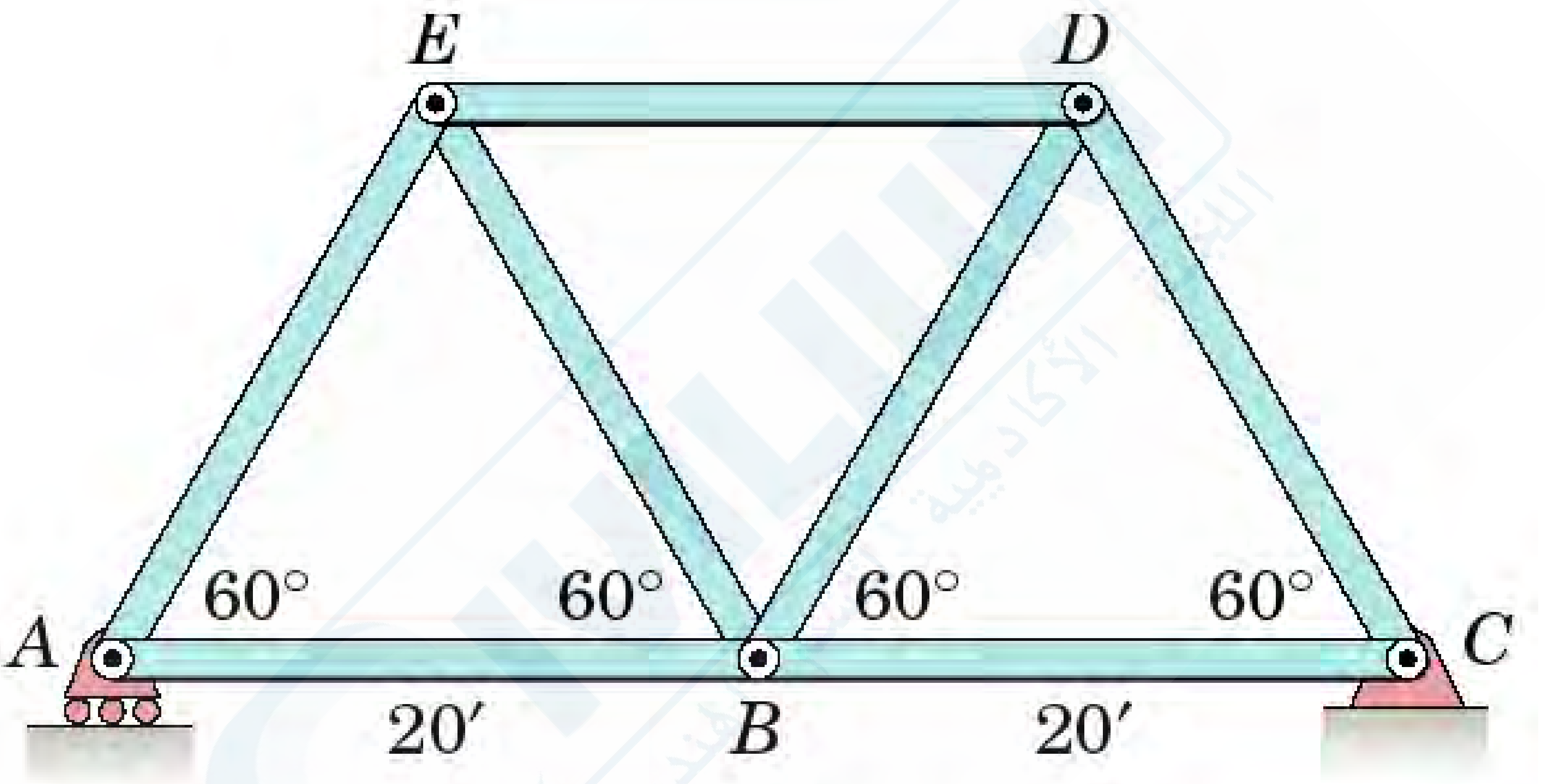


Problem 4/10

P 04 10

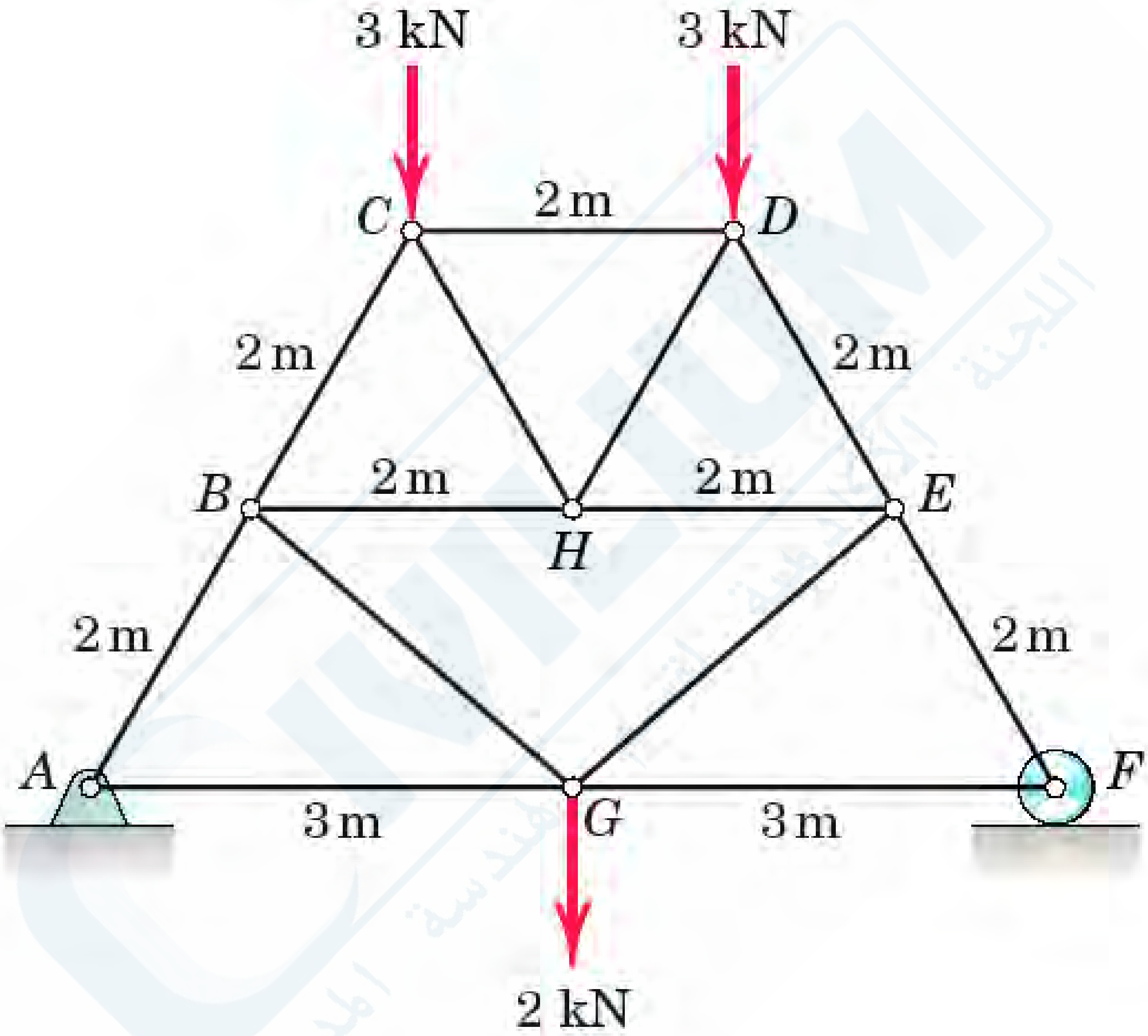


Problem 4/11



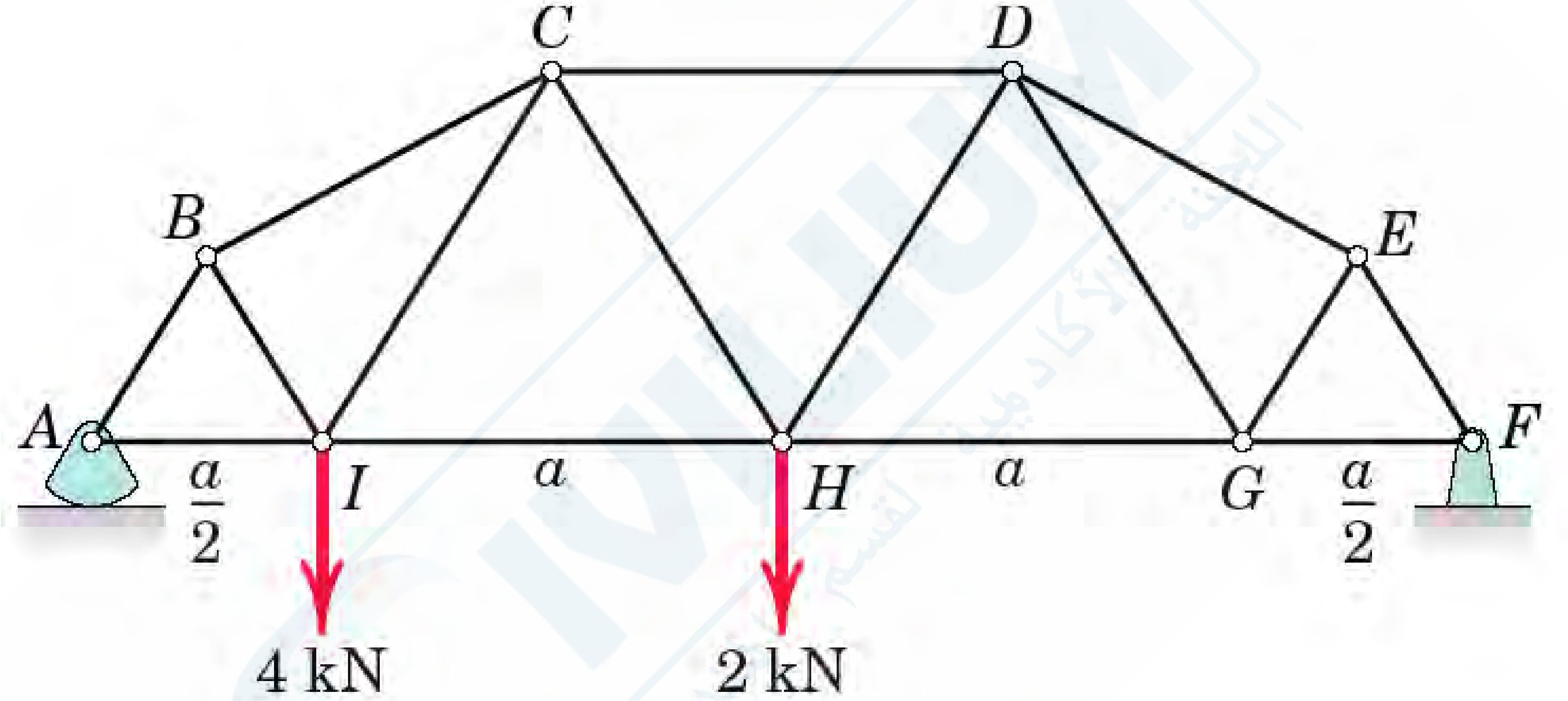
Problem 4/12

P_04_12



Problem 4/15

P_04_15



Problem 4/16

P_04_16

Text Illustrations

To Accompany

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Appendix D: Useful Tables.

CIVILIUM
اللجنة الأكاديمية لقسم الهندسة المدنية

TABLE D/1 PHYSICAL PROPERTIES

Density (kg/m^3) and *specific weight* (lb/ft^3)

	kg/m^3	lb/ft^3		kg/m^3	lb/ft^3
Air*	1.2062	0.07530	Lead	11 370	710
Aluminum	2 690	168	Mercury	13 570	847
Concrete (av.)	2 400	150	Oil (av.)	900	56
Copper	8 910	556	Steel	7 830	489
Earth (wet, av.)	1 760	110	Titanium	3 080	192
(dry, av.)	1 280	80	Water (fresh)	1 000	62.4
Glass	2 590	162	(salt)	1 030	64
Gold	19 300	1205	Wood (soft pine)	480	30
Ice	900	56	(hard oak)	800	50
Iron (cast)	7 210	450			

* At 20°C (68°F) and atmospheric pressure

Coefficients of friction

(The coefficients in the following table represent typical values under normal working conditions. Actual coefficients for a given situation will depend on the exact nature of the contacting surfaces. A variation of 25 to 100 percent or more from these values could be expected in an actual application, depending on prevailing conditions of cleanliness, surface finish, pressure, lubrication, and velocity.)

	TYPICAL VALUES OF COEFFICIENT OF FRICTION	
CONTACTING SURFACE	STATIC, μ_s	KINETIC, μ_k
Steel on steel (dry)	0.6	0.4
Steel on steel (greasy)	0.1	0.05
Teflon on steel	0.04	0.04
Steel on babbitt (dry)	0.4	0.3
Steel on babbitt (greasy)	0.1	0.07
Brass on steel (dry)	0.5	0.4
Brake lining on cast iron	0.4	0.3
Rubber tires on smooth pavement (dry)	0.9	0.8
Wire rope on iron pulley (dry)	0.2	0.15
Hemp rope on metal	0.3	0.2
Metal on ice		0.02

D_01tbl

TABLE D/2 SOLAR SYSTEM CONSTANTS

Universal gravitational constant	$G = 6.673(10^{-11})\ m^3/(kg \cdot s^2)$
	$= 3.439(10^{-8})\ ft^4/(lbf\text{-}s^4)$
Mass of Earth	$m_e = 5.976(10^{24})$ kg
	$= 4.095(10^{23})\ lbf\text{-}s^2/ft$
Period of Earth's rotation (1 sidereal day)	= 23 h 56 min 4 s
	= 23.9344 h
Angular velocity of Earth	$\omega = 0.7292(10^{-4})$ rad/s
Mean angular velocity of Earth–Sun line	$\omega' = 0.1991(10^{-6})$ rad/s
Mean velocity of Earth's center about Sun	= 107 200 km/h
	= 66,610 mi/h

BODY	MEAN DISTANCE TO SUN km (mi)	ECCENTRICITY OF ORBIT e	PERIOD OF ORBIT solar days	MEAN DIAMETER km (mi)	MASS RELATIVE TO EARTH	SURFACE GRAVITATIONAL ACCELERATION m/s^2 (ft/s^2)	ESCAPE VELOCITY km/s (mi/s)
Sun	—	—	—	1 392 000 (865 000)	333 000	274 (898)	616 (383)
Moon	384 398* (238 854)*	0.055	27.32	3 476 (2 160)	0.0123	1.62 (5.32)	2.37 (1.47)
Mercury	57.3×10^6 (35.6×10^6)	0.206	87.97	5 000 (3 100)	0.054	3.47 (11.4)	4.17 (2.59)
Venus	108×10^6 (67.2×10^6)	0.0068	224.70	12 400 (7 700)	0.815	8.44 (27.7)	10.24 (6.36)
Earth	149.6×10^6 (92.96×10^6)	0.0167	365.26	12 742† (7 918)†	1.000	9.821‡ (32.22)‡	11.18 (6.95)
Mars	227.9×10^6 (141.6×10^6)	0.093	686.98	6 788 (4 218)	0.107	3.73 (12.3)	5.03 (3.13)

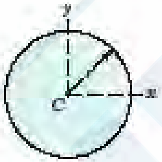
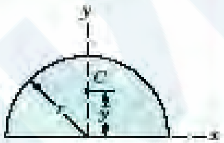
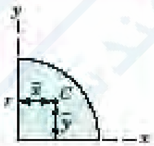
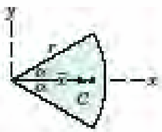
* Mean distance to Earth (center-to-center)

† Diameter of sphere of equal volume, based on a spheroidal Earth with a polar diameter of 12 714 km (7900 mi) and an equatorial diameter of 12 756 km (7926 mi)

‡ For nonrotating spherical Earth, equivalent to absolute value at sea level and latitude 37.5°

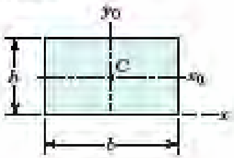
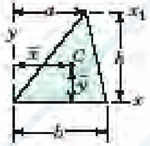
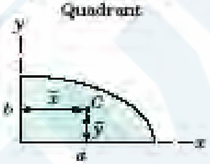
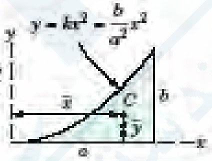
D_02tbl

TABLE D/3 PROPERTIES OF PLANE FIGURES

FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Arc Segment	$\bar{r} = \frac{r \sin \alpha}{\alpha}$	—
Quarter and Semicircular Arcs	$\bar{y} = \frac{2r}{\pi}$	—
Circular Area 	—	$I_x = I_y = \frac{\pi r^4}{4}$ $I_z = \frac{\pi r^4}{2}$
Semicircular Area 	$\bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{8}$ $\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{4}$
Quarter-Circular Area 	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$	$I_x = I_y = \frac{\pi r^4}{16}$ $\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4$ $I_z = \frac{\pi r^4}{8}$
Area of Circular Sector 	$\bar{x} = \frac{2}{3} \frac{r \sin \alpha}{\alpha}$	$I_x = \frac{r^4}{4}(\alpha - \frac{1}{2}\sin 2\alpha)$ $I_y = \frac{r^4}{4}(\alpha + \frac{1}{2}\sin 2\alpha)$ $I_z = \frac{1}{2} r^4 \alpha$

D 03tbl

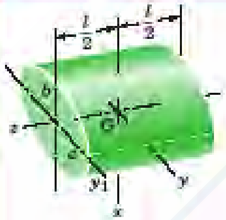
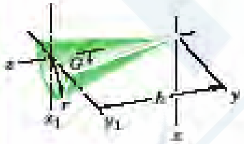
TABLE D/3 PROPERTIES OF PLANE FIGURES *Continued*

FIGURE	CENTROID	AREA MOMENTS OF INERTIA
Rectangular Area 	—	$I_x = \frac{bh^3}{3}$ $\bar{I}_x = \frac{bh^3}{12}$ $\bar{I}_z = \frac{bh}{12}(b^2+h^2)$
Triangular Area 	$\bar{x} = \frac{a+b}{3}$ $\bar{y} = \frac{h}{3}$	$I_x = \frac{bh^3}{12}$ $\bar{I}_x = \frac{bh^3}{36}$ $I_{x_1} = \frac{bh^3}{4}$
Area of Elliptical Quadrant 	$\bar{x} = \frac{4a}{3\pi}$ $\bar{y} = \frac{4b}{3\pi}$	$I_x = \frac{\pi ab^3}{16}$, $\bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)ab^3$ $I_y = \frac{\pi a^3 b}{16}$, $\bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^3 b$ $I_z = \frac{\pi ab}{16}(a^2+b^2)$
Subparabolic Area Area $A = \frac{ab}{3}$ 	$\bar{x} = \frac{3a}{4}$ $\bar{y} = \frac{3b}{10}$	$I_x = \frac{ab^3}{21}$ $I_y = \frac{a^3 b}{5}$ $I_z = ab\left(\frac{a^2}{5} + \frac{b^2}{21}\right)$
Parabolic Area Area $A = \frac{2ab}{3}$	$\bar{x} = \frac{3a}{8}$ $\bar{y} = \frac{3b}{5}$	$I_x = \frac{2ab^3}{7}$ $I_y = \frac{2a^3 b}{15}$ $I_z = 2ab\left(\frac{a^2}{15} + \frac{b^2}{7}\right)$

D_03tbl_cont

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS *Continued*

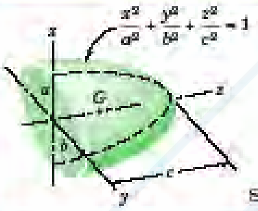
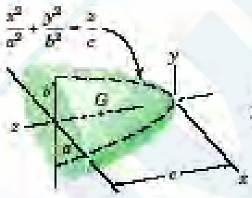
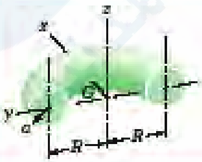
(m = mass of body shown)

BODY		MASS CENTER	MASS MOMENTS OF INERTIA
	Quarter-Circular Rod	$\bar{x} = \bar{y} = \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2}mr^2$ $I_{zz} = mr^2$
	Elliptical Cylinder	—	$I_{xx} = \frac{1}{4}ma^2 + \frac{1}{12}ml^2$ $I_{yy} = \frac{1}{4}mb^2 + \frac{1}{12}ml^2$ $I_{zz} = \frac{1}{4}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{4}mb^2 + \frac{1}{3}ml^2$
	Conical Shell	$\bar{z} = \frac{2h}{3}$	$I_{yy} = \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{y_1y_1} = \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{yy} = \frac{1}{4}mr^2 + \frac{1}{18}mh^2$
	Half Conical Shell	$\bar{x} = \frac{4r}{3\pi}$ $\bar{z} = \frac{2h}{3}$	$I_{xx} = I_{yy} = \frac{1}{4}mr^2 + \frac{1}{2}mh^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{4}mr^2 + \frac{1}{6}mh^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
	Right Circular Cone	$\bar{z} = \frac{3h}{4}$	$I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{y_1y_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{yy} = \frac{3}{20}mr^2 + \frac{3}{80}mh^2$

D_04_tbl_cont2

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS *Continued*

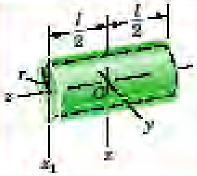
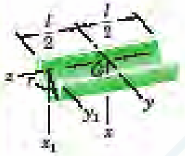
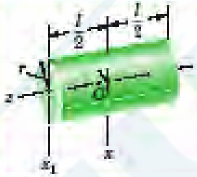
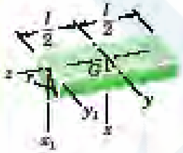
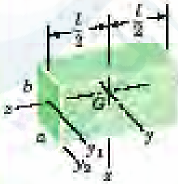
(m = mass of body shown)

BODY	MASS CENTER	MASS MOMENTS OF INERTIA
Half Cone	$\bar{x} = \frac{r}{\pi}$ $\bar{z} = \frac{3h}{4}$	$I_{xx} = I_{yy} = \frac{3}{20}mr^2 + \frac{3}{5}mh^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{3}{20}mr^2 + \frac{1}{10}mh^2$ $I_{zz} = \frac{3}{10}mr^2$ $\bar{I}_{zz} = \left(\frac{3}{10} - \frac{1}{\pi^2}\right)mr^2$
 Semiellipsoid	$\bar{z} = \frac{3c}{8}$	$I_{xx} = \frac{1}{5}m(b^2 + c^2)$ $I_{yy} = \frac{1}{5}m(a^2 + c^2)$ $I_{zz} = \frac{1}{5}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{5}m(b^2 + \frac{19}{64}c^2)$ $\bar{I}_{yy} = \frac{1}{5}m(a^2 + \frac{19}{64}c^2)$
 Elliptic Paraboloid	$\bar{z} = \frac{2c}{3}$	$I_{xx} = \frac{1}{6}mb^2 + \frac{1}{2}mc^2$ $I_{yy} = \frac{1}{6}ma^2 + \frac{1}{2}mc^2$ $I_{zz} = \frac{1}{6}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{1}{6}m(b^2 + \frac{1}{3}c^2)$ $\bar{I}_{yy} = \frac{1}{6}m(a^2 + \frac{1}{3}c^2)$
Rectangular Tetrahedron	$\bar{x} = \frac{a}{4}$ $\bar{y} = \frac{b}{4}$ $\bar{z} = \frac{c}{4}$	$I_{xx} = \frac{1}{10}m(b^2 + c^2)$ $I_{yy} = \frac{1}{10}m(a^2 + c^2)$ $I_{zz} = \frac{1}{10}m(a^2 + b^2)$ $\bar{I}_{xx} = \frac{3}{80}m(b^2 + c^2)$ $\bar{I}_{yy} = \frac{3}{80}m(a^2 + c^2)$ $\bar{I}_{zz} = \frac{3}{80}m(a^2 + b^2)$
 Half Torus	$\bar{x} = \frac{a^2 + 4R^2}{2\pi R}$	$I_{xx} = I_{yy} = \frac{1}{2}mR^2 + \frac{5}{8}ma^2$ $I_{zz} = mR^2 + \frac{3}{4}ma^2$

D_04_tbl_cont3

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS

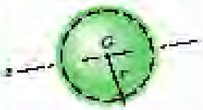
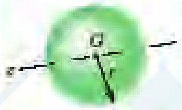
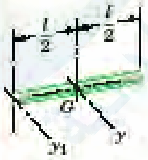
(m = mass of body shown)

BODY		MASS CENTER	MASS MOMENTS OF INERTIA
	Circular Cylindrical Shell	—	$I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = mr^2$
	Half Cylindrical Shell	$\bar{x} = \frac{2r}{\pi}$	$I_{xx} = I_{yy}$ $= \frac{1}{2}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{2}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = mr^2$ $\bar{I}_{zz} = \left(1 - \frac{4}{\pi^2}\right)mr^2$
	Circular Cylinder	—	$I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$
	Semicylinder	$\bar{x} = \frac{4r}{3\pi}$	$I_{xx} = I_{yy}$ $= \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = I_{y_1y_1}$ $= \frac{1}{4}mr^2 + \frac{1}{3}ml^2$ $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
	Rectangular Parallelepiped	—	$I_{xx} = \frac{1}{12}m(a^2 + l^2)$ $I_{yy} = \frac{1}{12}m(b^2 + l^2)$ $I_{zz} = \frac{1}{12}m(a^2 + b^2)$ $I_{y_1y_1} = \frac{1}{12}mb^2 + \frac{1}{3}ml^2$ $I_{y_2y_2} = \frac{1}{3}m(b^2 + l^2)$

D_04tbl

TABLE D/4 PROPERTIES OF HOMOGENEOUS SOLIDS *Continued*

(m = mass of body shown)

BODY		MASS CENTER	MASS MOMENTS OF INERTIA
	Spherical Shell	—	$I_{zz} = \frac{2}{3}mr^2$
	Hemispherical Shell	$\bar{x} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{5}{12}mr^2$
	Sphere	—	$I_{zz} = \frac{2}{5}mr^2$
	Hemisphere	$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{83}{320}mr^2$
	Uniform Slender Rod	—	$I_{yy} = \frac{1}{12}ml^2$ $I_{y_1y_1} = \frac{1}{3}ml^2$

D_04tbl_cont

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 1 : Introduction to Statics.

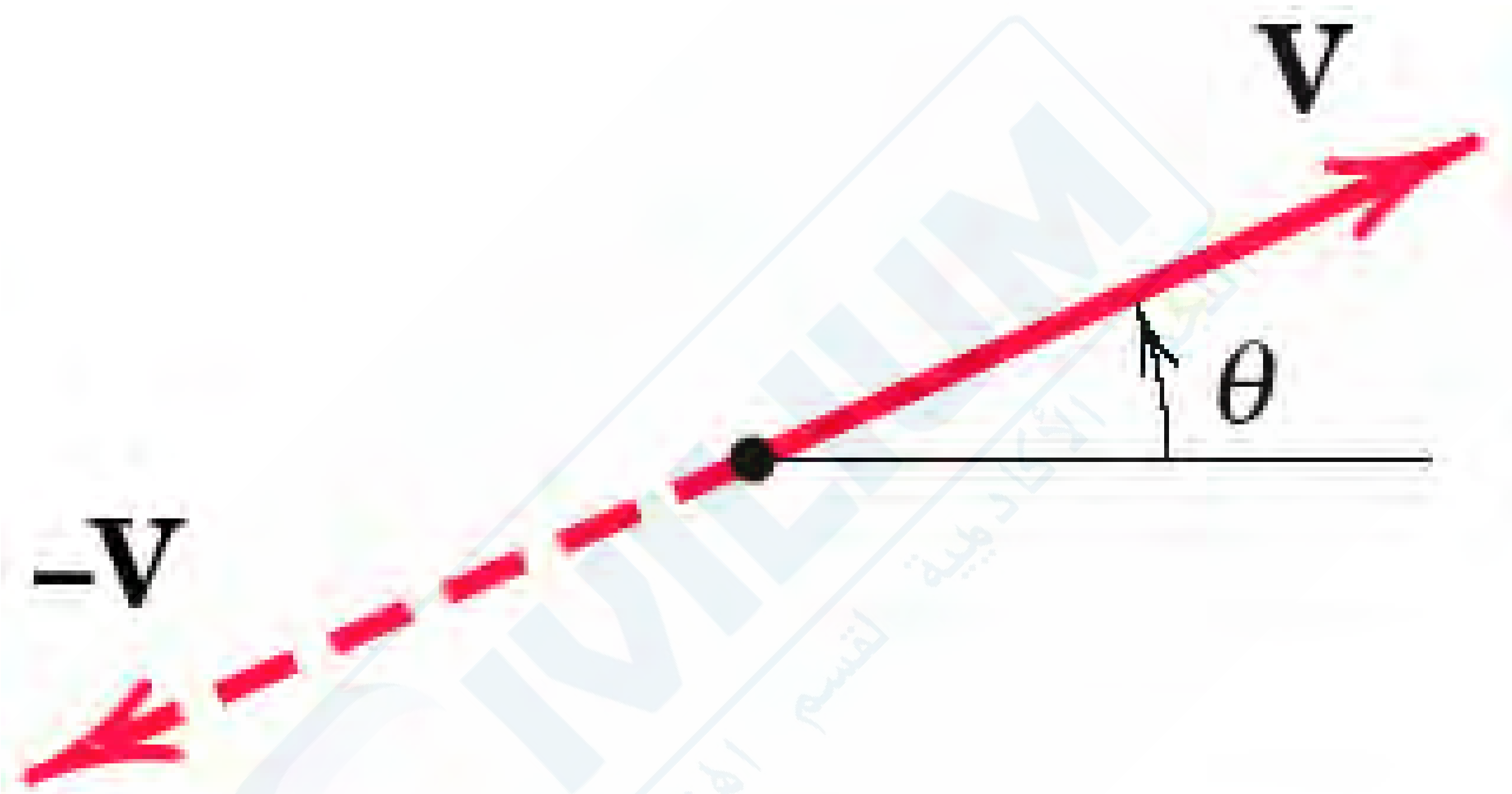


Figure 1/1

01_01

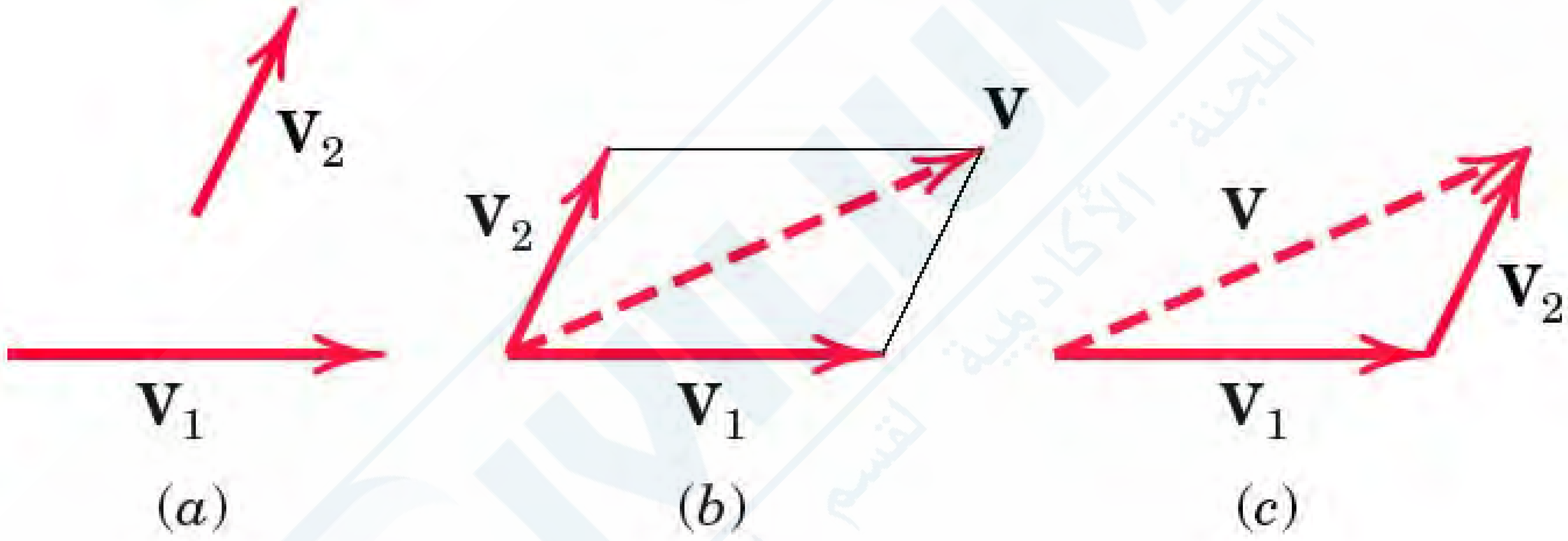


Figure 1/2

01_02

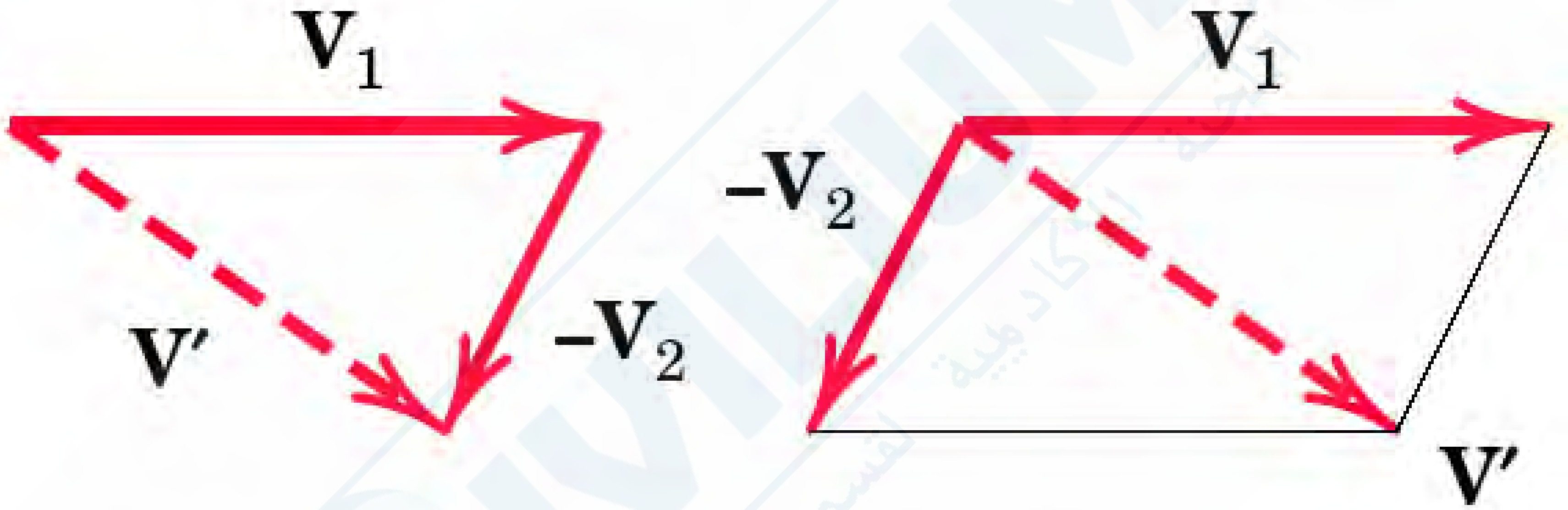


Figure 1/3

01_03

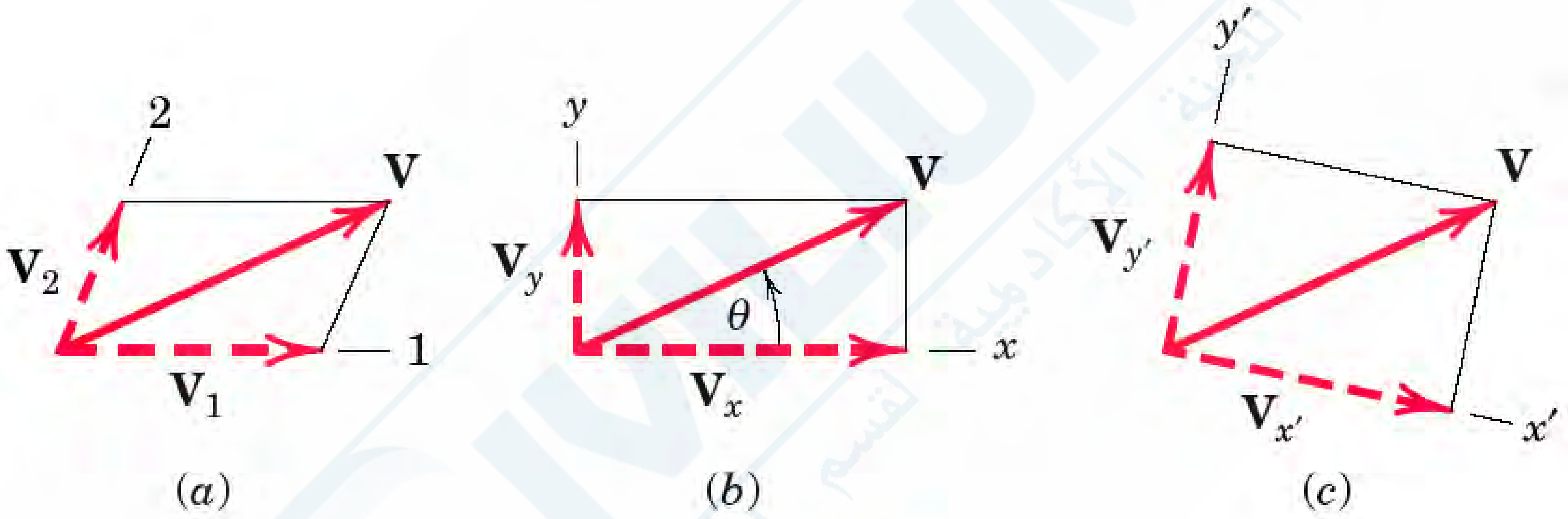


Figure 1/4

01_04

CIVILIUM

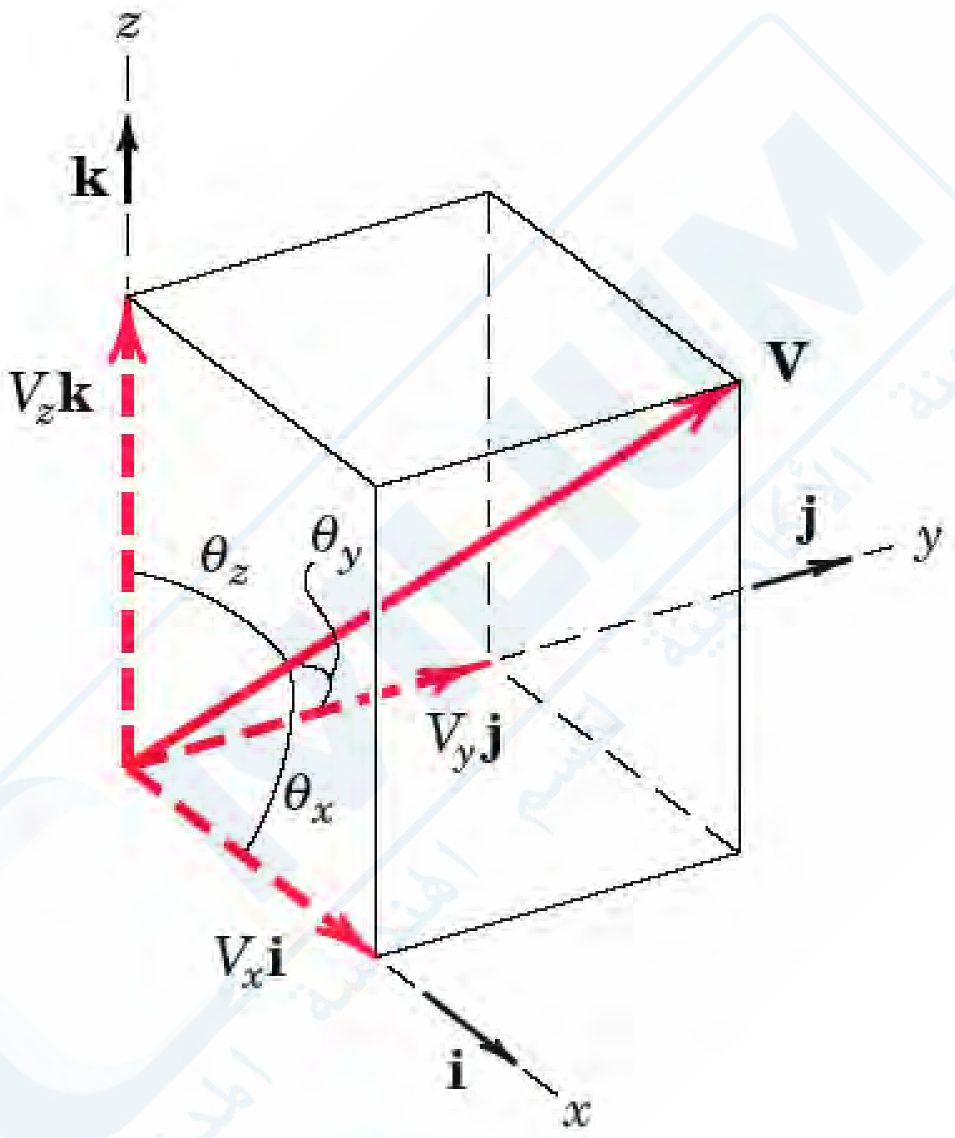


Figure 1/5

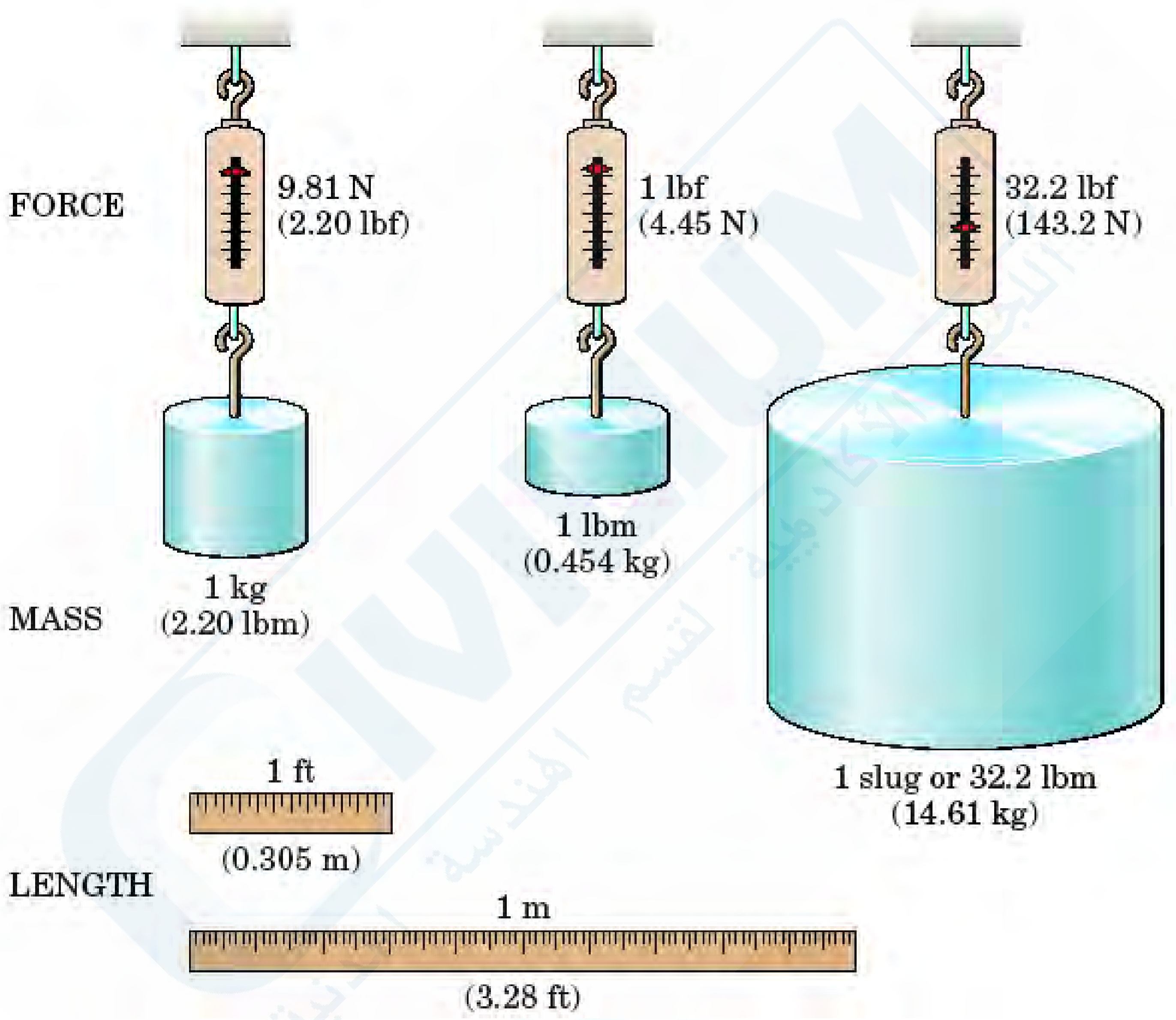

FORCE
9.81 N
(2.20 lbf)
1 lbf
(4.45 N)
32.2 lbf
(143.2 N)
MASS
1 kg
(2.20 lbm)
1 lbm
(0.454 kg)
1 slug or 32.2 lbm
(14.61 kg)
LENGTH
1 ft
(0.305 m)
1 m
(3.28 ft)

Figure 1/6

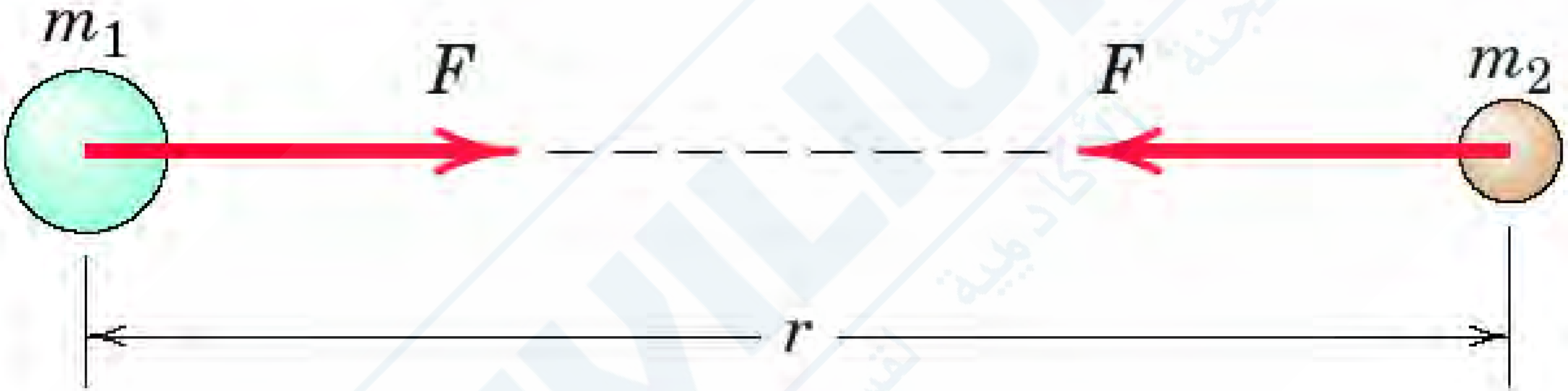


Figure 1/7

01_07

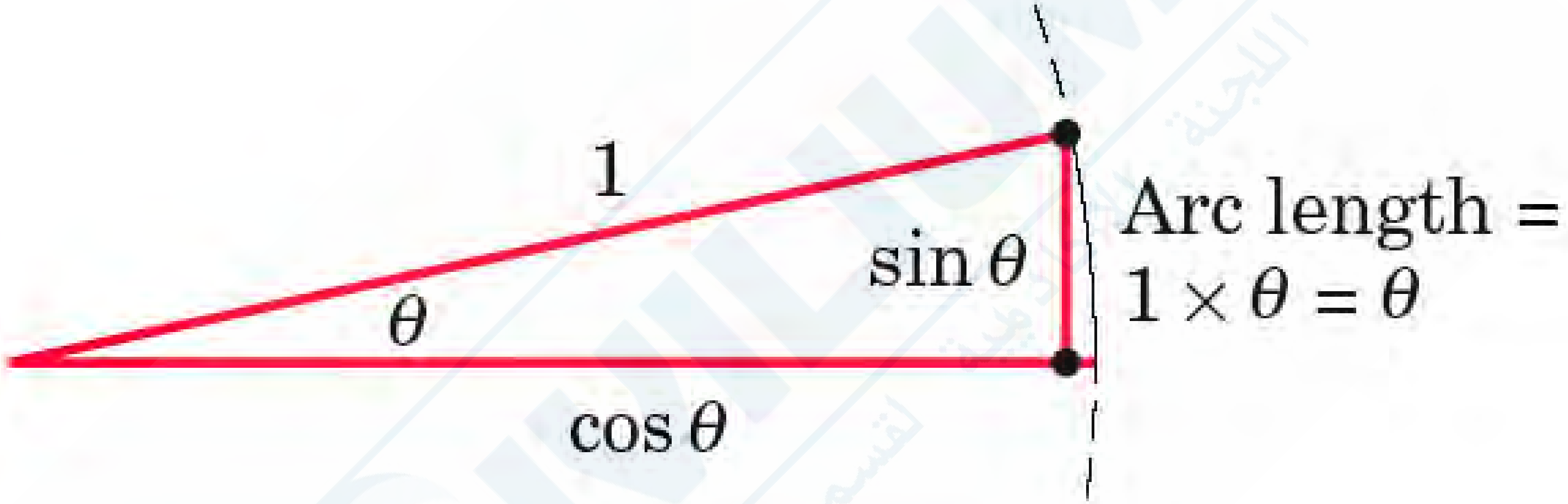
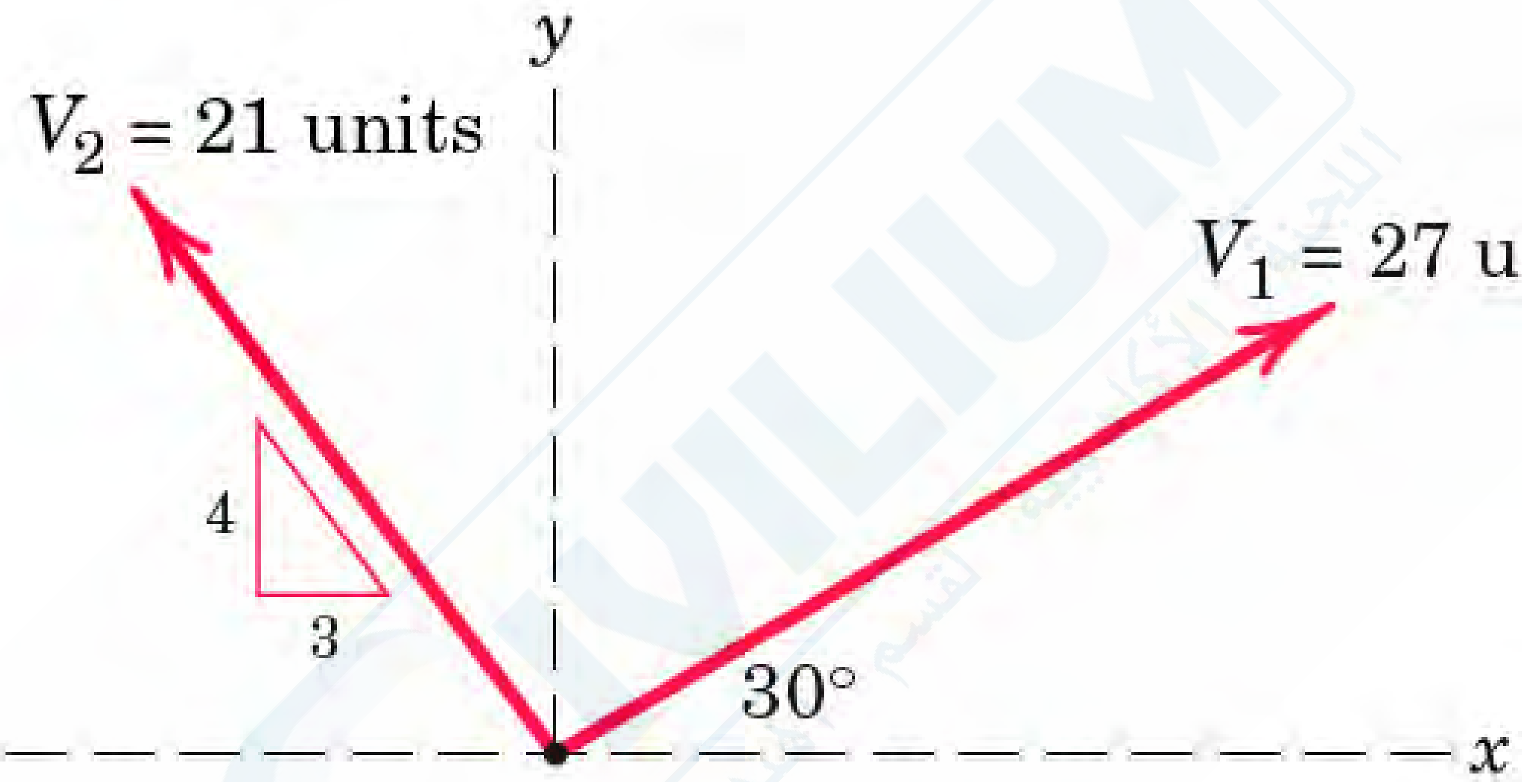


Figure 1/8

01_08

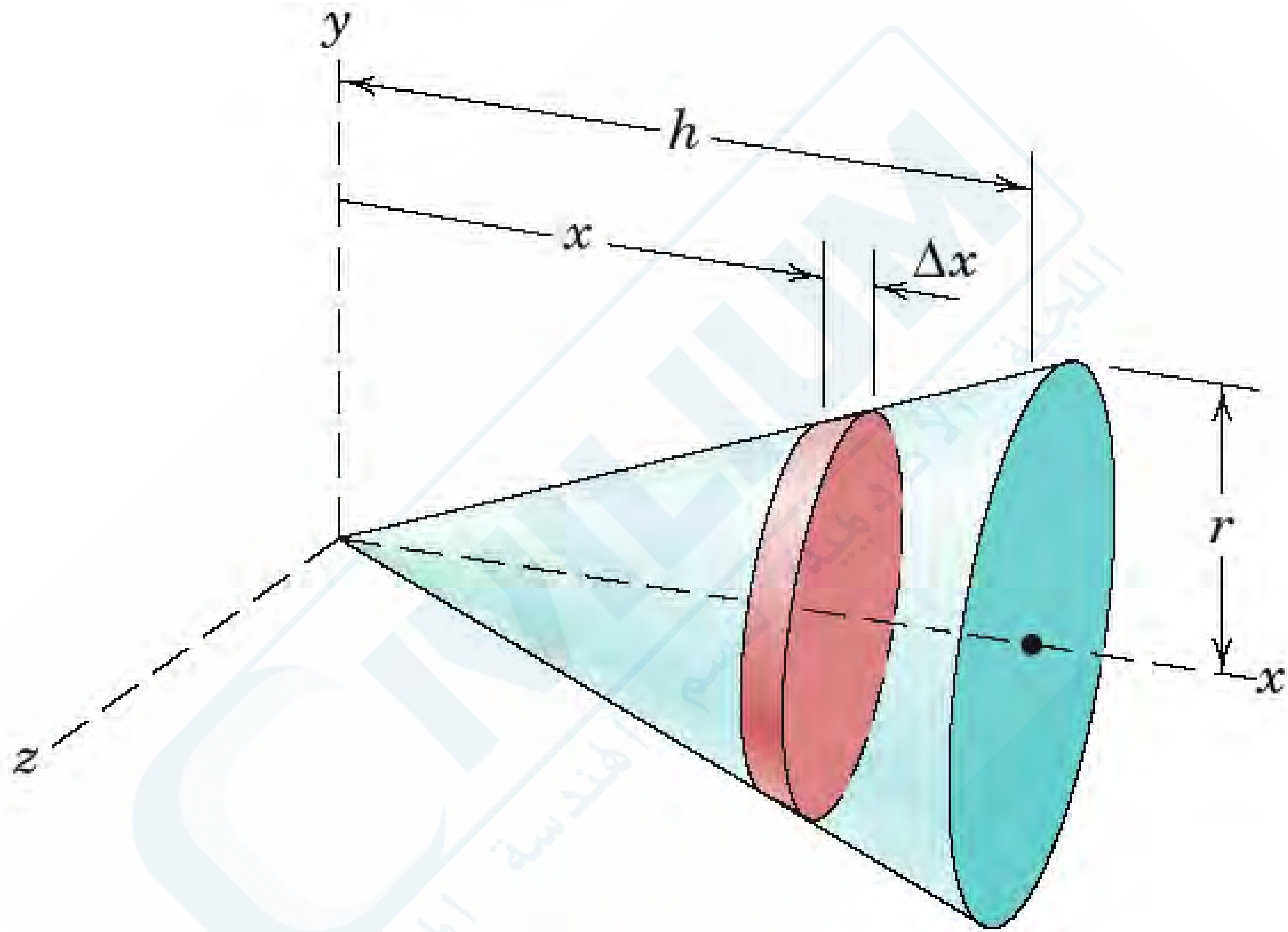


Problem 1/2

P_01_02

Problem 1/9

P_01_09



Problem 1/10

P_01_10

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 2 : Force Systems.

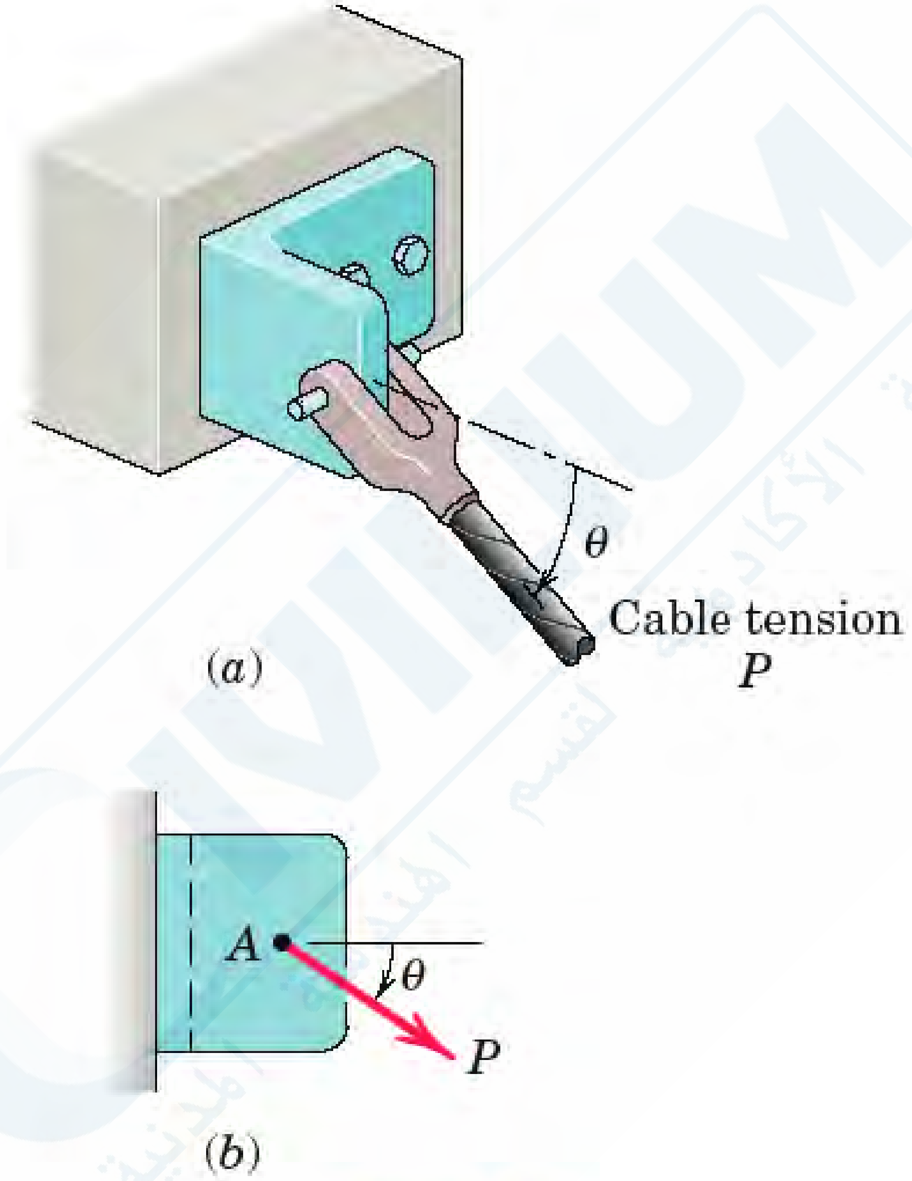


Figure 2/1

CIVIUM
اللجنة الأكاديمية لقسم الهندسة المدنية

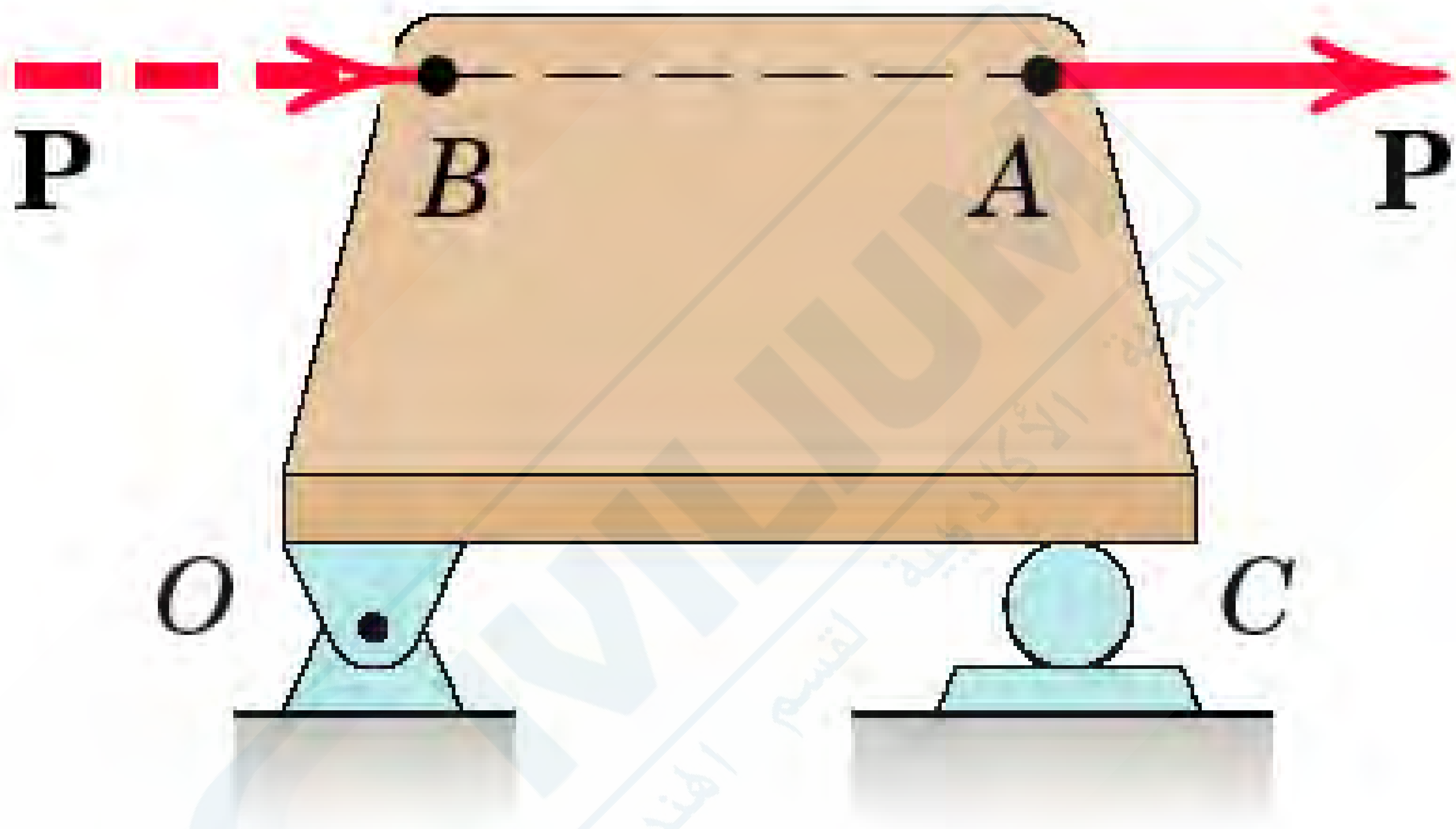


Figure 2/2

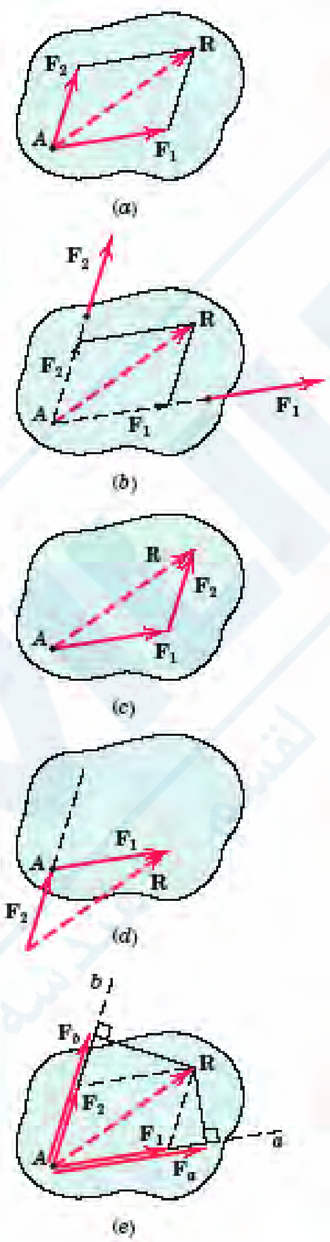


Figure 2/3

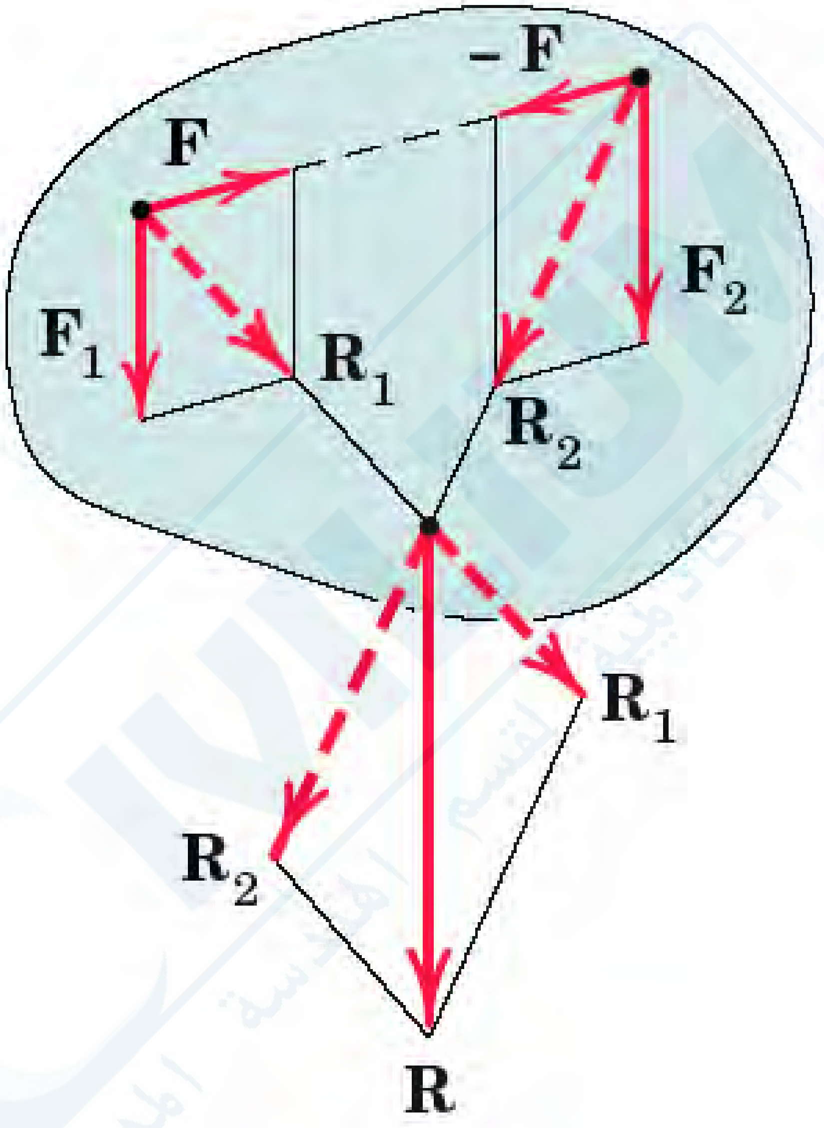


Figure 2/4

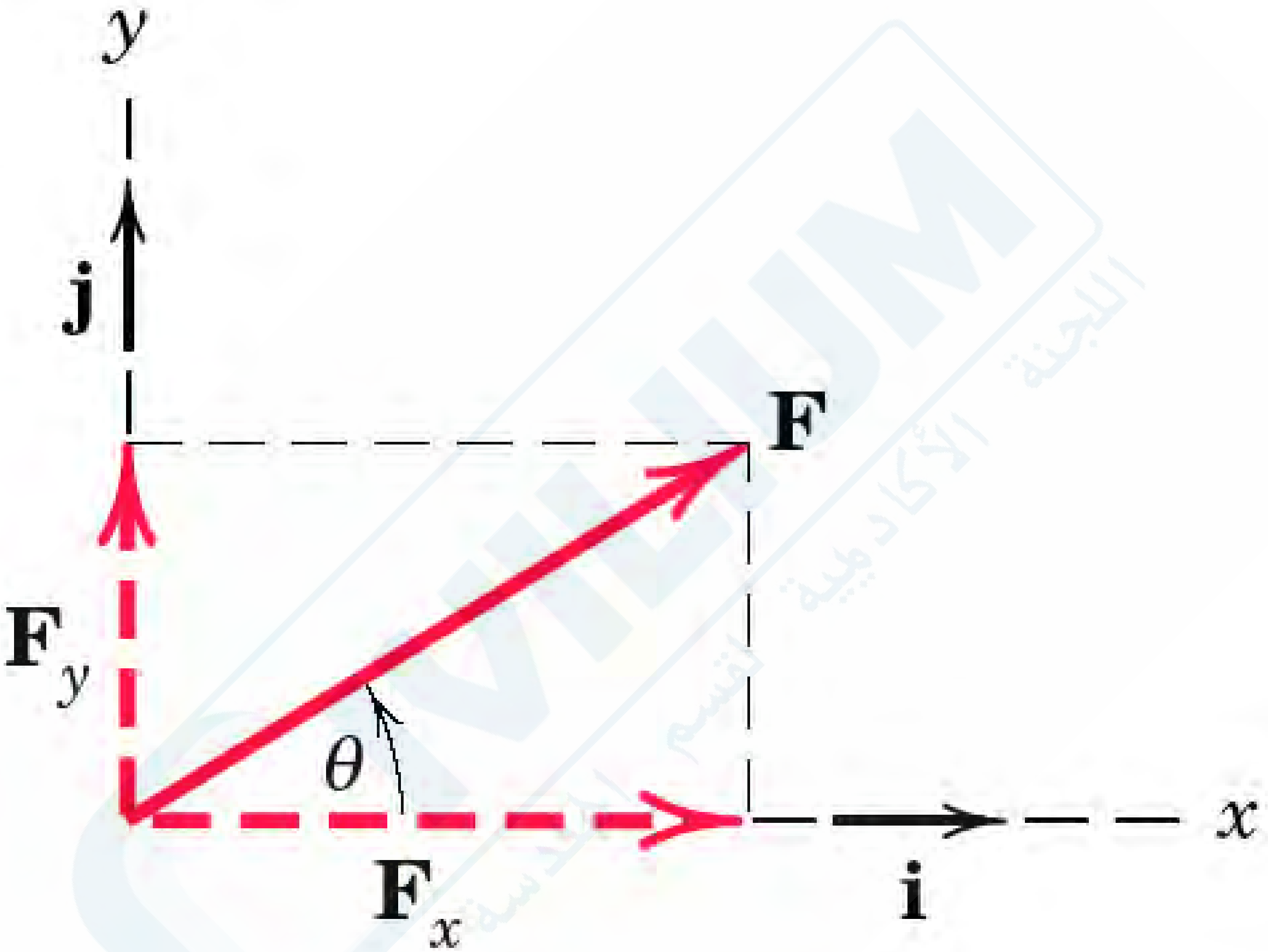


Figure 2/5

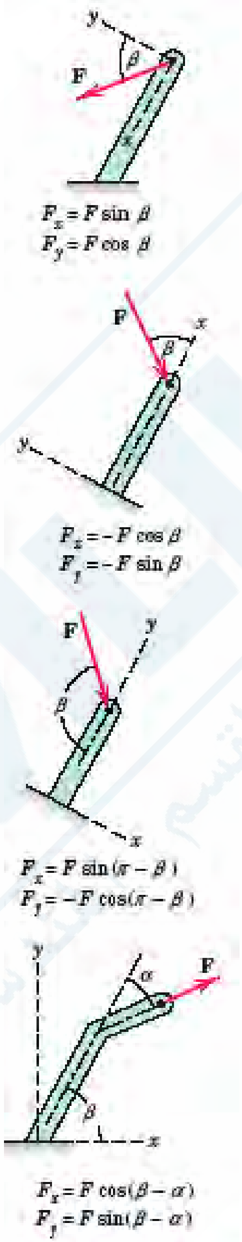


Figure 2/6

CIVILIUM اللجنة الأكاديمية لقسم الهندسة المدنية

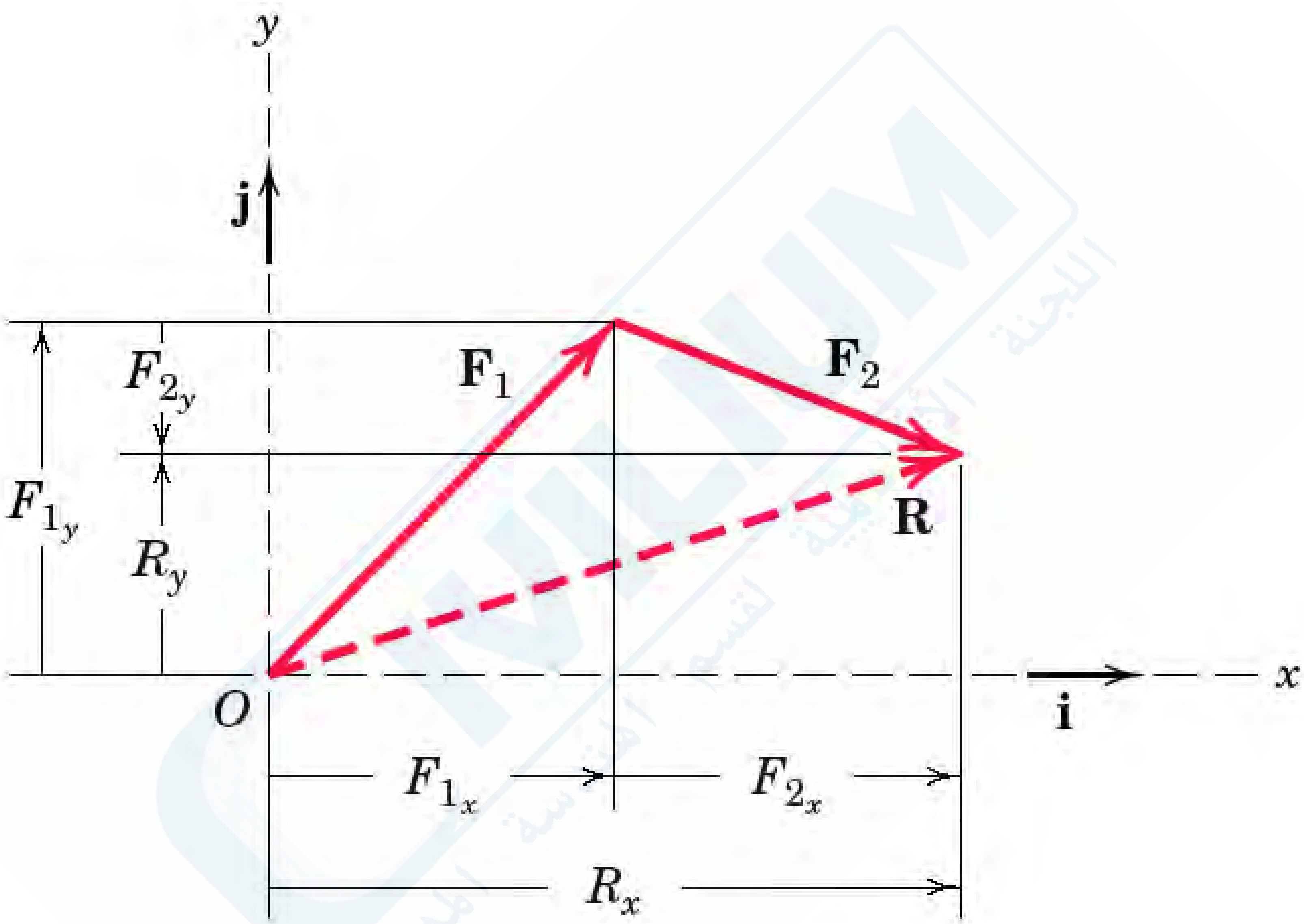


Figure 2/7

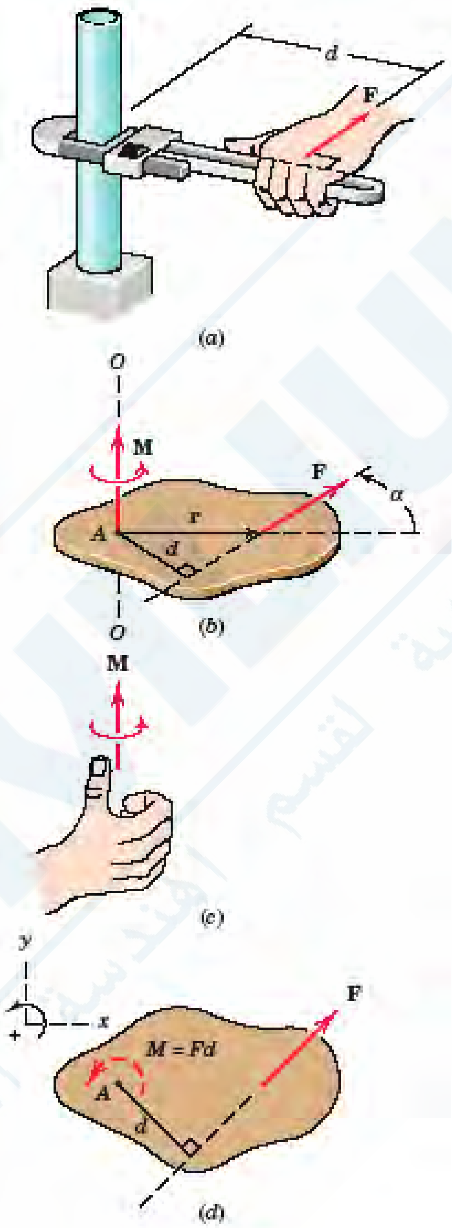


Figure 2/8

اللجنة الأكاديمية لقسم الهندسة المدنية

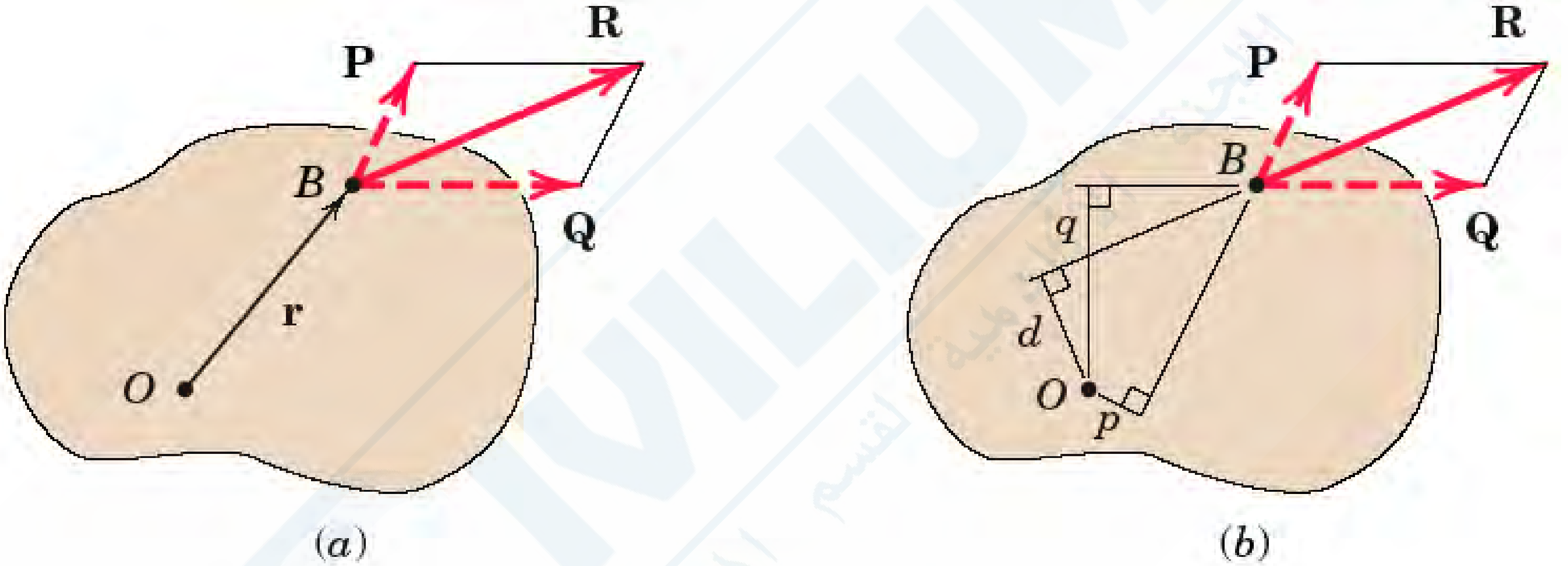


Figure 2/9

02_09

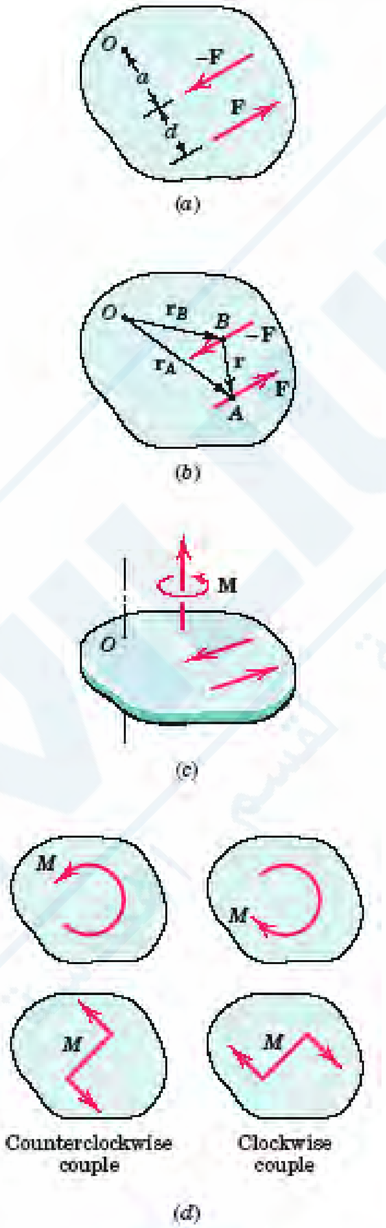

O
a
d
–F
F
(a)
O
r_B
B
–F
r
r_A
F
A
(b)
M
O
(c)
M
M
M
M
Counterclockwise couple
Clockwise couple
(d)

Figure 2/10

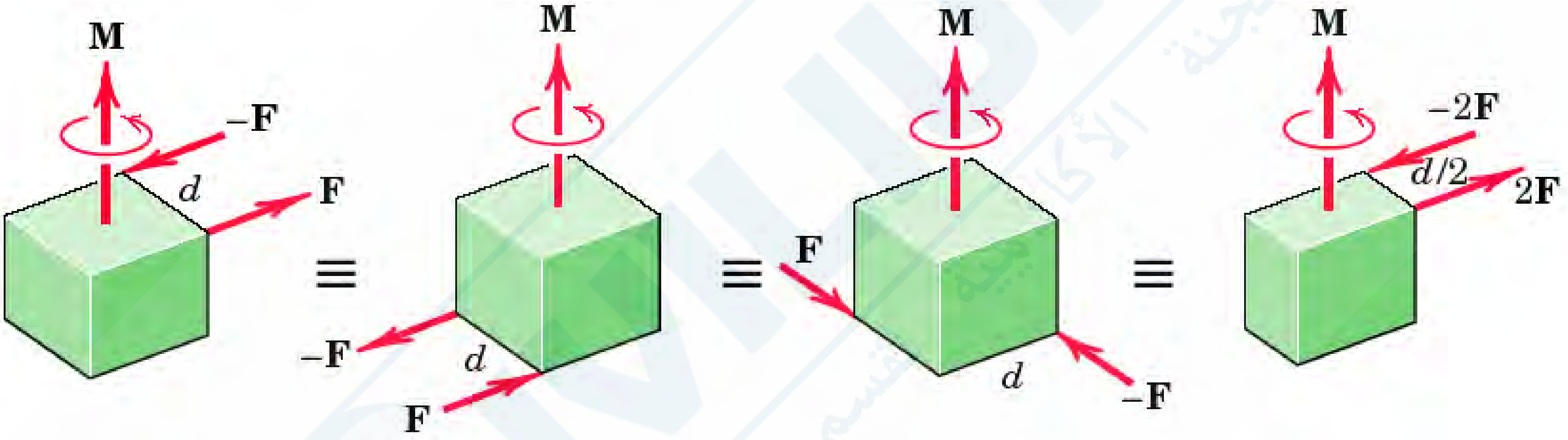


Figure 2/11

02_11

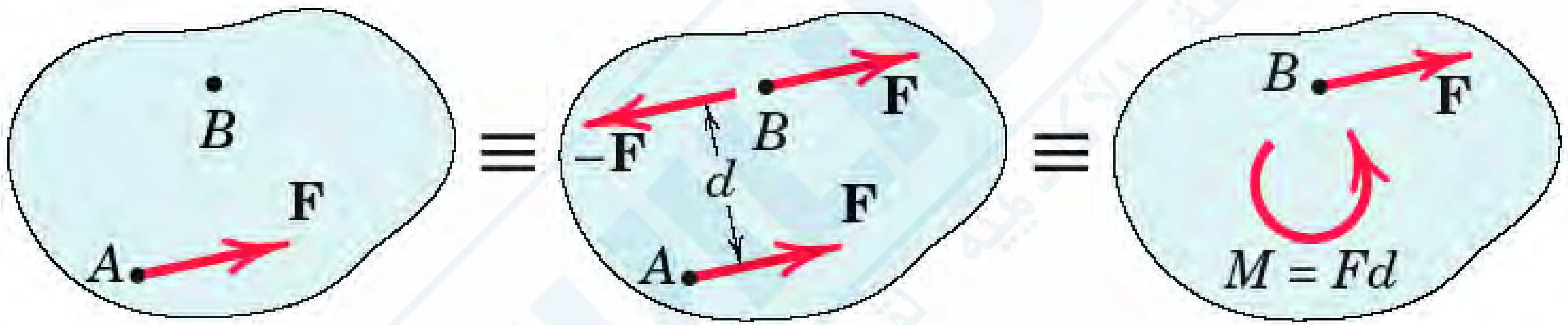


Figure 2/12

02_12

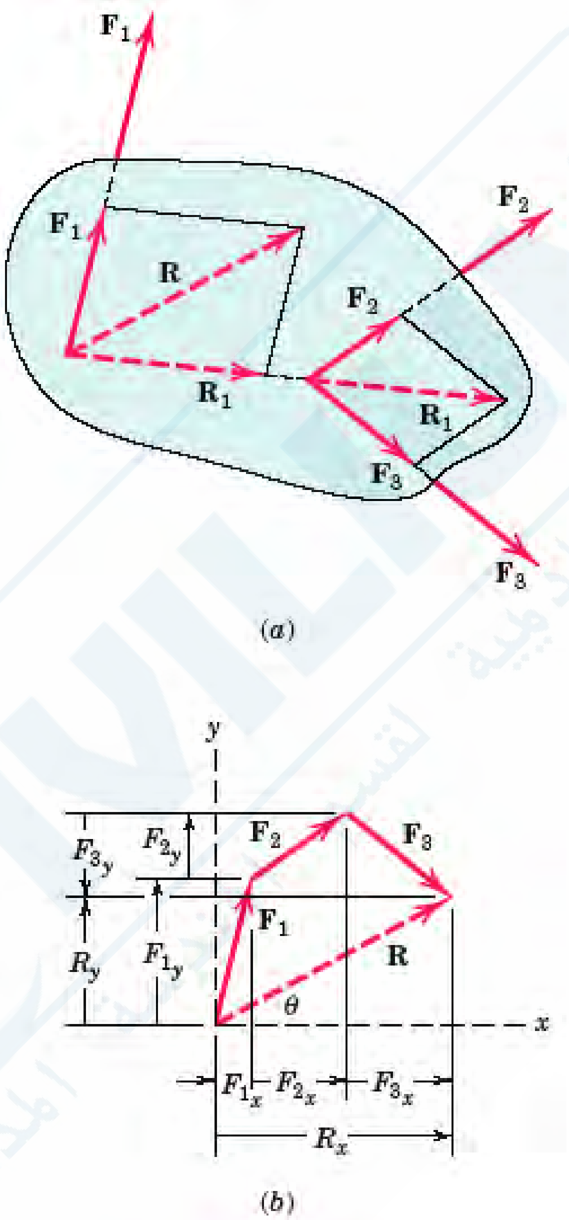

F1
F1
R
R1
F2
F2
R1
F3
F3
(a)
y
F3y
F2y
F2
F3
F1
Ry
F1y
R
θ
x
F1x
F2x
F3x
Rx
(b)

Figure 2/13

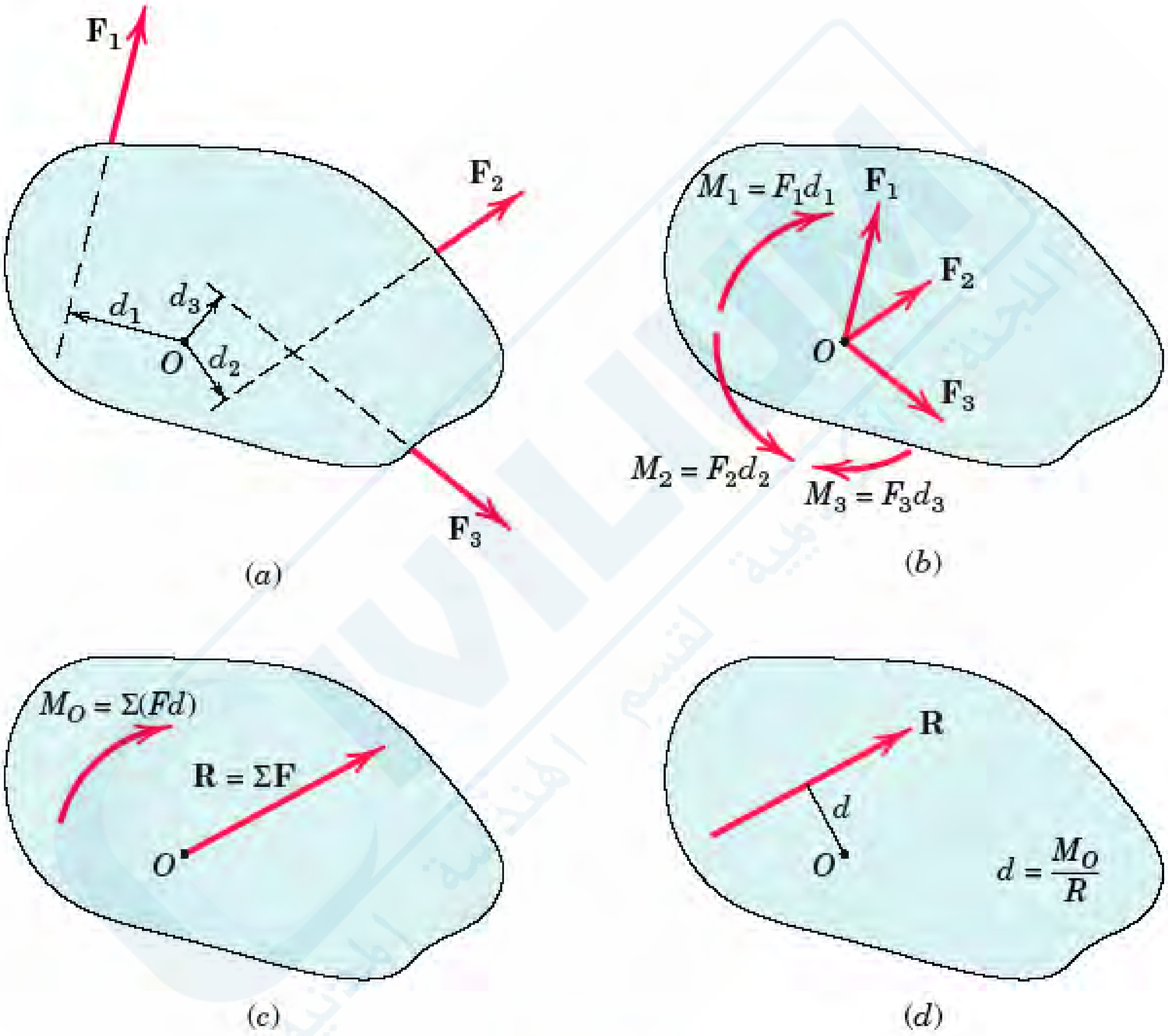

F1
F2
F3
d1
d3
d2
O
(a)
M1 = F1d1
F1
F2
F3
O
M2 = F2d2
M3 = F3d3
(b)
MO = Σ(Fd)
R = ΣF
O
(c)
R
d
O
d = MO/R
(d)

Figure 2/14

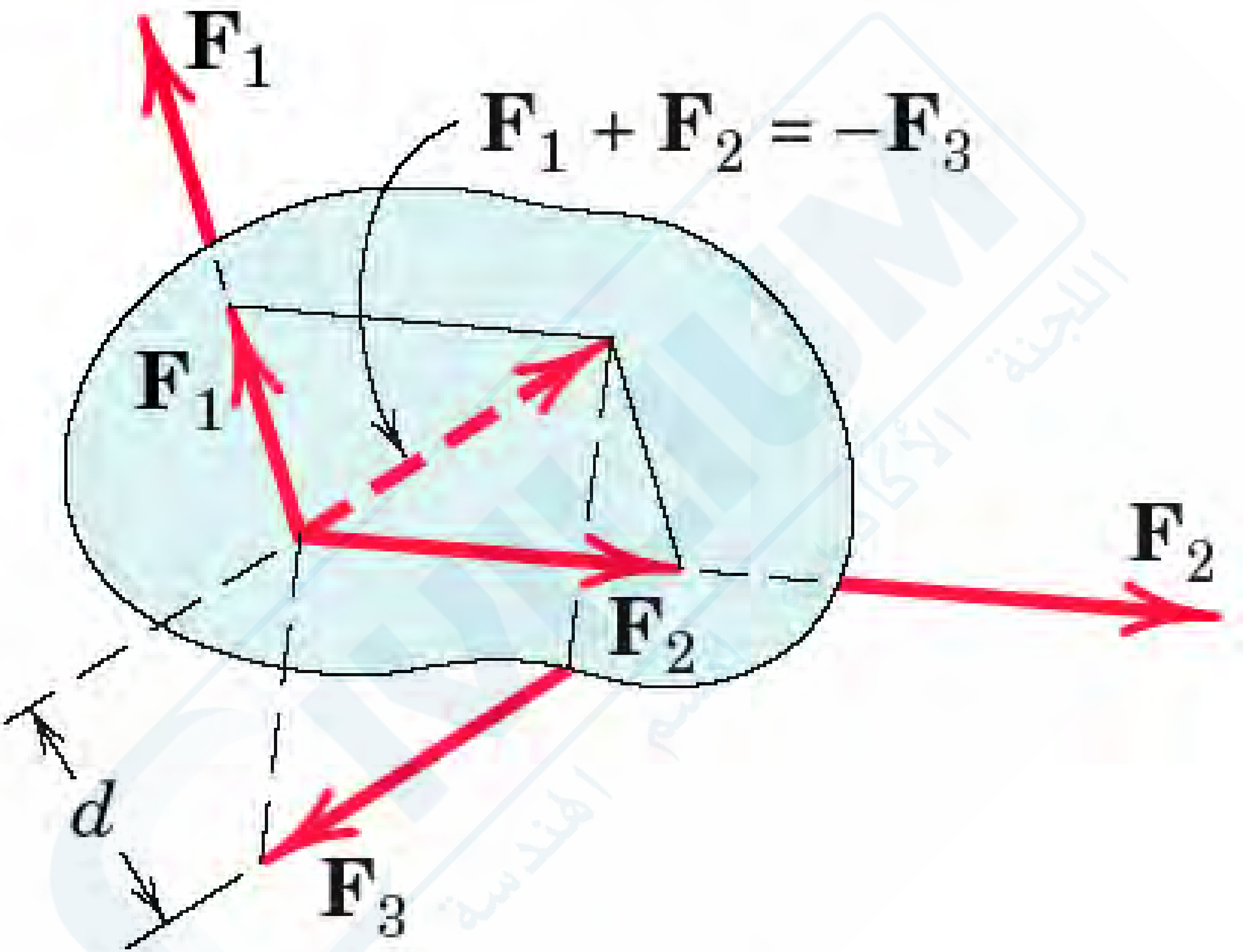


Figure 2/15

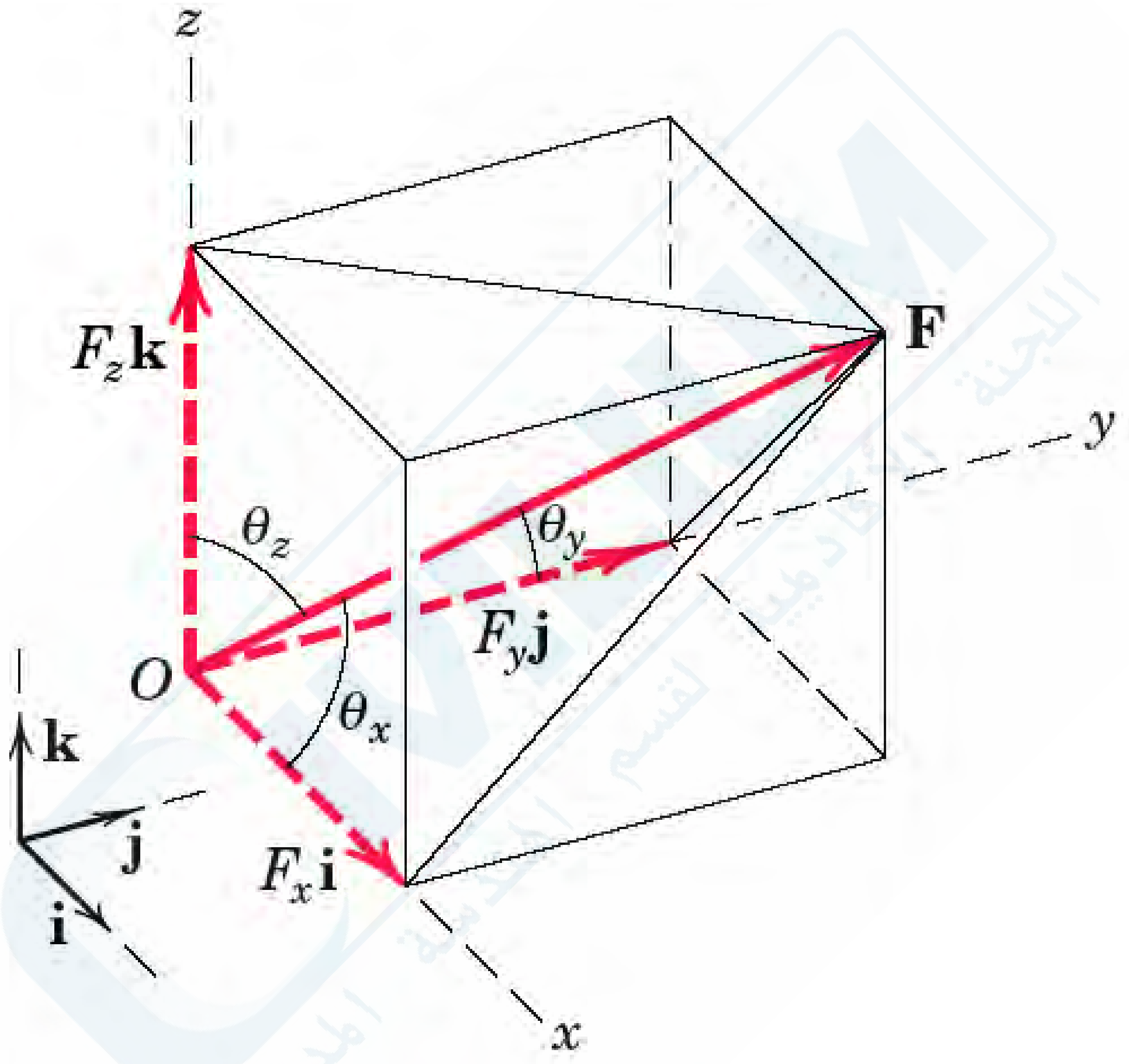

z
F
y
$F_z\mathbf{k}$
θ_z
θ_y
$F_y\mathbf{j}$
O
θ_x
k
j
i
$F_x\mathbf{i}$
x

Figure 2/16

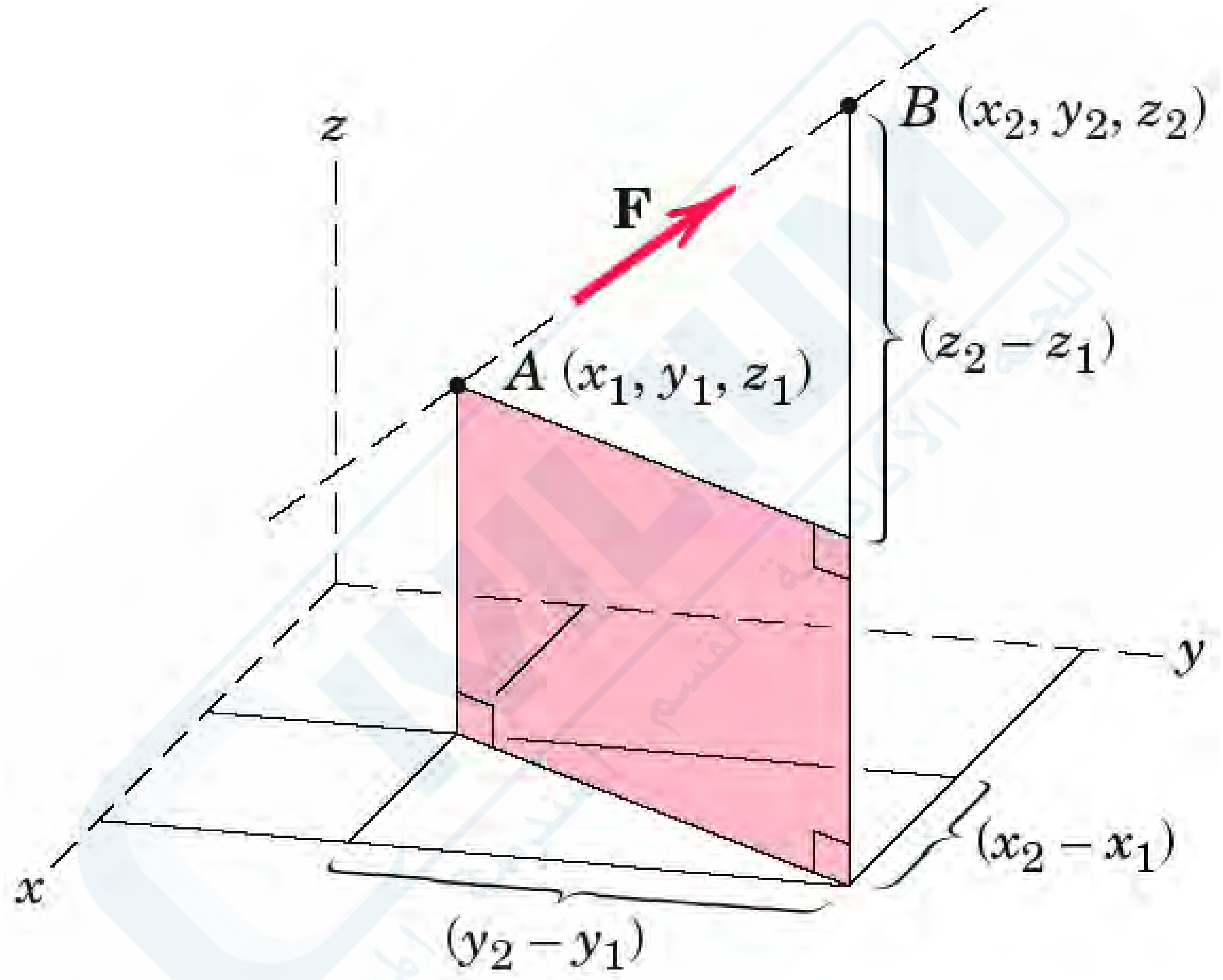


Figure 2/17

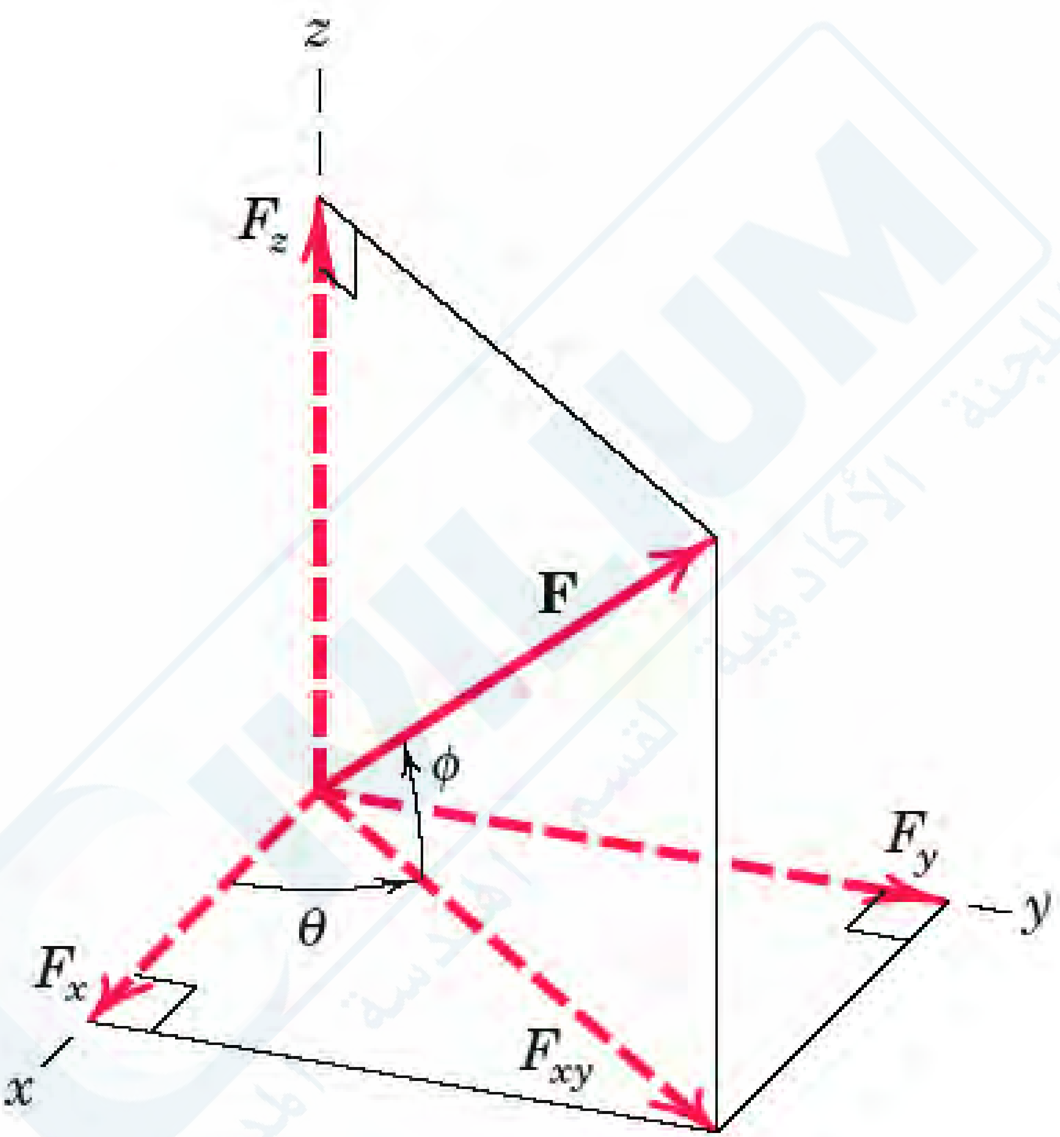


Figure 2/18

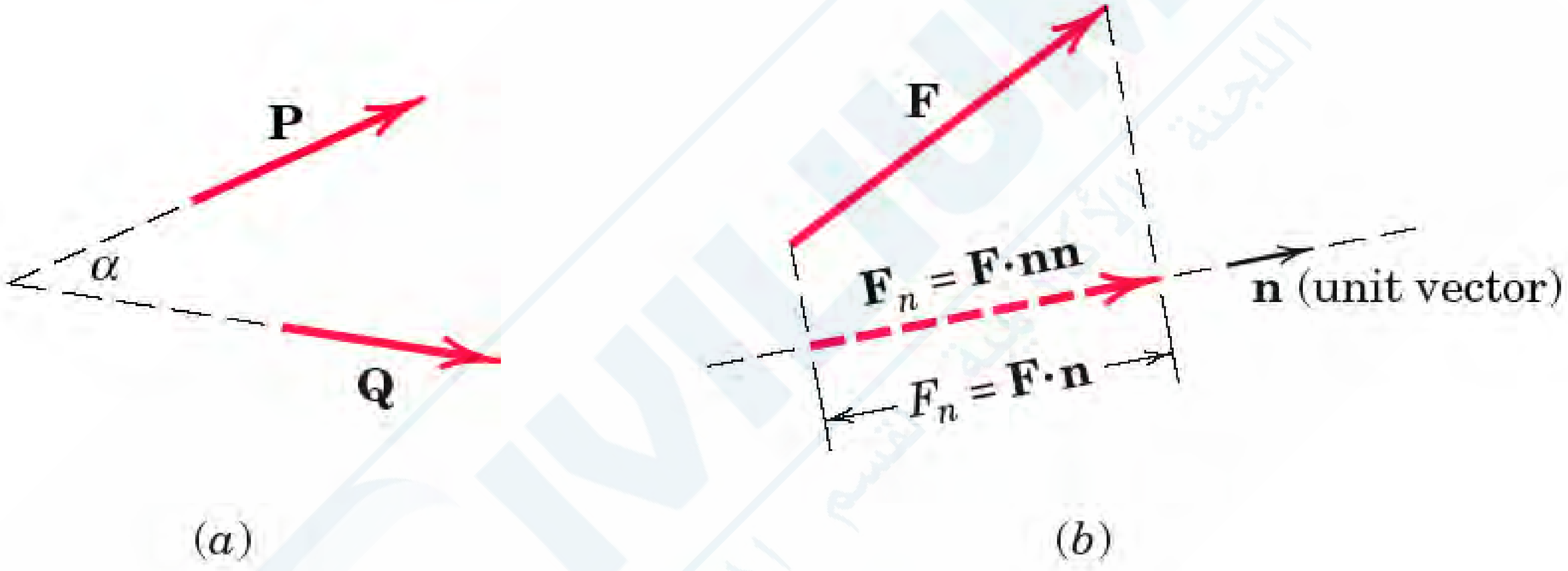


Figure 2/19

02_19

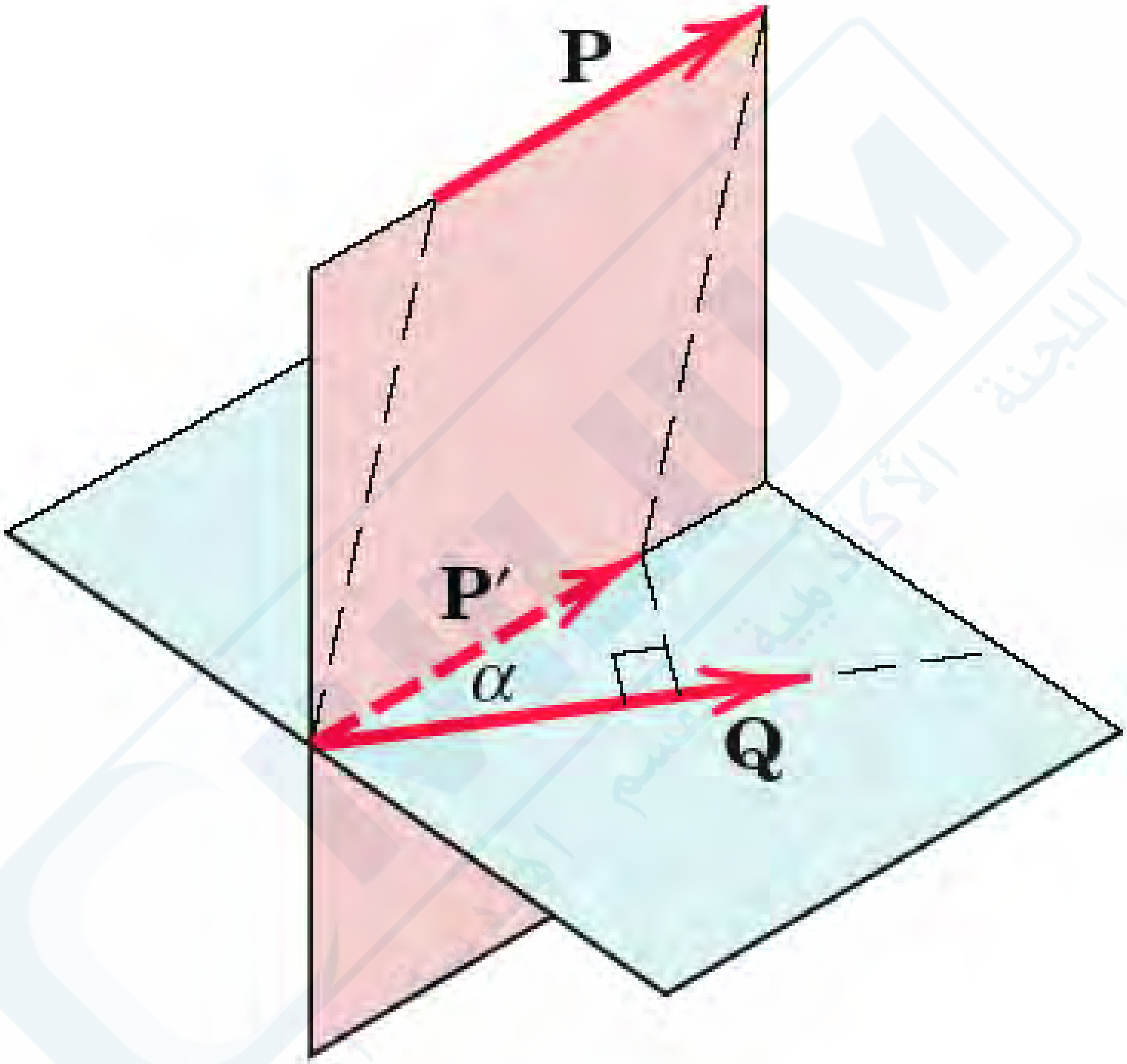


Figure 2/20

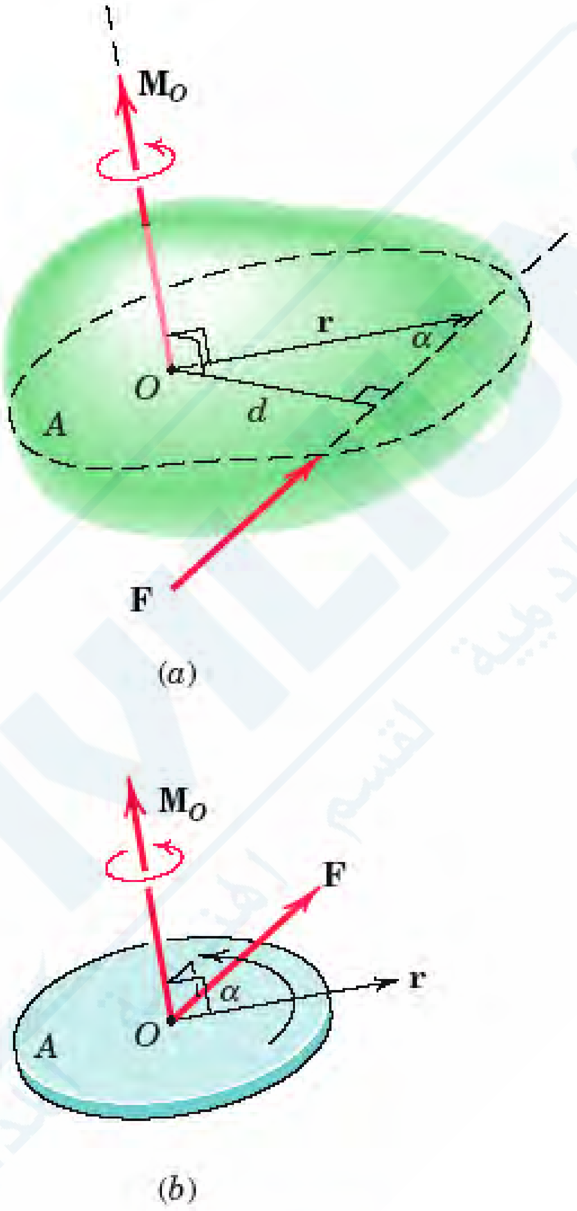


Figure 2/21

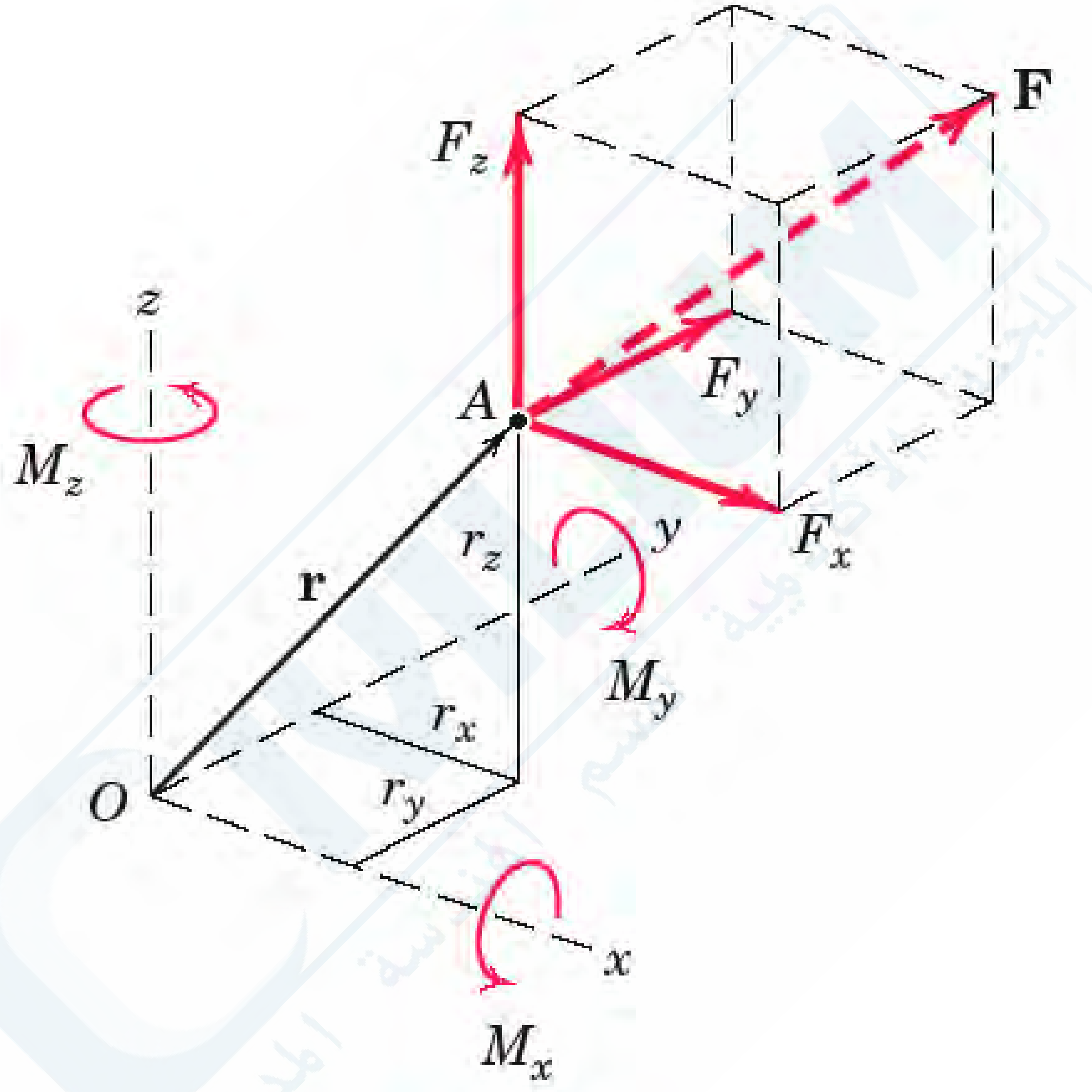


Figure 2/22

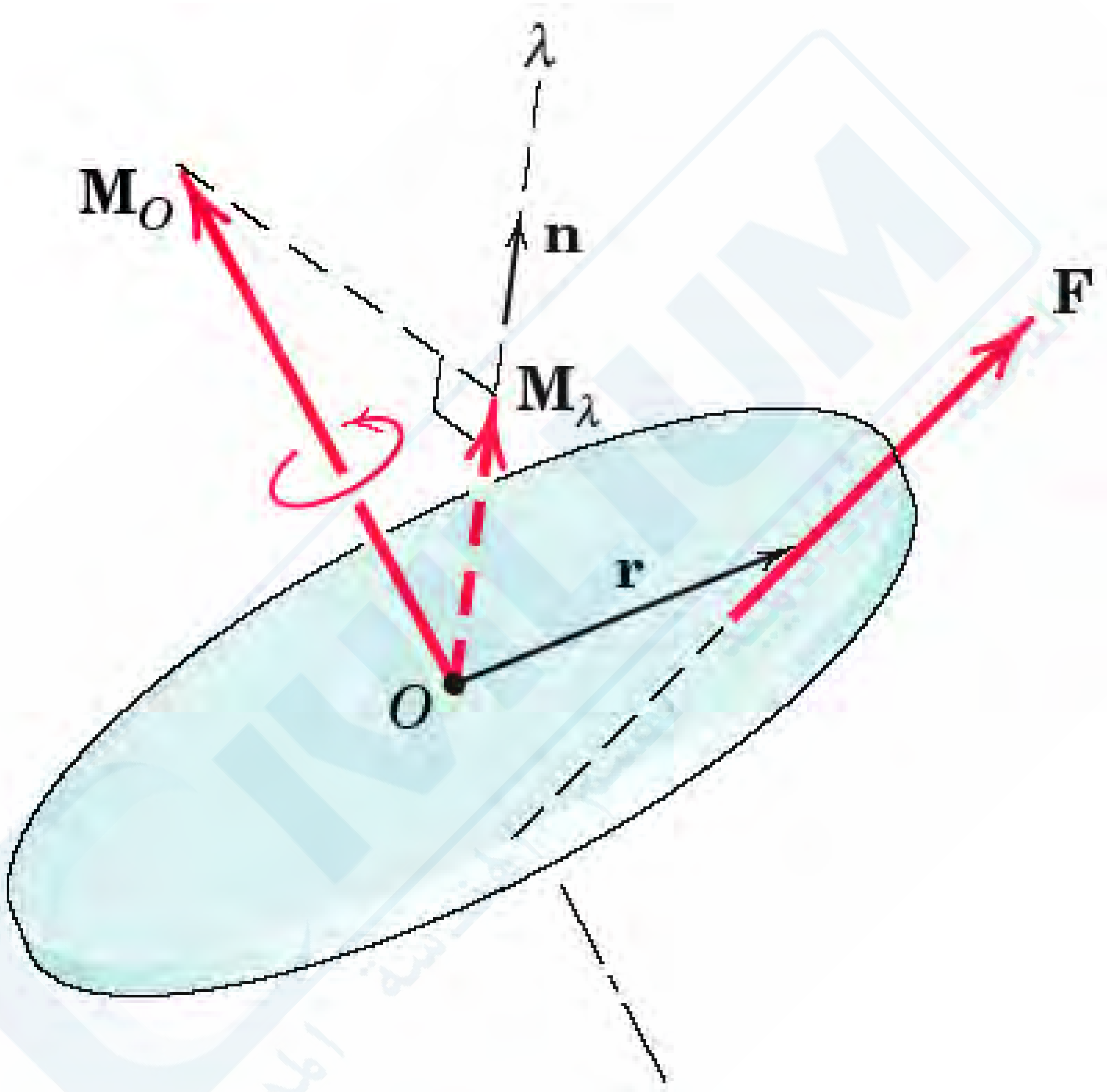


Figure 2/23

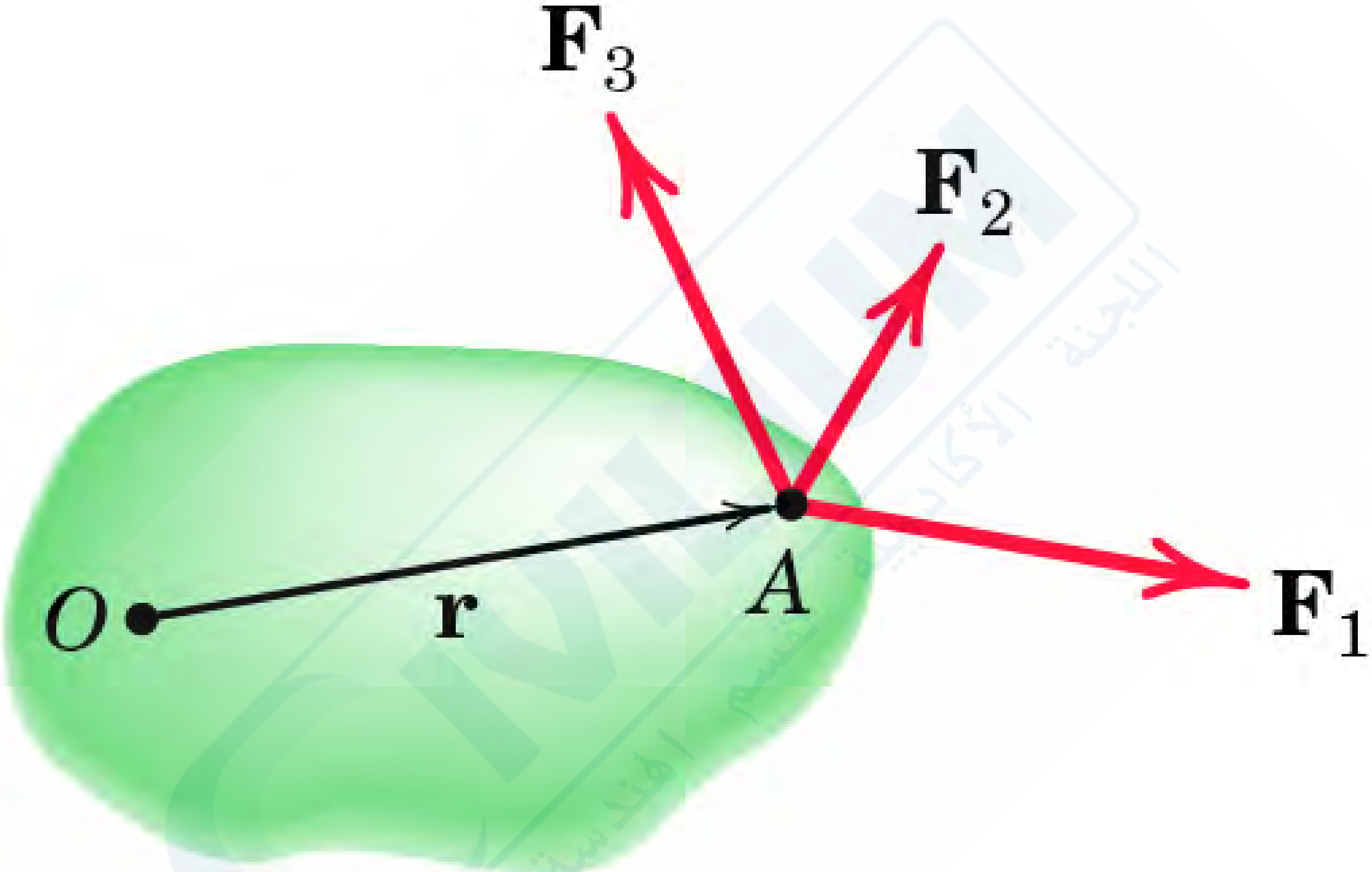


Figure 2/24

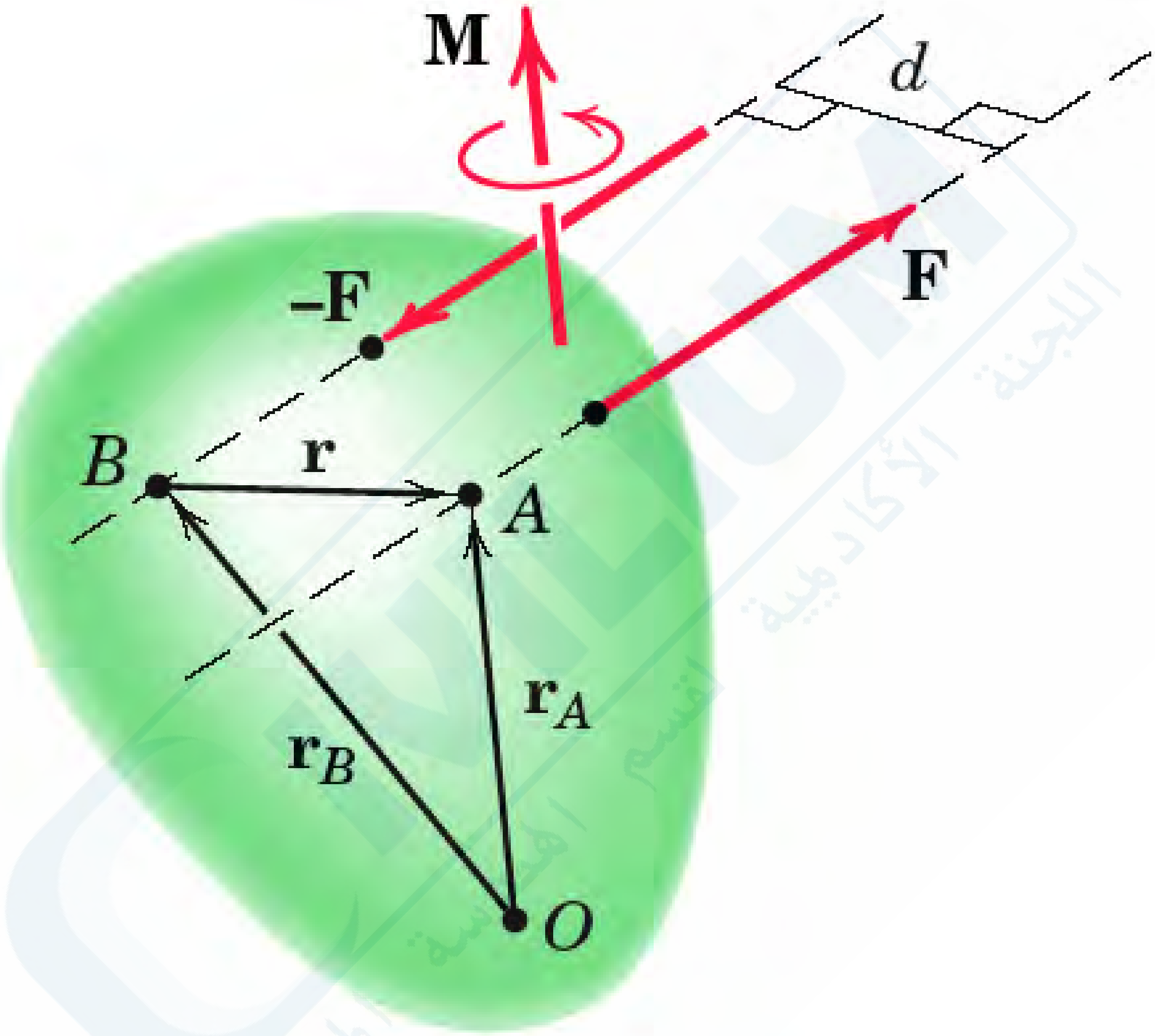


Figure 2/25

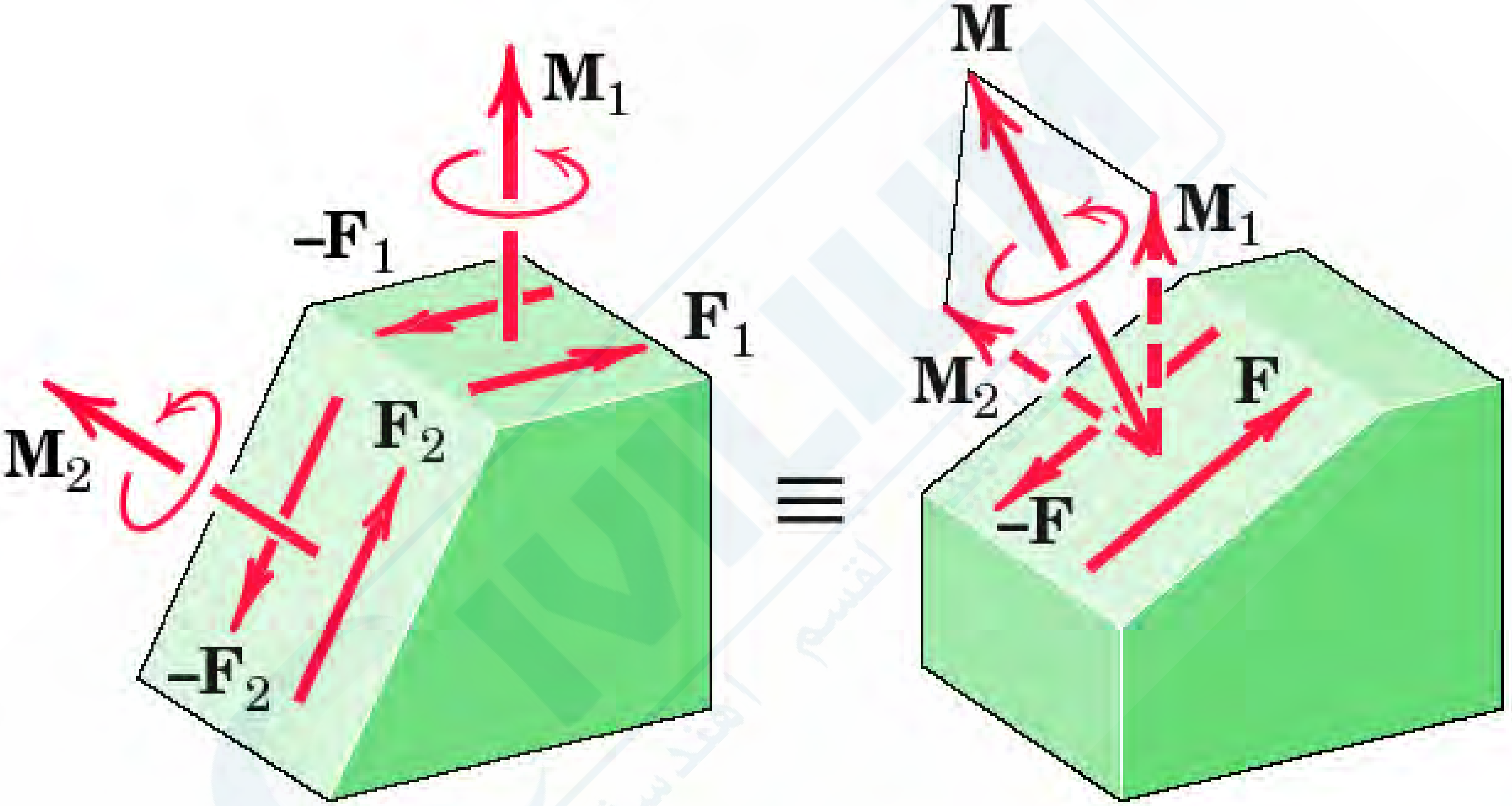


Figure 2/26

02_26

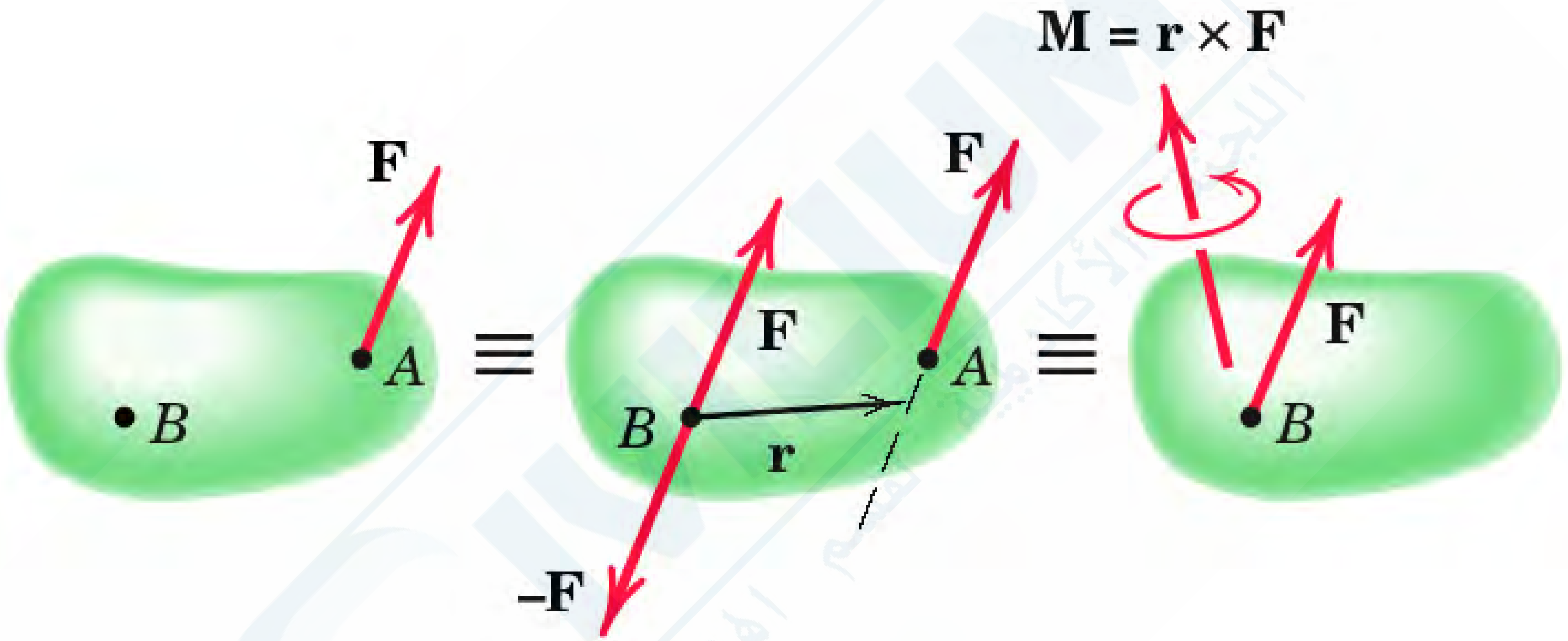


Figure 2/27

02_27

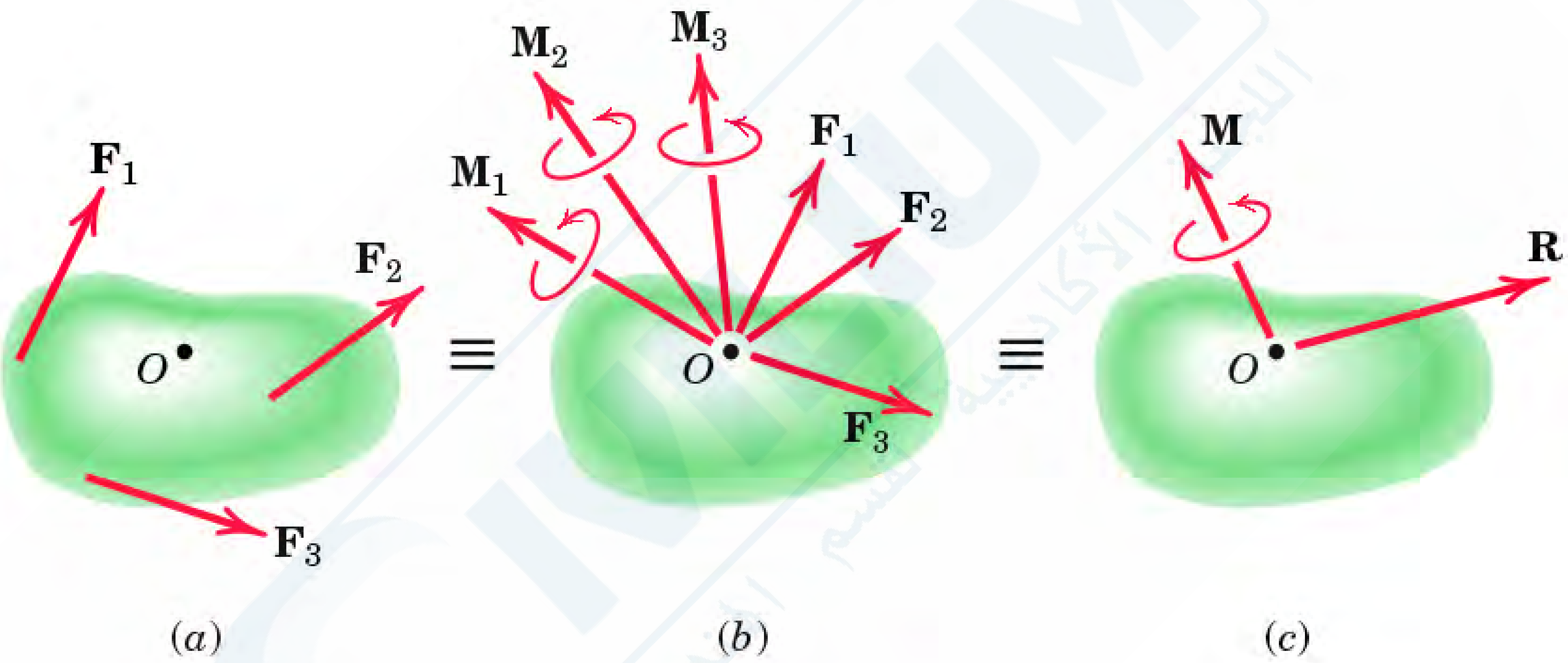


Figure 2/28

02_28

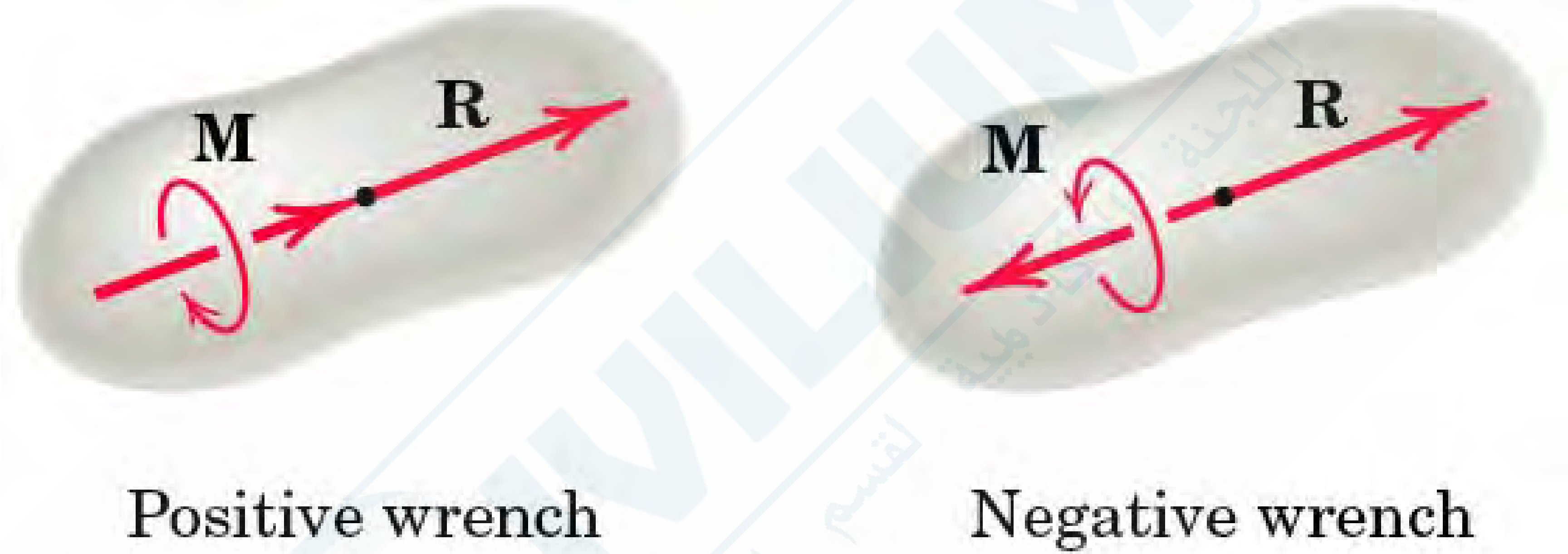


Figure 2/29

02_29

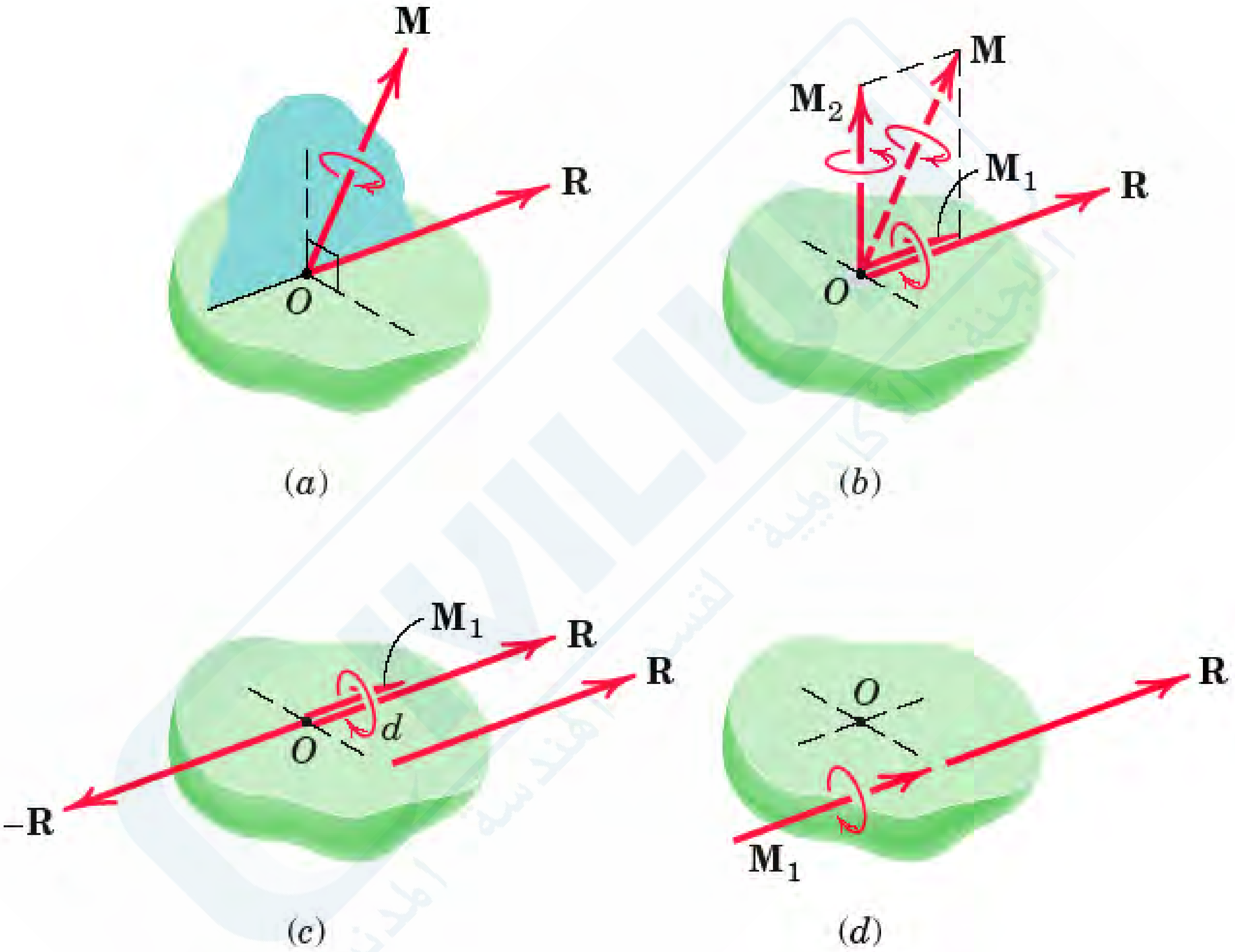
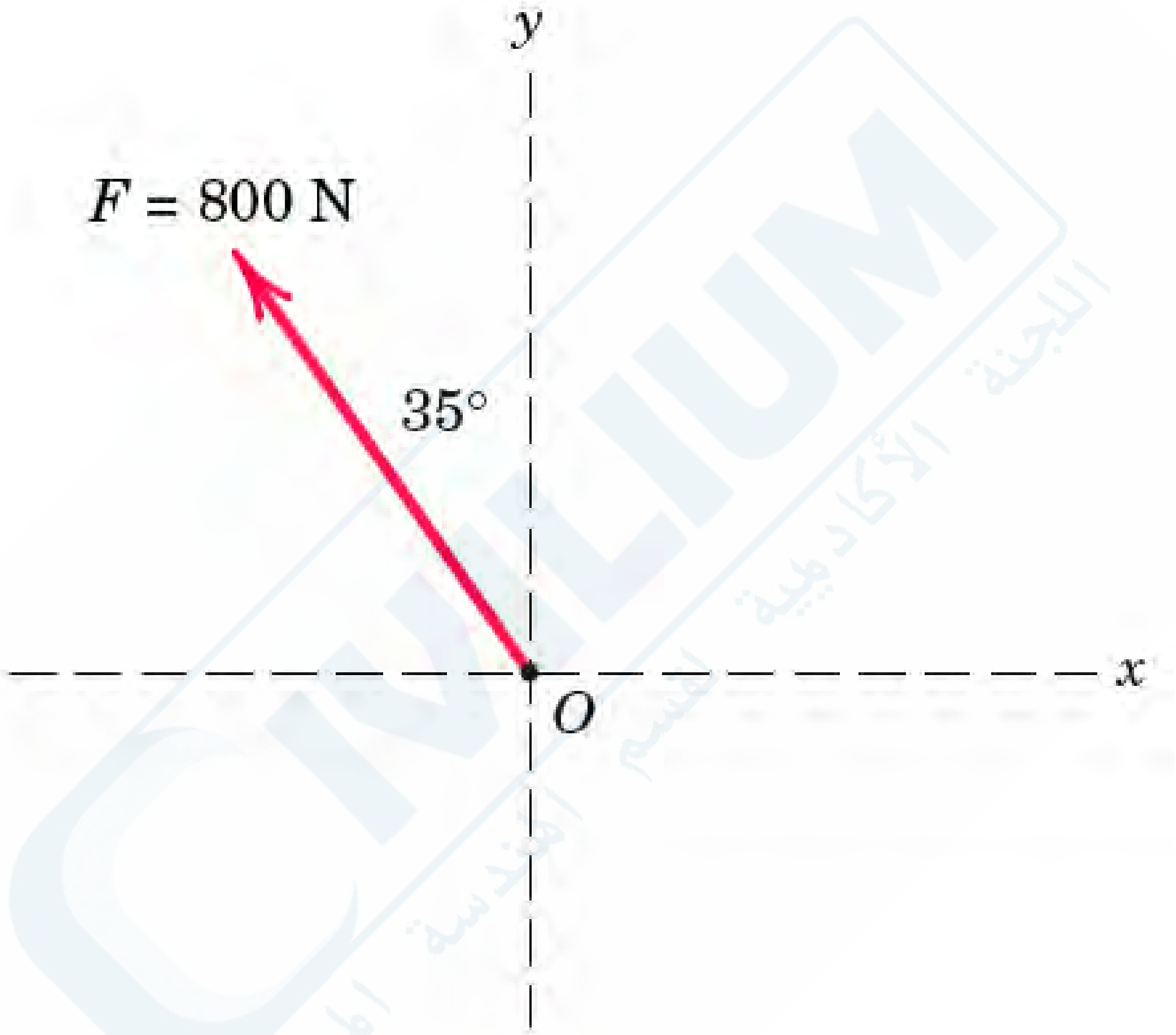
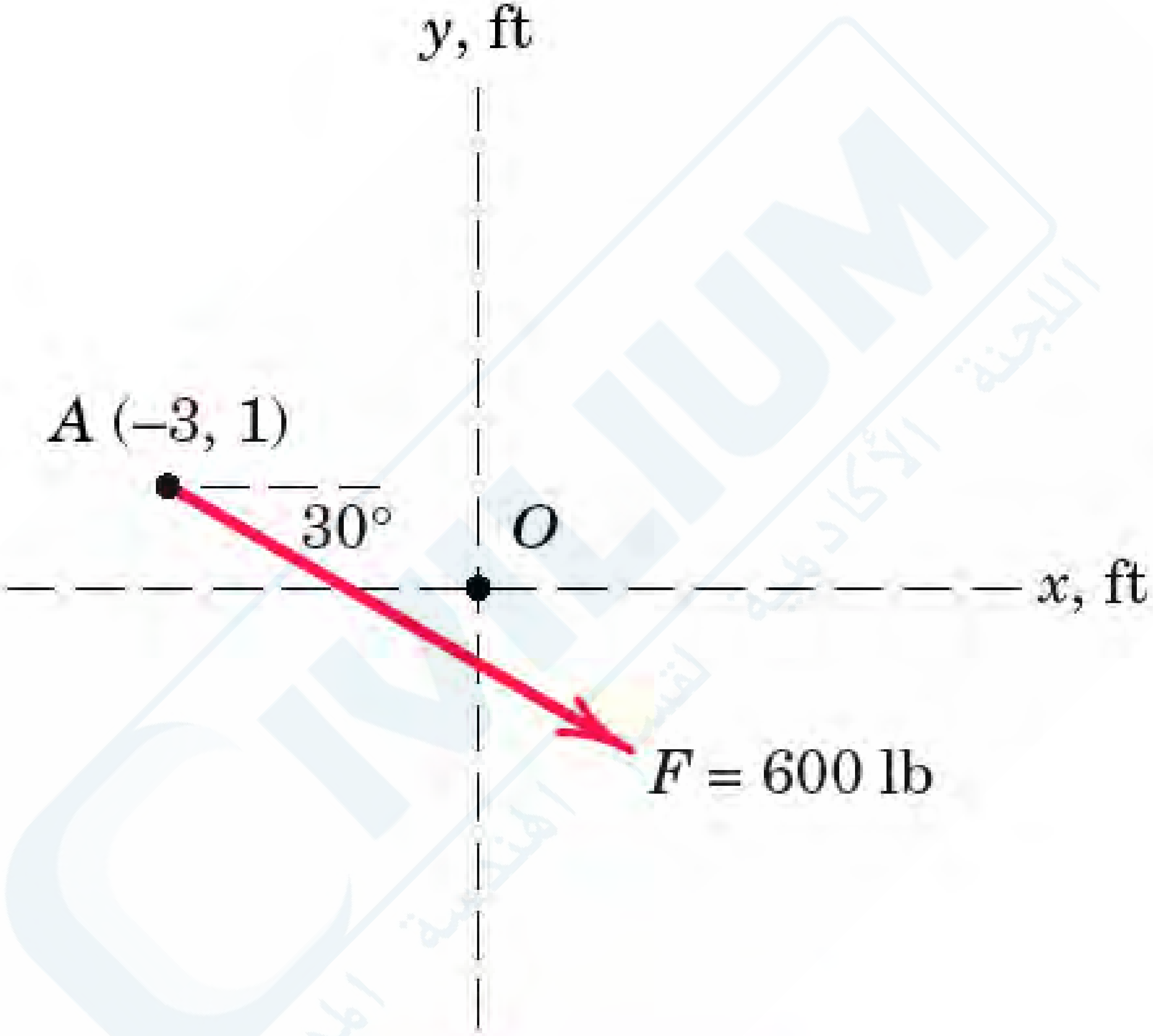


Figure 2/30



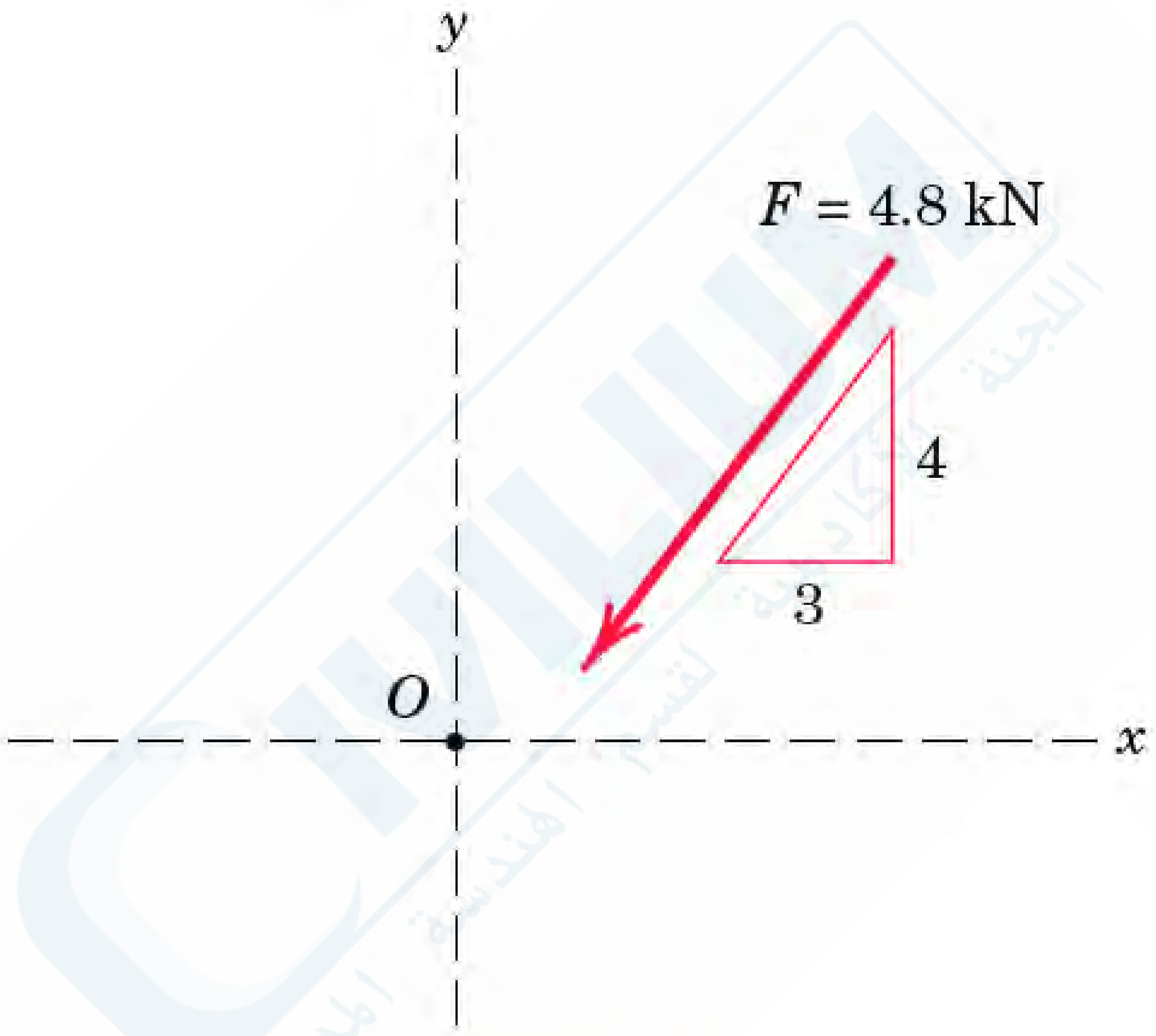
Problem 2/1

P 02 01



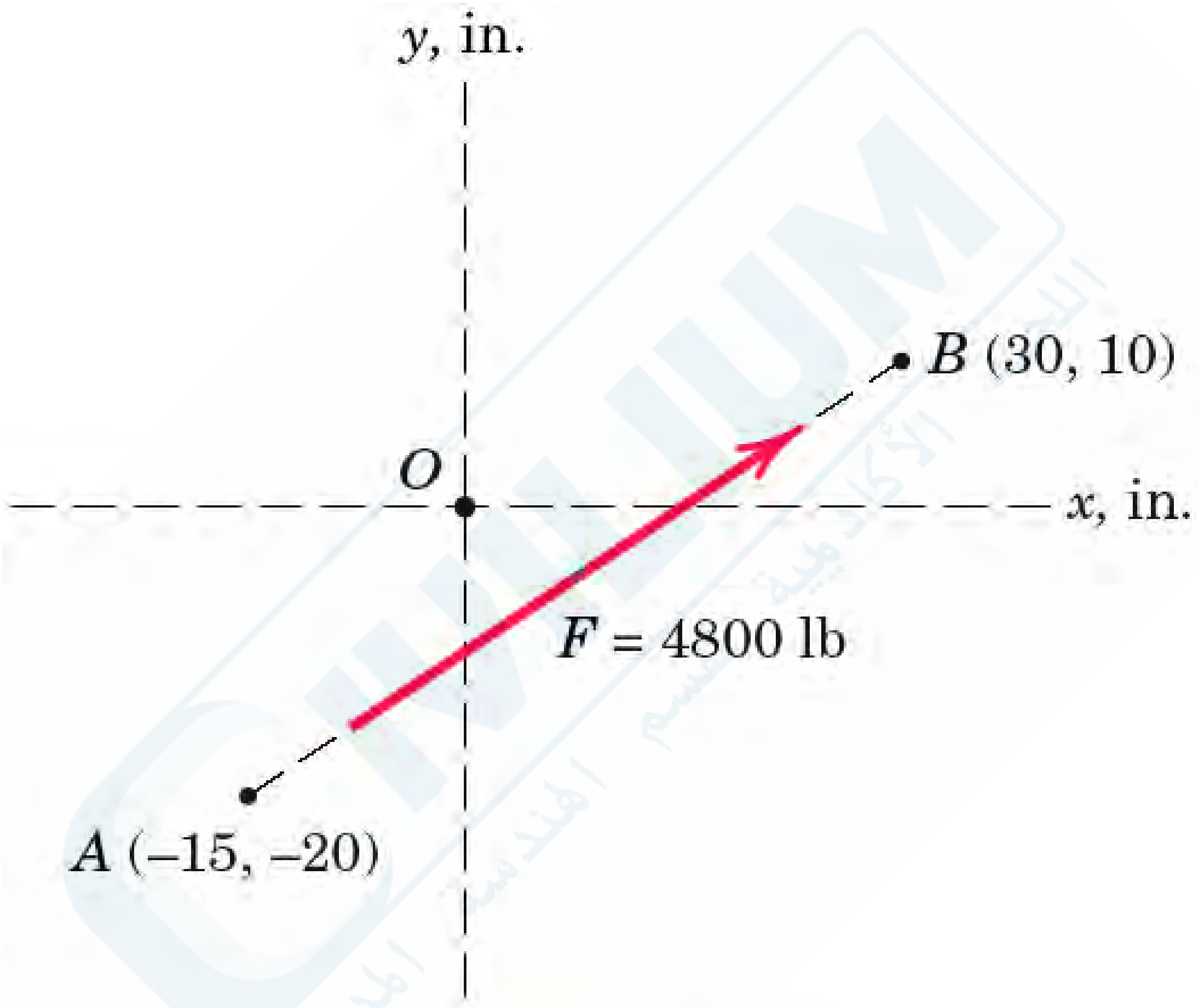
Problem 2/2

P 02 02

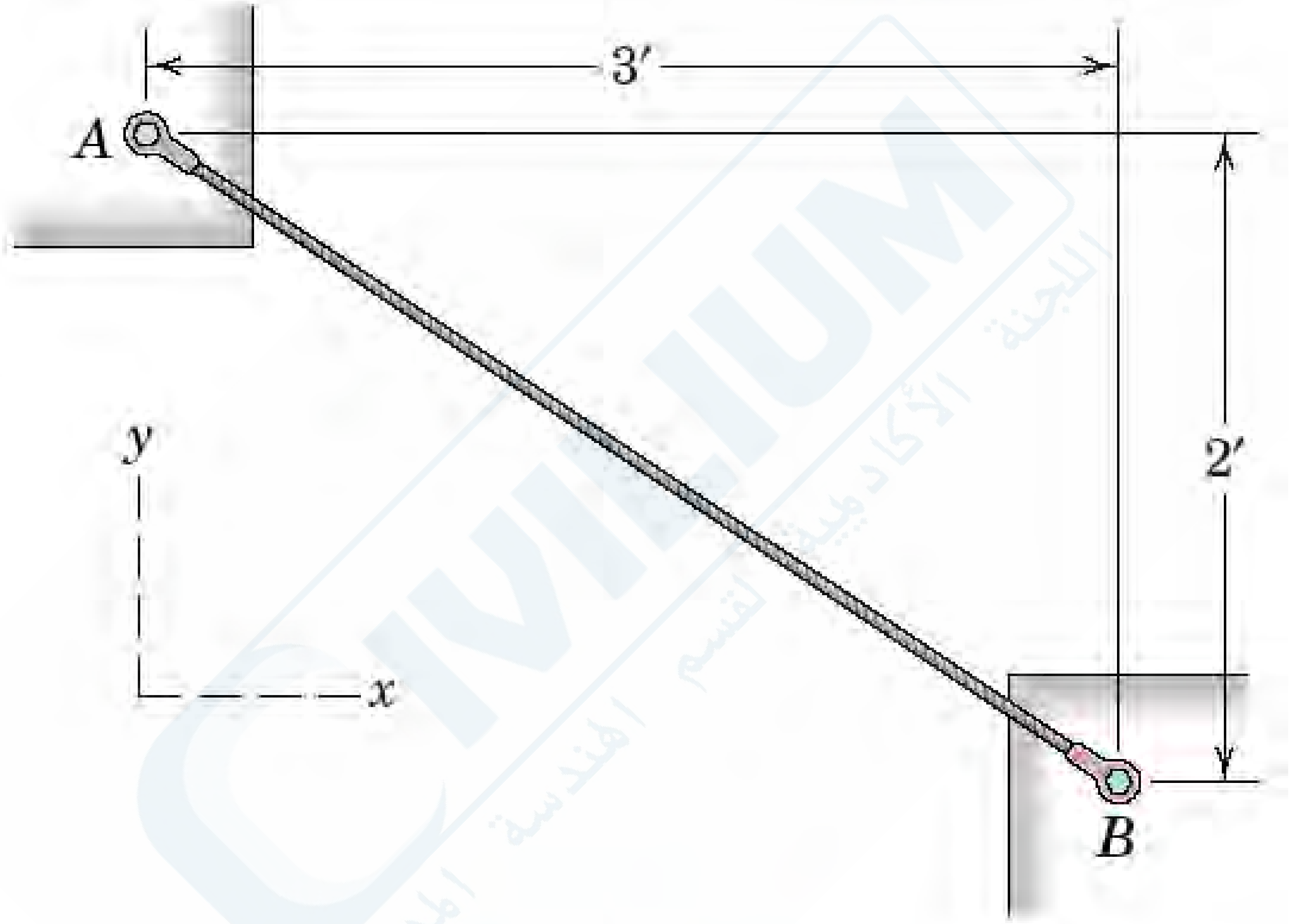


Problem 2/3

P_02_03

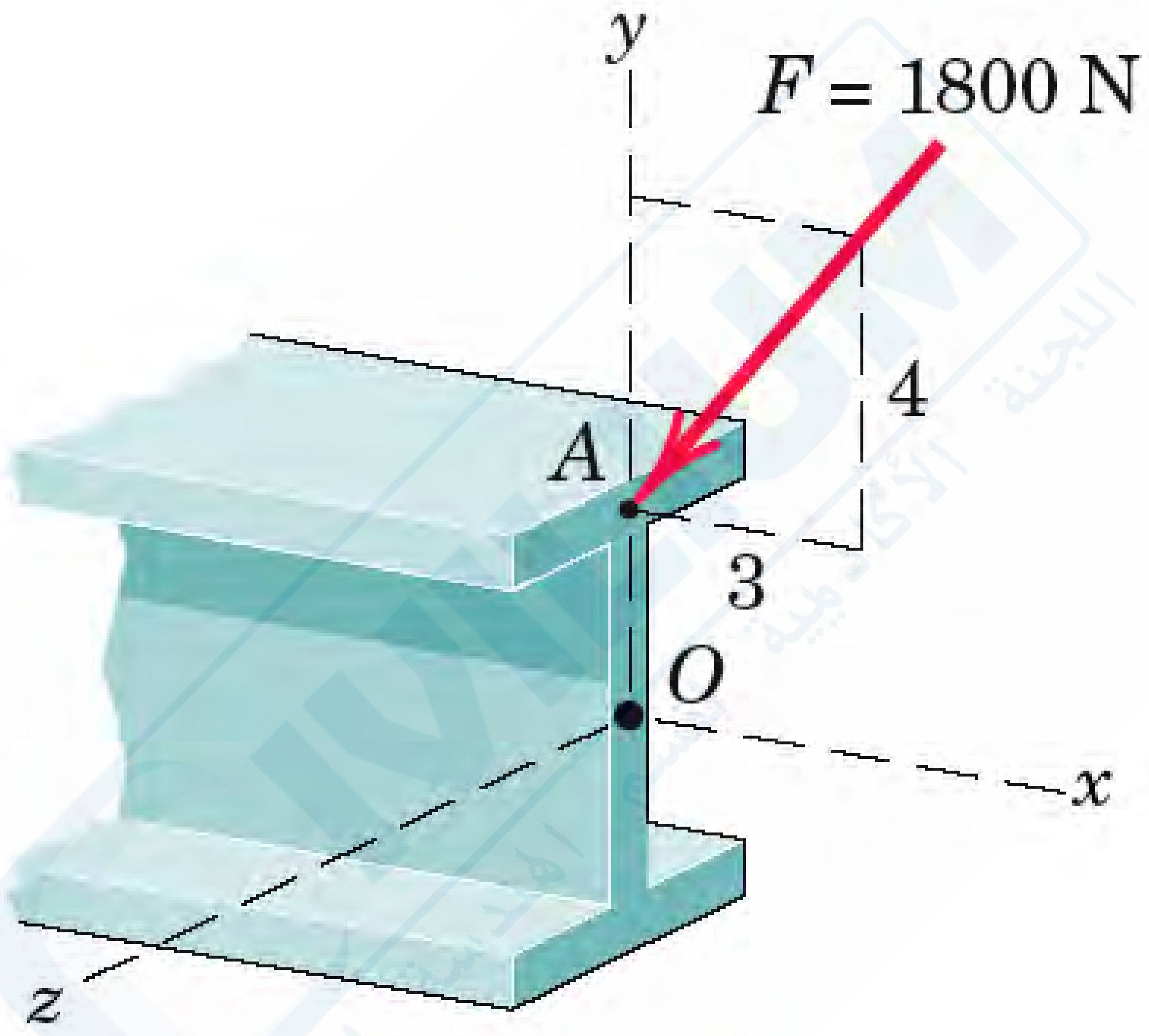


Problem 2/4



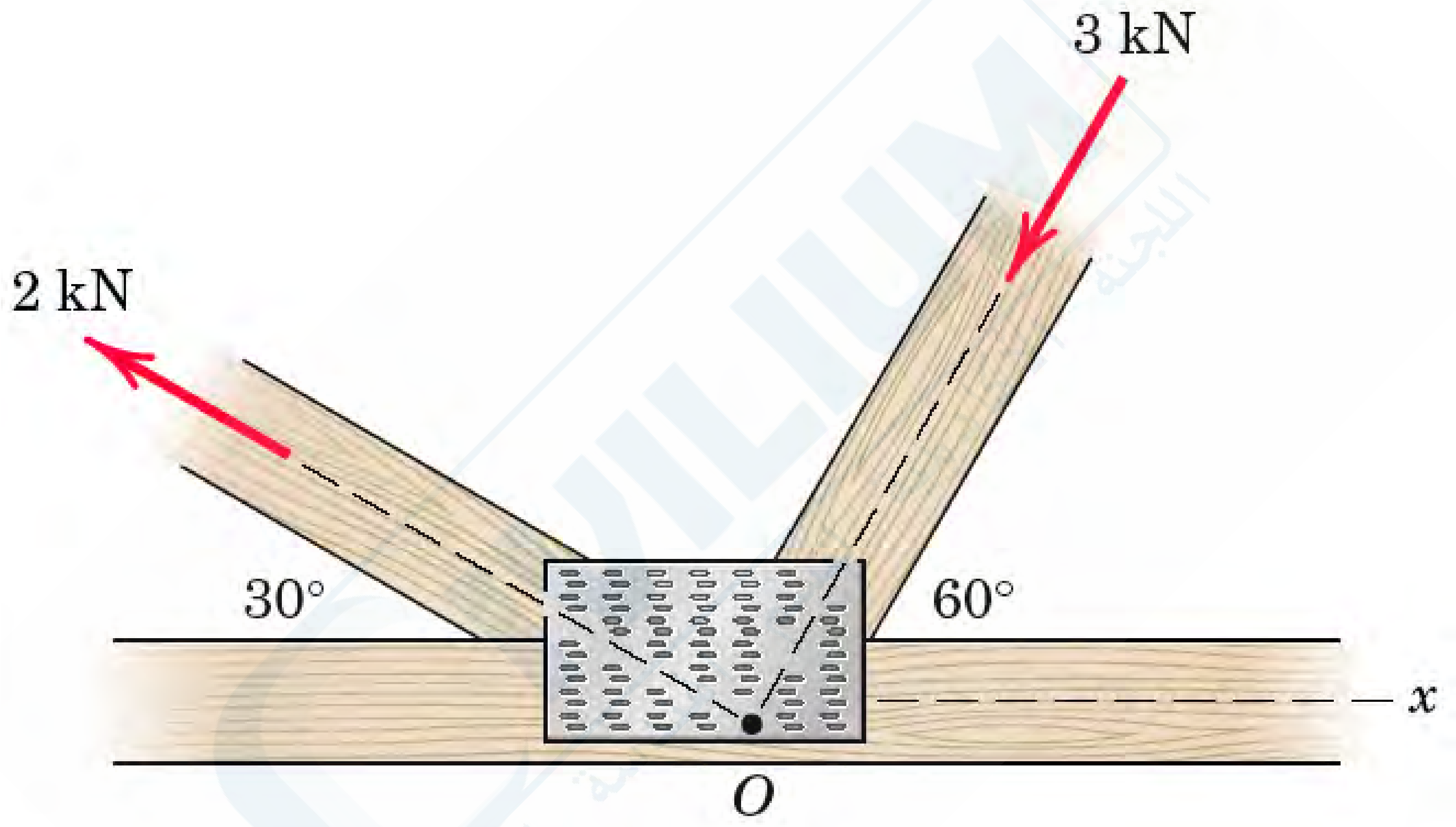
Problem 2/5

P_02_05



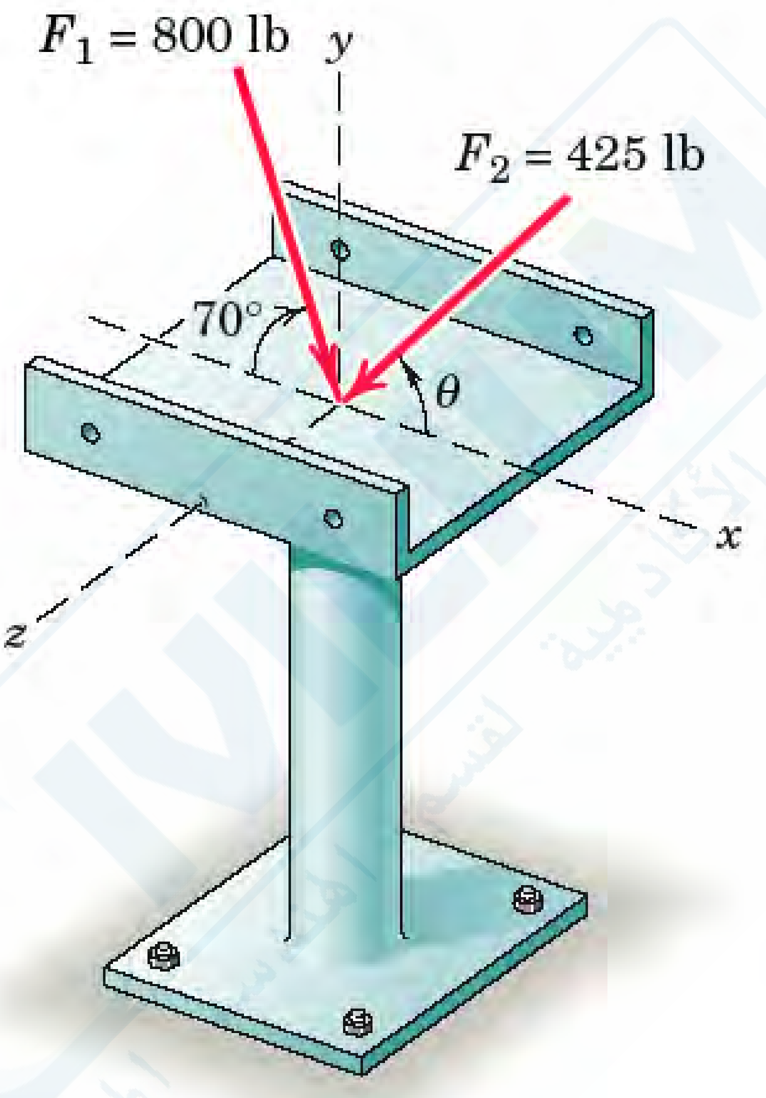
Problem 2/6

P_02_06



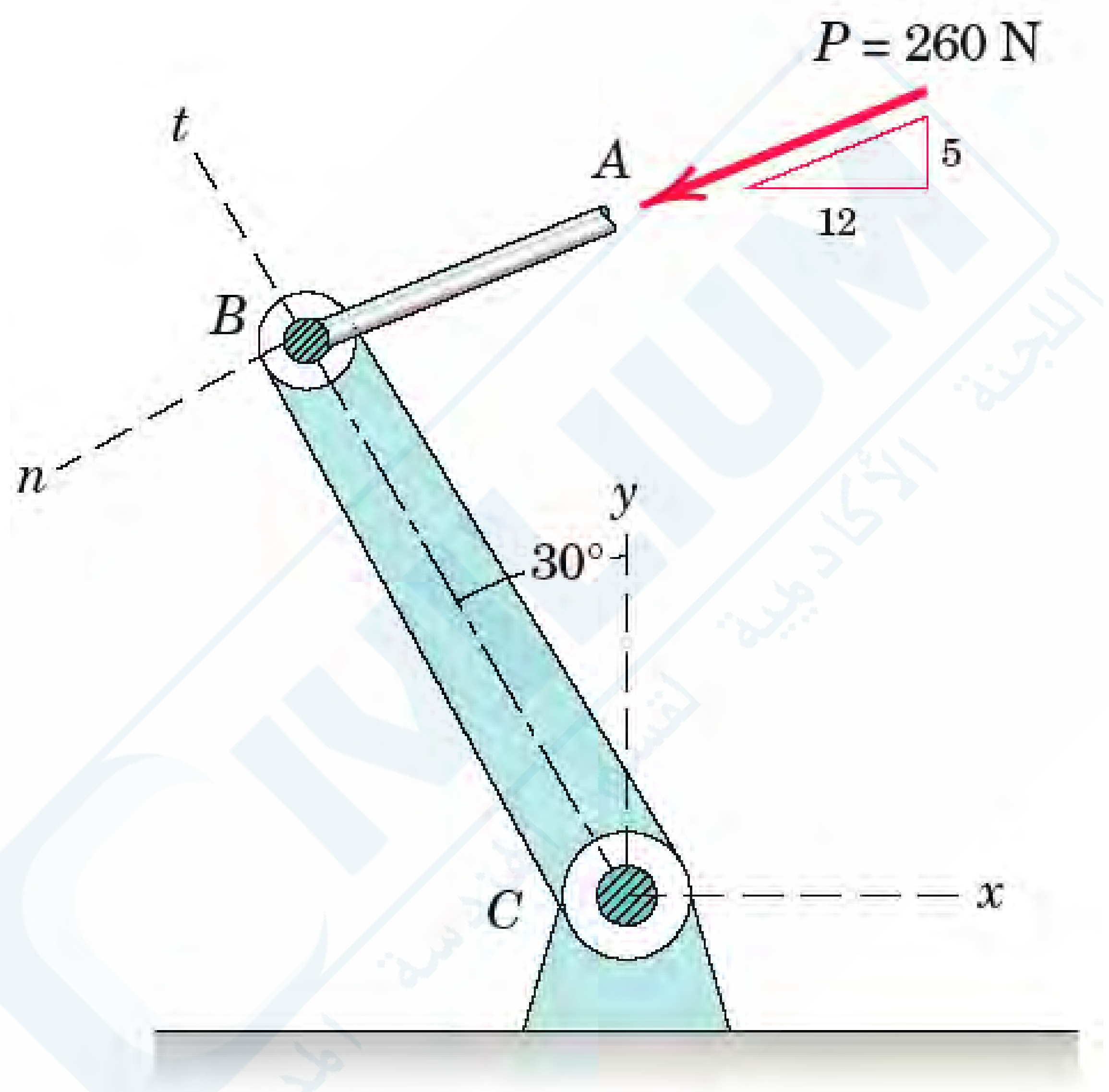
Problem 2/7

P_02_07



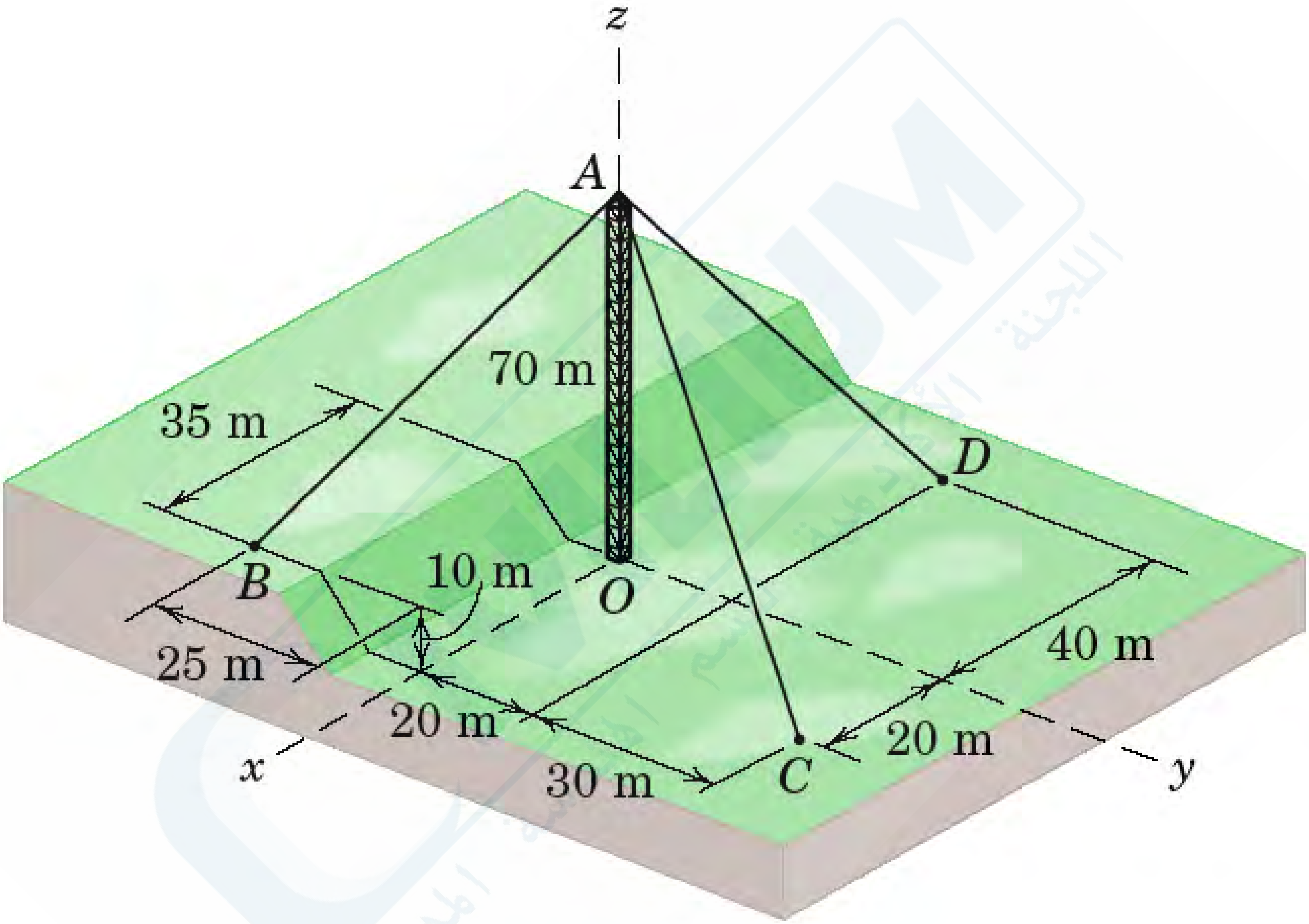
Problem 2/8

P 02 08



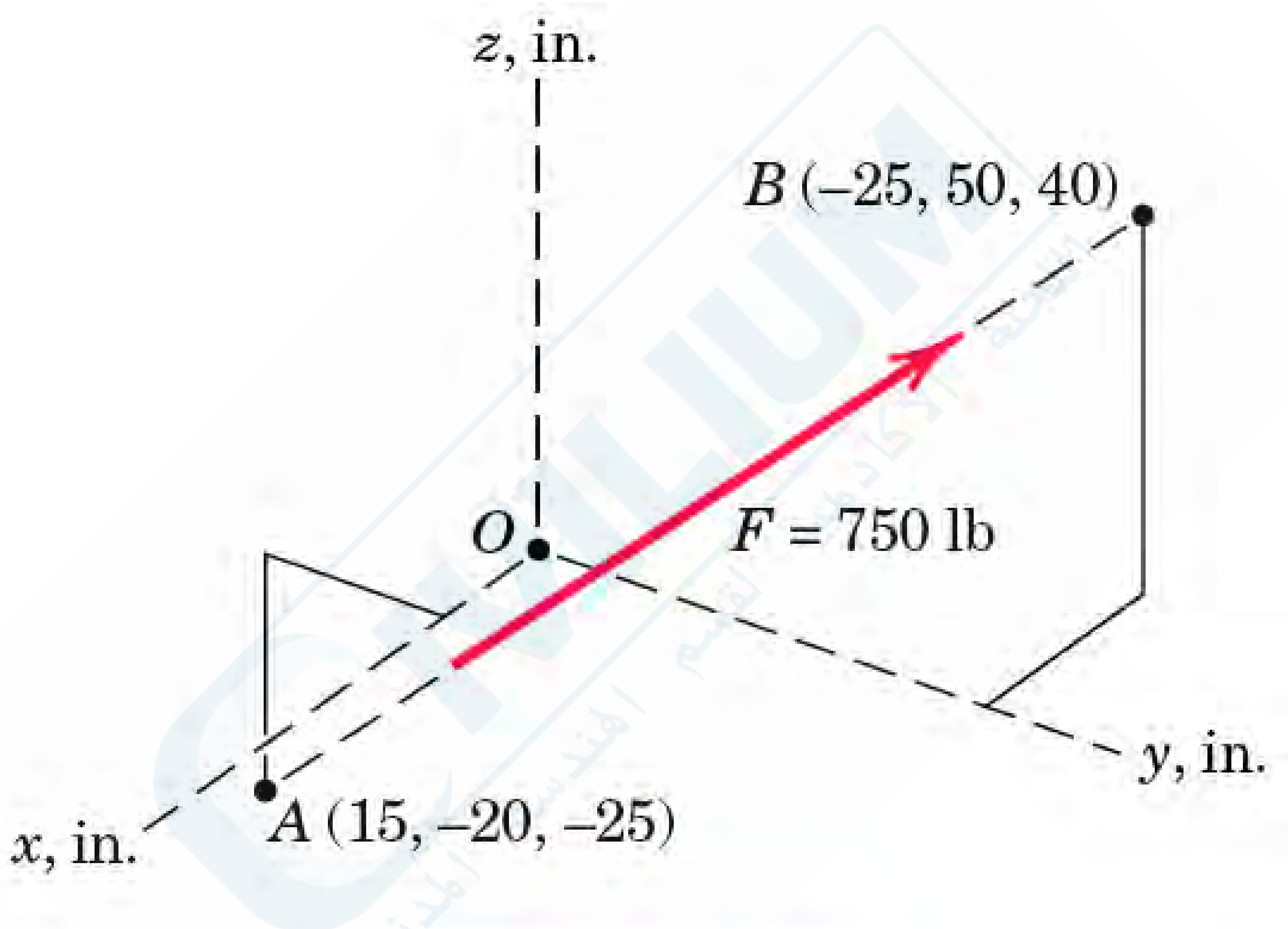
Problem 2/9

P_02_09



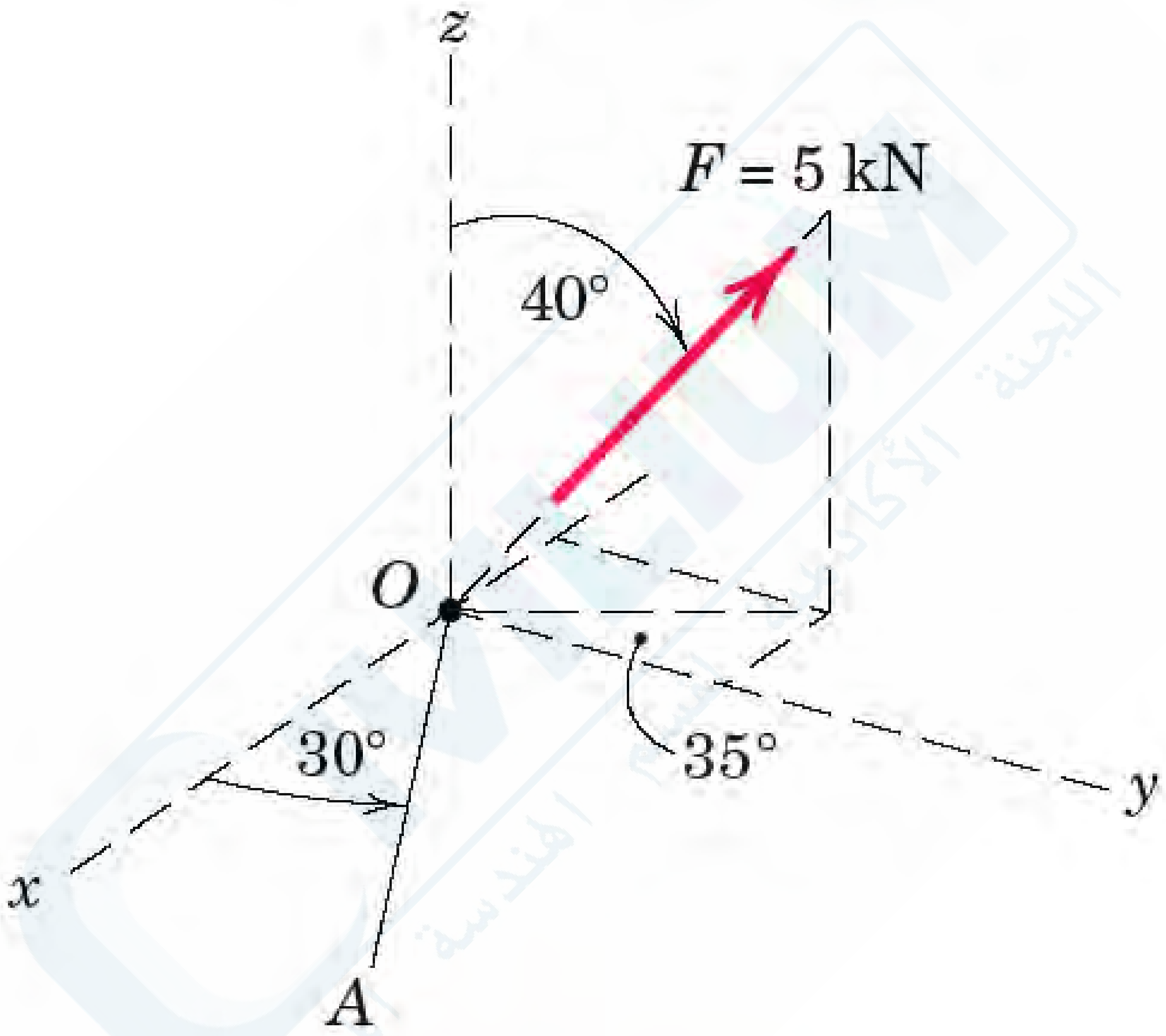
Problem 2/100

P 02 100



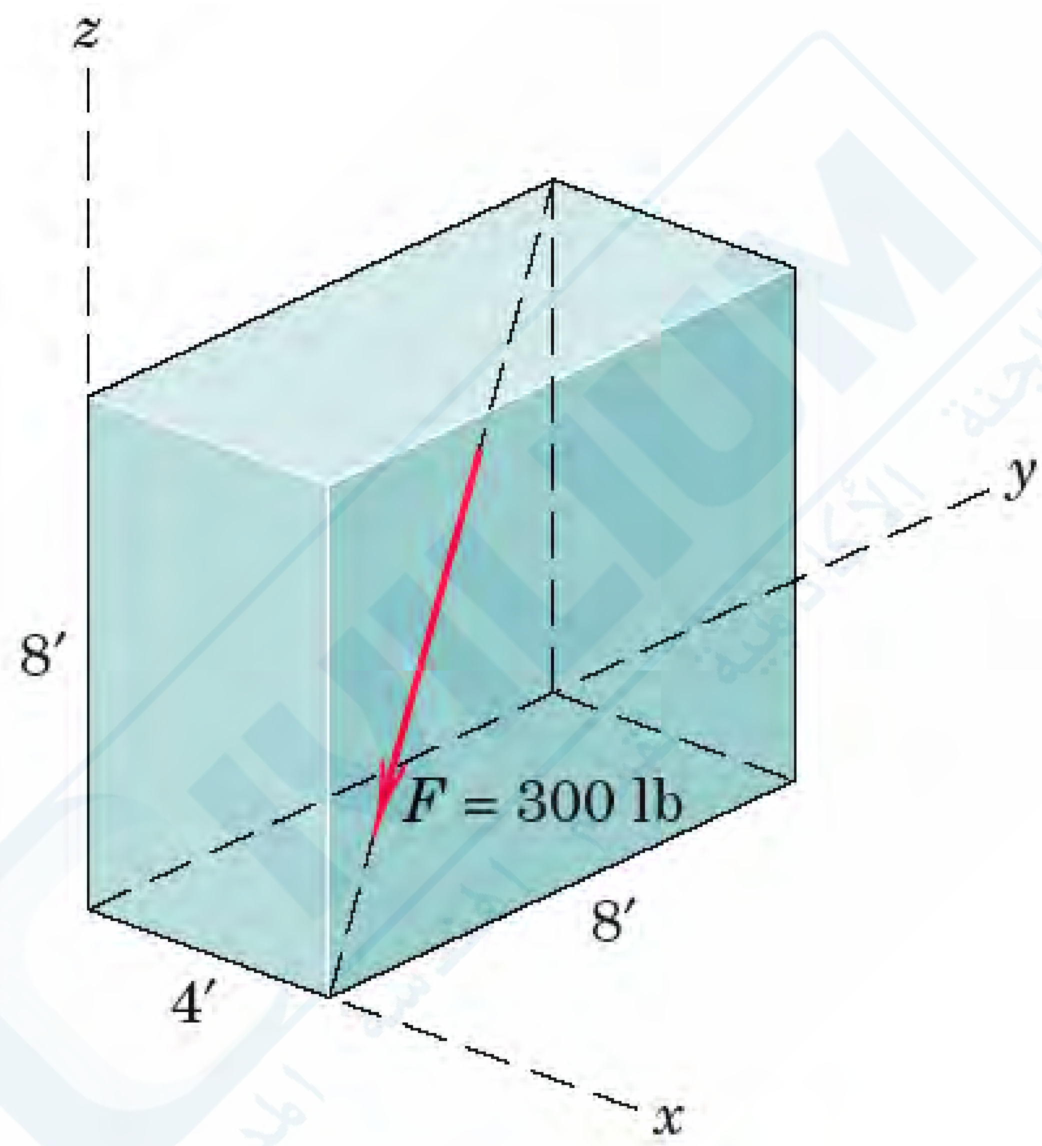
Problem 2/101

P 02 101



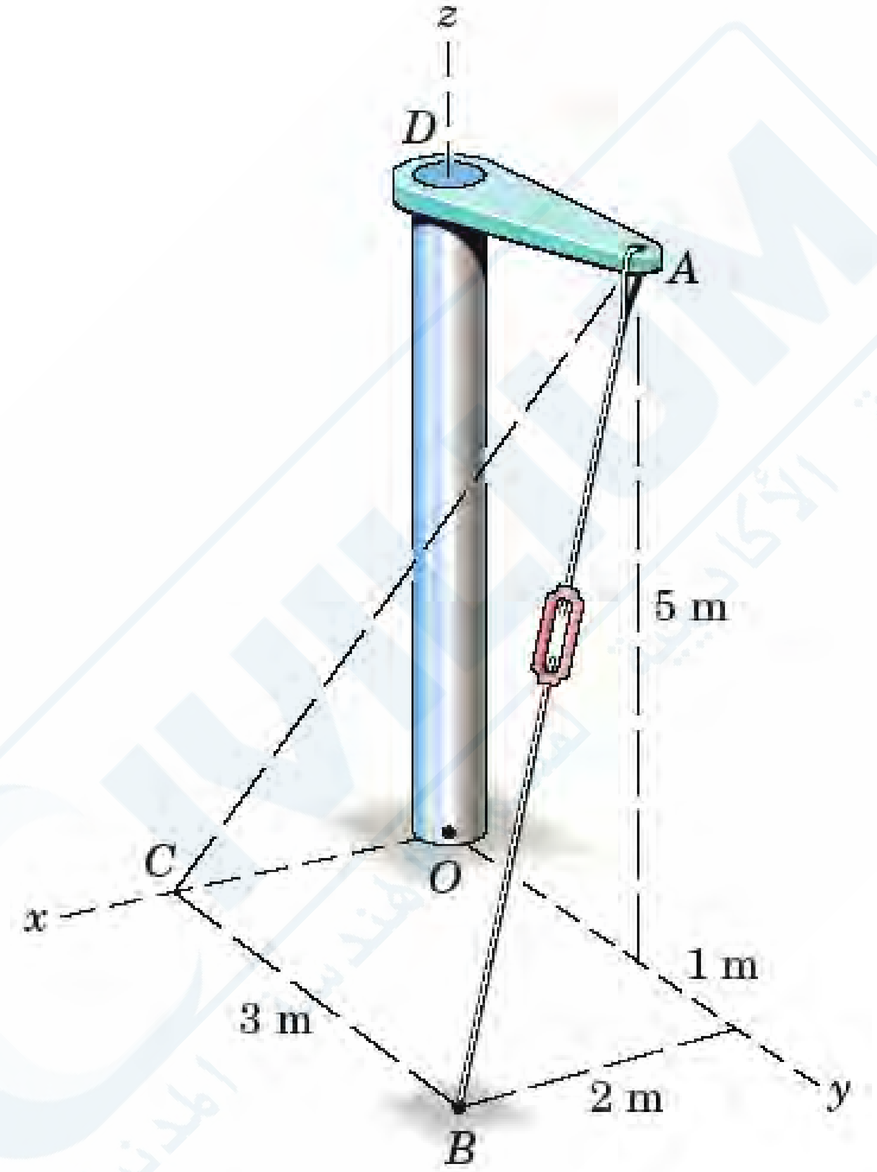
Problem 2/102

P 02 102



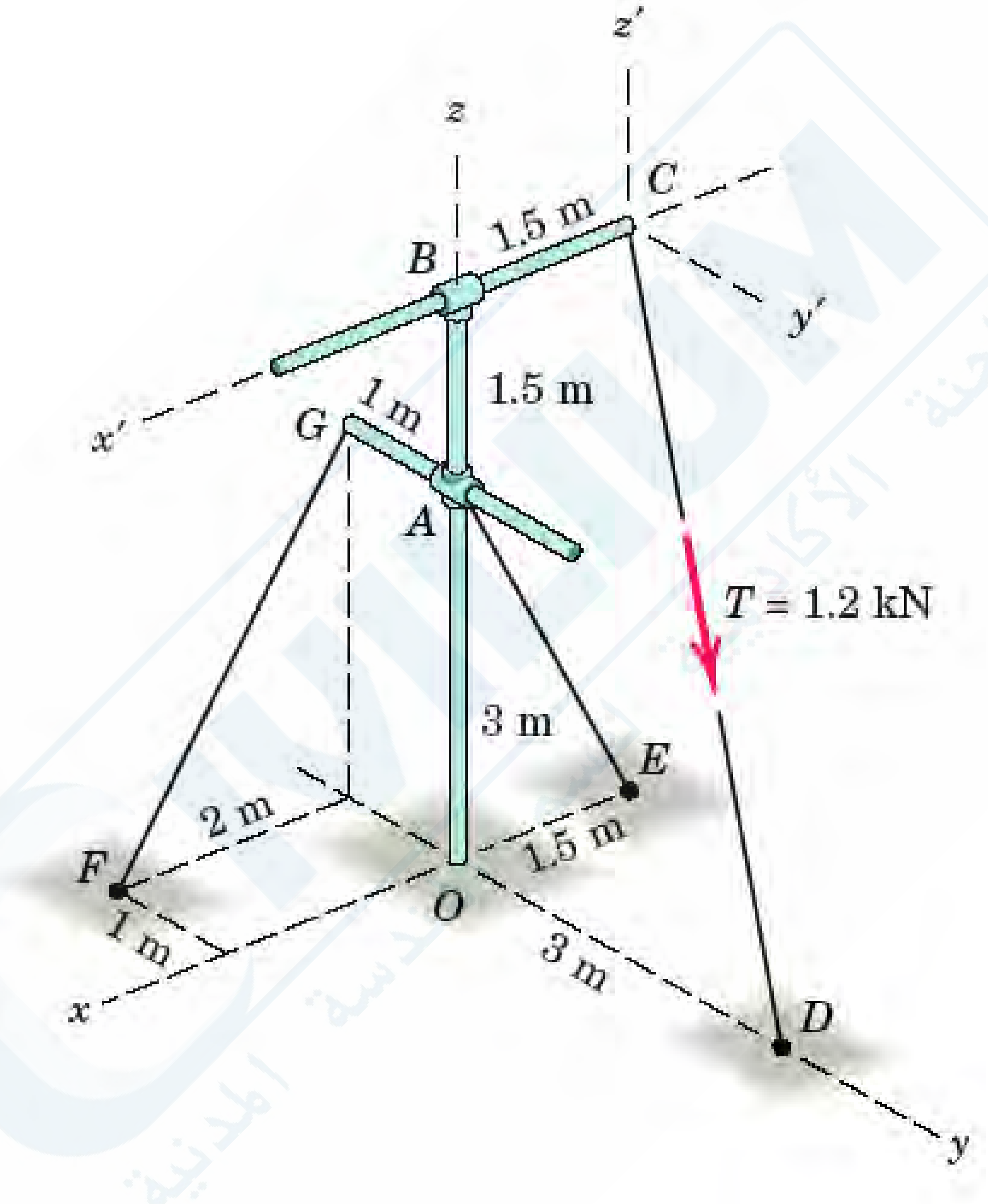
Problem 2/103

P 02 103



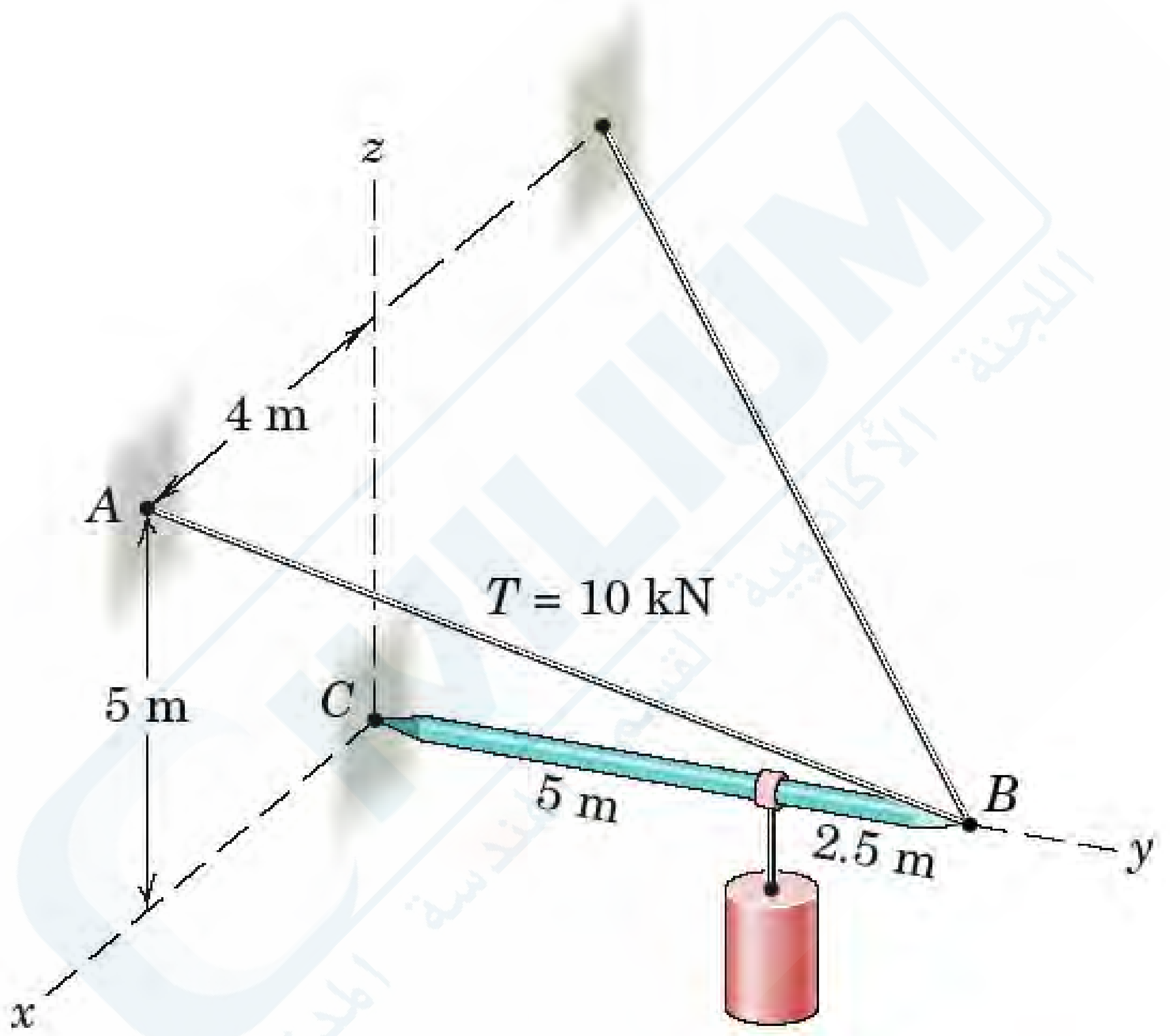
Problem 2/104

P 02 104



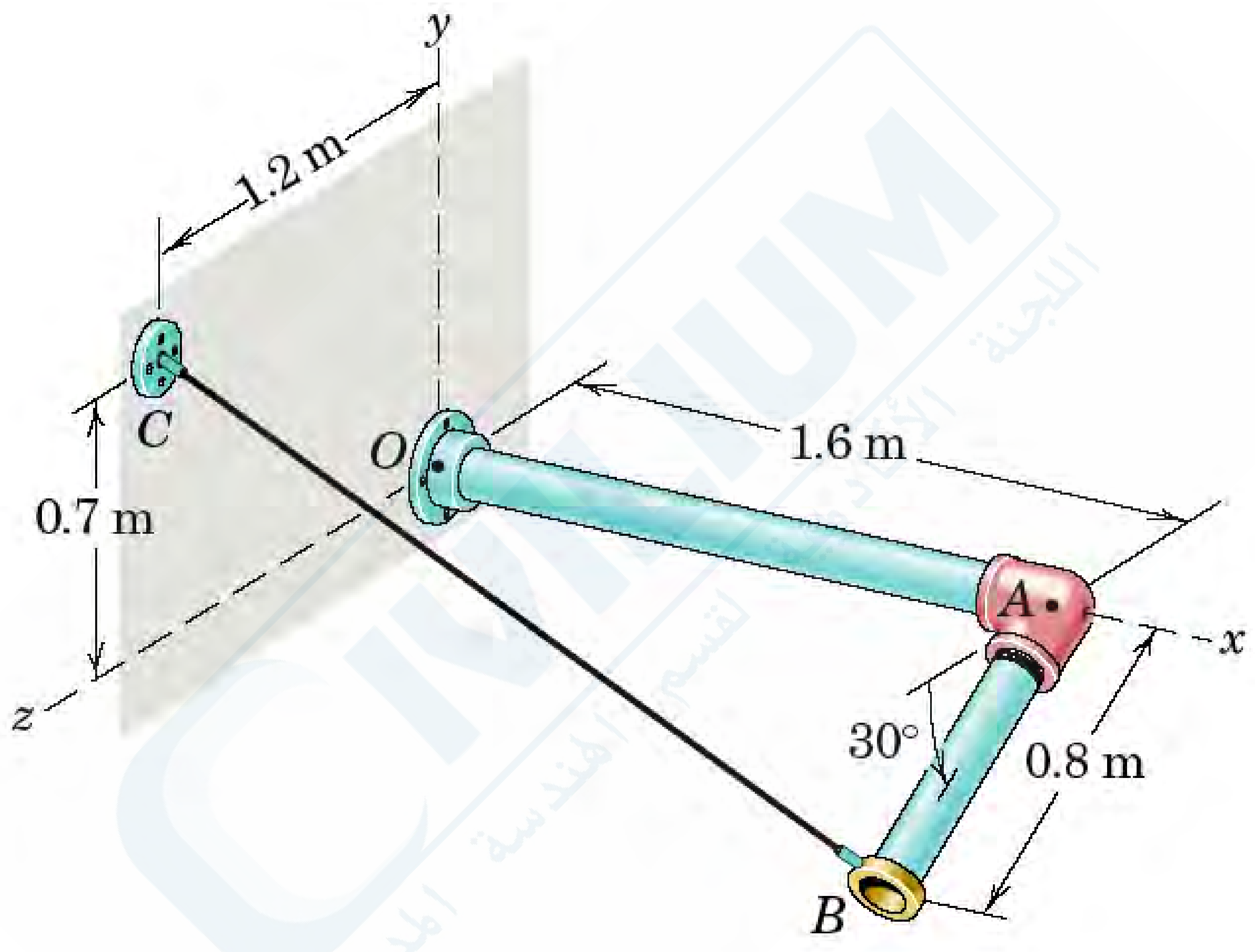
Problem 2/105

P_02_105



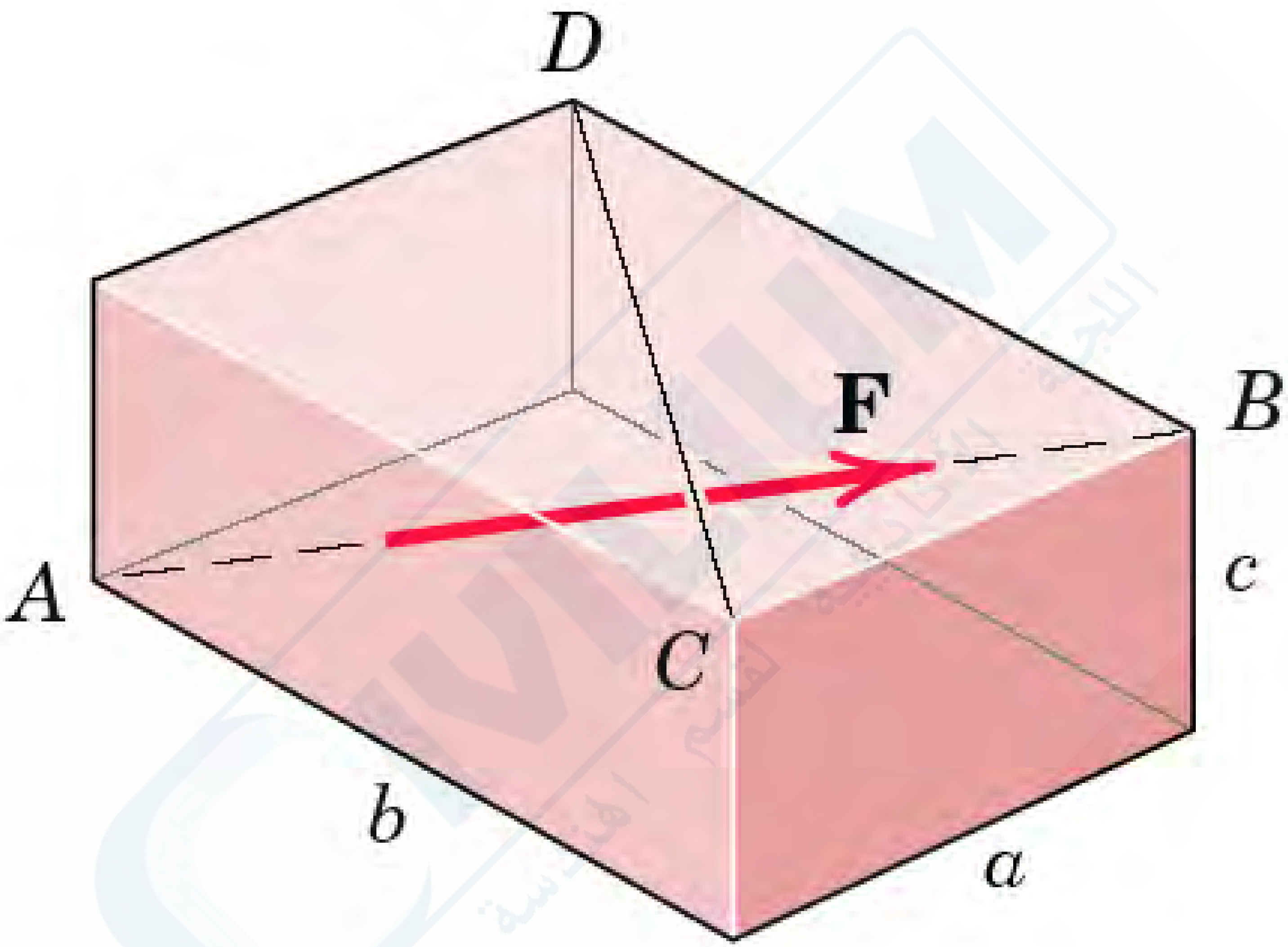
Problem 2/107

P 02 107



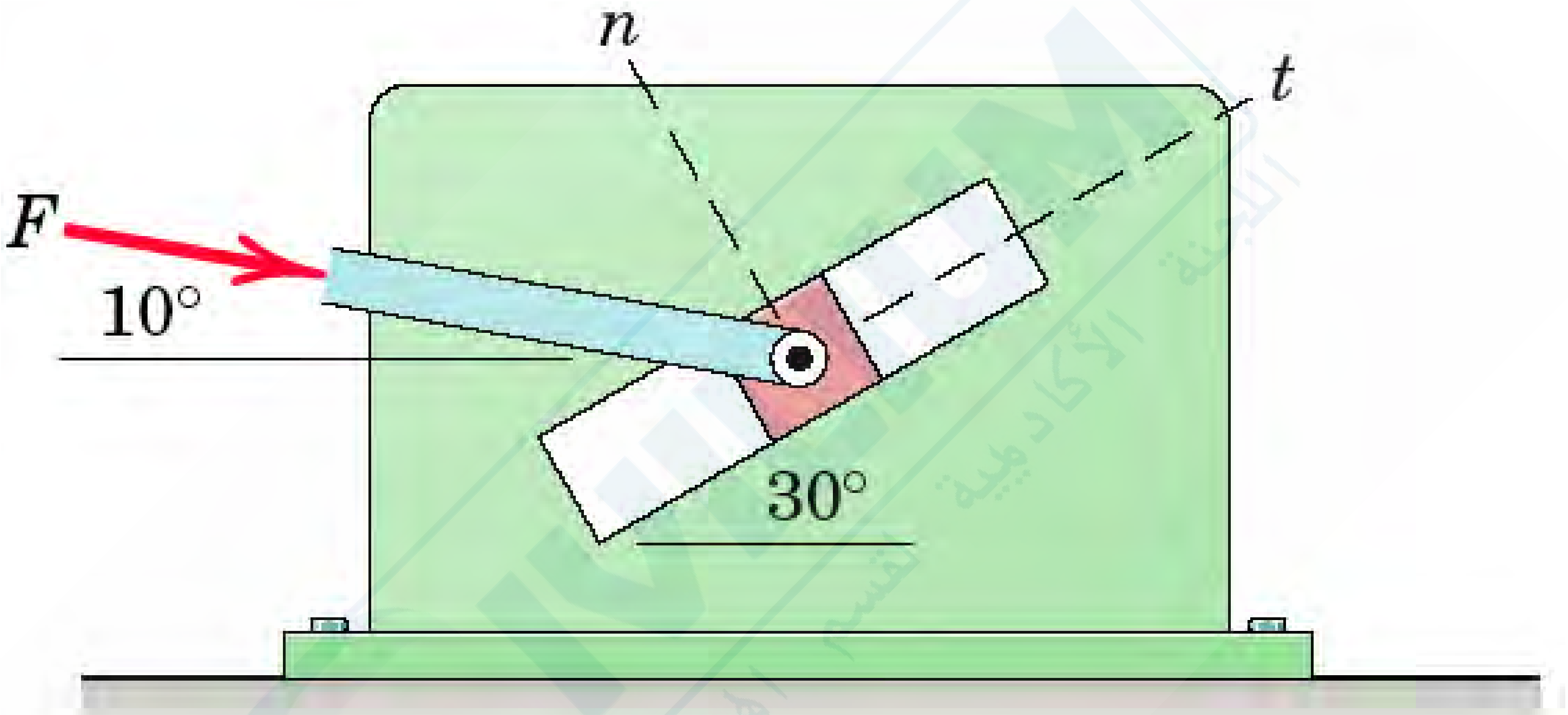
Problem 2/108

P 02 108



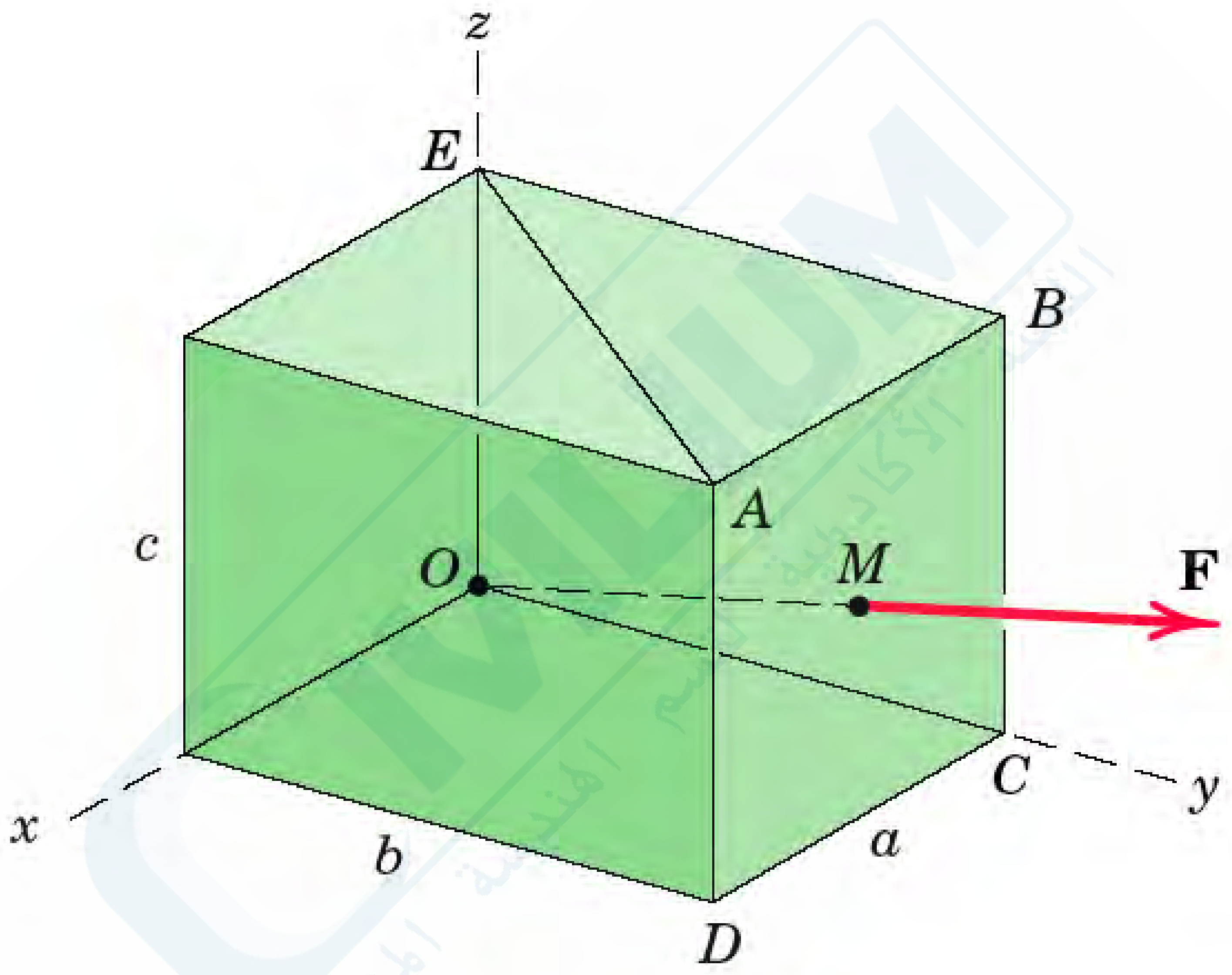
Problem 2/109

P_02_109



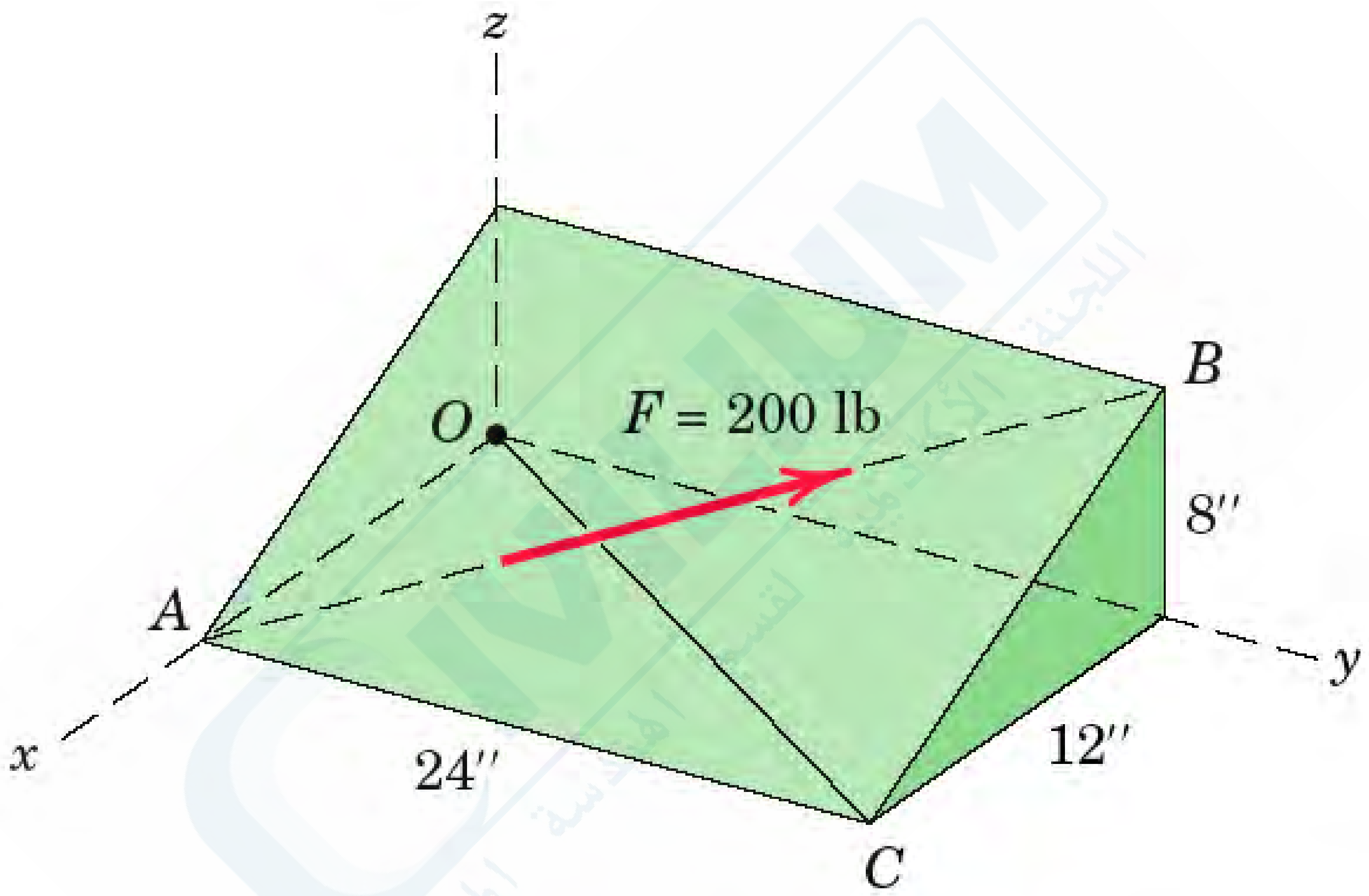
Problem 2/11

P_02_11



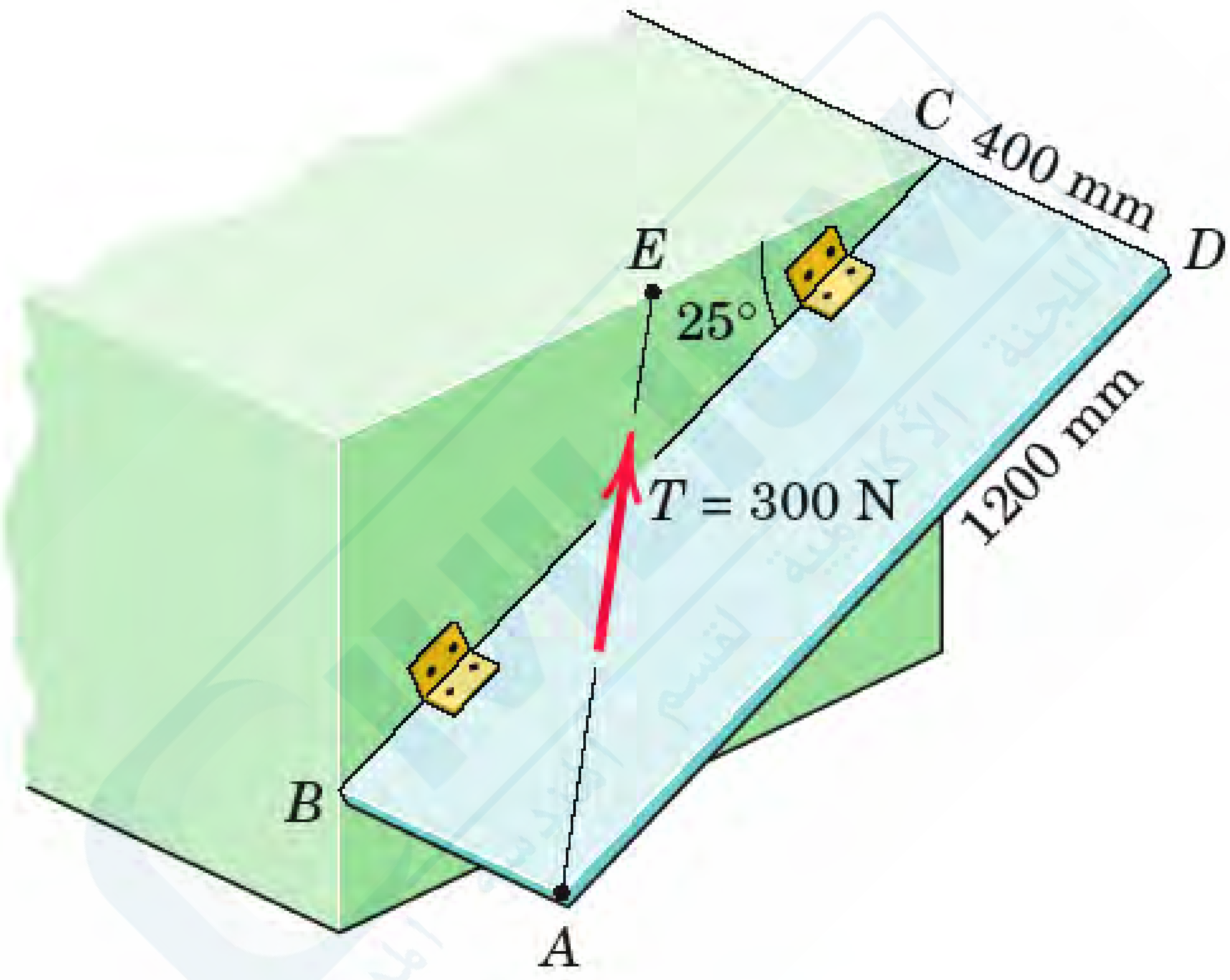
Problem 2/110

P 02 110



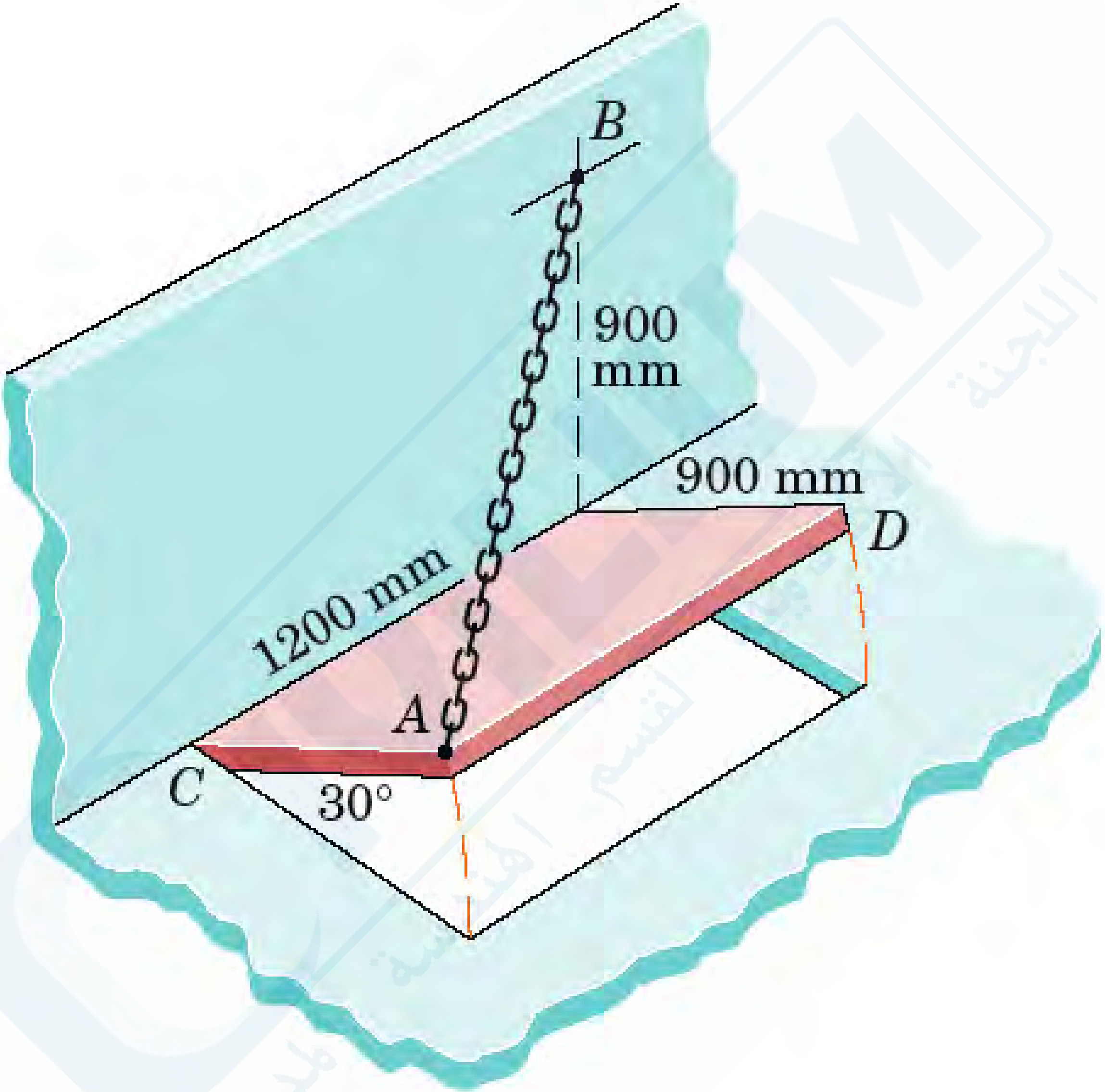
Problem 2/111

P_02_111



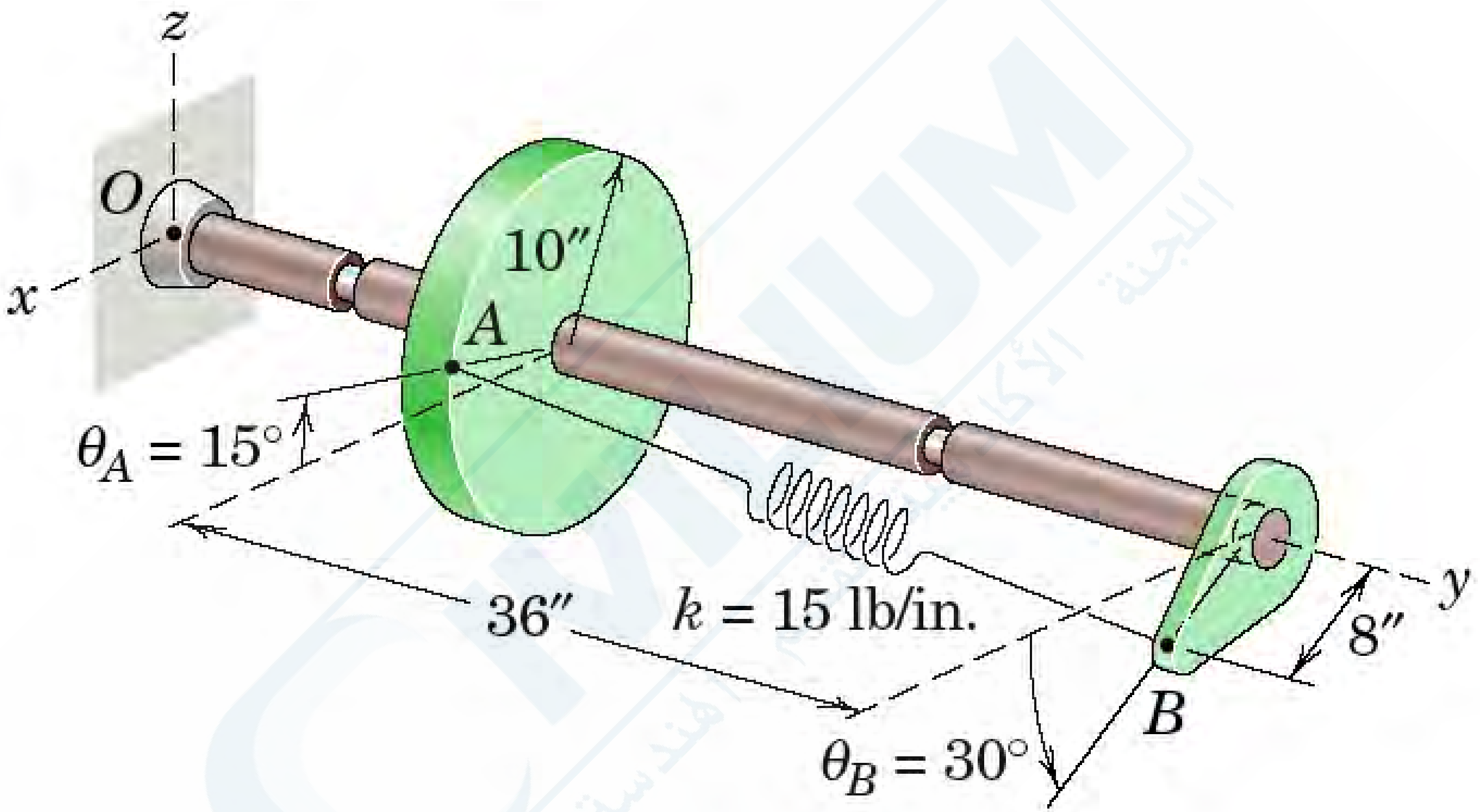
Problem 2/112

P 02 112



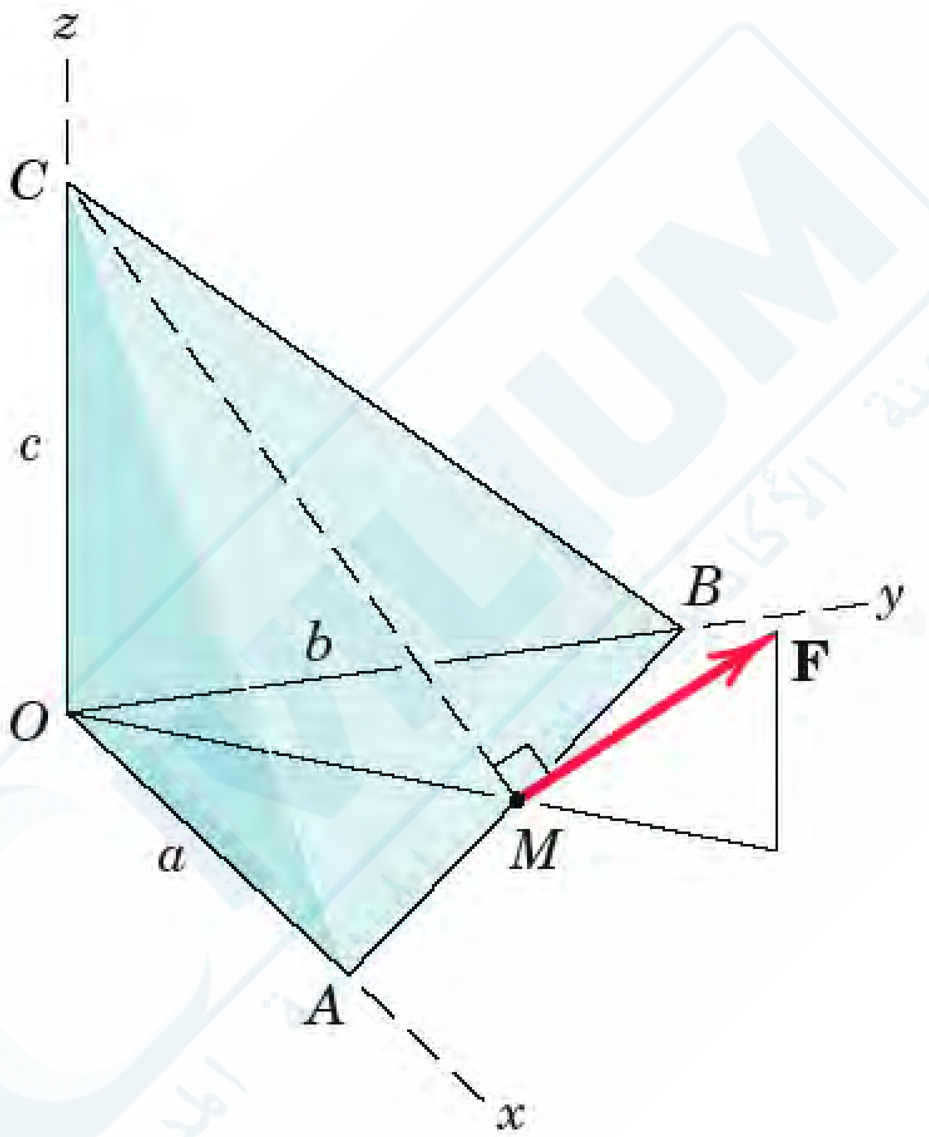
Problem 2/113

P 02 113



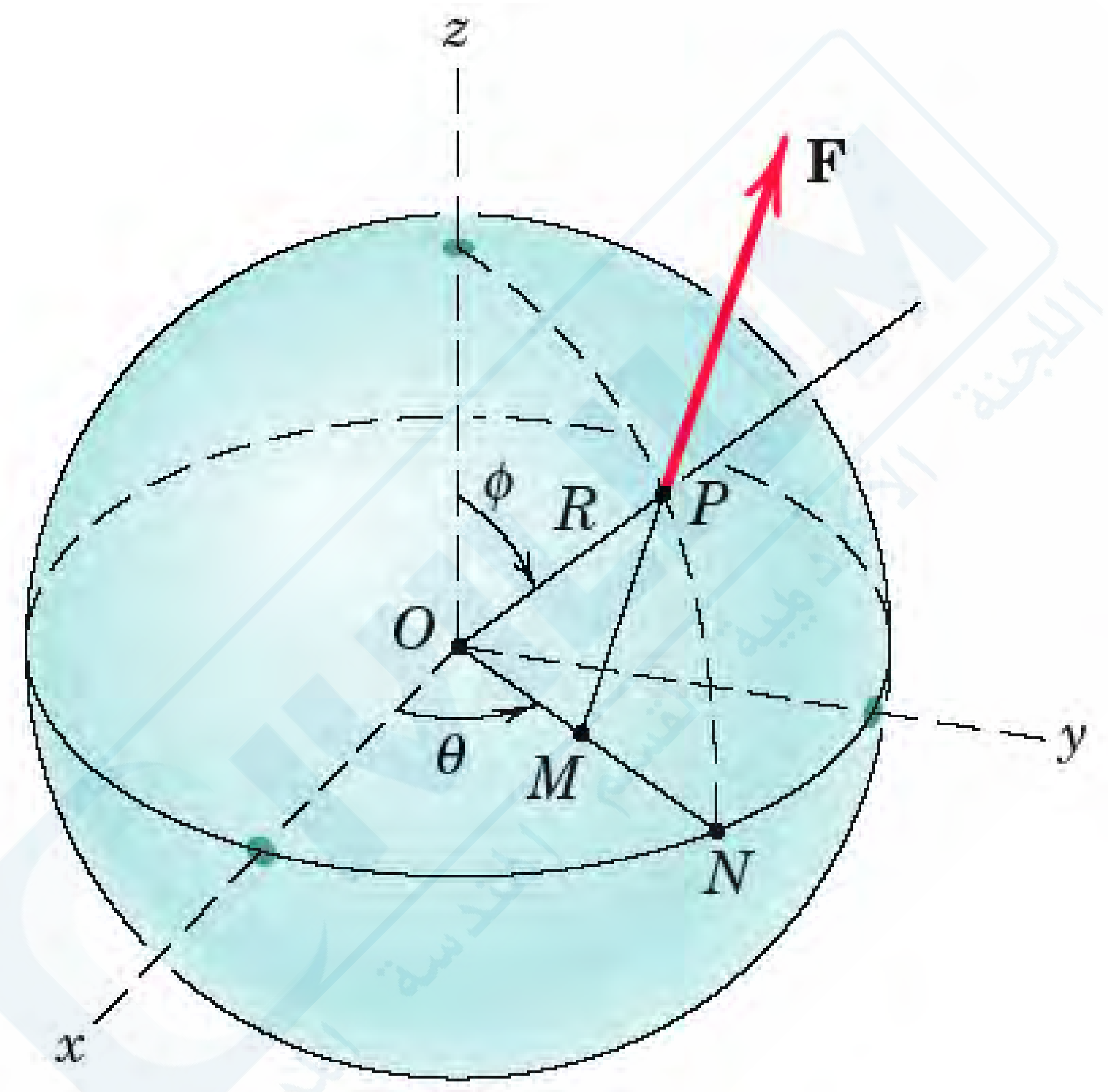
Problem 2/114

P_02_114



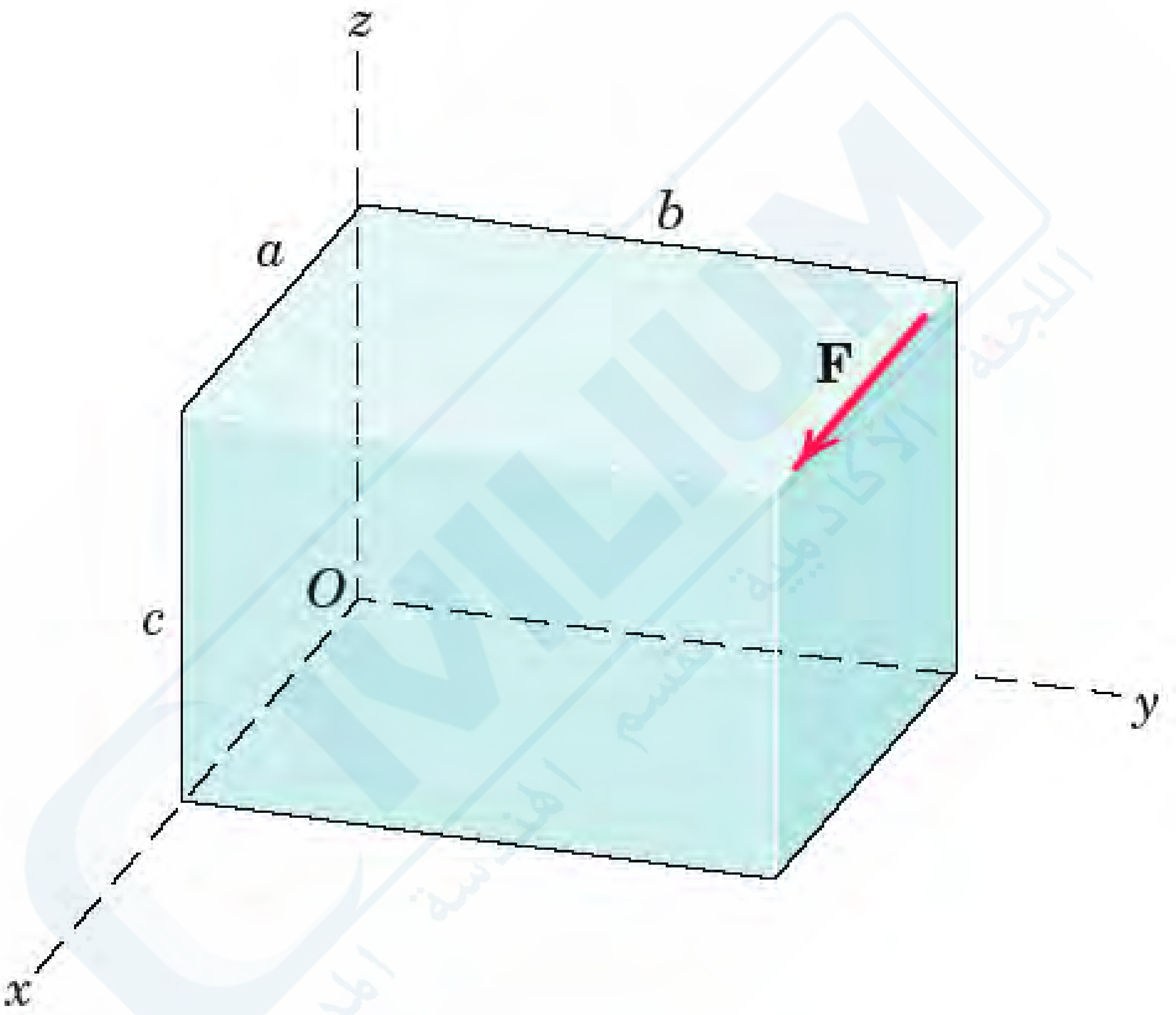
Problem 2/115

P 02 115



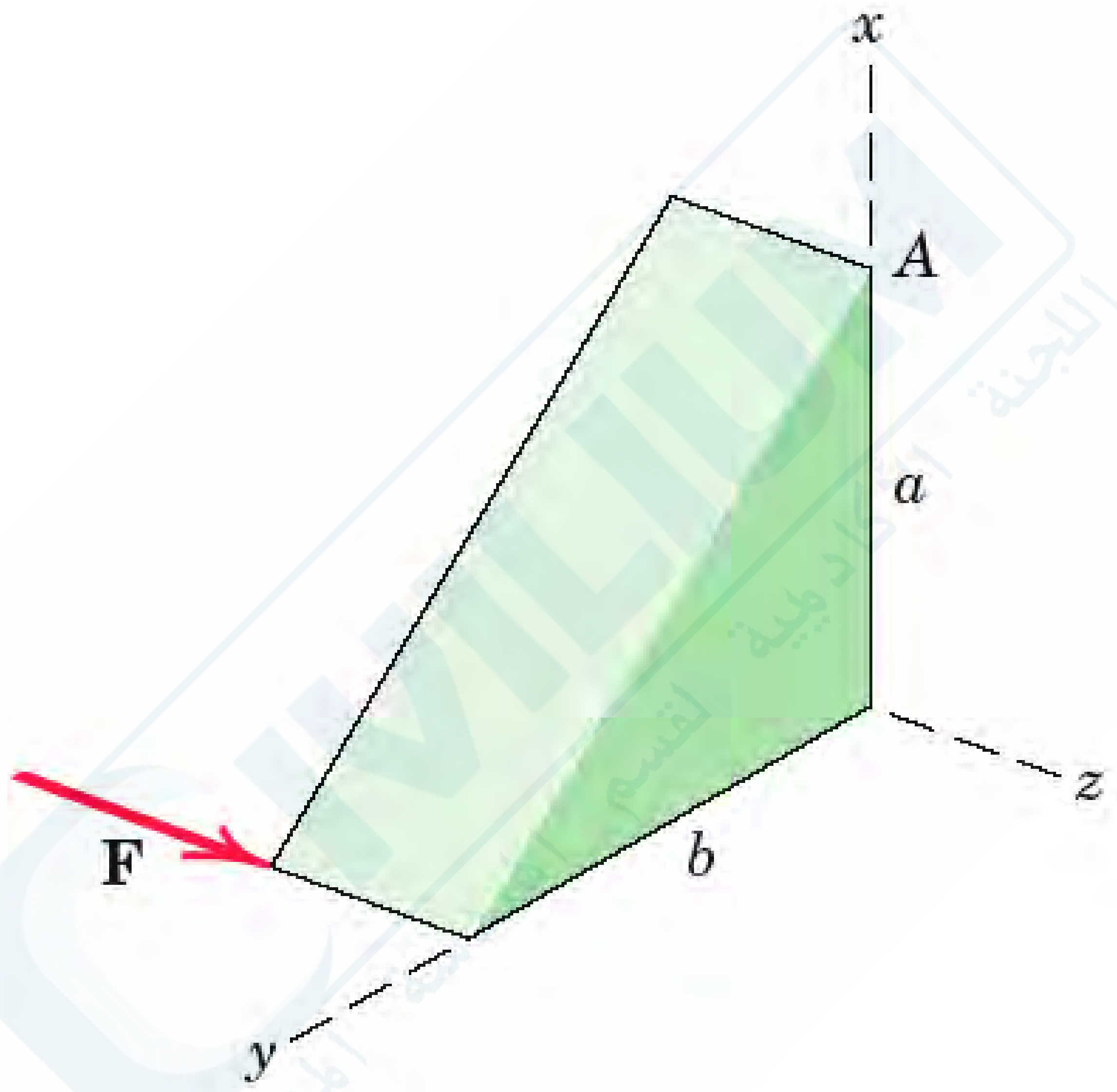
Problem 2/116

P 02 116

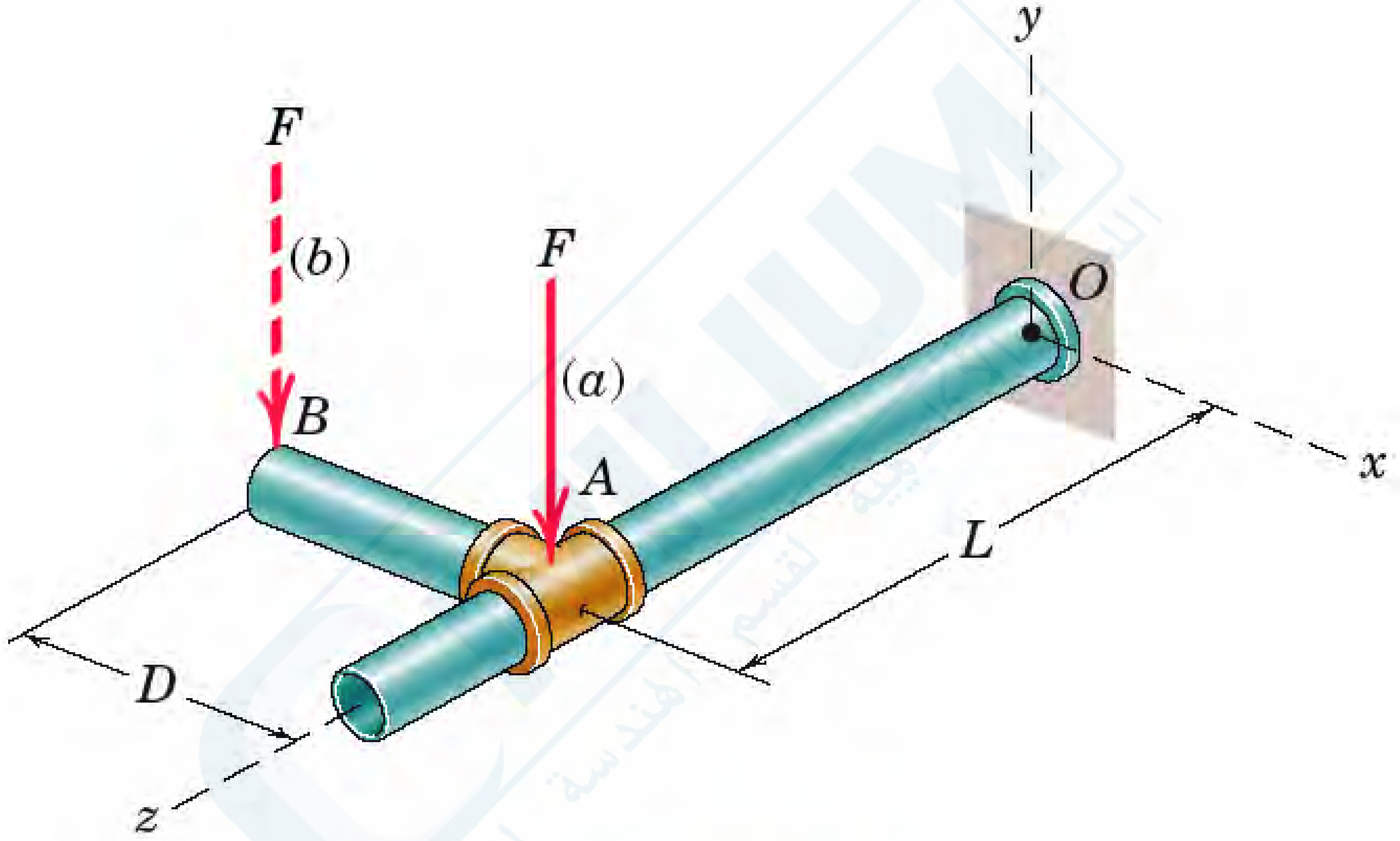


Problem 2/117

P 02 117

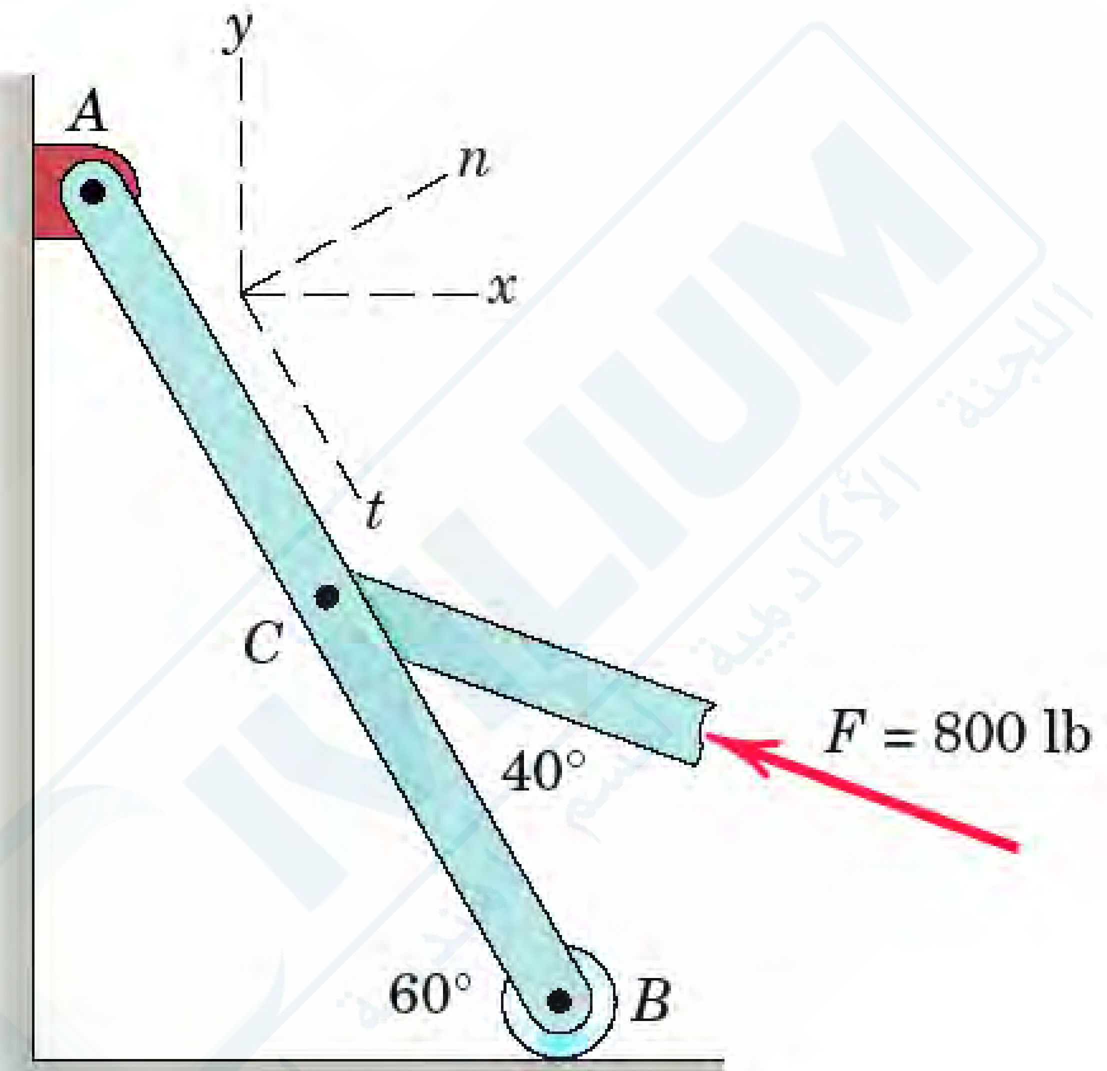


Problem 2/118

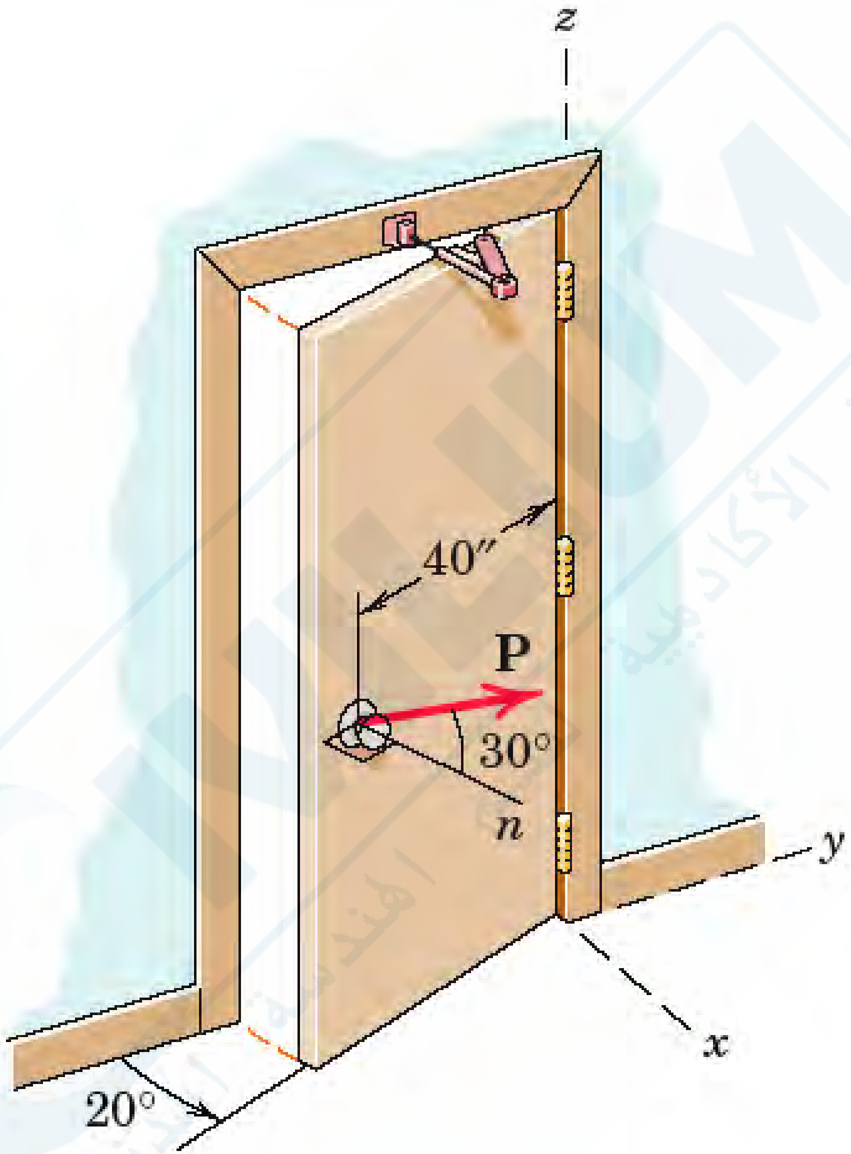


Problem 2/119

P_02_119

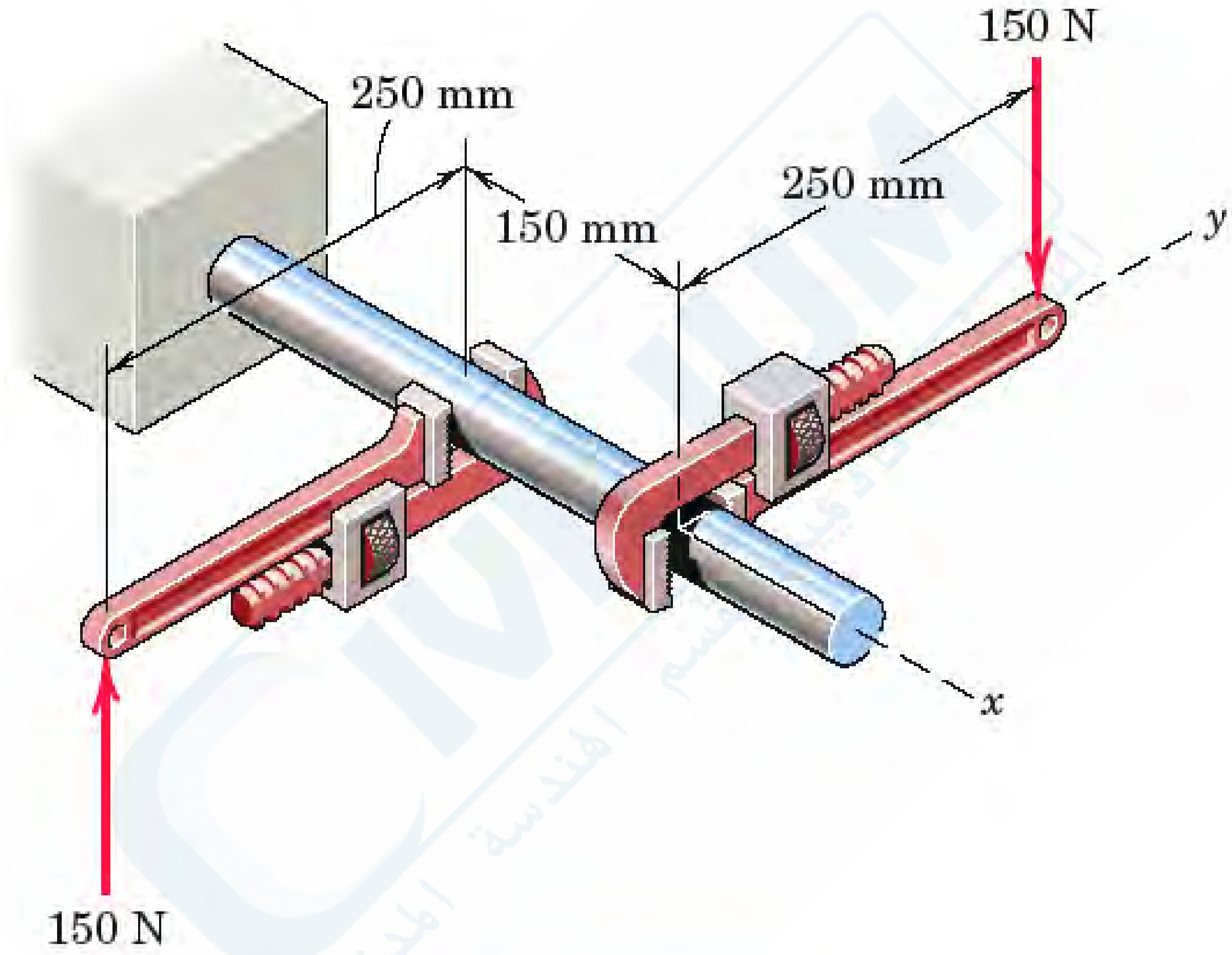


Problem 2/12



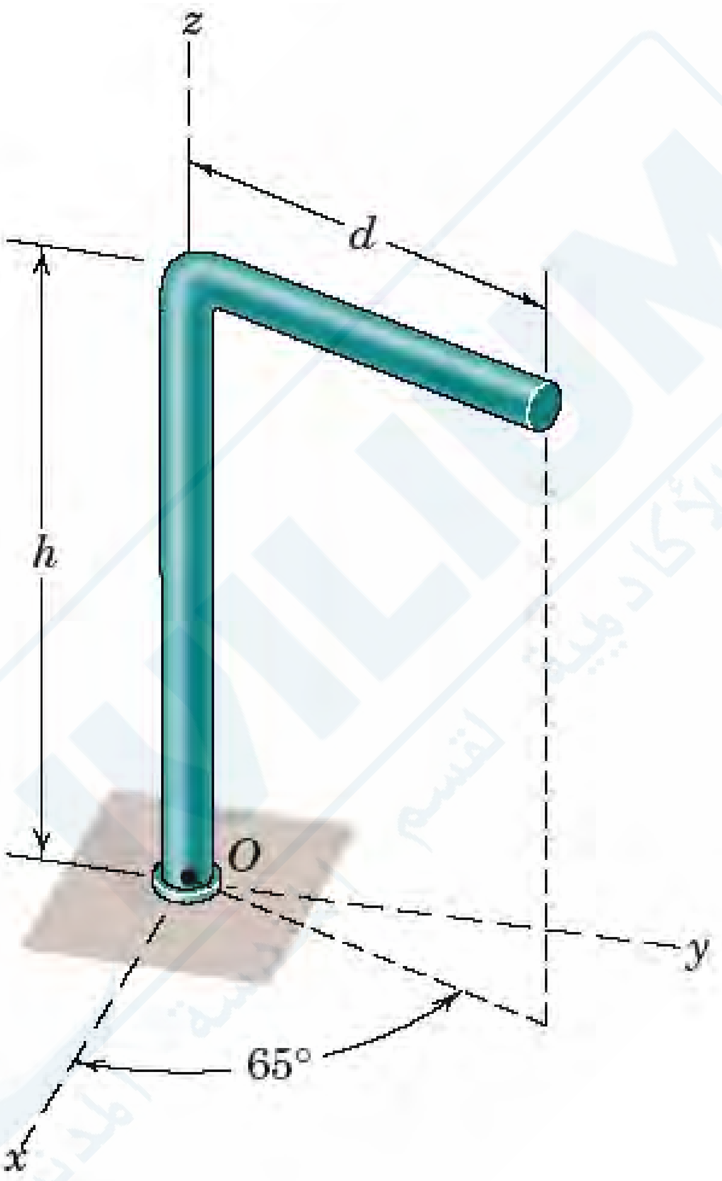
Problem 2/120

P 02 120



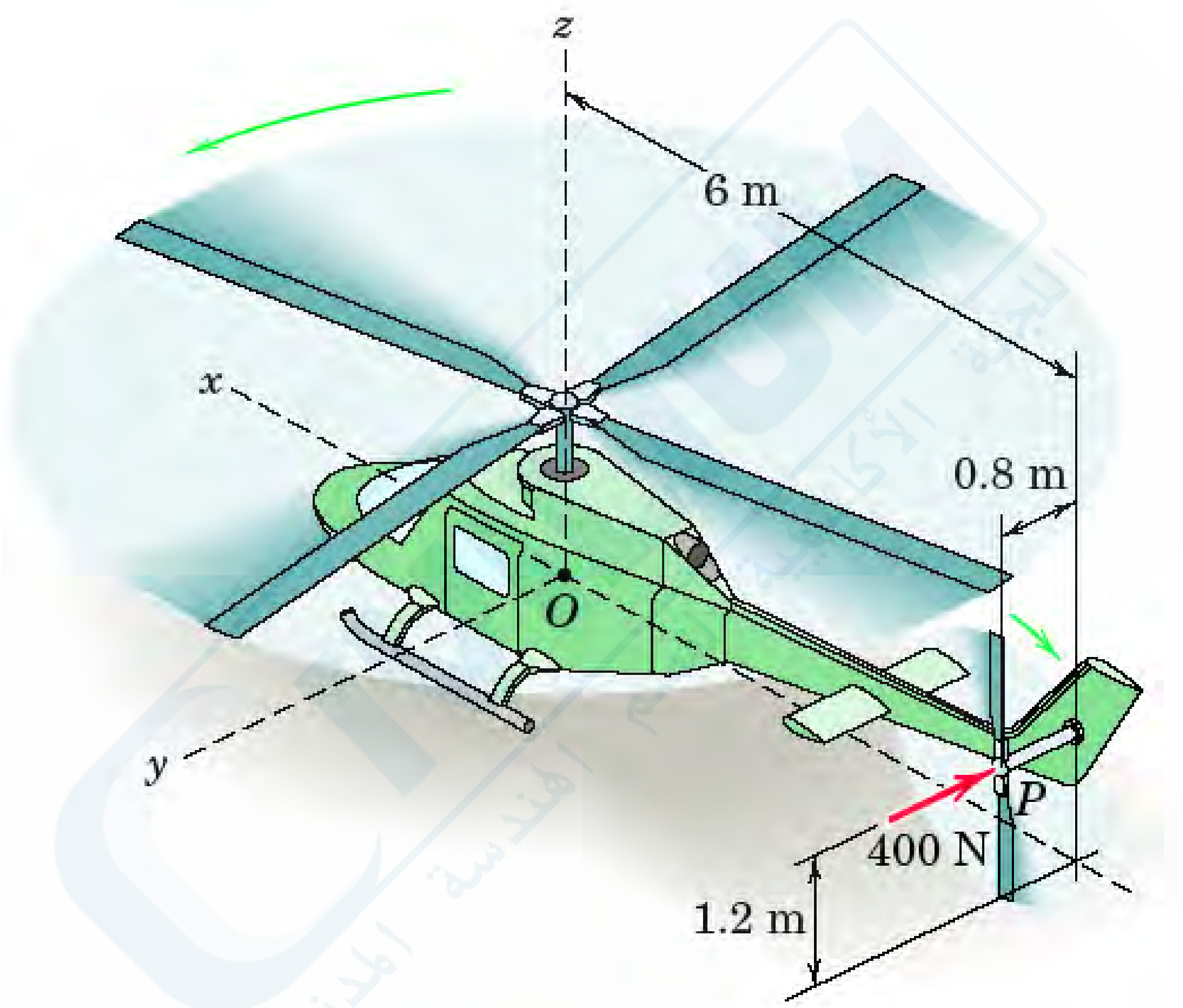
Problem 2/121

P 02 121



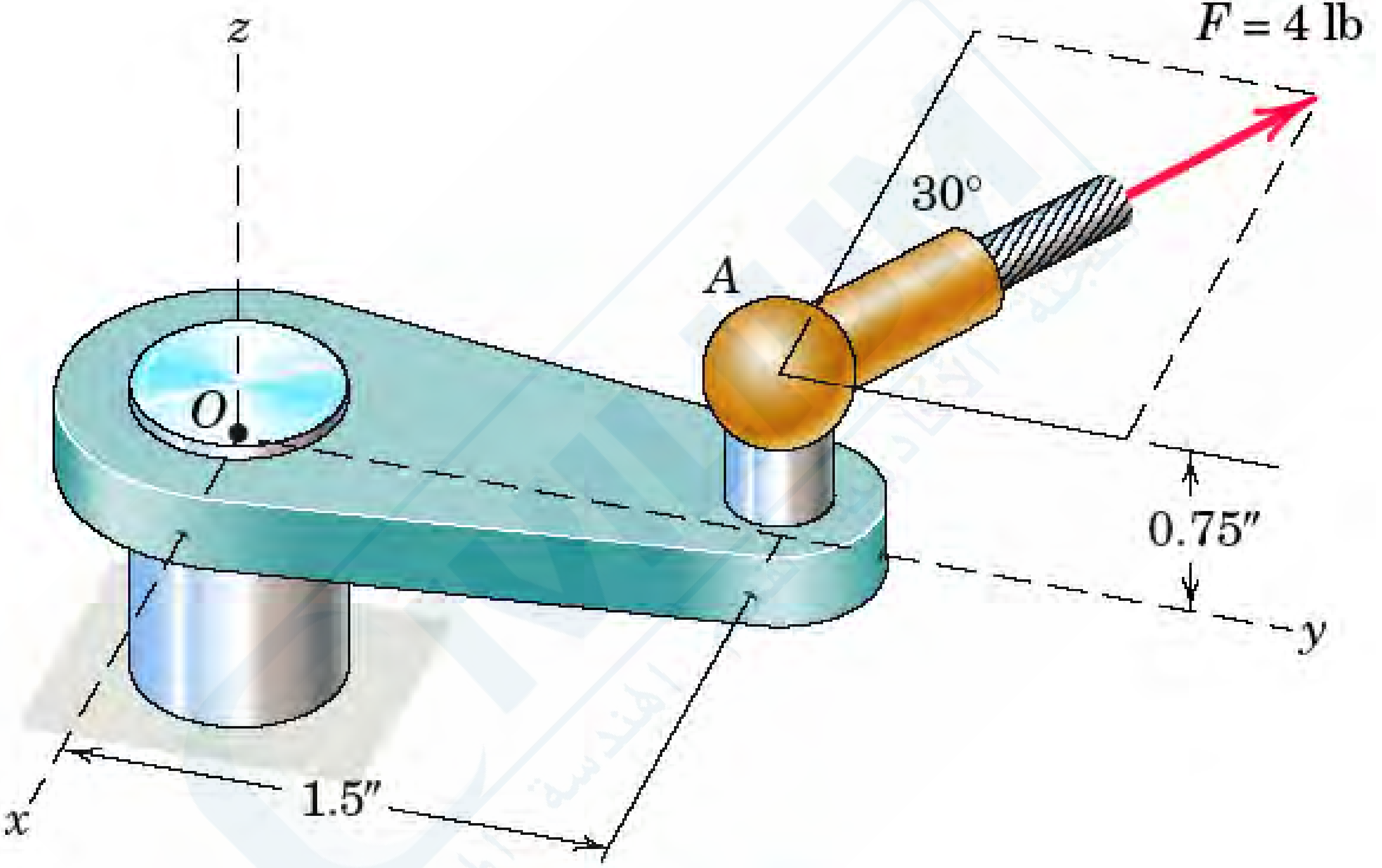
Problem 2/122

P 02 122



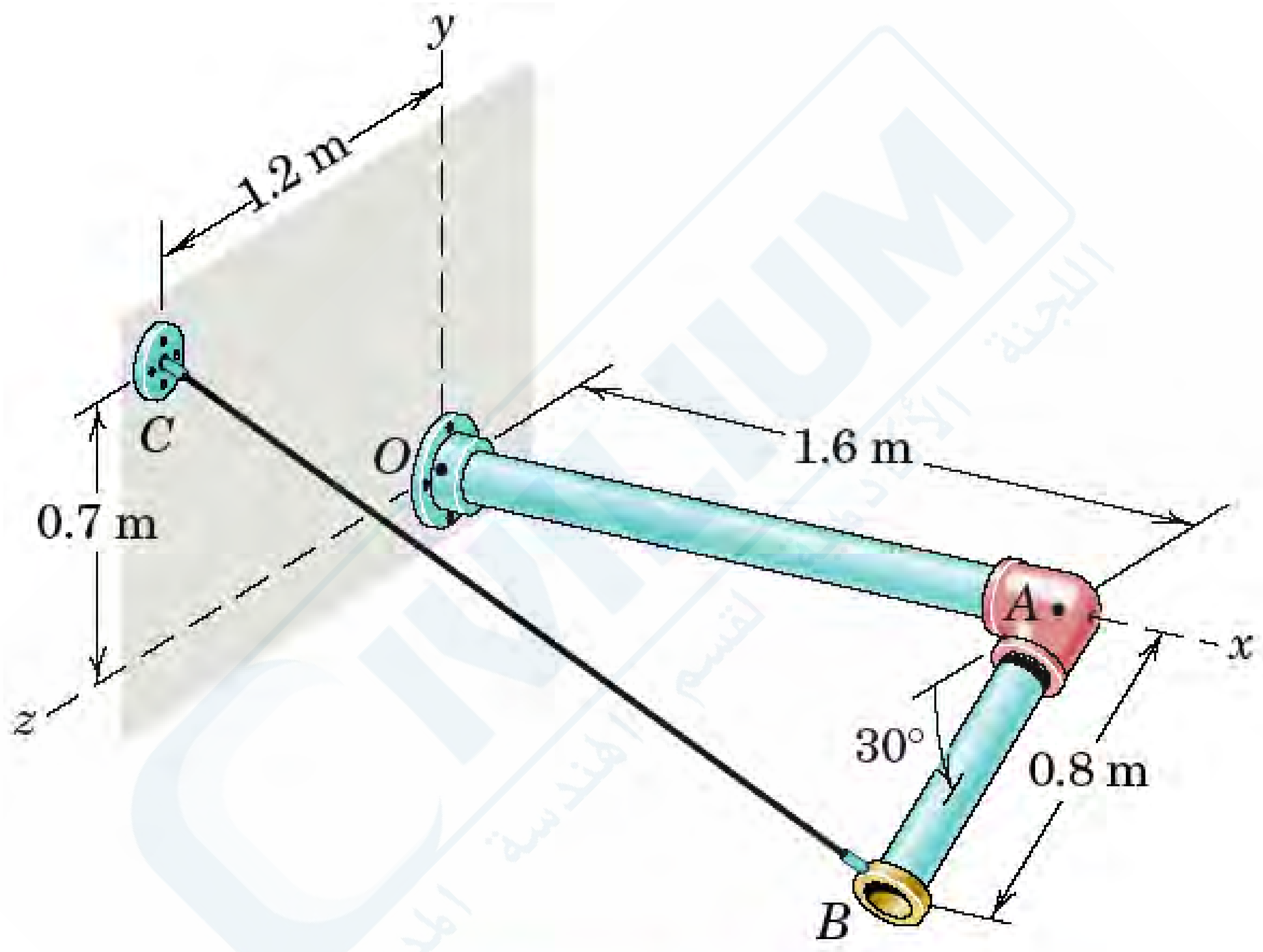
Problem 2/123

P 02 123



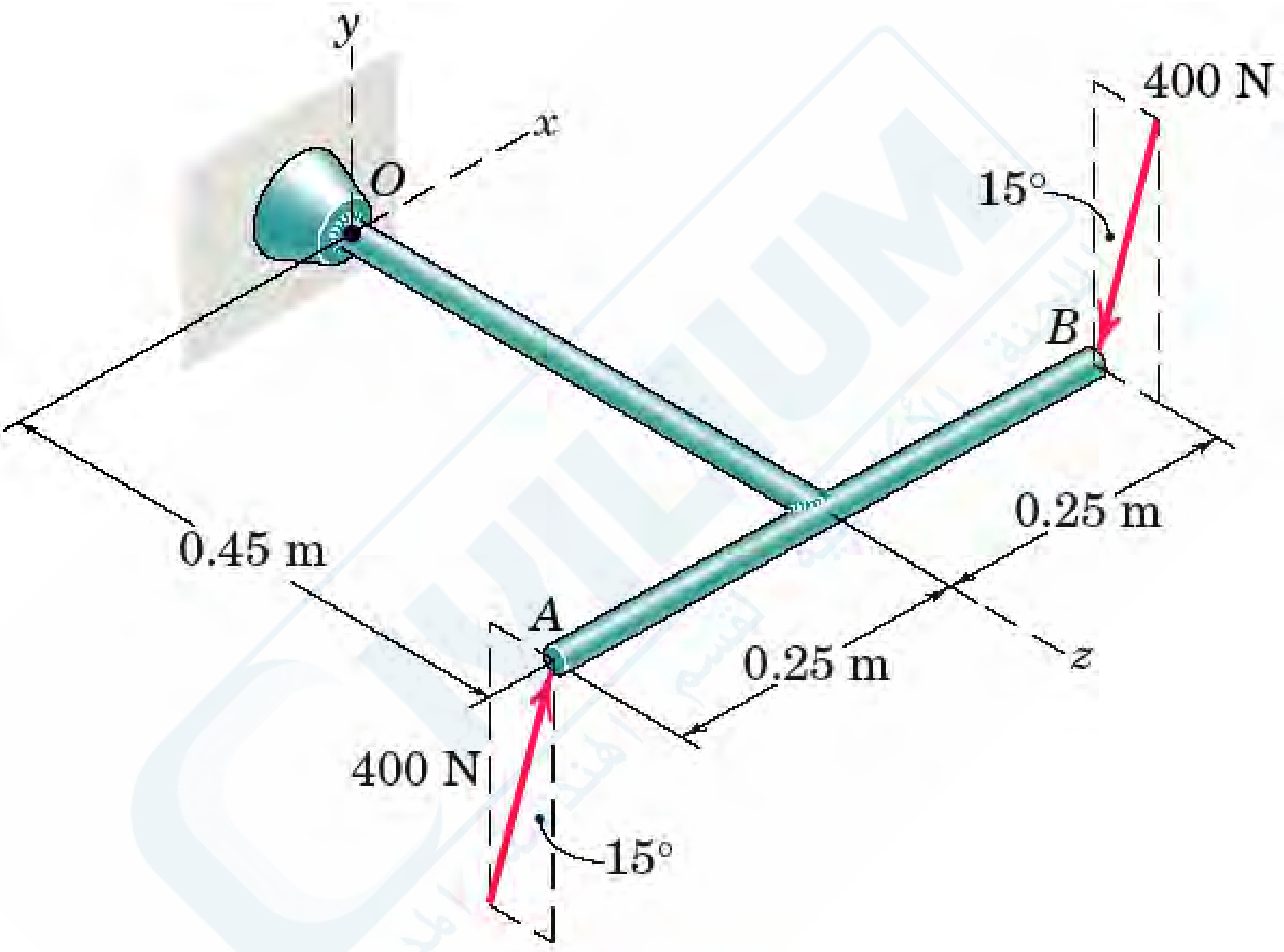
Problem 2/124

P_02_124



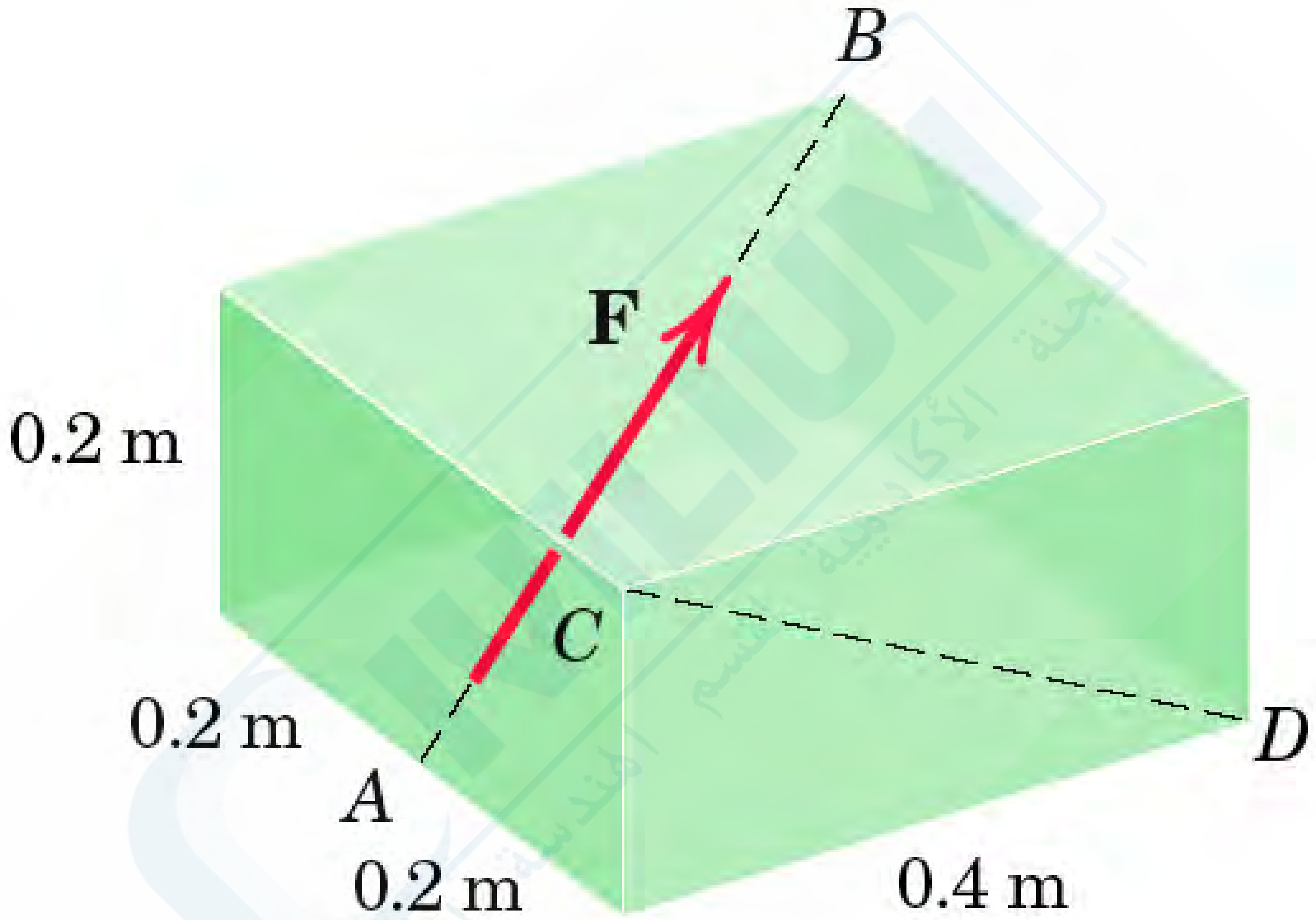
Problem 2/125

P 02 125



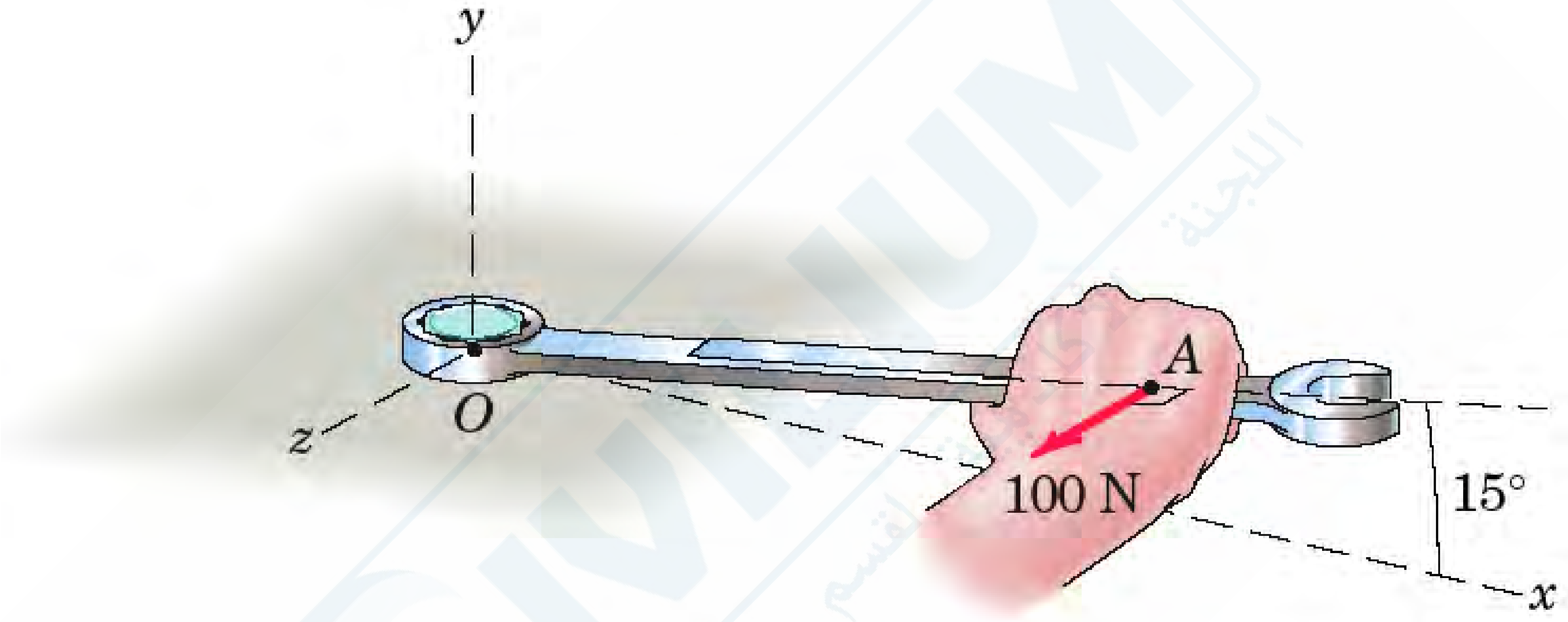
Problem 2/126

P 02 126



Problem 2/127

P 02 127

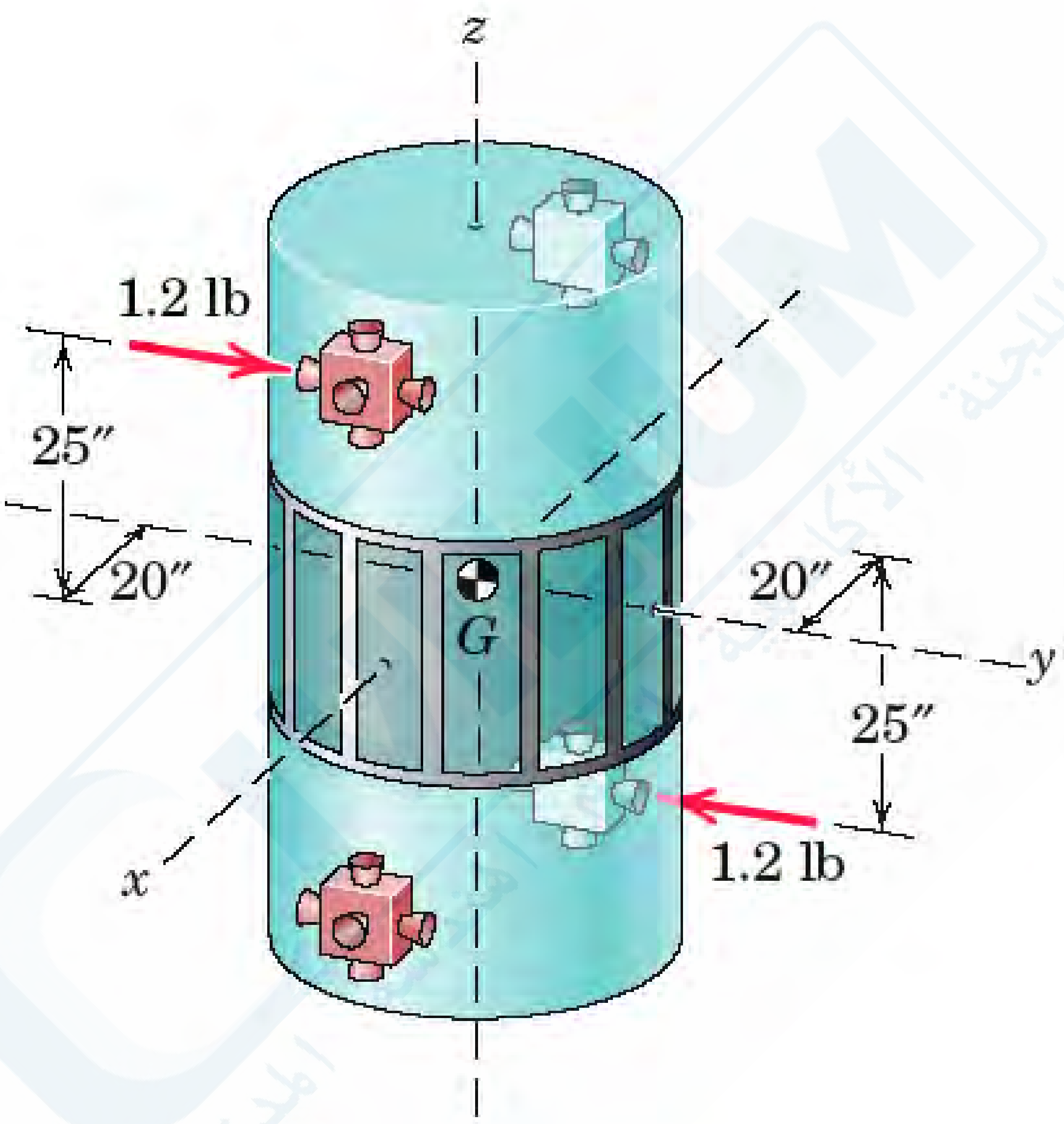


$\overline{OA} = 185$ mm

Problem 2/128

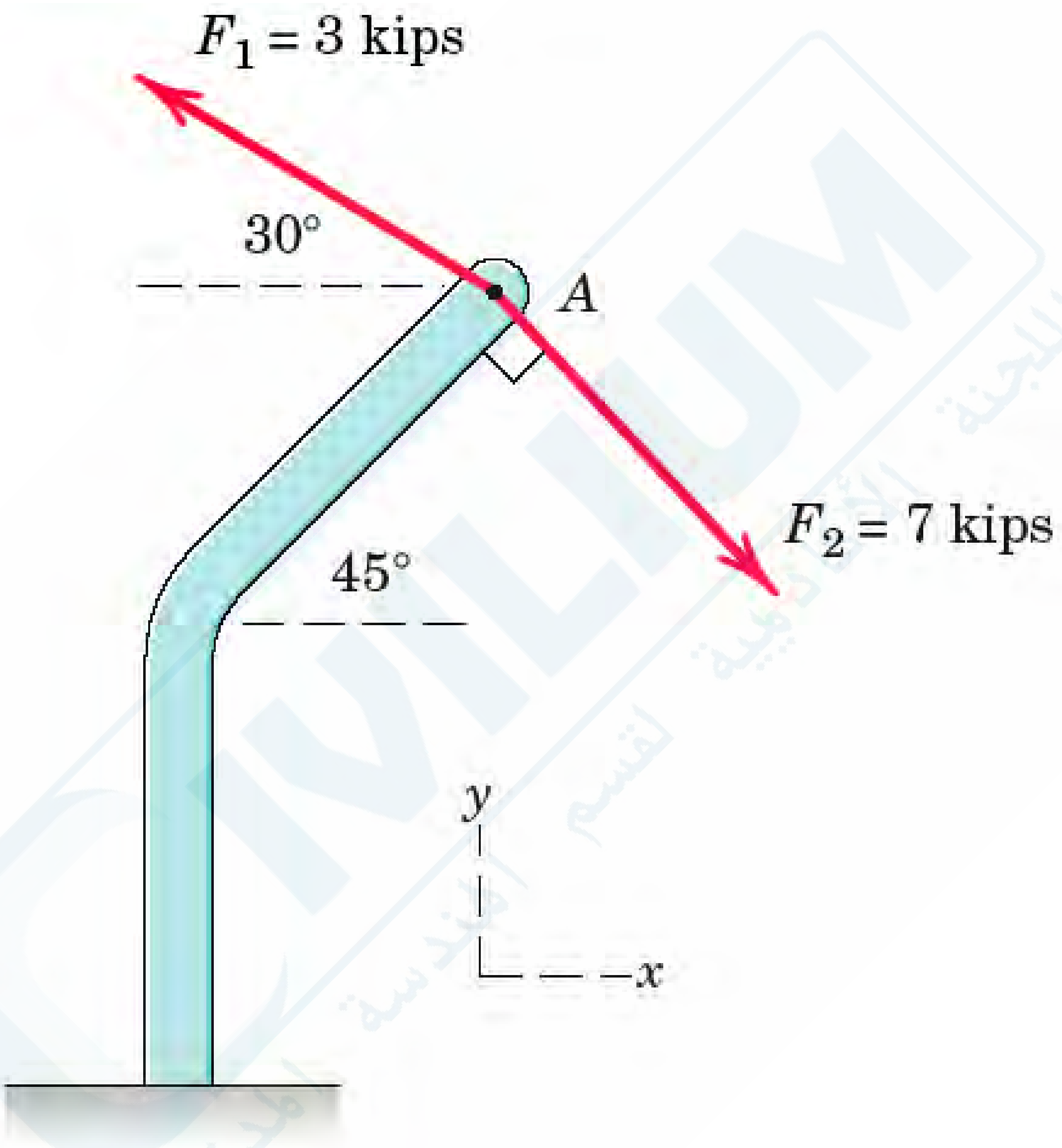
P_02_128

CIVILIUM



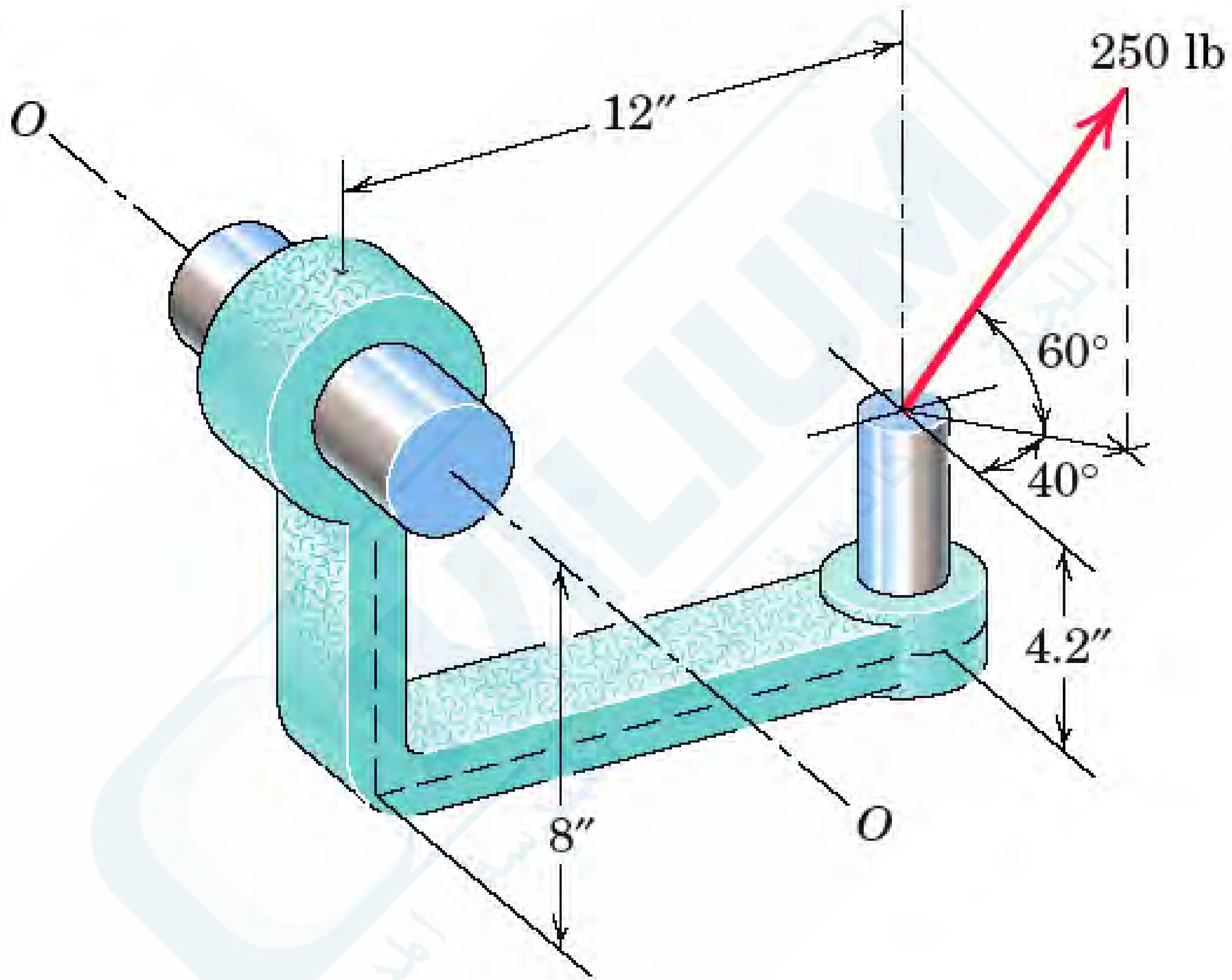
Problem 2/129

P 02 129



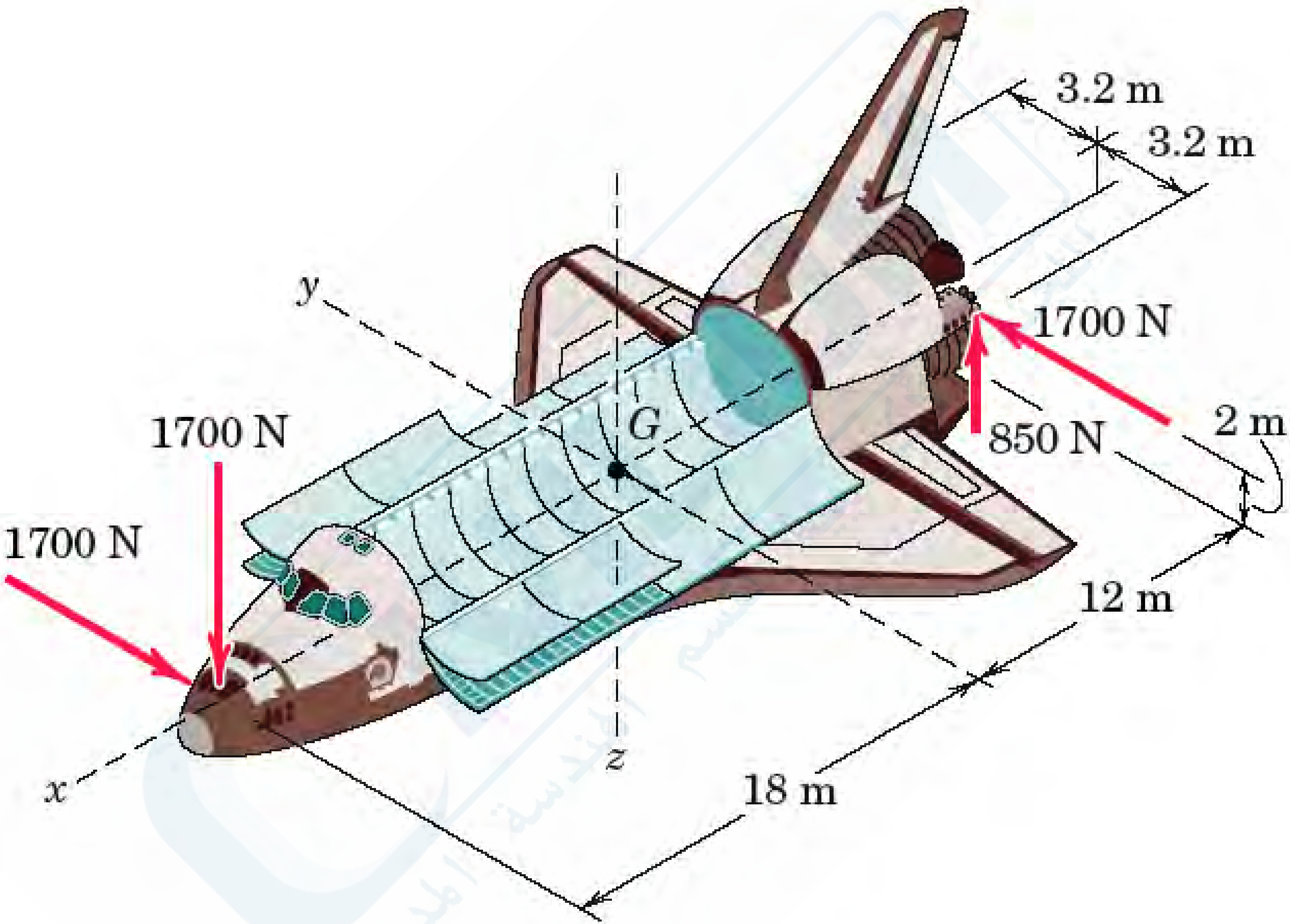
Problem 2/13

P 02 13



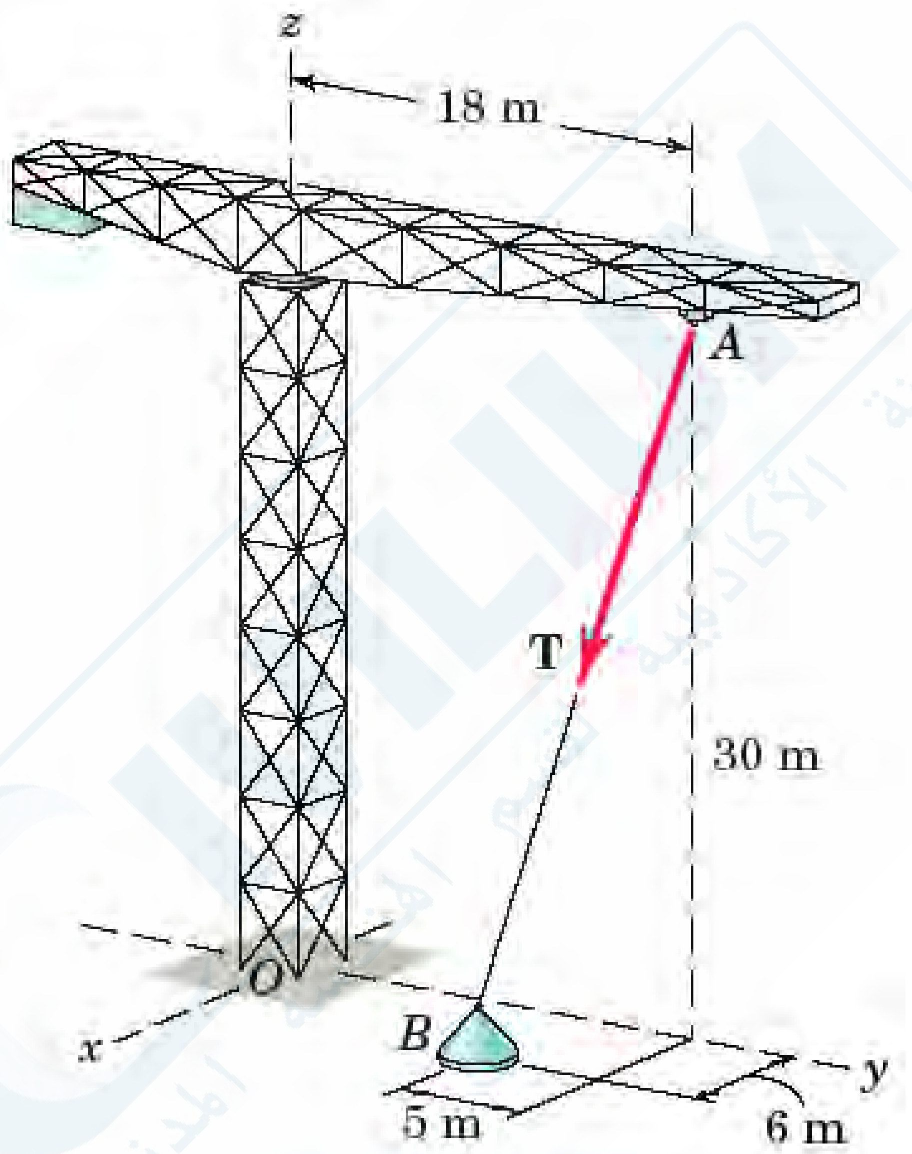
Problem 2/130

P_02_130



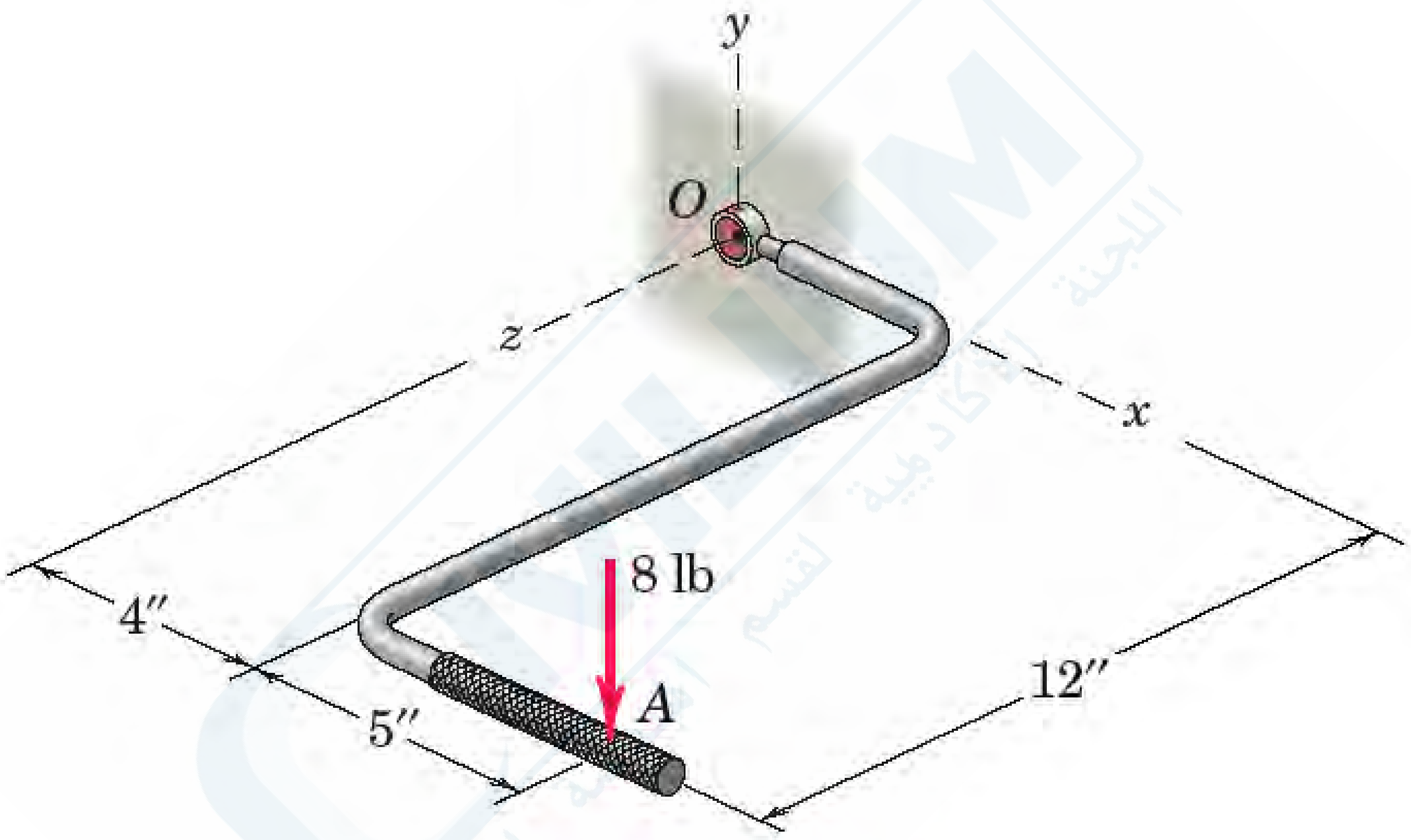
Problem 2/131

P 02 131



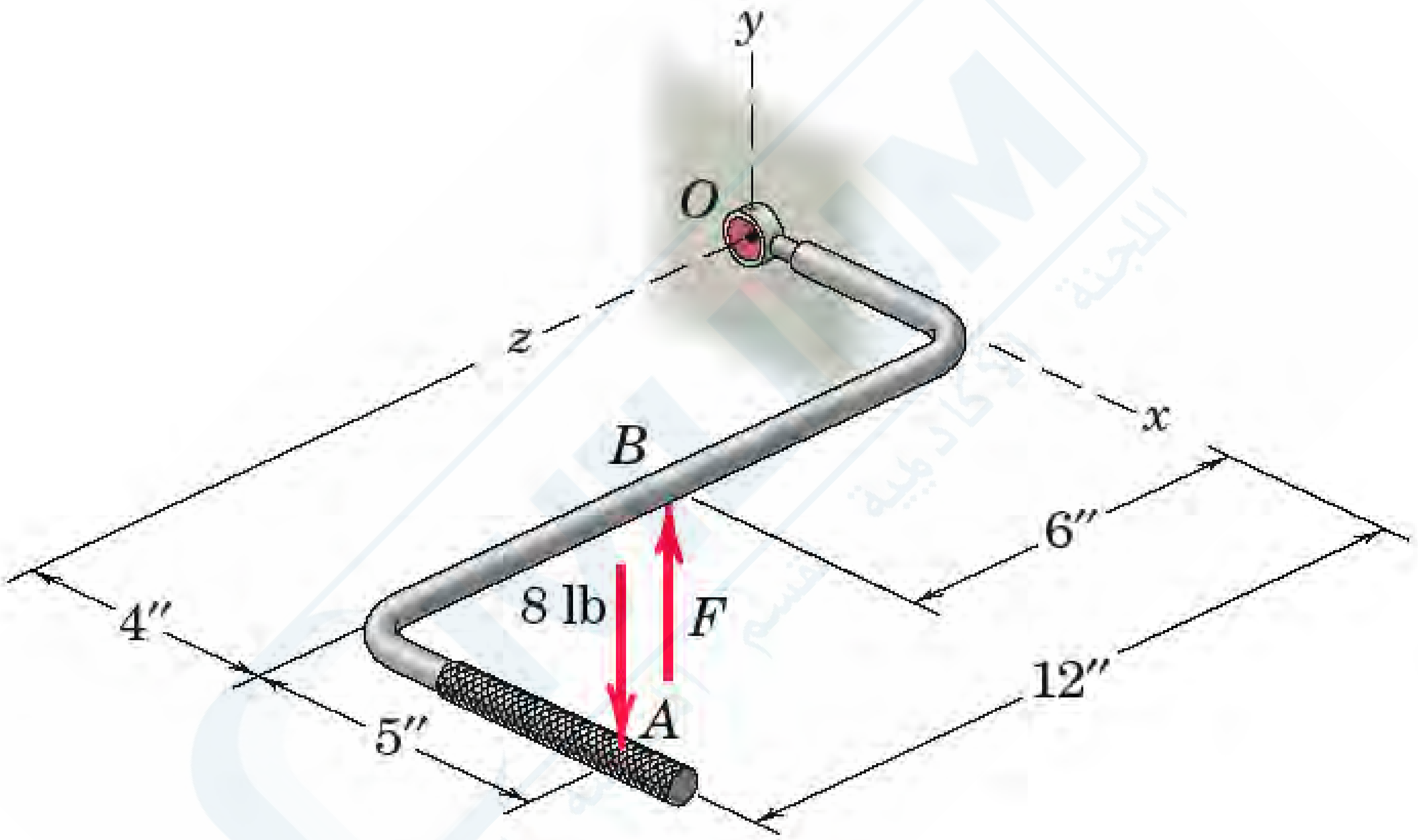
Problem 2/132

P 02 132



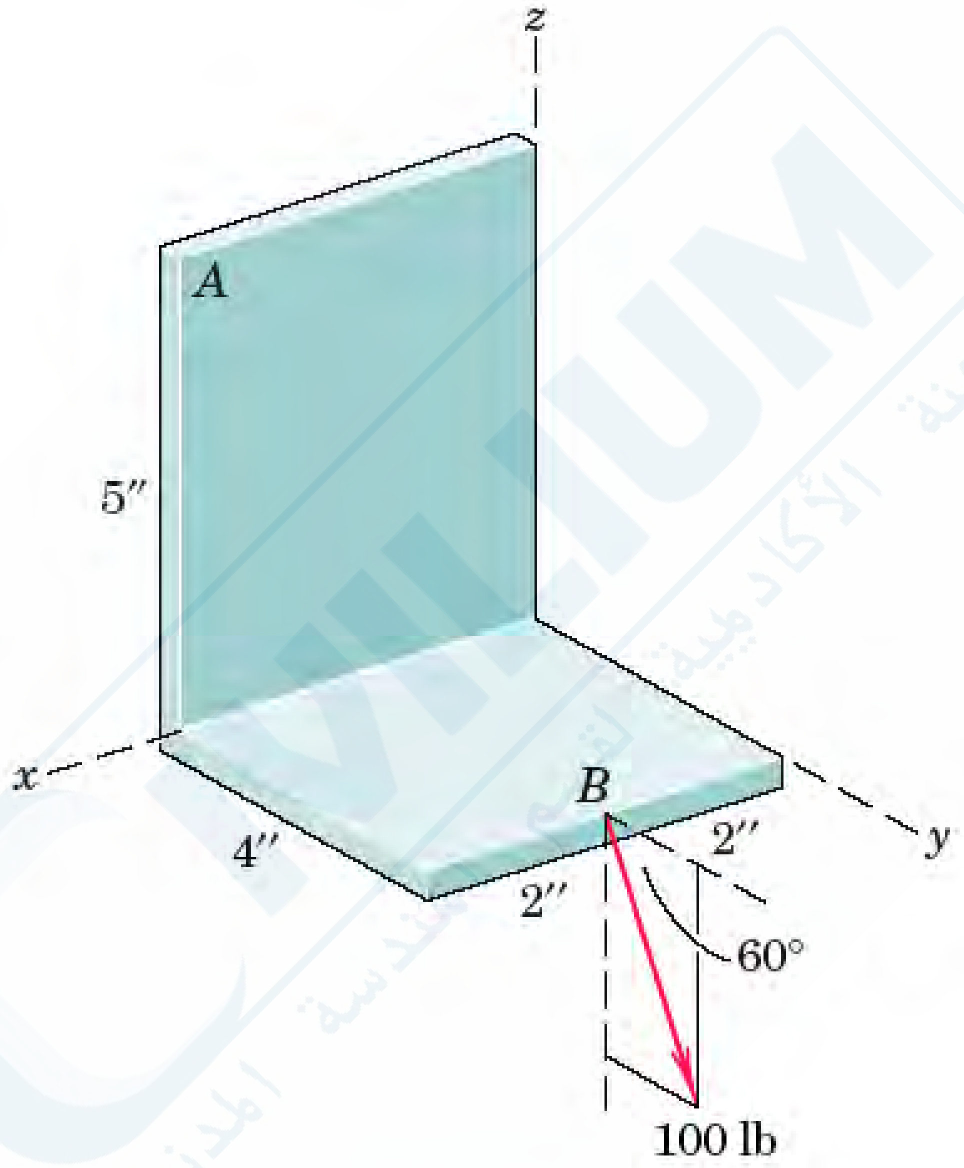
Problem 2/133

P_02_133



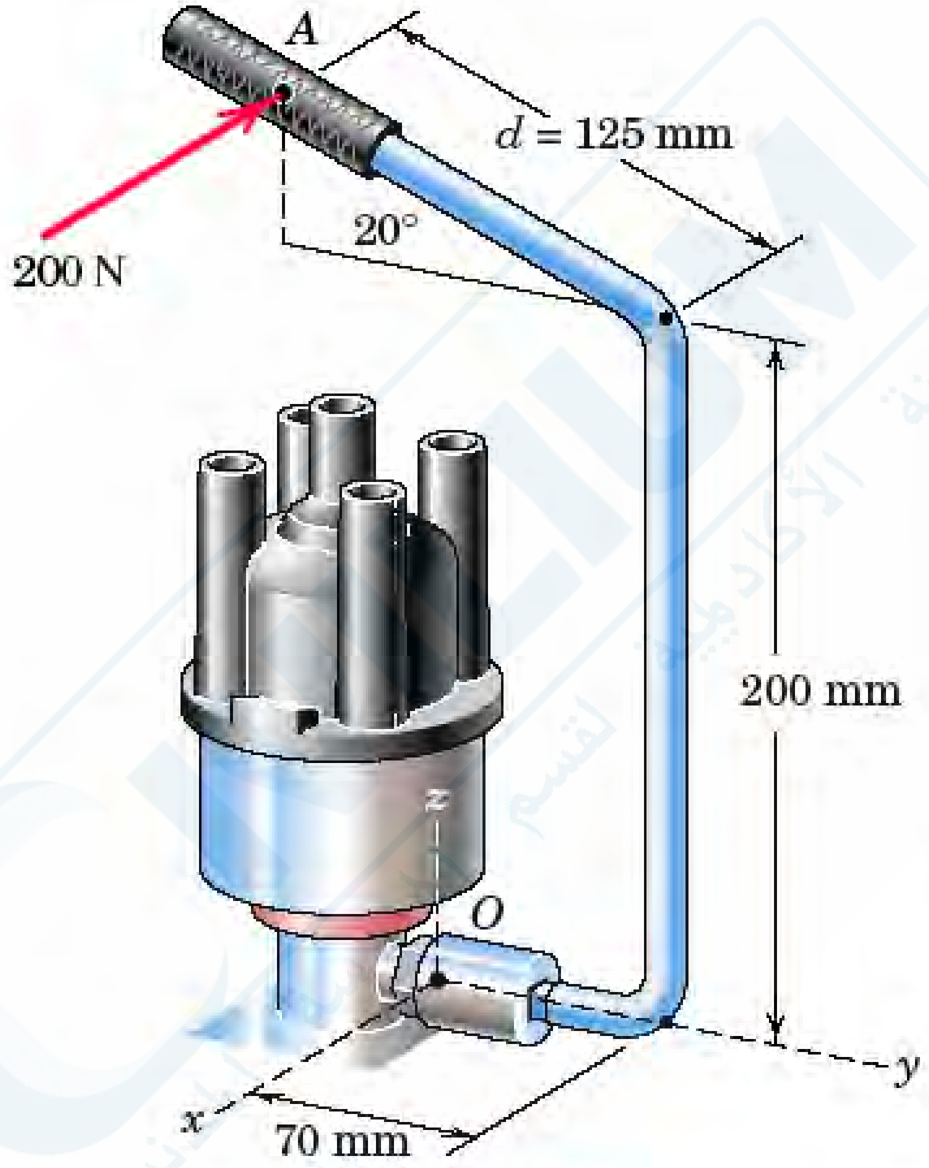
Problem 2/134

P_02_134



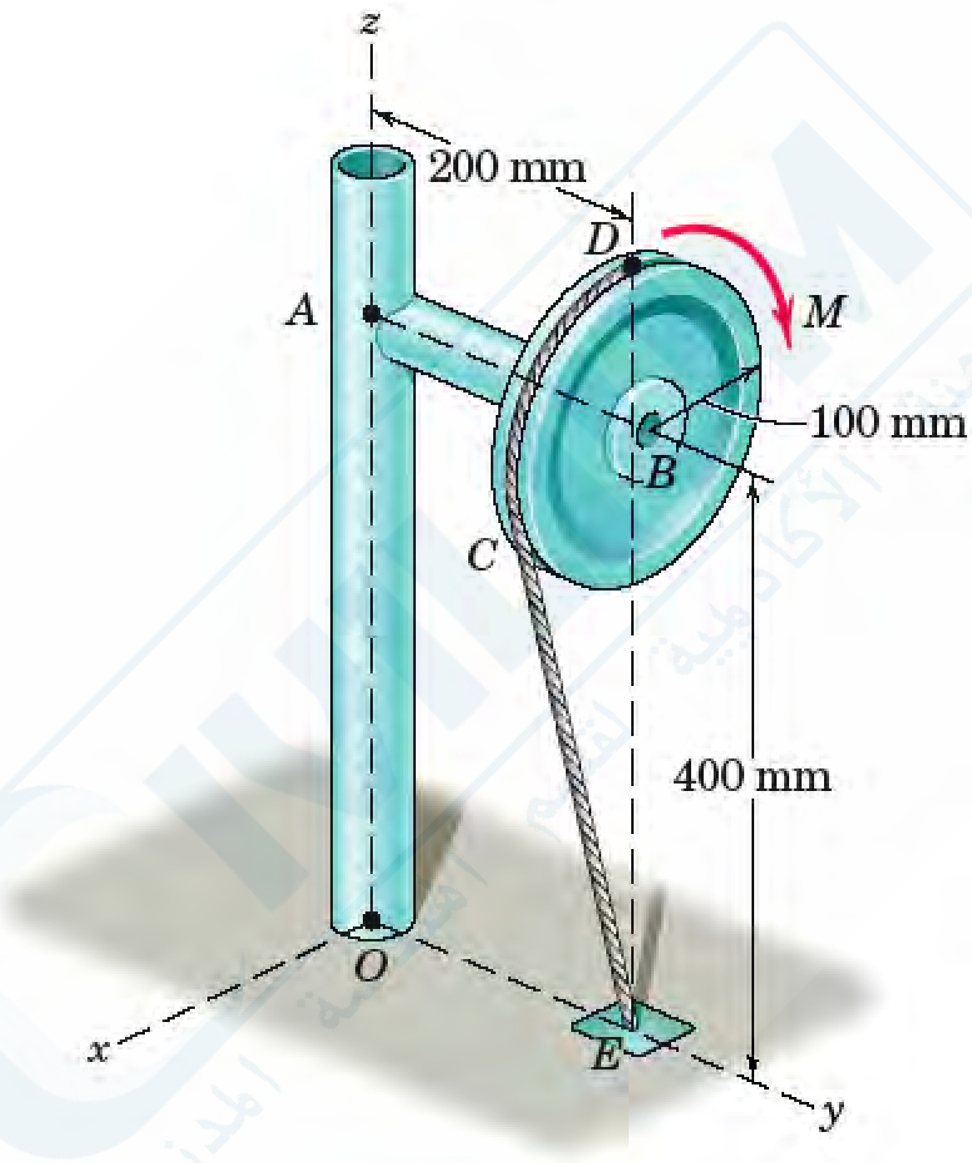
Problem 2/135

P_02_135



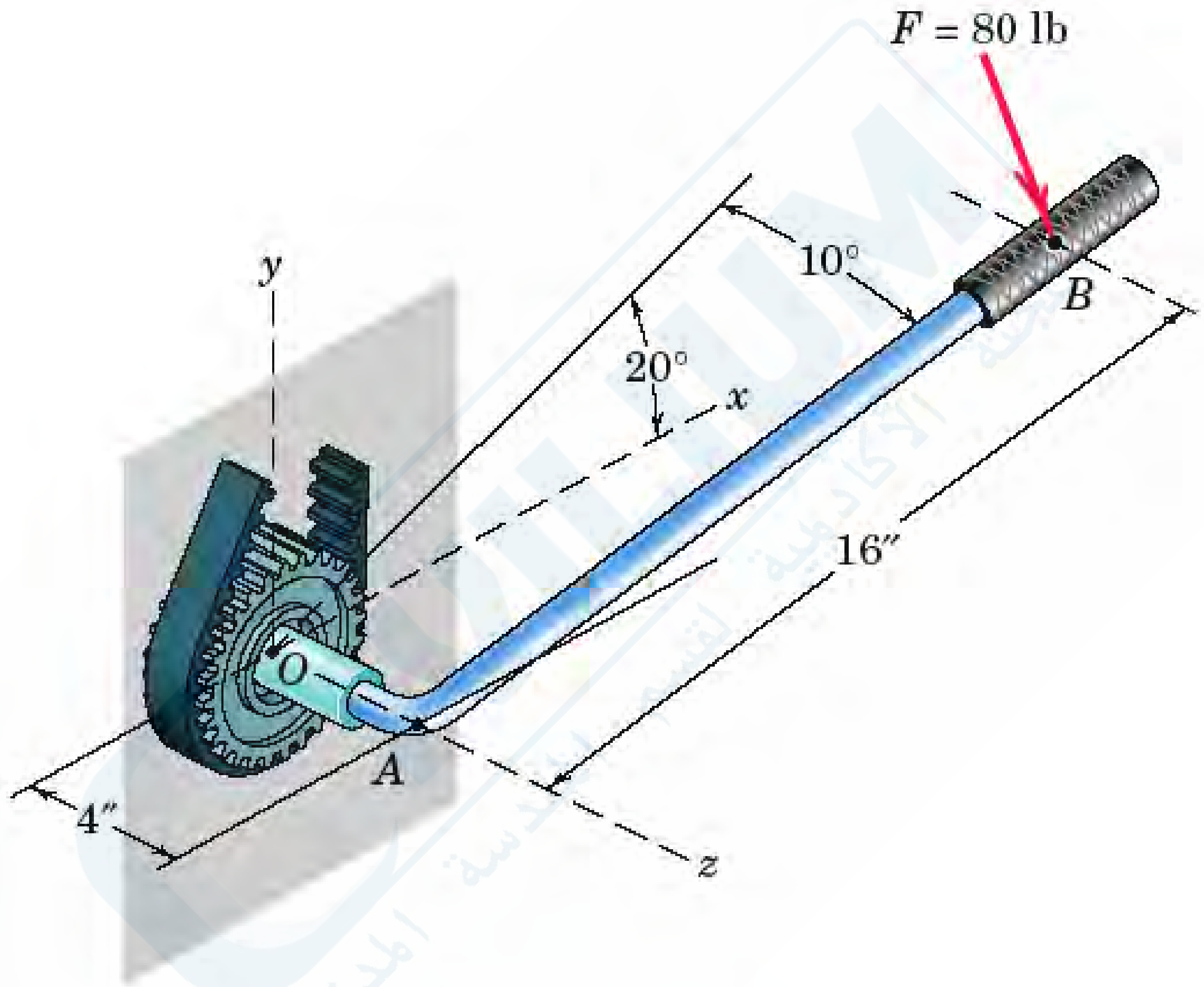
Problem 2/136

P 02 136



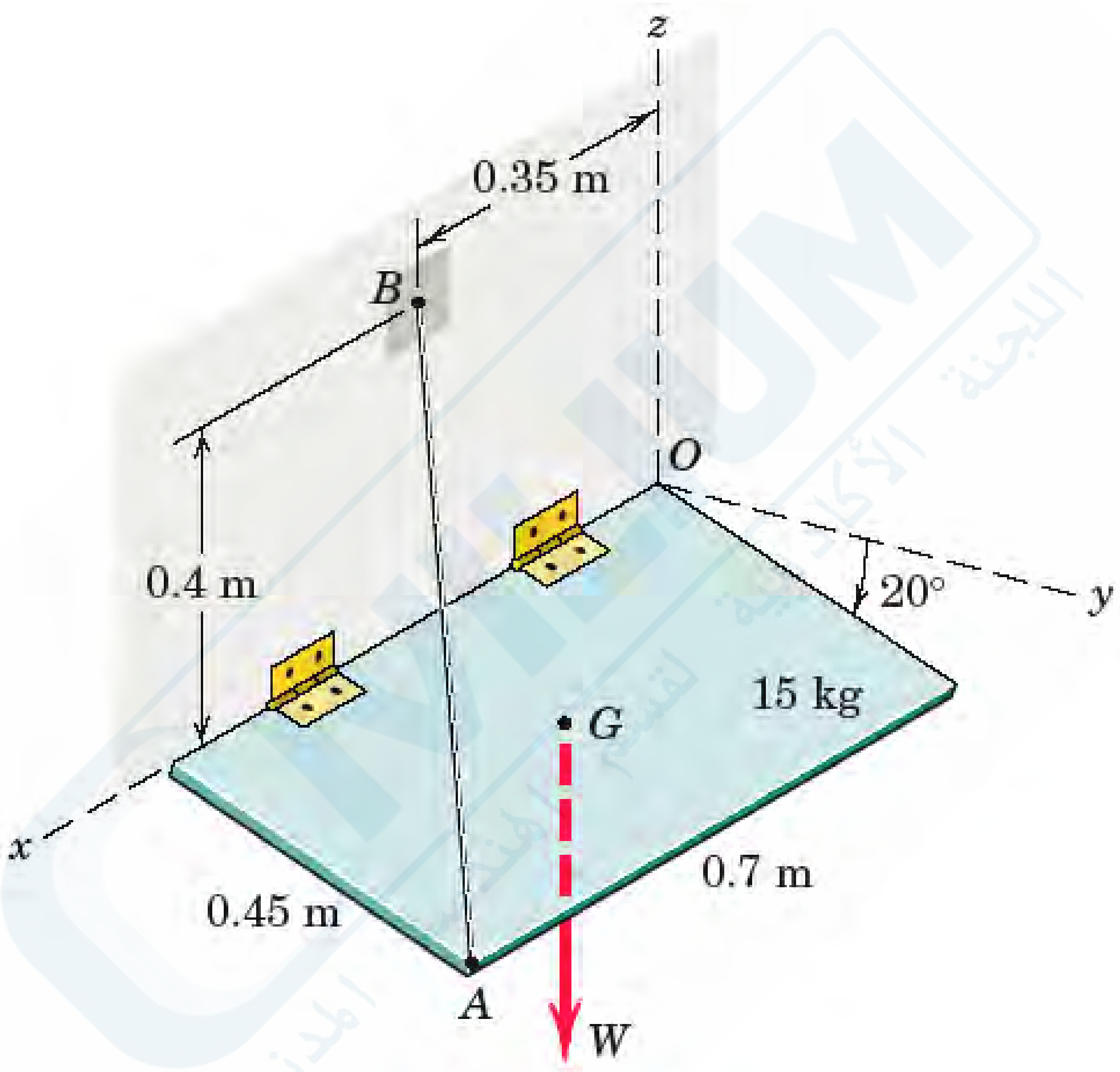
Problem 2/137

P_02_137



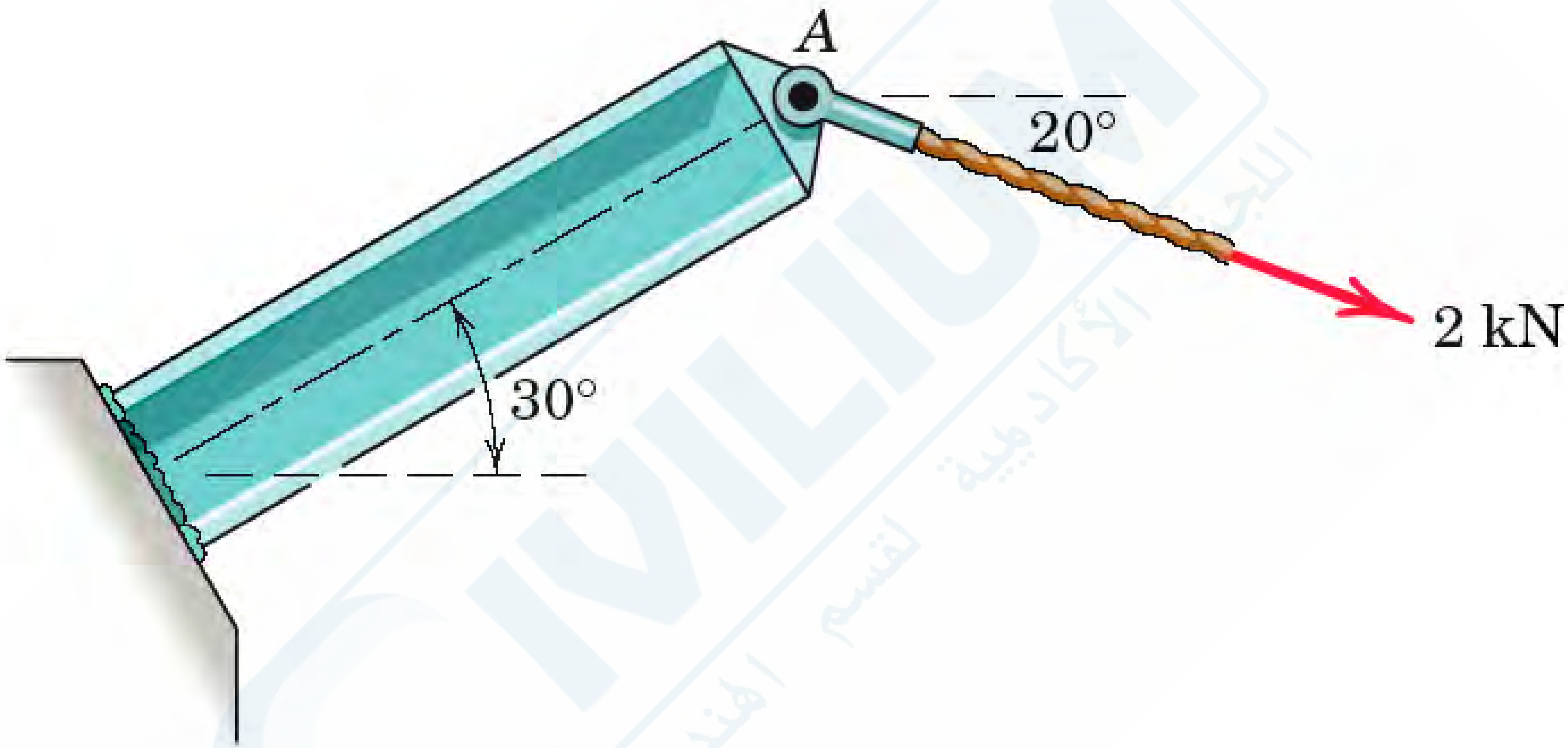
Problem 2/138

P 02 138



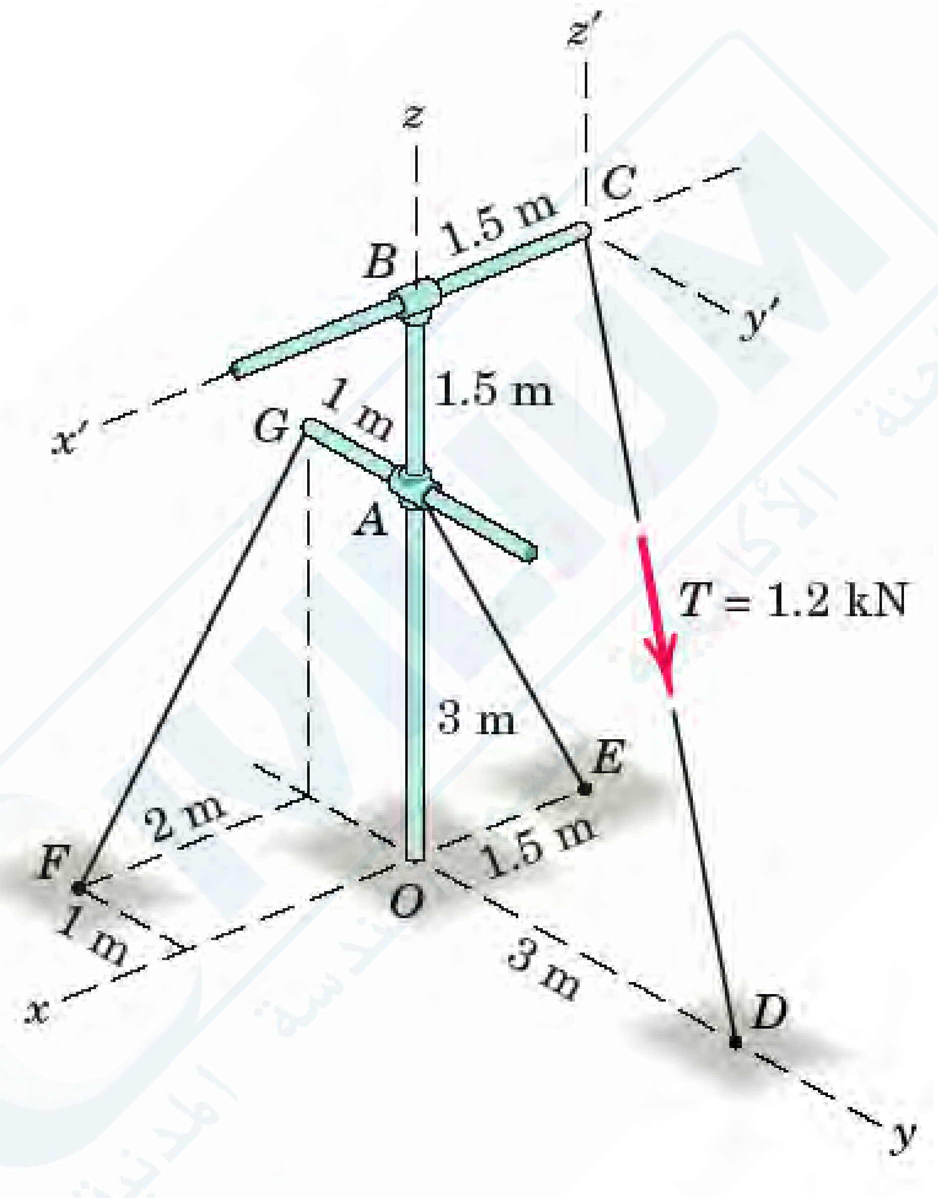
Problem 2/139

P 02 139



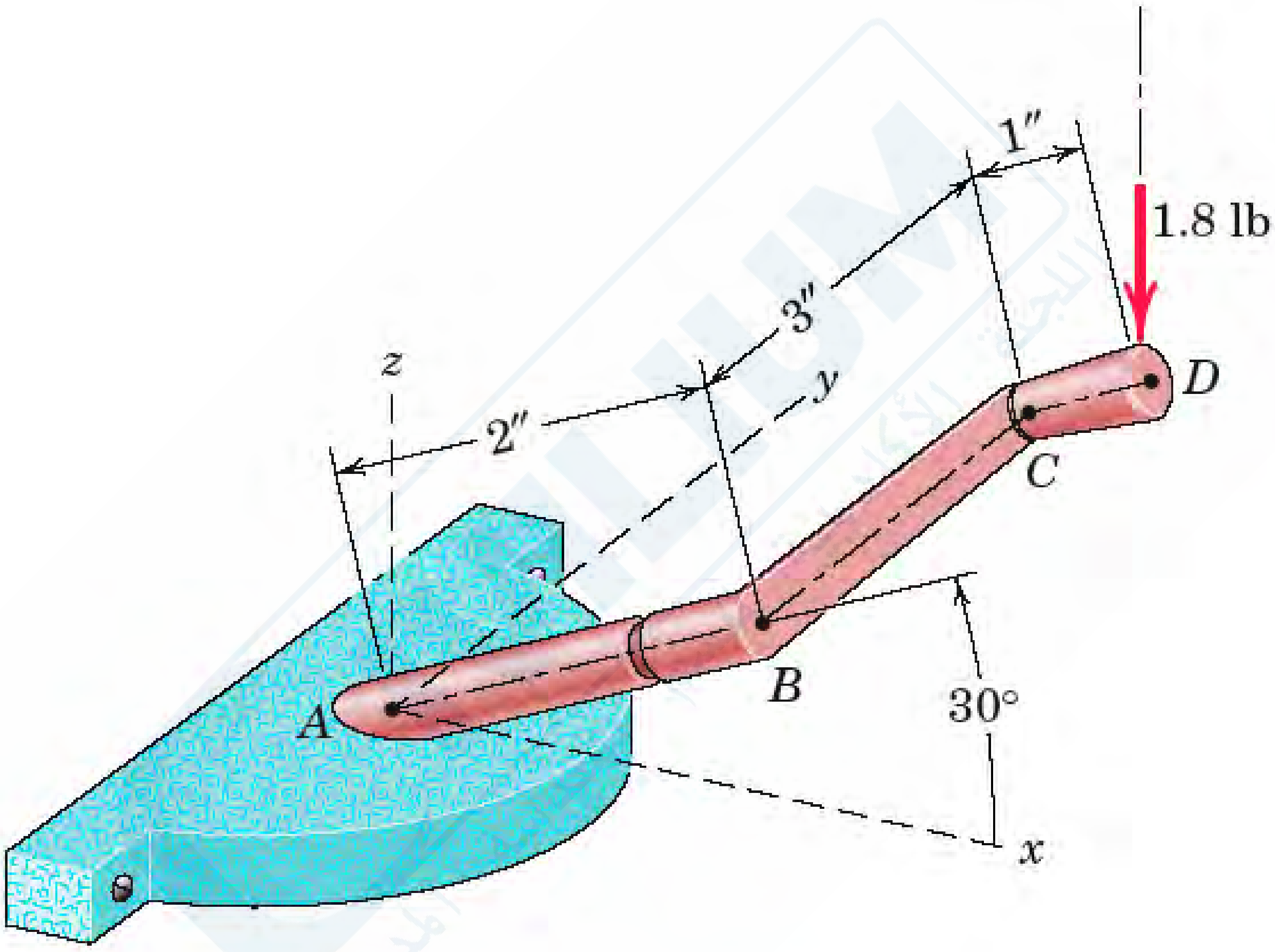
Problem 2/14

P_02_14



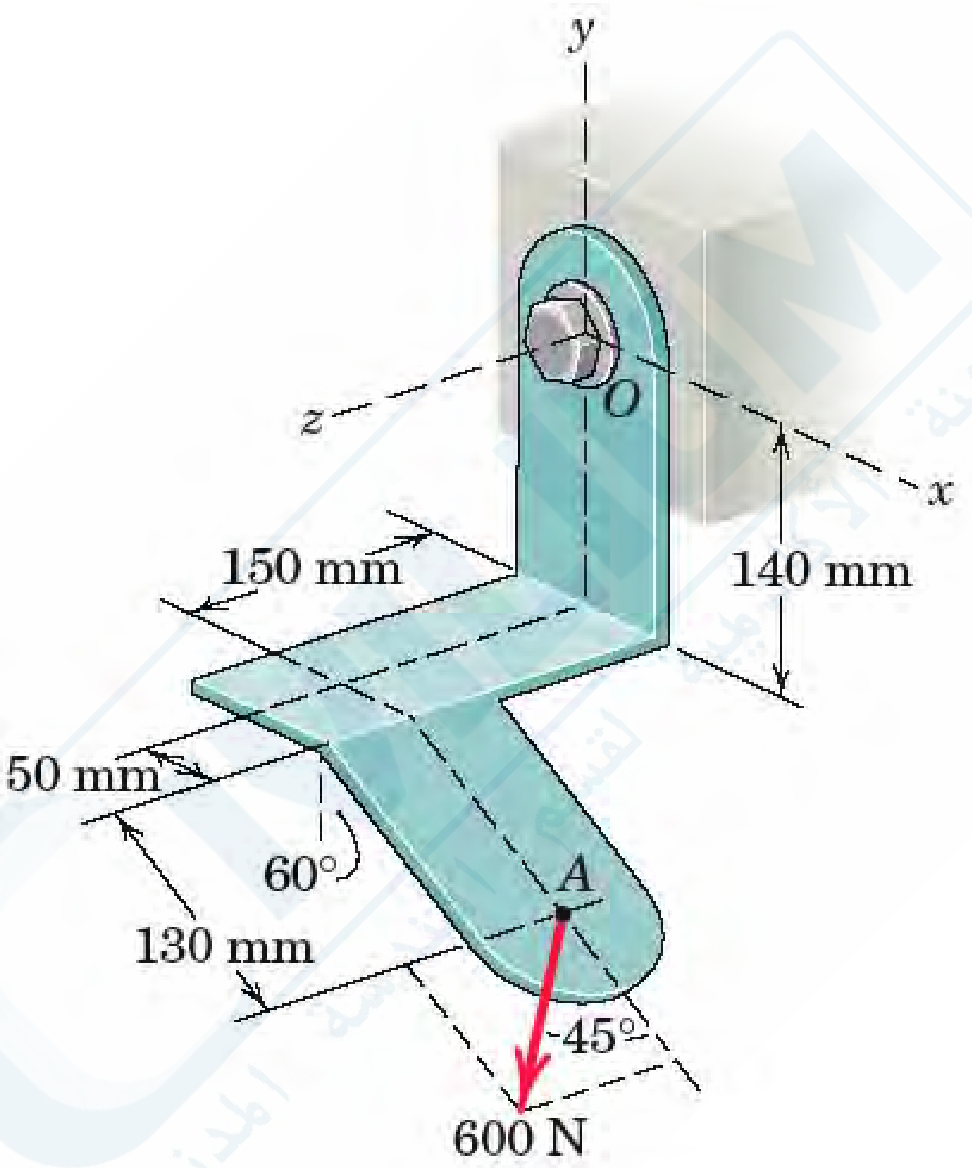
Problem 2/140

P 02 140



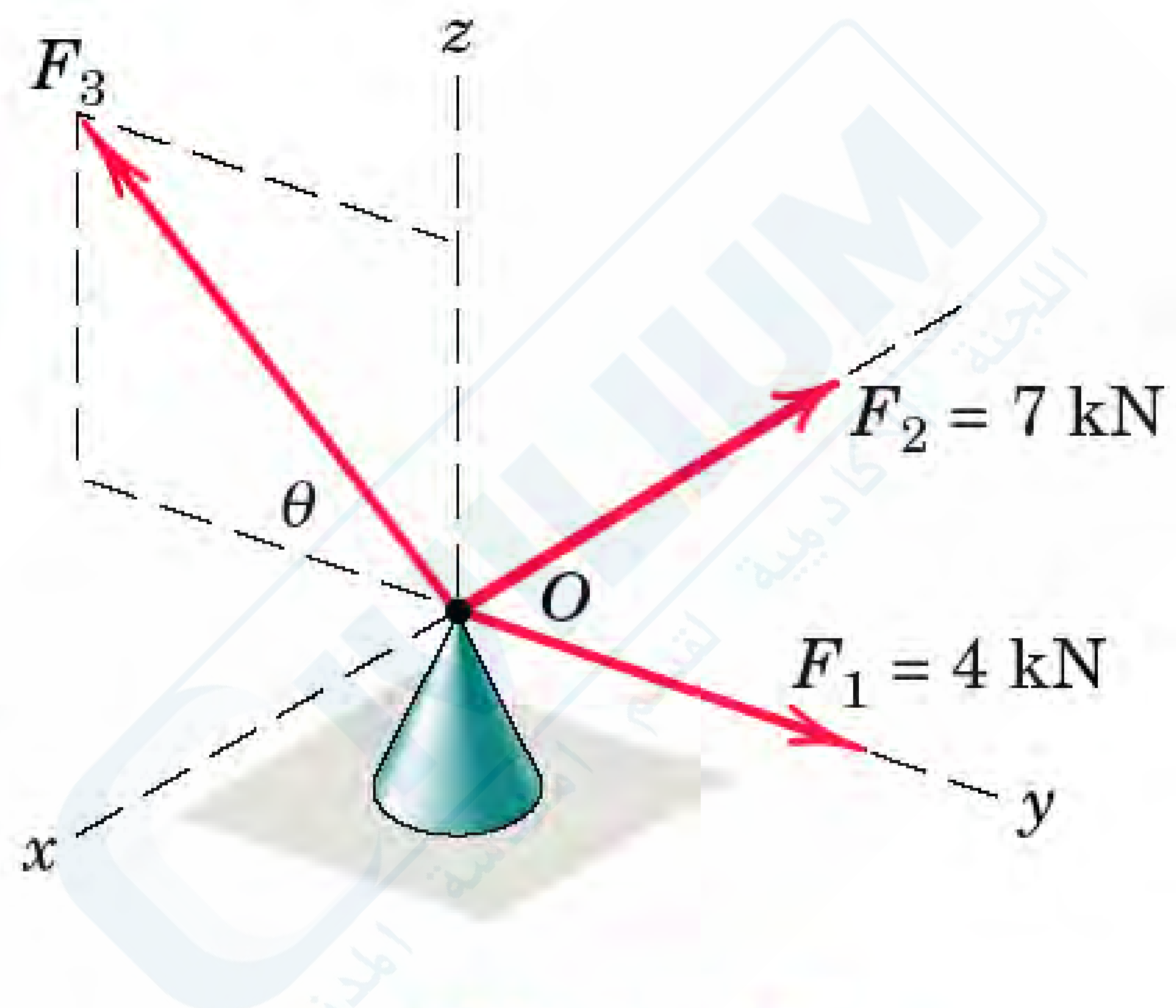
Problem 2/141

P 02 141



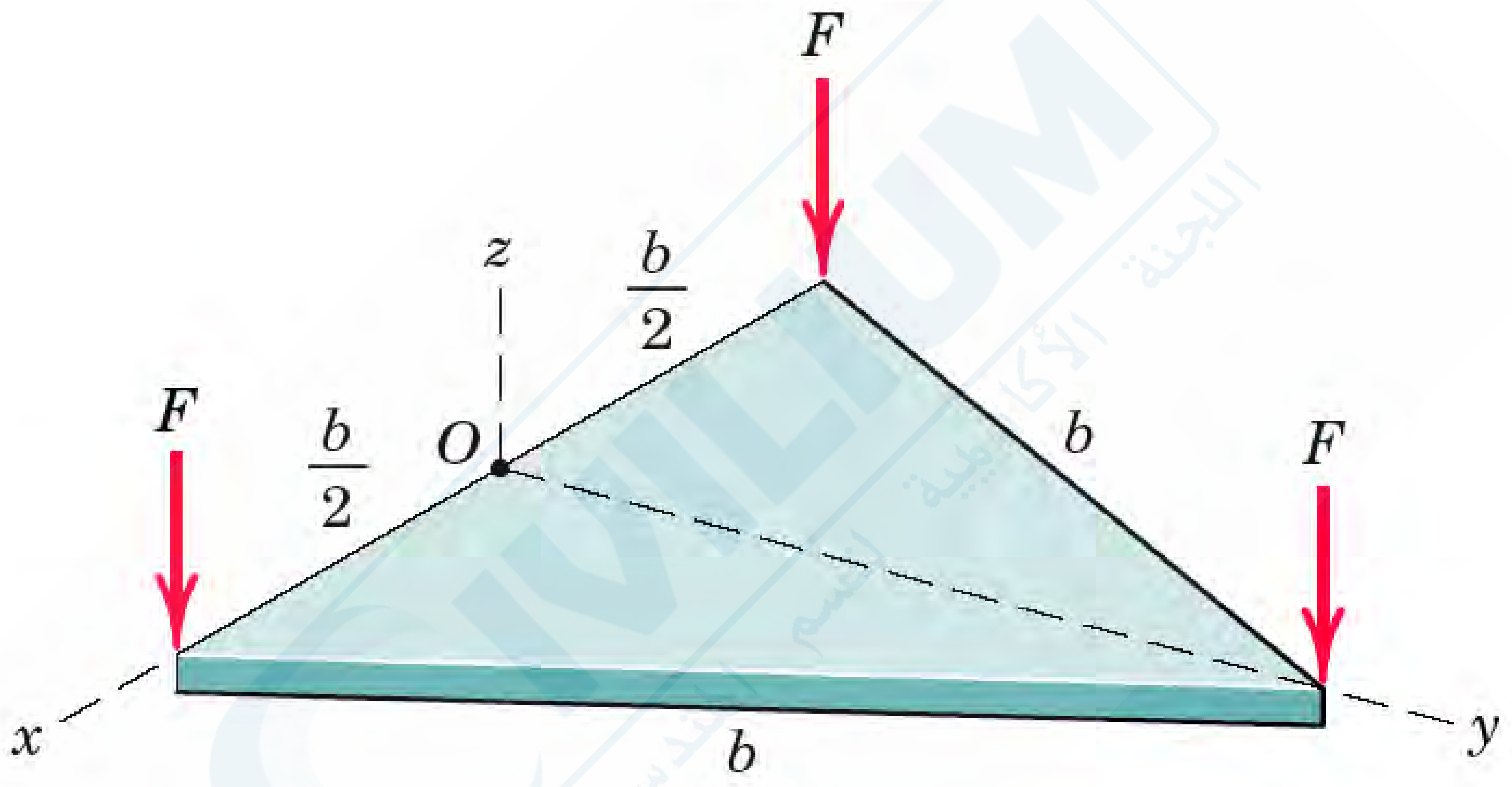
Problem 2/142

P 02 142



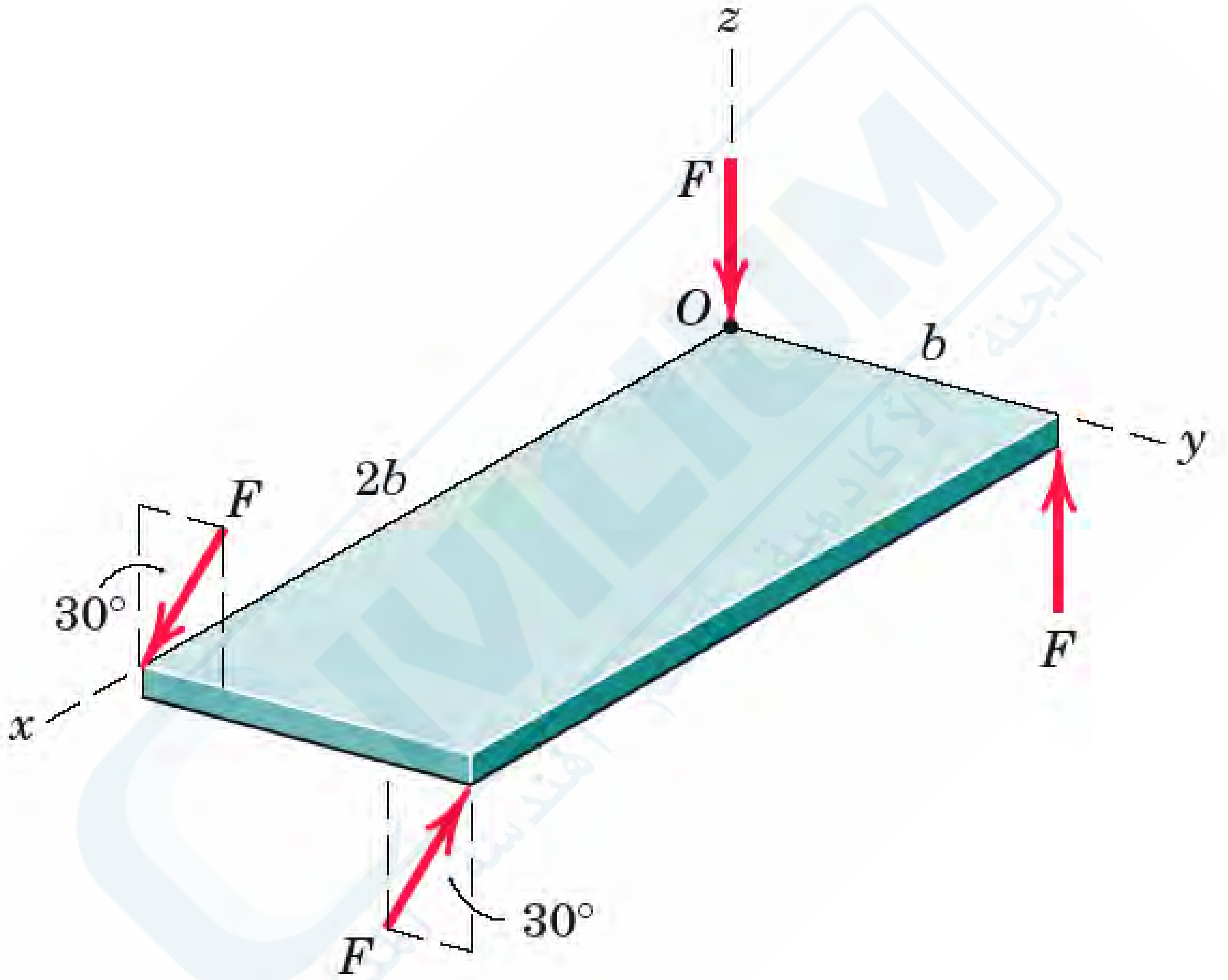
Problem 2/143

P 02 143



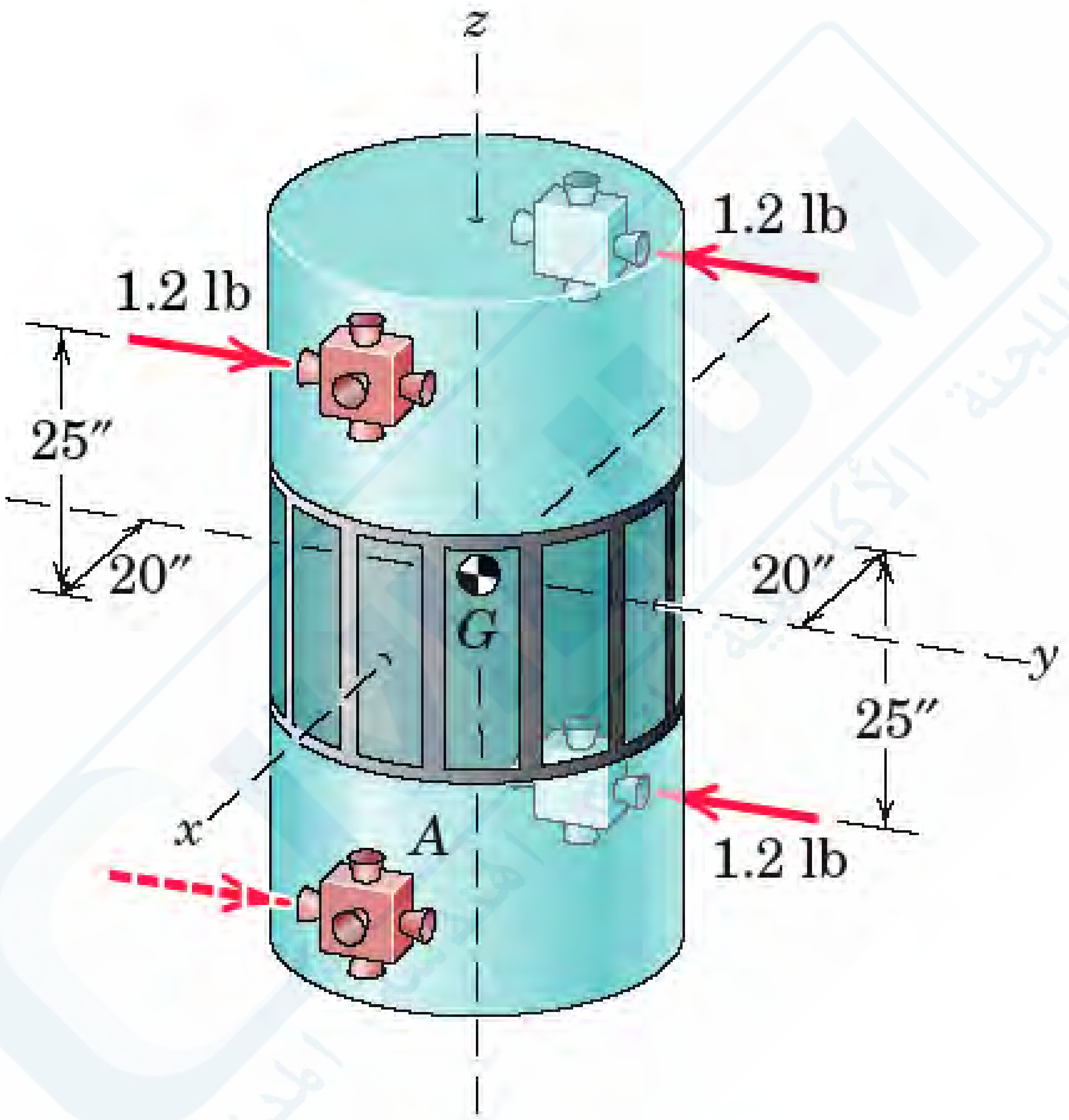
Problem 2/144

P_02_144



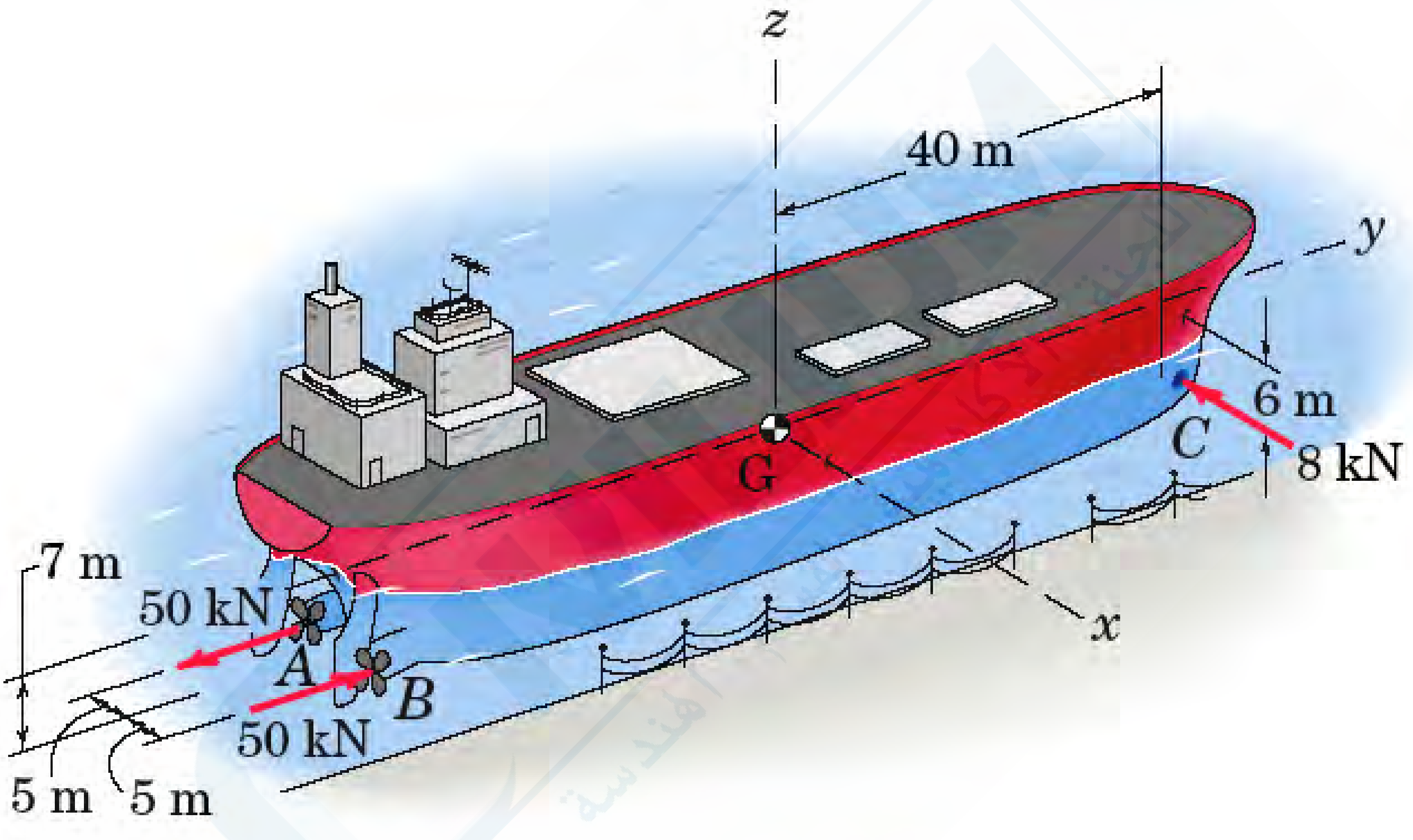
Problem 2/145

P 02 145



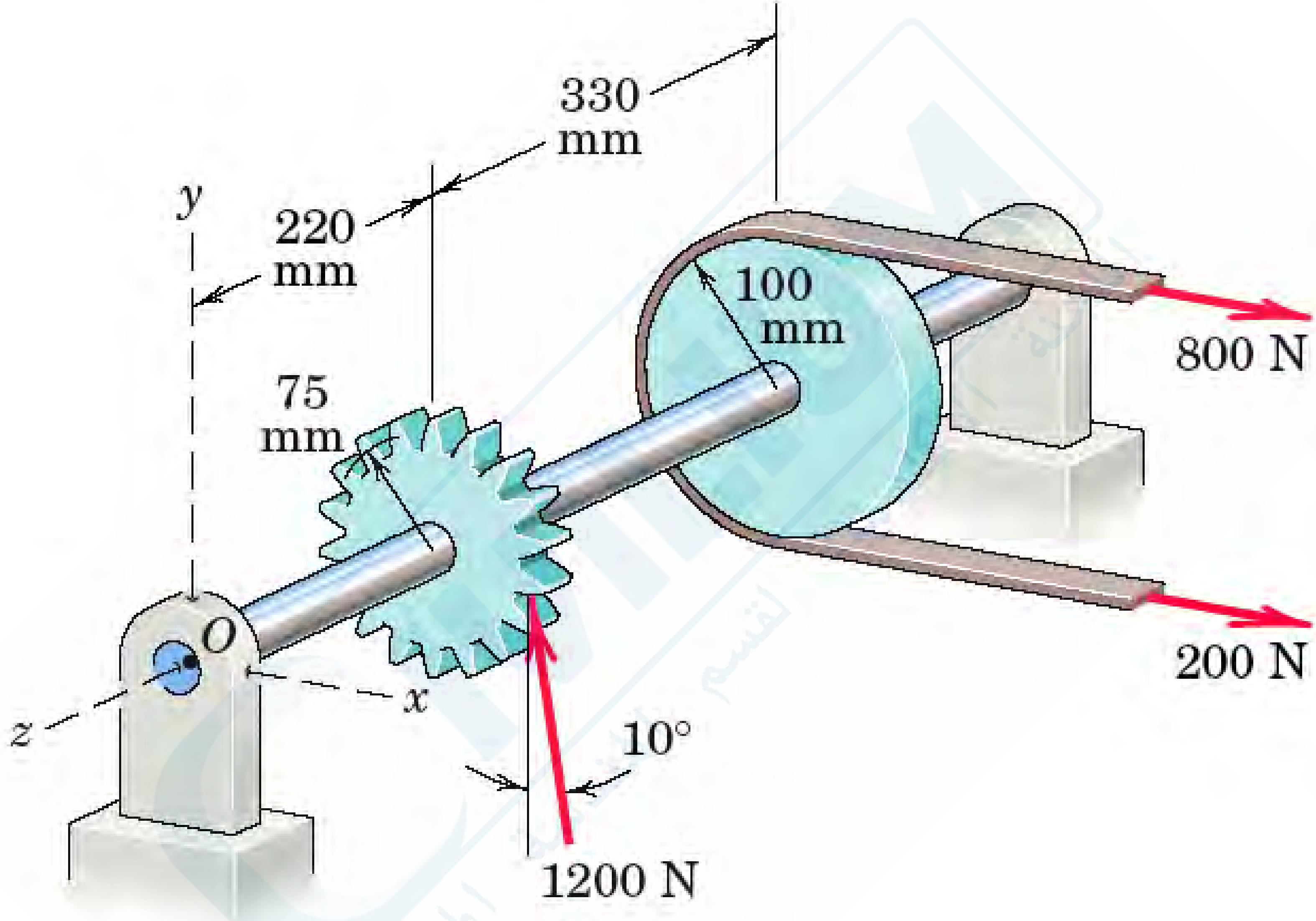
Problem 2/146

P_02_146



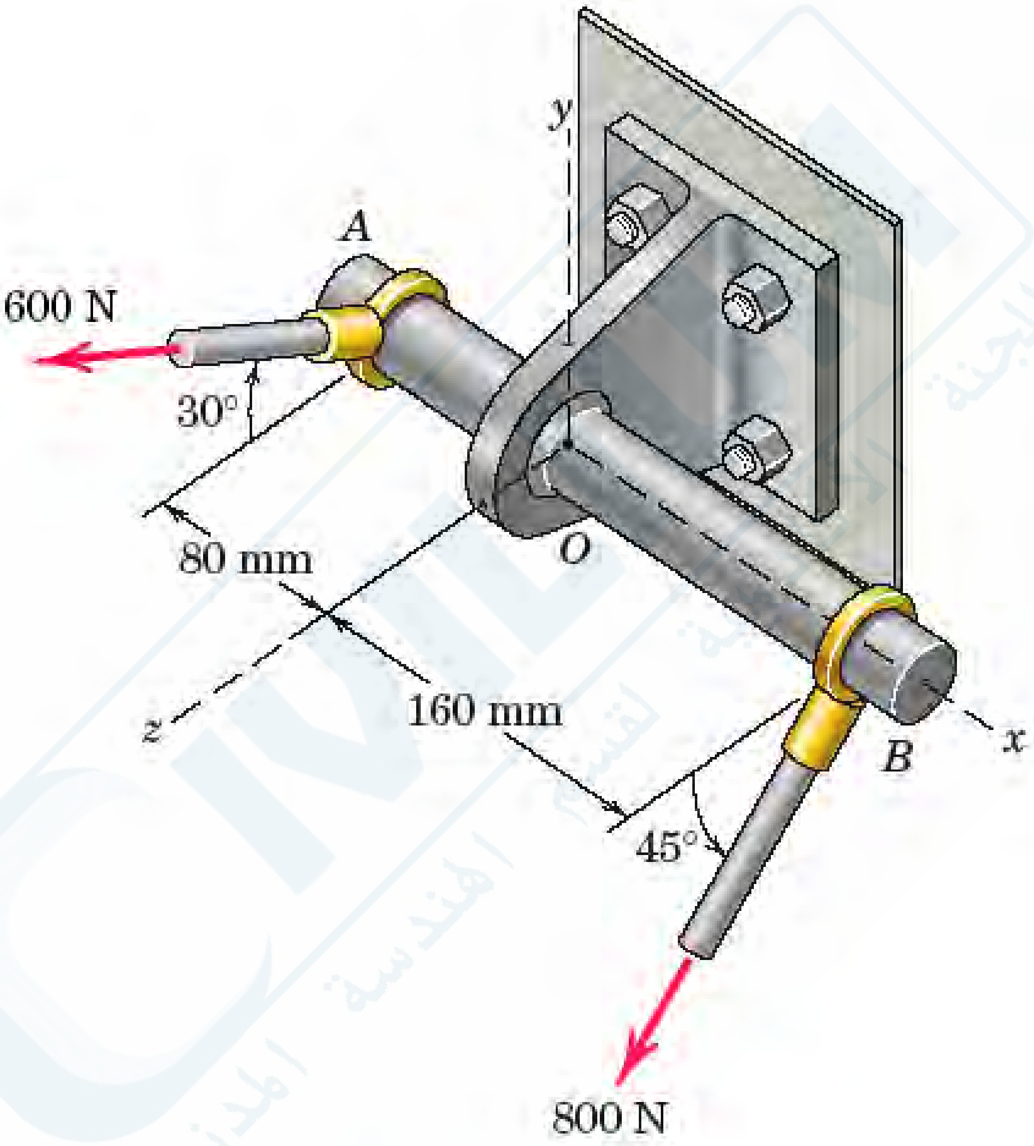
Problem 2/147

P_02_147



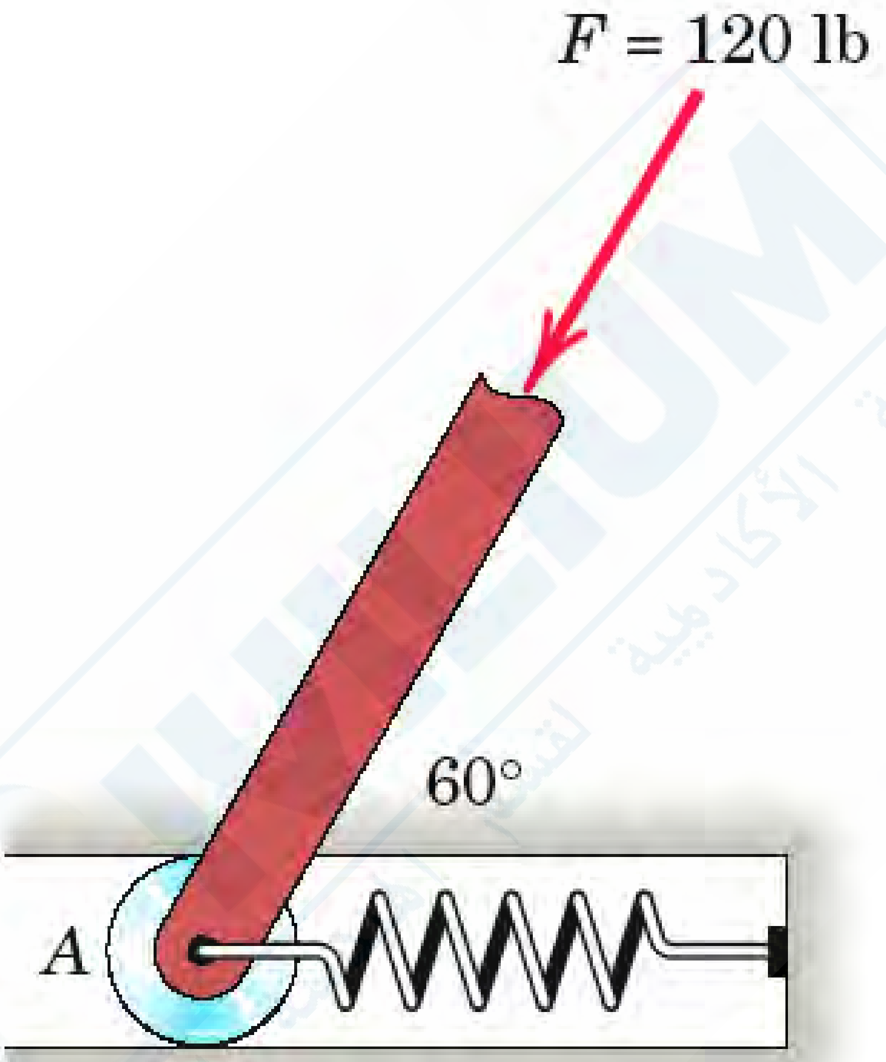
Problem 2/148

P 02 148

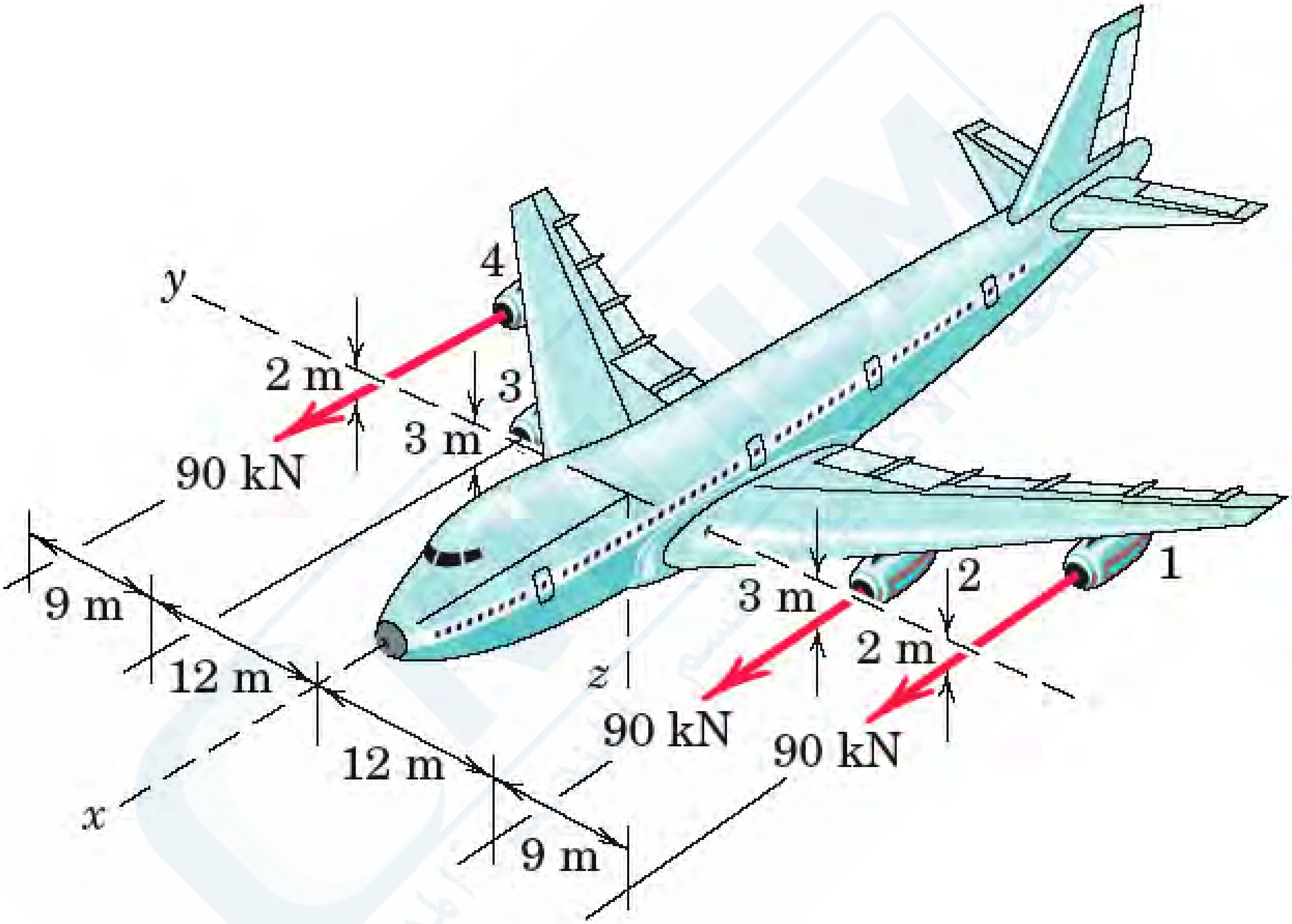


Problem 2/149

P_02_149

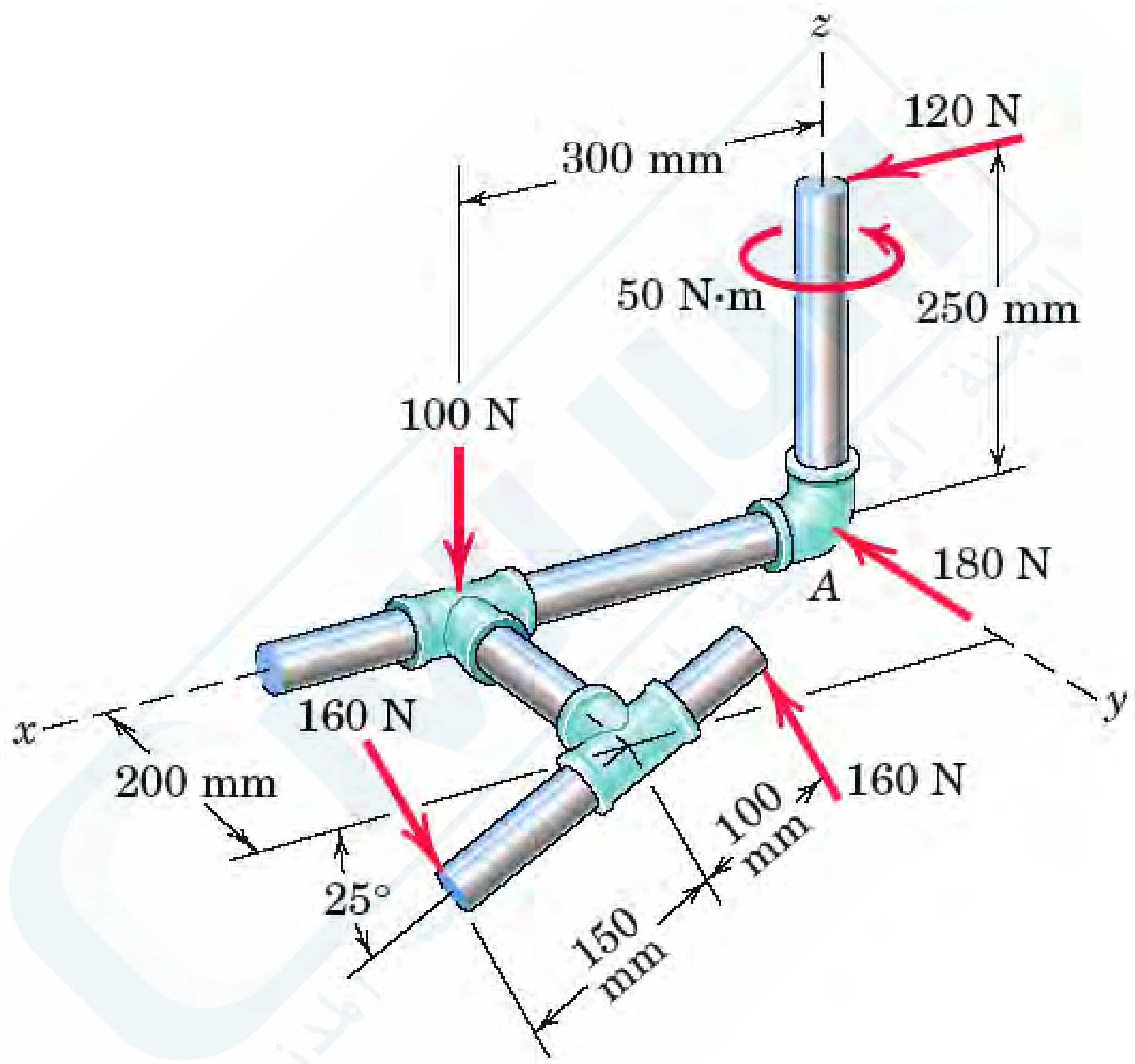


Problem 2/15



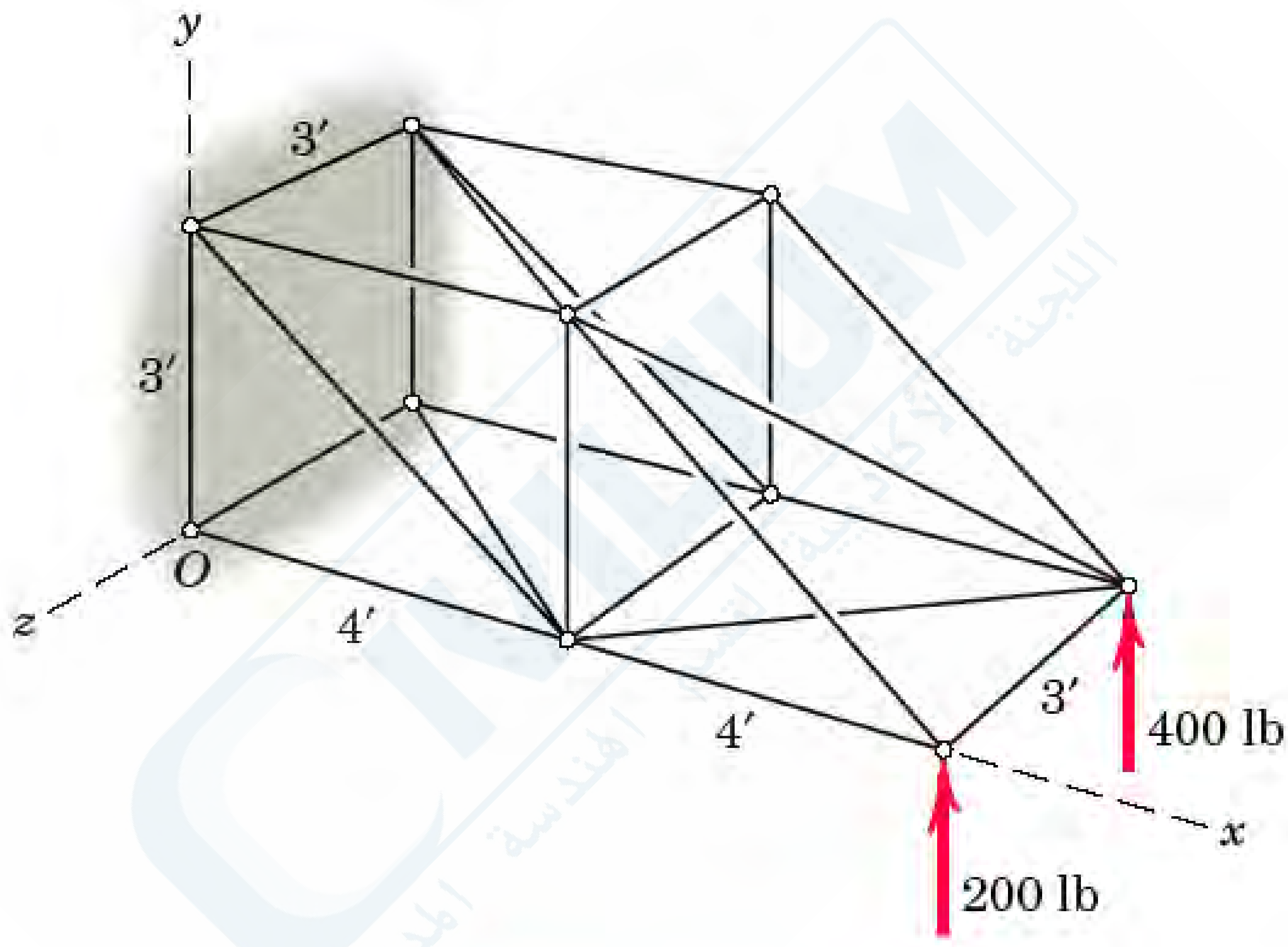
Problem 2/150

P 02 150



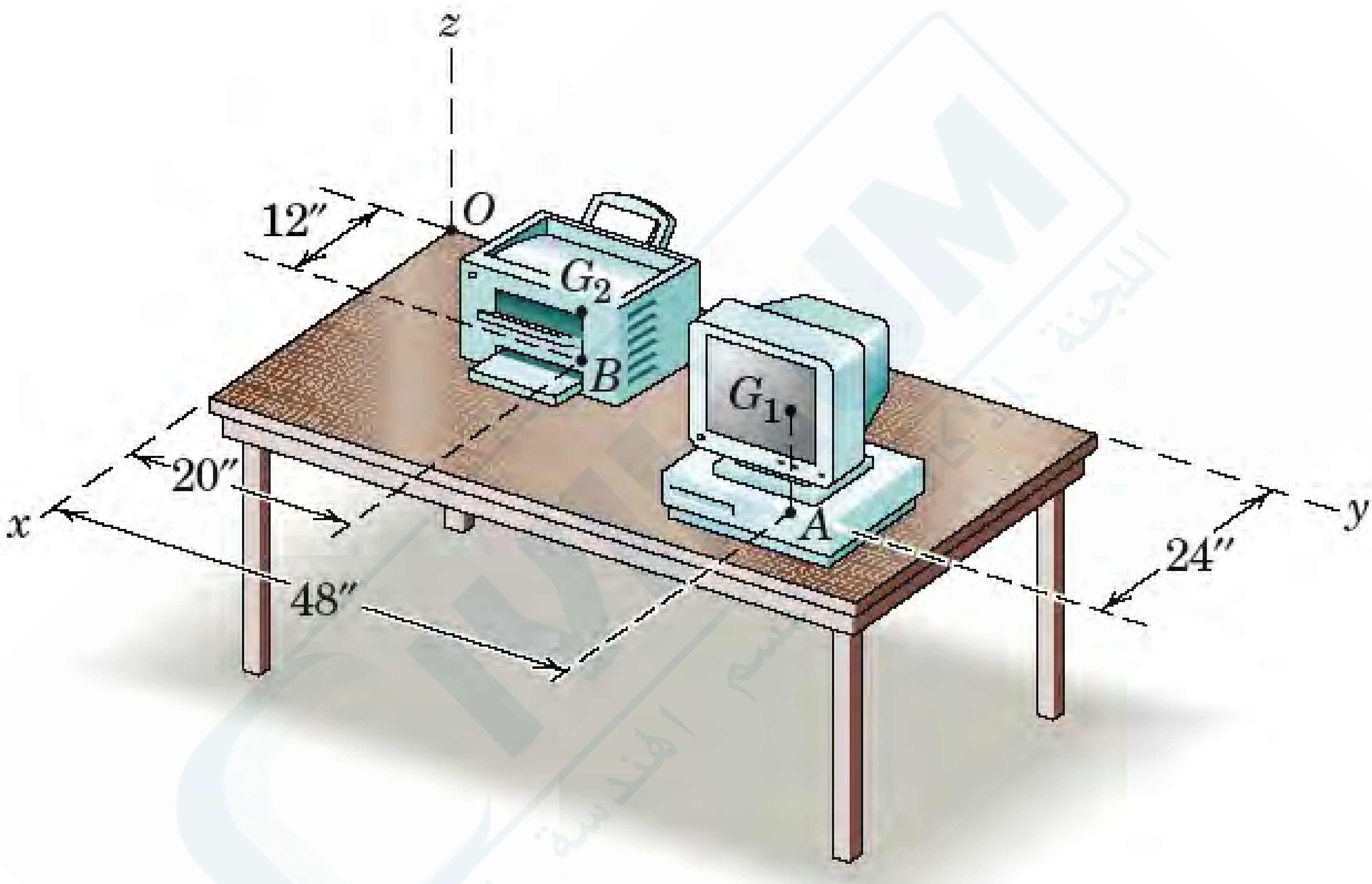
Problem 2/151

P 02 151



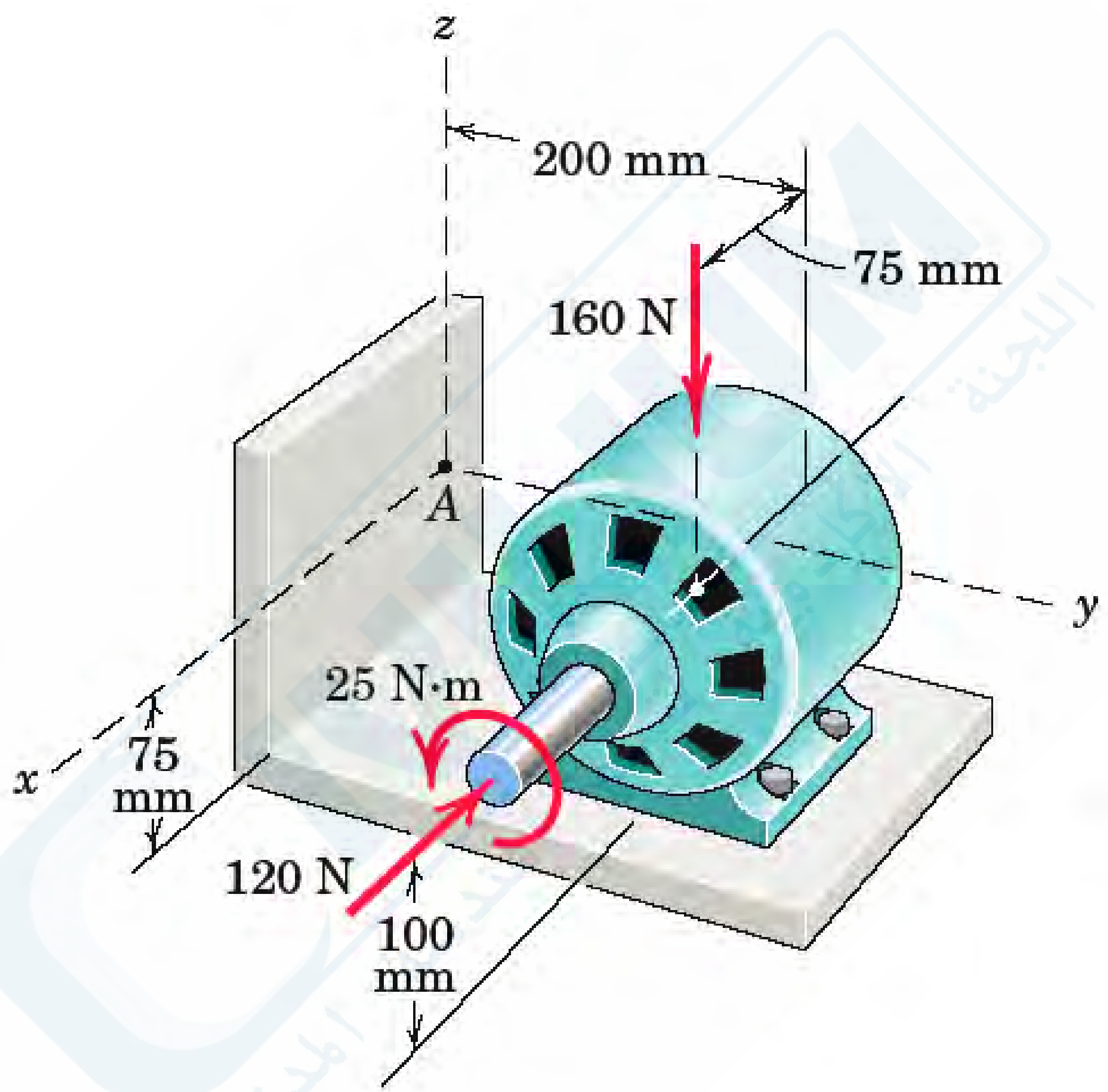
Problem 2/152

P 02 152



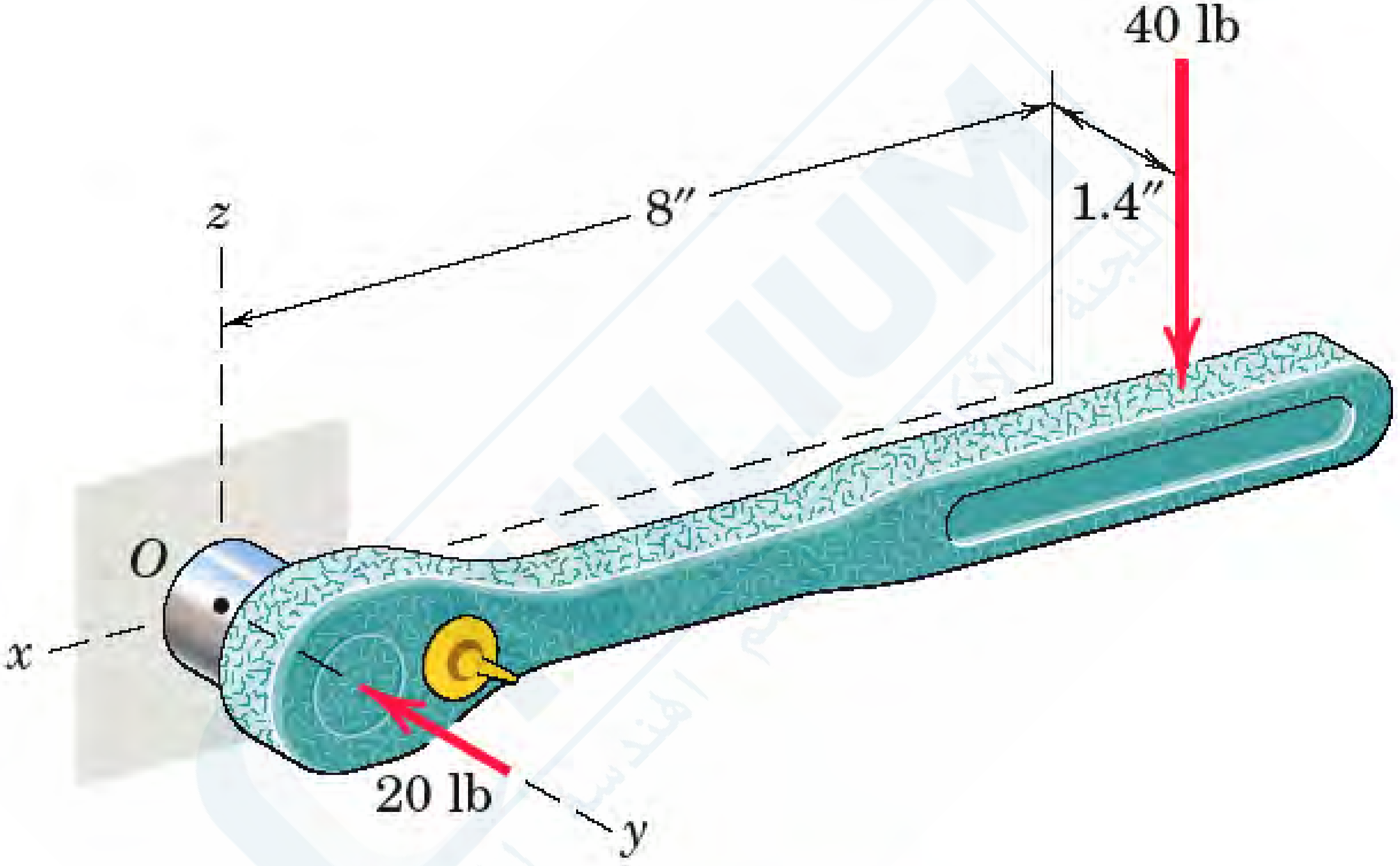
Problem 2/153

P 02 153



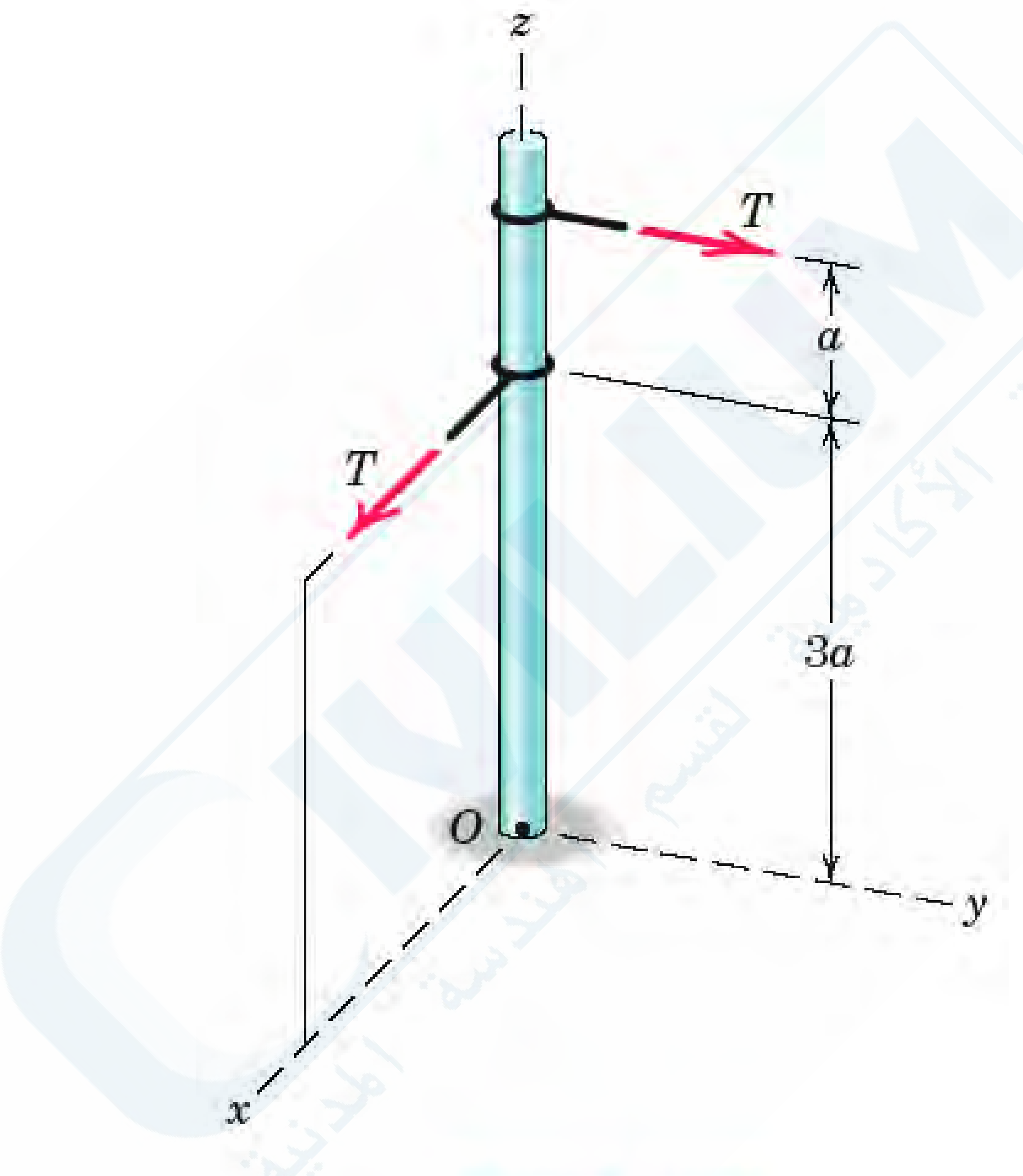
Problem 2/154

P 02 154



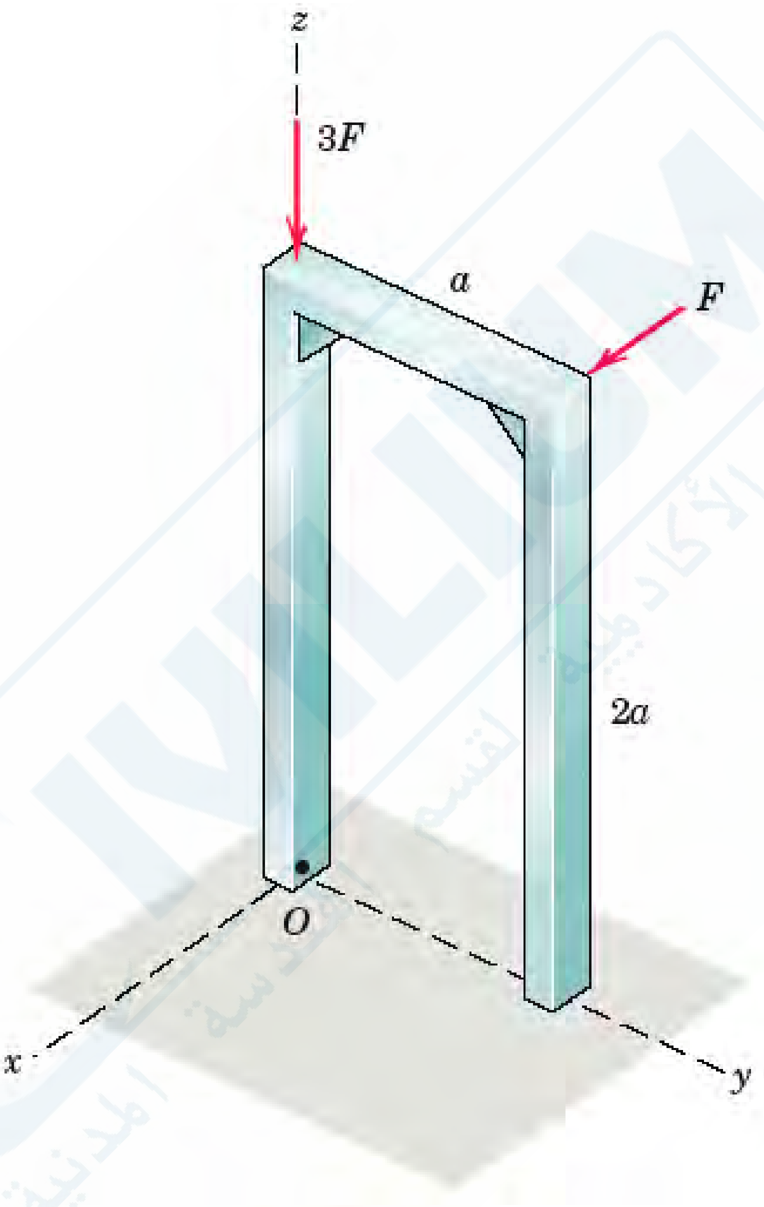
Problem 2/155

P_02_155



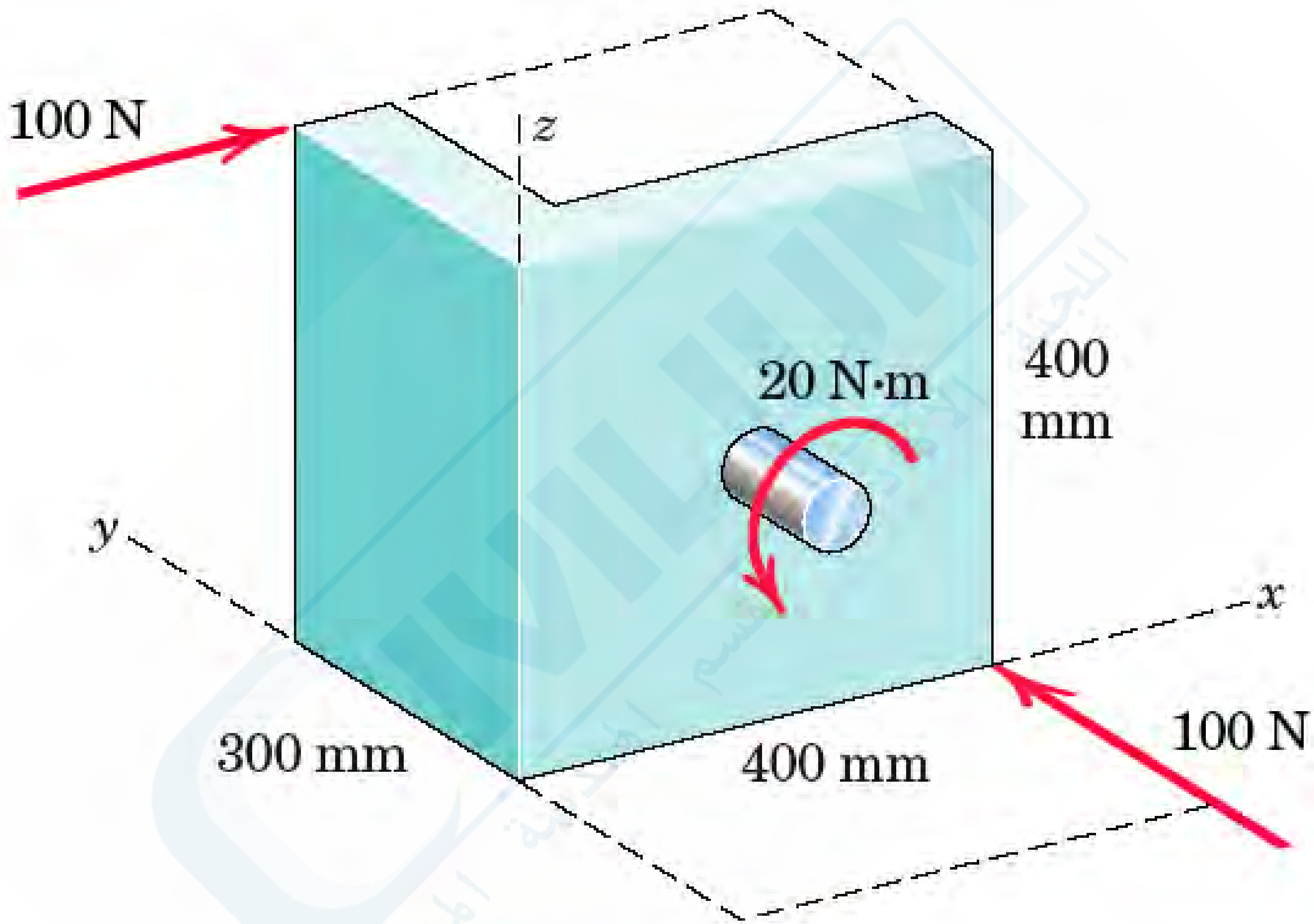
Problem 2/156

P 02 156



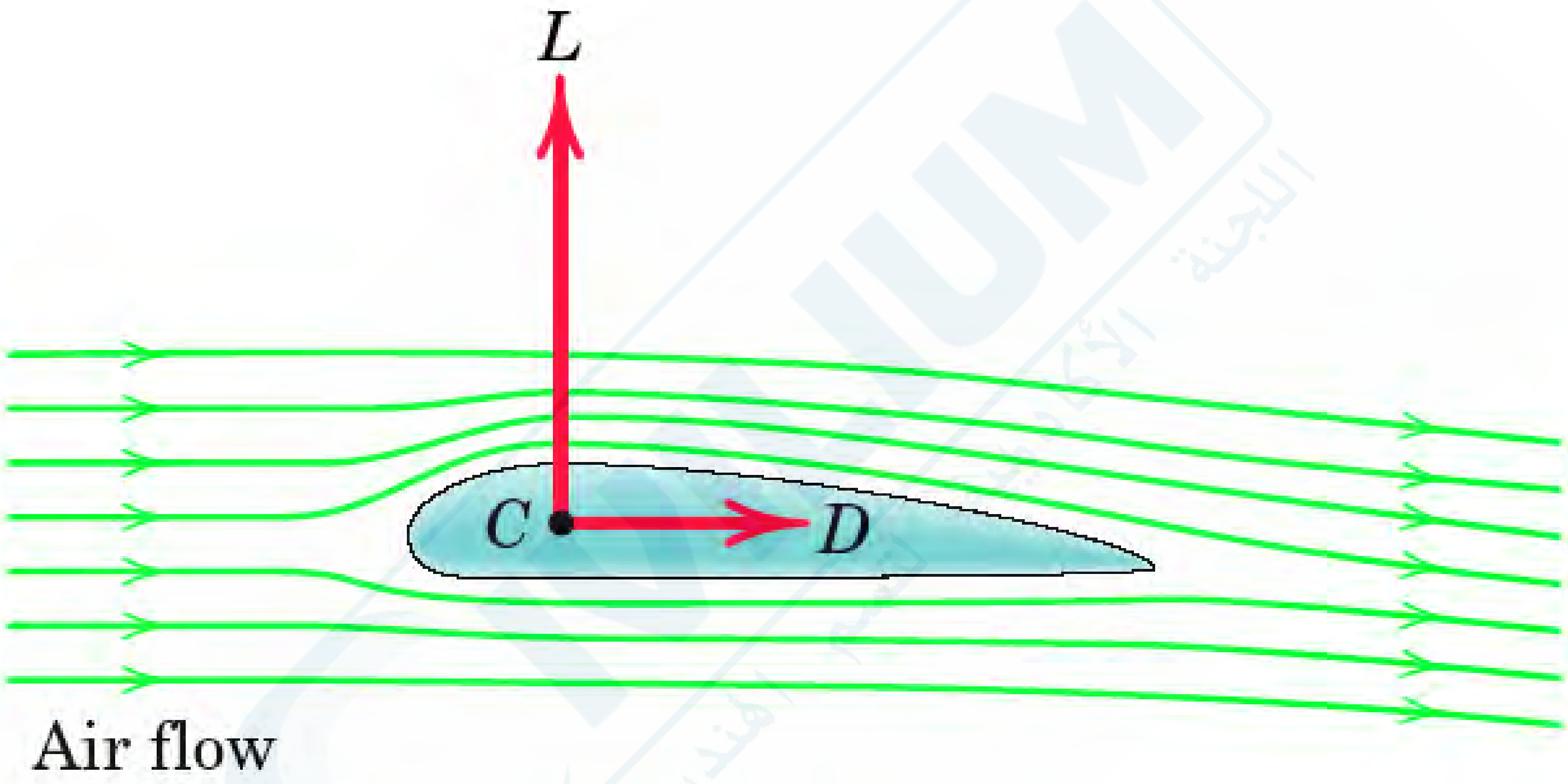
Problem 2/157

P_02_157



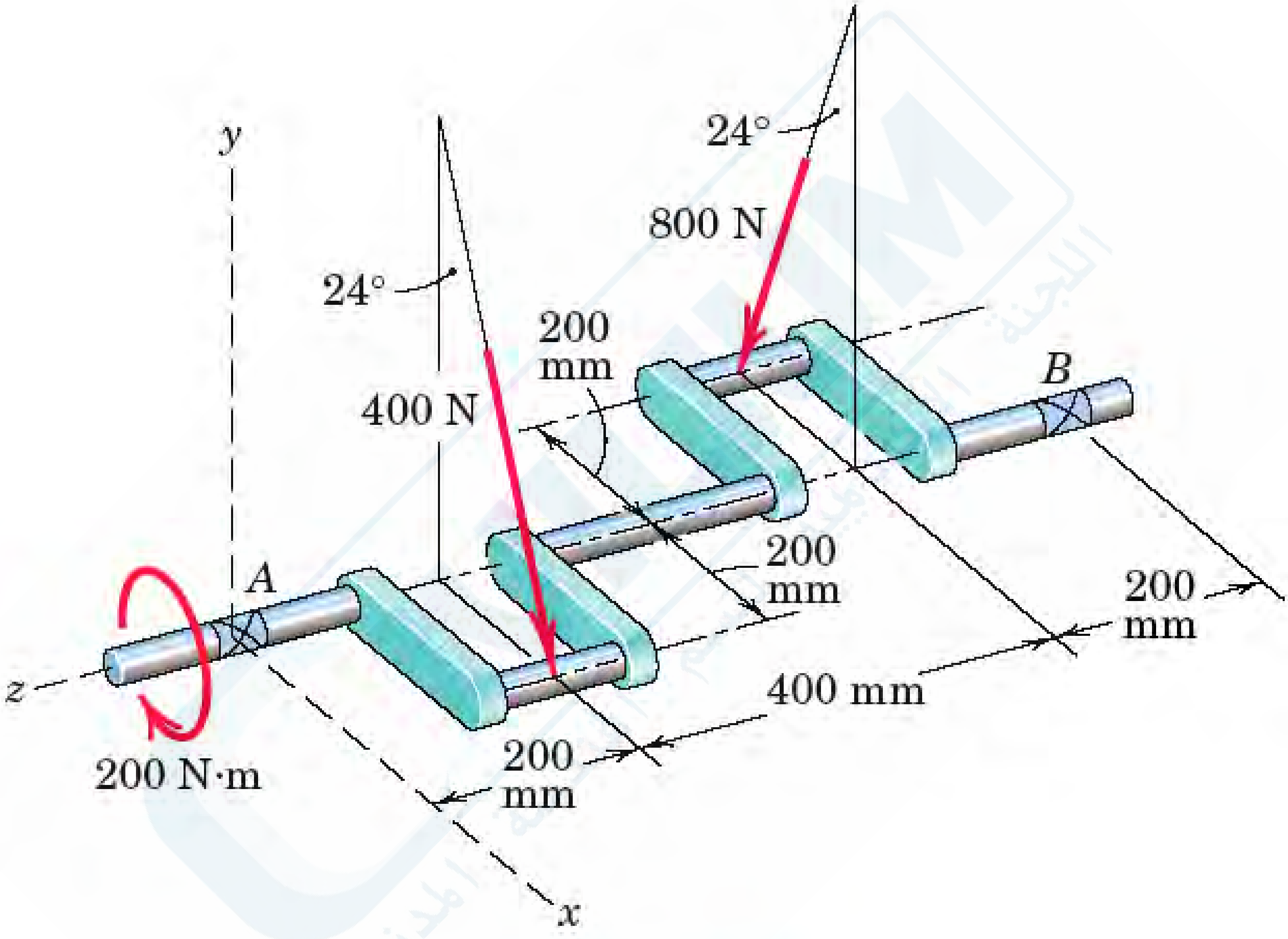
Problem 2/159

P_02_159



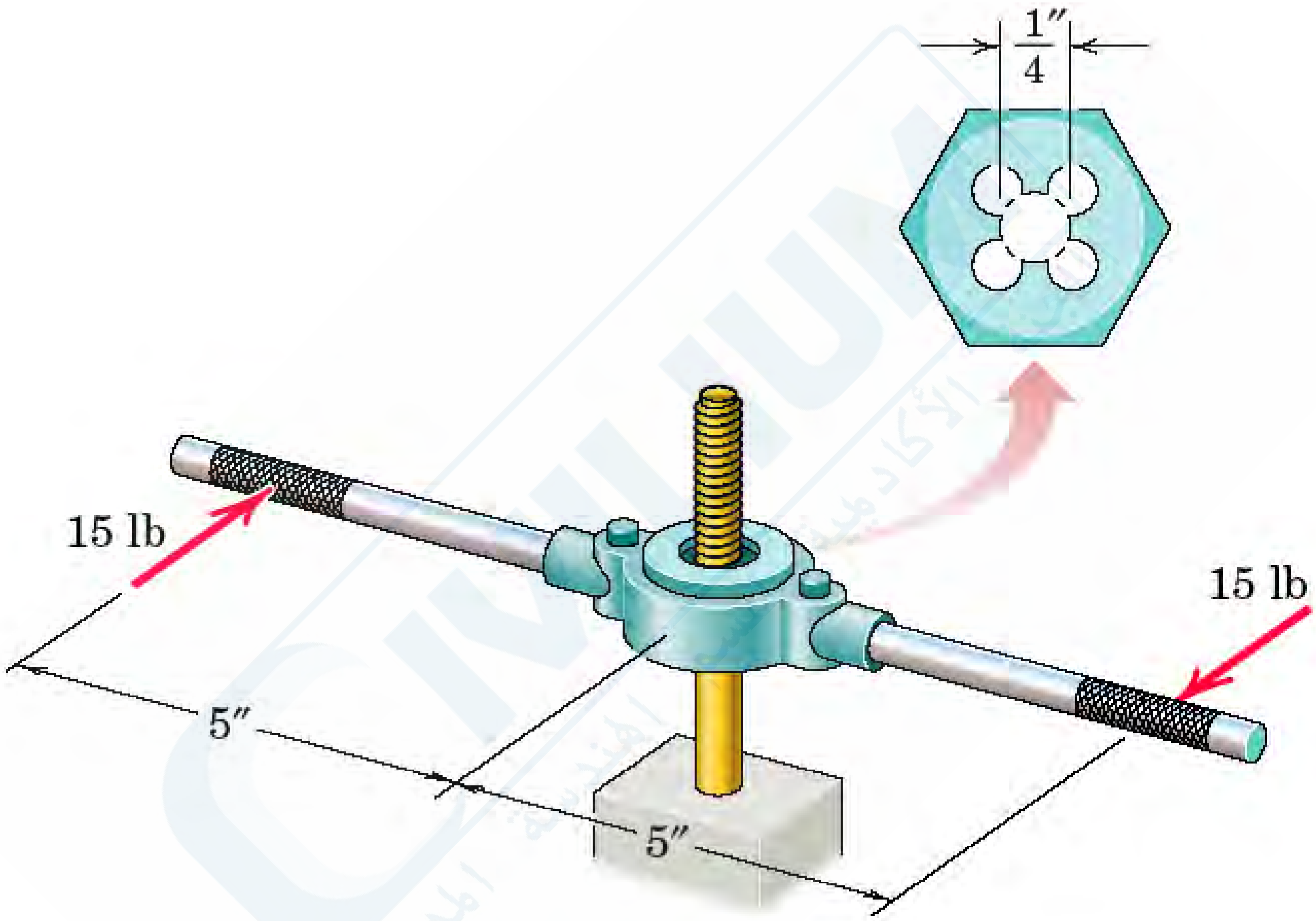
Problem 2/16

P_02_16



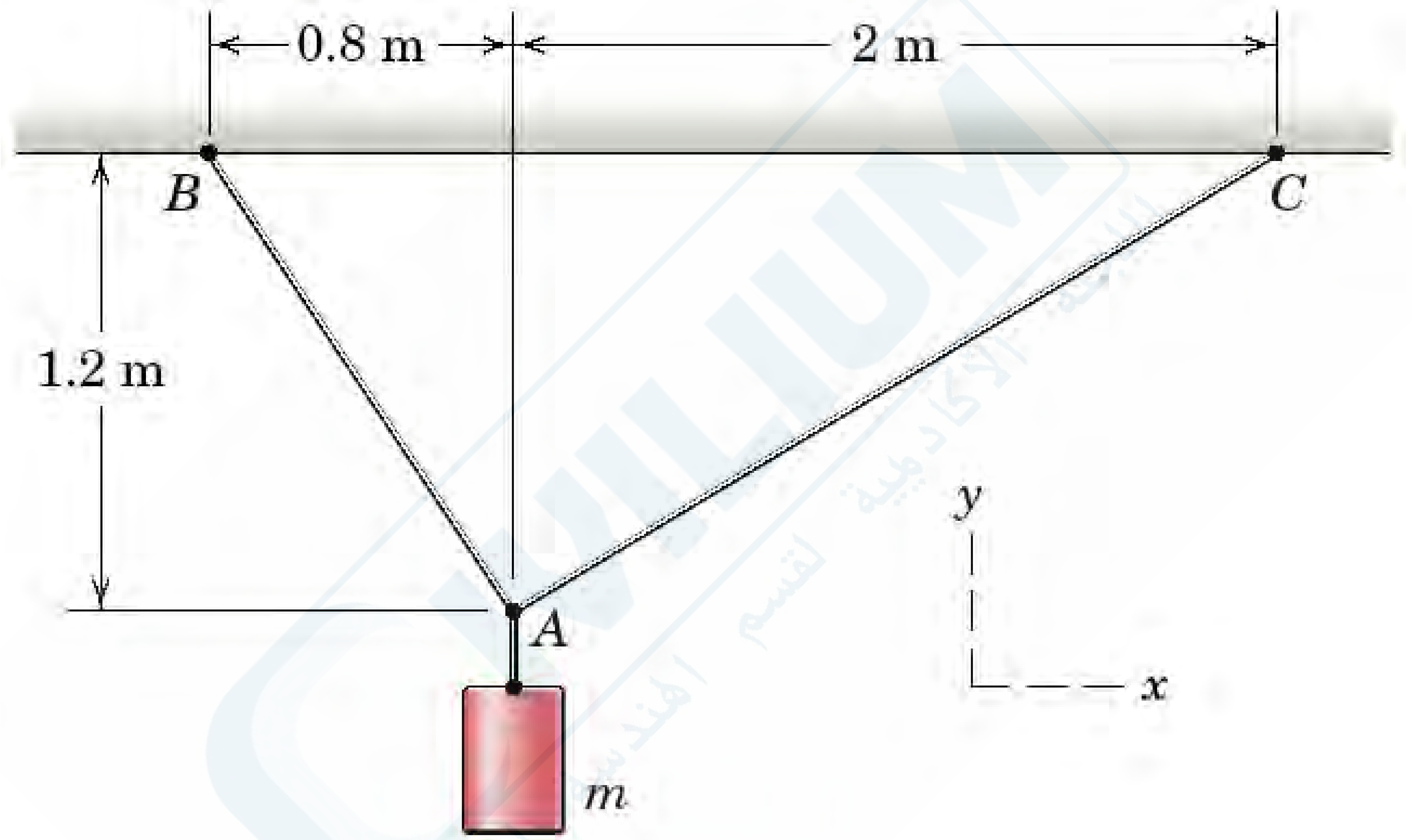
Problem 2/160

P 02 160



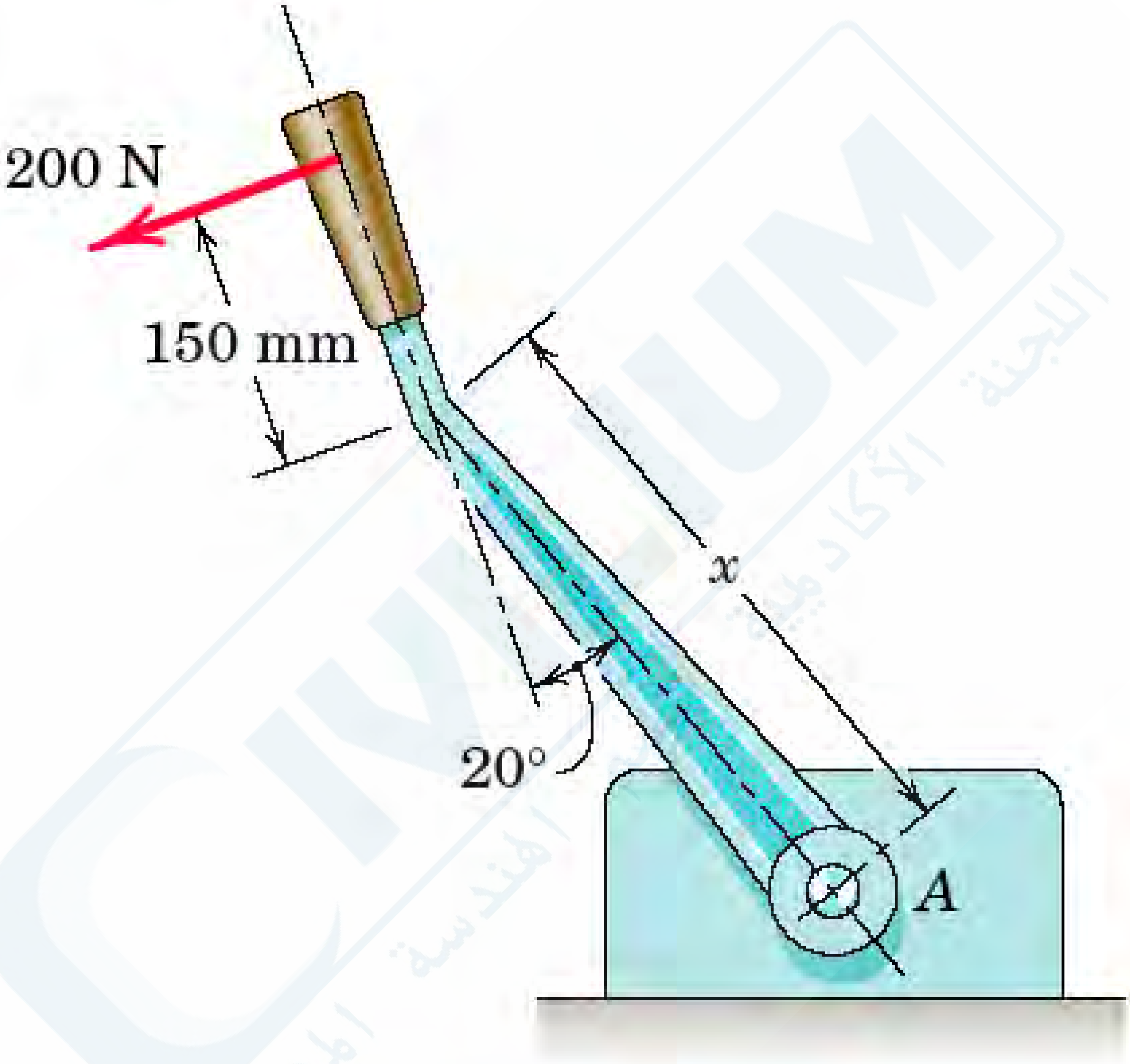
Problem 2/161

P 02 161



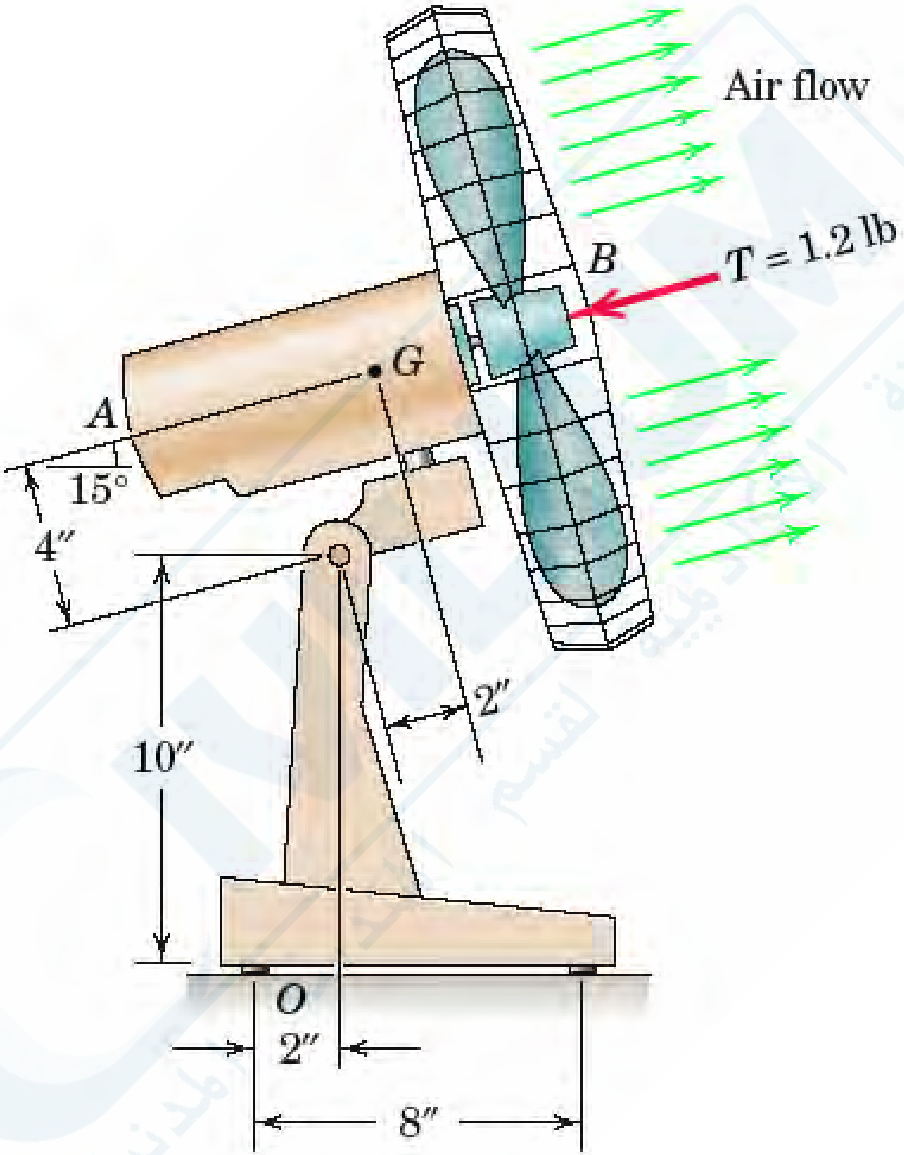
Problem 2/162

P_02_162



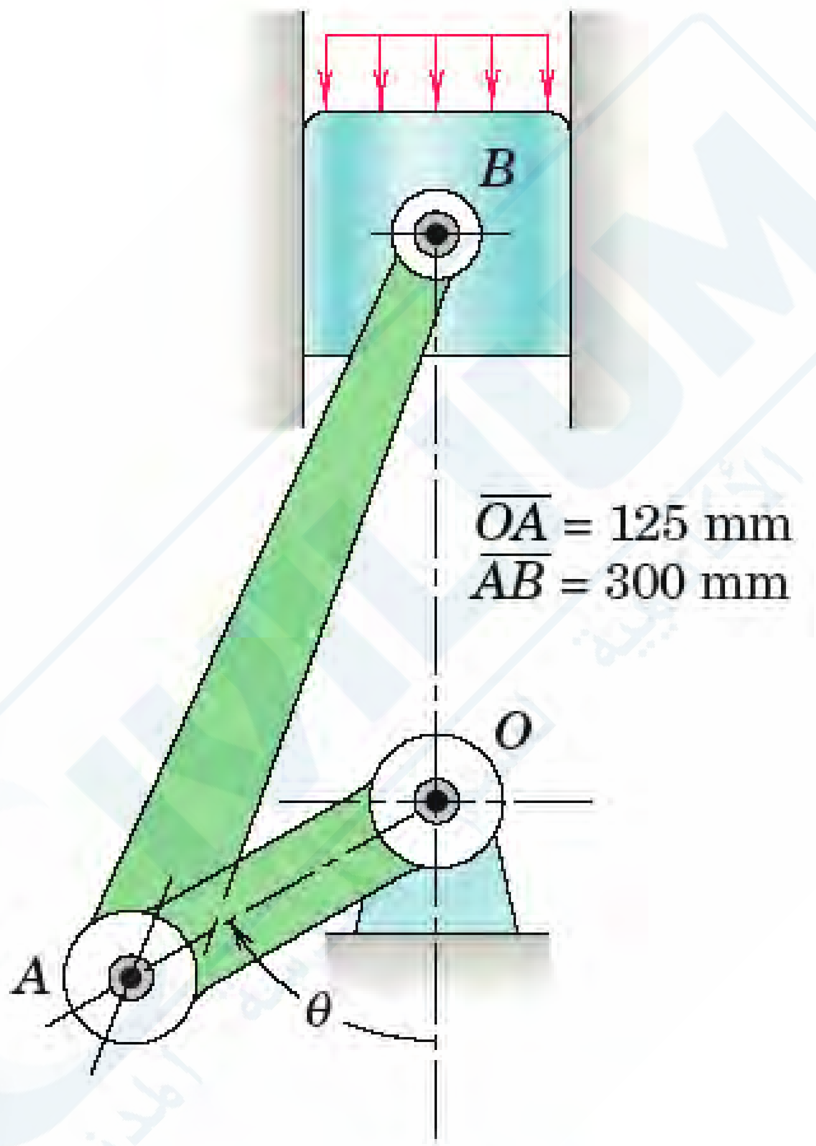
Problem 2/163

P 02 163



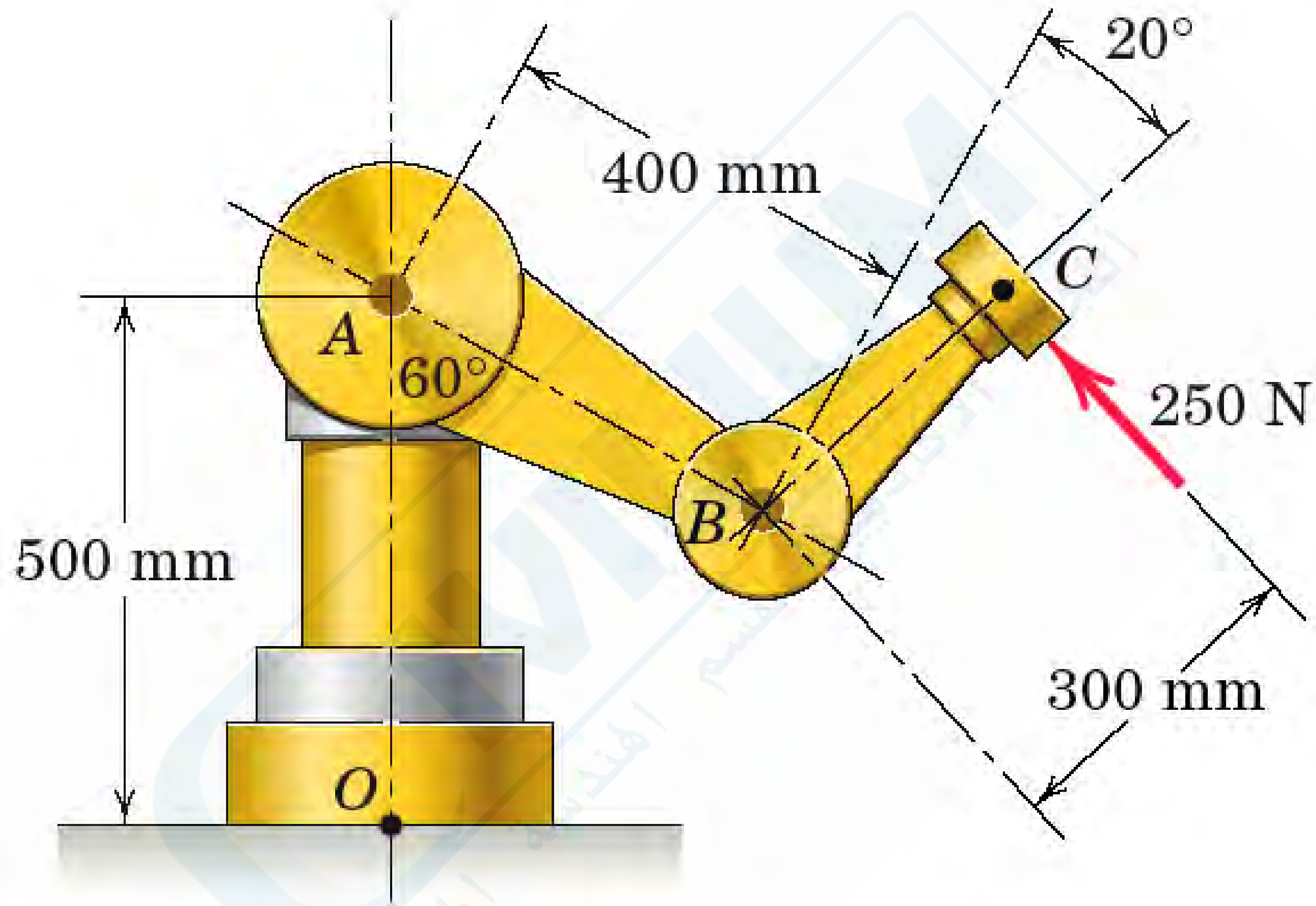
Problem 2/164

P 02 164



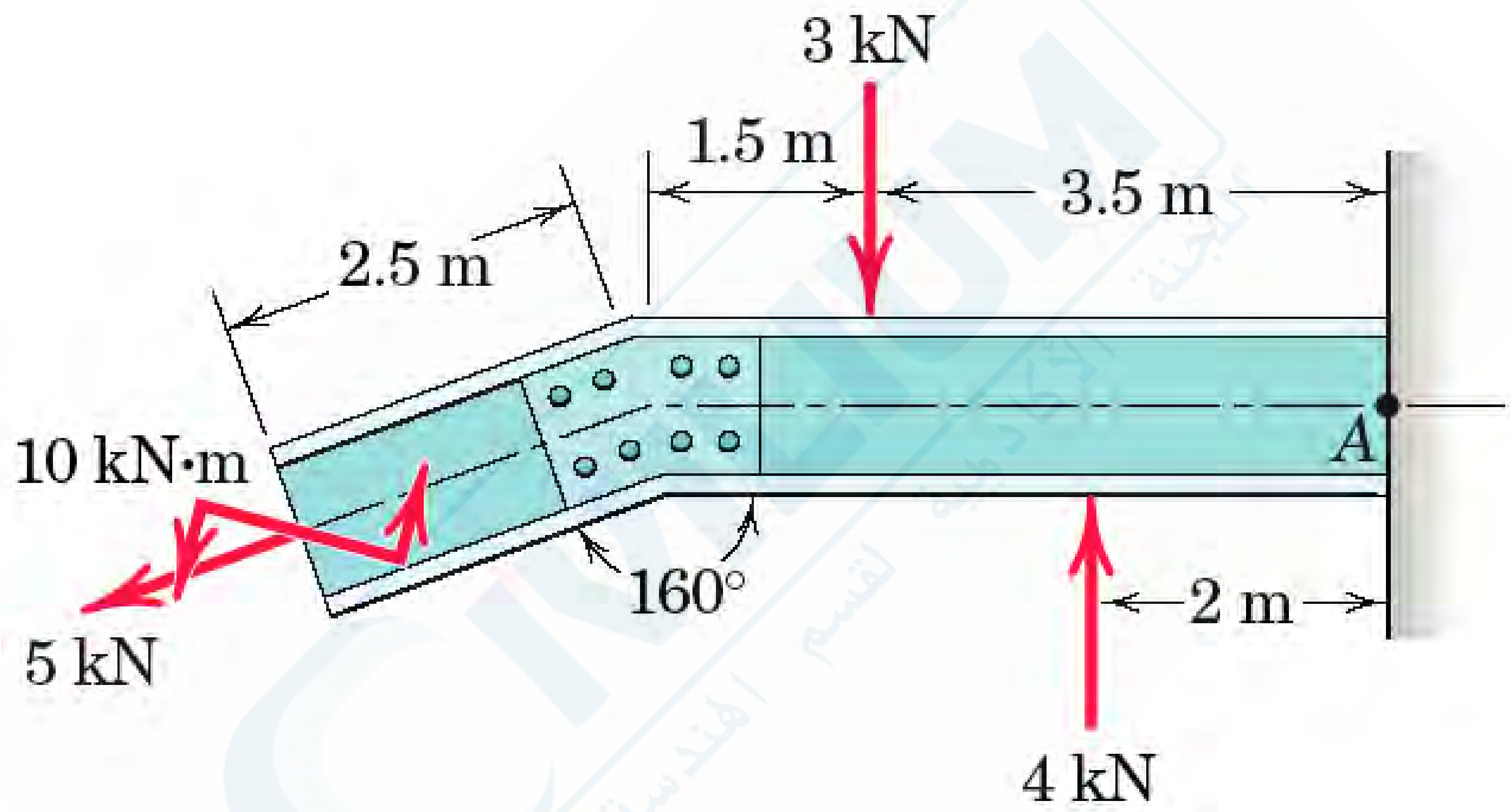
Problem 2/165

P_02_165



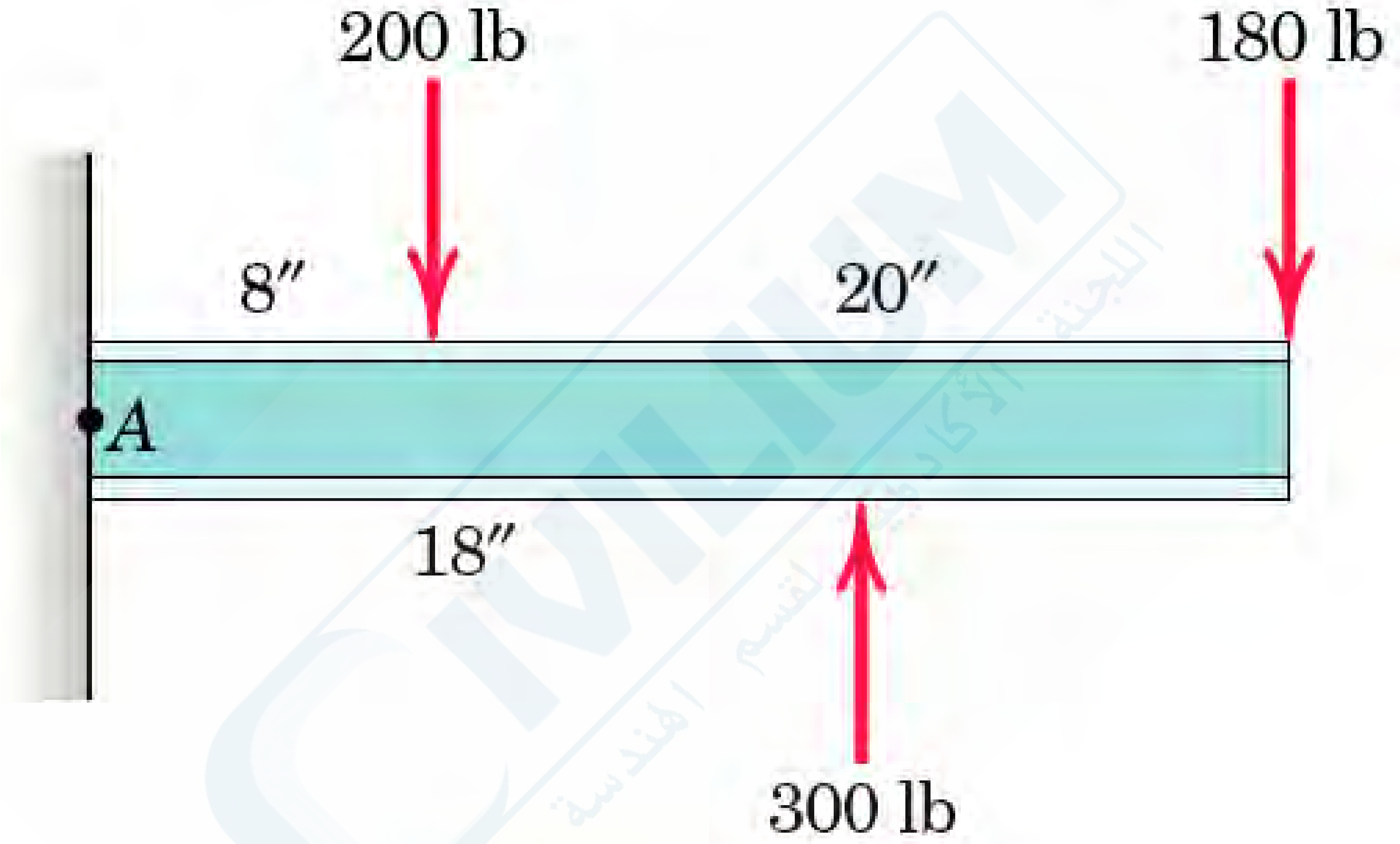
Problem 2/166

P 02 166



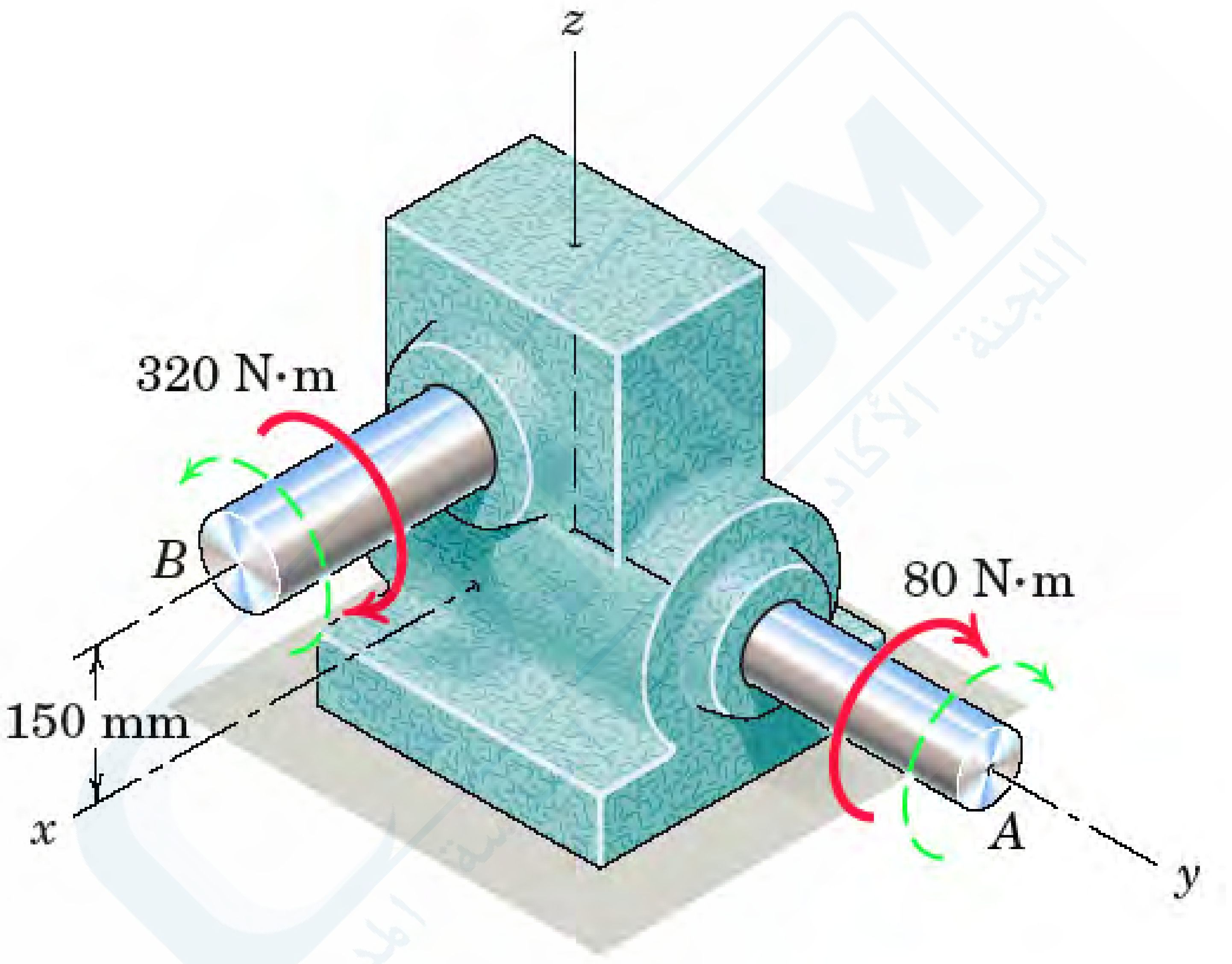
Problem 2/167

P_02_167



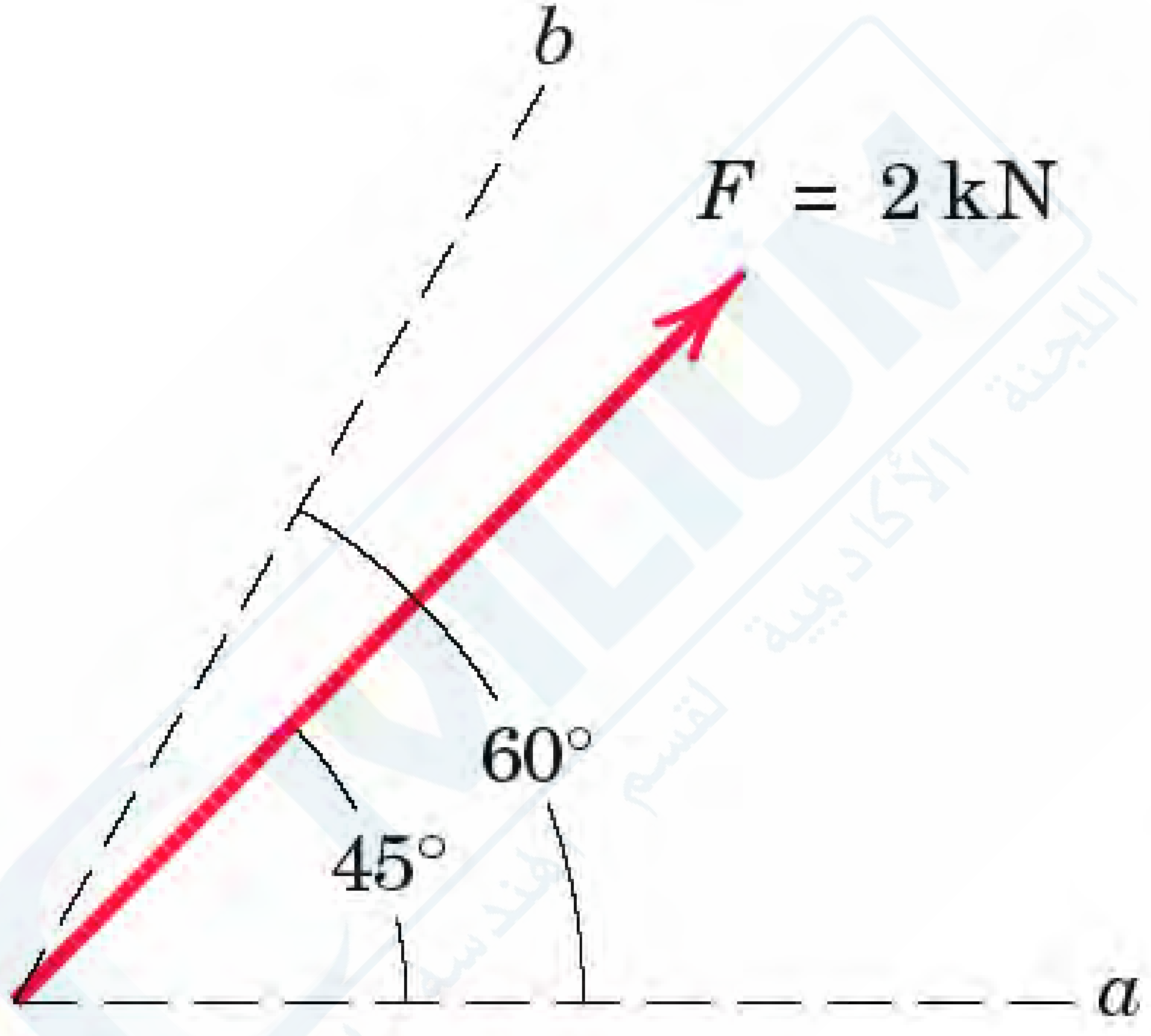
Problem 2/168

P_02_168

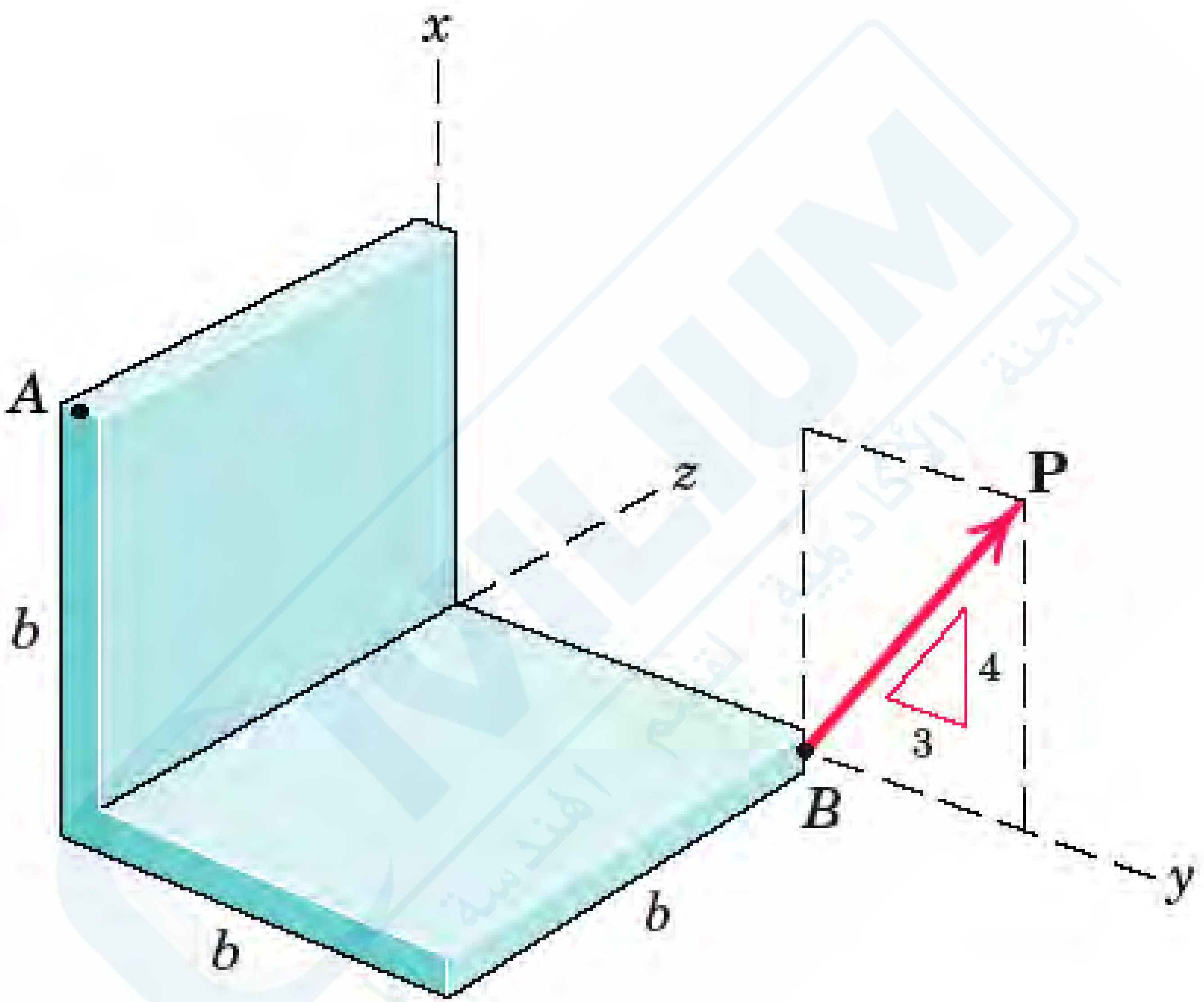


Problem 2/169

P 02 169

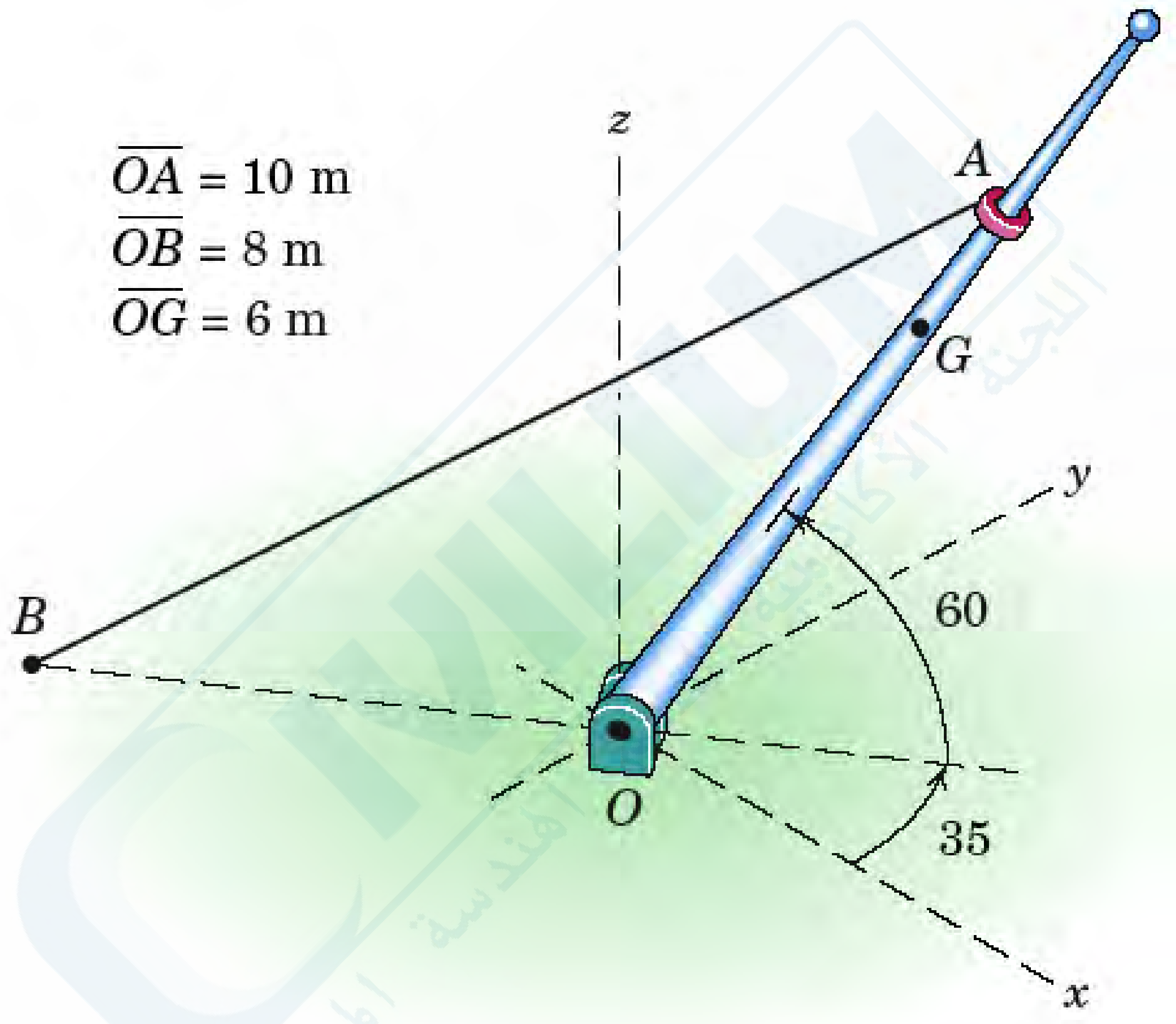


Problem 2/17



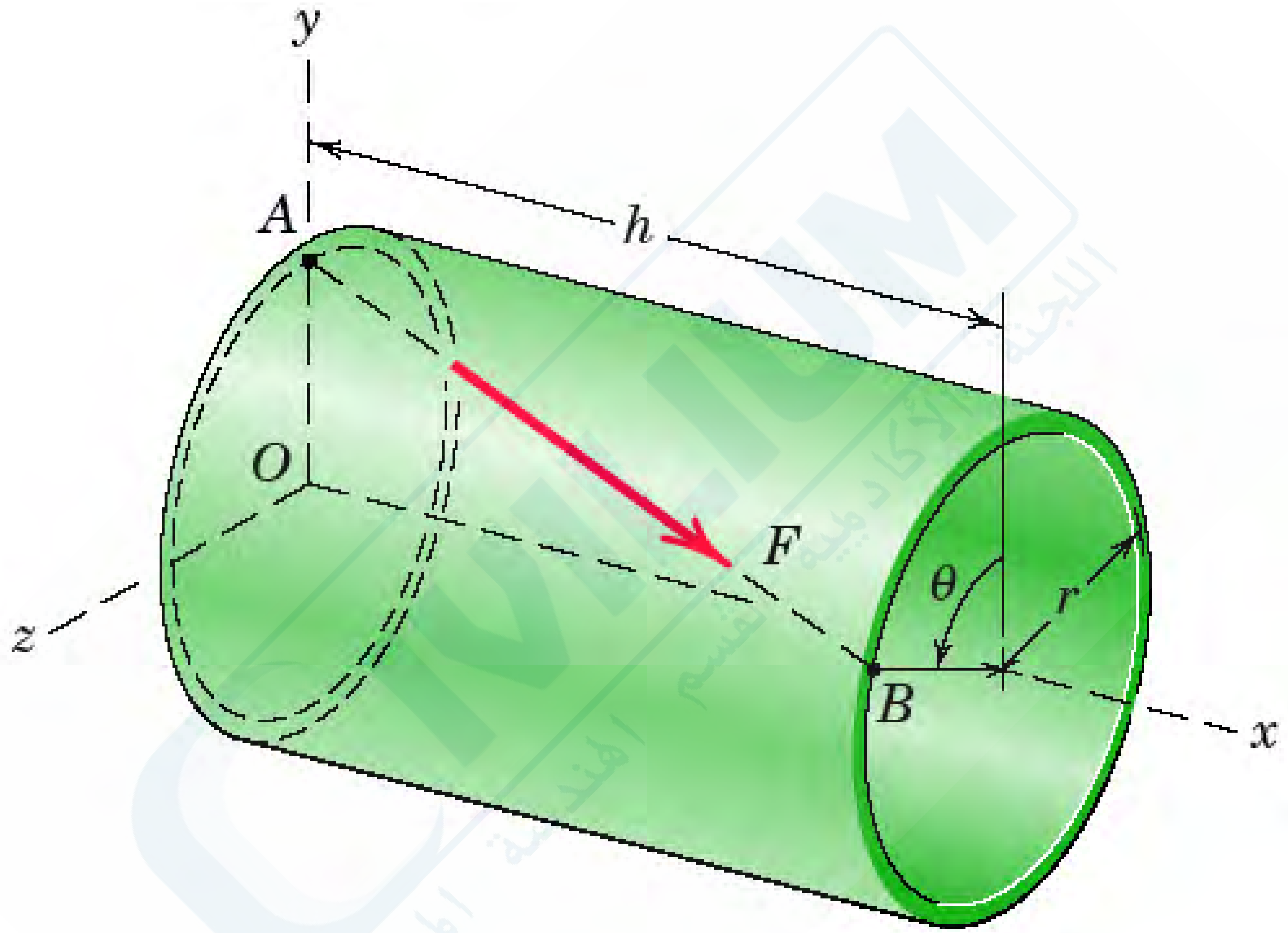
Problem 2/170

P 02 170



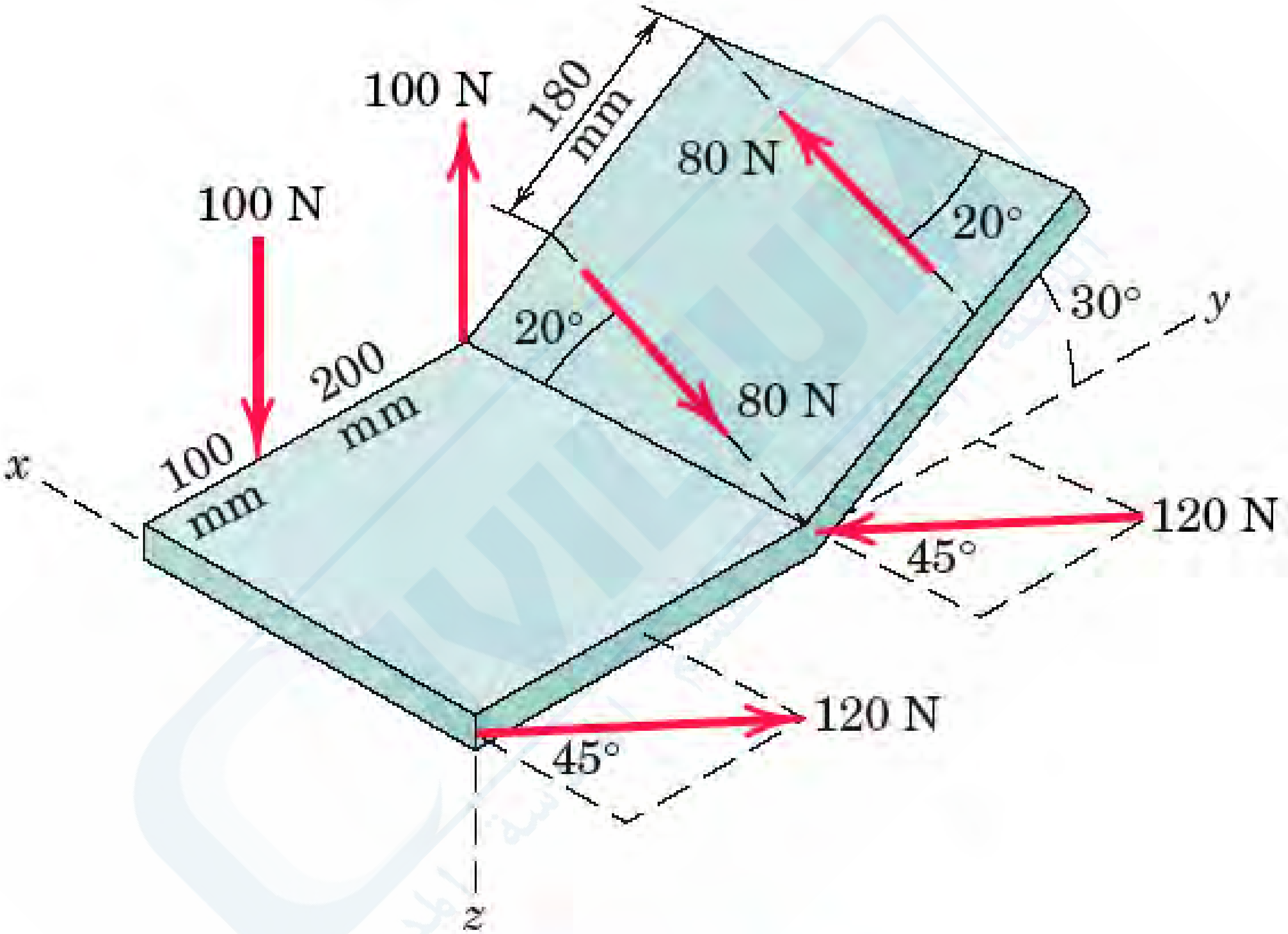
Problem 2/171

P_02_171

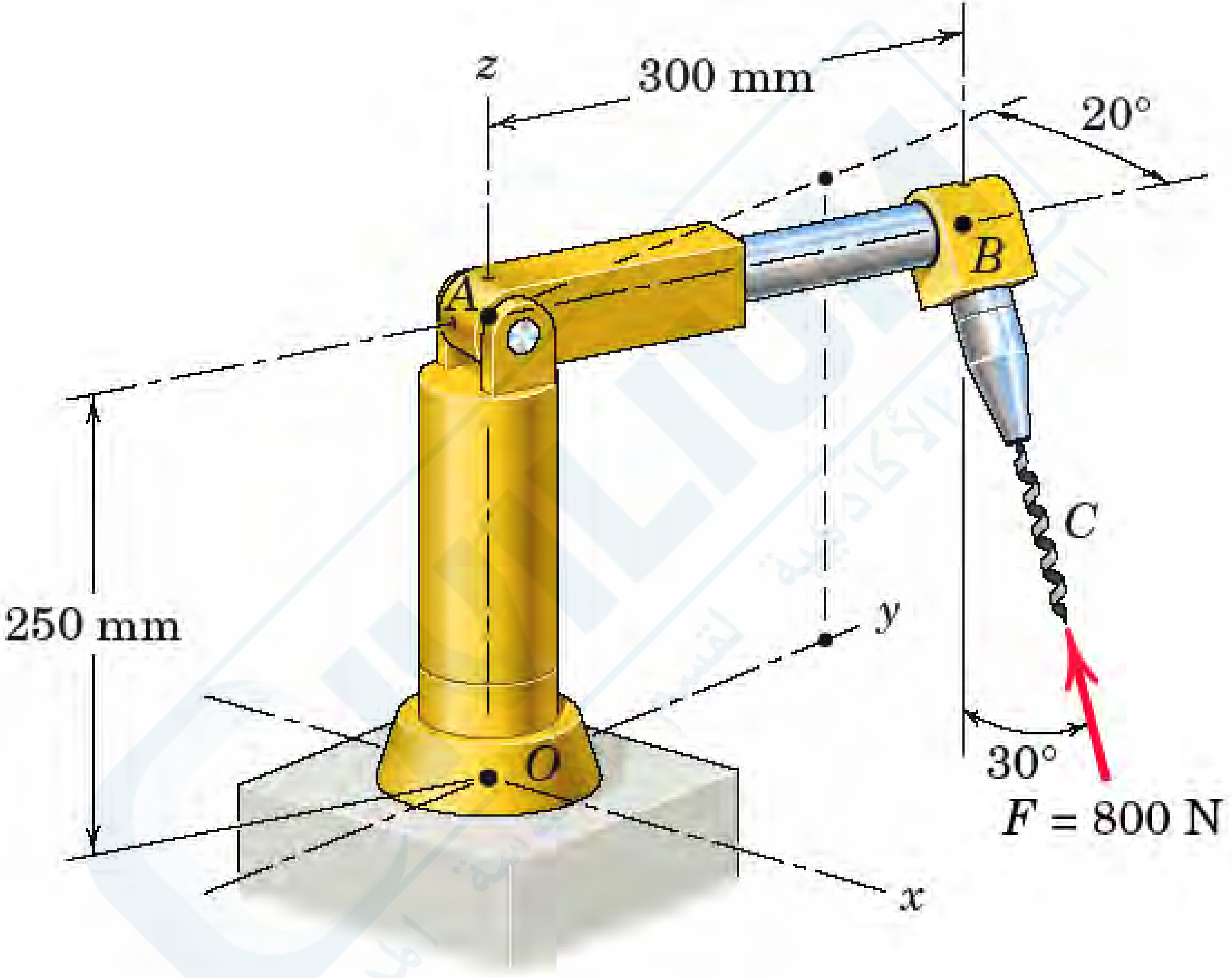


Problem 2/172

P 02 172

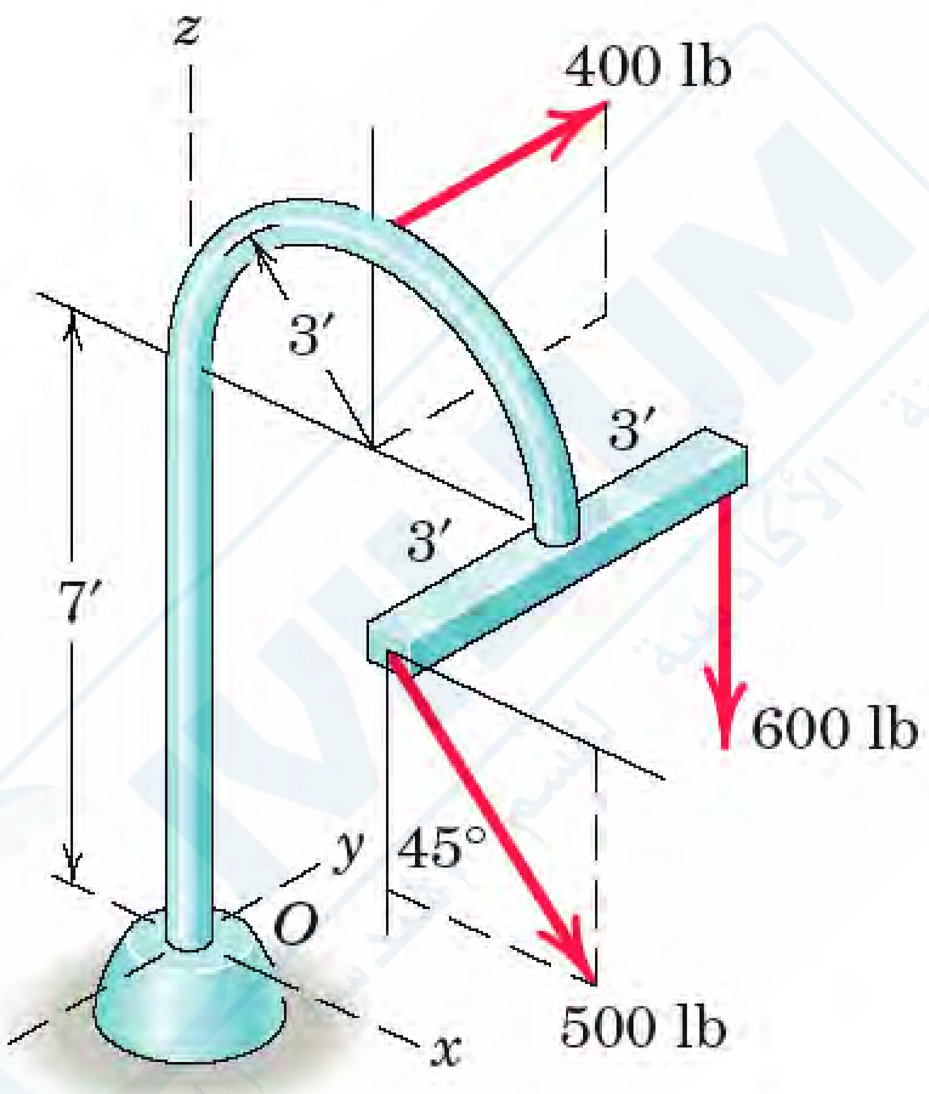


Problem 2/173



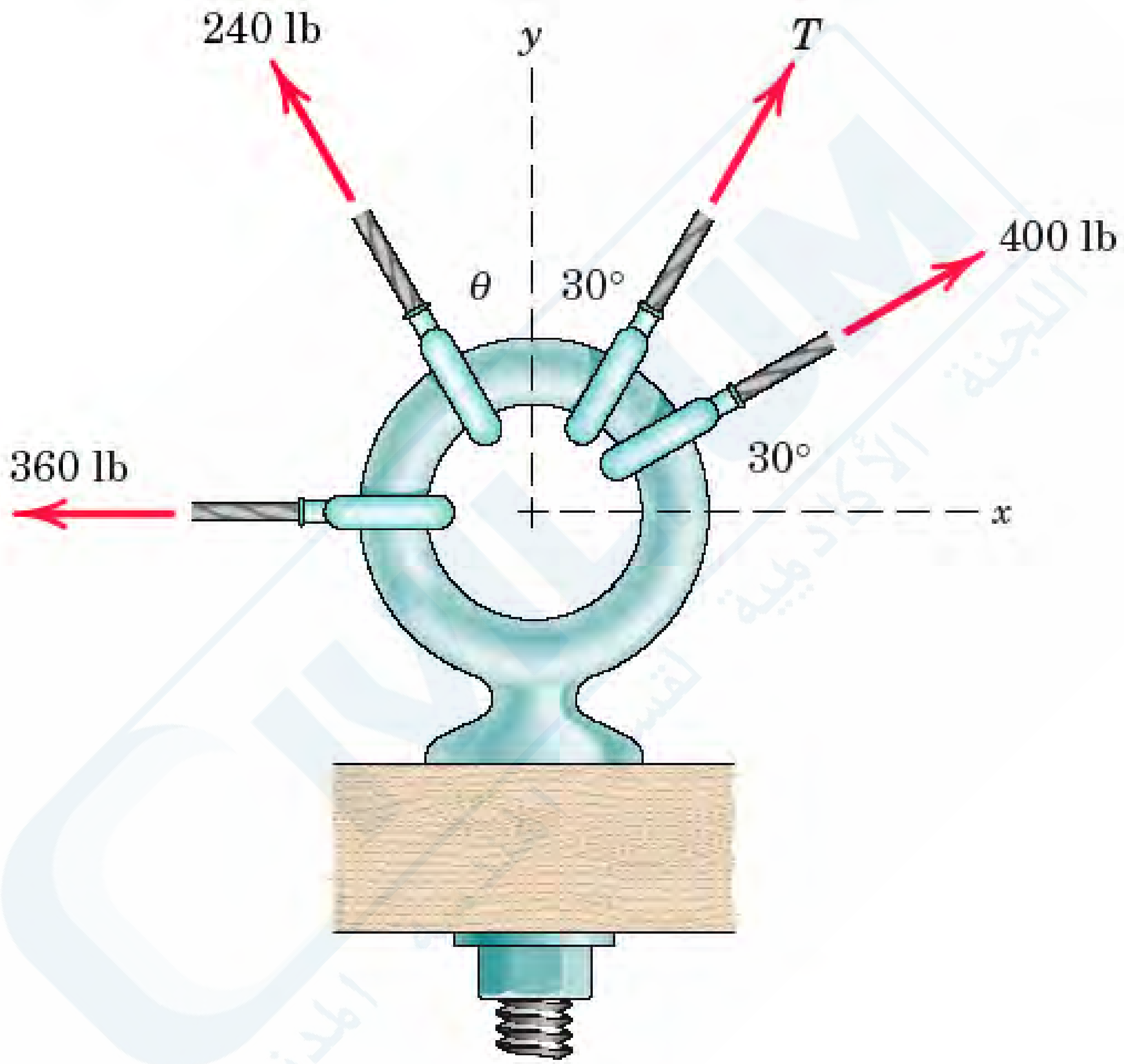
Problem 2/174

P 02 174



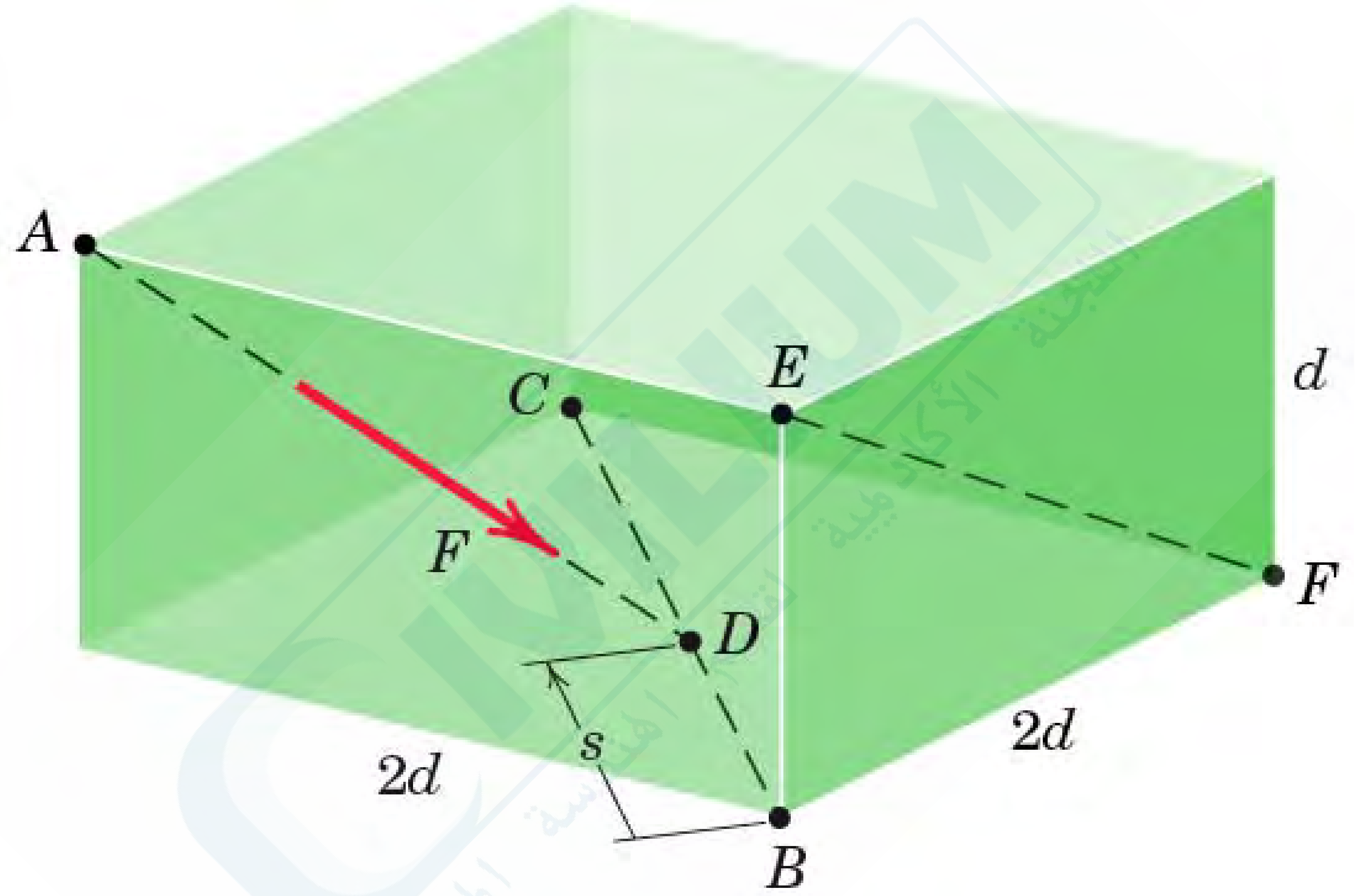
Problem 2/175

P_02_175



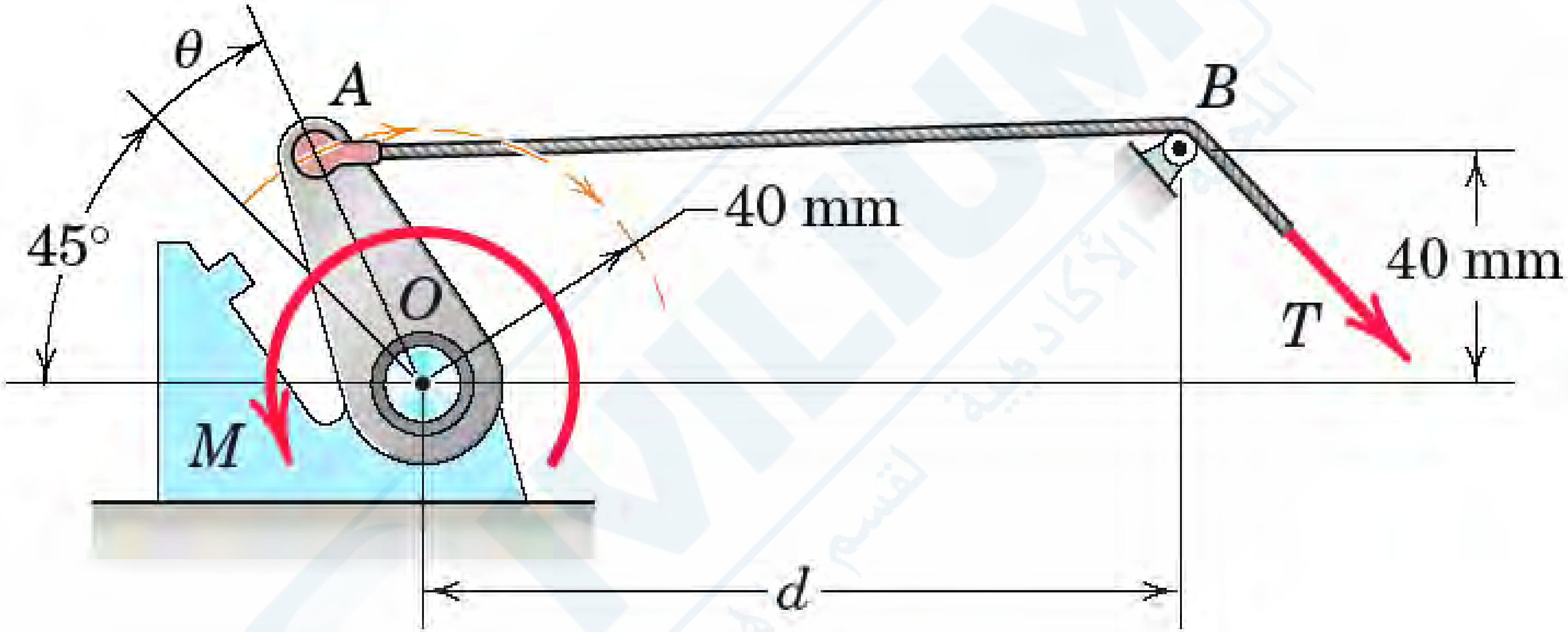
Problem 2/176

P 02 176



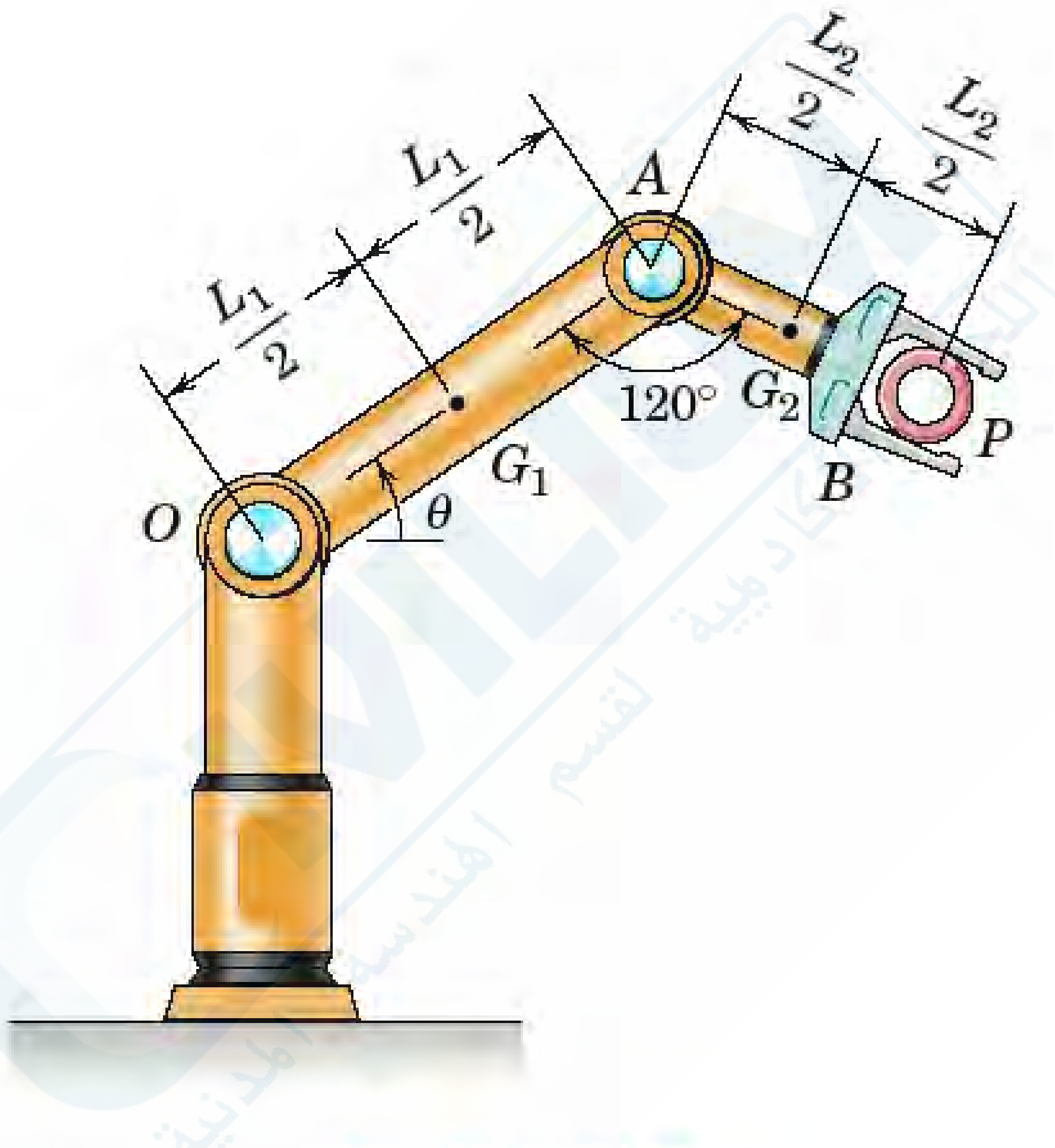
Problem 2/177

P 02 177



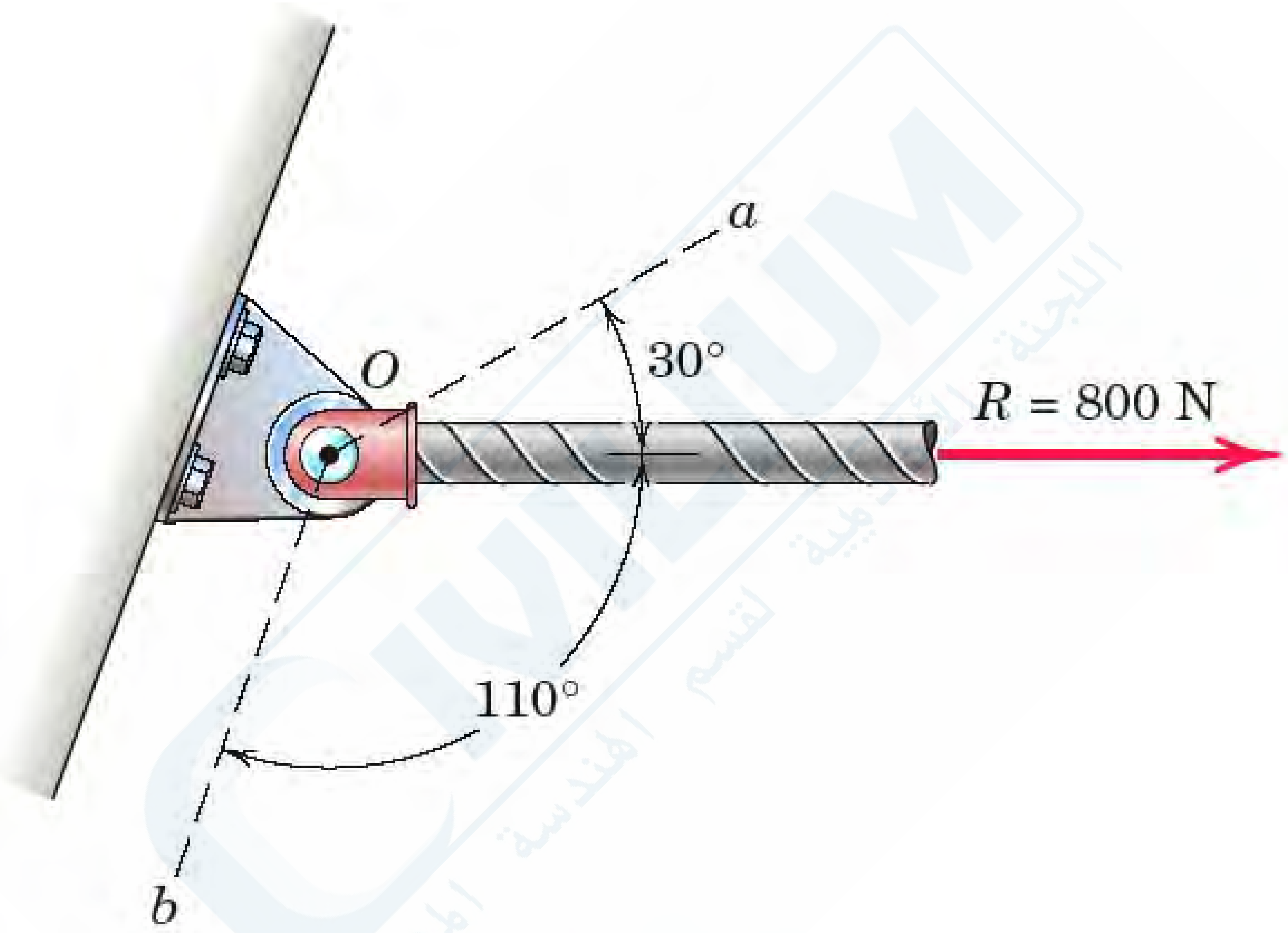
Problem 2/178

P_02_178

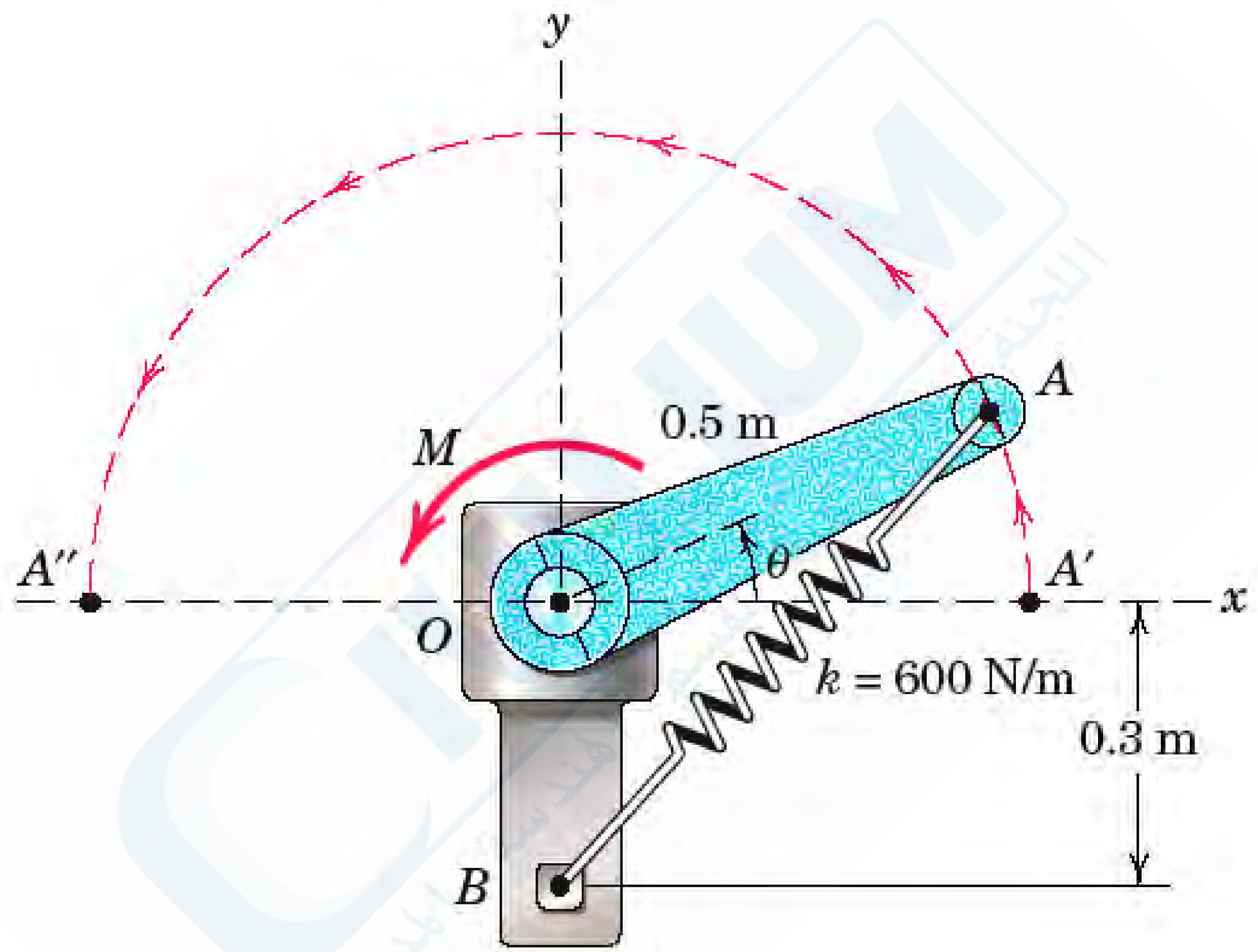


Problem 2/179

P_02_179

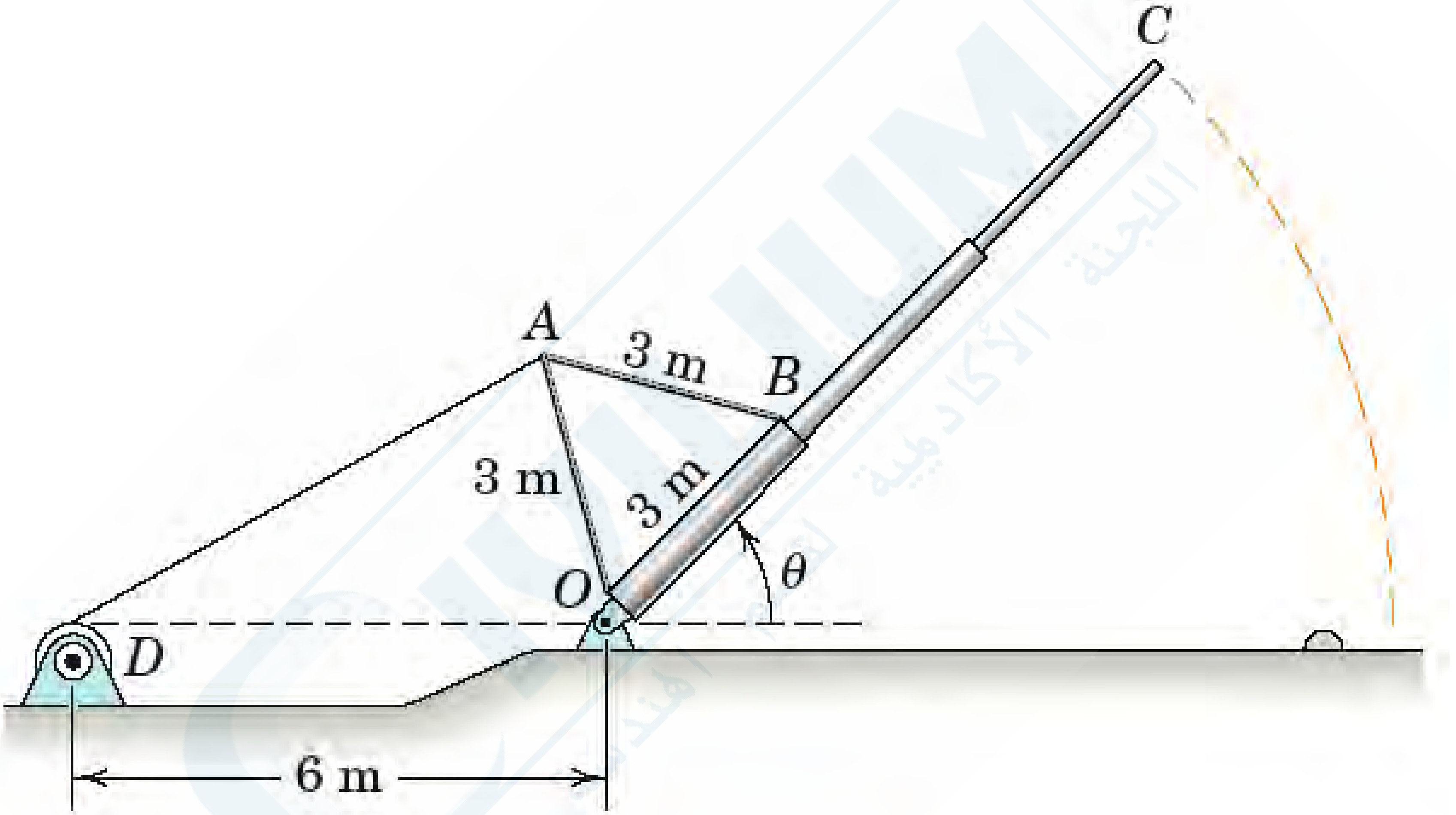


Problem 2/18



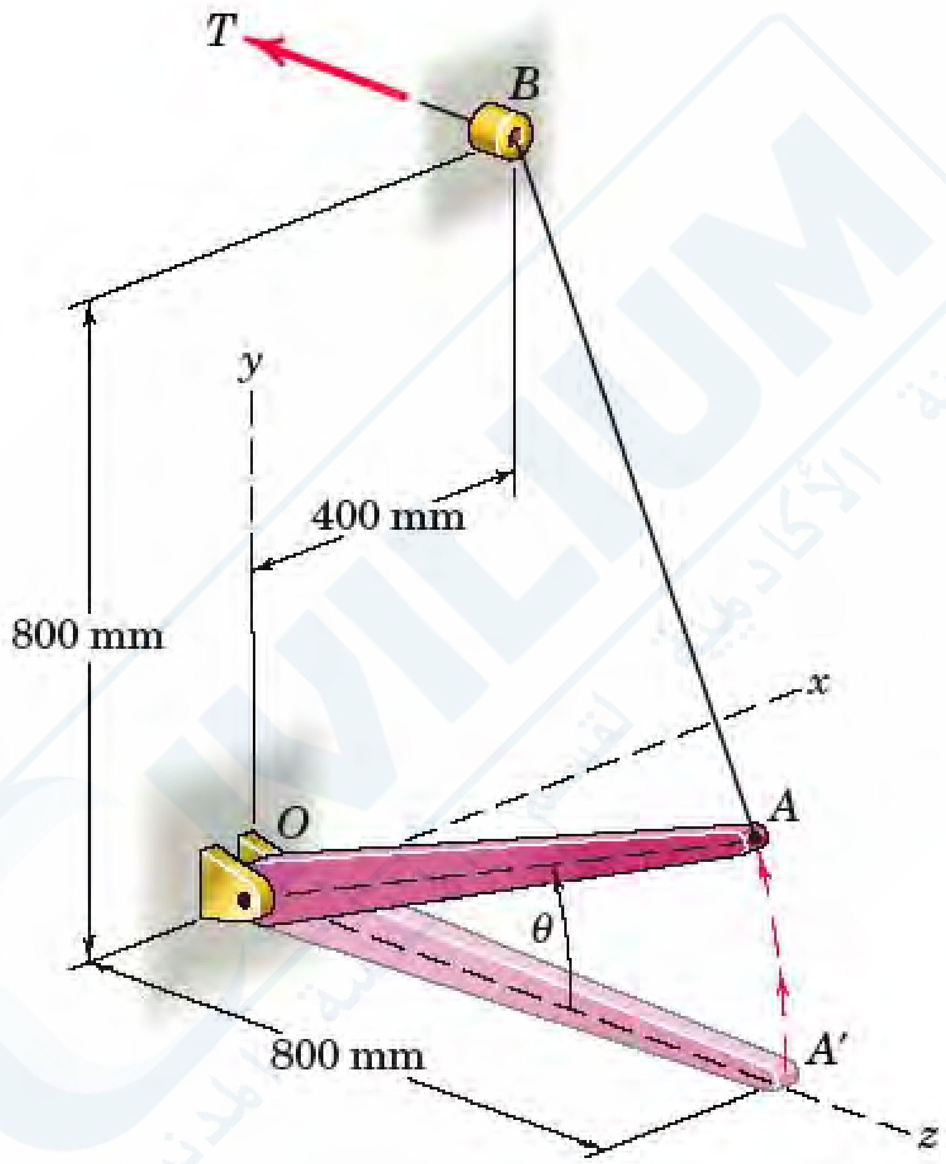
Problem 2/180

P 02 180



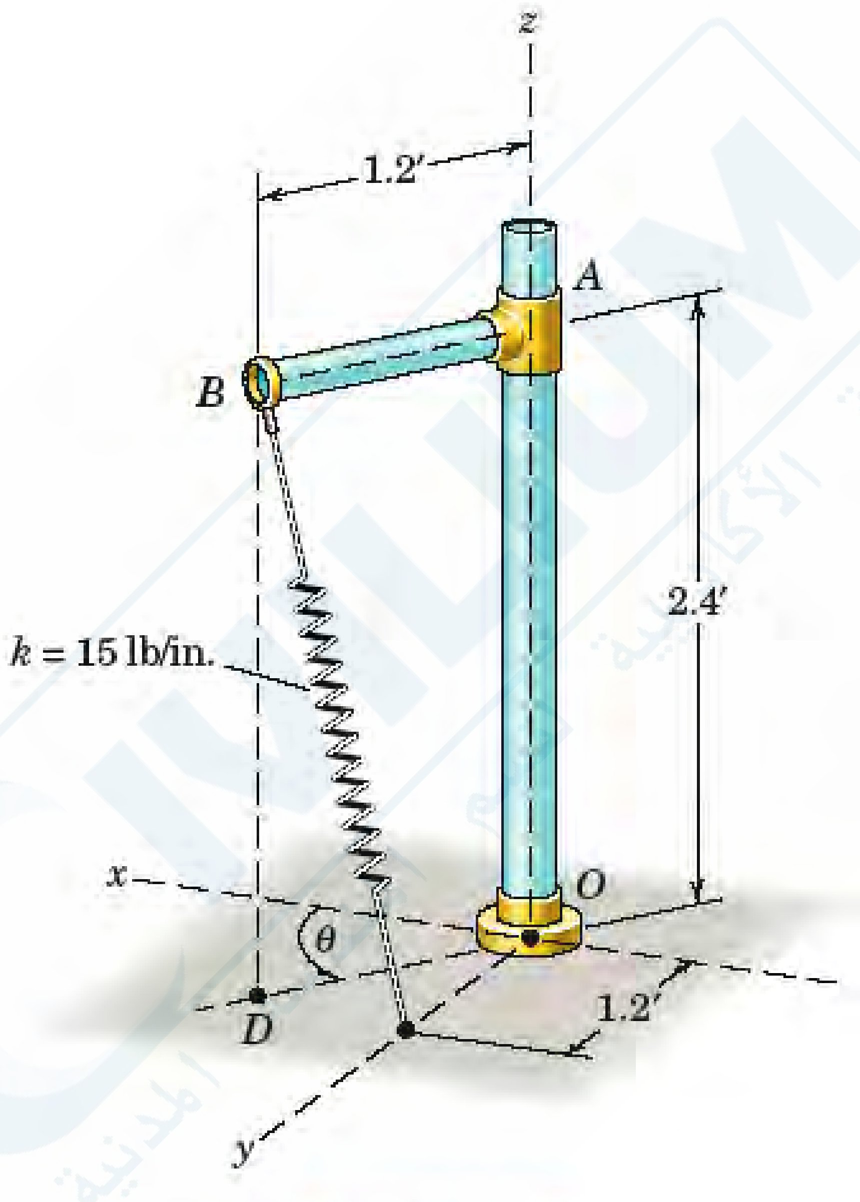
Problem 2/181

P_02_181



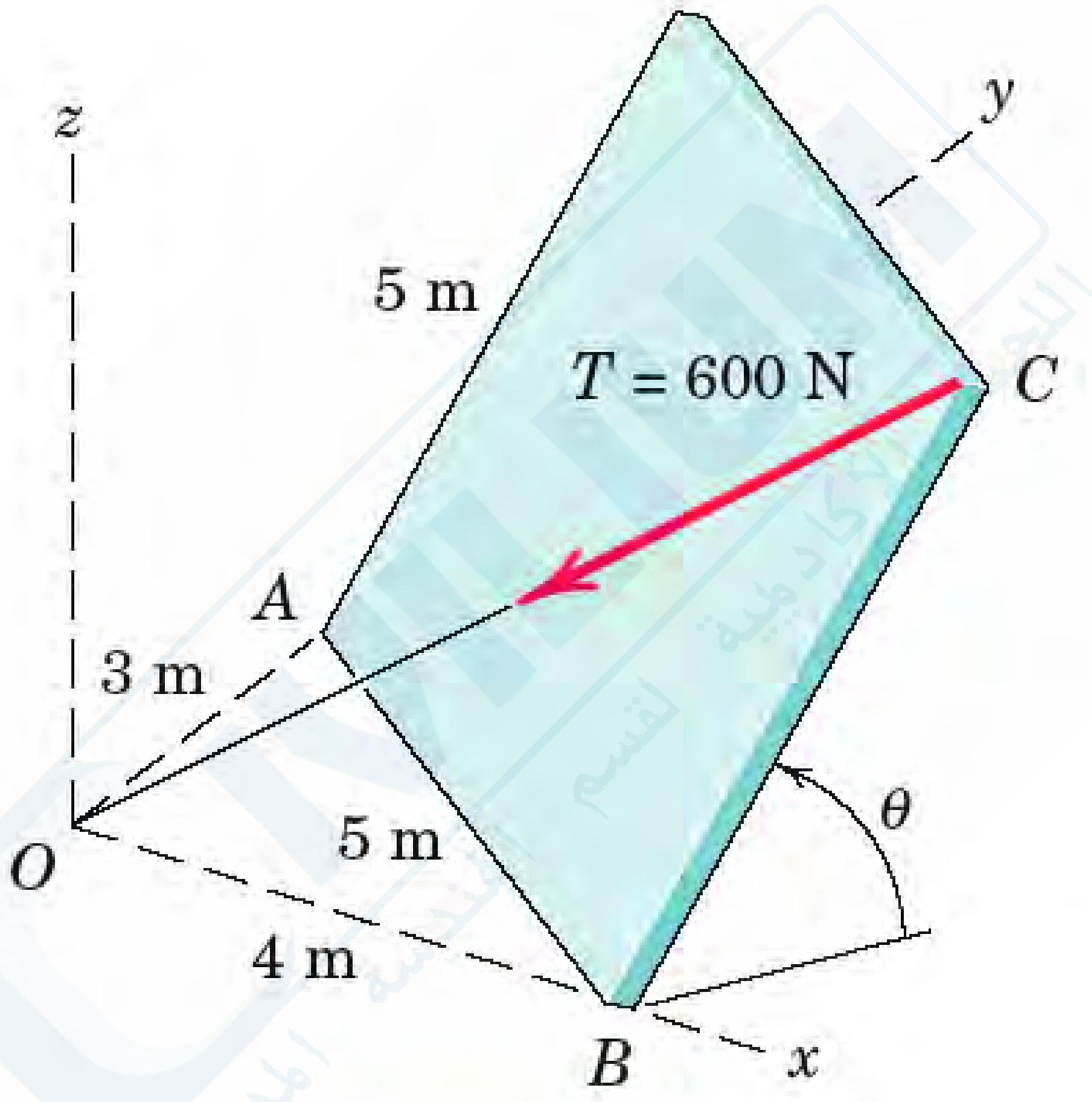
Problem 2/182

P 02 182



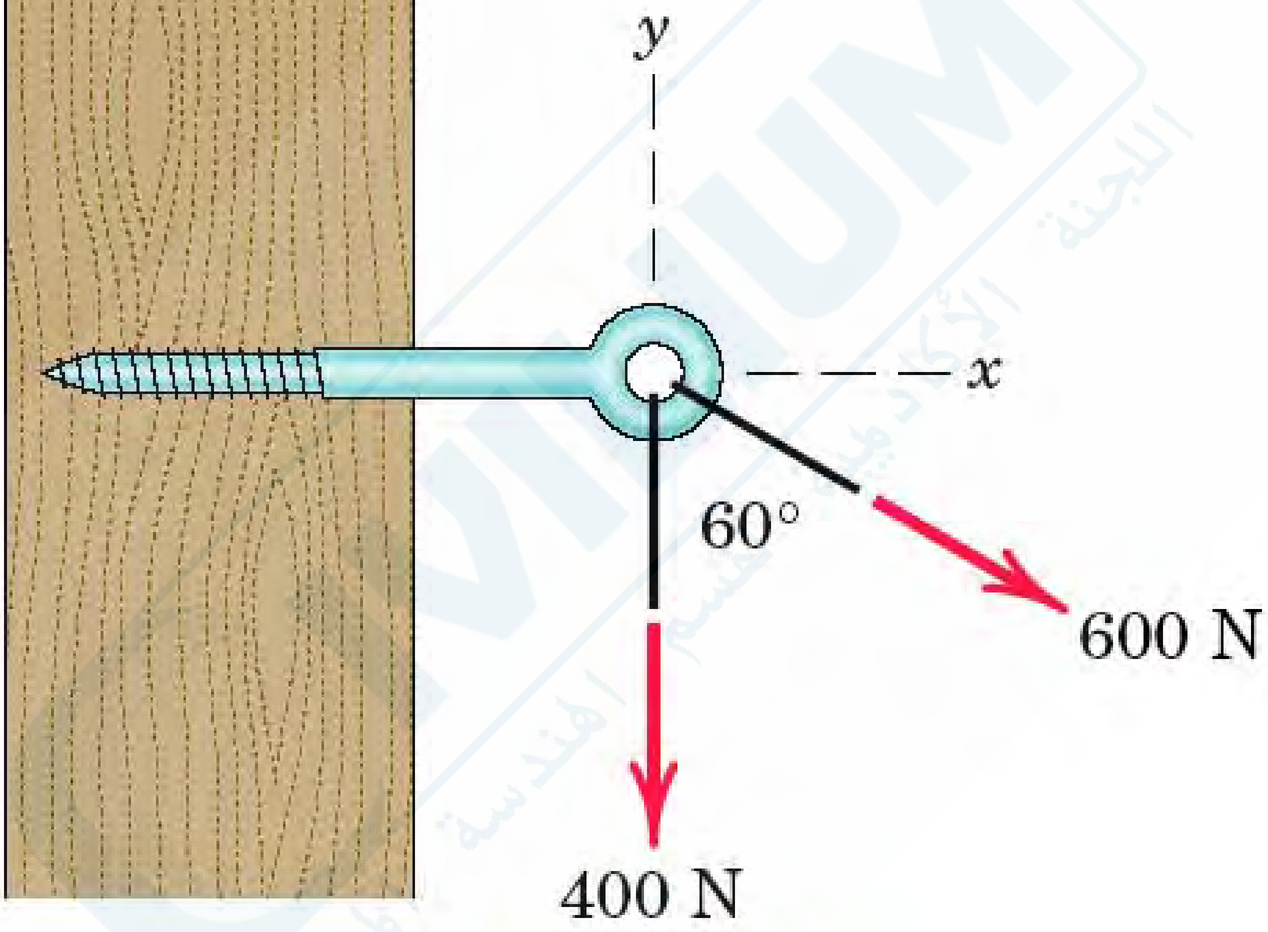
Problem 2/183

P 02 183



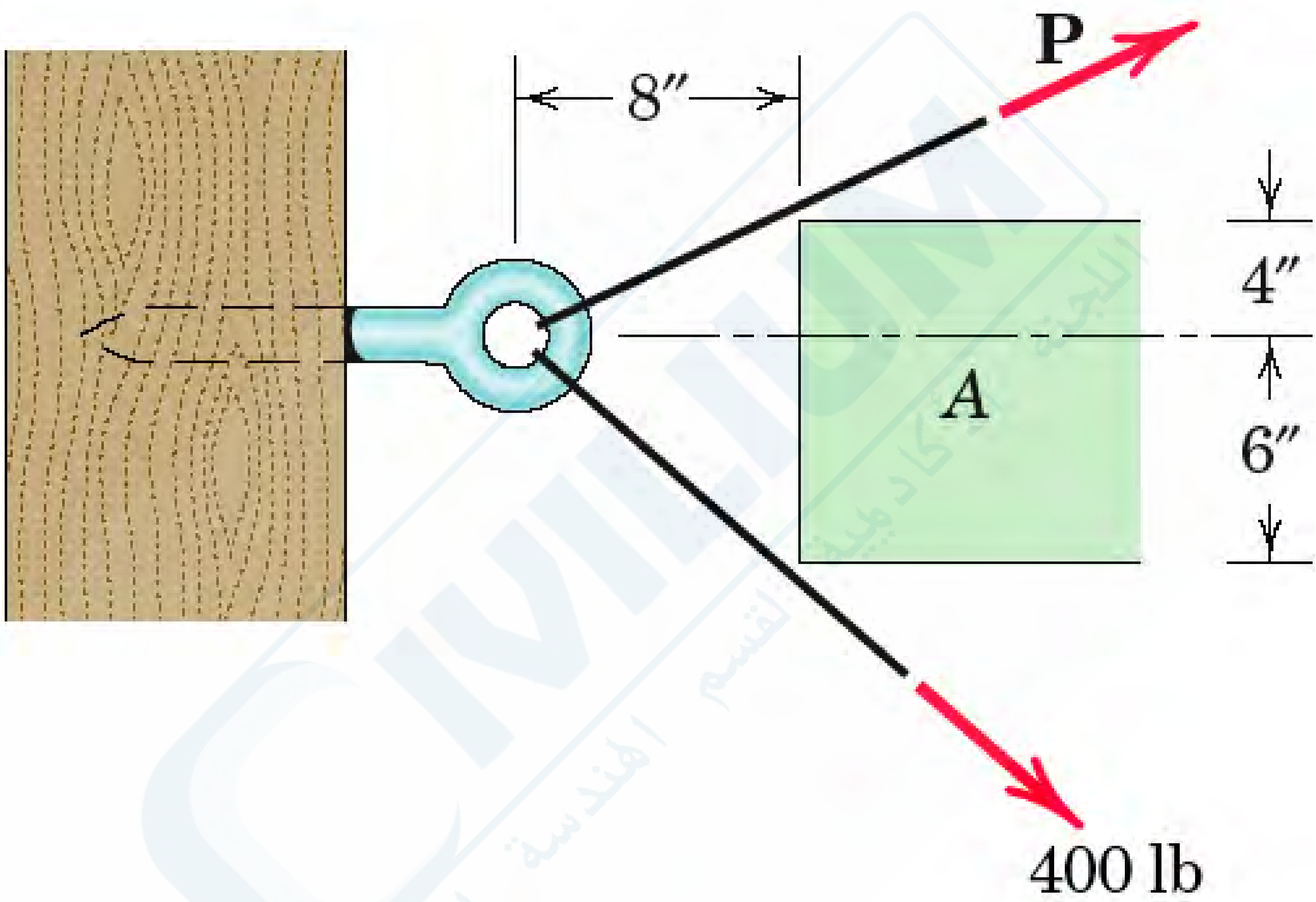
Problem 2/184

P 02 184

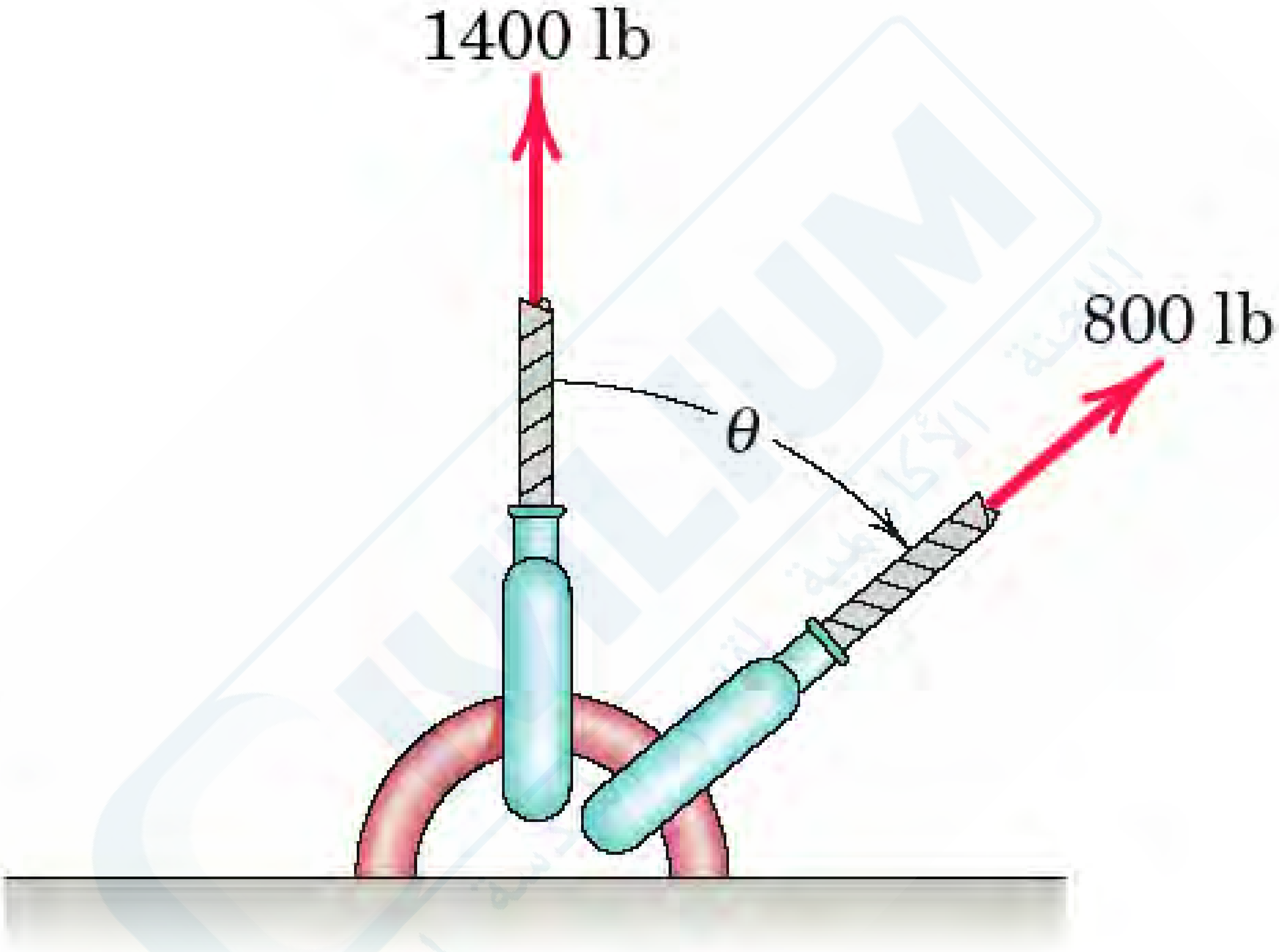


Problem 2/19

P_02_19

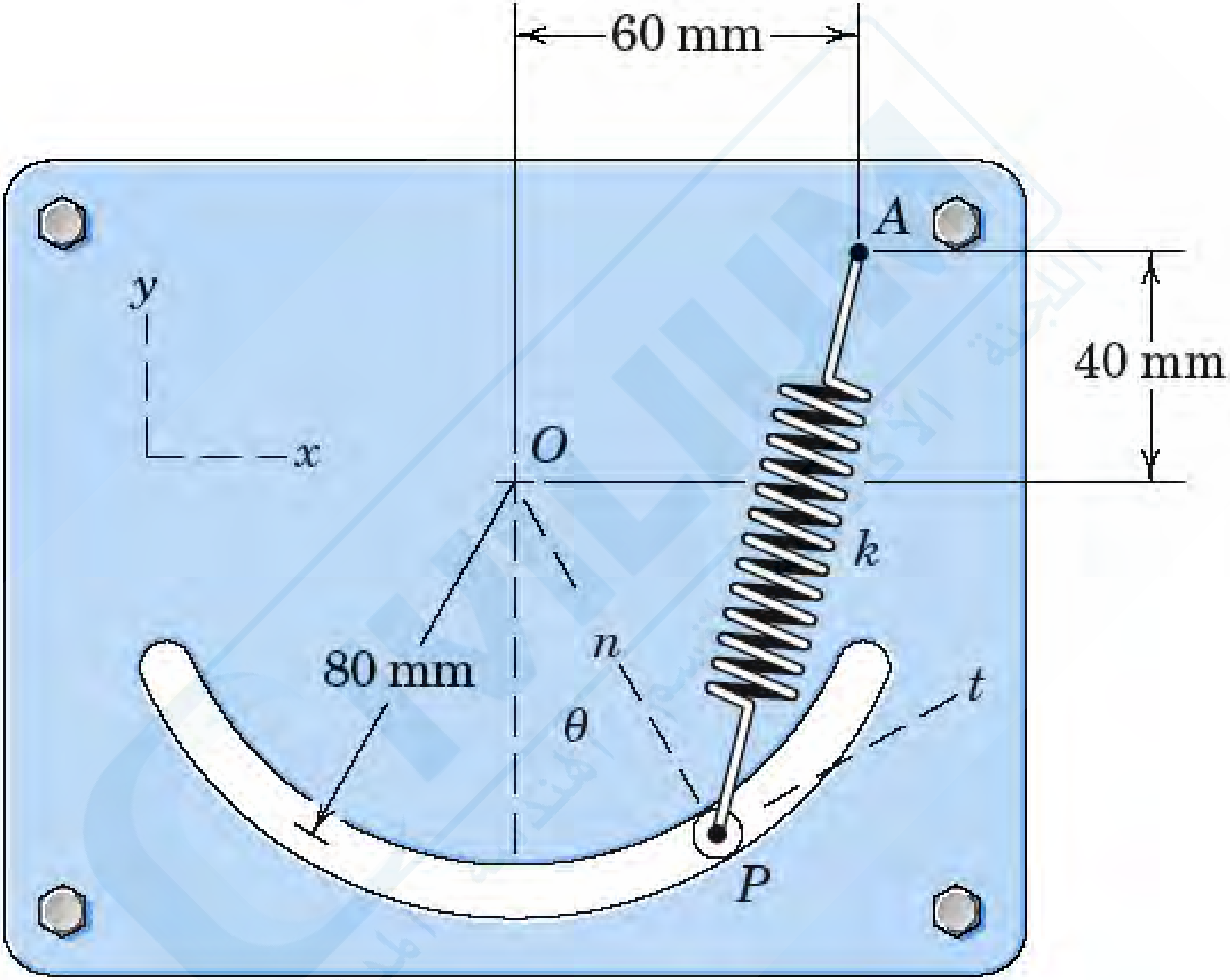


Problem 2/20



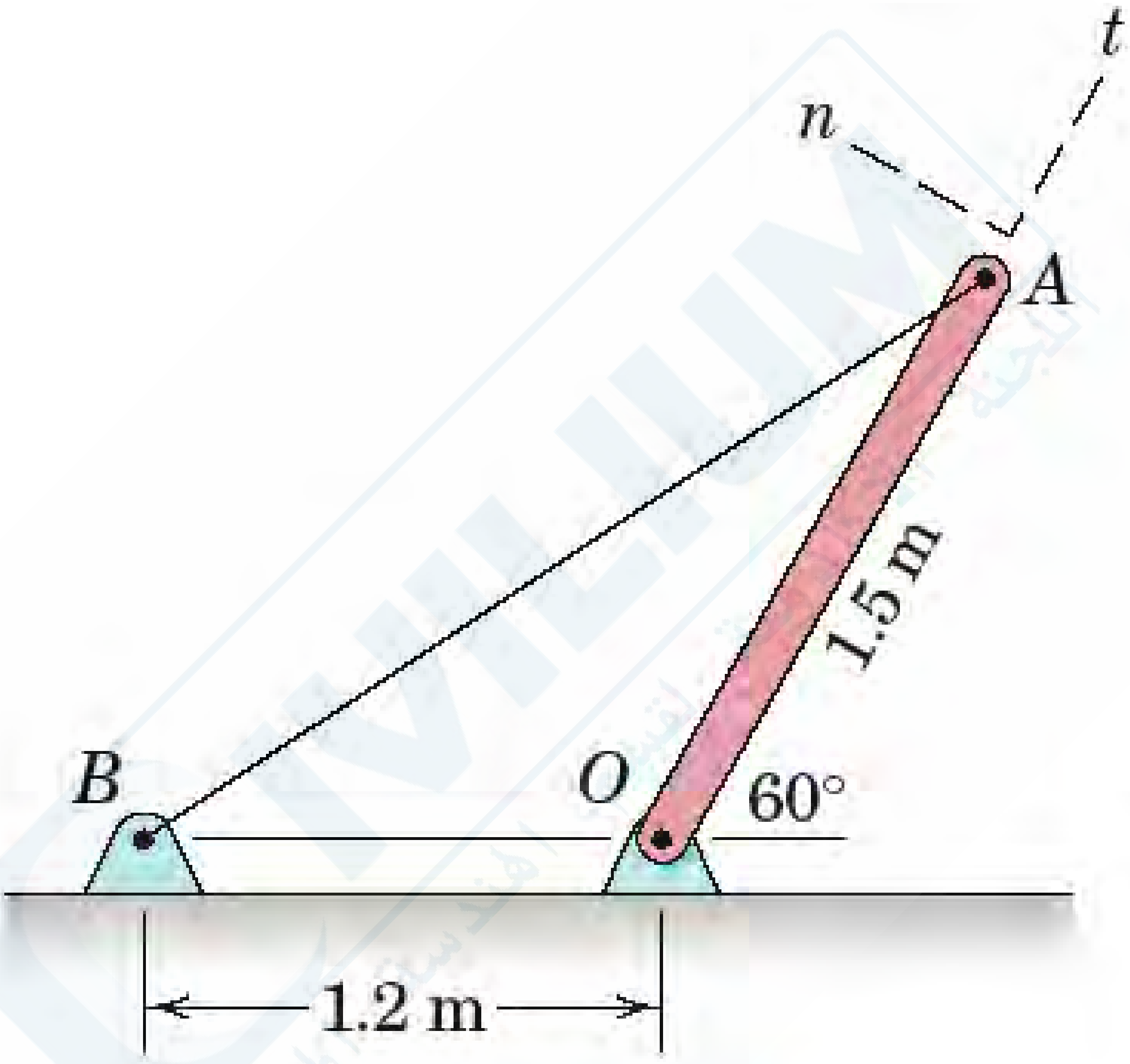
Problem 2/21

P 02 21



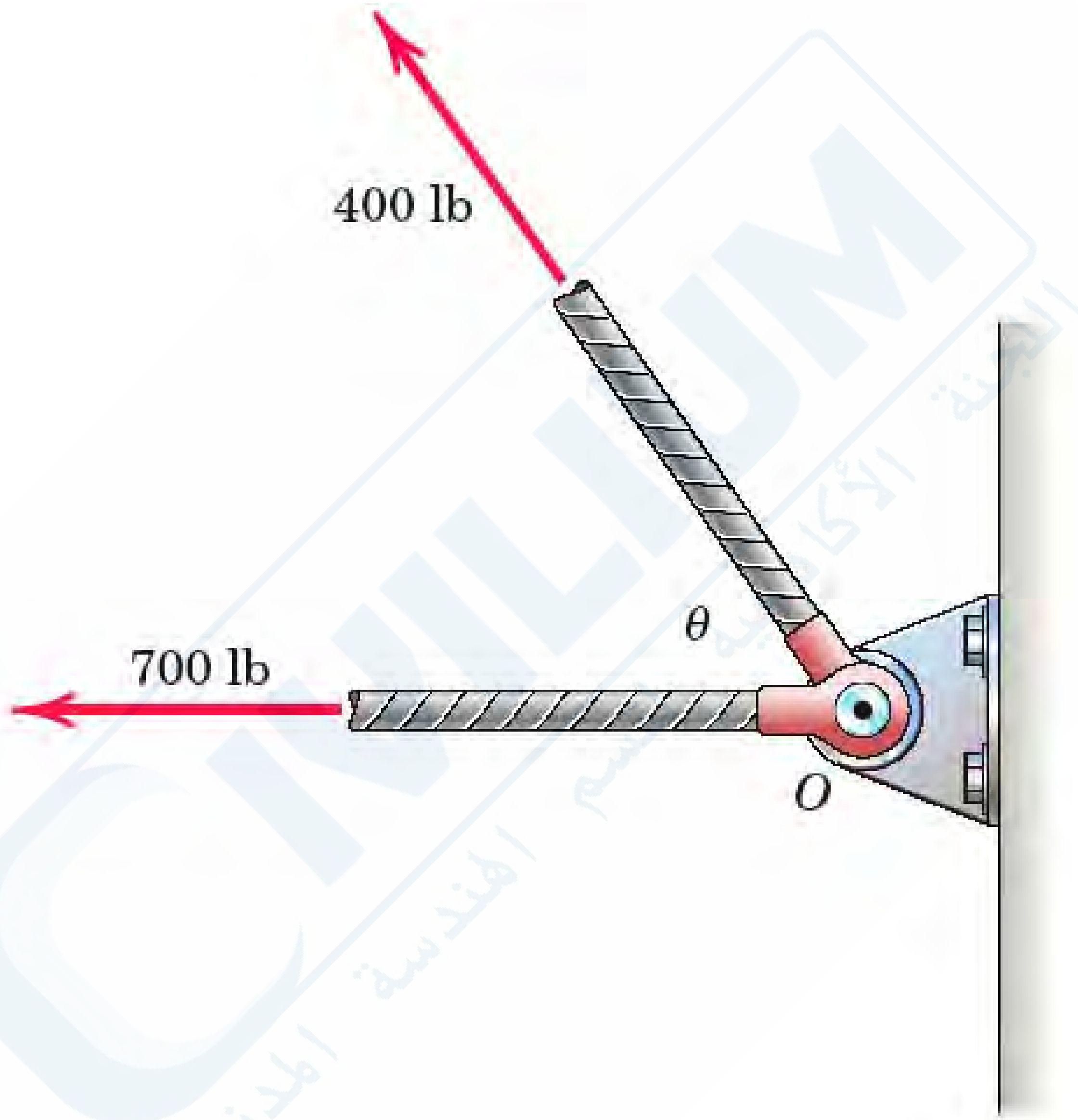
Problem 2/22

P 02 22



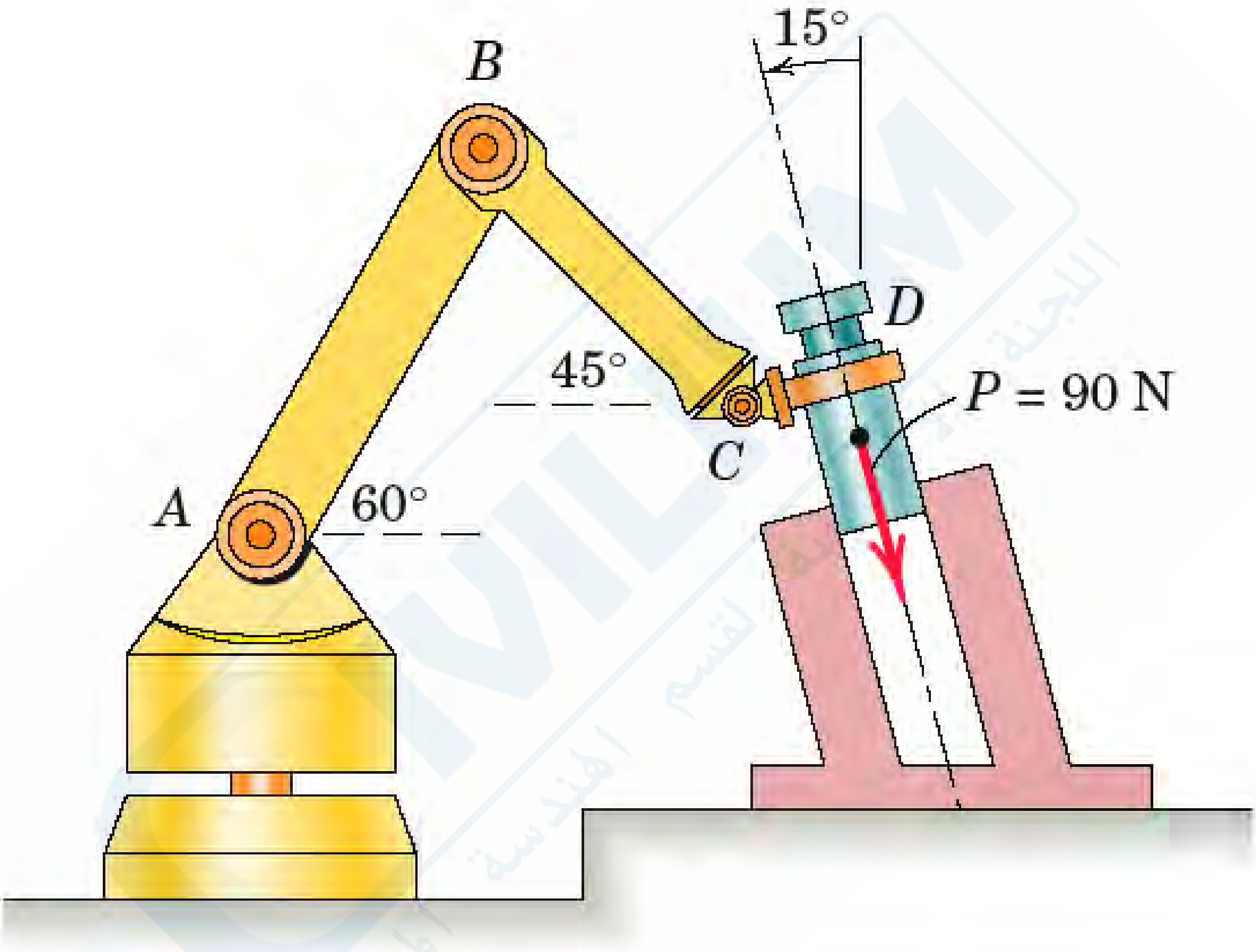
Problem 2/24

P_02_24



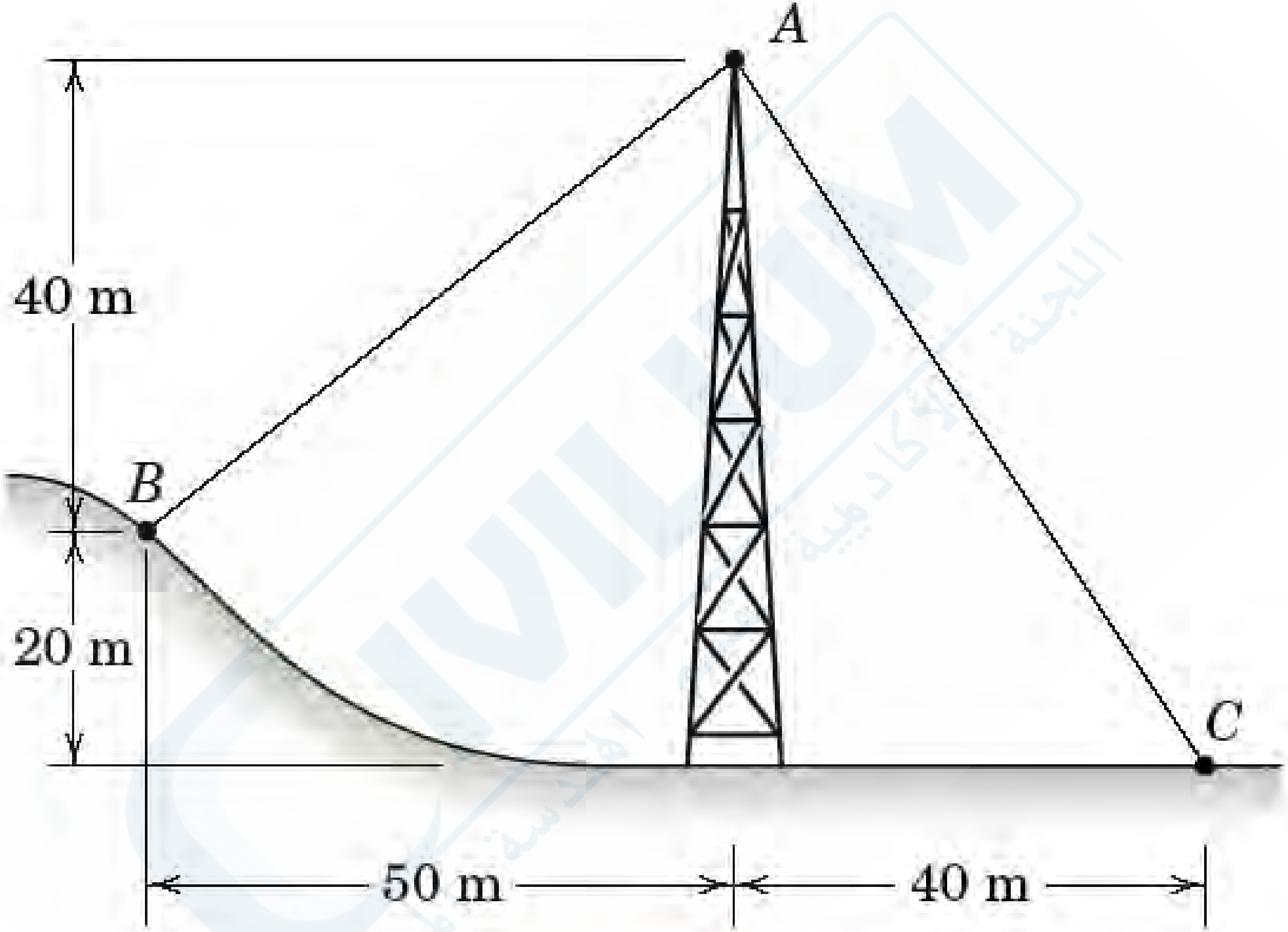
Problem 2/25

P 02 25



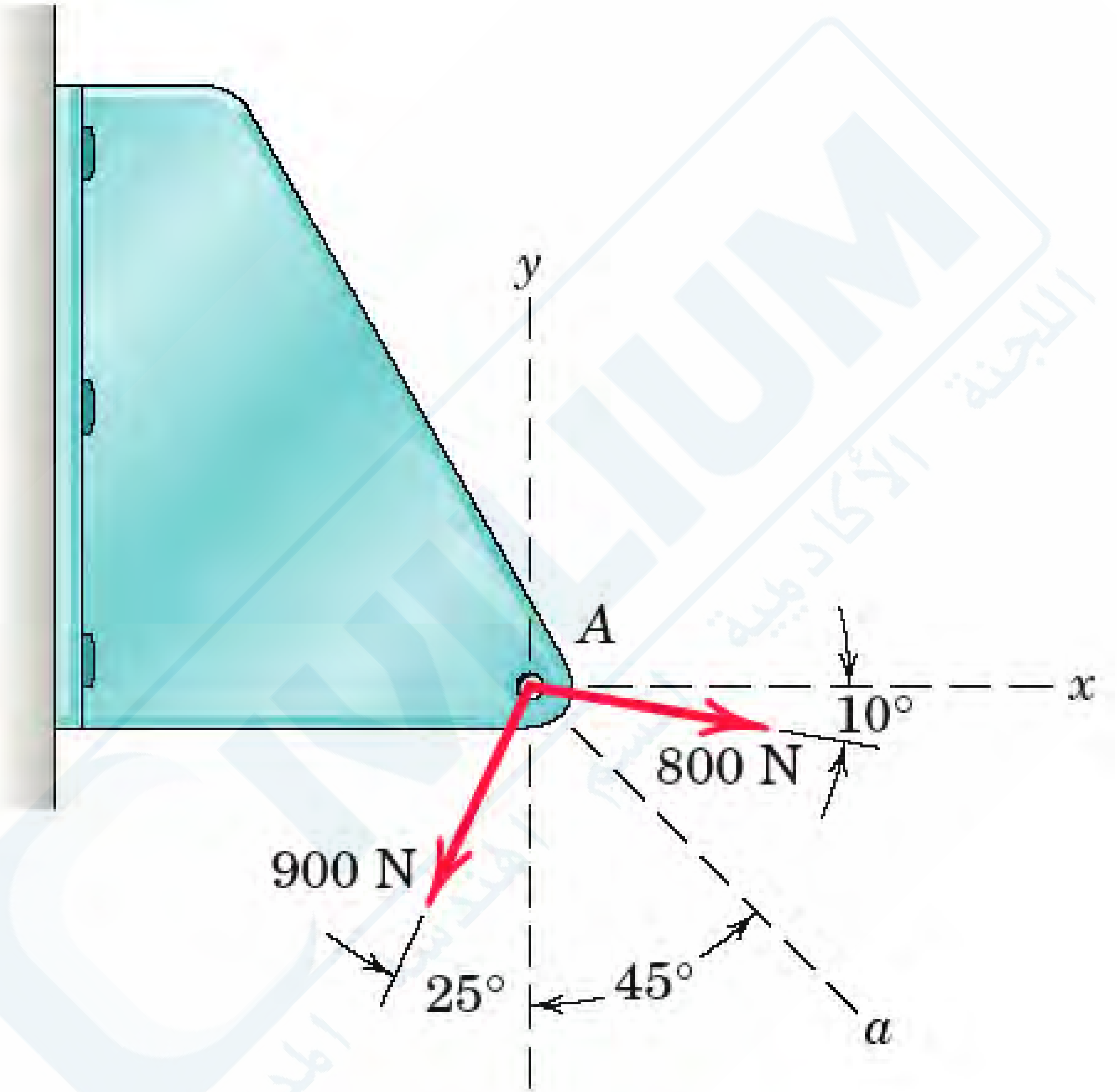
Problem 2/26

P 02 26

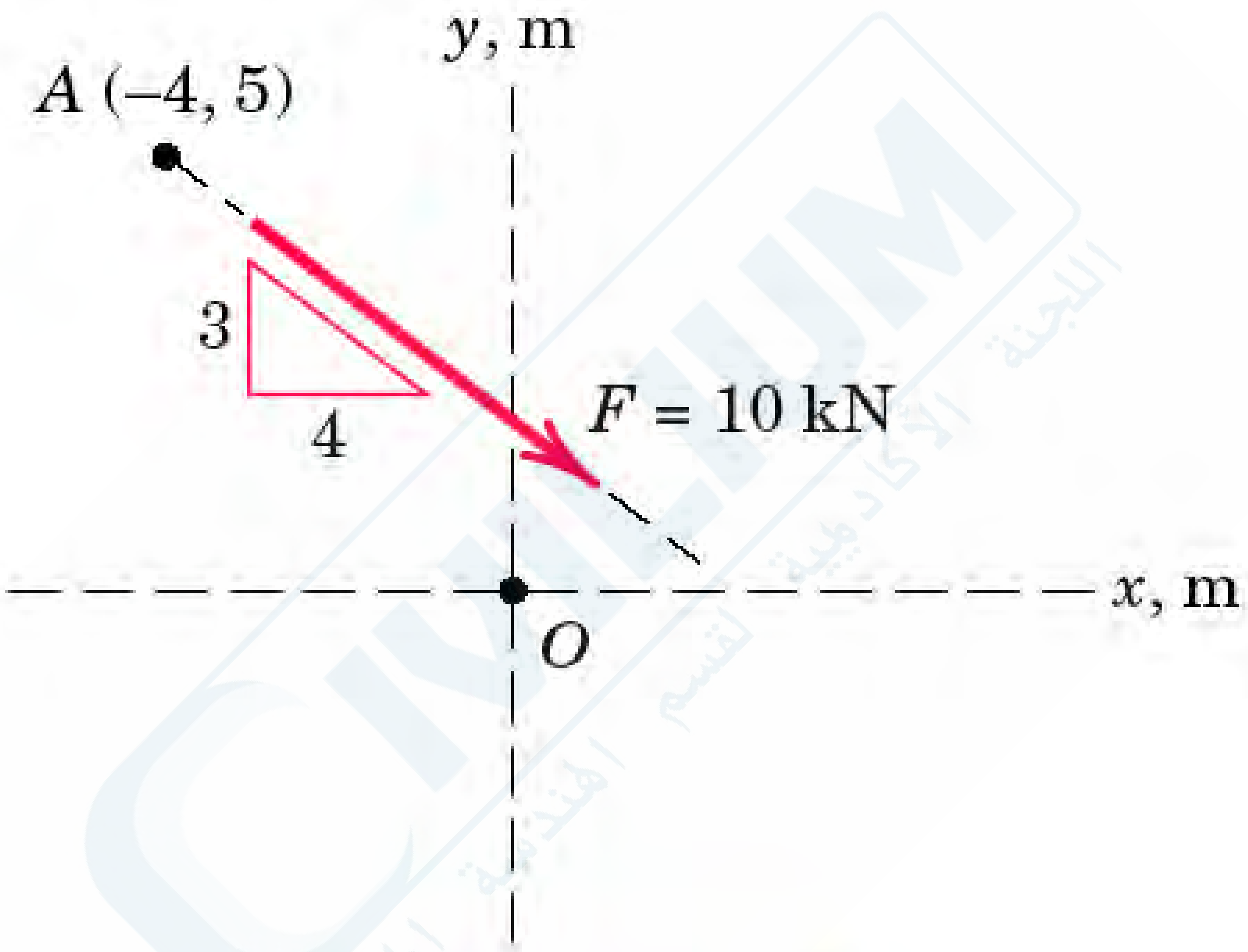


Problem 2/27

P 02 27

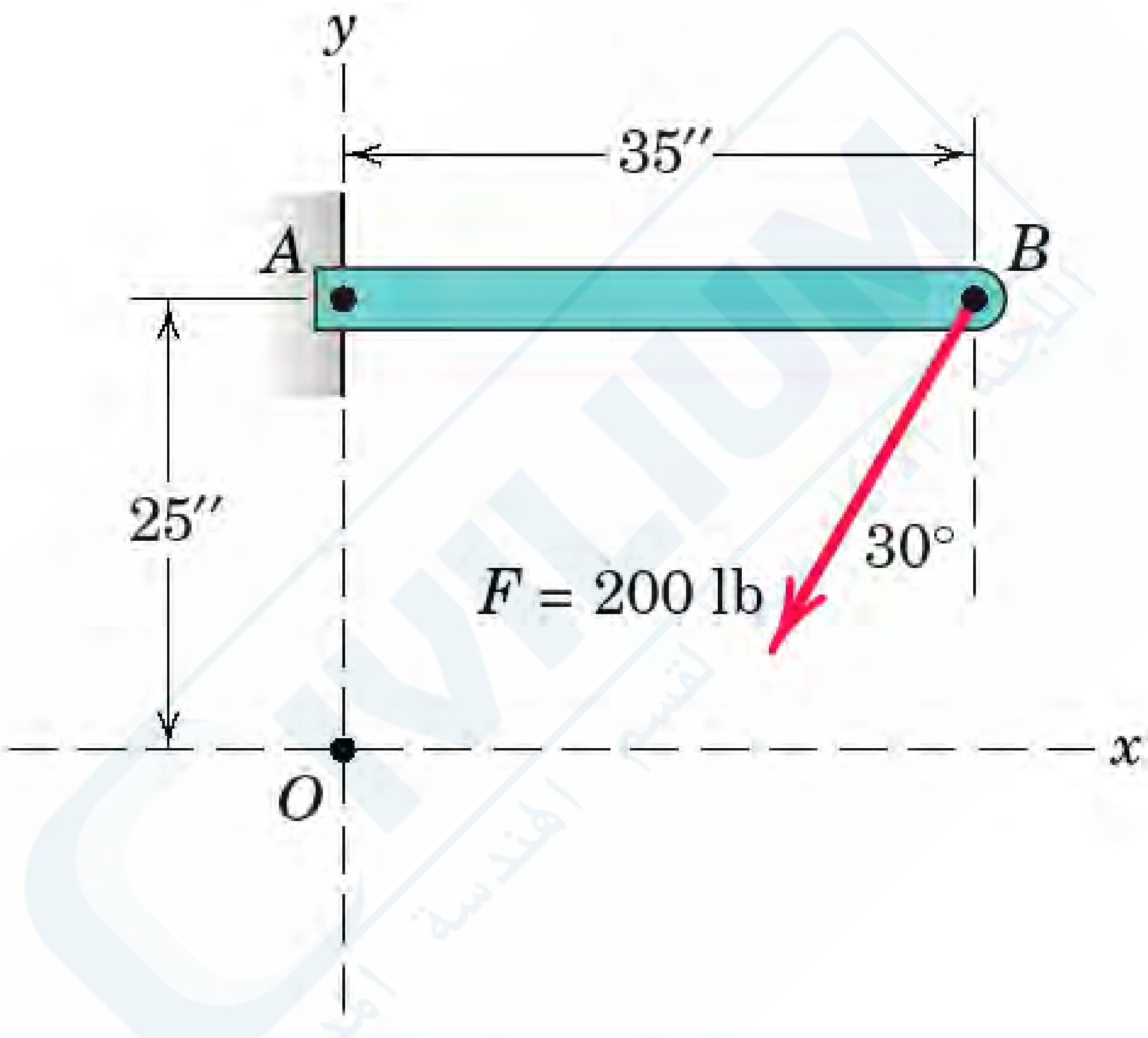


Problem 2/28



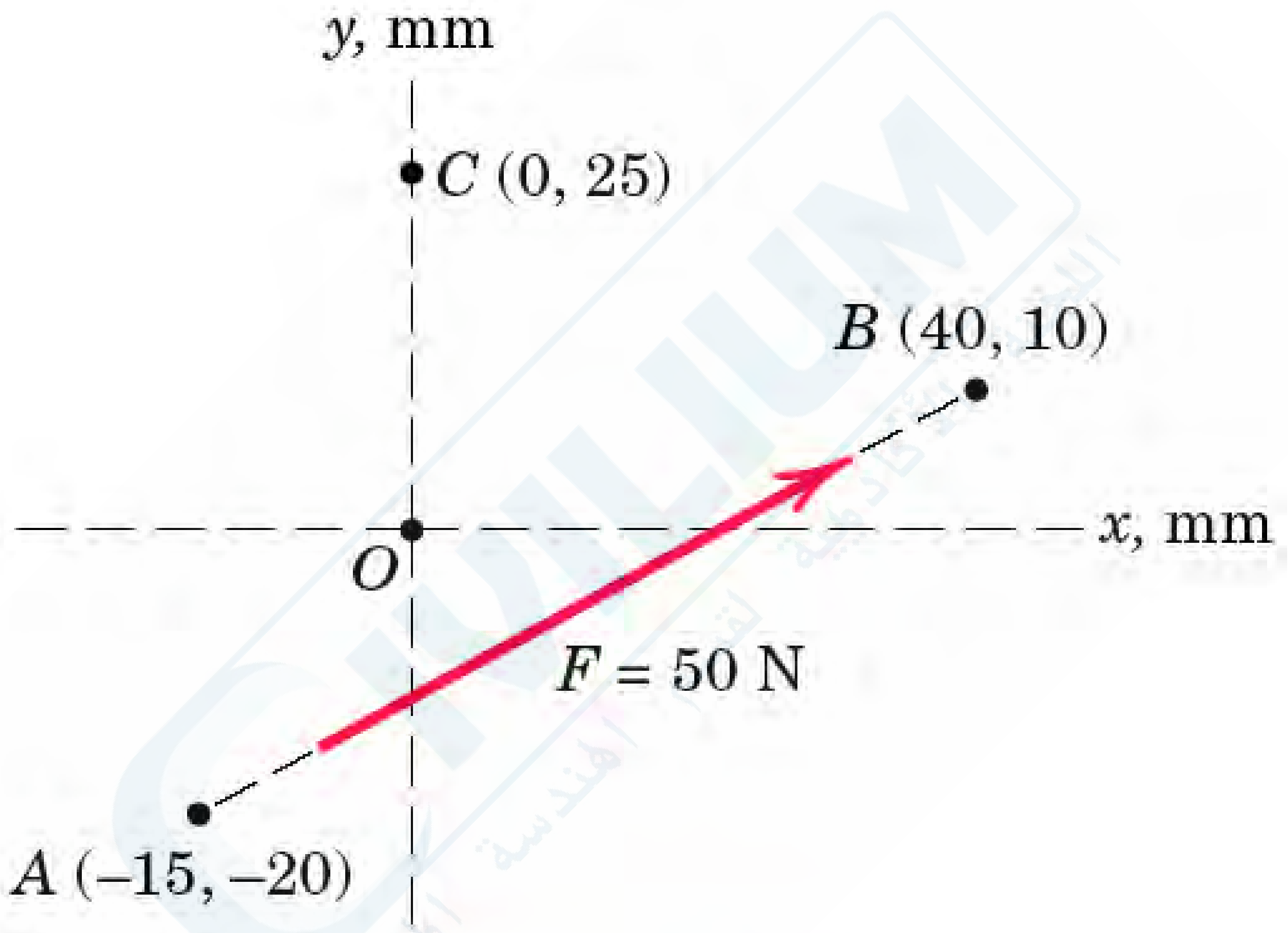
Problem 2/29

P 02 29



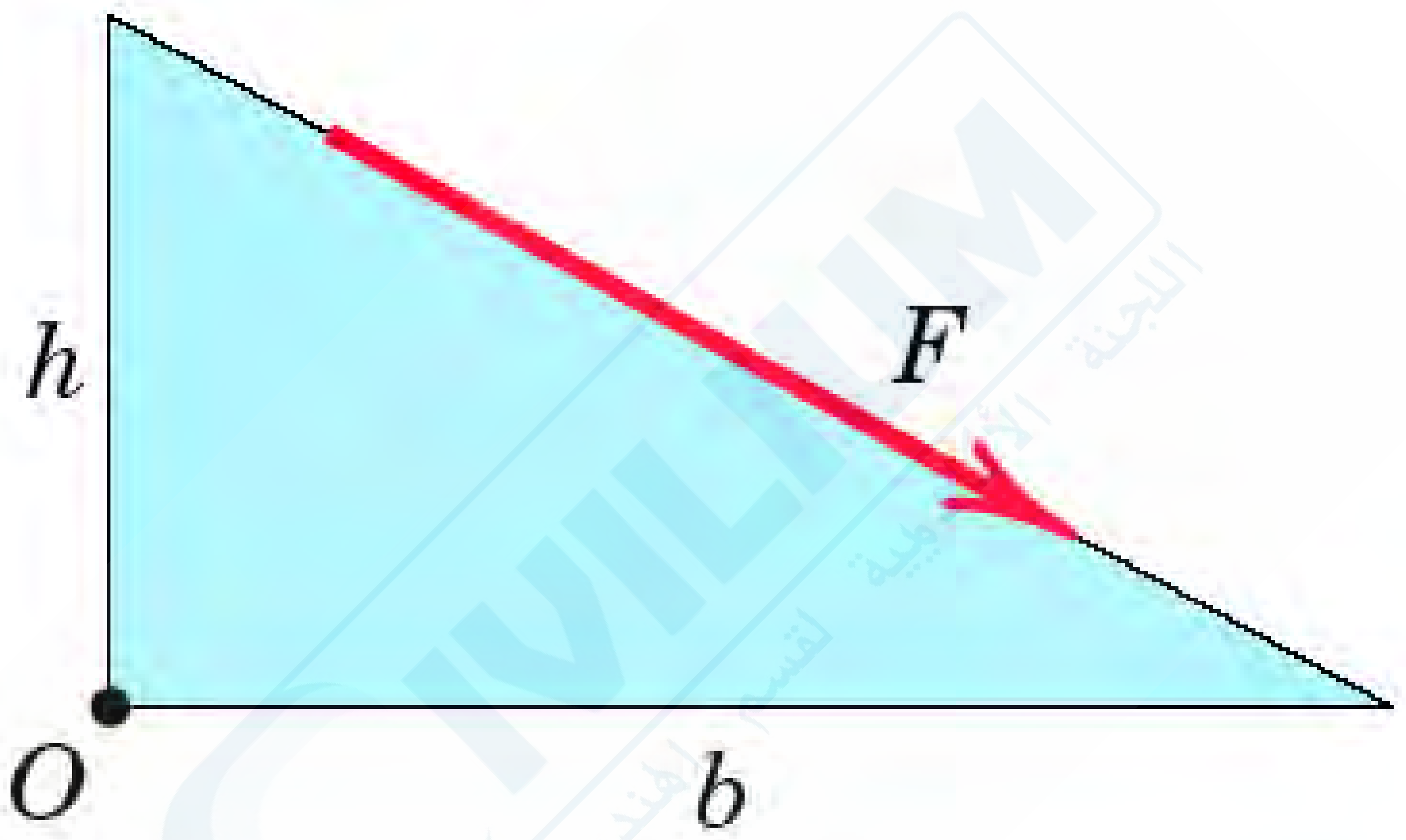
Problem 2/30

P 02 30



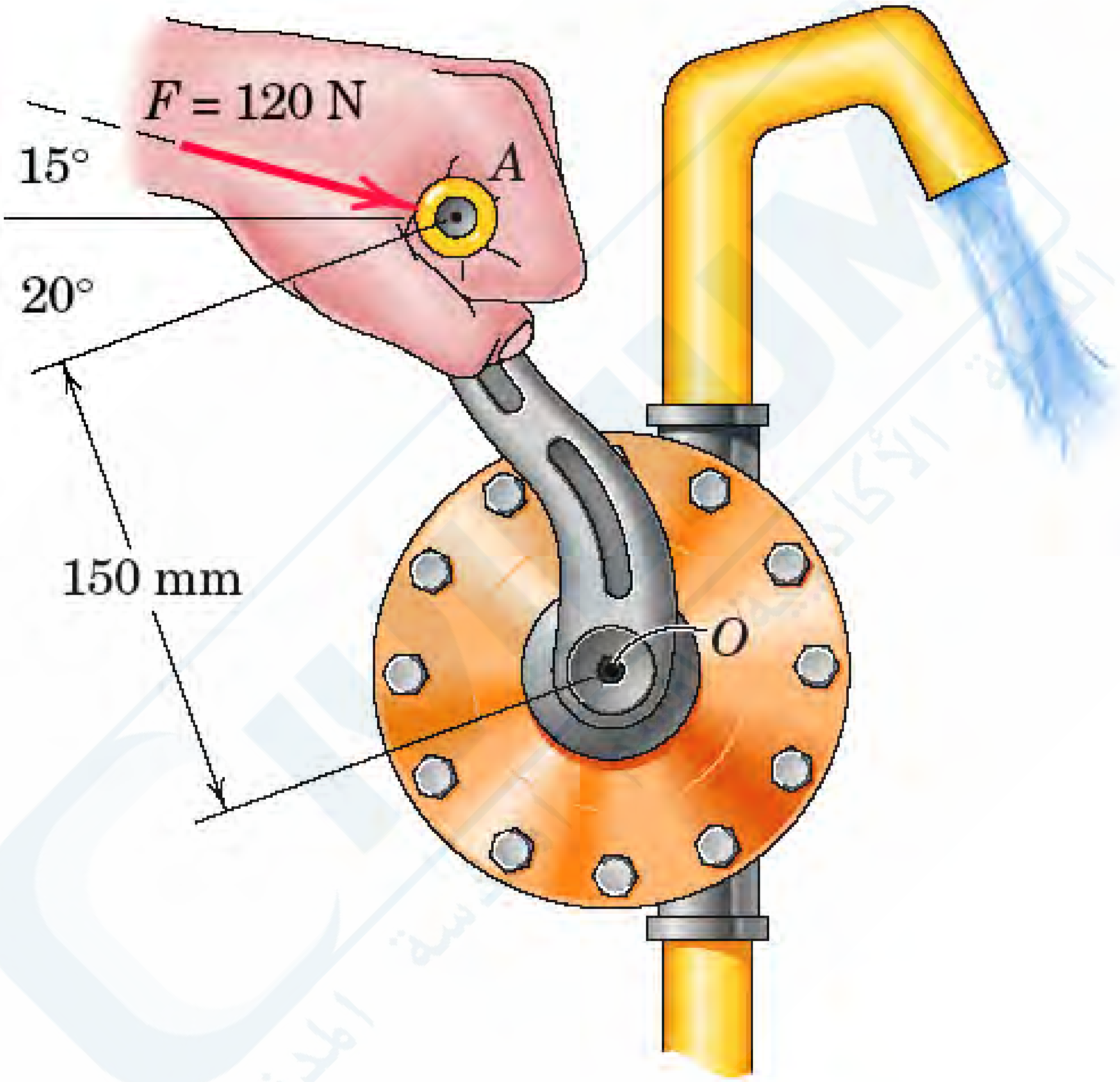
Problem 2/31

P 02 31



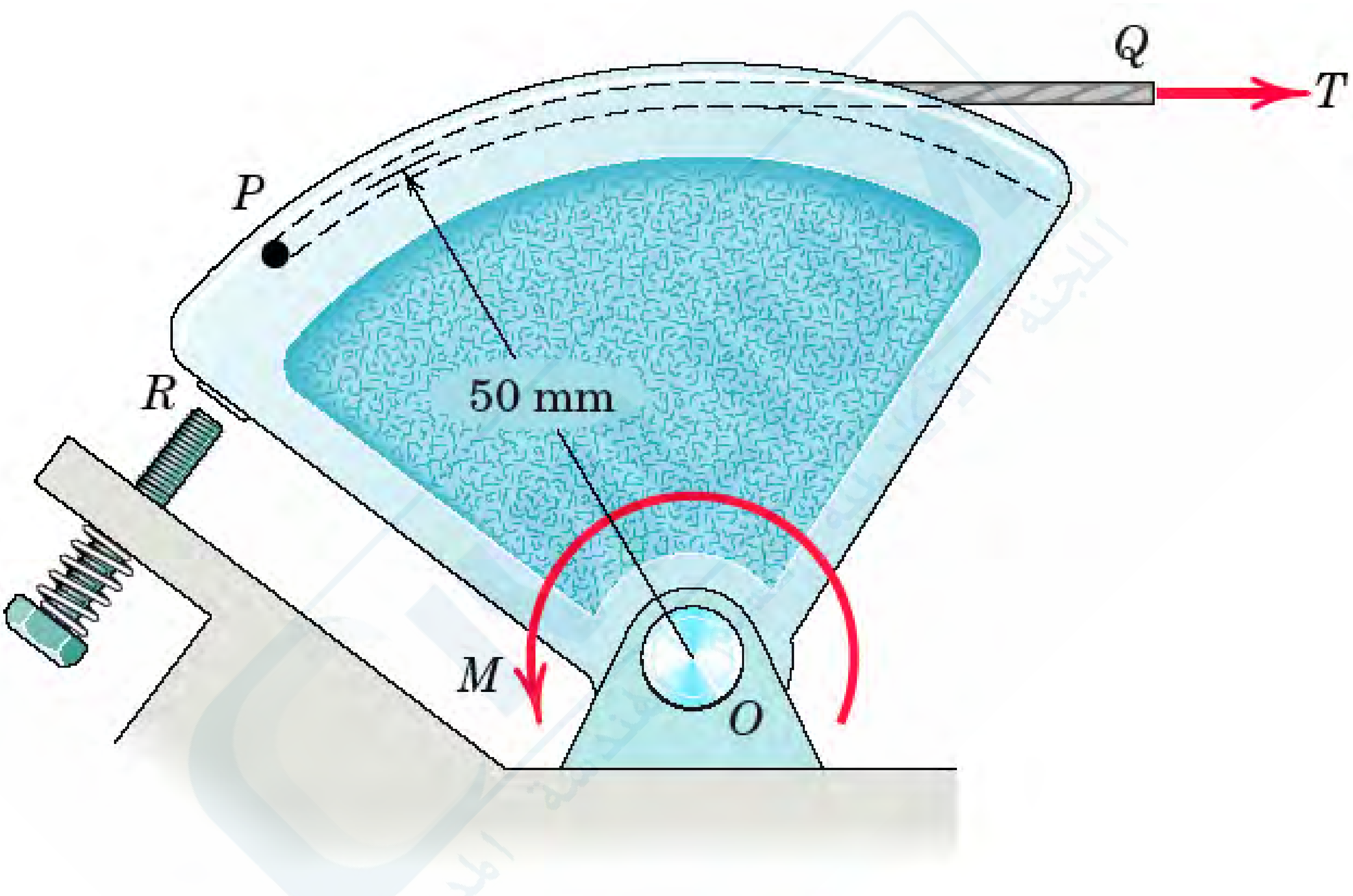
Problem 2/32

P 02 32



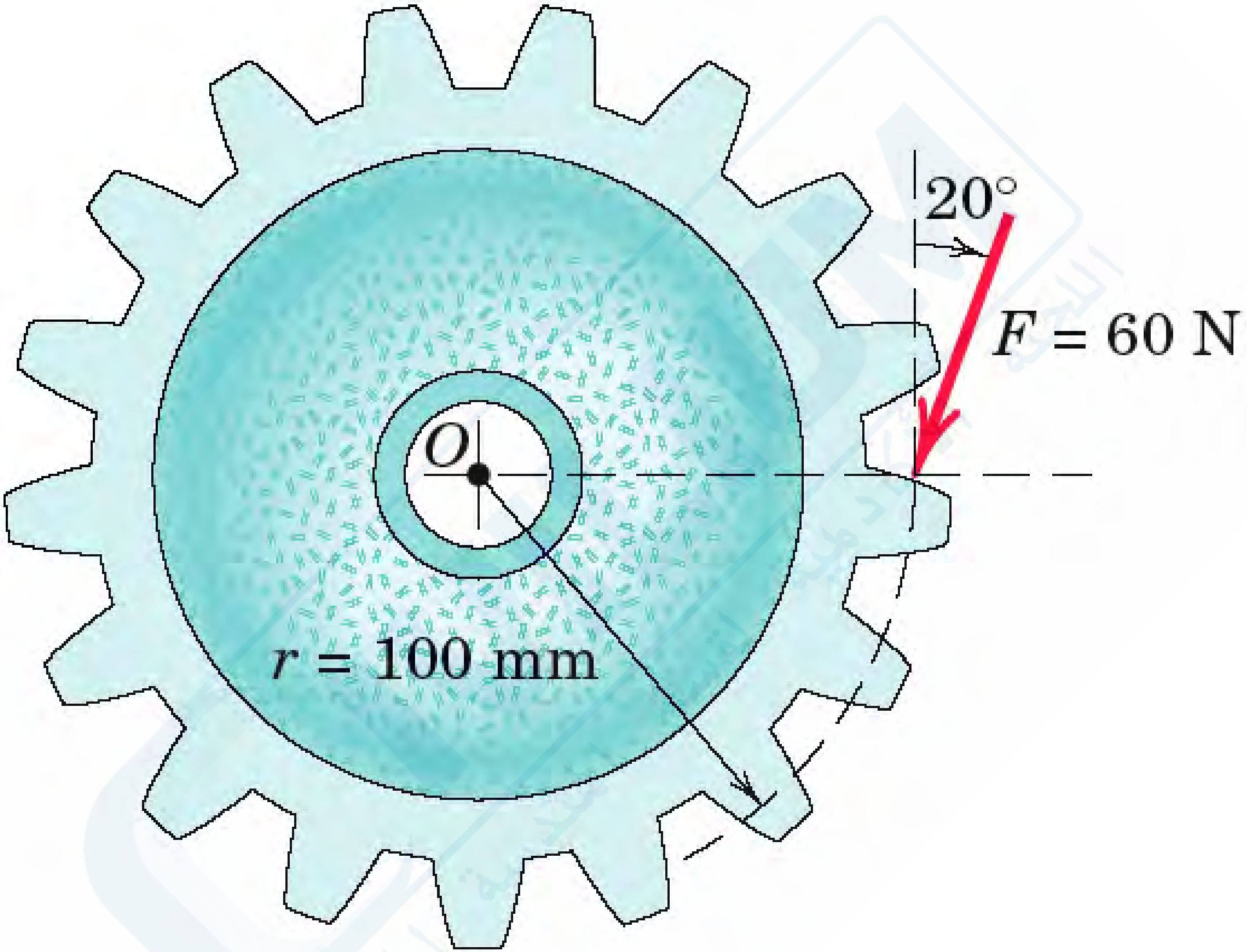
Problem 2/33

P 02 33



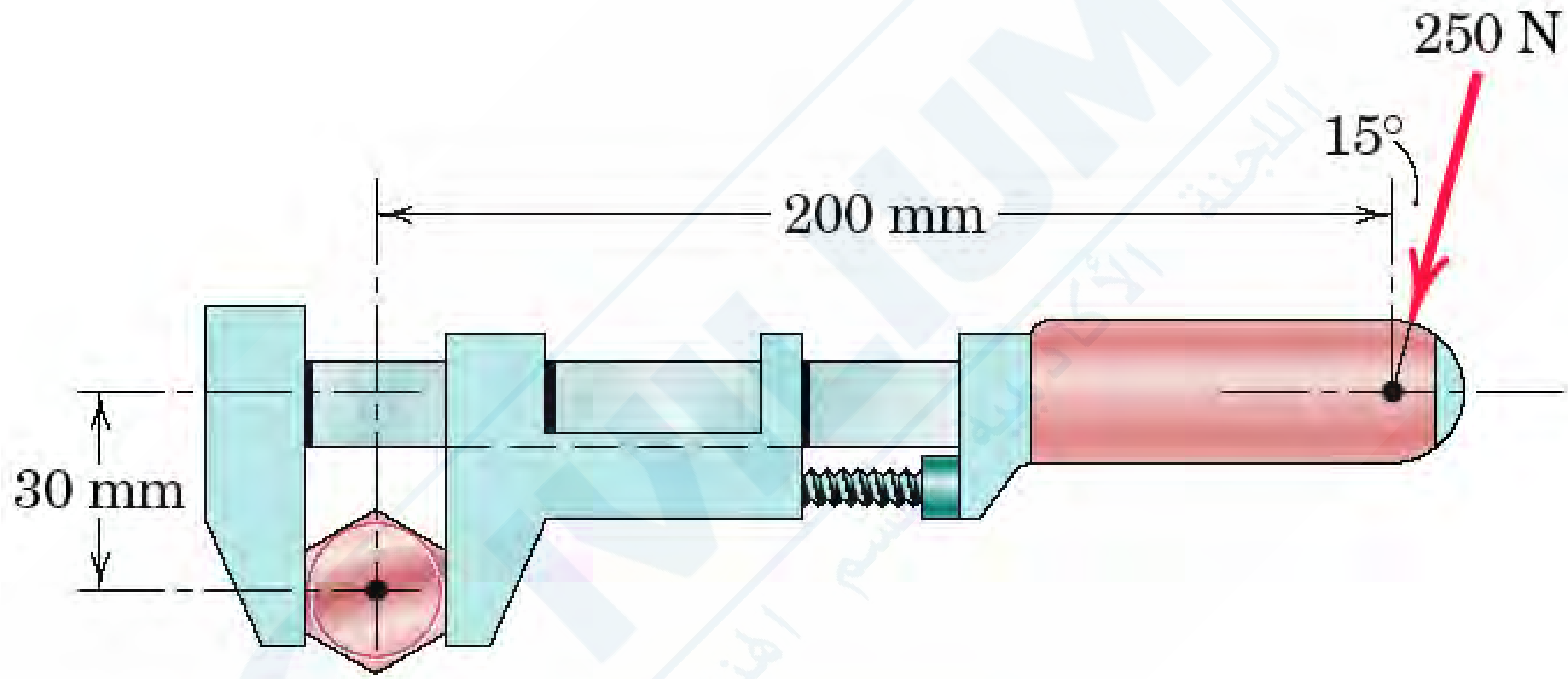
Problem 2/34

P_02_34



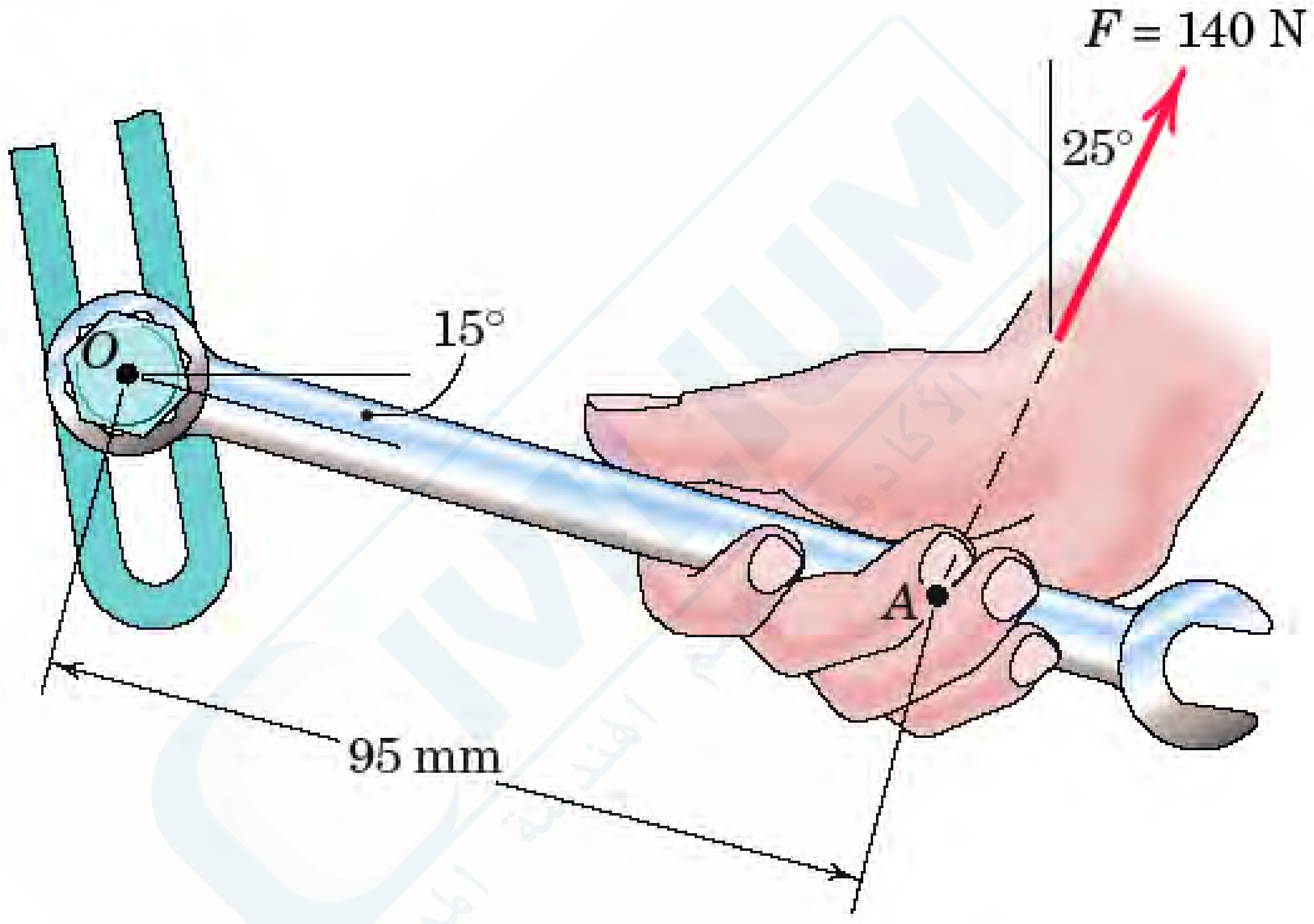
Problem 2/35

P 02 35

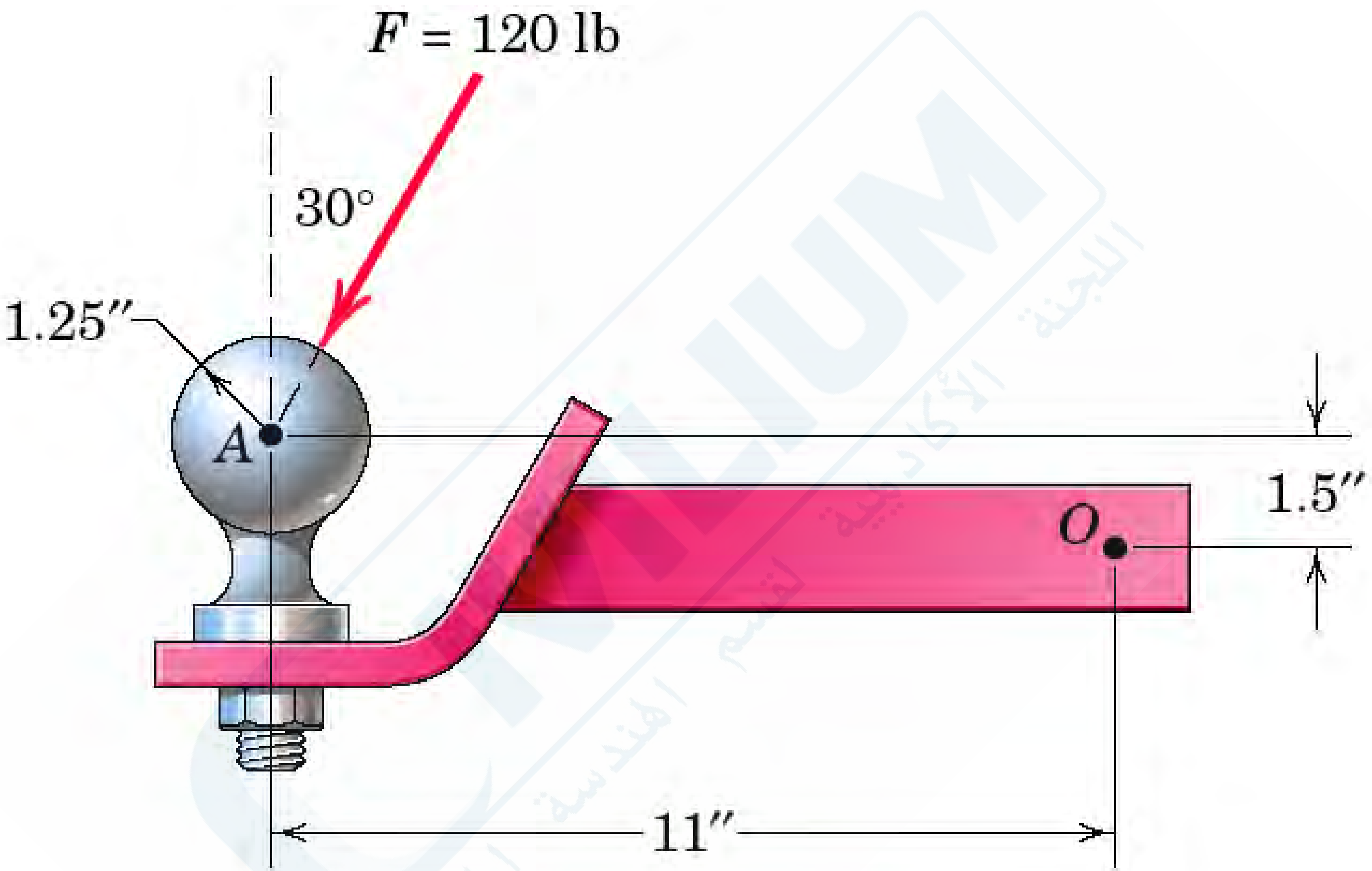


Problem 2/36

P_02_36

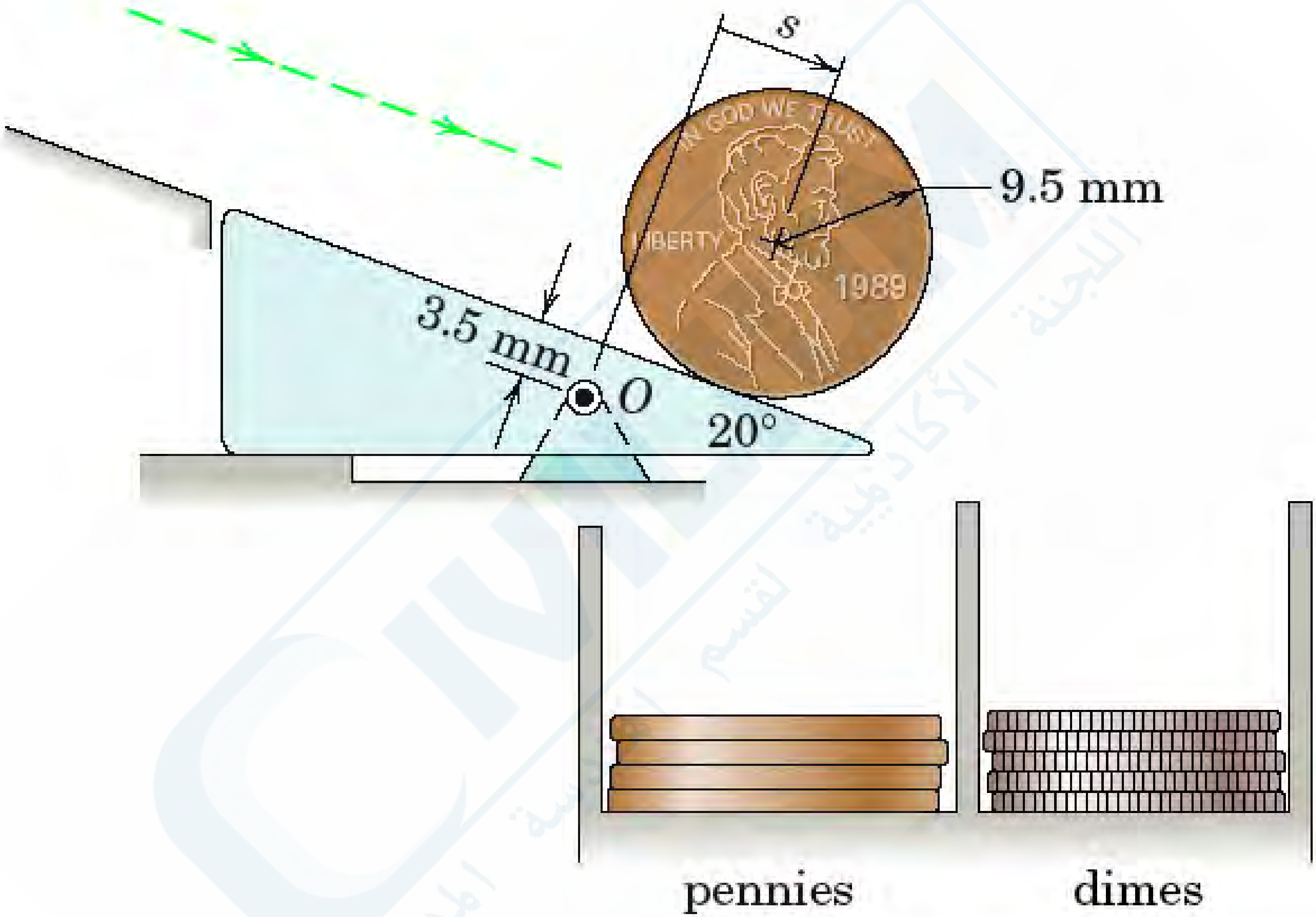


Problem 2/37

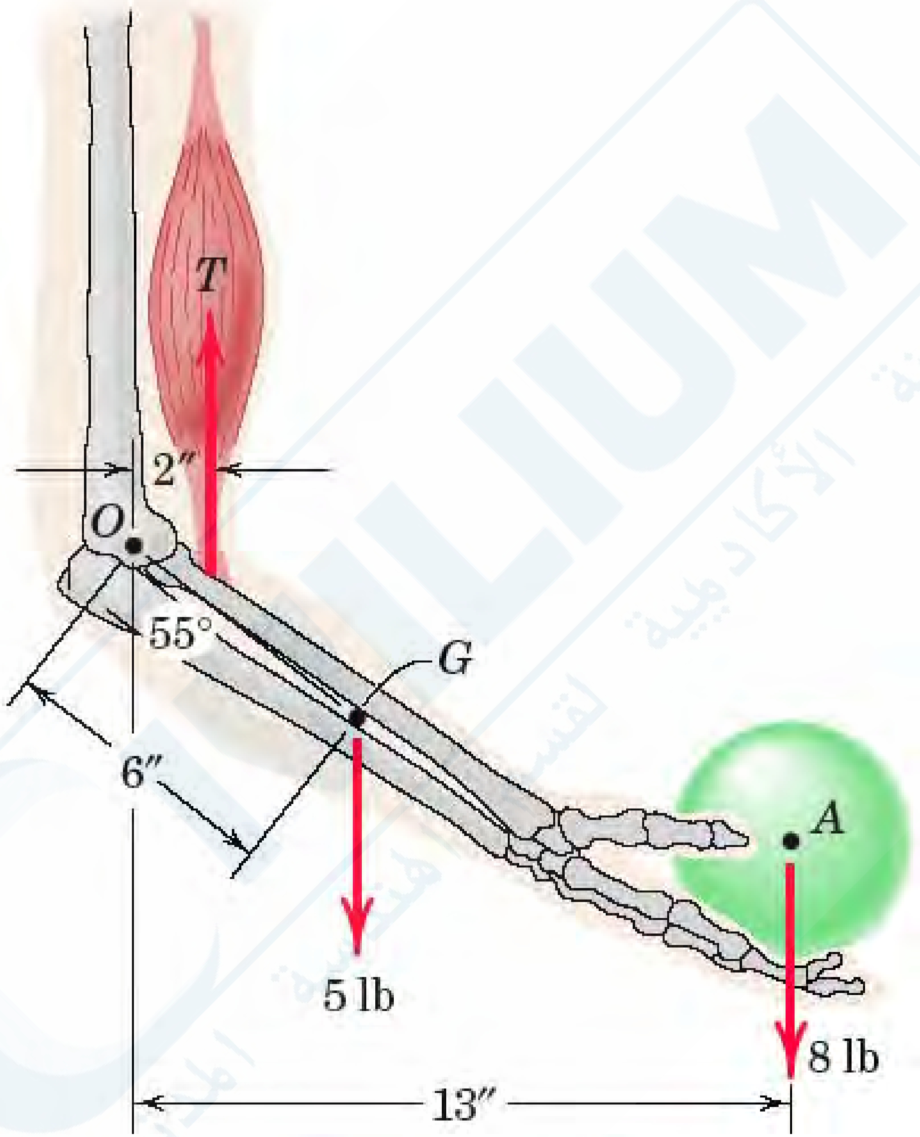


Problem 2/38

P_02_38

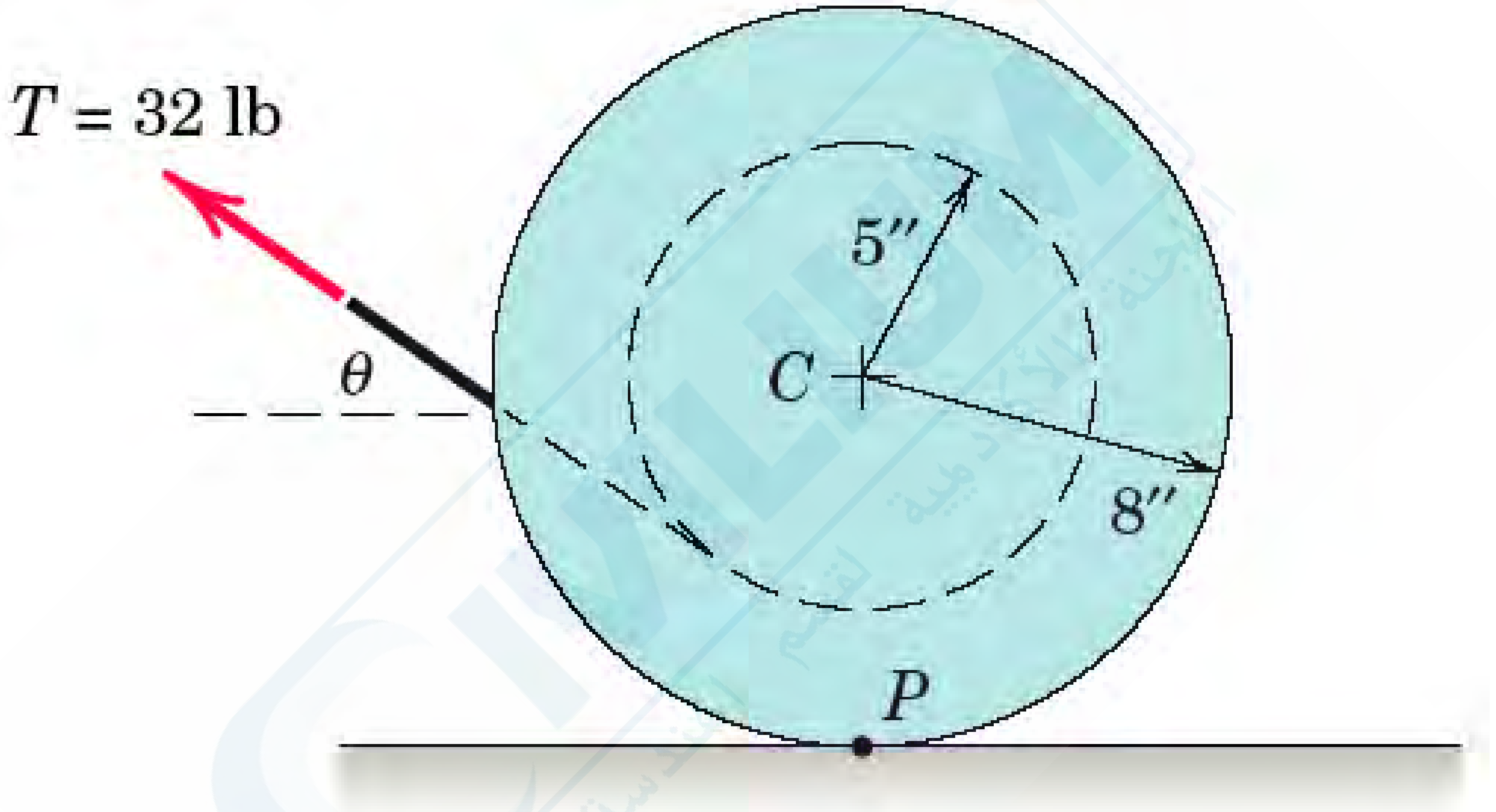


Problem 2/39



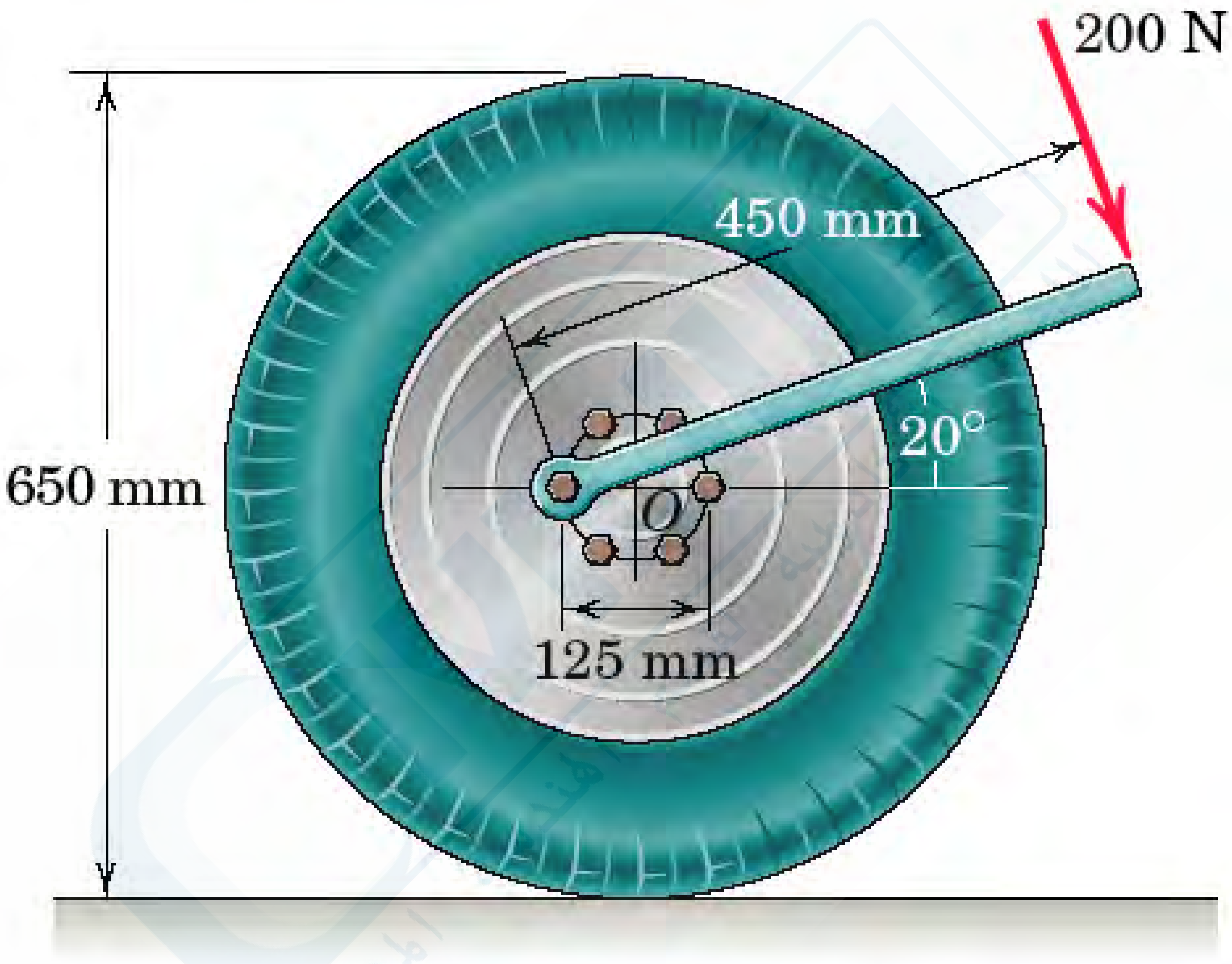
Problem 2/40

P 02 40



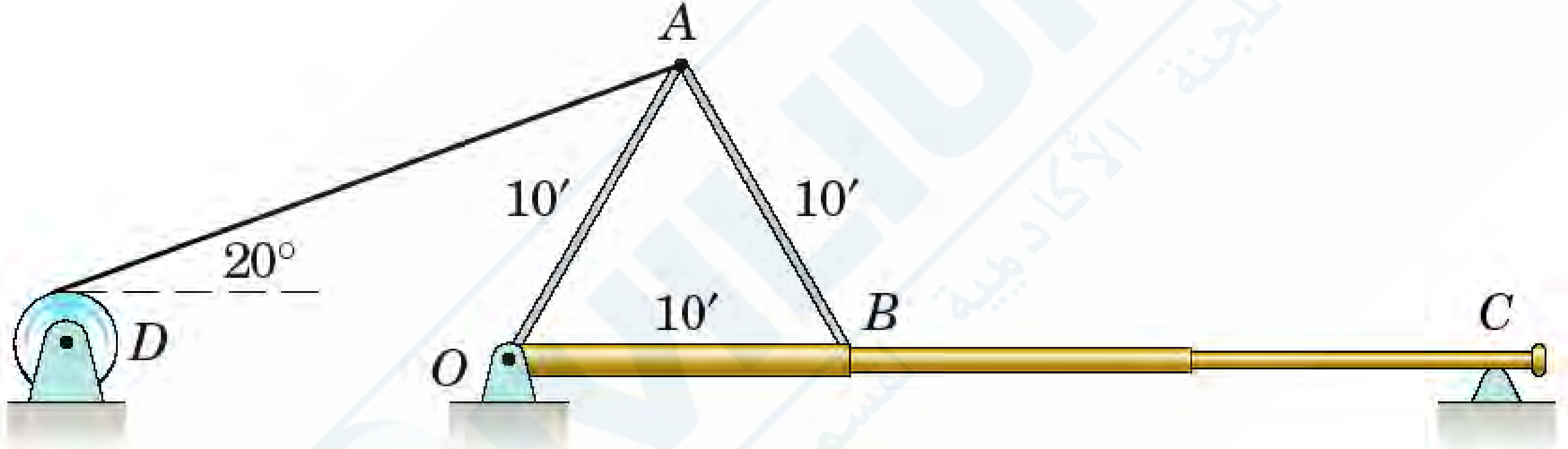
Problem 2/41

P_02_41



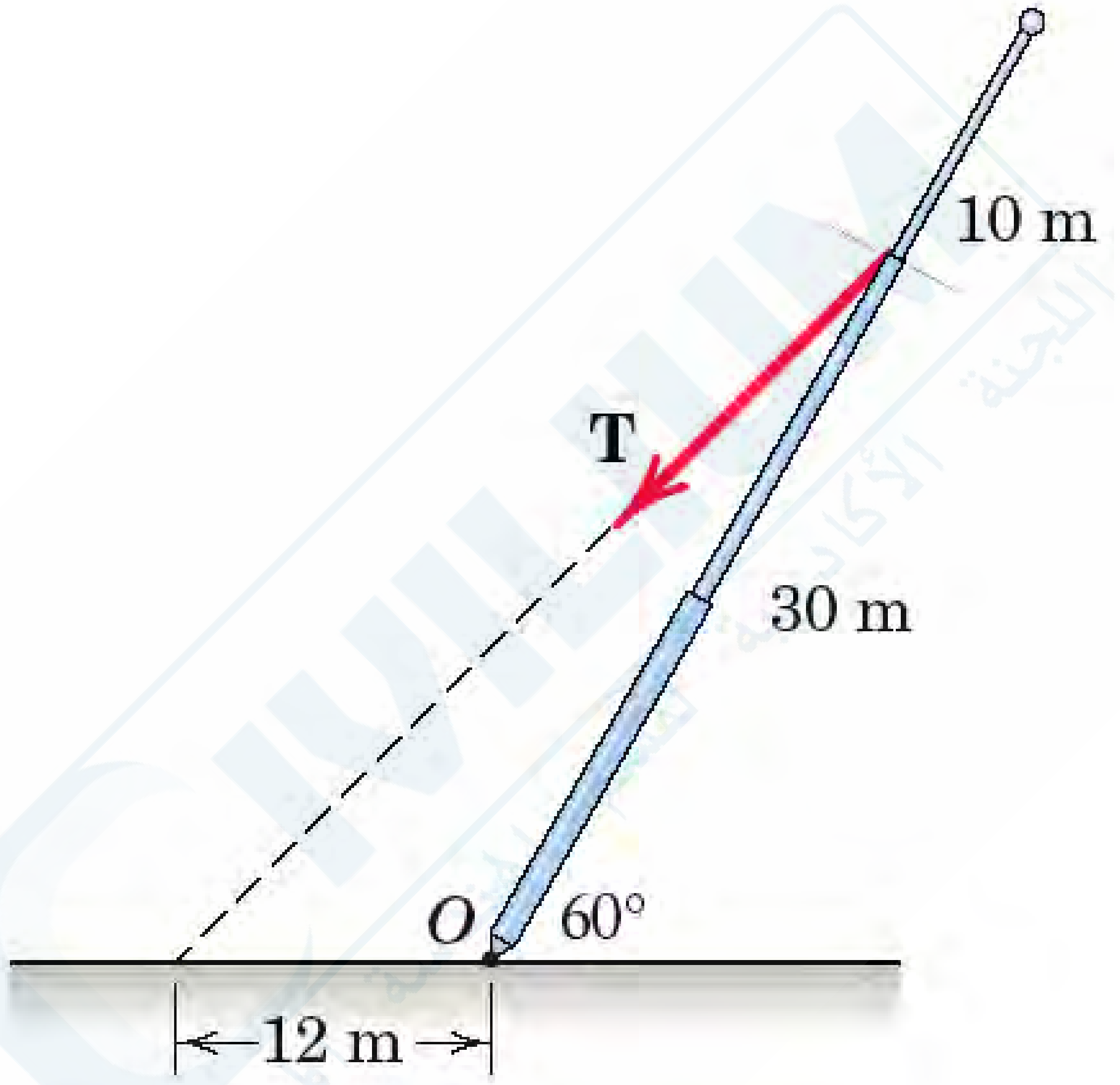
Problem 2/42

P_02_42



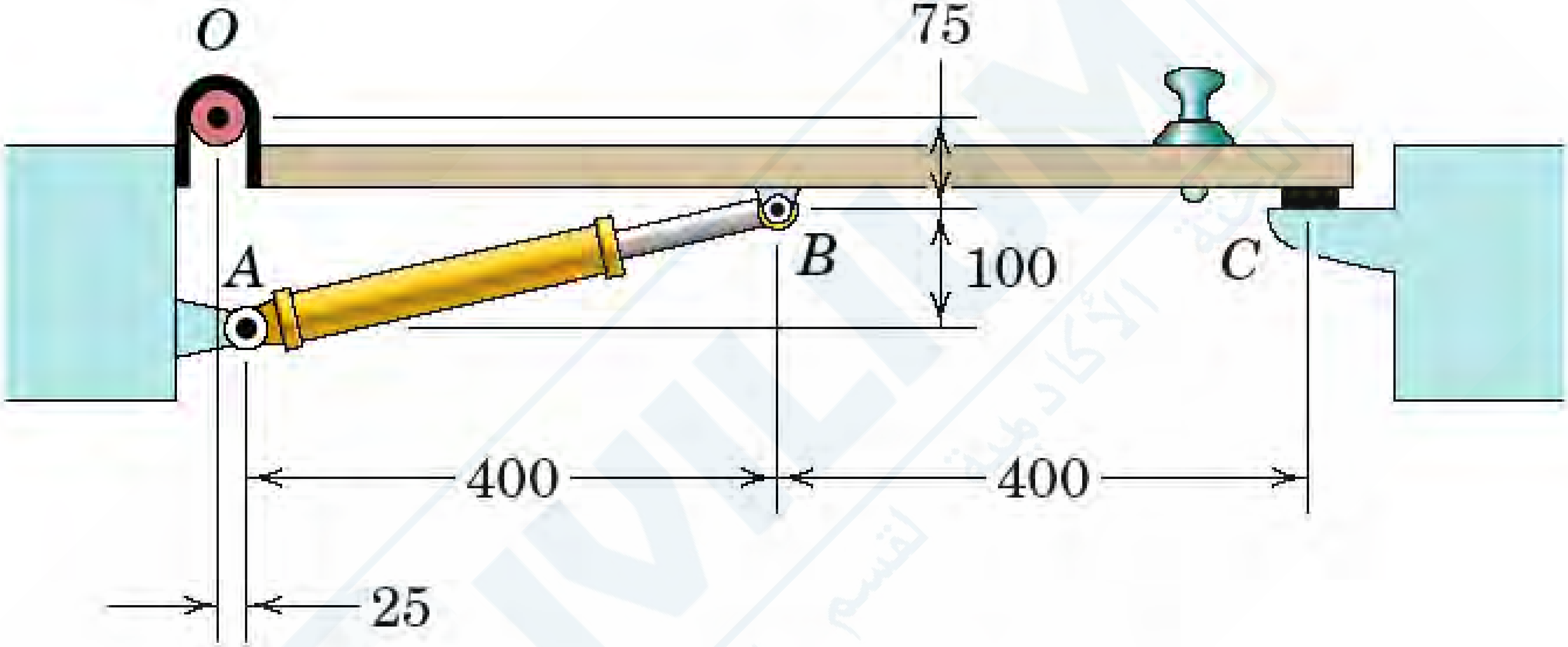
Problem 2/43

P_02_43



Problem 2/45

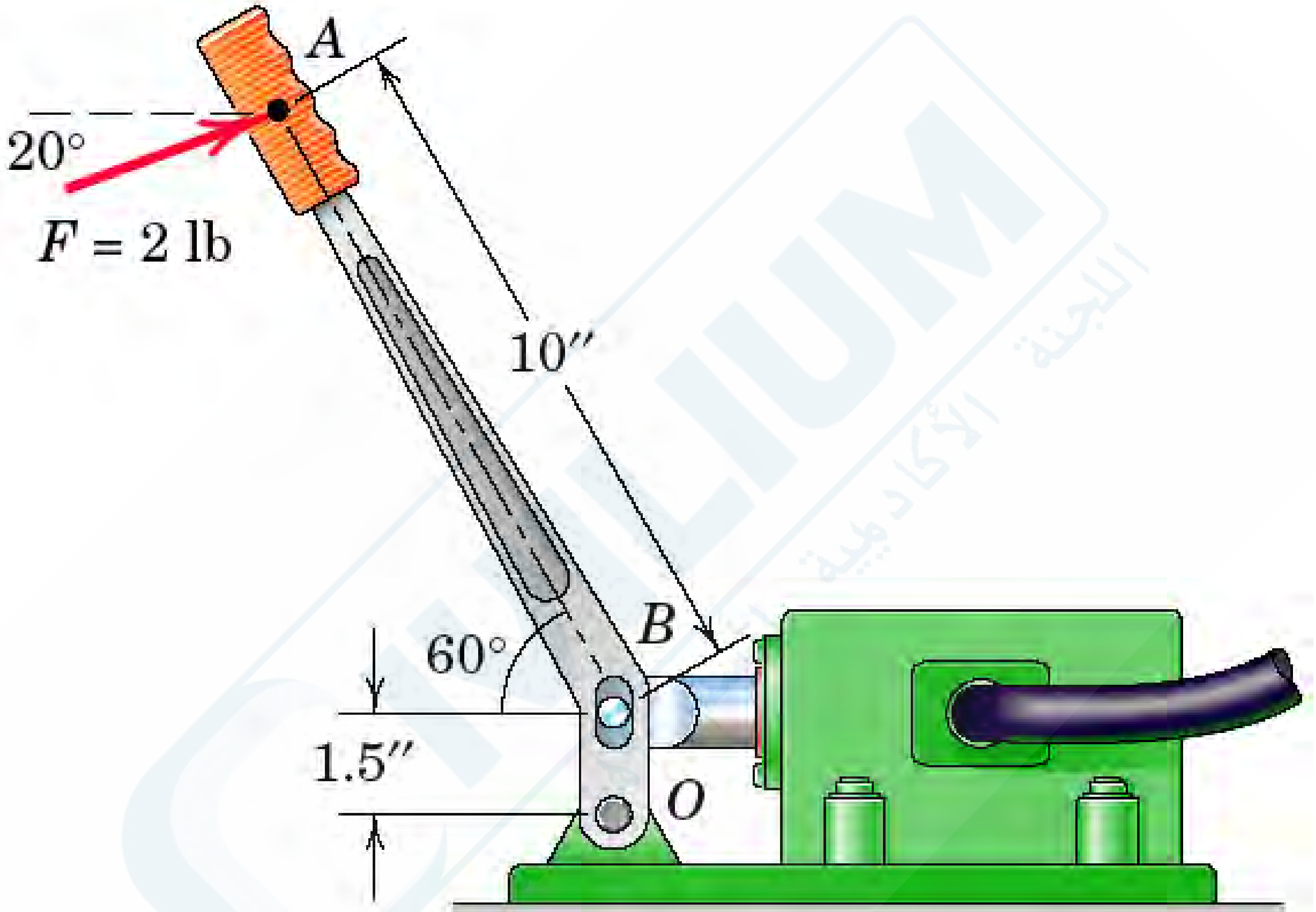
P_02_45



Dimensions in millimeters

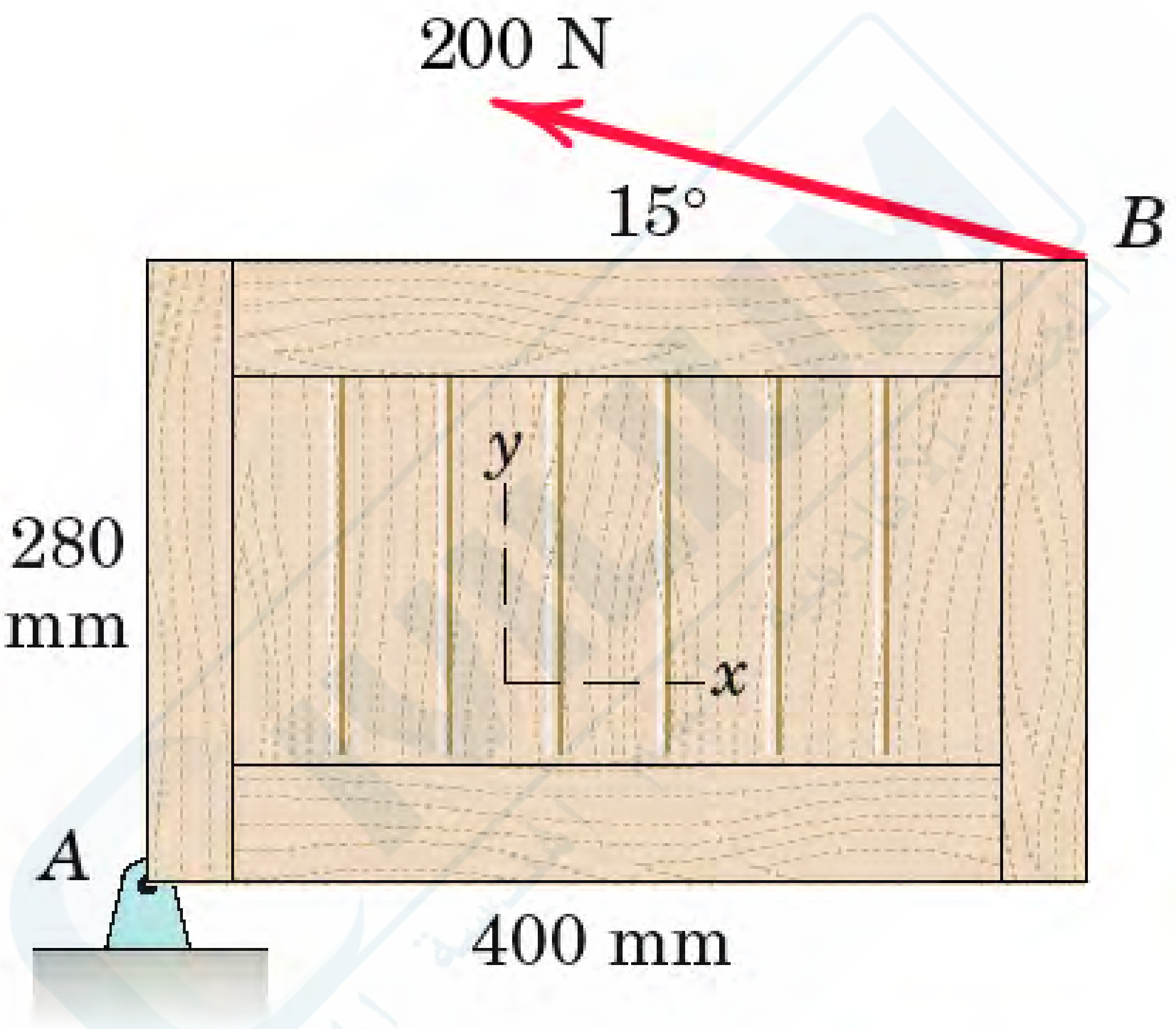
Problem 2/46

P_02_46



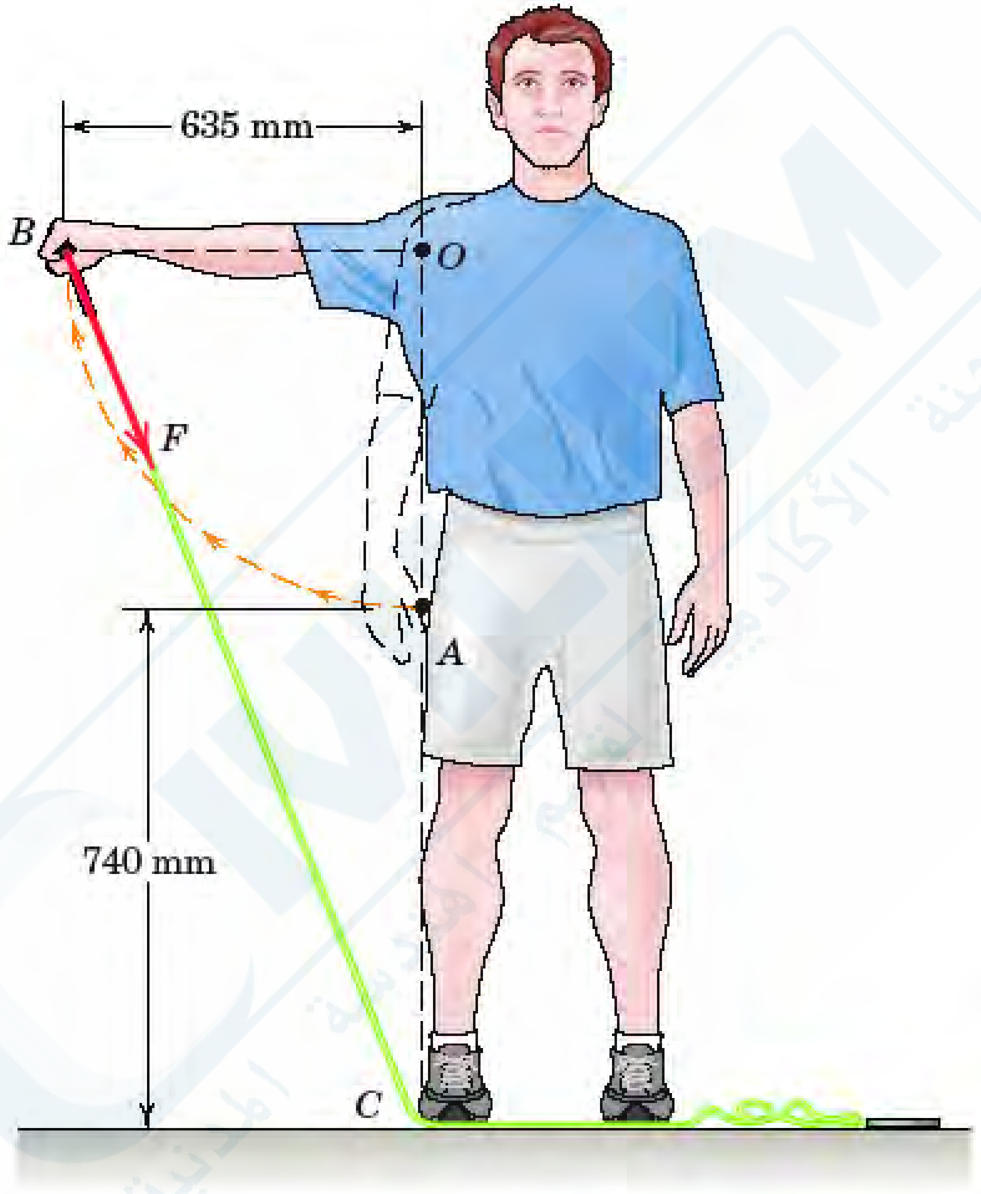
Problem 2/47

P 02 47



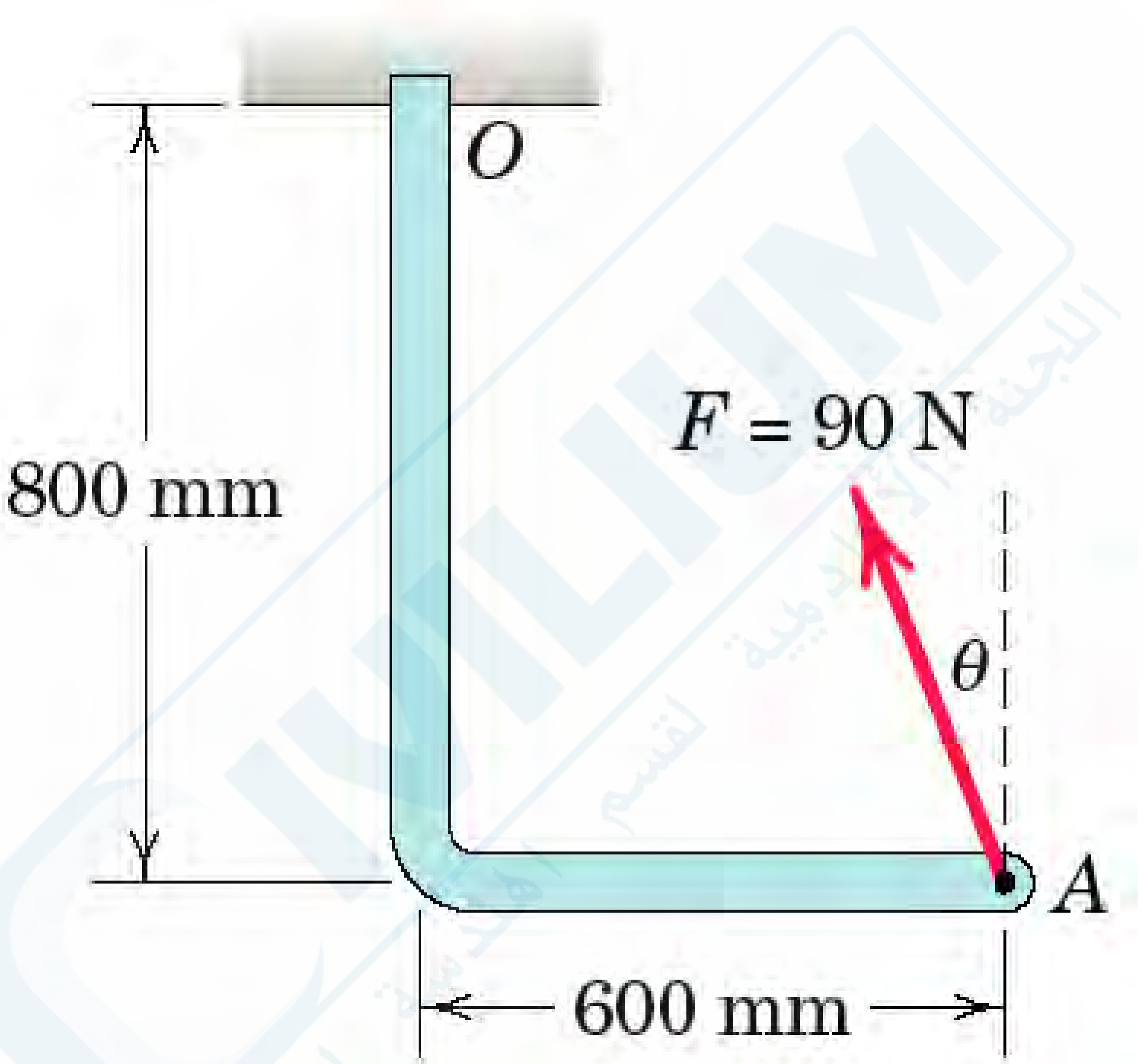
Problem 2/48

P_02_48



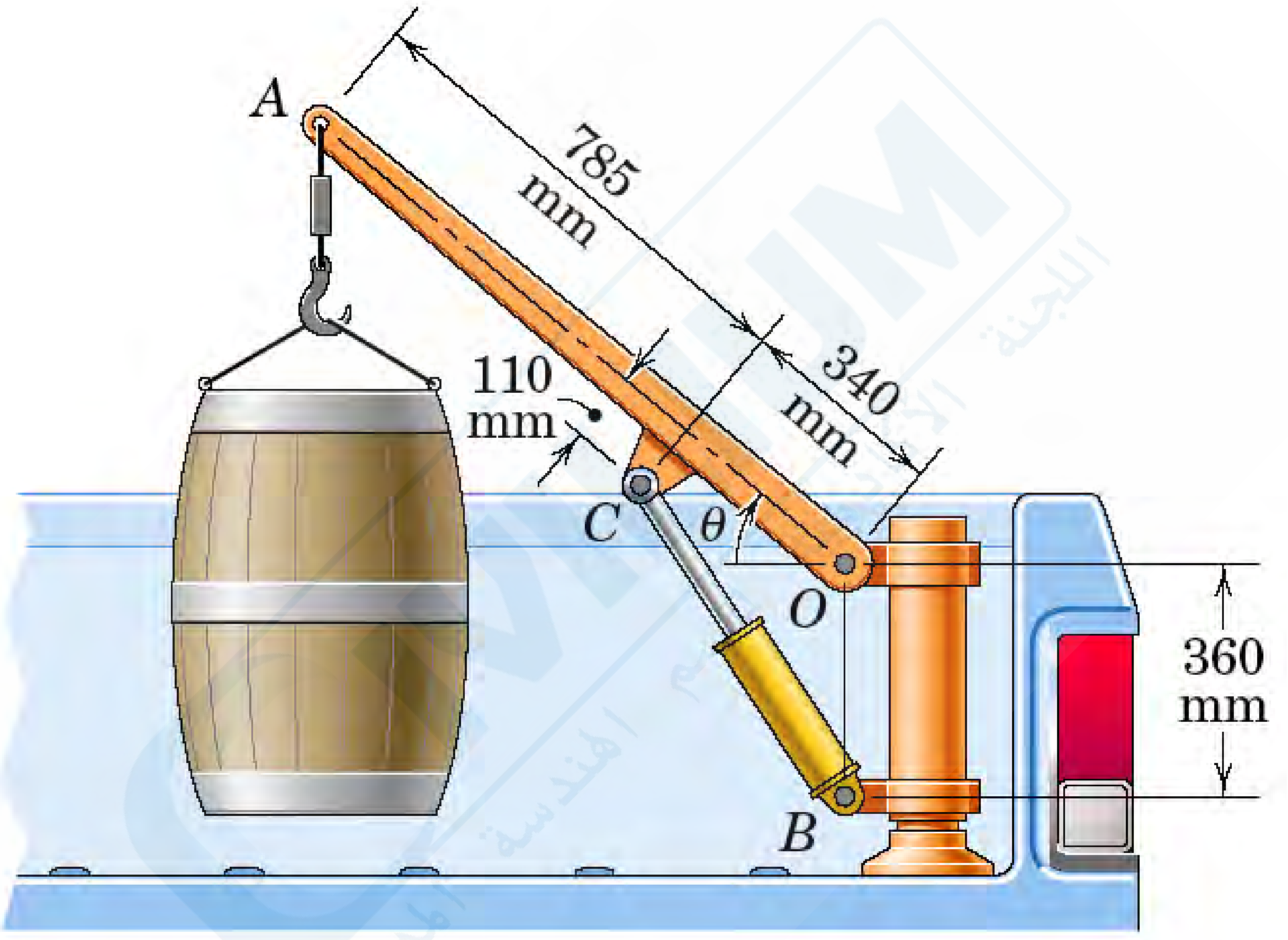
Problem 2/49

P 02 49



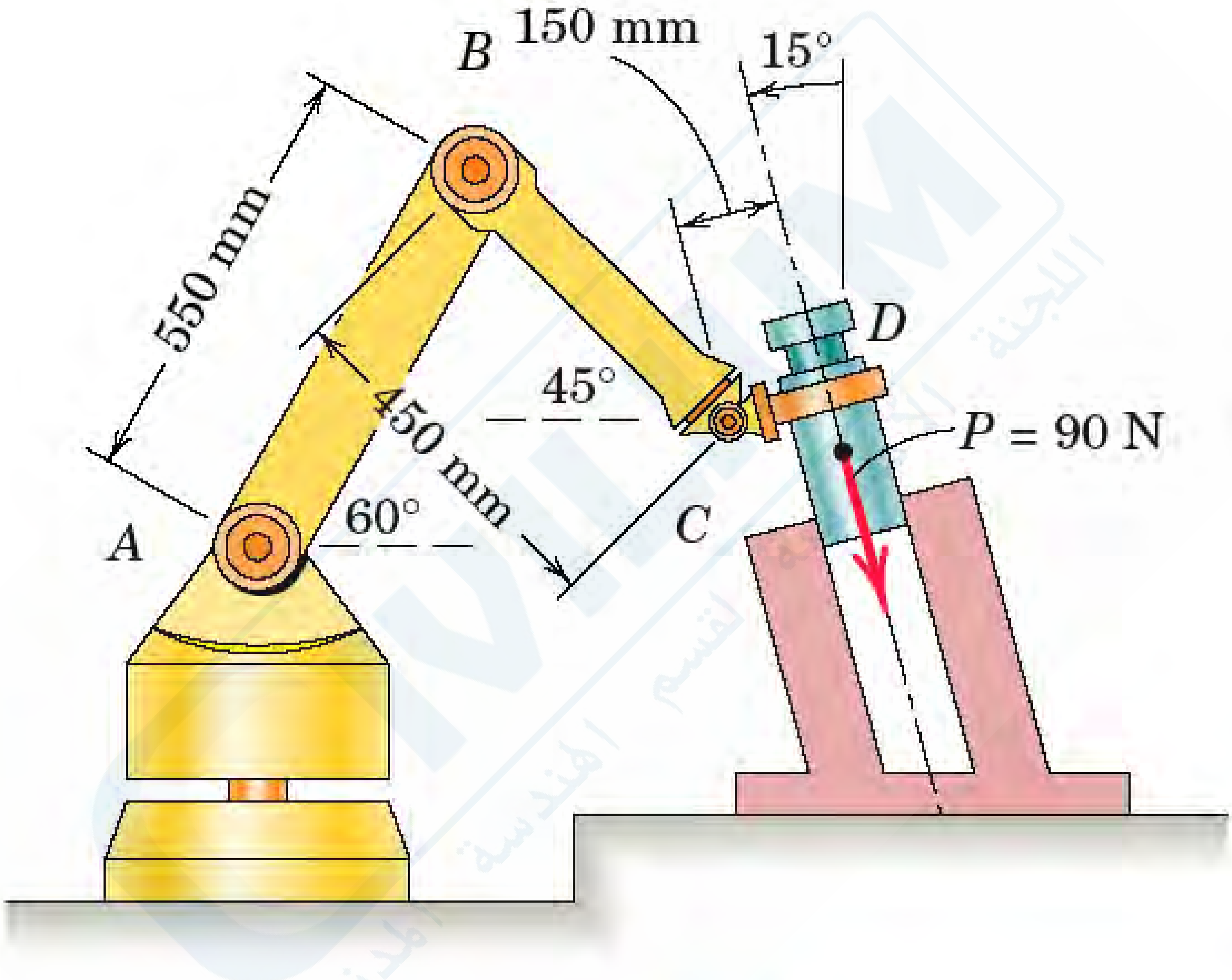
Problem 2/50

P_02_50



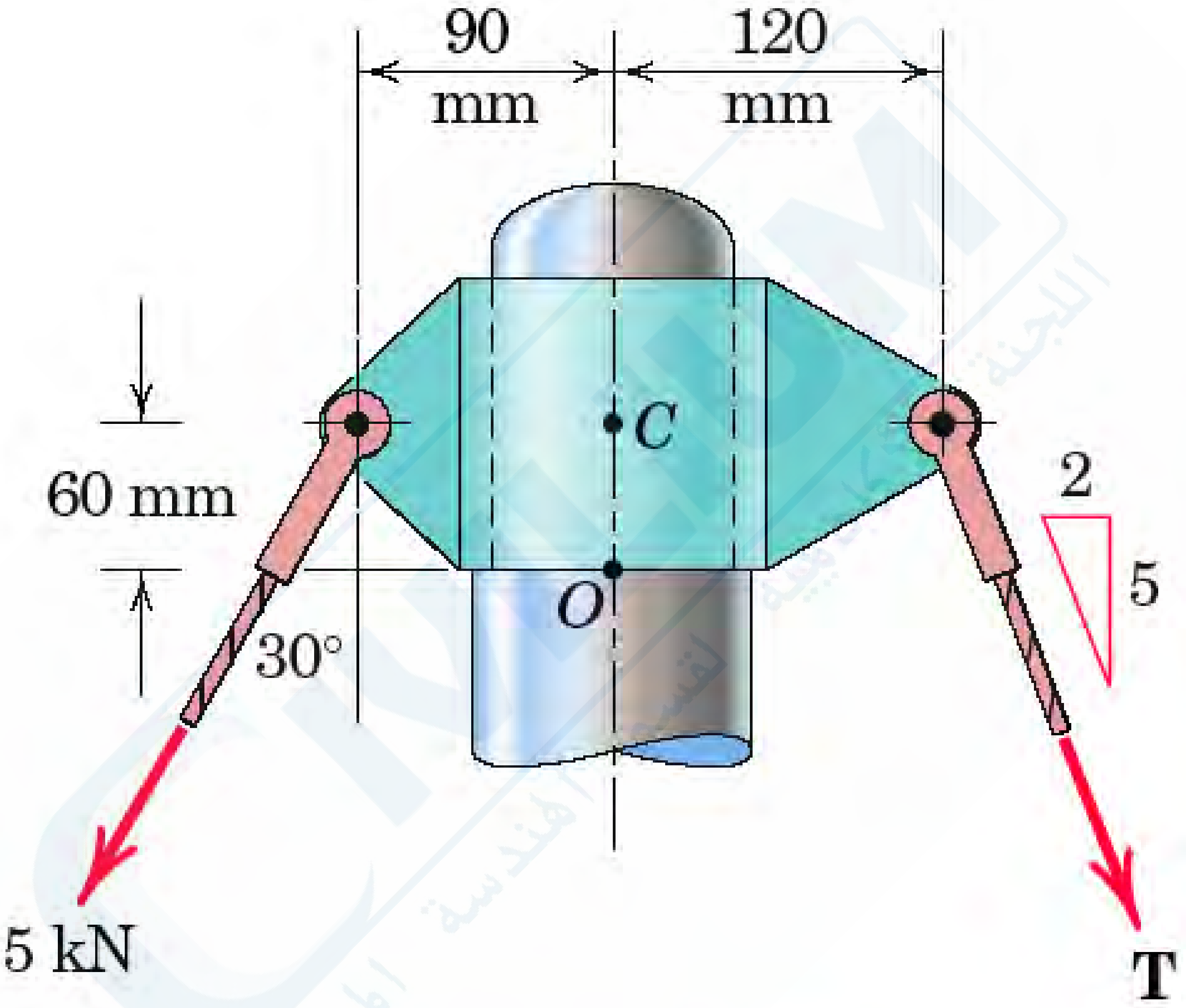
Problem 2/51

P_02_51



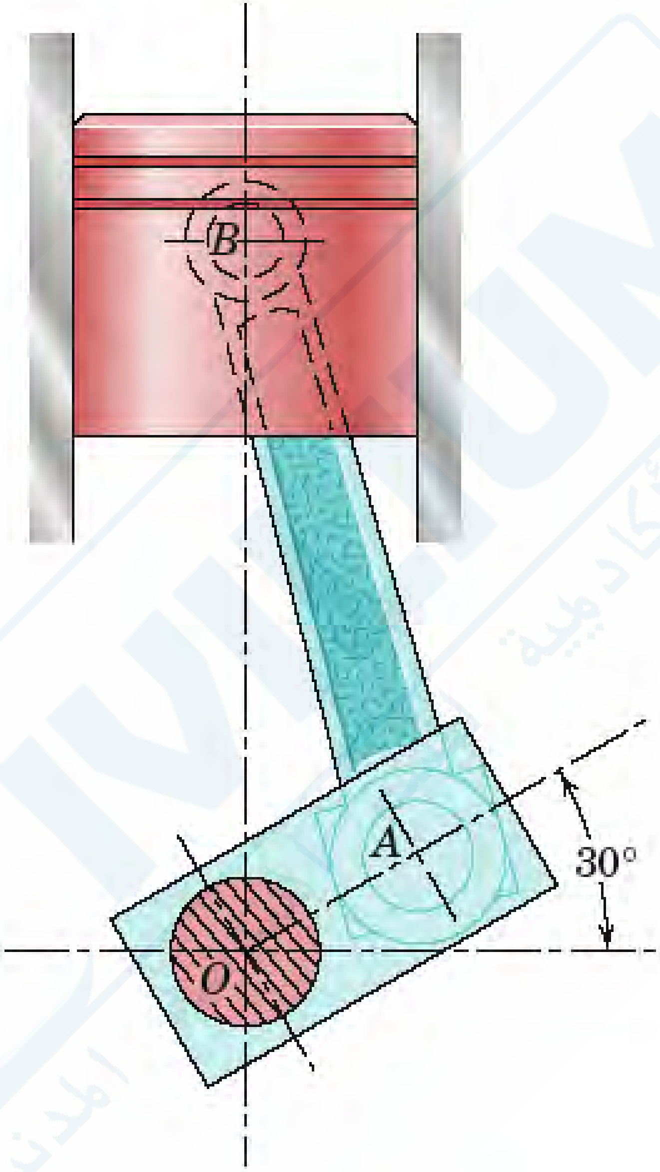
Problem 2/52

P 02 52



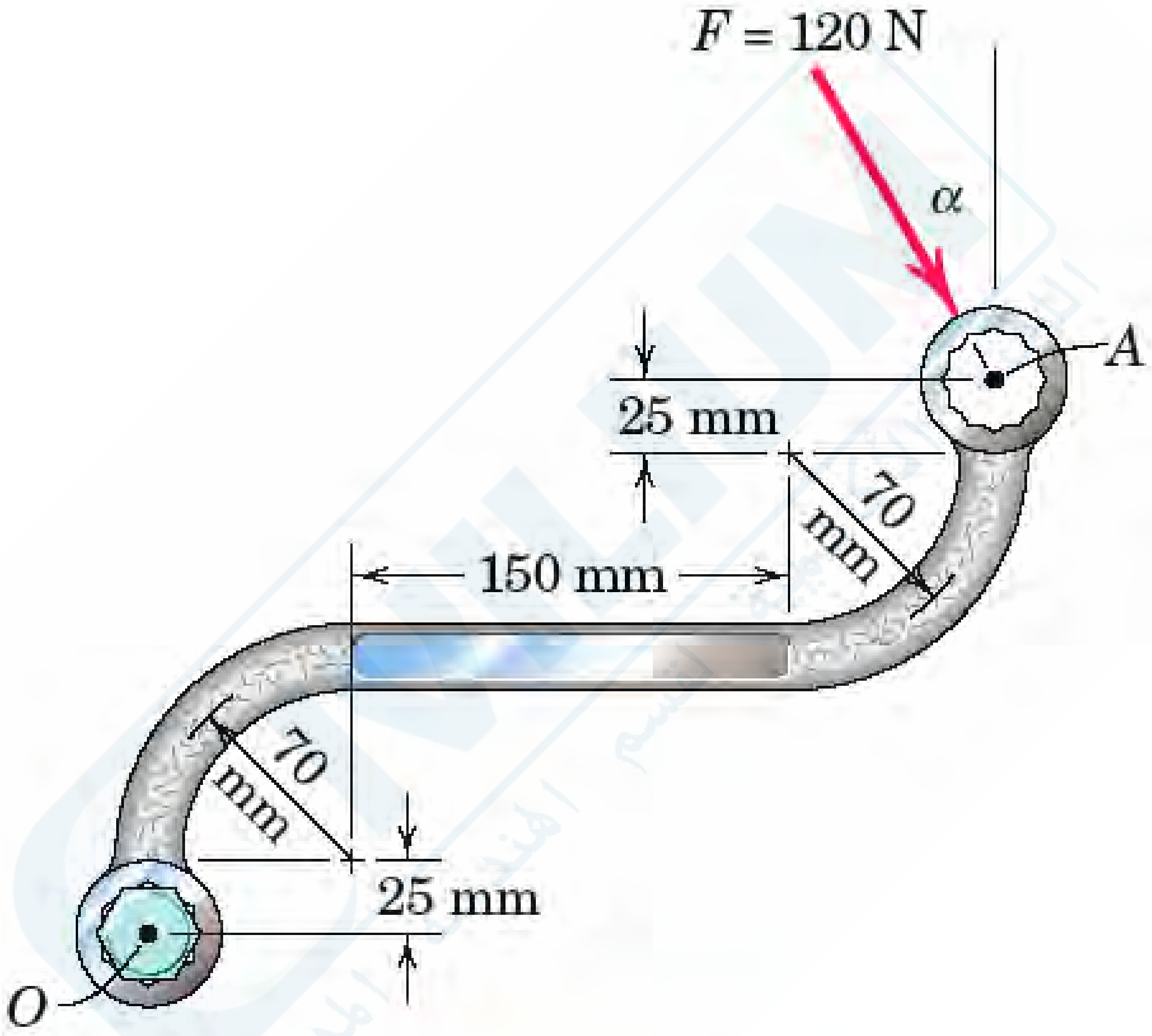
Problem 2/53

P 02 53



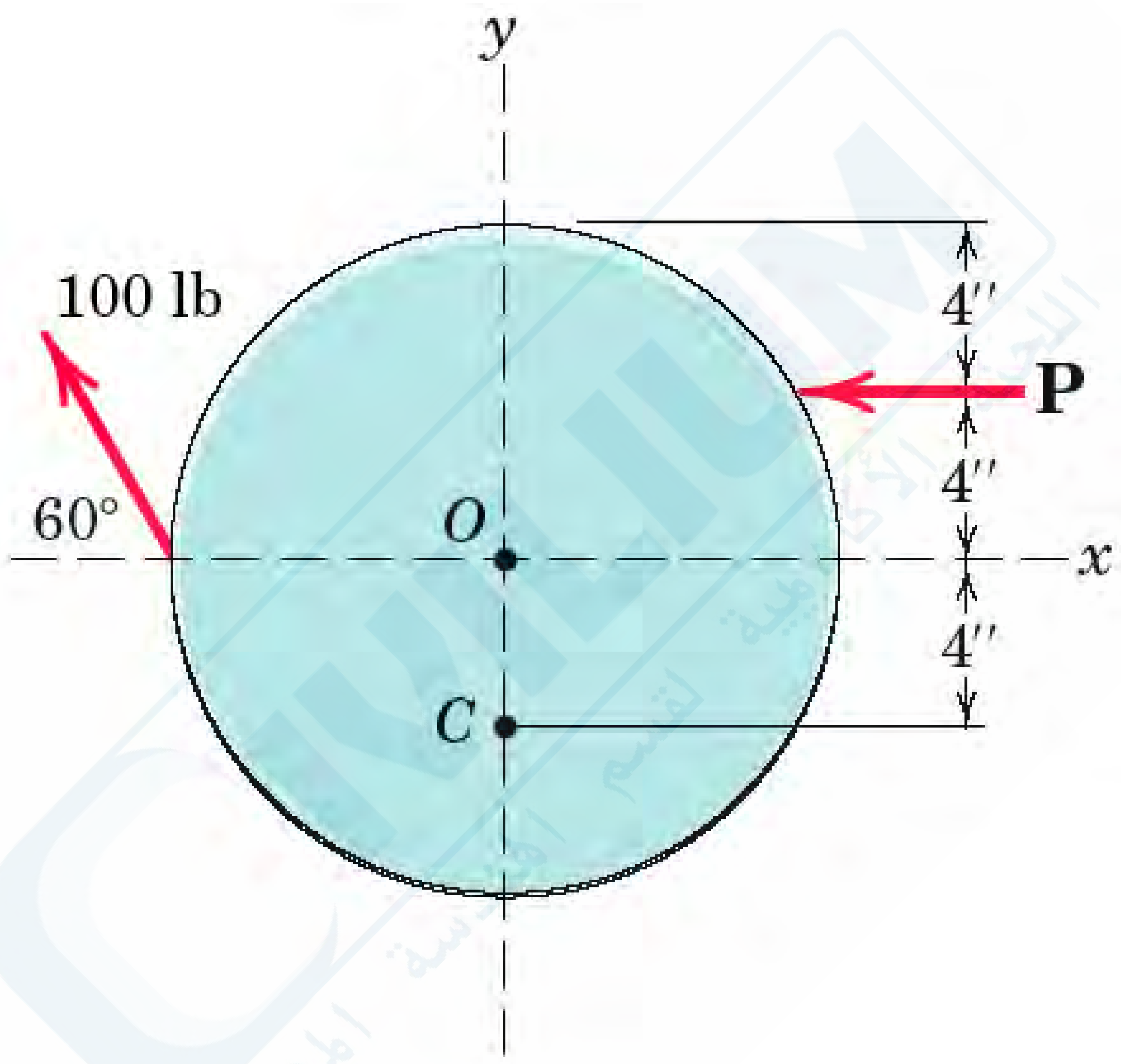
Problem 2/54

P_02_54



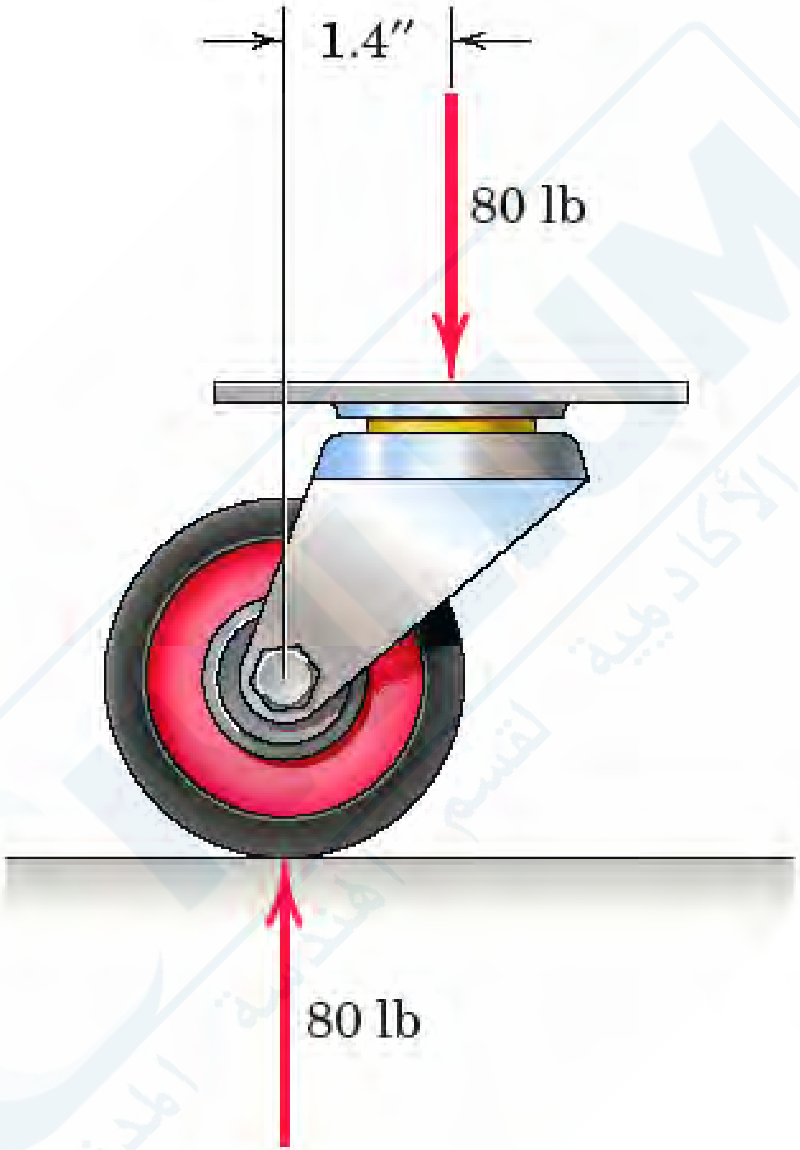
Problem 2/55

P_02_55



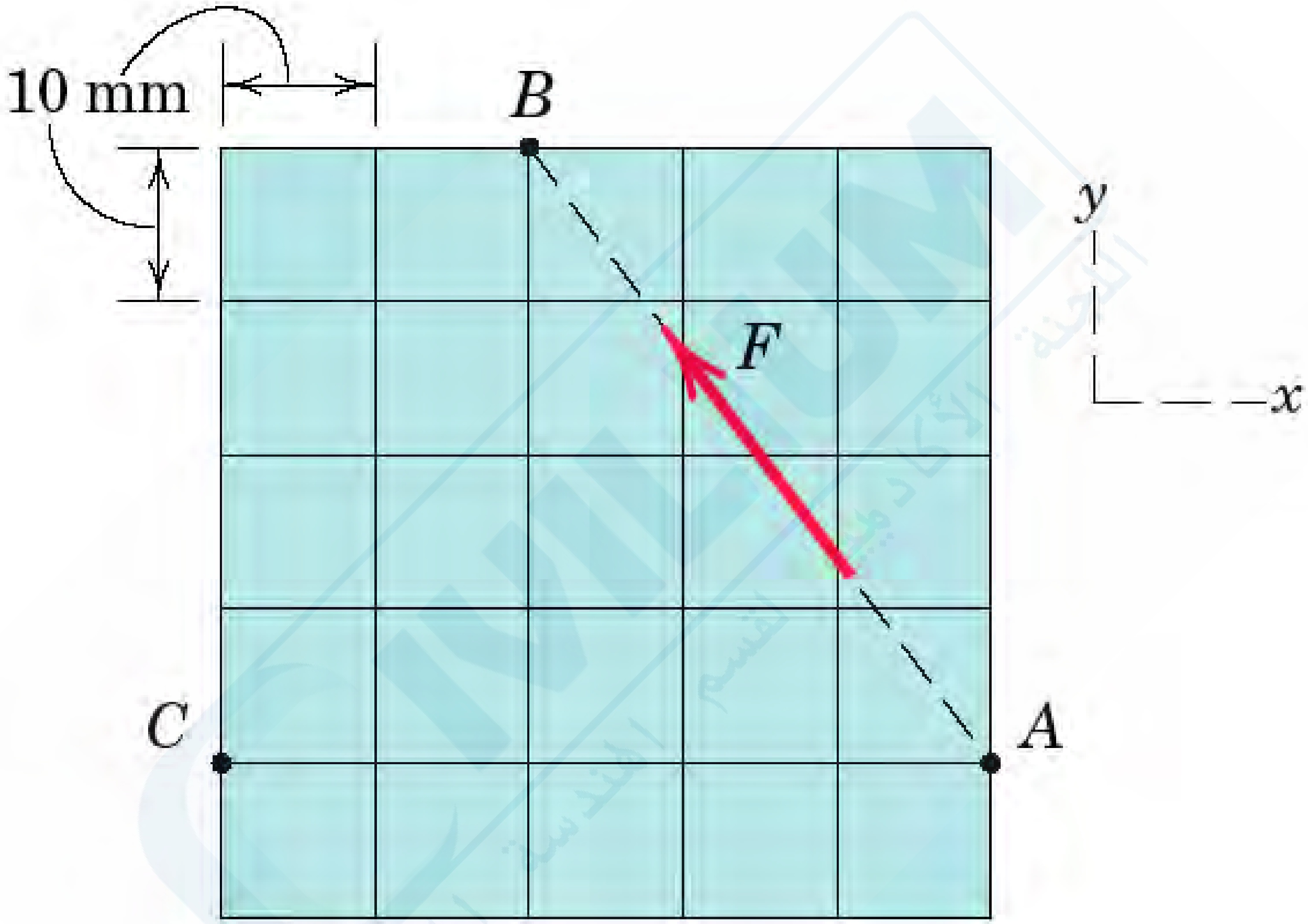
Problem 2/56

P_02_56



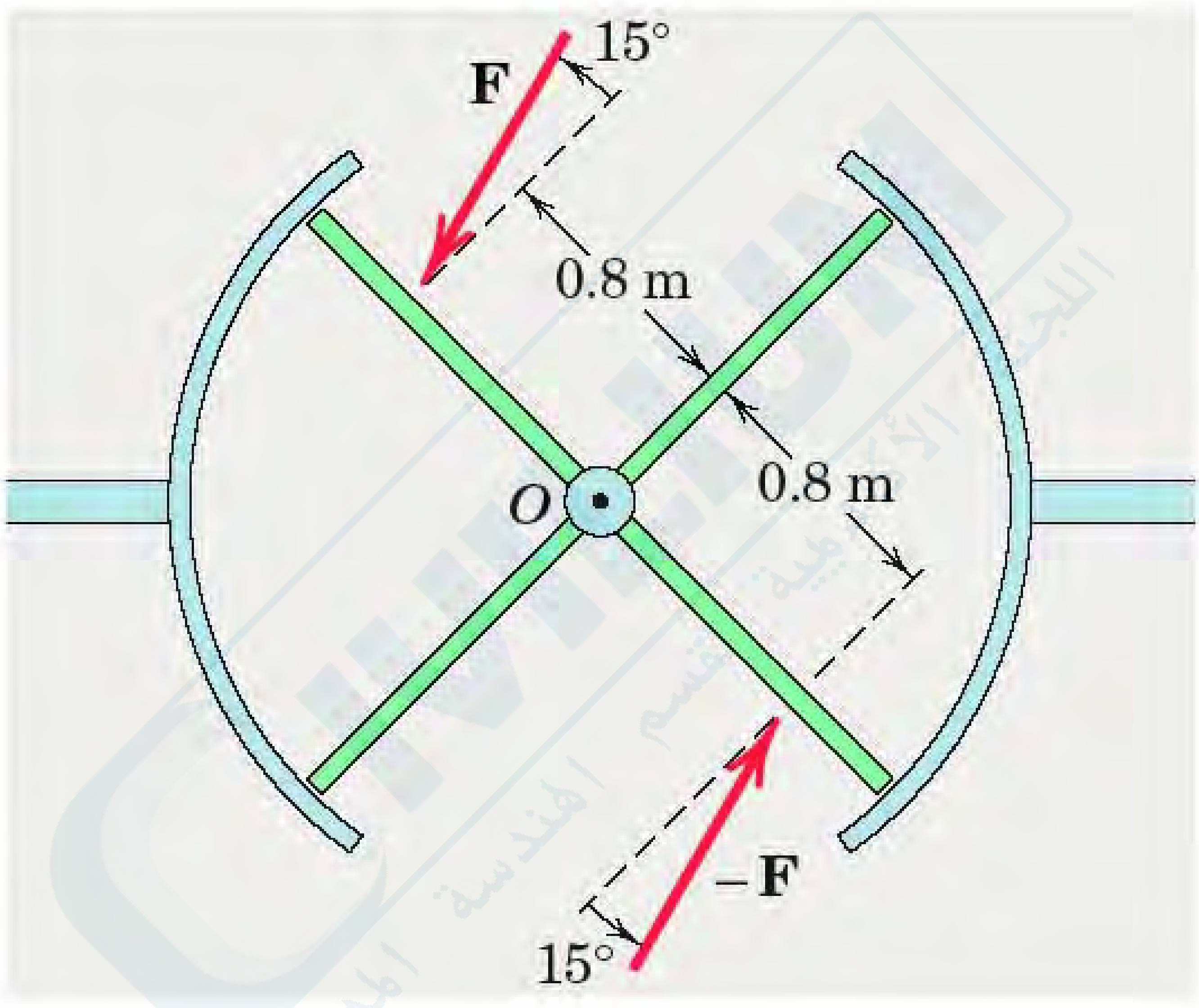
Problem 2/57

P_02_57



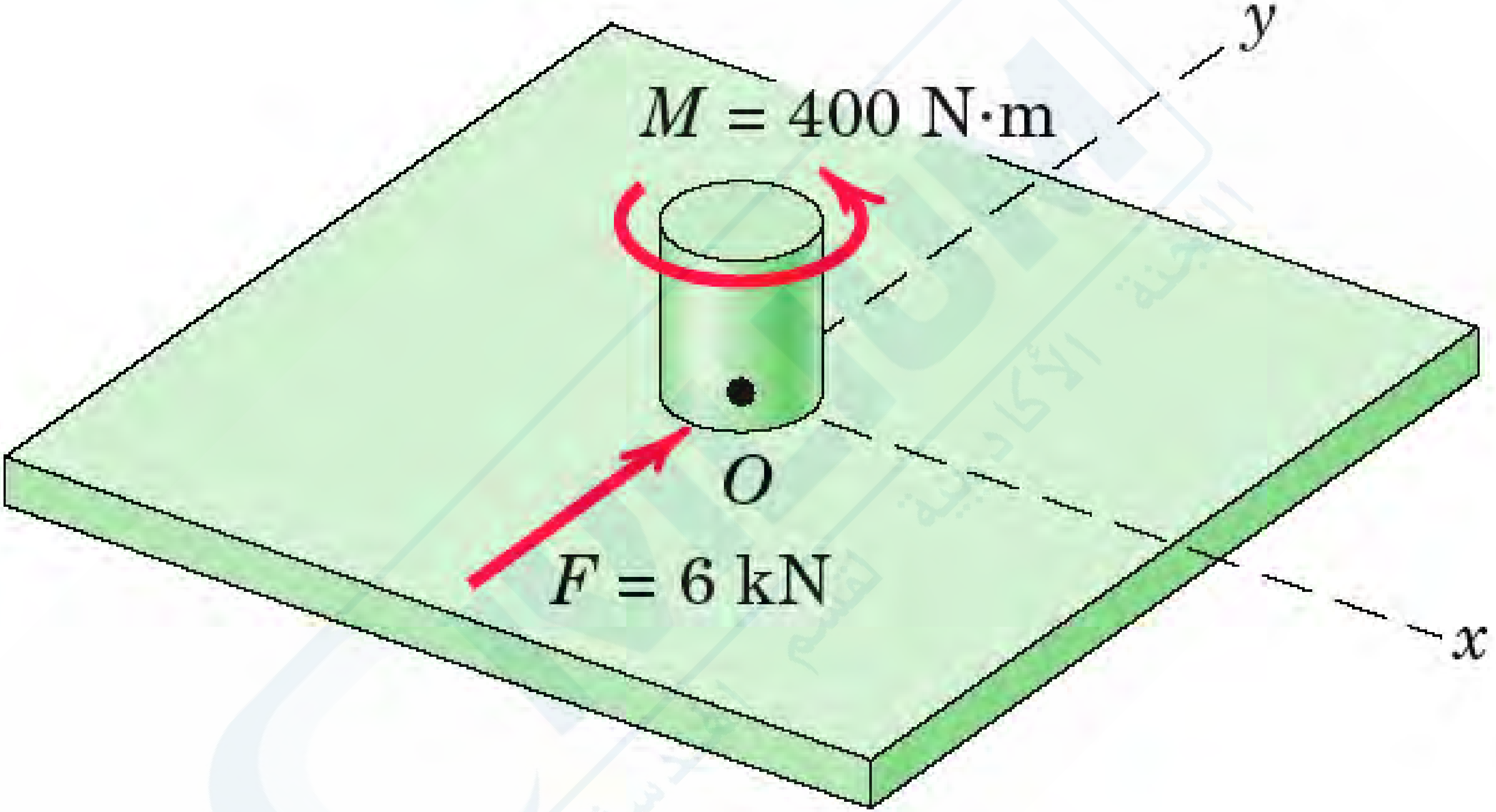
Problem 2/58

P_02_58



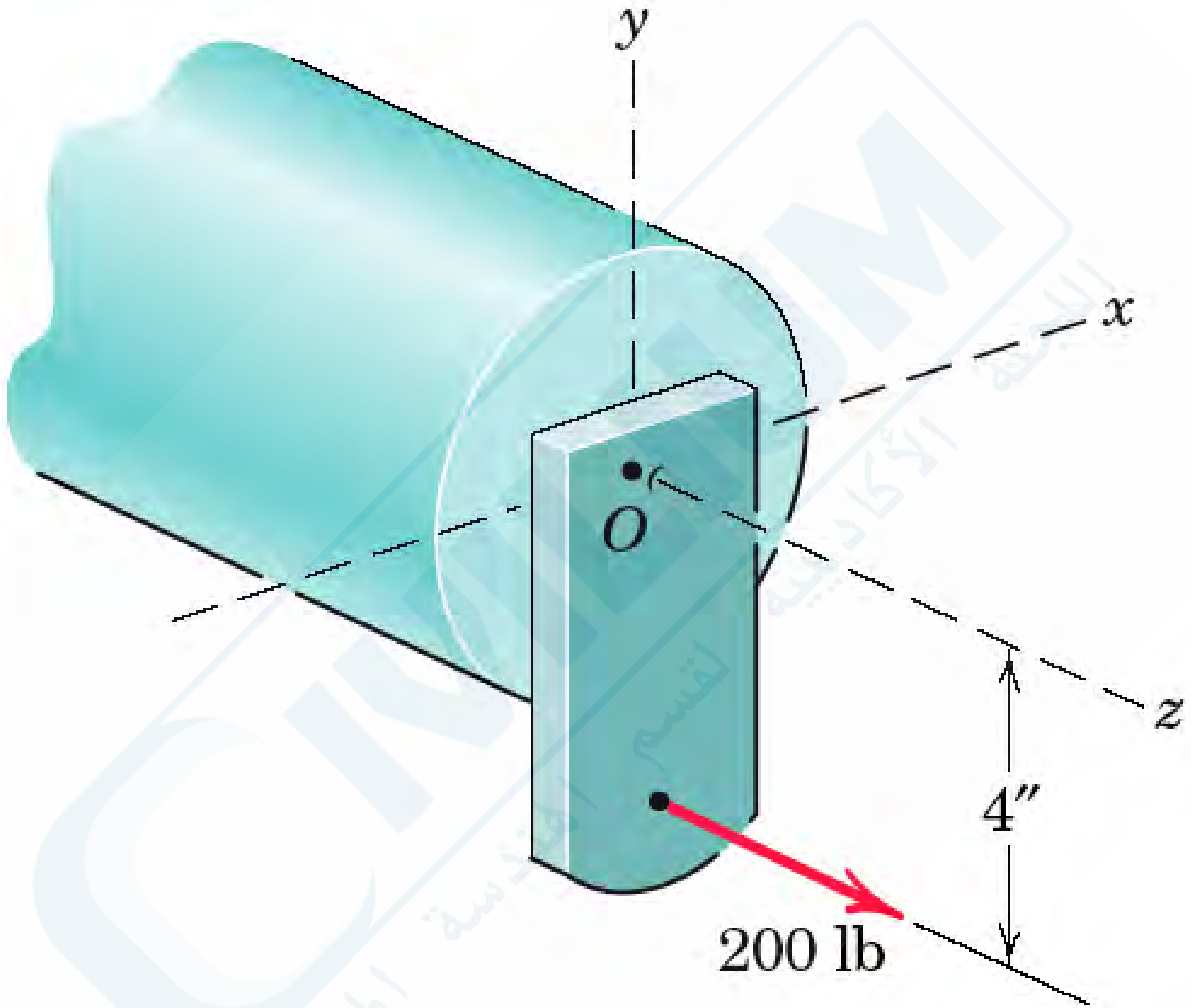
Problem 2/59

P_02_59



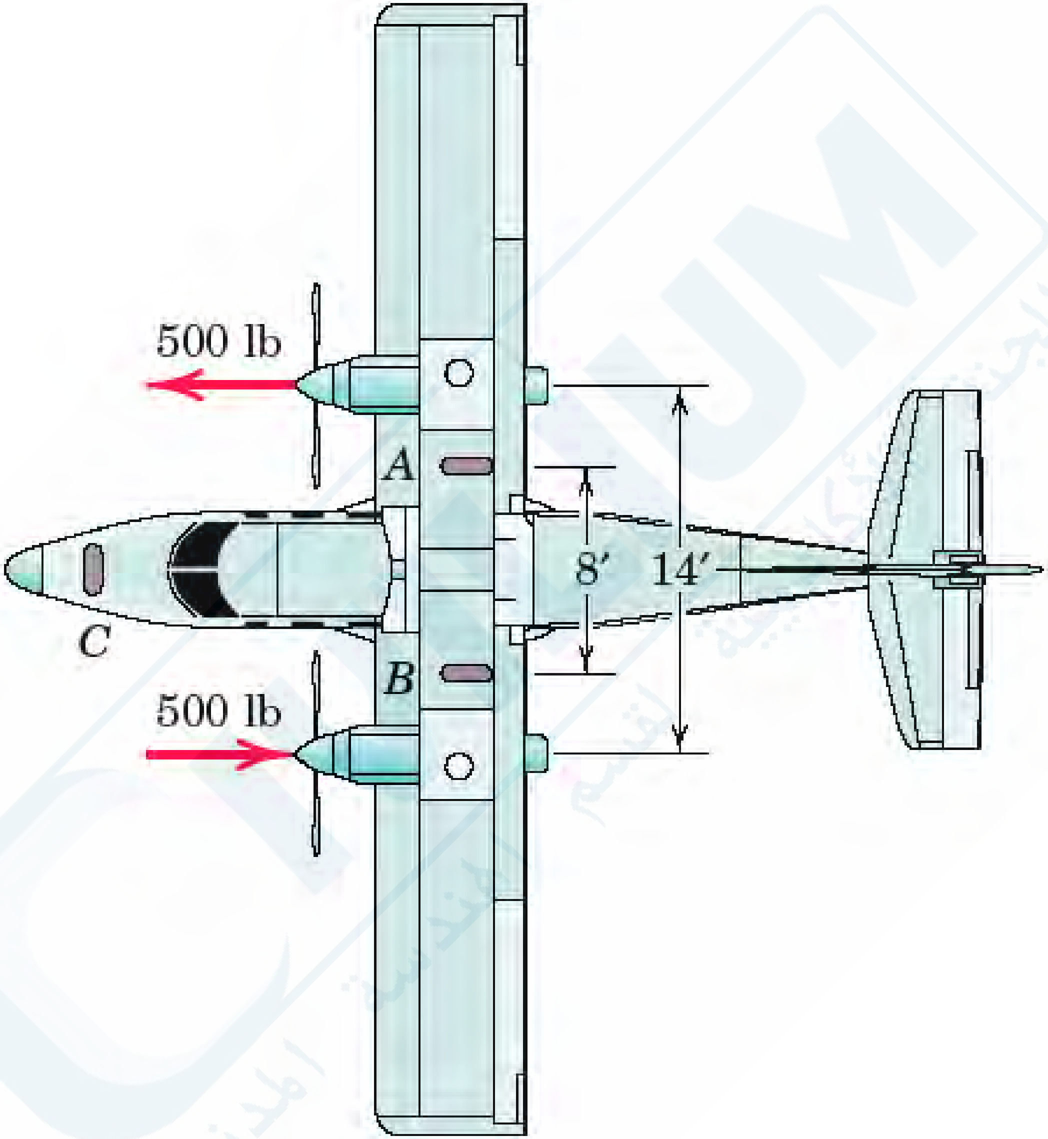
Problem 2/60

P_02_60



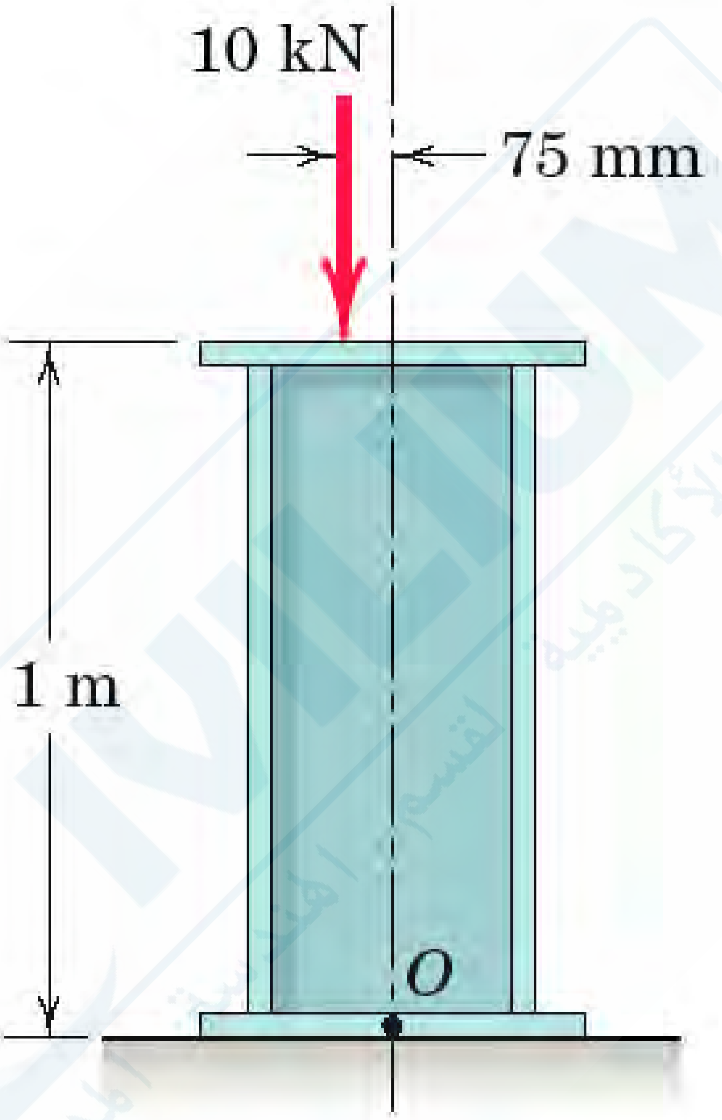
Problem 2/61

P_02_61



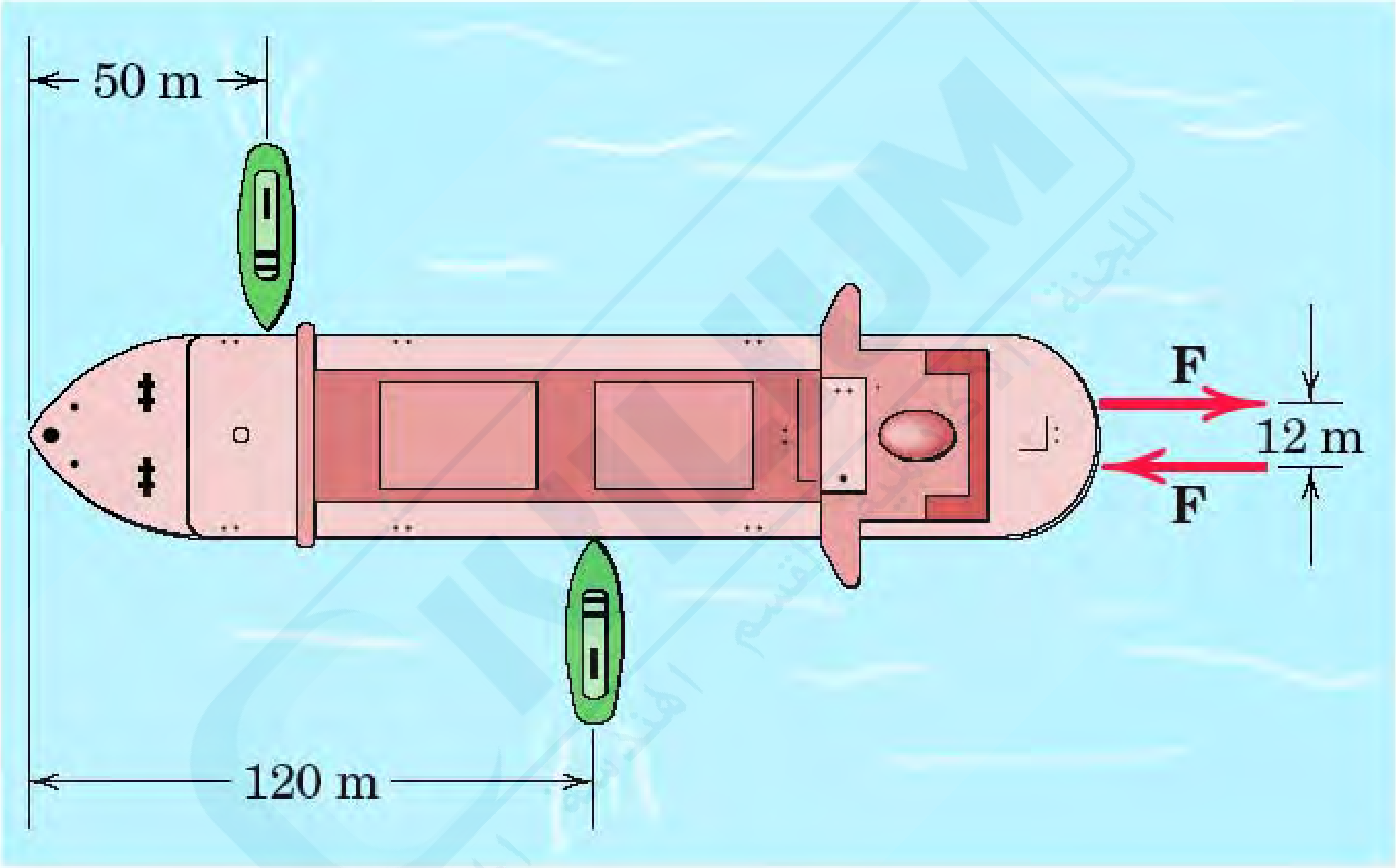
Problem 2/62

P 02 62



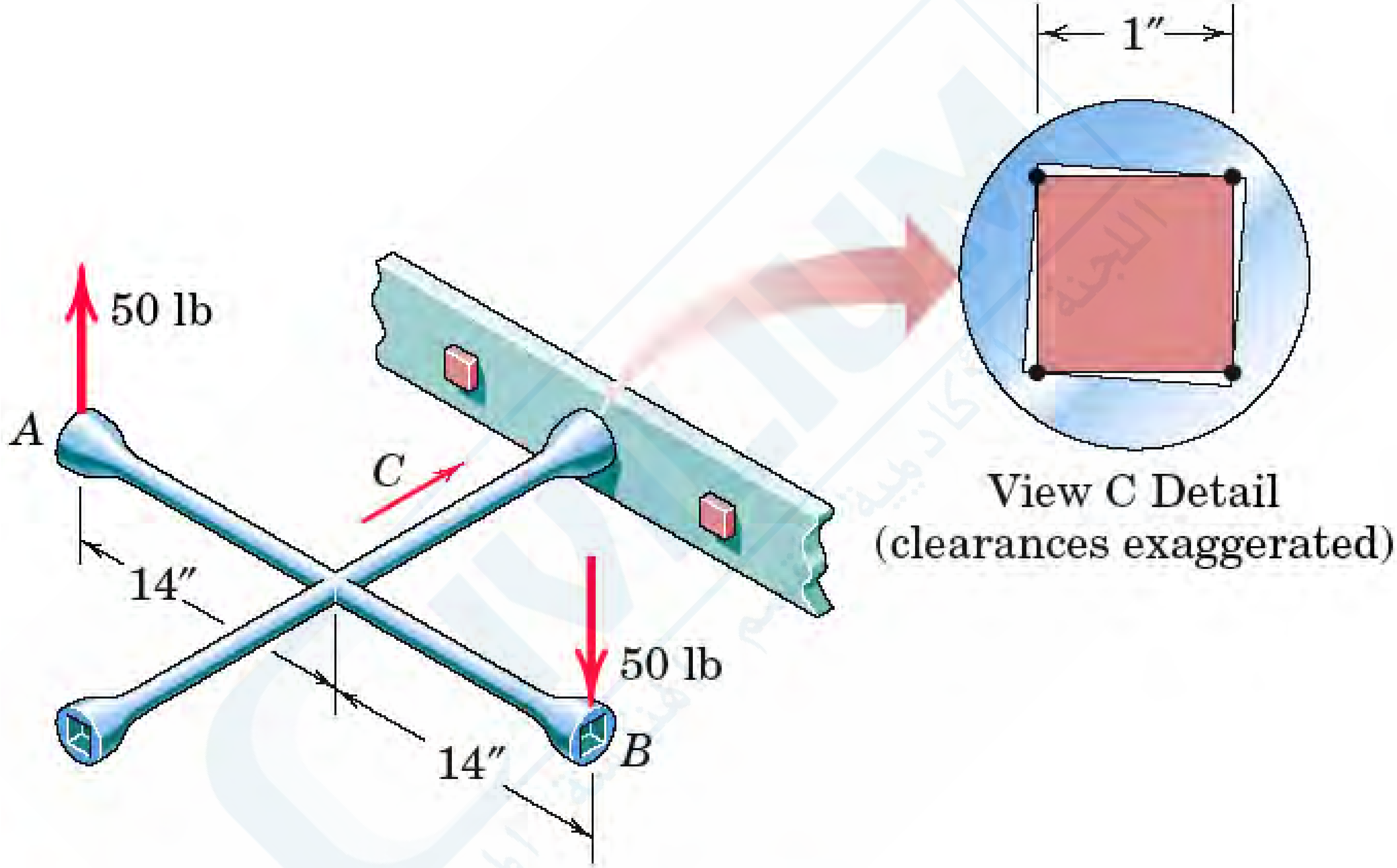
Problem 2/63

P_02_63



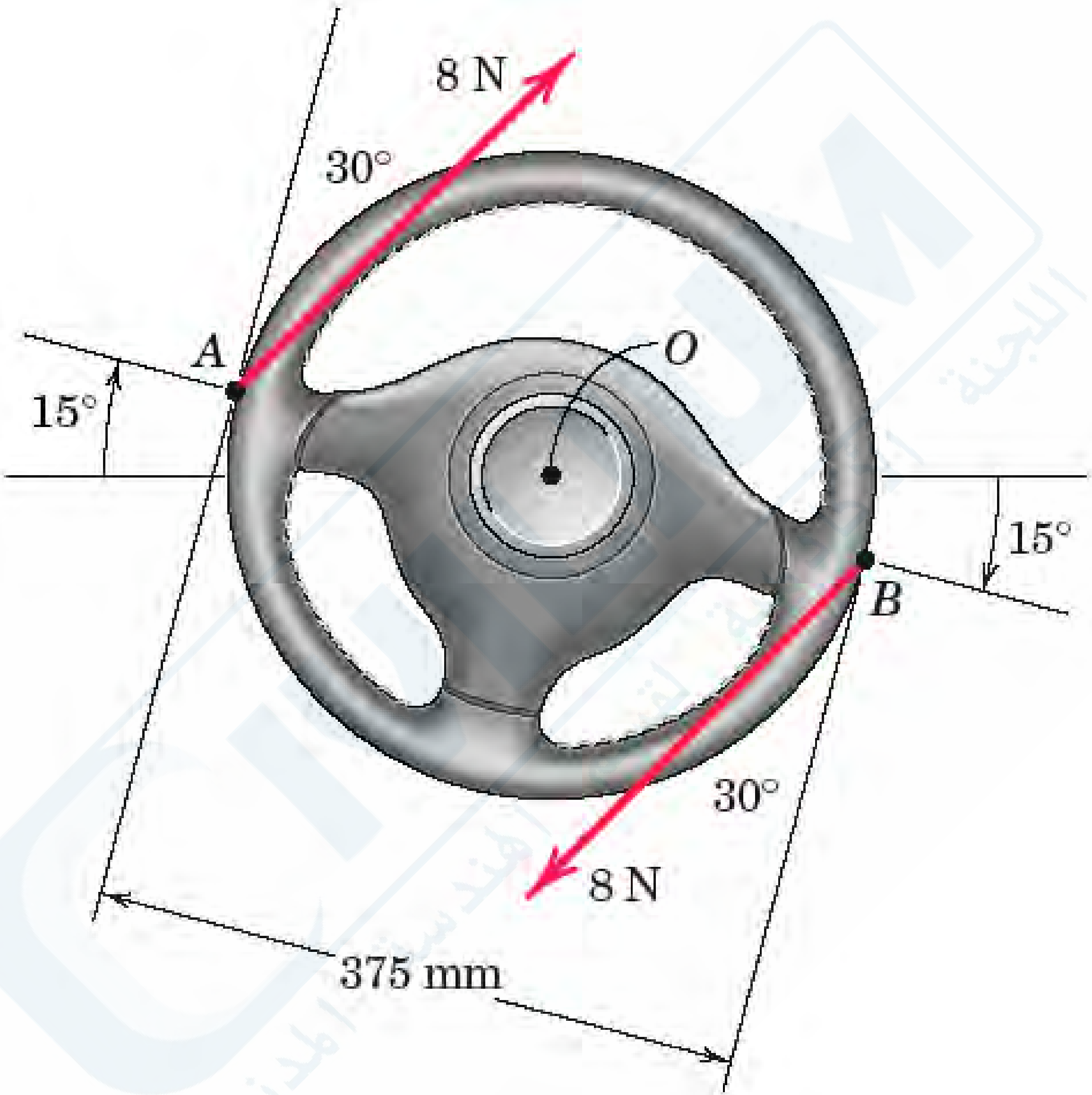
Problem 2/64

P_02_64



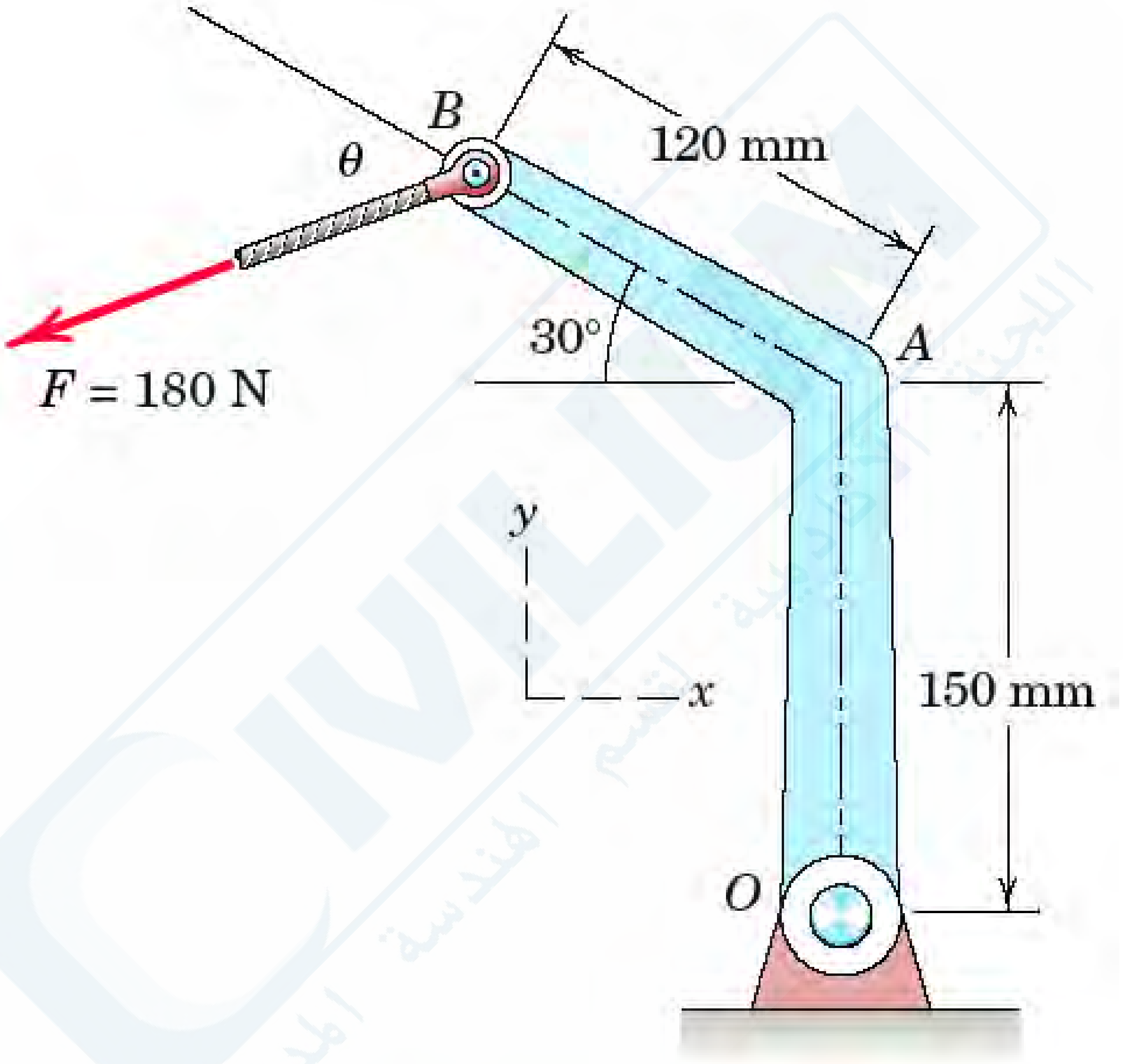
Problem 2/65

P_02_65



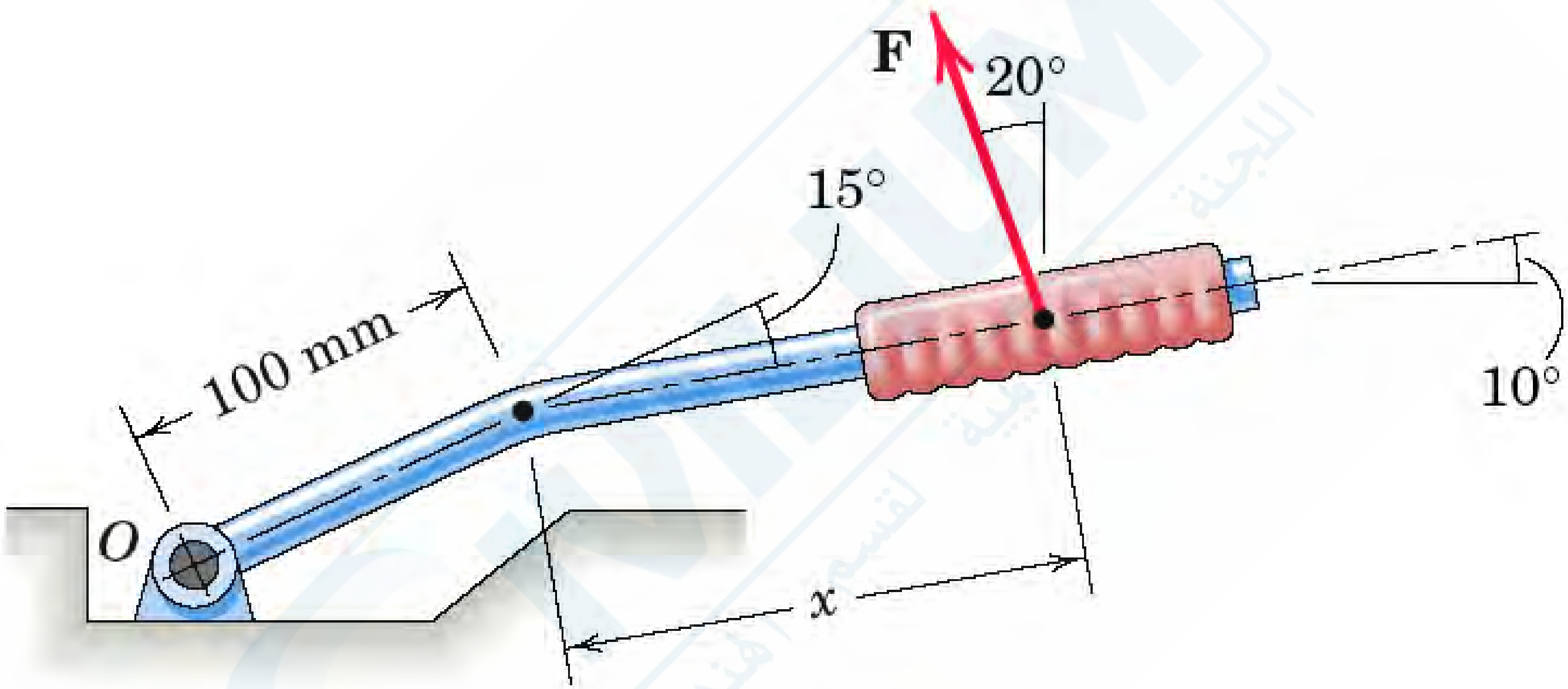
Problem 2/66

P_02_66



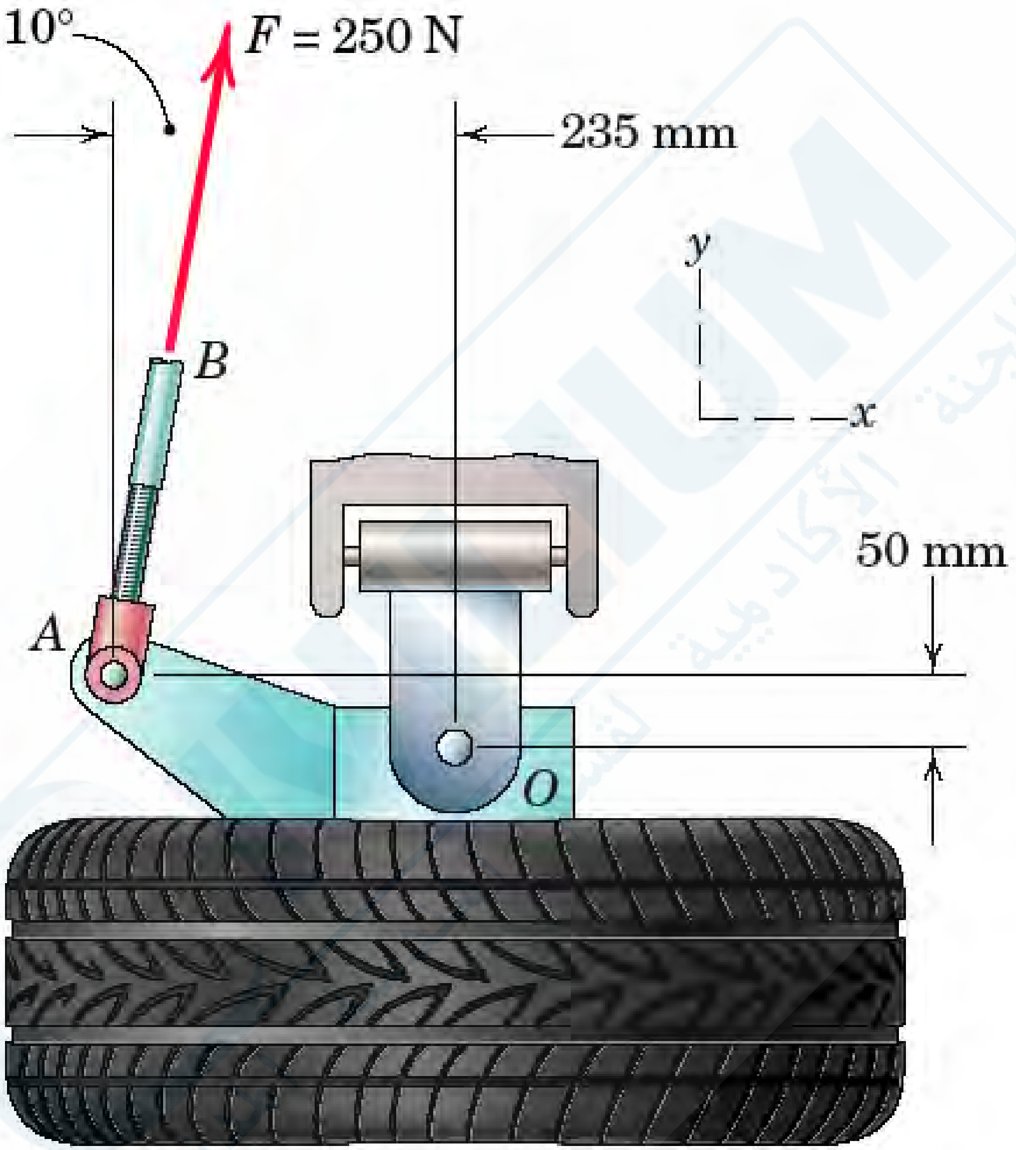
Problem 2/67

P 02 67



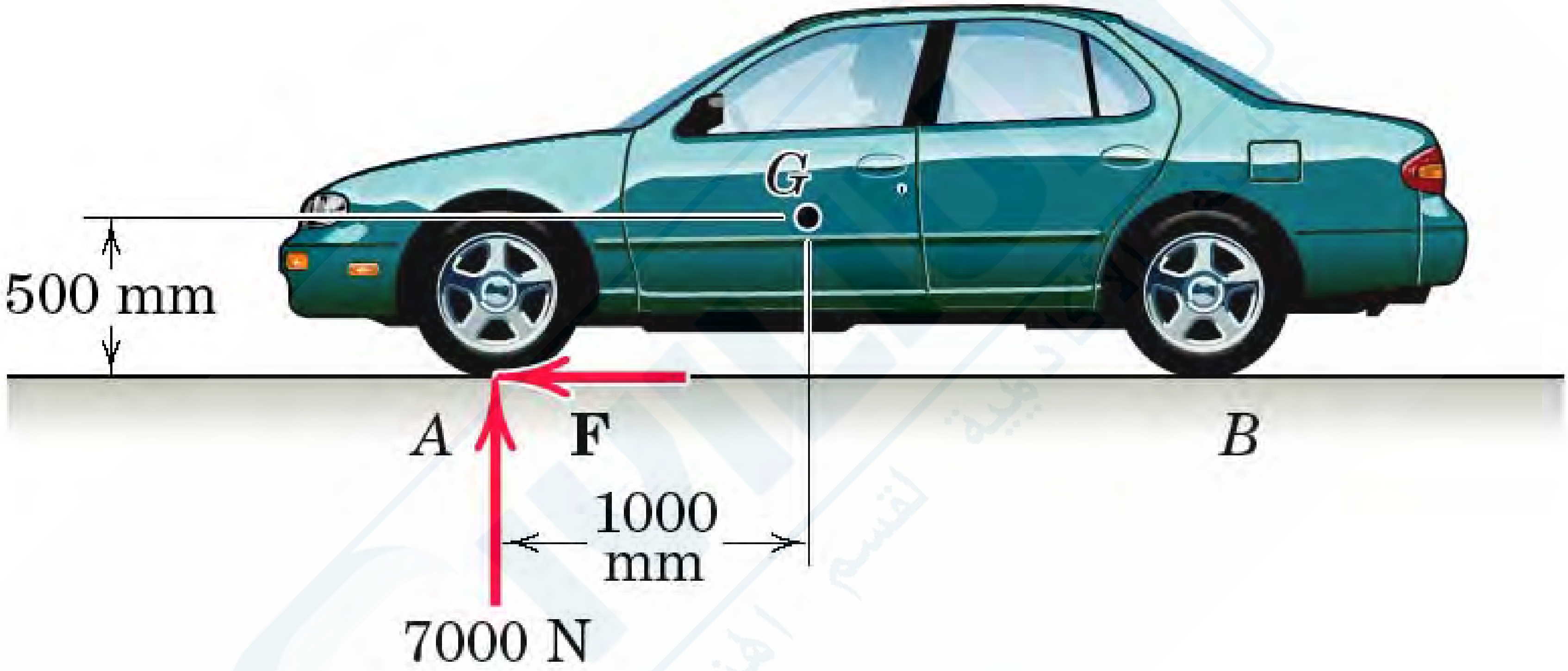
Problem 2/68

P_02_68



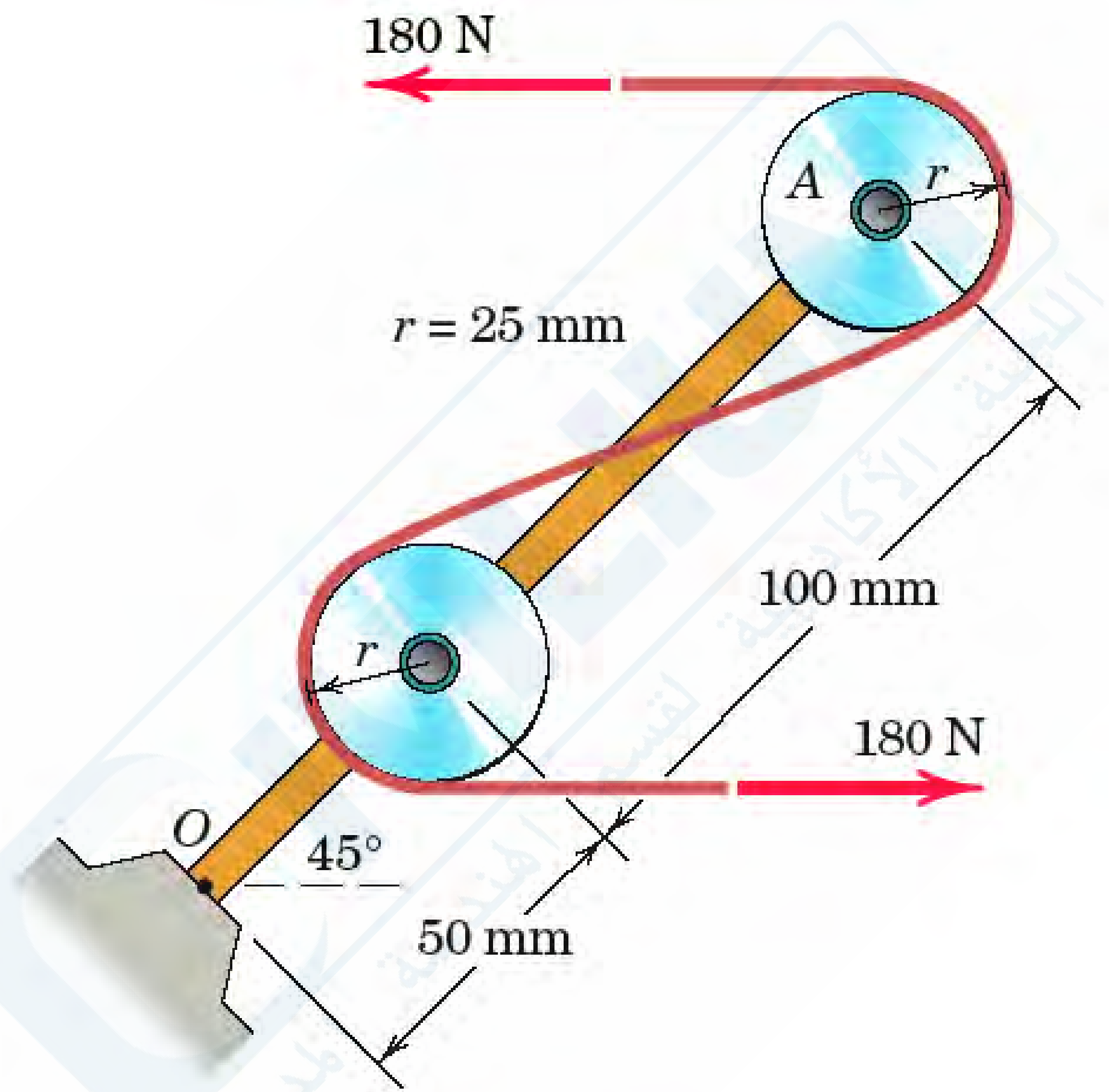
Problem 2/69

P_02_69



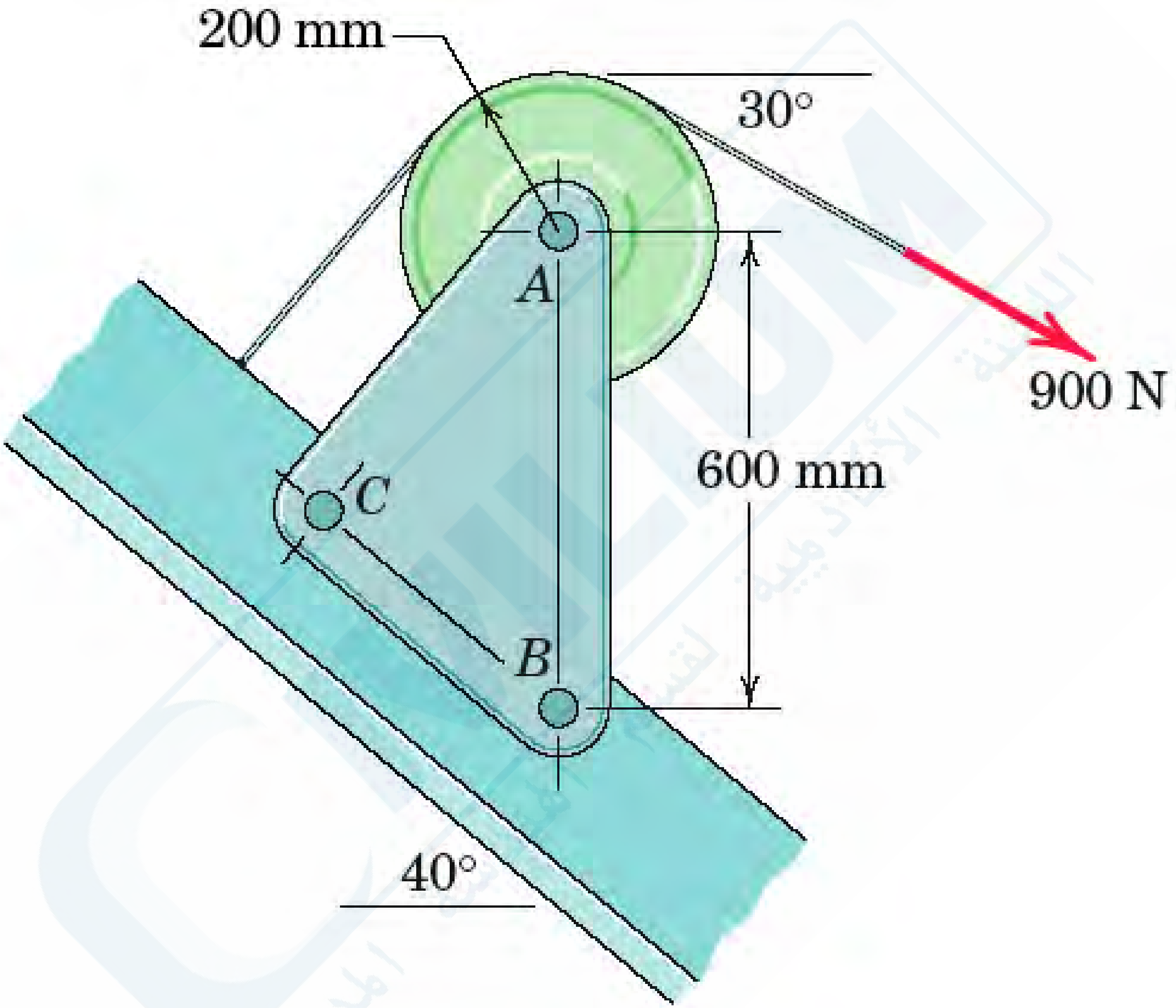
Problem 2/70

P_02_70



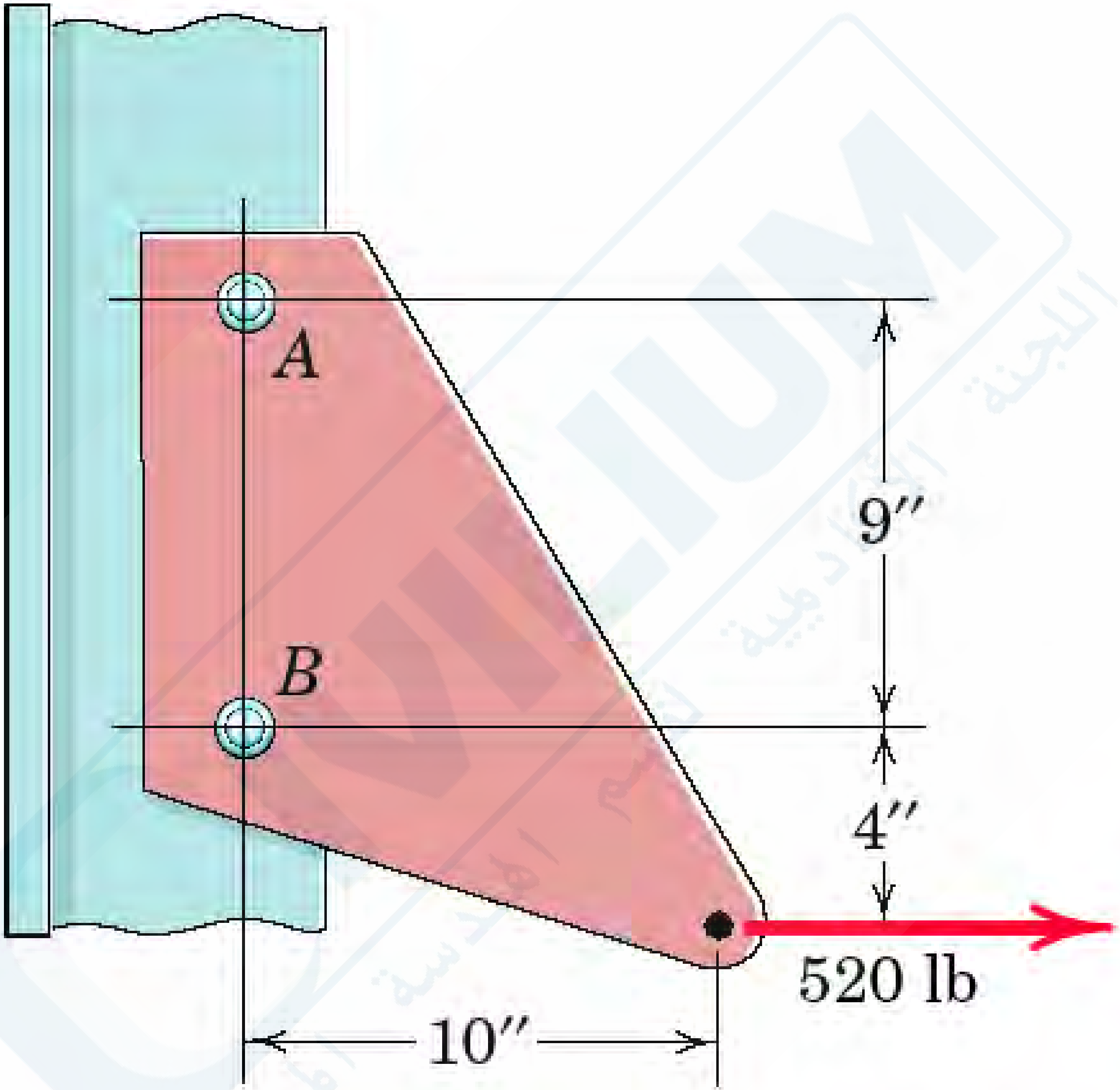
Problem 2/71

P_02_71



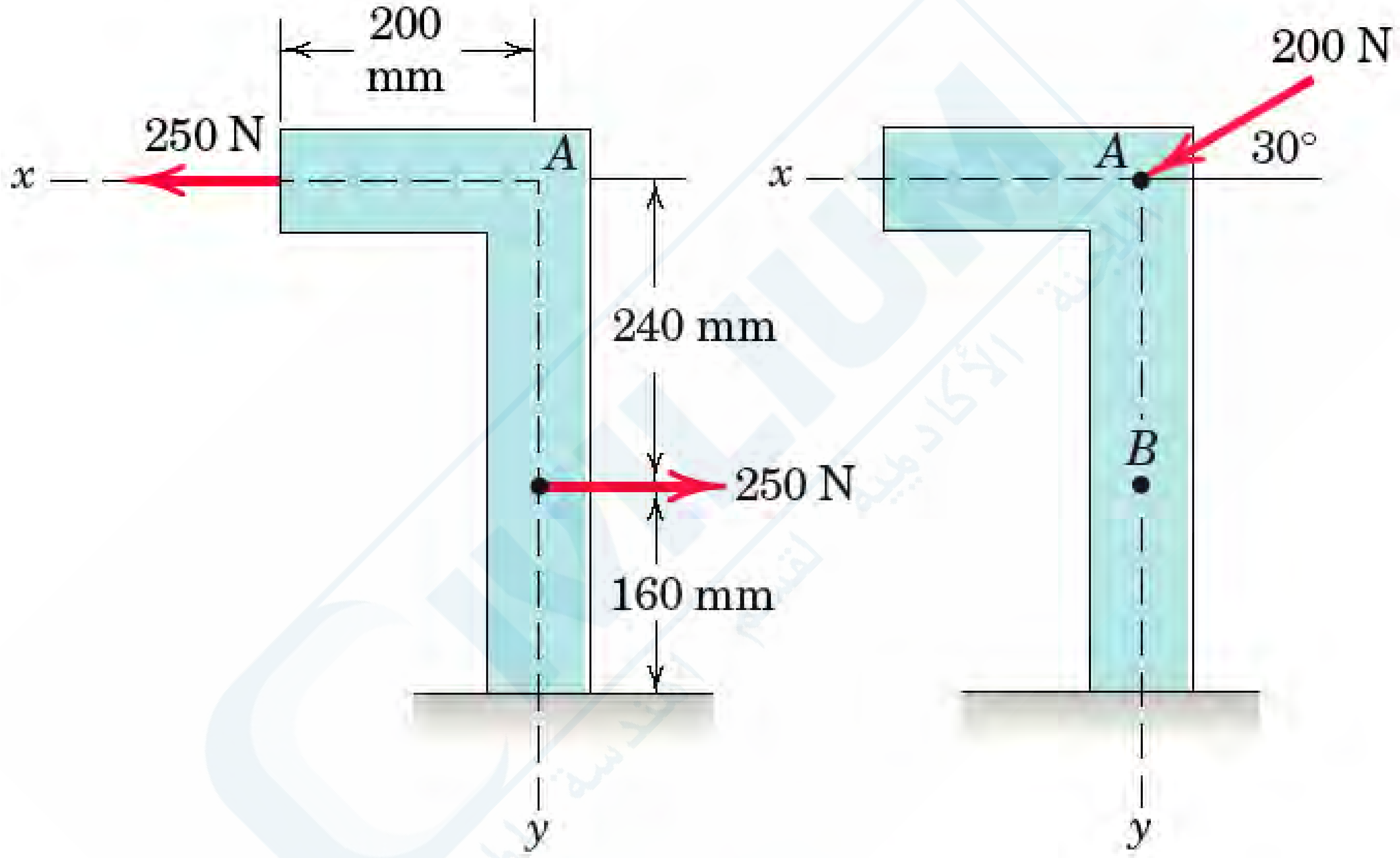
Problem 2/72

P 02 72



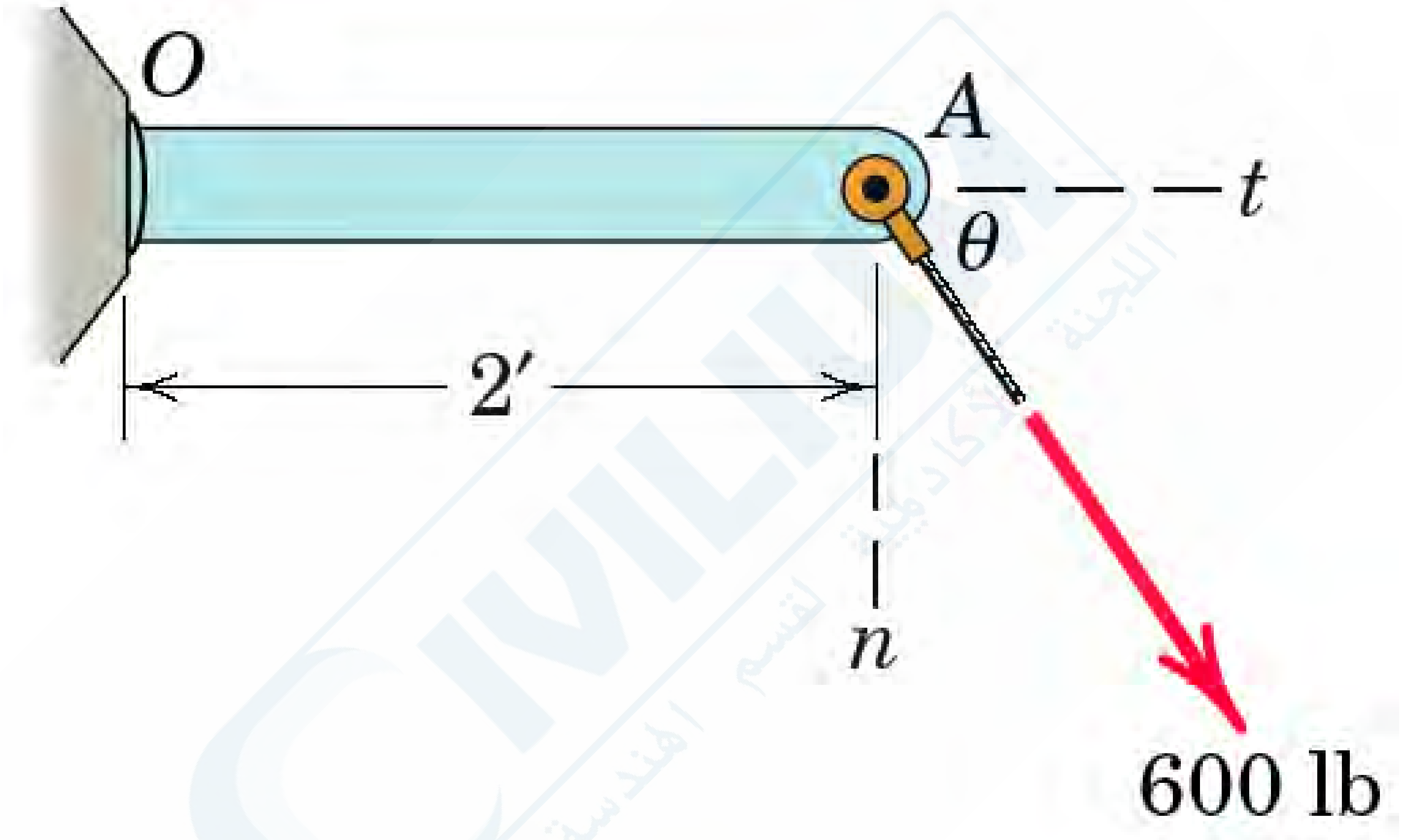
Problem 2/73

P 02 73



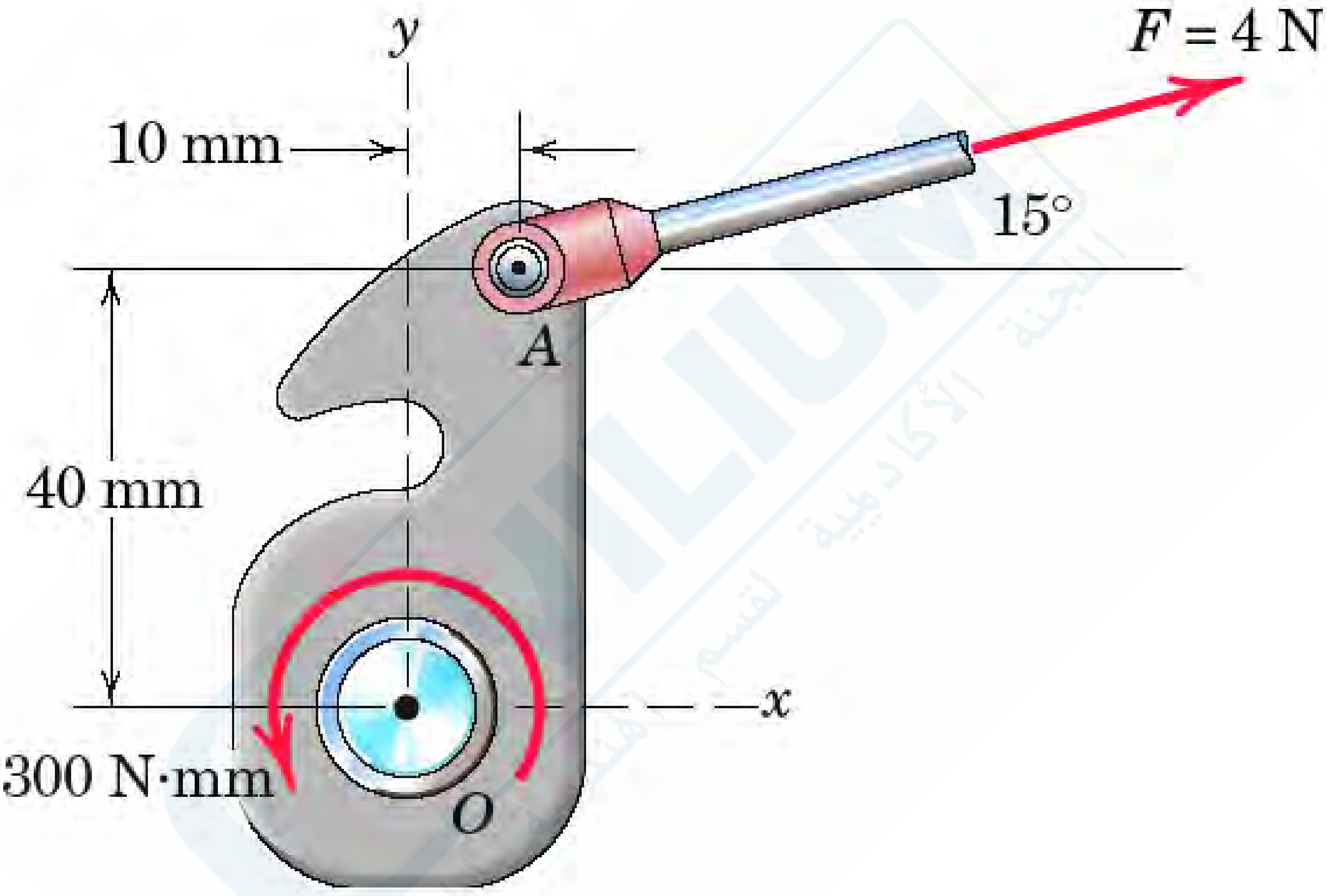
Problem 2/74

P_02_74



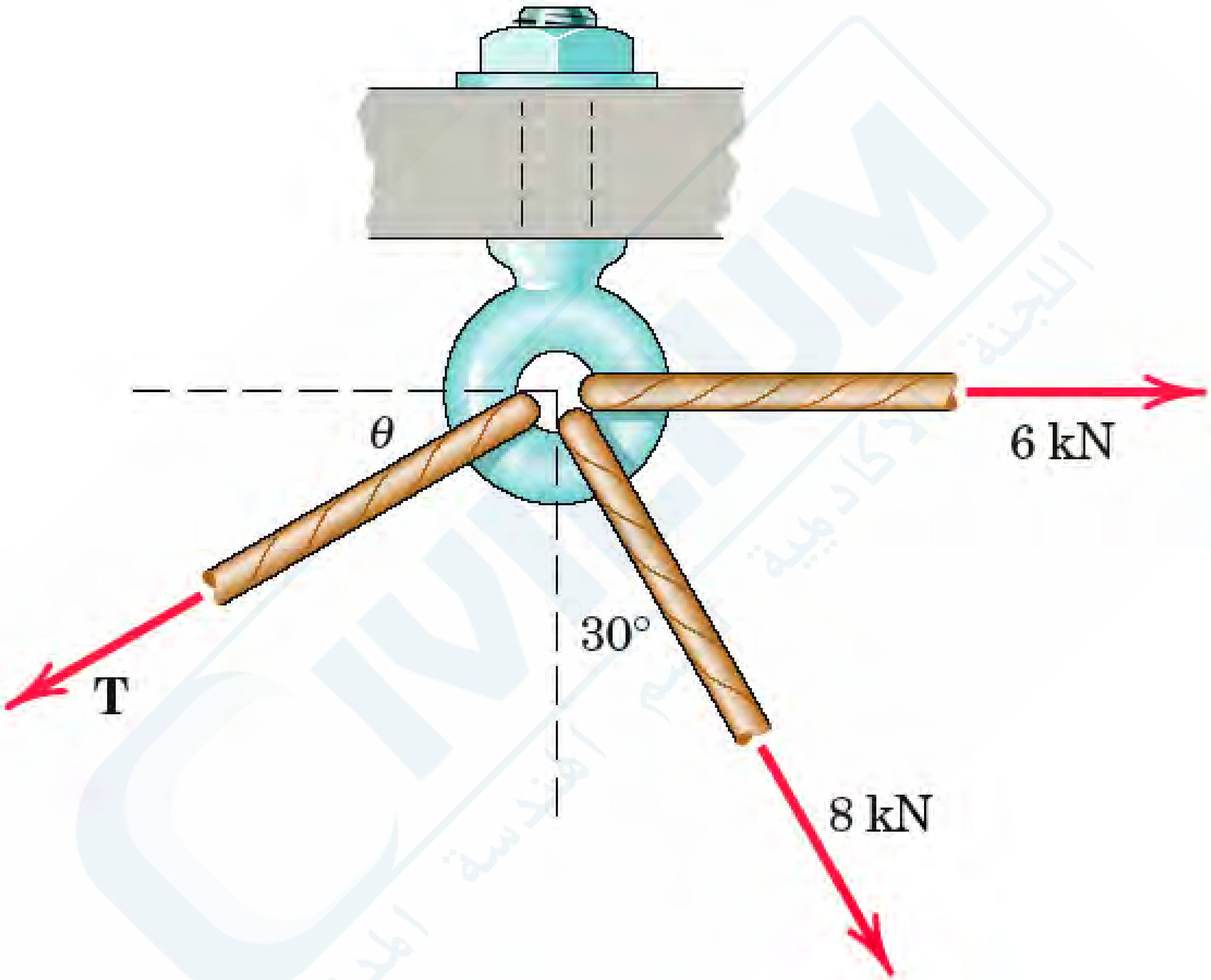
Problem 2/75

P_02_75



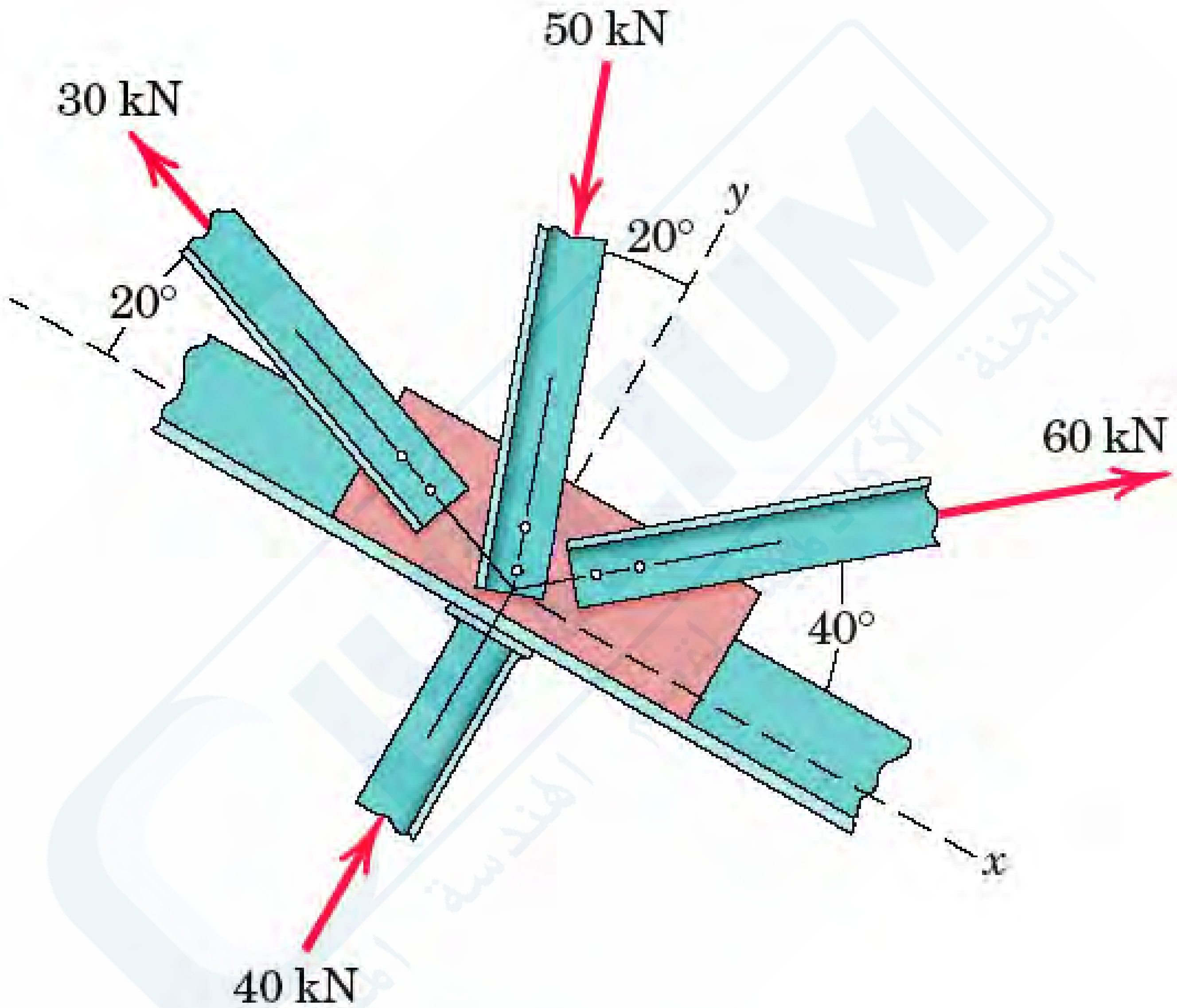
Problem 2/76

P 02 76



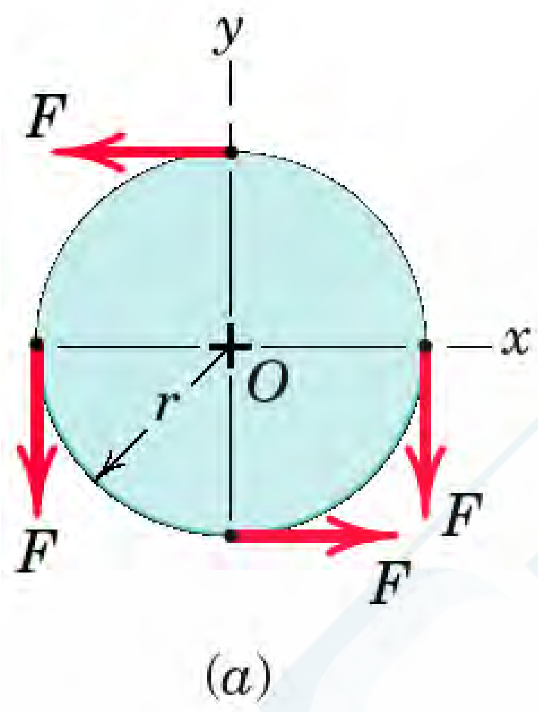
Problem 2/77

P 02 77



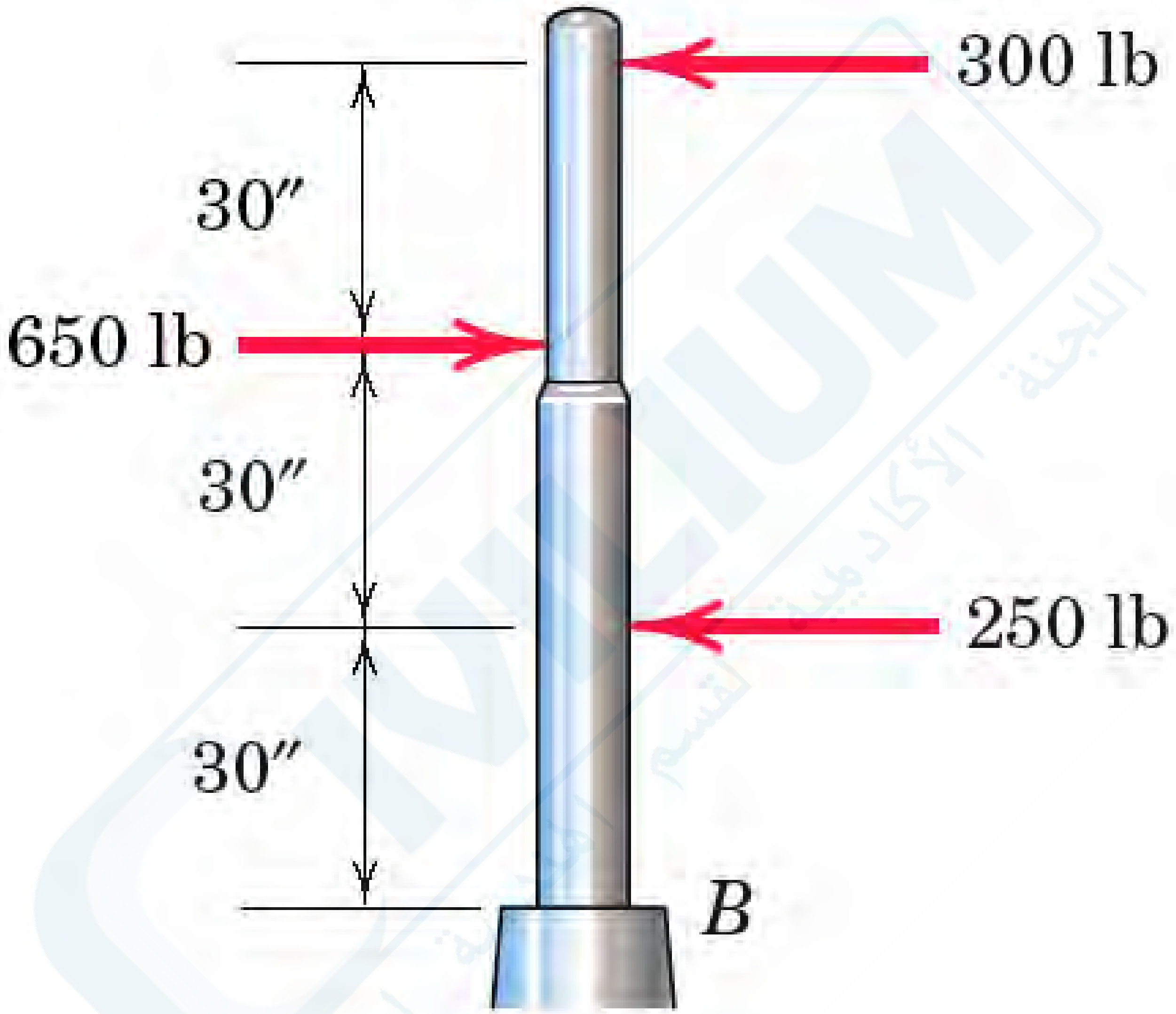
Problem 2/78

P 02 78



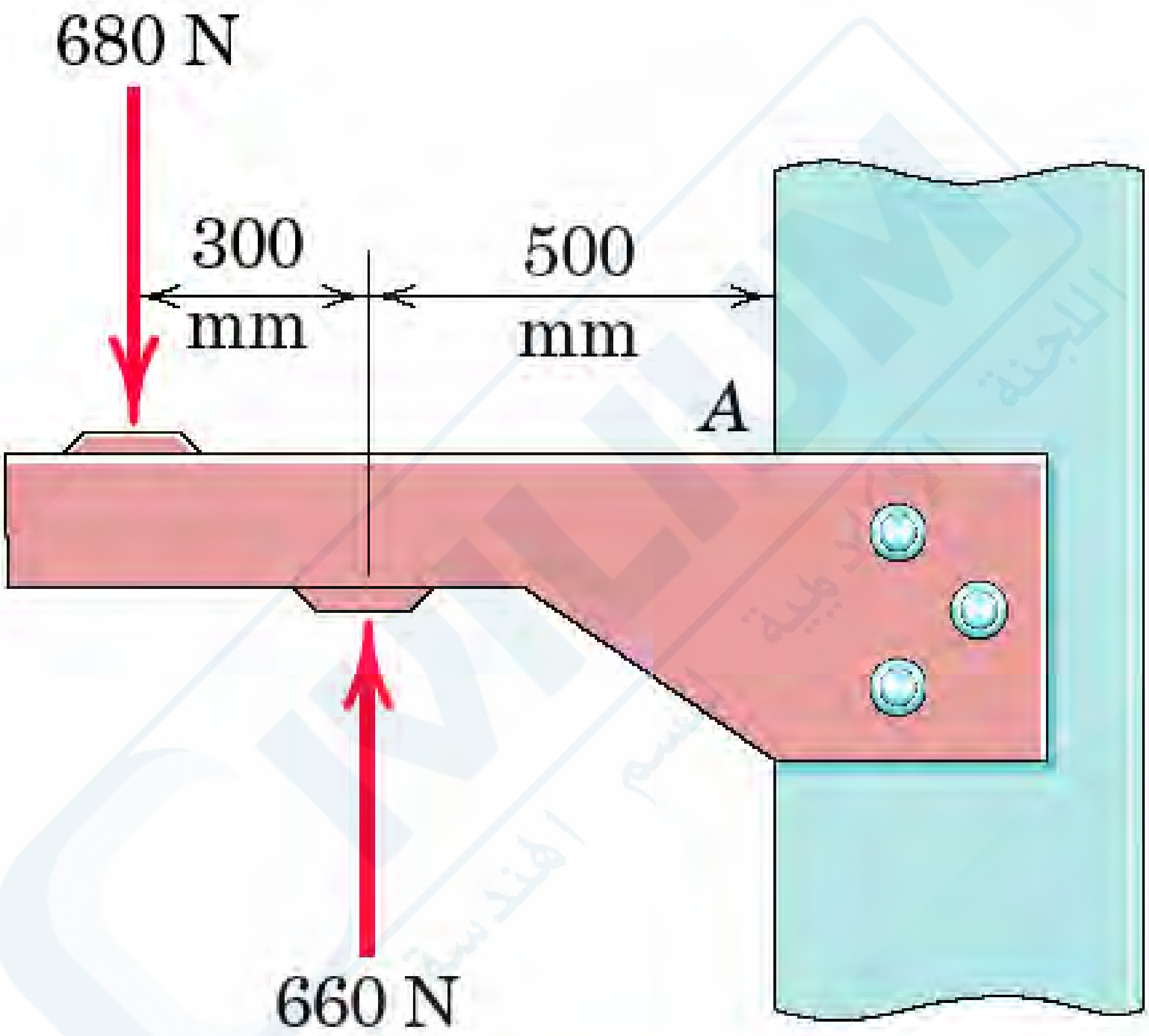
Problem 2/79

P_02_79

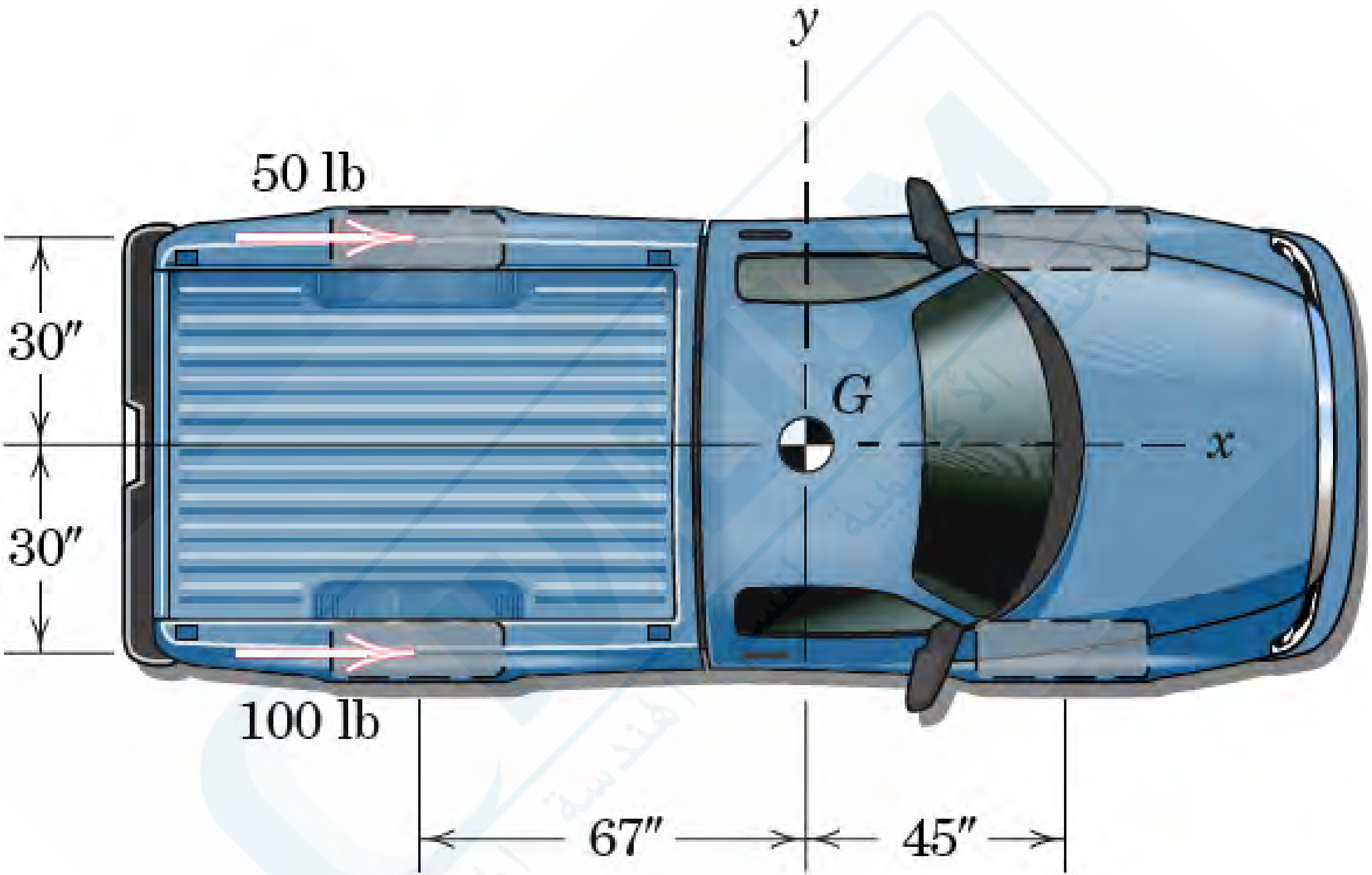


Problem 2/80

P 02 80

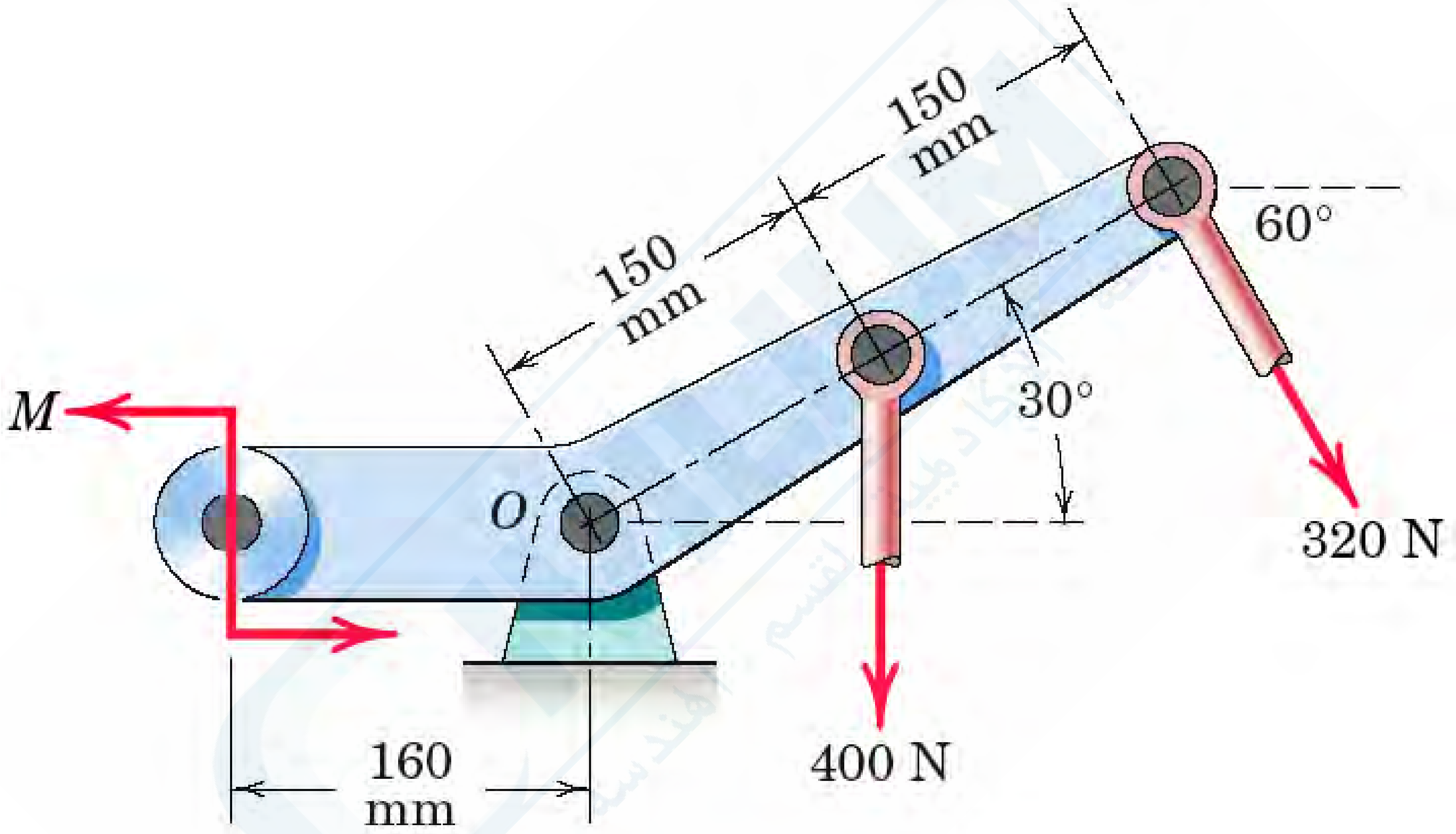


Problem 2/81



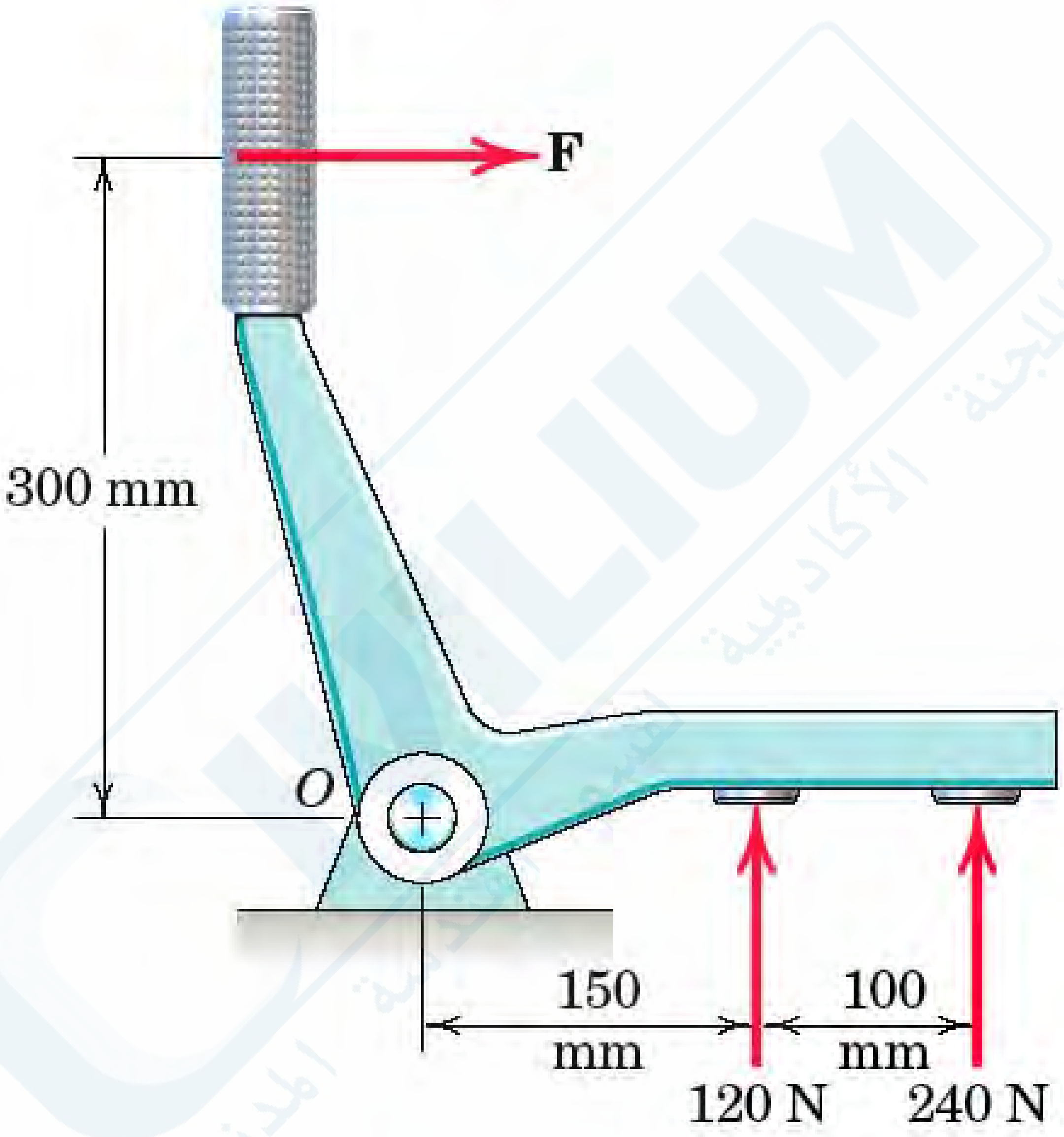
Problem 2/82

P_02_82



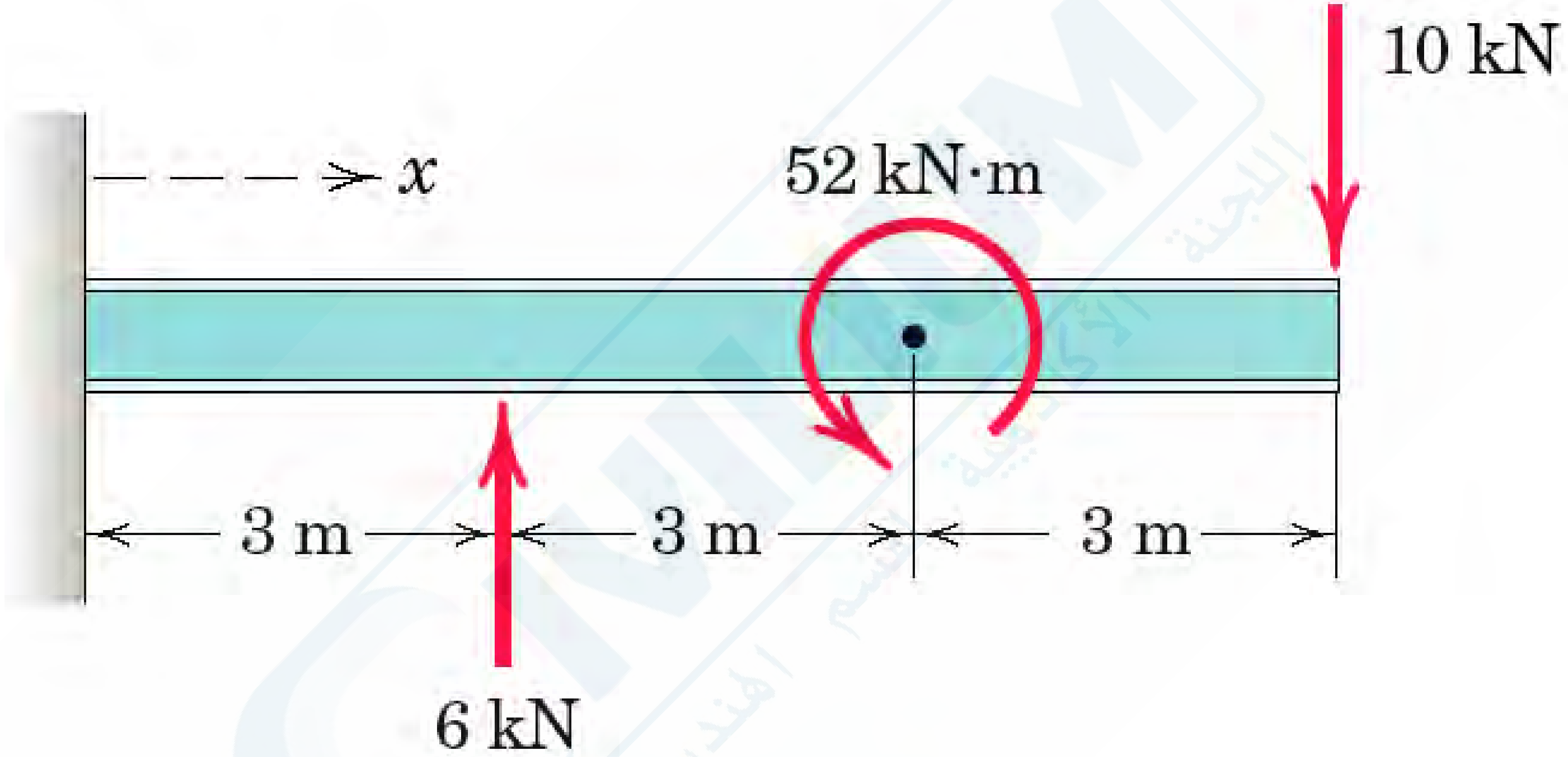
Problem 2/83

P_02_83



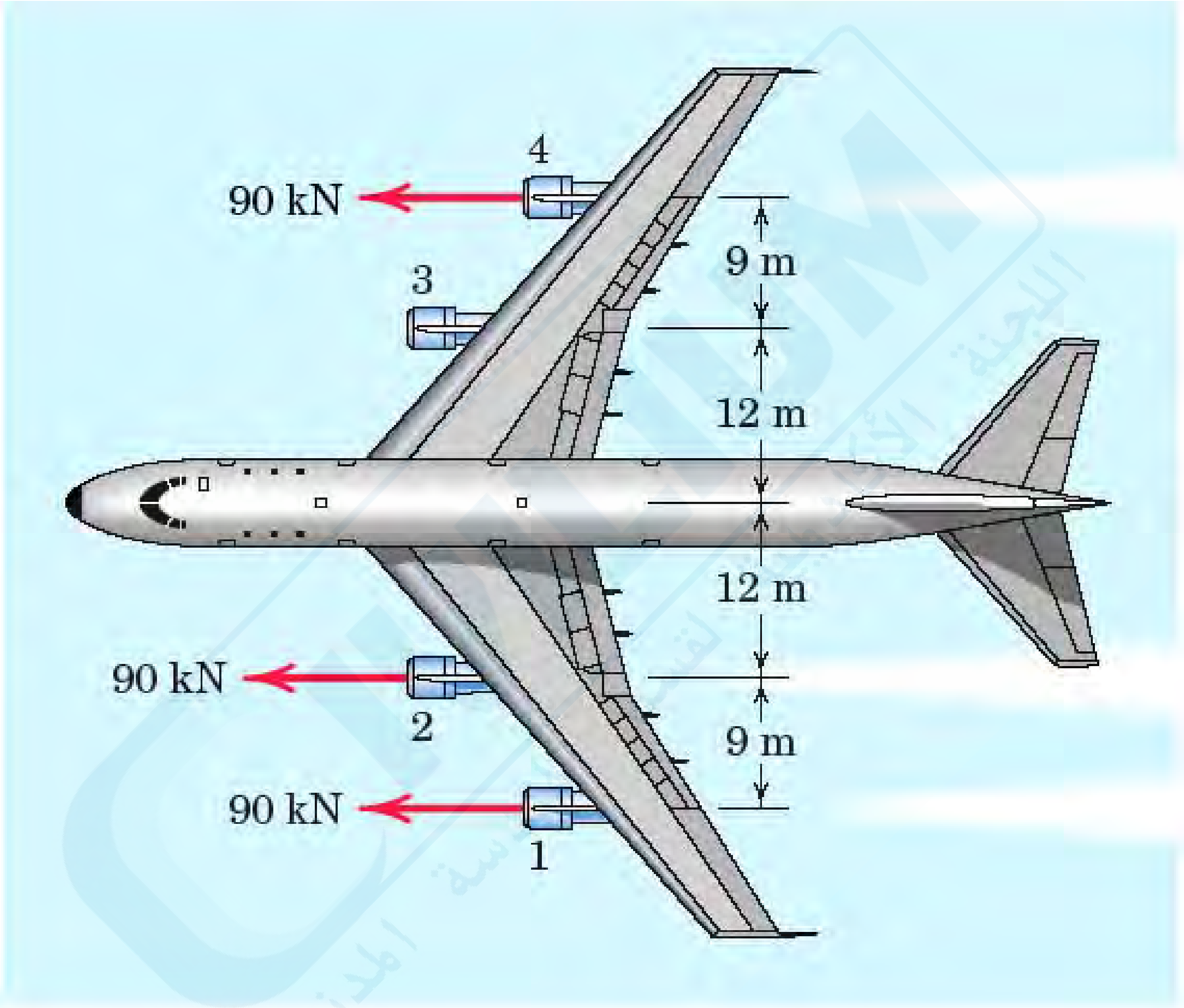
Problem 2/84

P 02 84



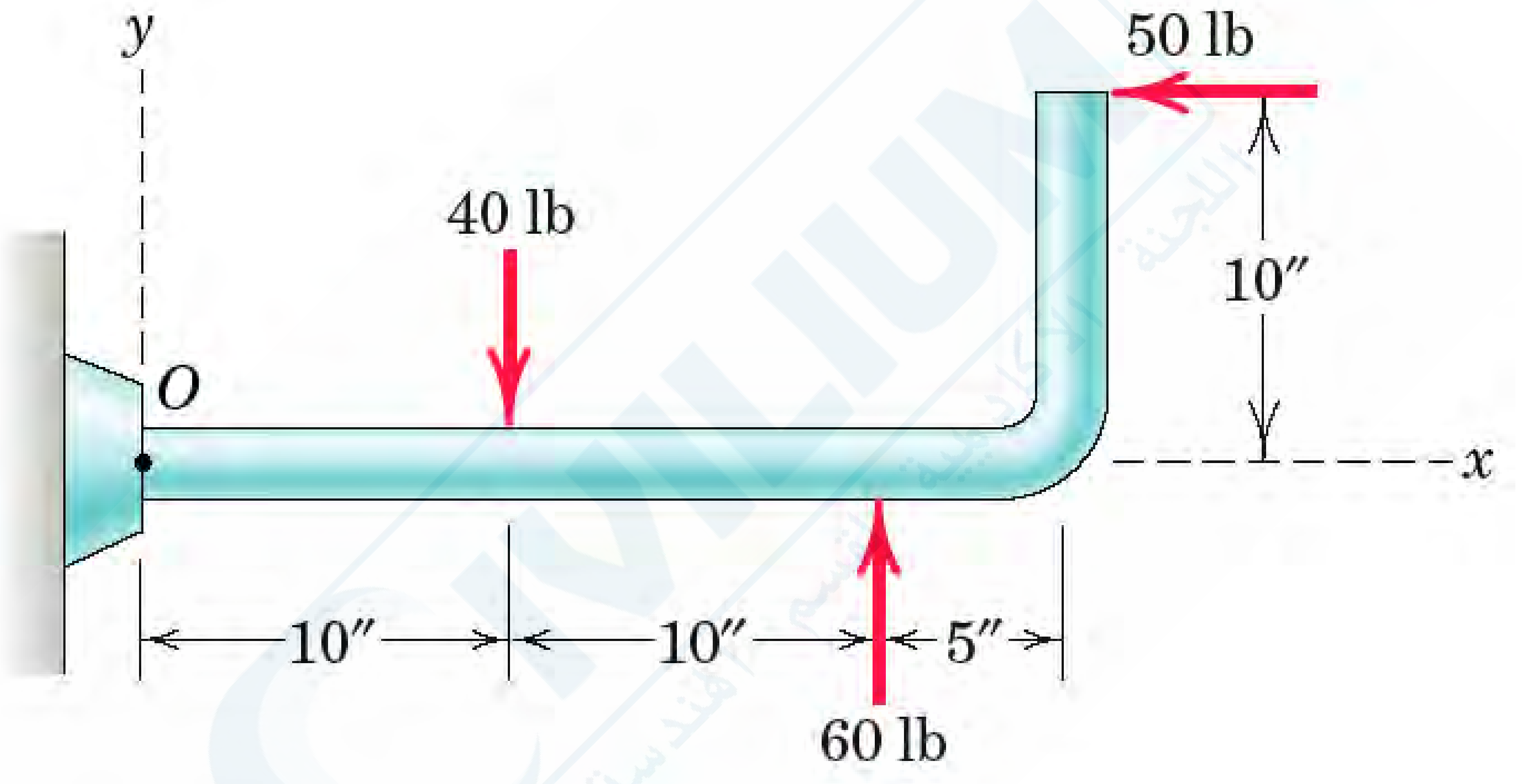
Problem 2/85

P_02_85



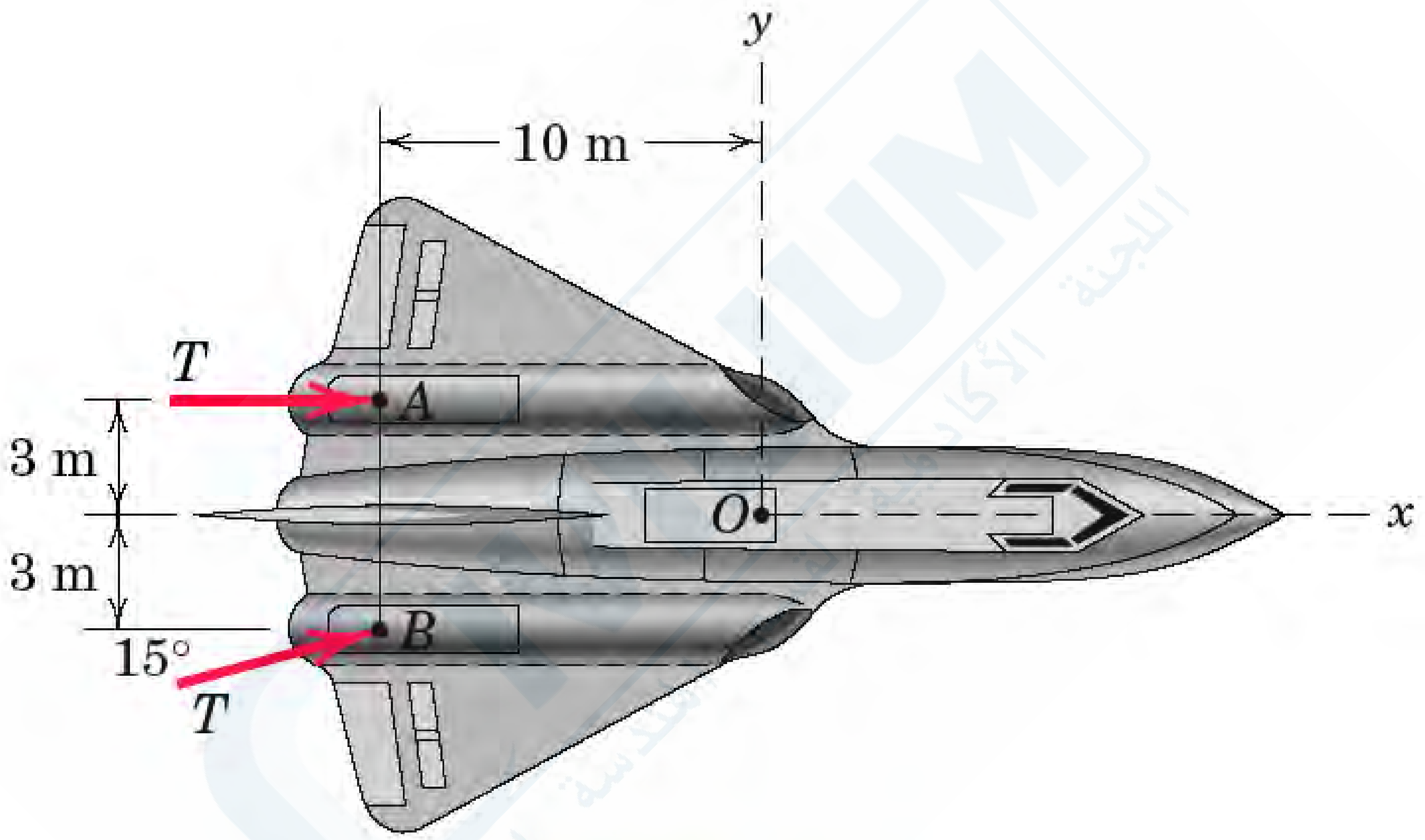
Problem 2/86

P 02 86



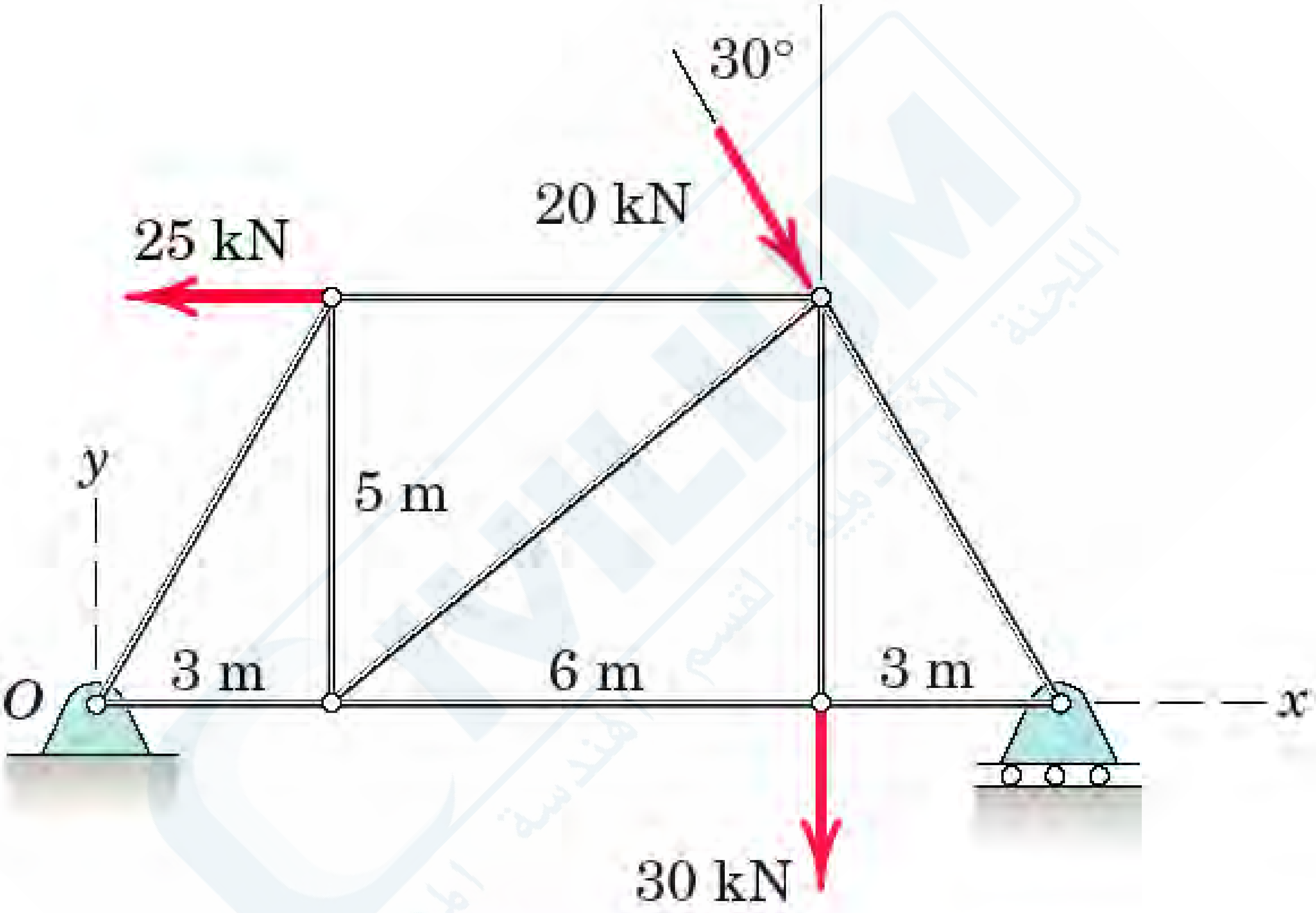
Problem 2/87

P_02_87



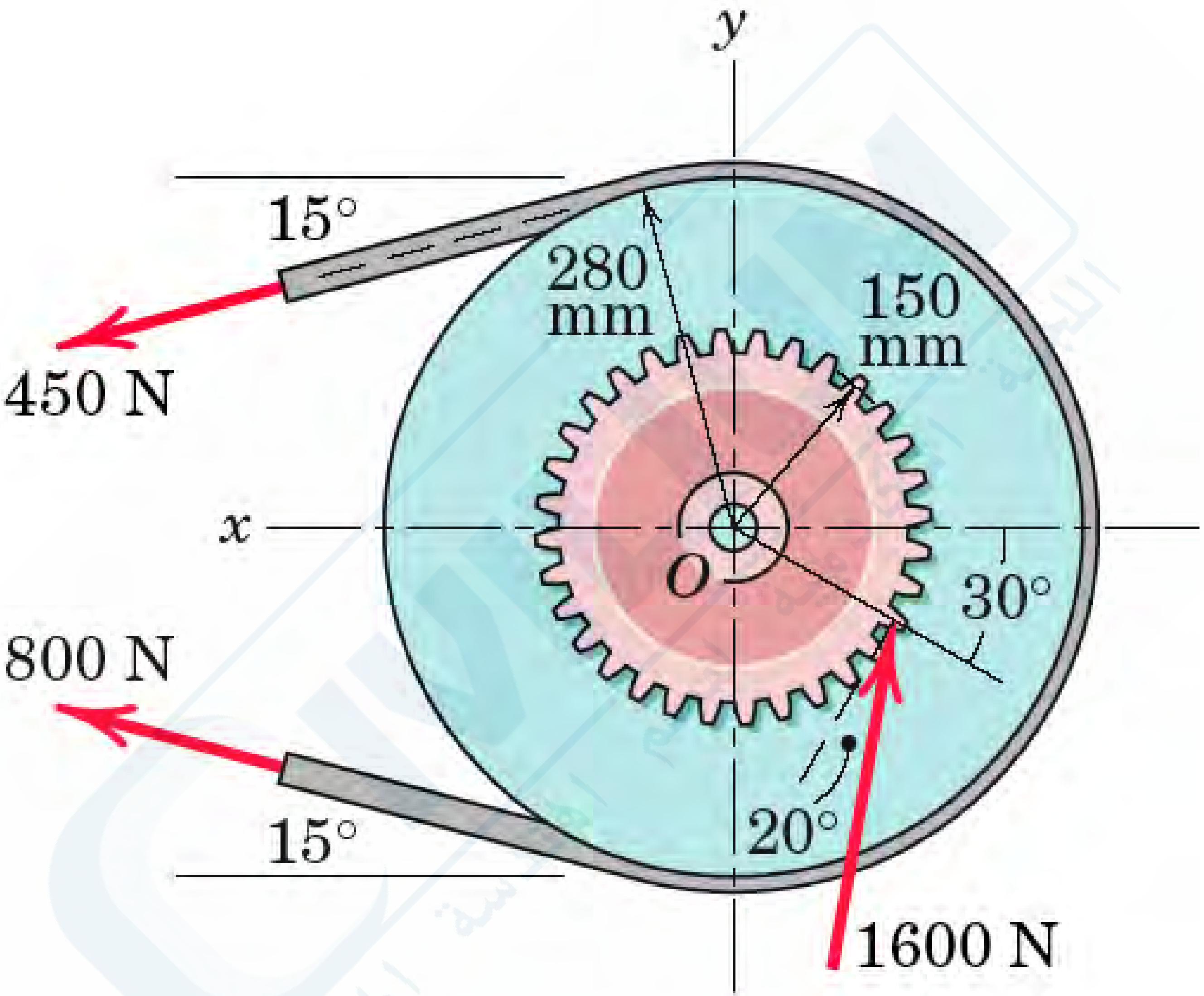
Problem 2/88

P_02_88



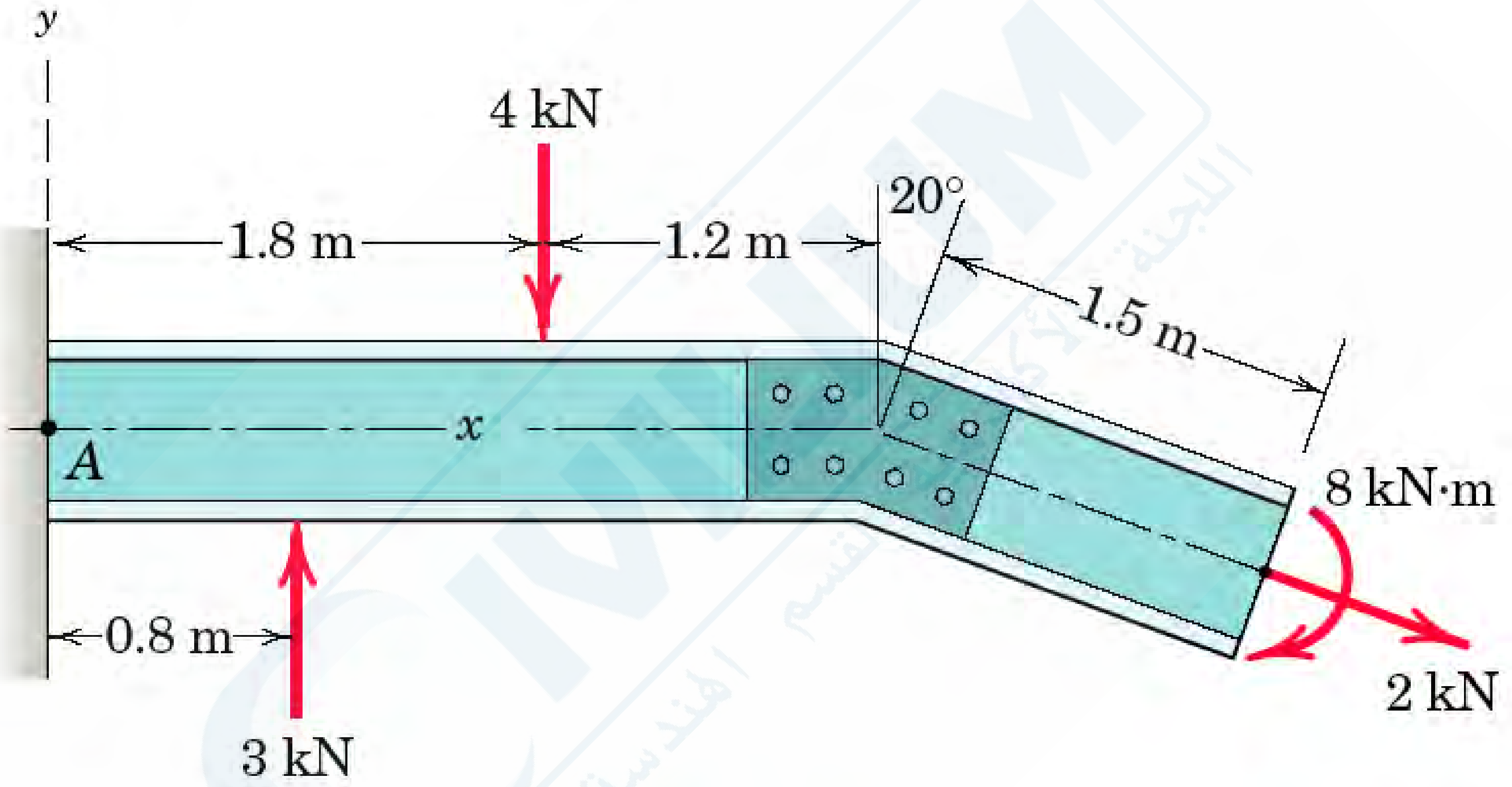
Problem 2/89

P_02_89



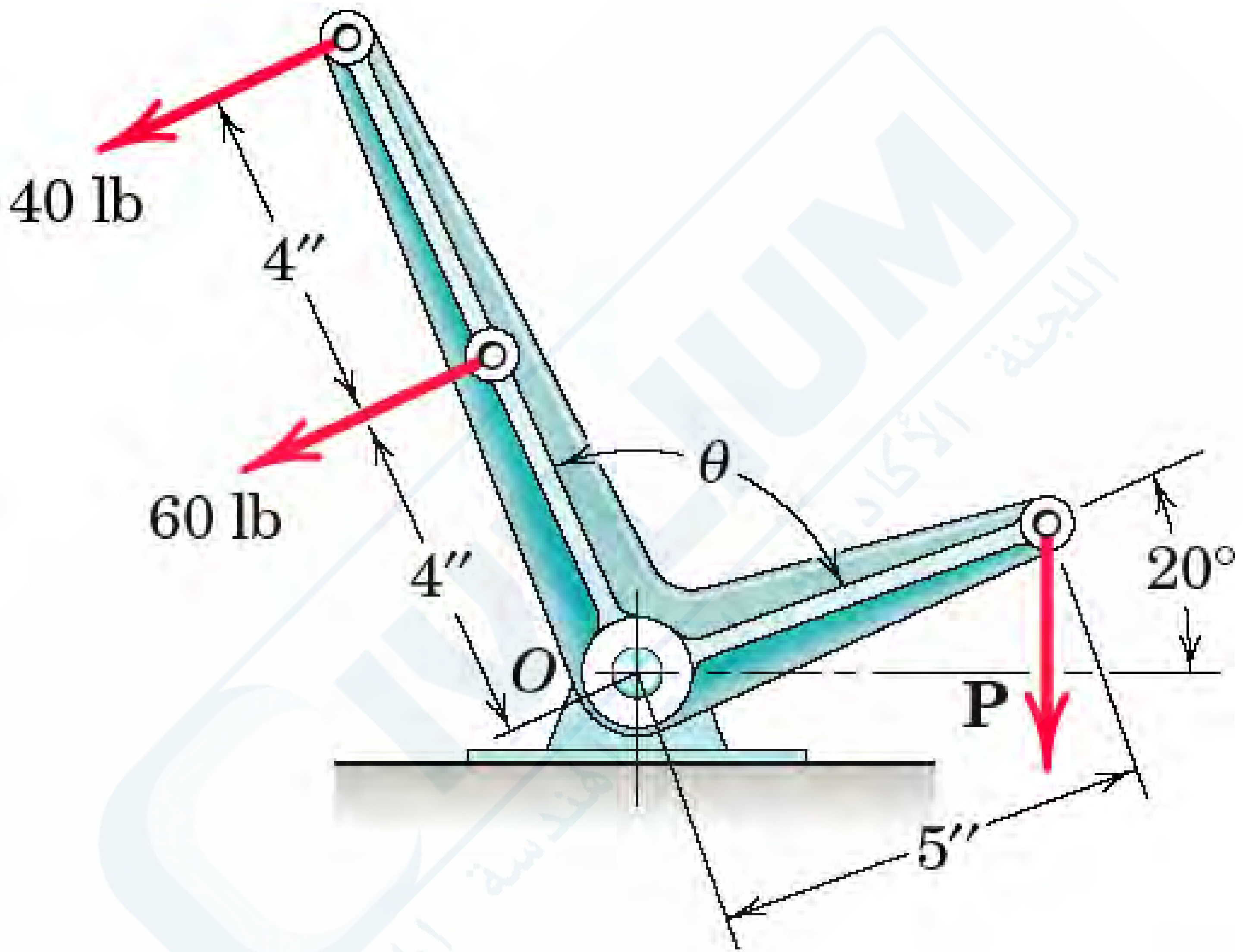
Problem 2/90

P 02 90

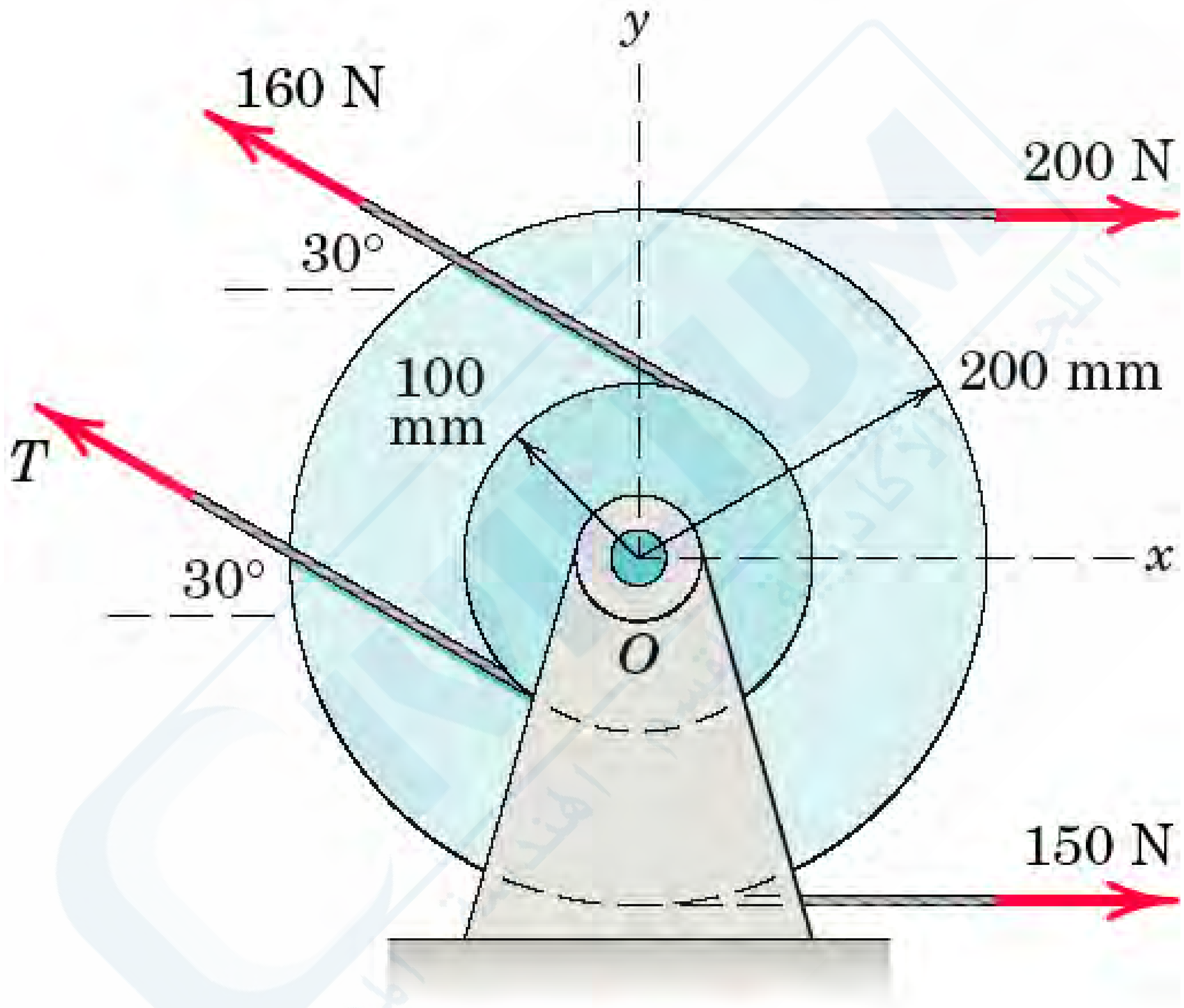


Problem 2/91

P_02_91

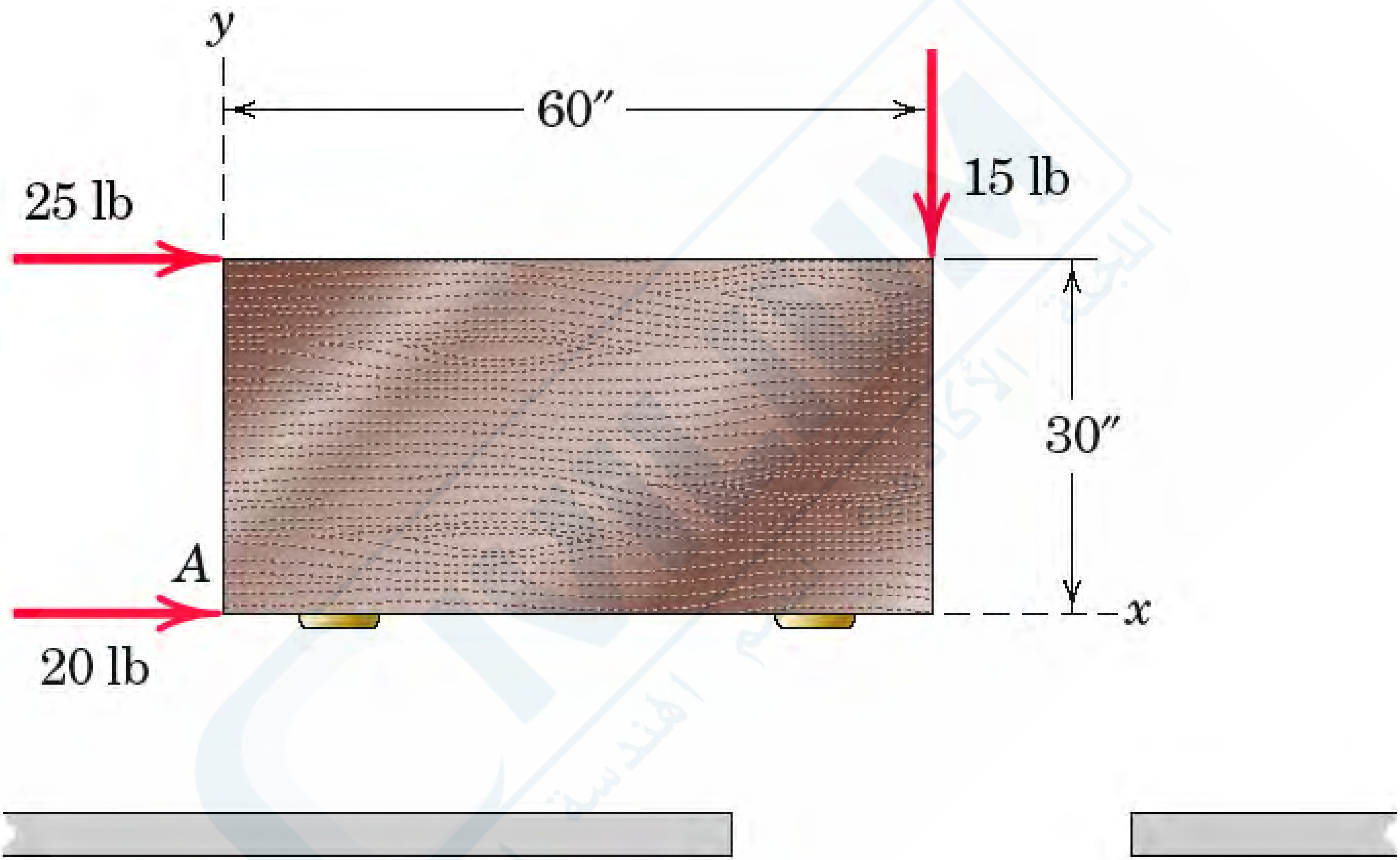


Problem 2/92



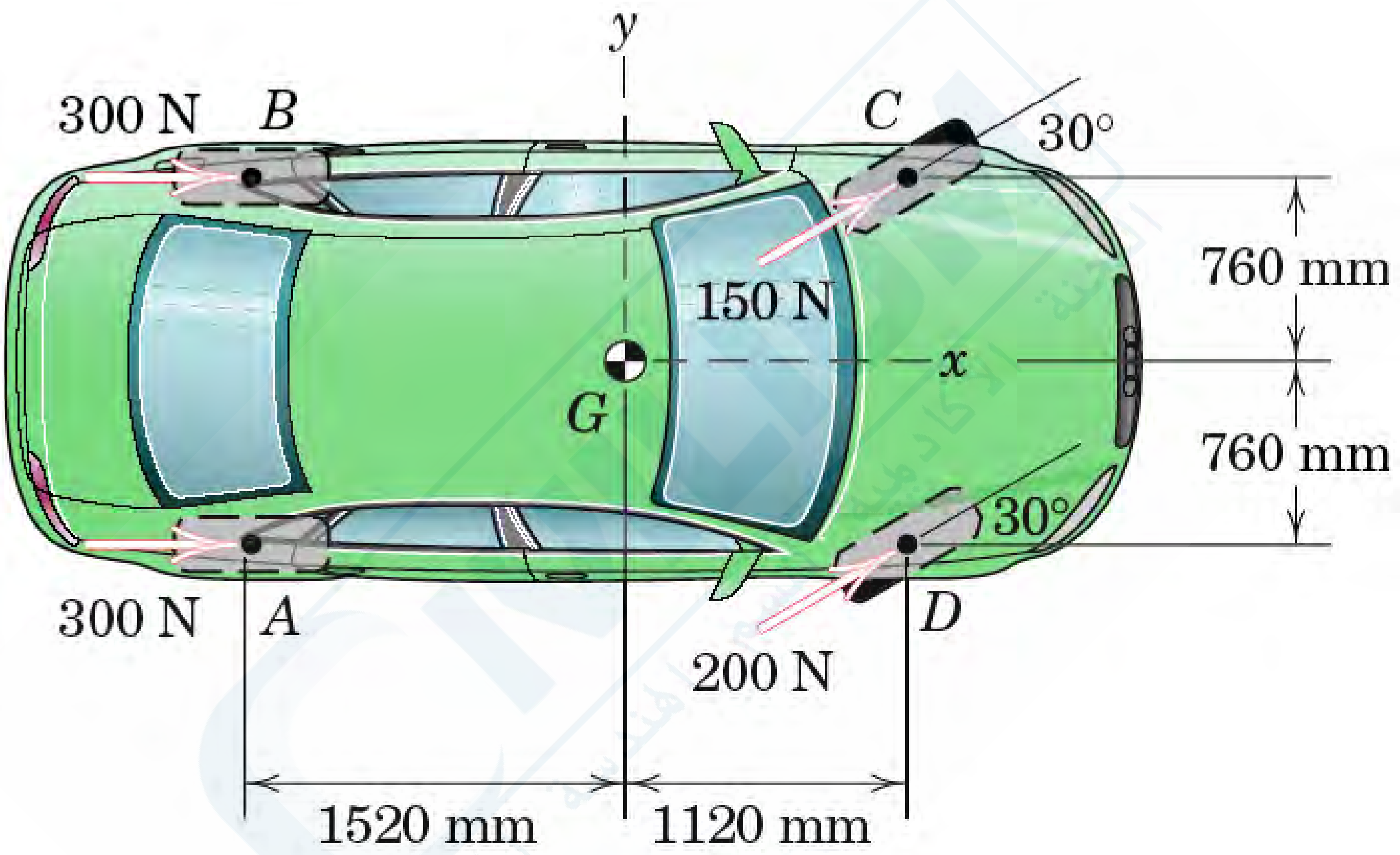
Problem 2/93

P_02_93



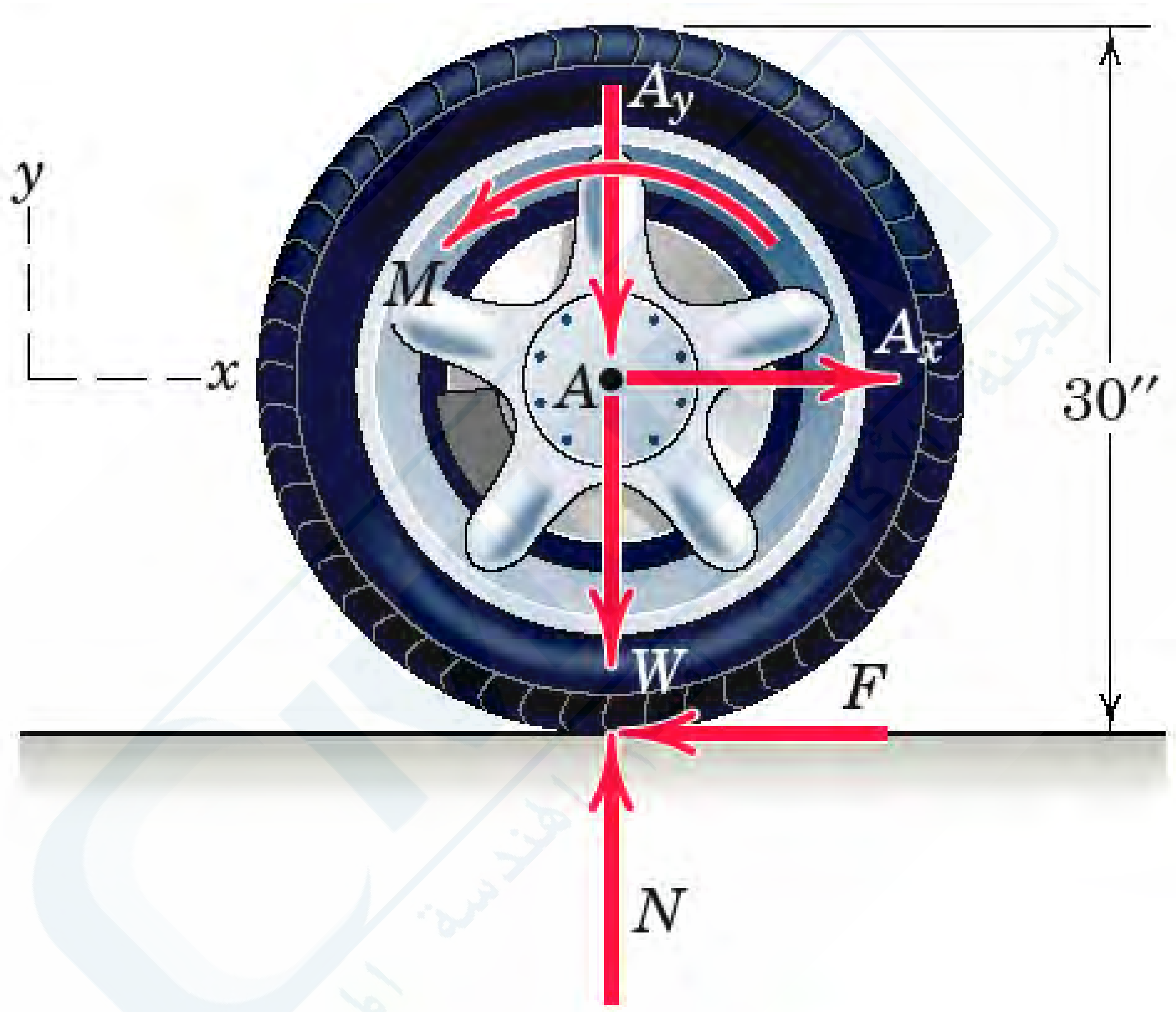
Problem 2/94

P_02_94

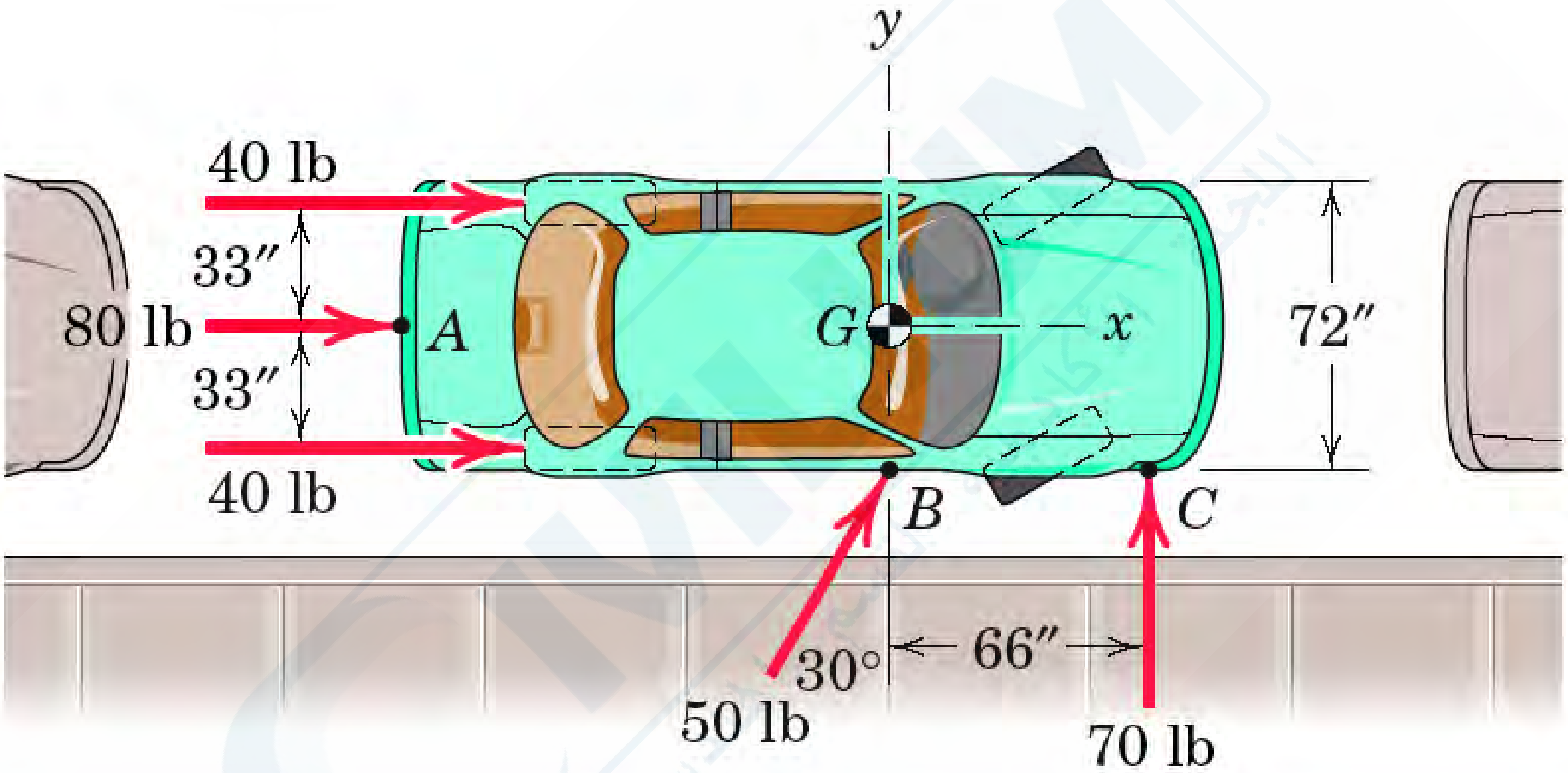


Problem 2/95

P_02_95

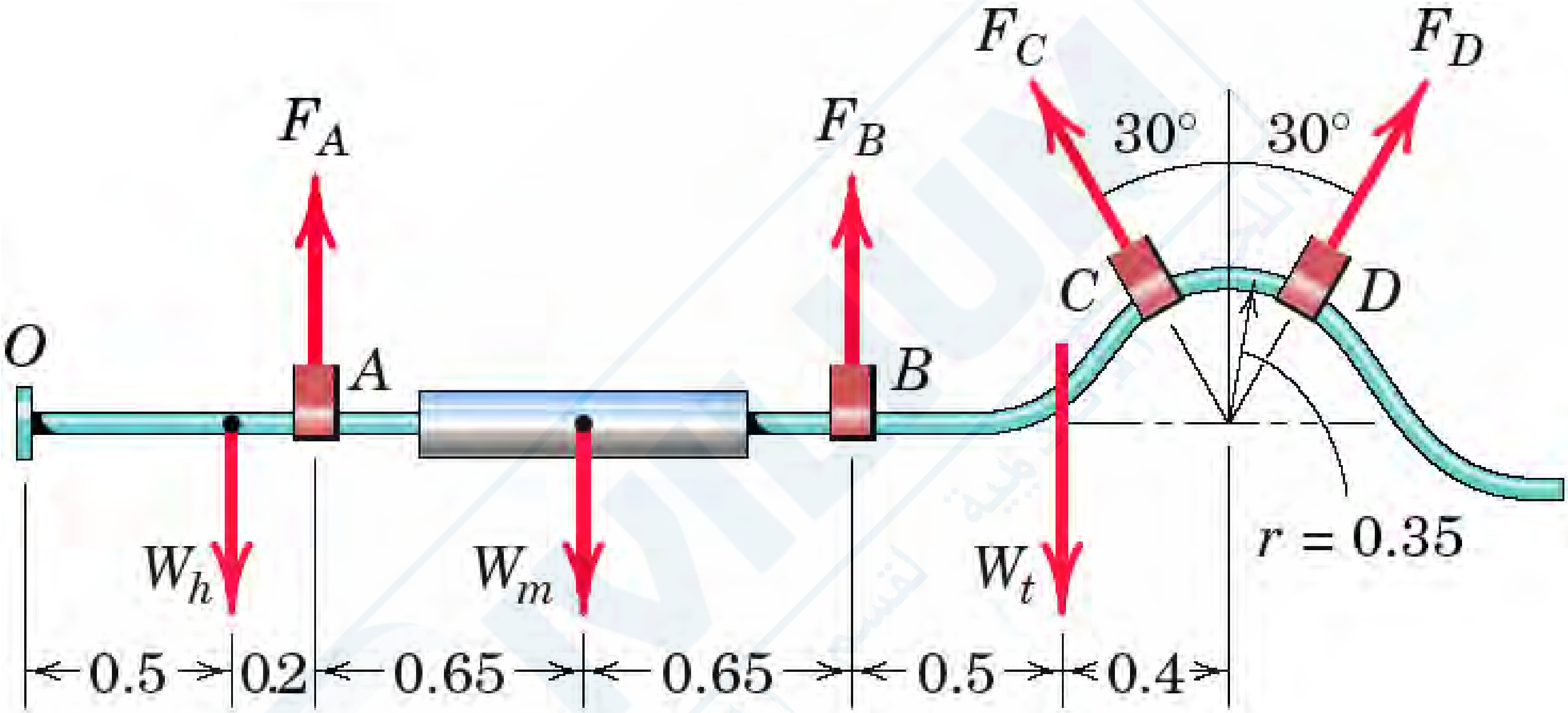


Problem 2/96



Problem 2/97

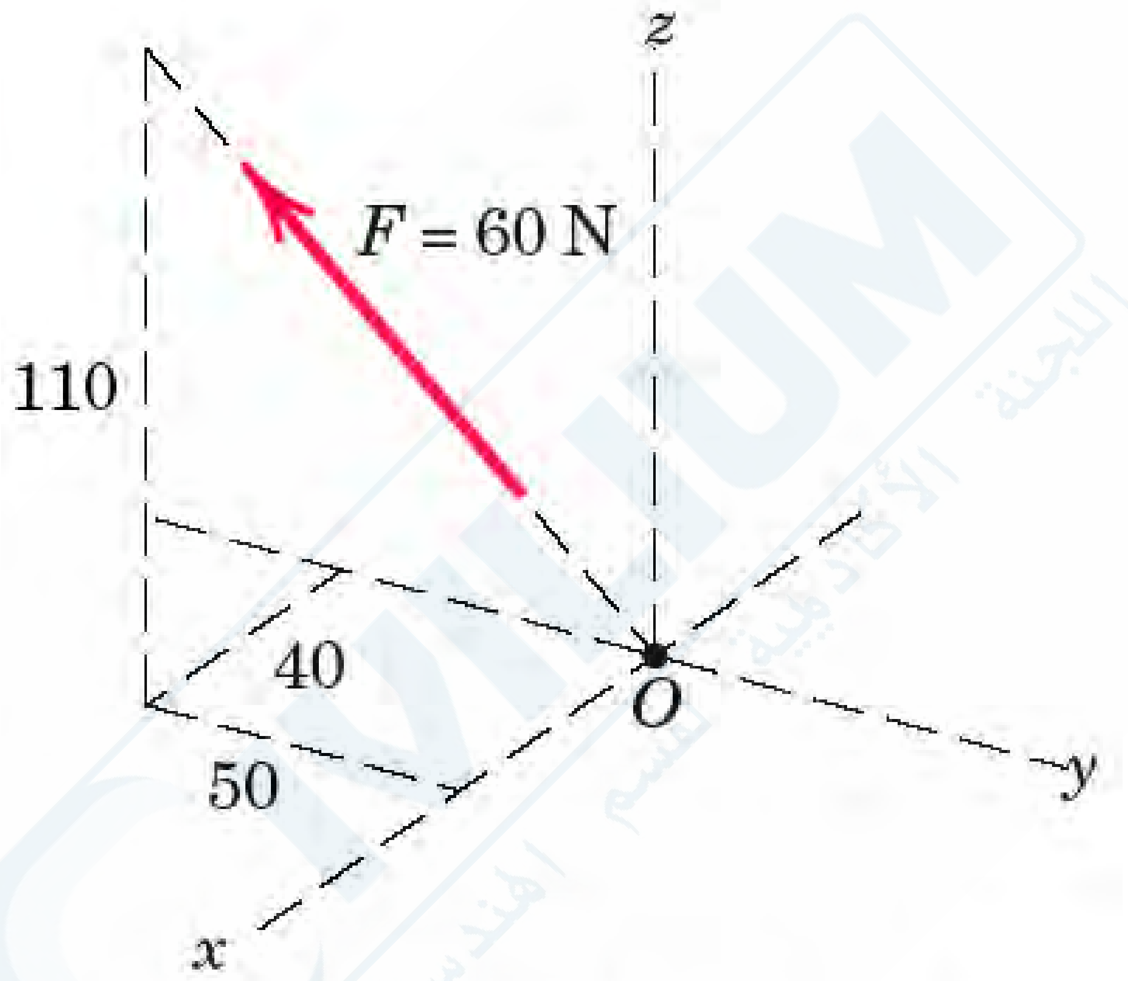
P_02_97



Dimensions in meters

Problem 2/98

P_02_98



Dimensions in millimeters

Problem 2/99

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 3 : Equilibrium.

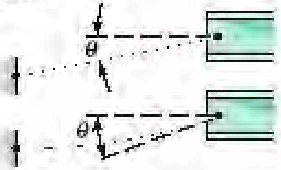
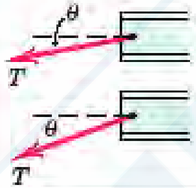
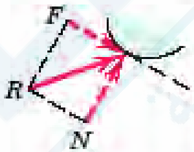
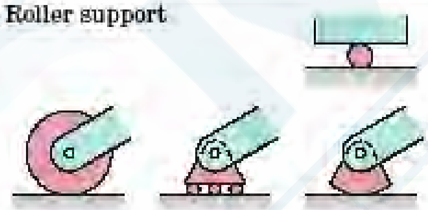
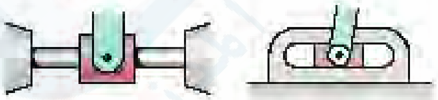
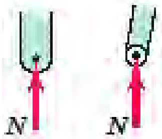
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces	Contact force is compressive and is normal to the surface.
3. Rough surfaces	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 	Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

Figure 3/1

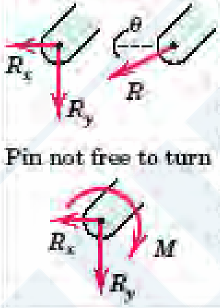
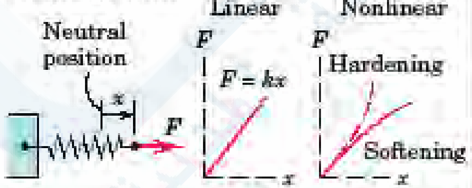
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS *(cont.)*	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
6. Pin connection	 A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support	A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction	The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Figure 3/1, continued

03_01 cont

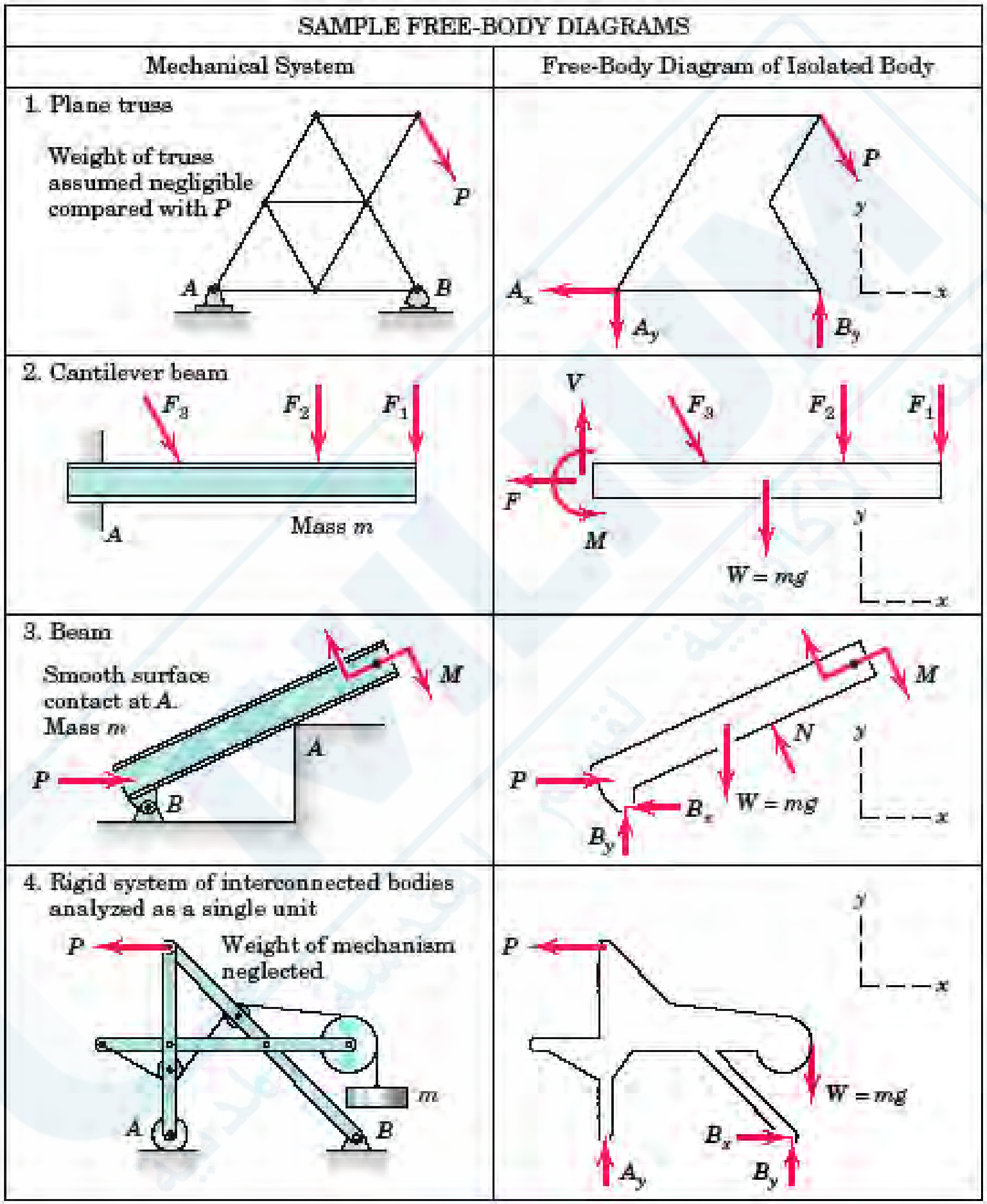

SAMPLE FREE-BODY DIAGRAMS
Mechanical System
Free-Body Diagram of Isolated Body
1. Plane truss
Weight of truss assumed negligible compared with P
A
B
P
A_x
A_y
B_y
2. Cantilever beam
F_3
F_2
F_1
Mass m
V
F
M
W = mg
3. Beam
Smooth surface contact at A.
Mass m
M
N
B_x
B_y
4. Rigid system of interconnected bodies analyzed as a single unit
Weight of mechanism neglected
m
y
x

Figure 3/2

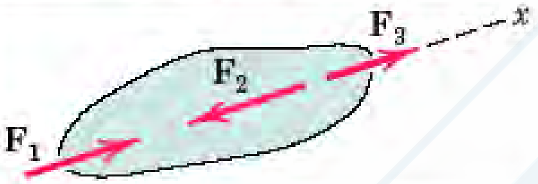
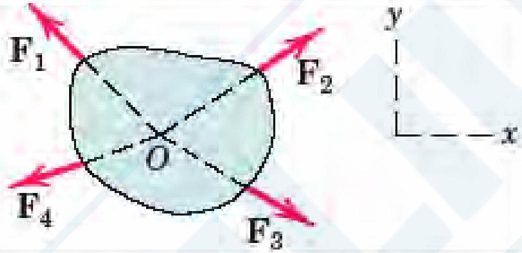
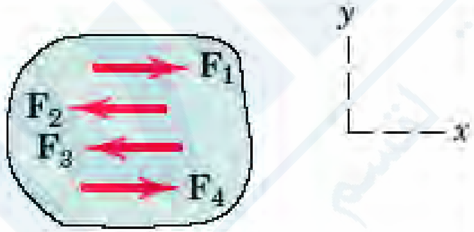
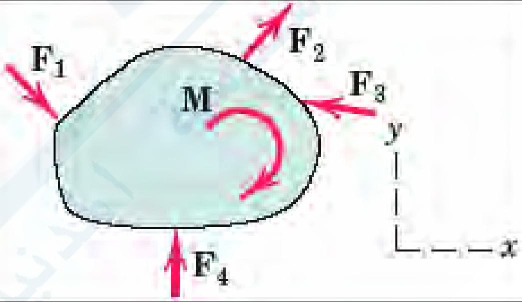
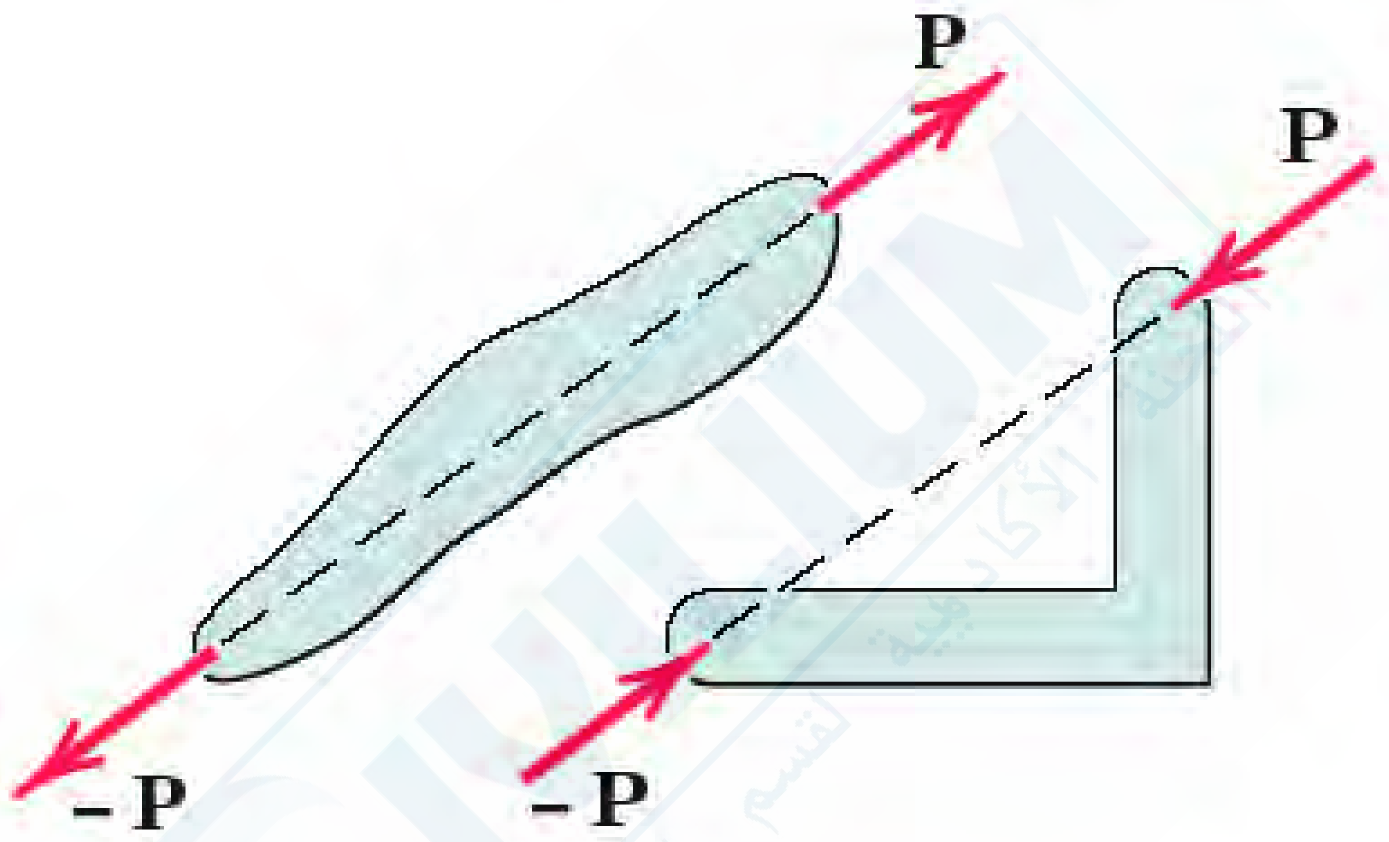
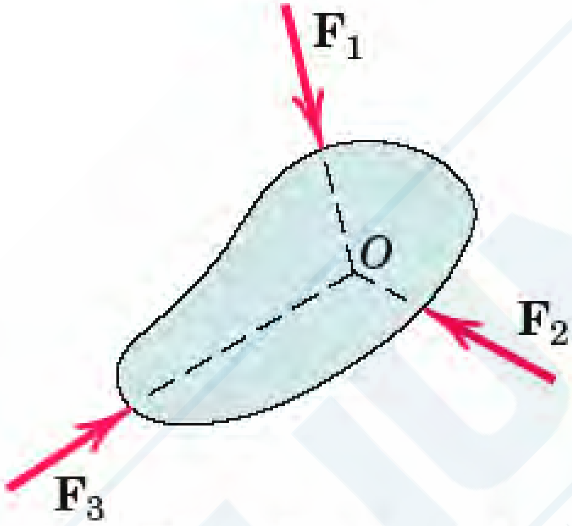
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

Figure 3/3

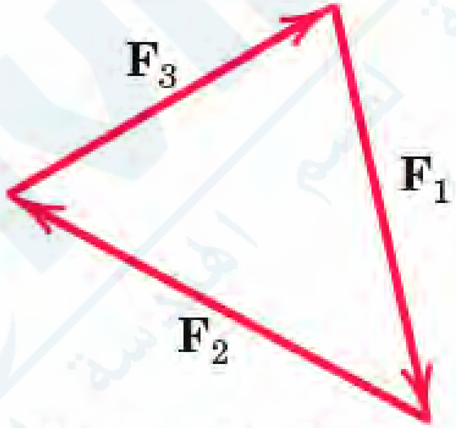


Two-force members

Figure 3/4

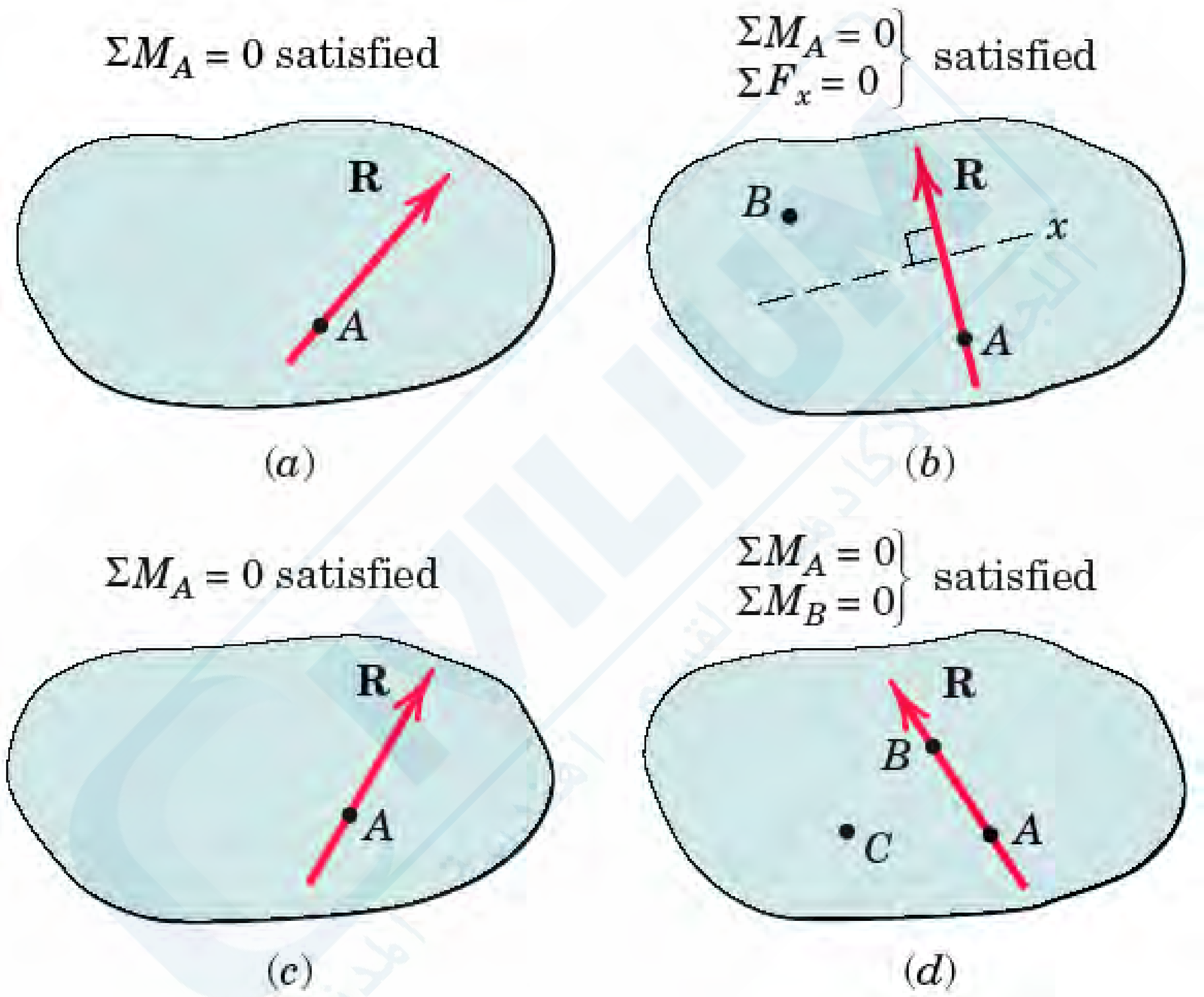


(*a*) Three-force member



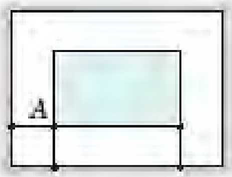
(*b*) Closed polygon satisfies $\Sigma \mathbf{F} = \mathbf{0}$

Figure 3/5

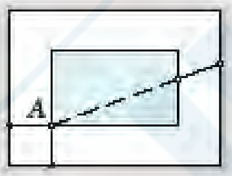


ΣM_A = 0 satisfied
R
A
(a)
ΣM_A = 0
ΣF_x = 0
satisfied
R
B
x
A
(b)
ΣM_A = 0 satisfied
R
A
(c)
ΣM_A = 0
ΣM_B = 0
satisfied
R
B
A
C
(d)

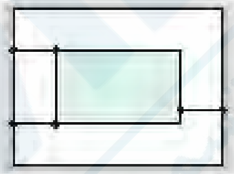
Figure 3/6



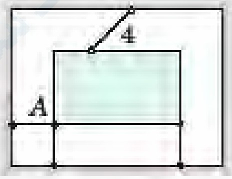
(a) Complete fixity
Adequate constraints



(b) Incomplete fixity
Partial constraints

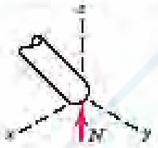
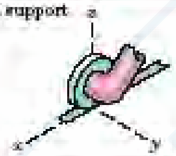
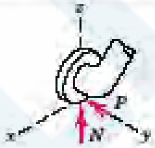
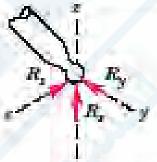
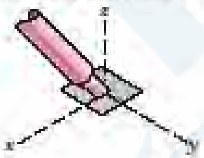
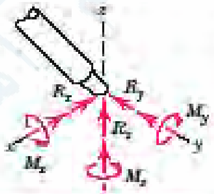
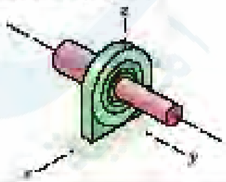
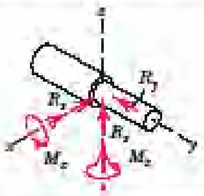


(c) Incomplete fixity
Partial constraints



(d) Excessive fixity
Redundant constraint

Figure 3/7

MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Member in contact with smooth surface, or ball-supported member	 Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface	The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint 	 A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.
4. Ball-and-socket joint	 A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.
5. Fixed connection (embedded or welded) 	 In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
6. Thrust-bearing support 	 Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

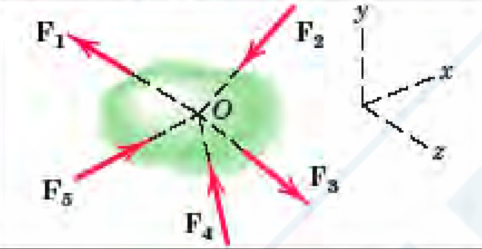
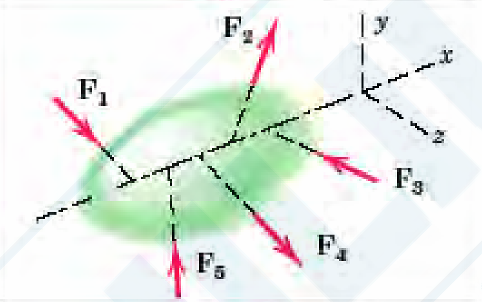
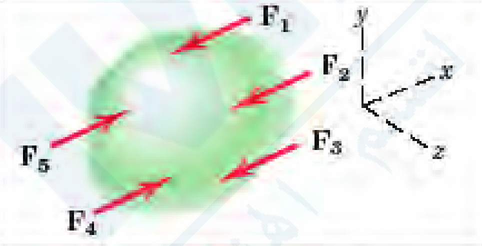
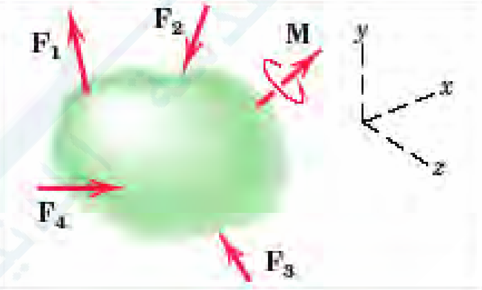
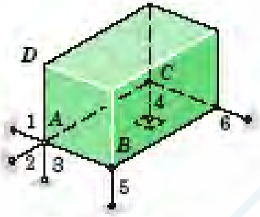
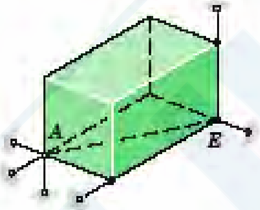
CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	**Free-Body Diagram**	**Independent Equations**
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$ $\Sigma F_y = 0$ $\quad \Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\quad \Sigma M_x = 0$ $\Sigma F_y = 0$ $\quad \Sigma M_y = 0$ $\Sigma F_z = 0$ $\quad \Sigma M_z = 0$

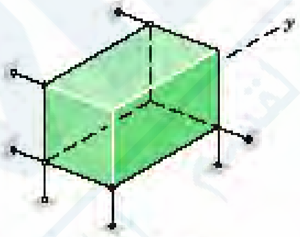
Figure 3/9



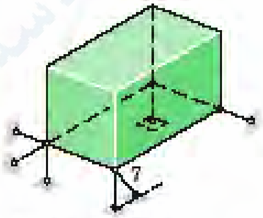
(a) Complete fixity
Adequate constraints



(b) Incomplete fixity
Partial constraints



(c) Incomplete fixity
Partial constraints



(d) Excessive fixity
Redundant constraints

Figure 3/10

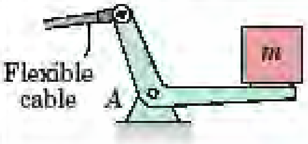
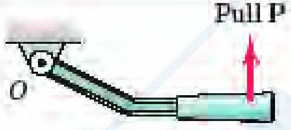
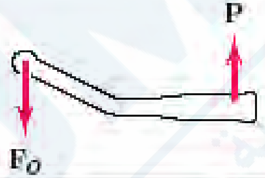
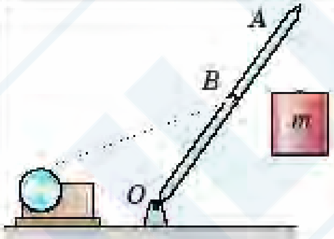
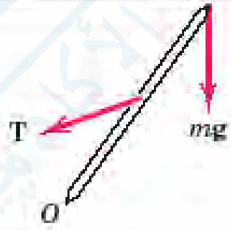
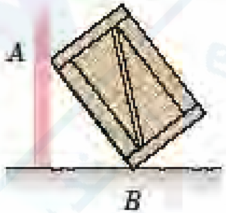
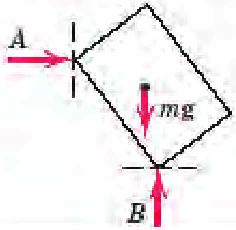
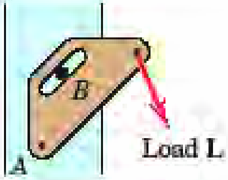
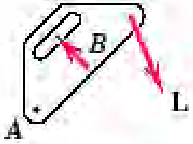
	Body	Incomplete *FBD*
1. Bell crank supporting mass m with pin support at A.		
2. Control lever applying torque to shaft at O.		
3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.		
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

Figure 3/A

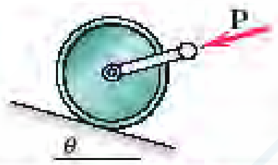
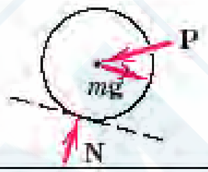
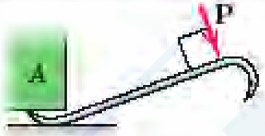
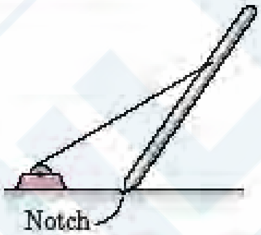
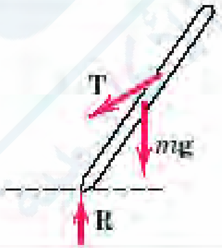
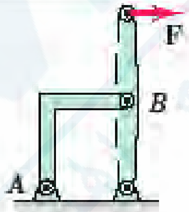
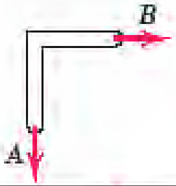
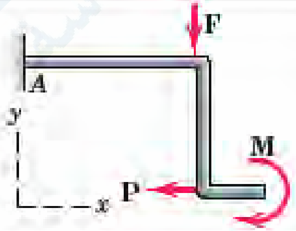
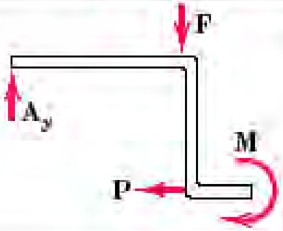
<table>
<tr><th></th><th>Body</th><th>Wrong or Incomplete FBD</th></tr>
<tr><td>1. Lawn roller of mass m being pushed up incline θ.</td><td>
</td><td>
</td></tr>
<tr><td>2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.</td><td>
</td><td>
</td></tr>
<tr><td>3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.</td><td>
</td><td>
</td></tr>
<tr><td>4. Supporting angle bracket for frame; pin joints.</td><td>
</td><td>
</td></tr>
<tr><td>5. Bent rod welded to support at A and subjected to two forces and couple.</td><td>
</td><td>
</td></tr>
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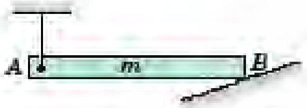
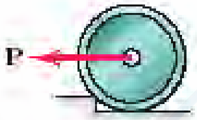
Figure 3/8

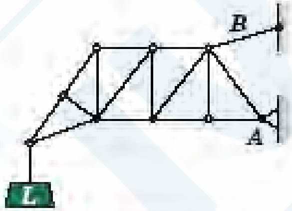
1. Uniform horizontal bar of mass m suspended by vertical cable at A and supported by rough inclined surface at B.



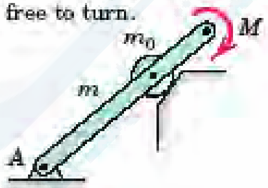
2. Wheel of mass m on verge of being rolled over curb by pull **P**.



3. Loaded truss supported by pin joint at A and by cable at B.



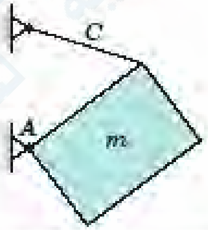
4. Uniform bar of mass m and roller of mass m_0 taken together. Subjected to couple M and supported as shown. Roller is free to turn.



5. Uniform grooved wheel of mass m supported by a rough surface and by action of horizontal cable.

6. Bar, initially horizontal but deflected under load **L**. Pinned to rigid support at each end.

7. Uniform heavy plate of mass m supported in vertical plane by cable C and hinge A.



8. Entire frame, pulleys, and contacting cable to be isolated as a single unit.

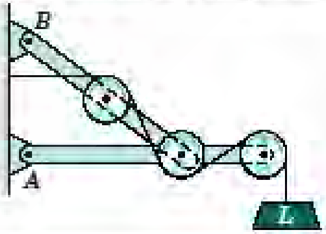
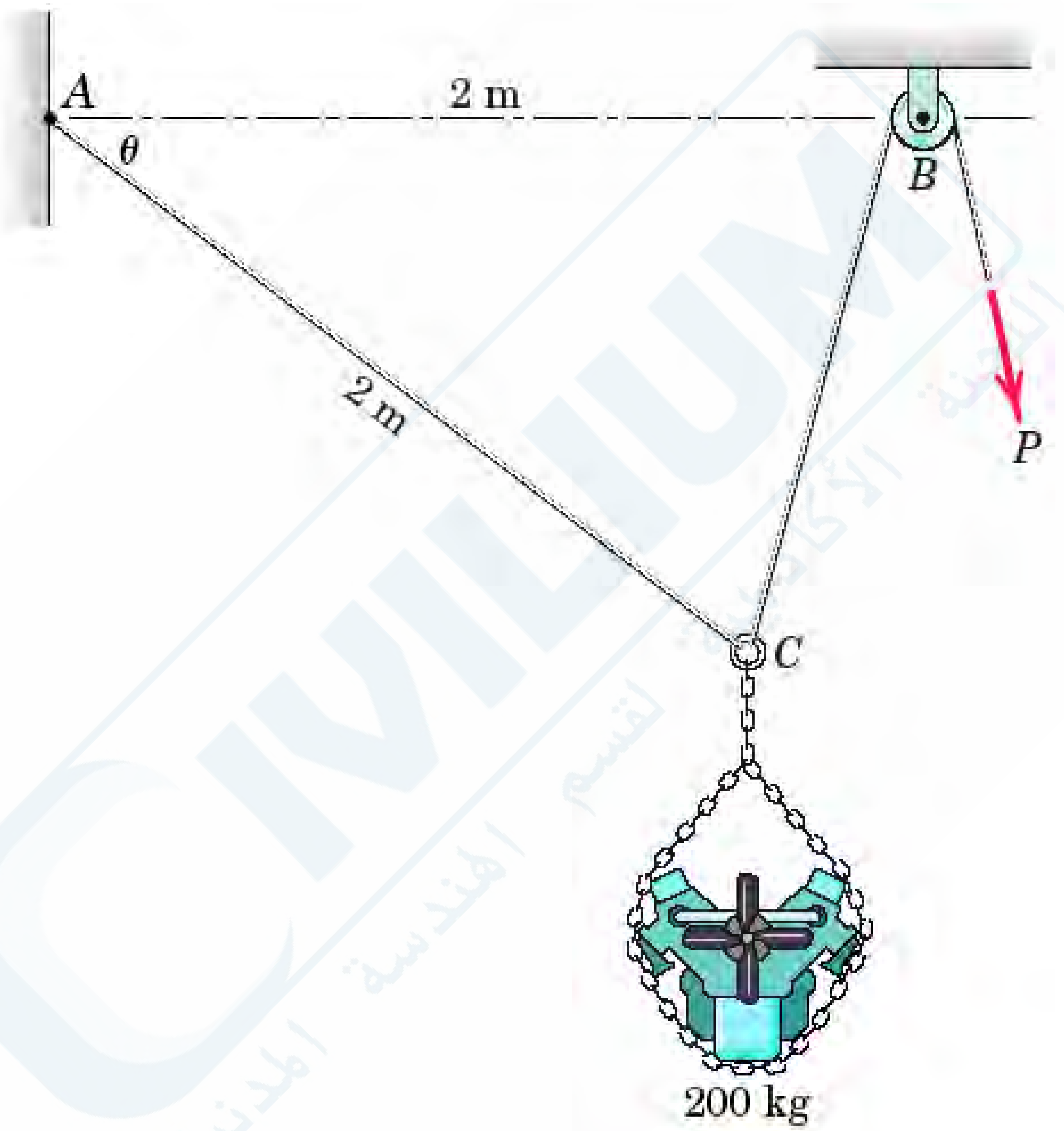
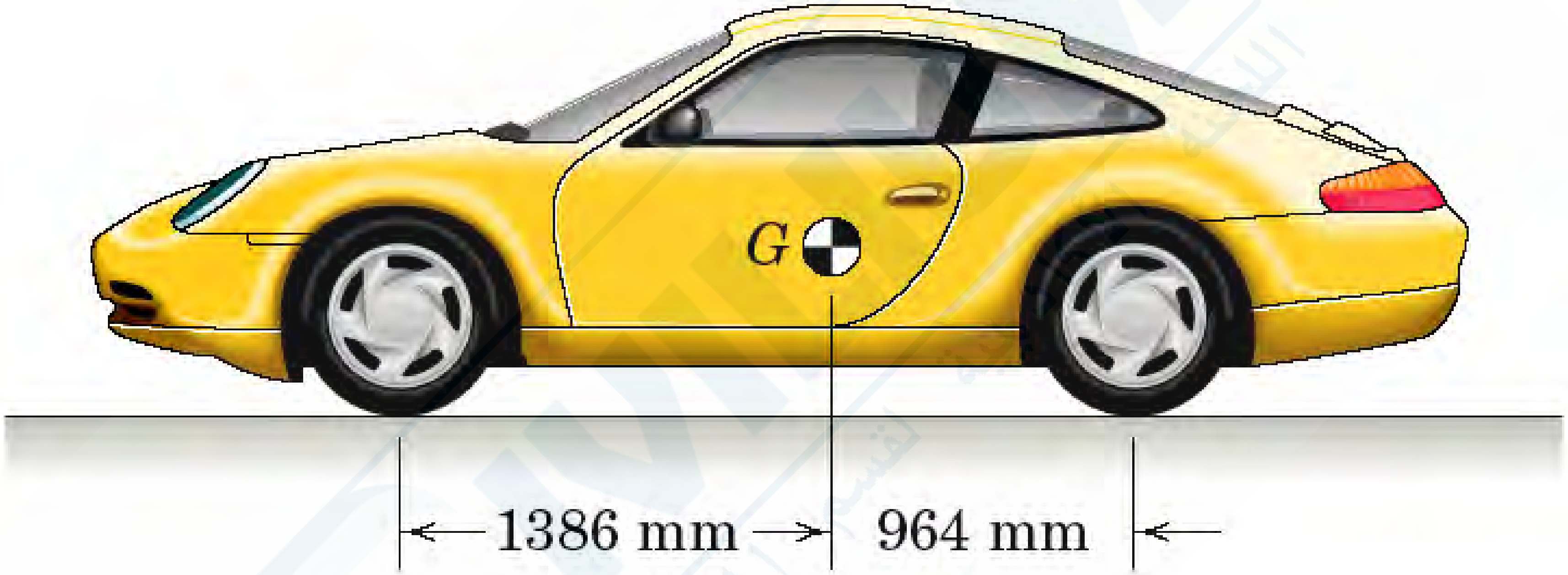


Figure 1/C



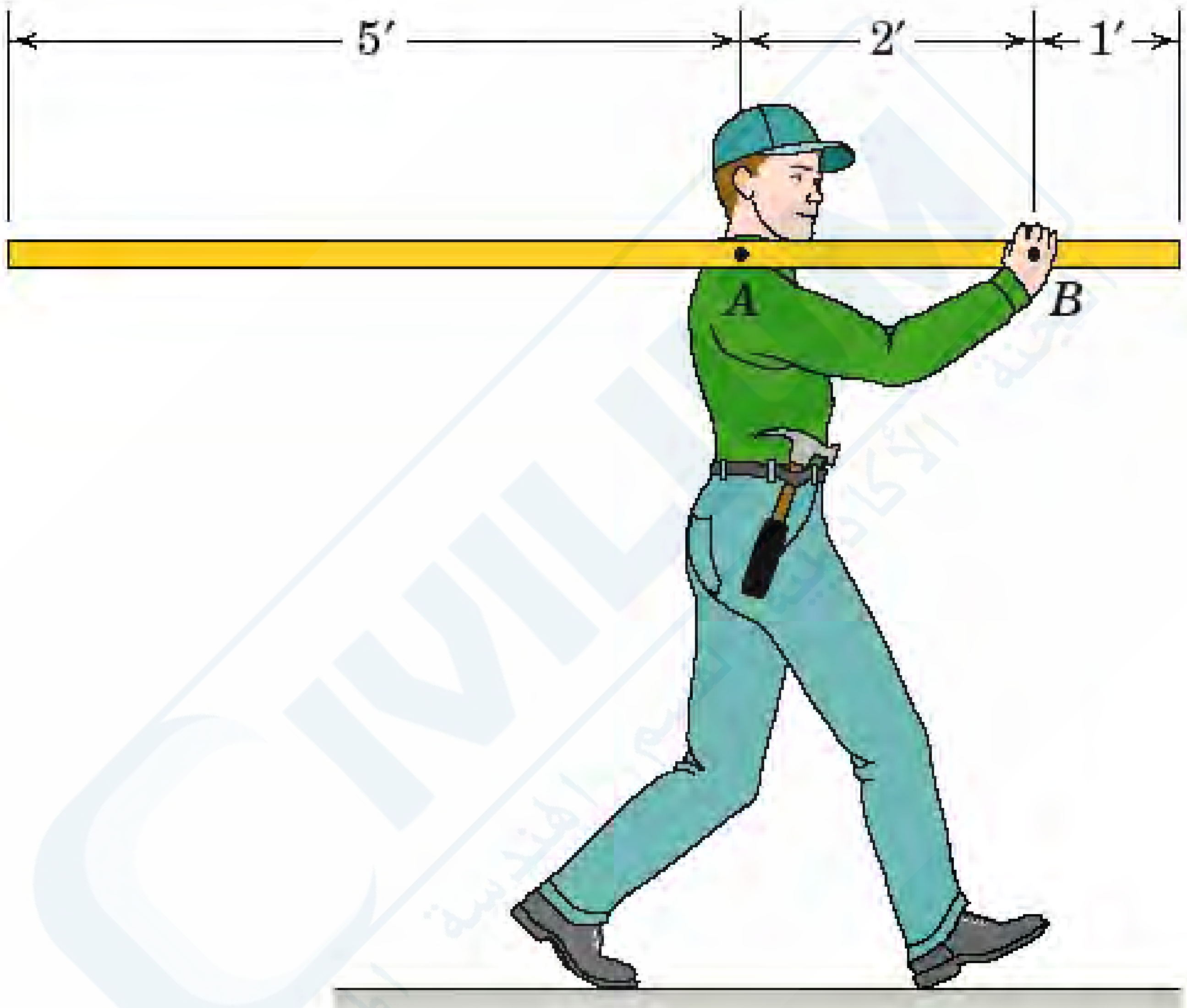
Problem 3/1

P 03 01

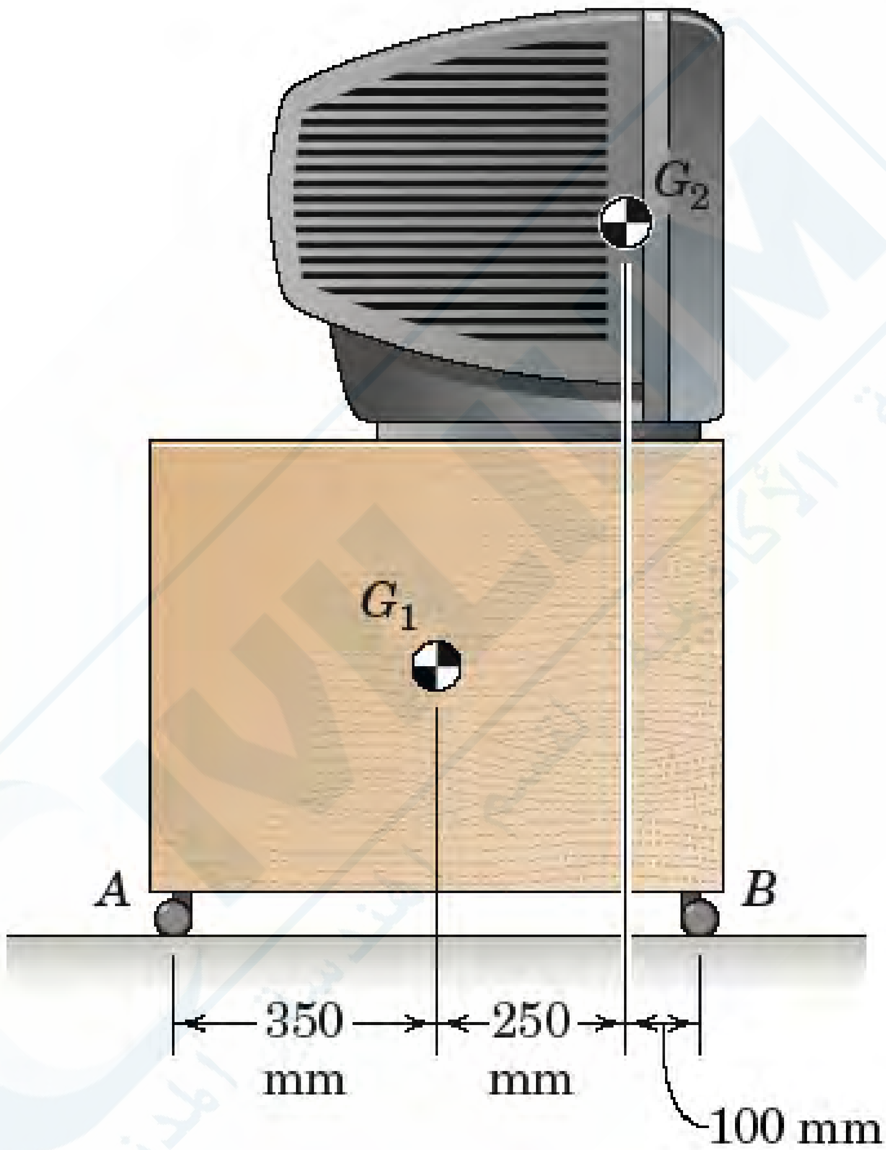


Problem 3/2

P_03_02

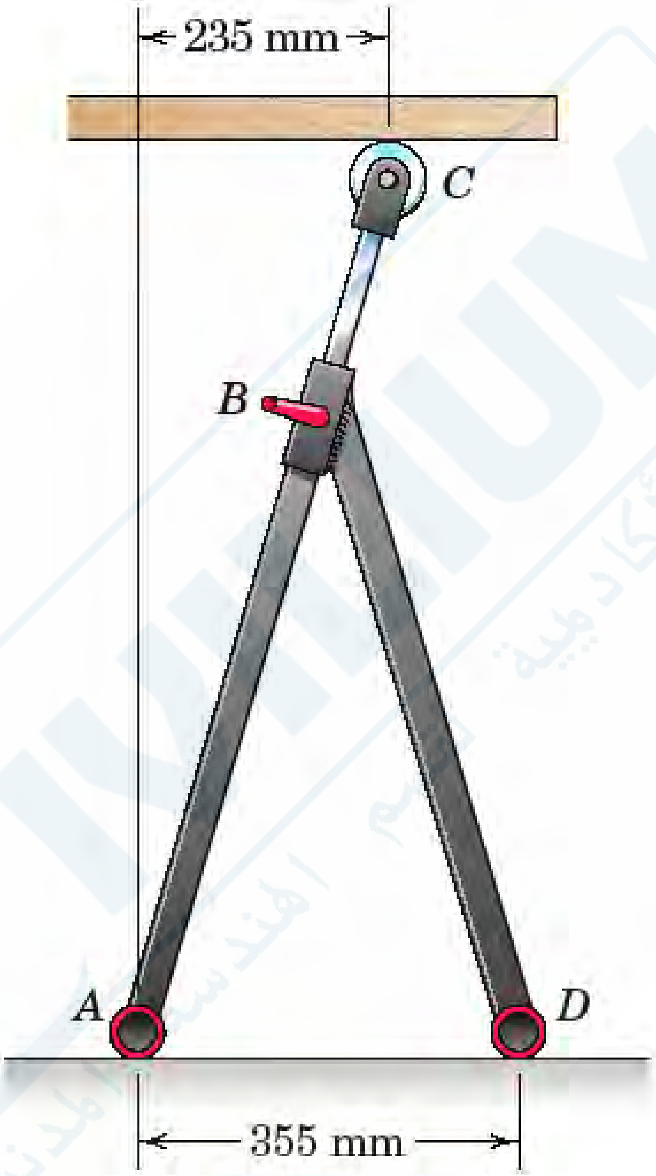


Problem 3/3



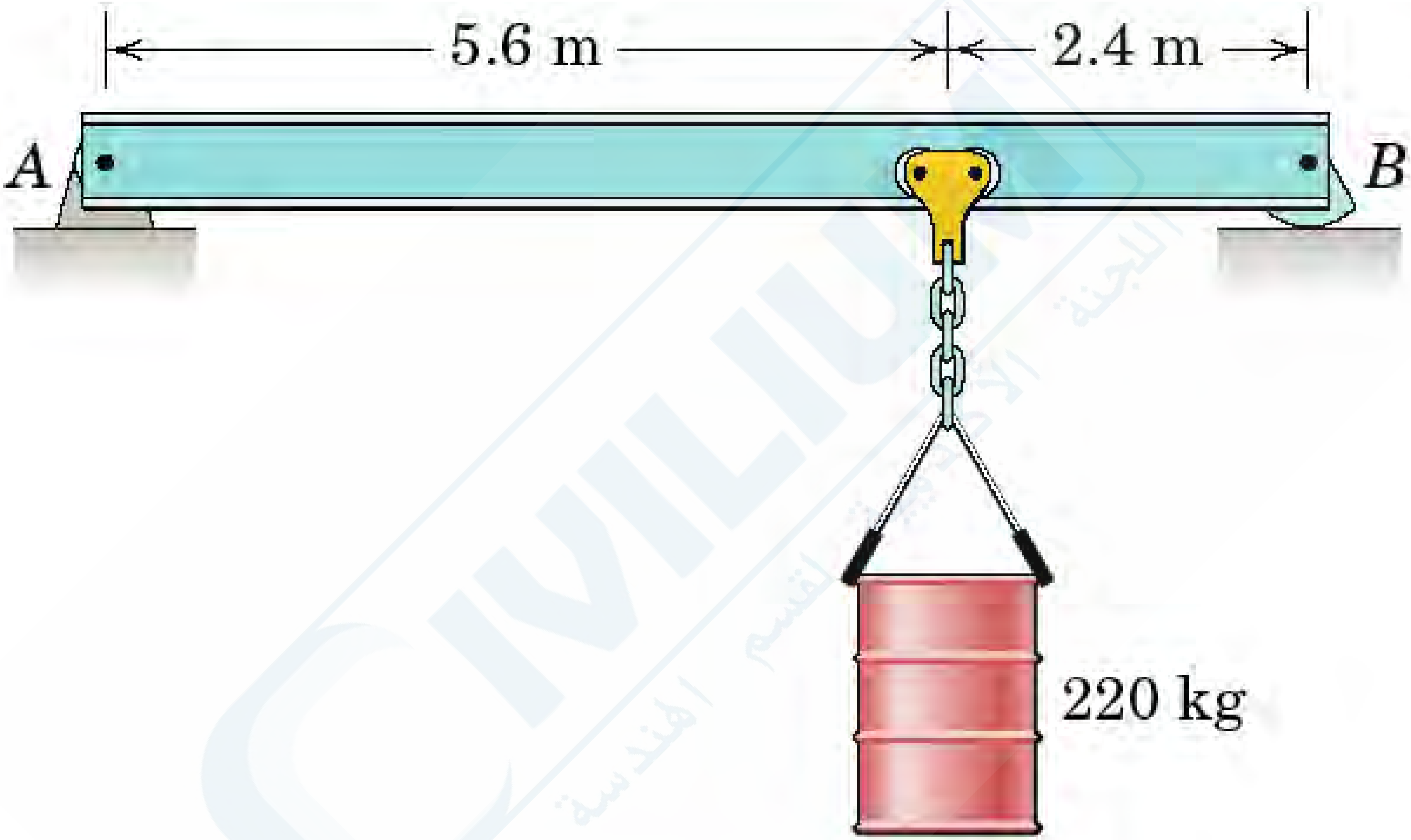
Problem 3/4

P 03 04



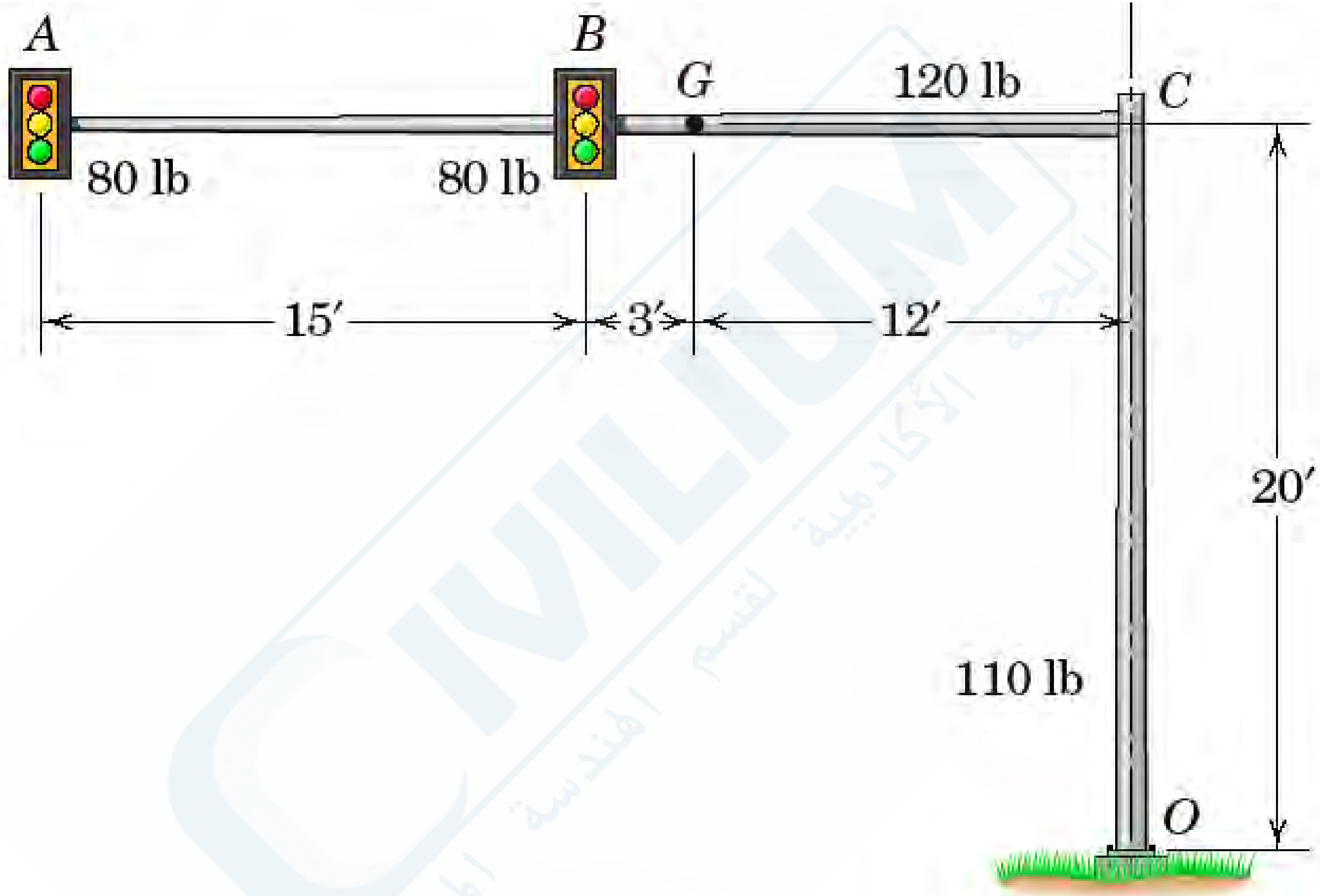
Problem 3/5

P_03_05

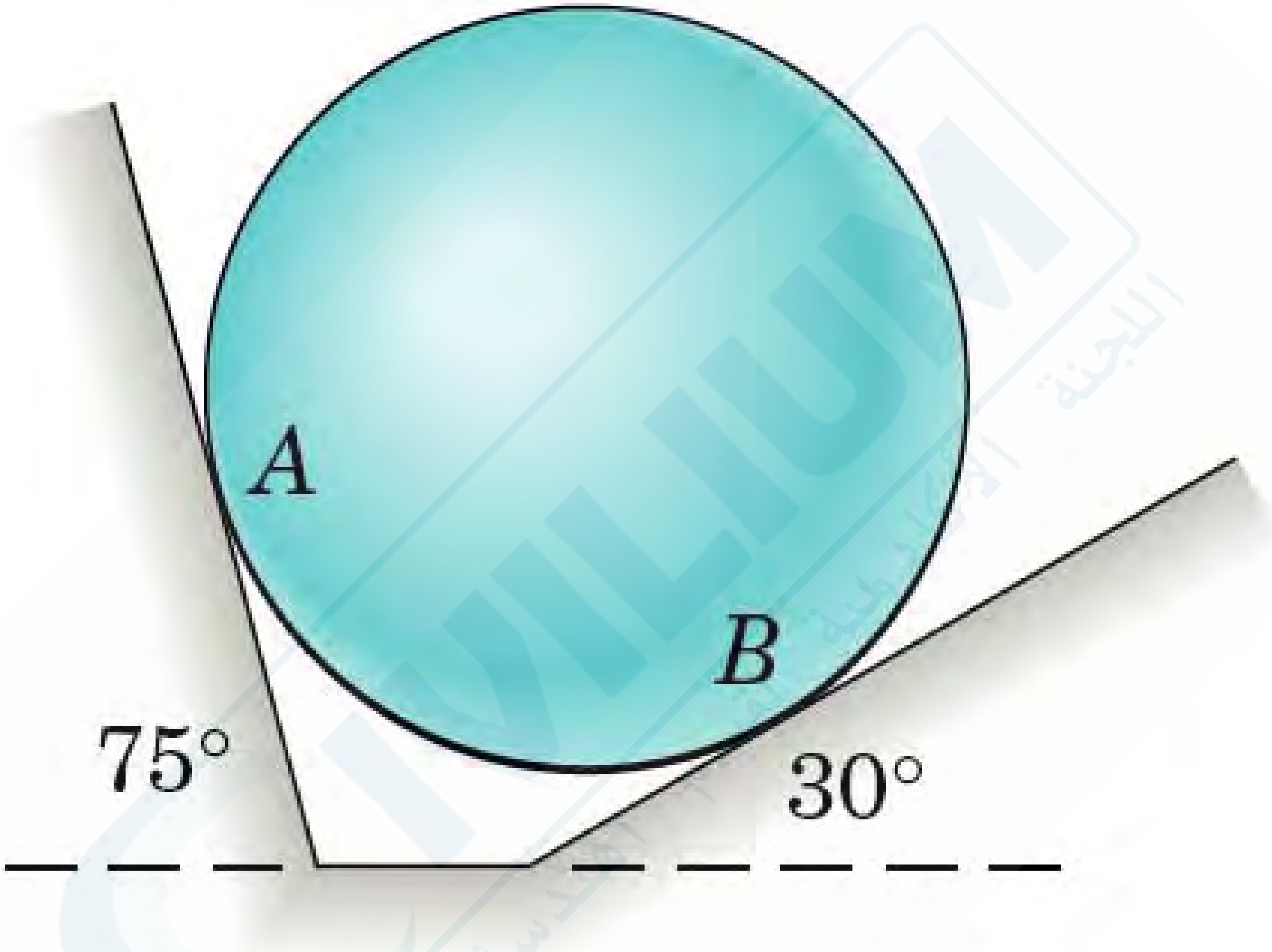


Problem 3/6

P_03_06

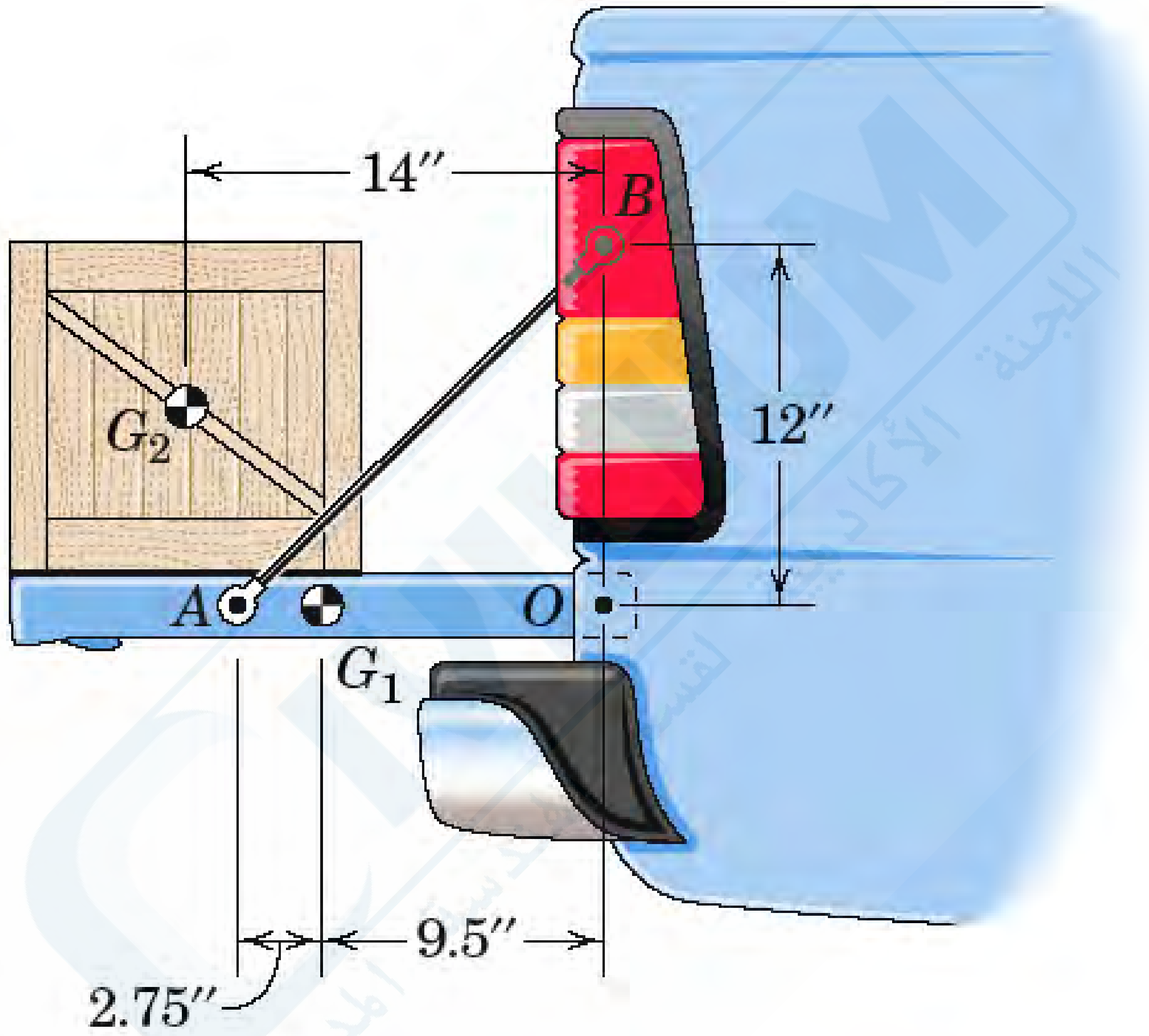


Problem 3/7



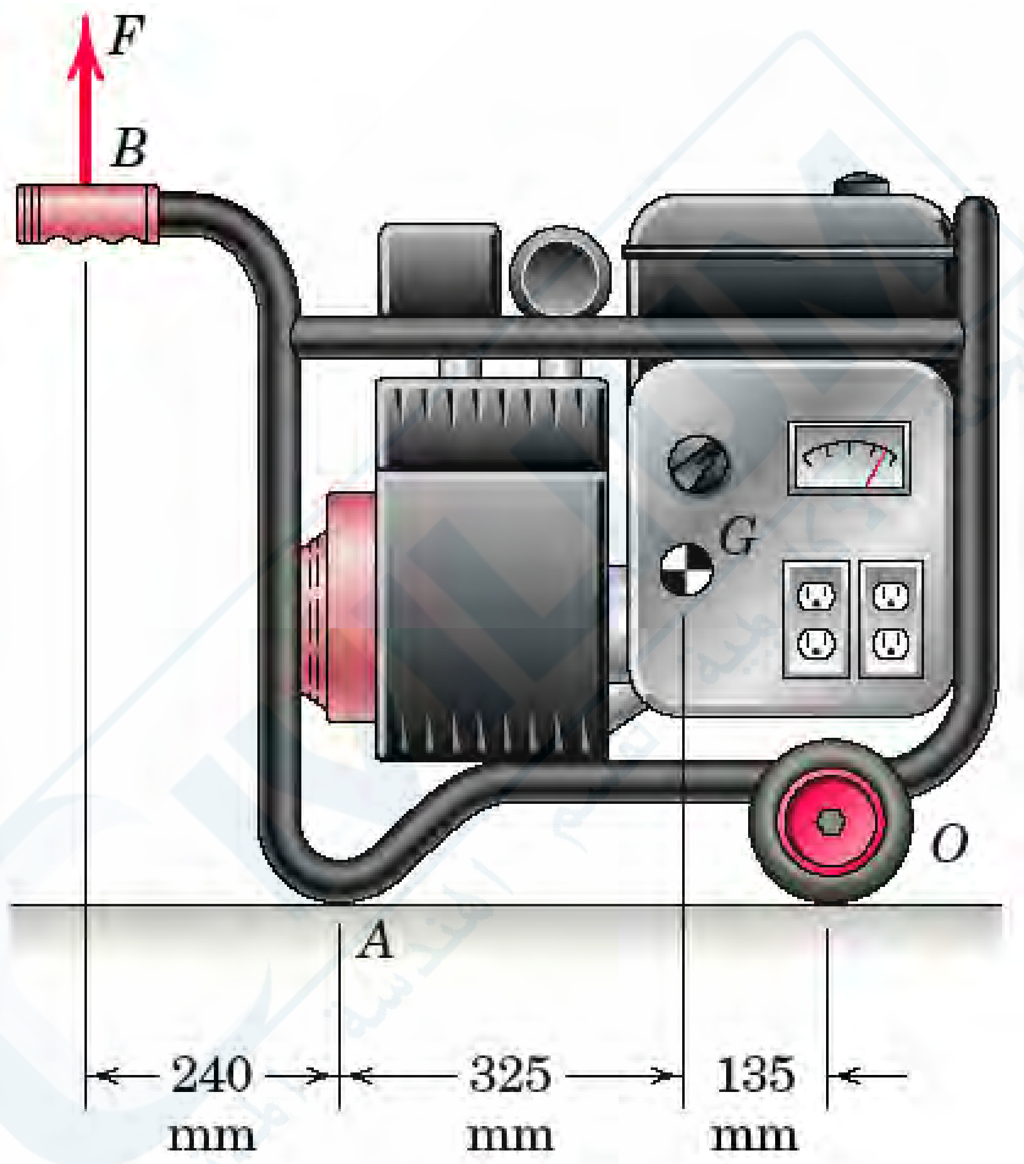
Problem 3/8

P_03_08



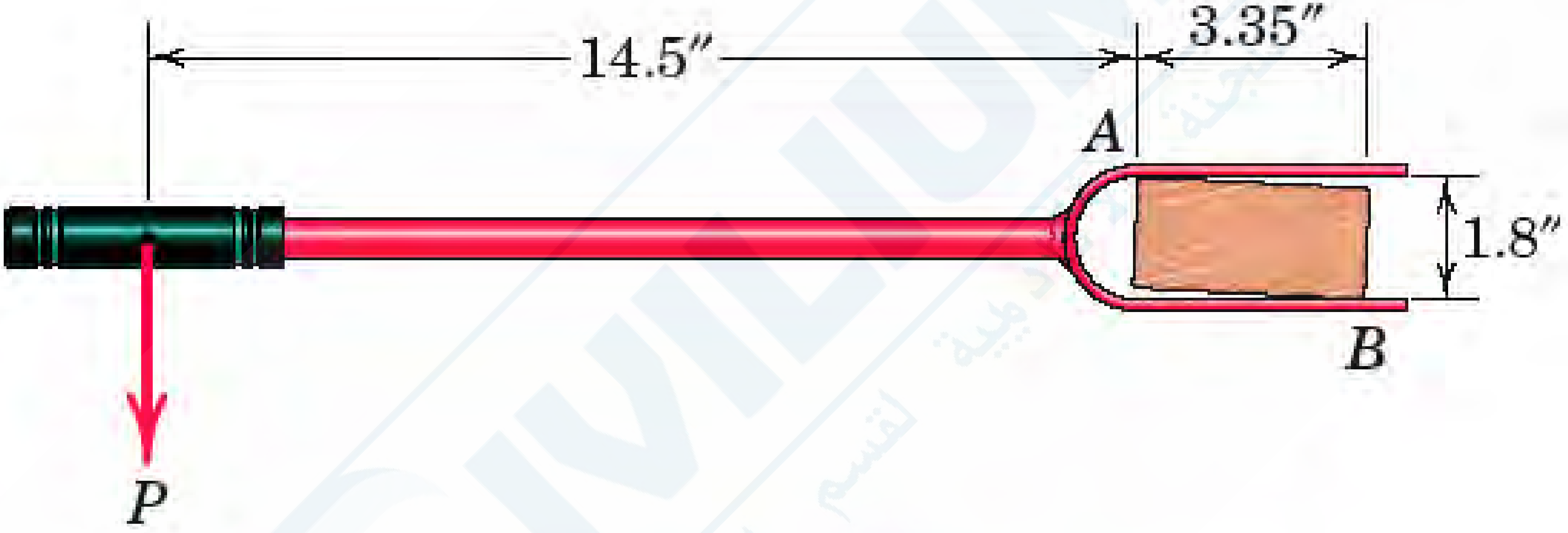
Problem 3/9

P_03_09



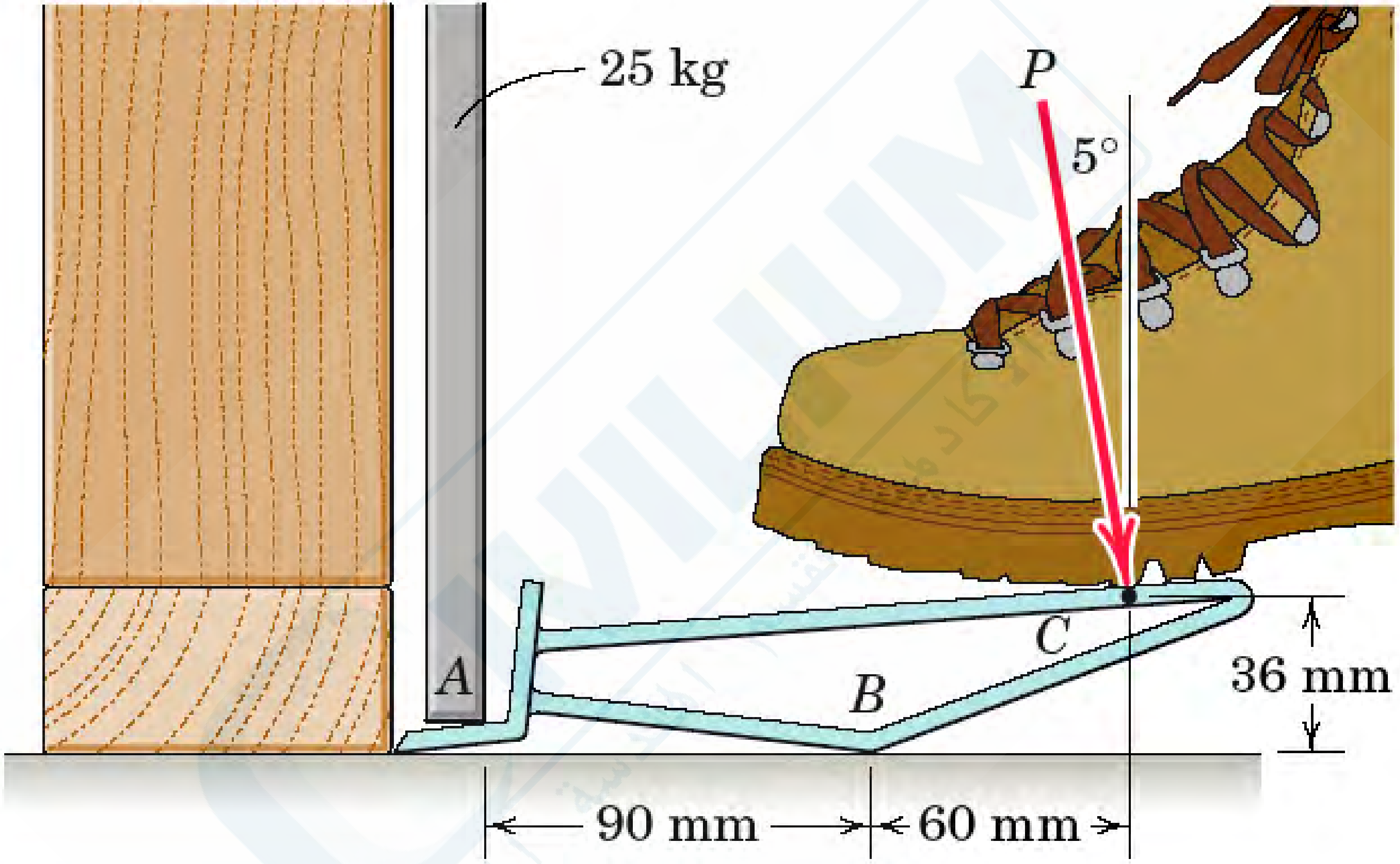
Problem 3/10

P 03 10



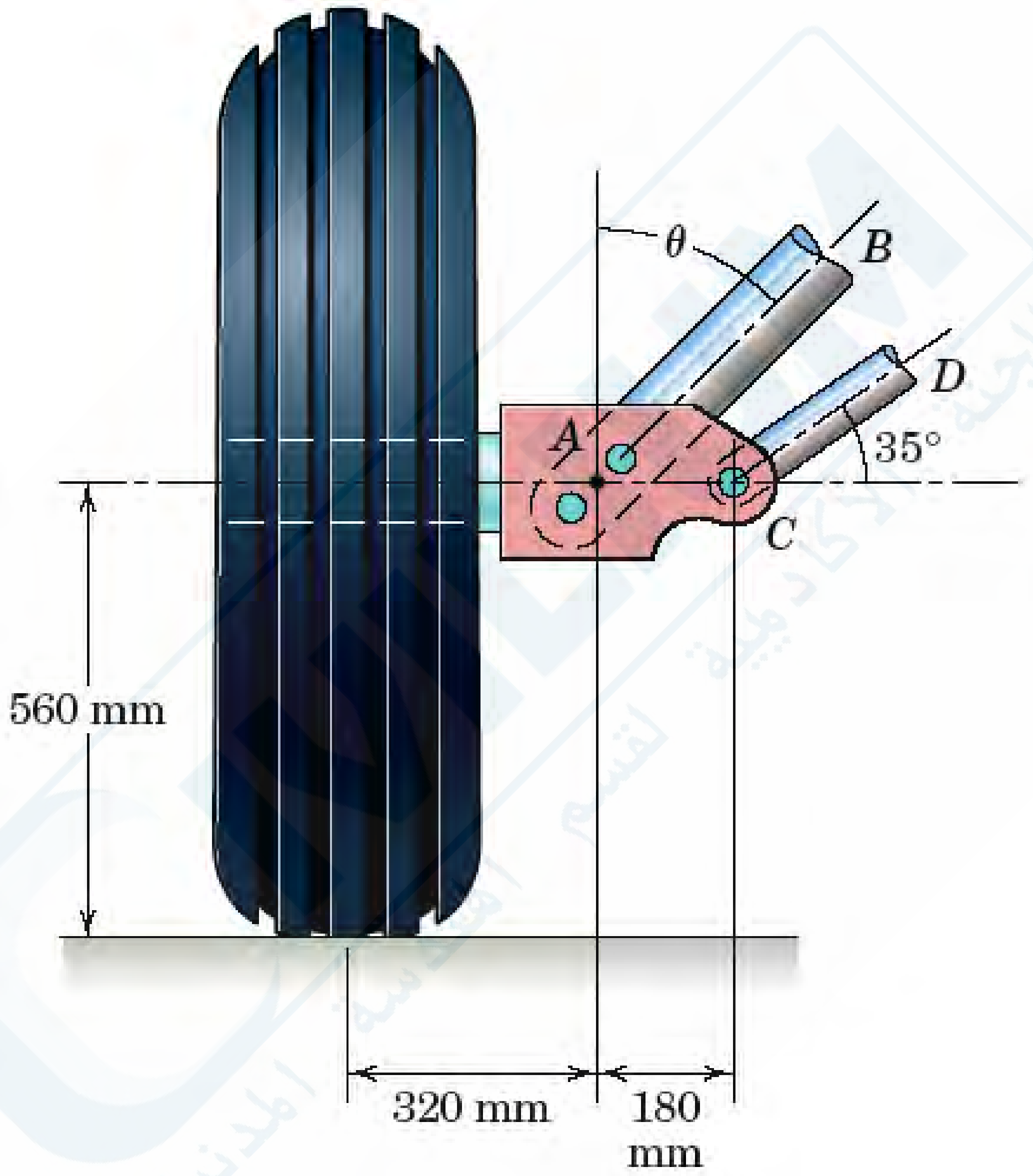
Problem 3/100

P_03_100



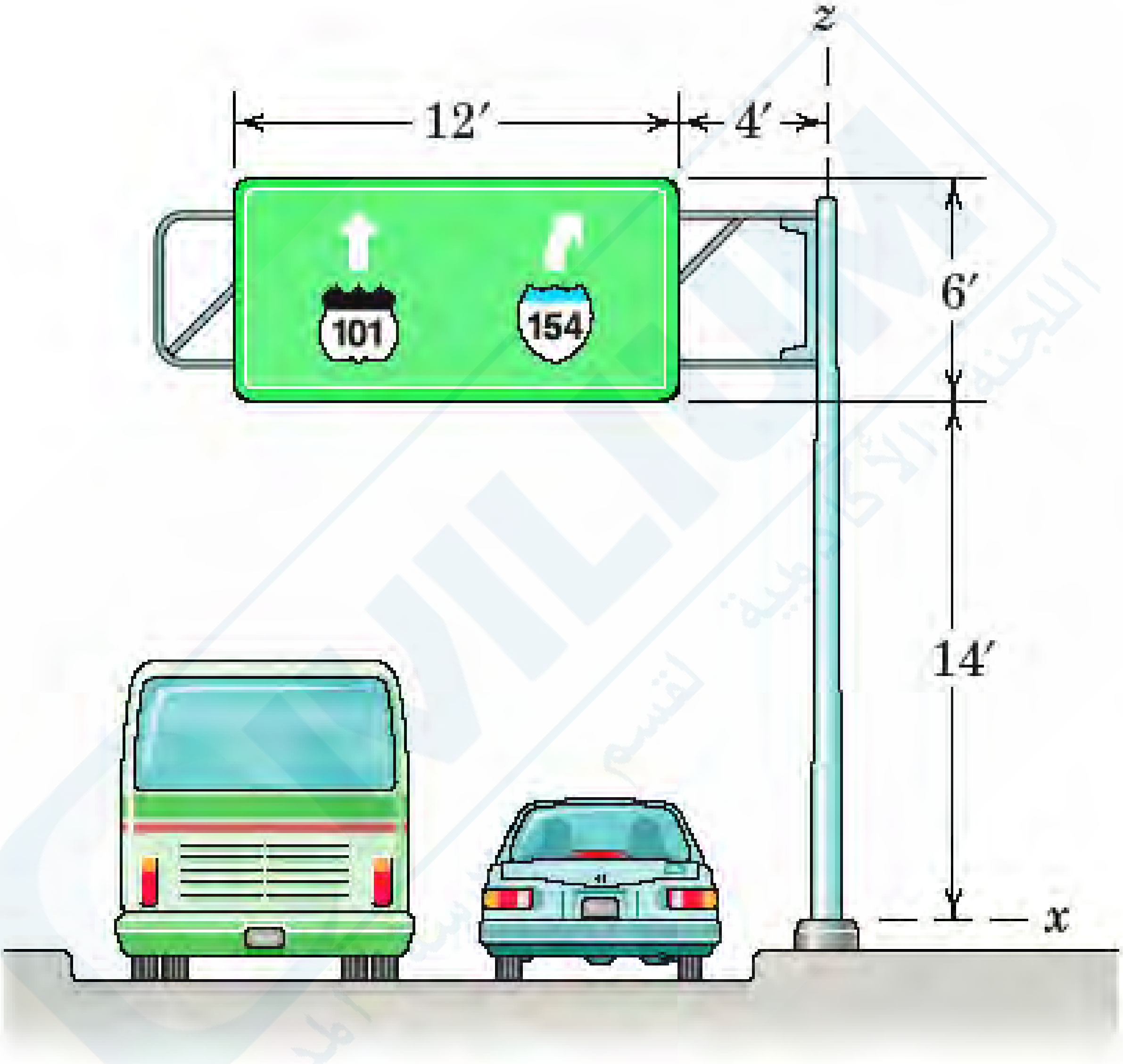
Problem 3/101

P_03_101



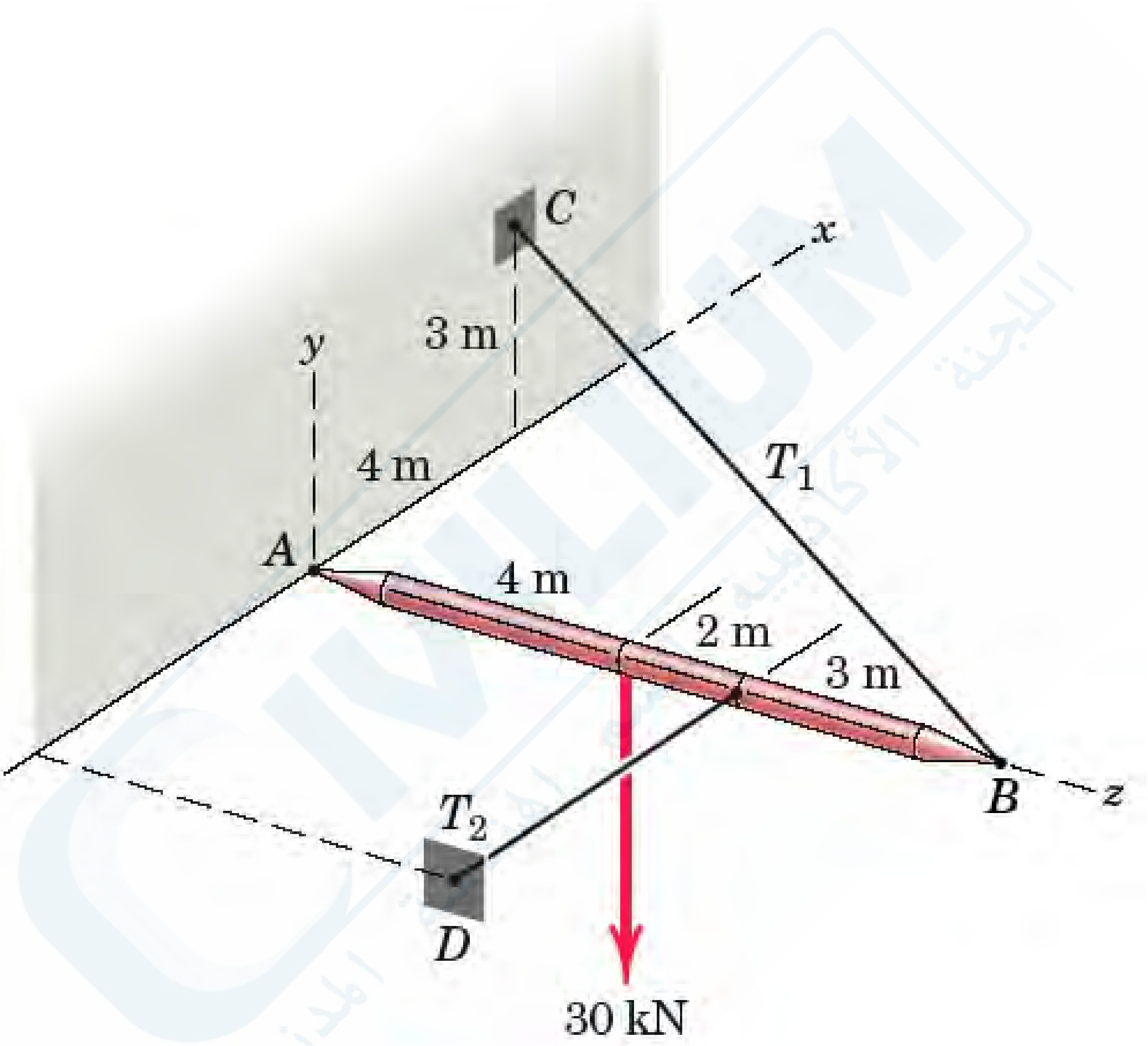
Problem 3/102

P 03 102



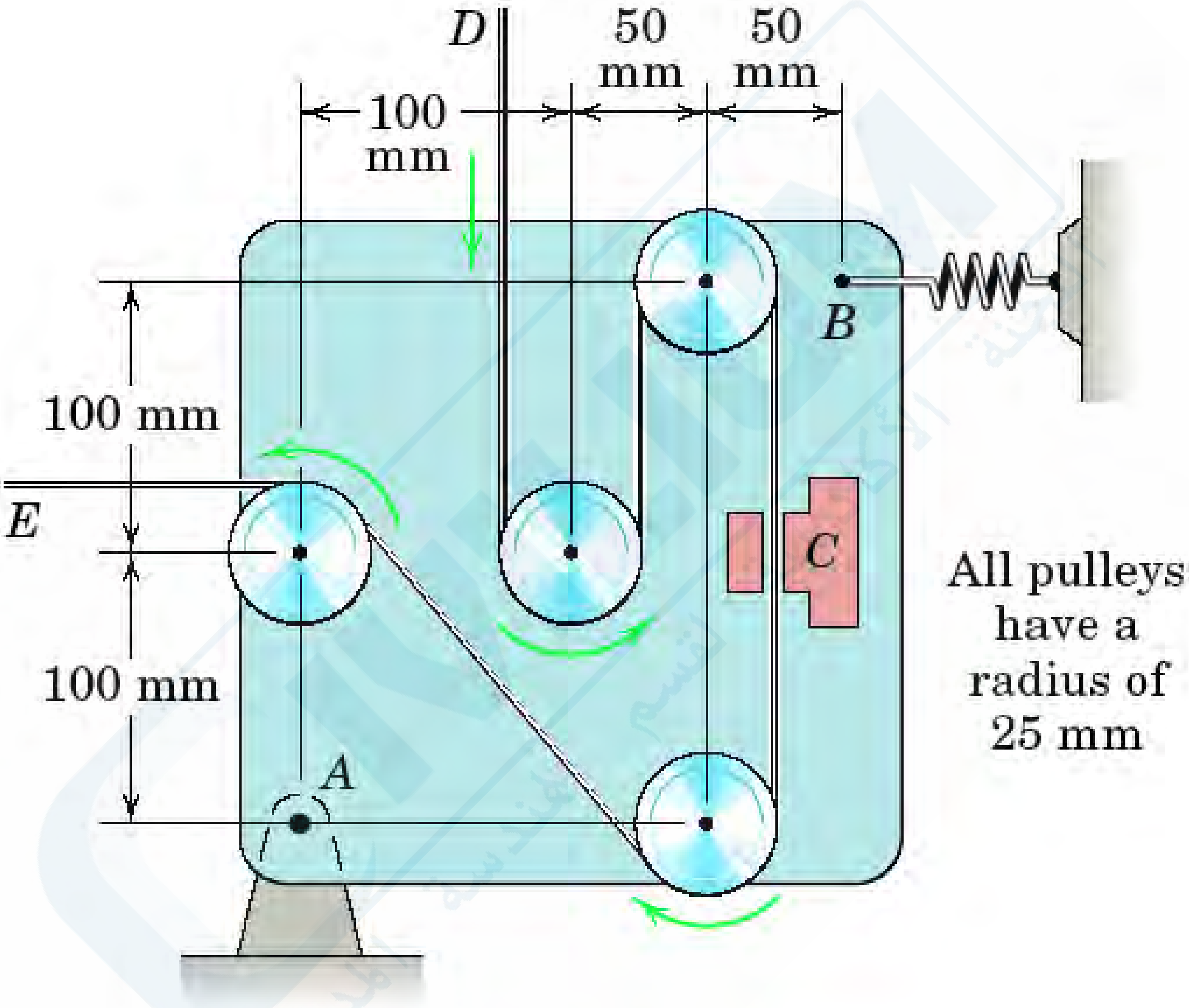
Problem 3/103

P_03_103



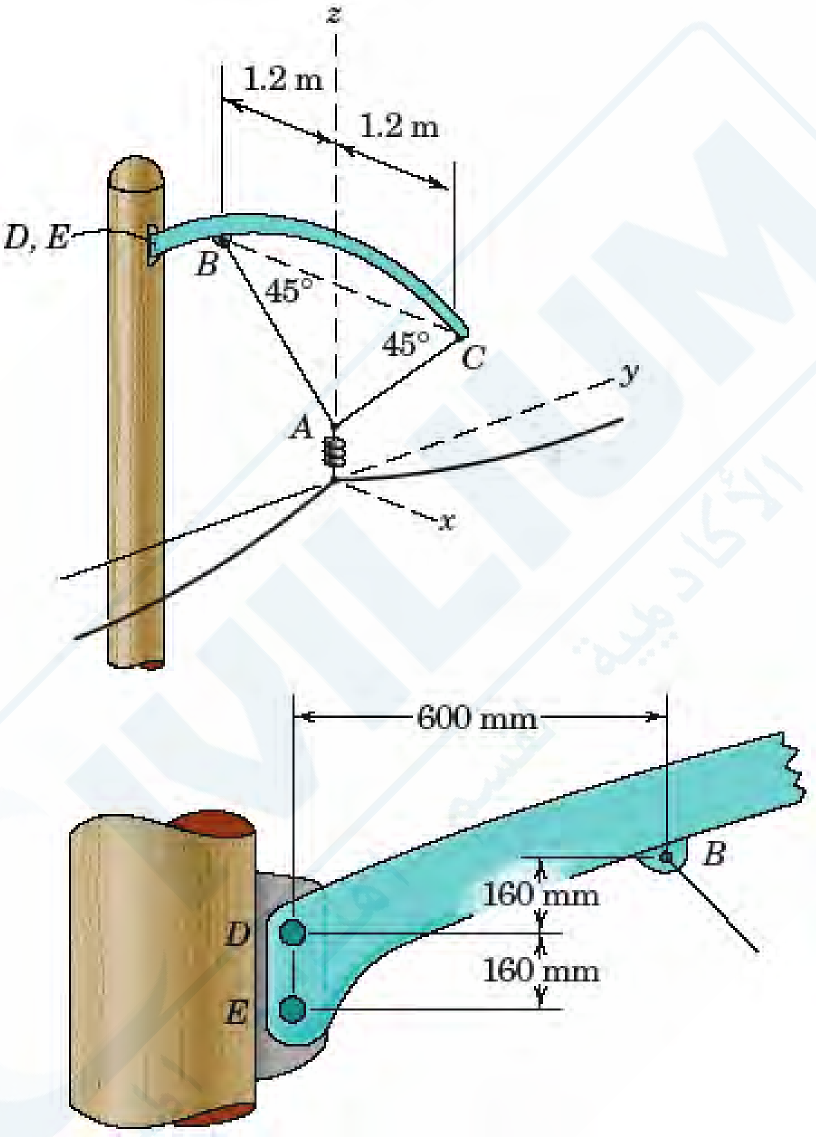
Problem 3/104

P 03 104



Problem 3/105

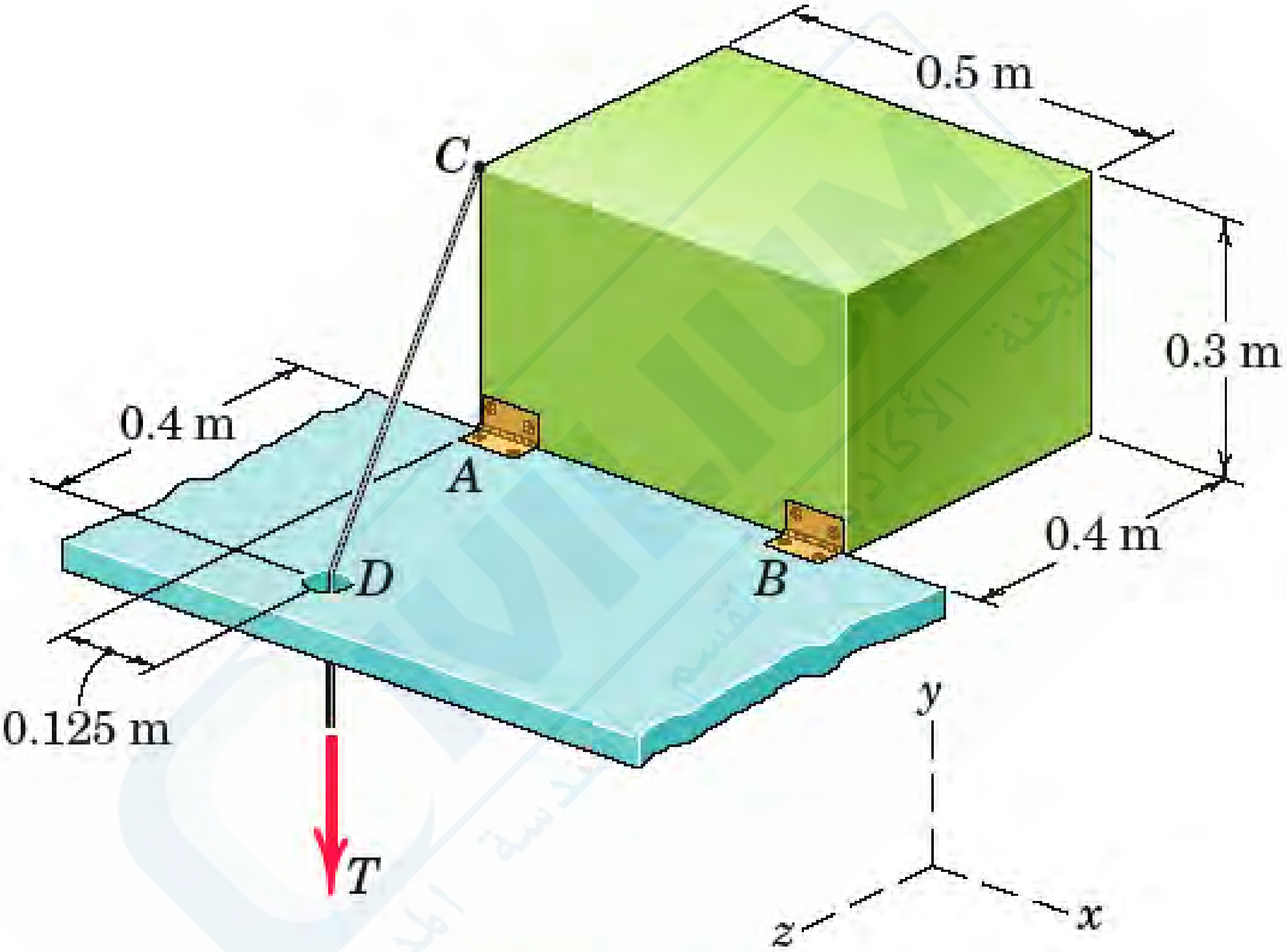
P 03 105



Detail of arm attachment

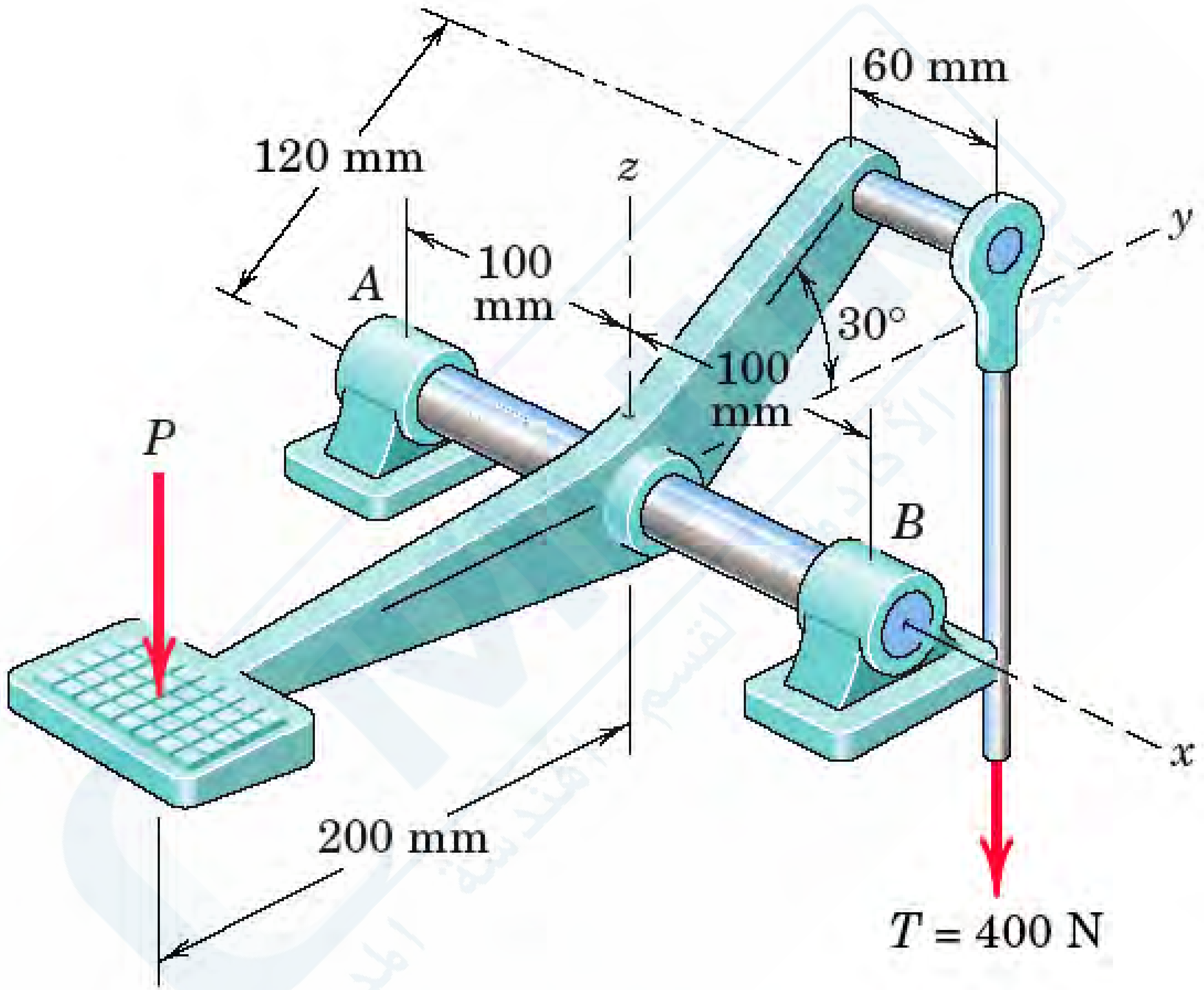
Problem 3/106

P 03 106



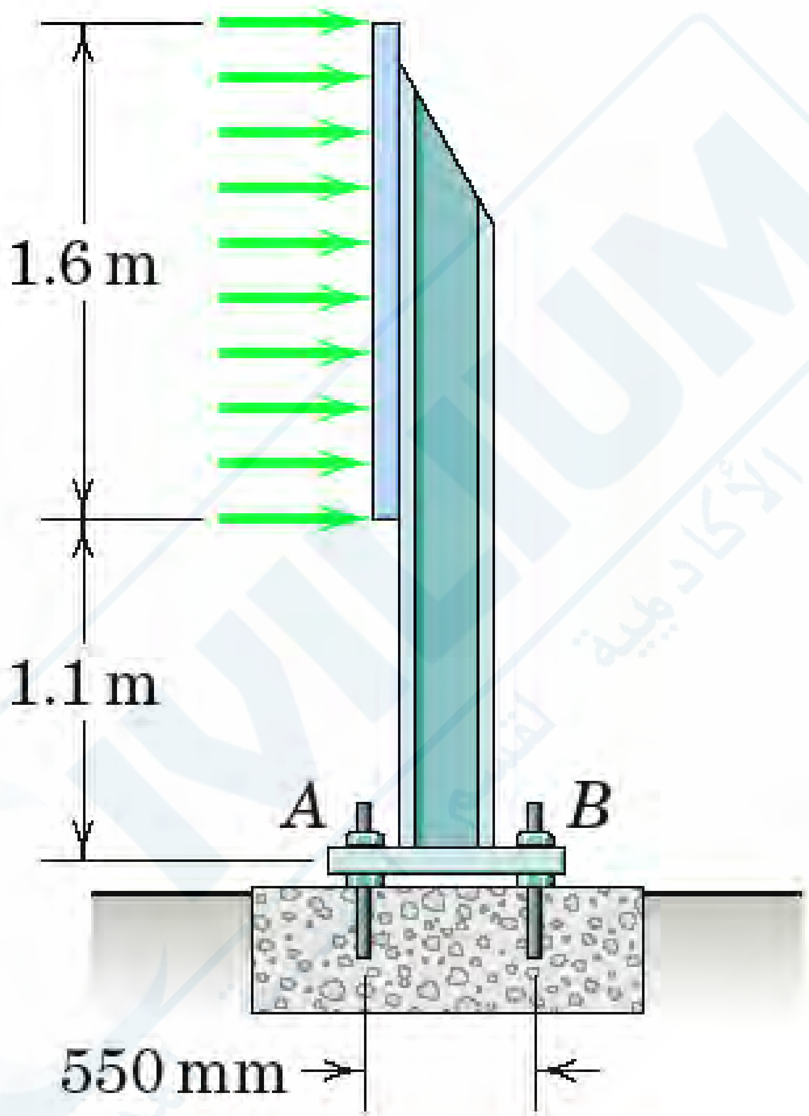
Problem 3/107

P 03 107



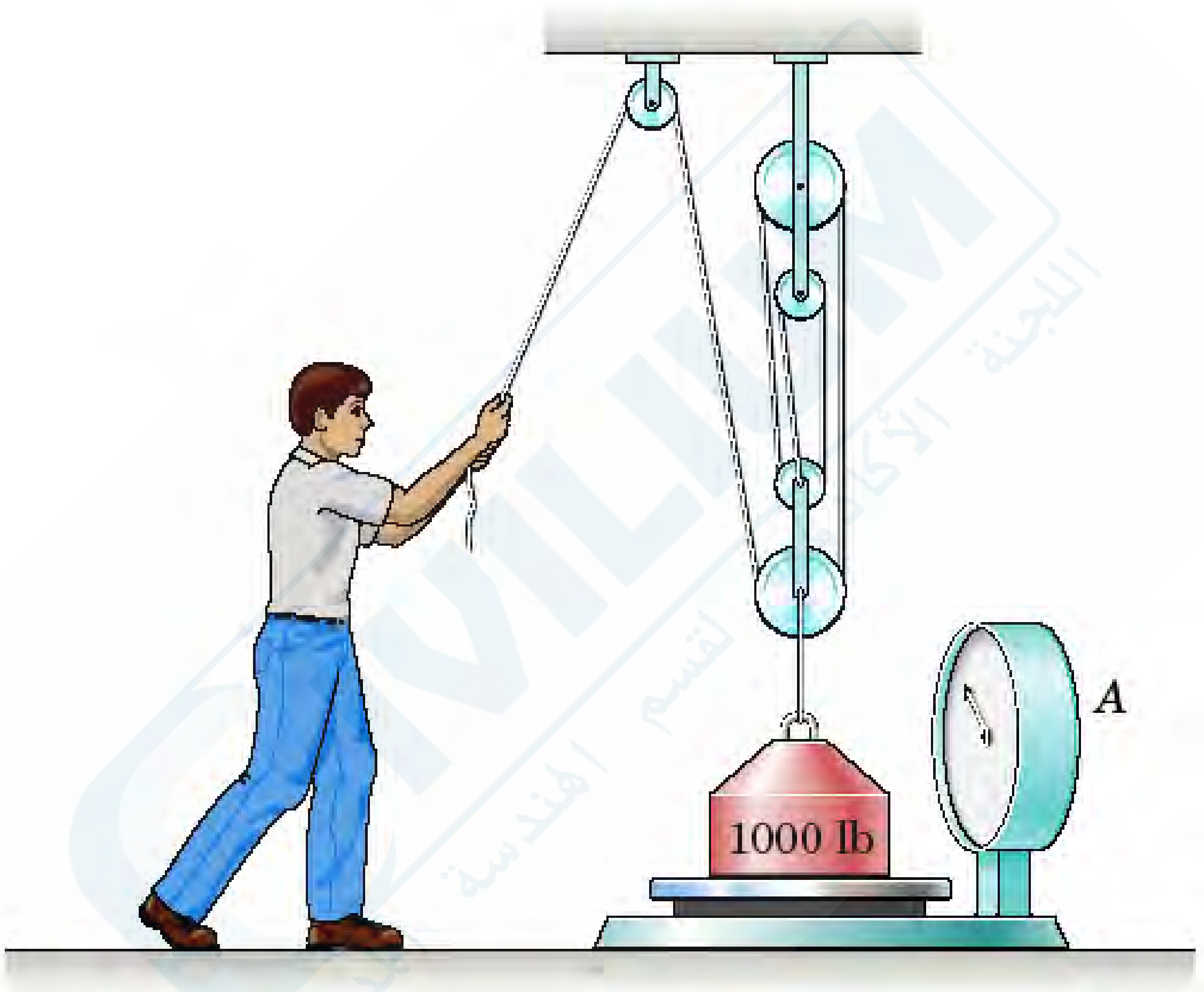
Problem 3/108

P 03 108



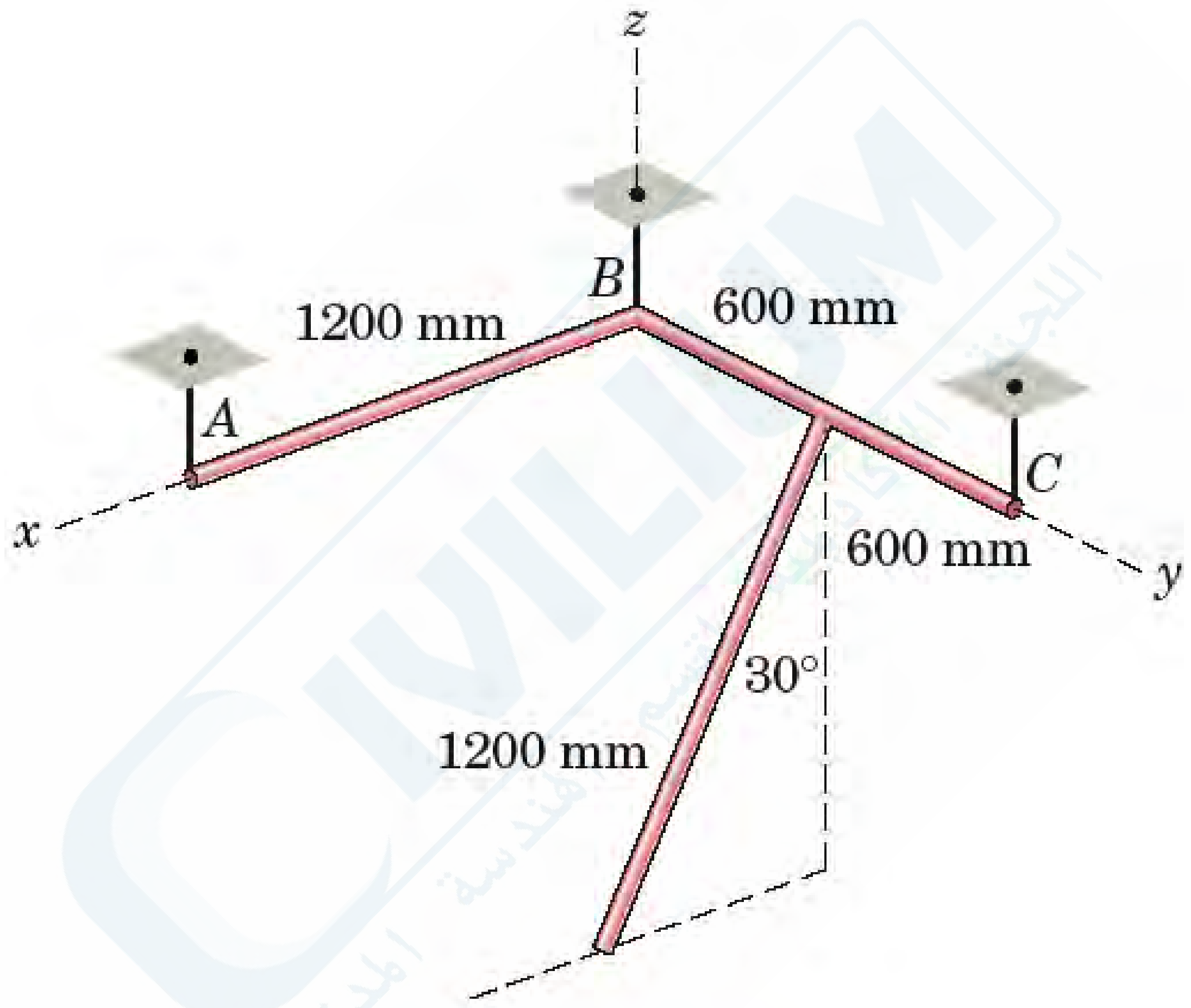
Problem 3/109

P 03 109



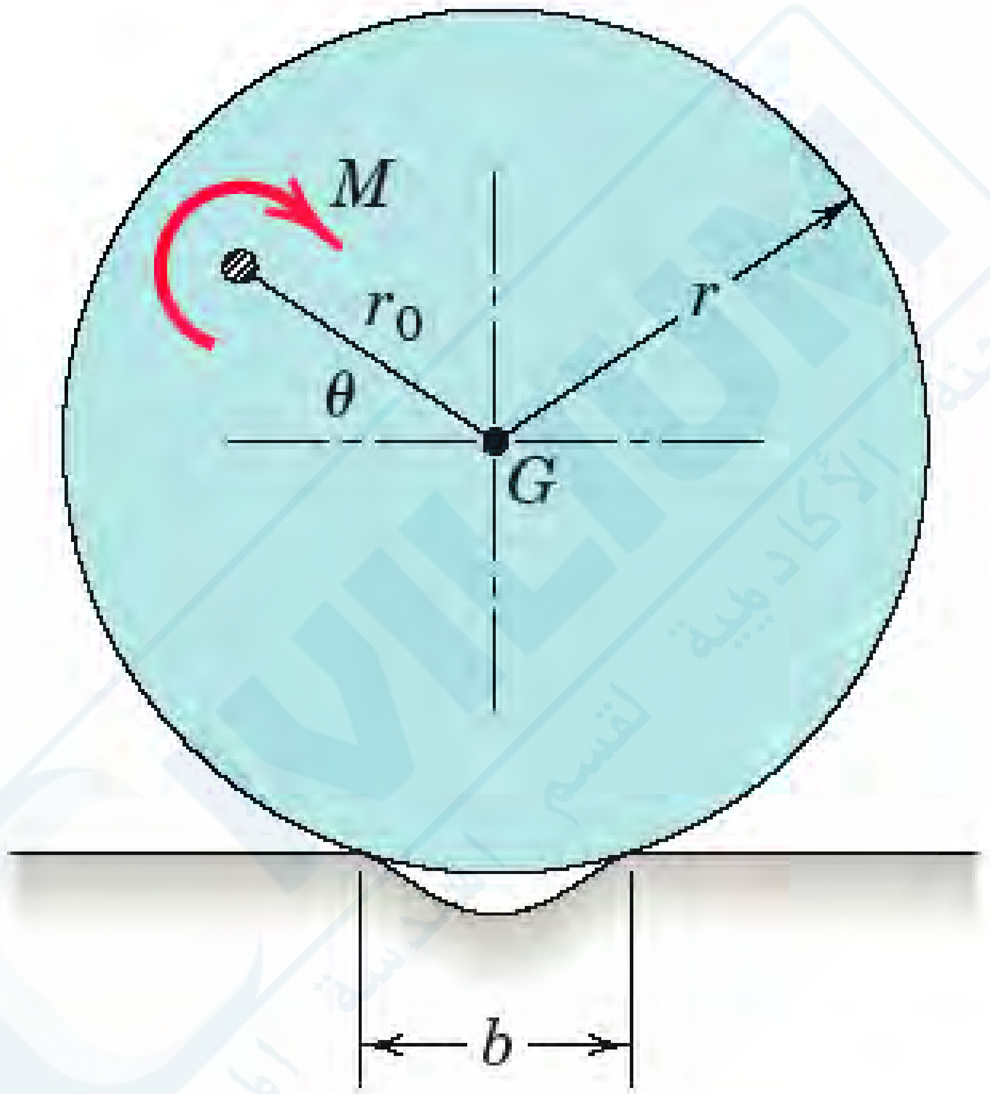
Problem 3/11

P 03 11



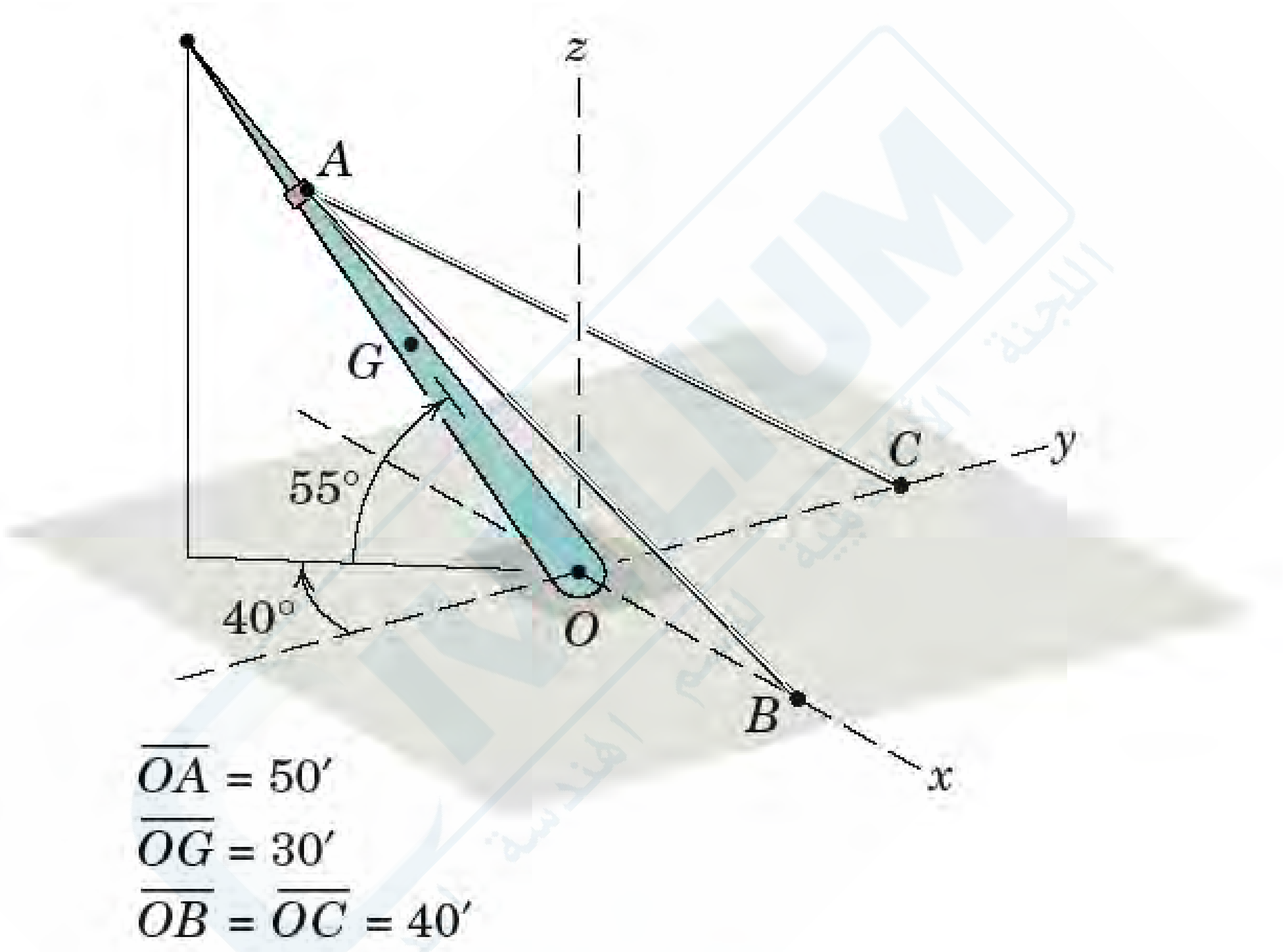
Problem 3/110

P_03_110



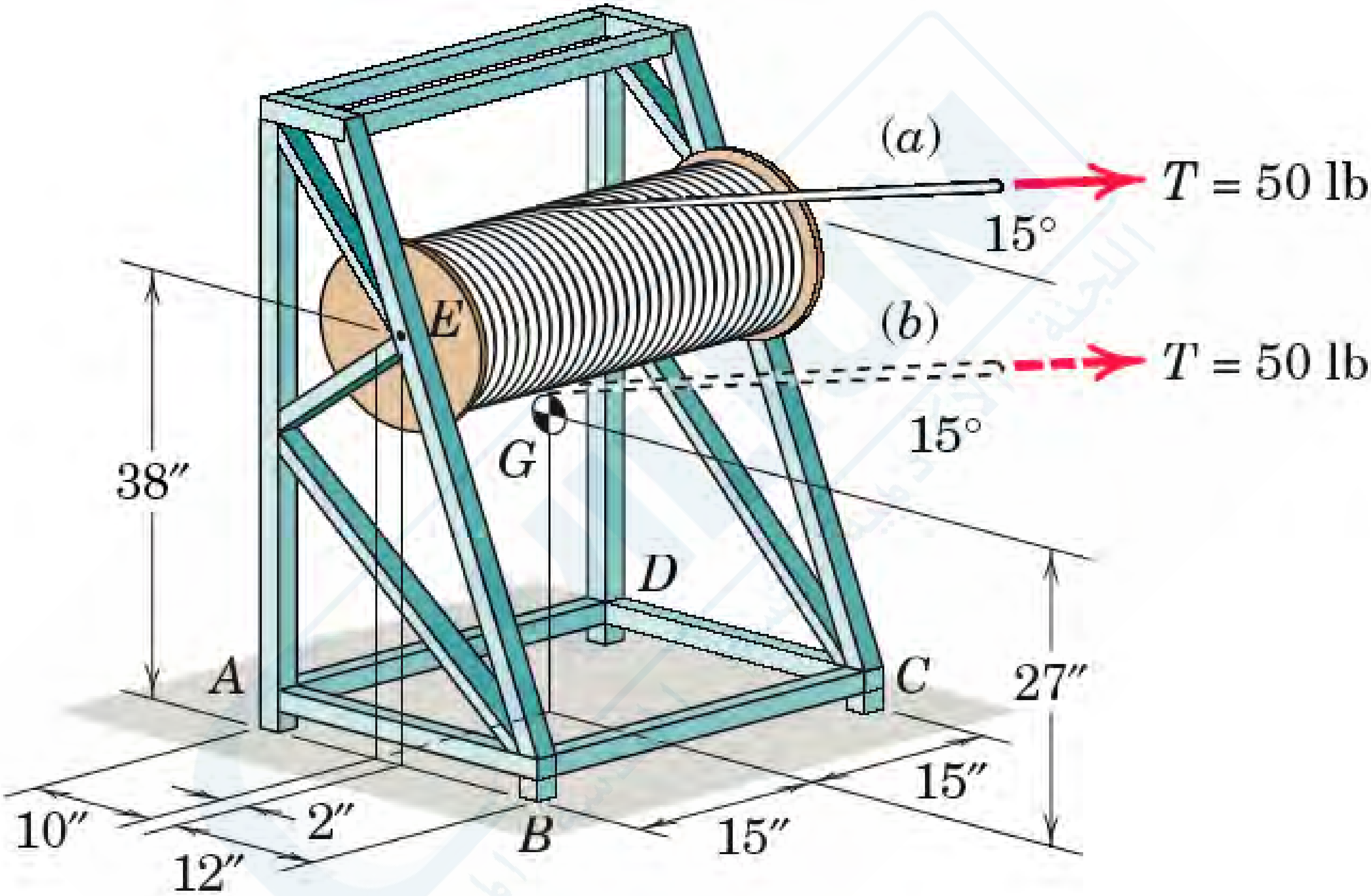
Problem 3/111

P 03 111



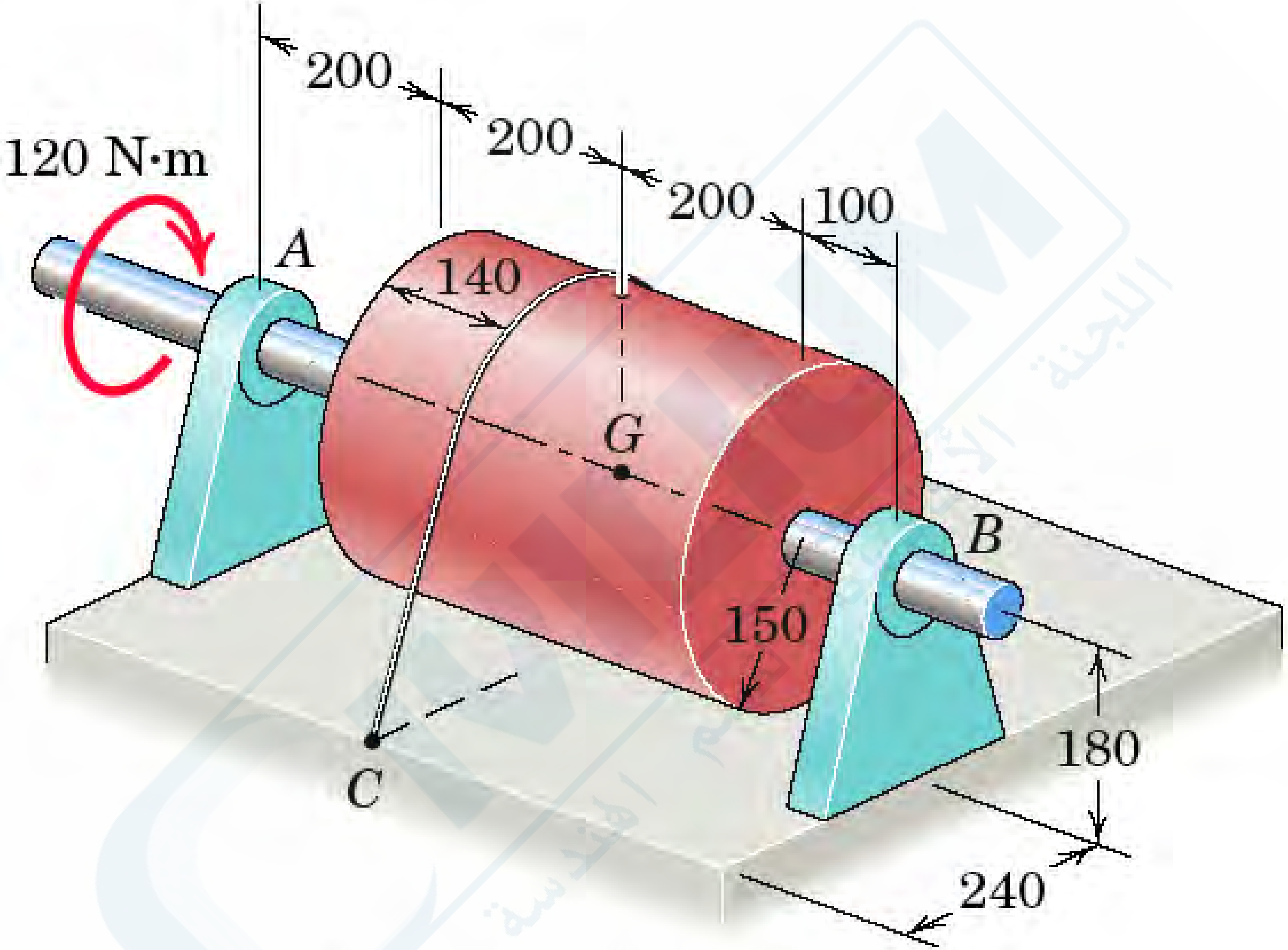
Problem 3/112

P 03 112



Problem 3/113

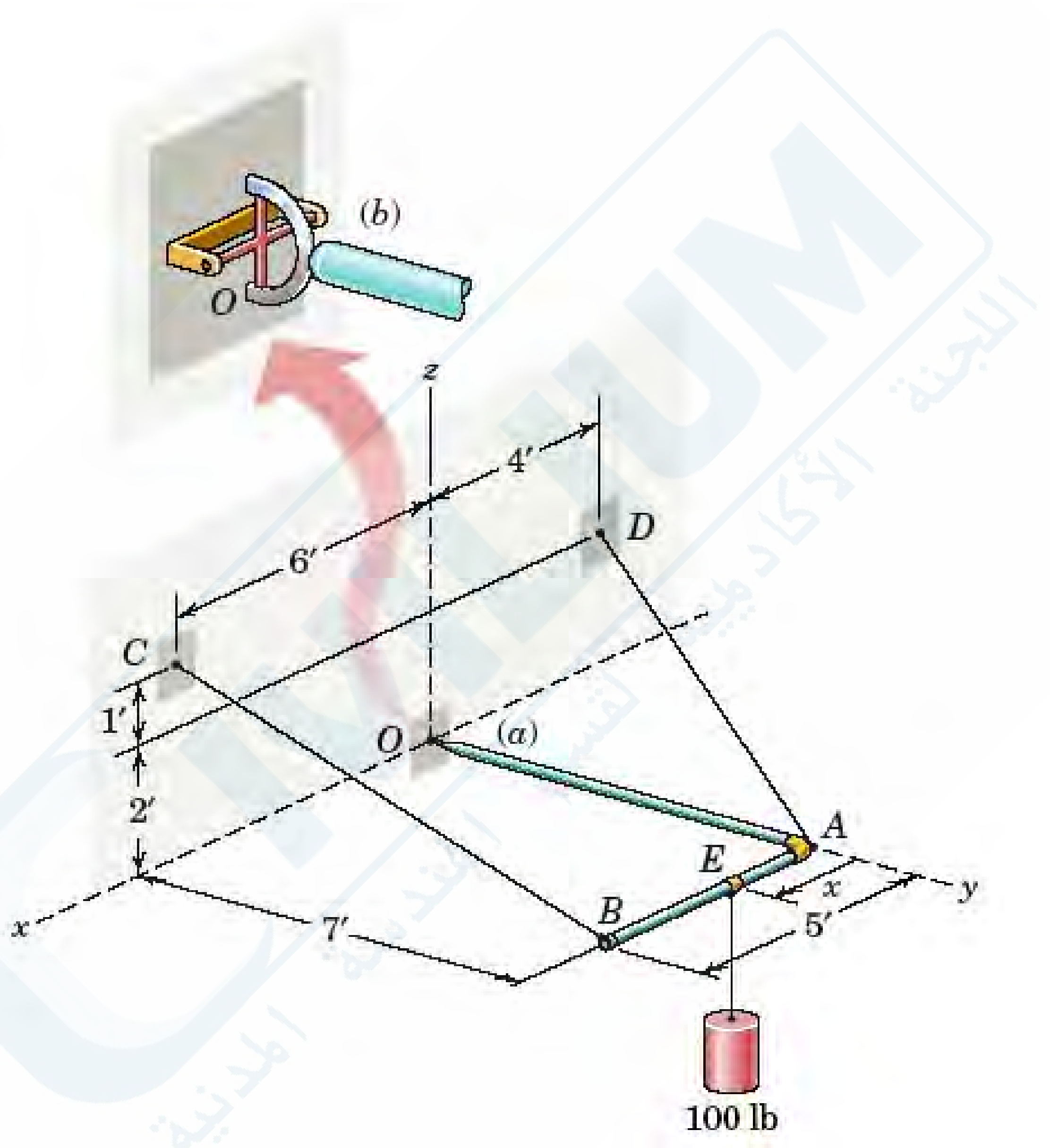
P 03 113



Dimensions in millimeters

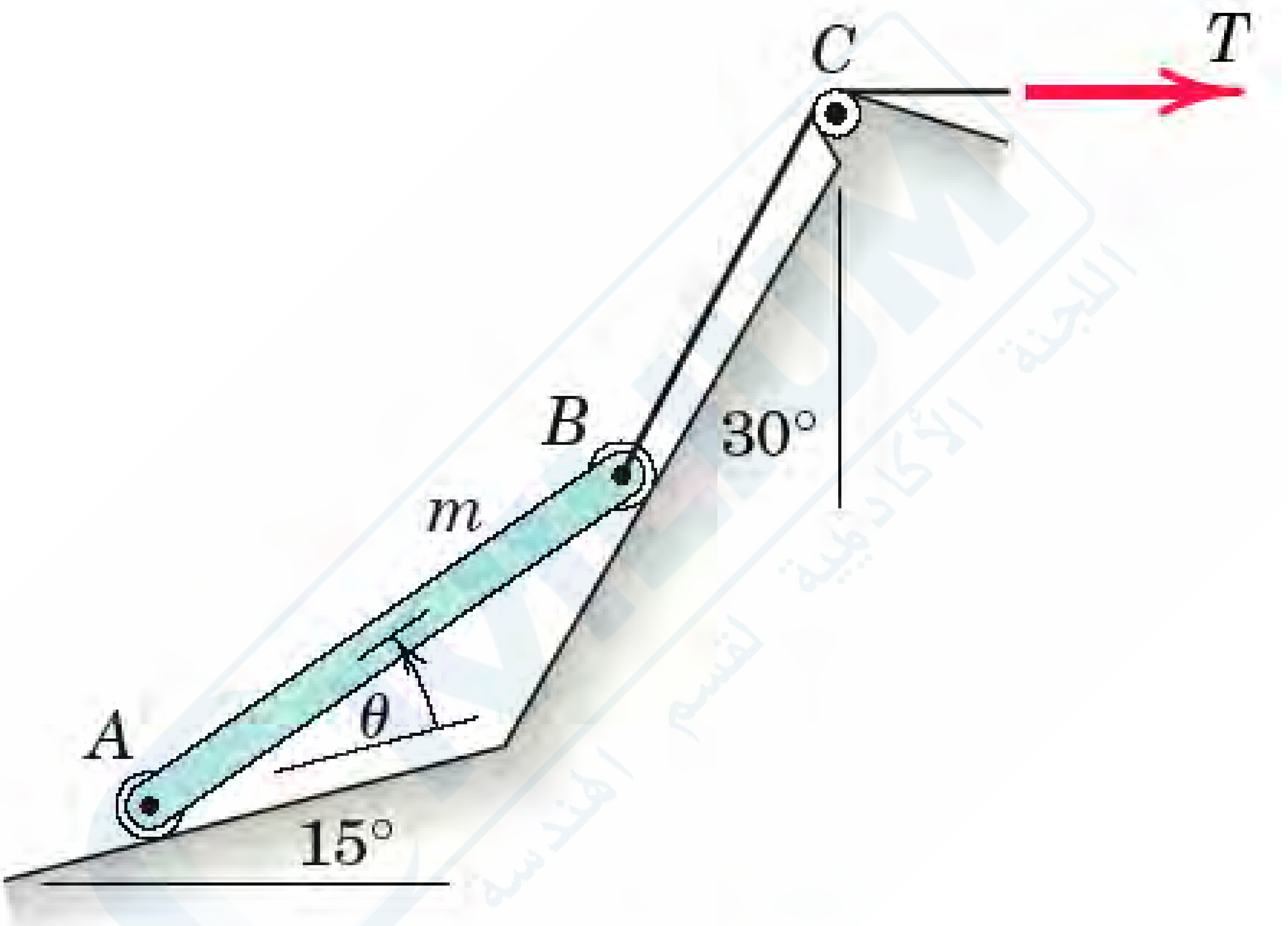
Problem 3/114

P 03 114



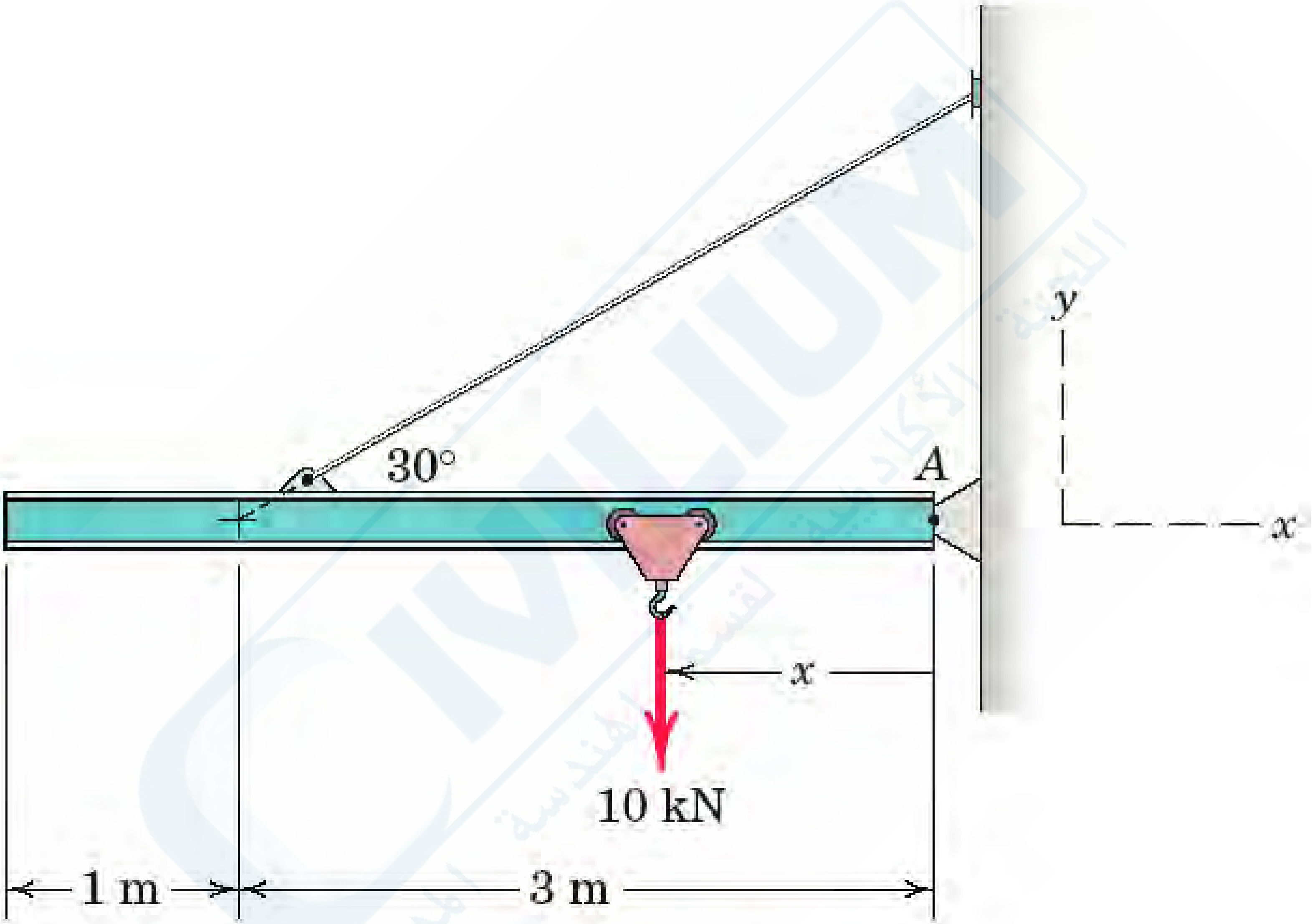
Problem 3/115

P 03 115



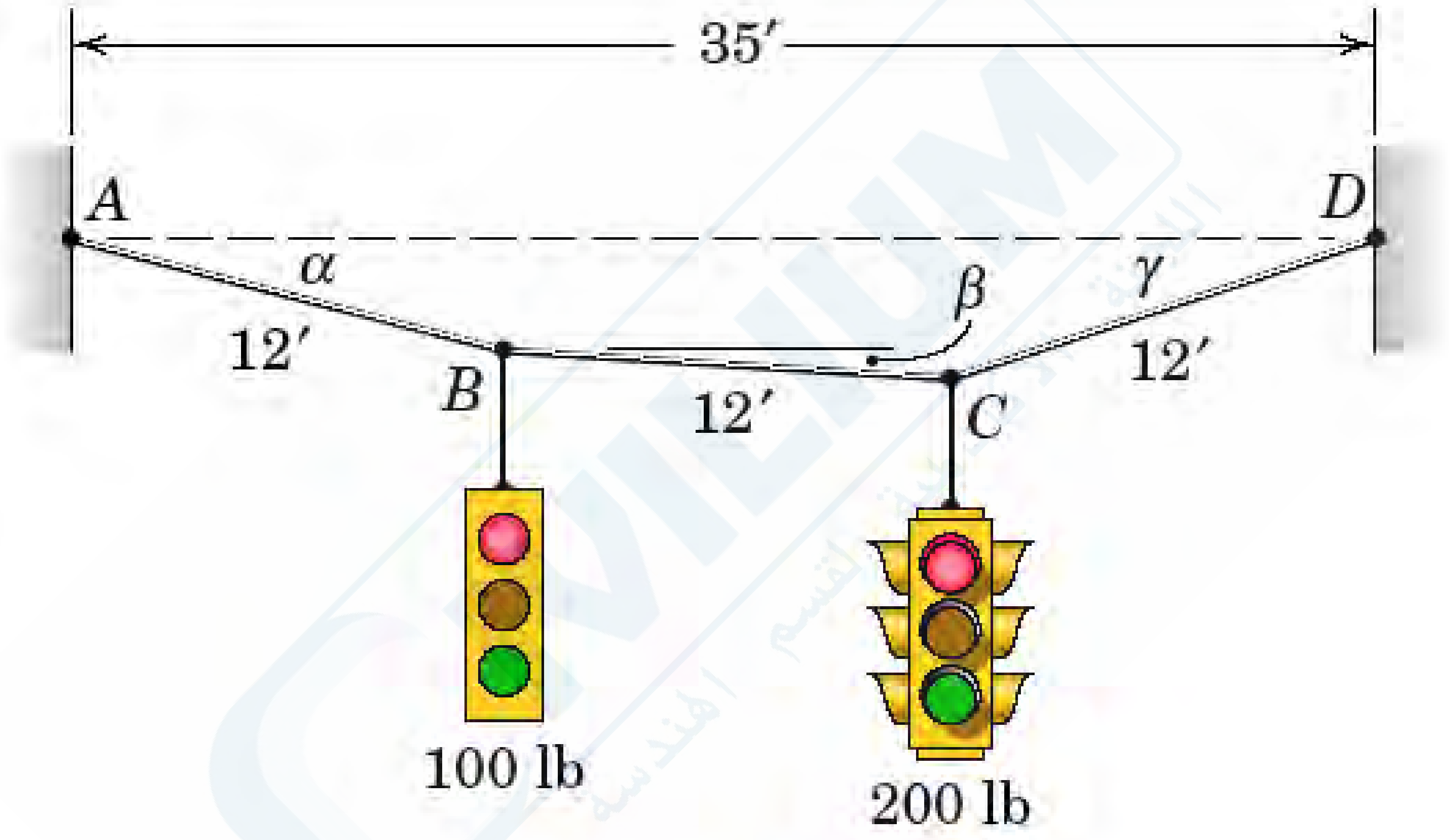
Problem 3/116

P 03 116



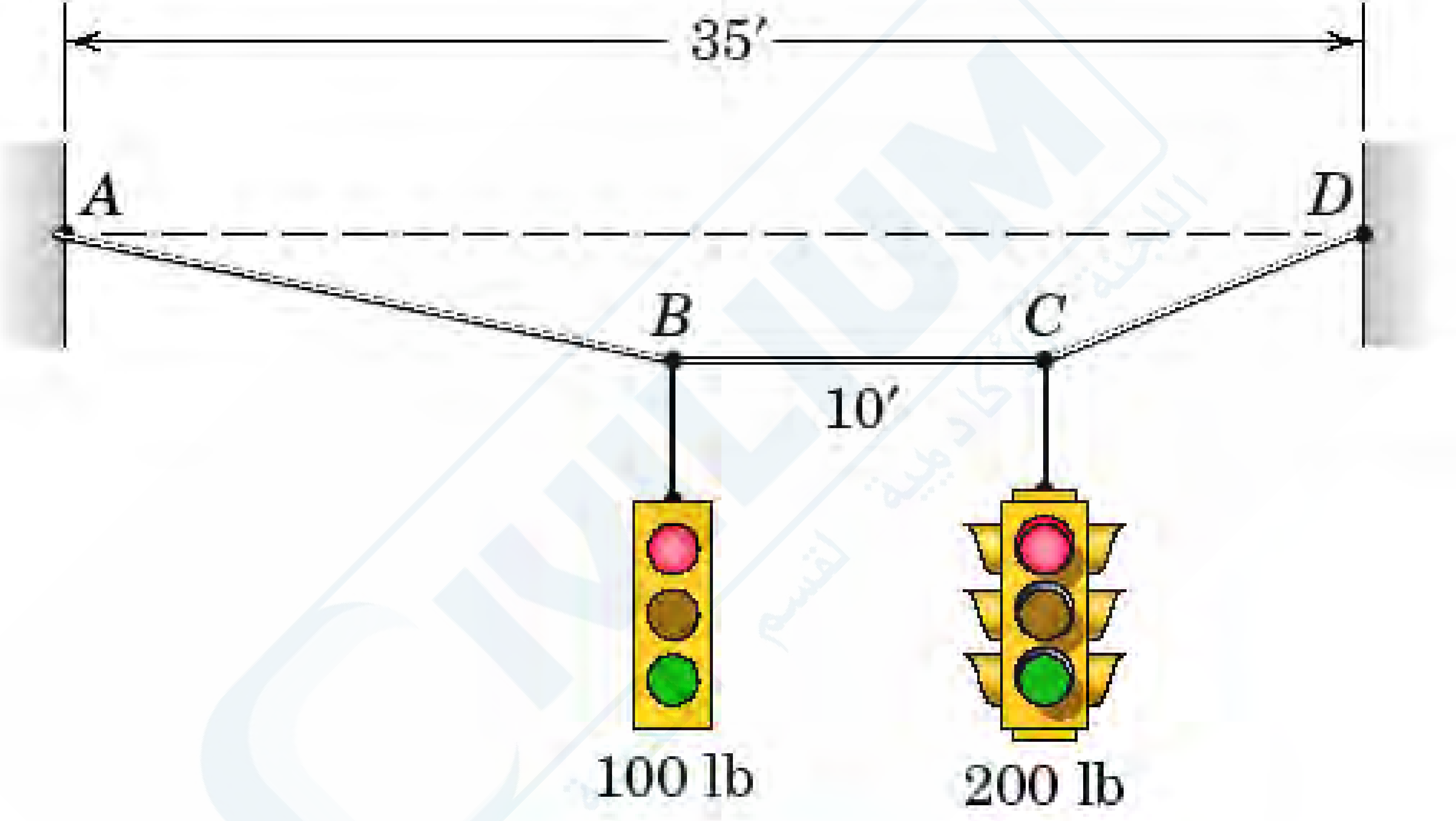
Problem 3/117

P_03_117



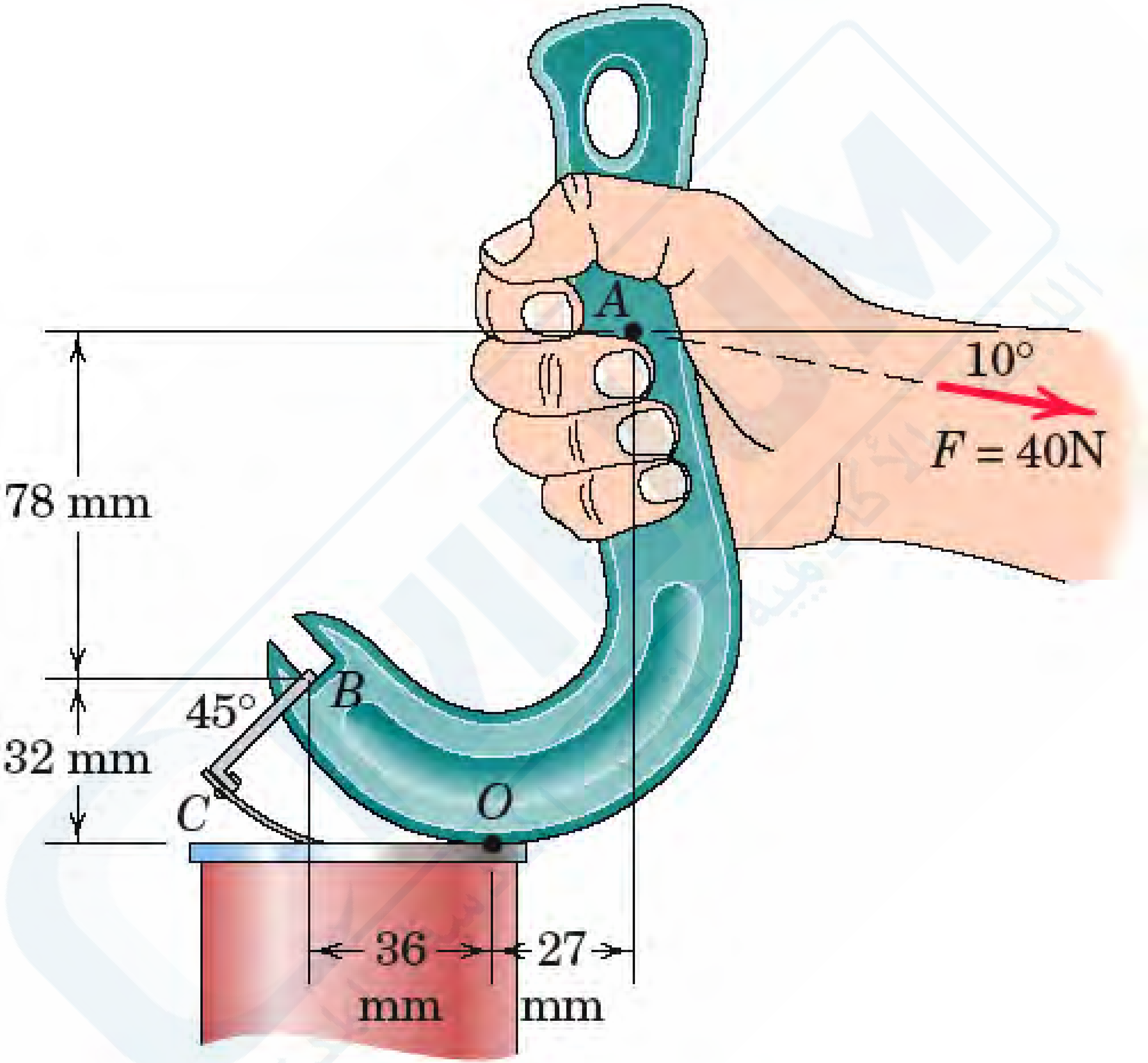
Problem 3/118

P_03_118



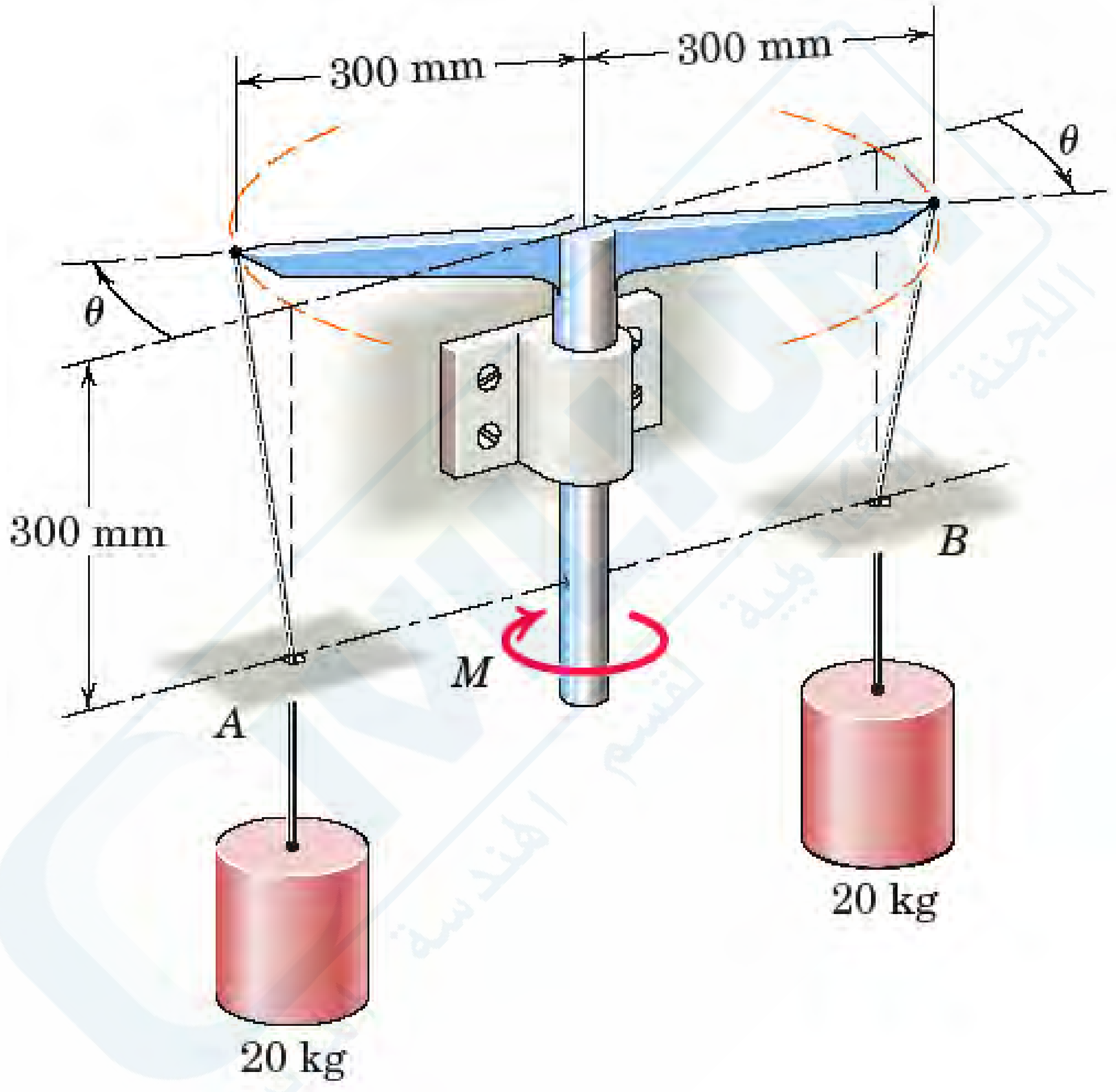
Problem 3/119

P_03_119



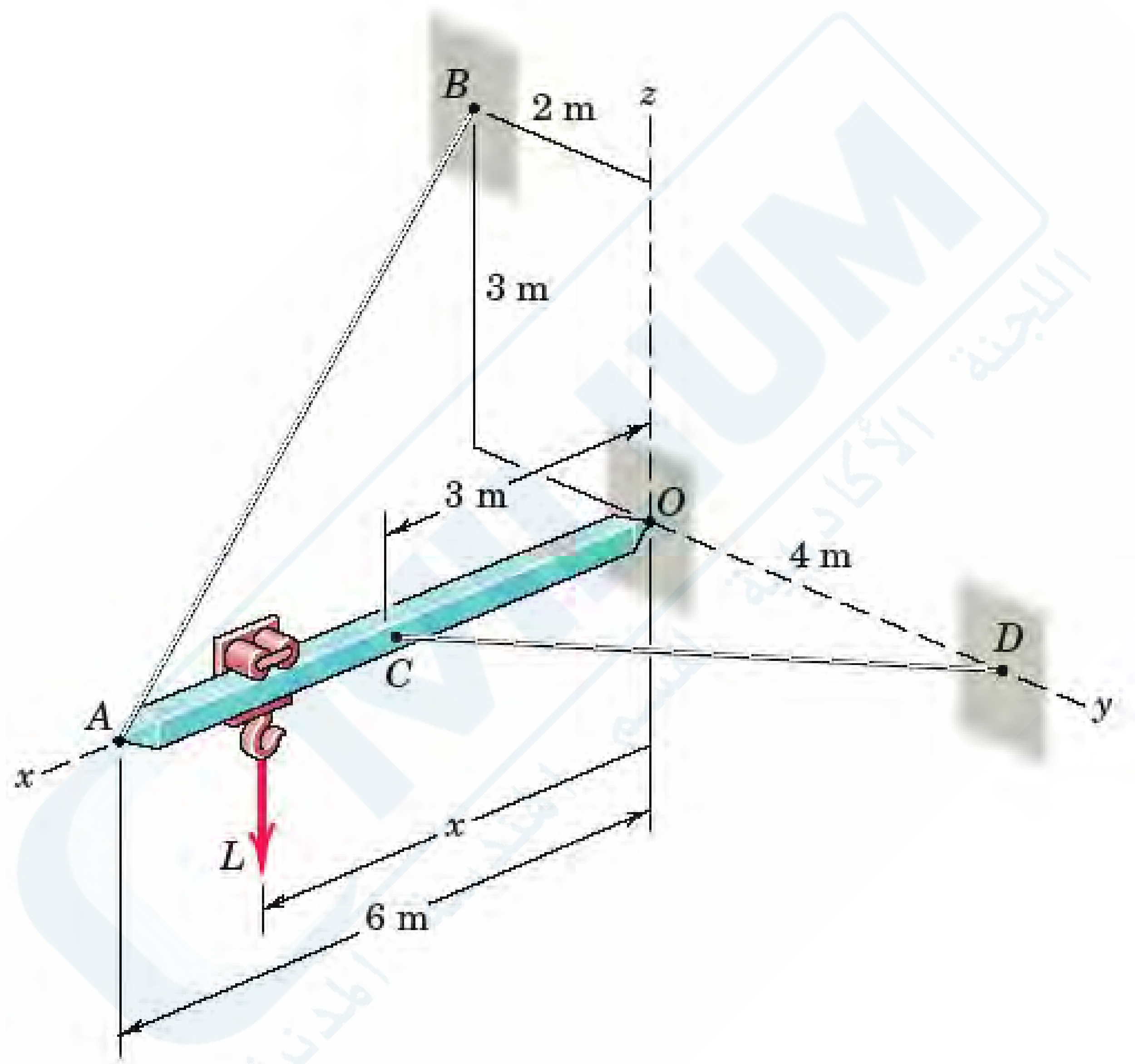
Problem 3/12

P 03 12



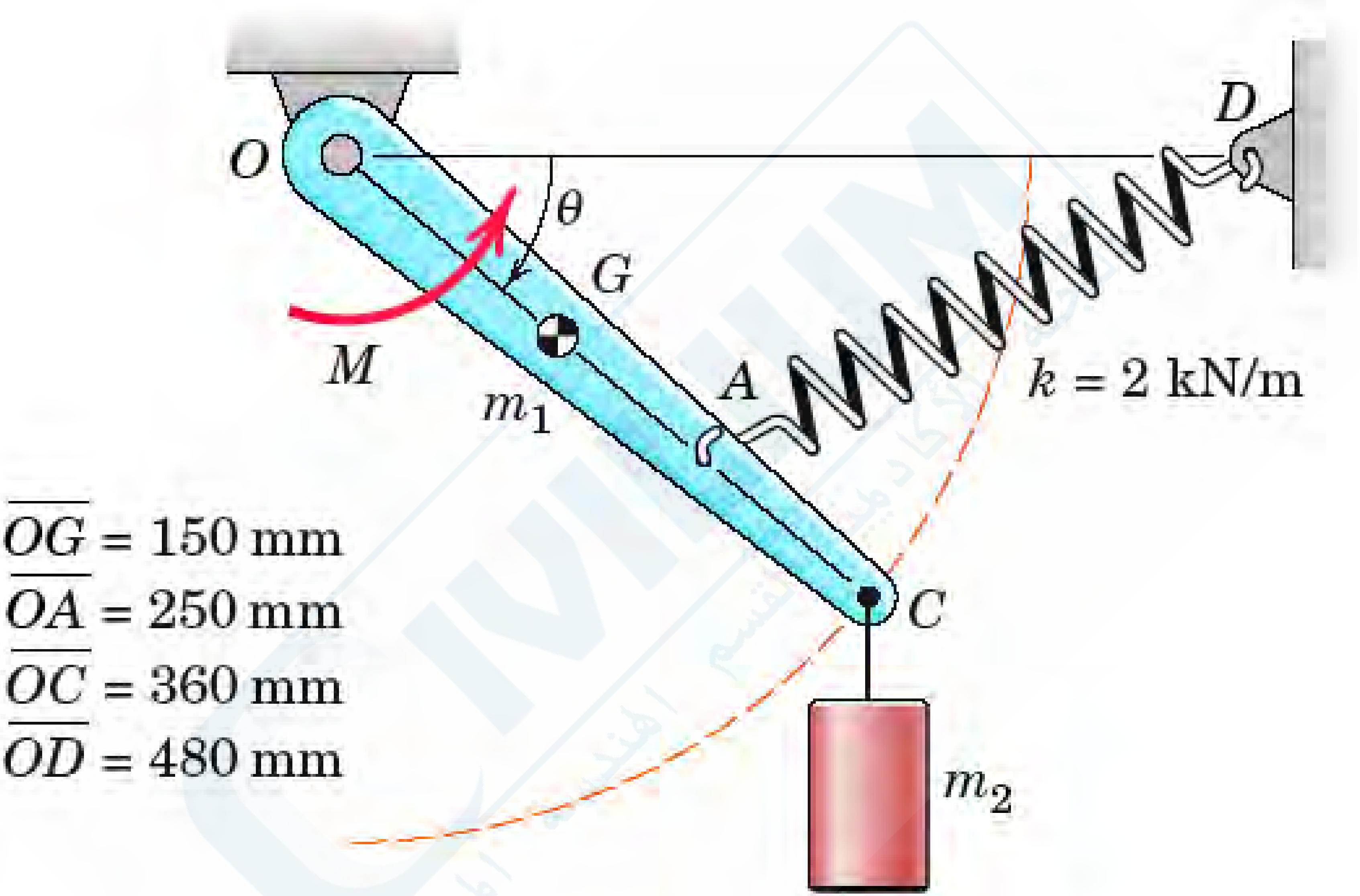
Problem 3/120

P 03 120



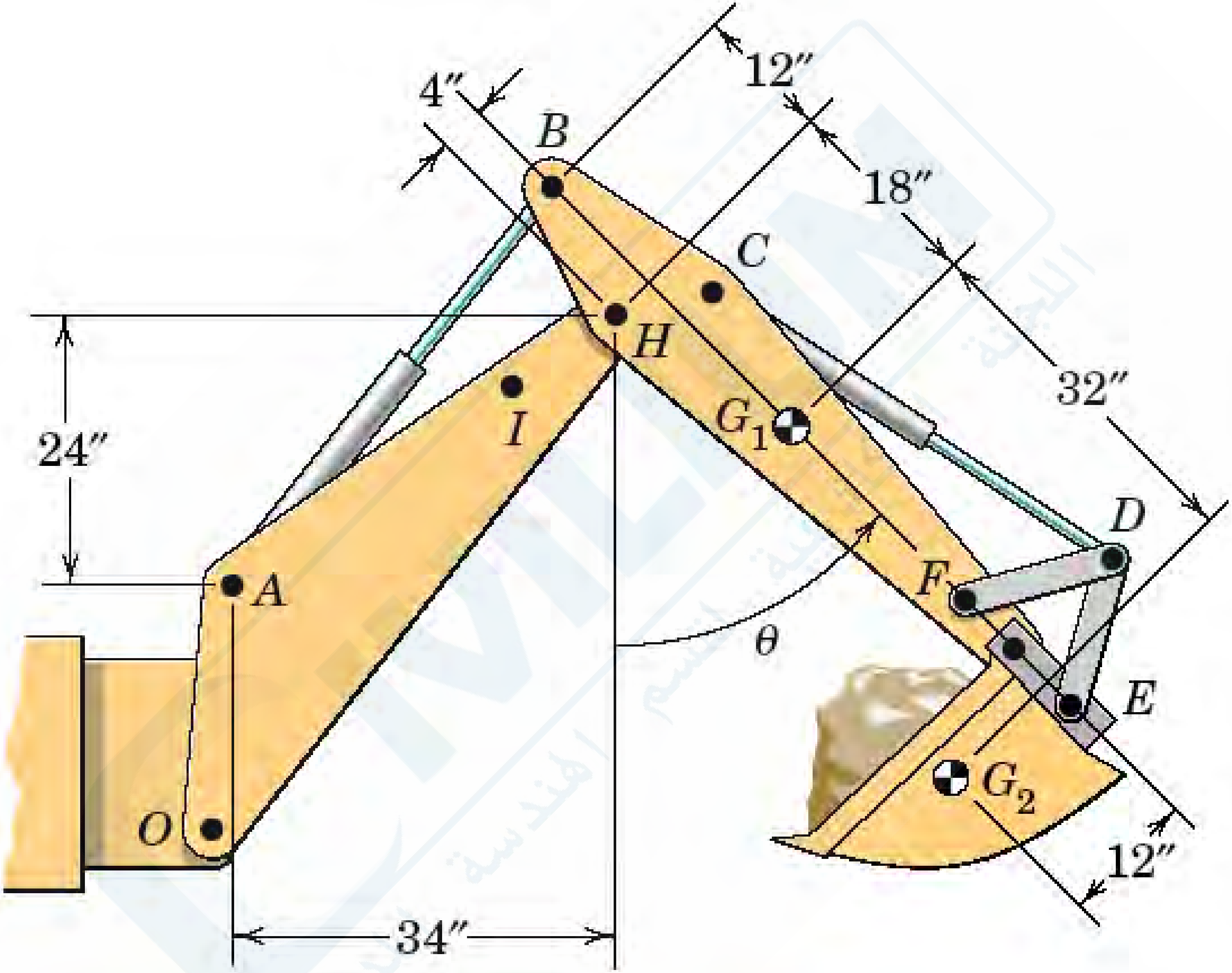
Problem 3/121

P 03 121



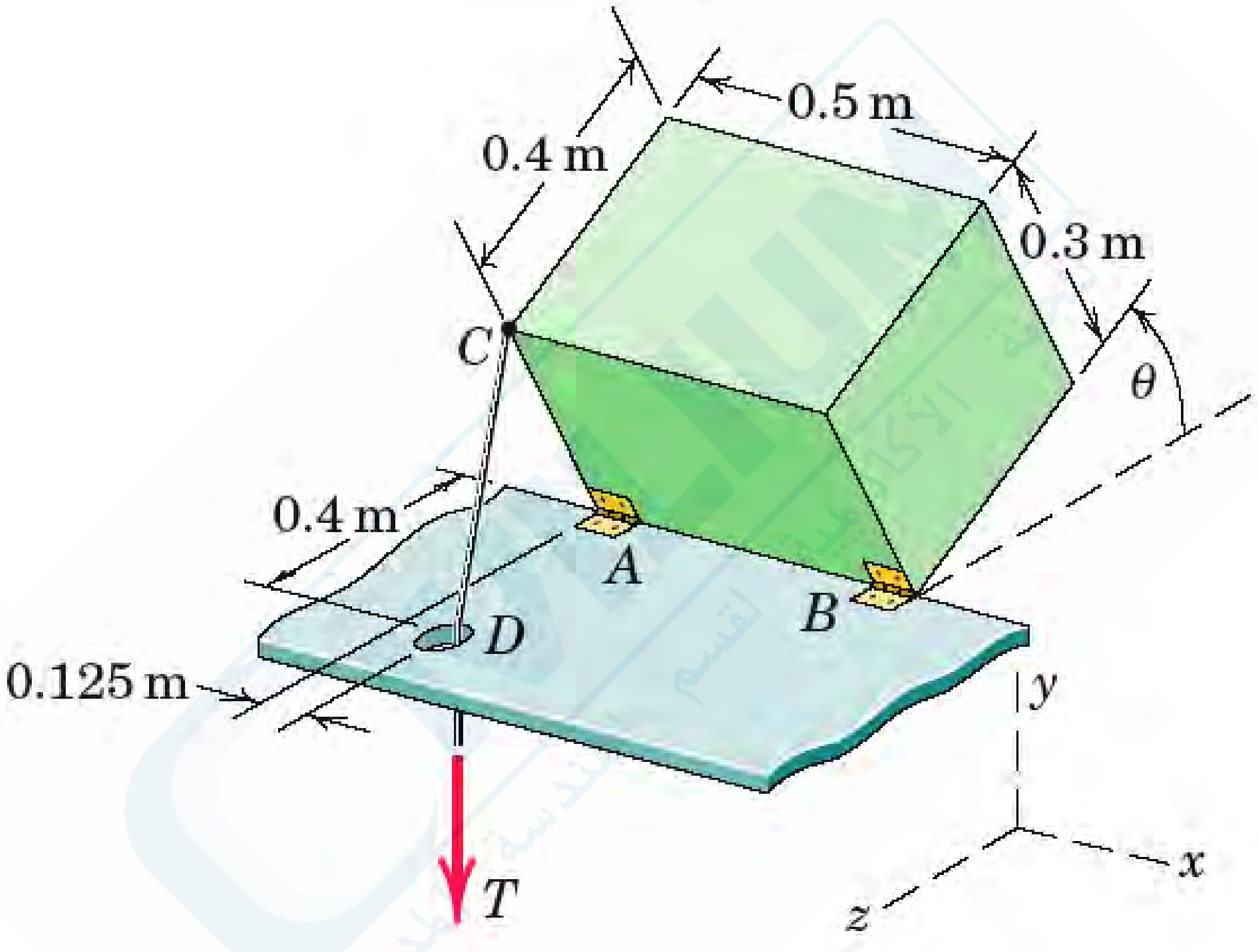
Problem 3/122

P 03 122



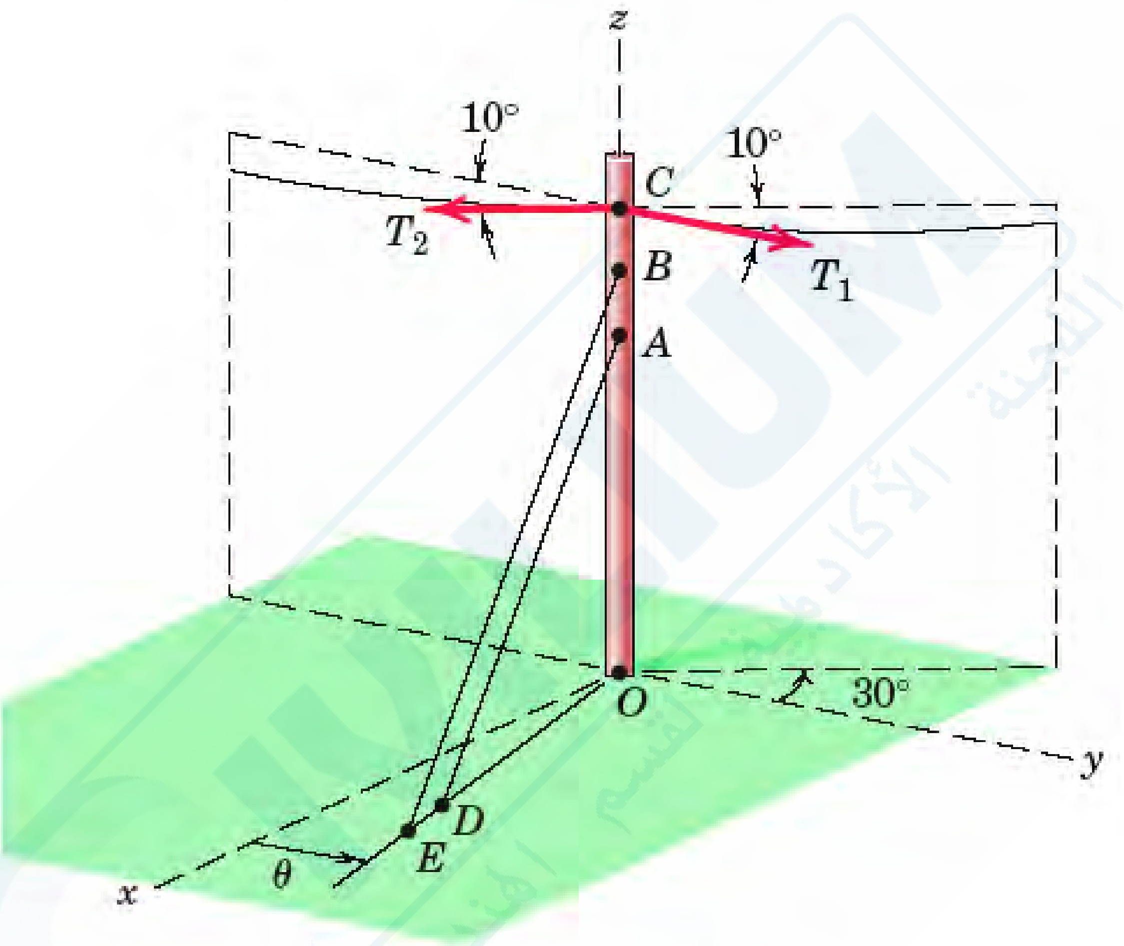
Problem 3/123

P 03 123



Problem 3/124

P 03 124



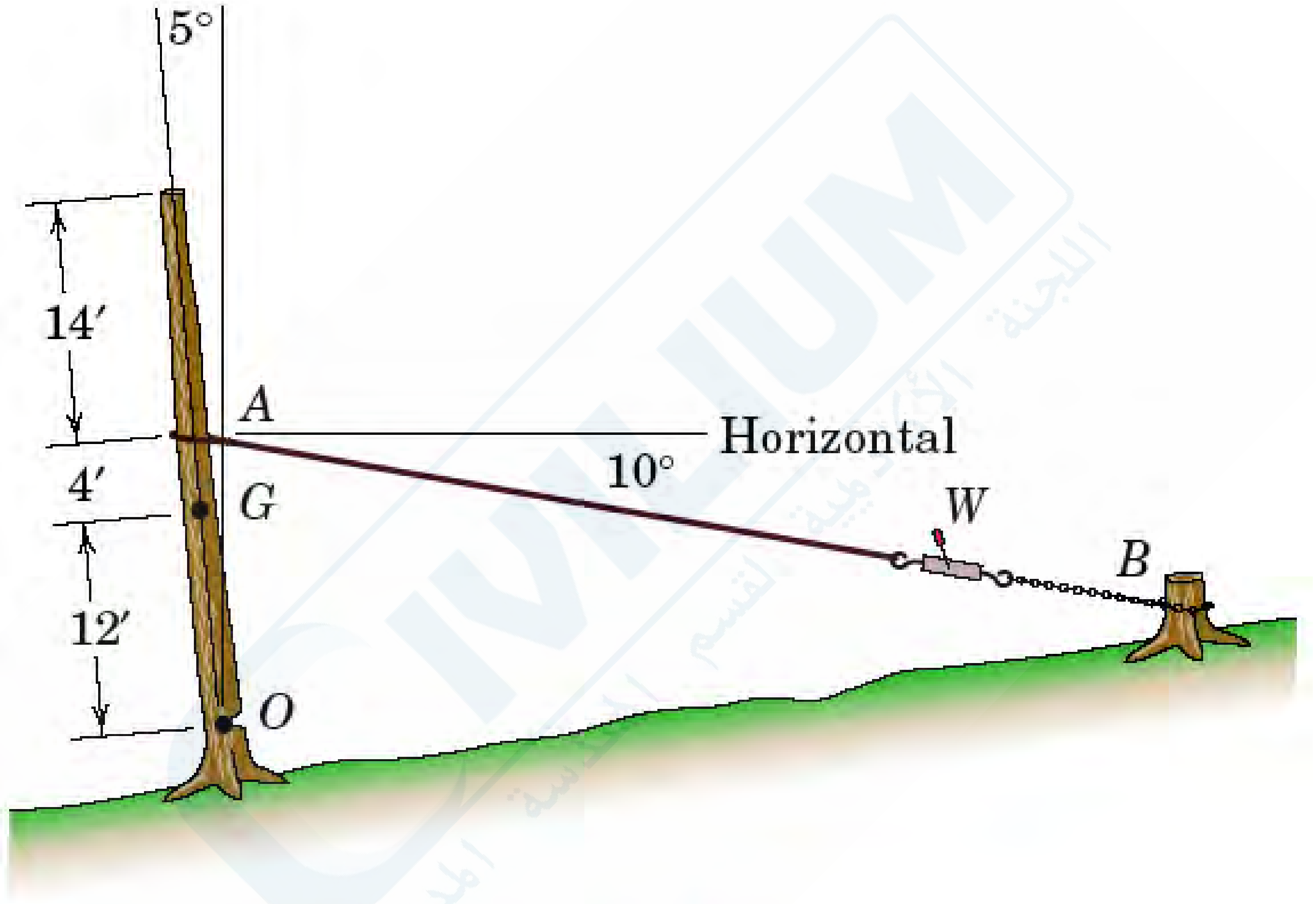
$\overline{OA} = 9$ m $\qquad \overline{OD} = 8$ m

$\overline{OB} = 11$ m $\qquad \overline{OE} = 10$ m

$\overline{OC} = 13$ m

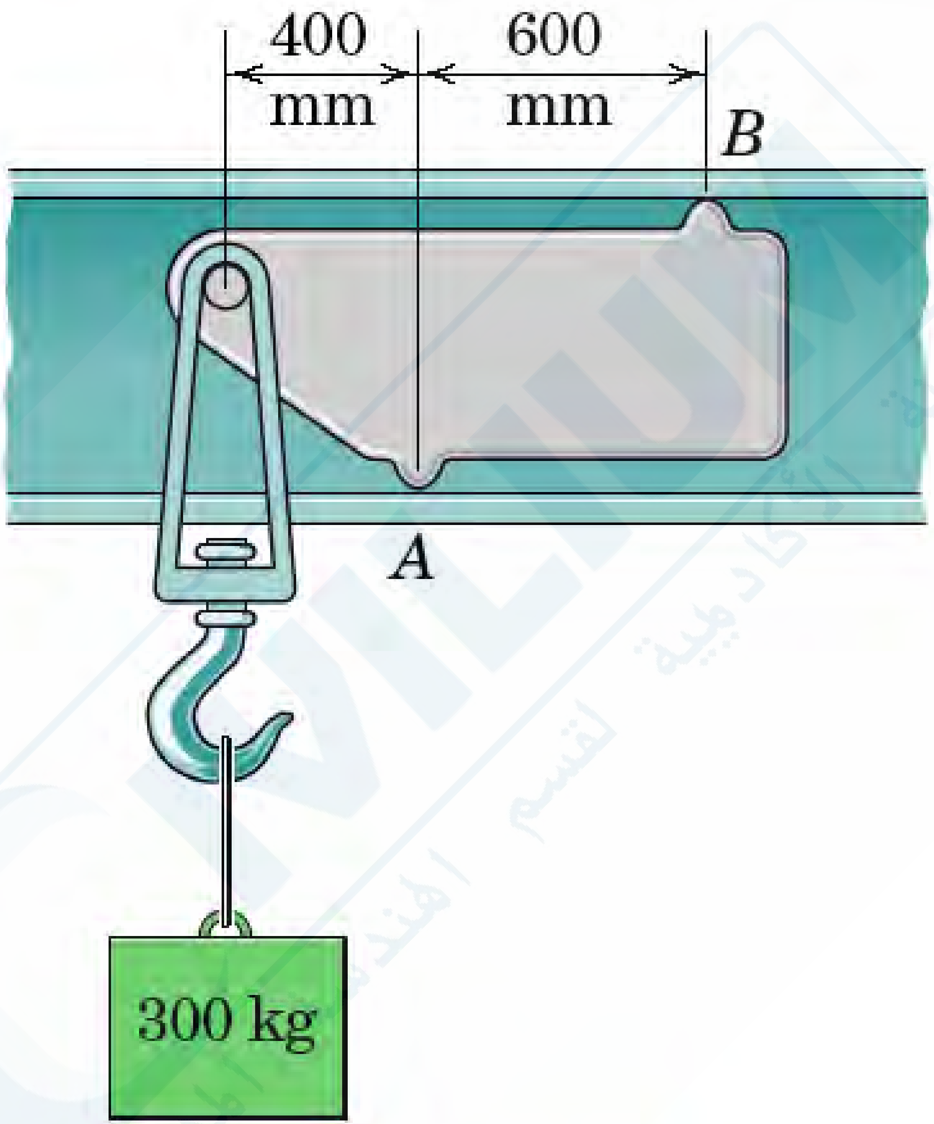
Problem 3/125

P 03 125



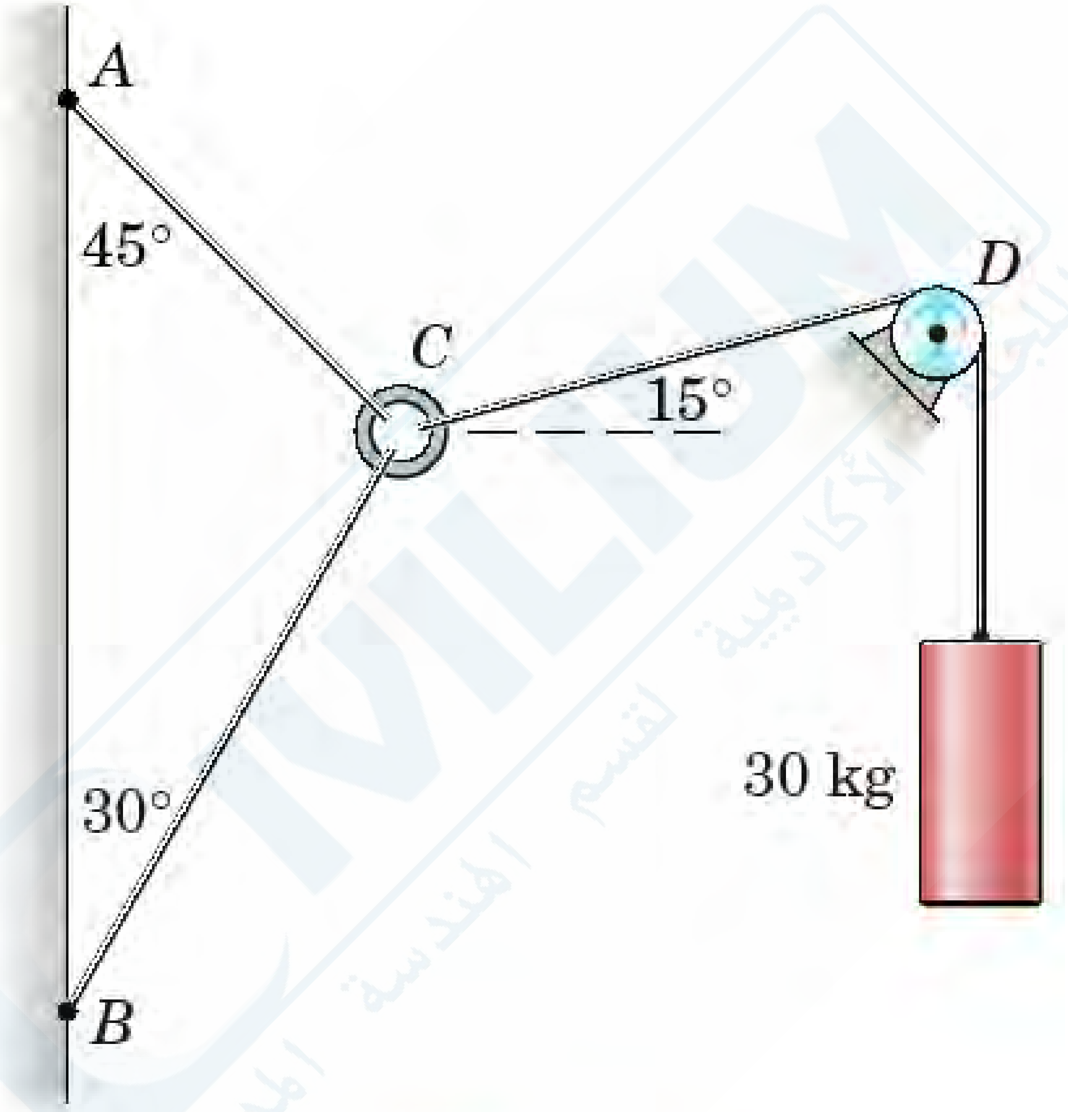
Problem 3/13

P 03 13



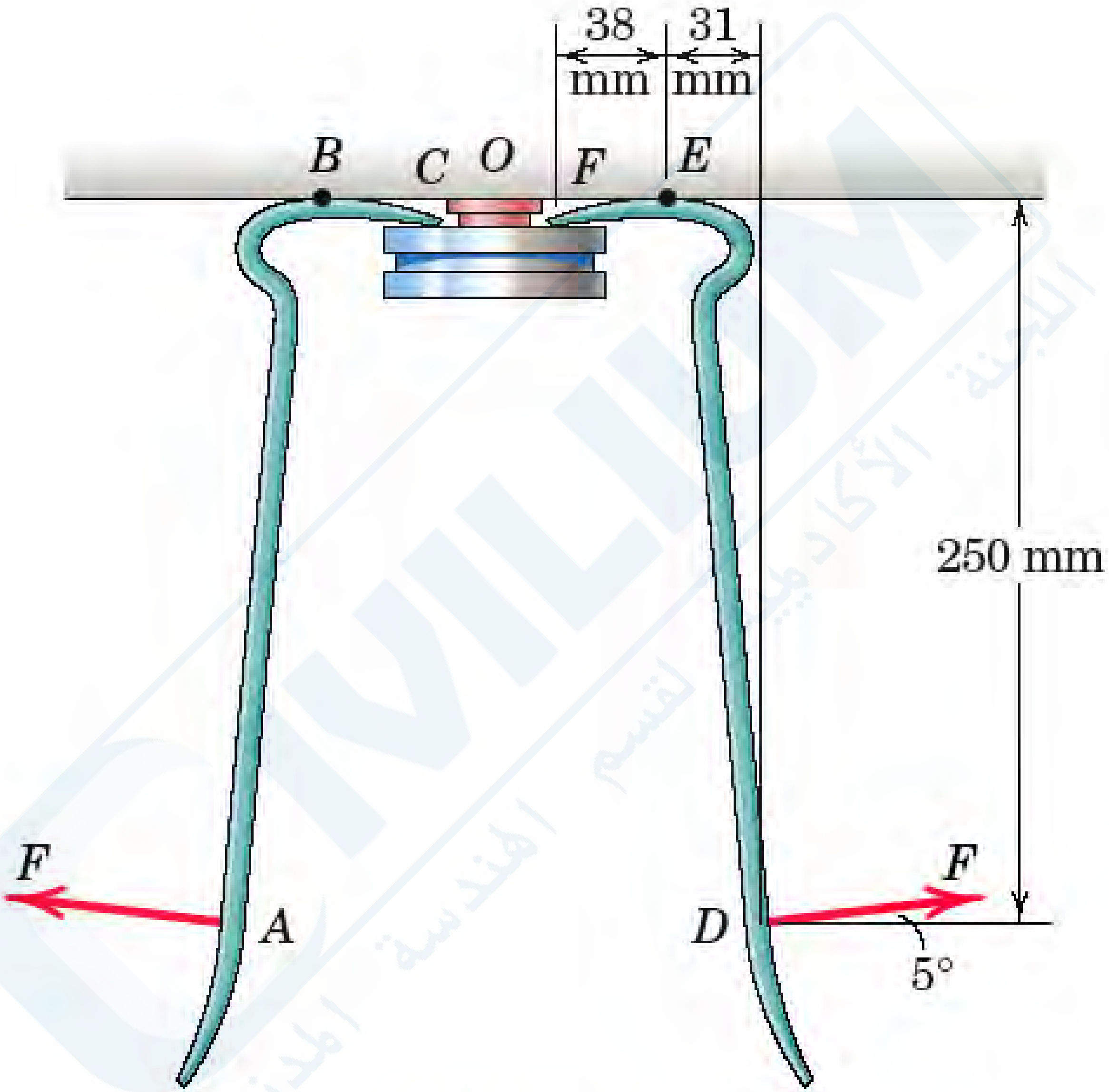
Problem 3/14

P_03_14



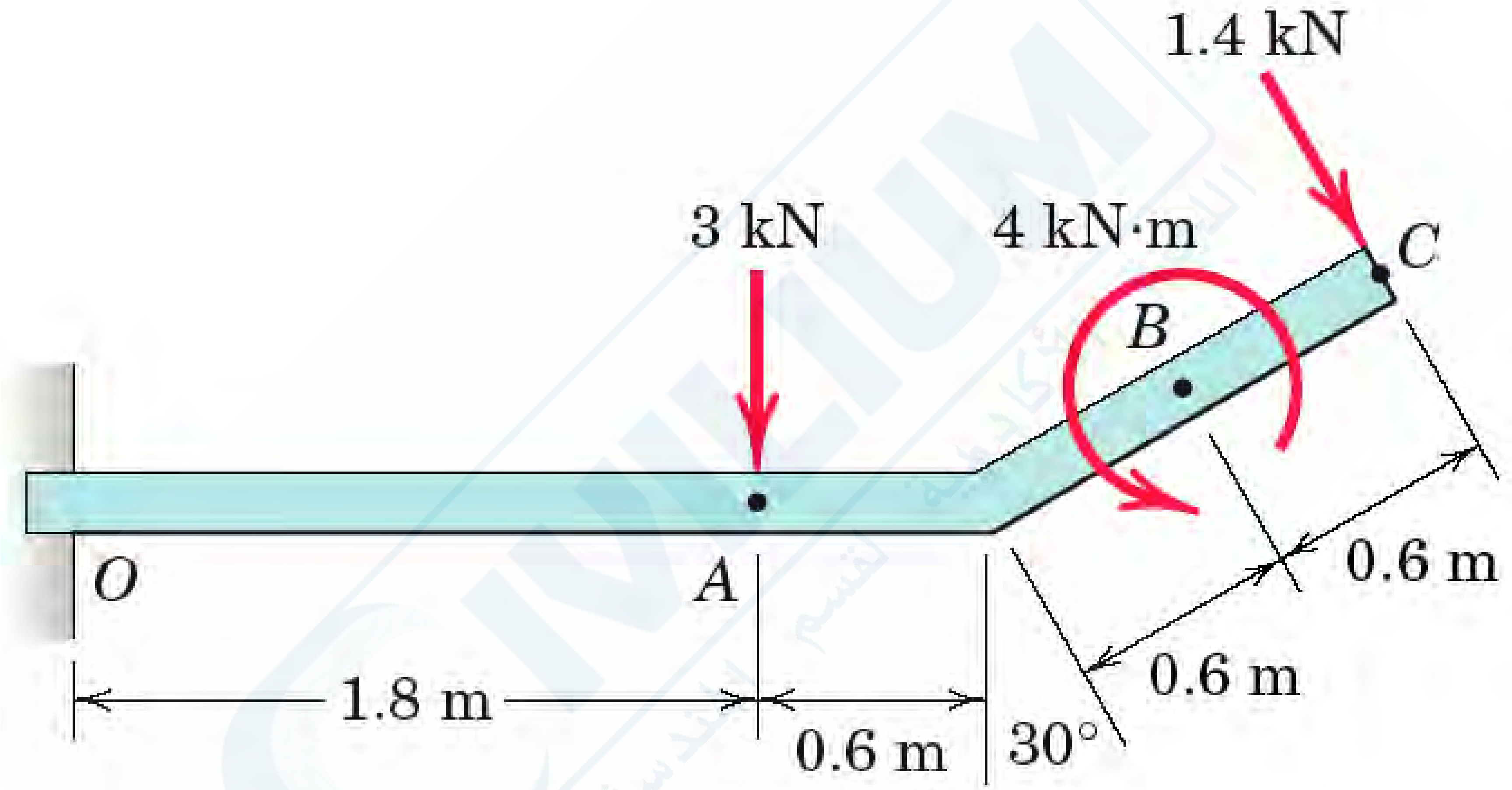
Problem 3/15

P_03_15



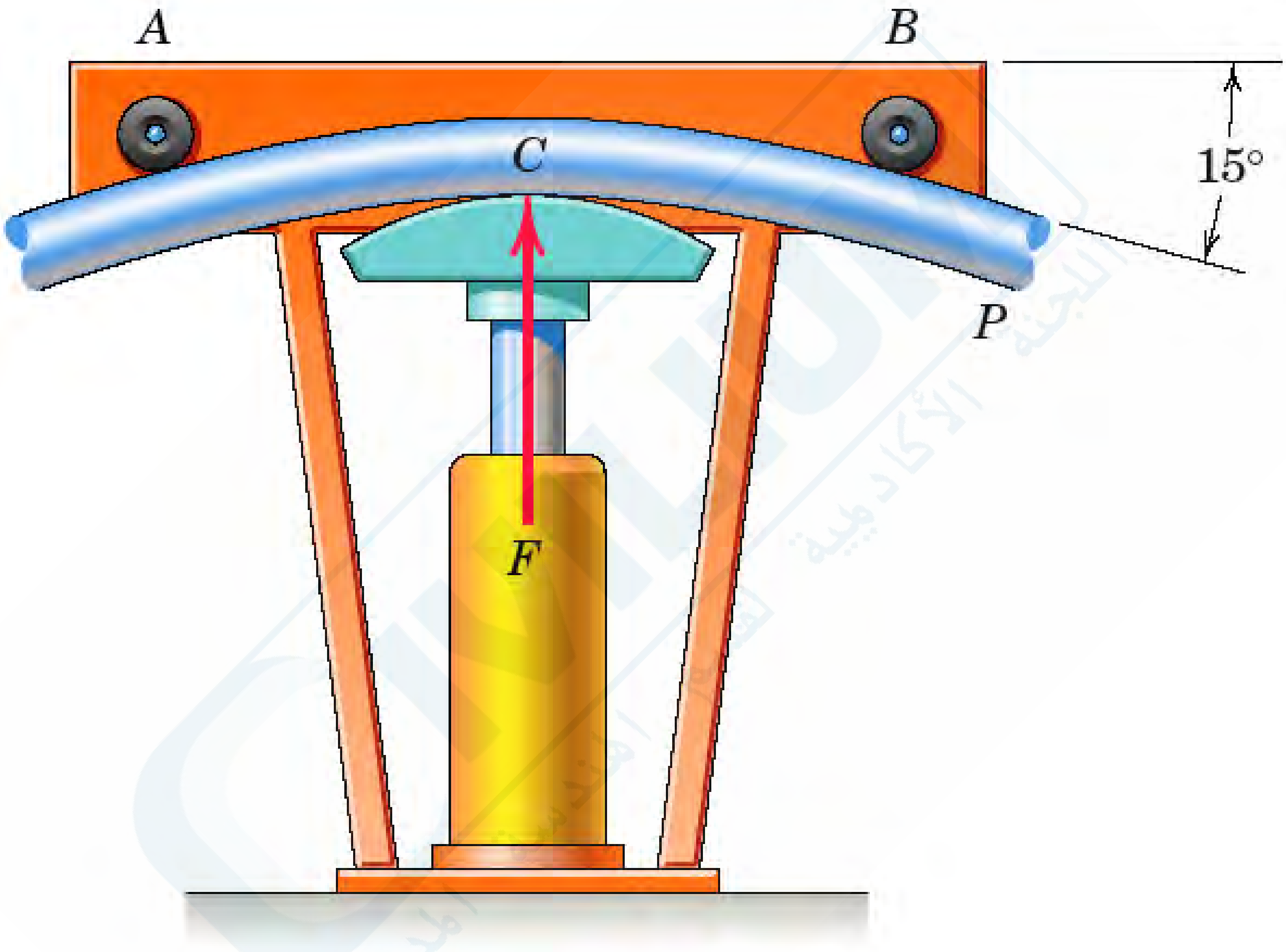
Problem 3/16

P 03 16



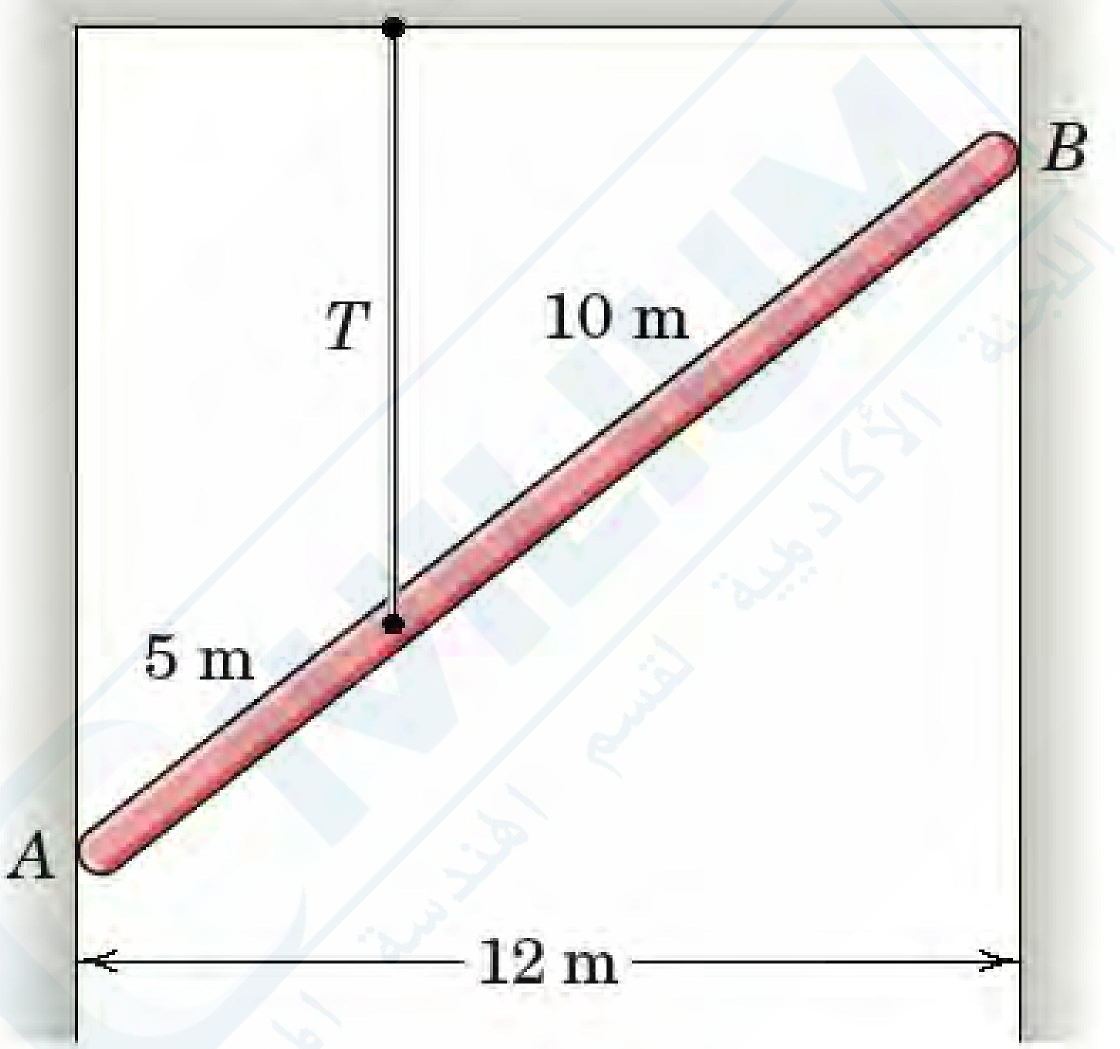
Problem 3/17

P_03_17



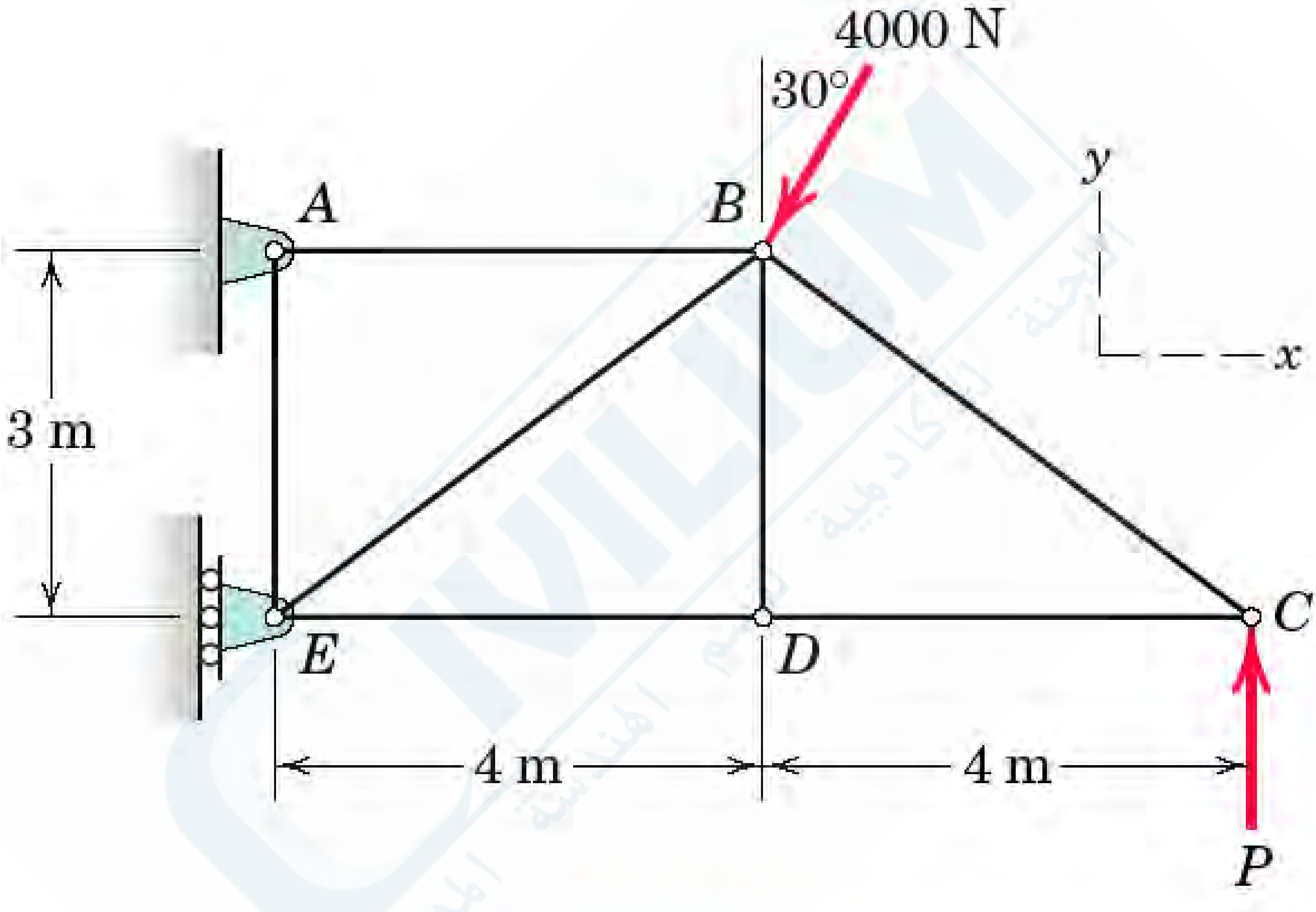
Problem 3/18

P 03 18



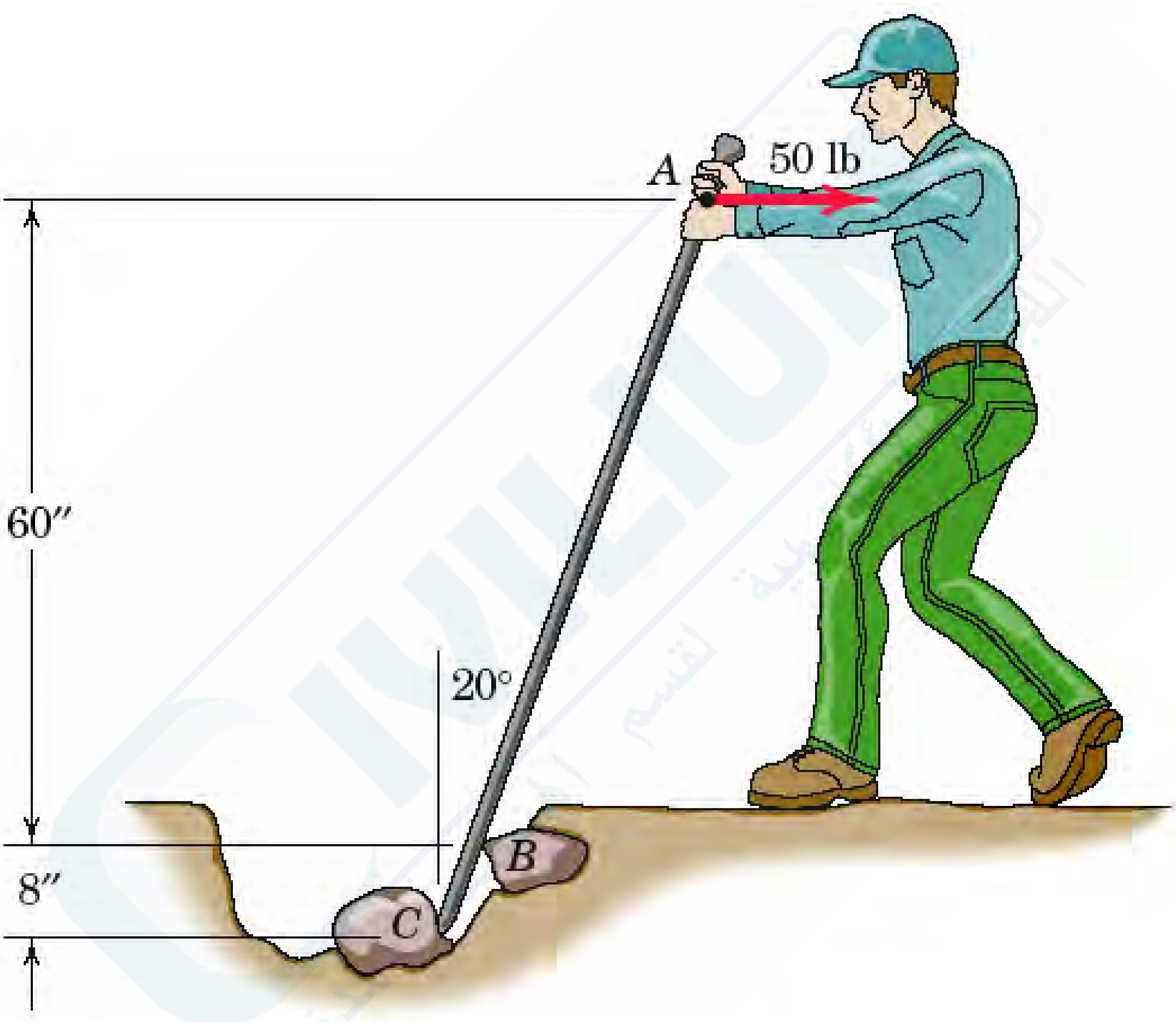
Problem 3/19

P_03_19



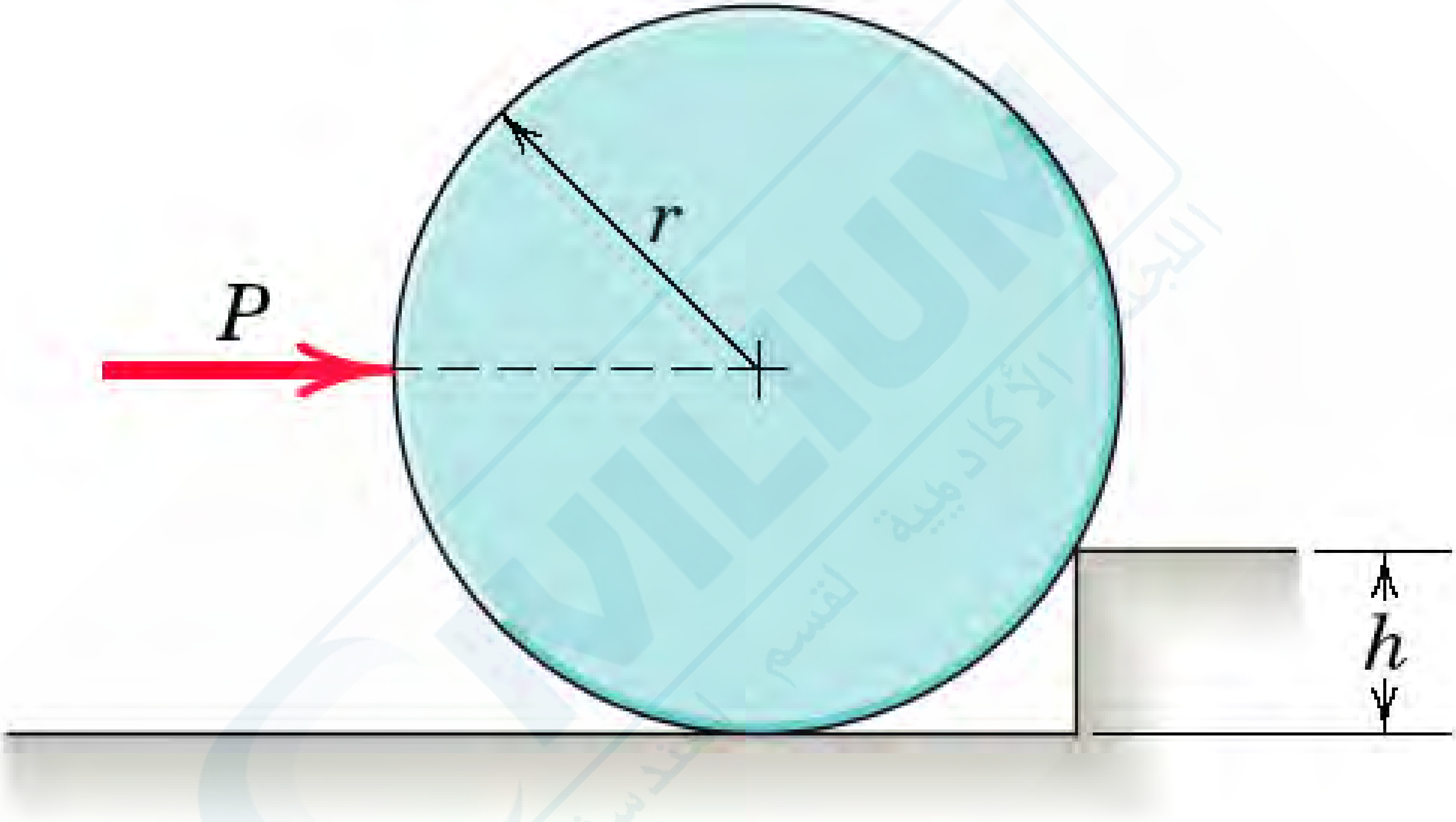
Problem 3/20

P_03_20



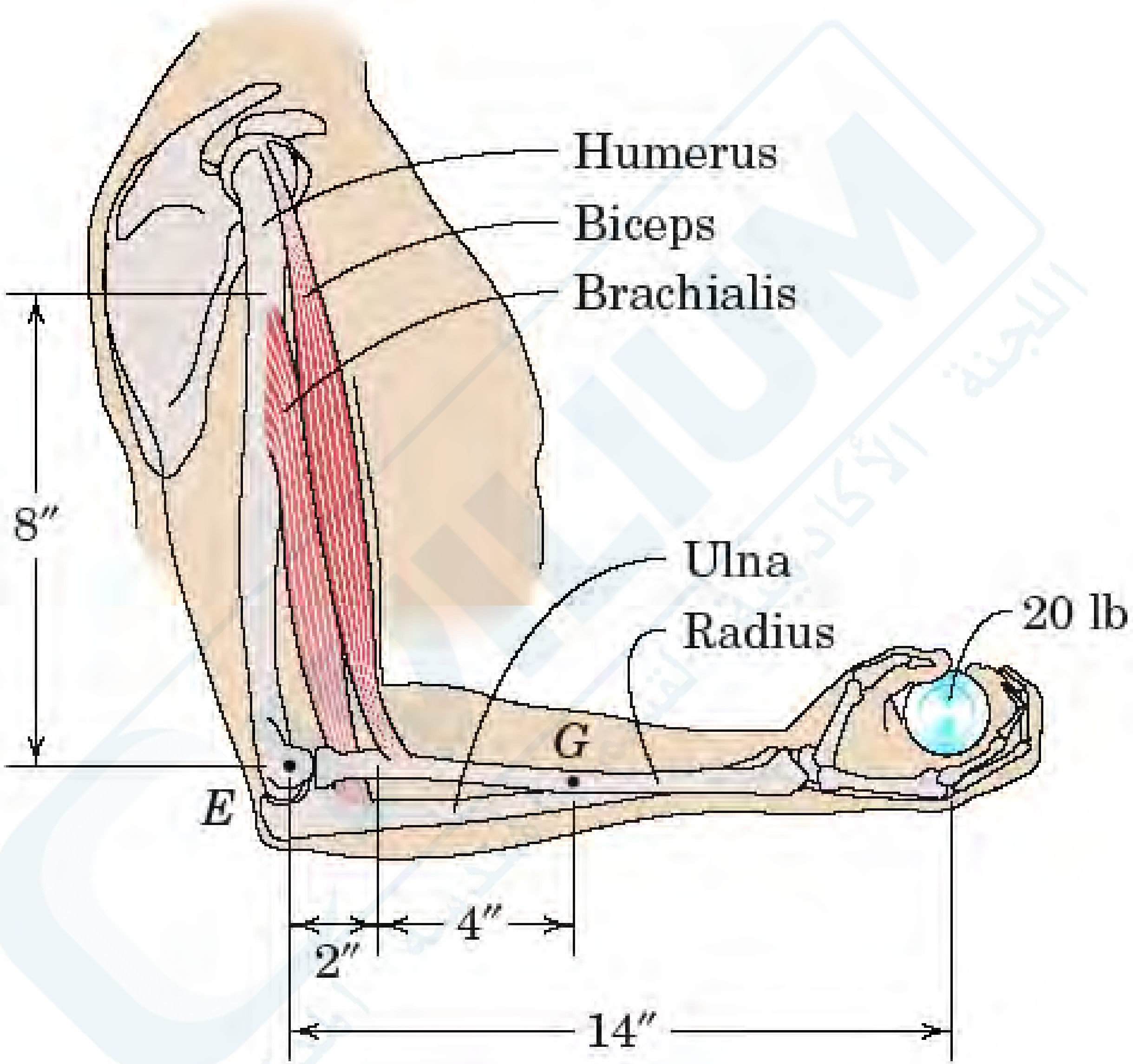
Problem 3/21

P_03_21



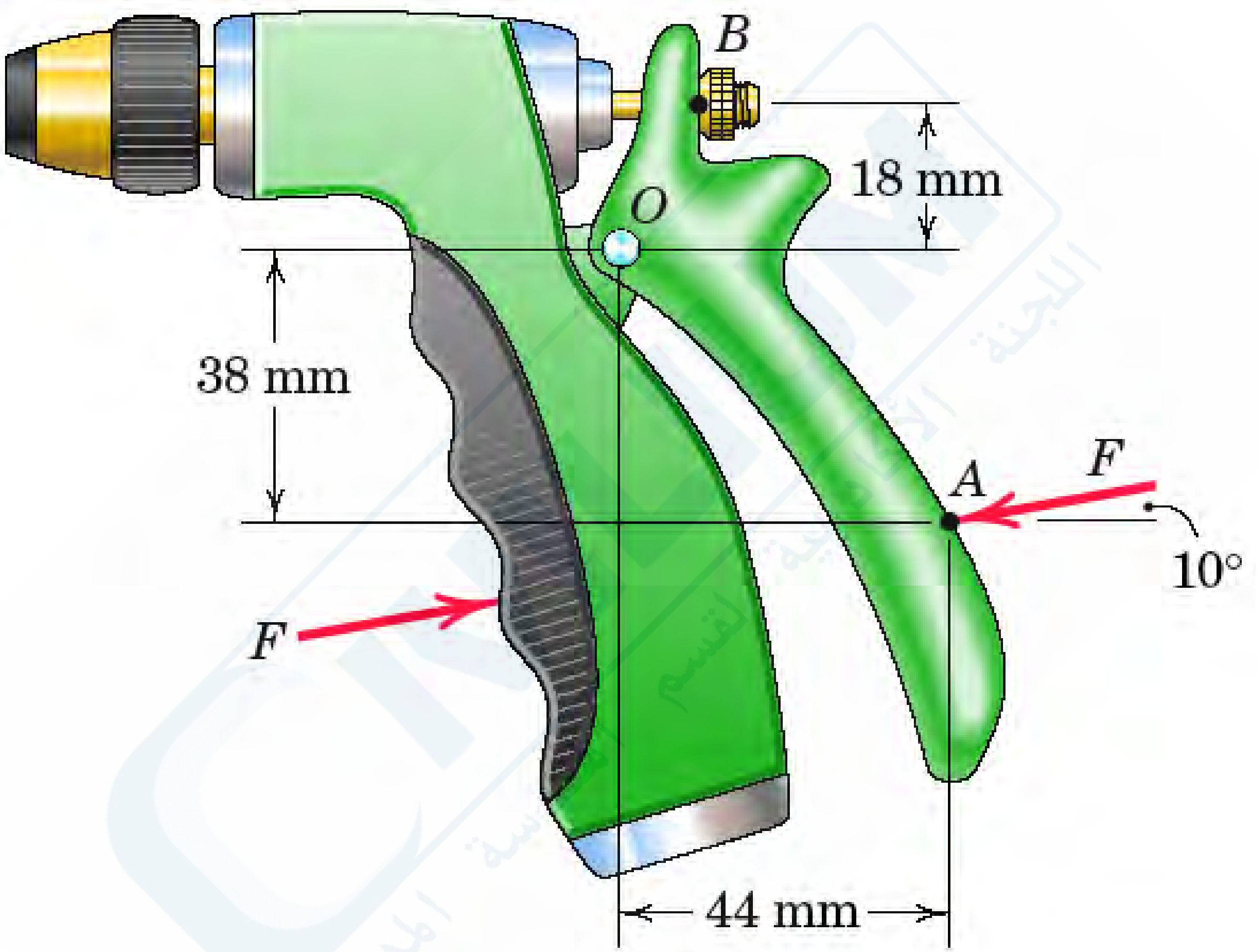
Problem 3/22

P_03_22



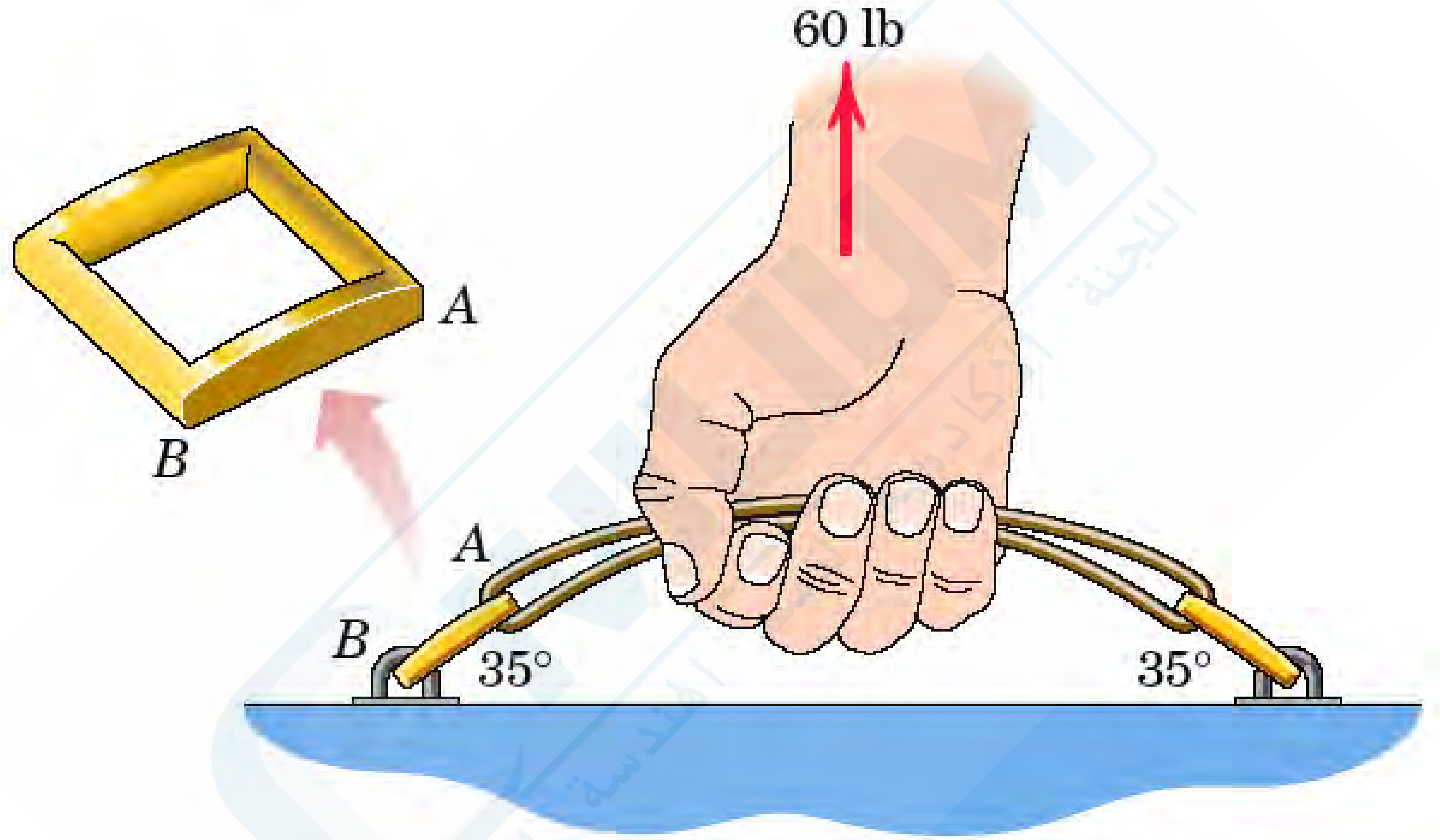
Problem 3/23

P_03_23( )



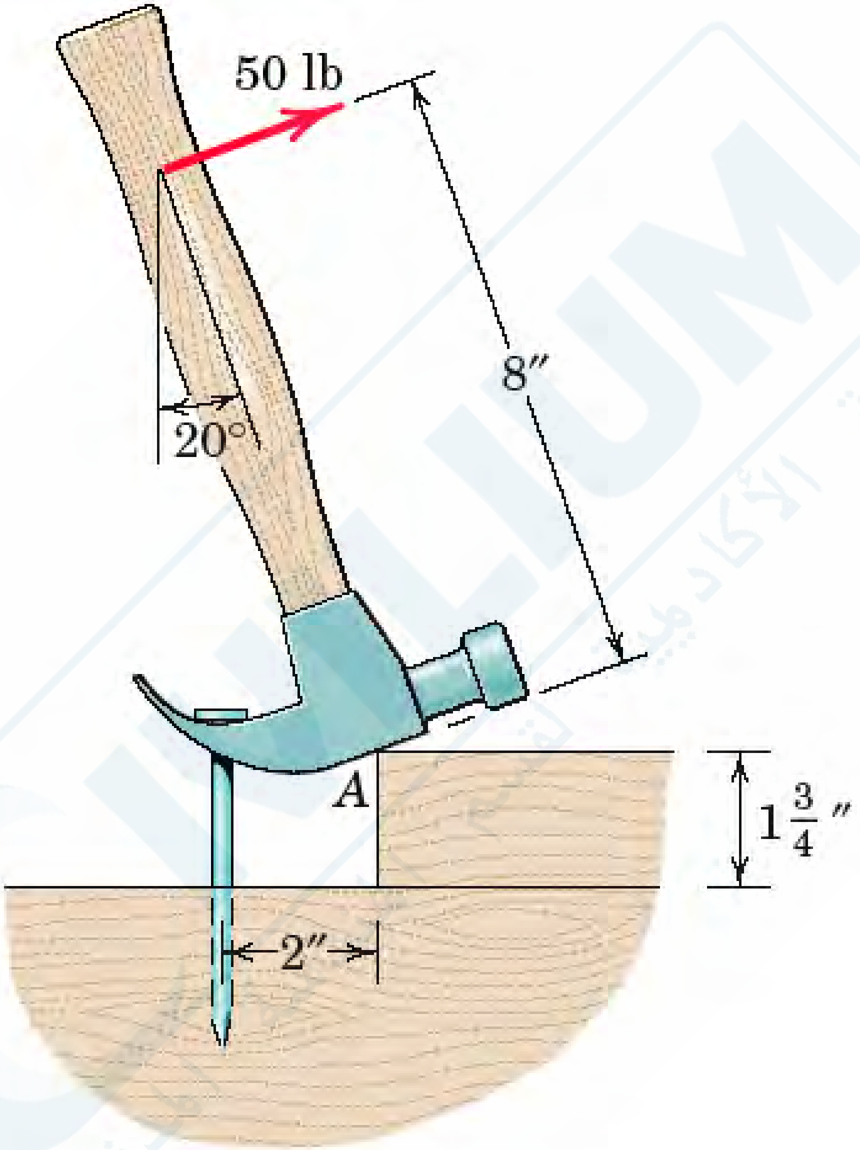
Problem 3/23

P 03 23



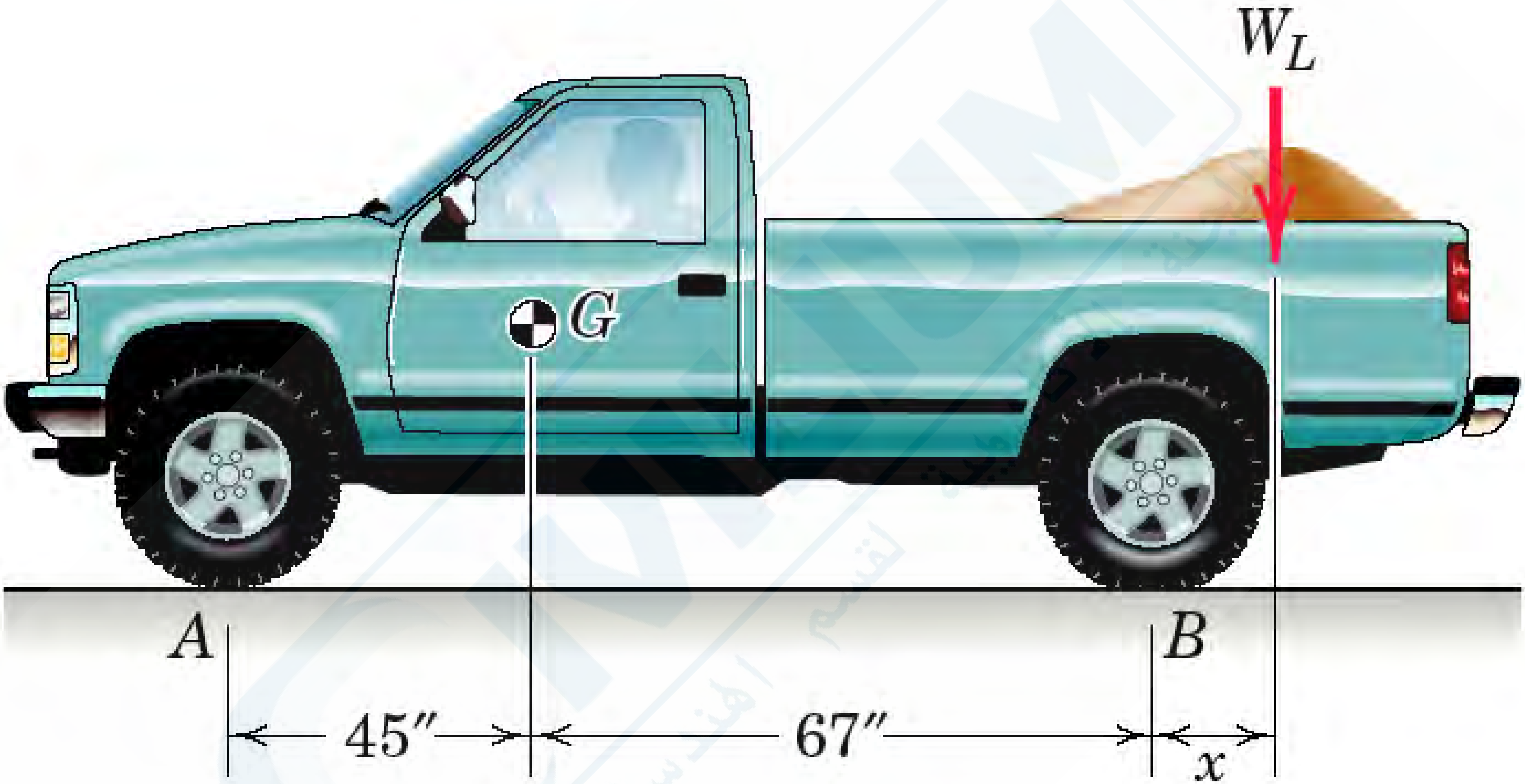
Problem 3/24

P_03_24



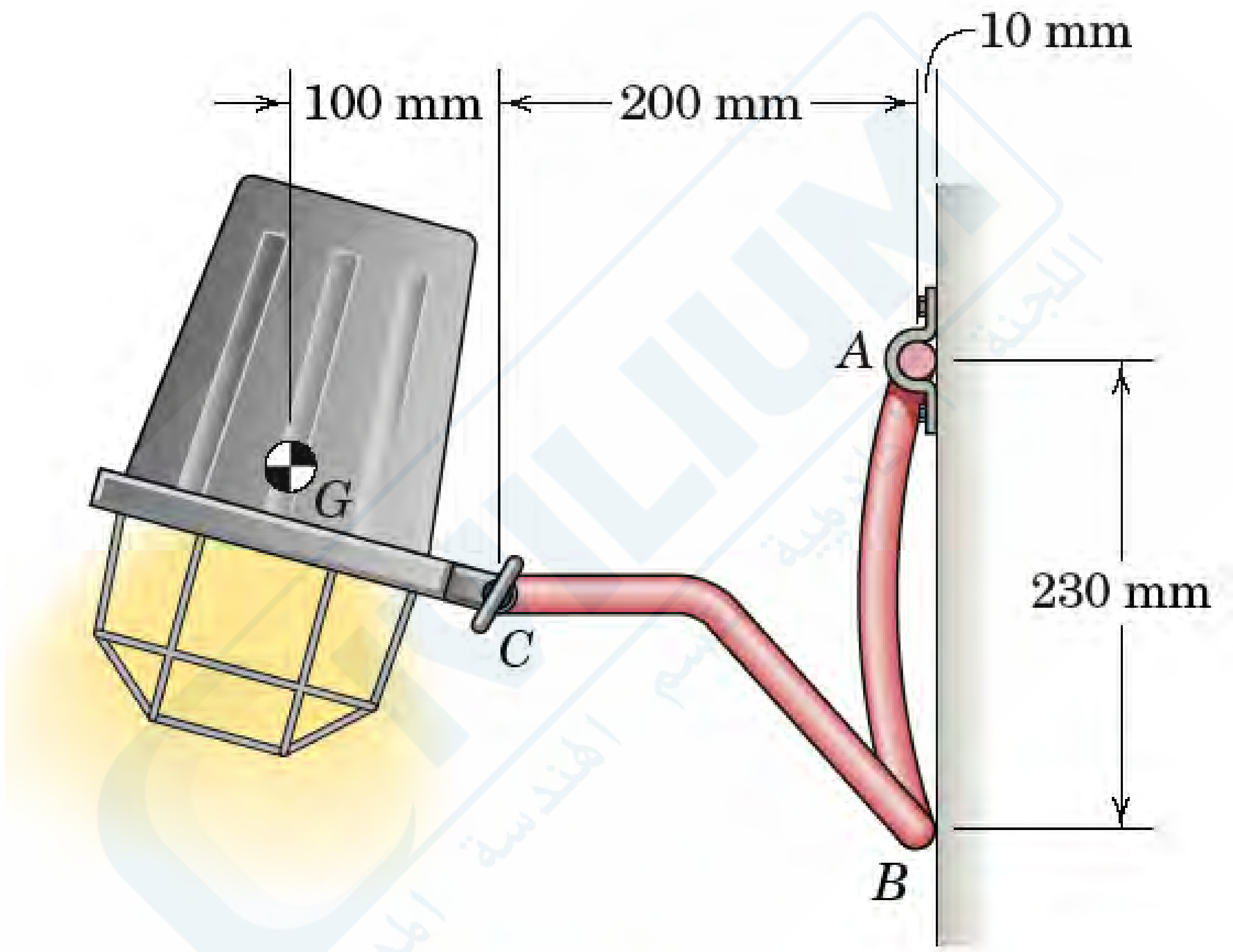
Problem 3/25

P_03_25



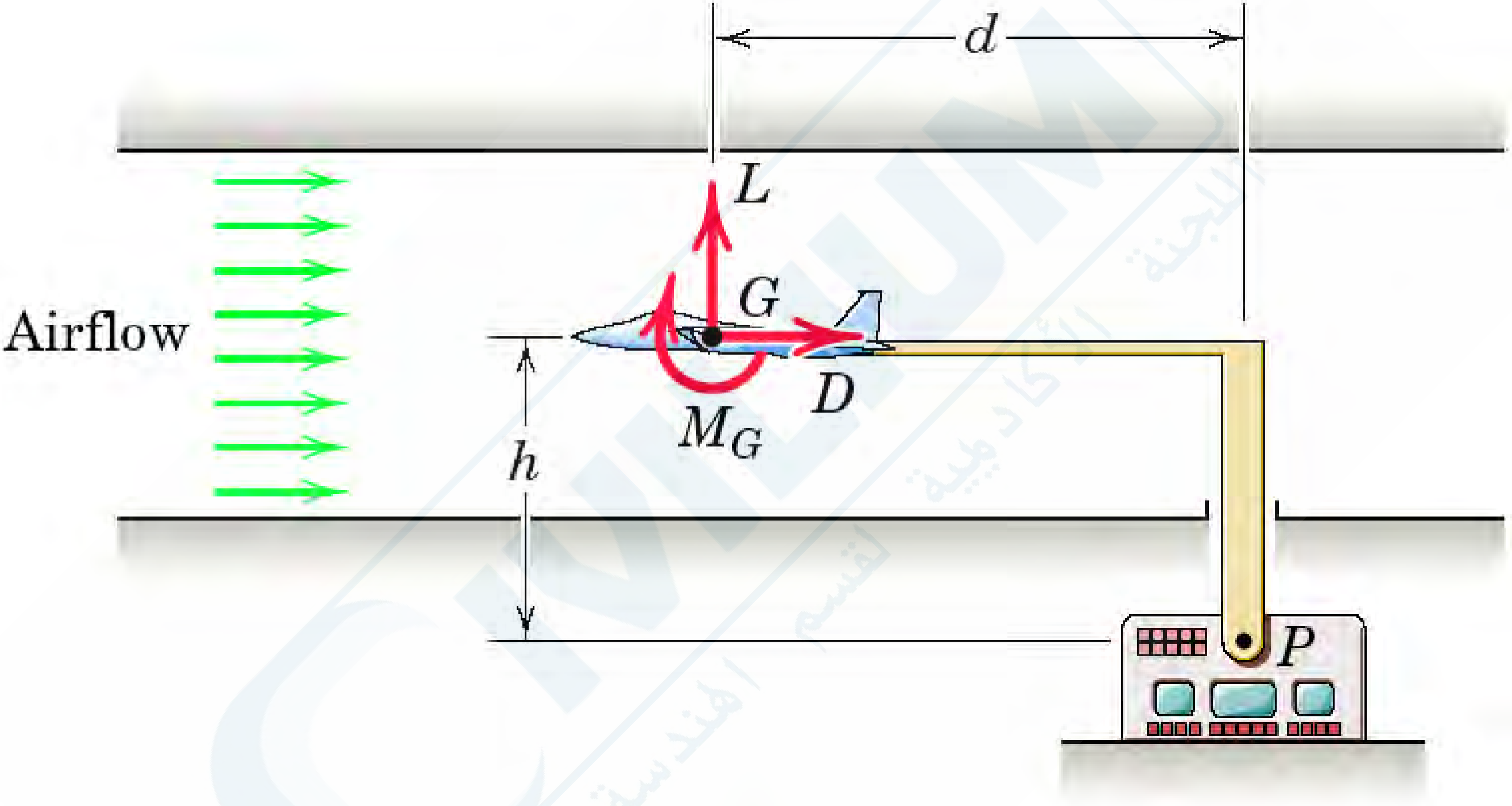
Problem 3/26

P_03_26



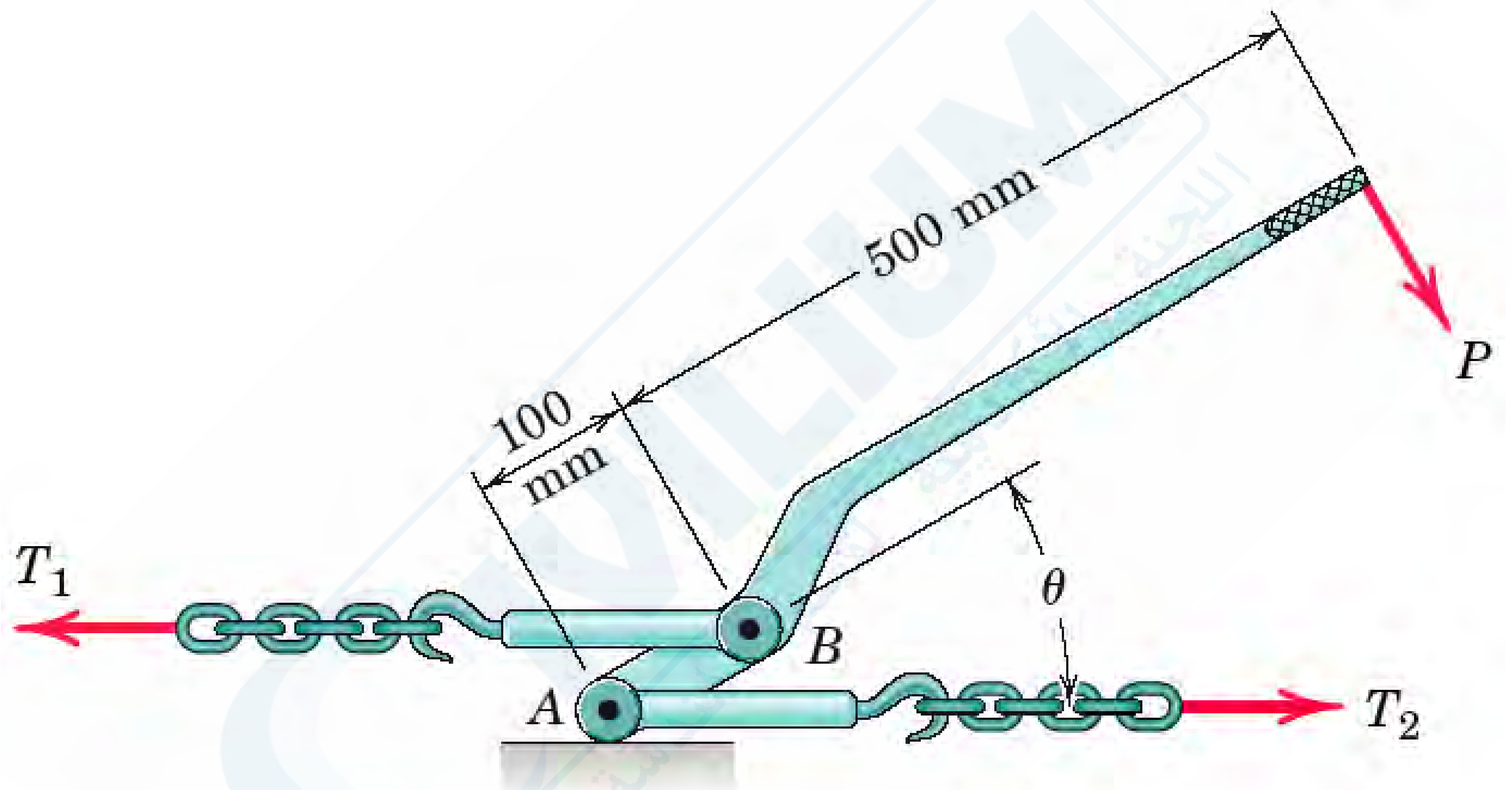
Problem 3/27

P 03 27



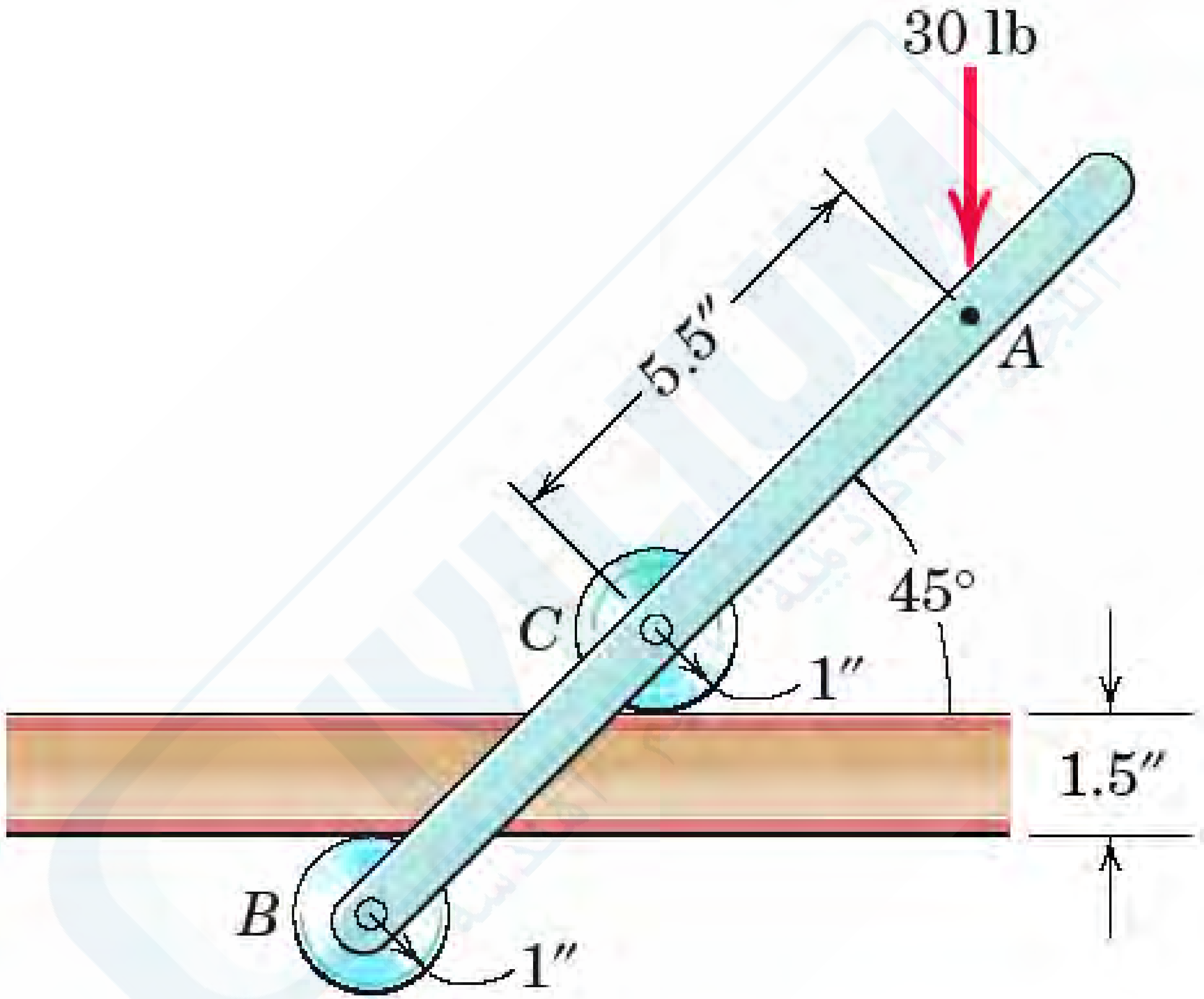
Problem 3/28

P_03_28



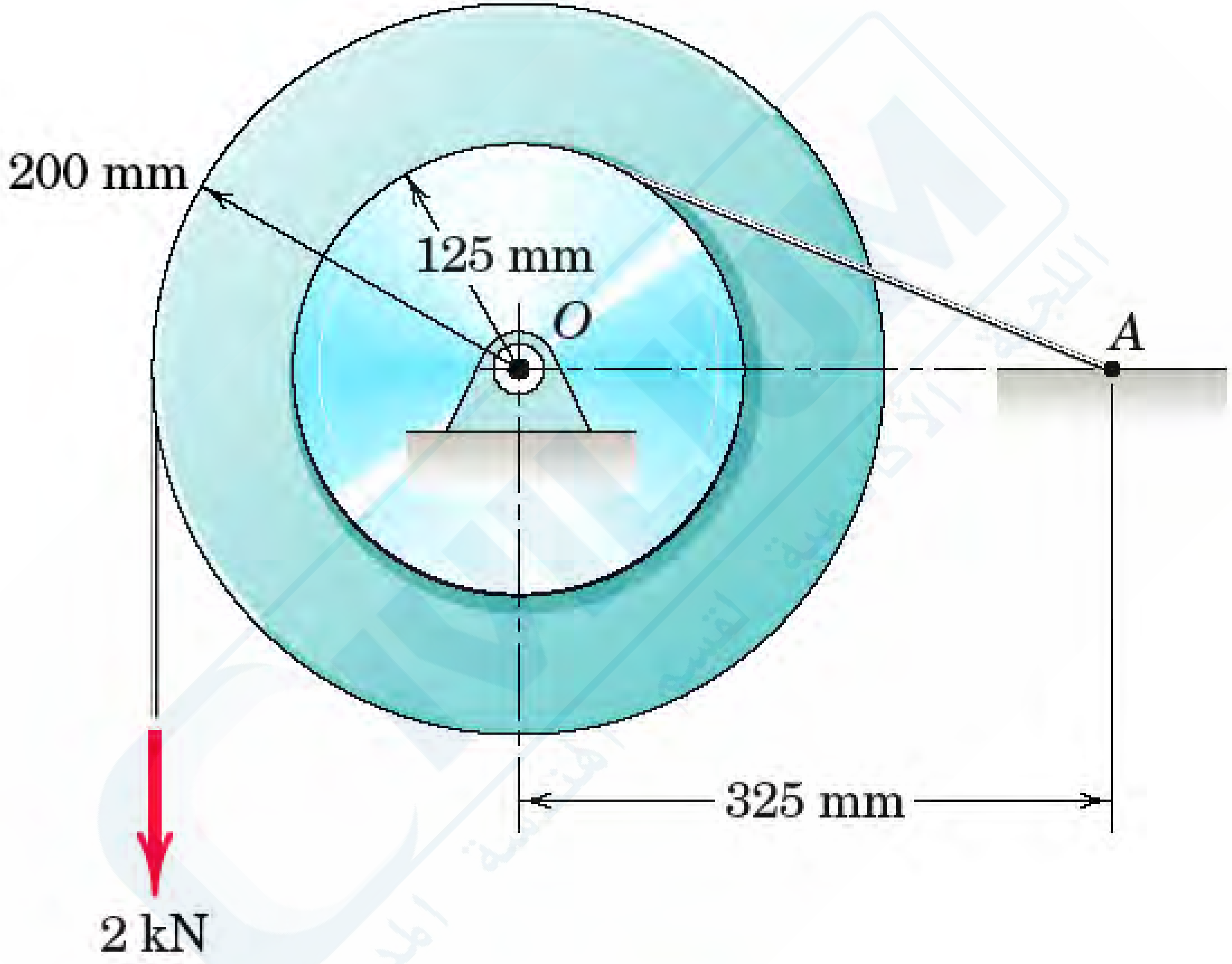
Problem 3/29

P_03_29



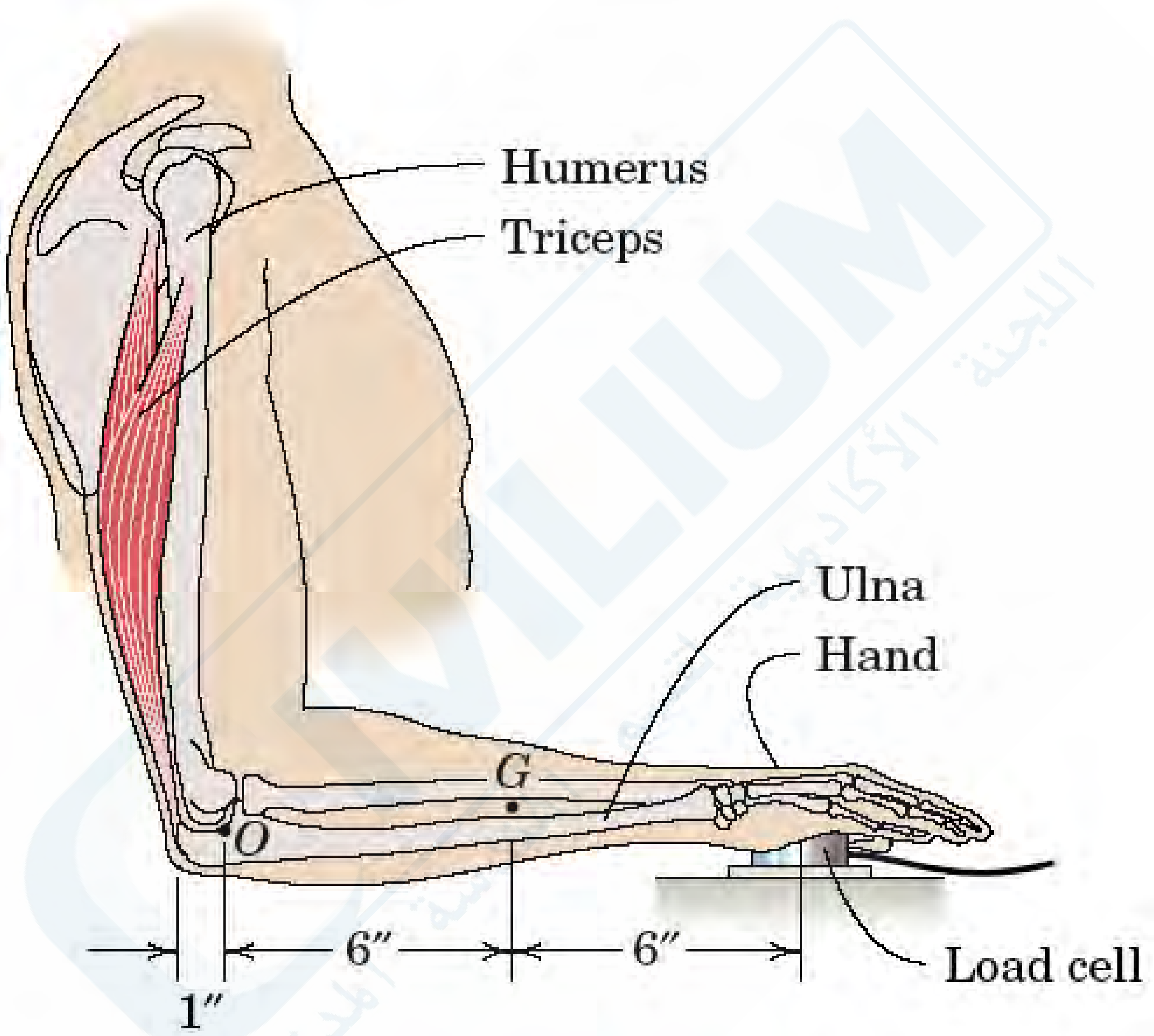
Problem 3/30

P_03_30



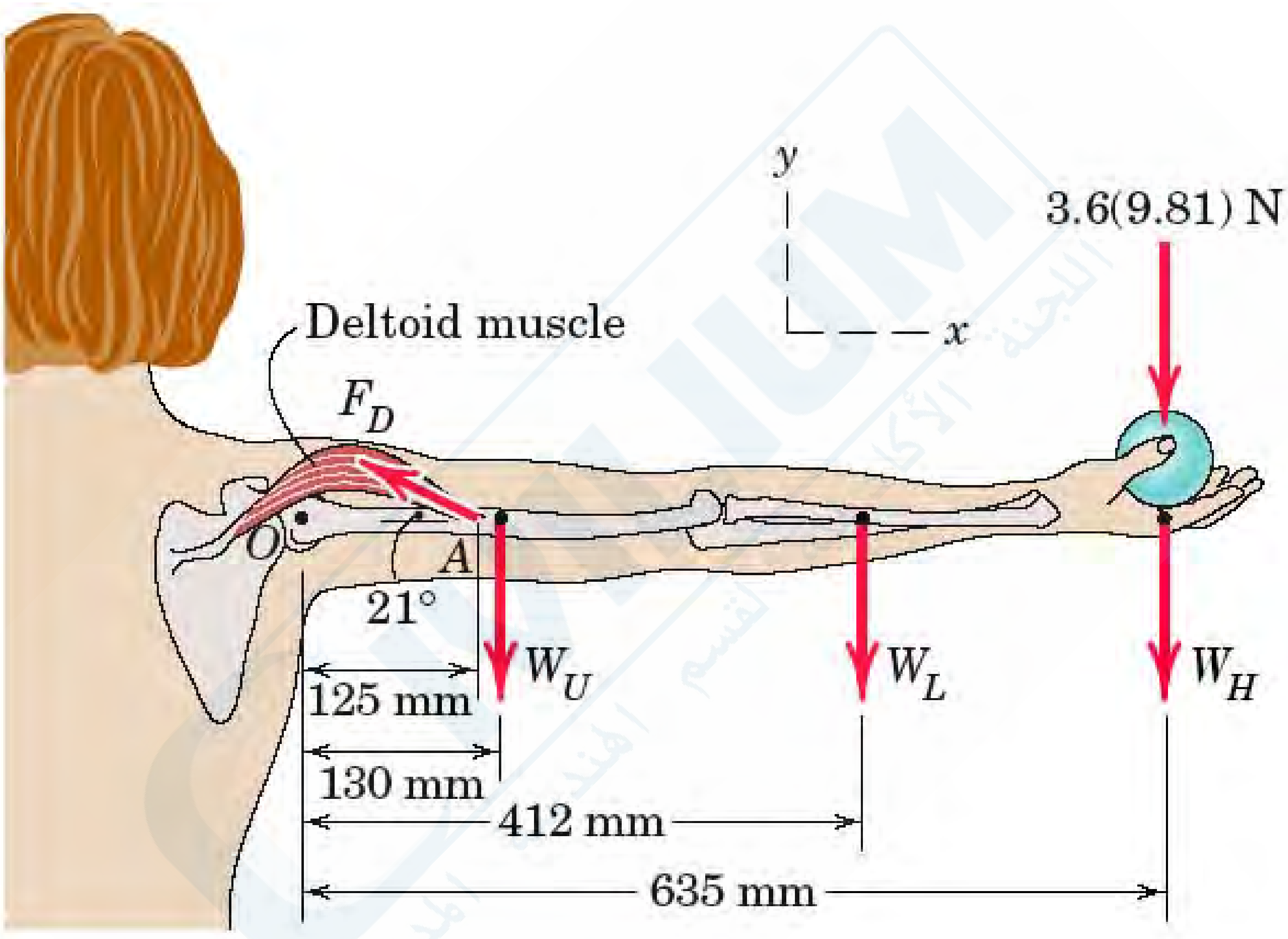
Problem 3/31

P_03_31



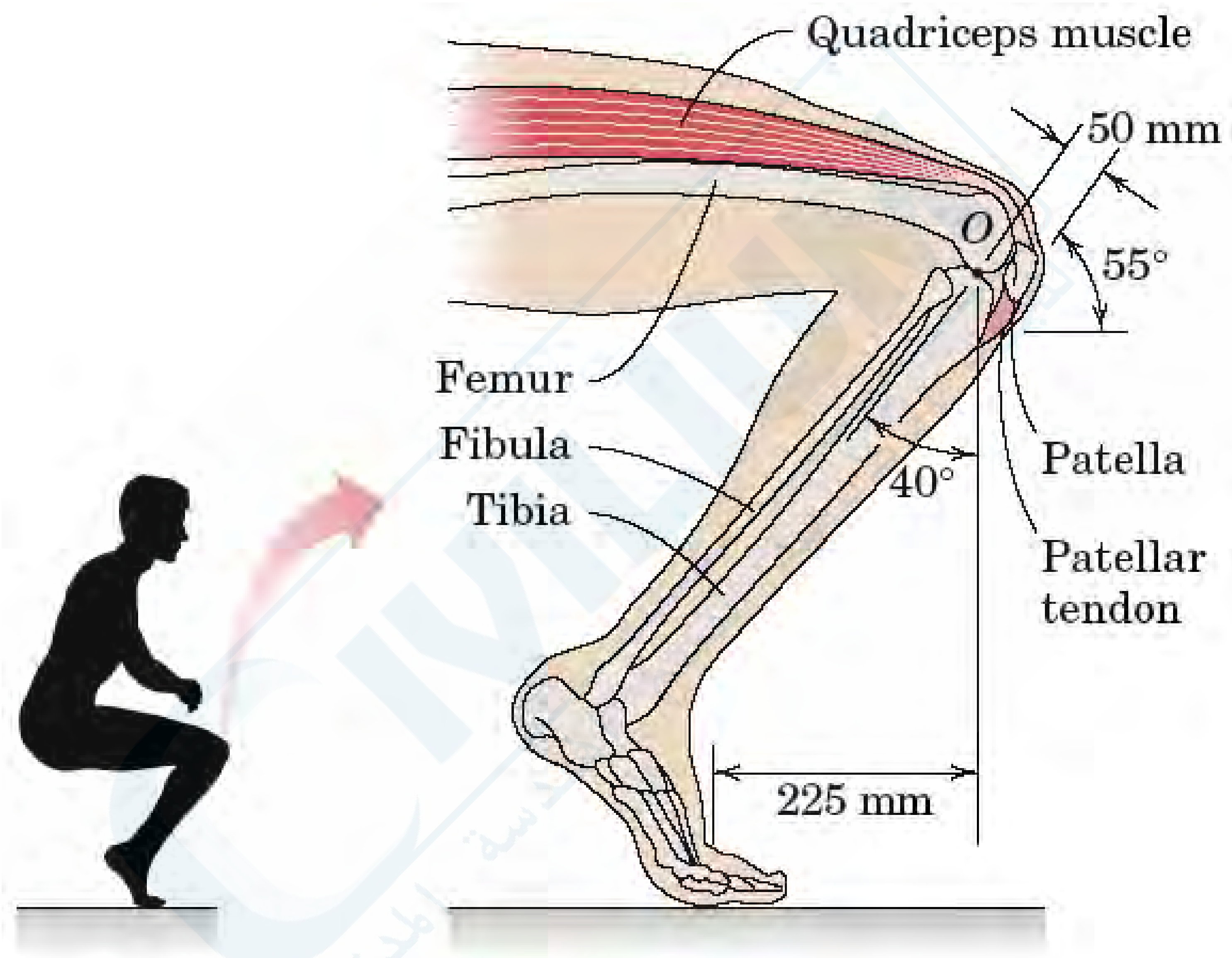
Problem 3/32

P 03 32

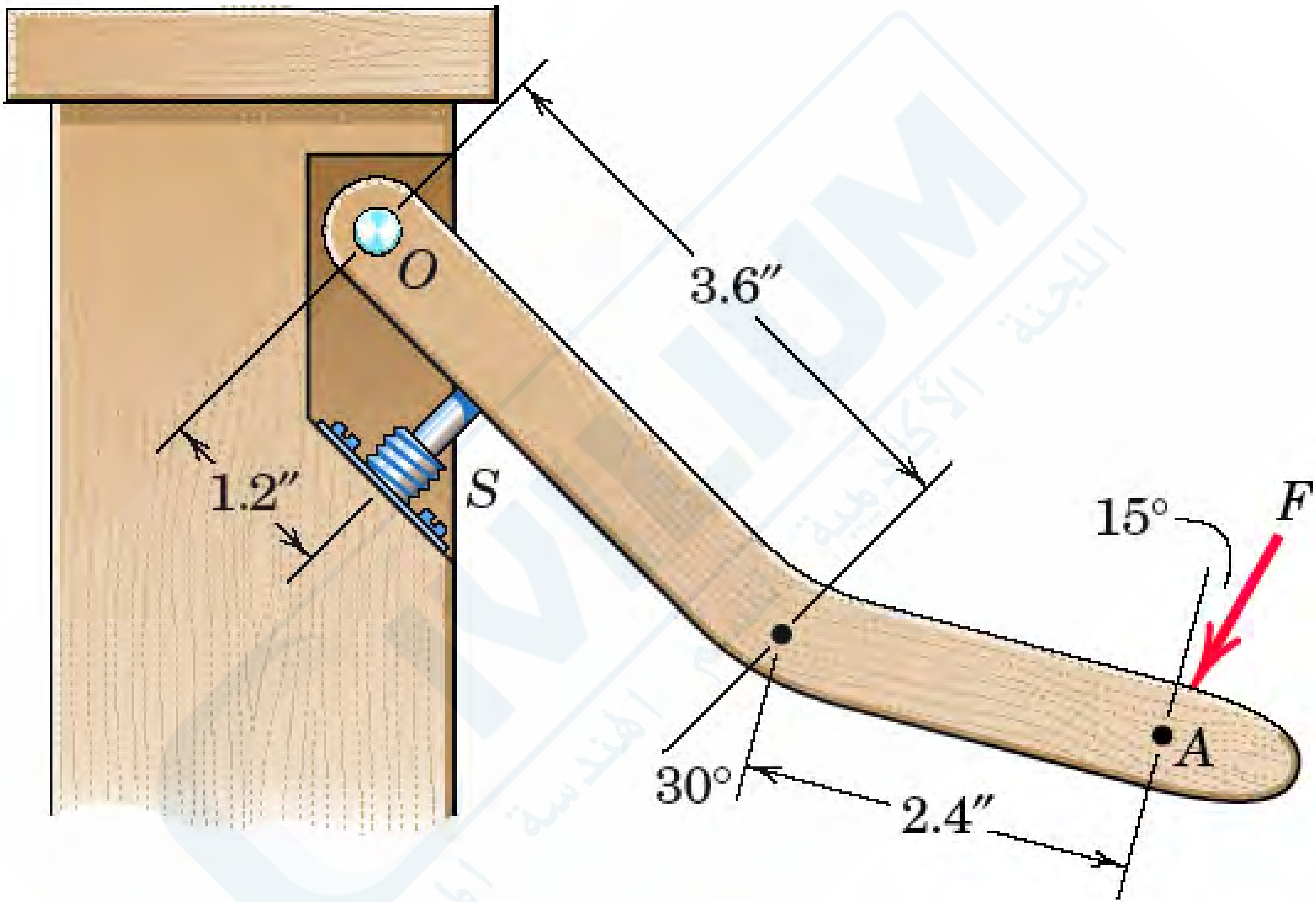


Problem 3/34

P 03 34

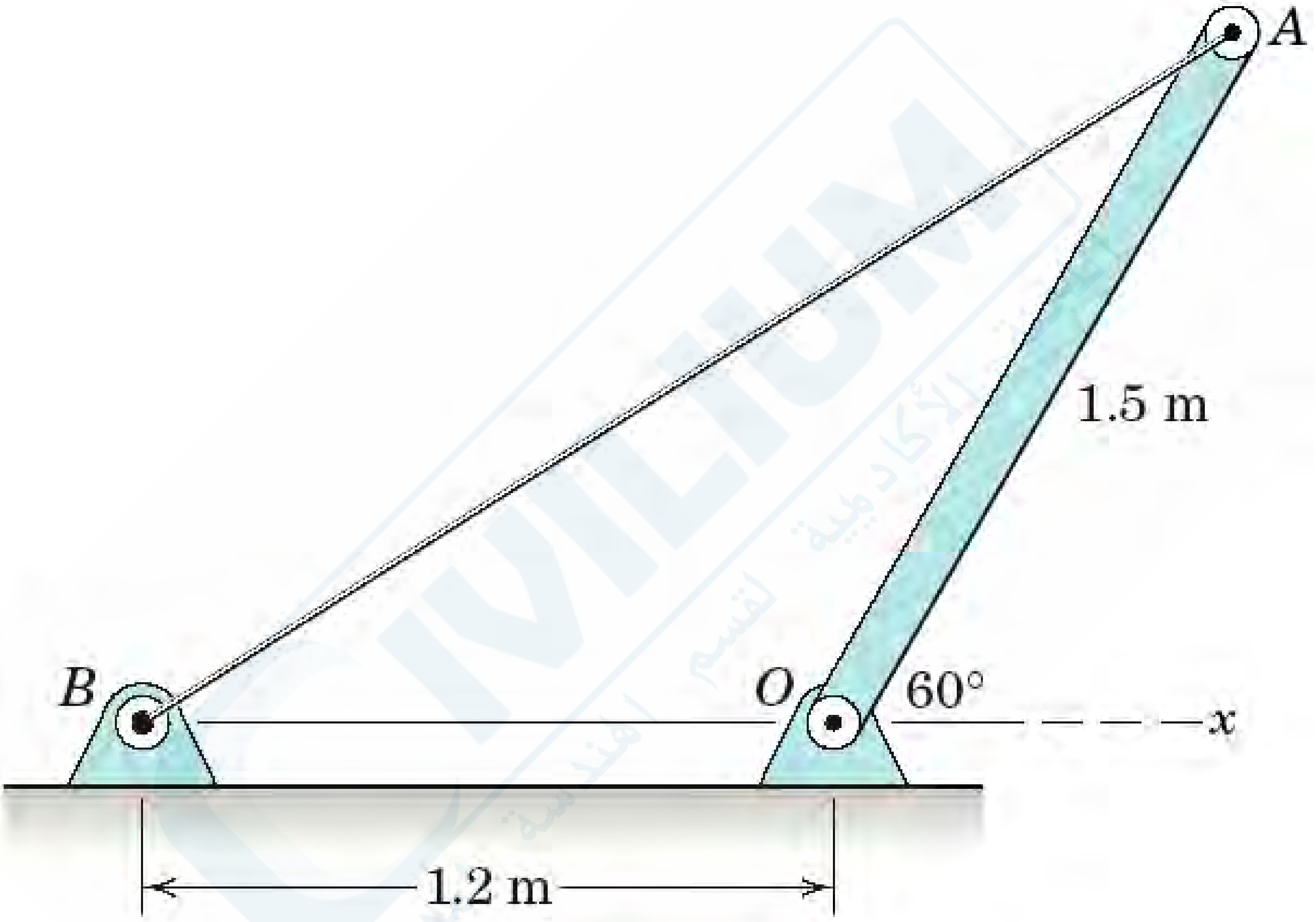

Quadriceps muscle
50 mm
O
55°
Femur
Fibula
Tibia
40°
Patella
Patellar
tendon
225 mm

Problem 3/35



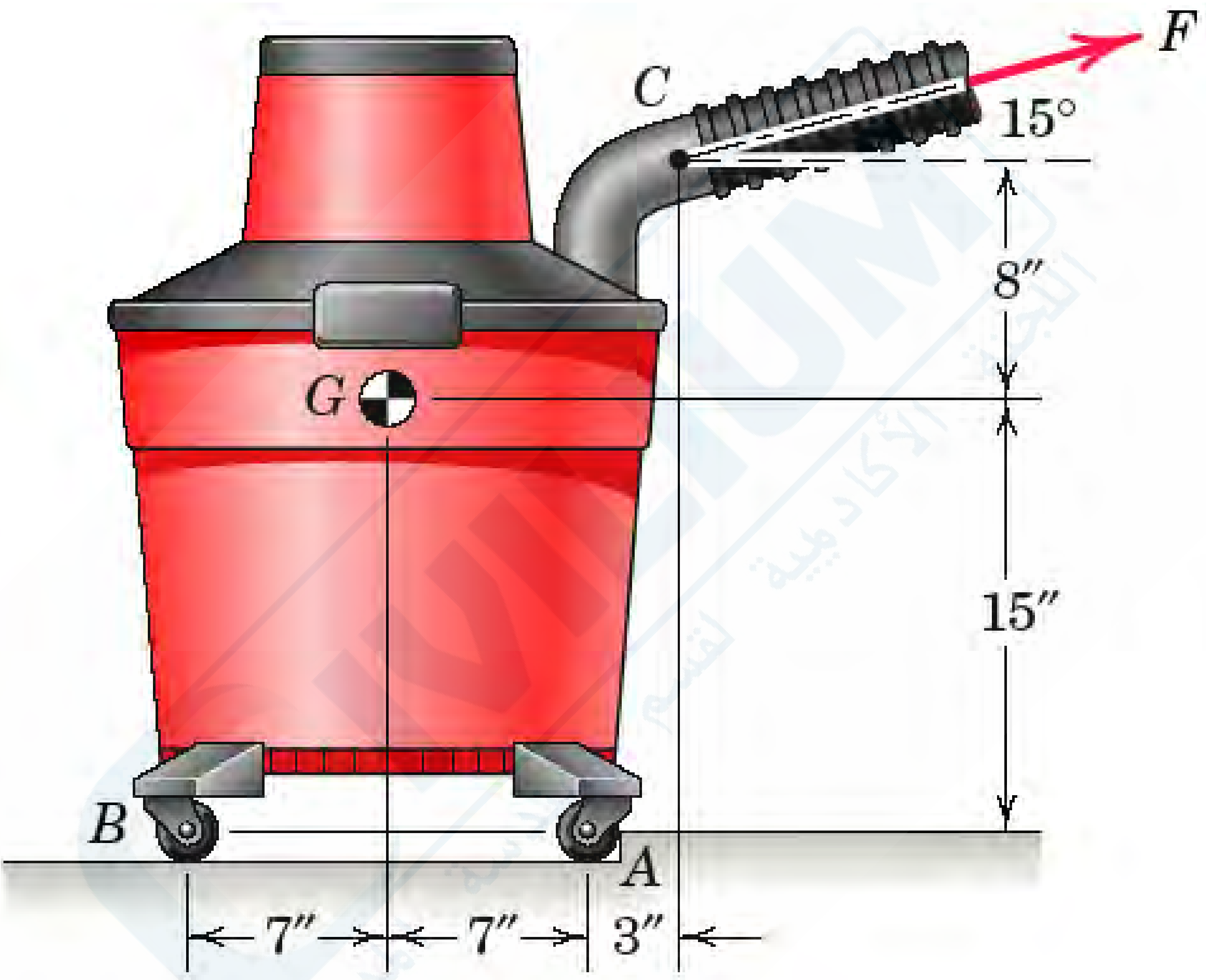
Problem 3/36

P 03 36



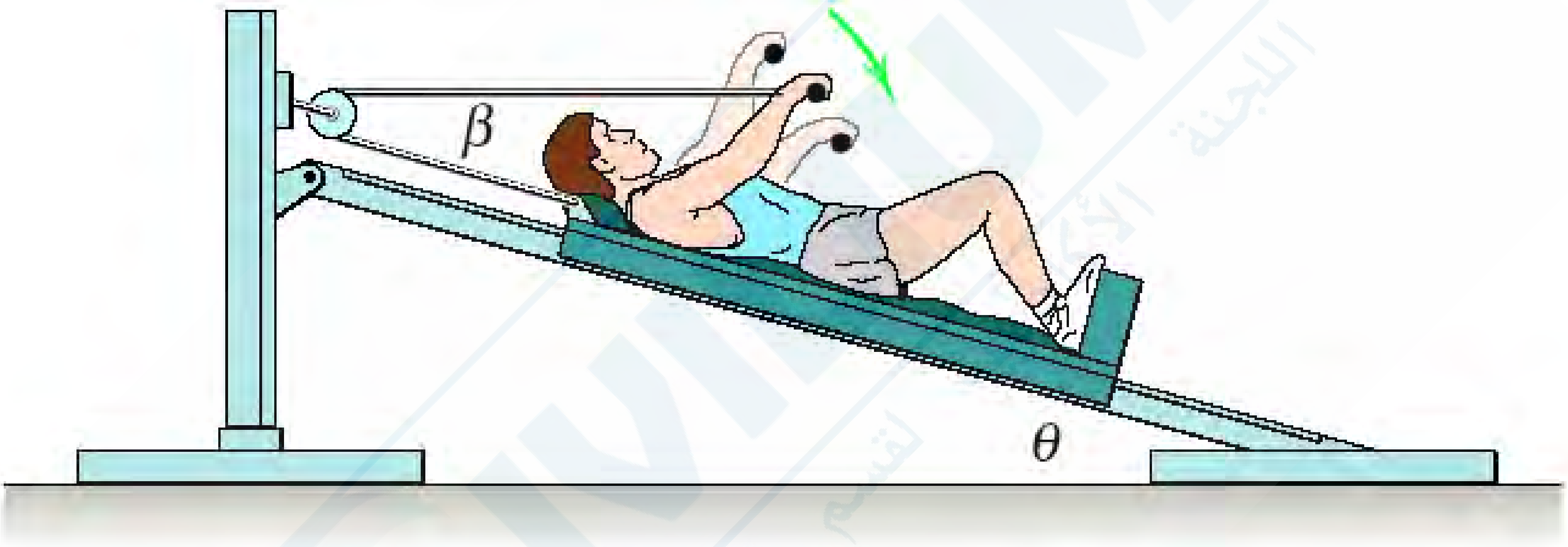
Problem 3/37

P_03_37



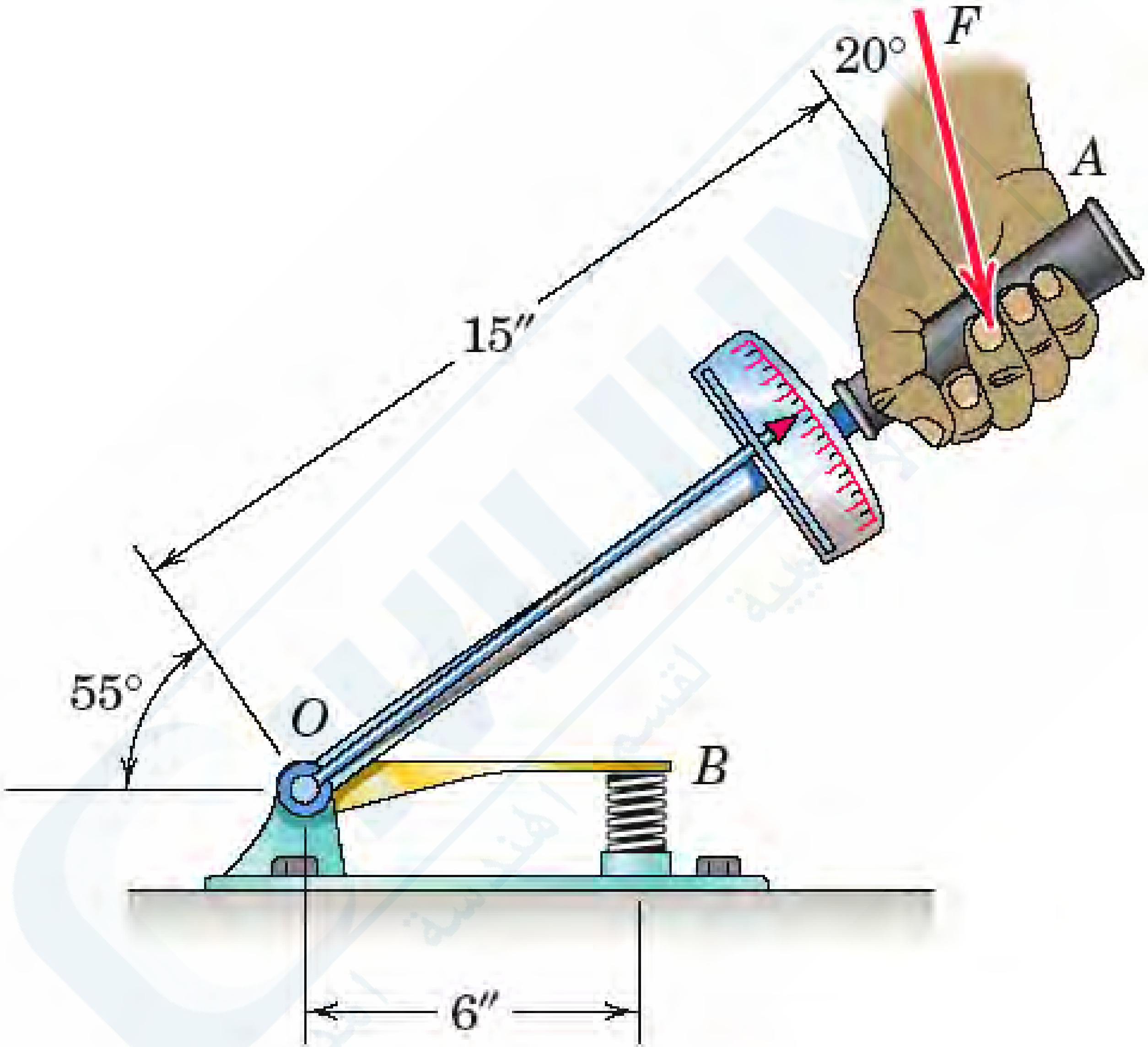
Problem 3/38

P_03_38



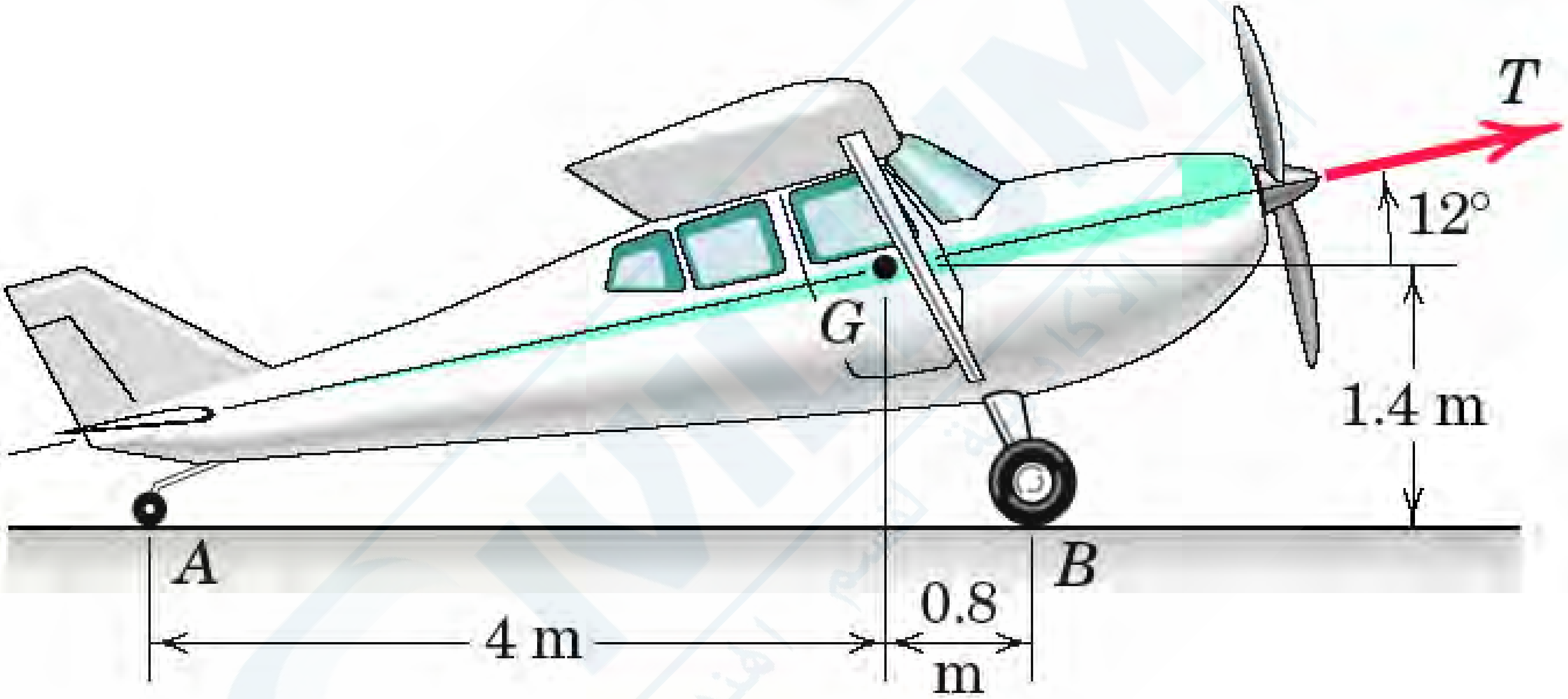
Problem 3/39

P_03_39



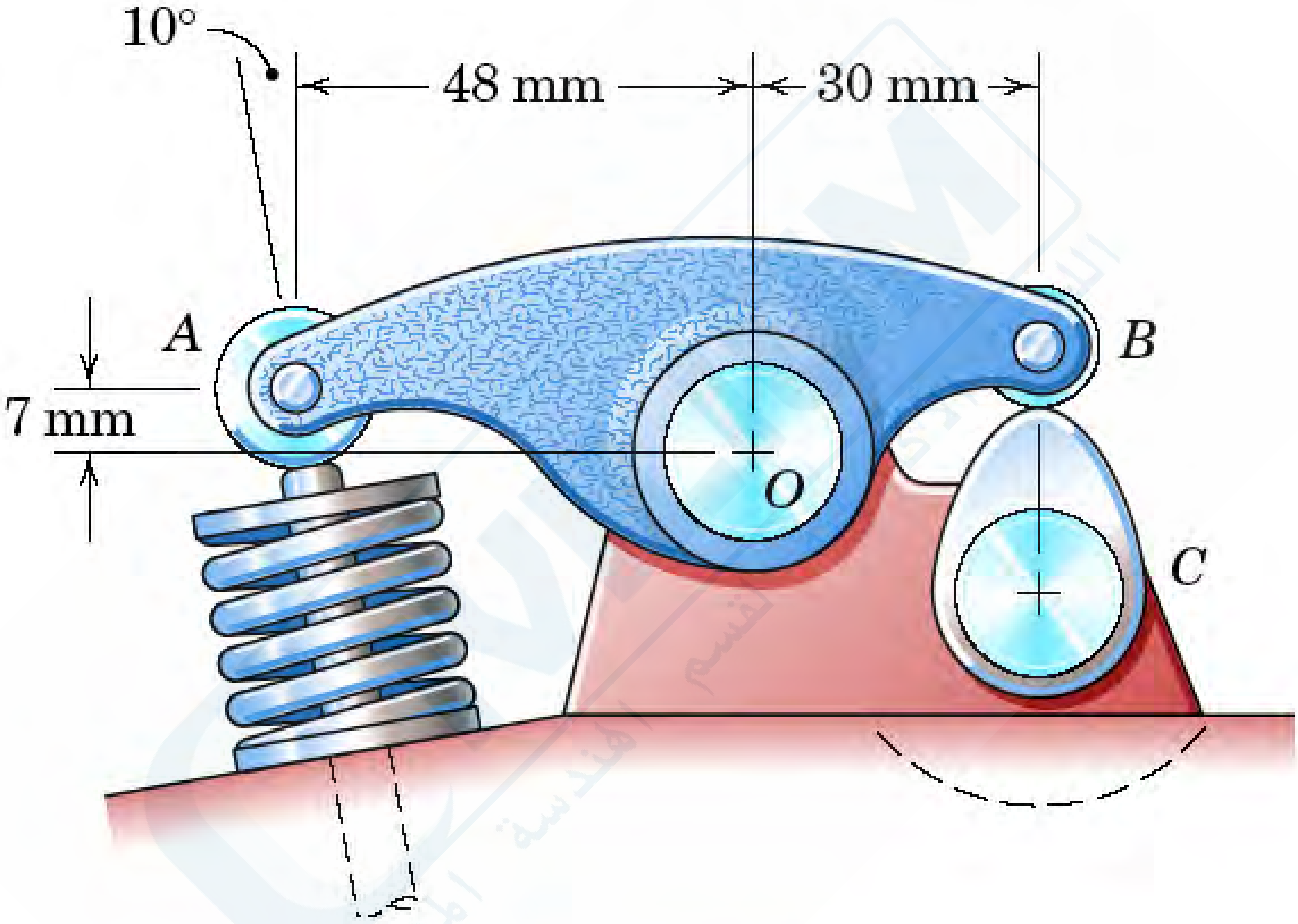
Problem 3/40

P_03_40



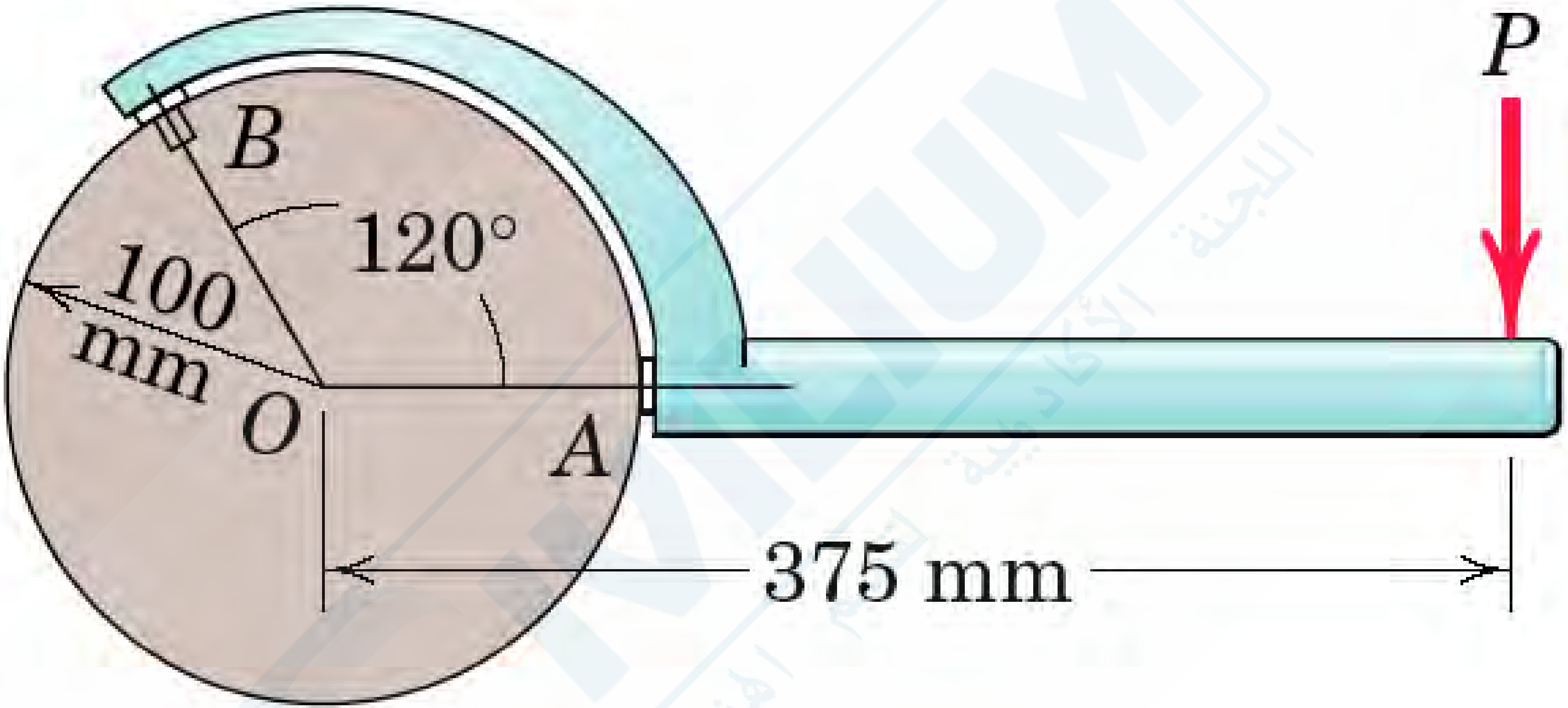
Problem 3/41

P_03_41



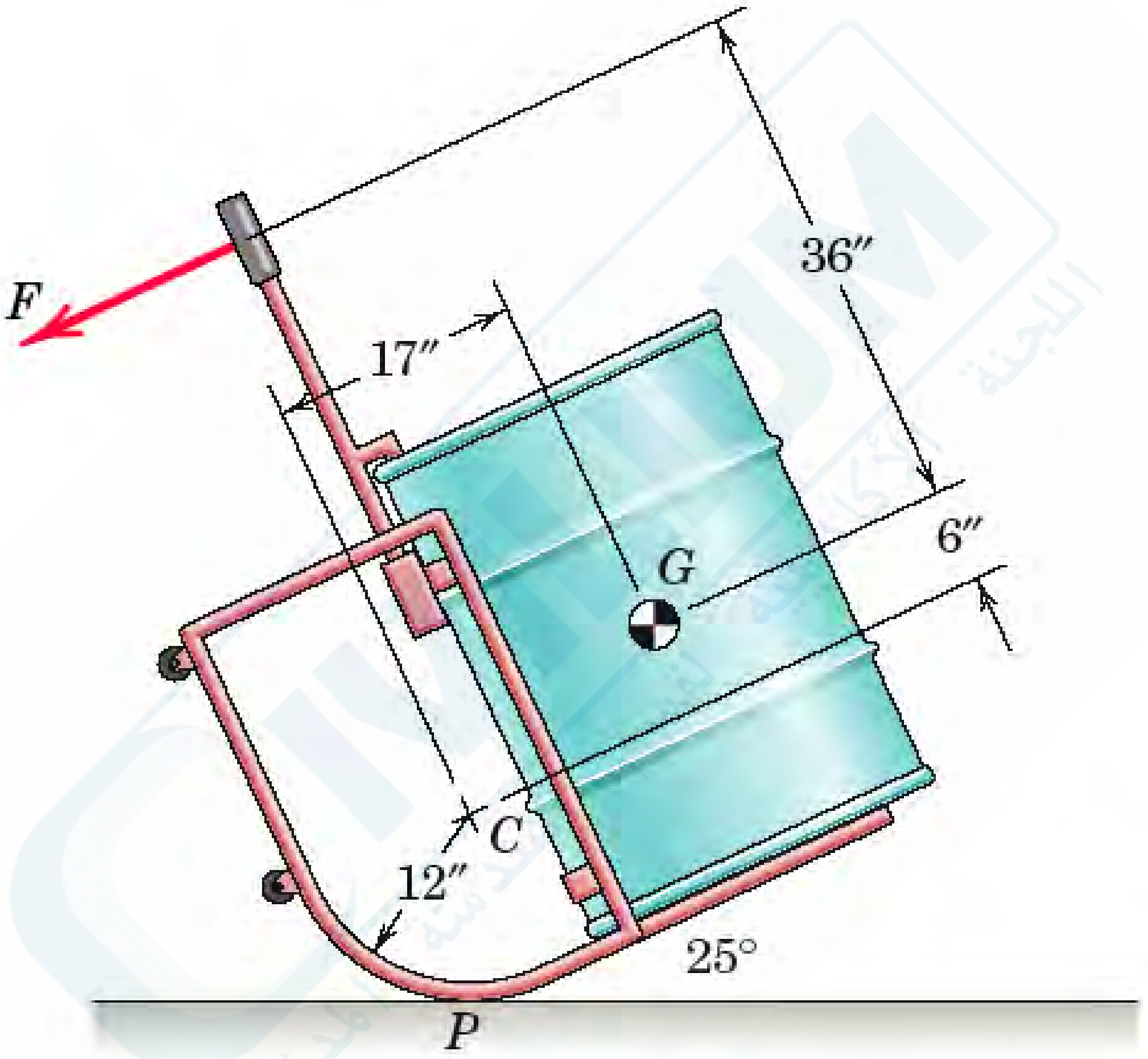
Problem 3/42

P 03 42



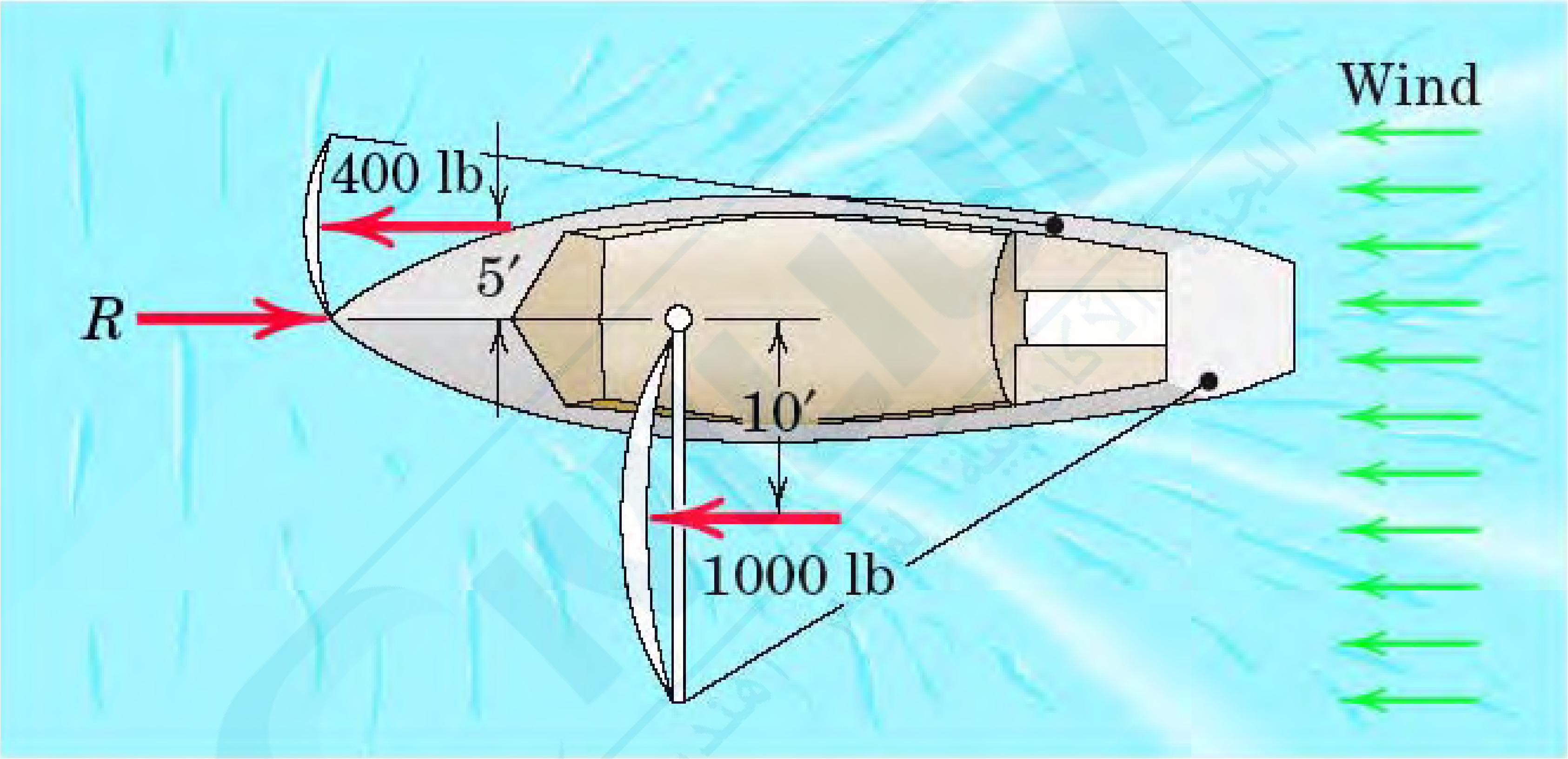
Problem 3/43

P_03_43



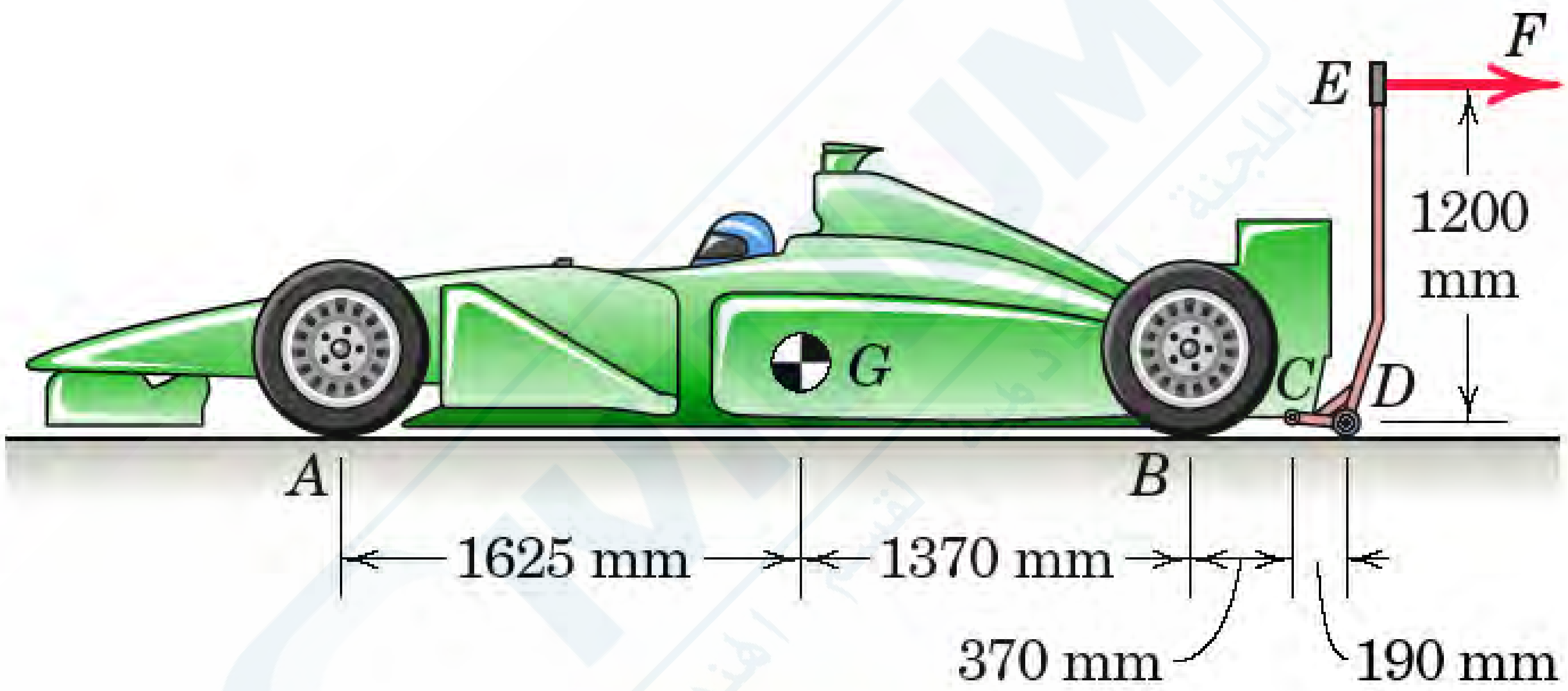
Problem 3/44

P_03_44



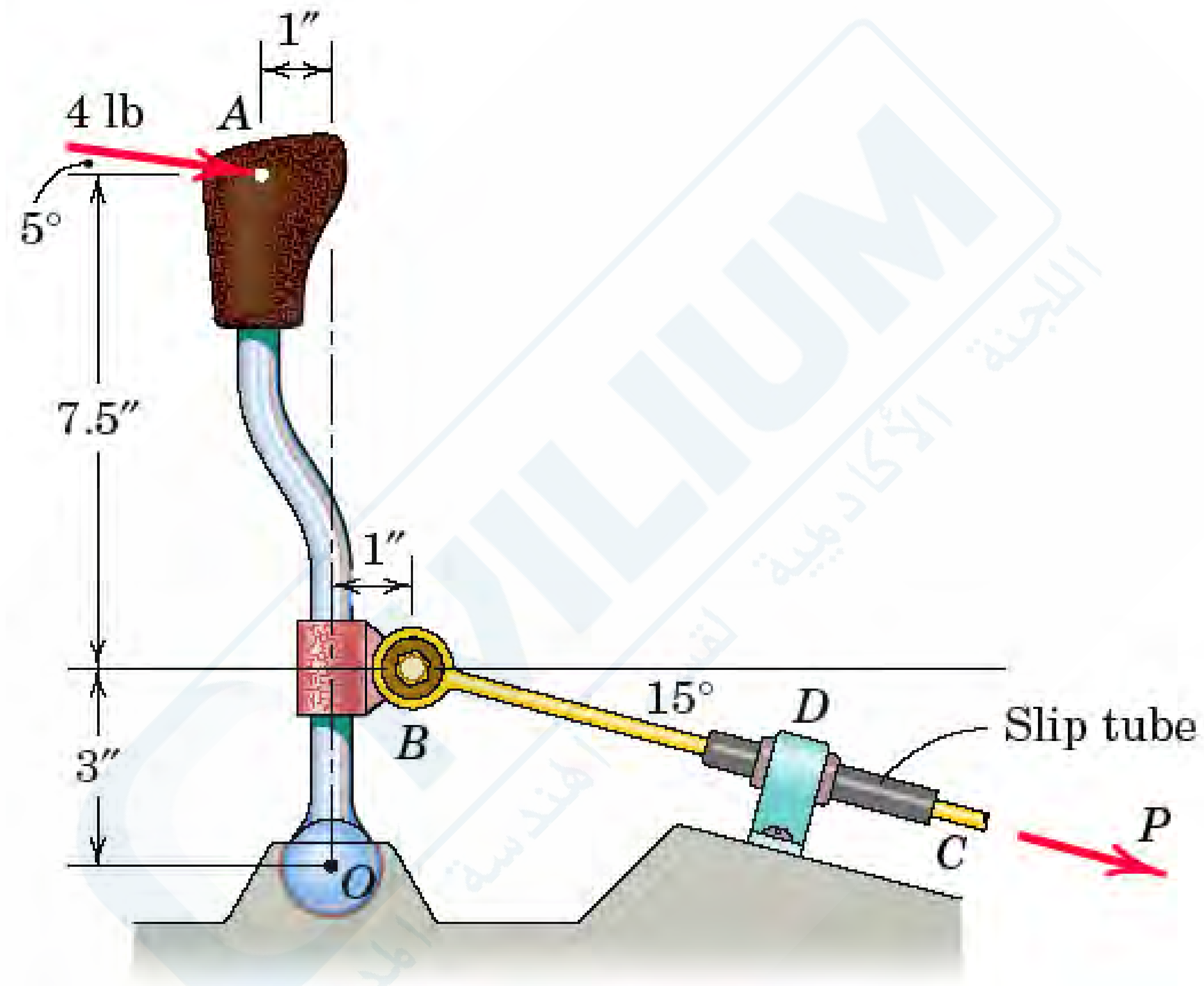
Problem 3/45

P_03_45



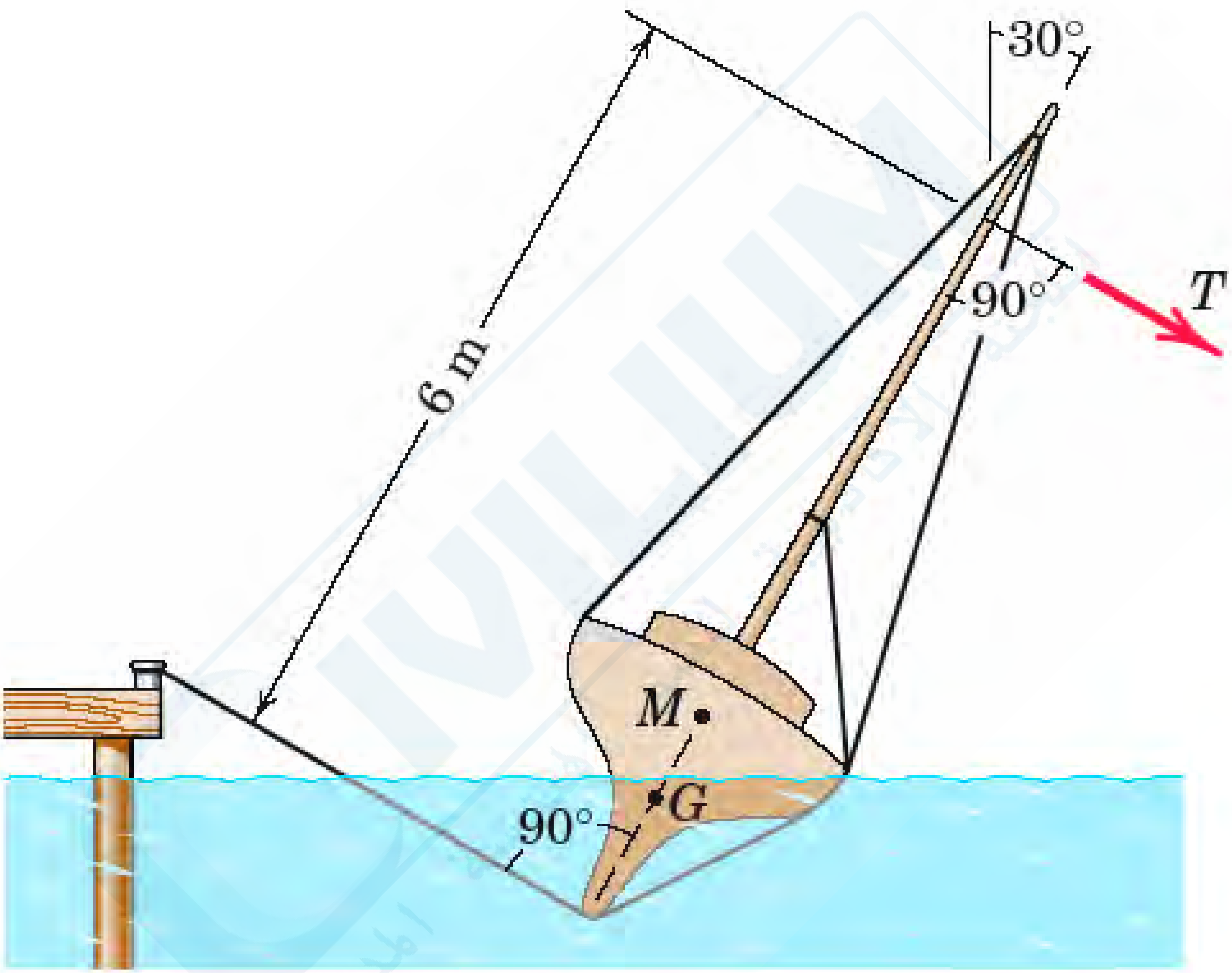
Problem 3/46

P_03_46



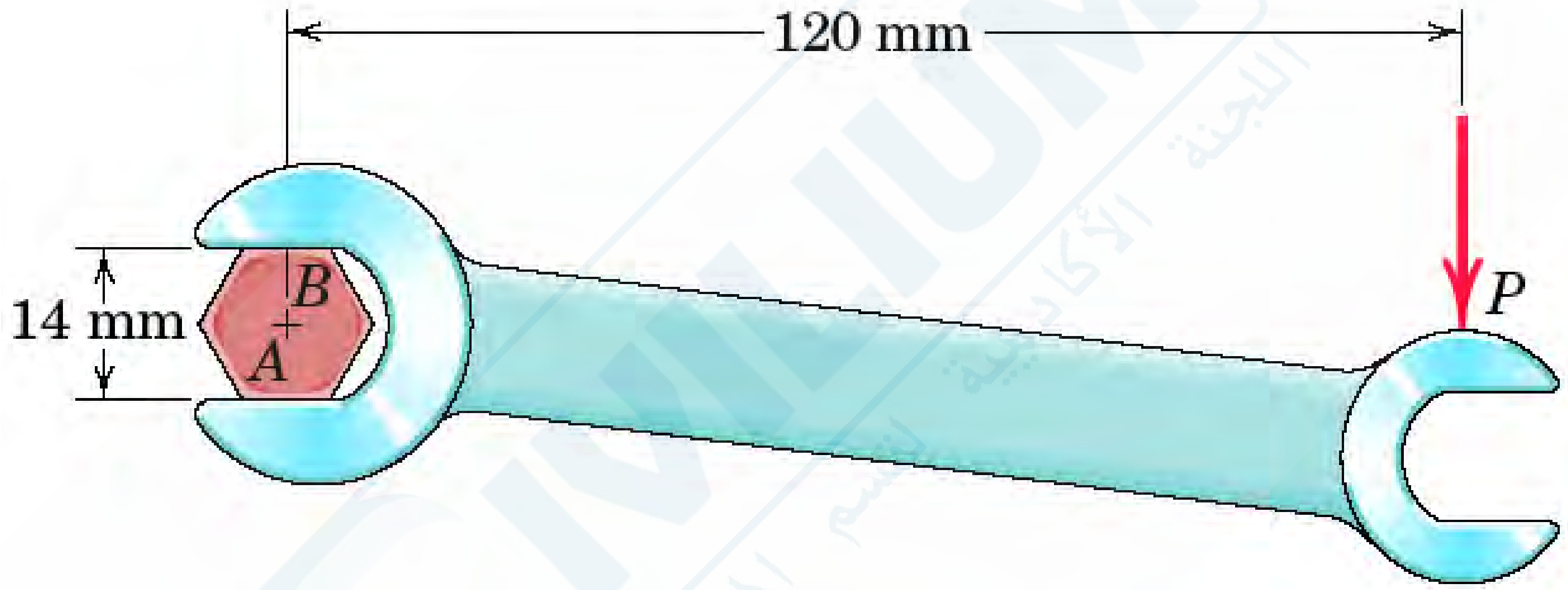
Problem 3/47

P 03 47



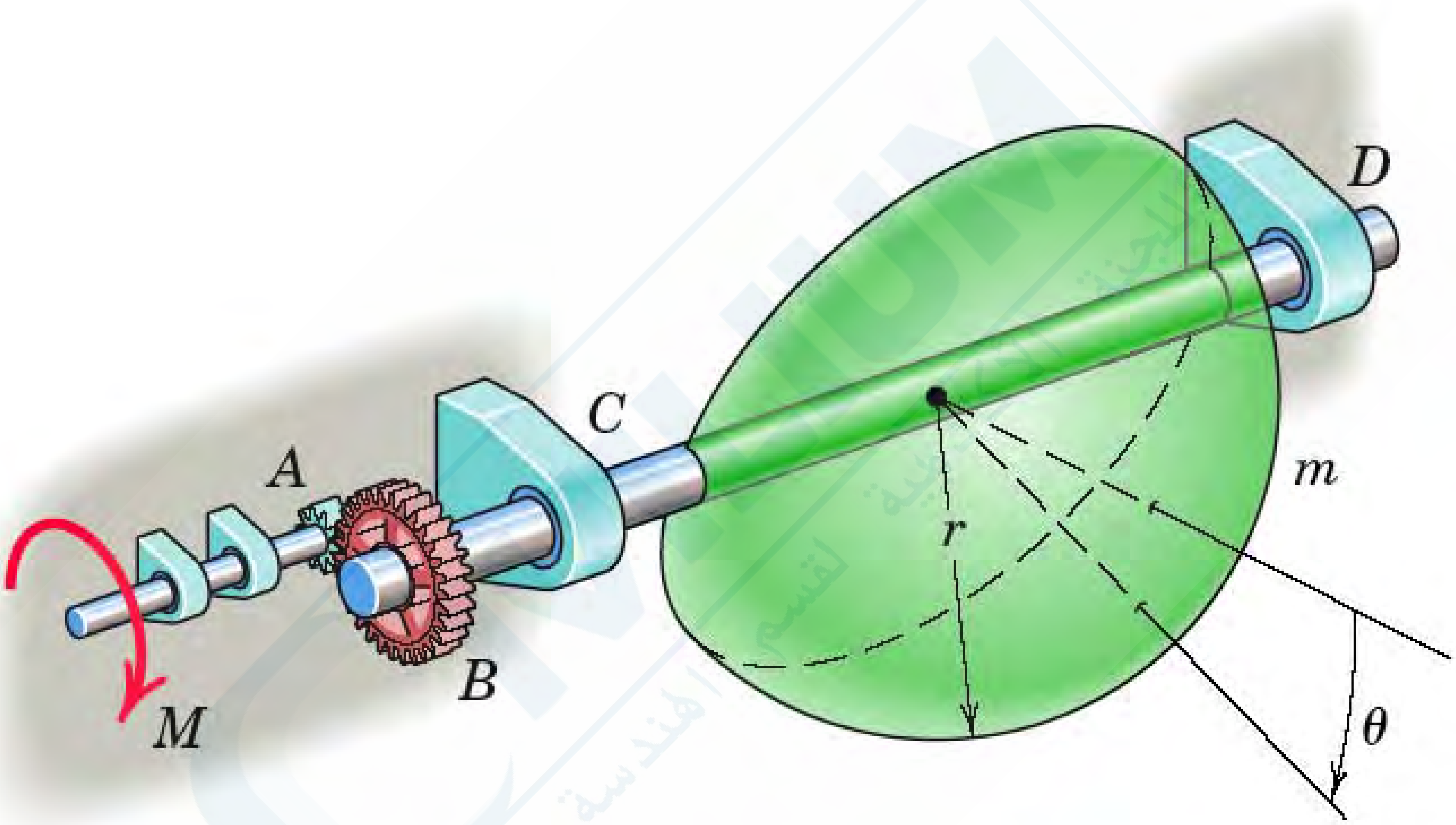
Problem 3/48

P_03_48



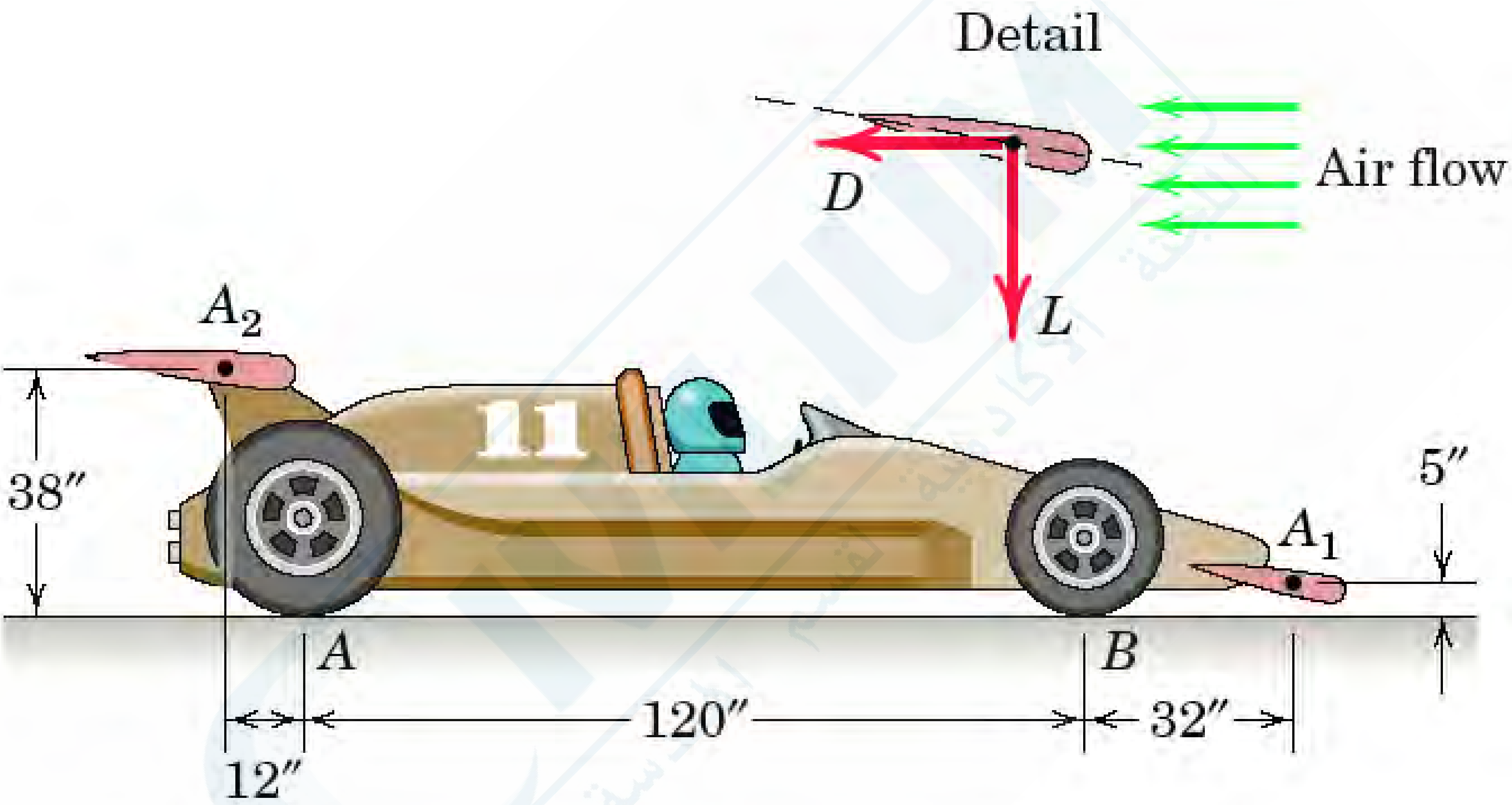
Problem 3/49

P_03_49



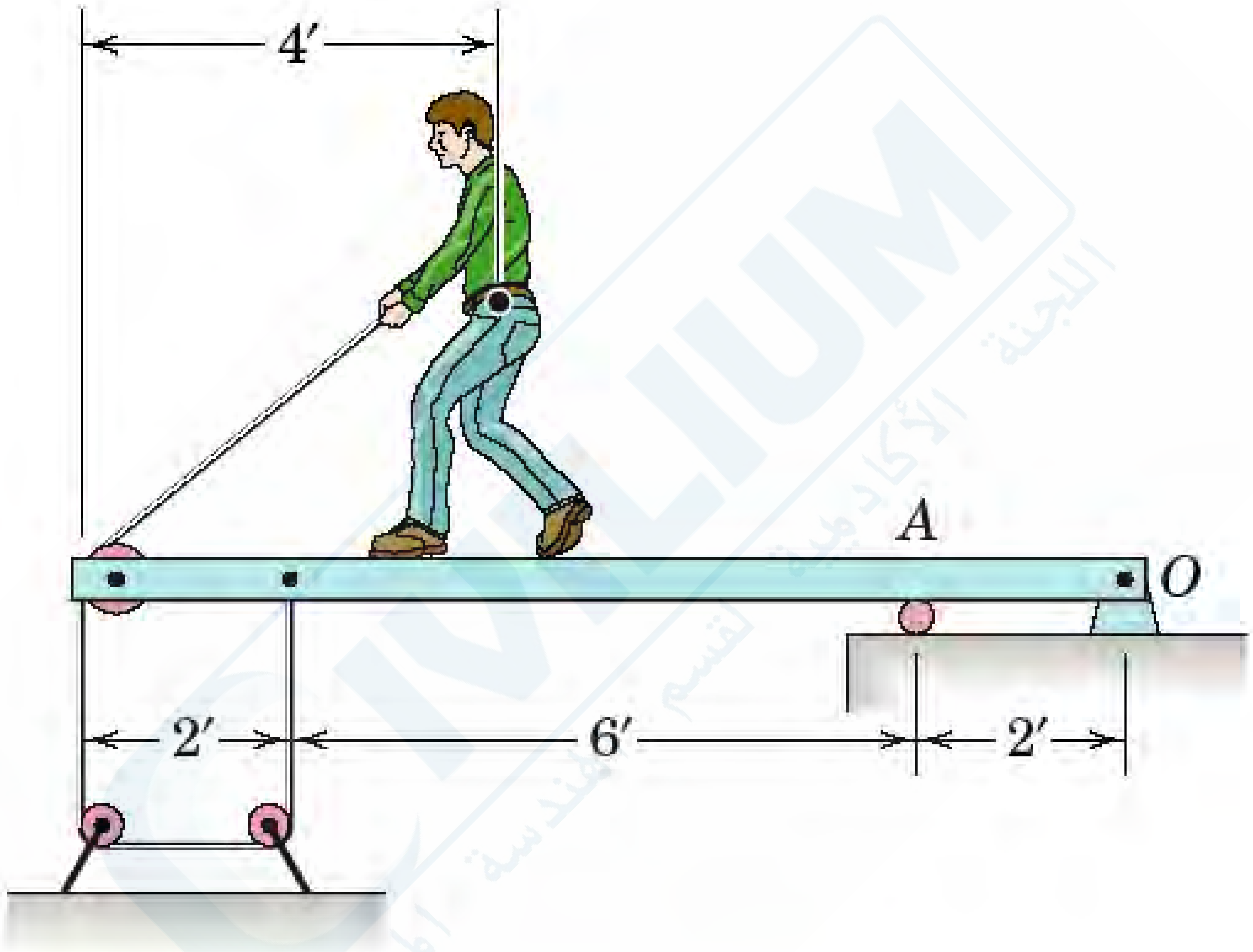
Problem 3/50

P_03_50



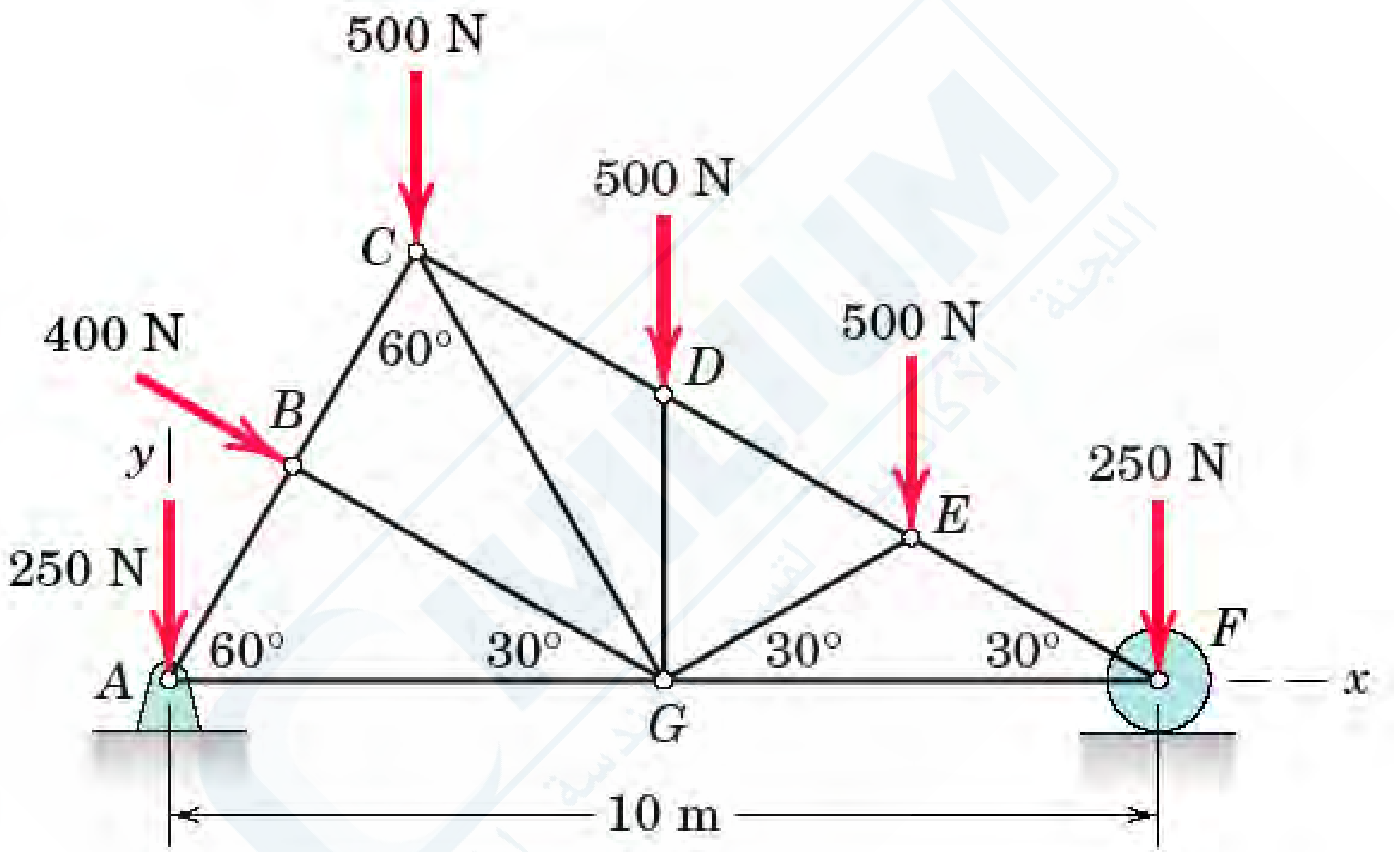
Problem 3/51

P_03_51



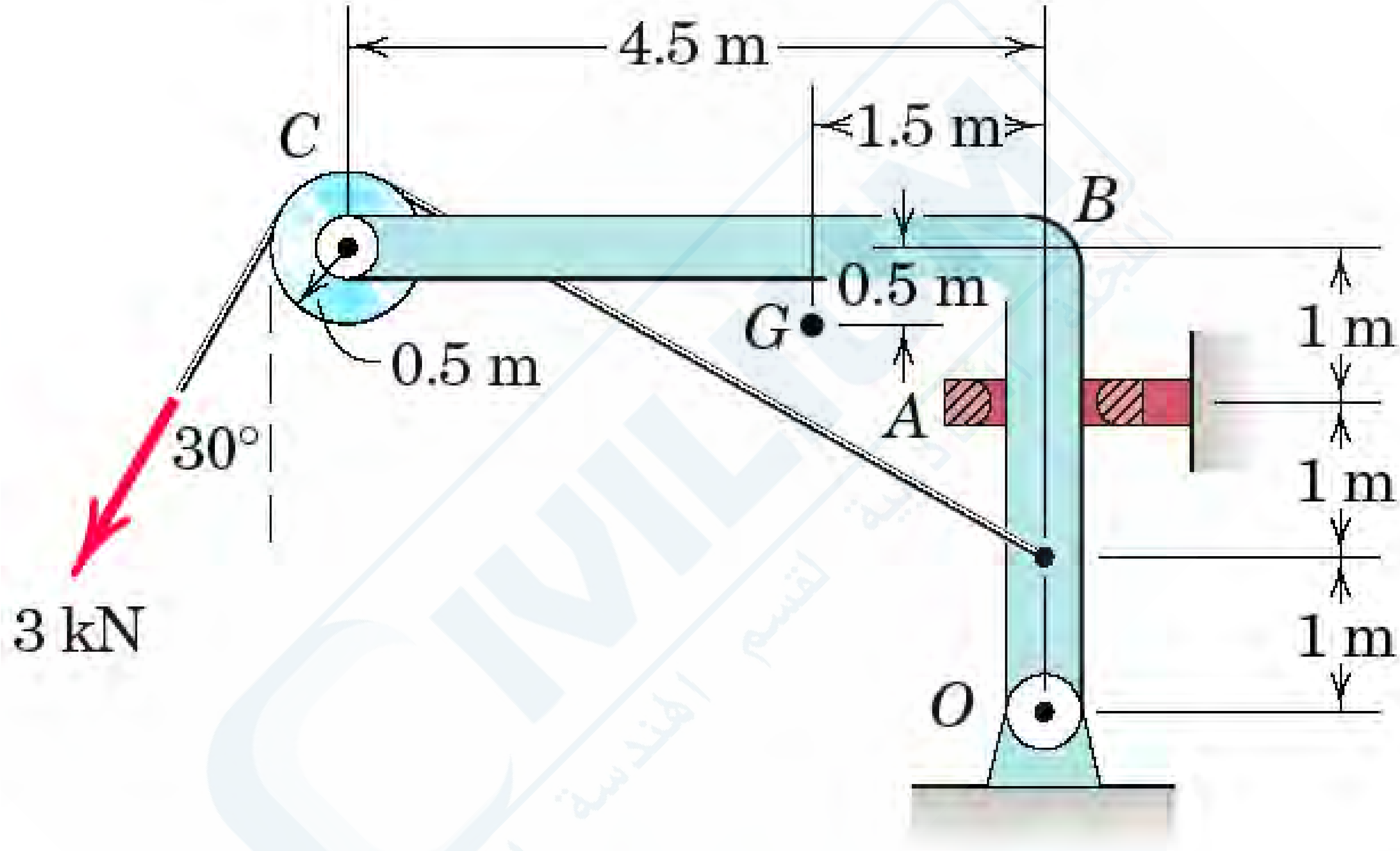
Problem 3/52

P 03 52



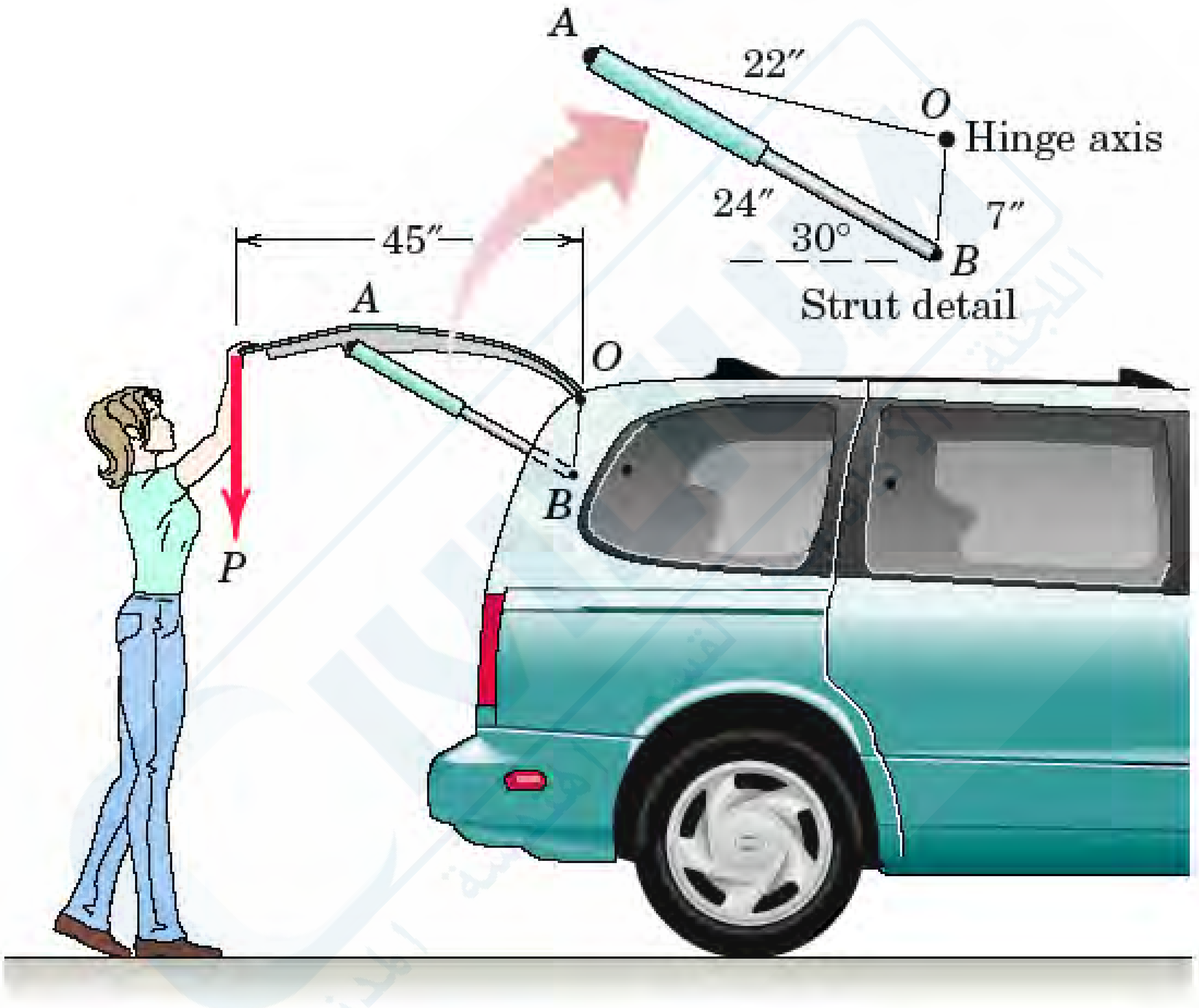
Problem 3/53

P_03_53



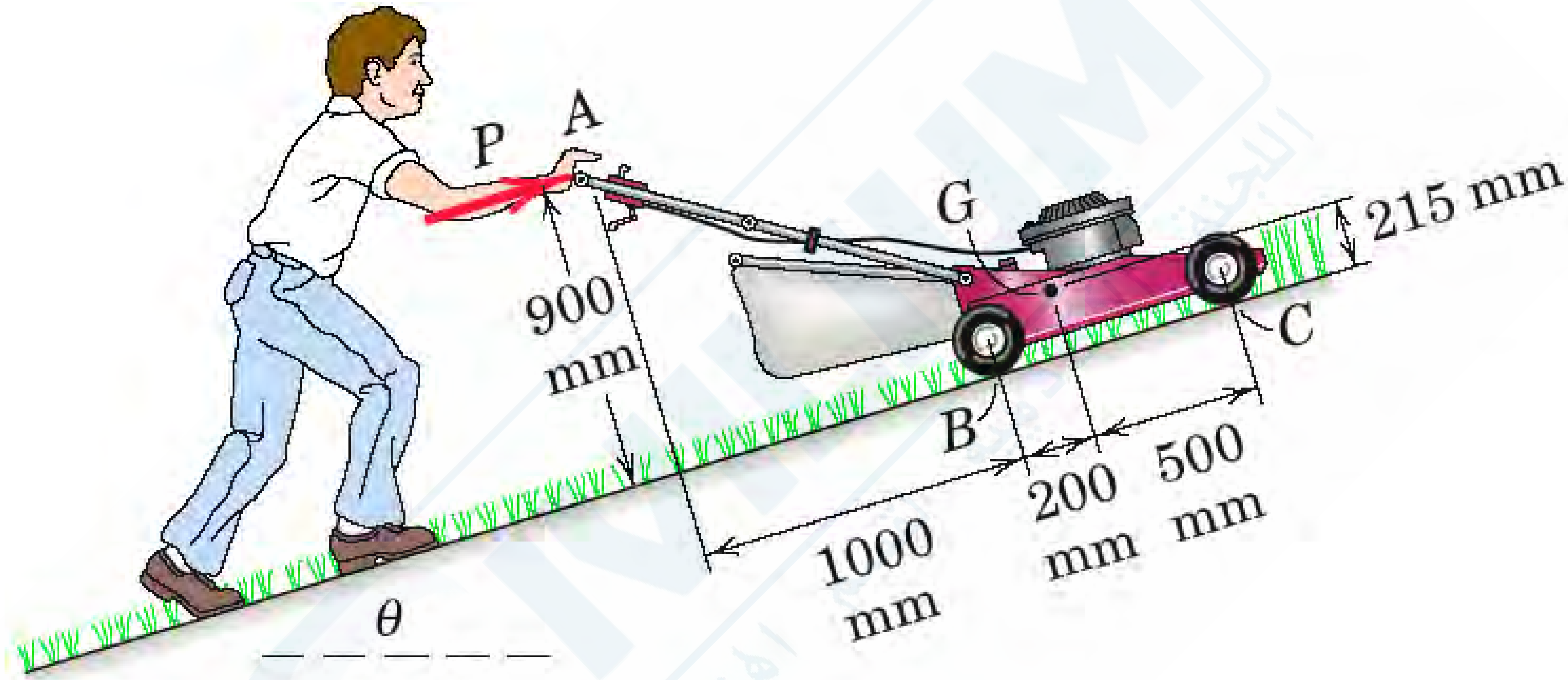
Problem 3/54

P_03_54



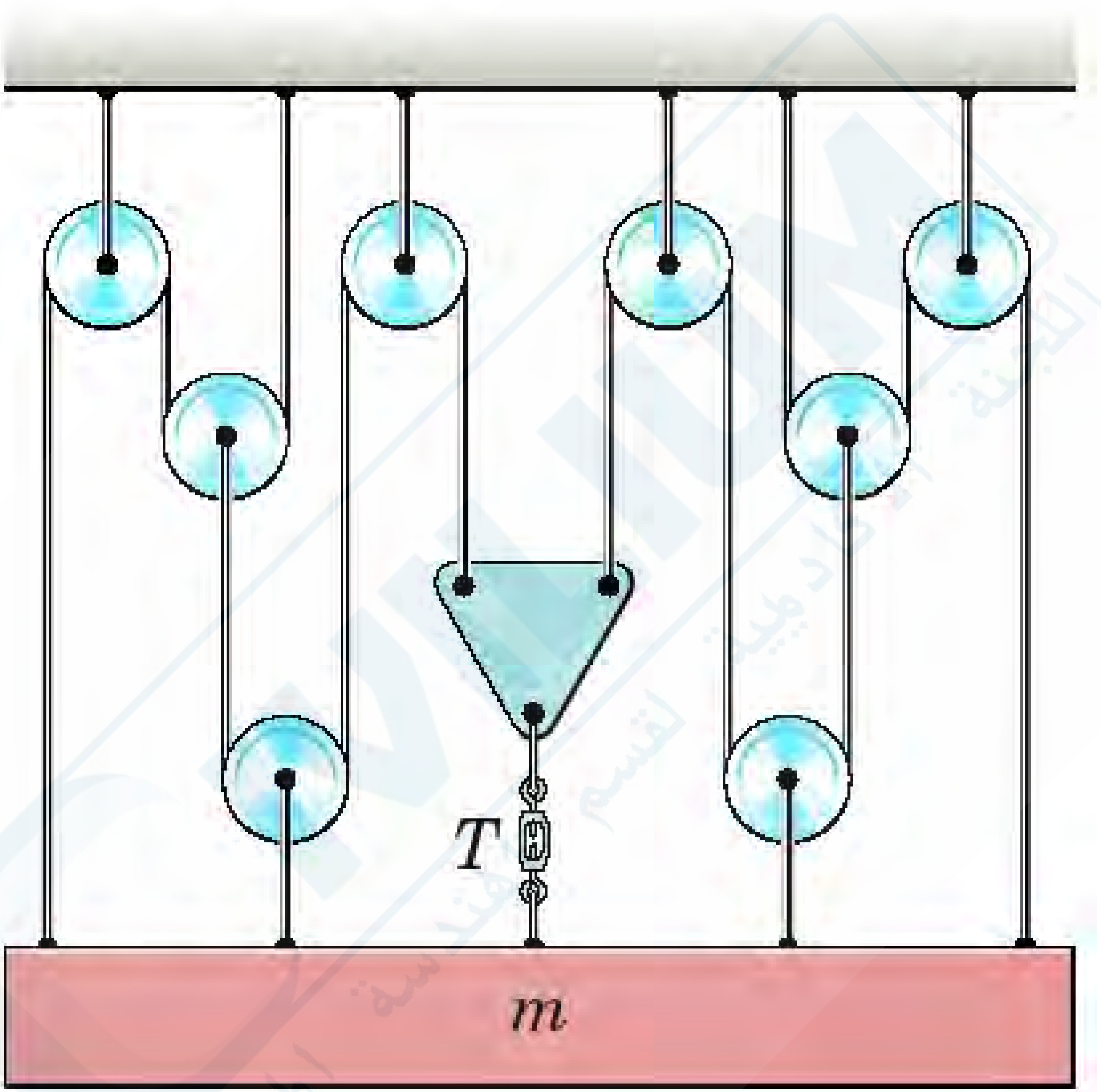
Problem 3/55

P_03_55



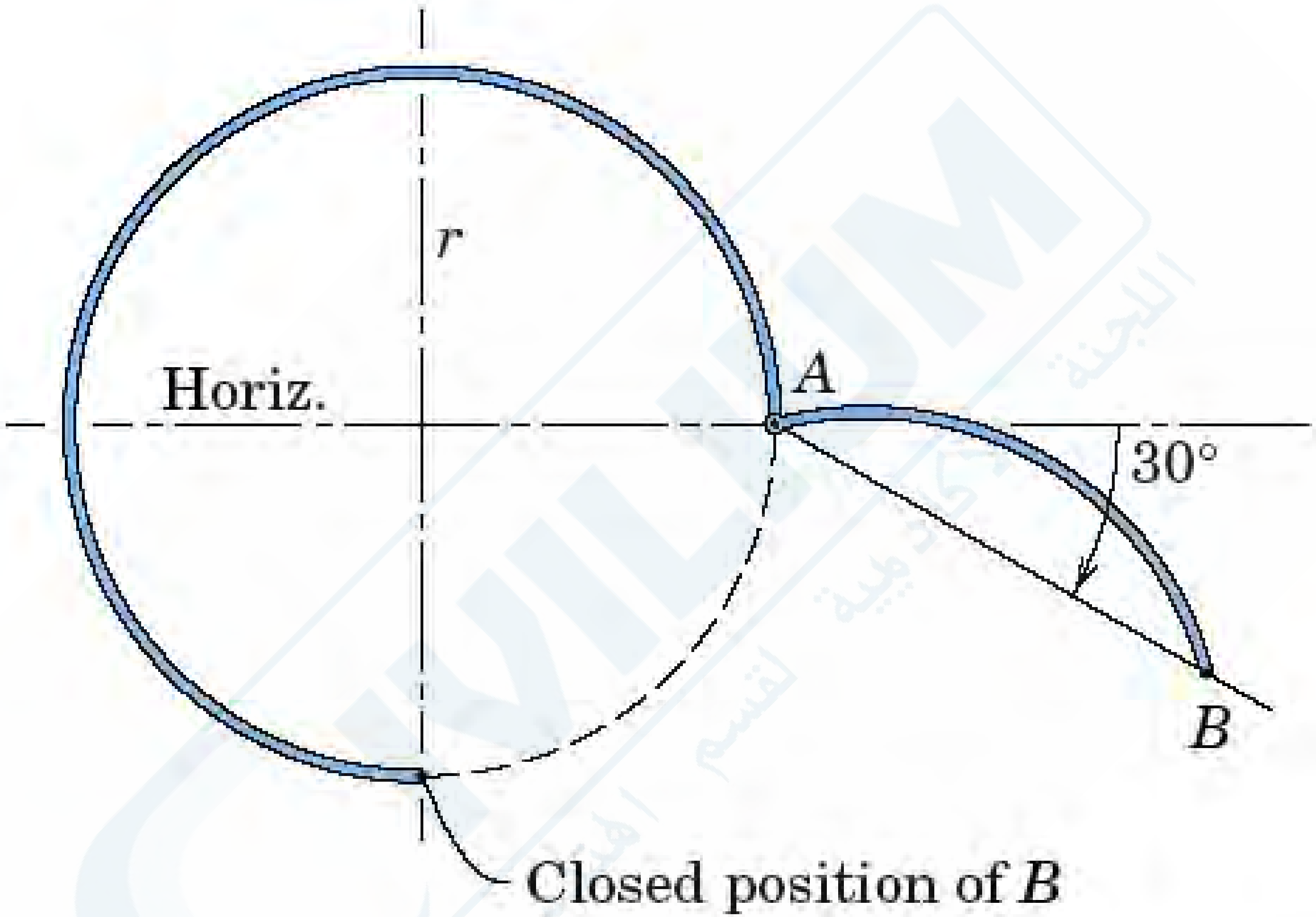
Problem 3/56

P_03_56



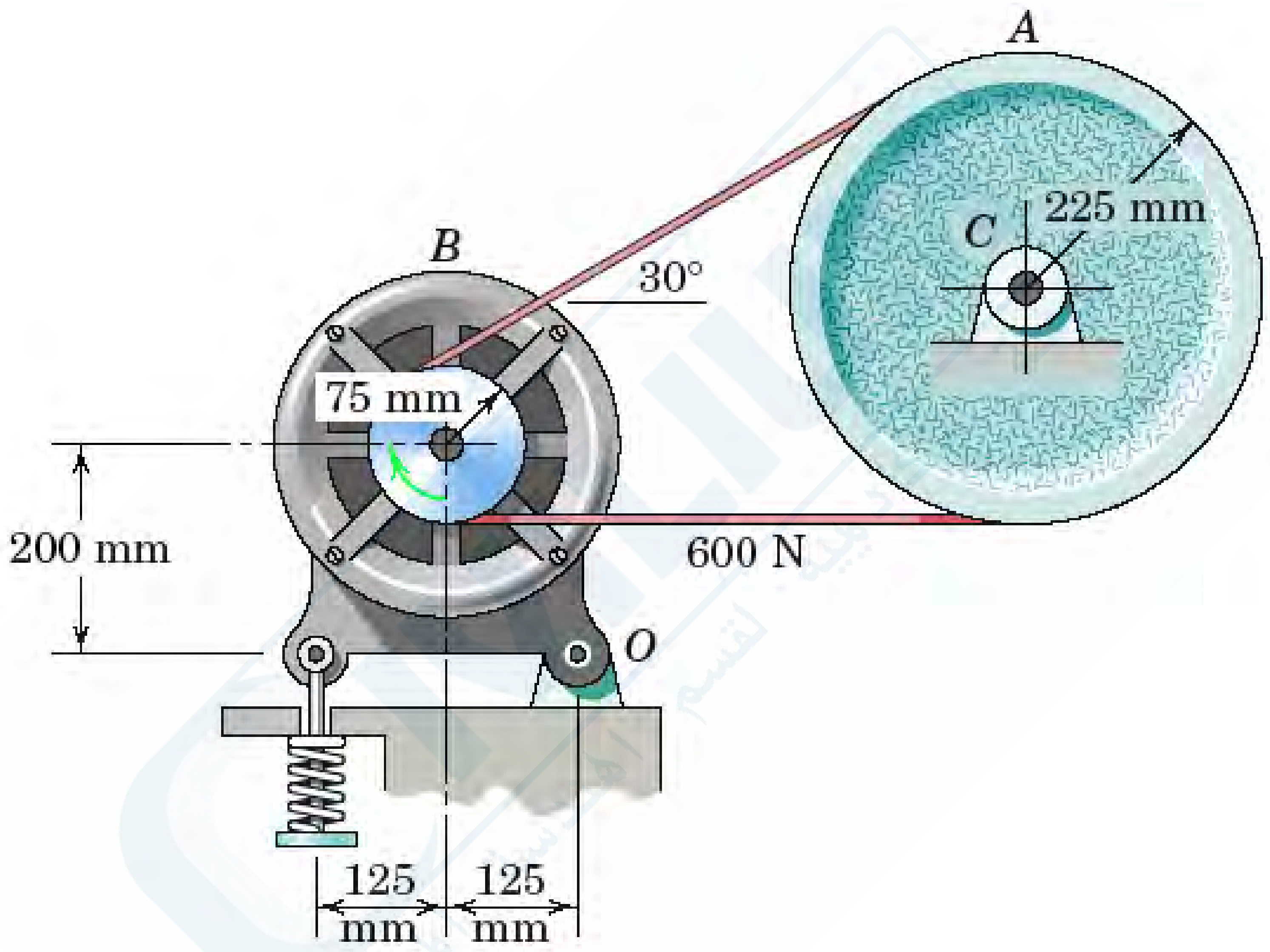
Problem 3/57

P_03_57



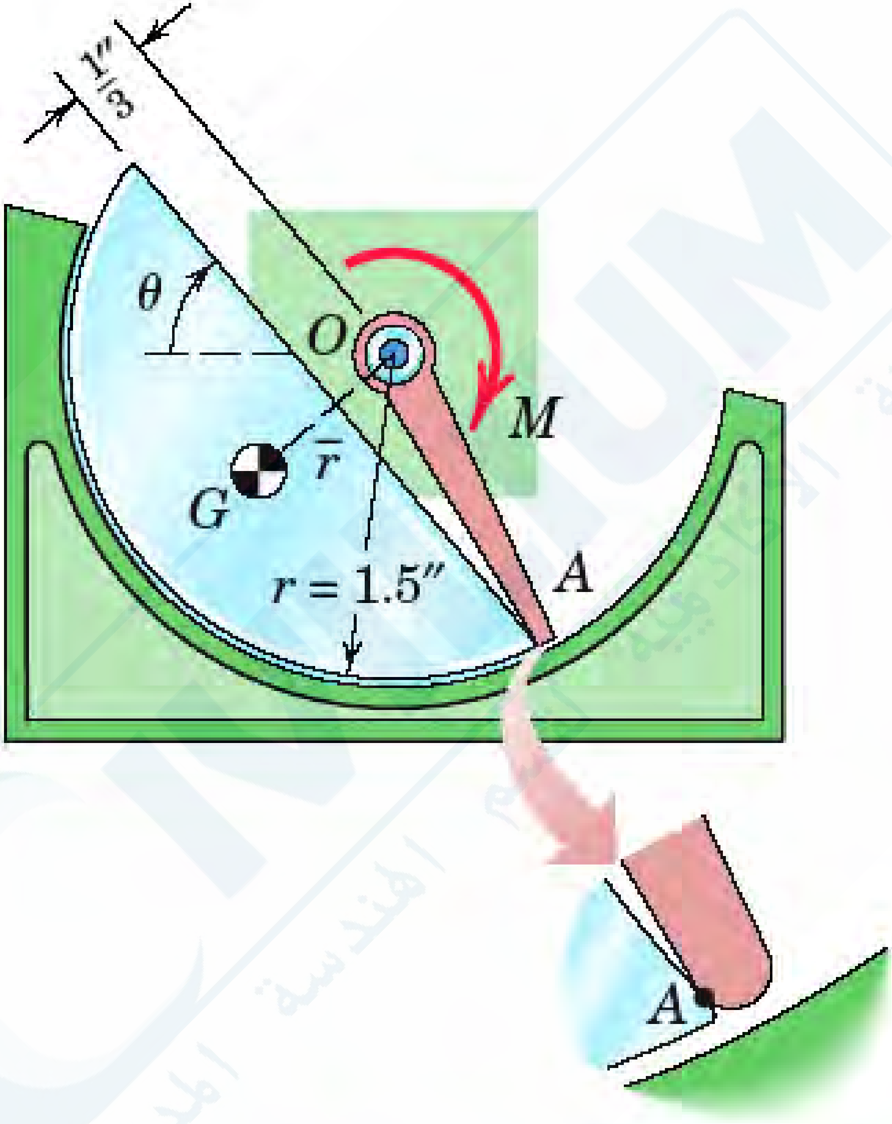
Problem 3/58

P_03_58



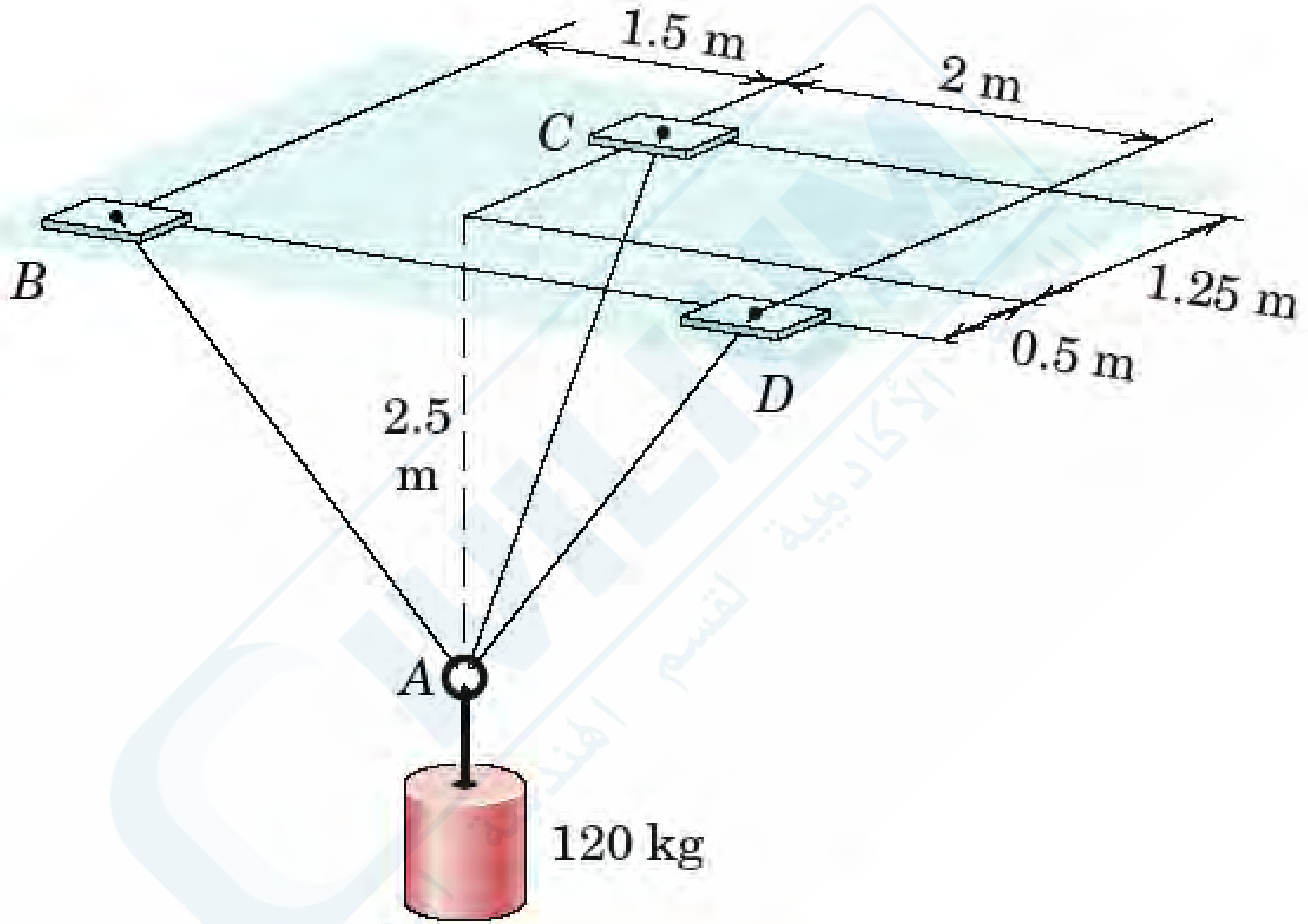
Problem 3/59

P 03 59

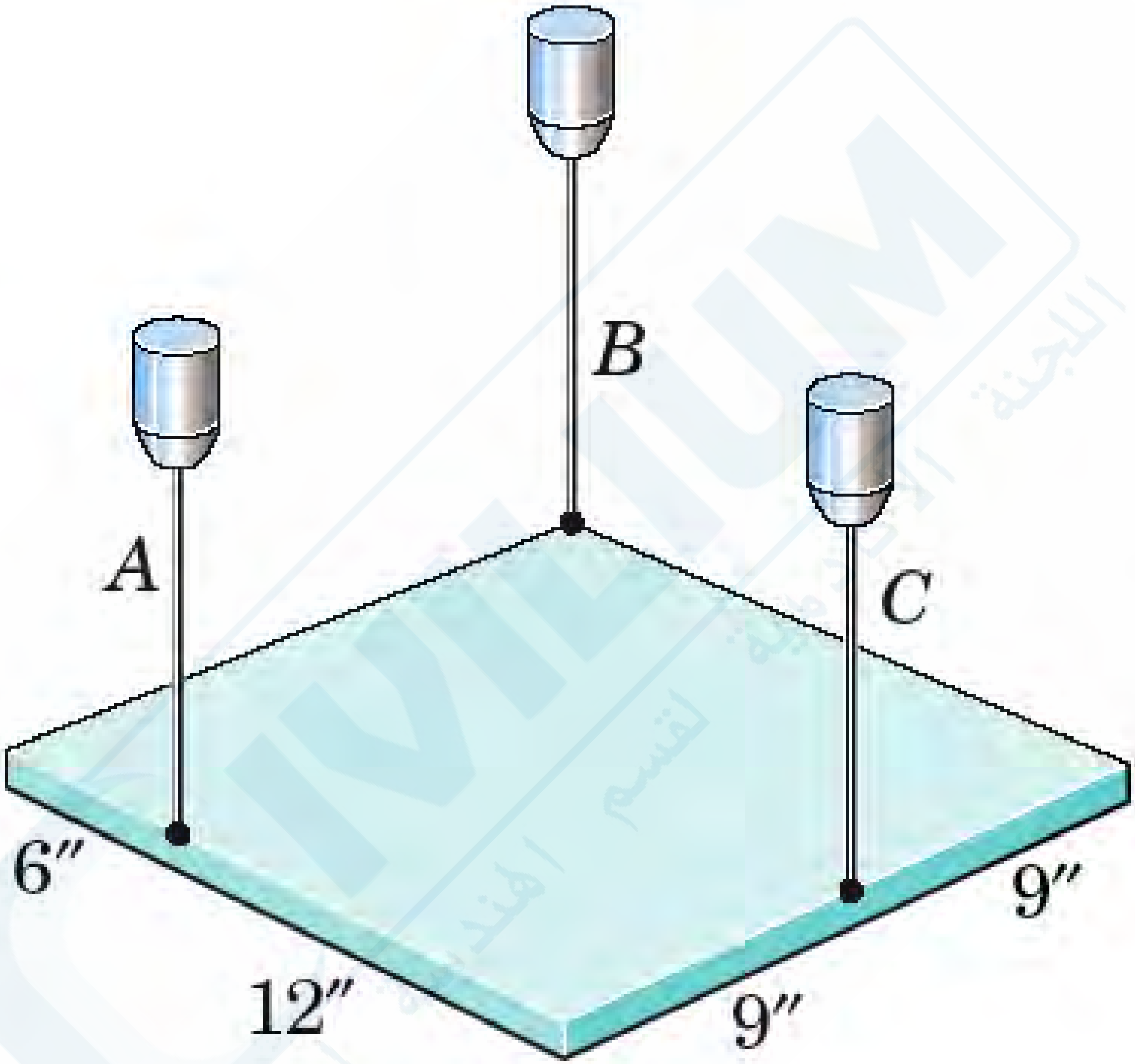


Problem 3/60

P_03_60

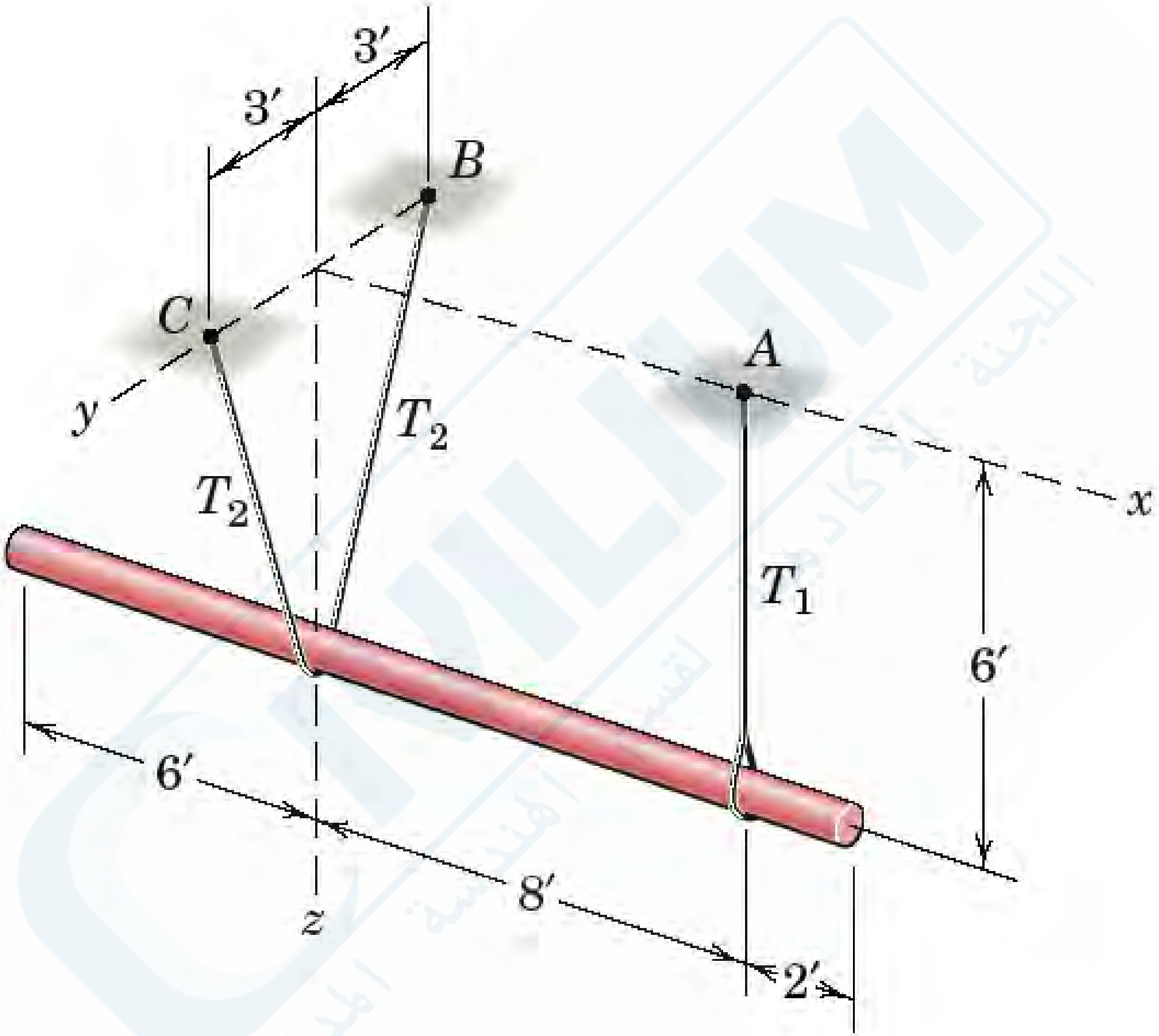


Problem 3/61



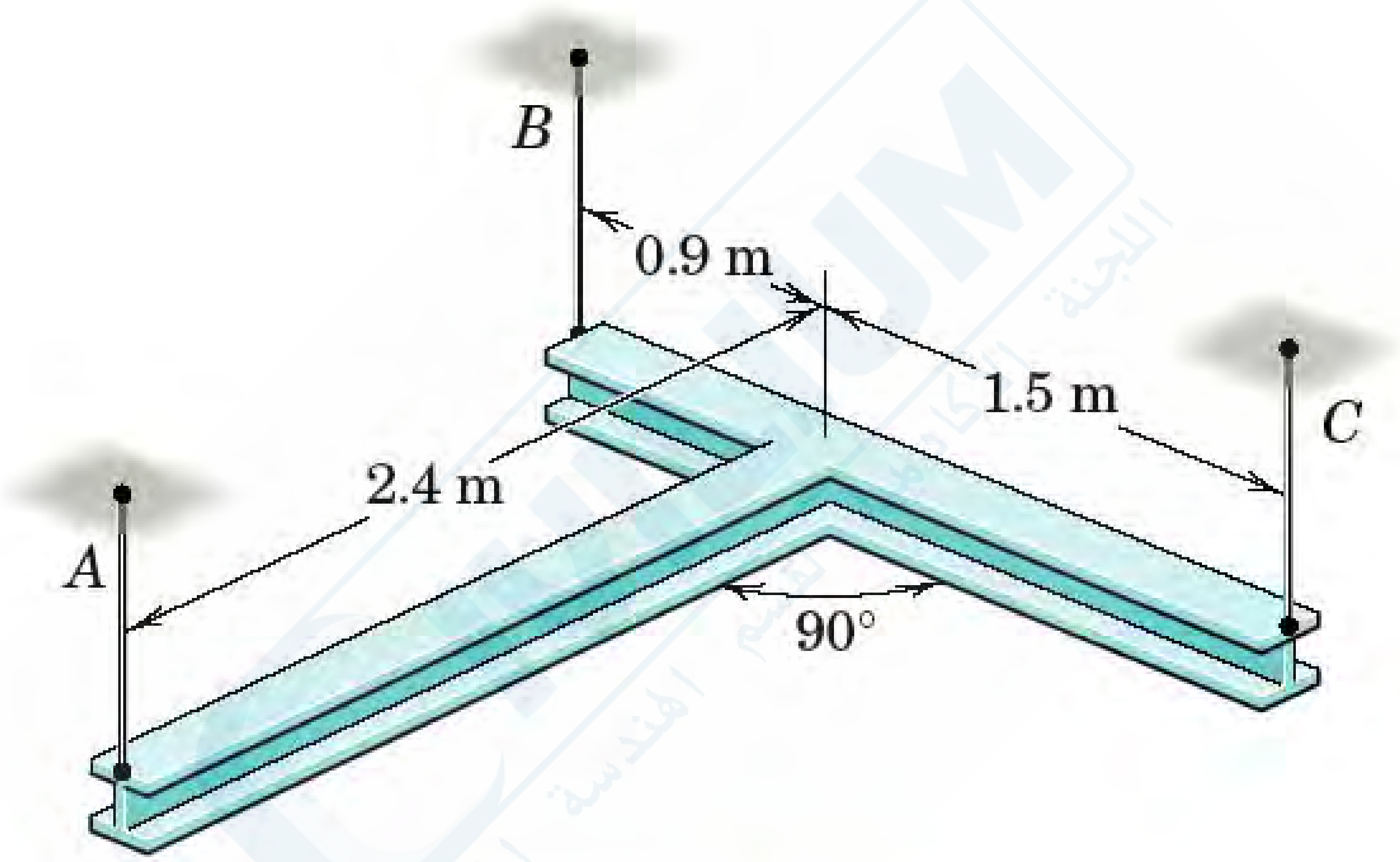
Problem 3/62

P_03_62



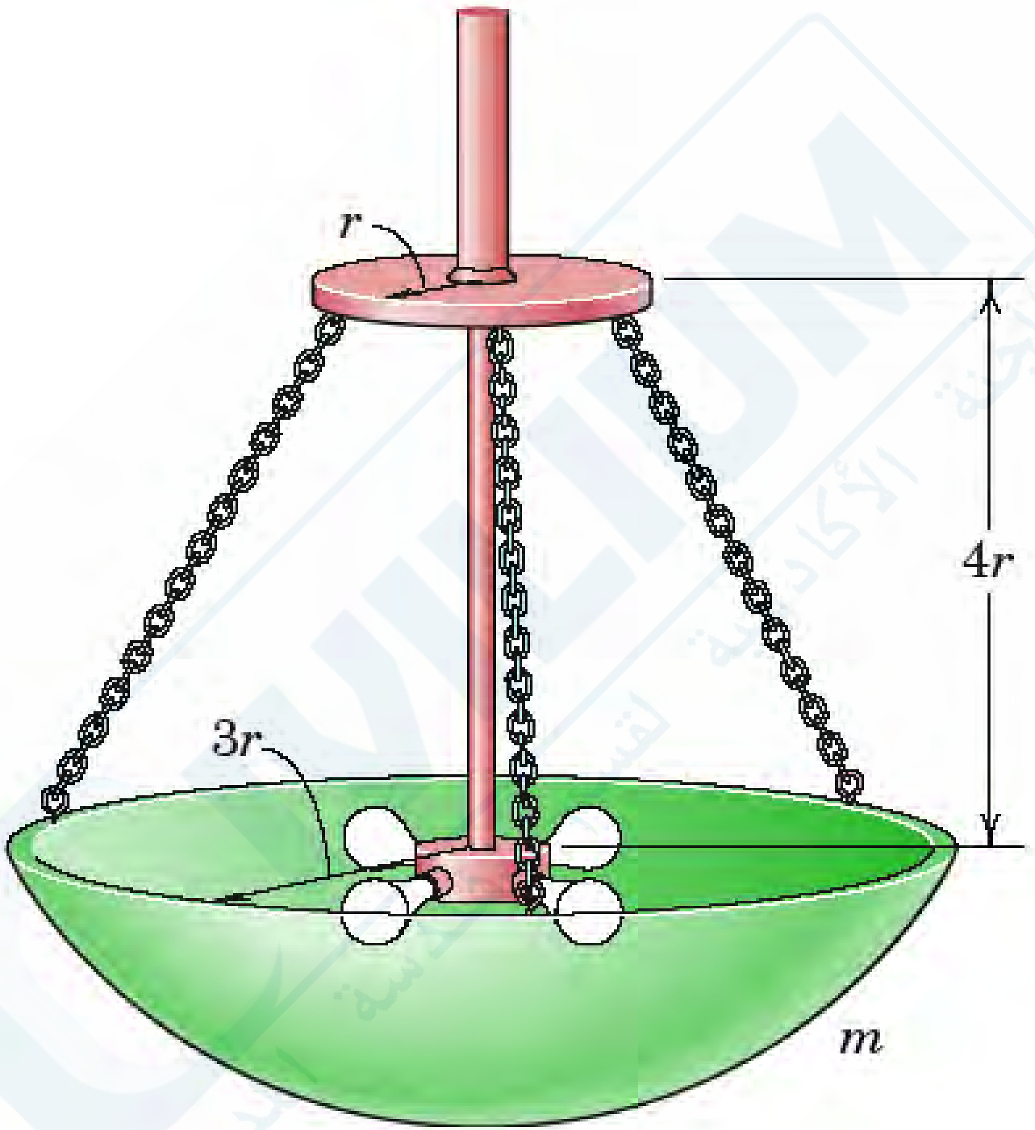
Problem 3/63

P 03 63



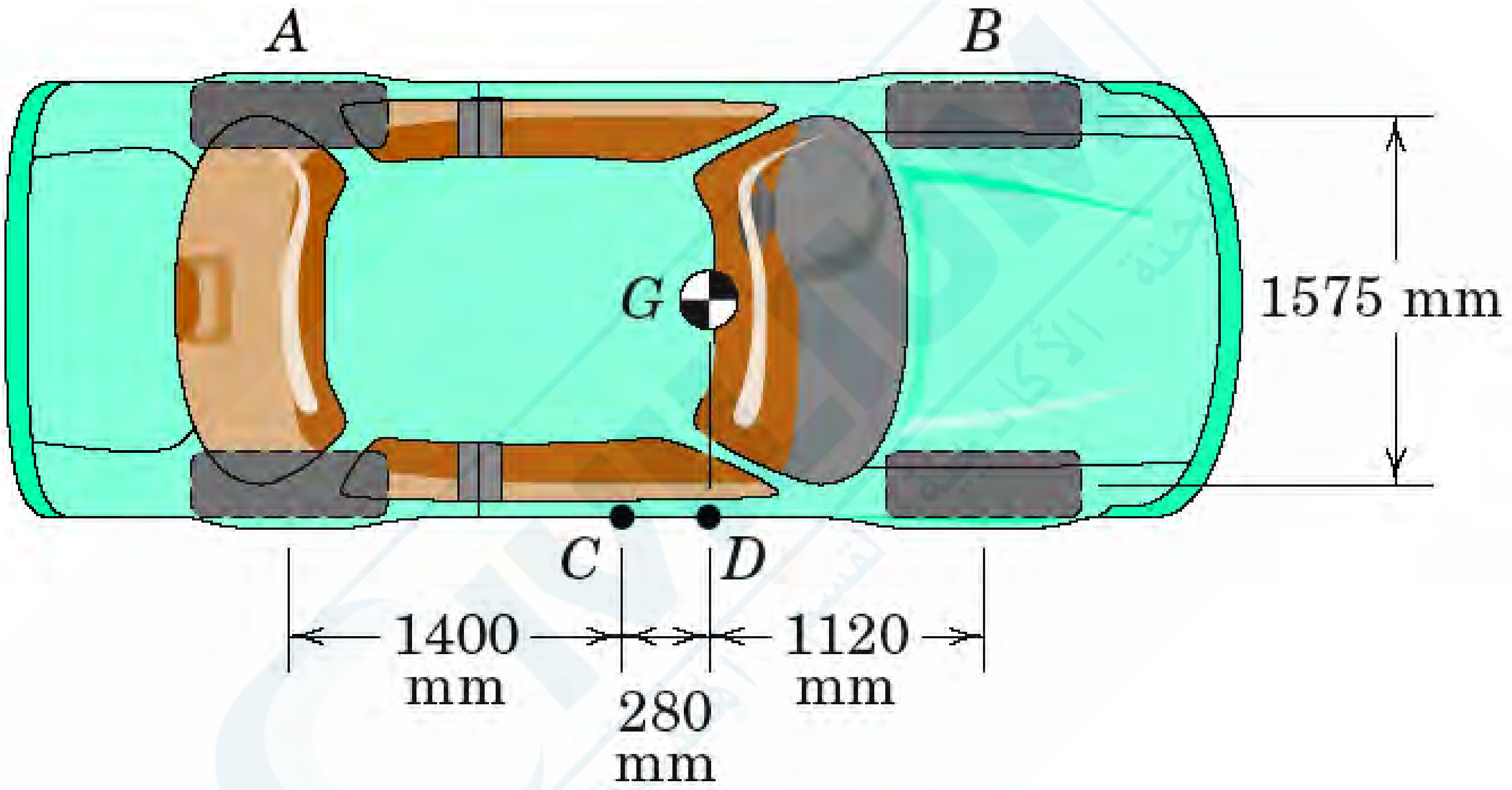
Problem 3/64

P_03_64



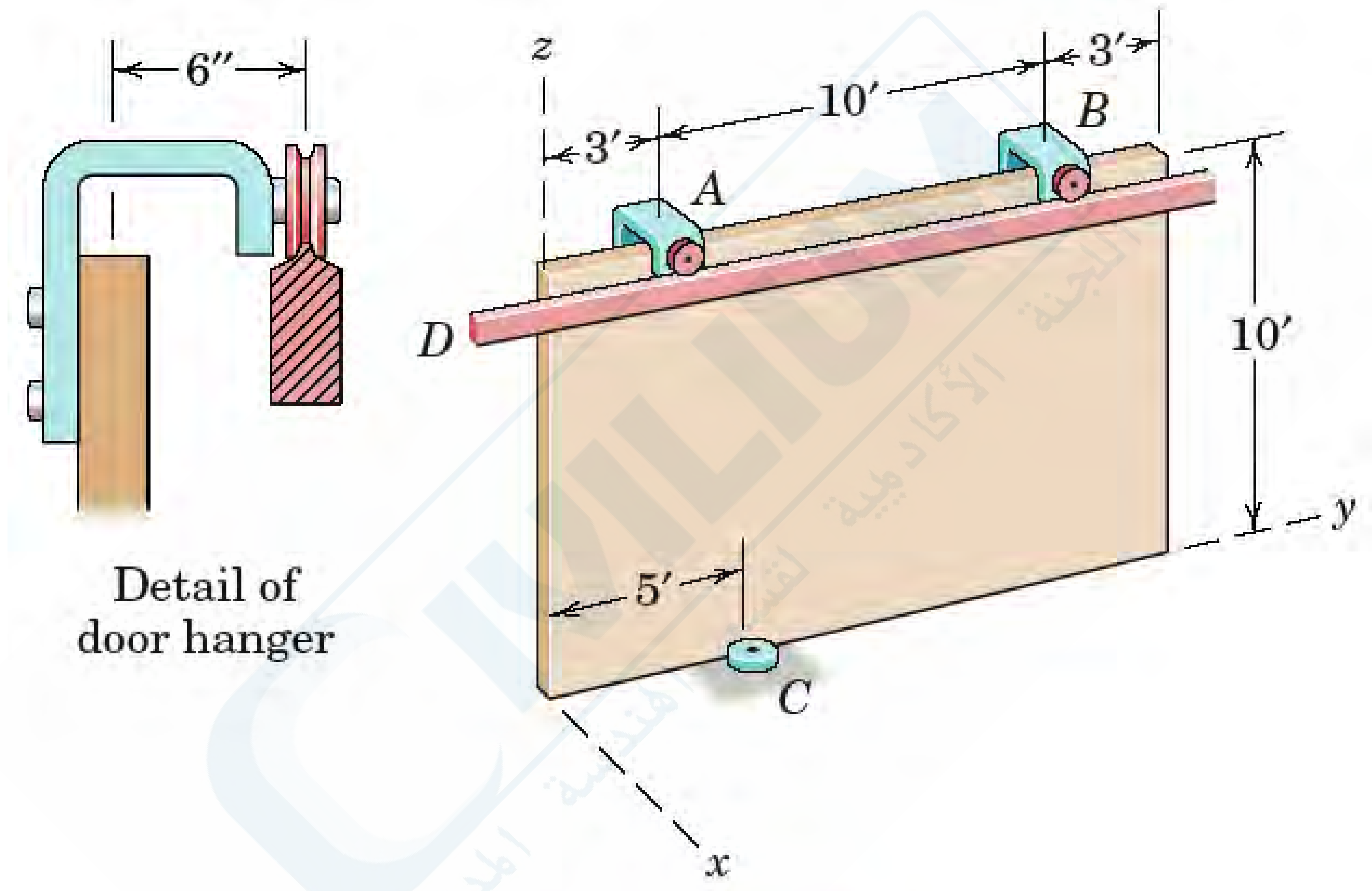
Problem 3/65

P 03 65



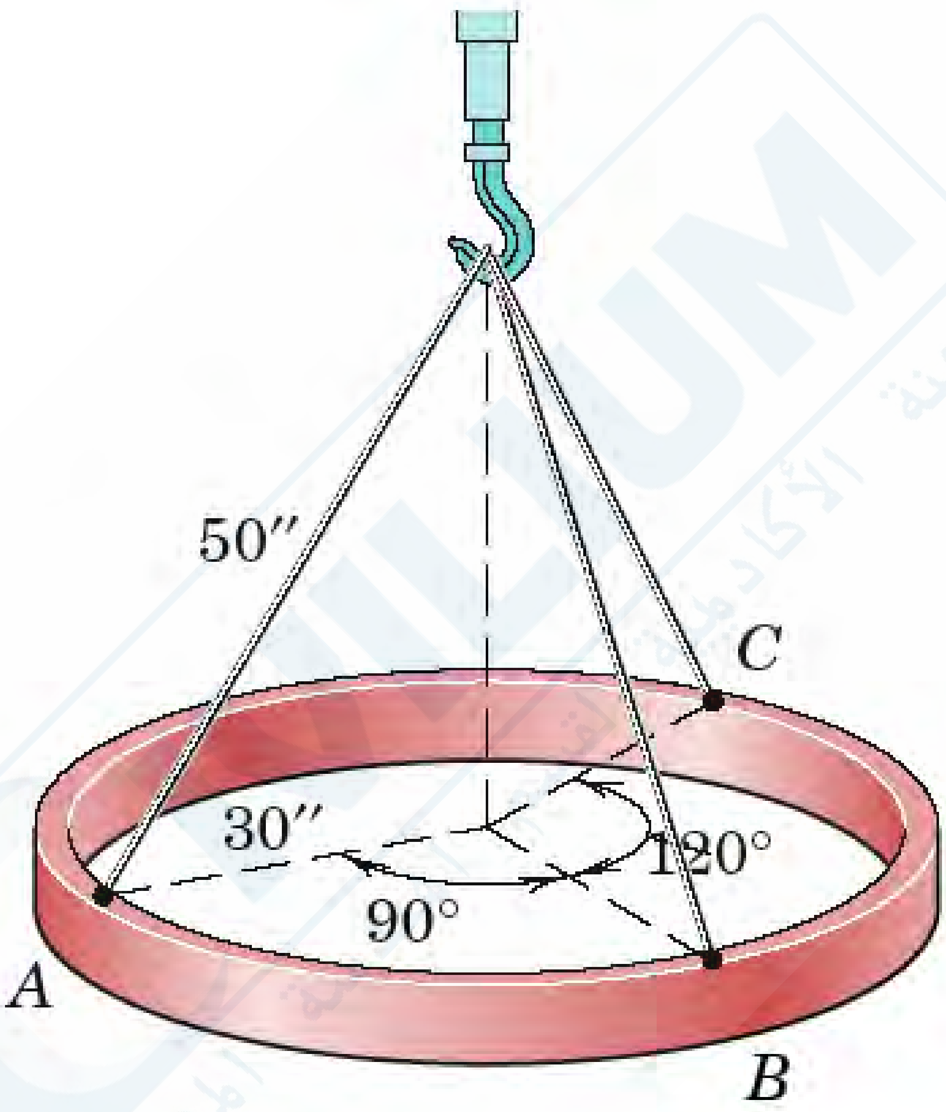
Problem 3/66

P_03_66



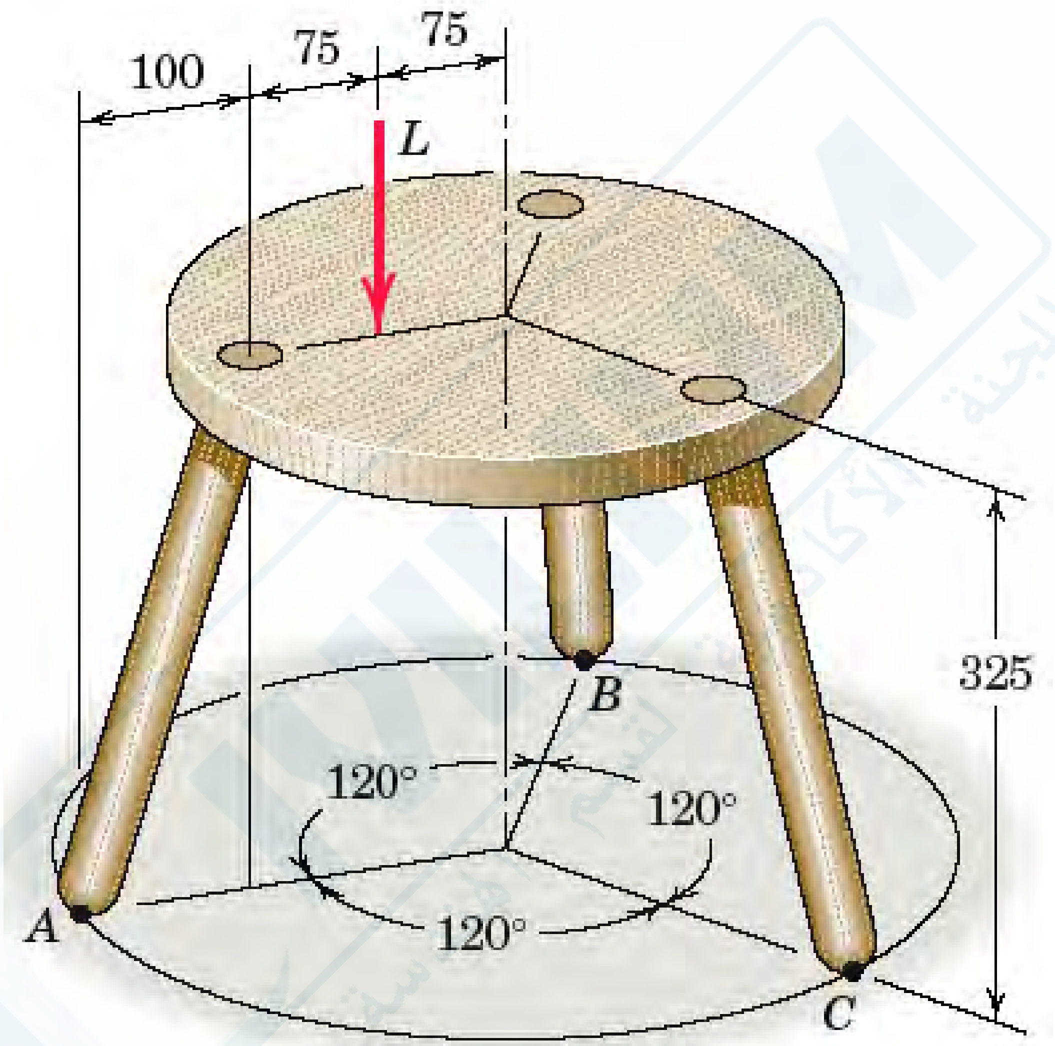
Problem 3/67

P_03_67



Problem 3/68

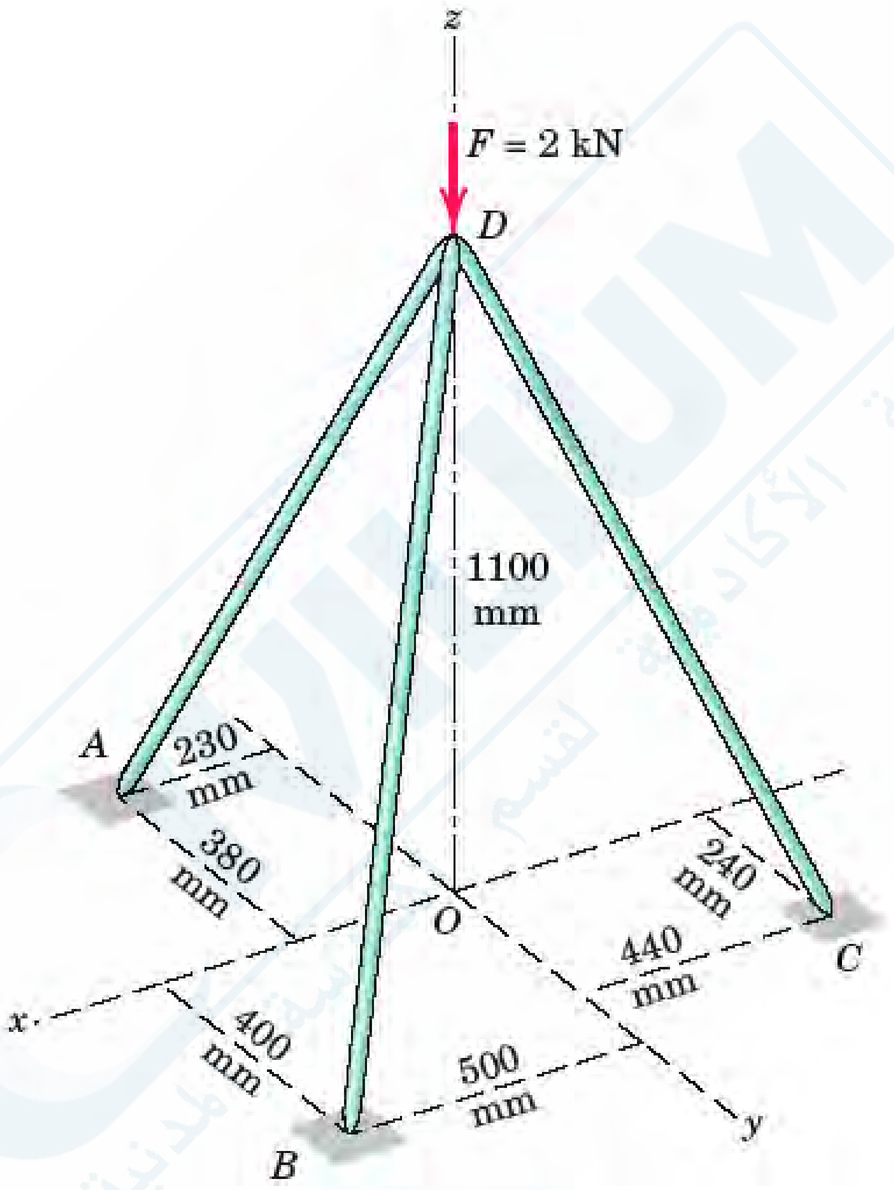
P 03 68



Dimensions in millimeters

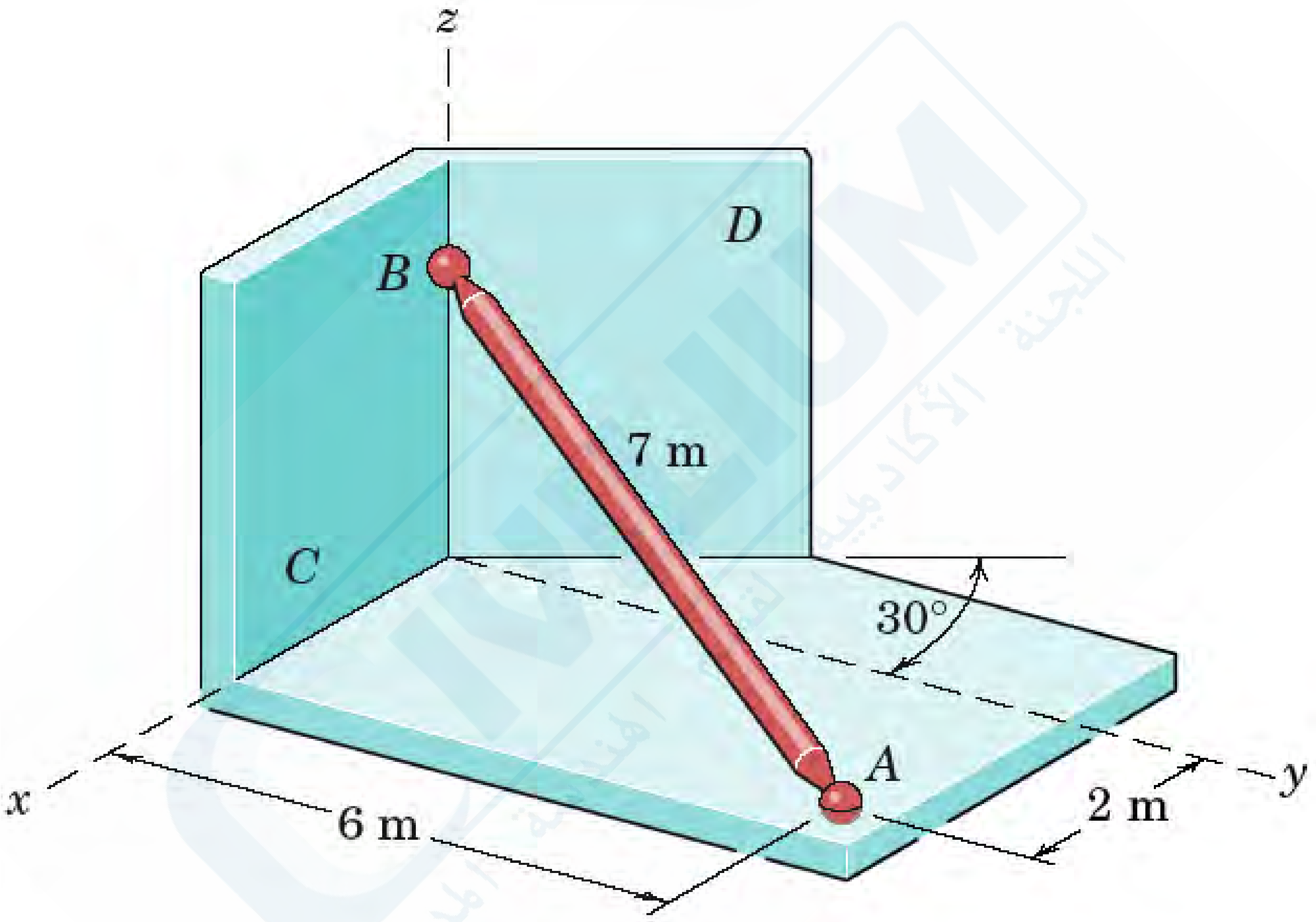
Problem 3/69

P 03 69



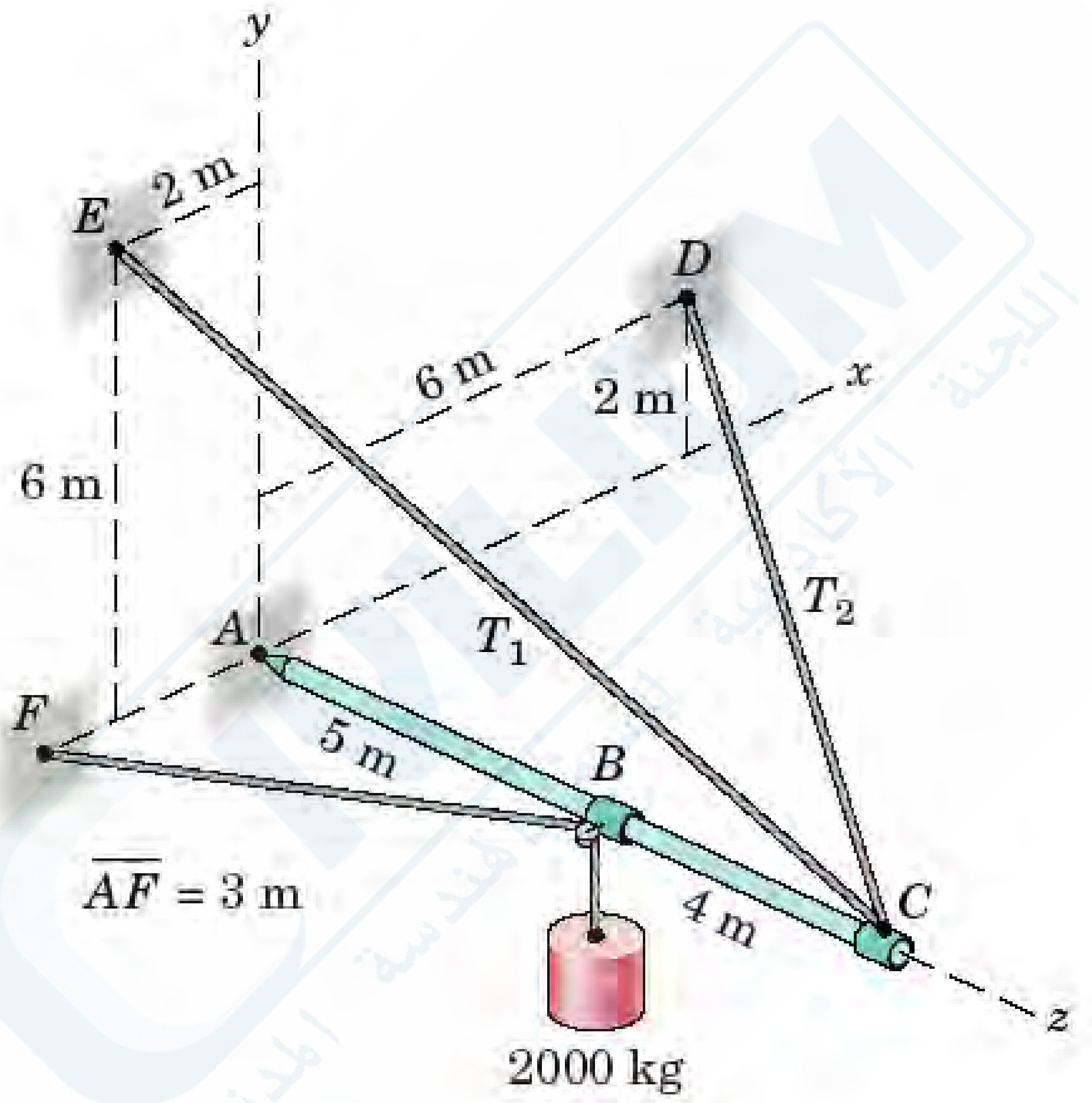
Problem 3/70

P 03 70



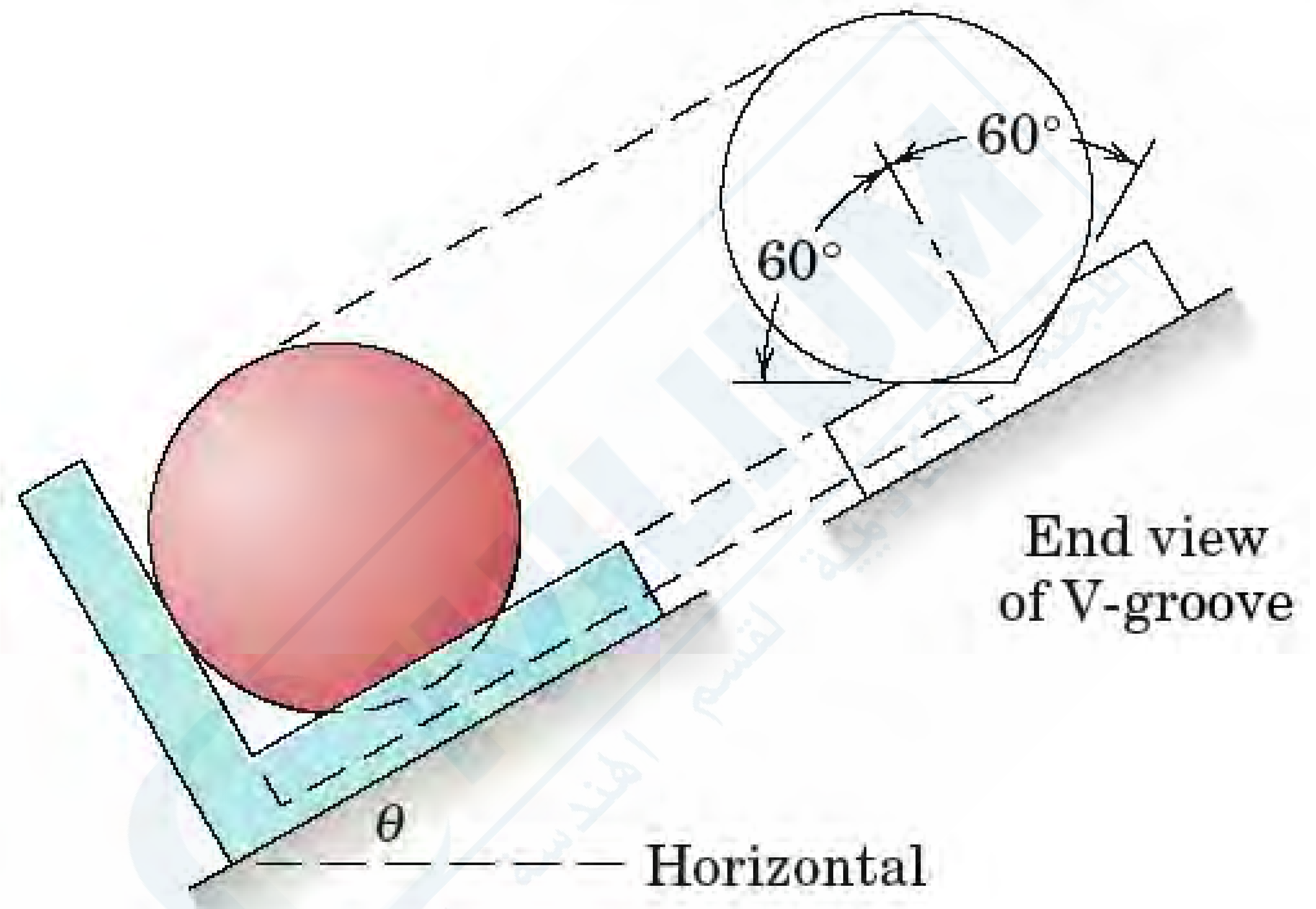
Problem 3/71

P 03 71



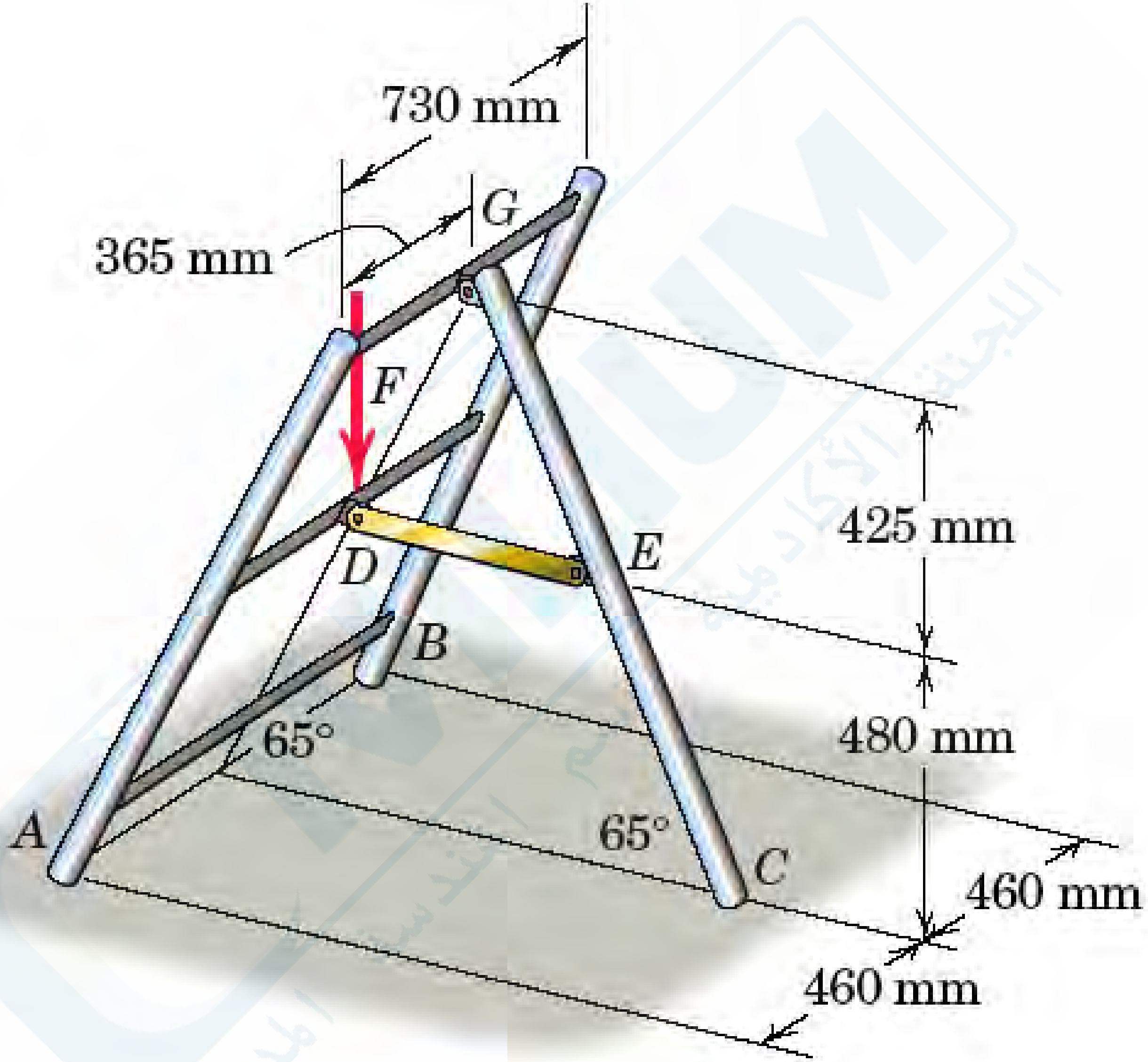
Problem 3/72

P_03_72



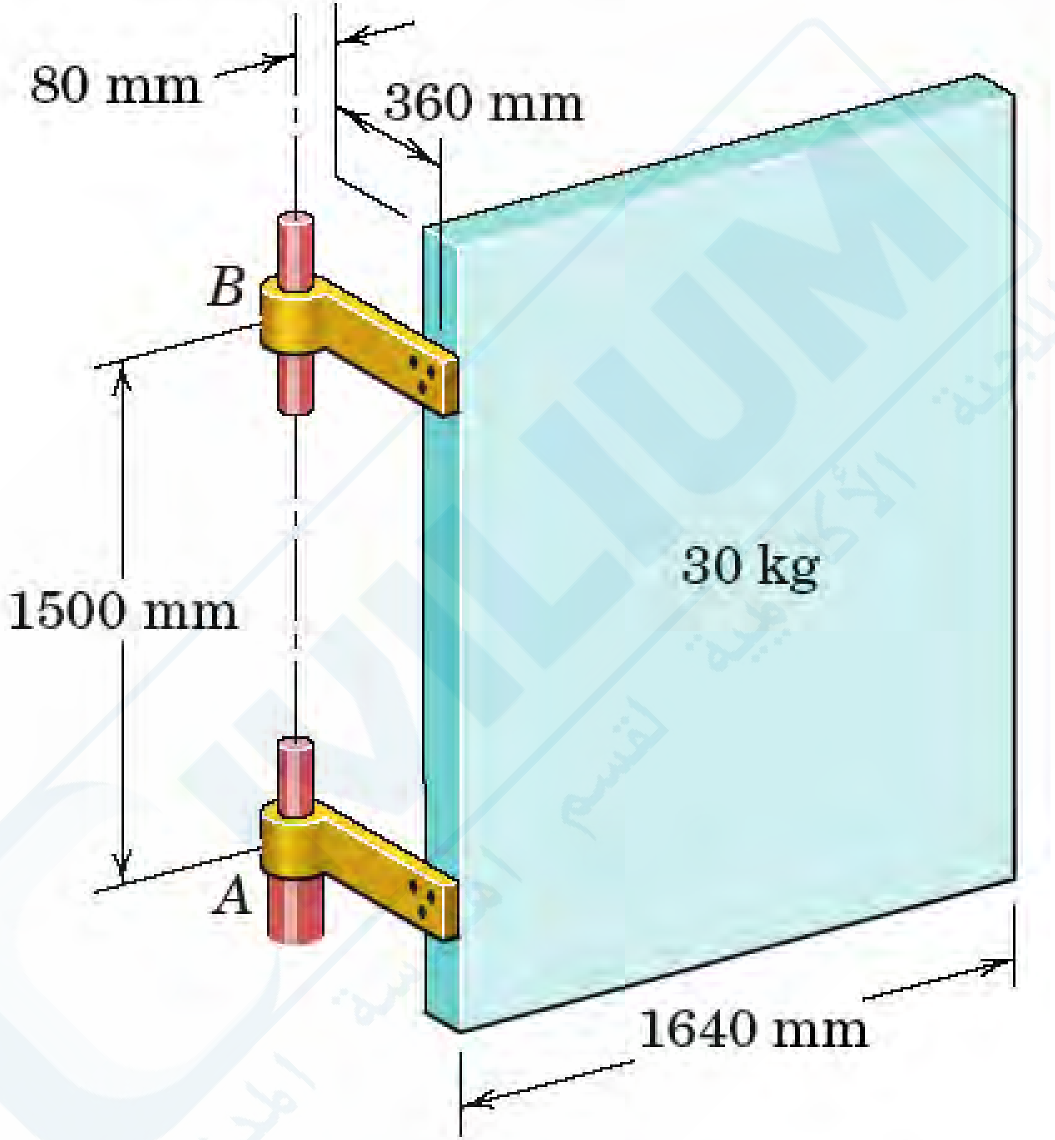
Problem 3/73

P 03 73



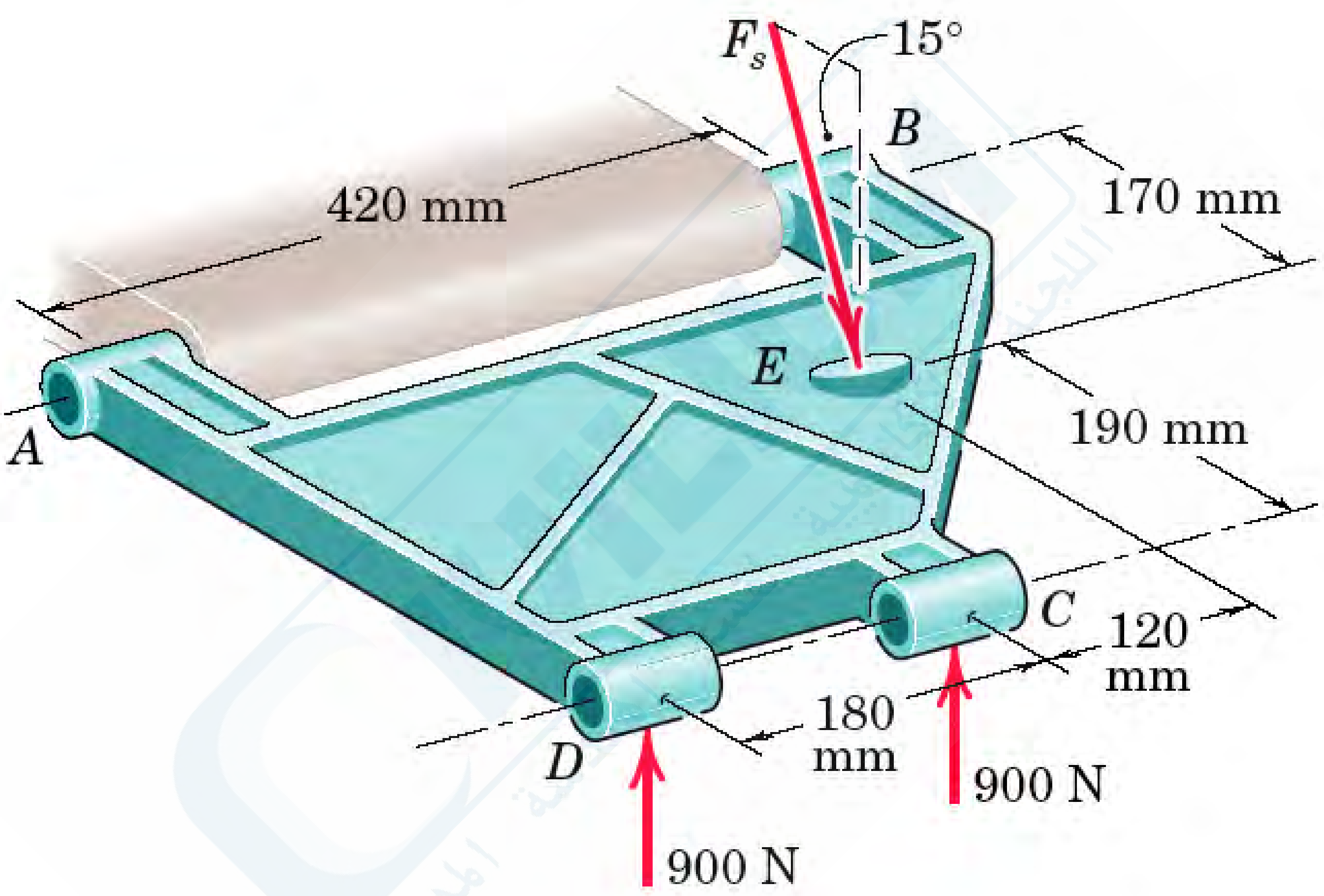
Problem 3/74

P_03_74



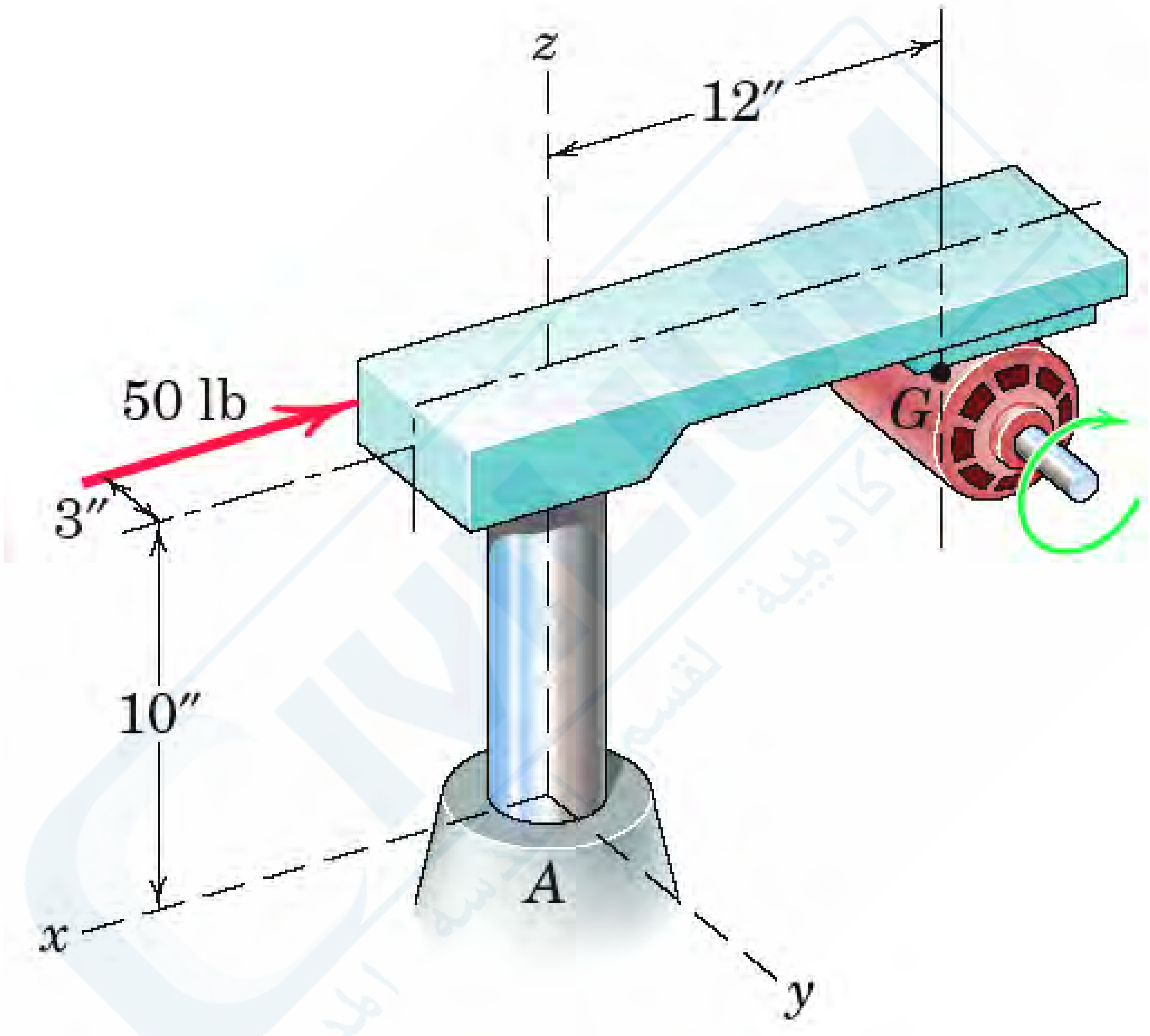
Problem 3/75

P_03_75



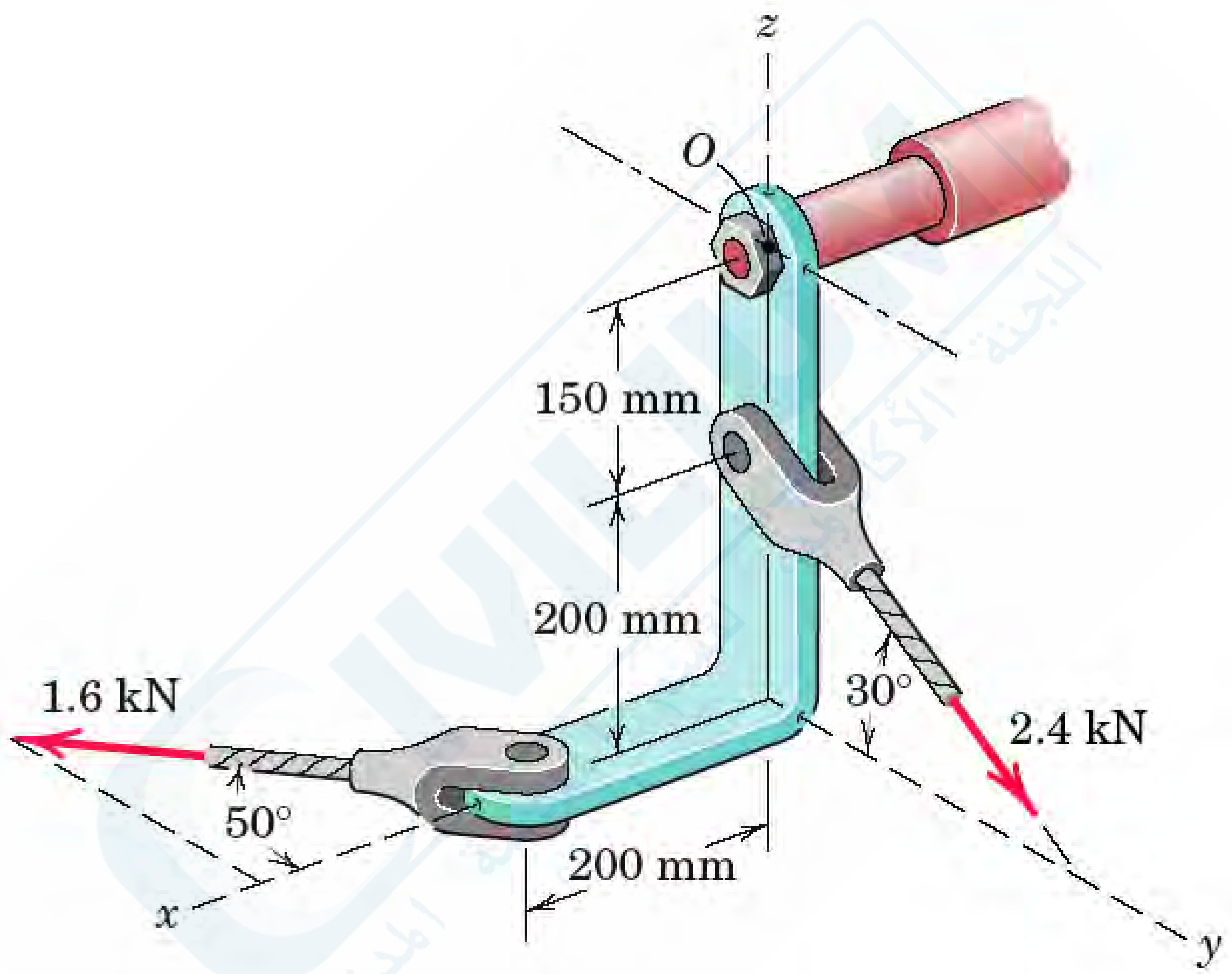
Problem 3/76

P 03 76



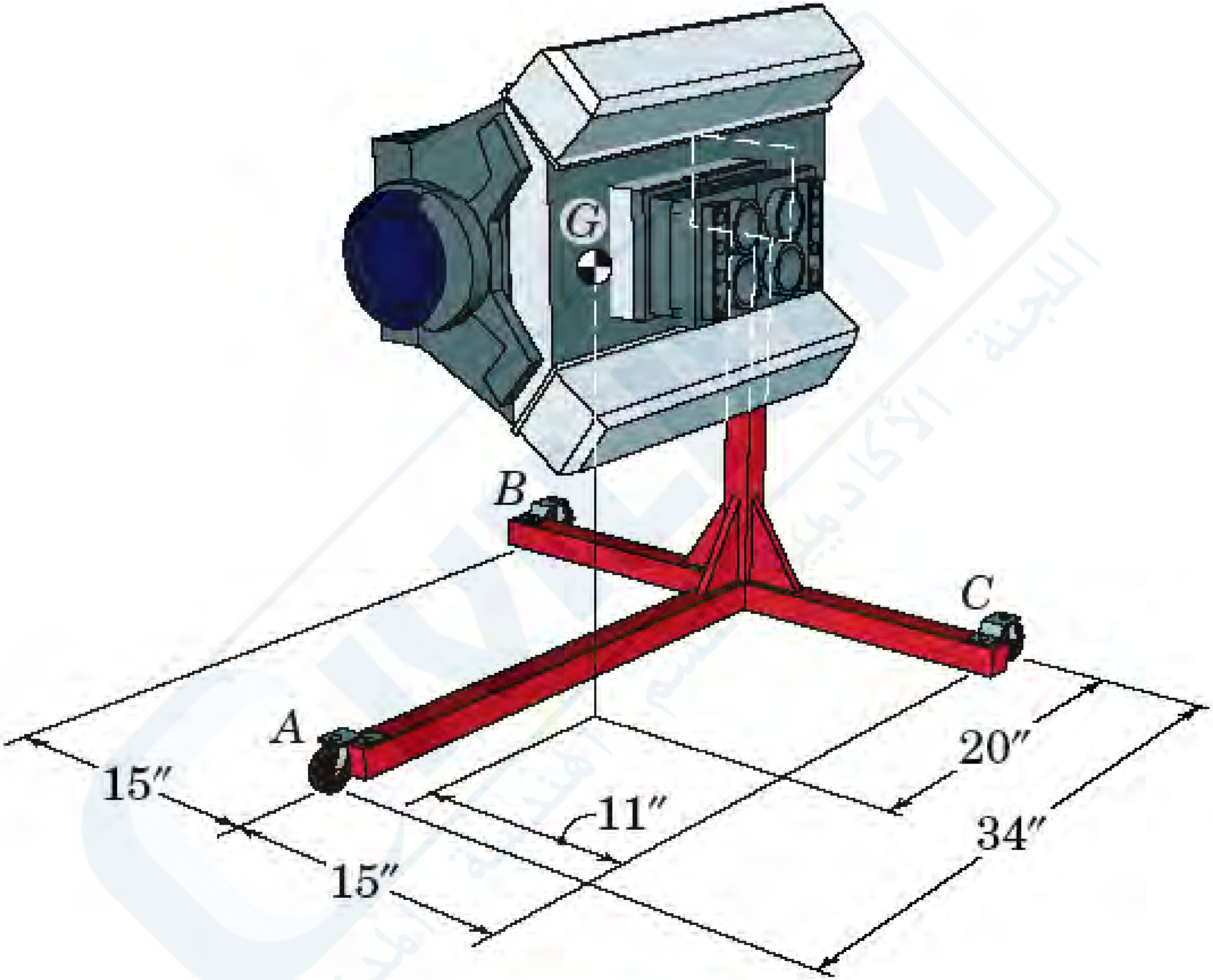
Problem 3/77

P_03_77



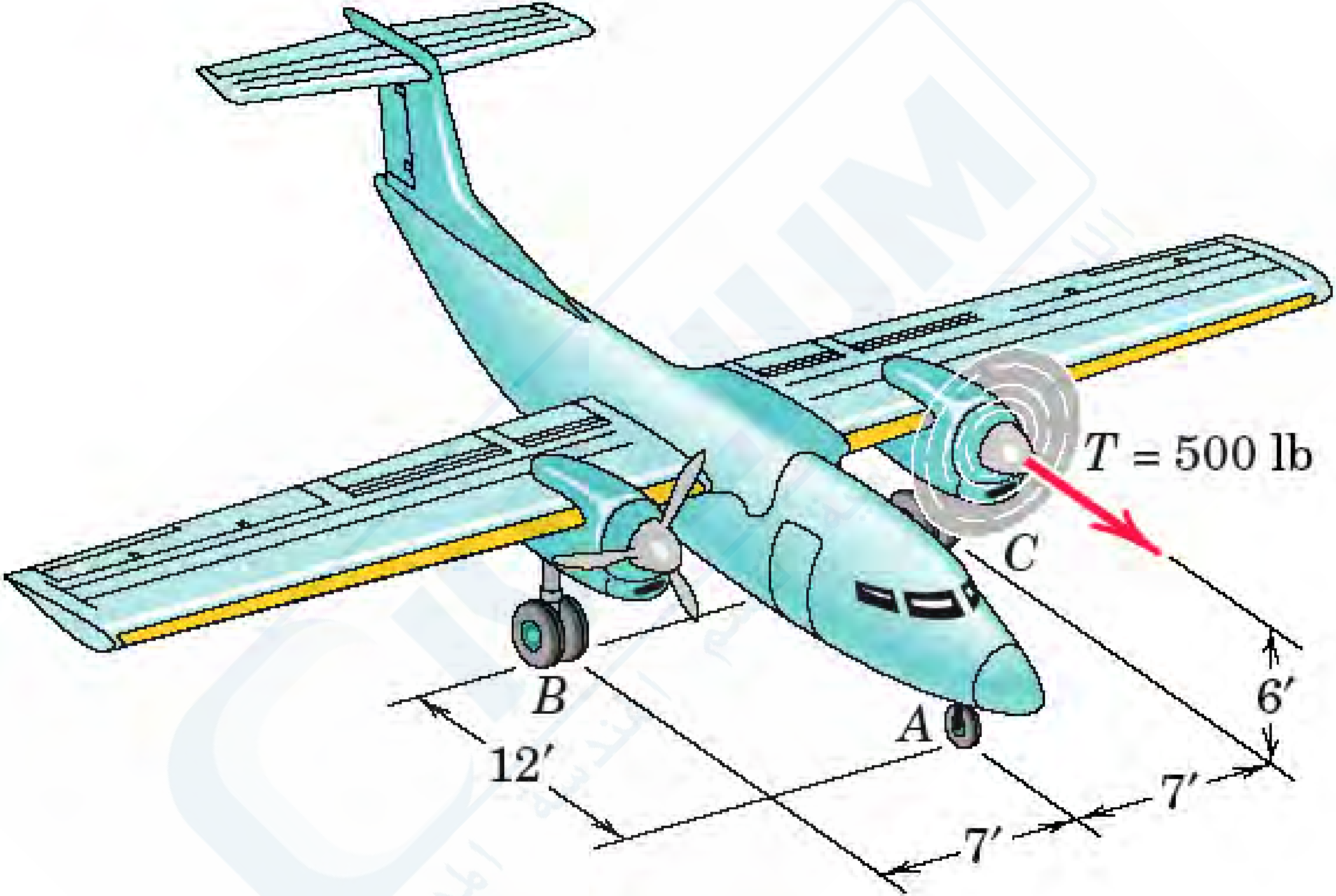
Problem 3/78

P 03 78



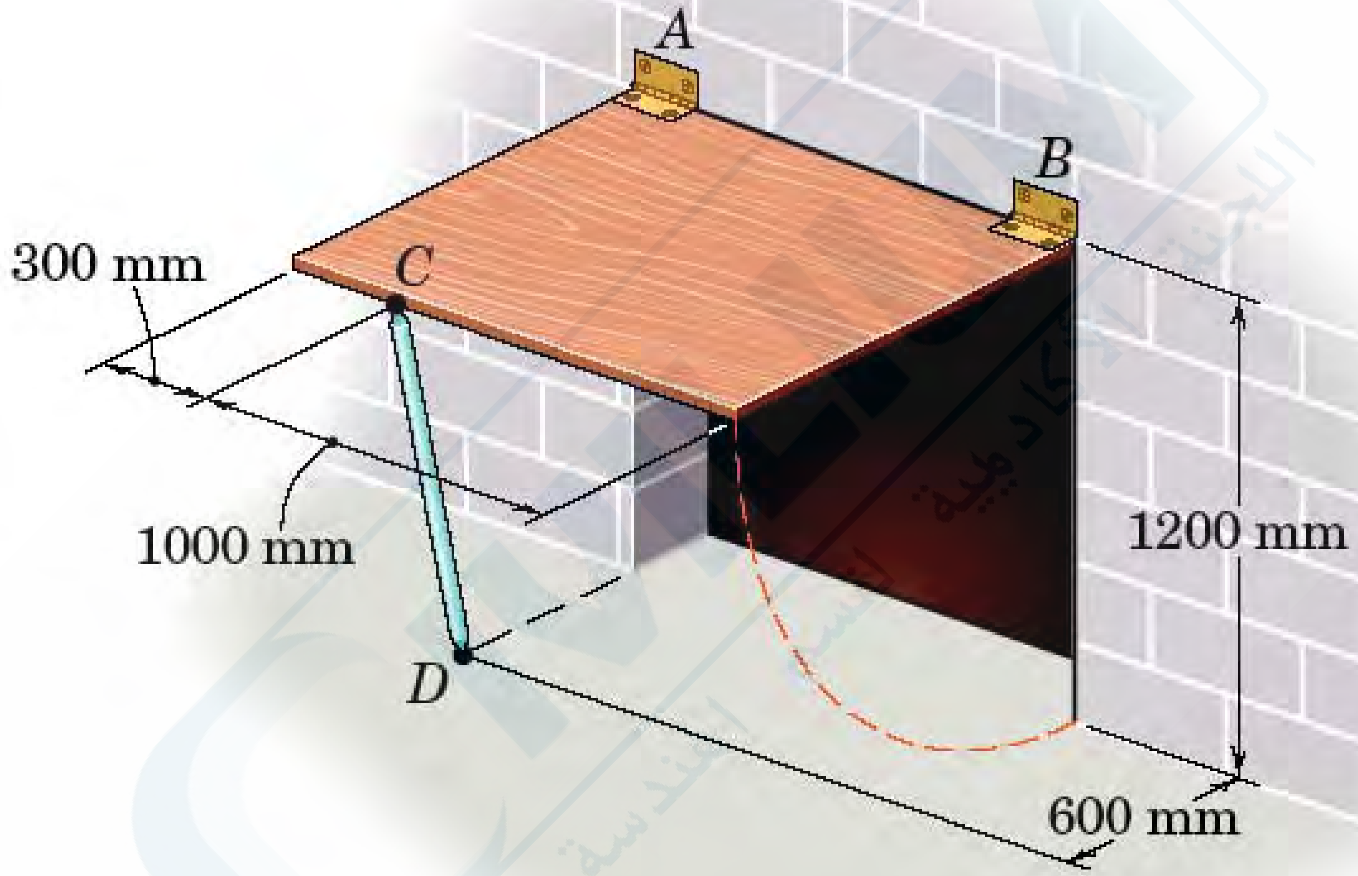
Problem 3/79

P 03 79



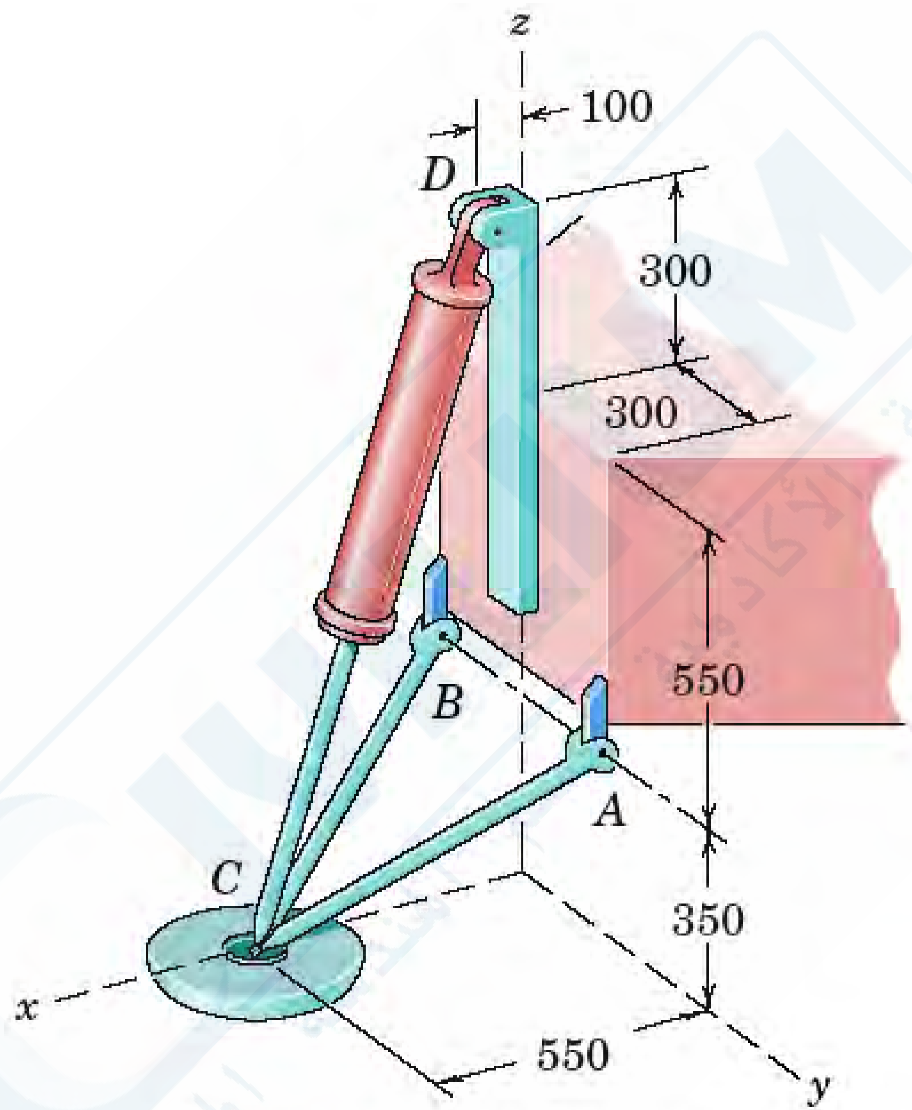
Problem 3/80

P_03_80



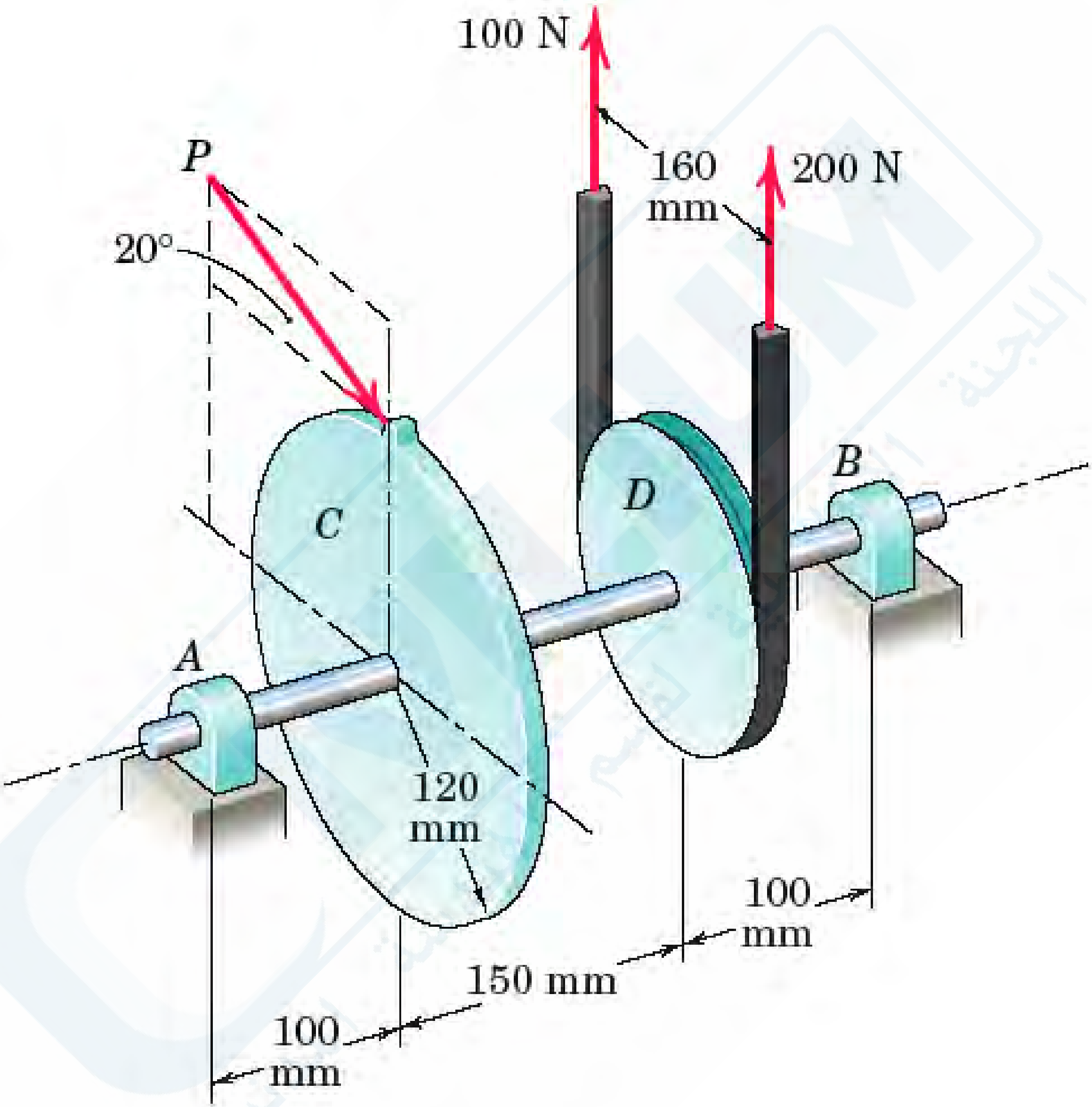
Problem 3/81

P 03 81



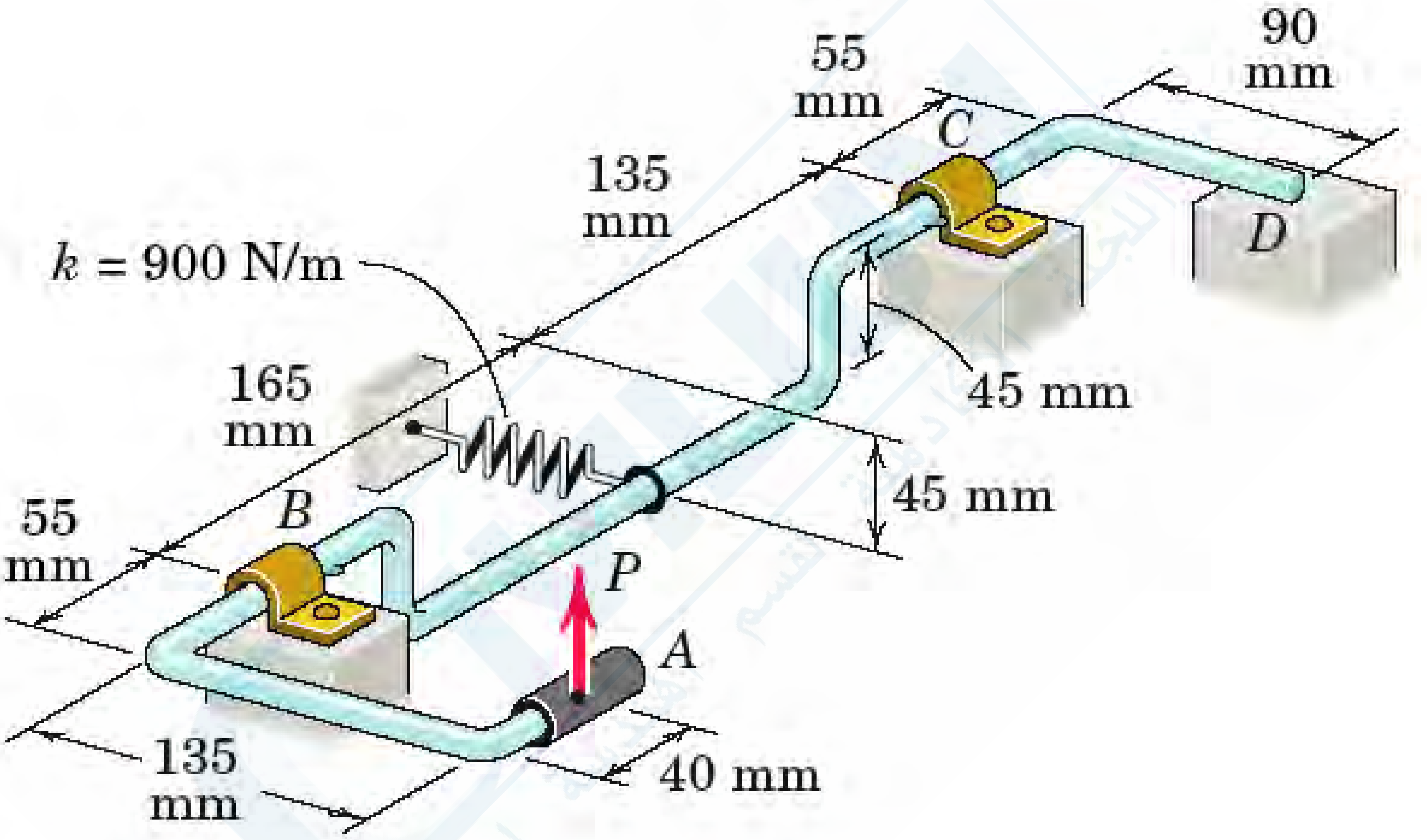
Dimensions in millimeters

Problem 3/82



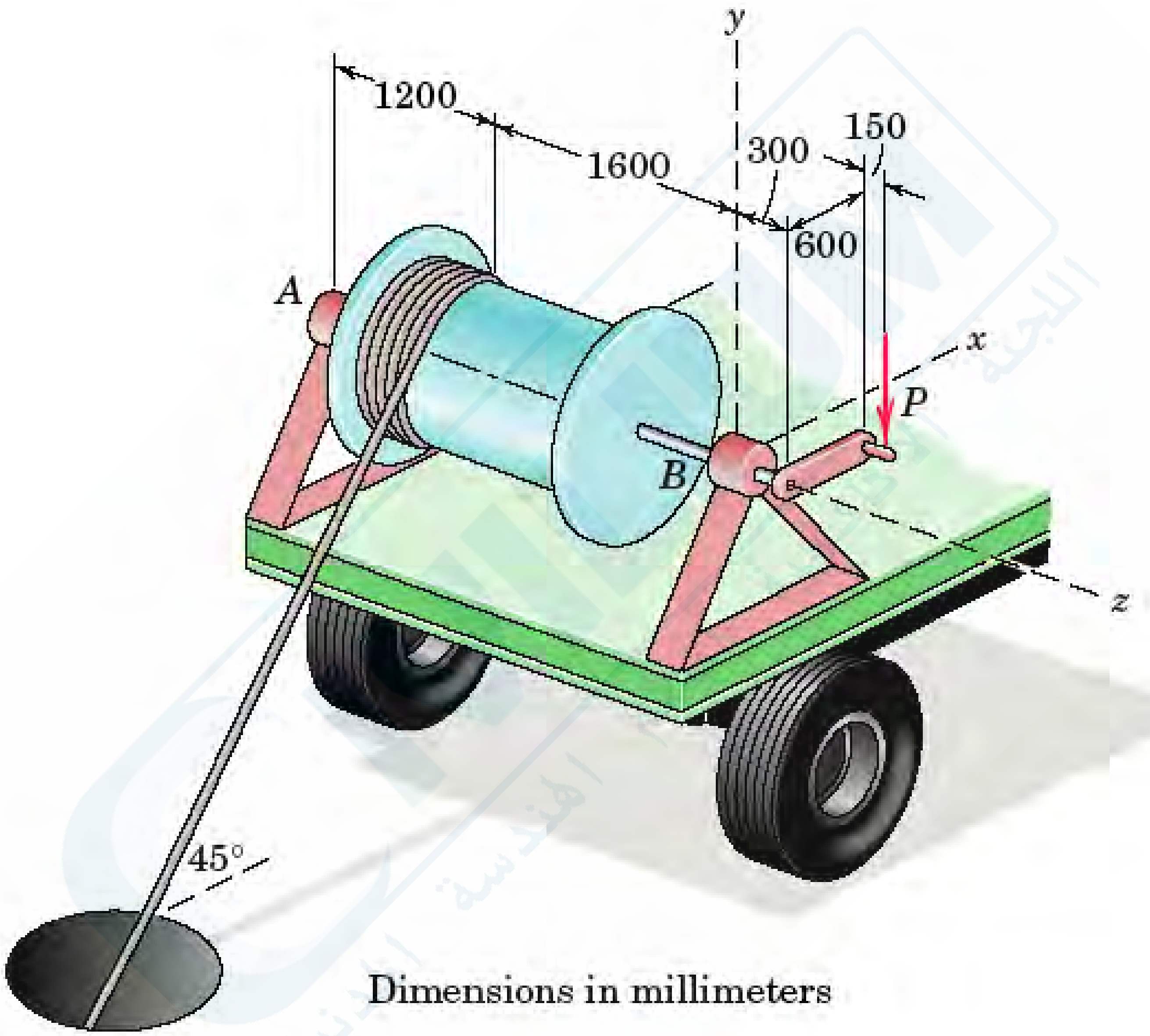
Problem 3/83

P 03 83



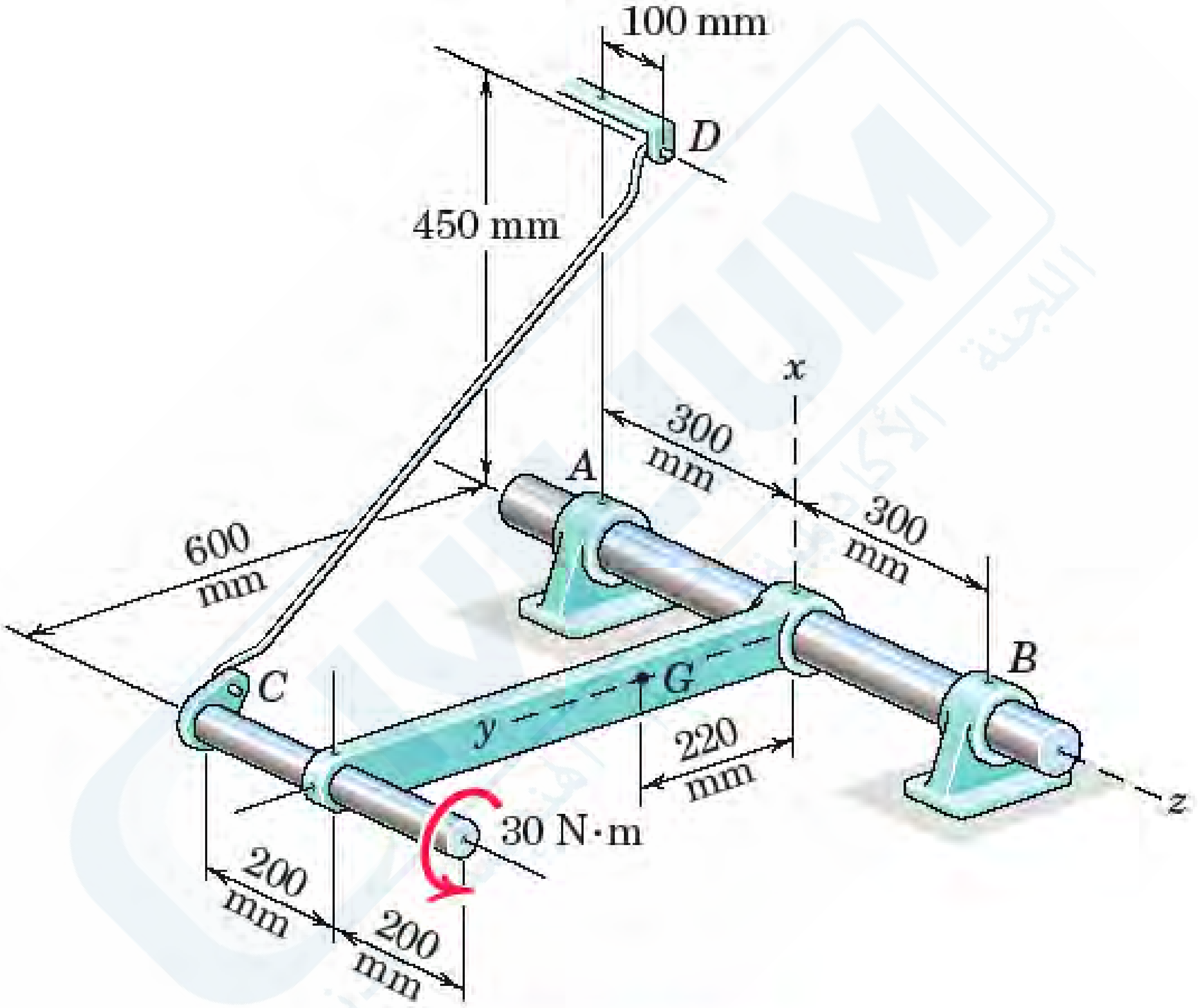
Problem 3/84

P_03_84



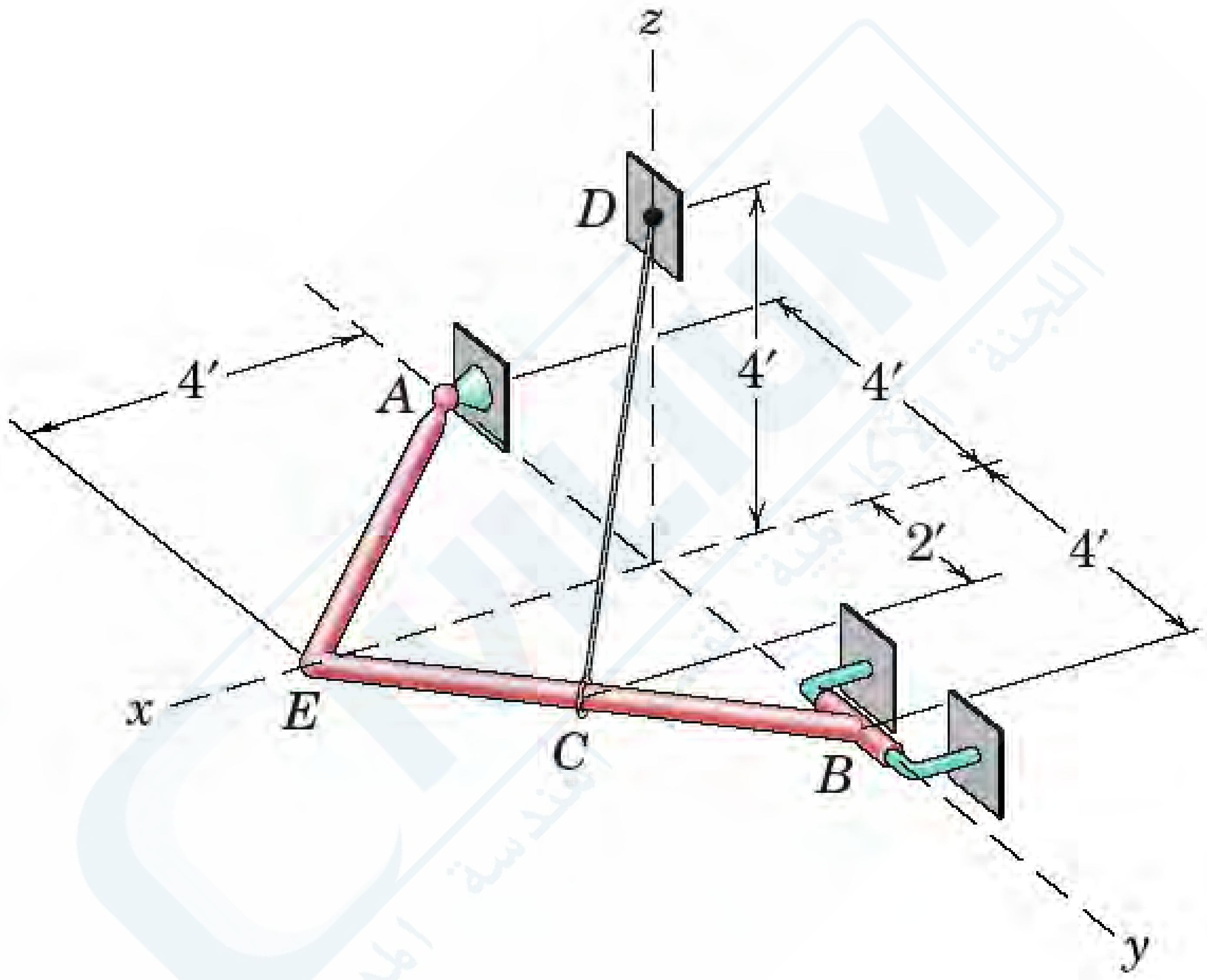
Dimensions in millimeters

Problem 3/85



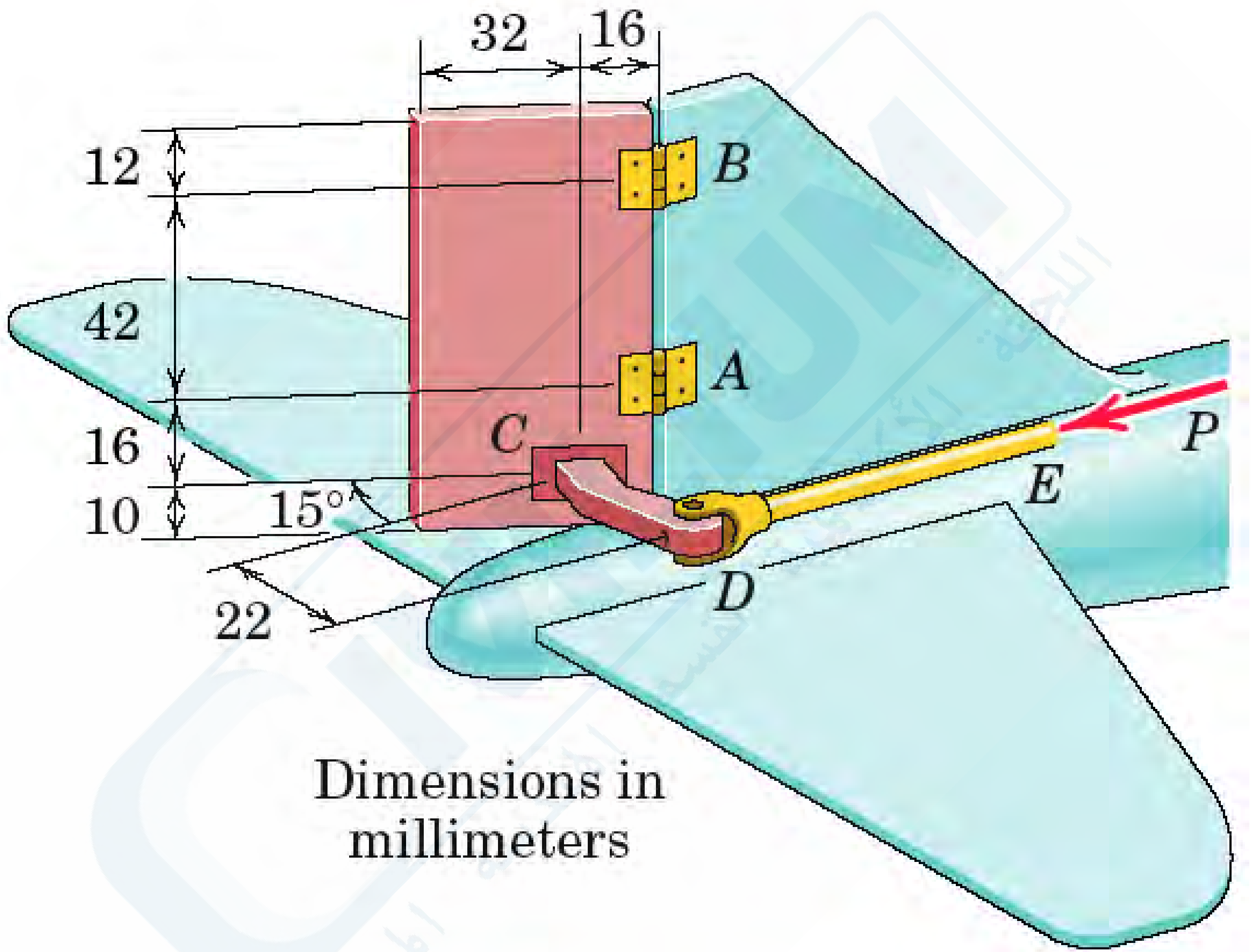
Problem 3/86

P_03_86

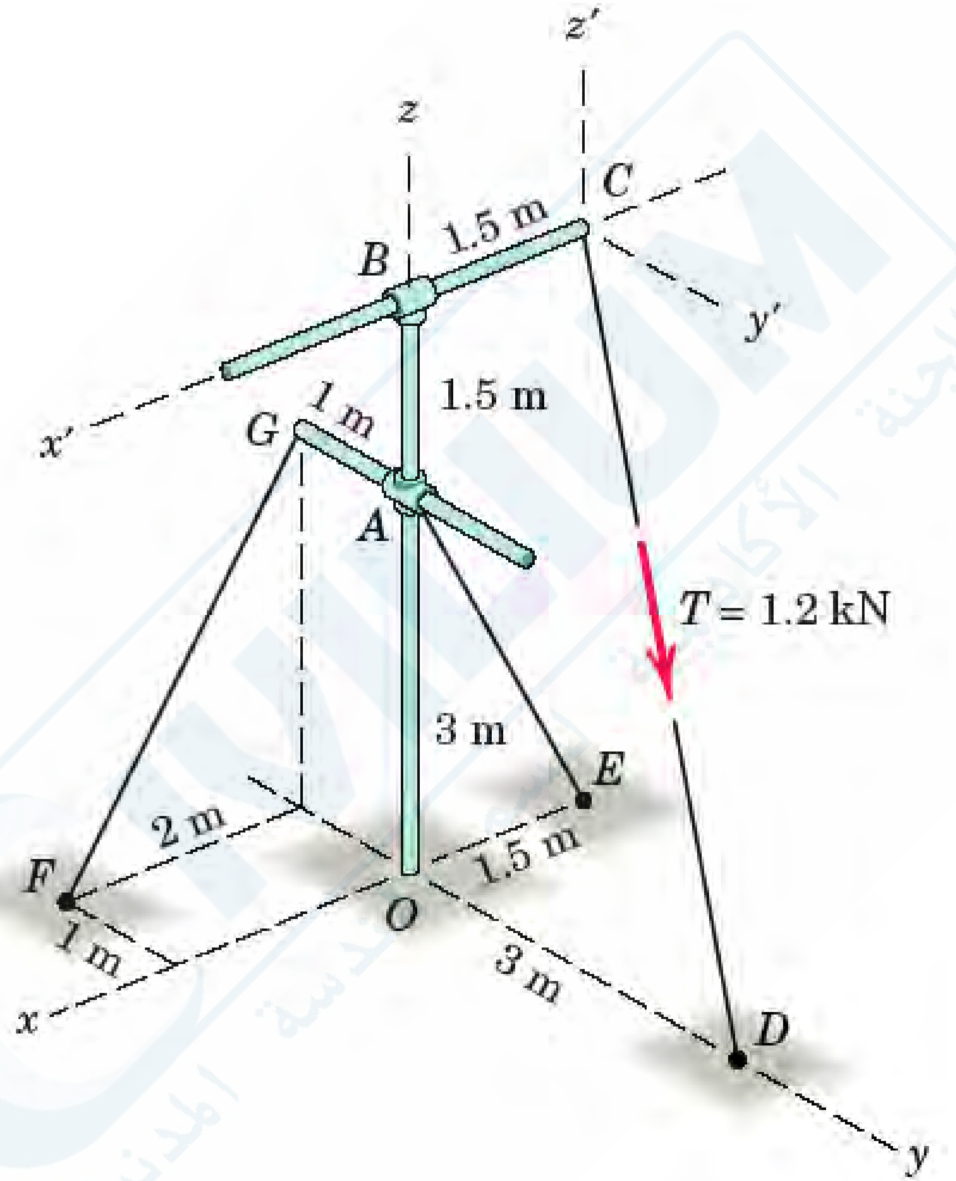


Problem 3/87

P 03 87

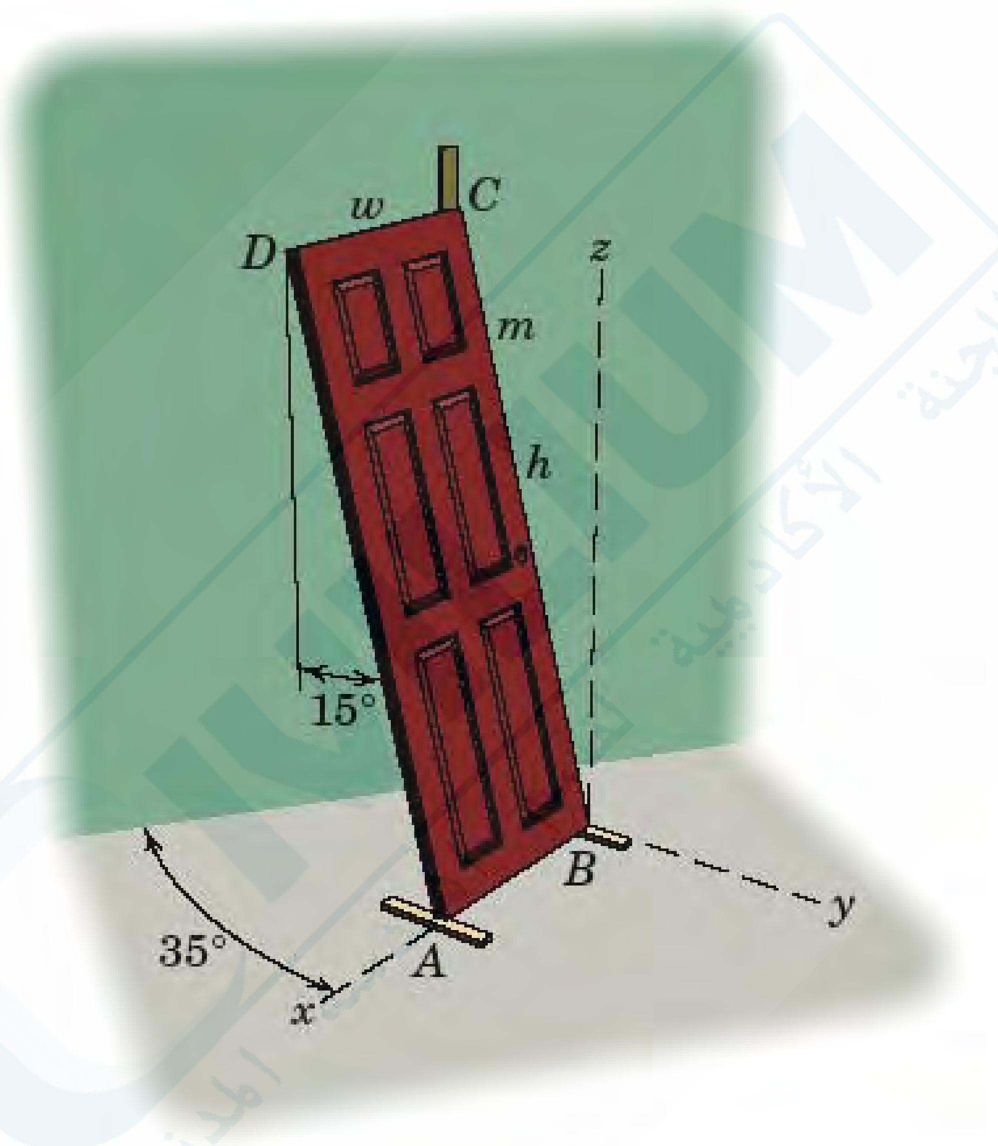


Problem 3/88



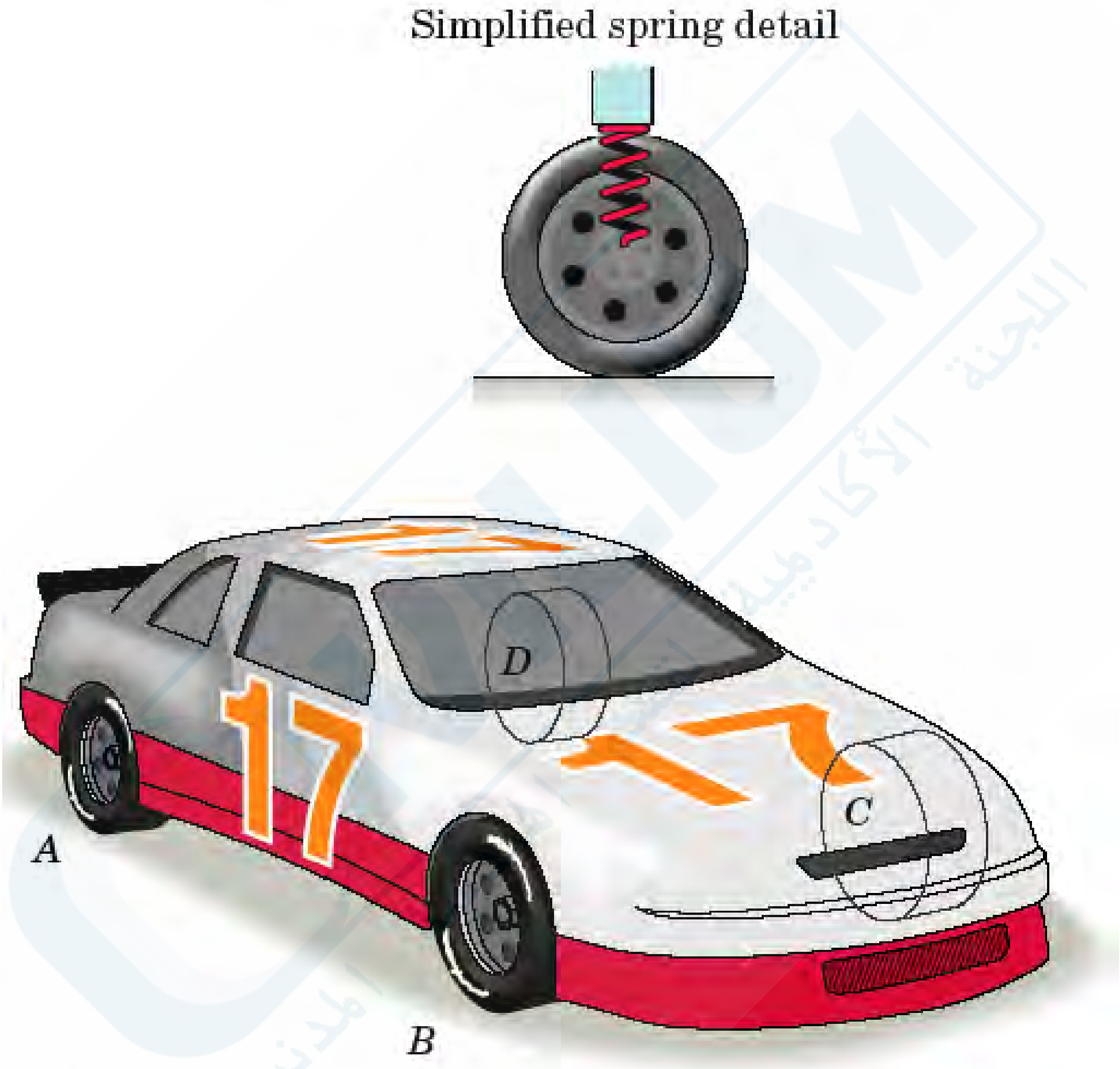
Problem 3/89

P 03 89

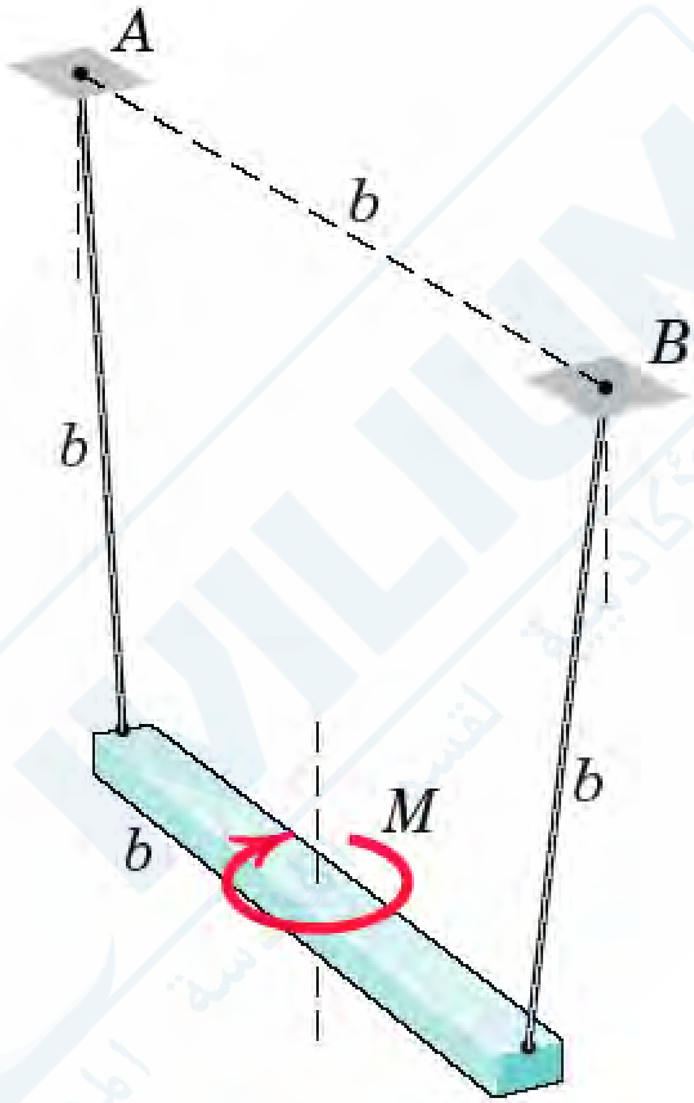


Problem 3/90

P_03_90

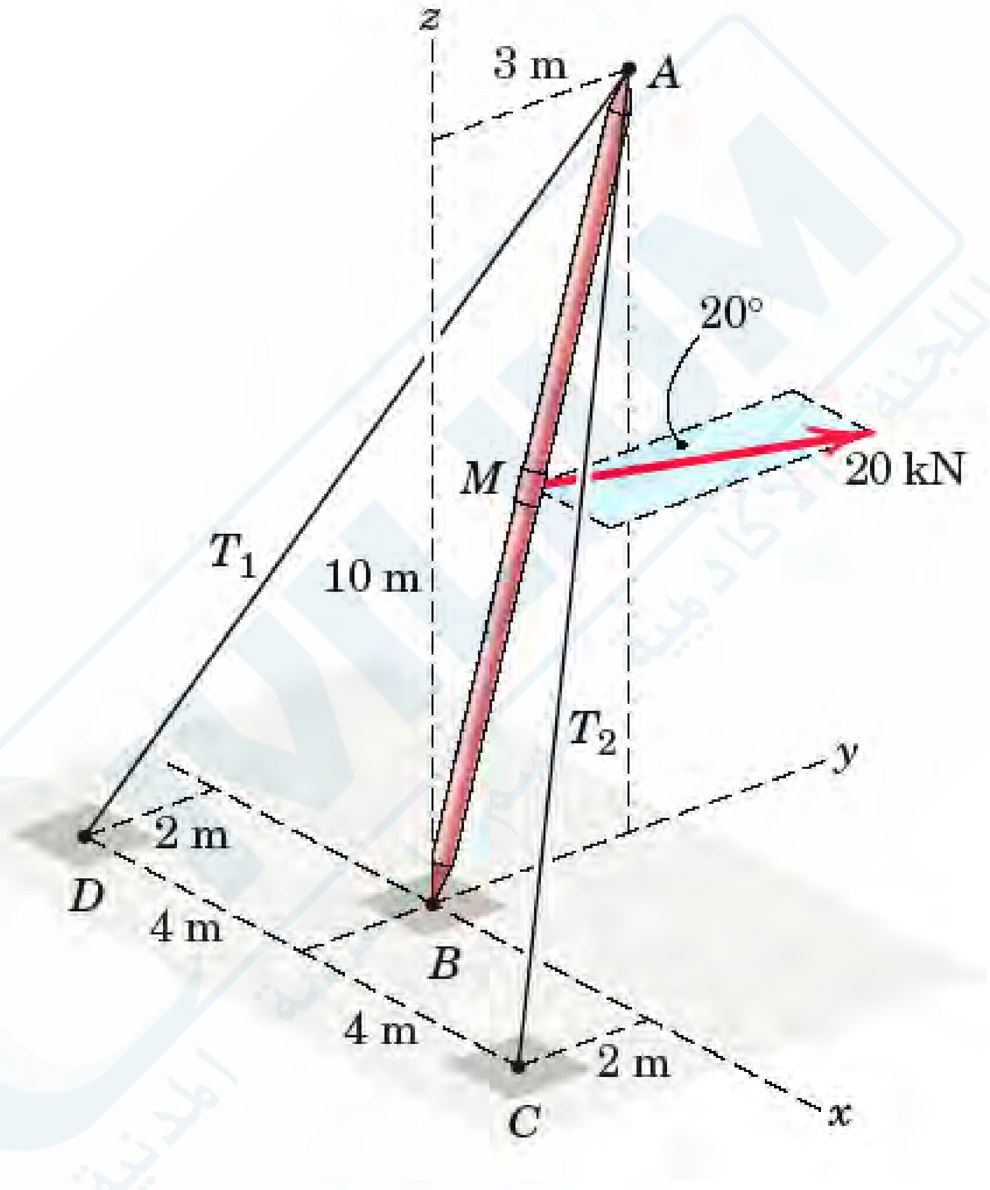


Problem 3/91



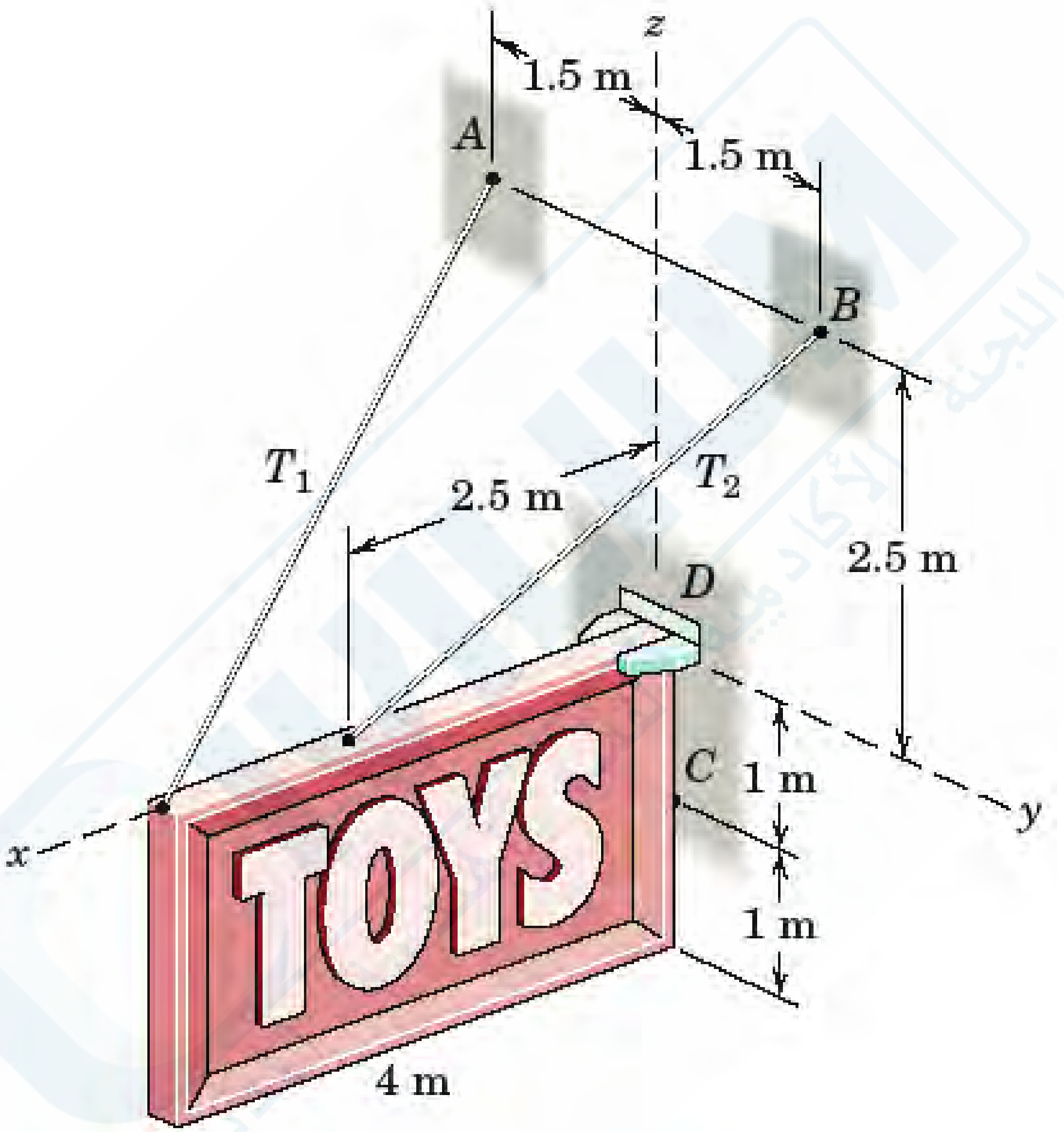
Problem 3/92

P_03_92



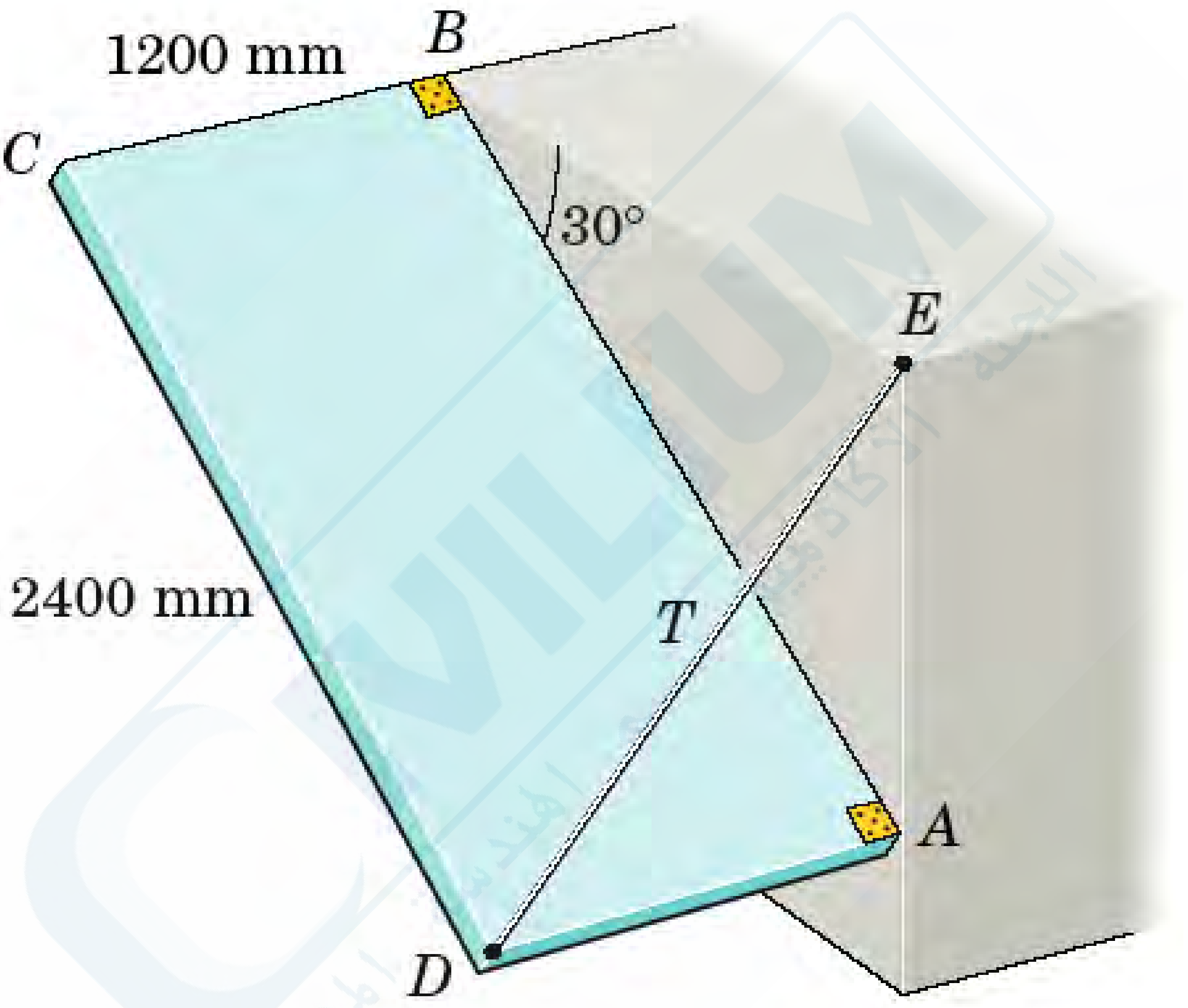
Problem 3/93

P 03 93



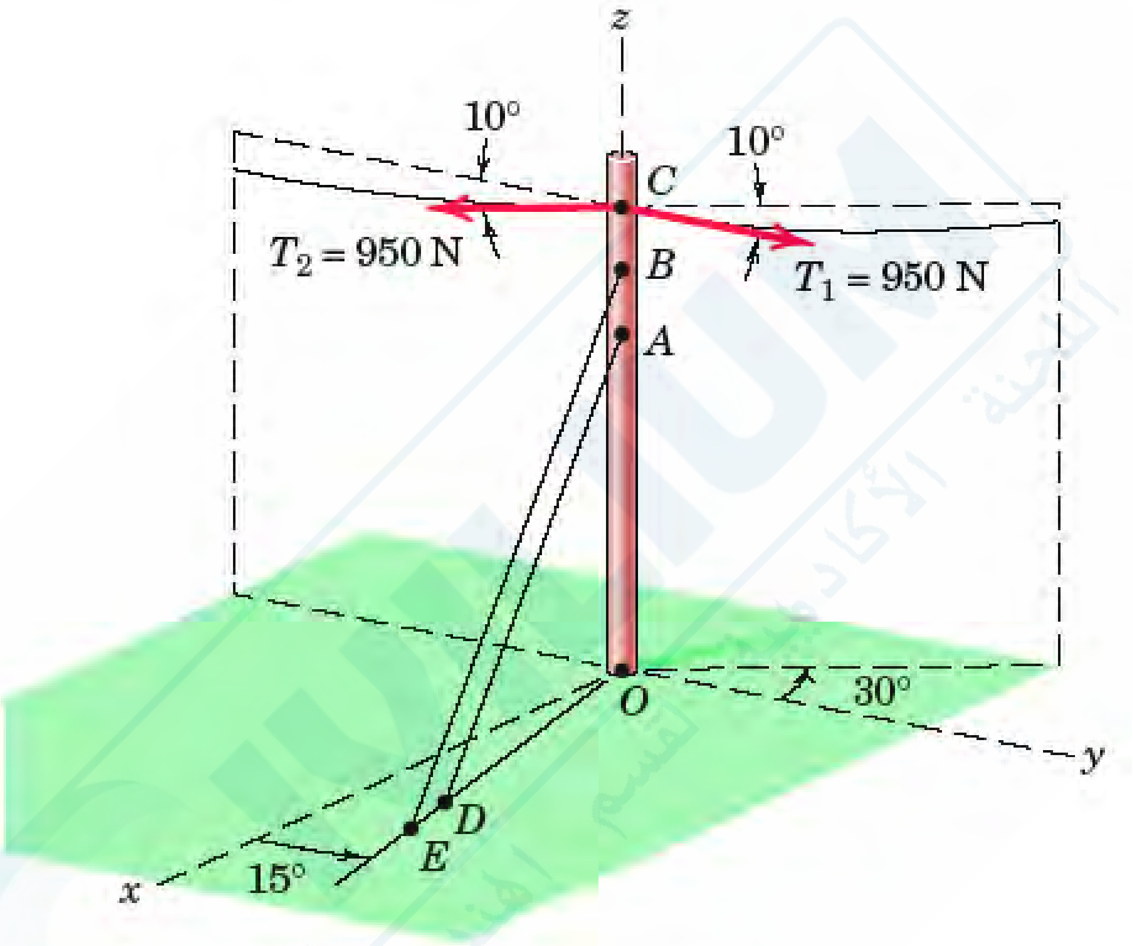
Problem 3/94

P 03 94



Problem 3/95

P_03_95



$\overline{OA} = 9$ m

$\overline{OB} = 11$ m

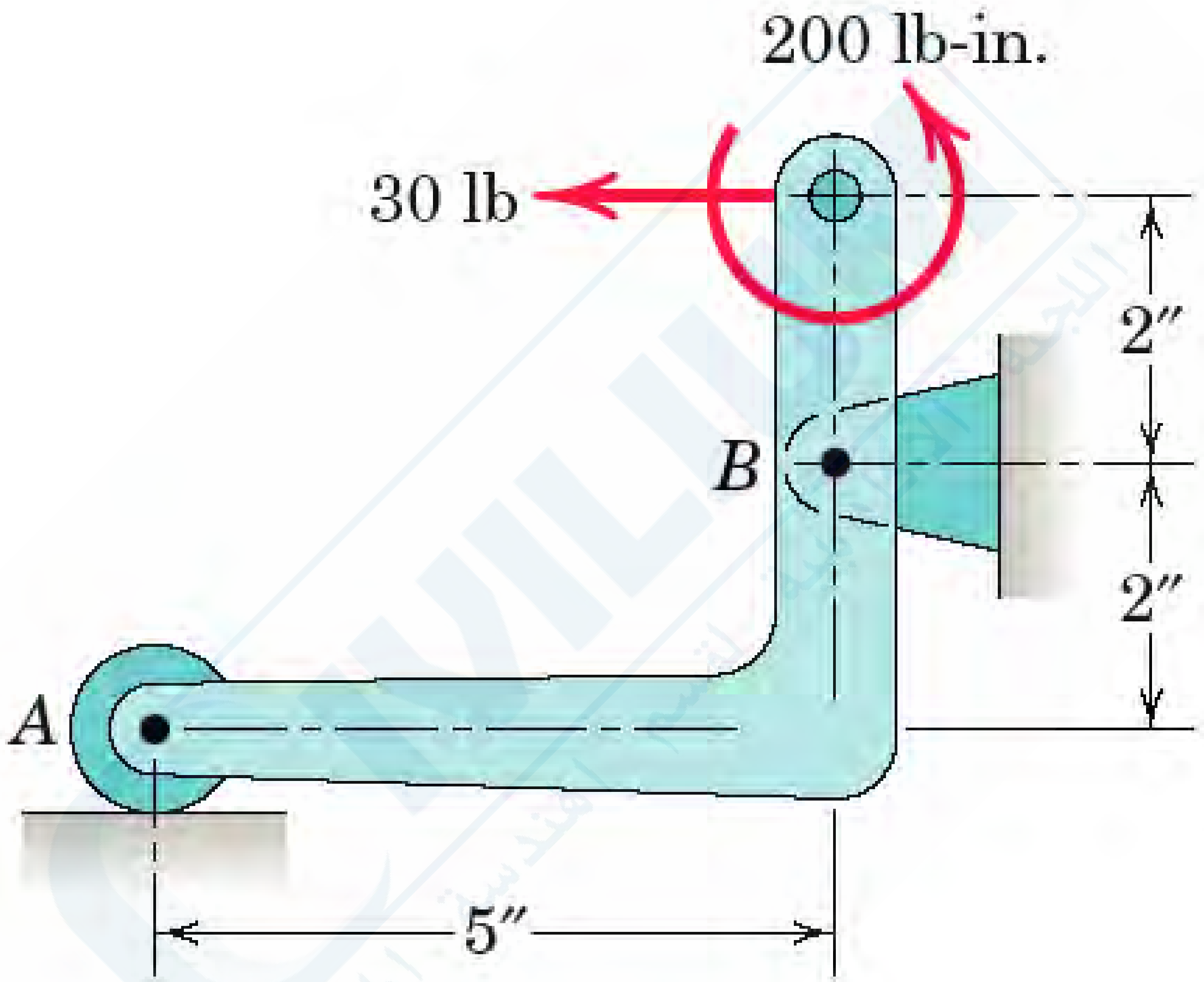
$\overline{OC} = 13$ m

$\overline{OD} = 8$ m

$\overline{OE} = 10$ m

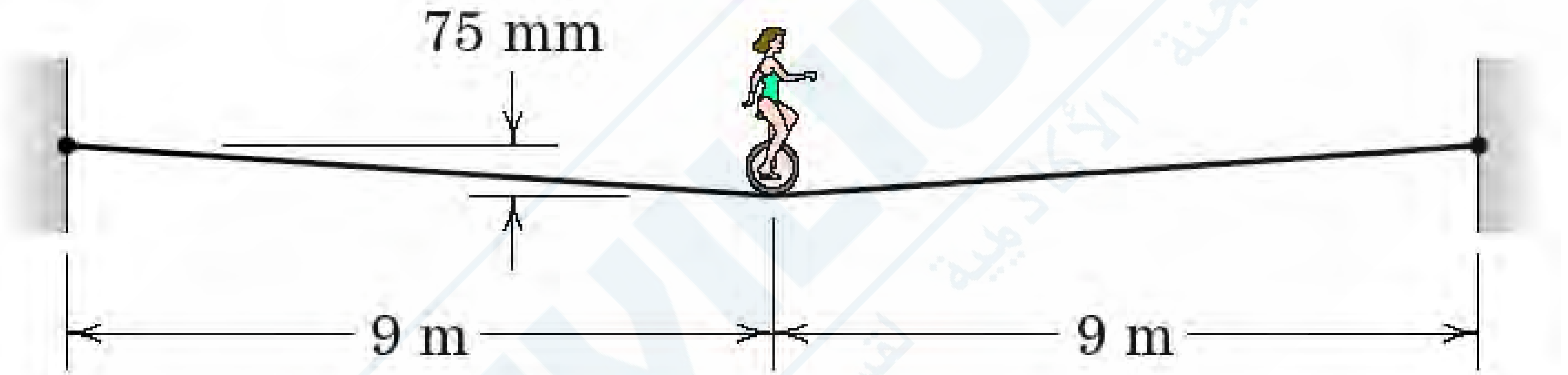
Problem 3/96

P 03 96



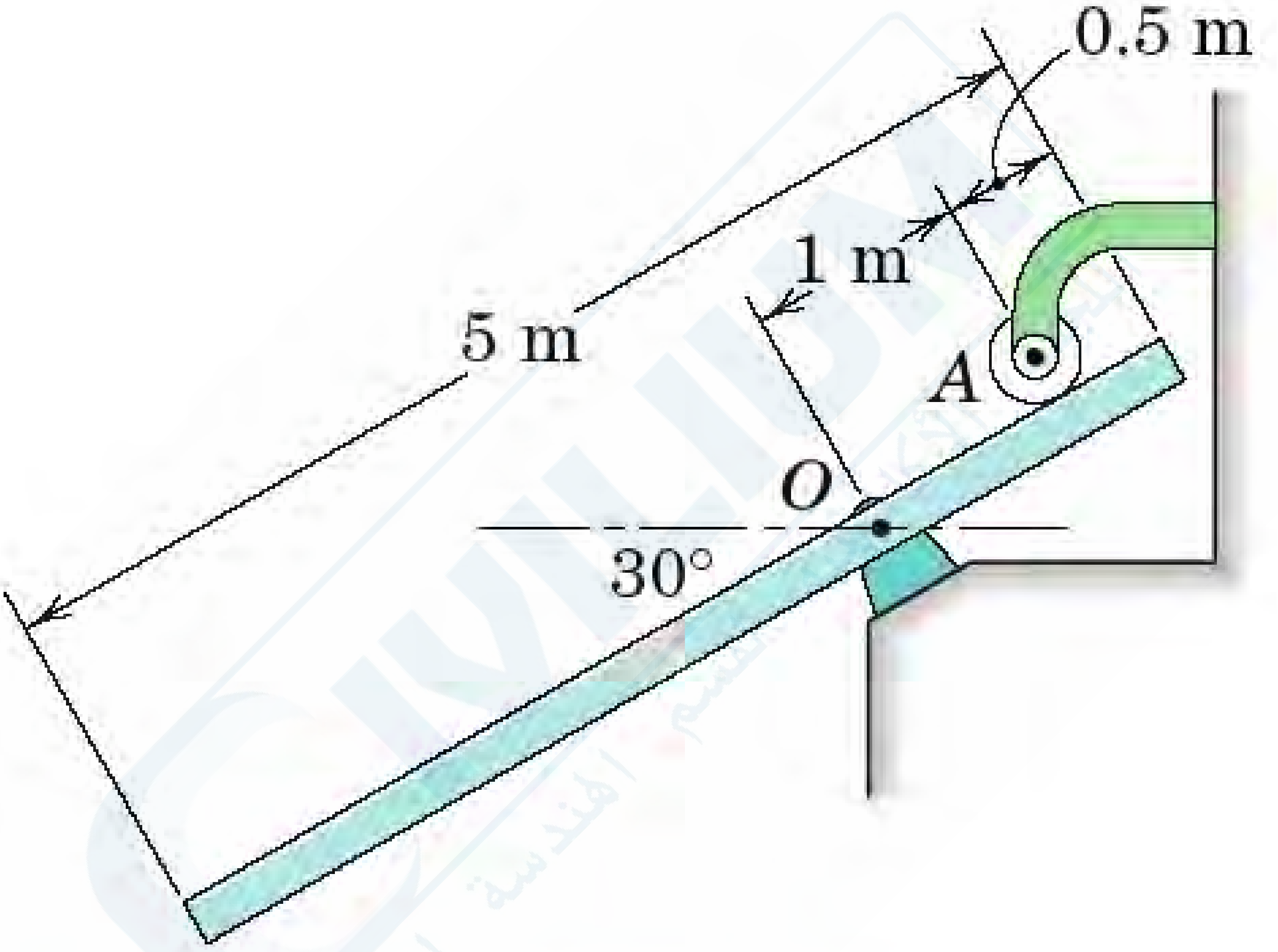
Problem 3/97

P_03_97



Problem 3/98

P_03_98



Problem 3/99

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 4 : Structures.

CIVILIUM
اللجنة الأكاديمية لقسم الهندسة المدنية

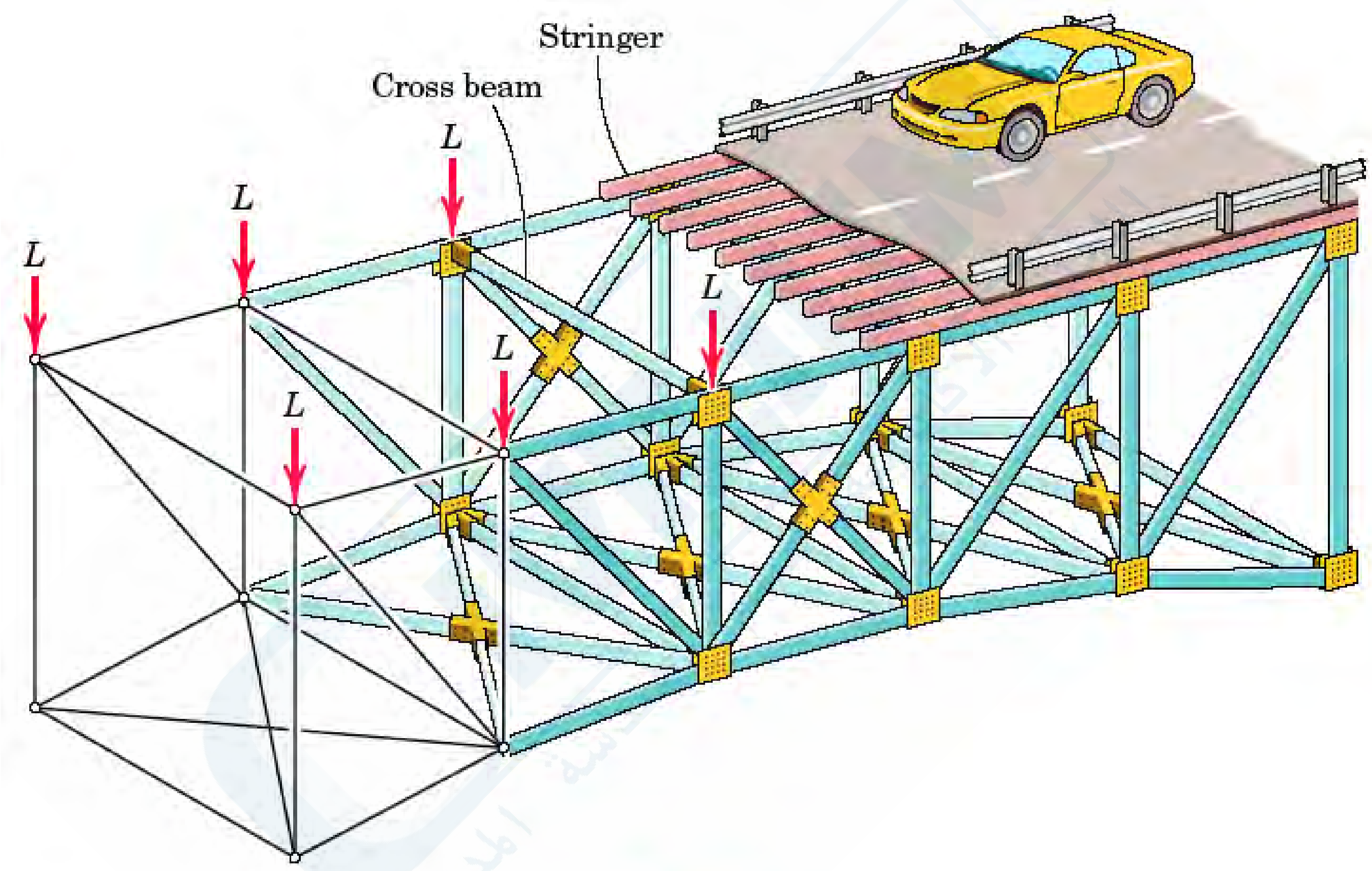
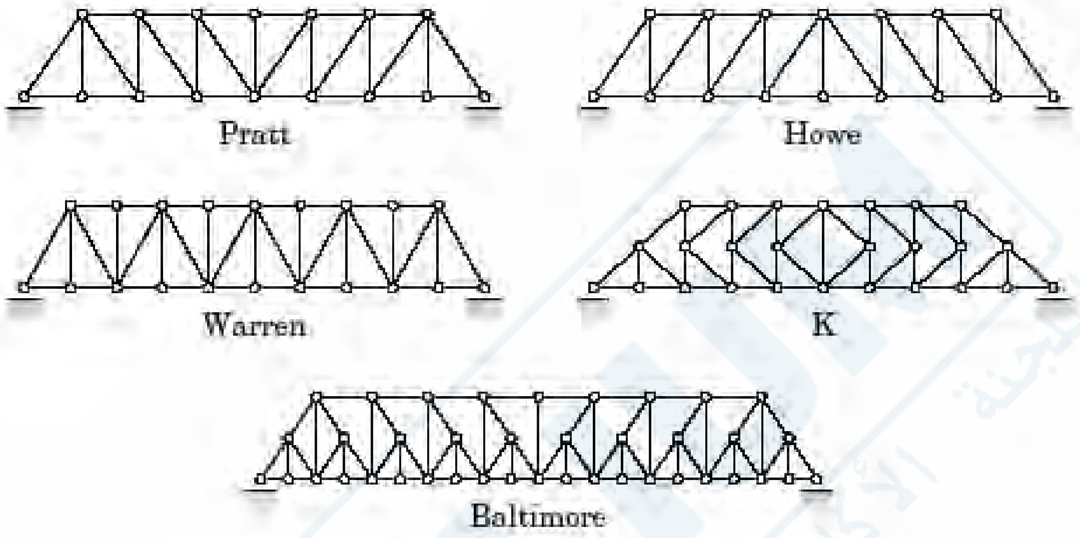
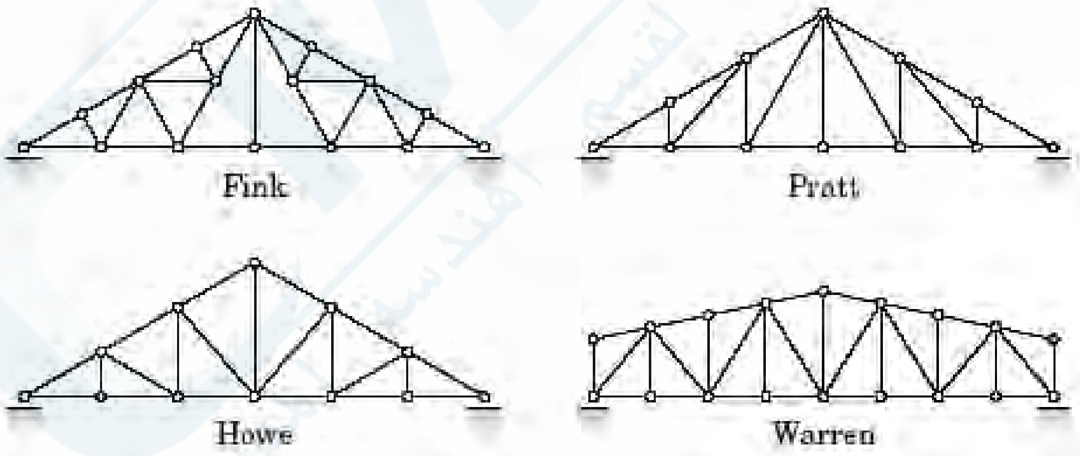


Figure 4/1

04_01



Commonly Used Bridge Trusses



Commonly Used Roof Trusses

Figure 4/1

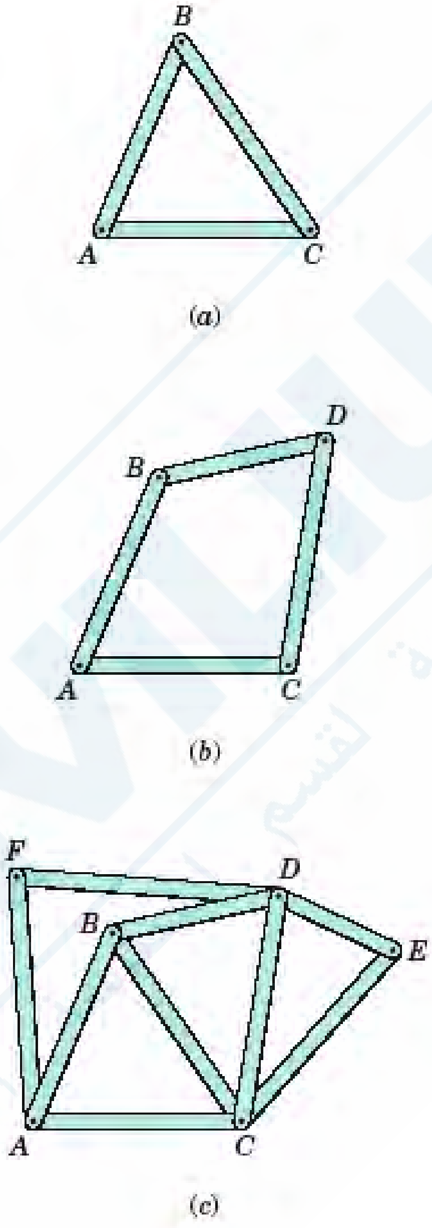
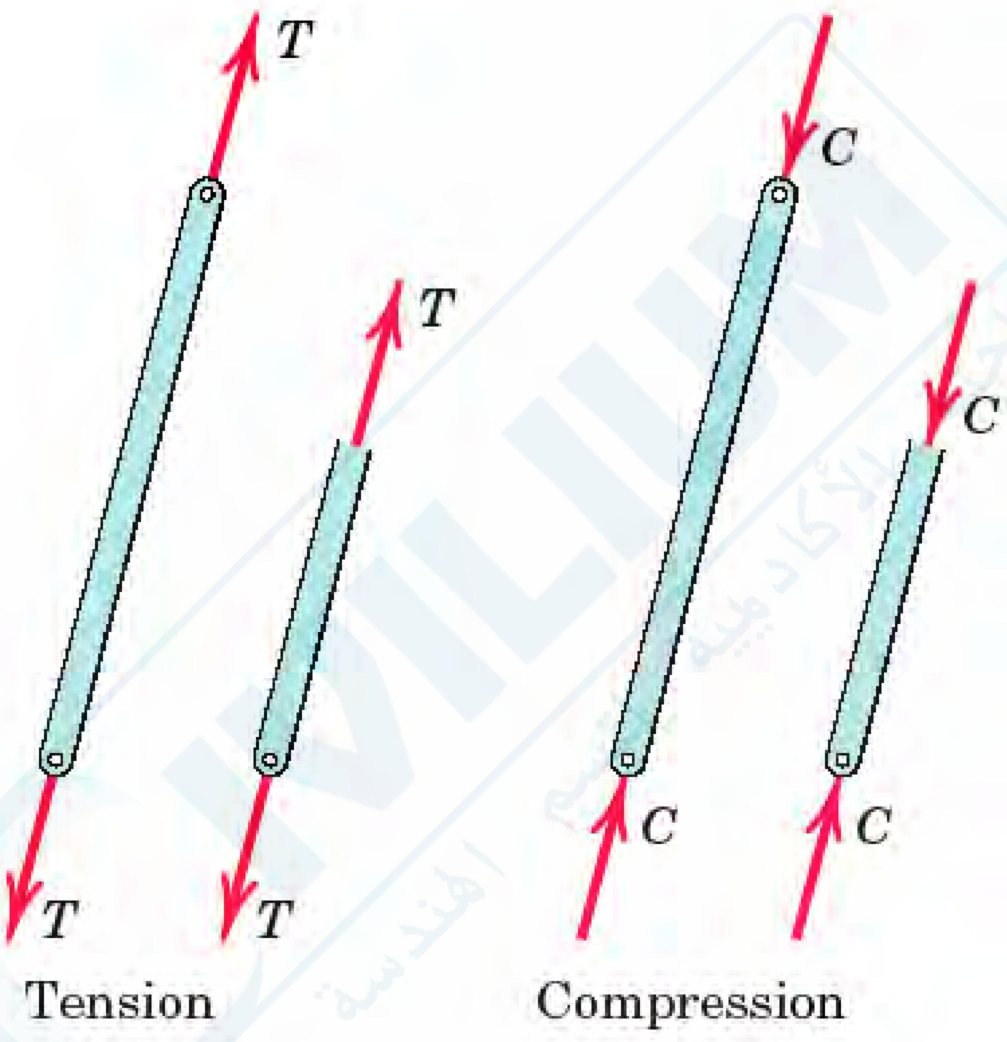


Figure 4/3



Two-Force Members

Figure 4/4

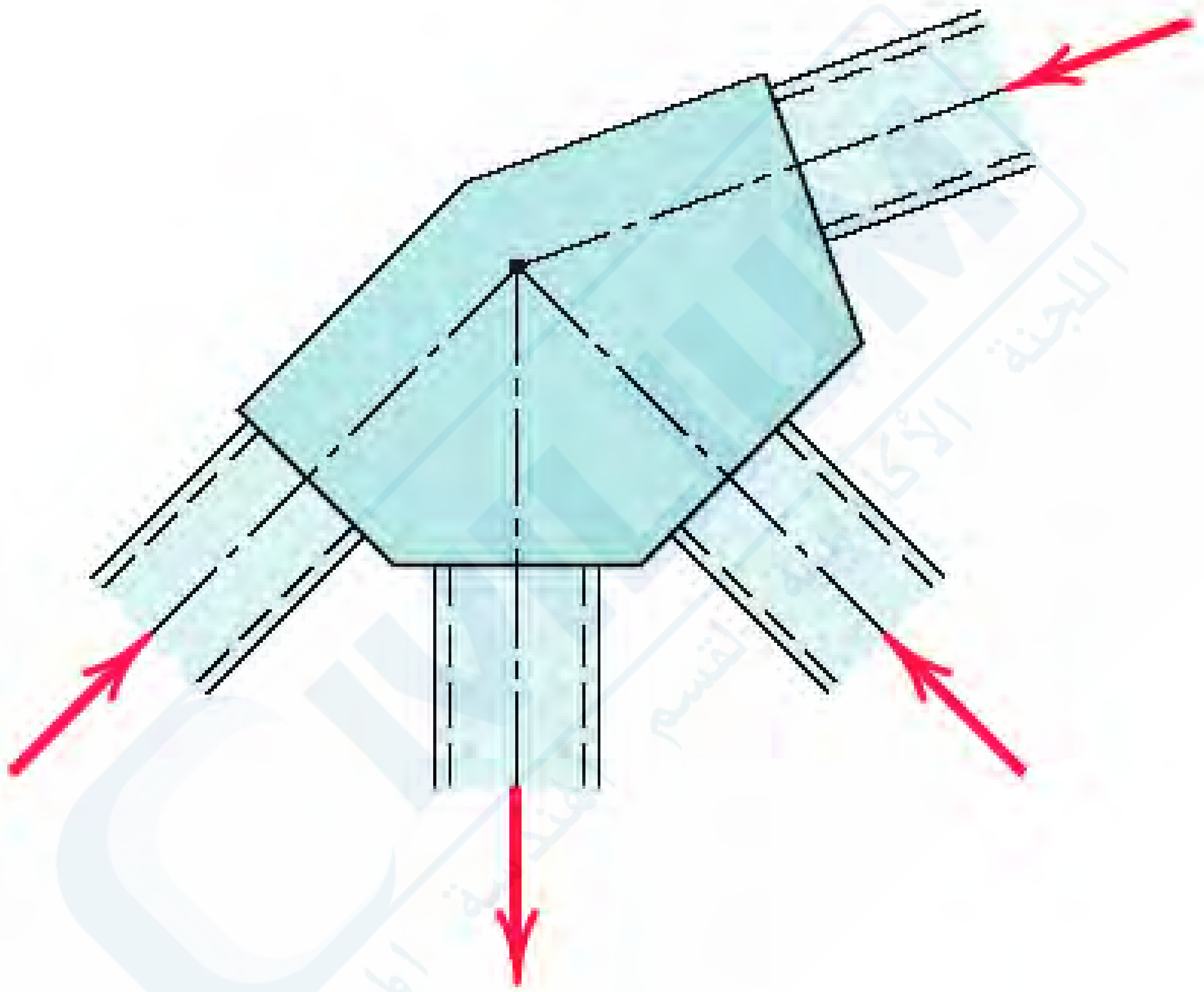


Figure 4/5

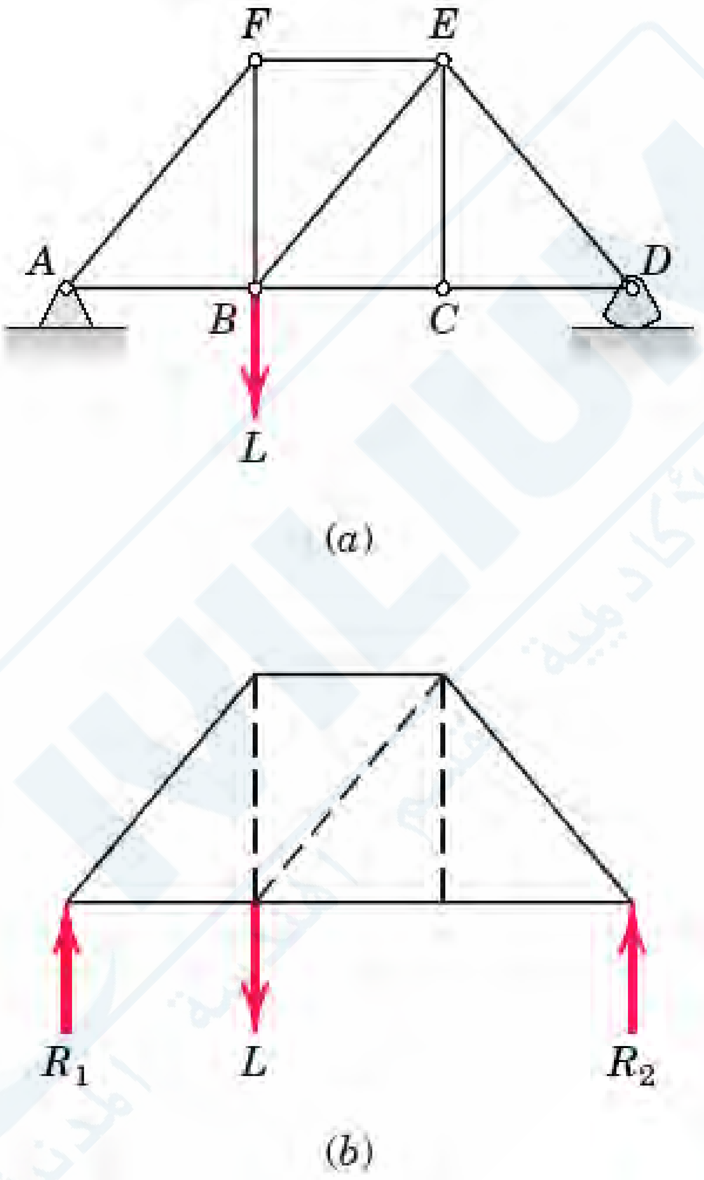


Figure 4/6

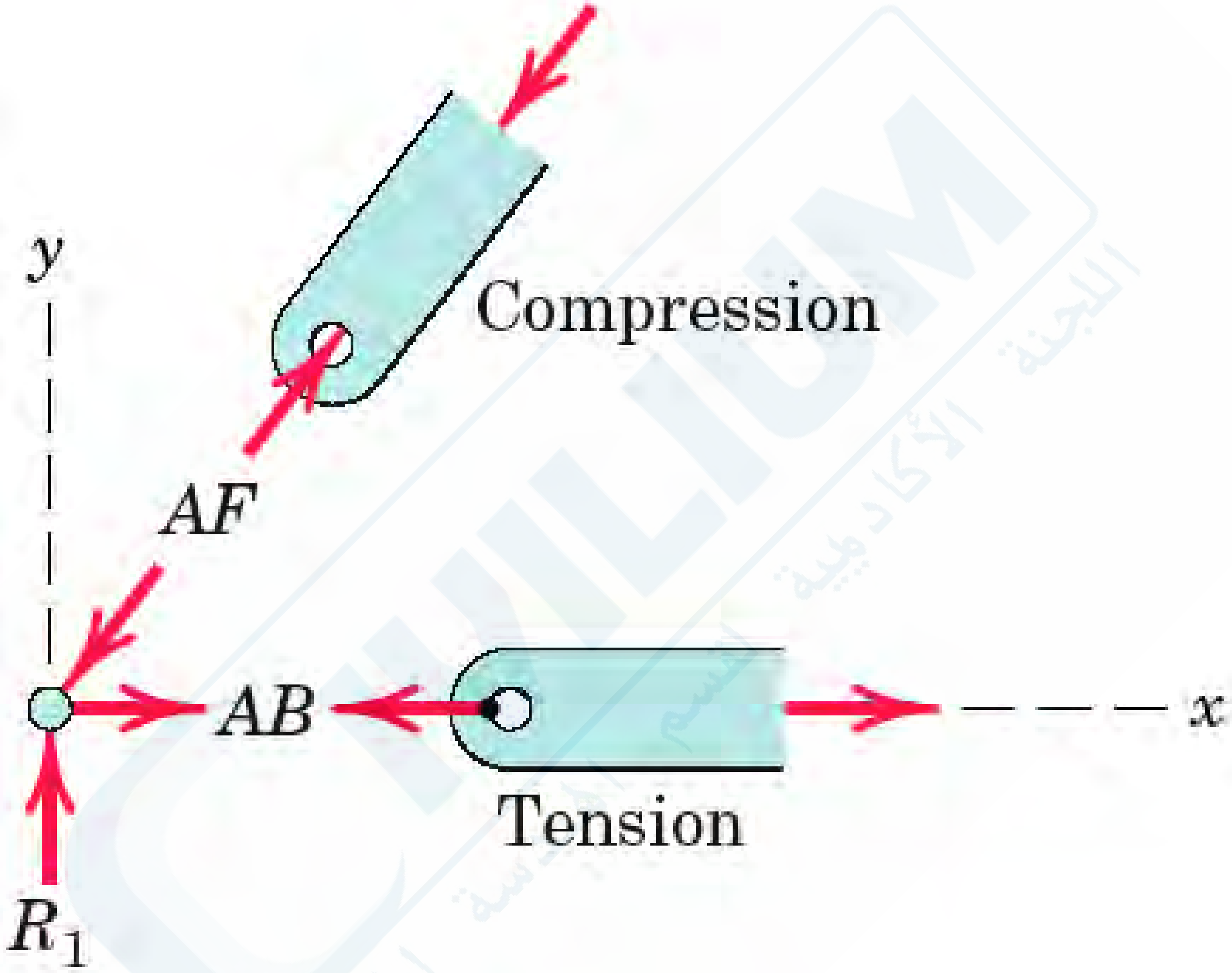
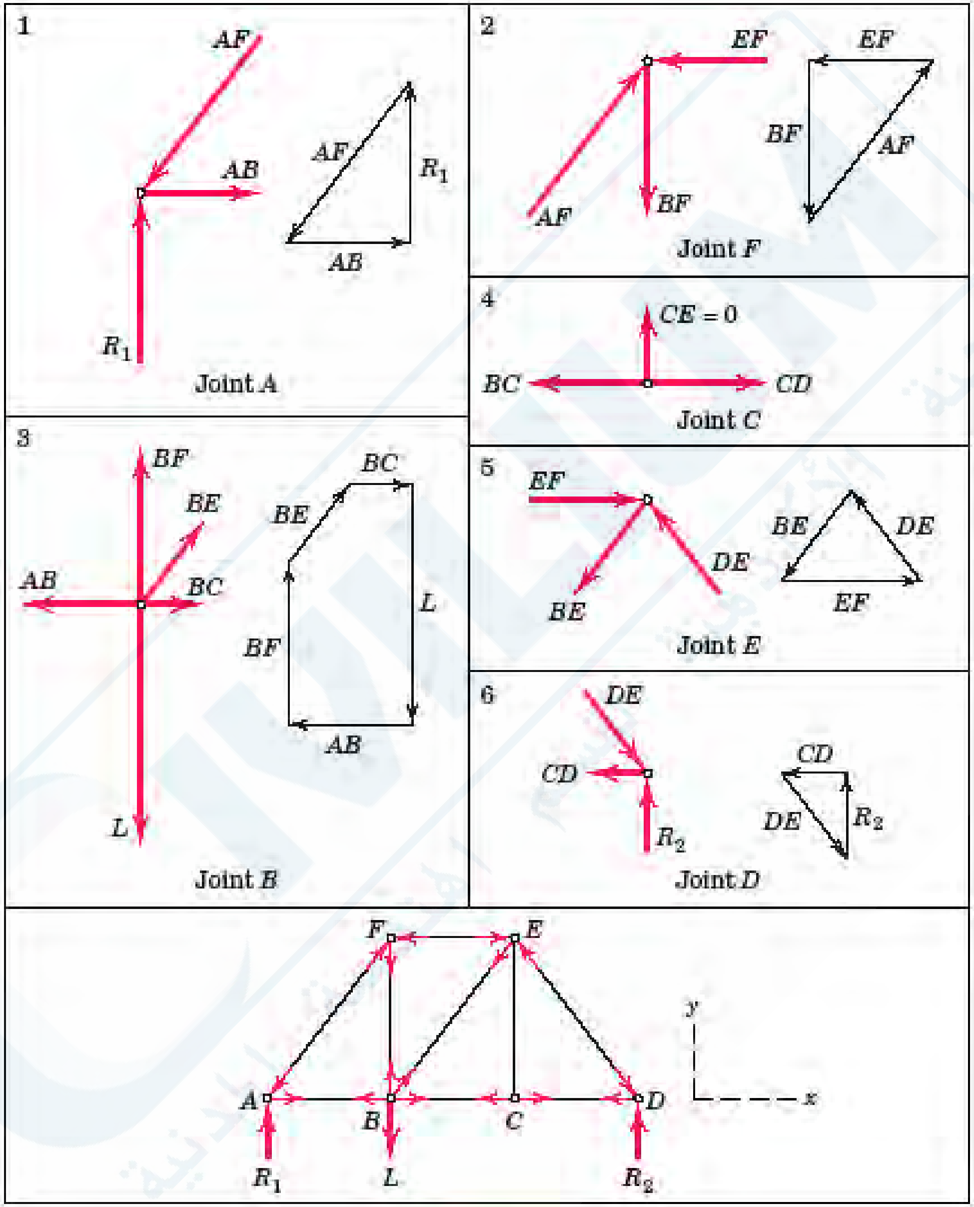


Figure 4/7



1
AF
AB
R1
Joint A
2
EF
BF
AF
Joint F
4
CE = 0
BC
CD
Joint C
3
BF
BE
BC
AB
L
Joint B
5
EF
BE
DE
Joint E
6
DE
CD
R2
Joint D
F
E
A
B
C
D
R1
L
R2
y
x

Figure 4/8

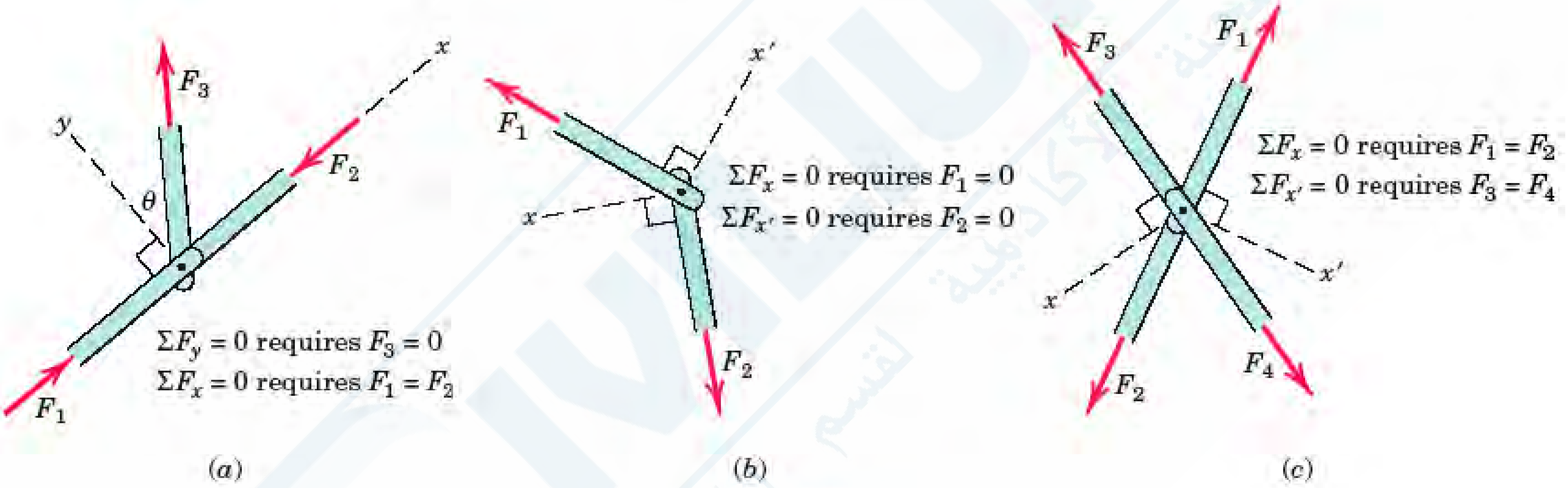


Figure 4/9

04_09

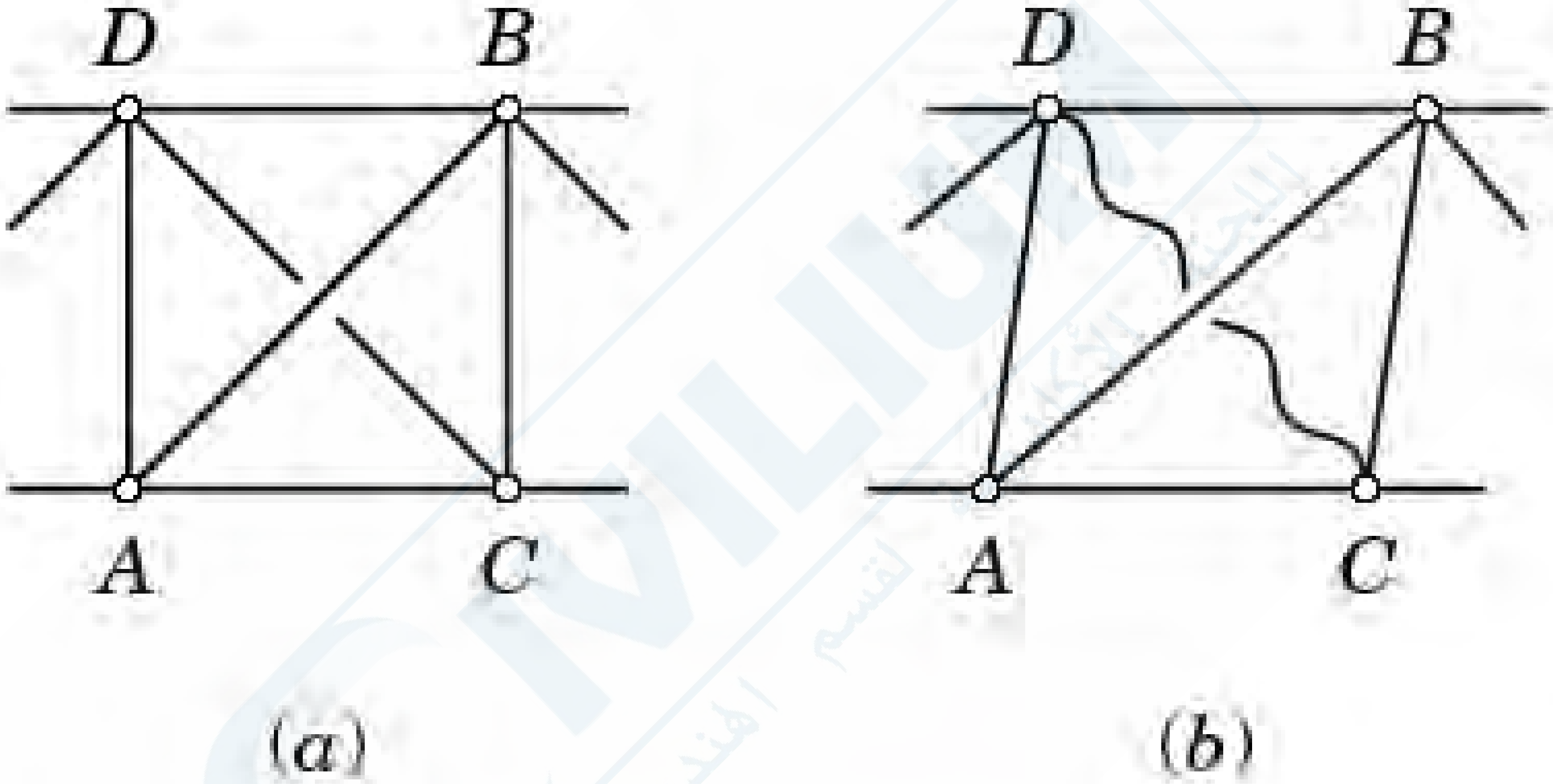


Figure 4/10

04_10

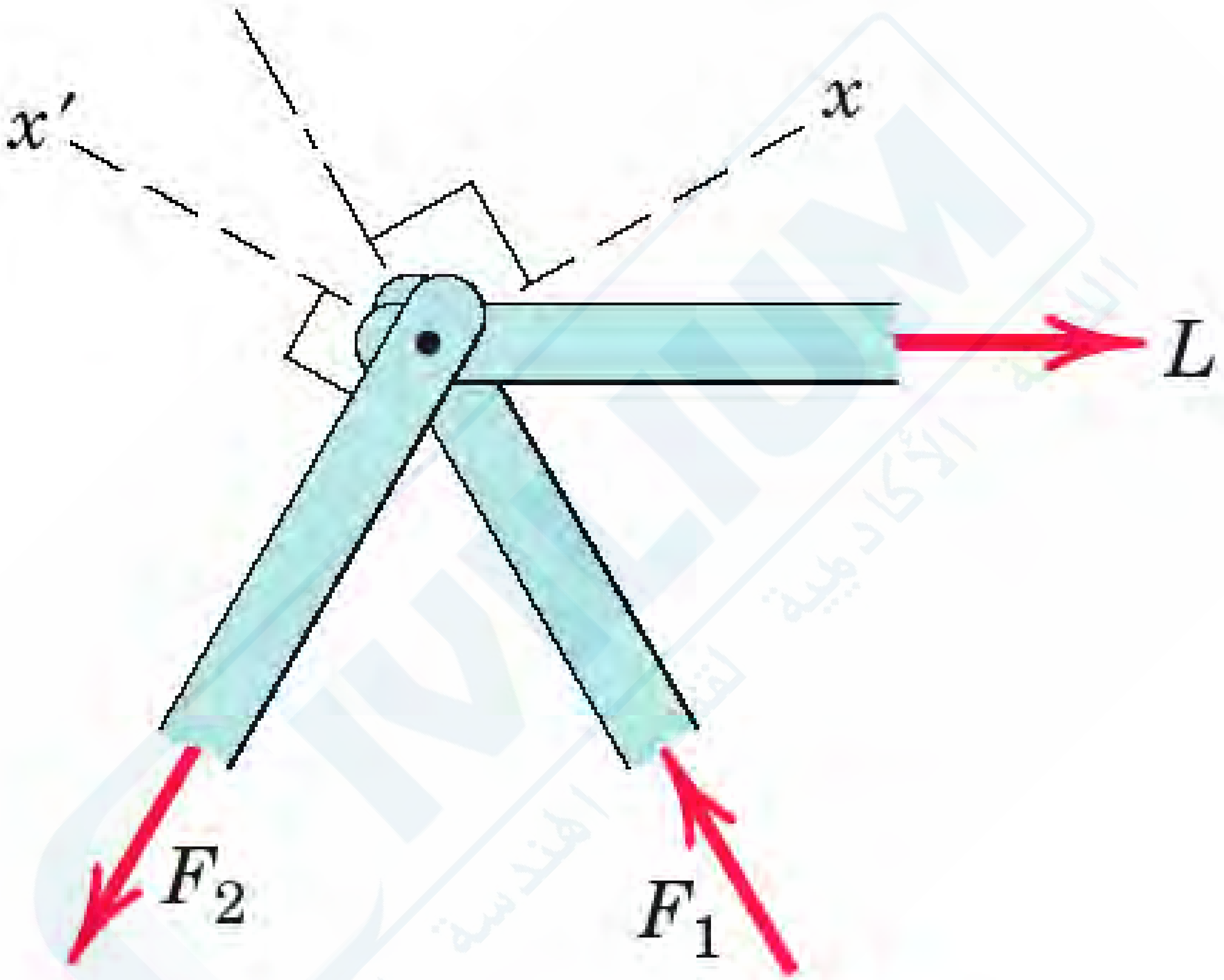
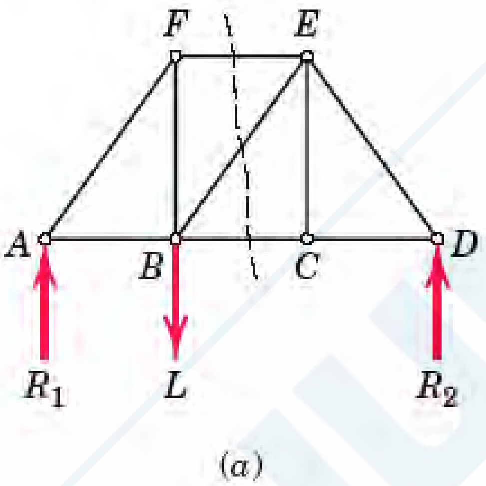


Figure 4/11



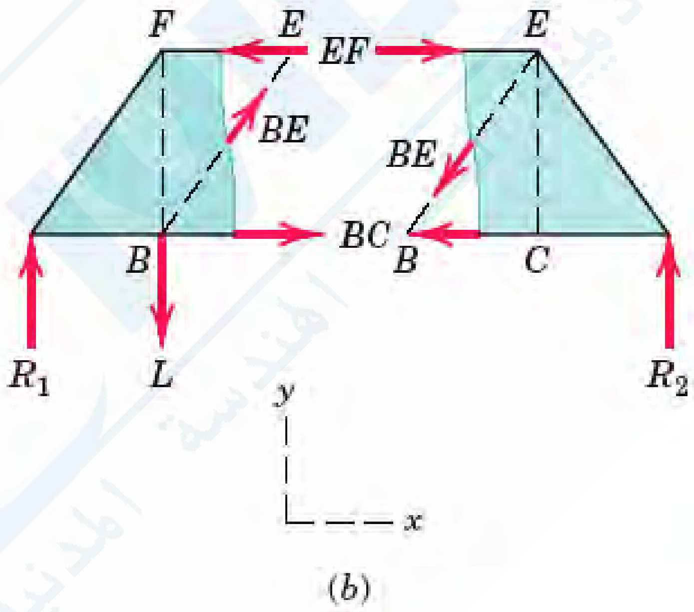


Figure 4/12

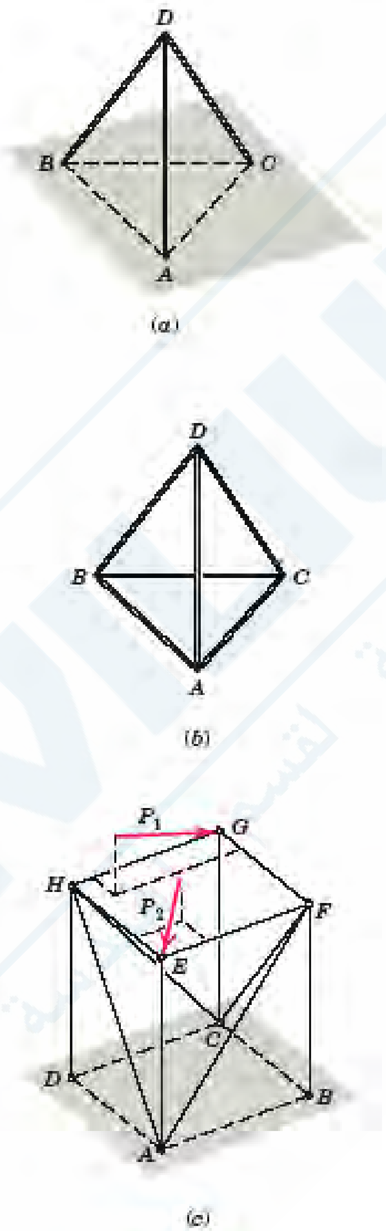


Figure 4/11

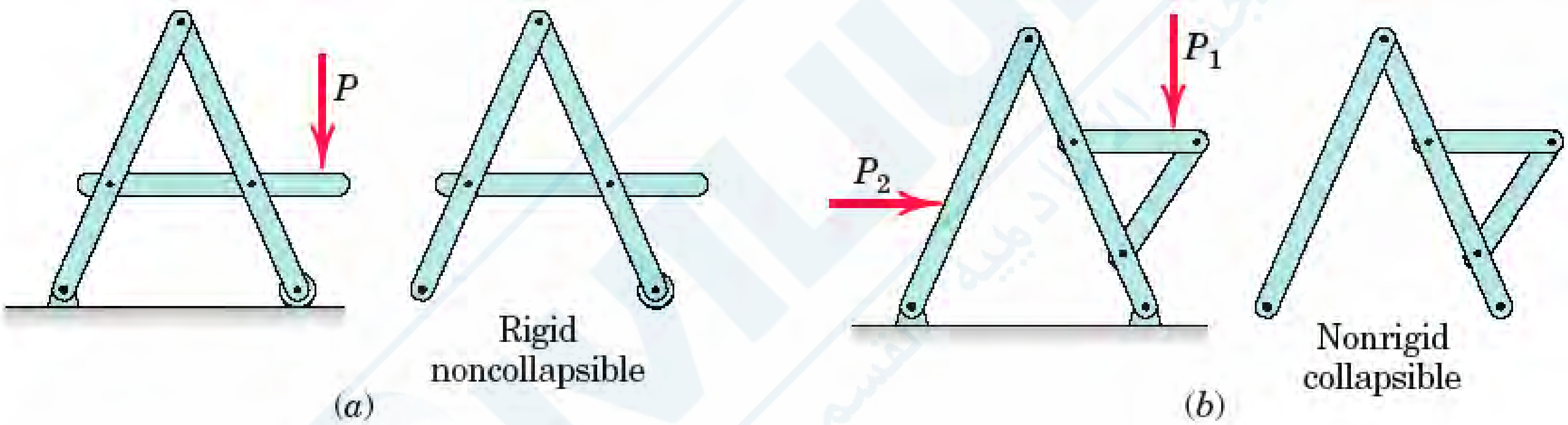


Figure 4/14

04_14

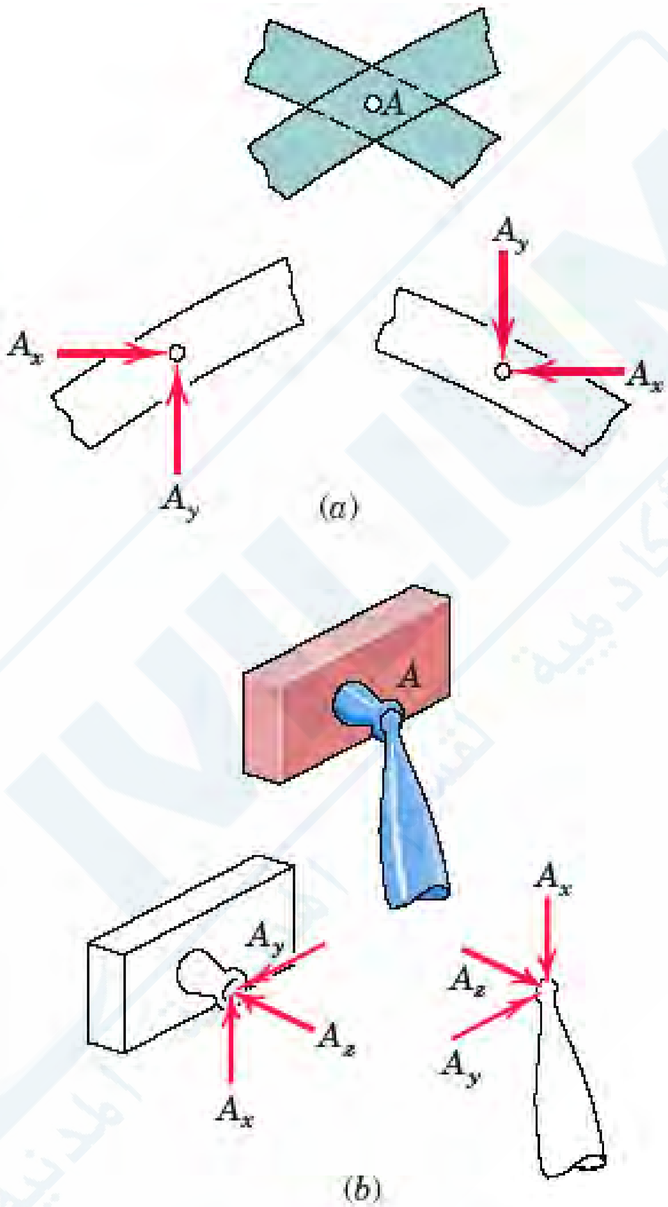


Figure 4/15

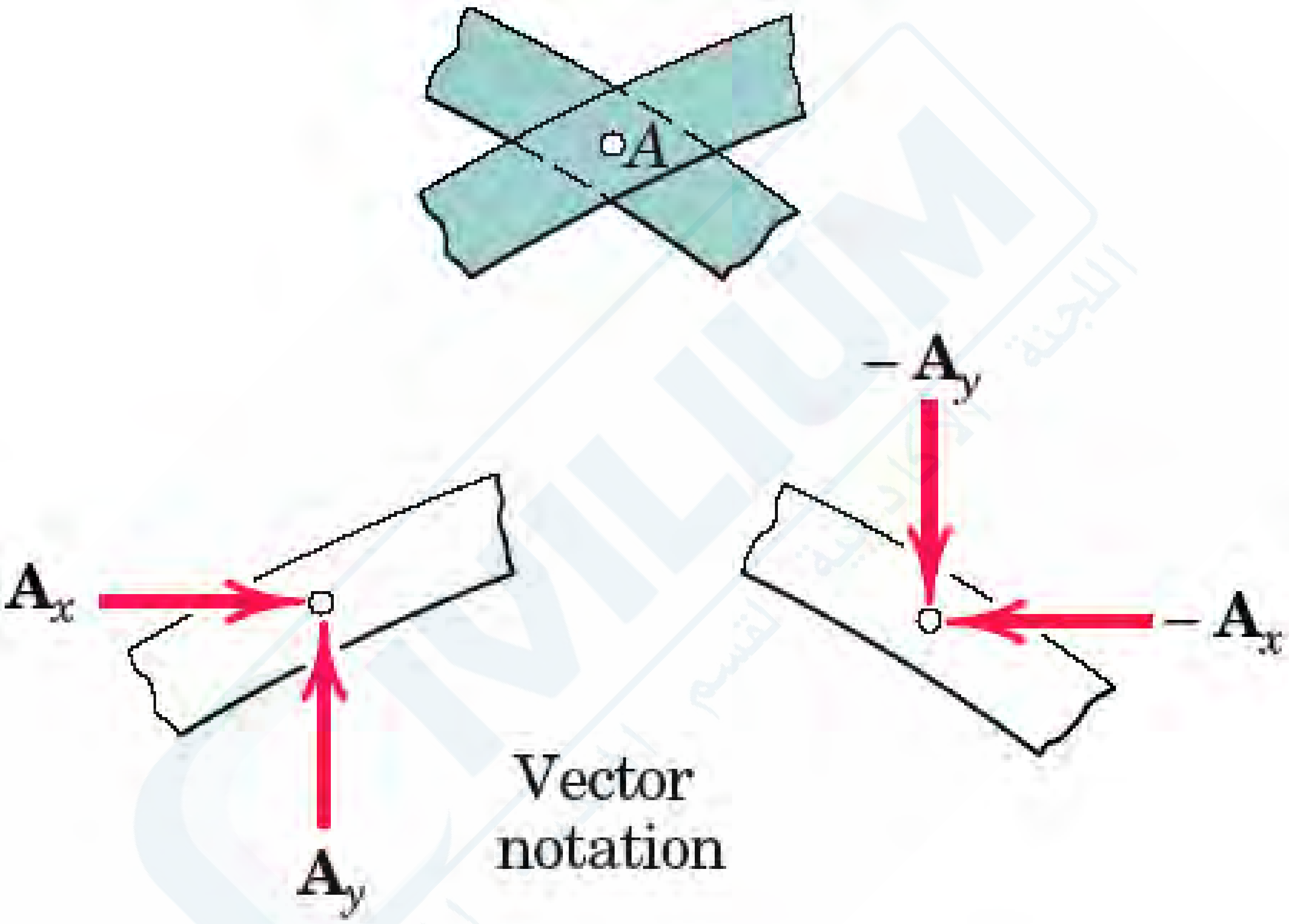
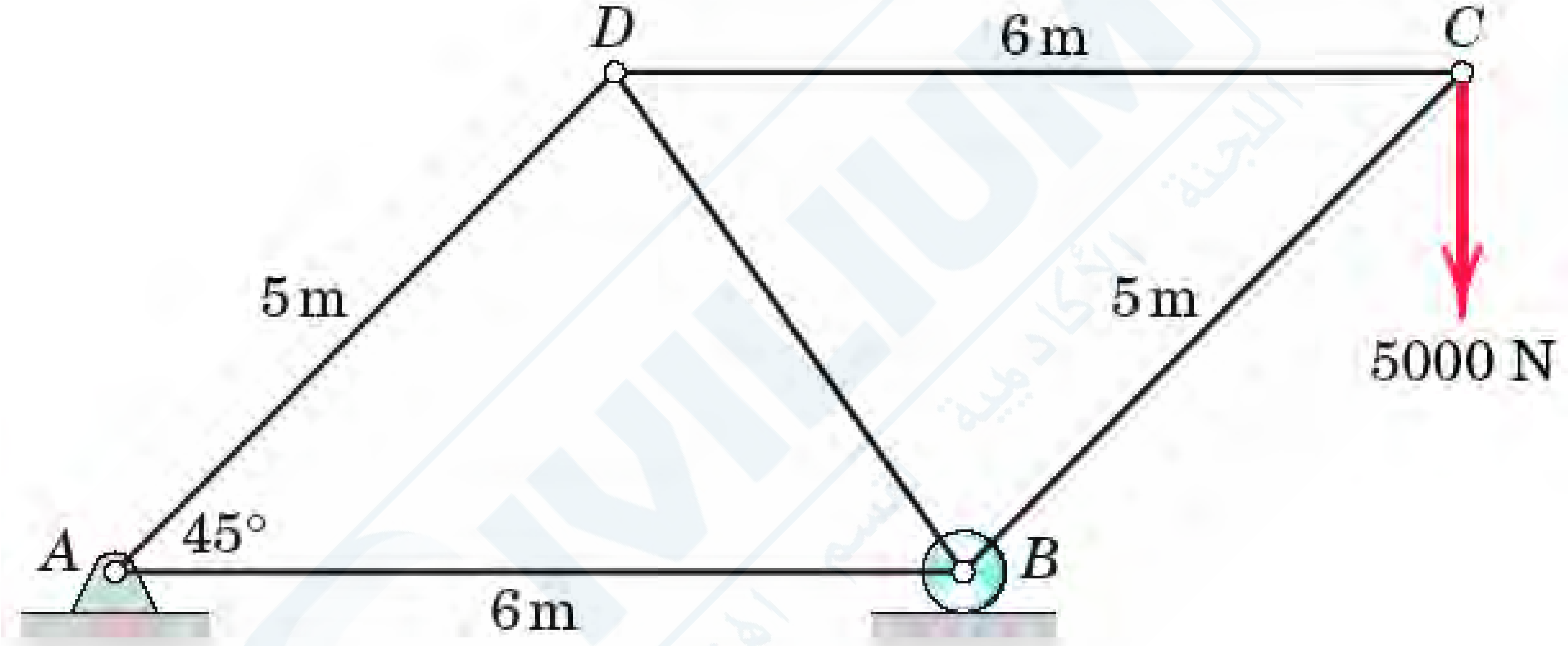
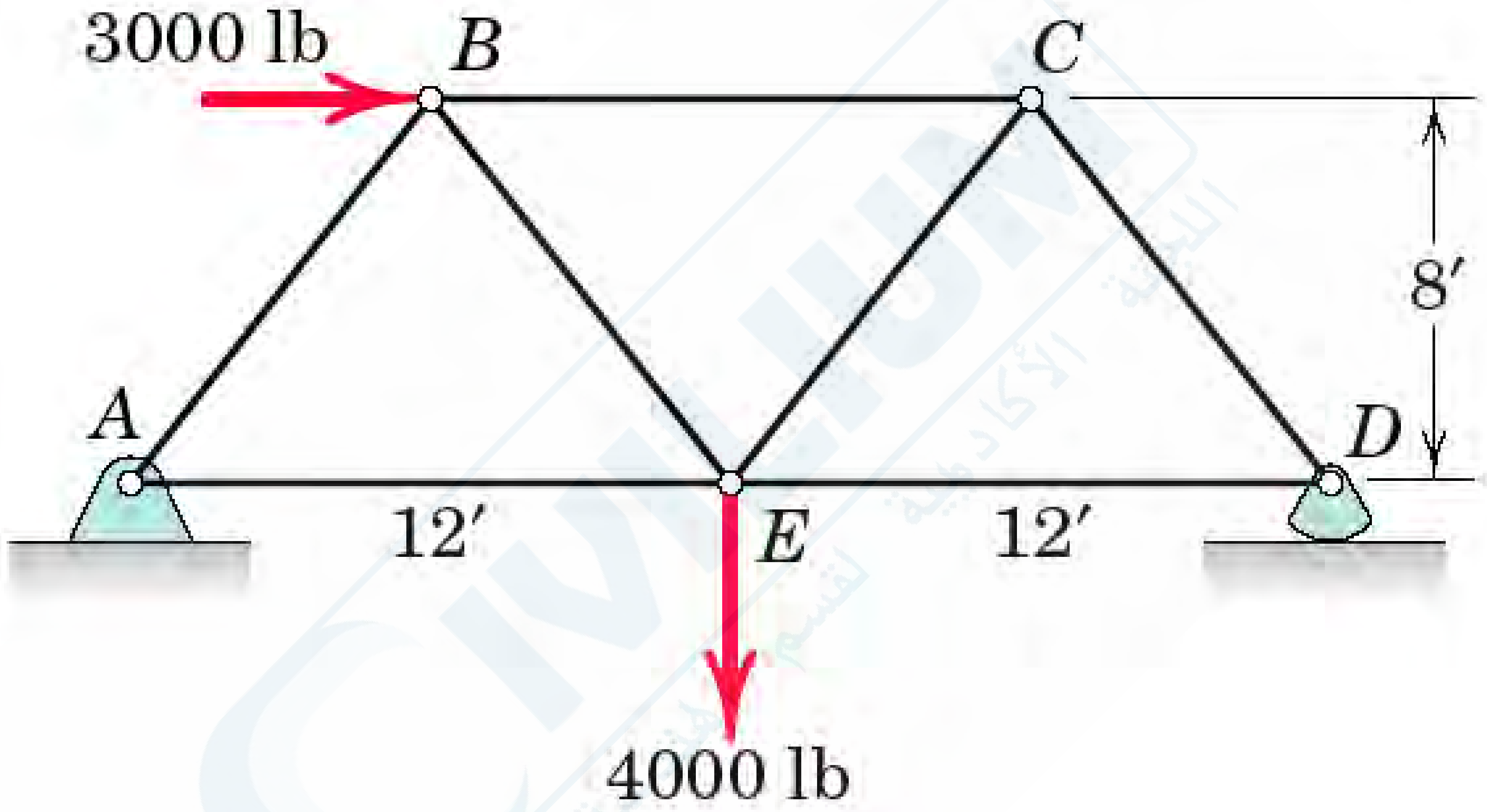


Figure 4/16



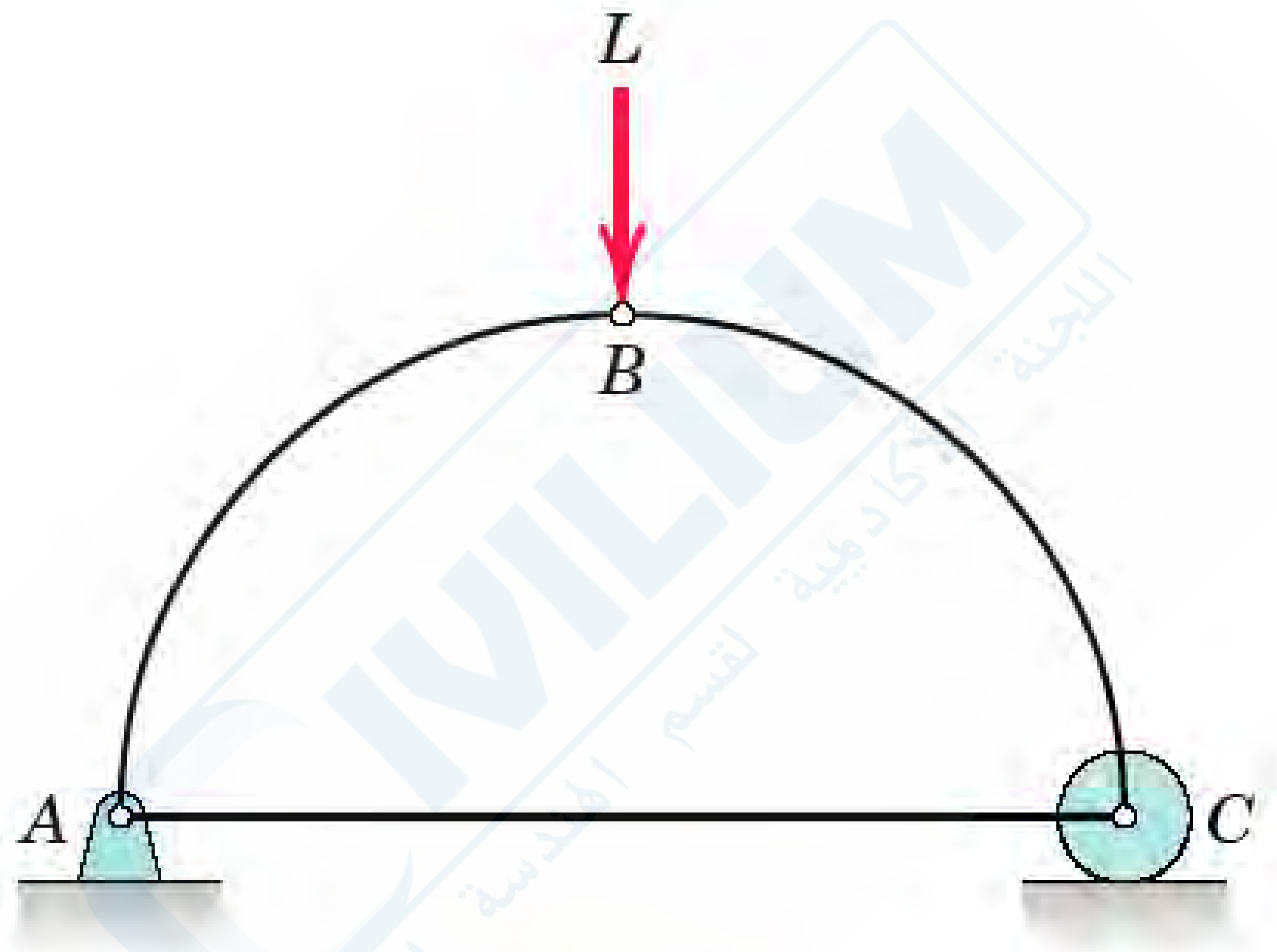
Problem 4/4

P_04_04



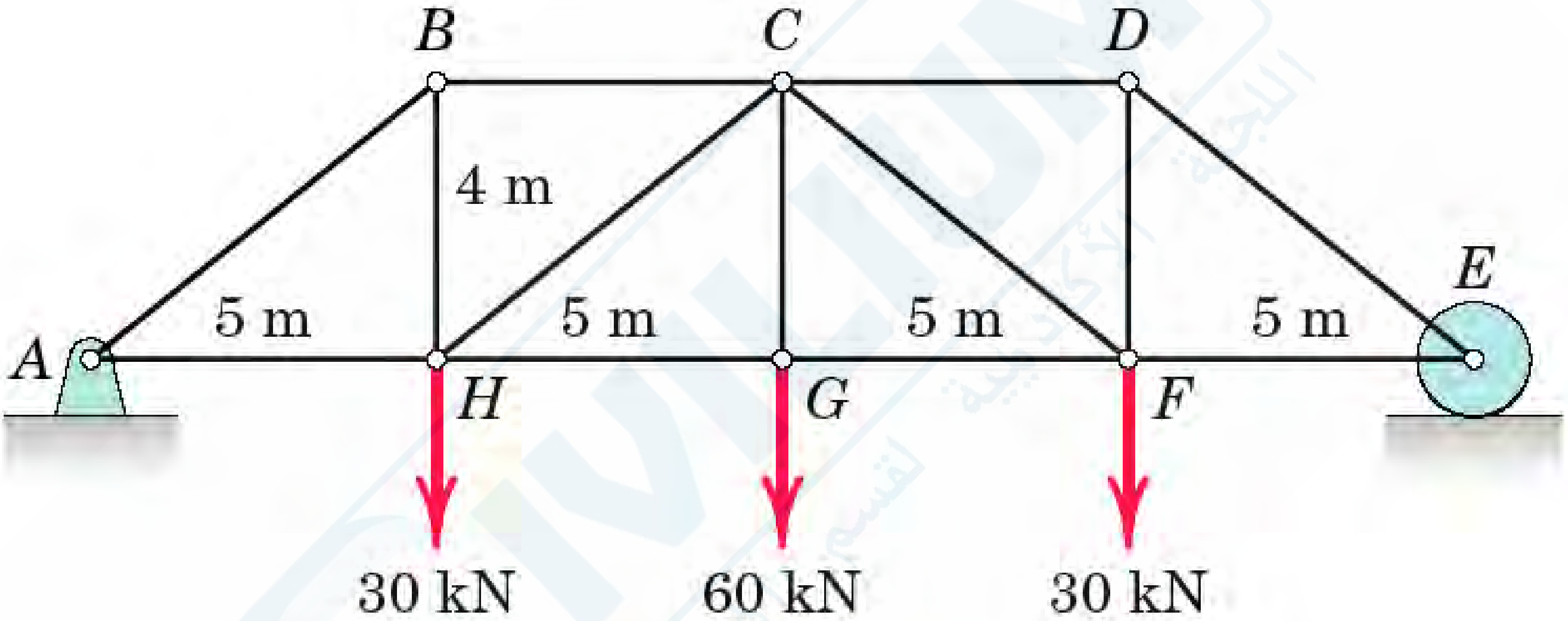
Problem 4/6

P_04_06



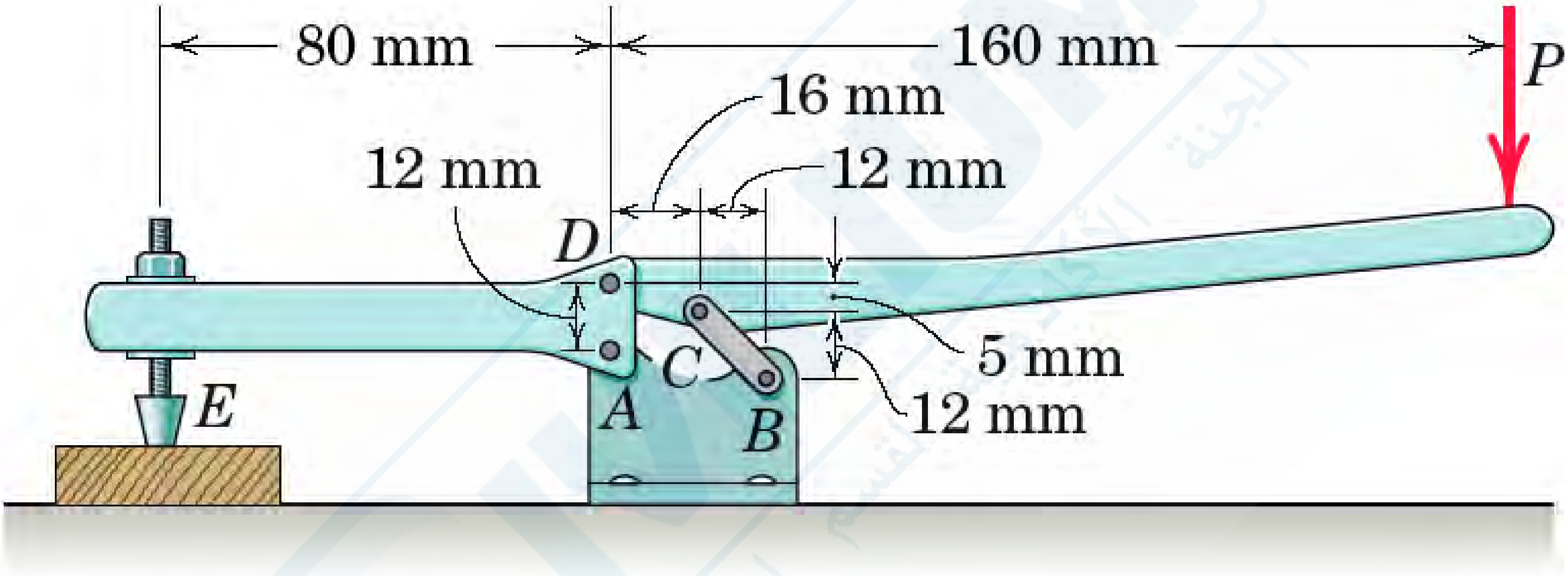
Problem 4/7

P_04_07



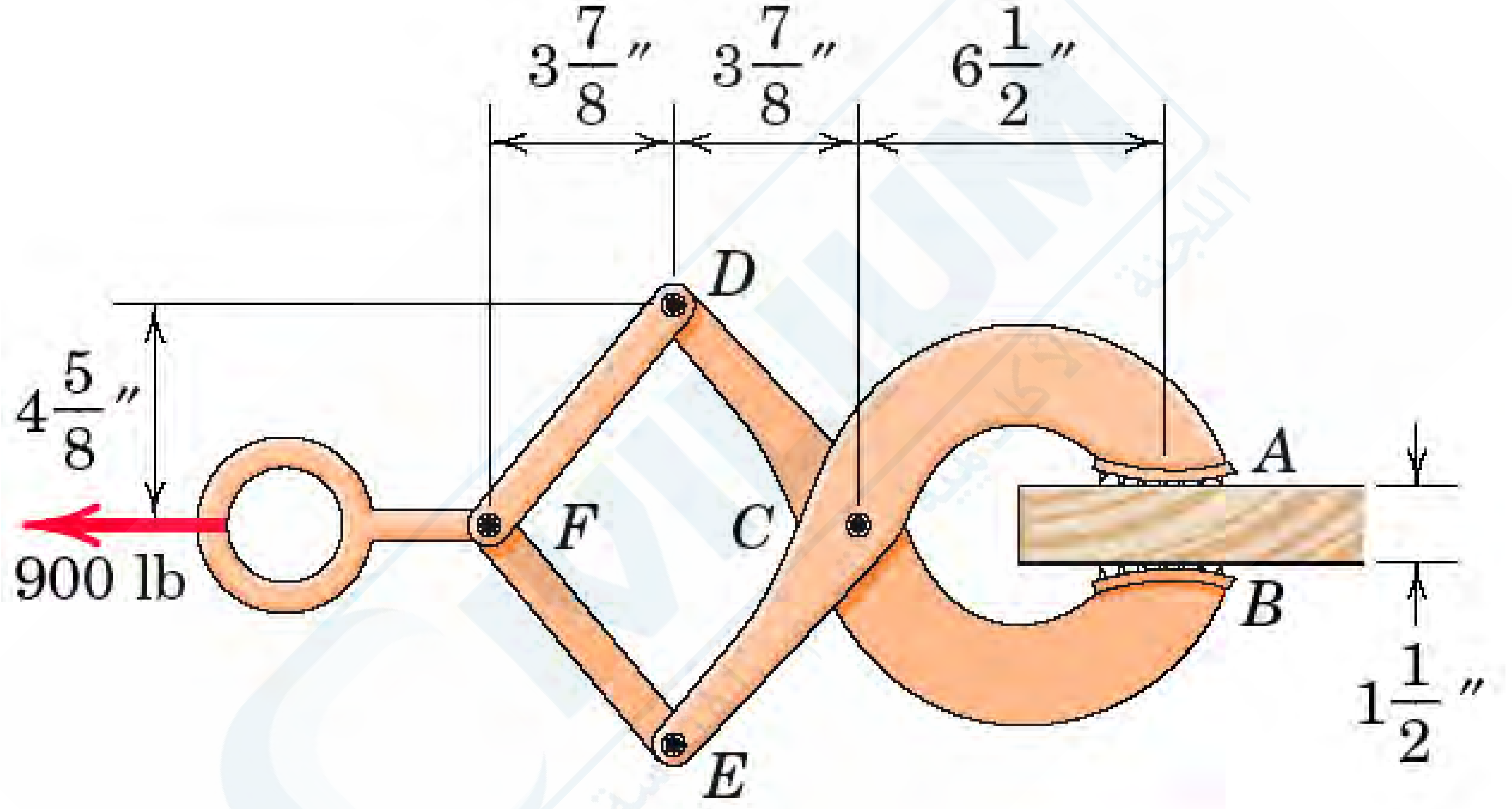
Problem 4/8

P_04_08



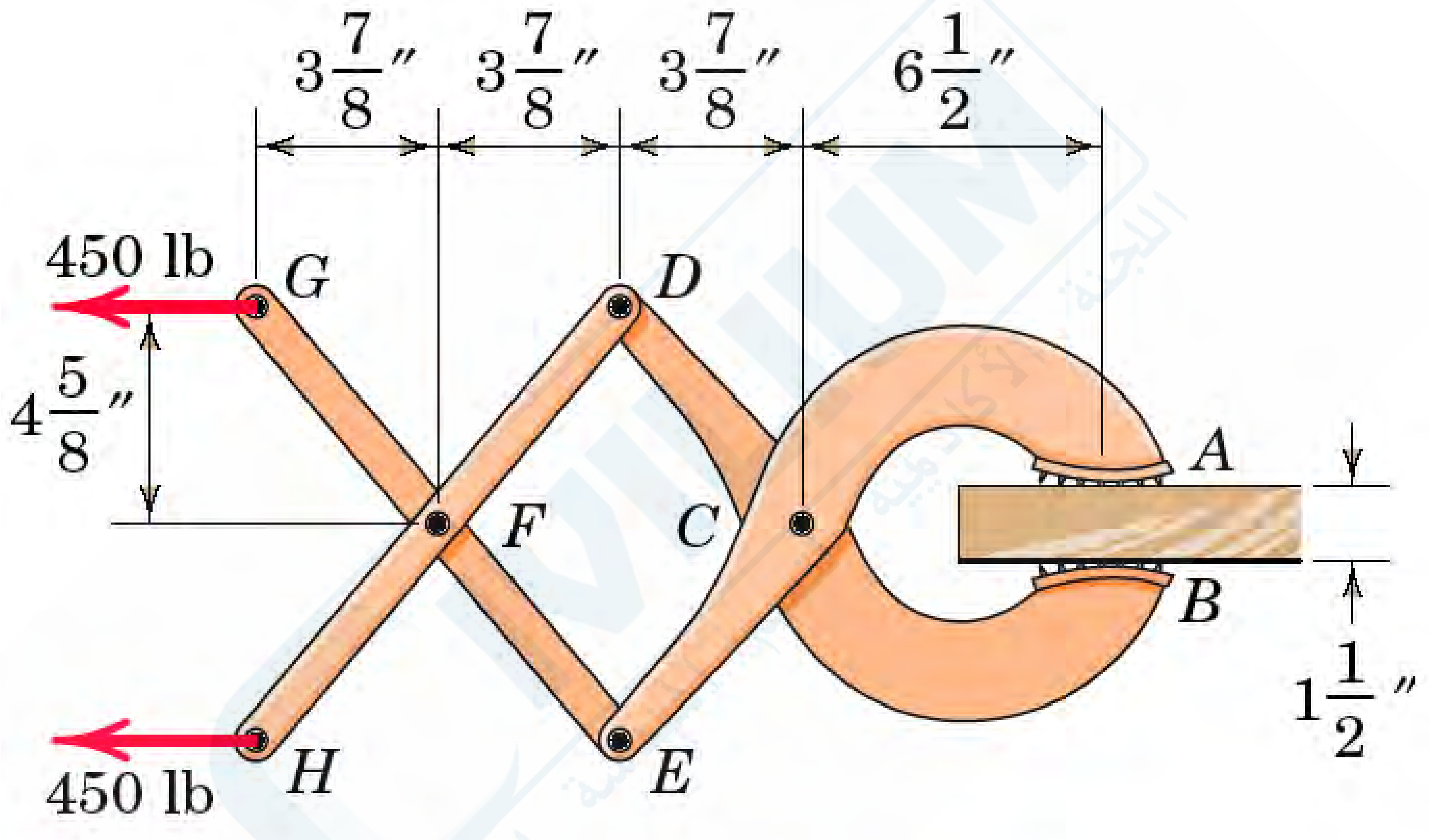
Problem 4/100

P_04_100



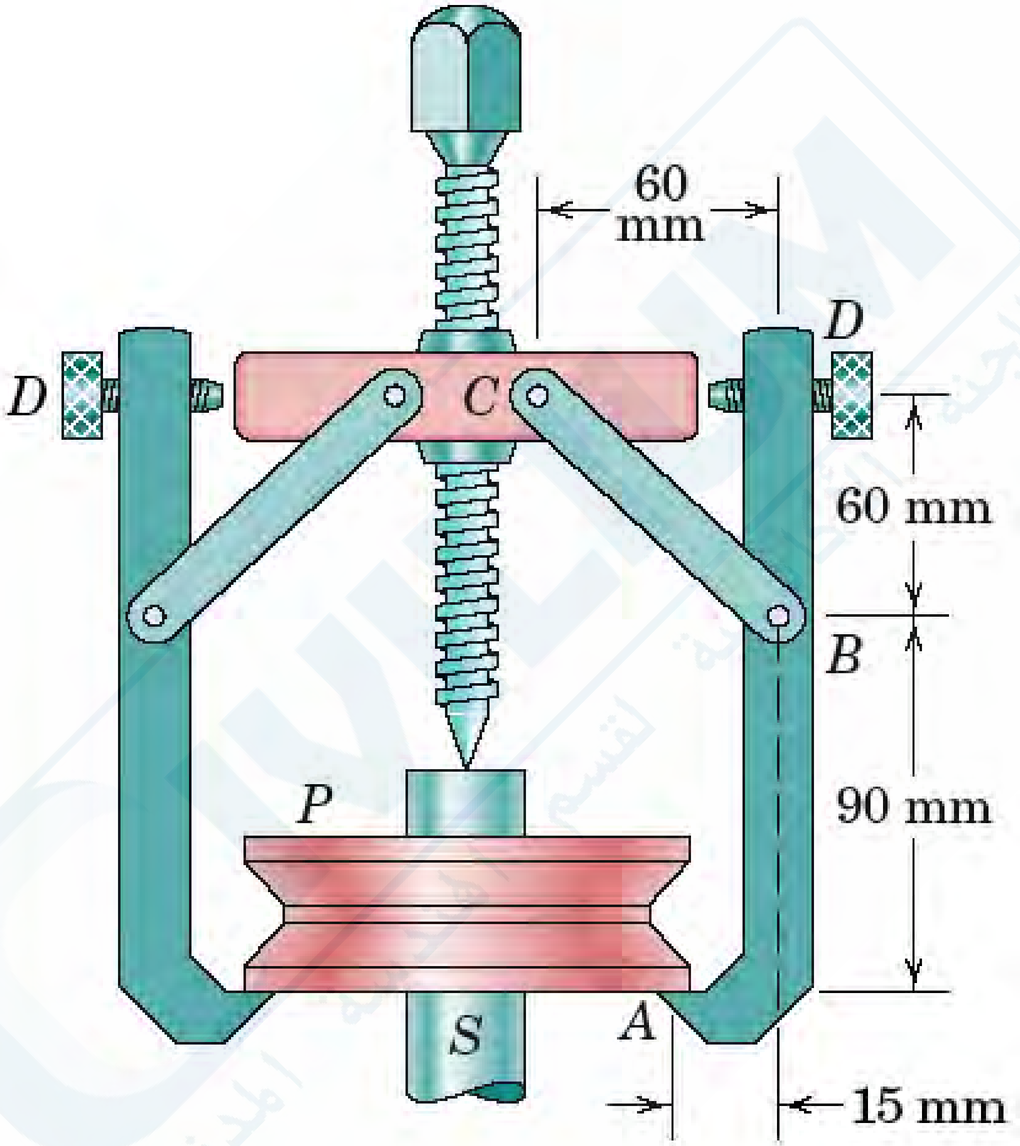
Problem 4/101

P_04_101



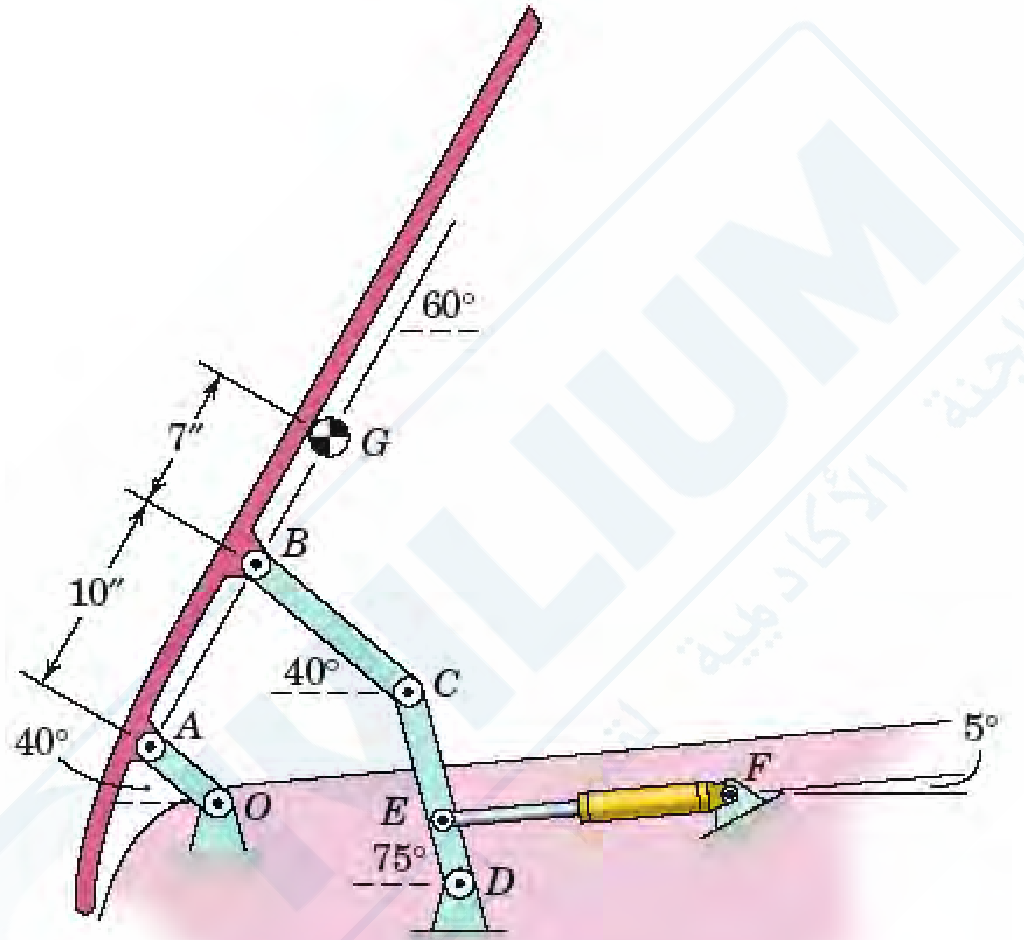
Problem 4/102

P_04_102



Problem 4/103

P 04 103

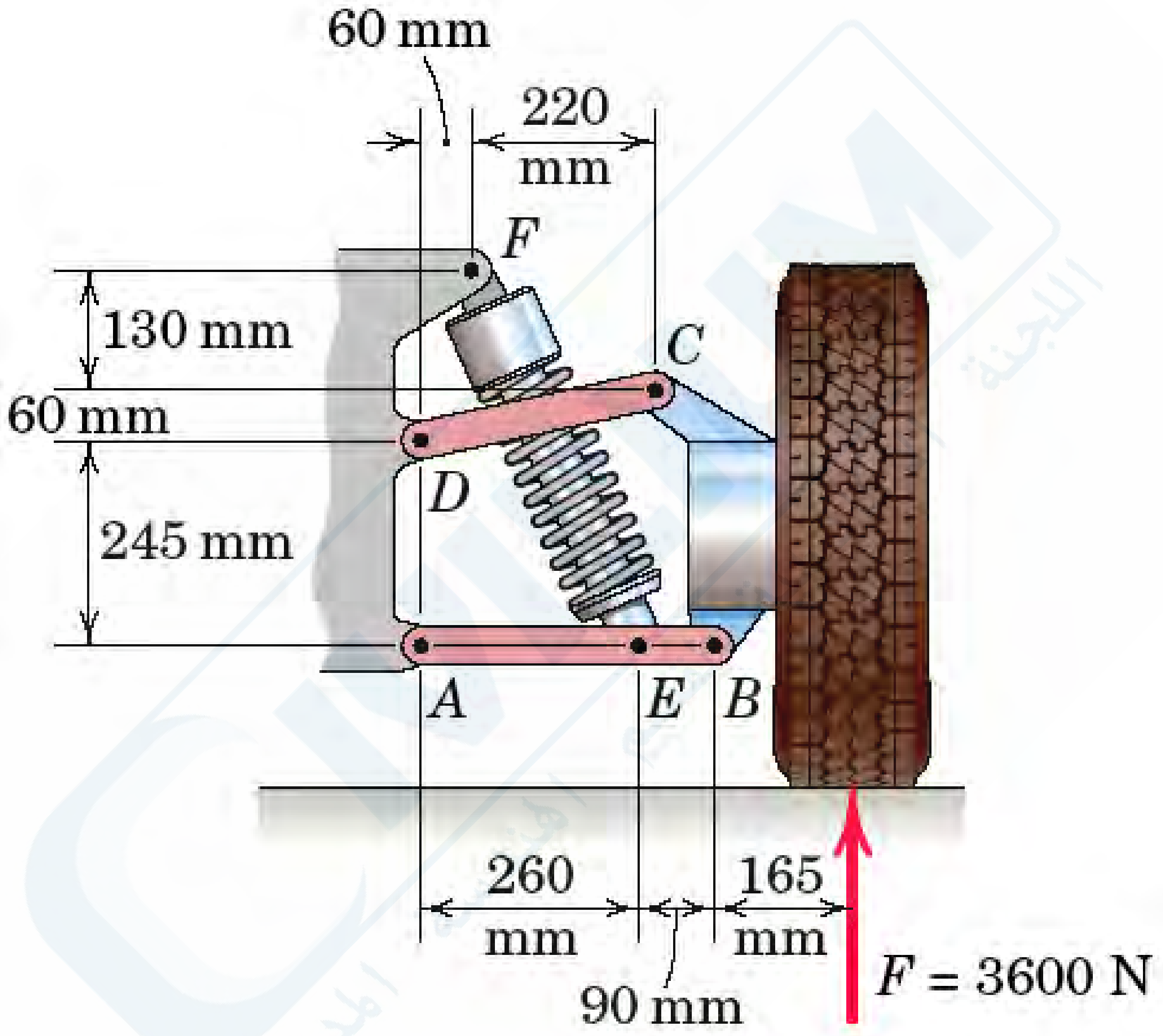


$\overline{AO} = 4''$ $\overline{DE} = 3''$

$\overline{BC} = \overline{CD} = 9''$

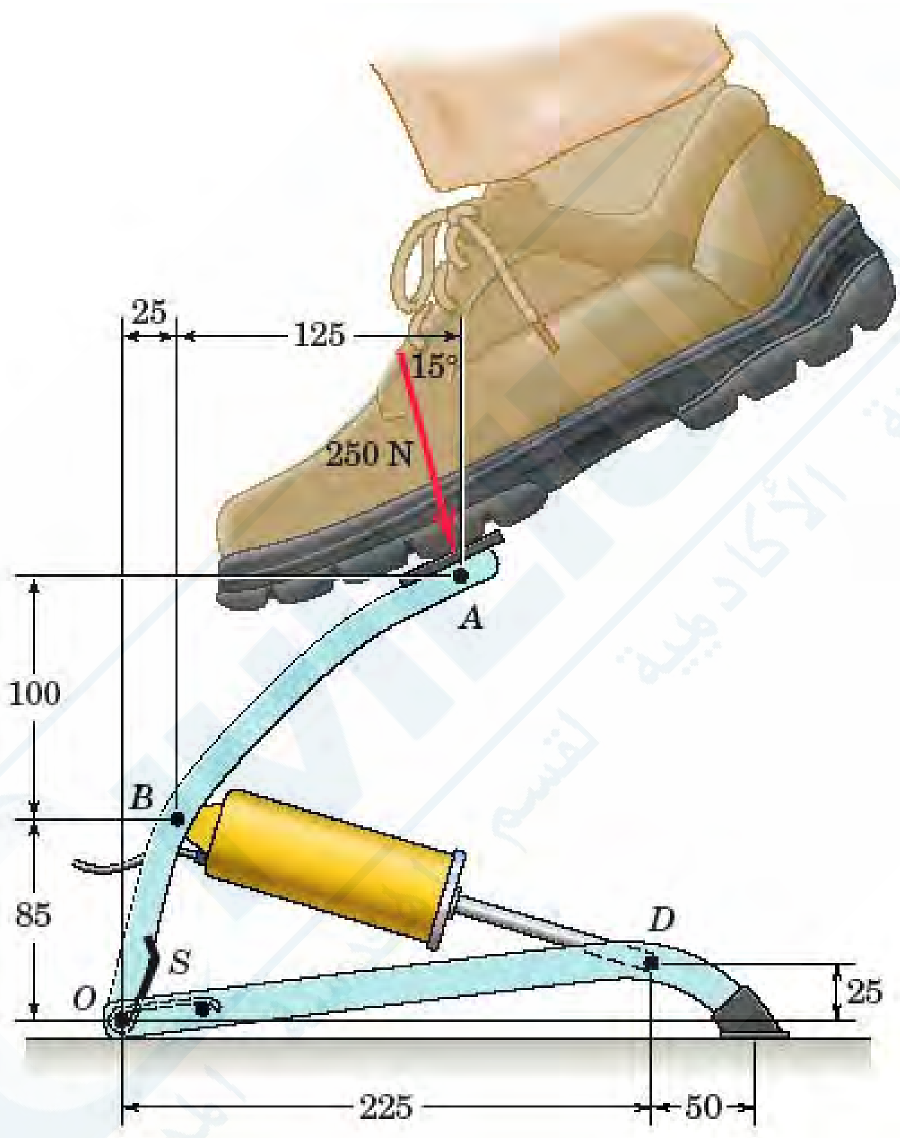
Problem 4/104

P 04 104



Problem 4/105

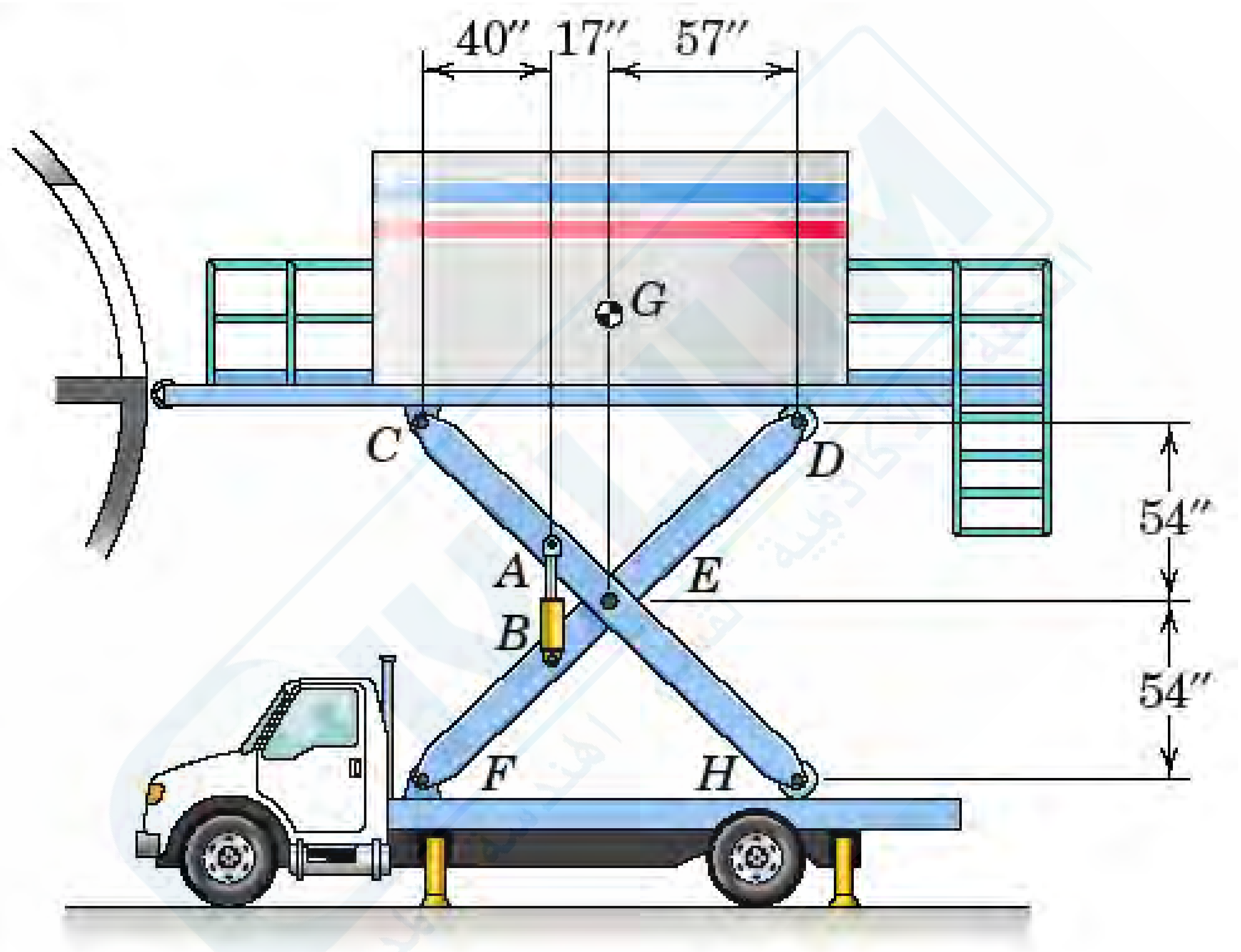
P 04 105



Dimensions in millimeters

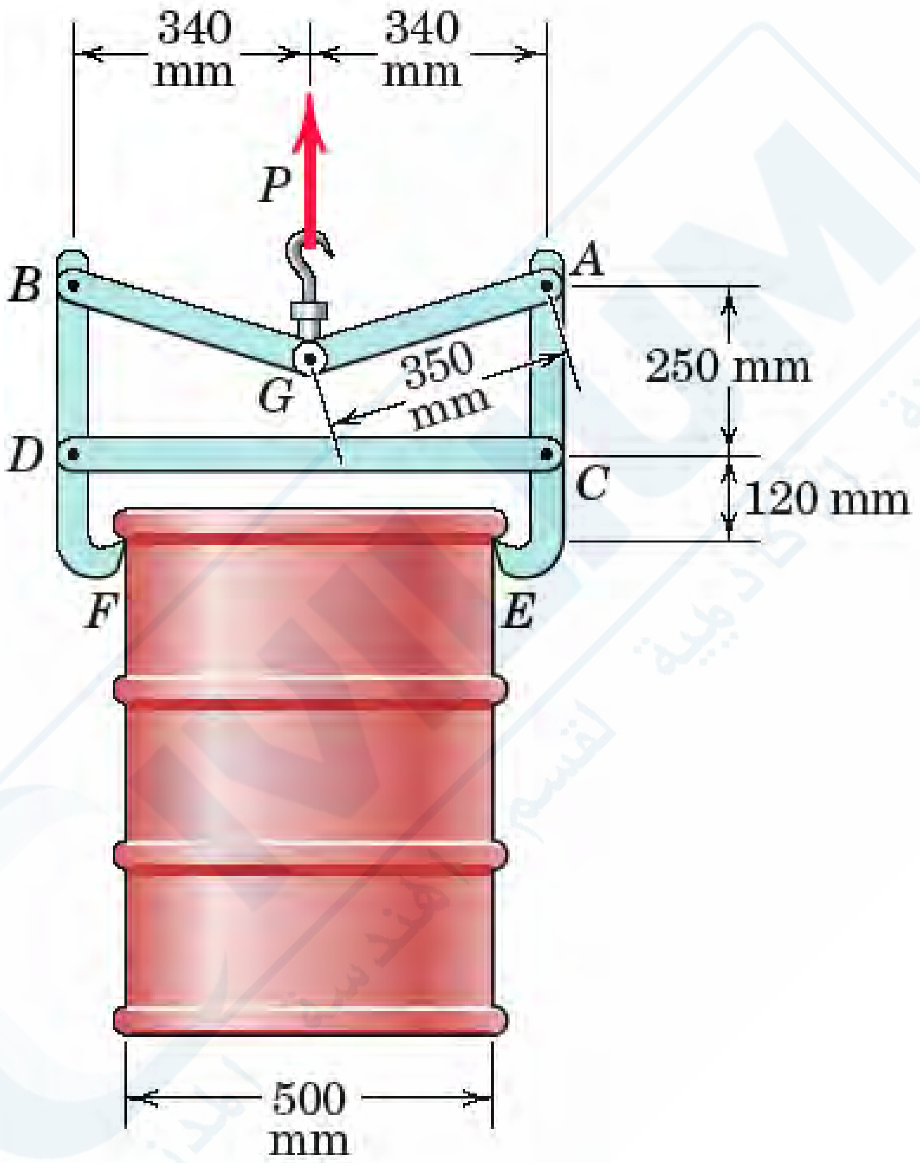
Problem 4/106

P 04 106



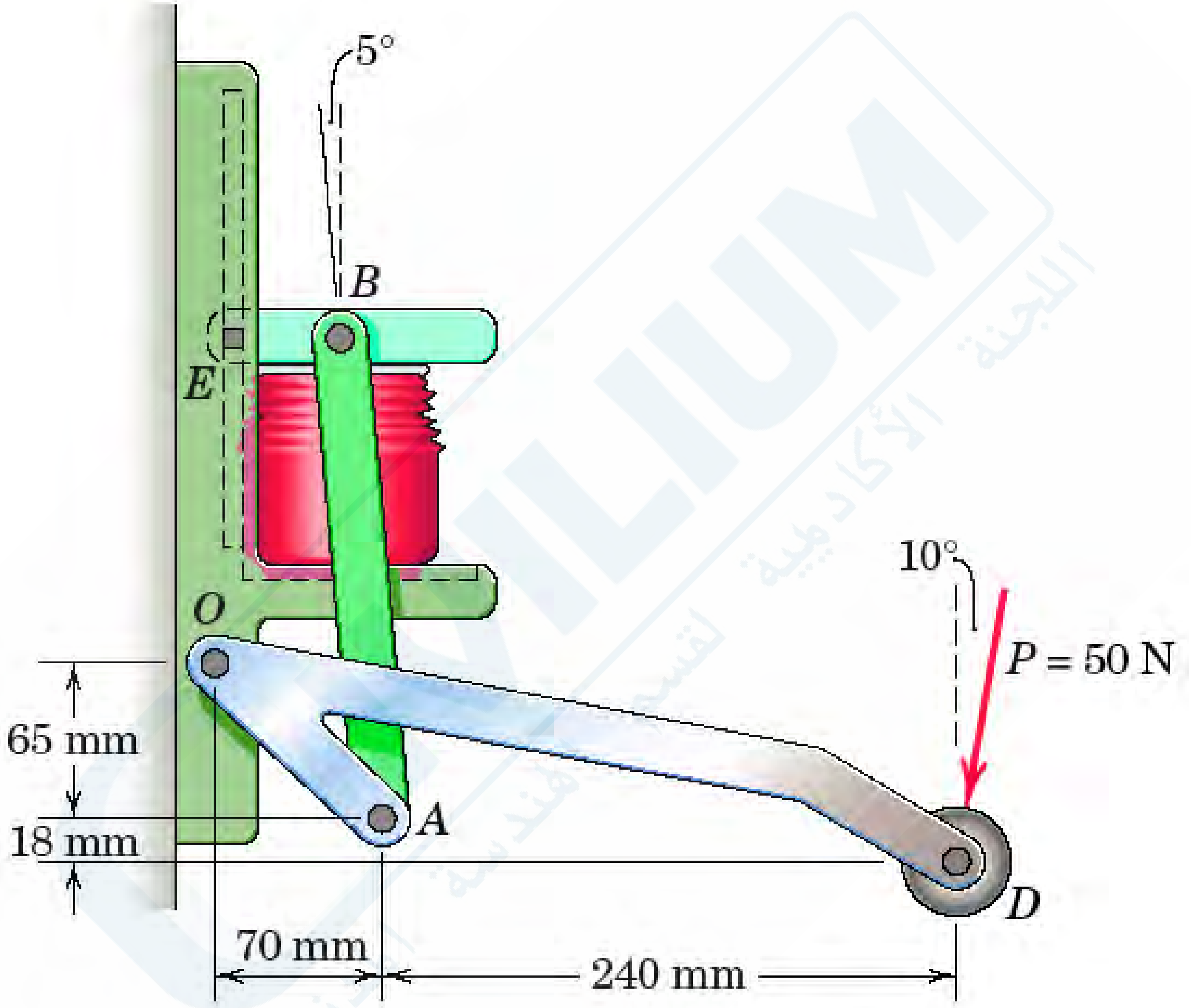
Problem 4/107

P_04_107



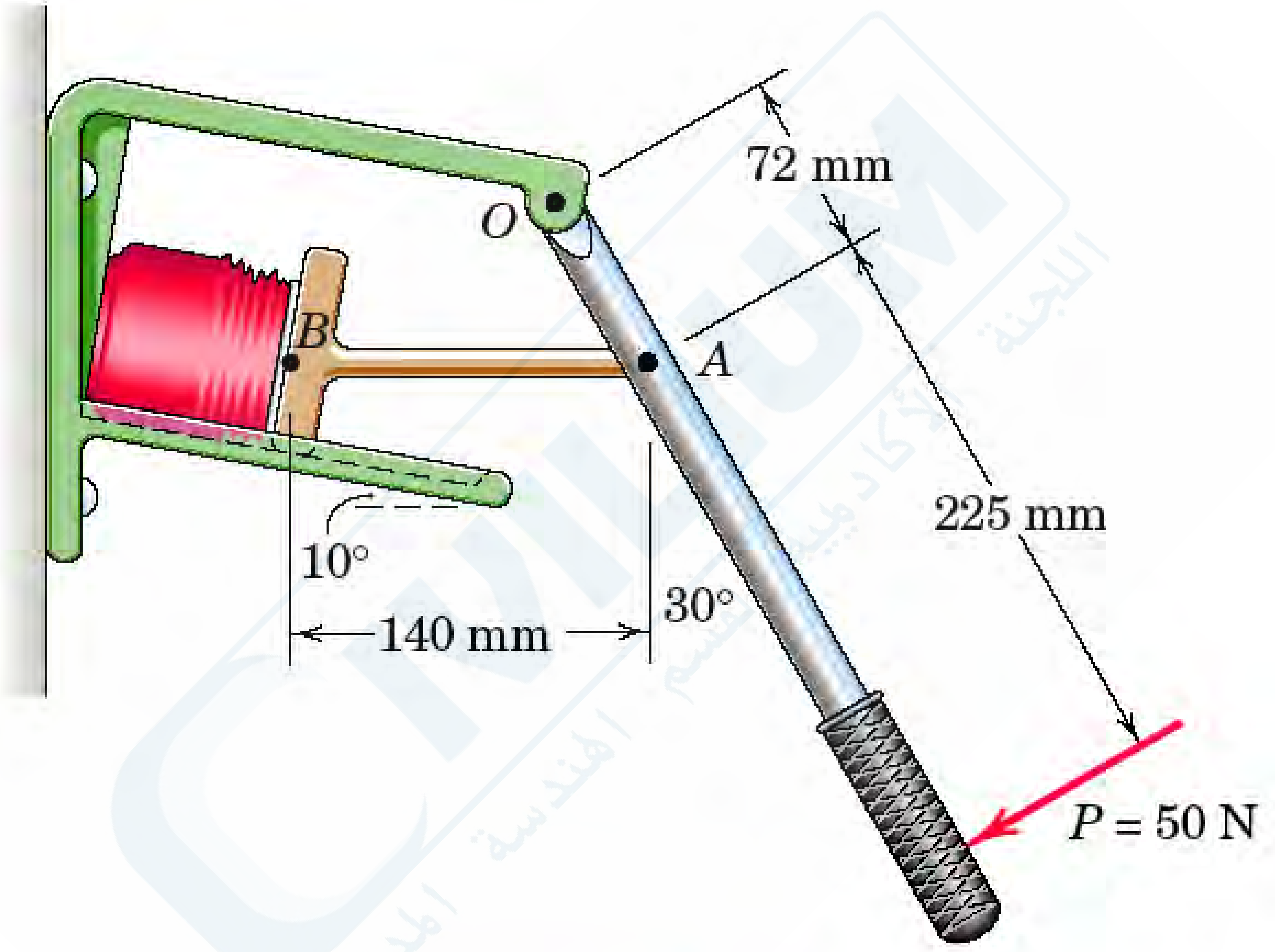
Problem 4/108

P 04 108



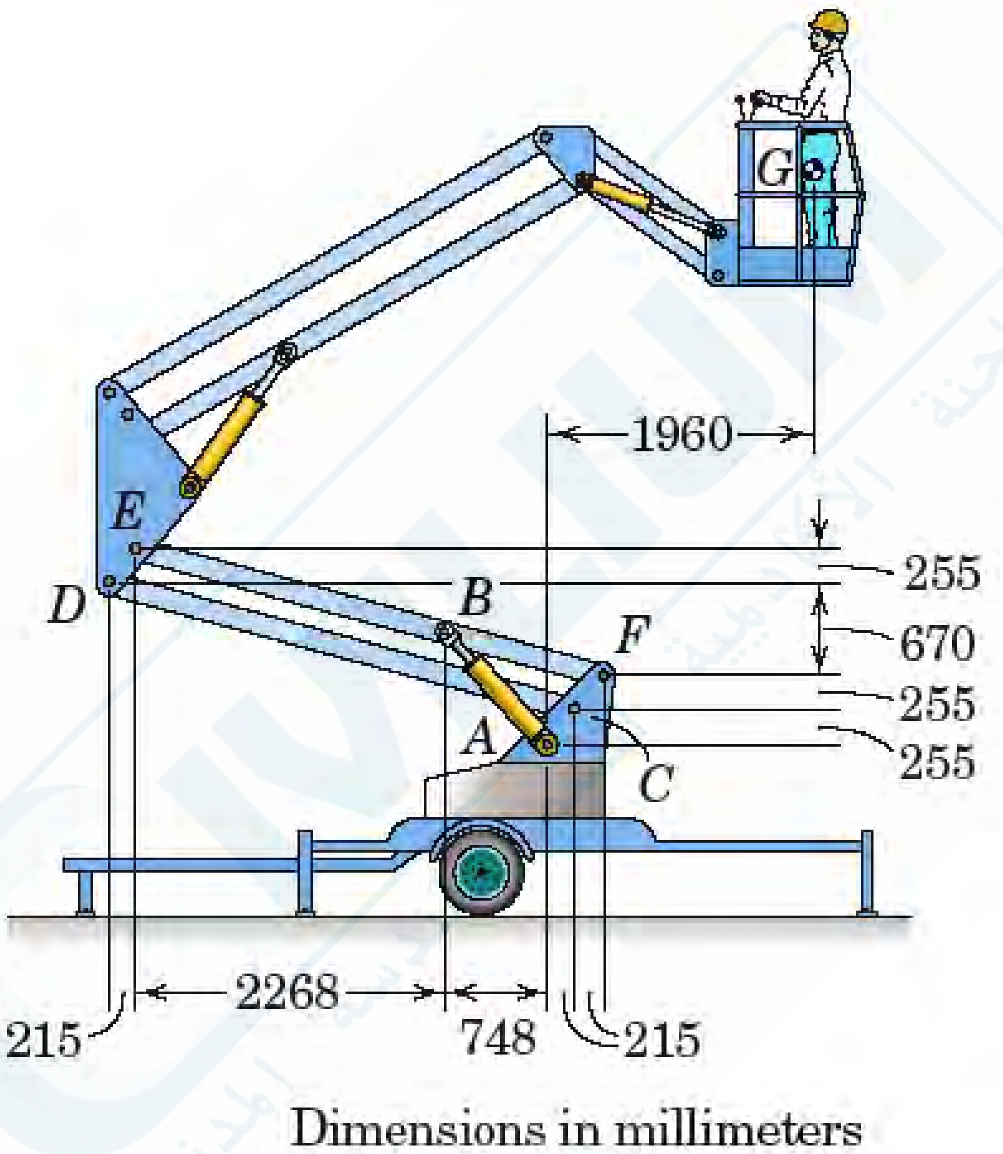
Problem 4/109

P_04_109



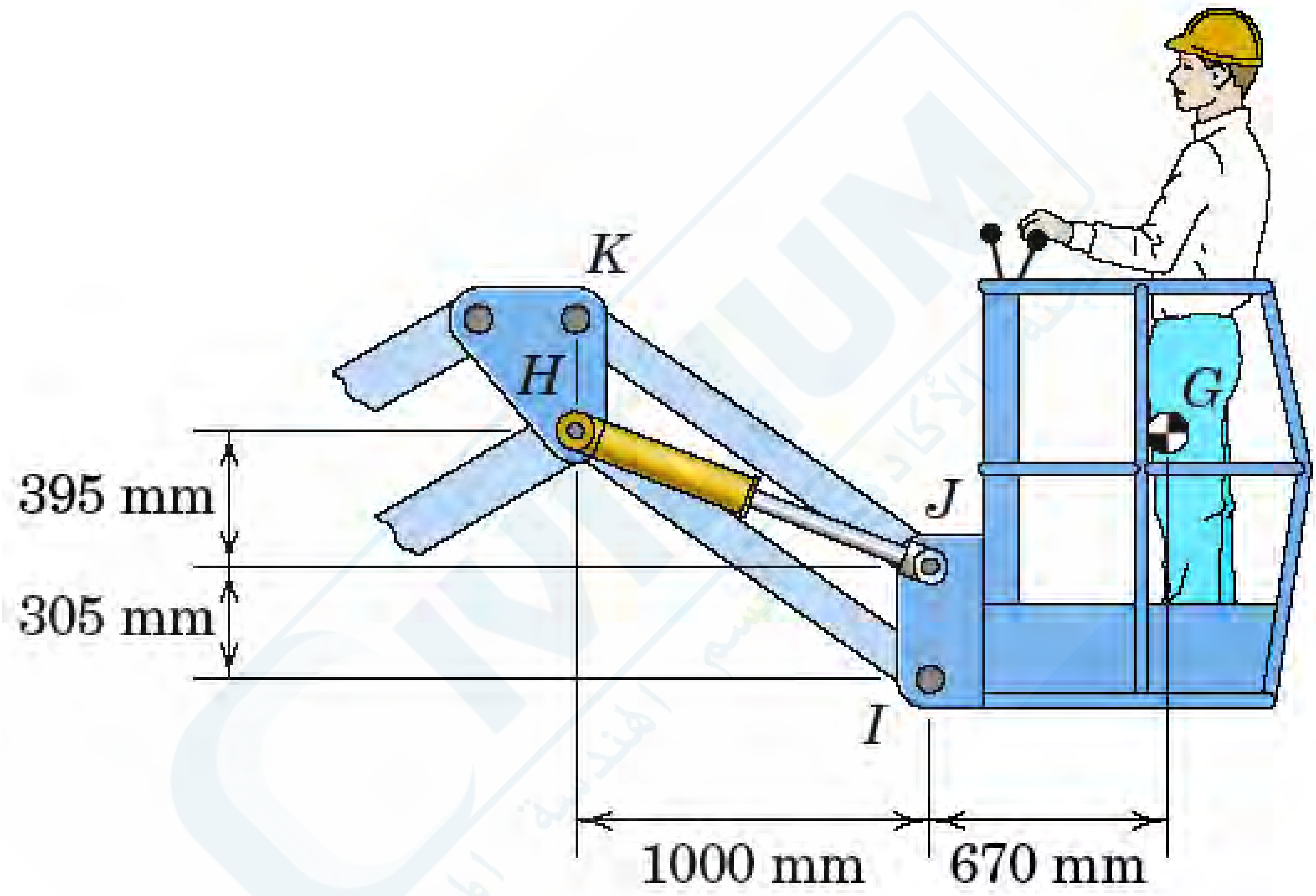
Problem 4/110

P 04 110



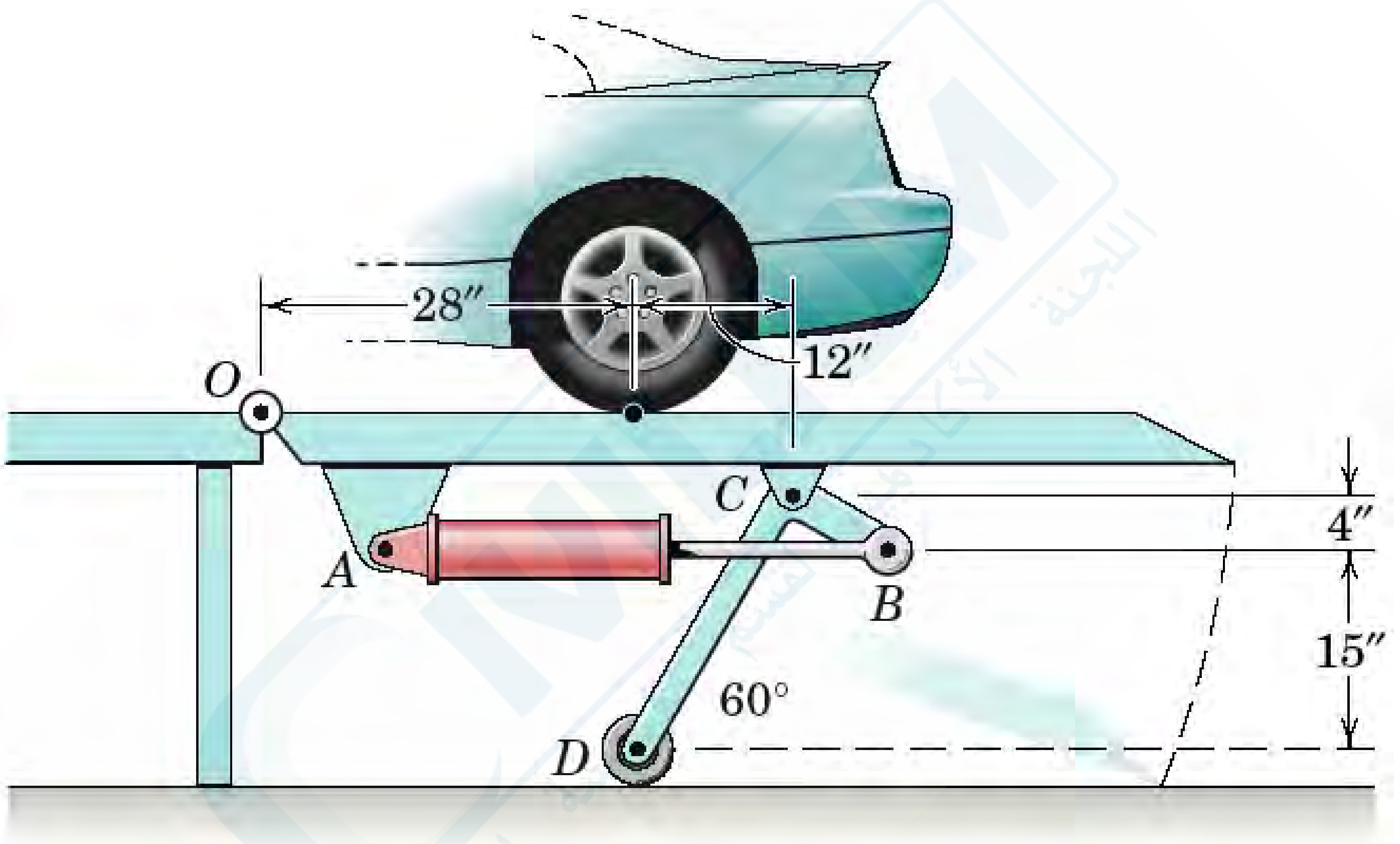
Problem 4/111

P 04 111



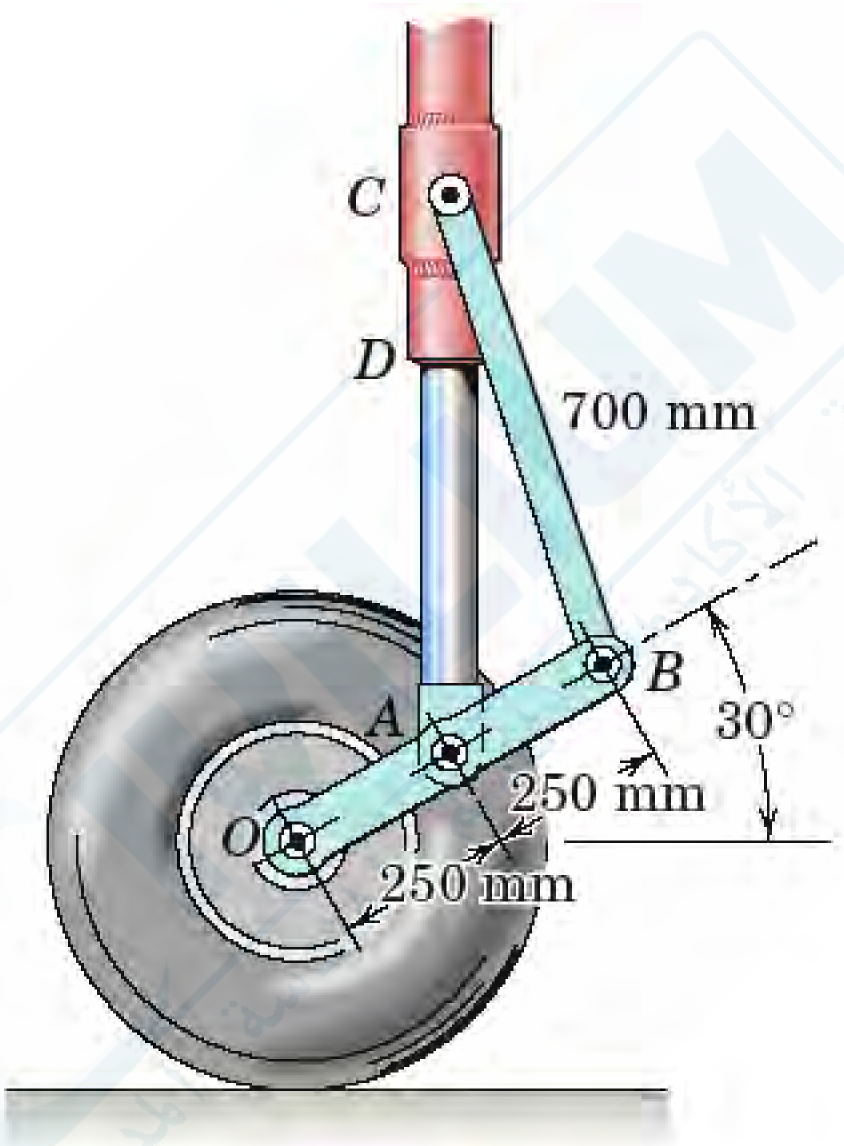
Problem 4/112

P 04 112



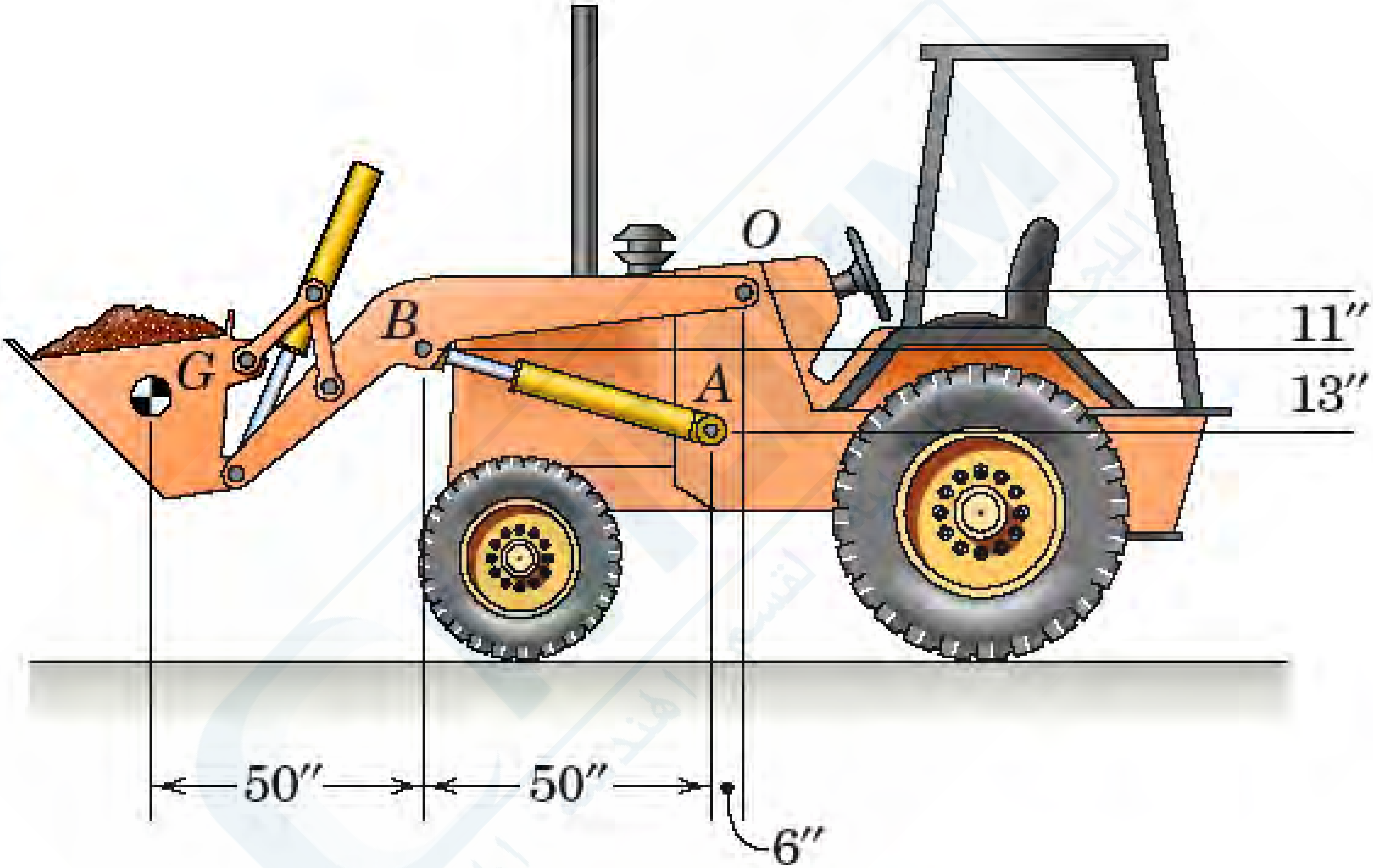
Problem 4/113

P_04_113



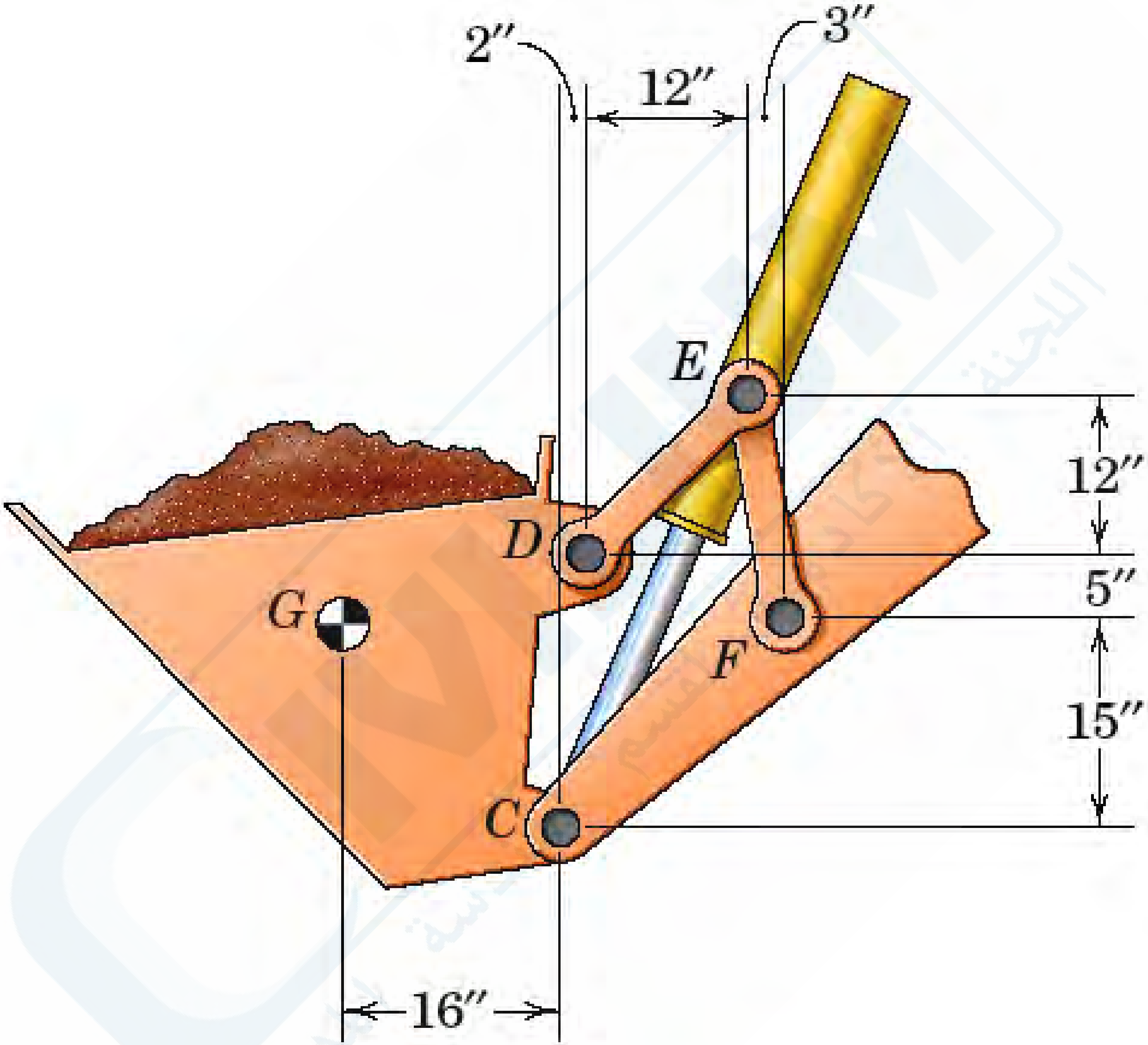
Problem 4/114

P_04_114



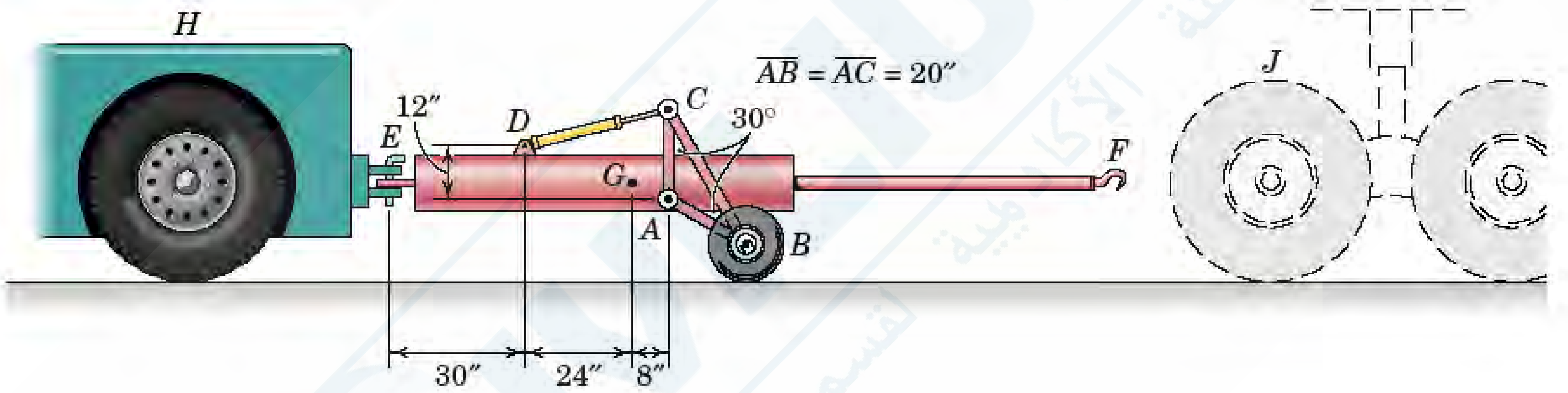
Problem 4/115

P_04_115



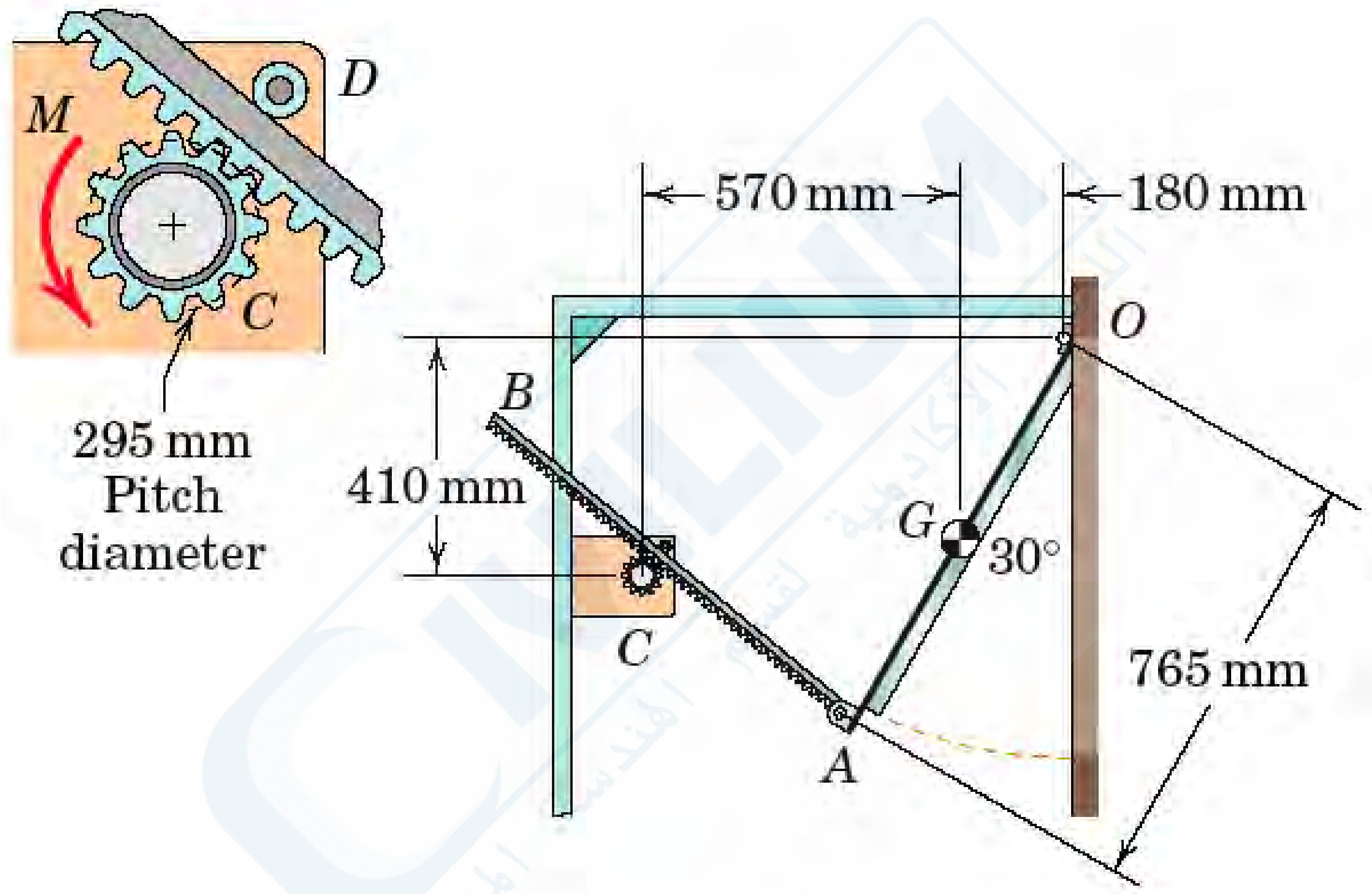
Problem 4/116

P_04_116

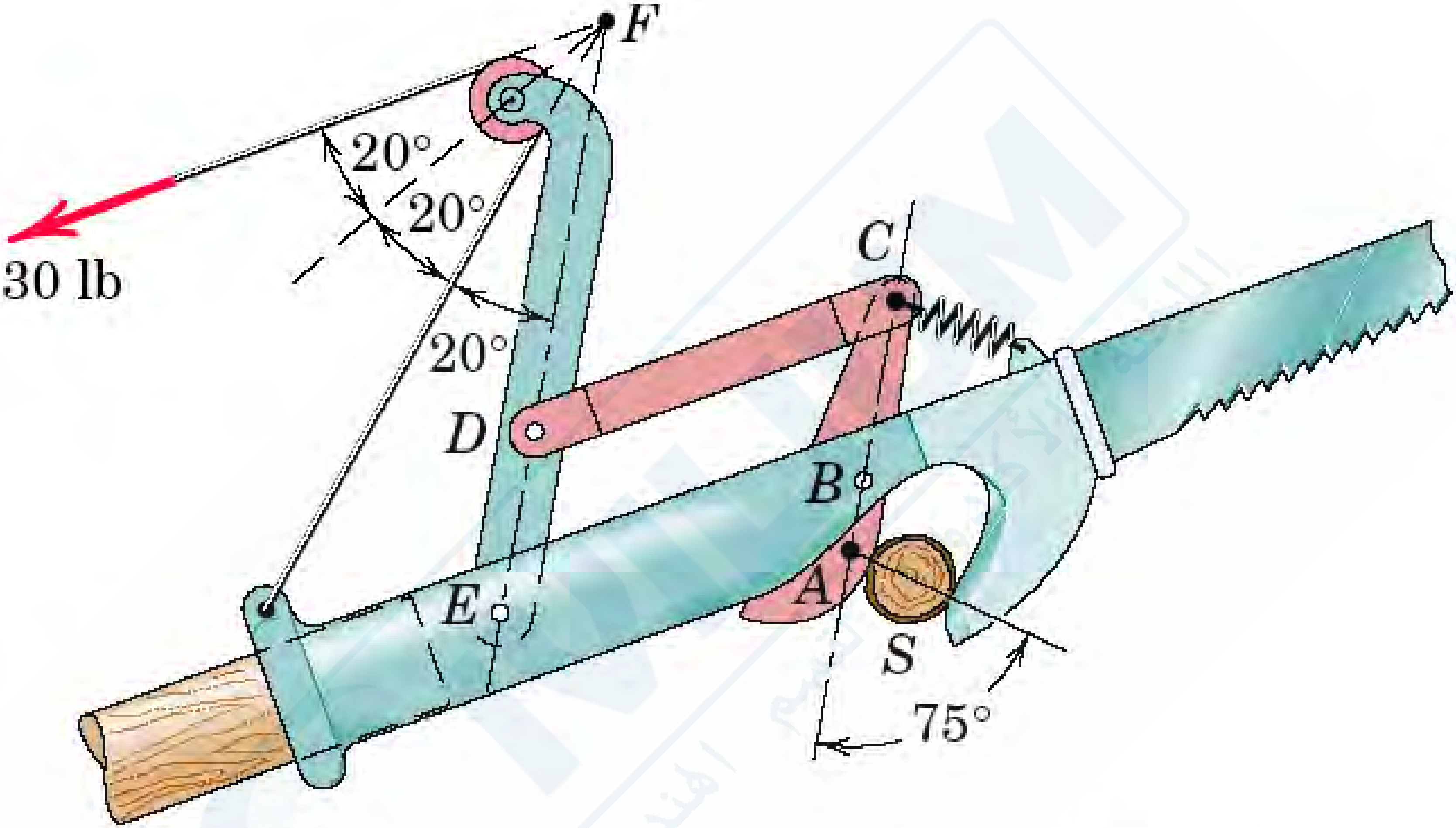


Problem 4/117

P_04_117

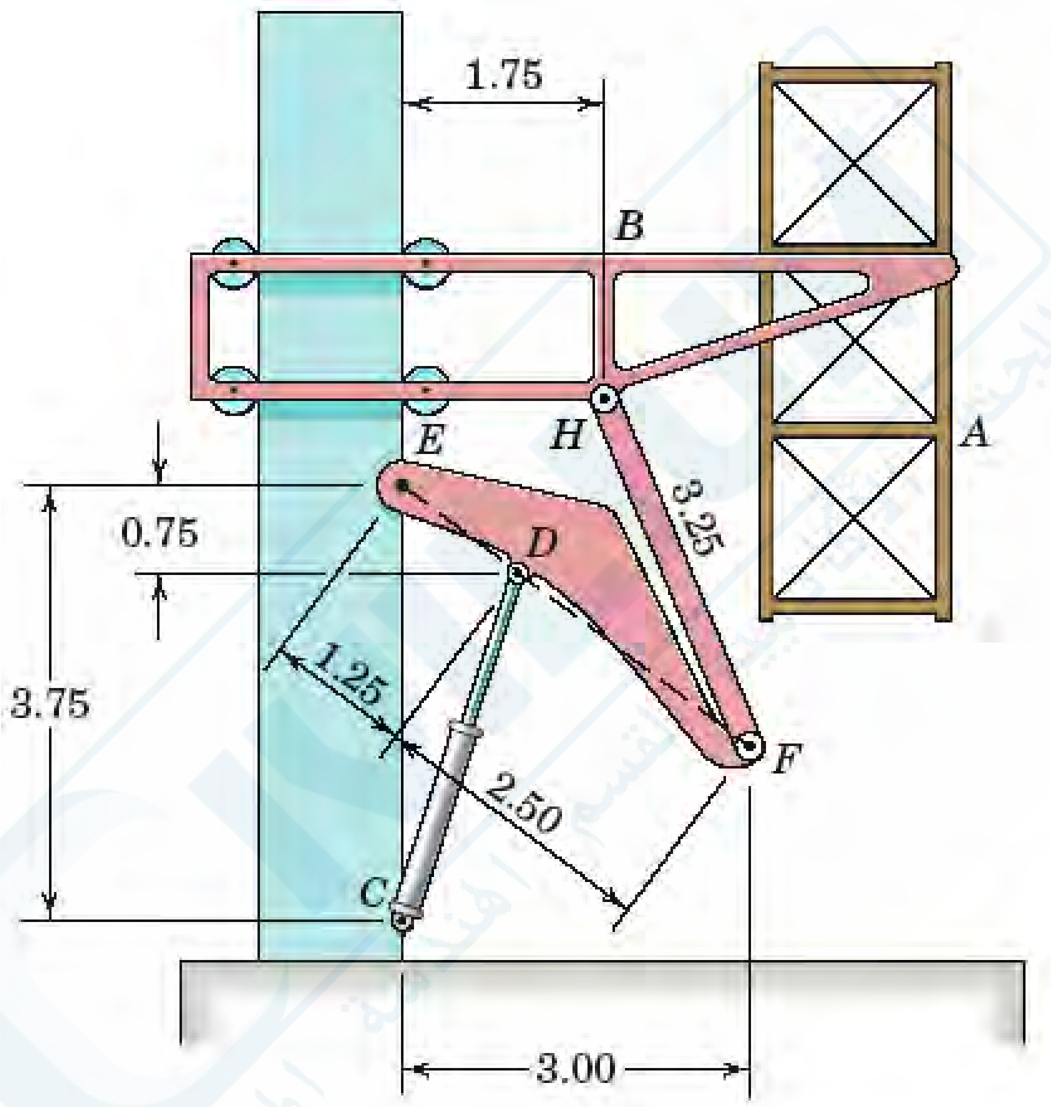


Problem 4/118



$\overline{AB} = 1''$, $\overline{BC} = \overline{ED} = 3''$, $\overline{EB} = \overline{DC} = 4\frac{1}{2}''$, $\overline{DF} = 6''$

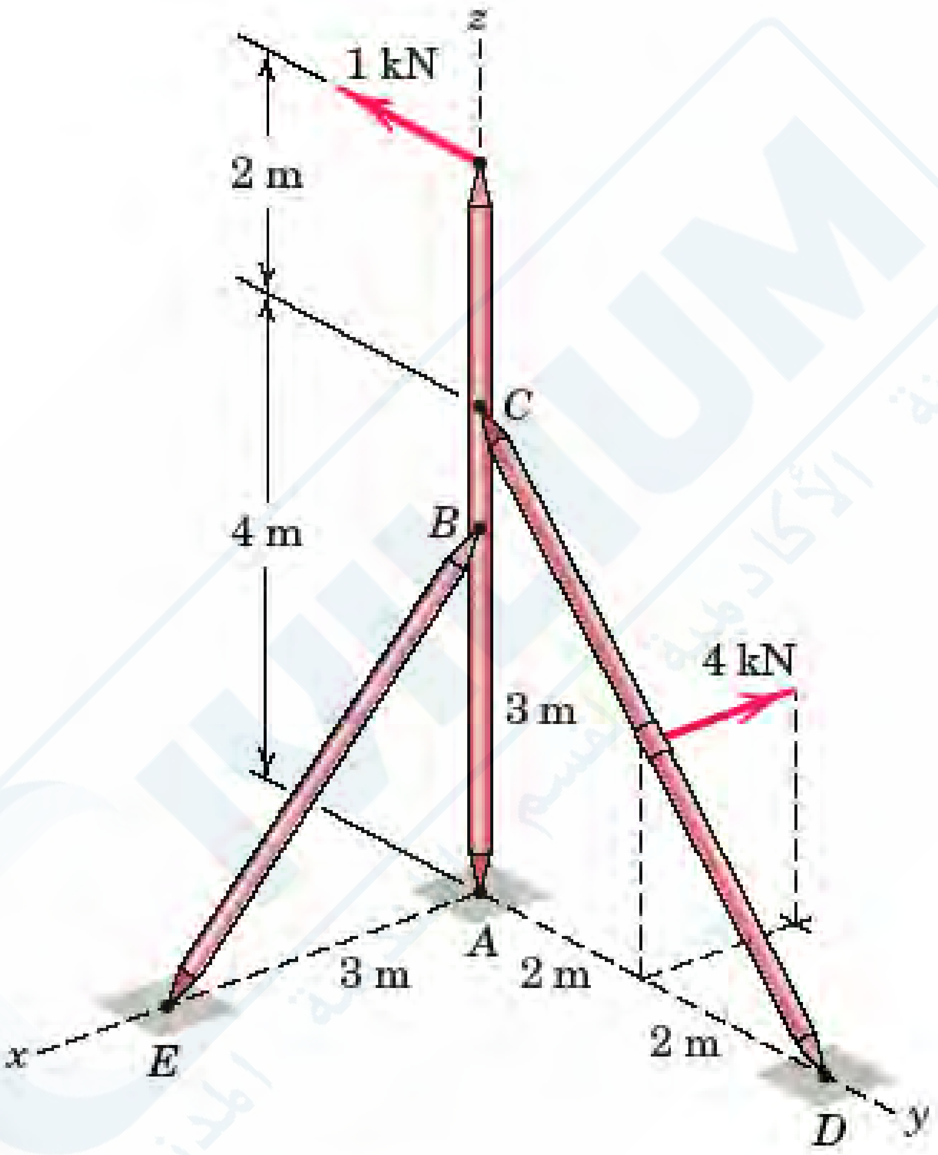
Problem 4/119



Dimensions in meters

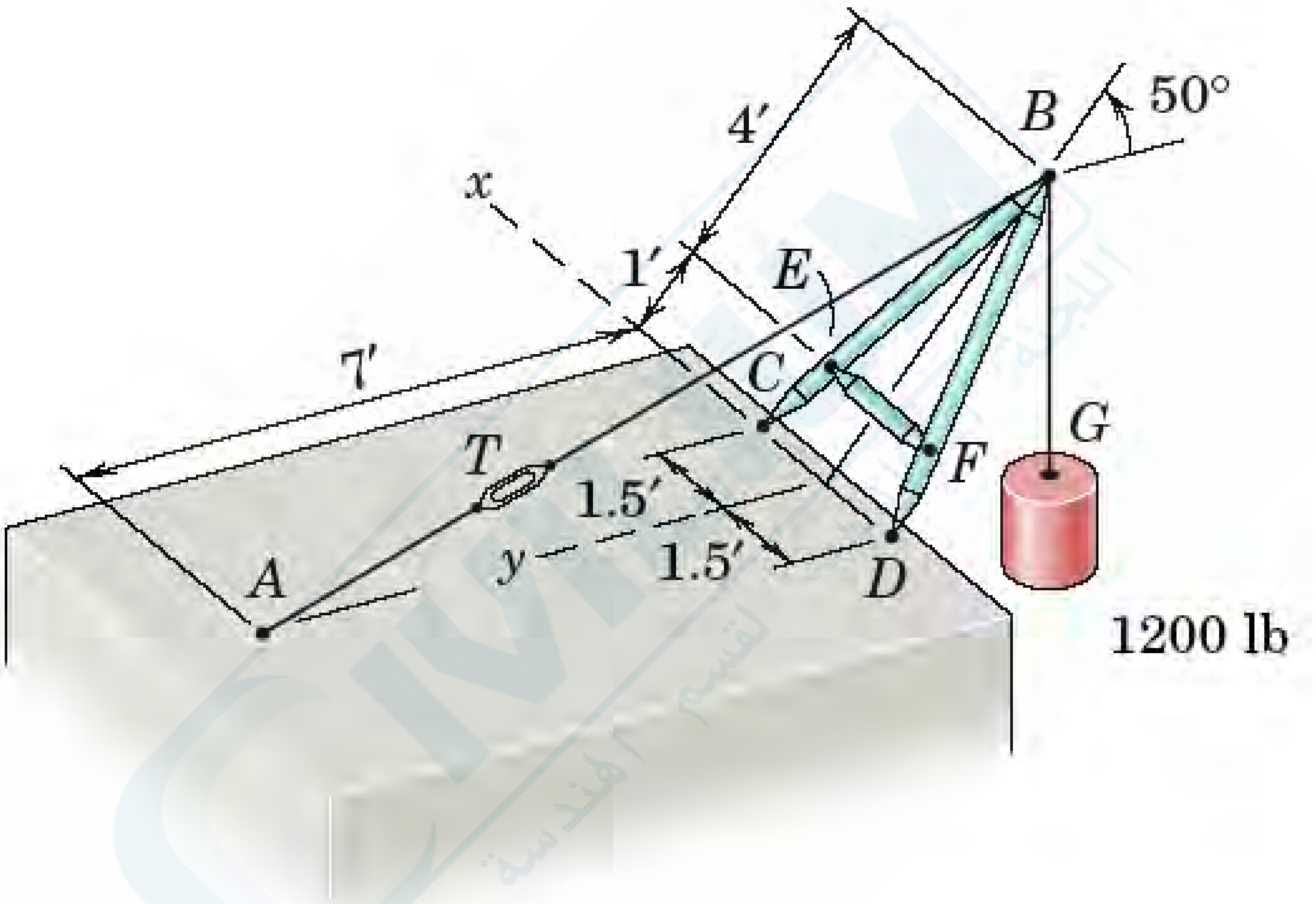
Problem 4/120

P 04 120



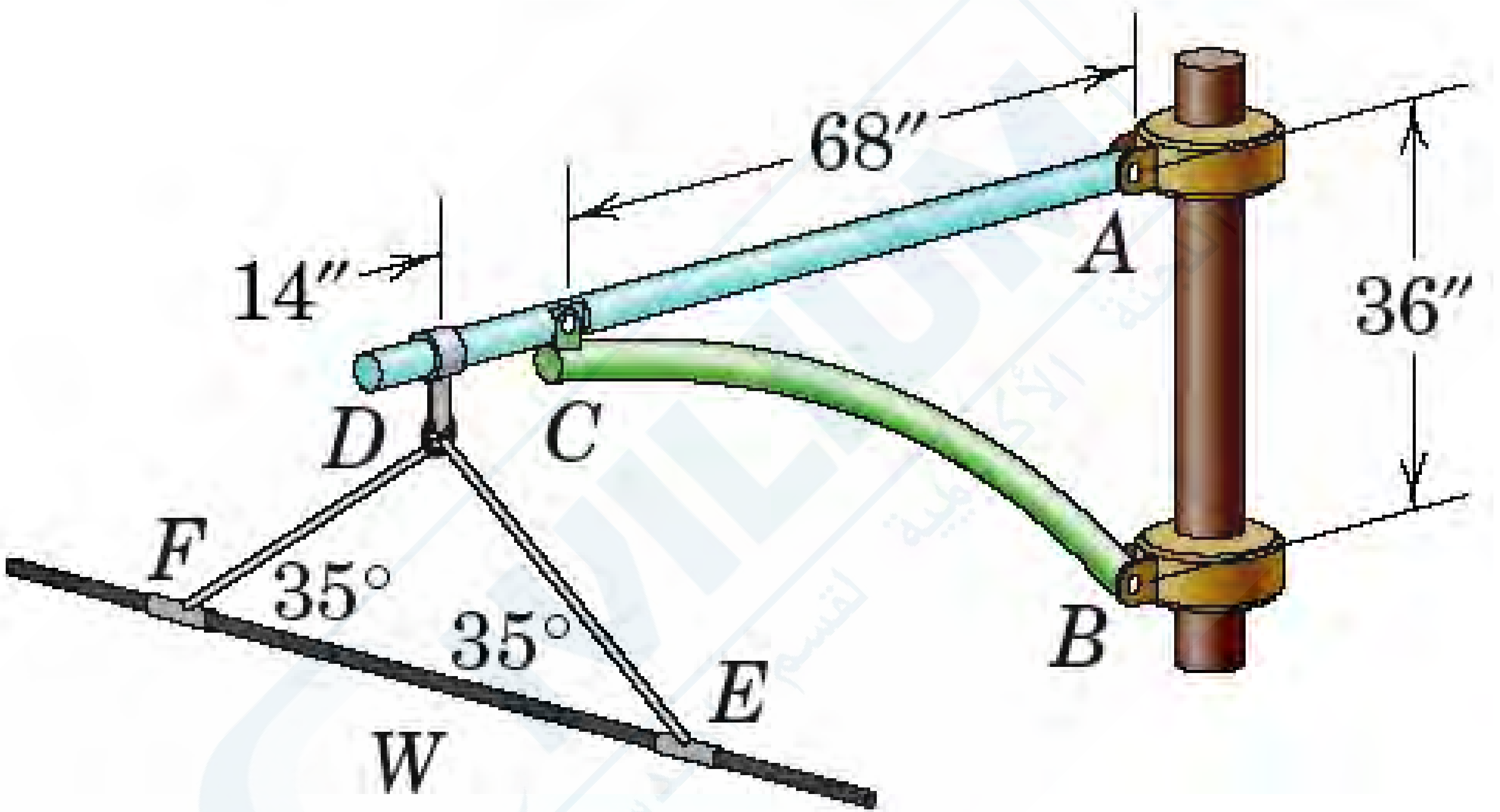
Problem 4/121

P_04_121



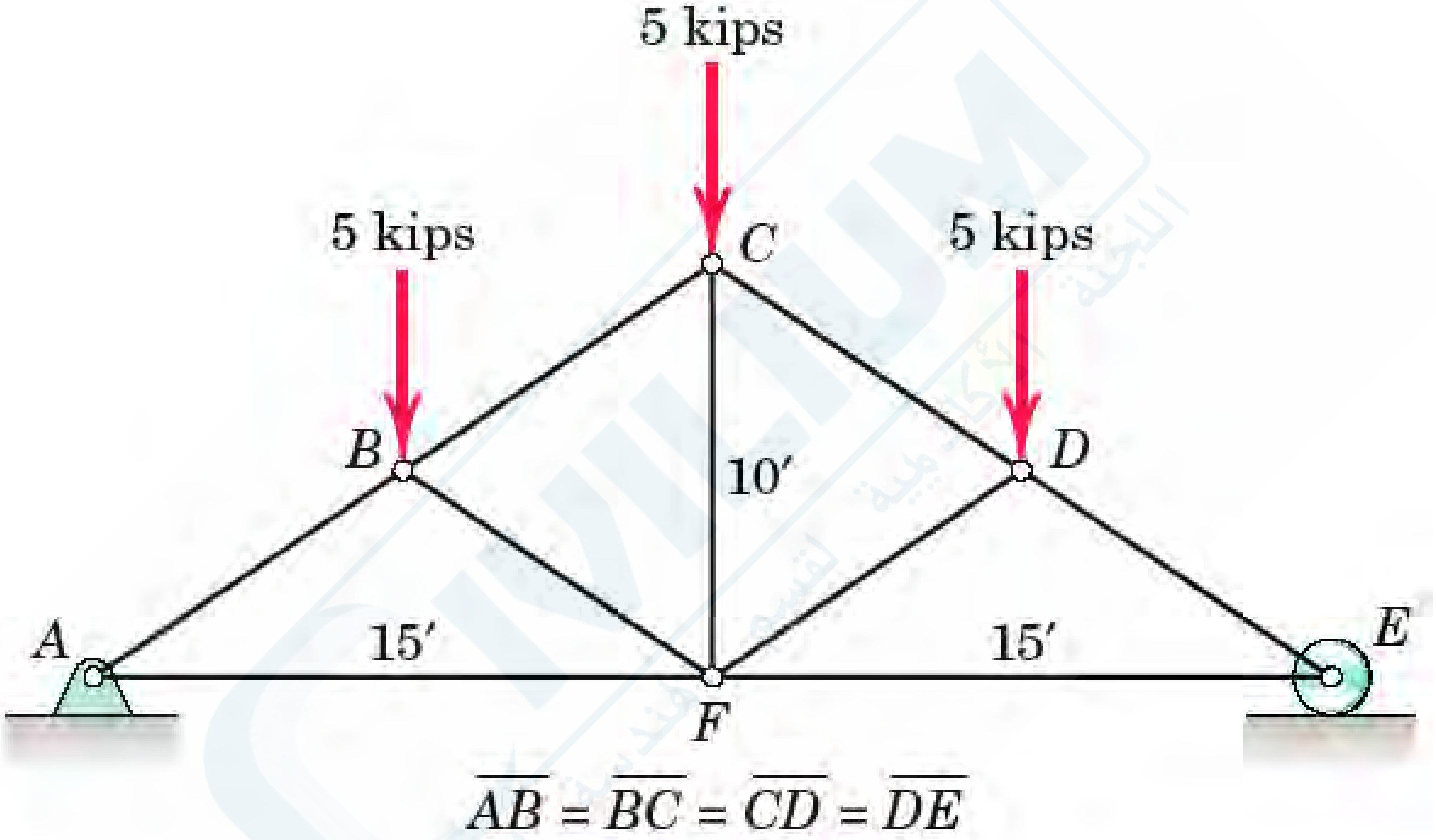
Problem 4/122

P 04 122



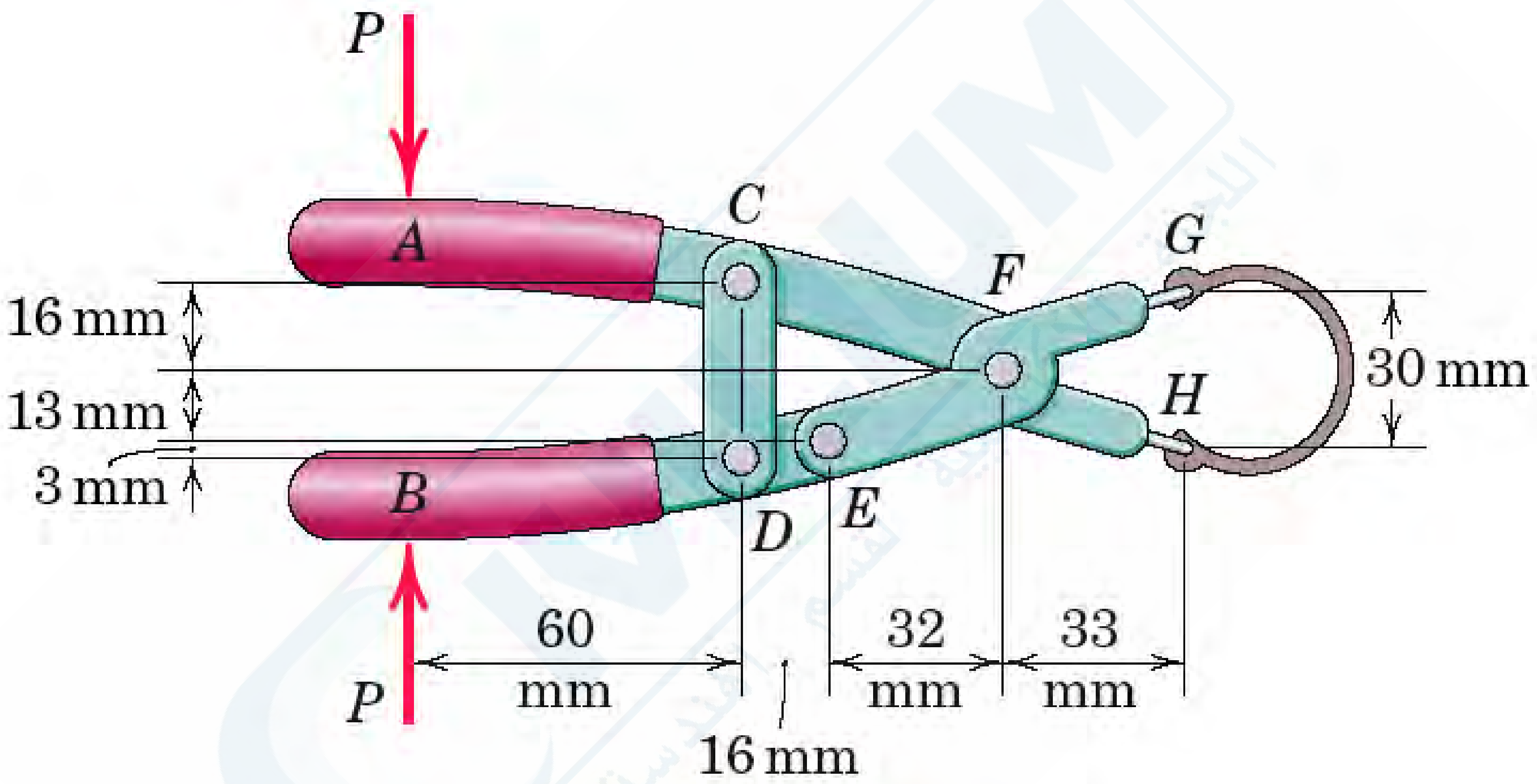
Problem 4/123

P_04_123



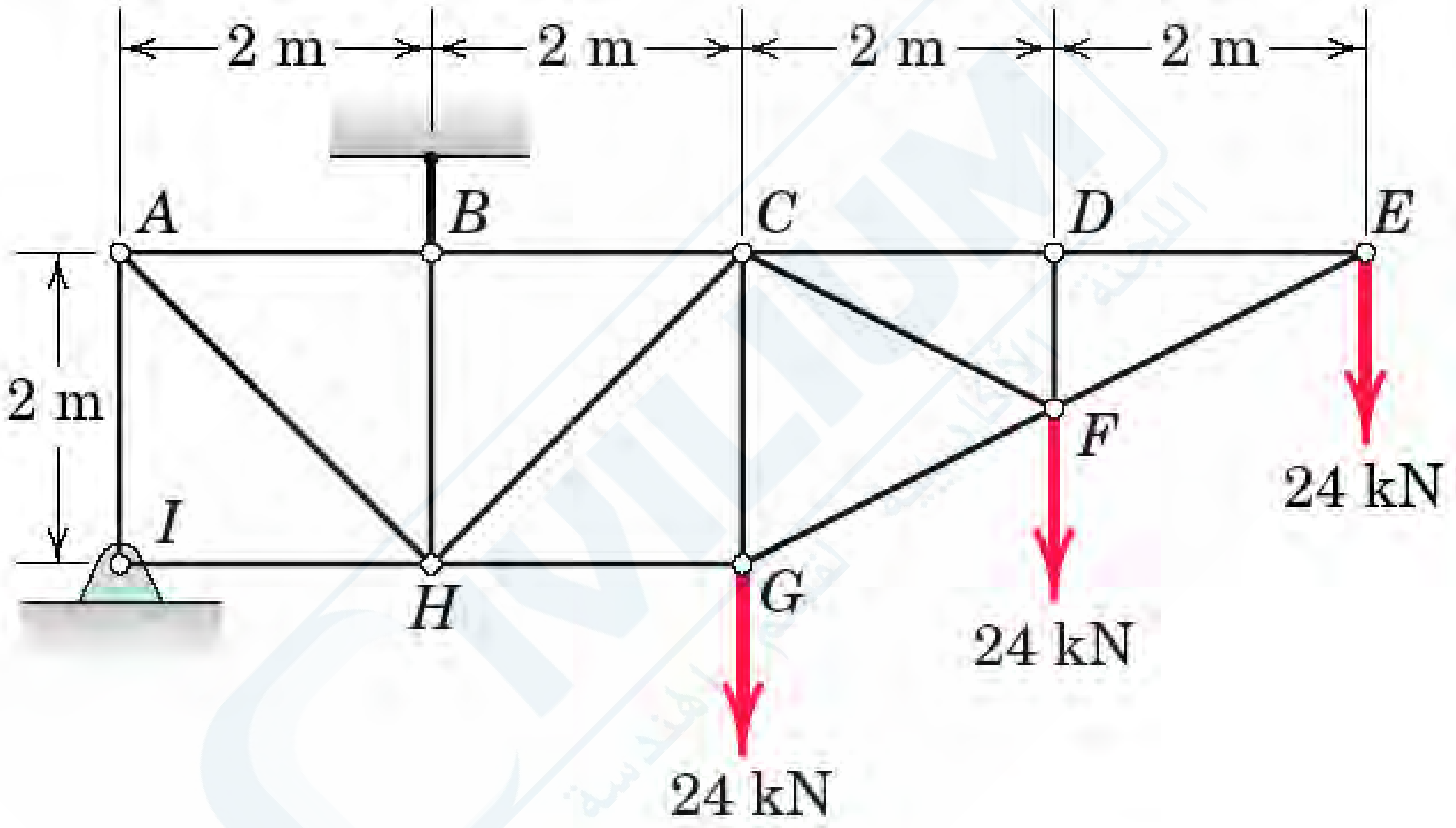
Problem 4/124

P_04_124



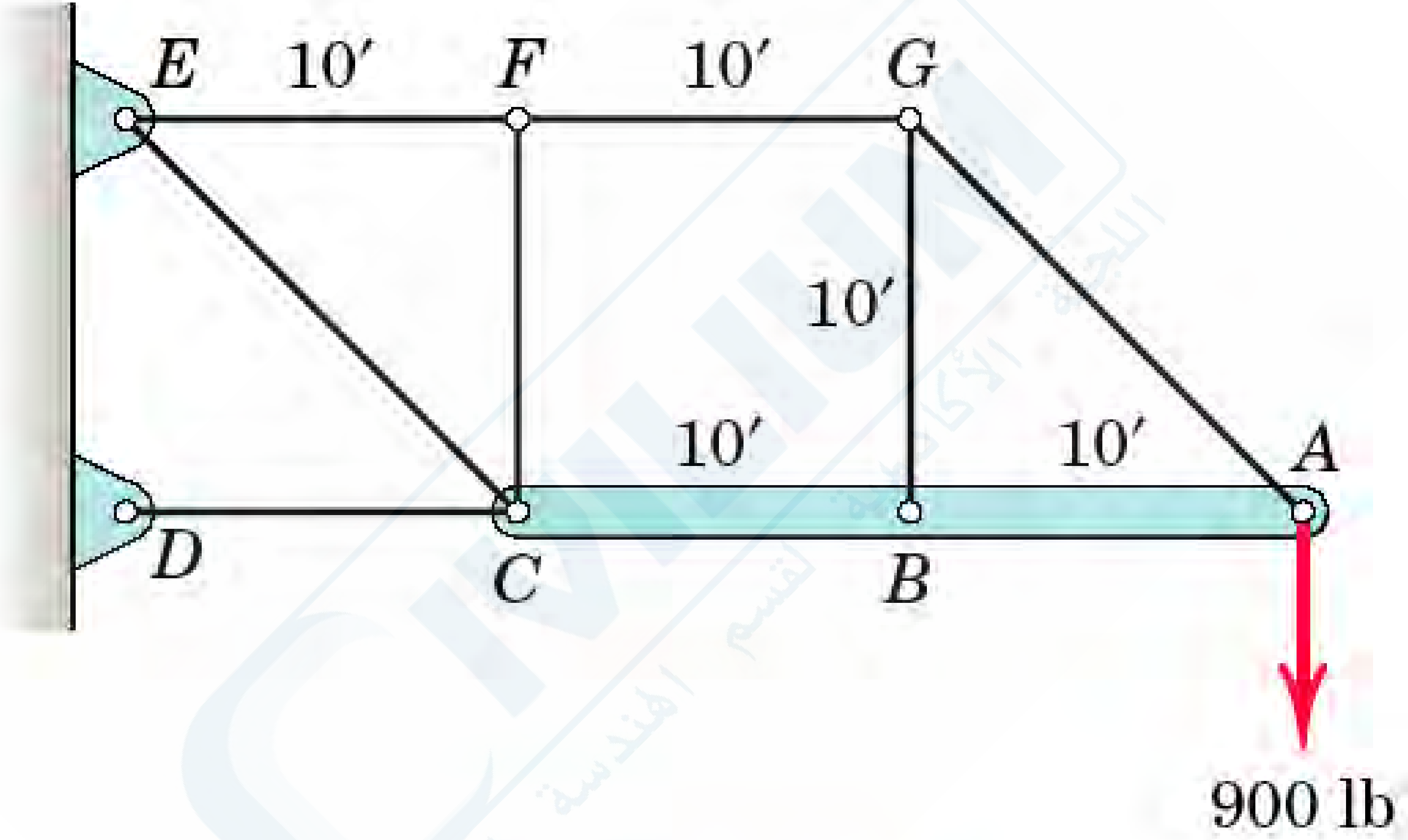
Problem 4/125

P_04_125



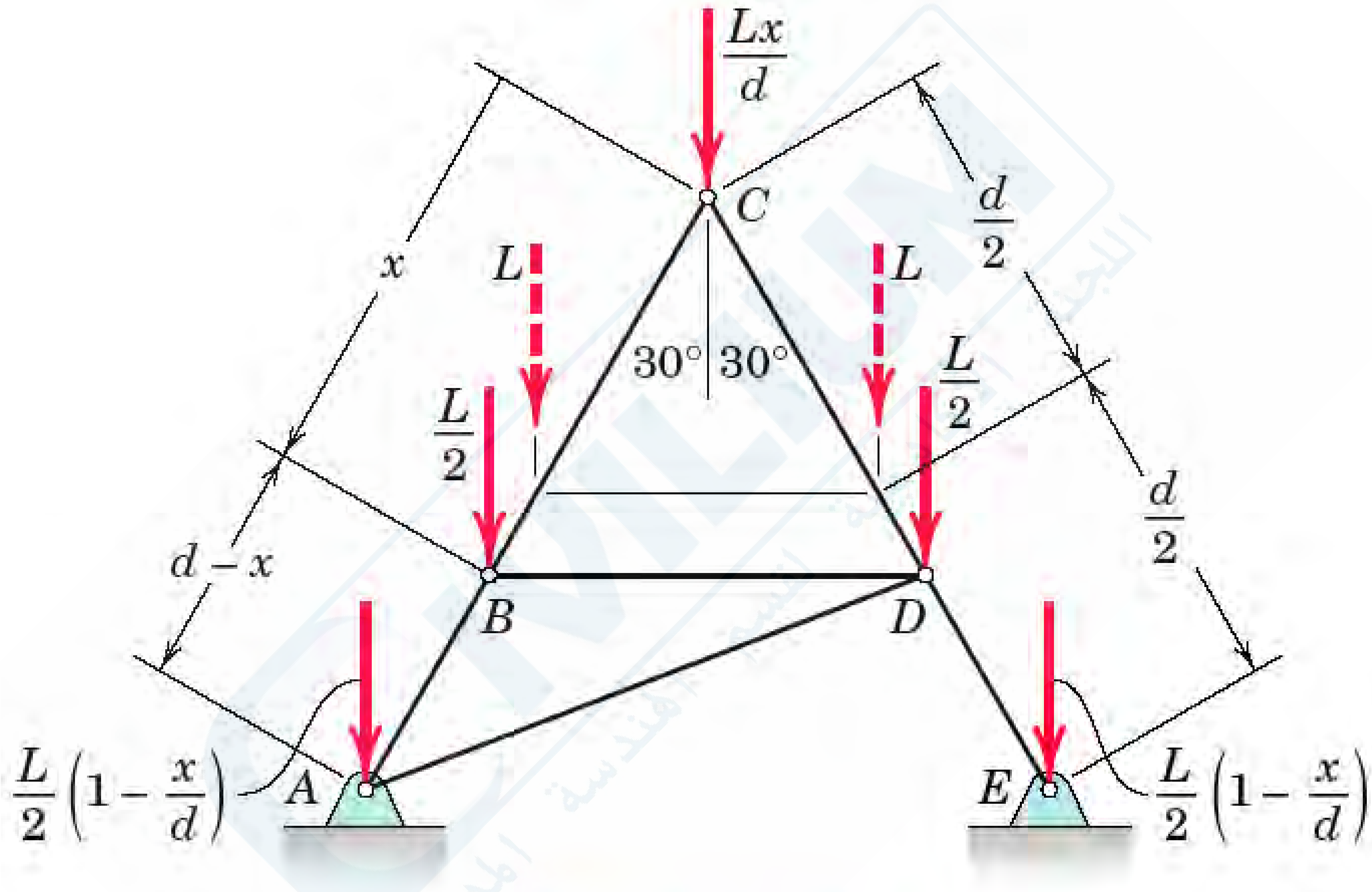
Problem 4/126

P_04_126



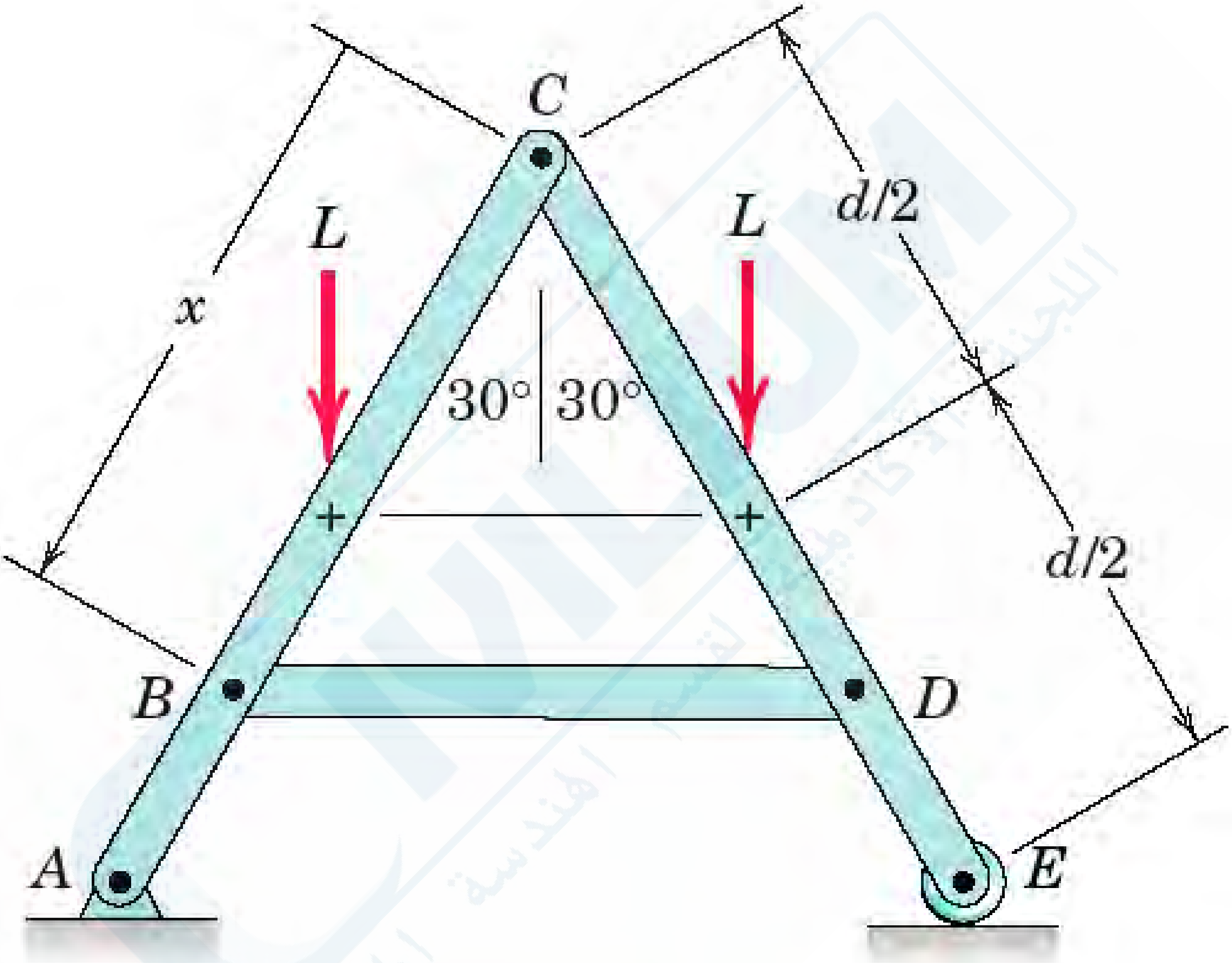
Problem 4/127

P_04_127



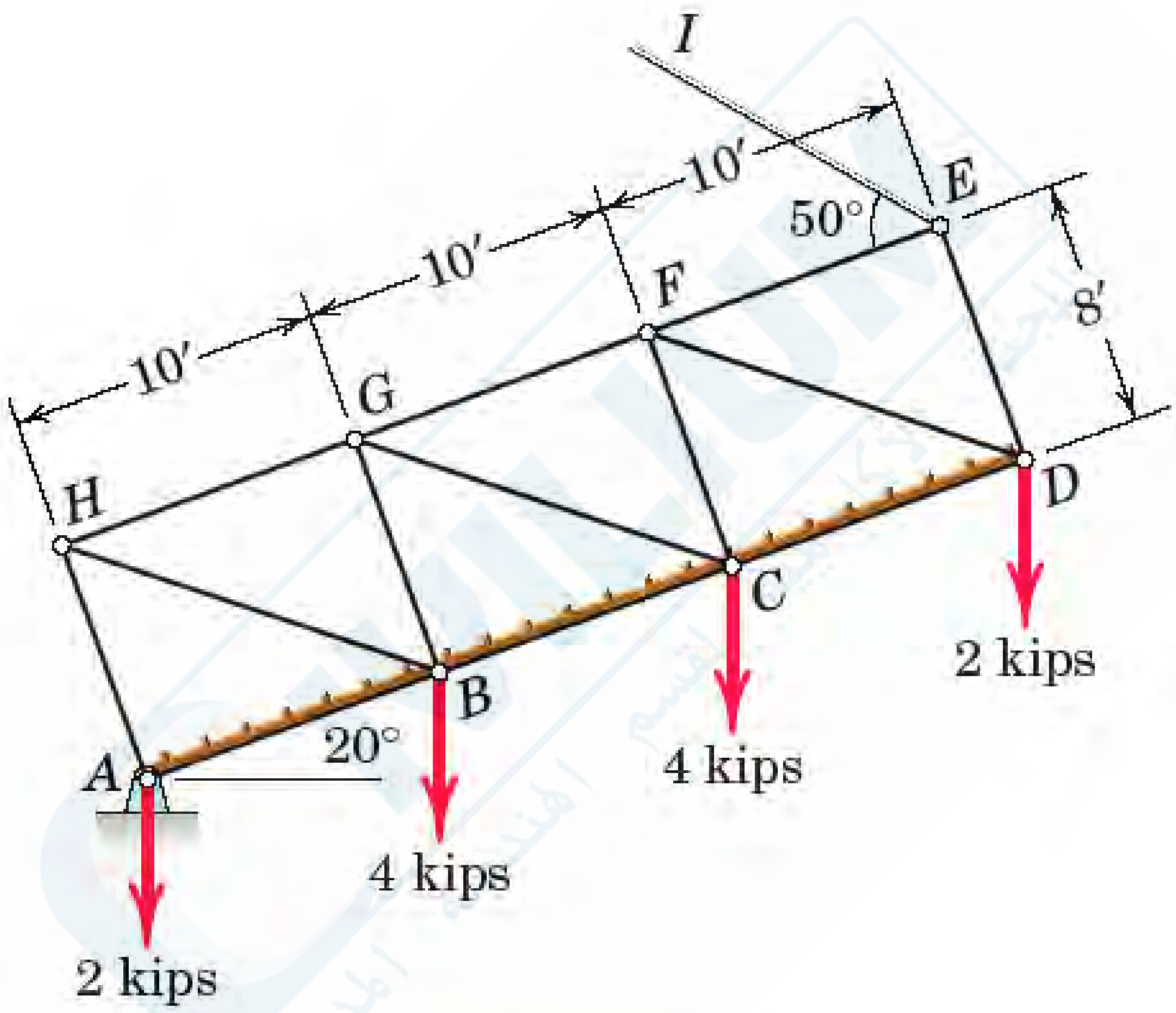
Problem 4/128

P_04_128



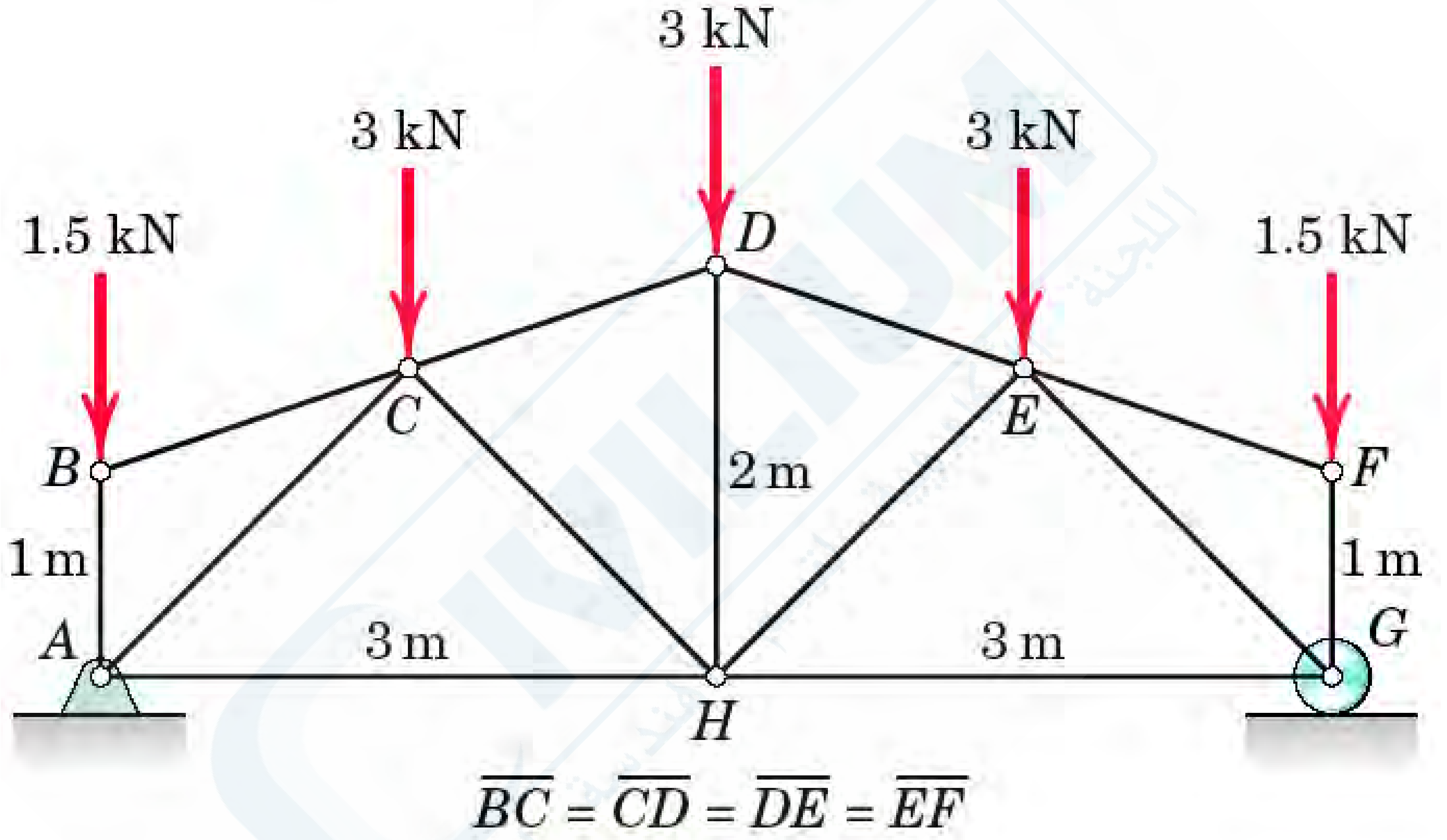
Problem 4/129

P 04 129



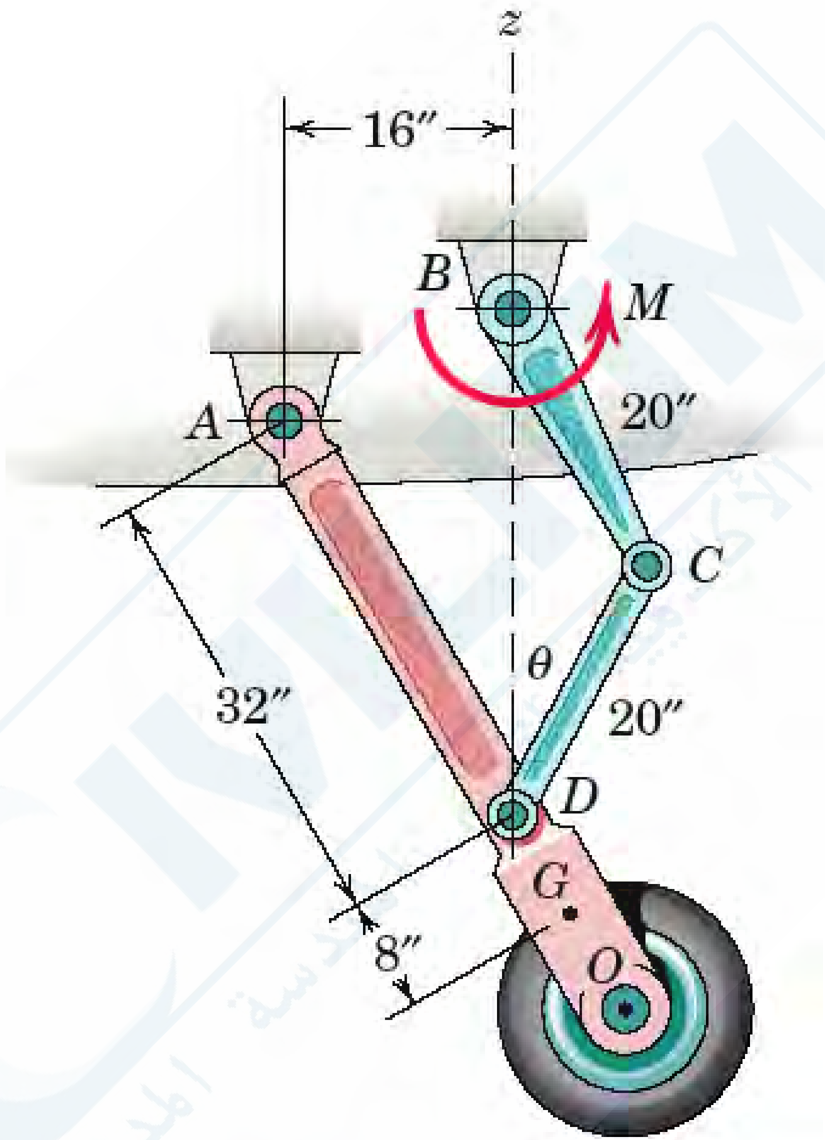
Problem 4/13

P 04 13



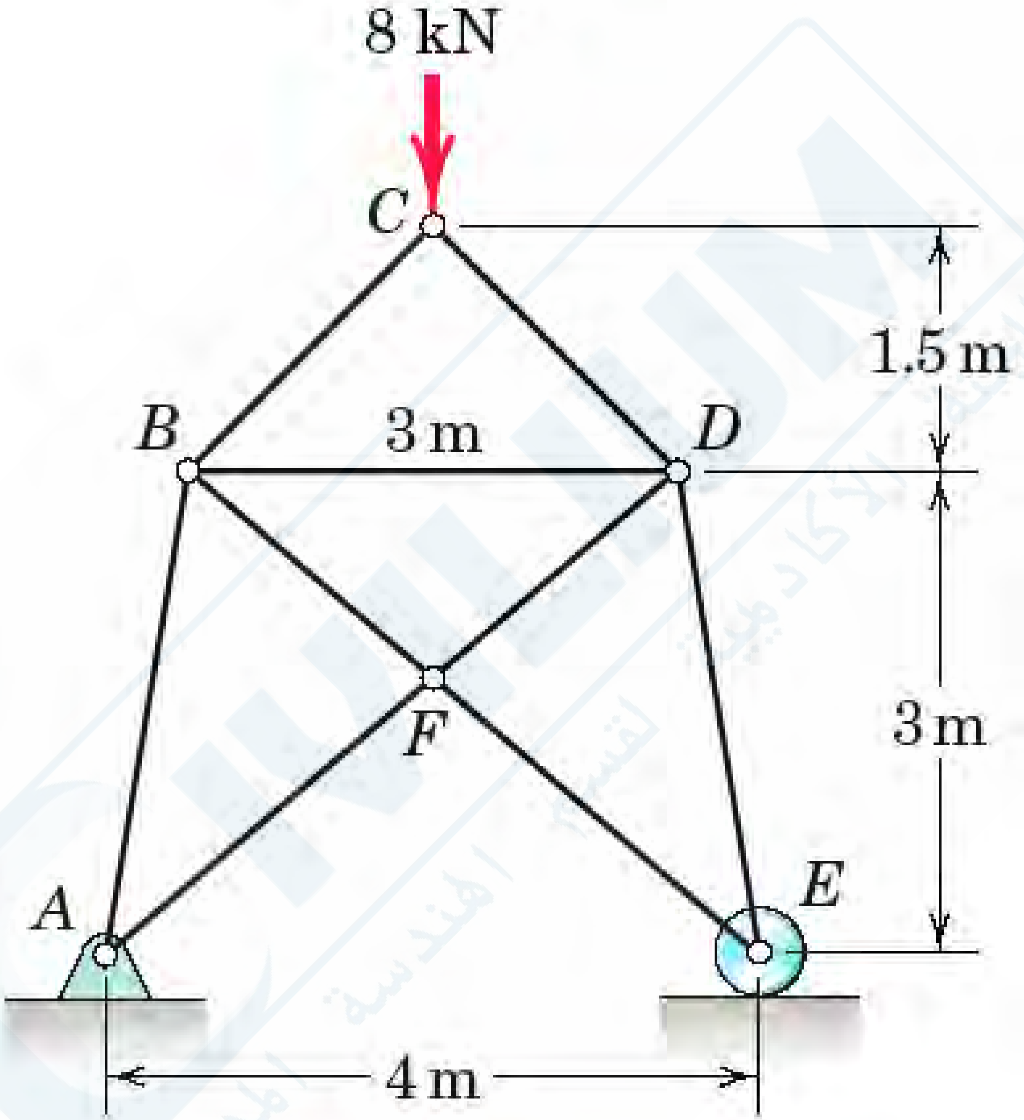
Problem 4/130

P_04_130



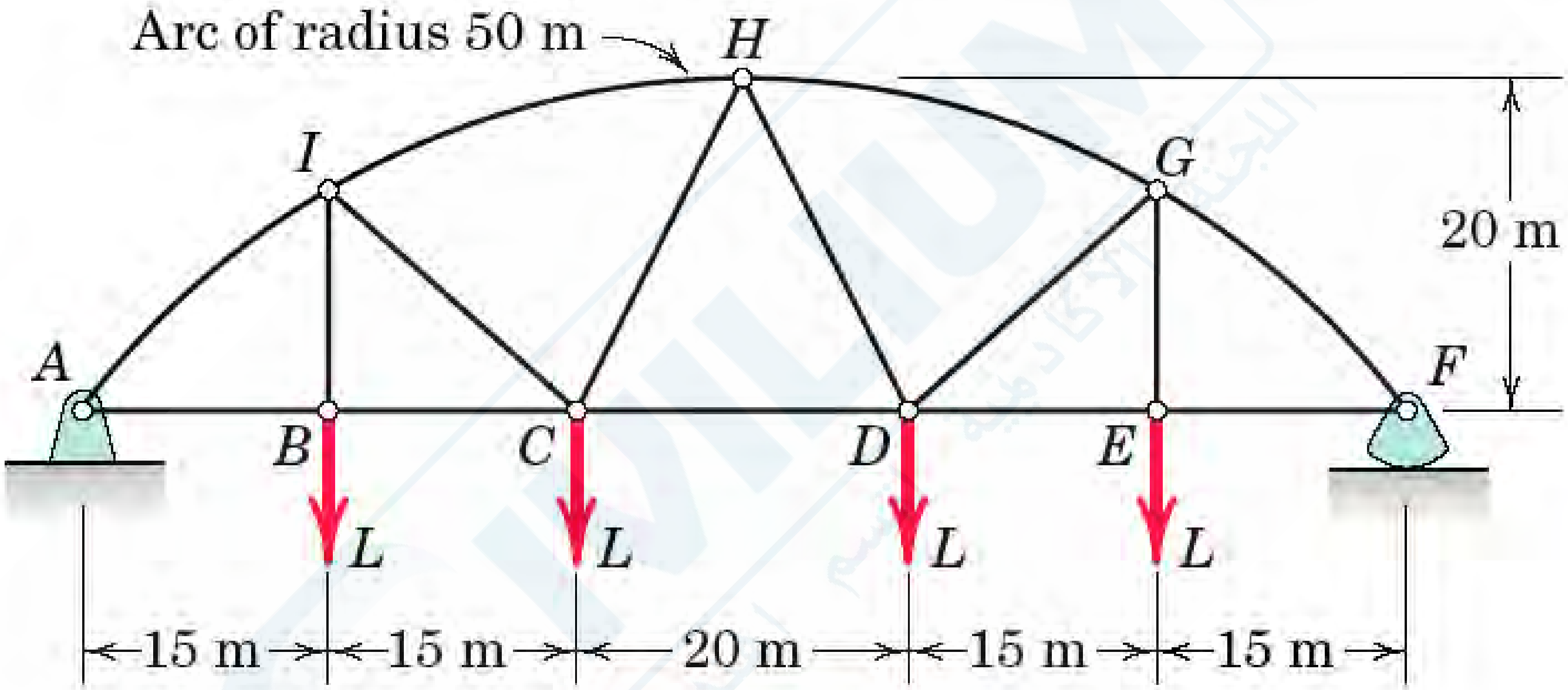
Problem 4/131

P 04 131



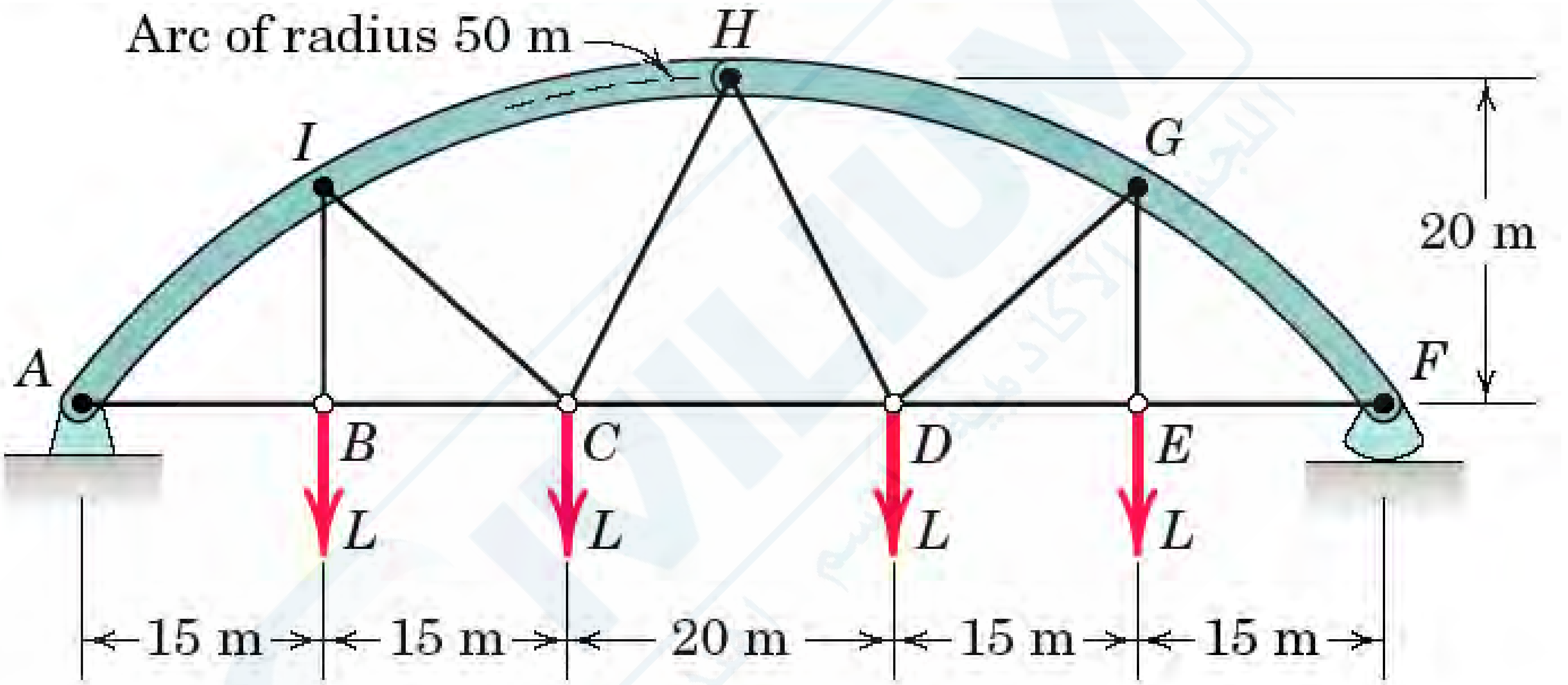
Problem 4/132

P 04 132



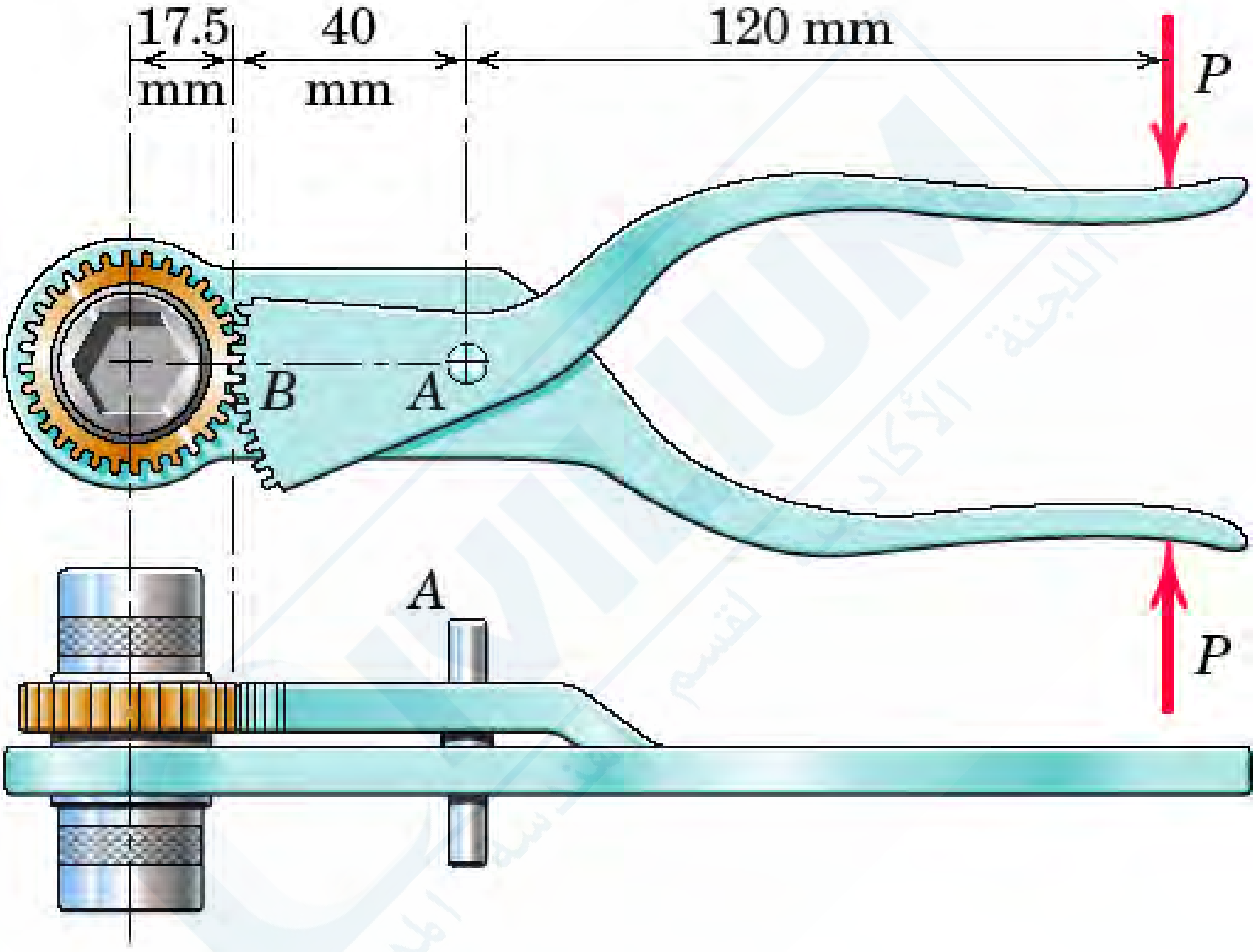
Problem 4/133

P_04_133



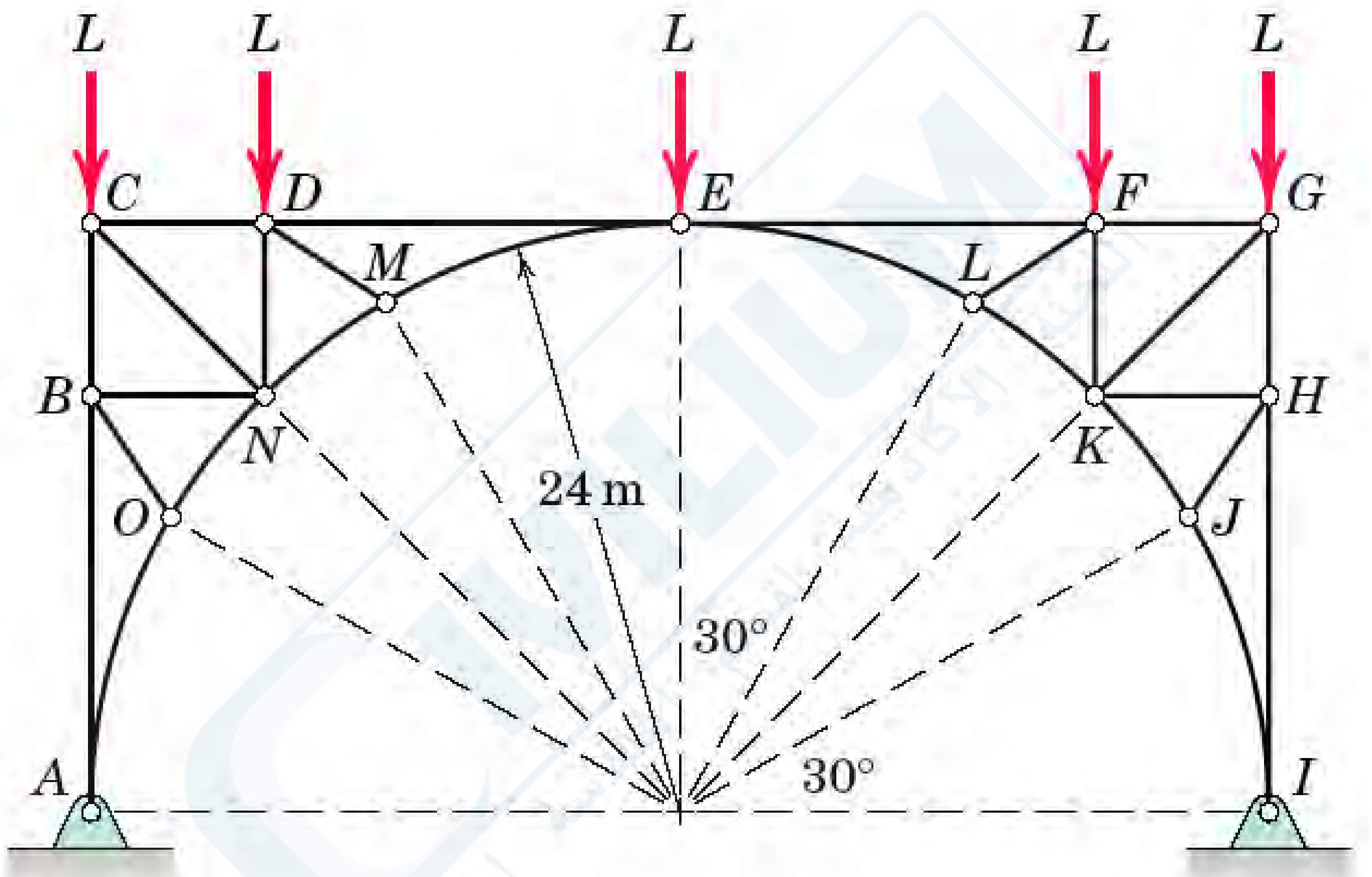
Problem 4/134

P_04_134



Problem 4/135

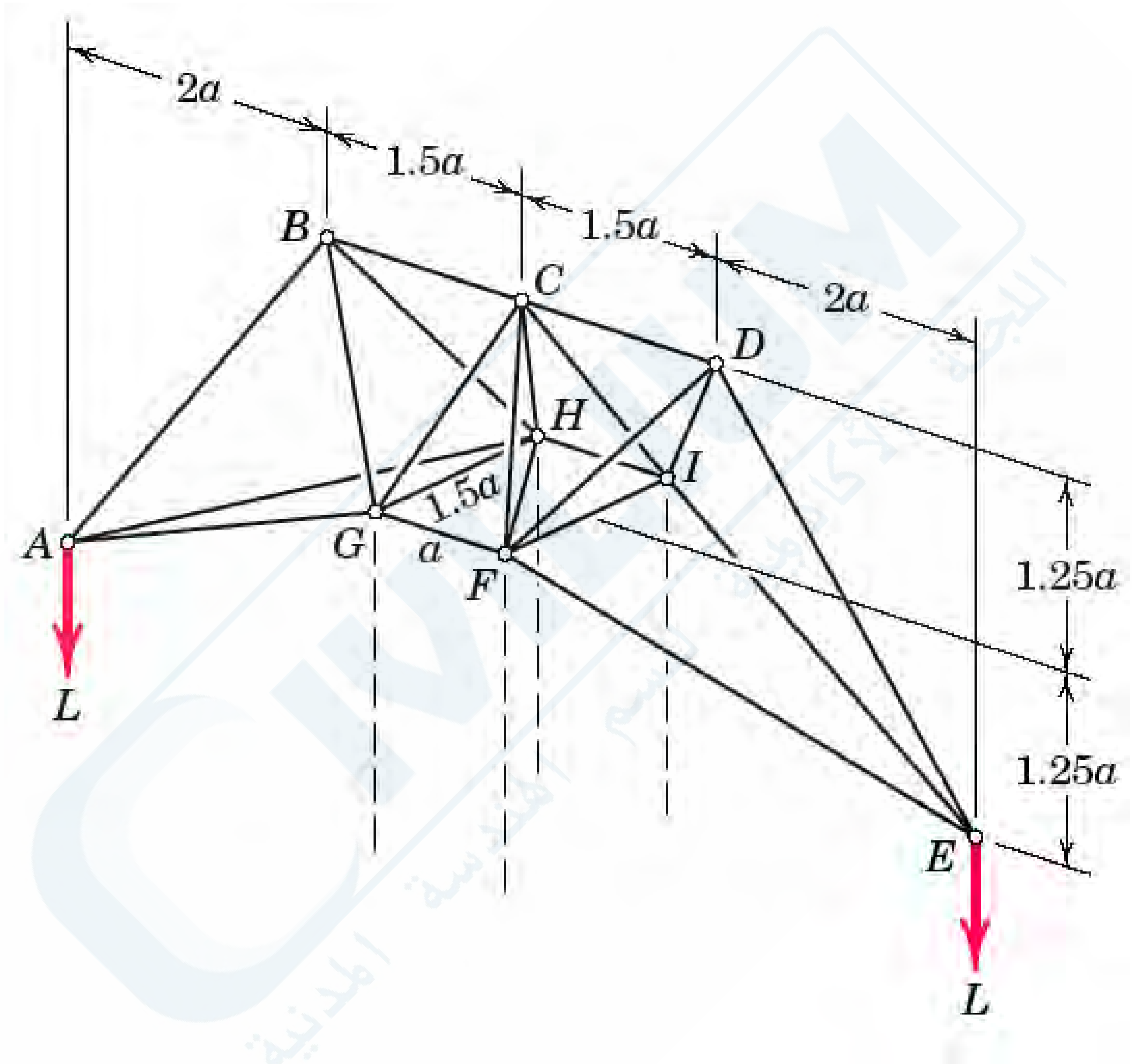
P_04_135



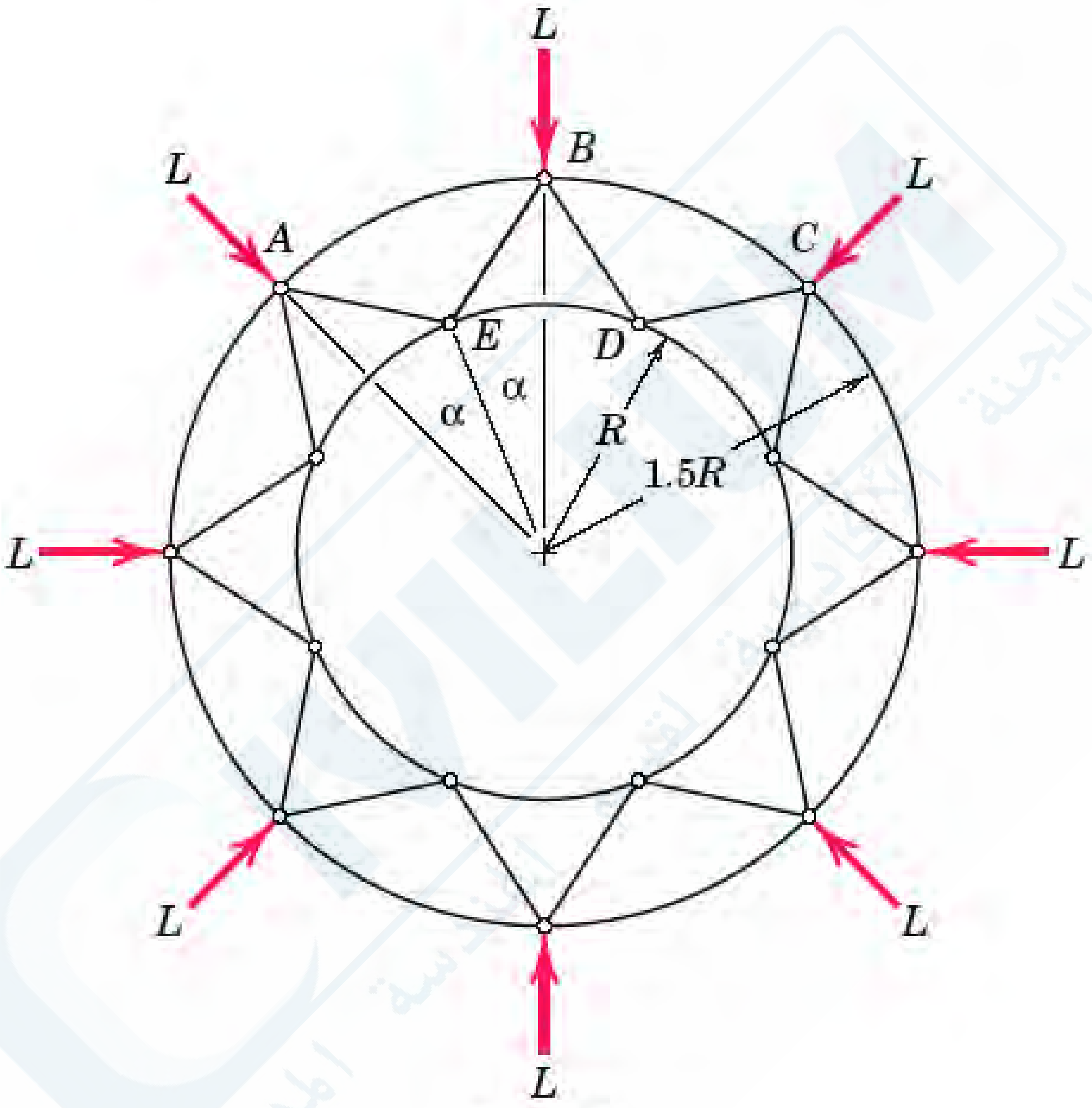
CIVILIUM

Problem 4/136

P 04 136



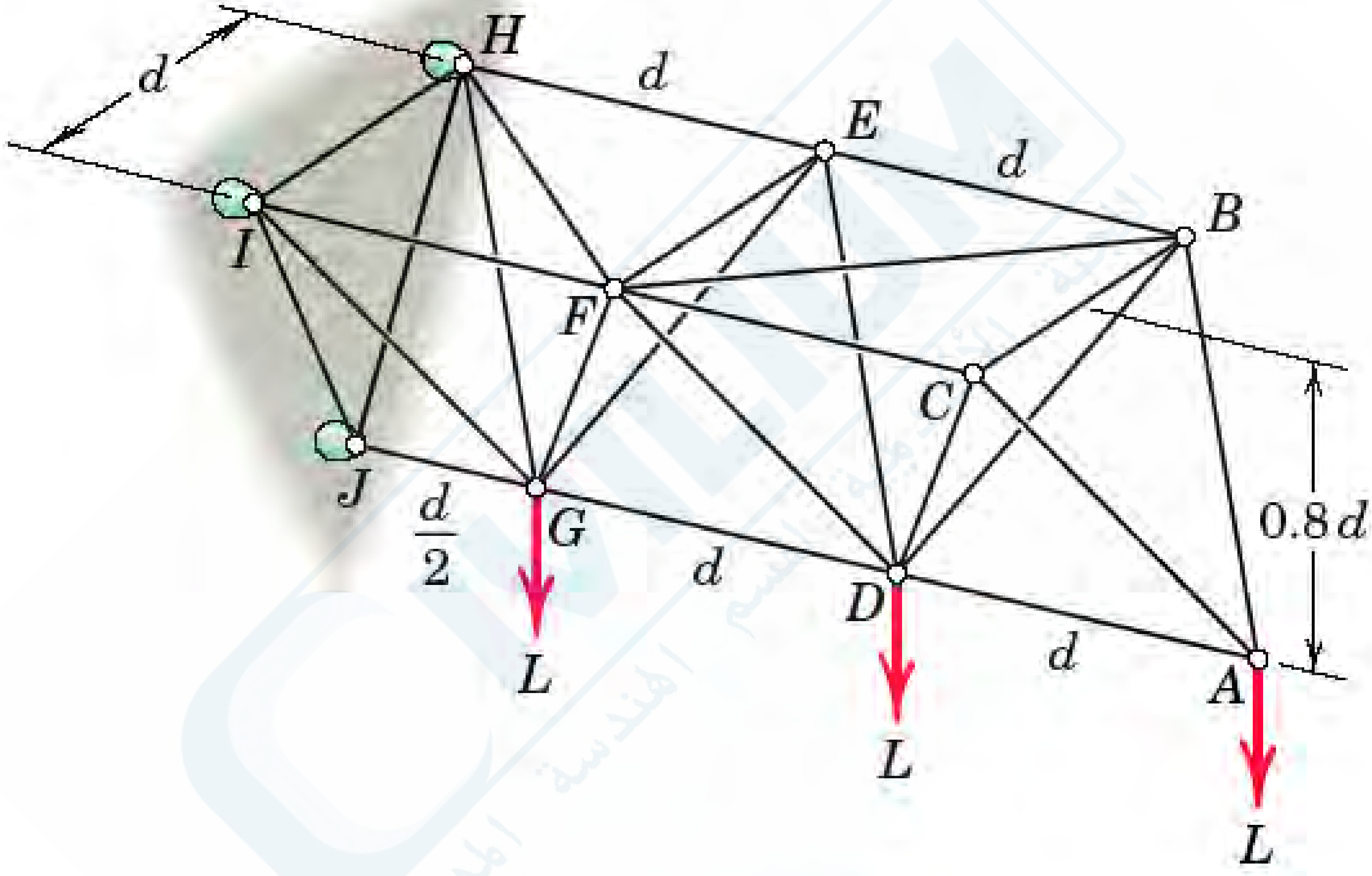
Problem 4/137



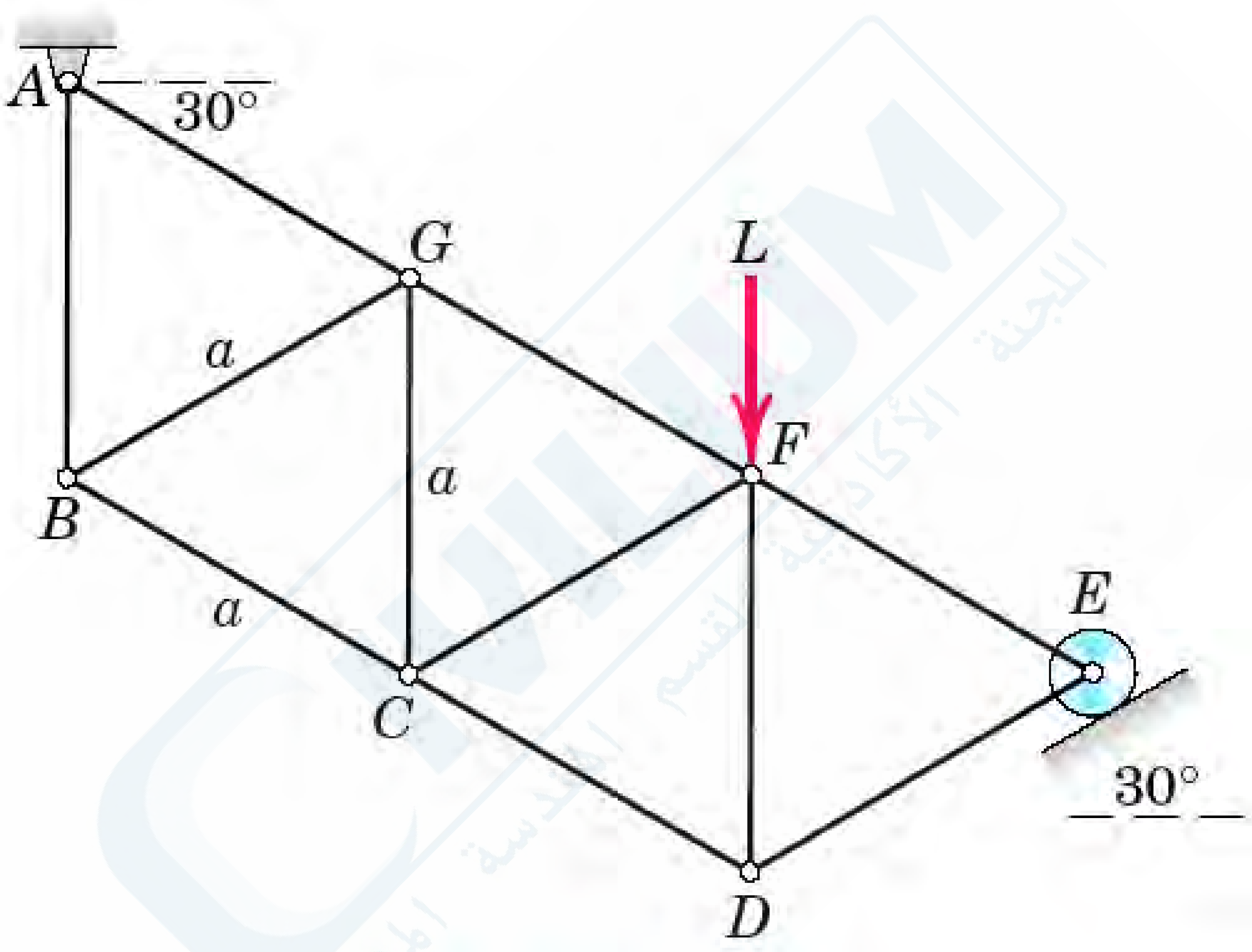
$\alpha = 22.5°$

Problem 4/138

P_04_138

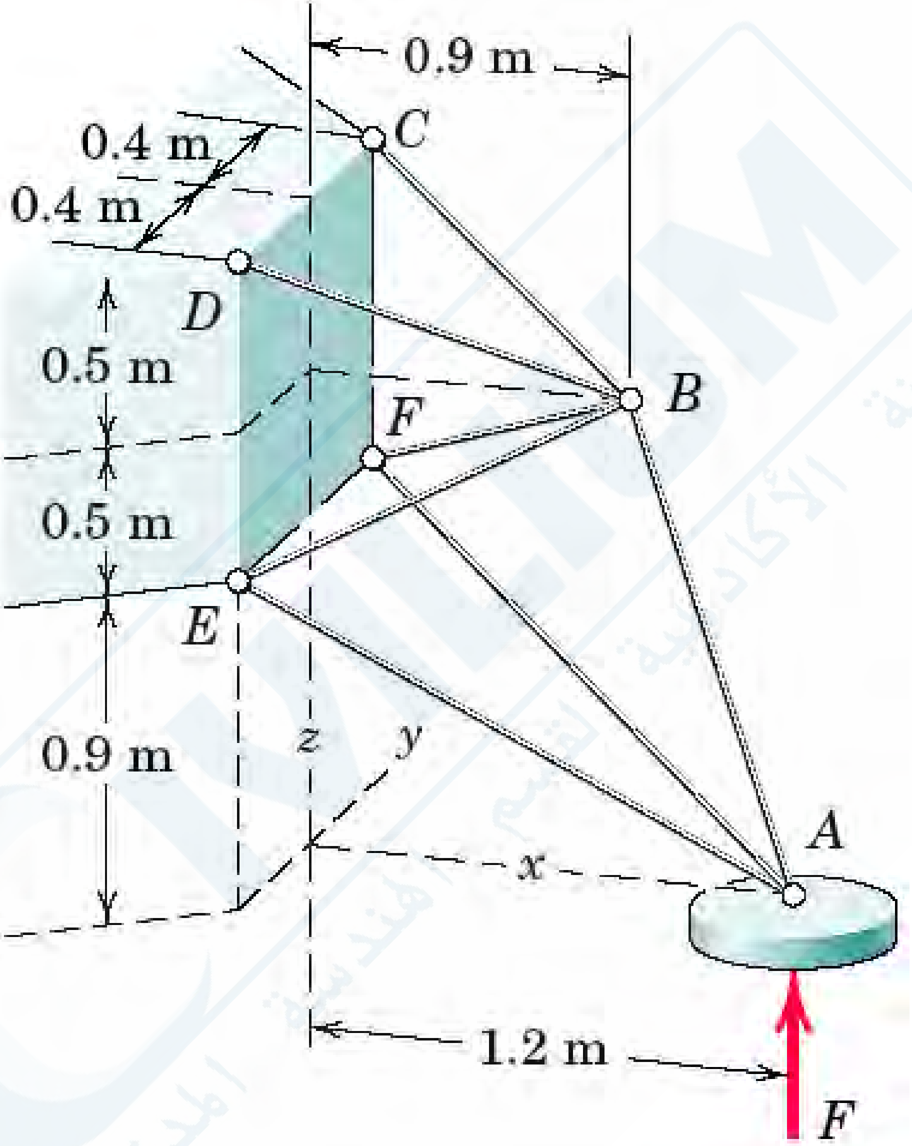


Problem 4/139



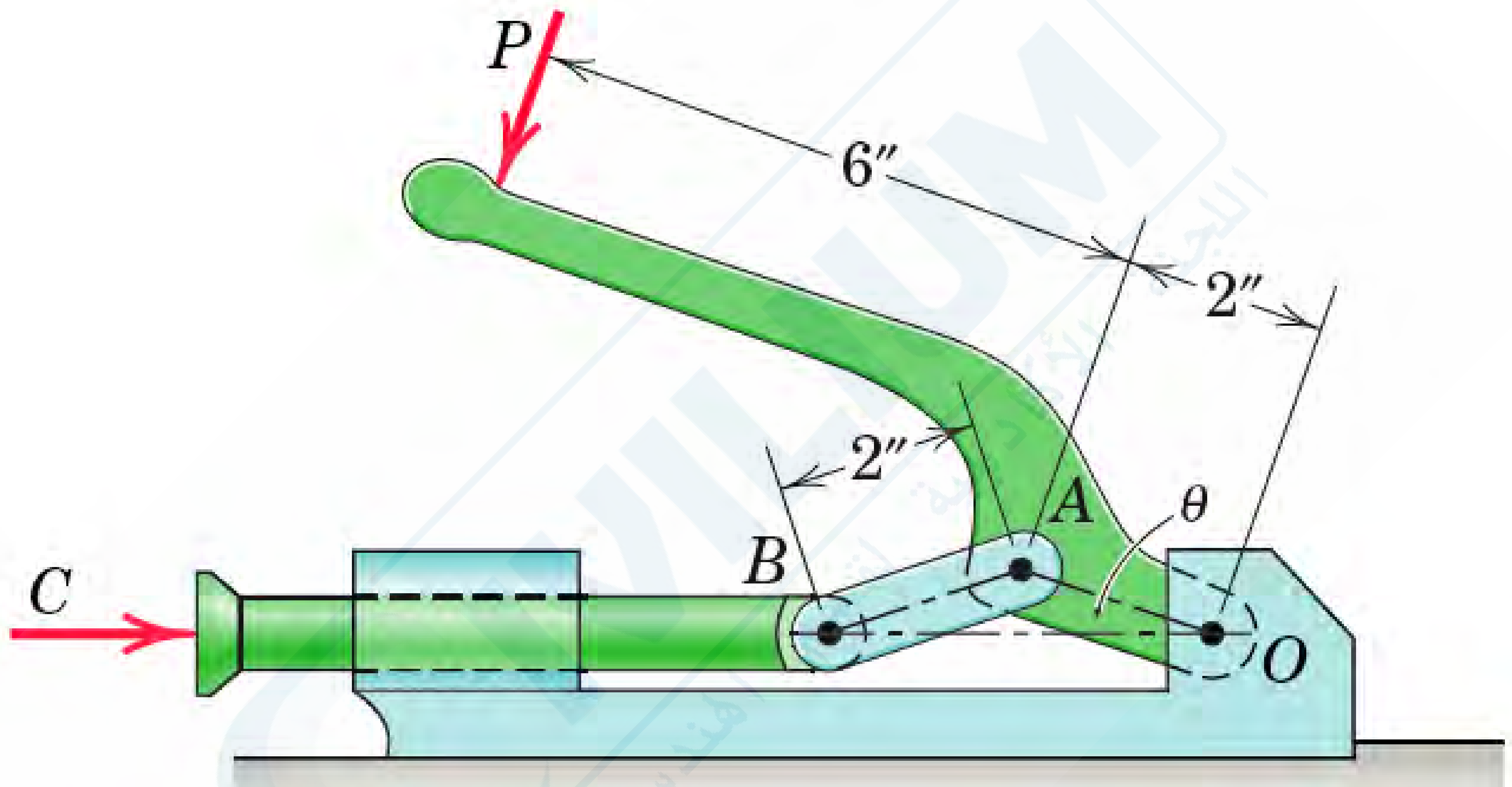
Problem 4/14

P 04 14



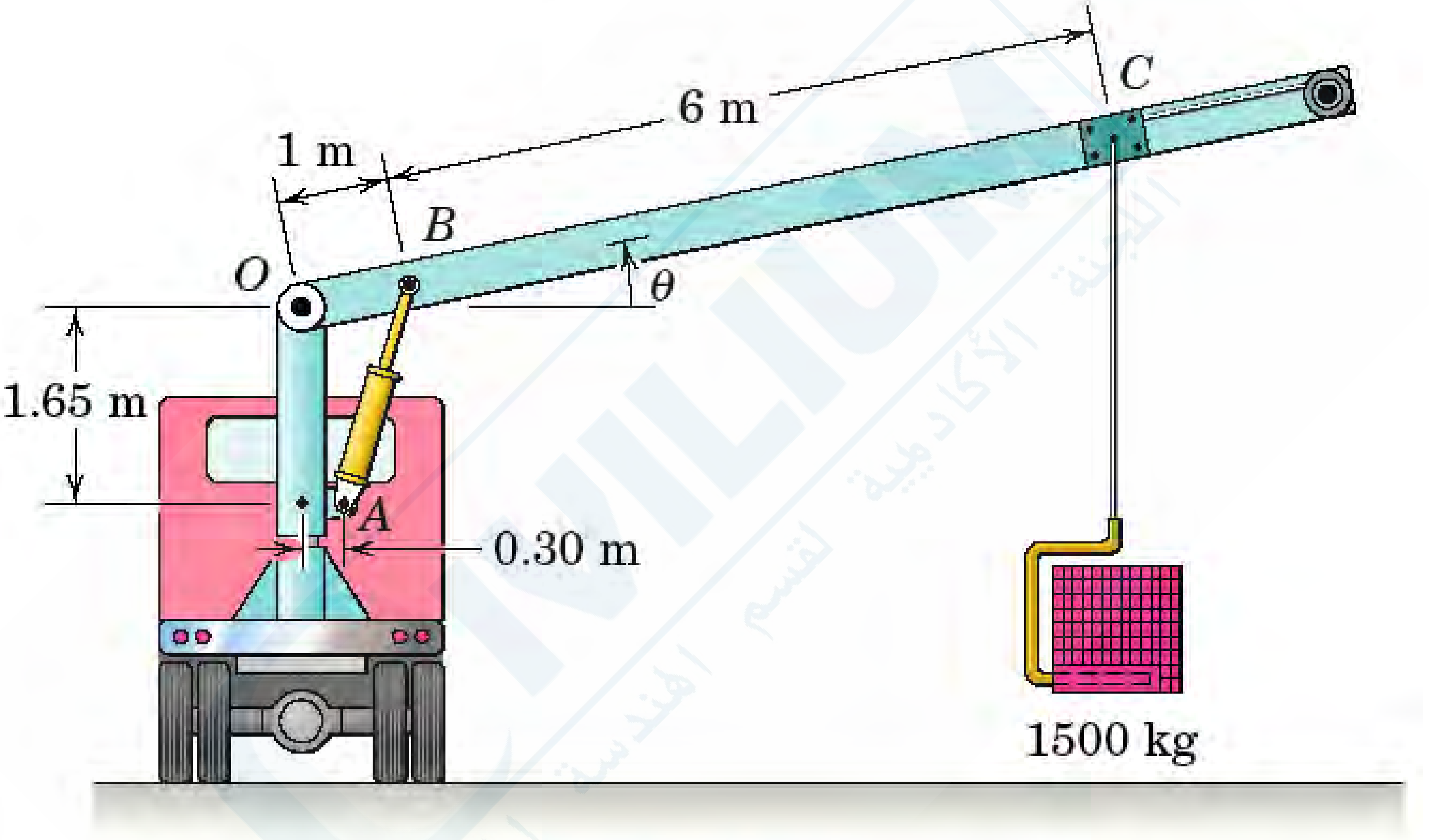
Problem 4/140

P 04 140



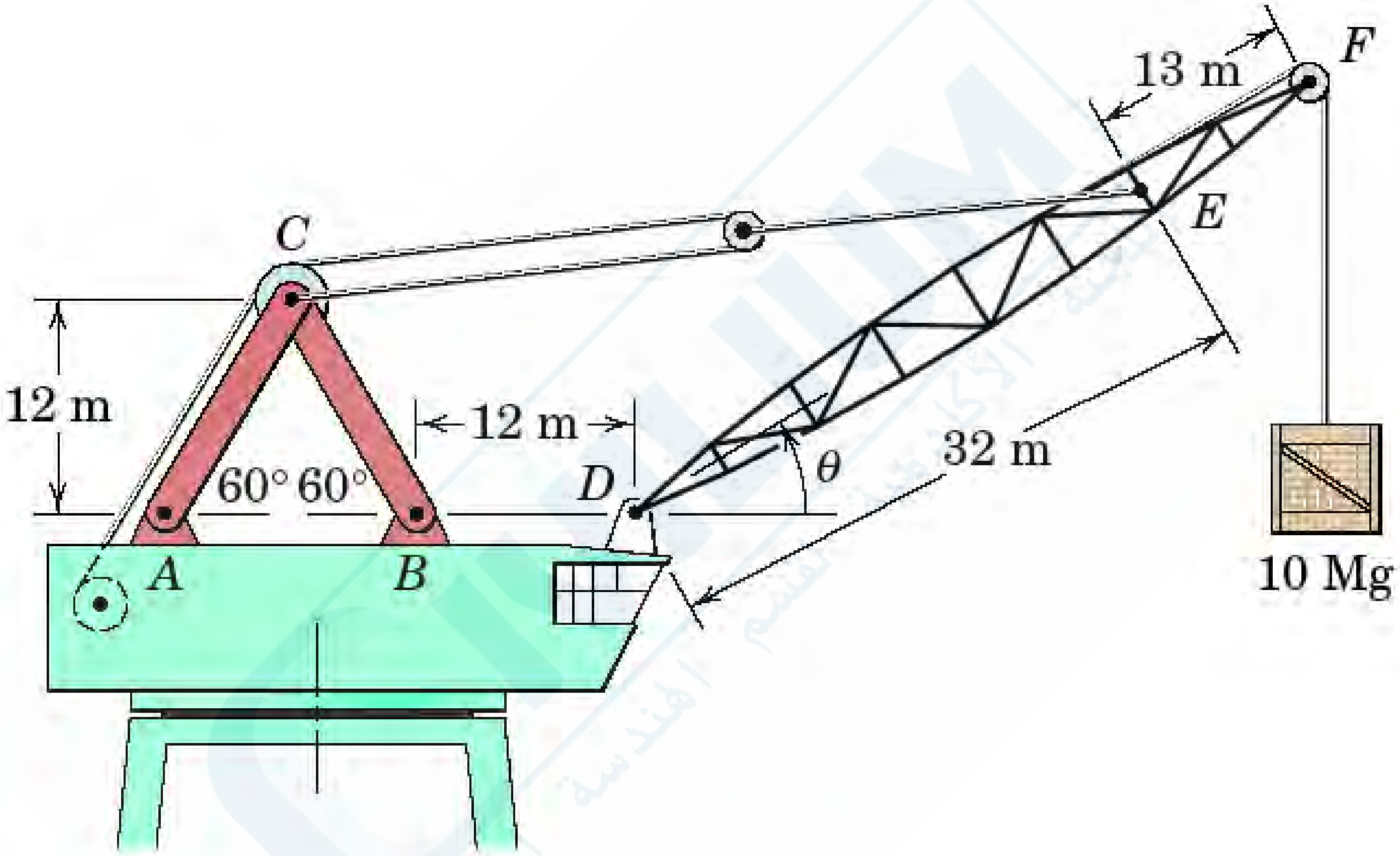
Problem 4/141

P_04_141



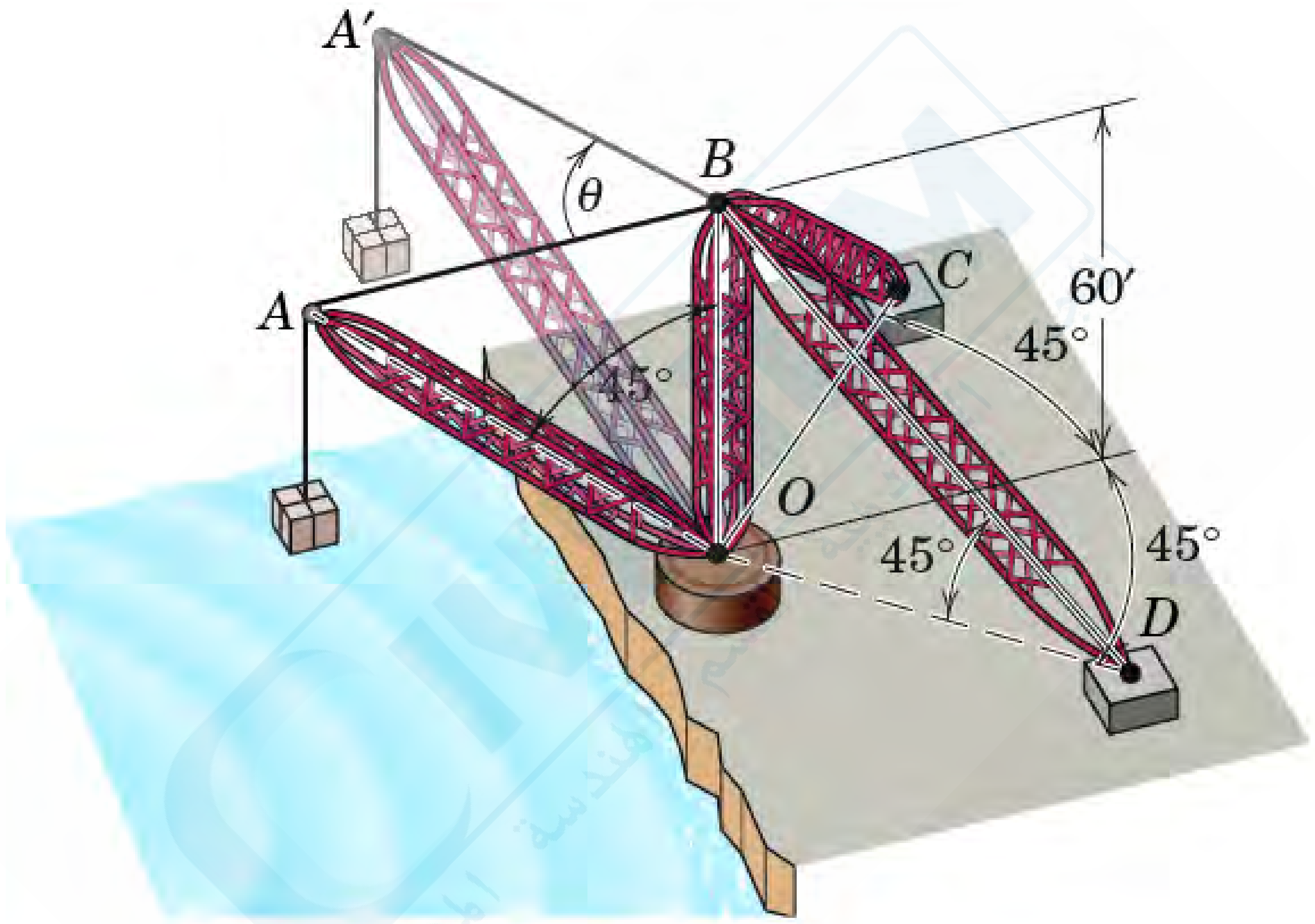
Problem 4/142

P_04_142



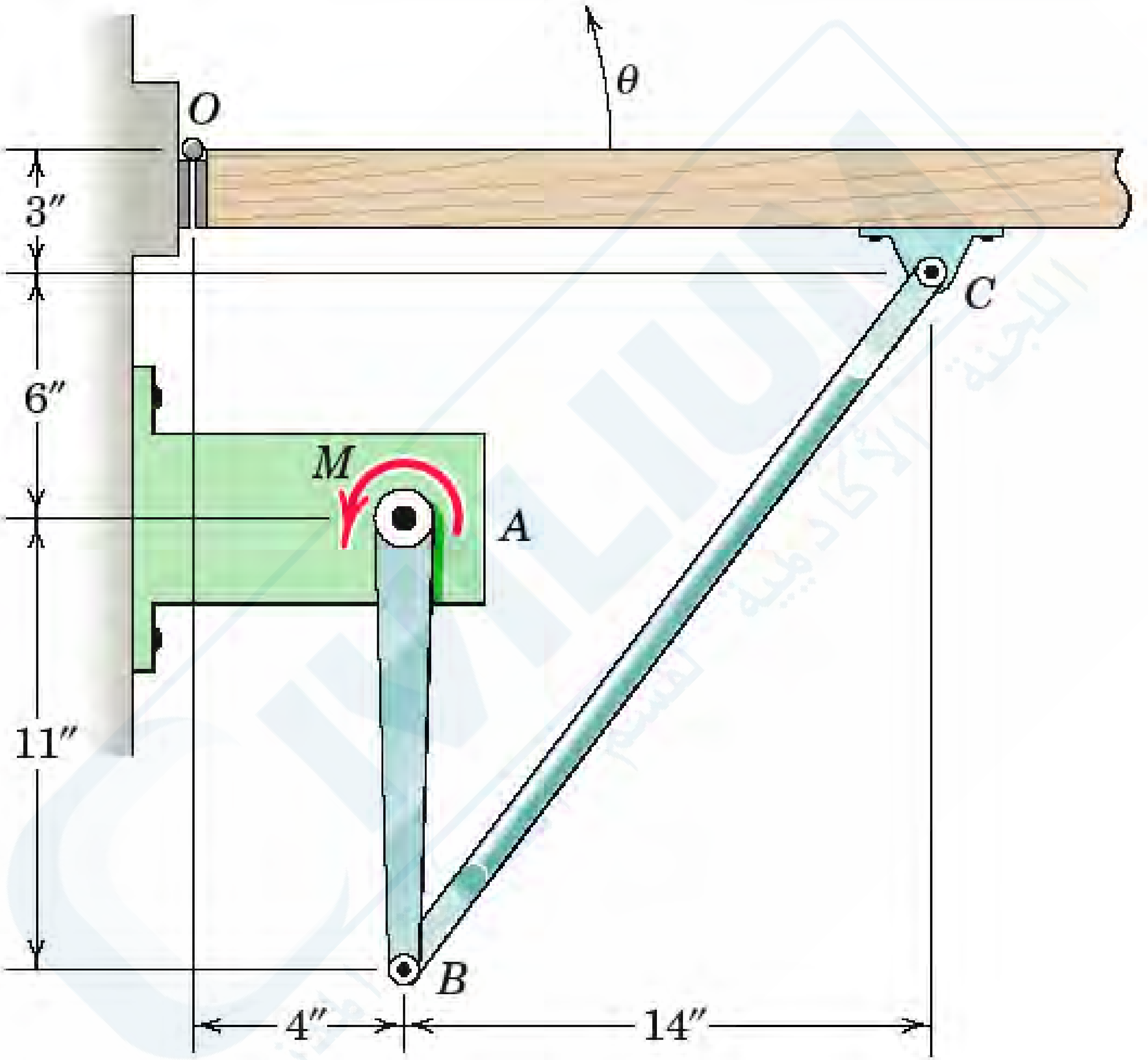
Problem 4/143

P_04_143



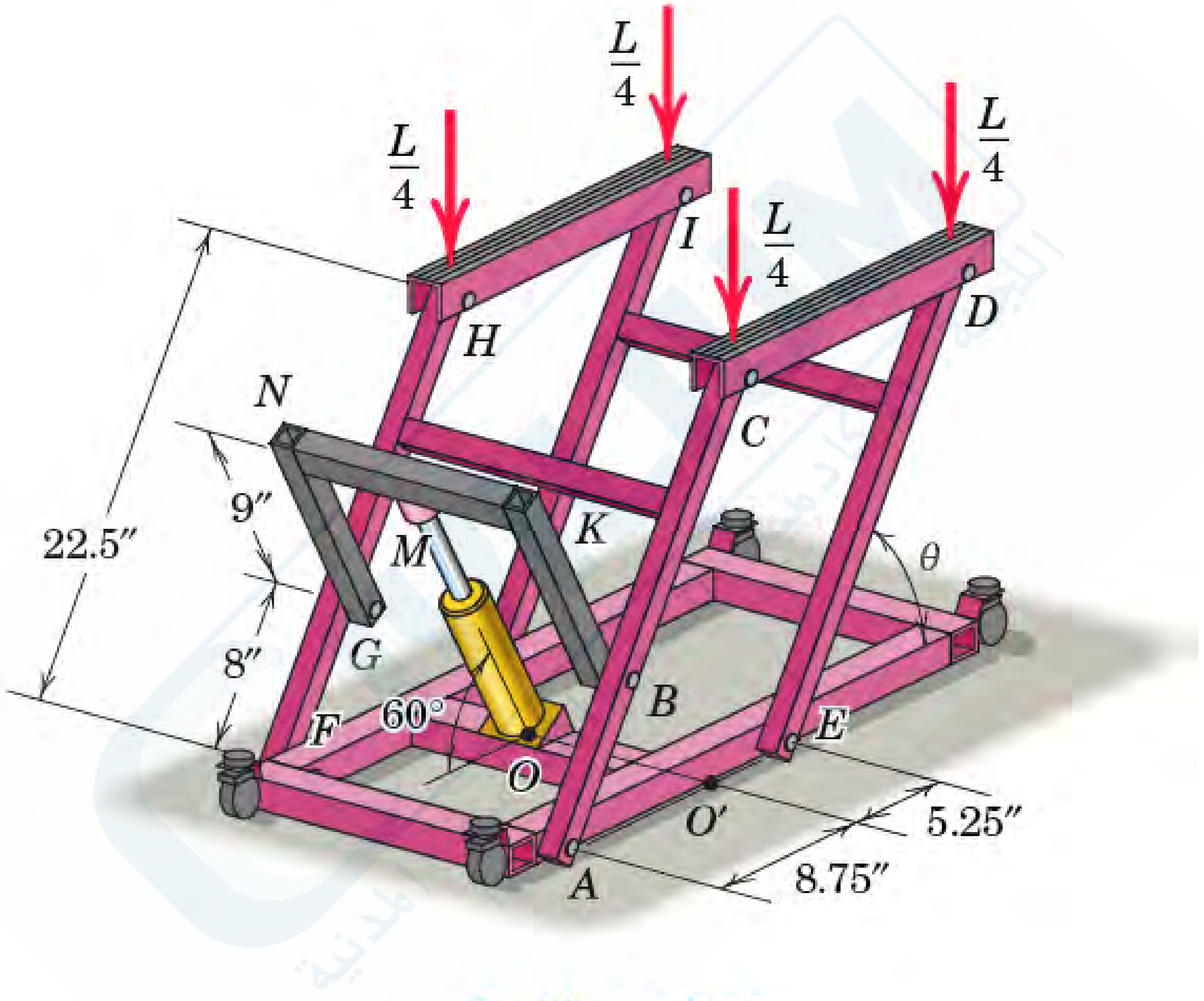
Problem 4/144

P 04 144



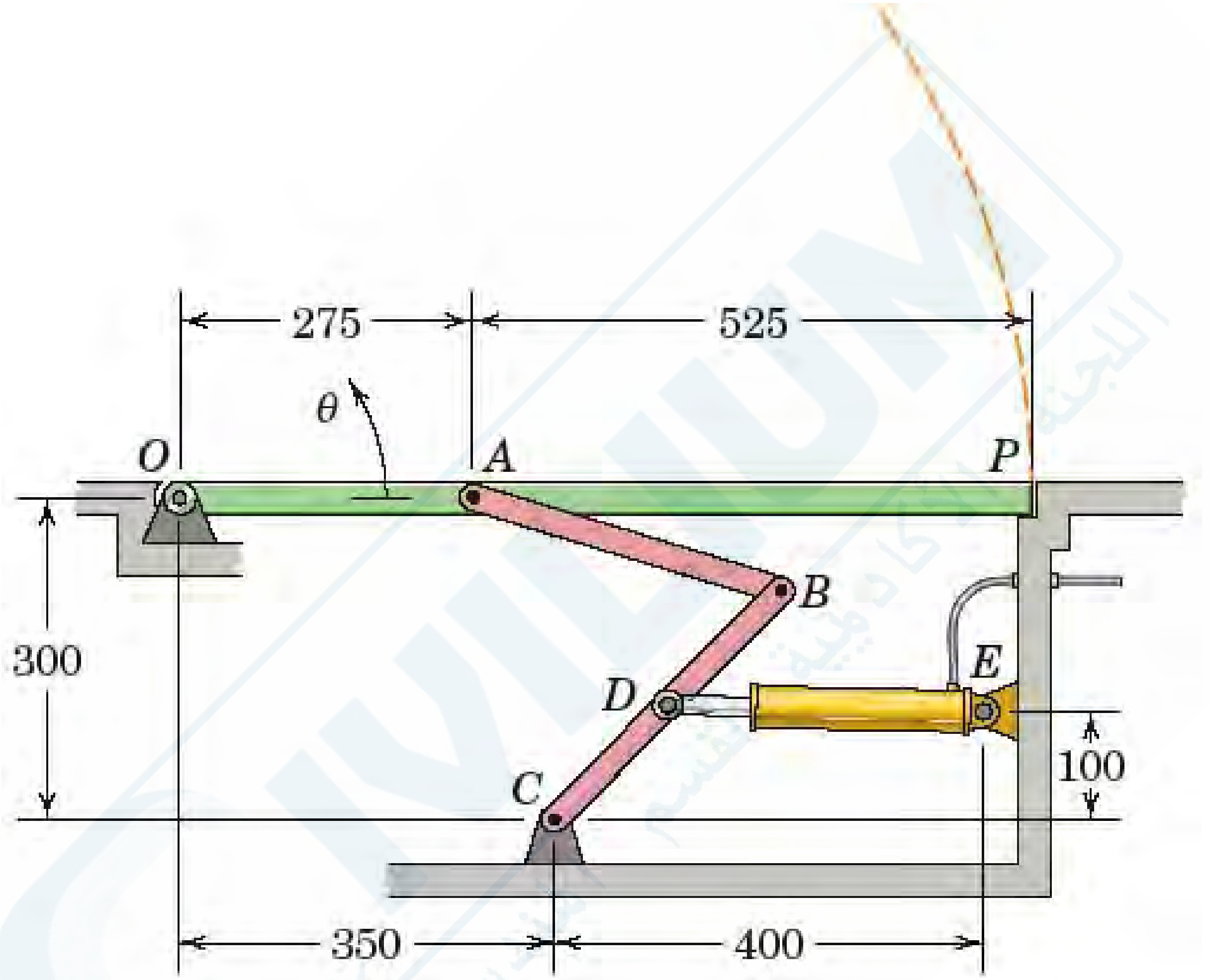
Problem 4/145

P_04_145



Problem 4/146

P 04 146

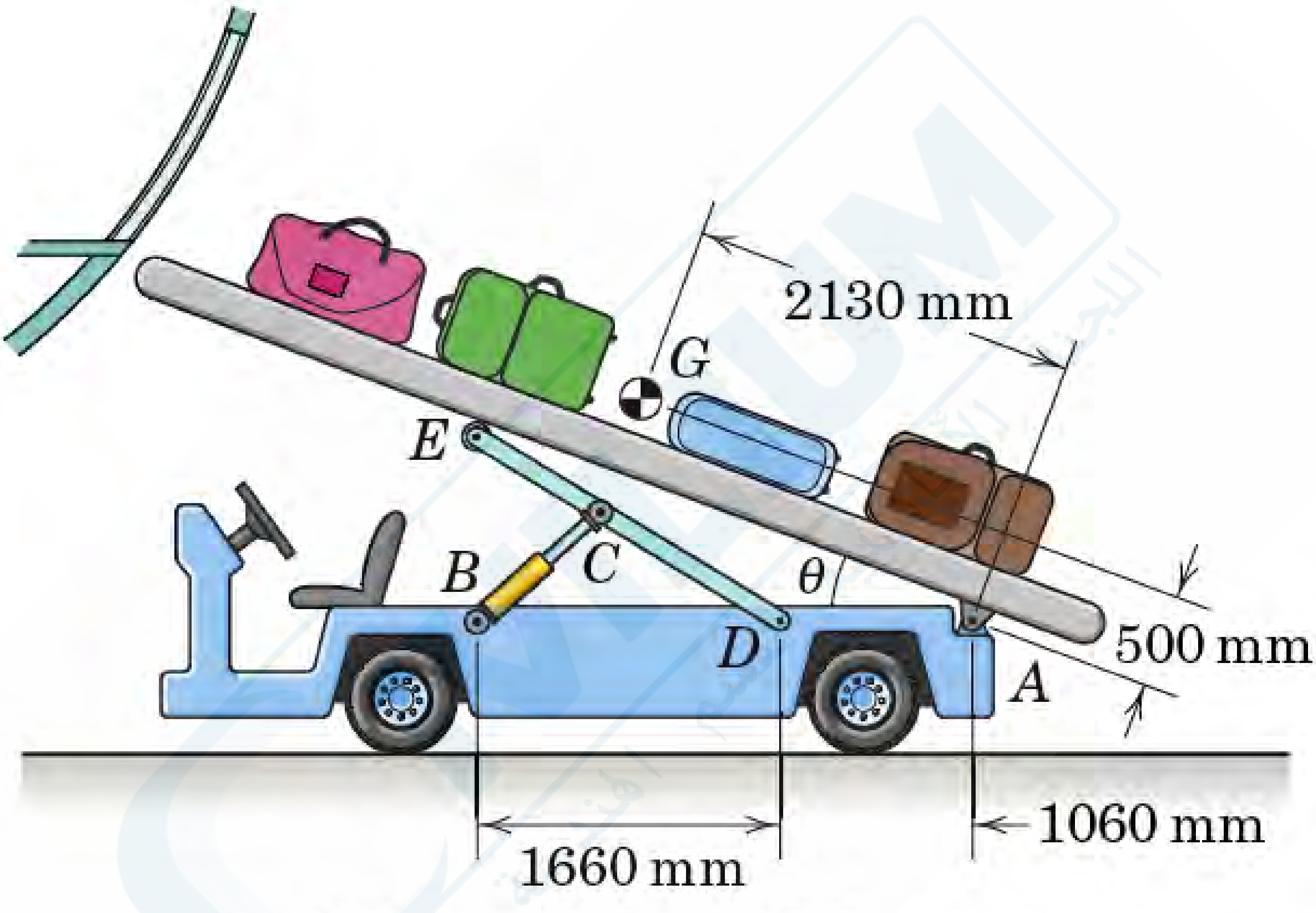


$\overline{AB} = 300$ $\overline{CD} = \overline{DB} = 150$

Dimensions in millimeters

Problem 4/147

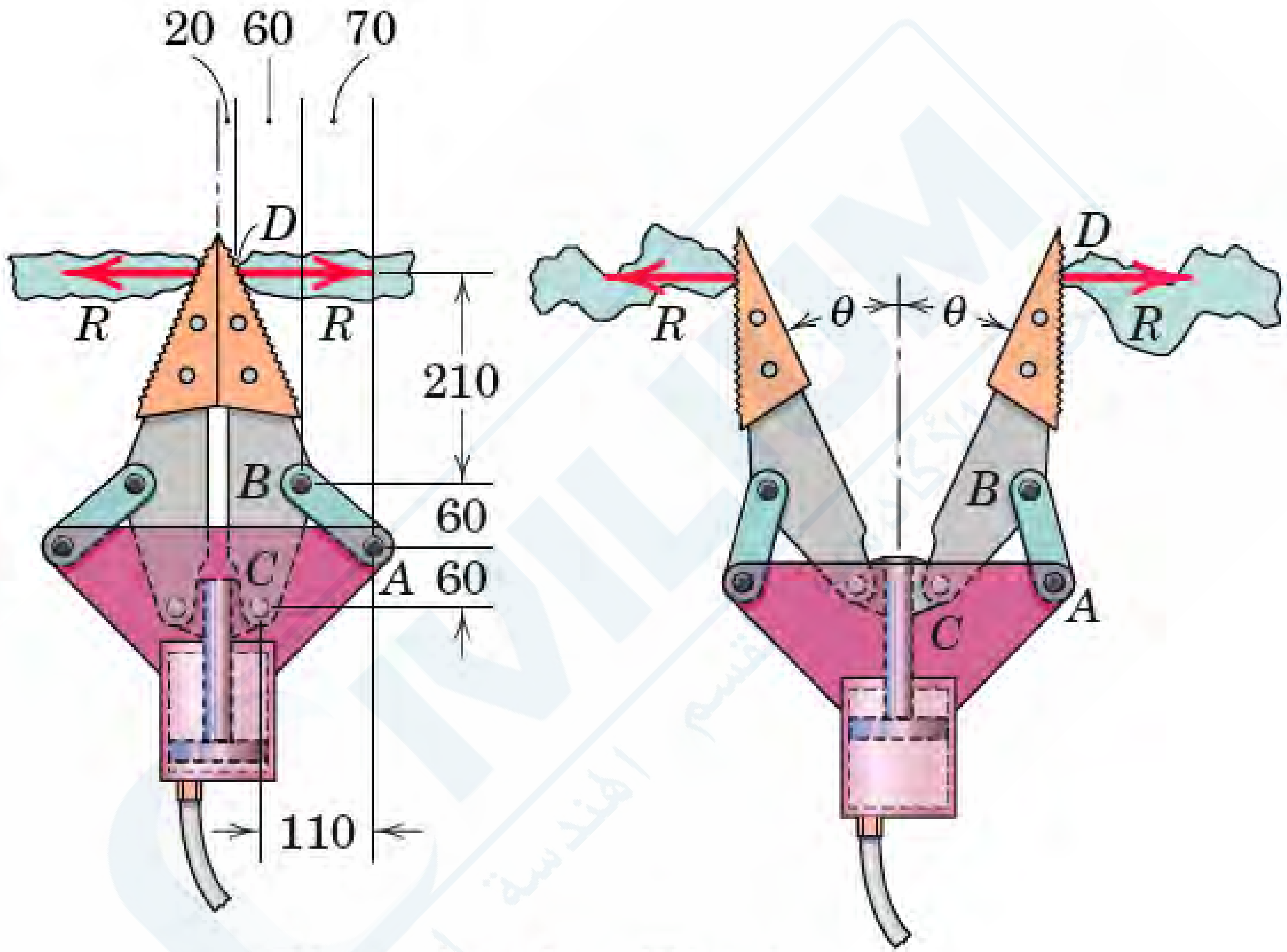
P_04_147



$\overline{DE} = 1945$ mm $\quad \overline{CD} = 1150$ mm

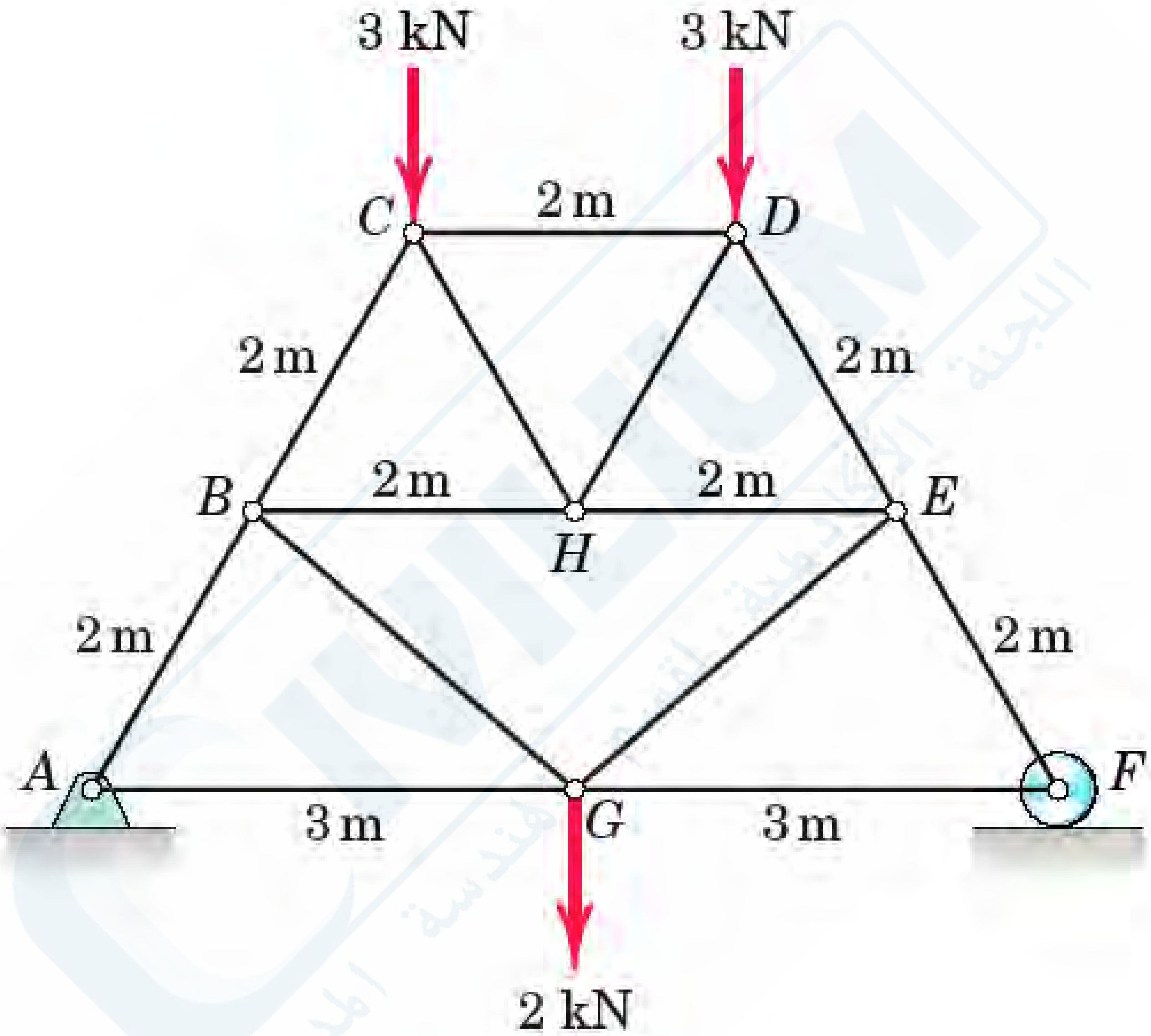
Problem 4/148

P 04 148



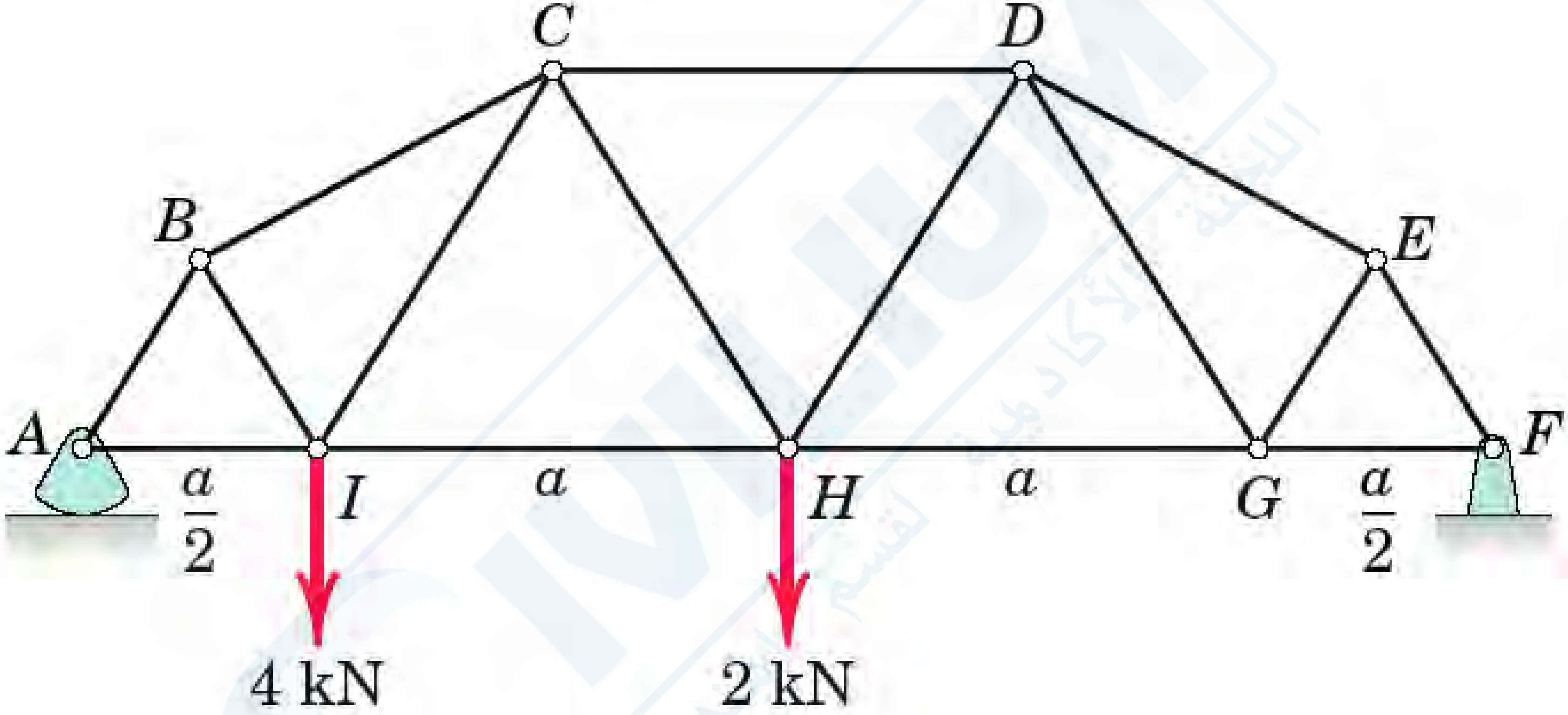
Dimensions in millimeters

Problem 4/149



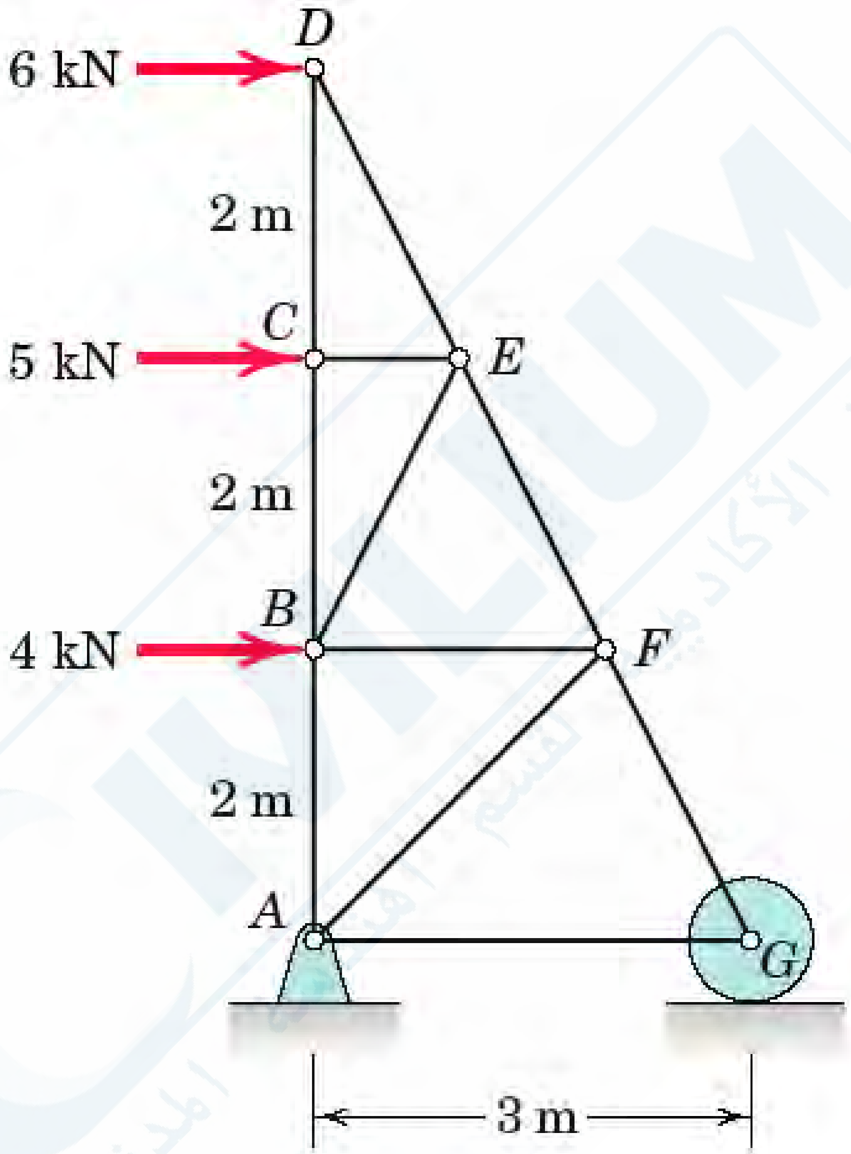
Problem 4/15

P 04 15



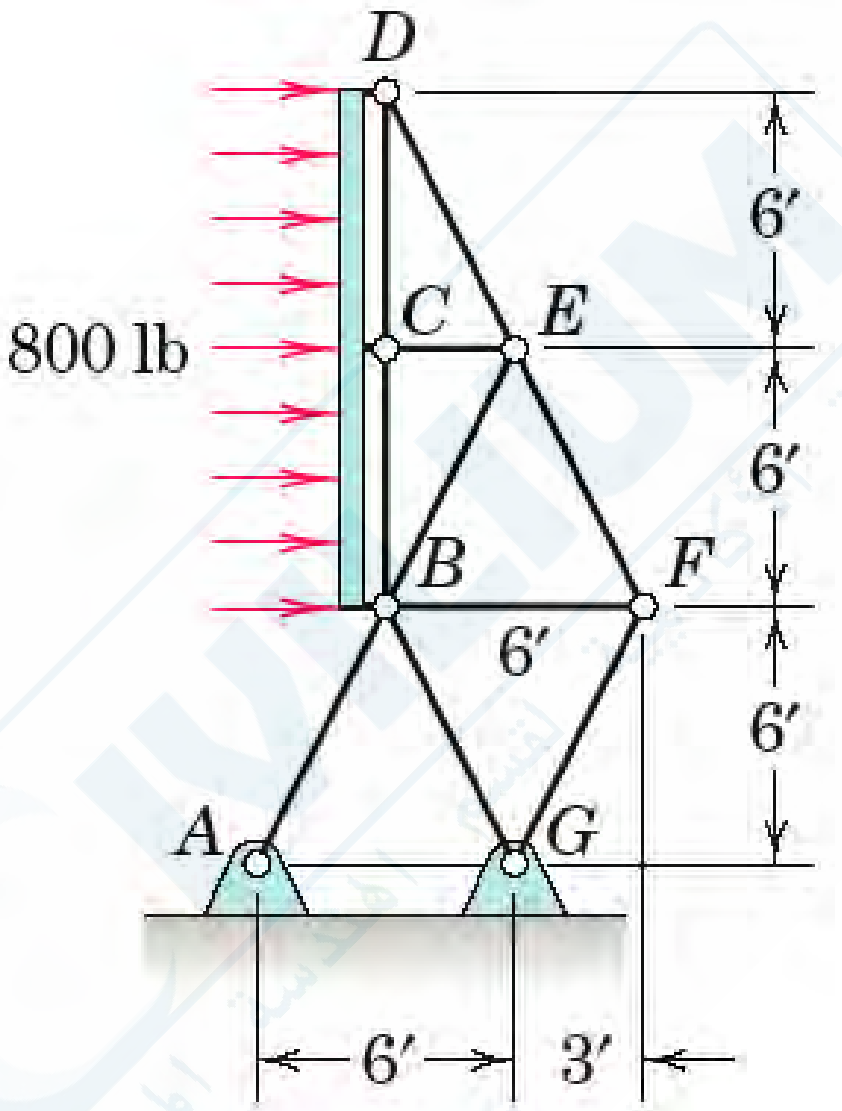
Problem 4/16

P_04_16



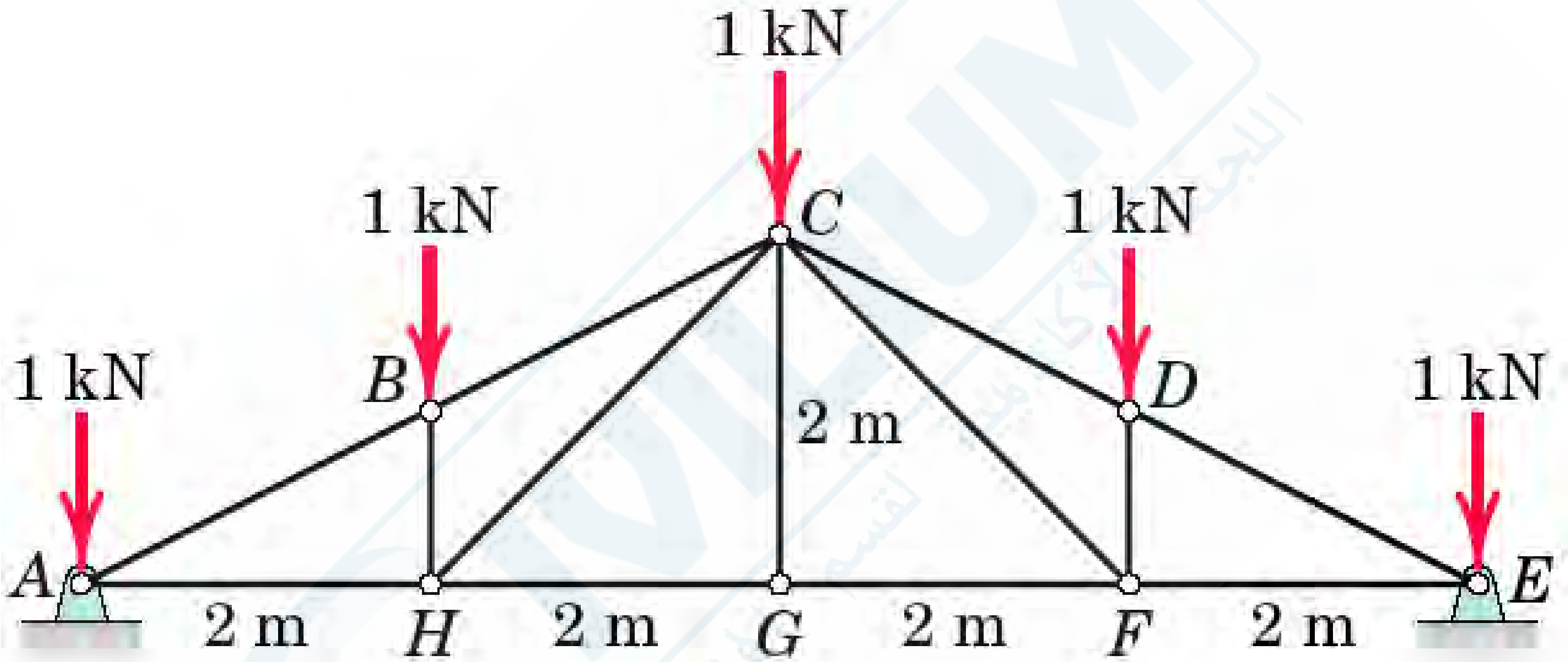
Problem 4/17

P_04_17



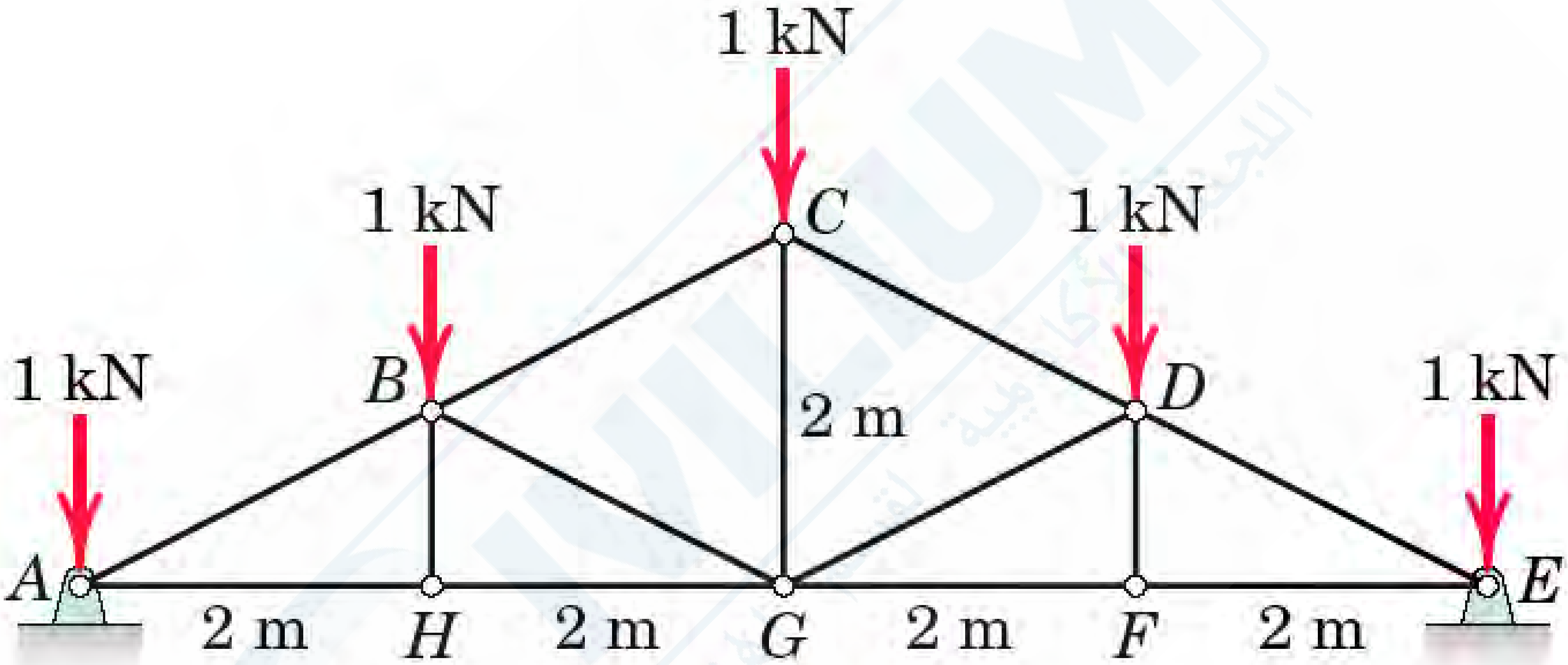
Problem 4/18

P_04_18



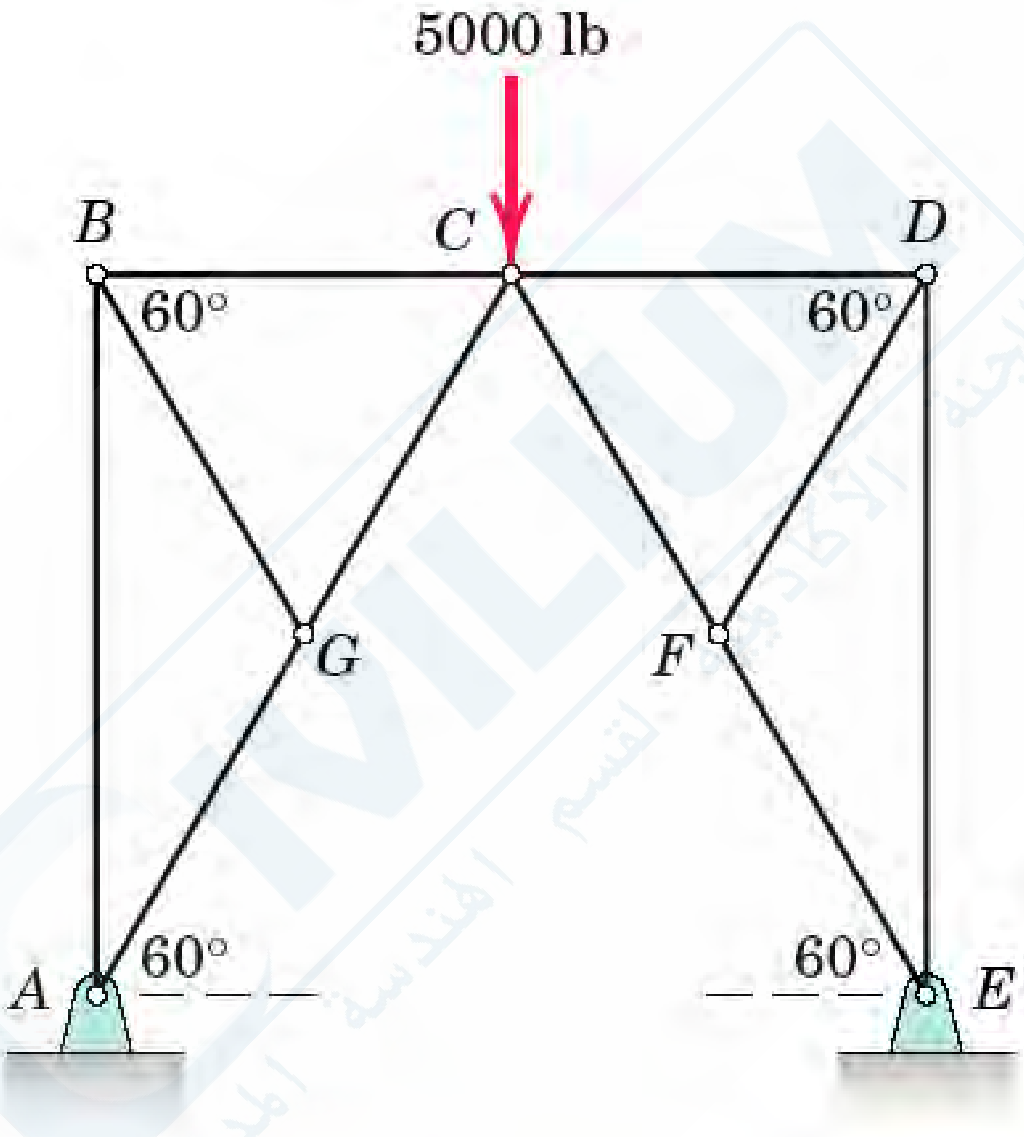
Problem 4/19

P_04_19



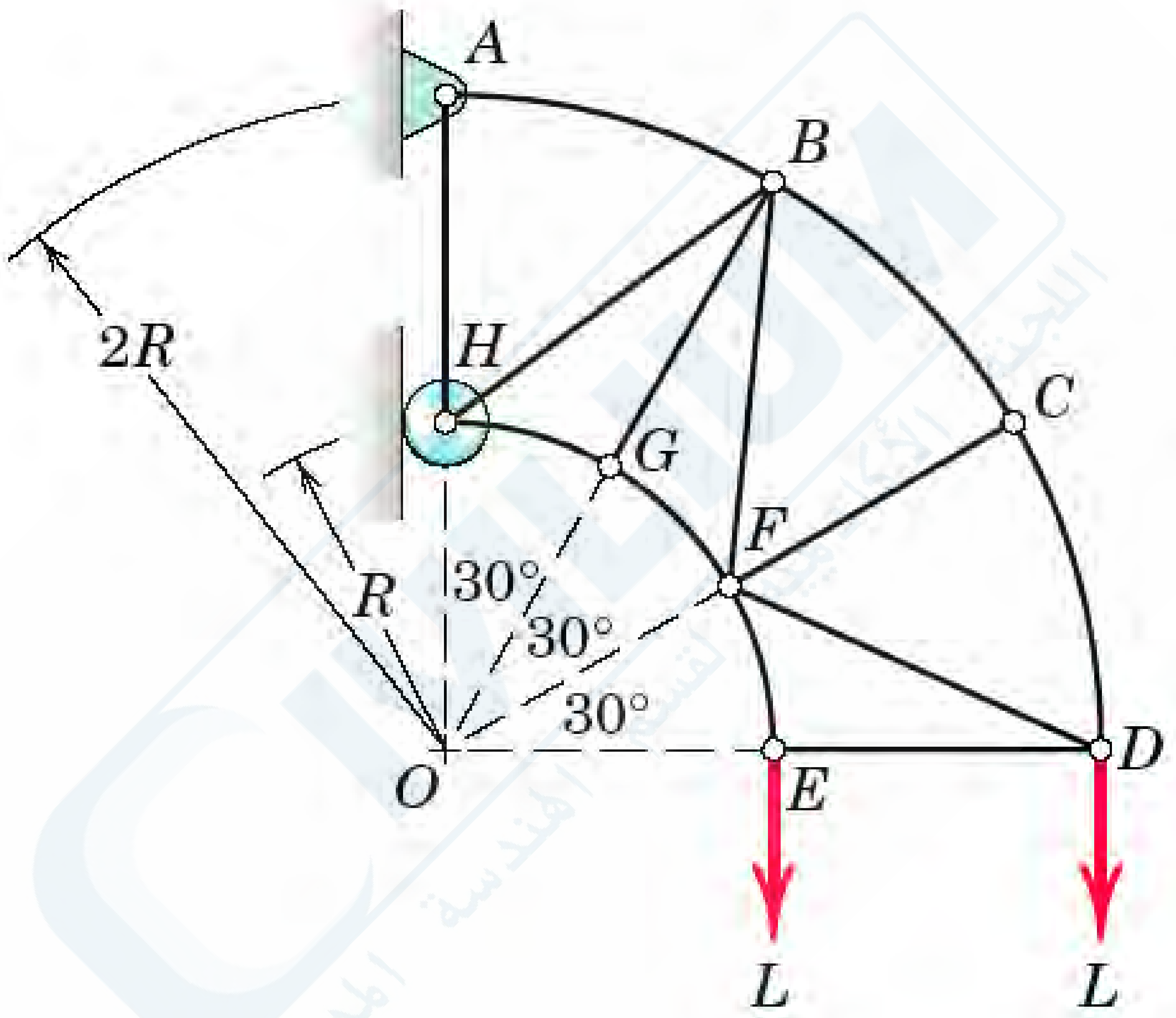
Problem 4/20

P_04_20



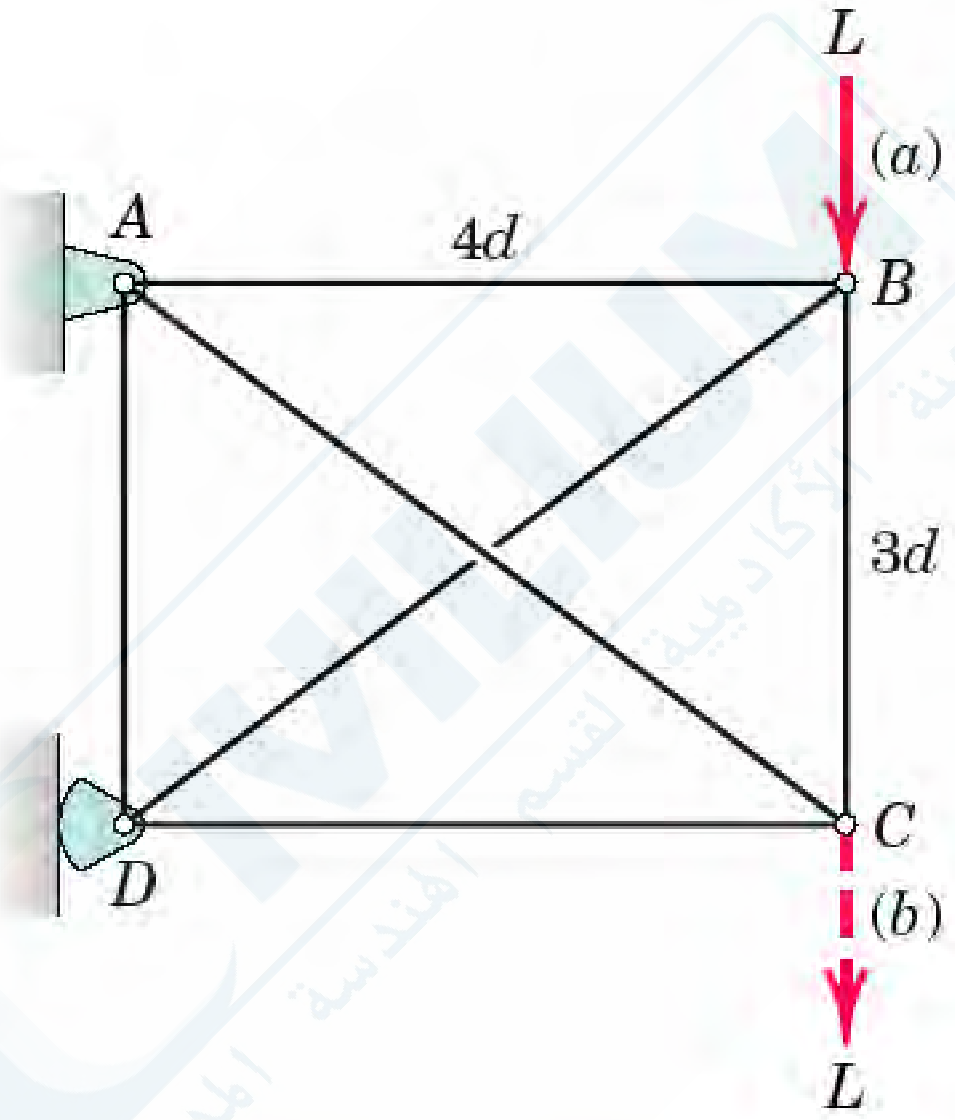
Problem 4/21

P_04_21



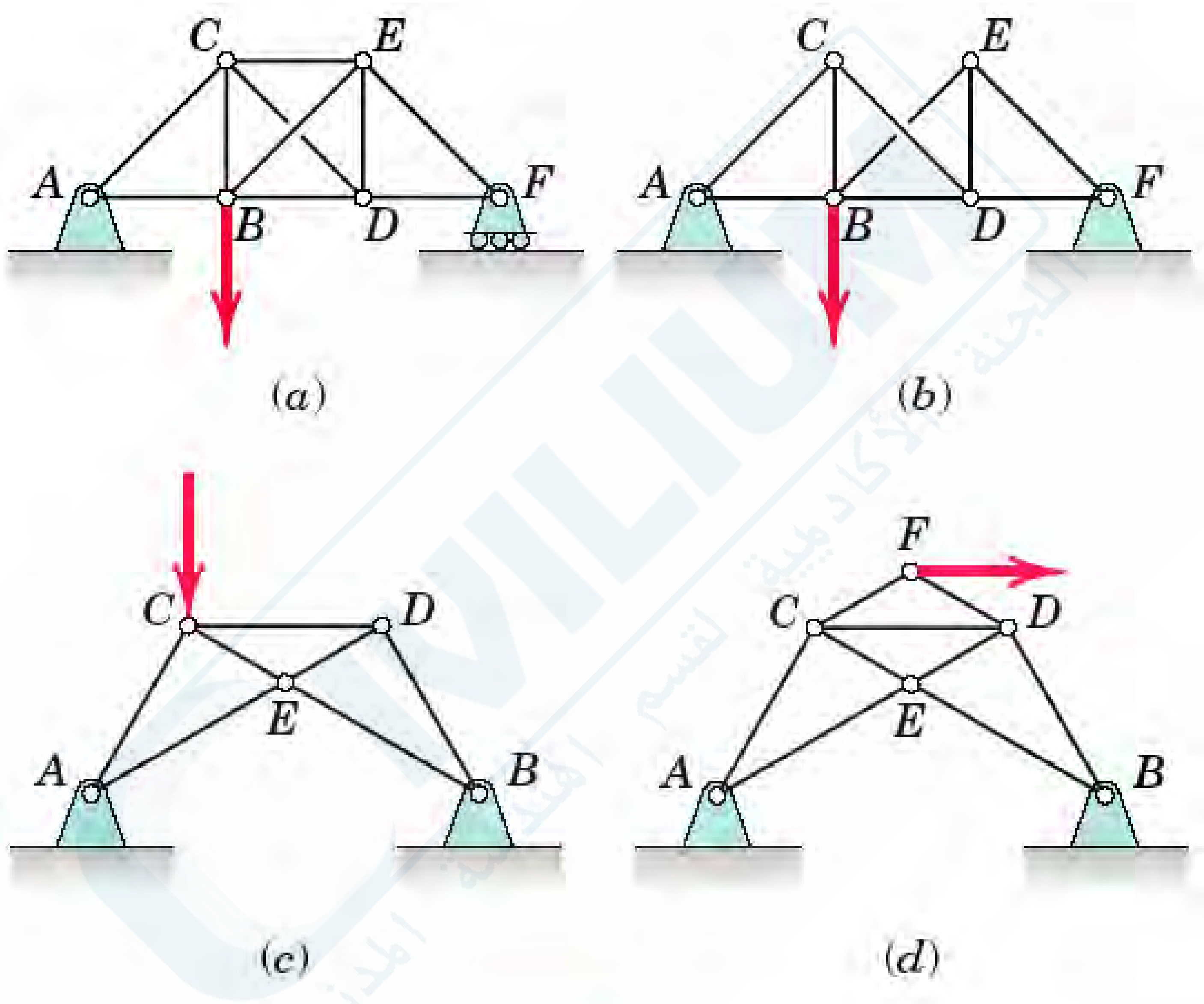
Problem 4/22

P_04_22

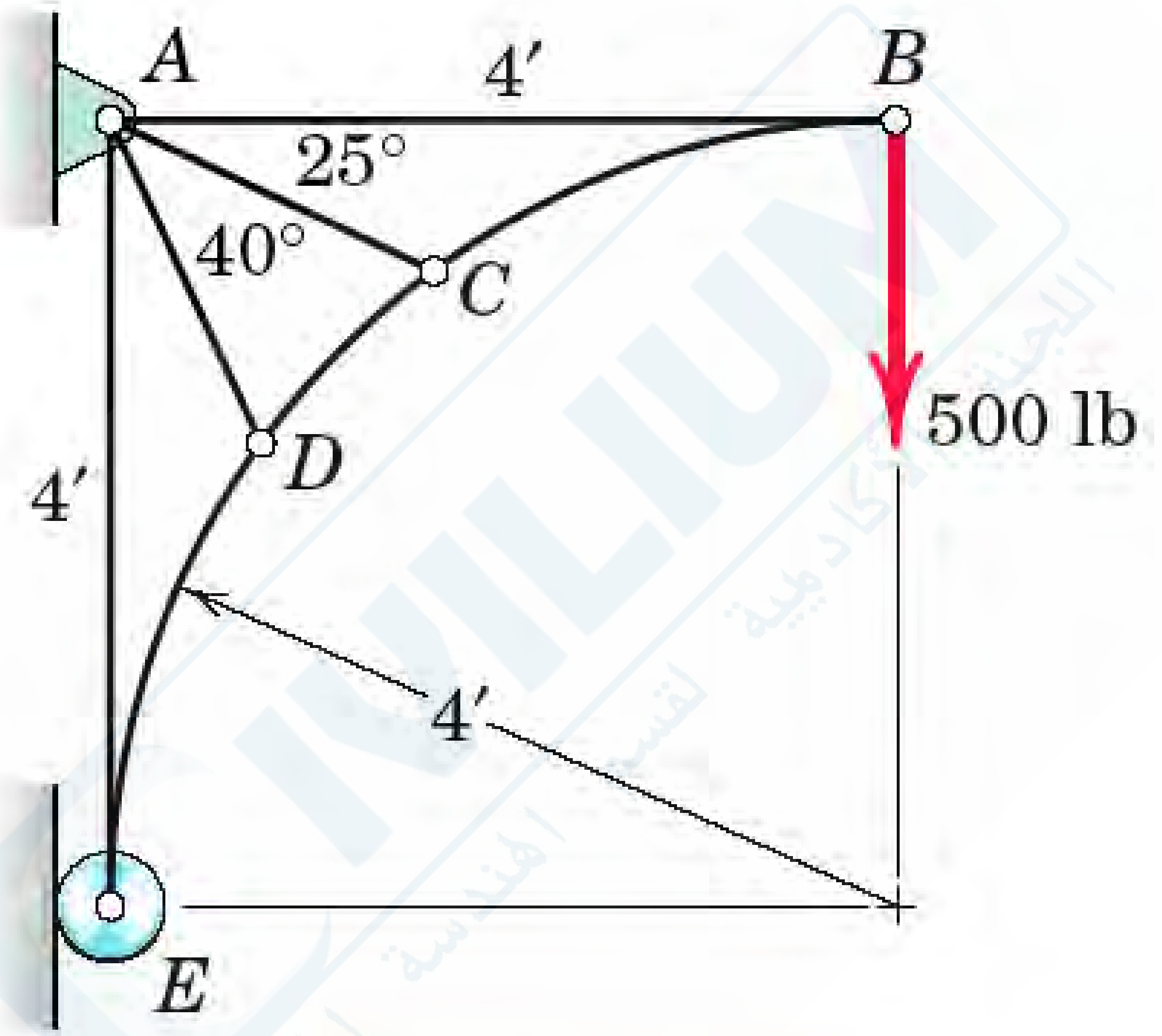


Problem 4/23

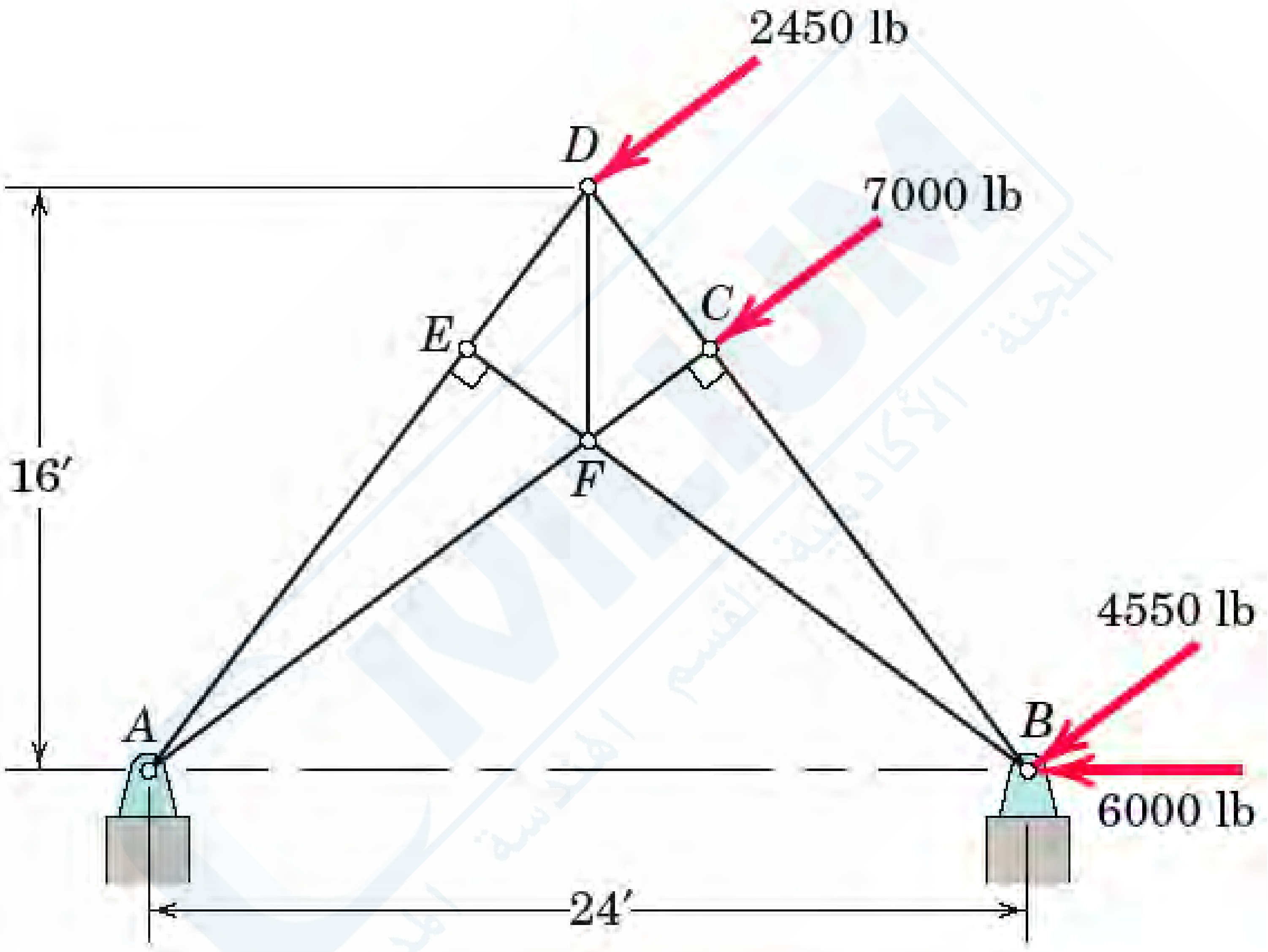
P 04 23



Problem 4/24

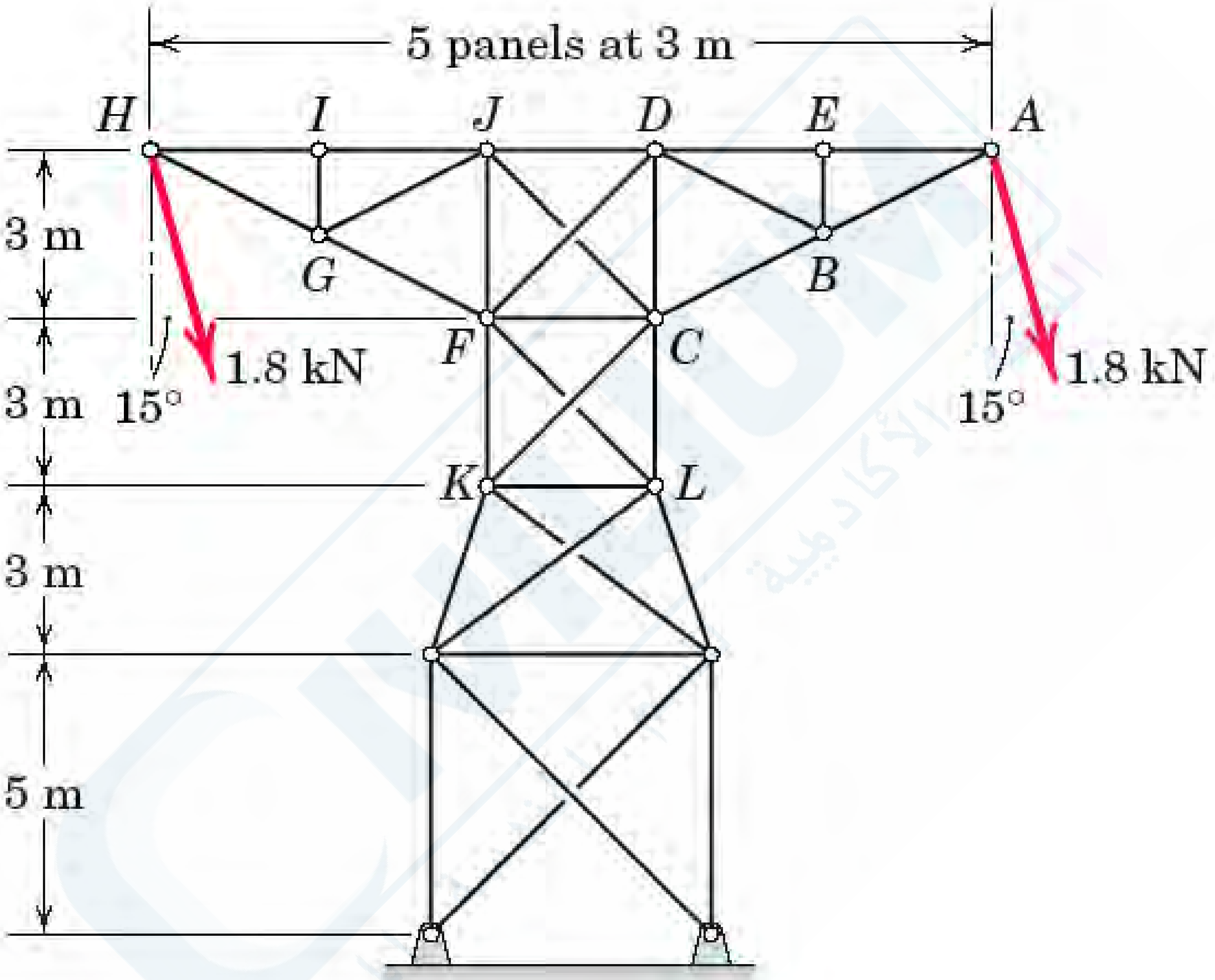


Problem 4/25



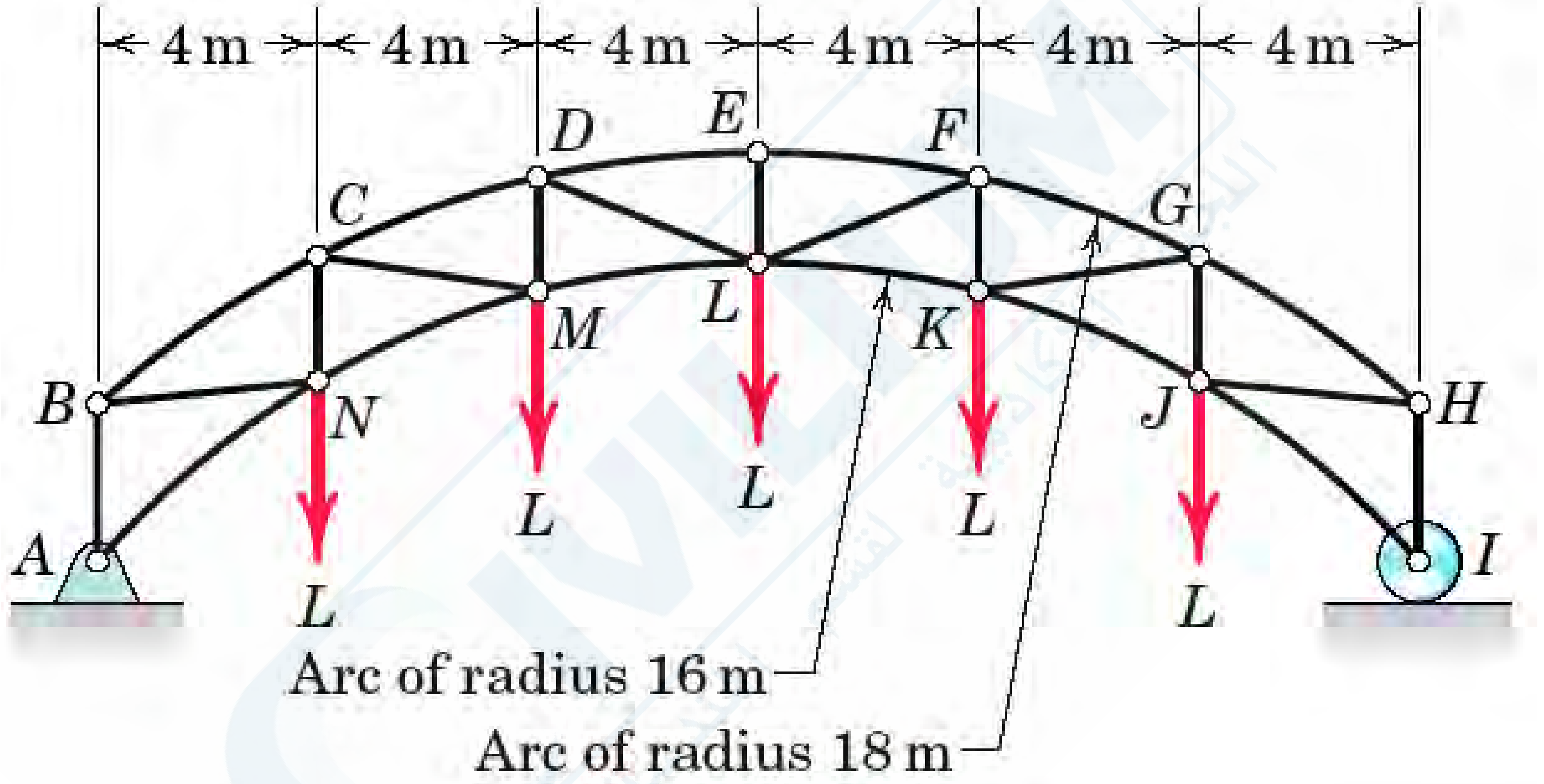
Problem 4/26

P 04 26



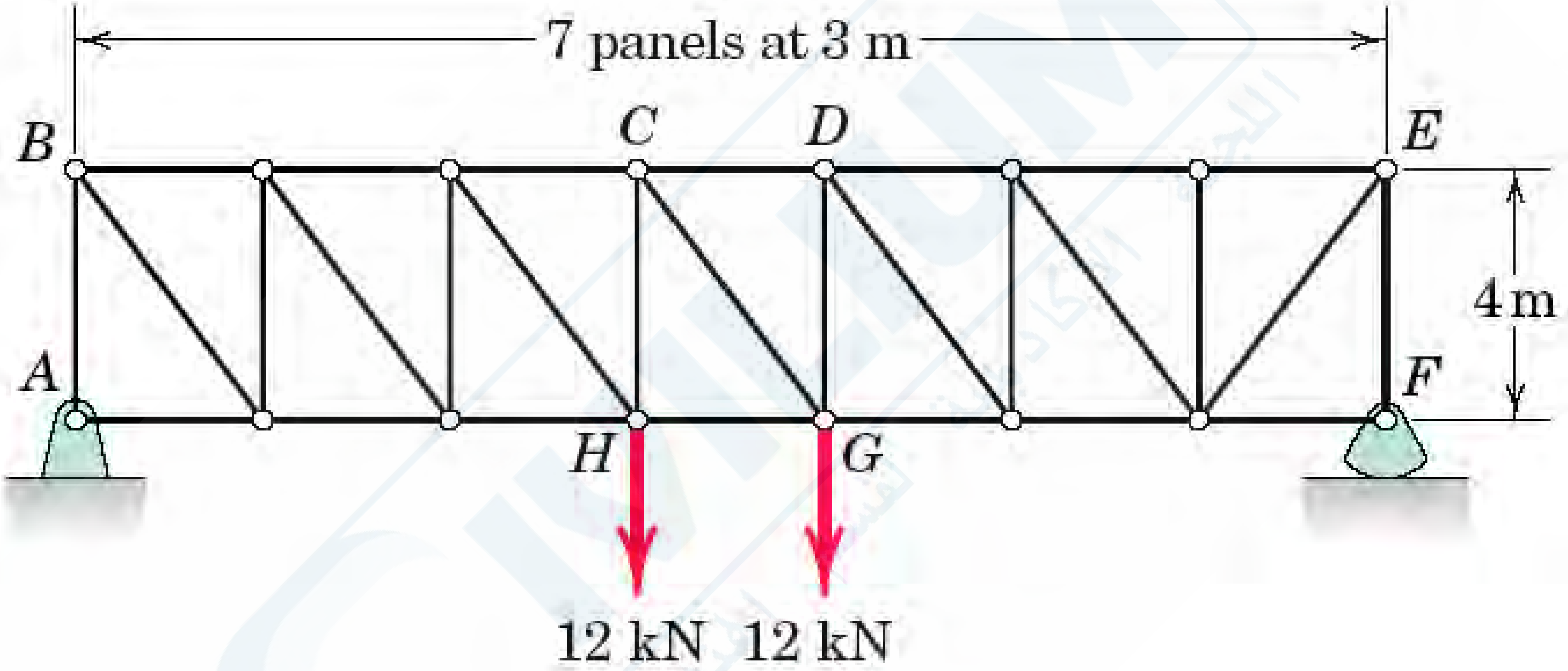
Problem 4/27

P 04 27



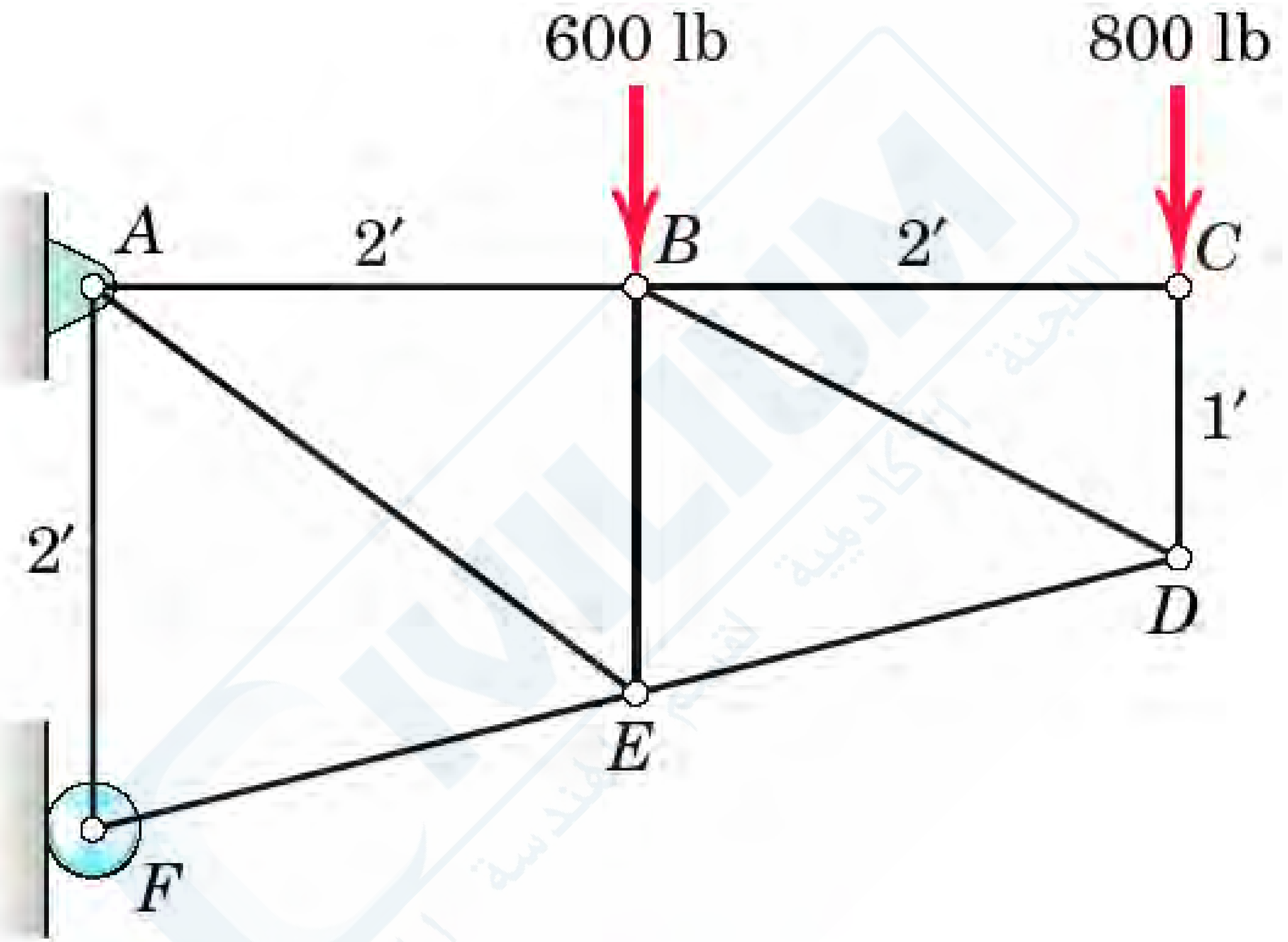
Problem 4/28

P_04_28



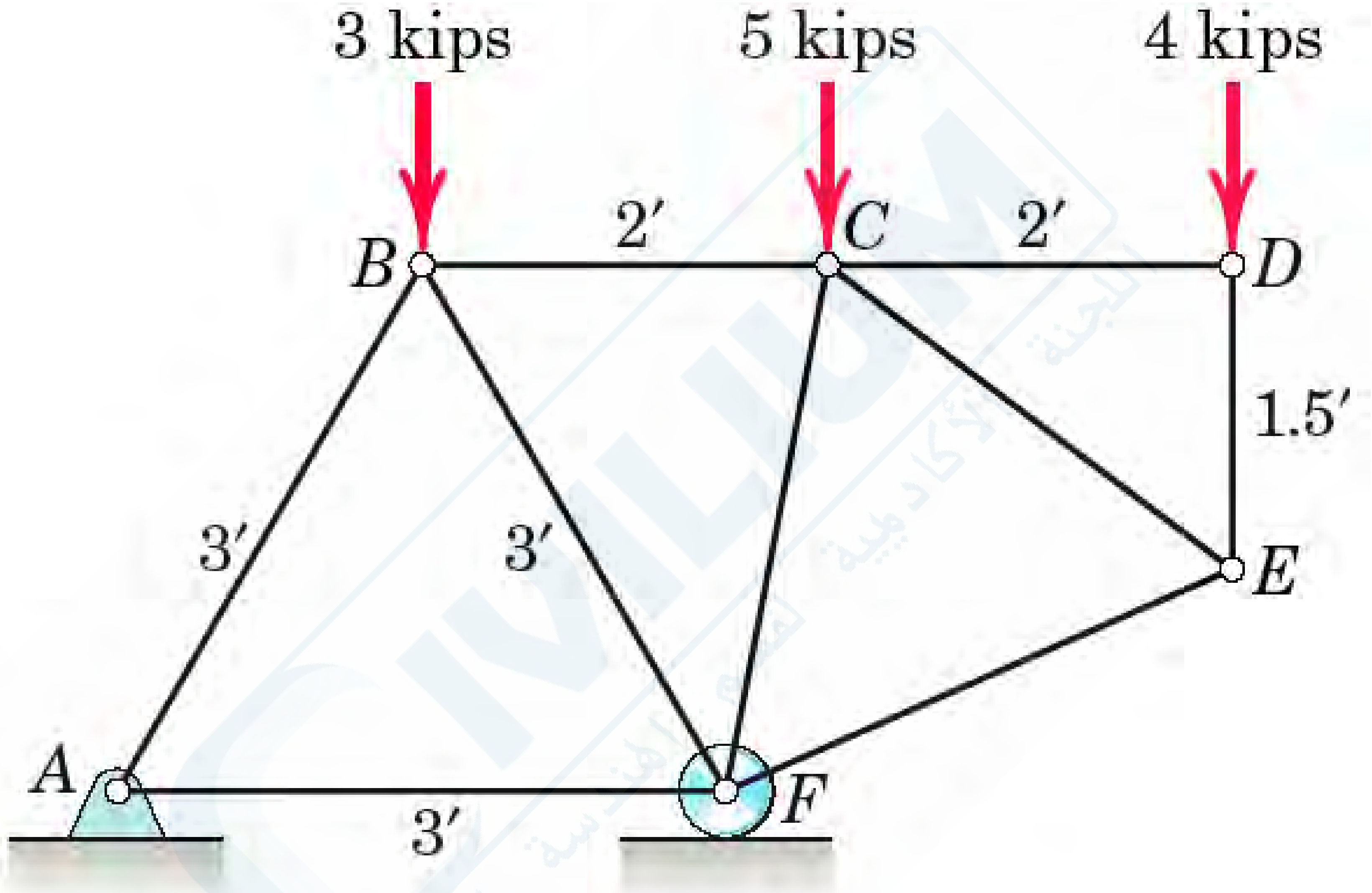
Problem 4/29

P_04_29



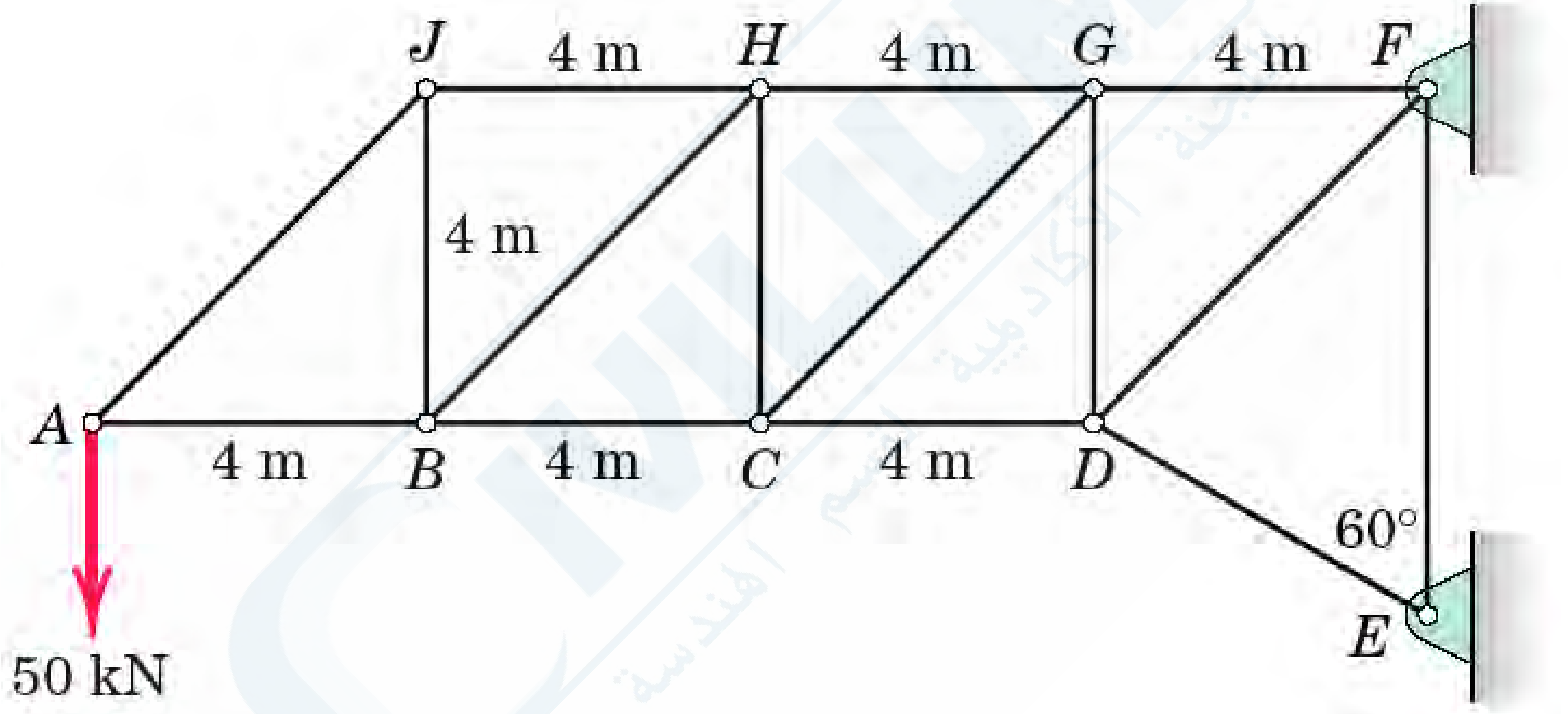
Problem 4/30

P_04_30



Problem 4/31

P_04_31



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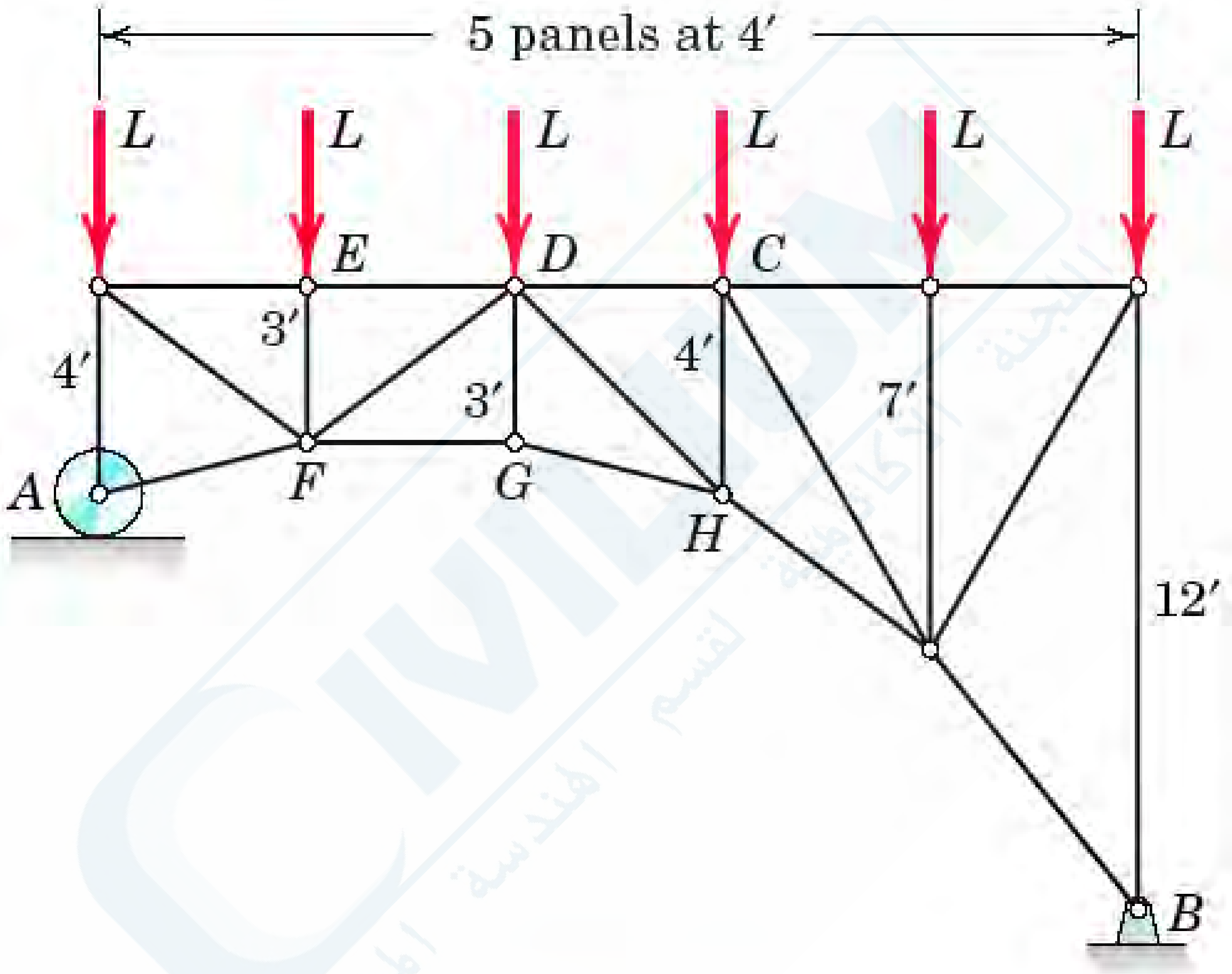
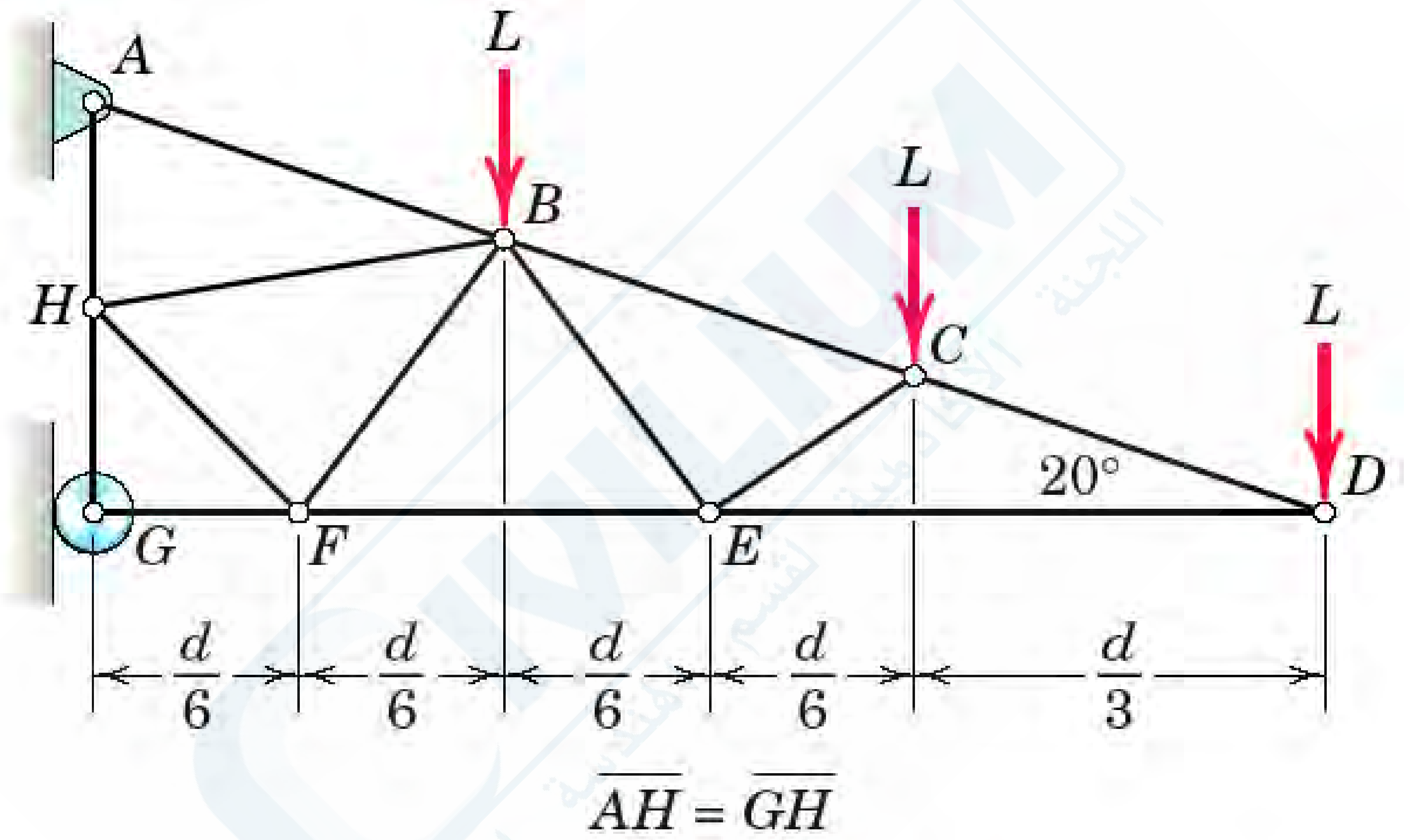


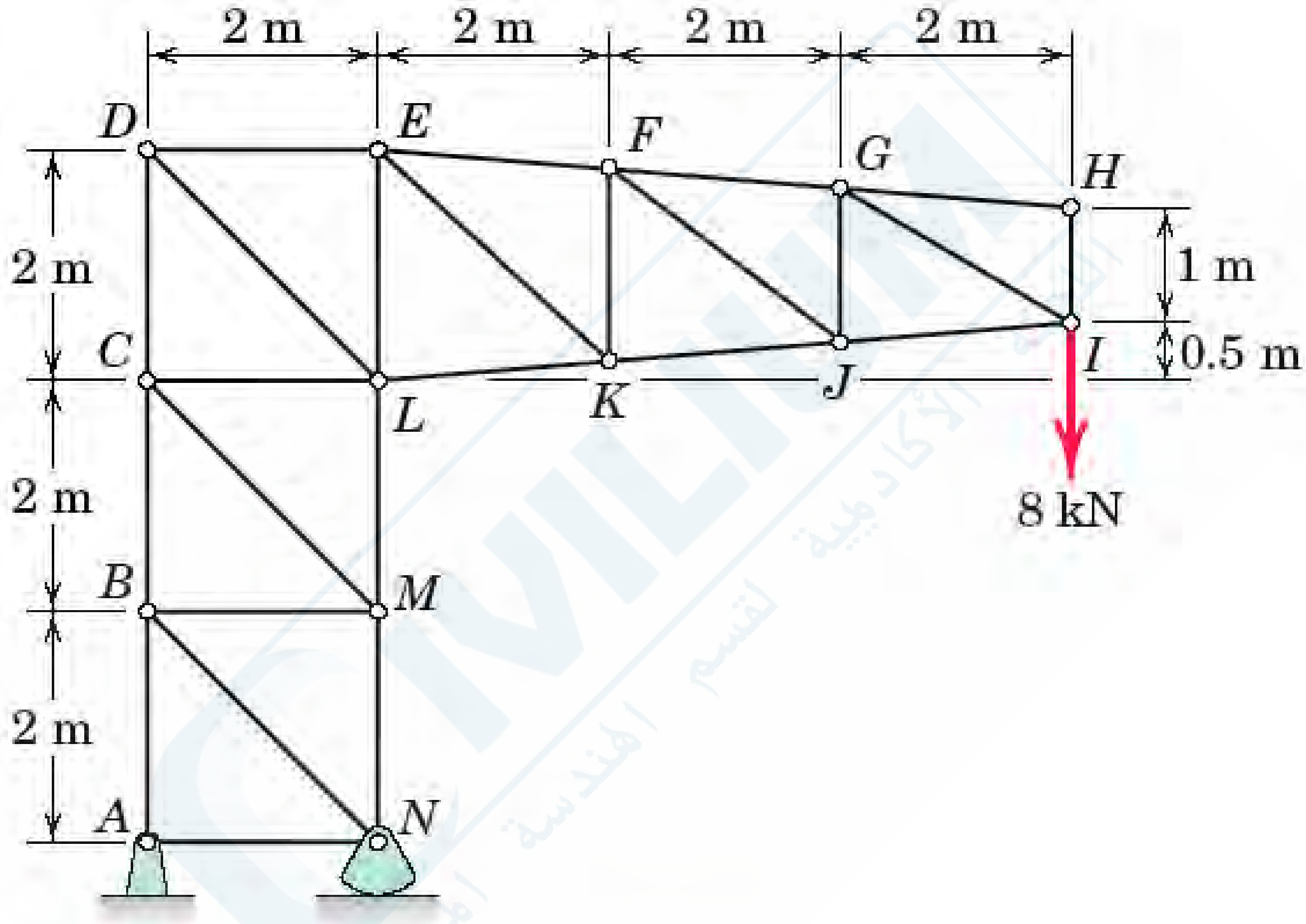
Problem 4/33

P 04 33

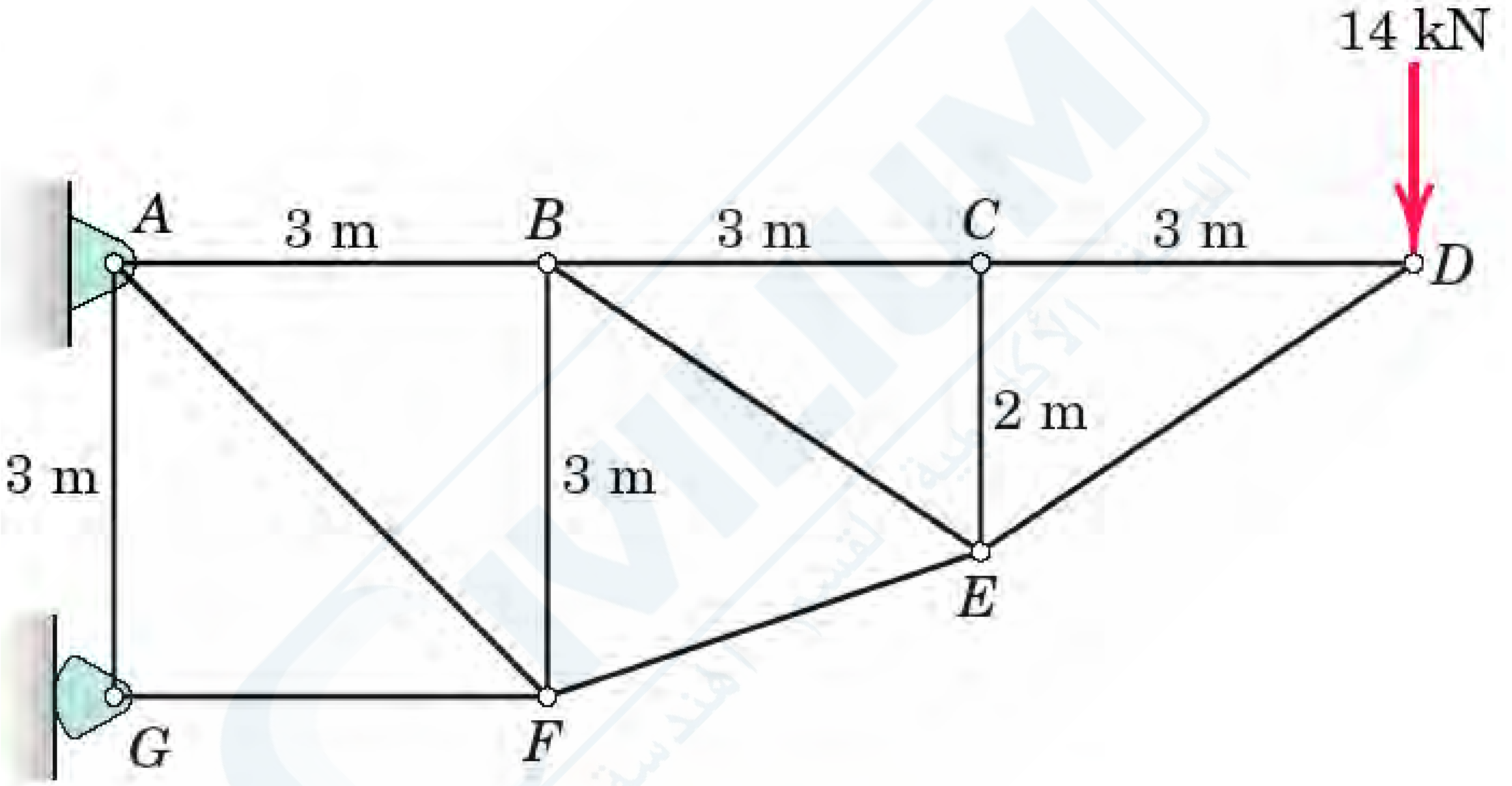


Problem 4/34

P_04_34

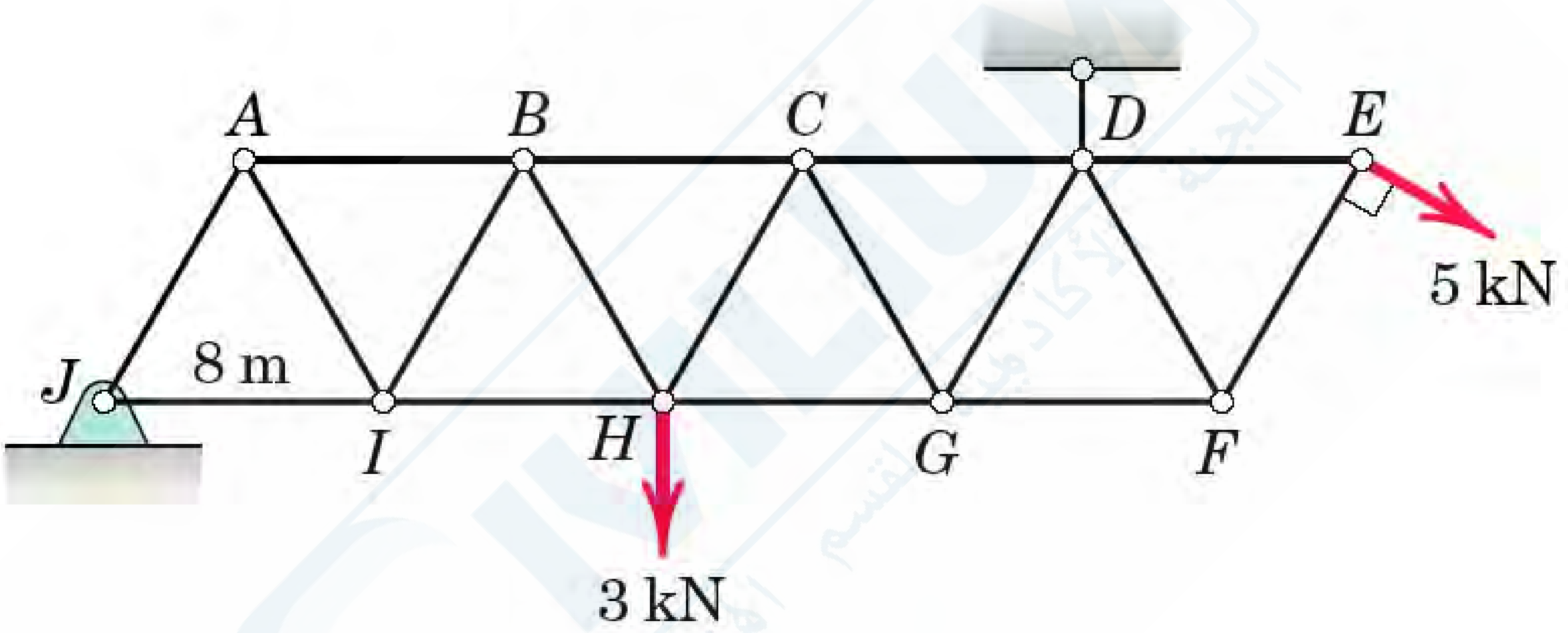


Problem 4/35



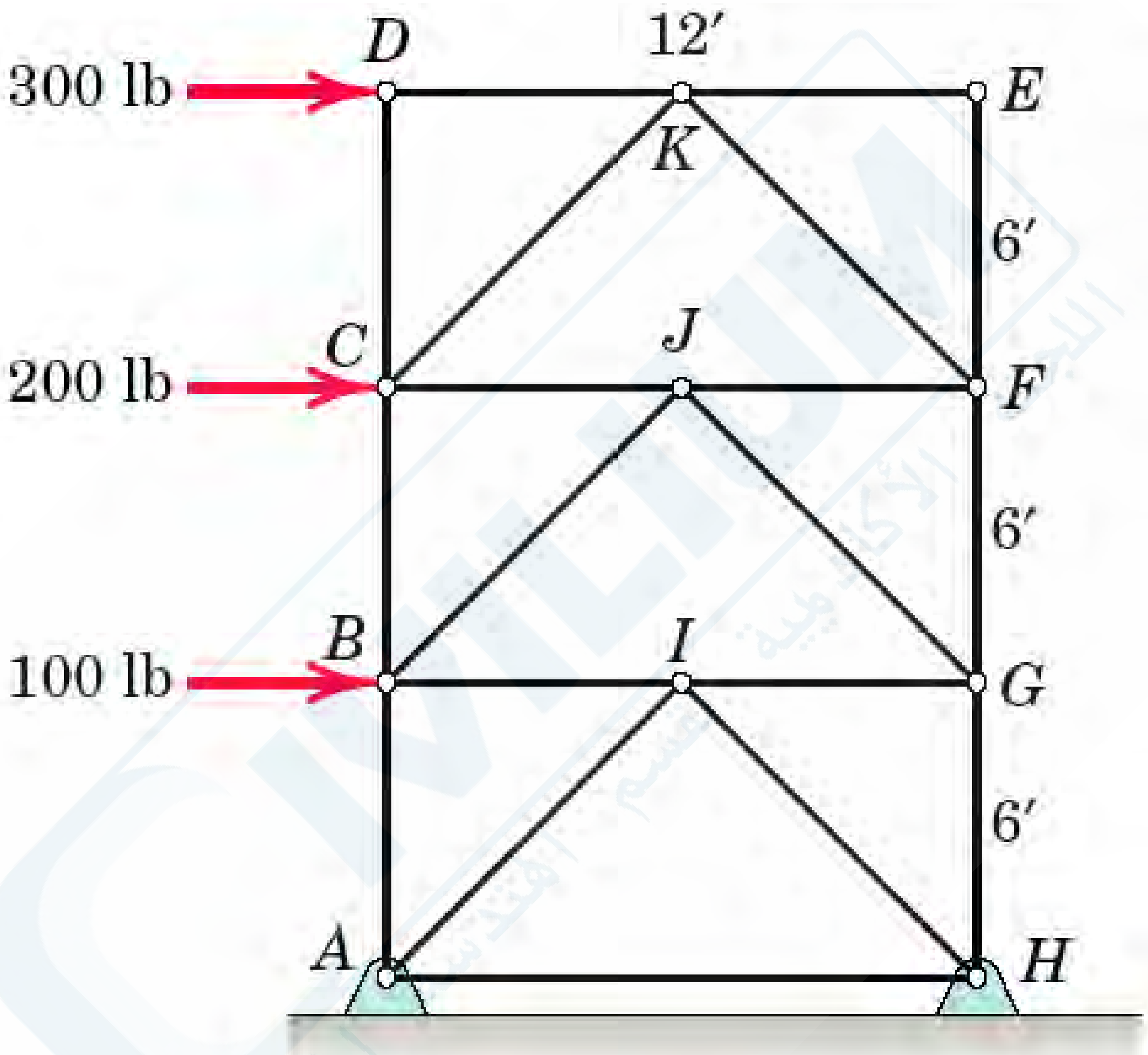
Problem 4/36

P_04_36



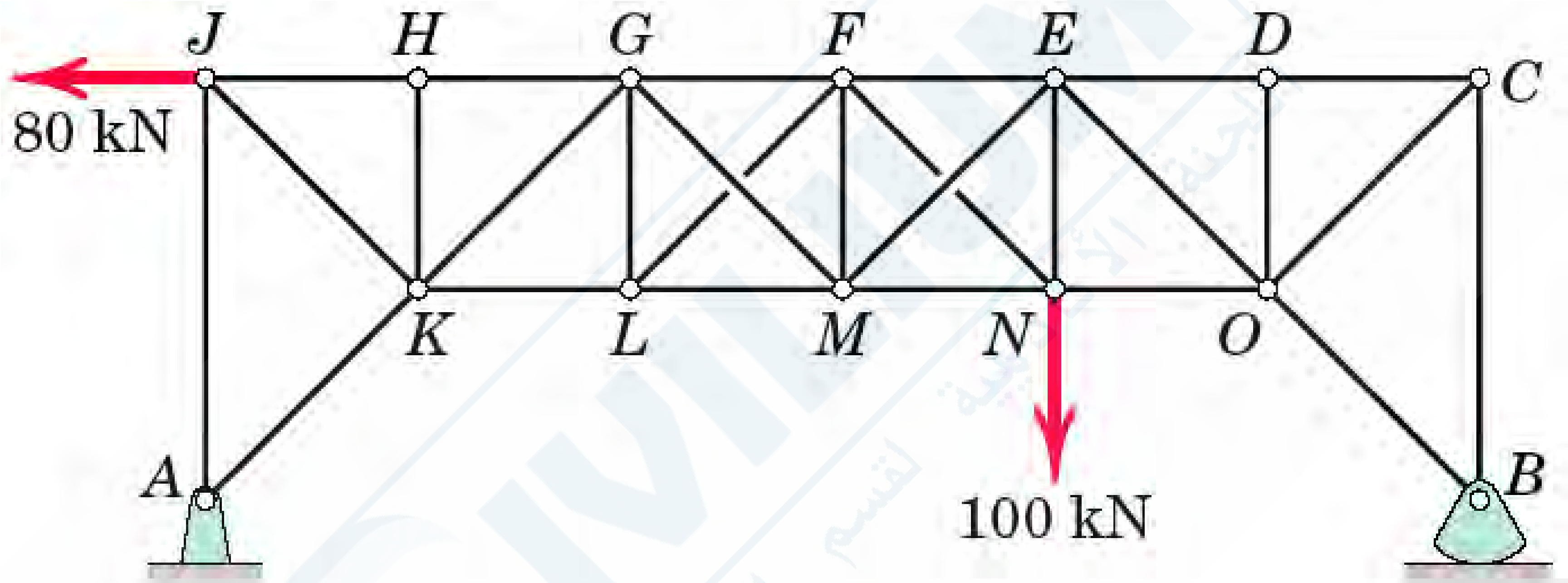
Problem 4/37

P_04_37



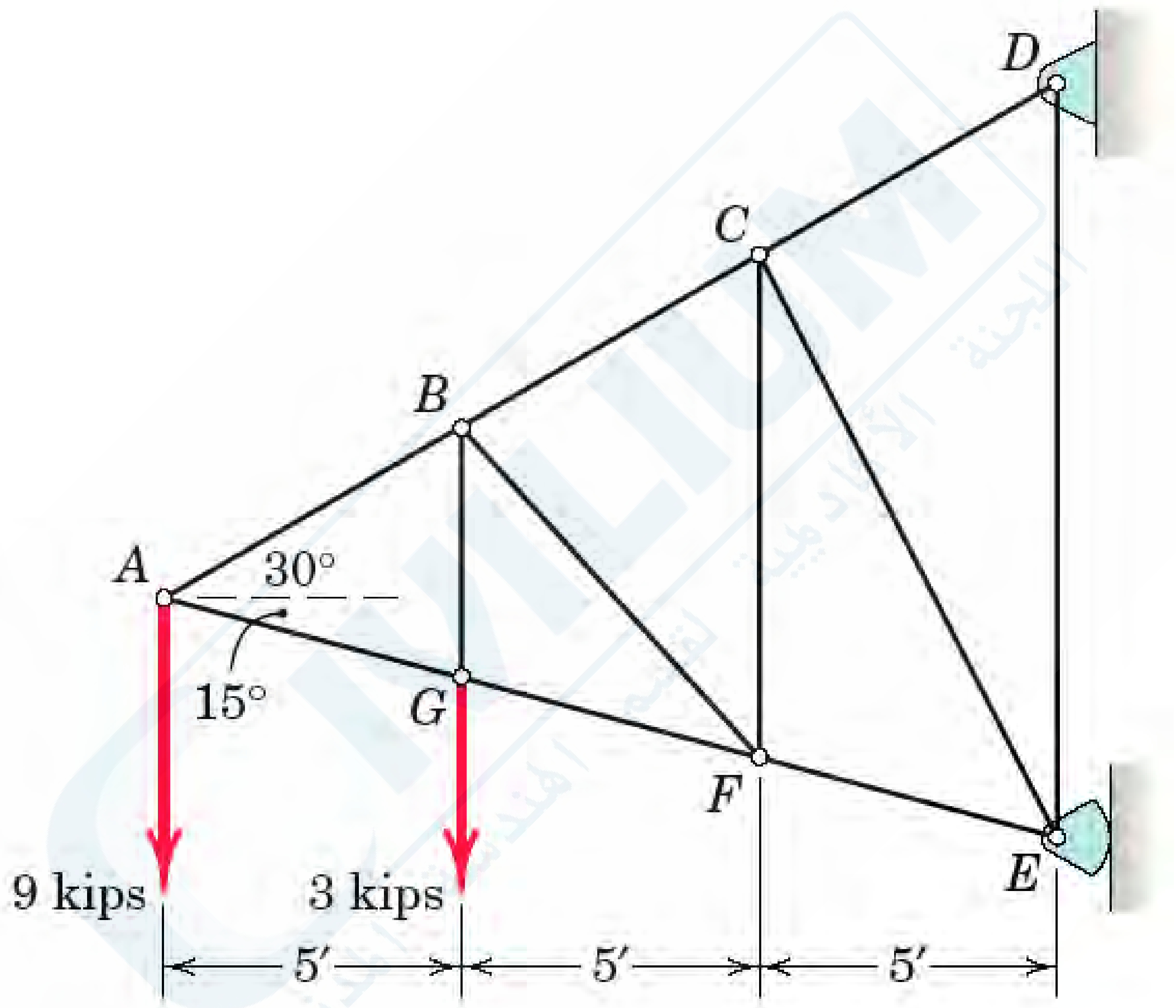
Problem 4/38

P_04_38



Problem 4/39

P_04_39



Problem 4/40

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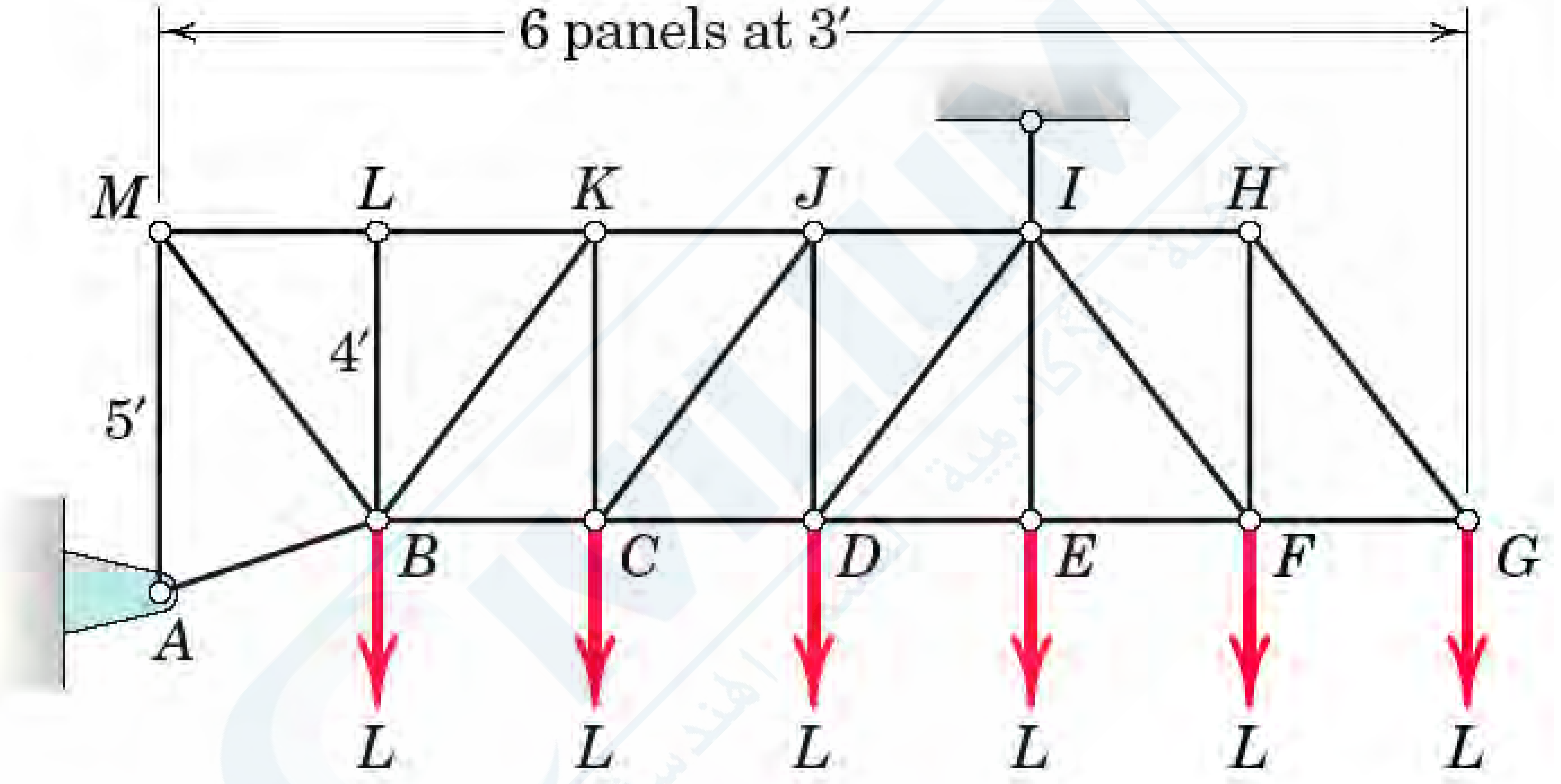
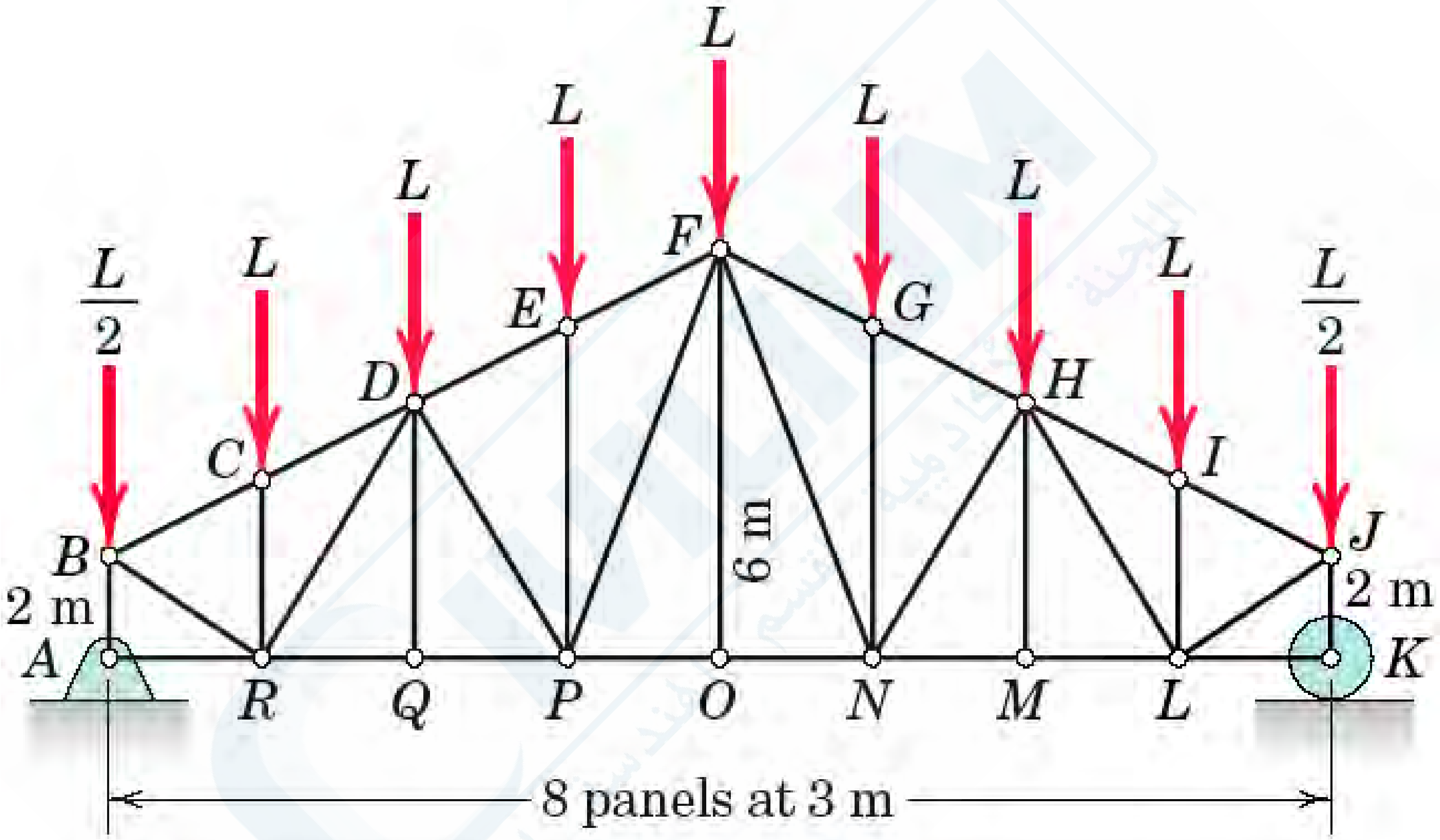


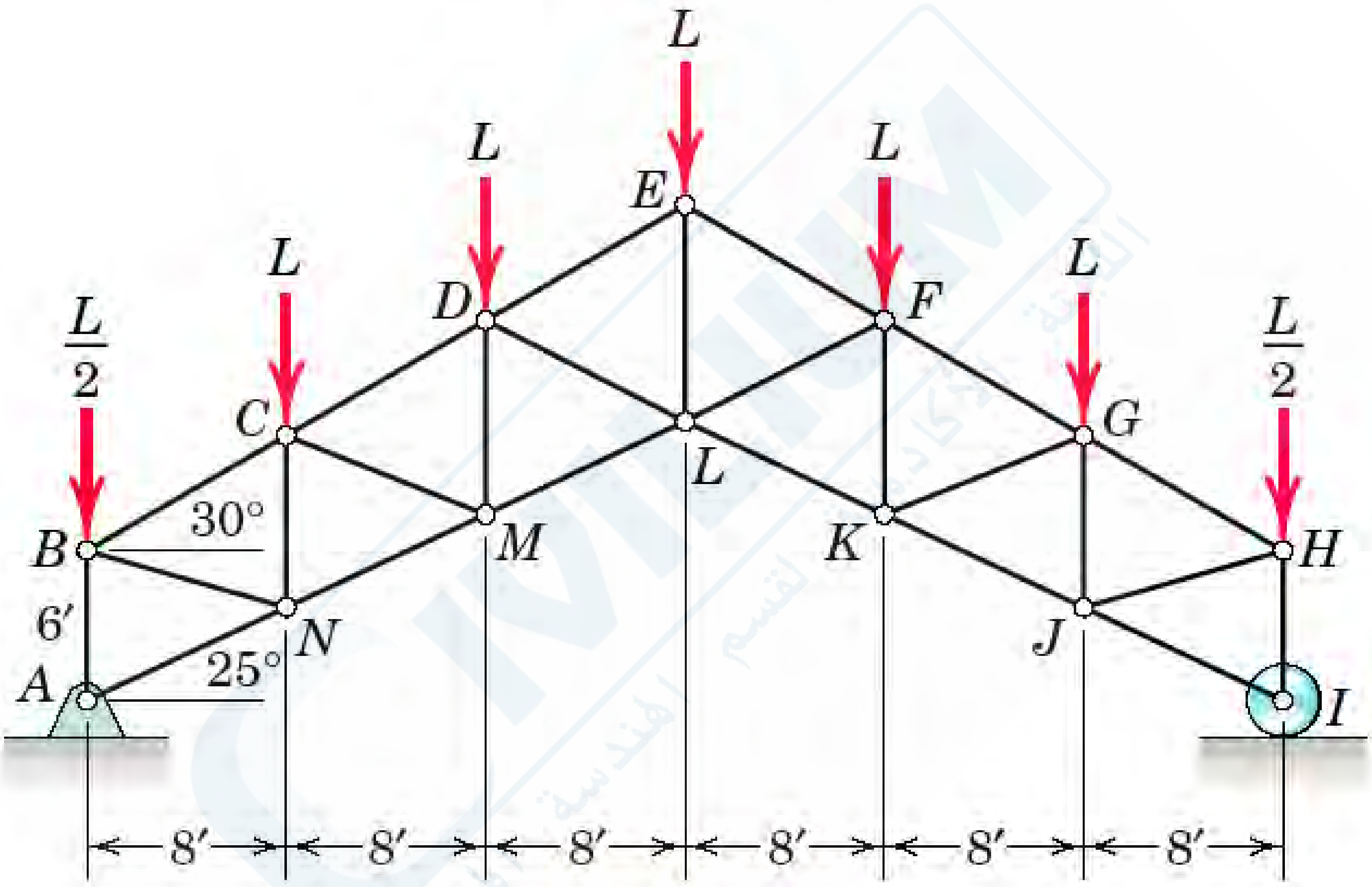
Problem 4/41

P_04_41



Problem 4/42

P_04_42



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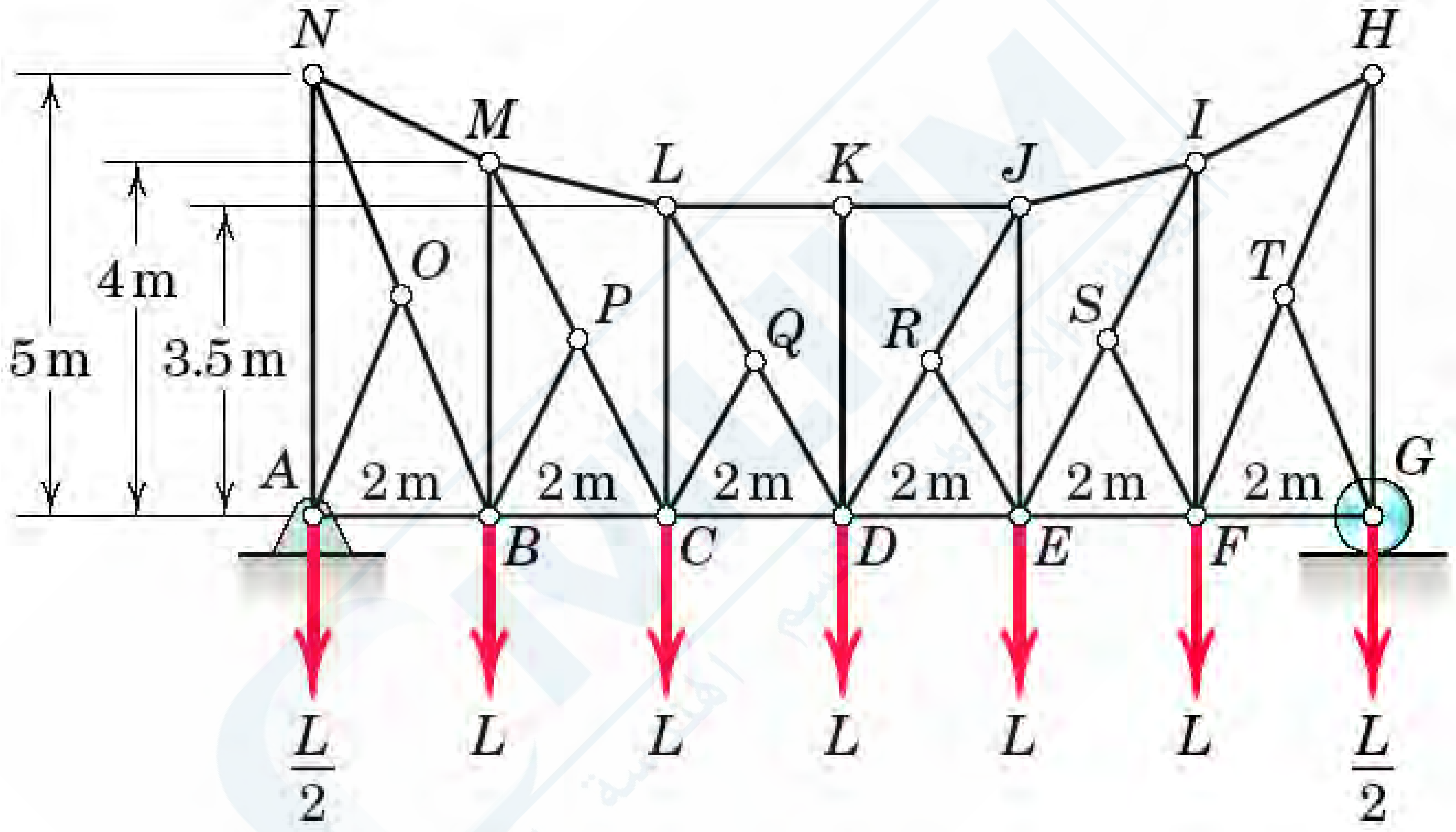
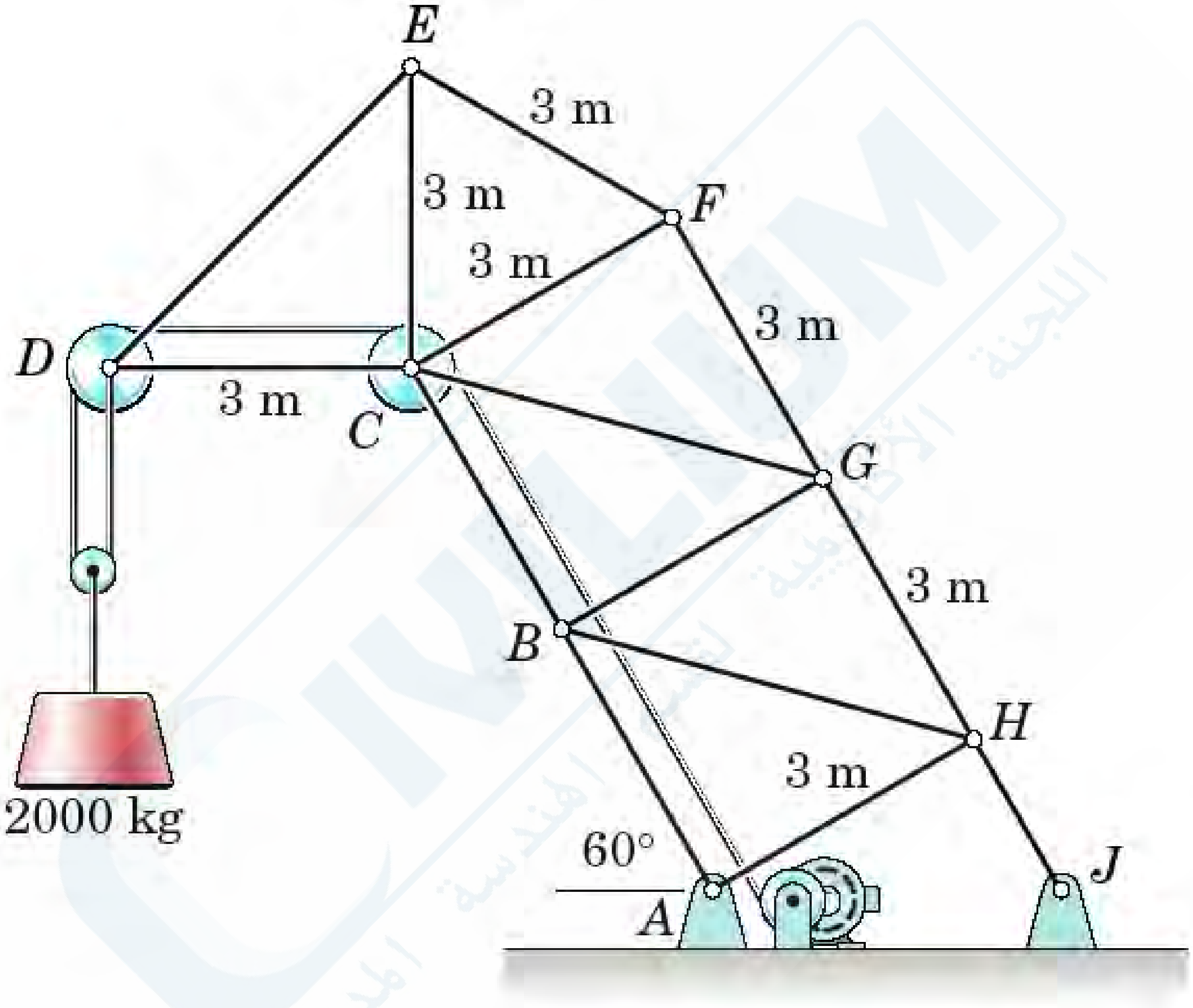
P_04_43



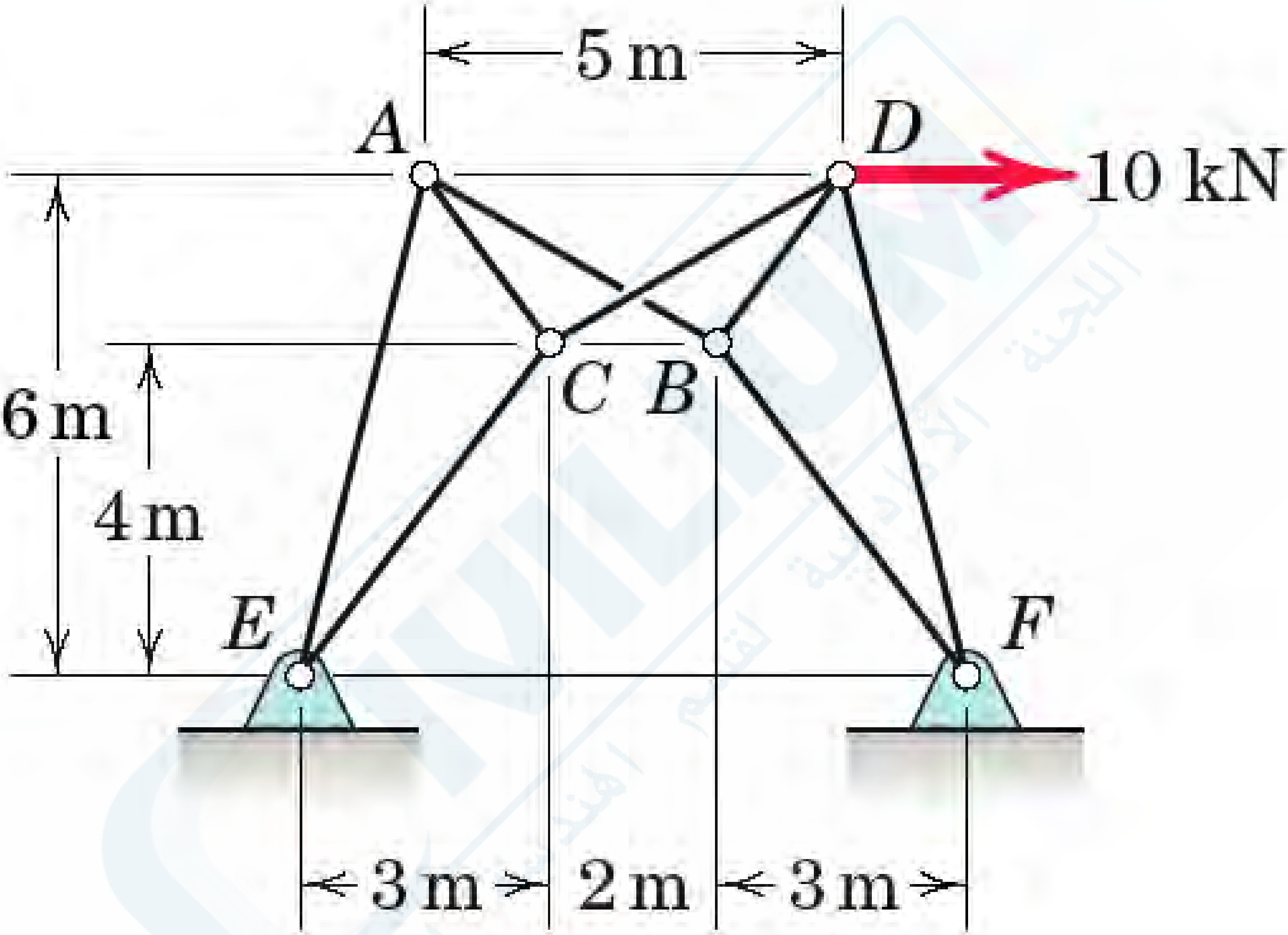
Problem 4/44

P_04_44



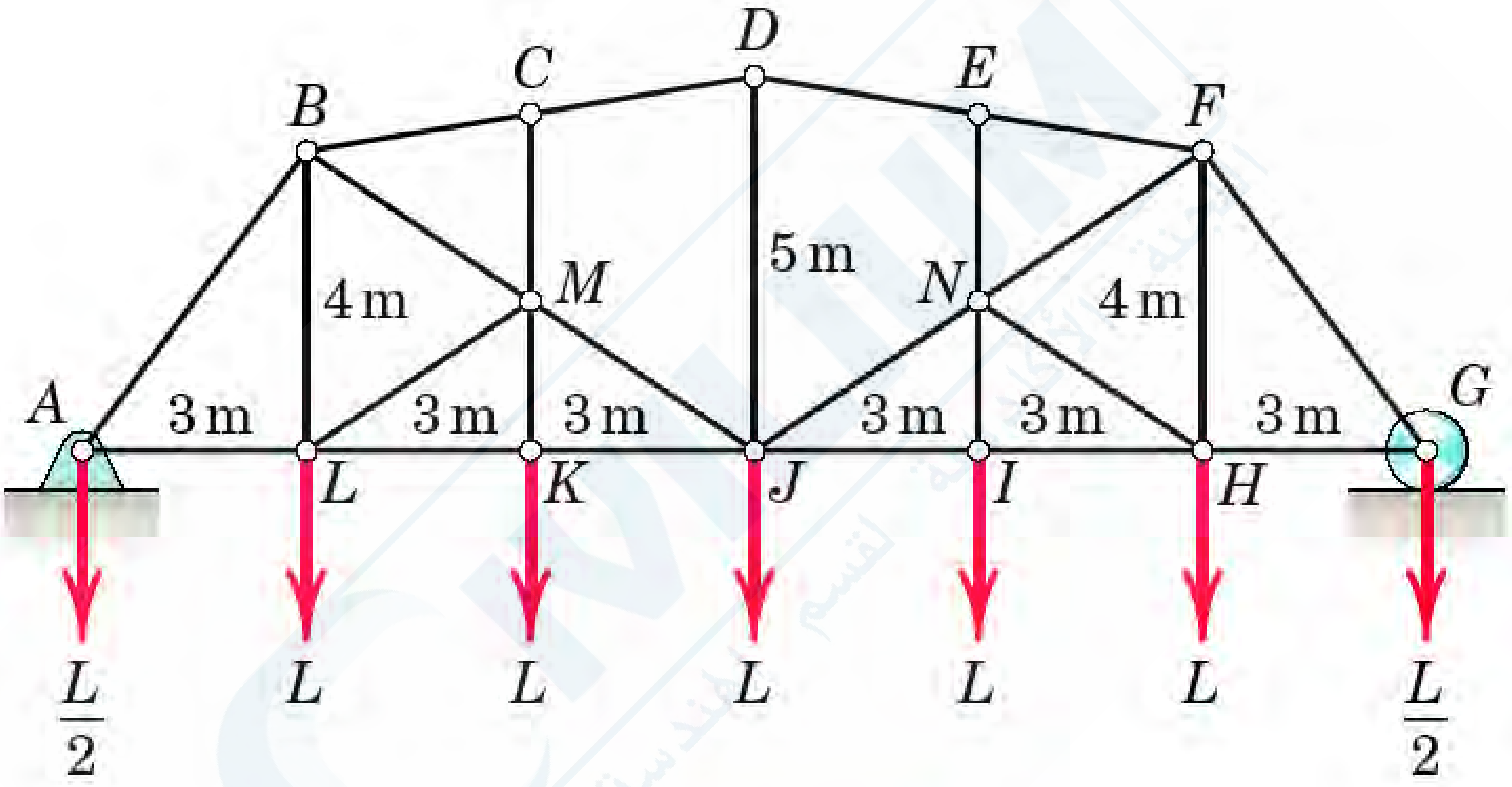
Problem 4/45

P 04 45



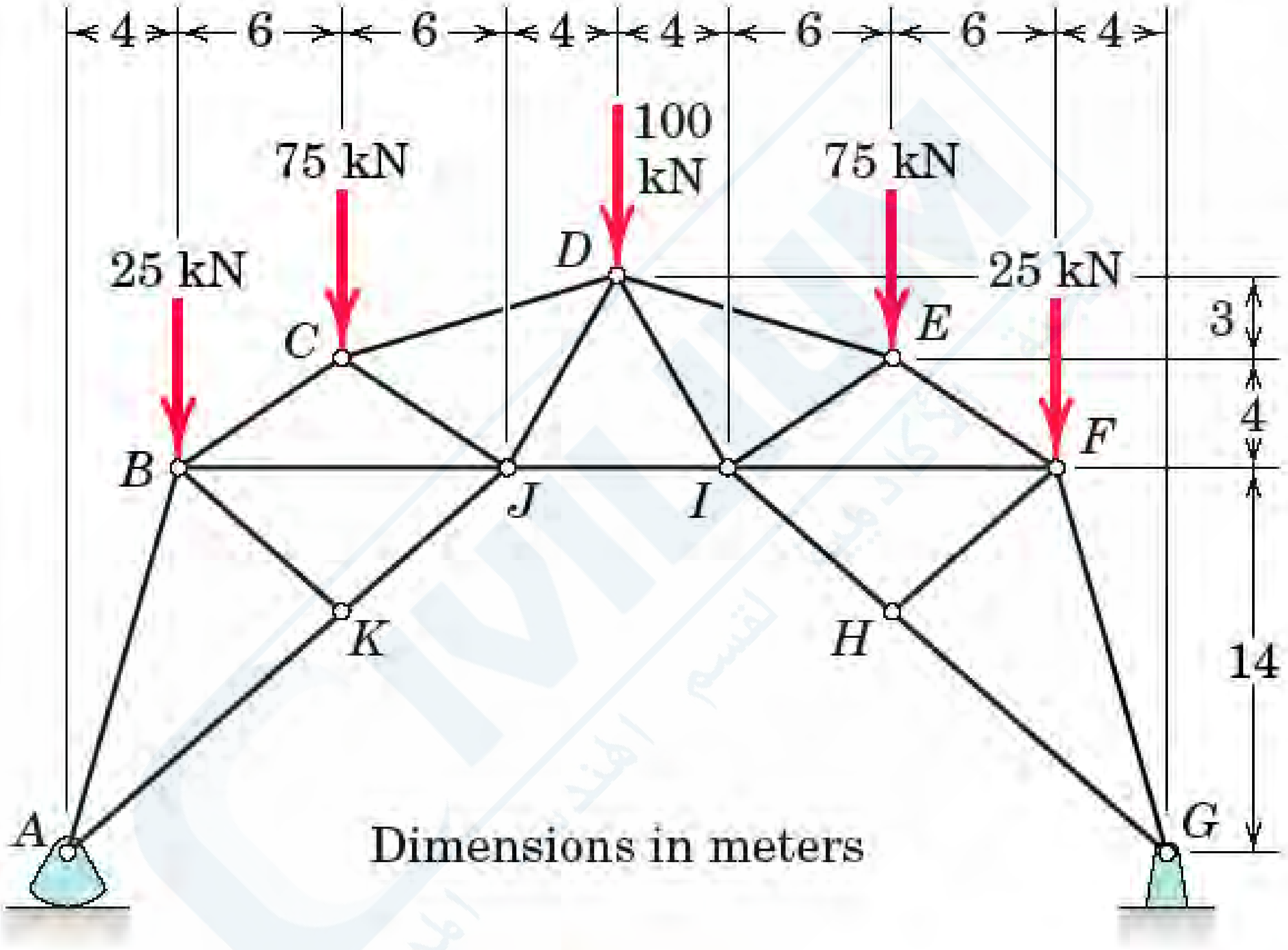
Problem 4/46

P_04_46



Problem 4/47

P_04_47


4
6
6
4
4
6
6
4
25 kN
75 kN
100
kN
75 kN
25 kN
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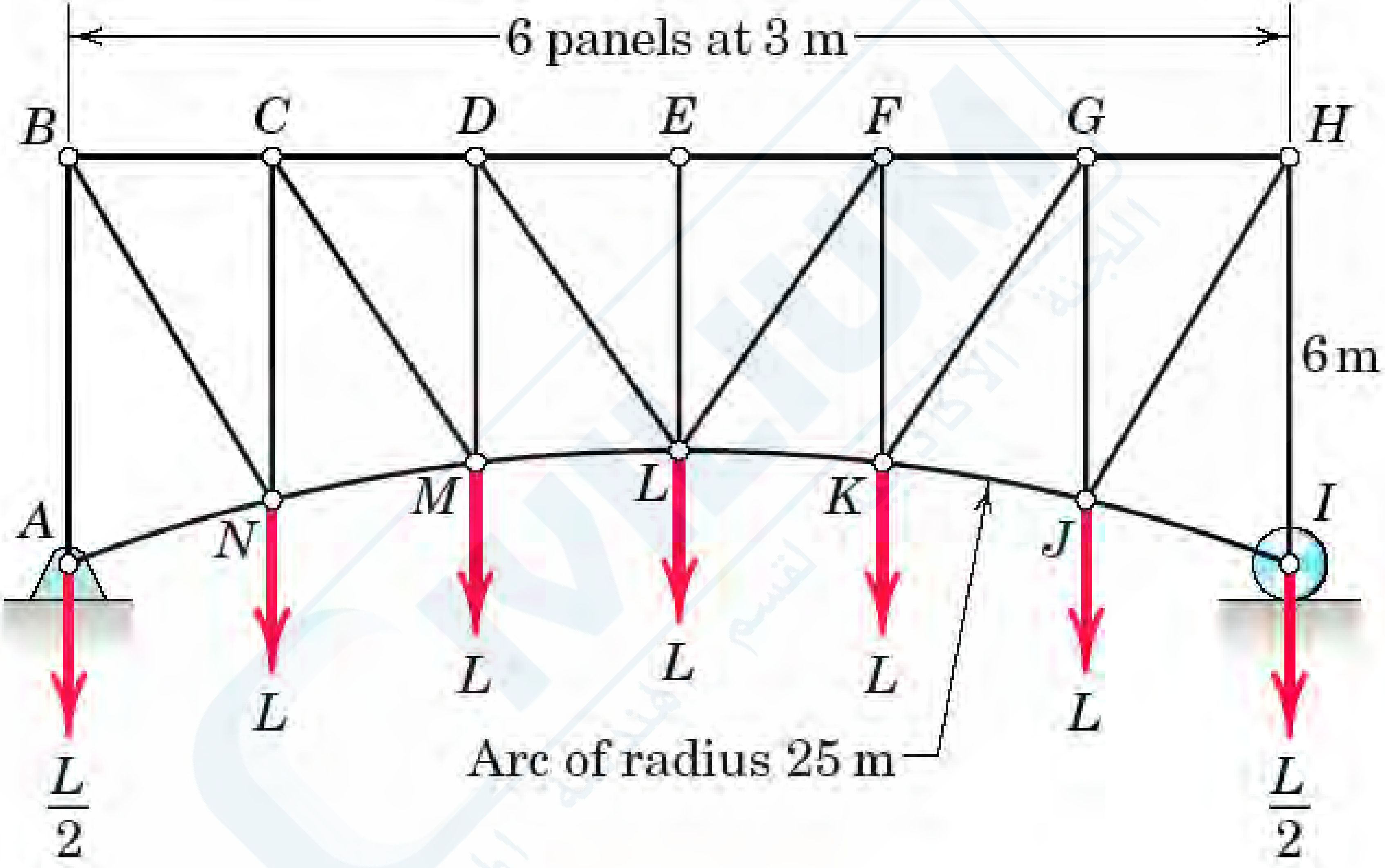
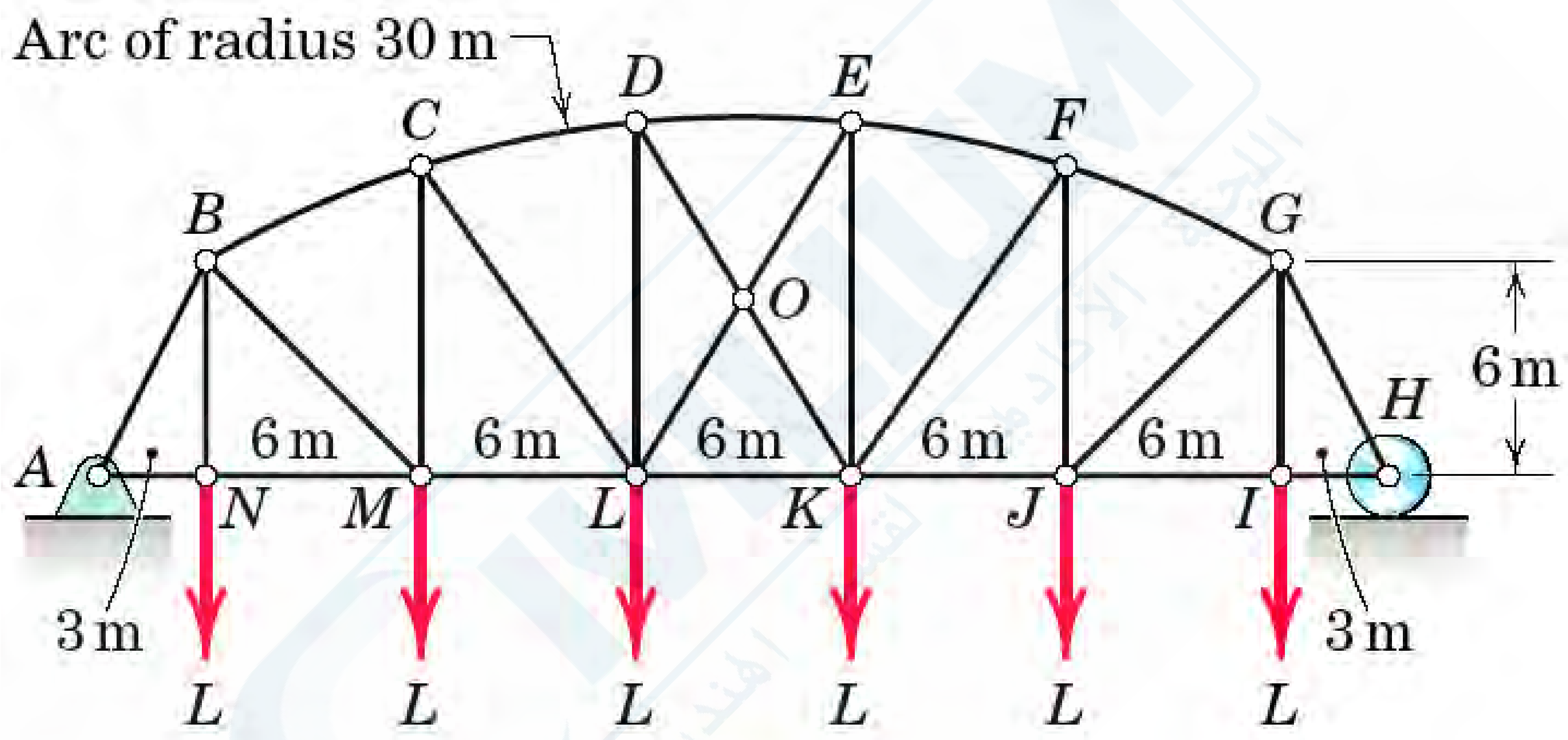
B
C
D
E
F
G
H
I
J
K
3
4
14
Dimensions in meters

Problem 4/48



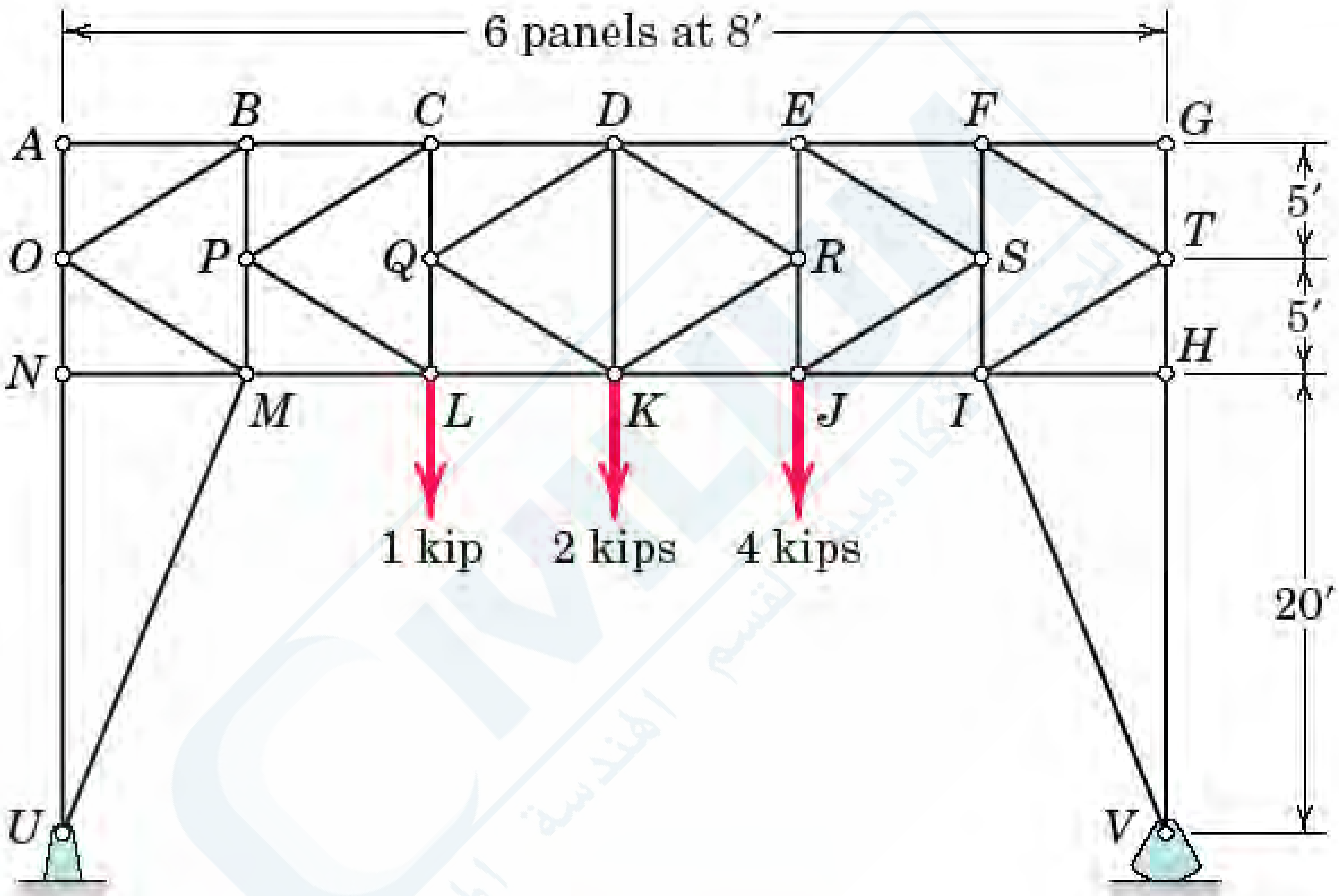
Problem 4/49

P_04_49



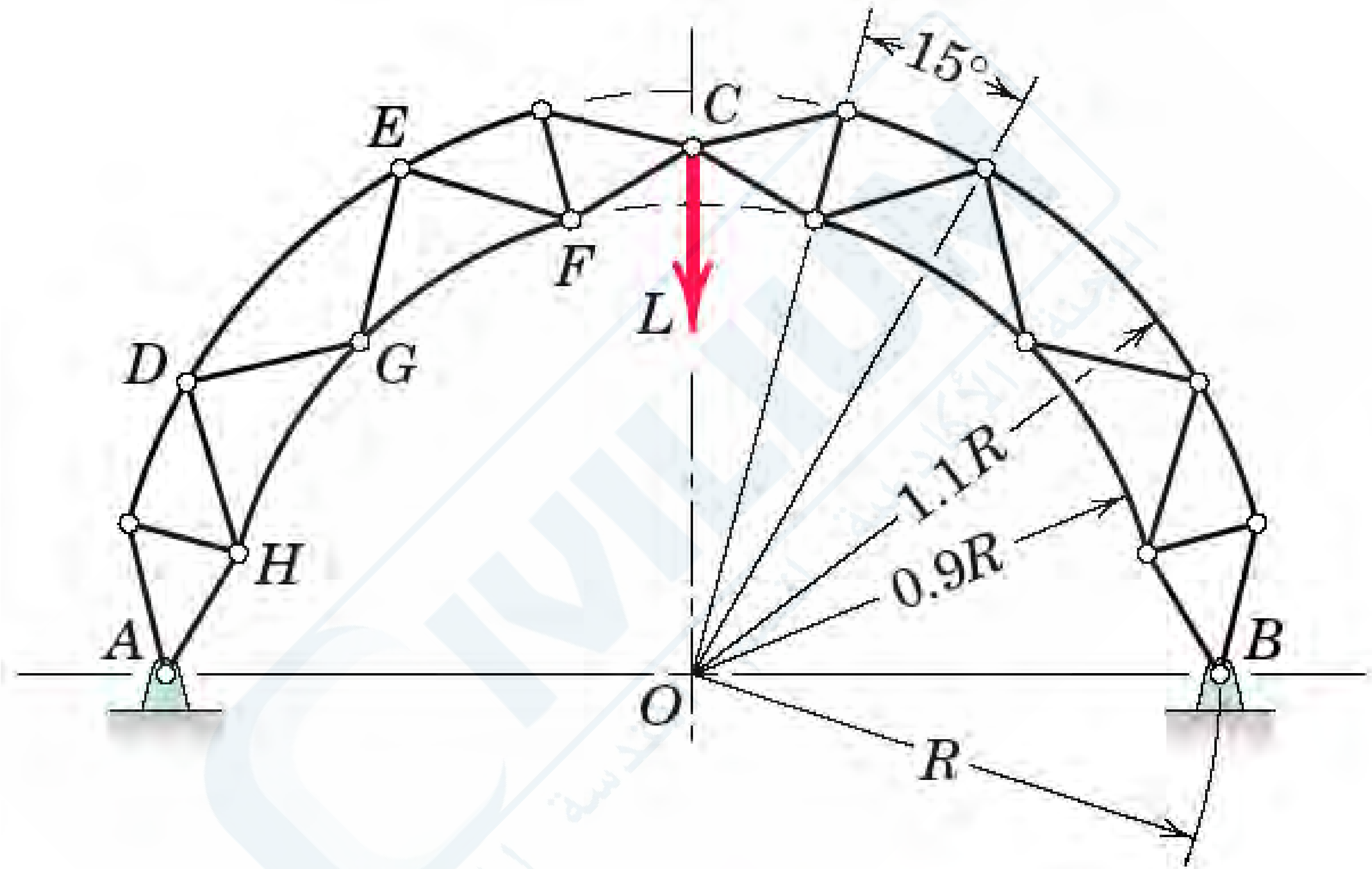
Problem 4/50

P_04_50



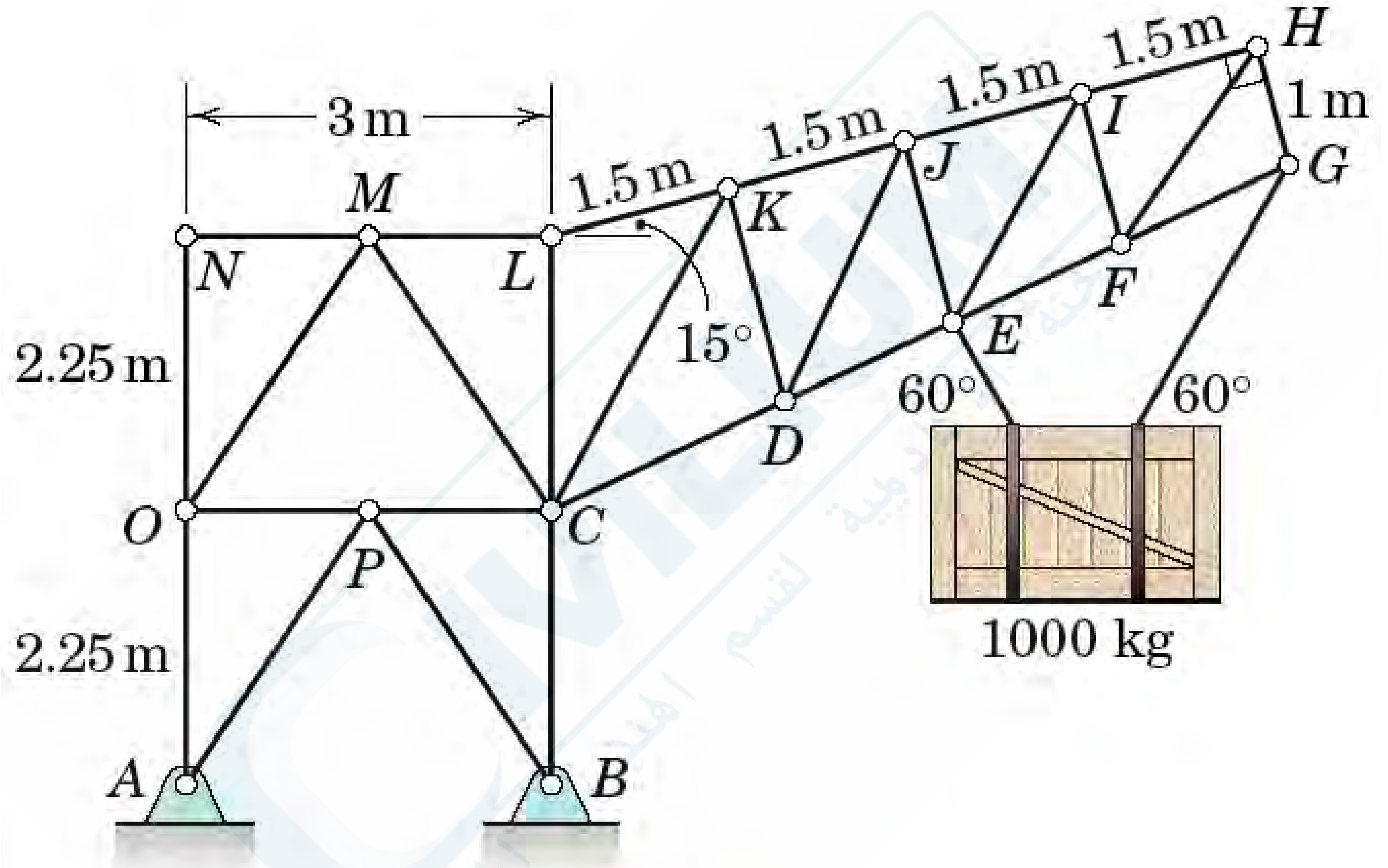
Problem 4/51

P 04 51



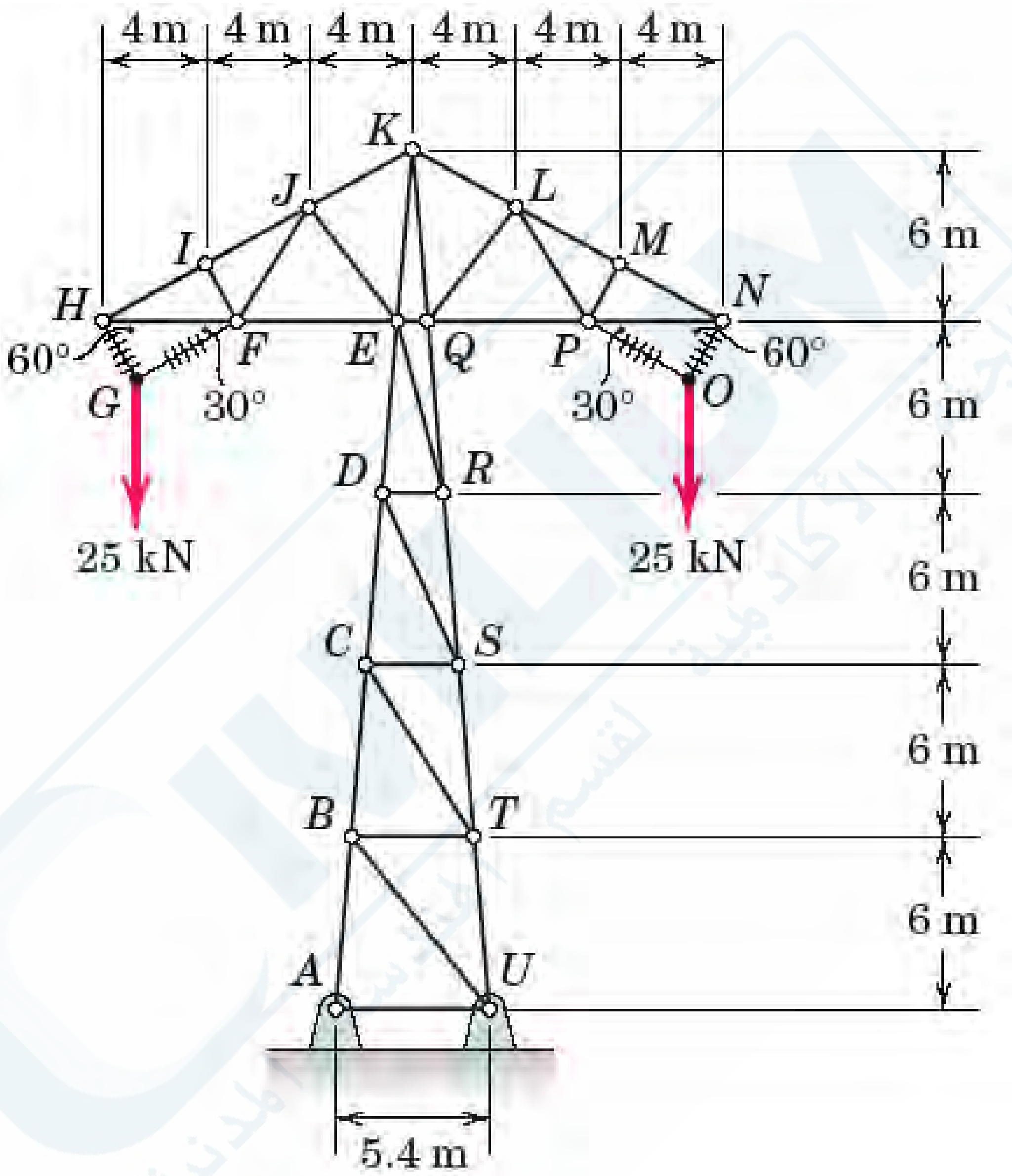
Problem 4/52

P_04_52



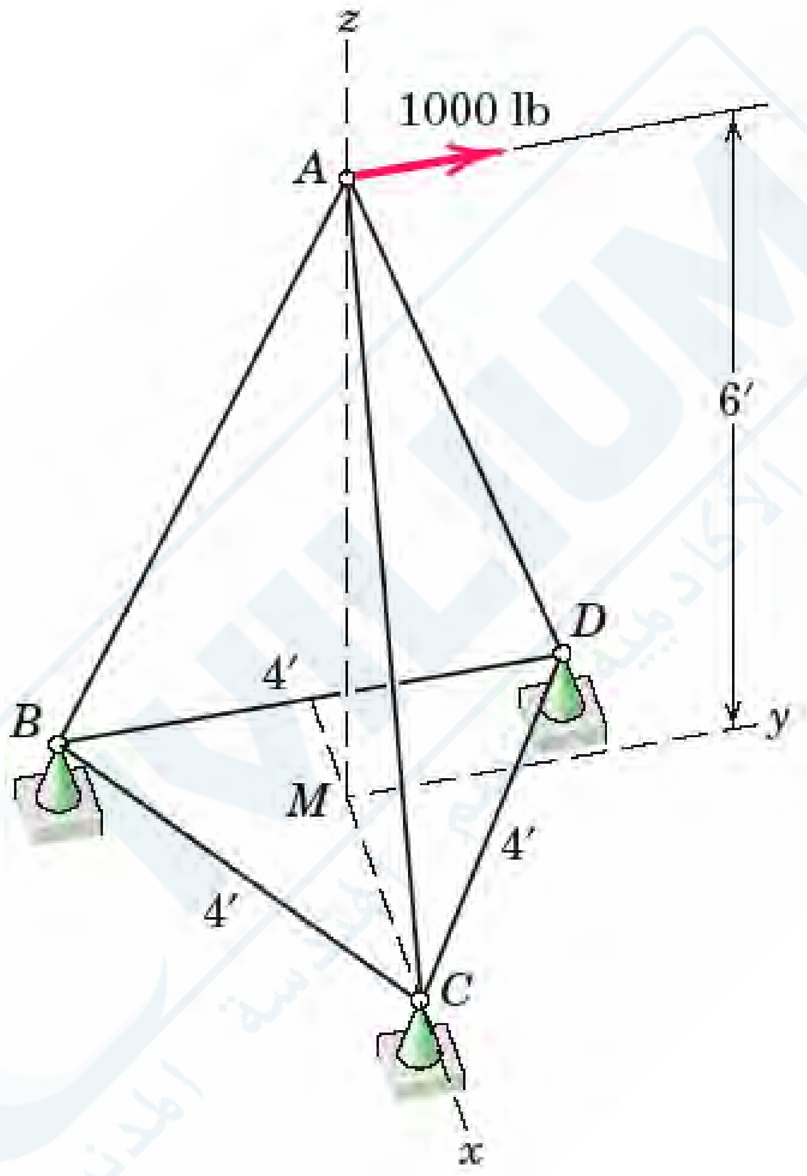
Problem 4/53

P 04 53



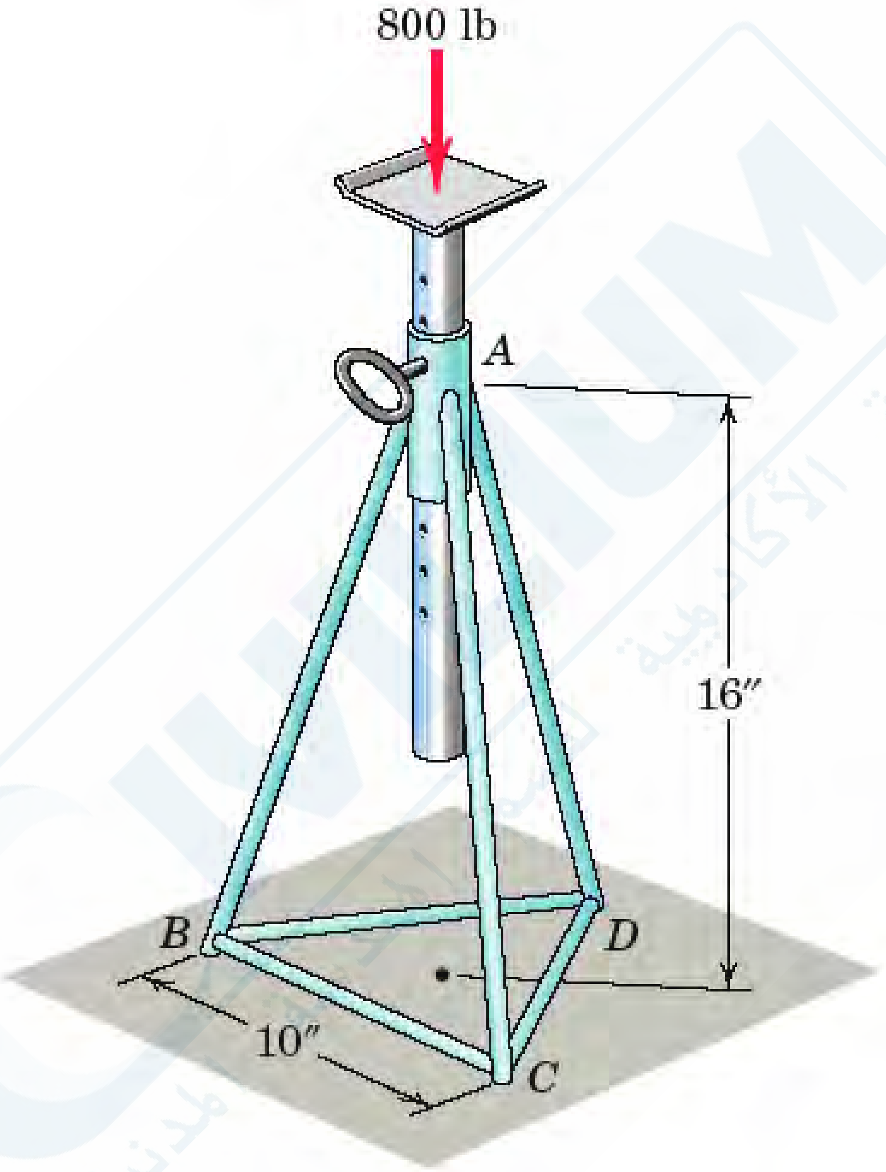
Problem 4/54

P 04 54



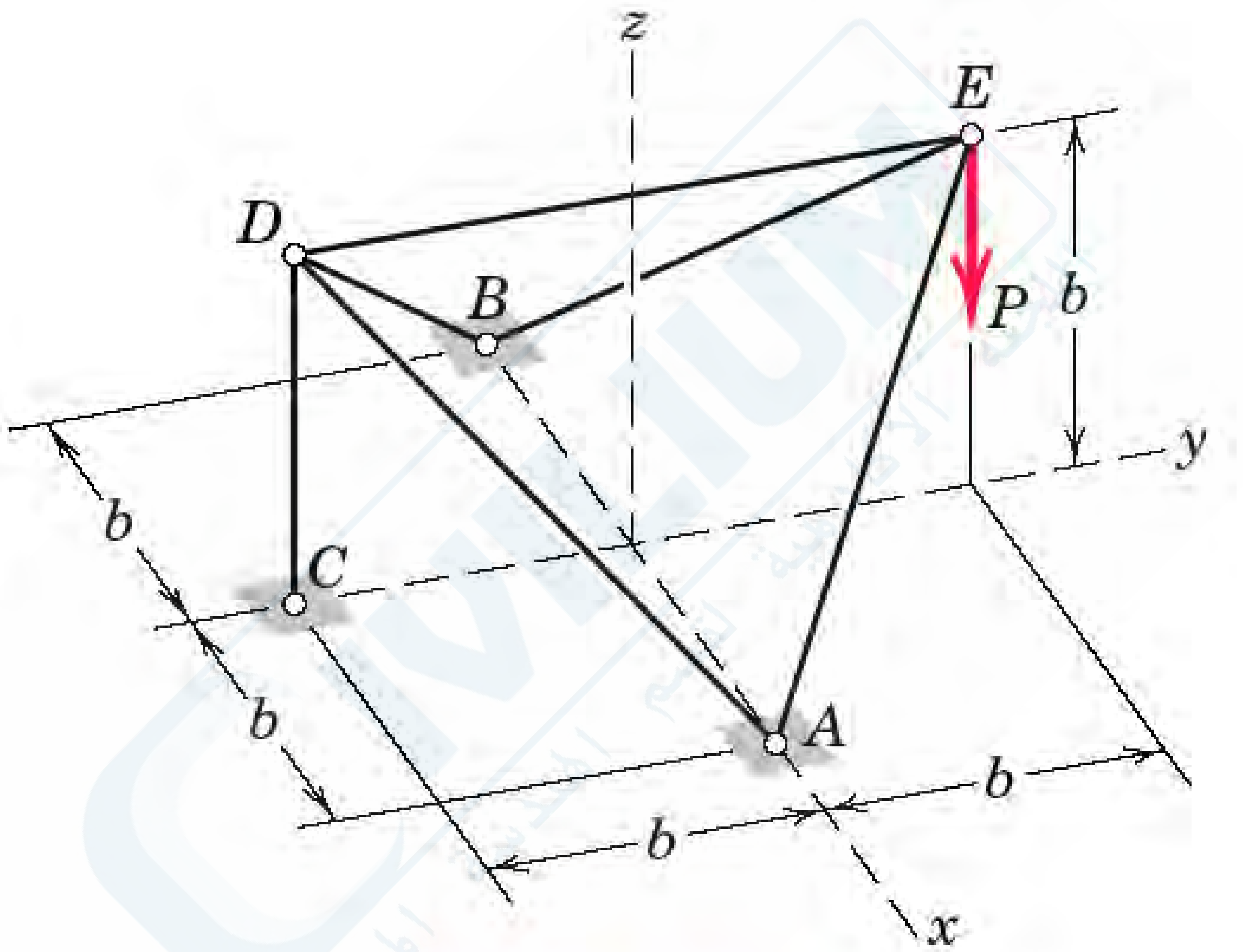
Problem 4/55

P 04 55



Problem 4/56

P_04_56



Problem 4/57

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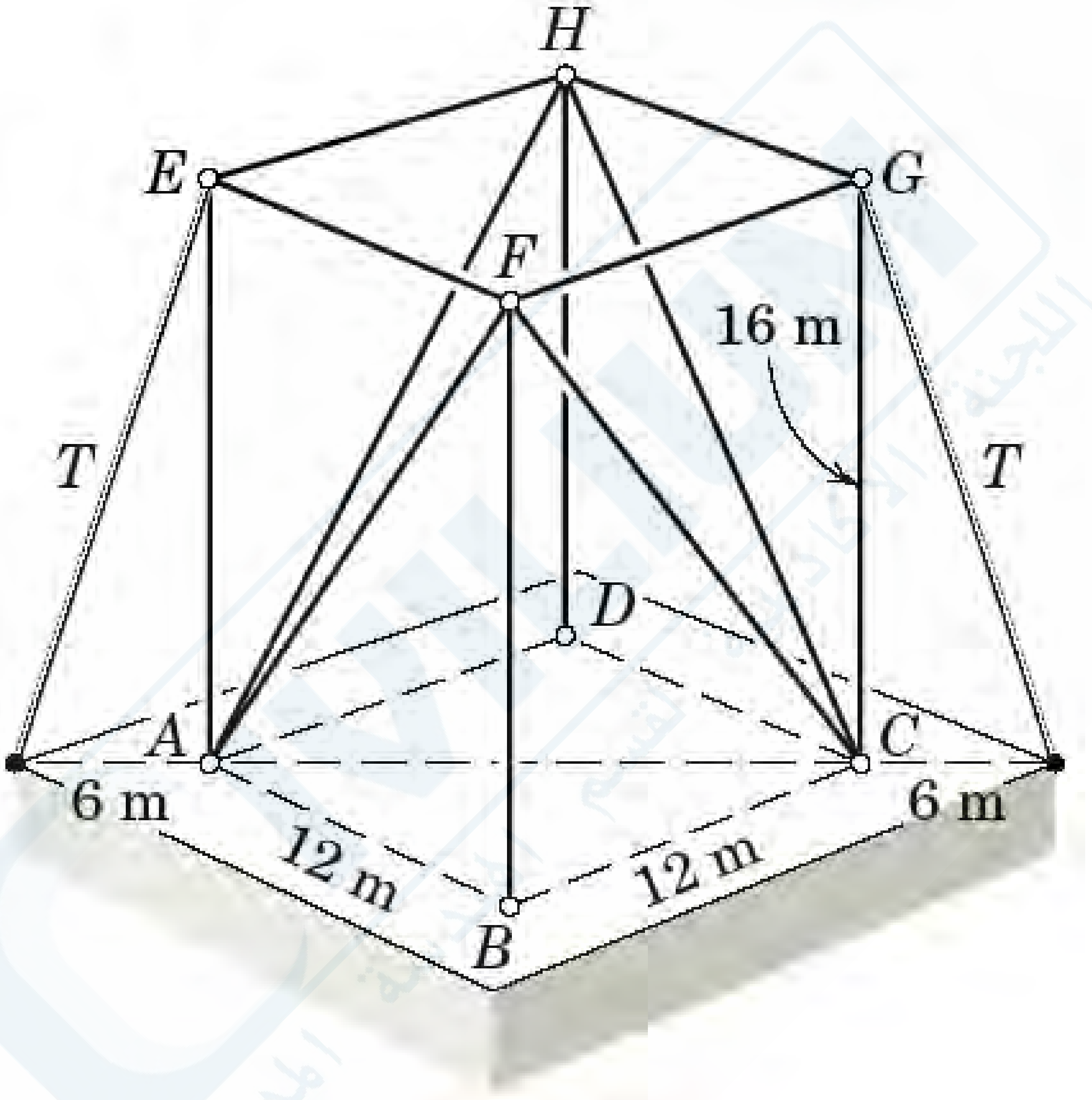
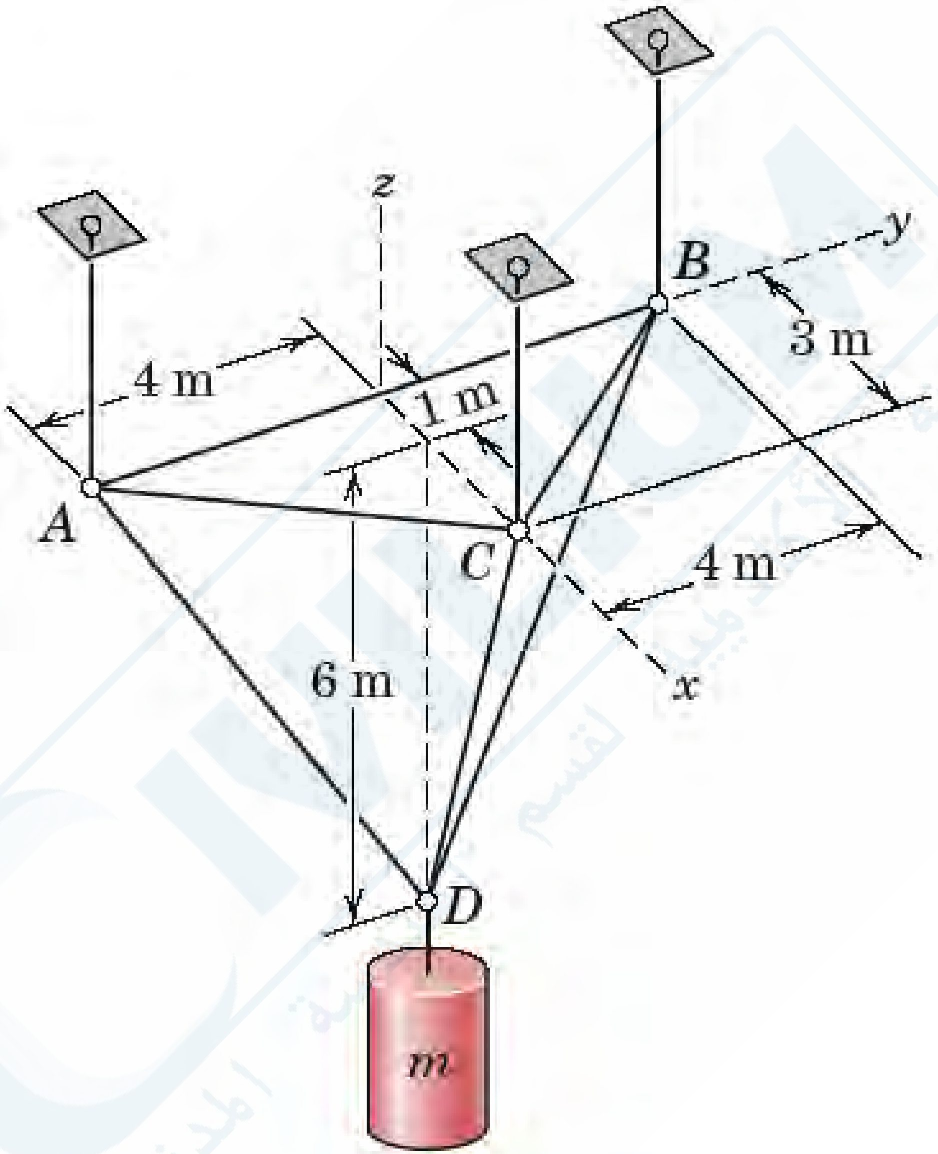


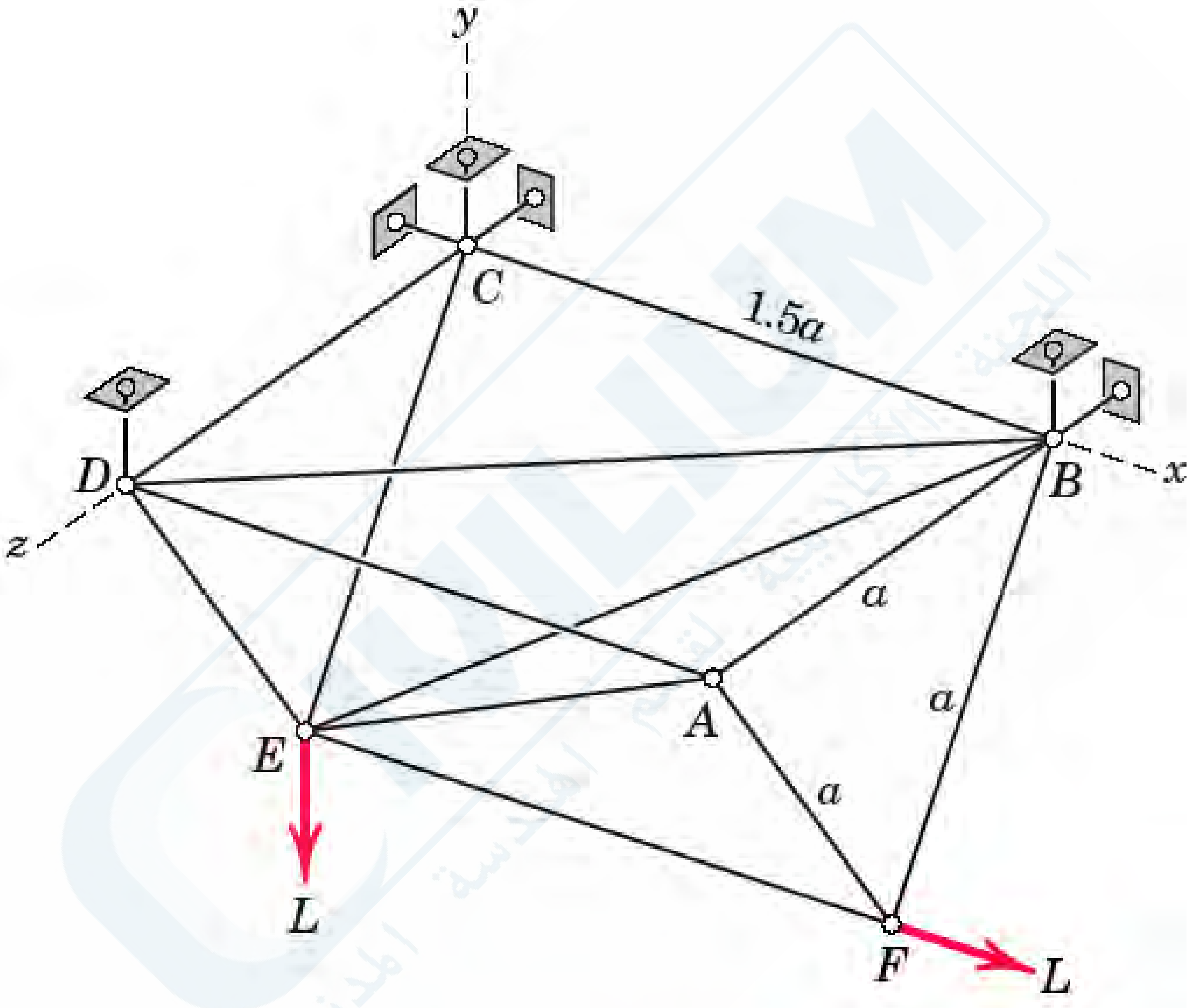
Problem 4/58

P_04_58



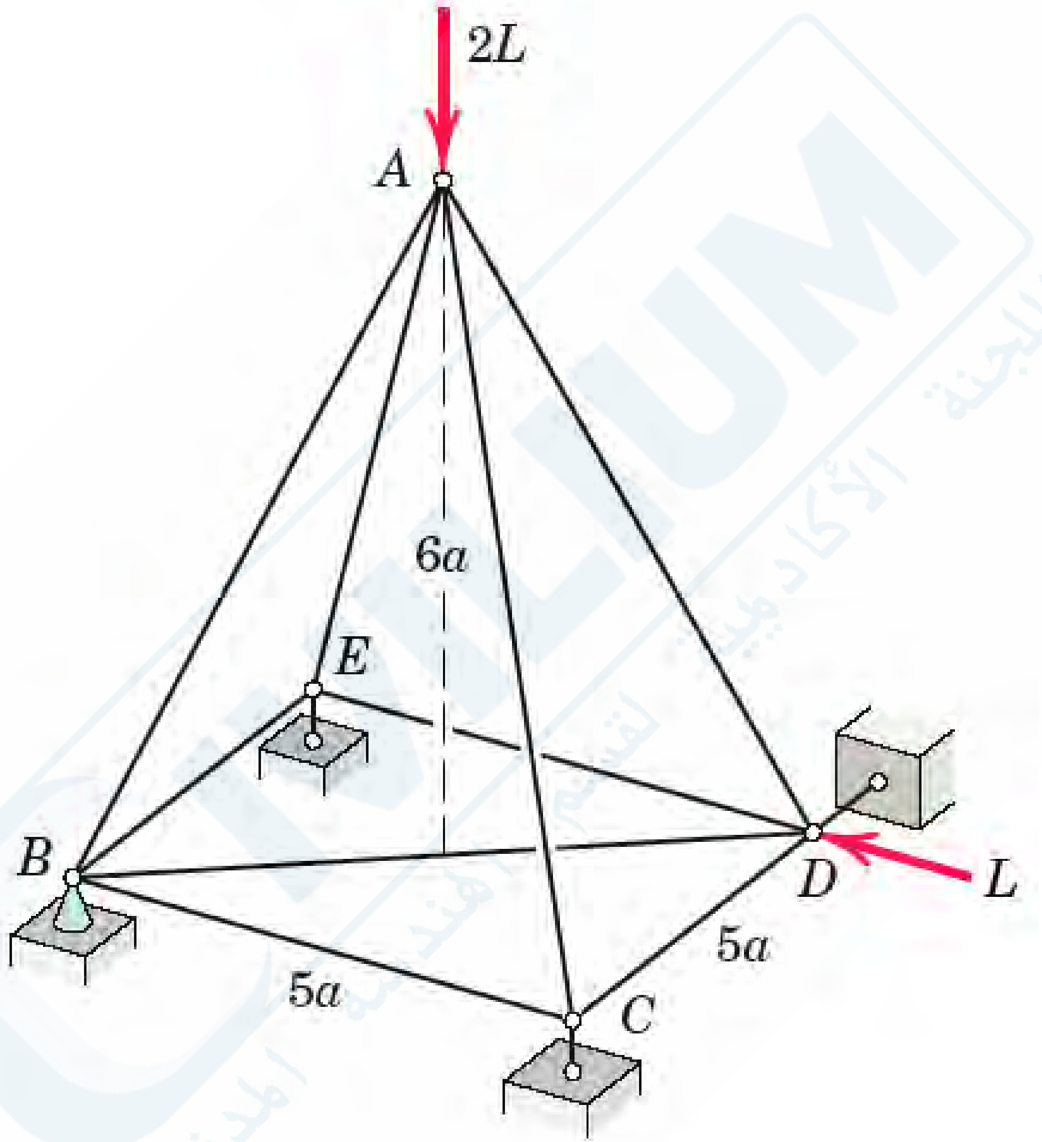
Problem 4/59

P_04_59



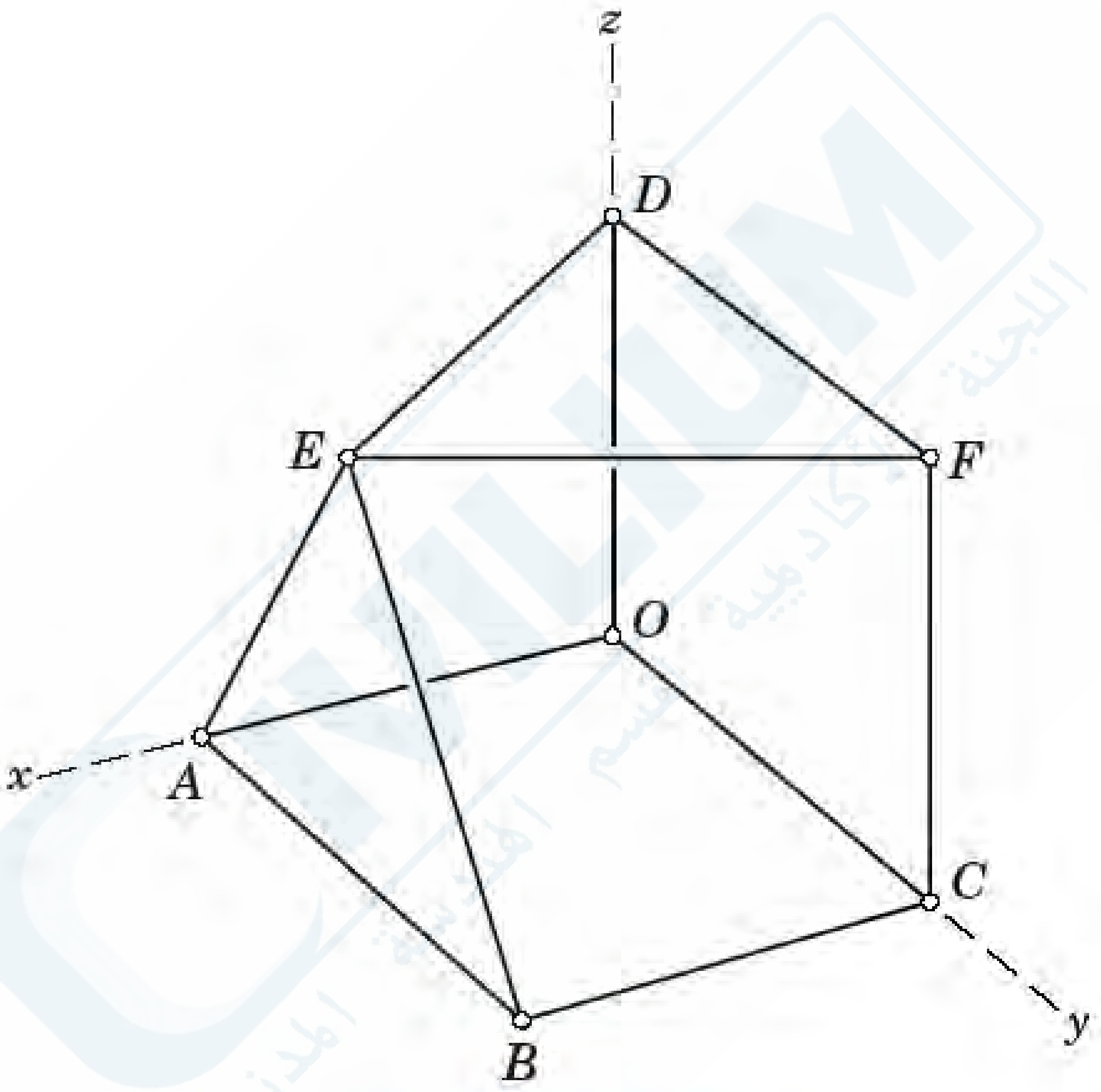
Problem 4/60

P_04_60



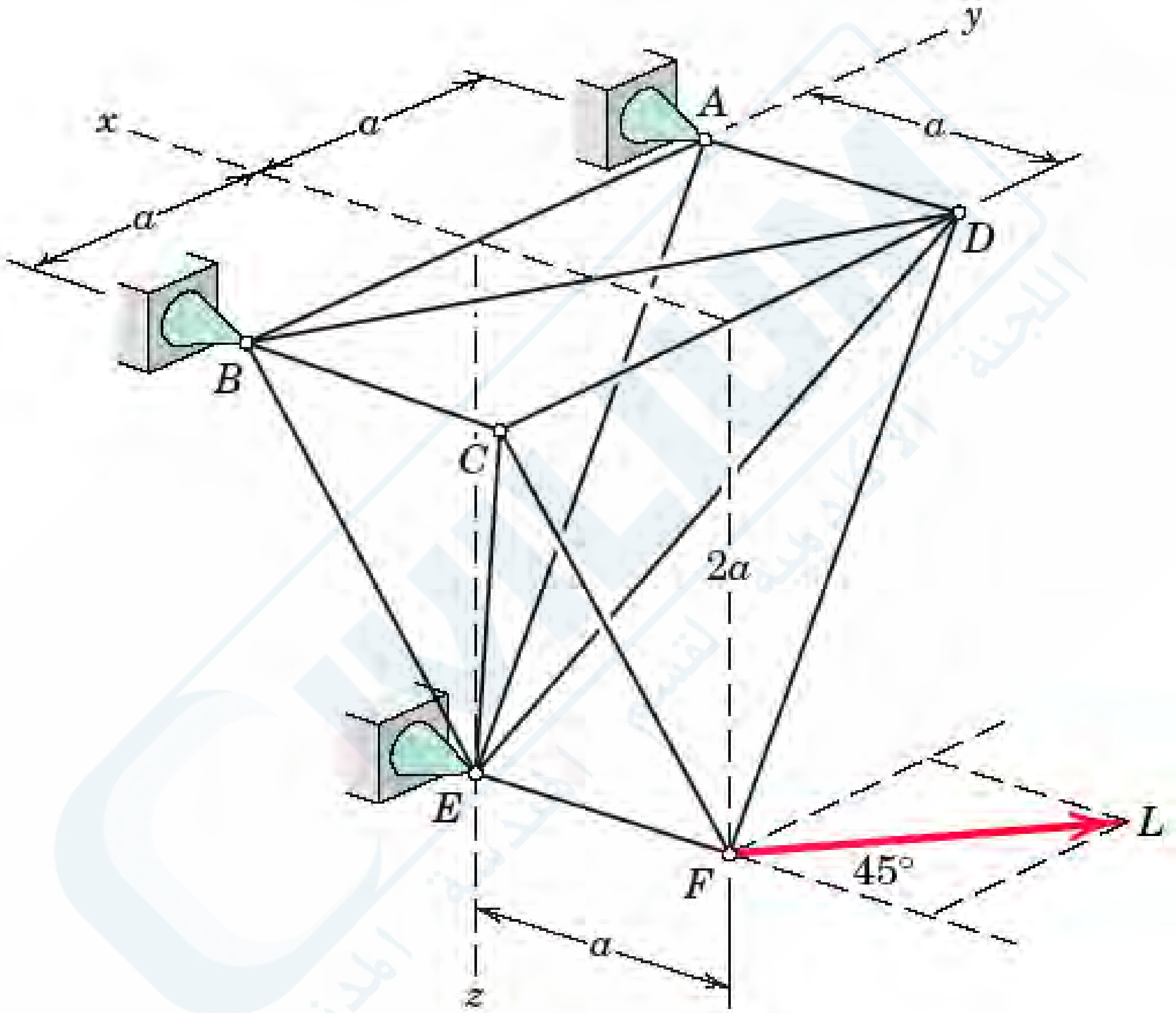
Problem 4/61

P 04 61



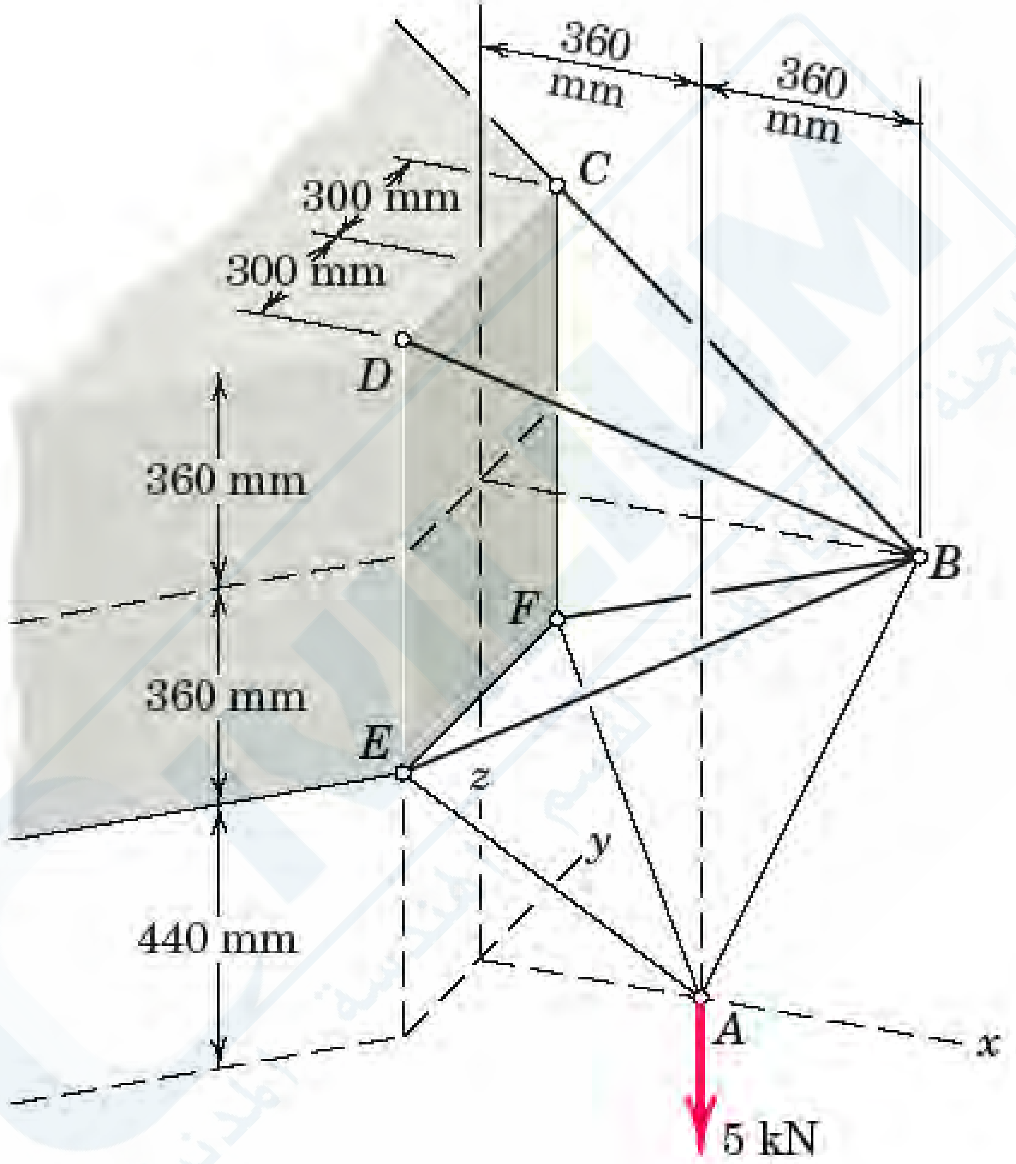
Problem 4/62

P_04_62



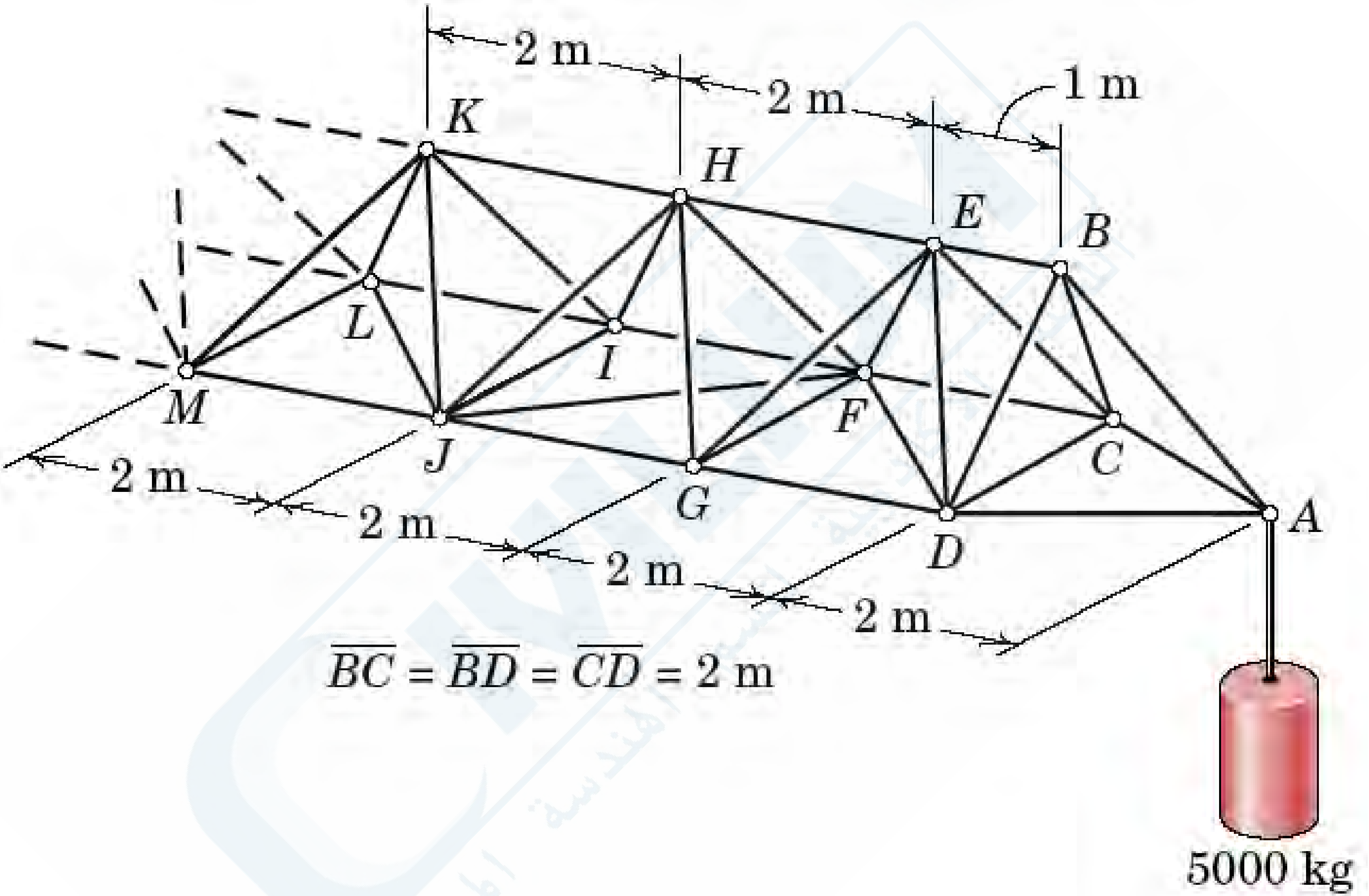
Problem 4/63

P 04 63



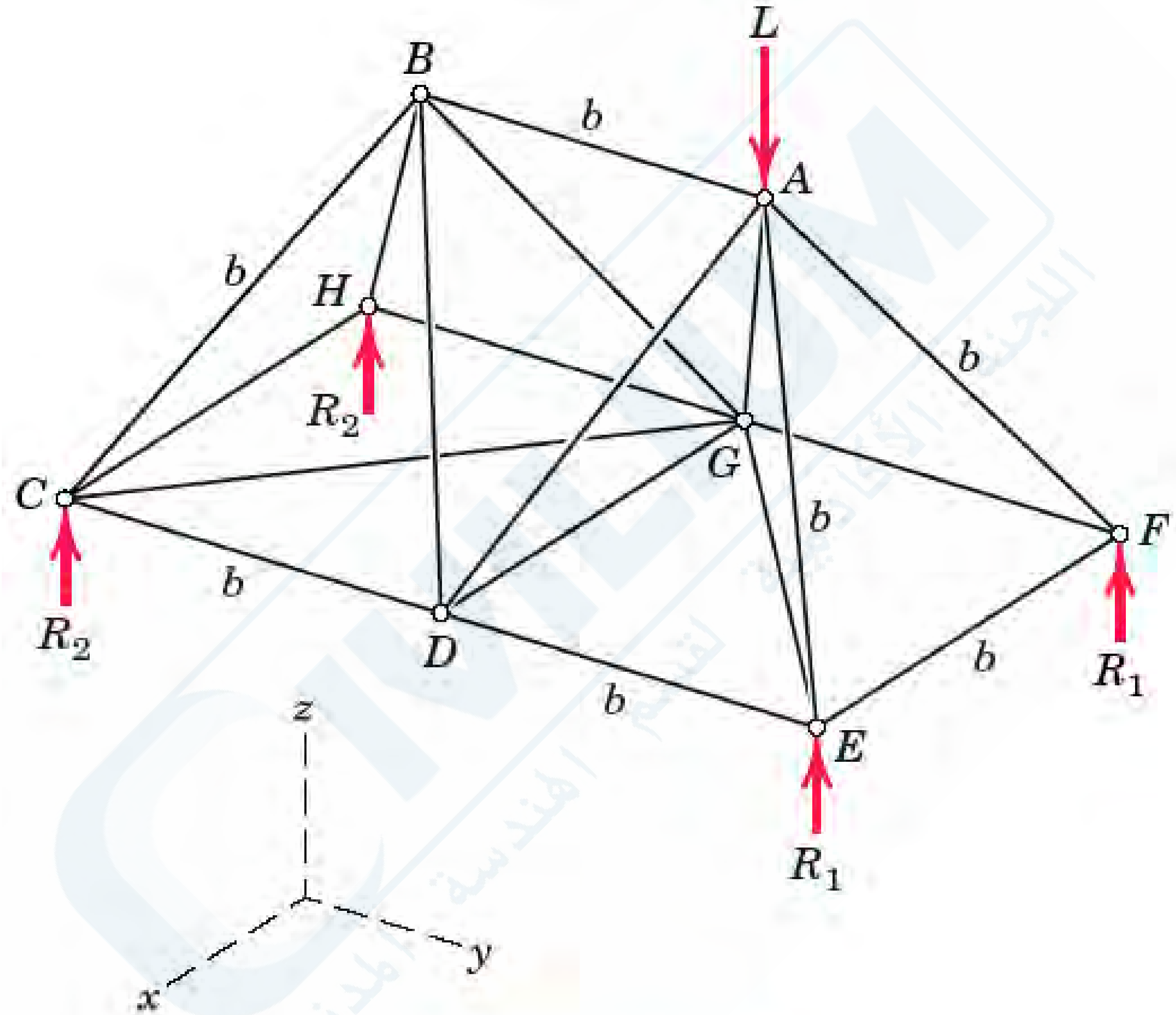
Problem 4/64

P_04_64



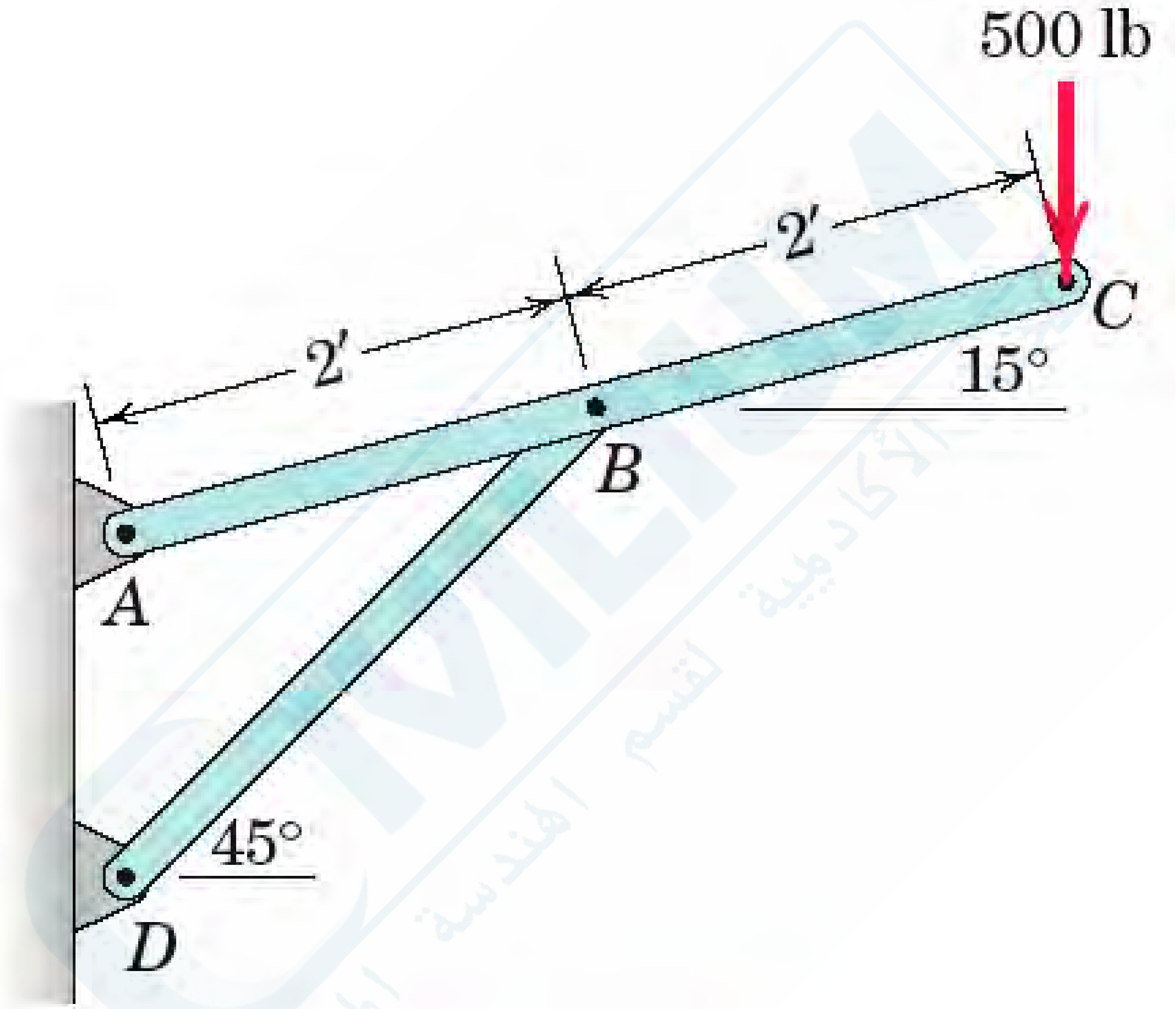
Problem 4/65

P 04 65



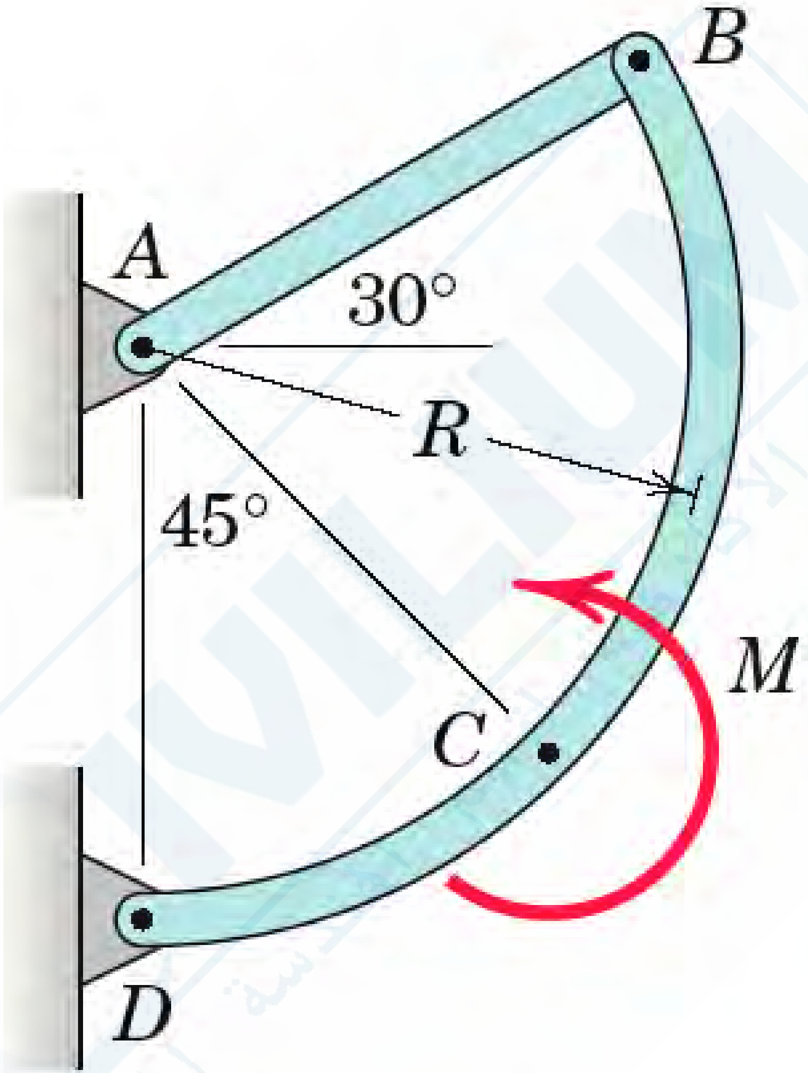
Problem 4/66

P_04_66



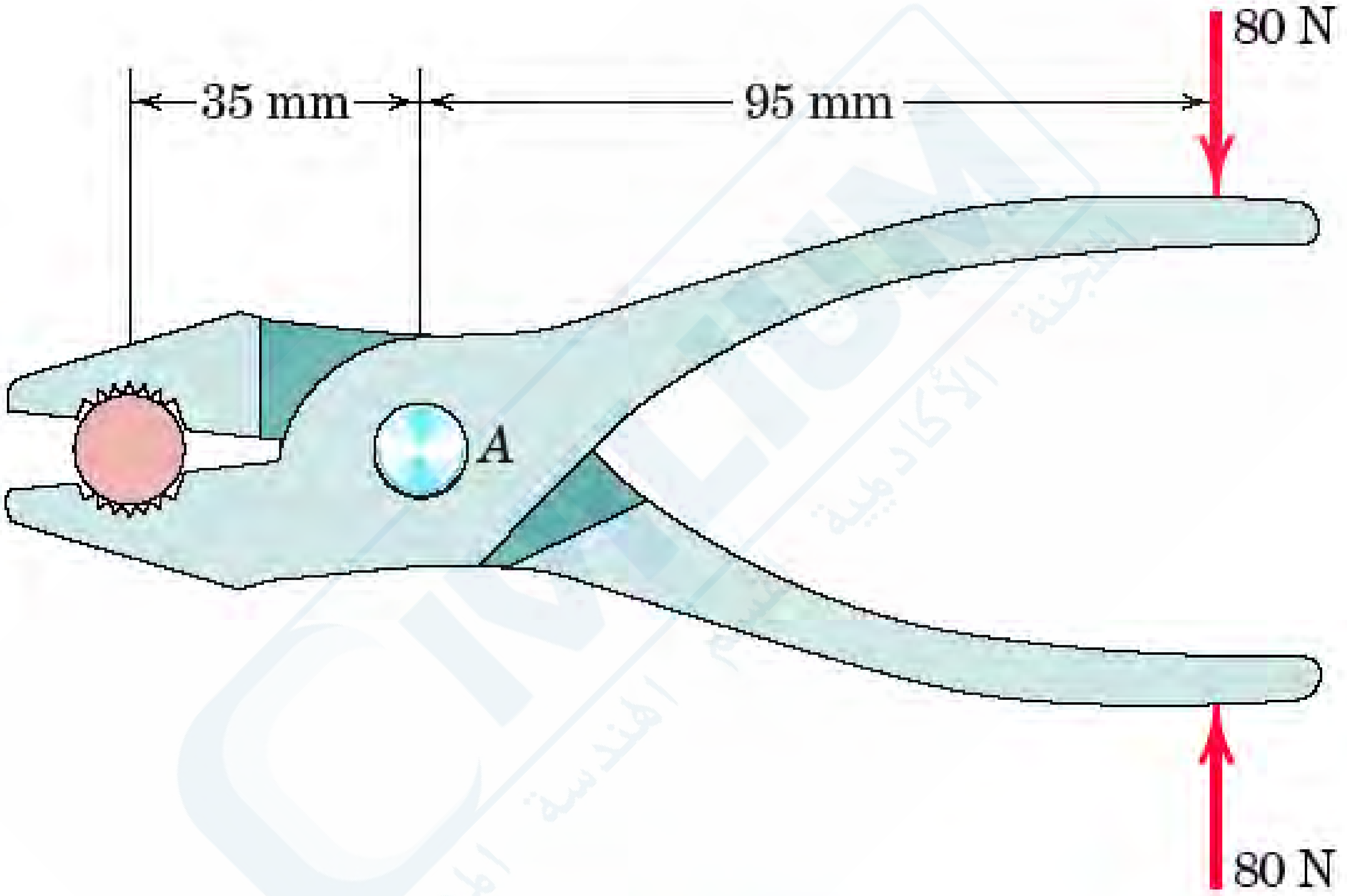
Problem 4/67

P_04_67



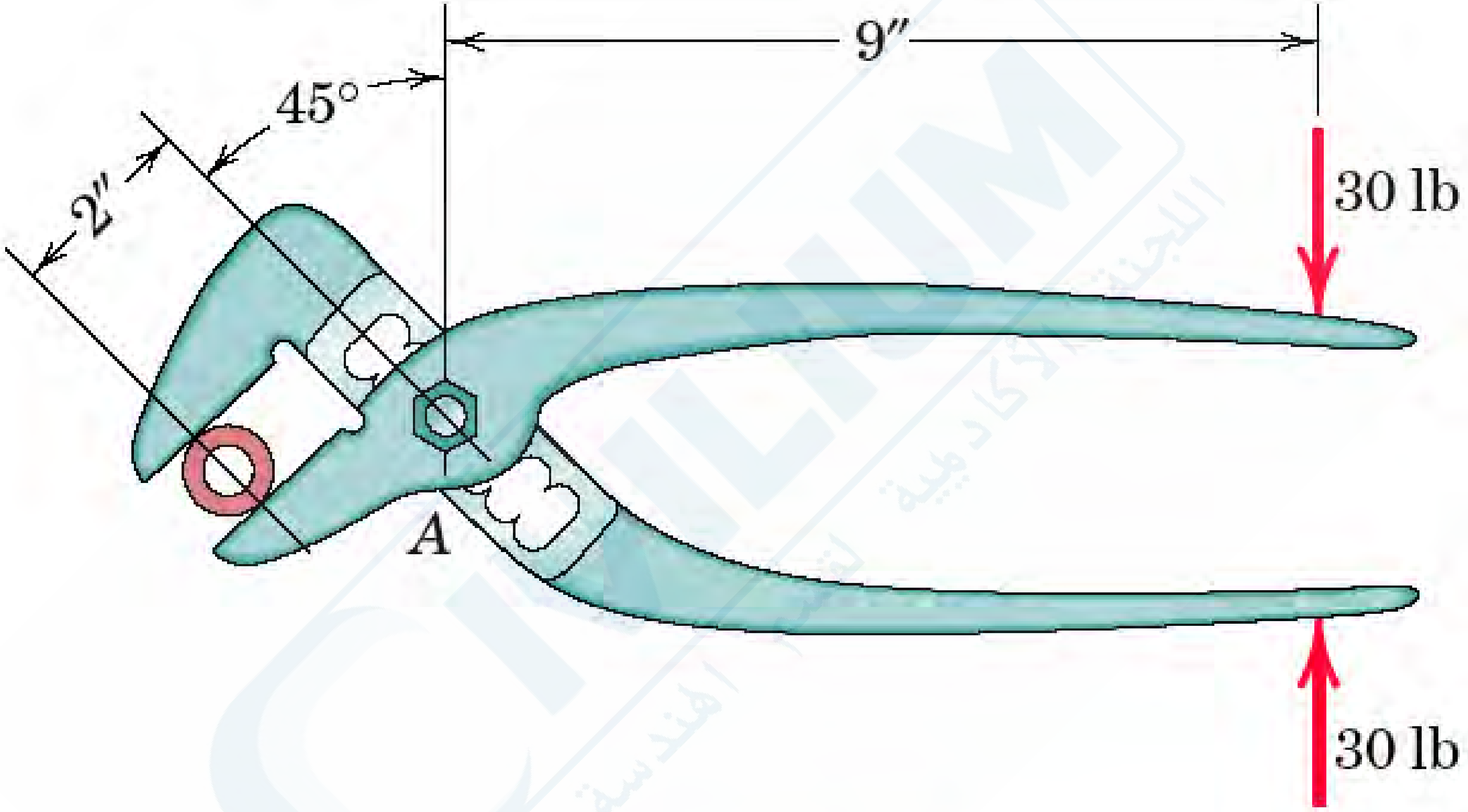
Problem 4/68

P_04_68



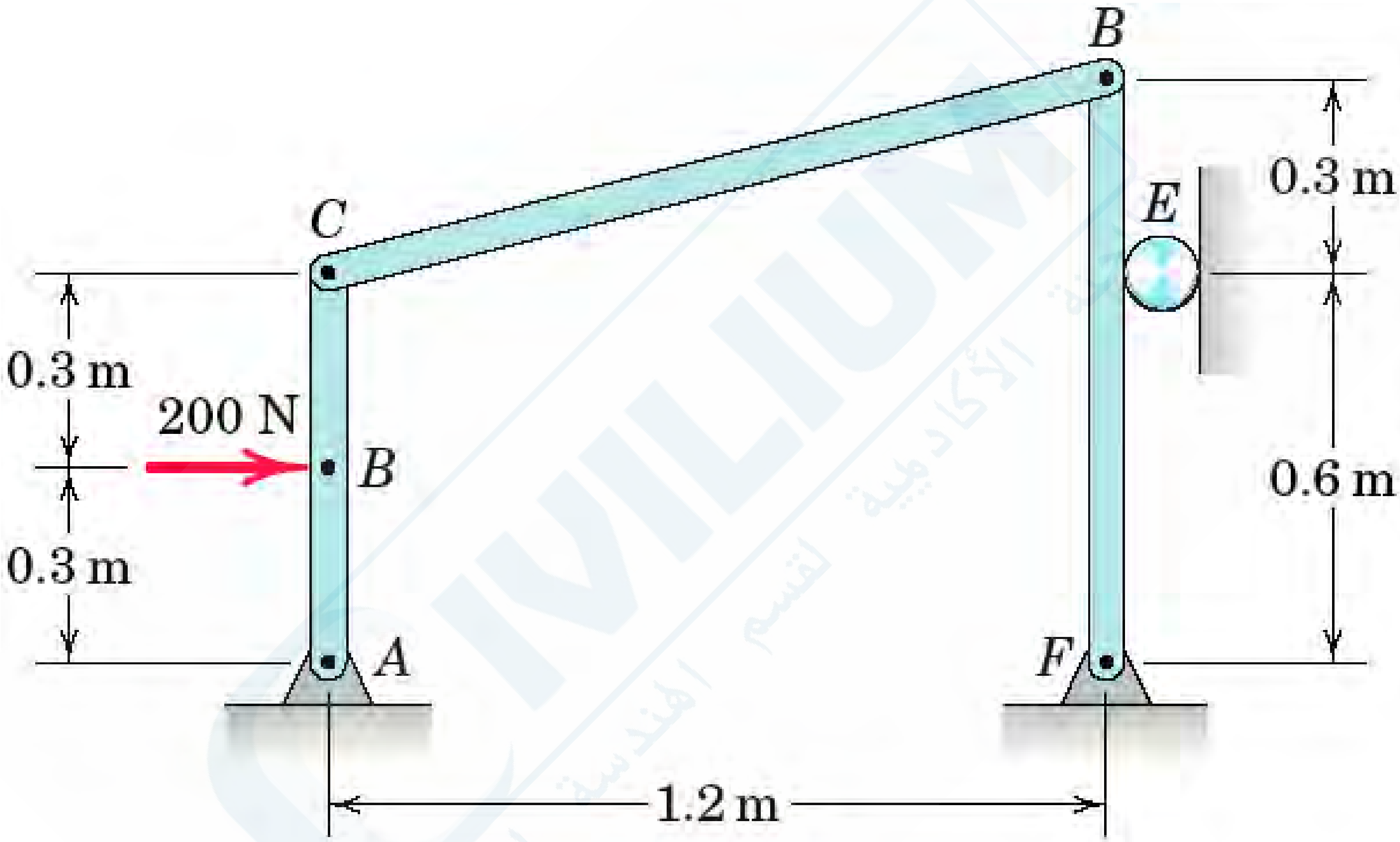
Problem 4/69

P 04 69



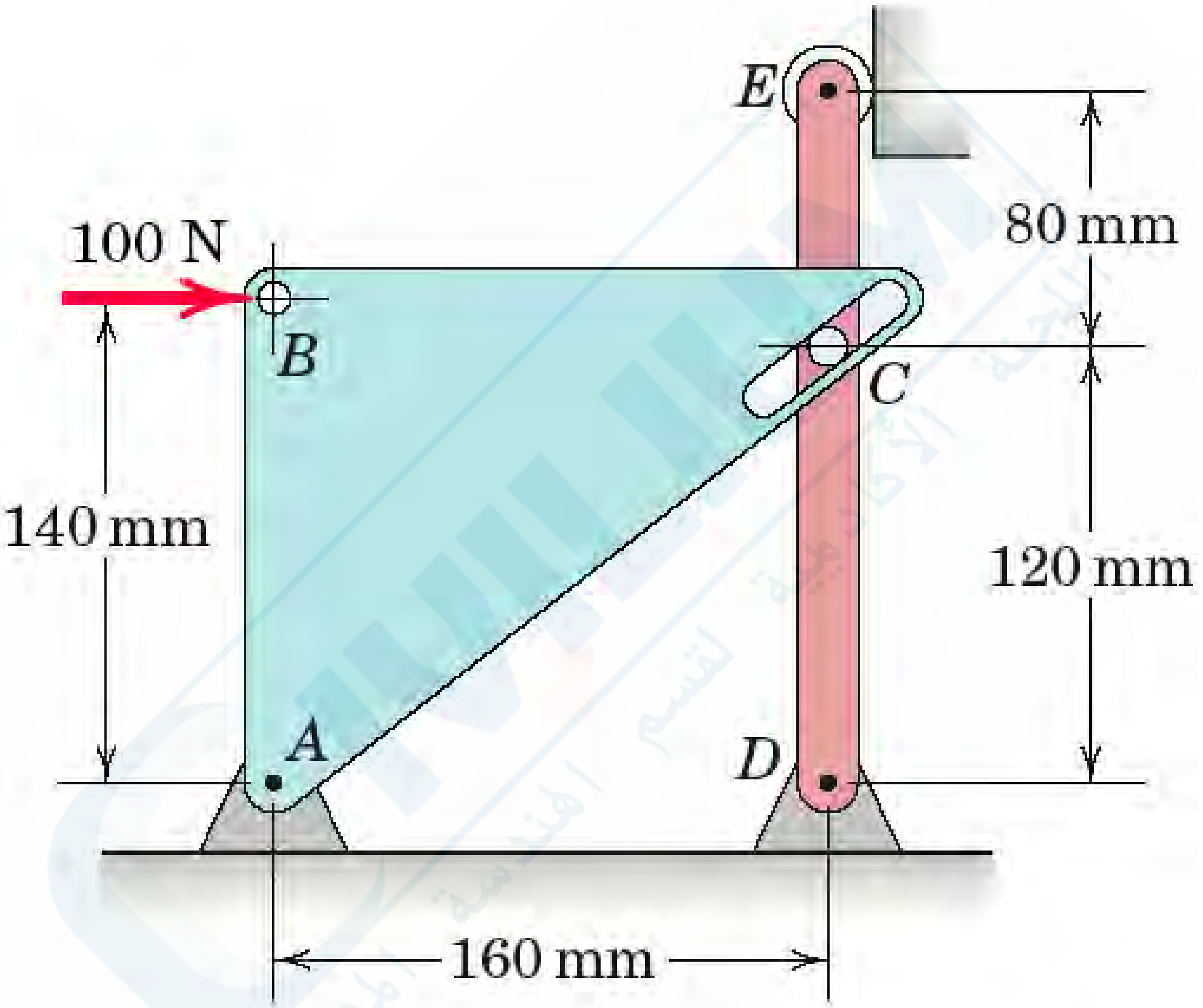
Problem 4/70

P_04_70



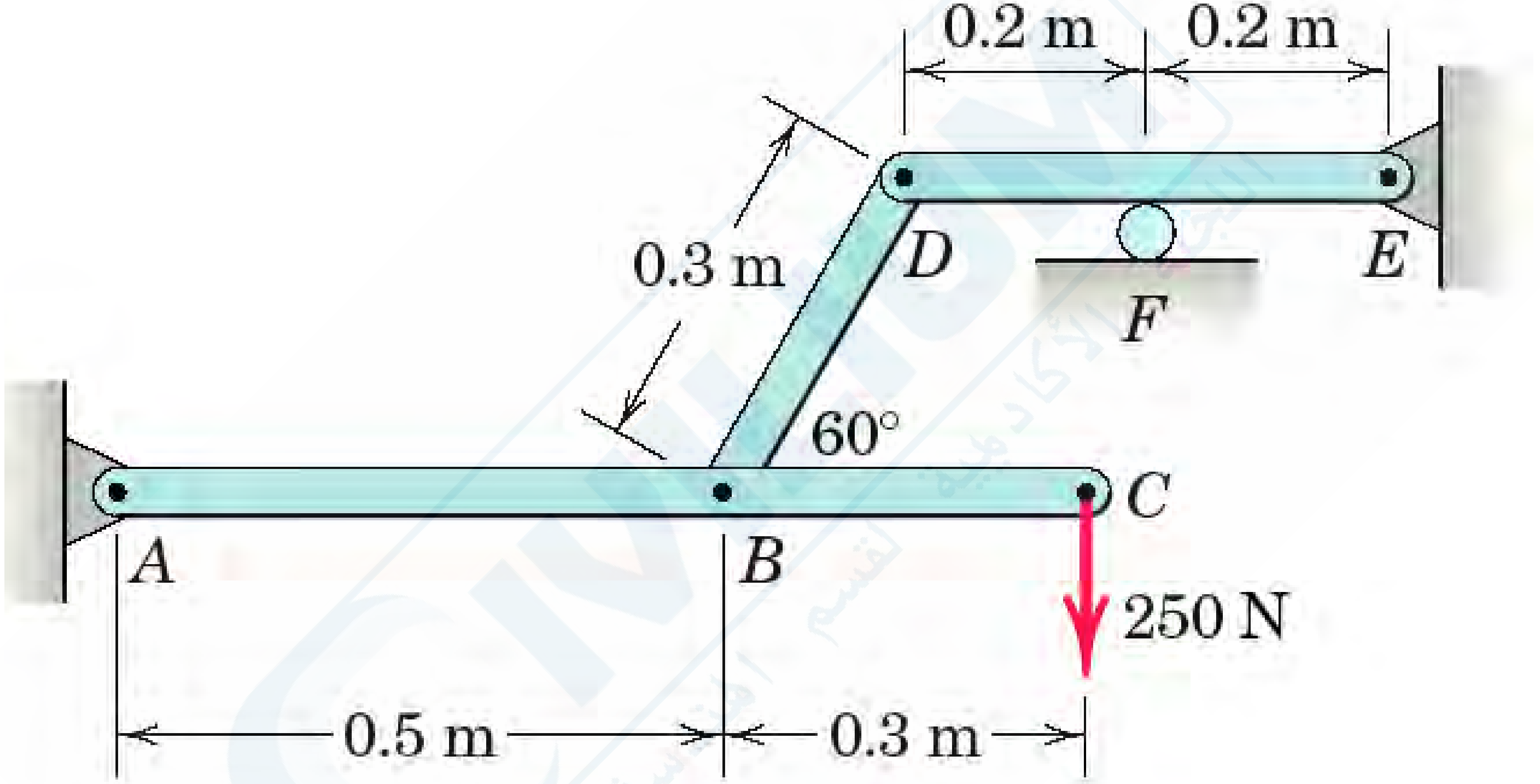
Problem 4/71

P_04_71



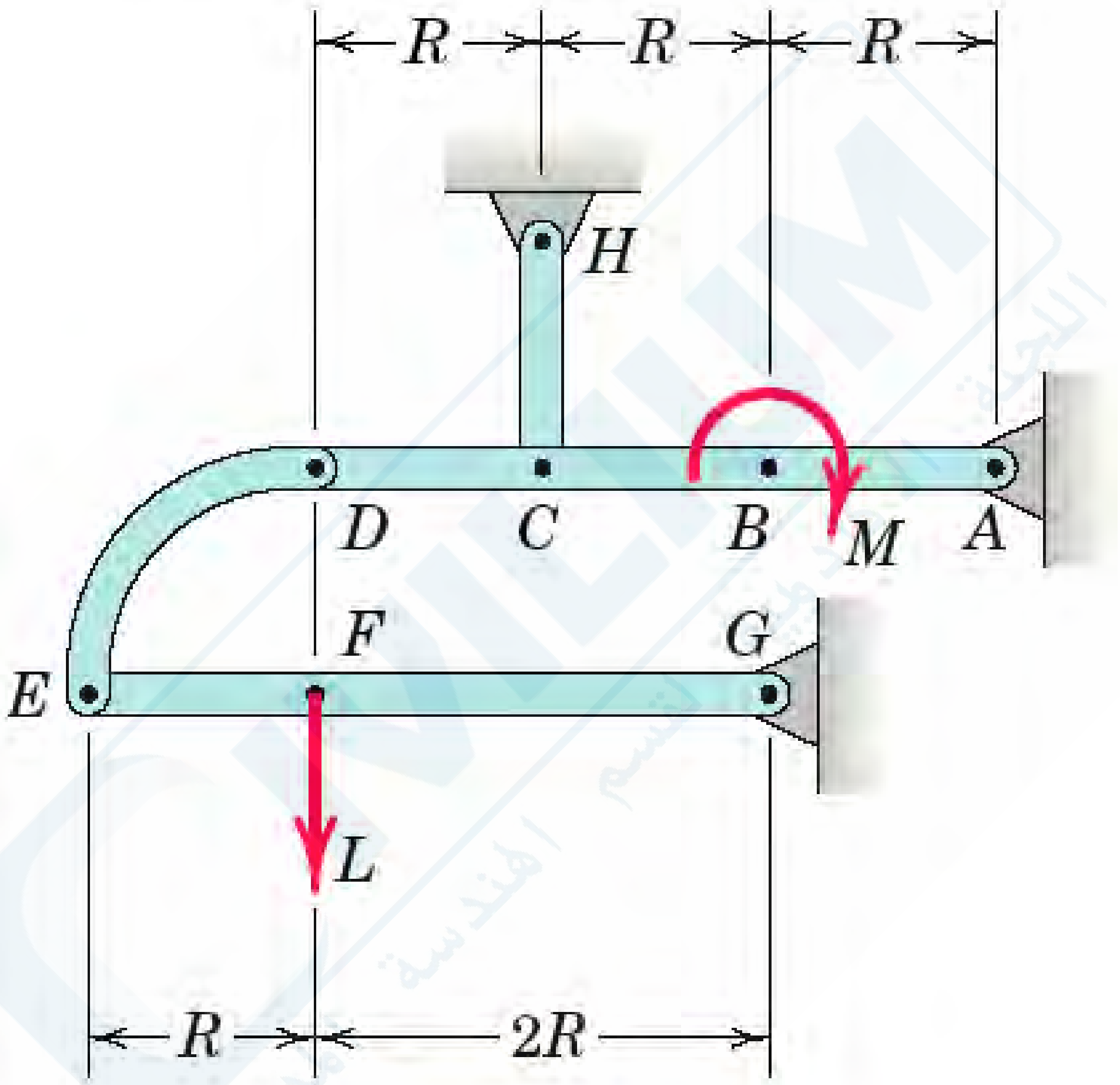
Problem 4/72

P 04 72



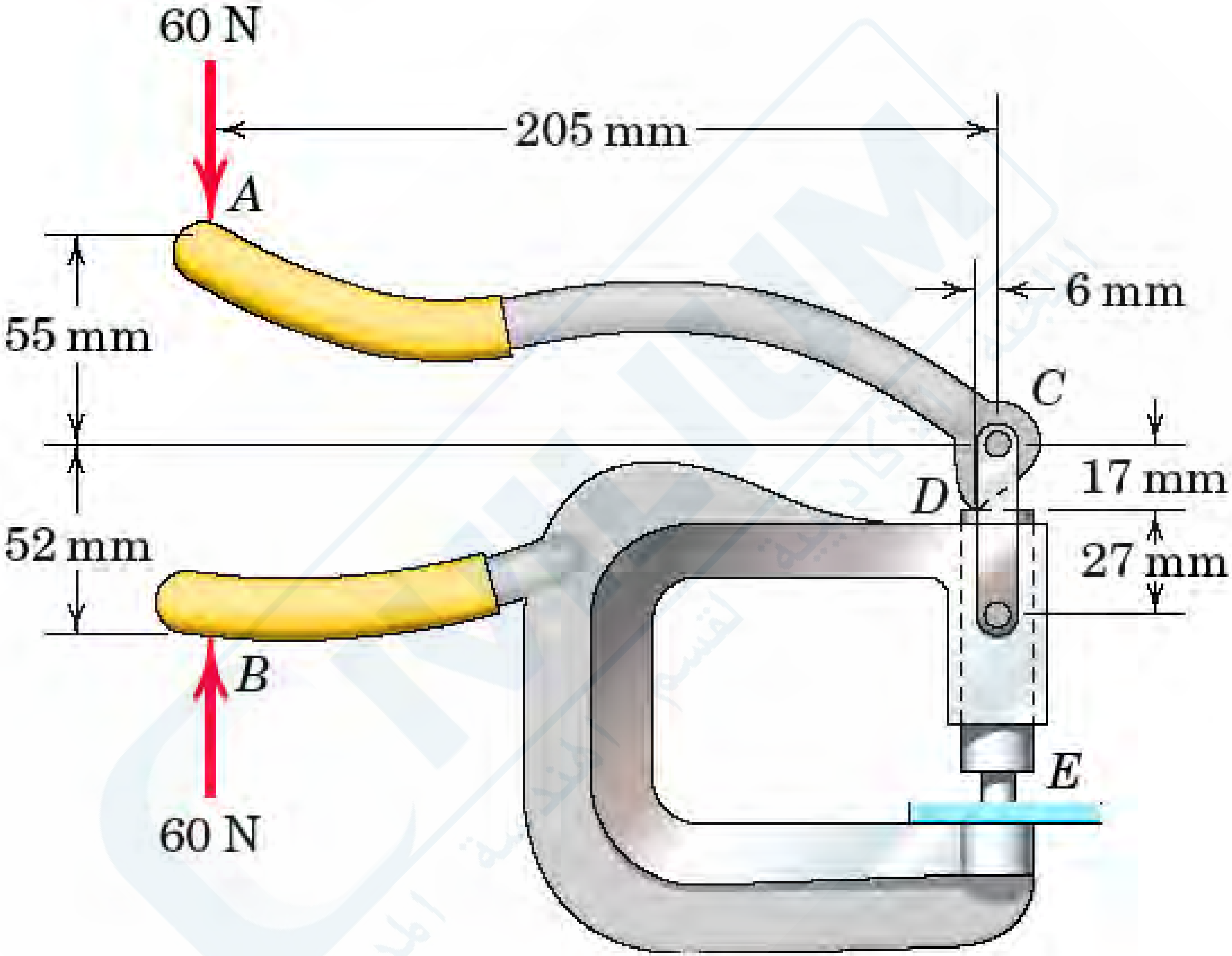
Problem 4/73

P_04_73



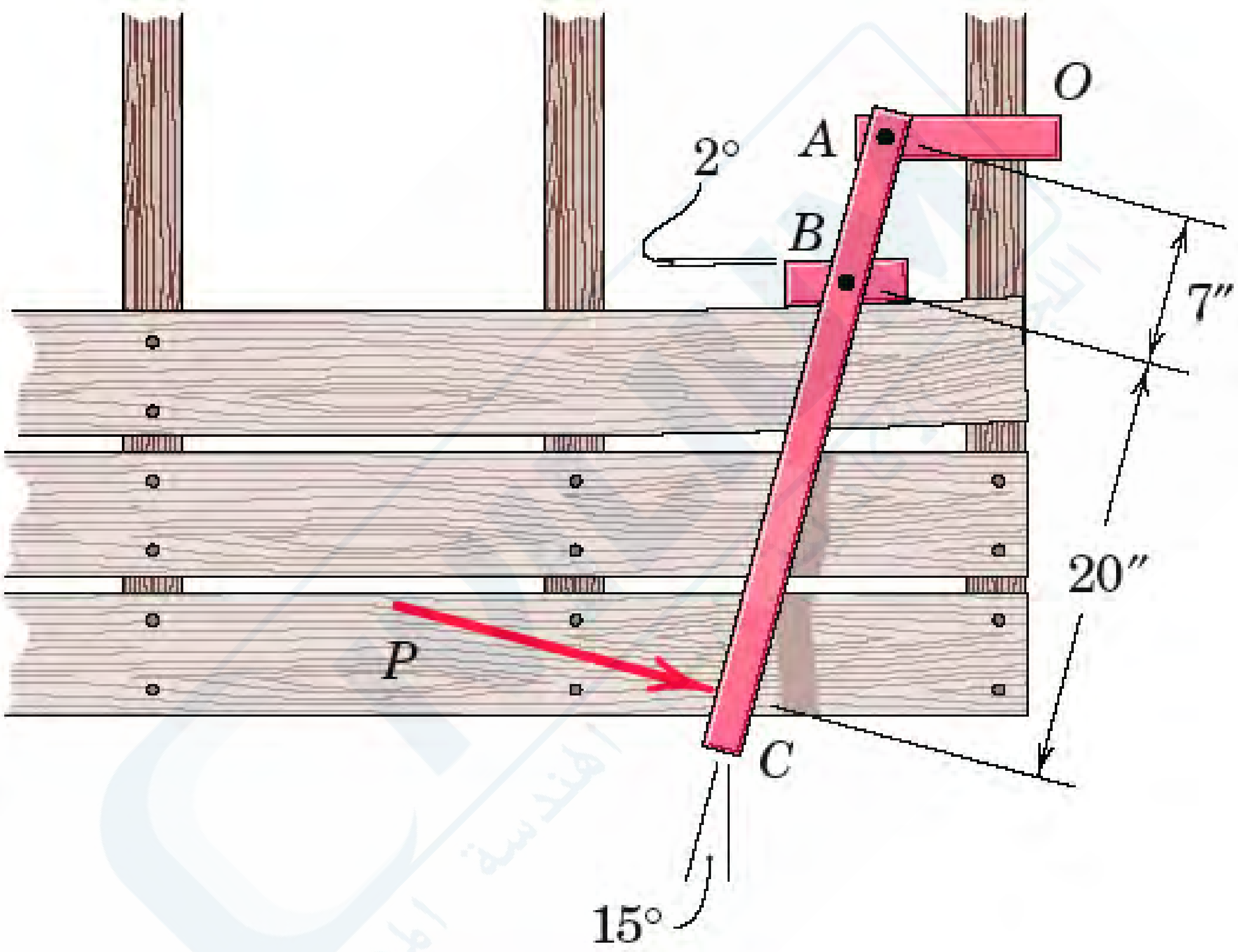
Problem 4/74

P_04_74



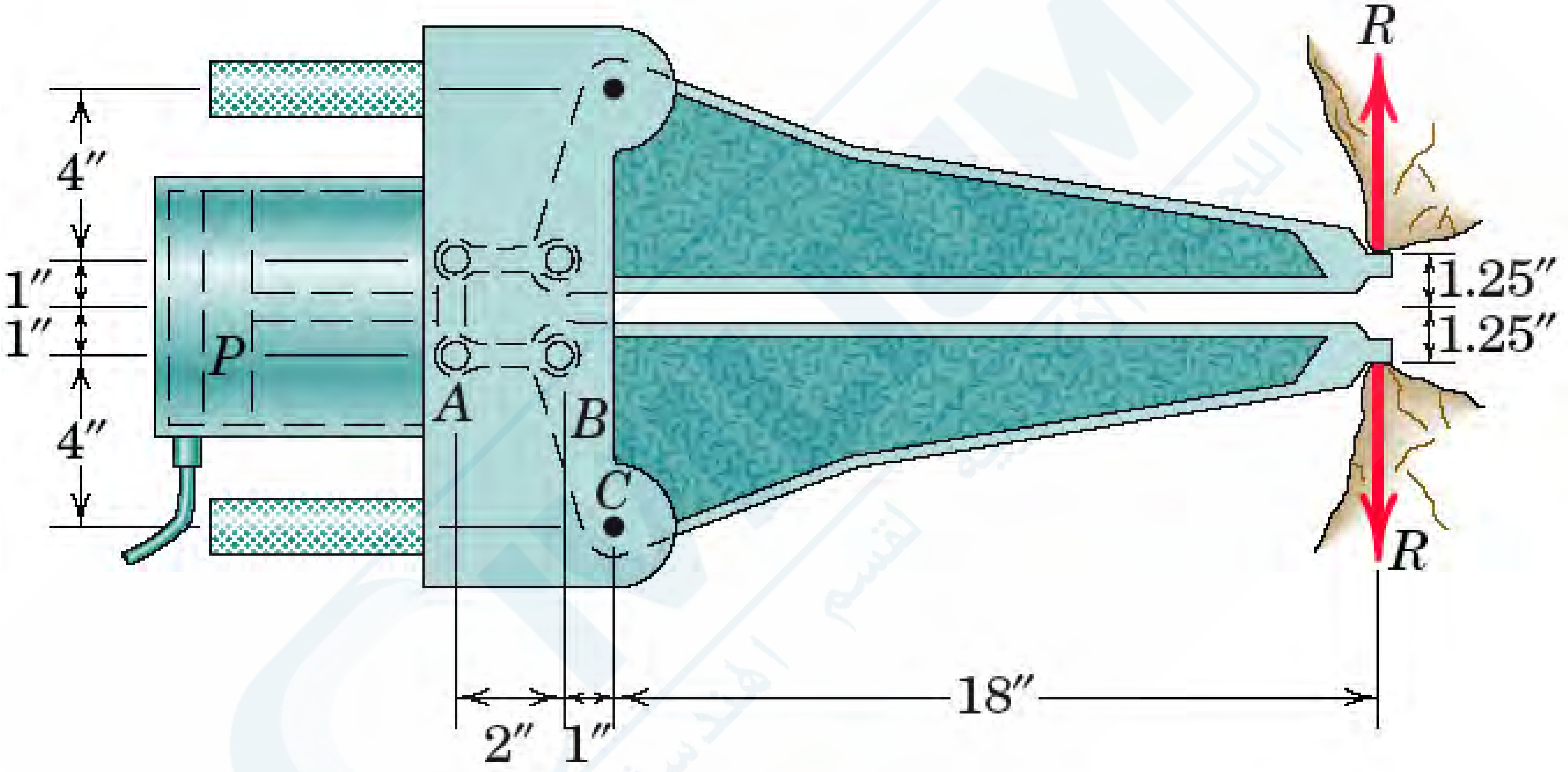
Problem 4/75

P_04_75



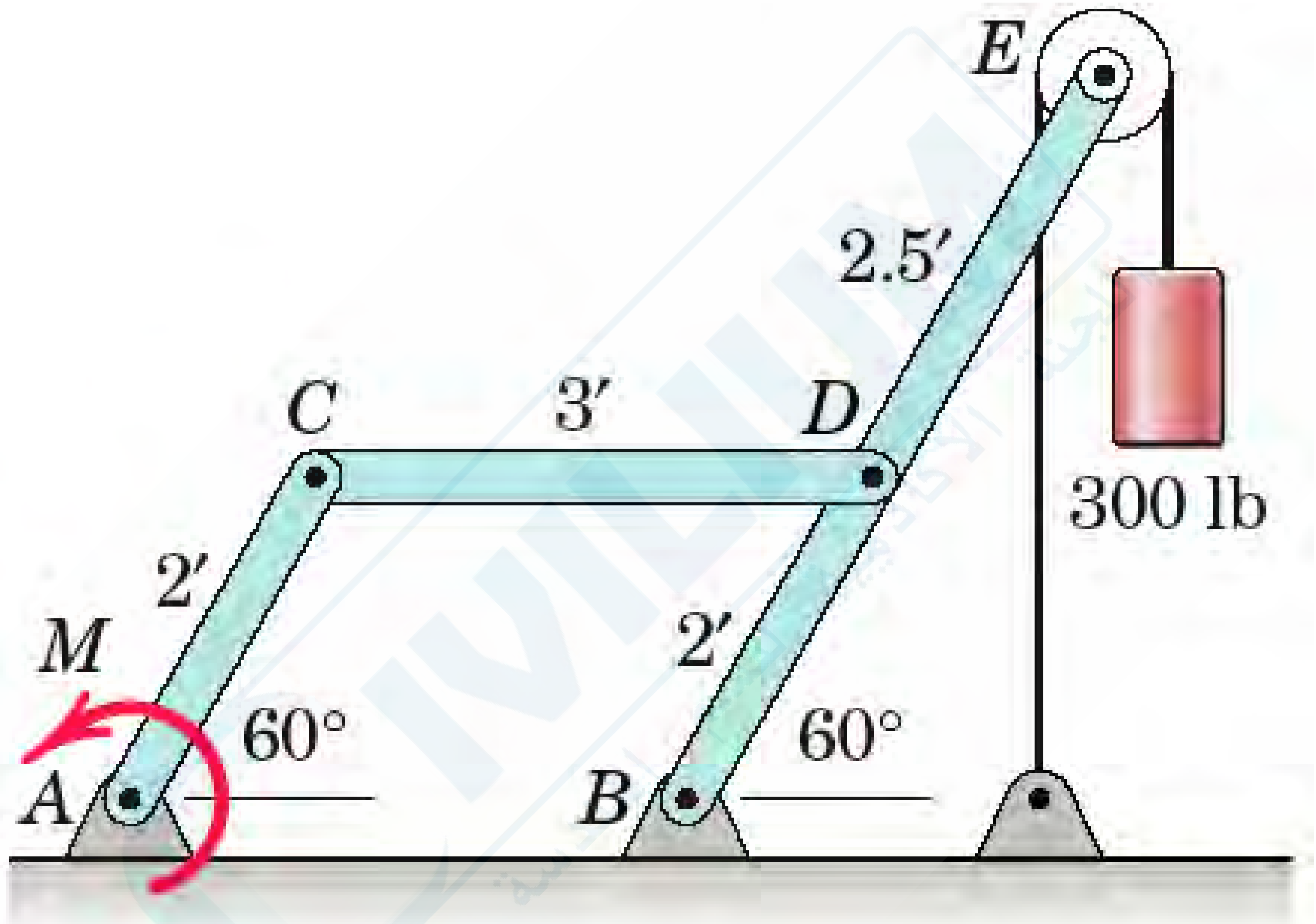
Problem 4/76

P_04_76



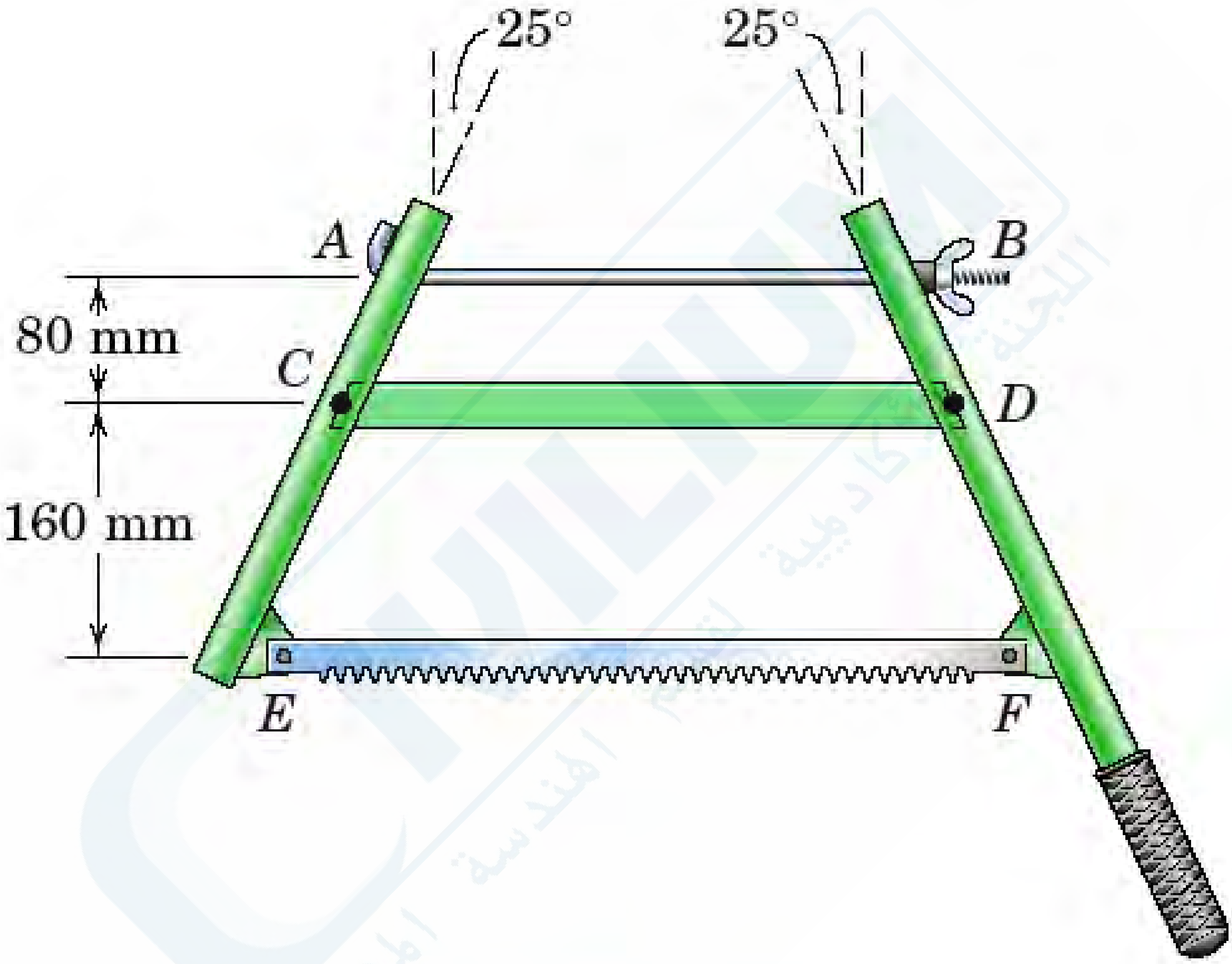
Problem 4/77

P_04_77



Problem 4/78

P 04 78



Problem 4/79

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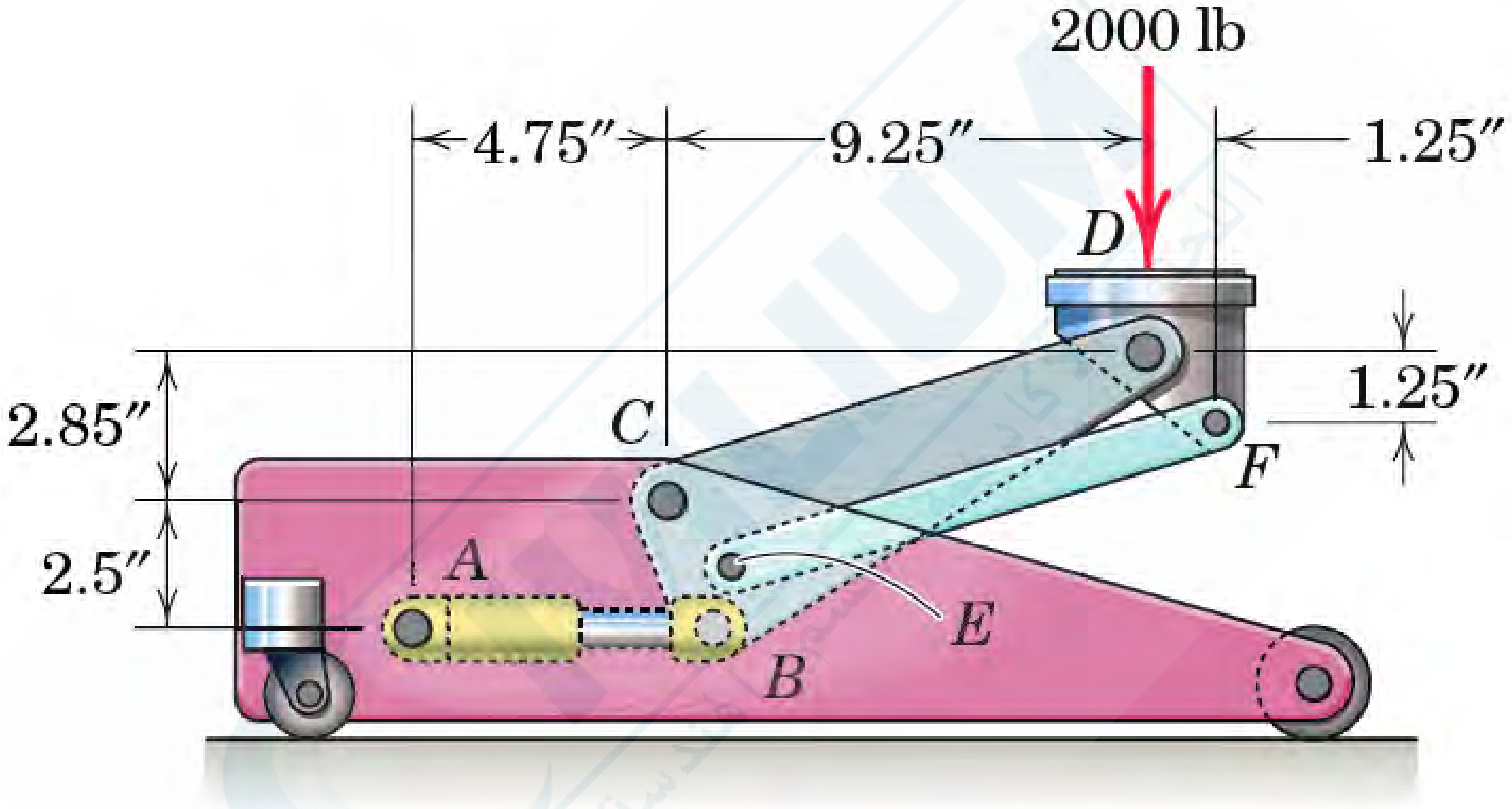

Problem 4/80

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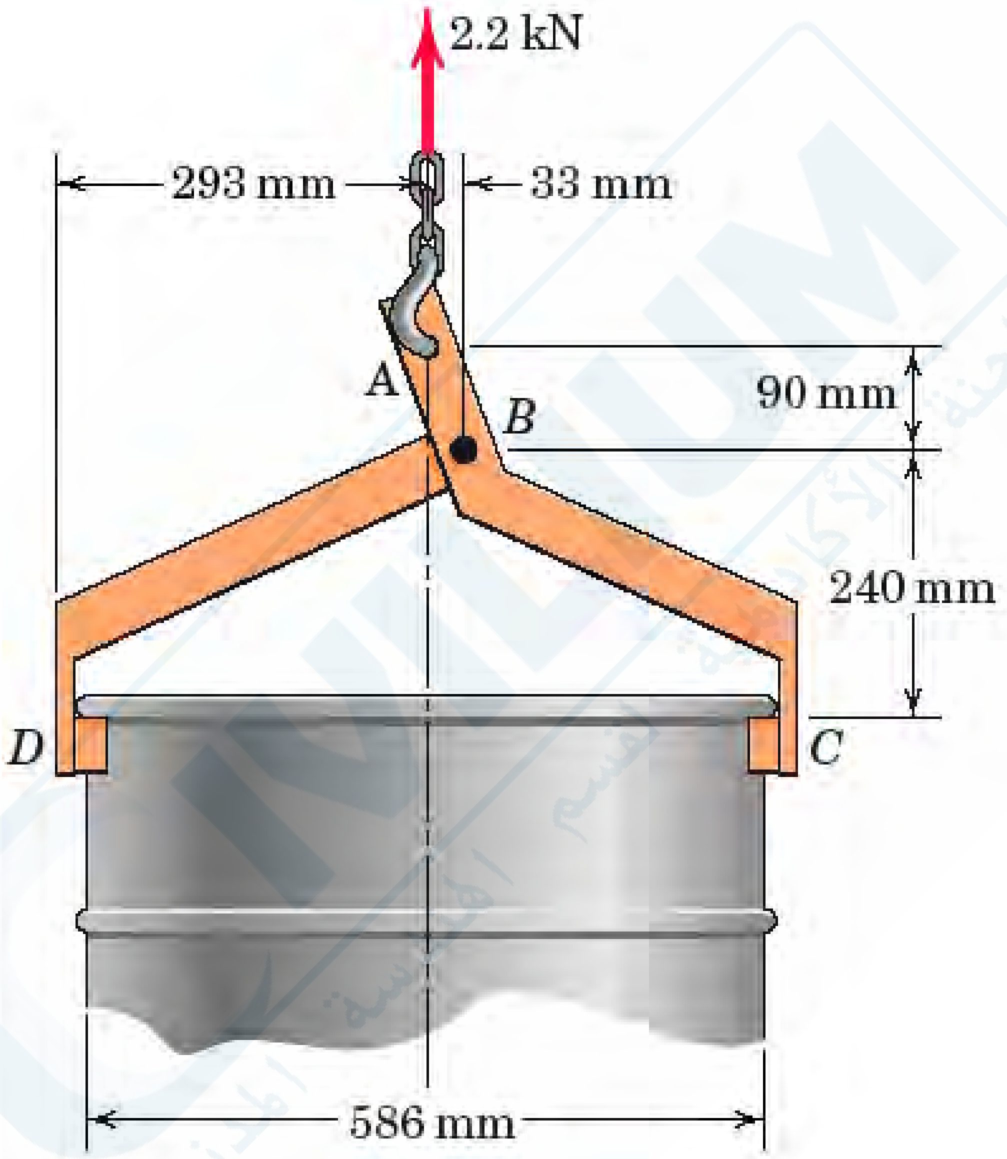
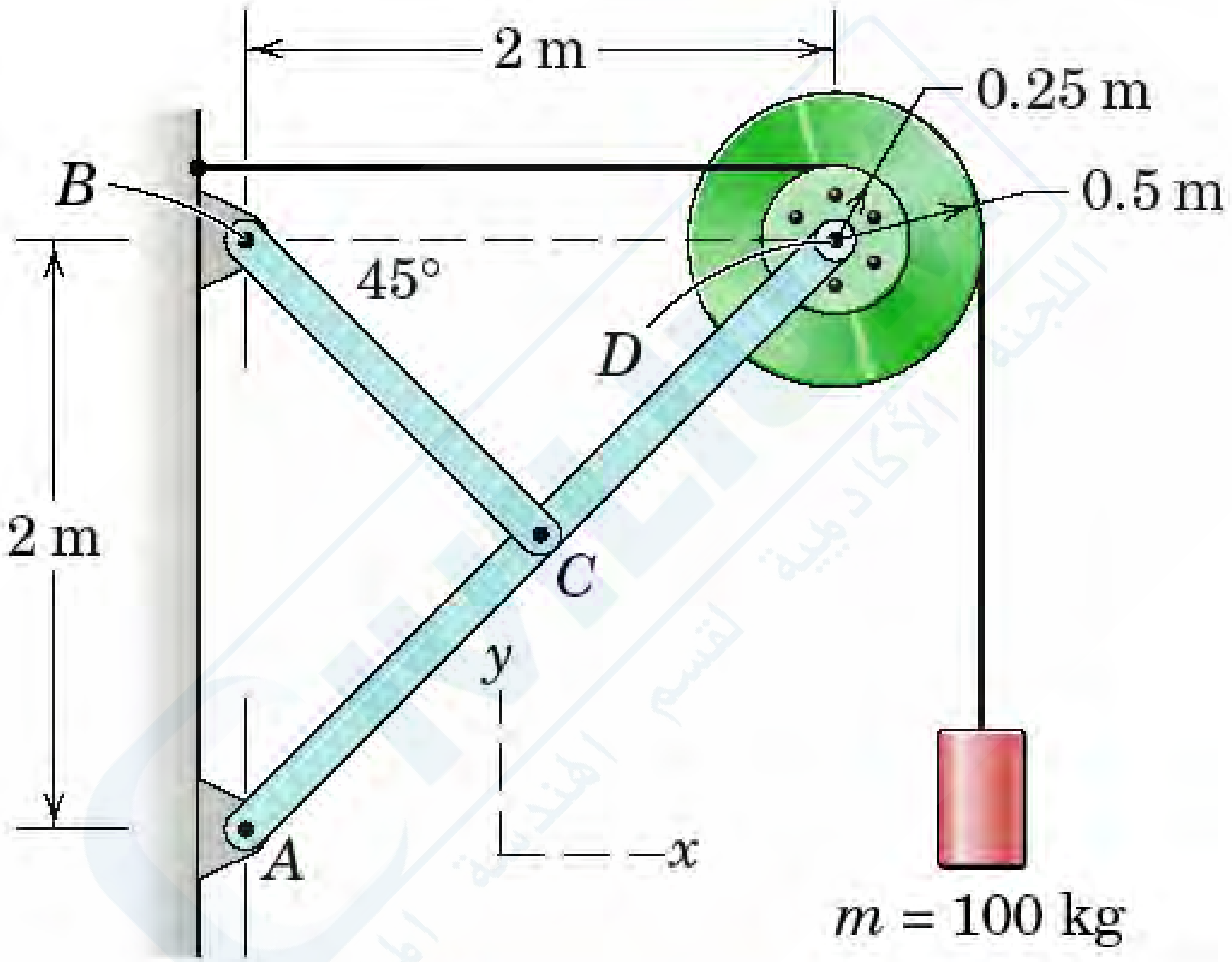


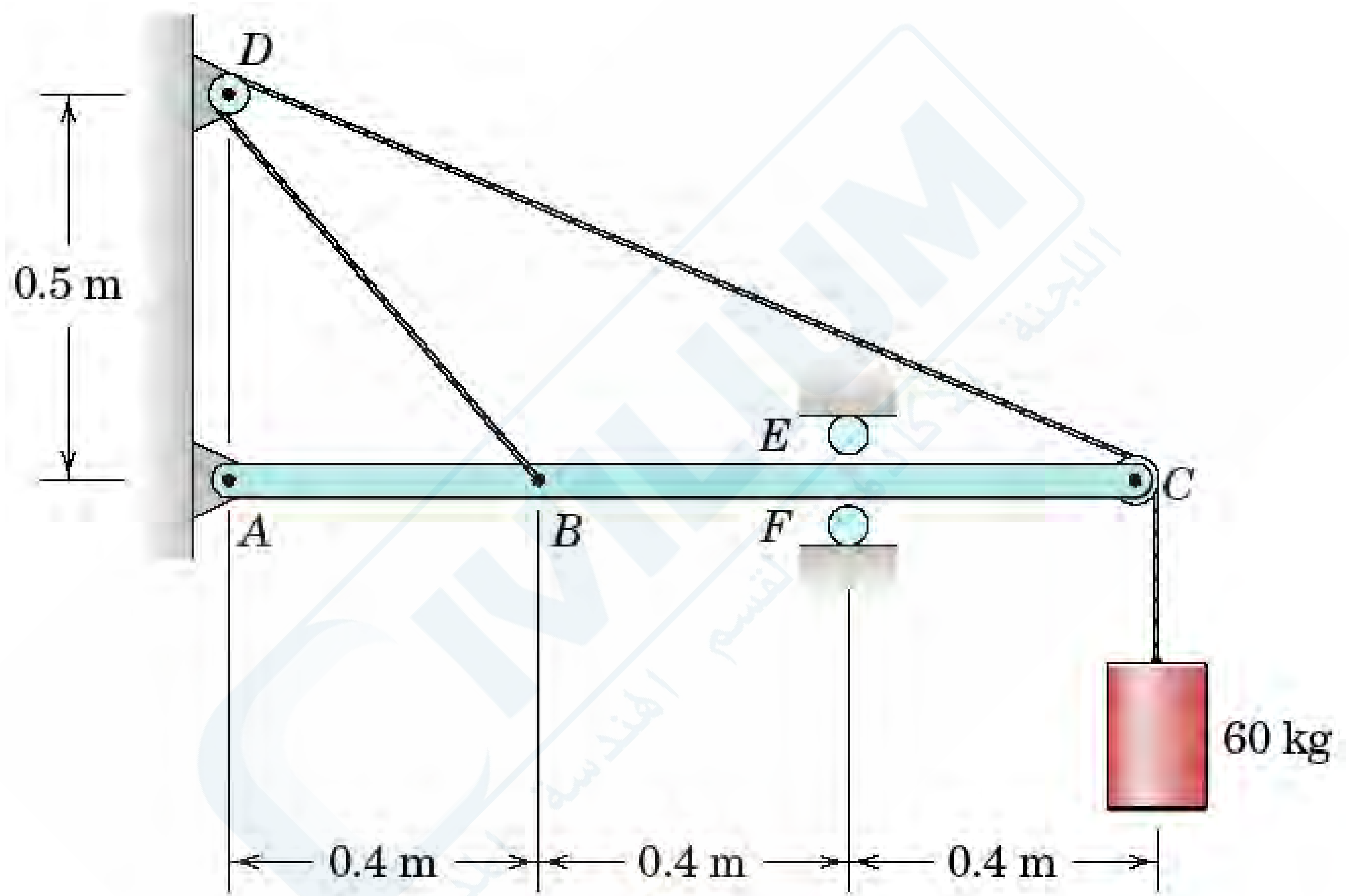
Problem 4/81

P 04 81

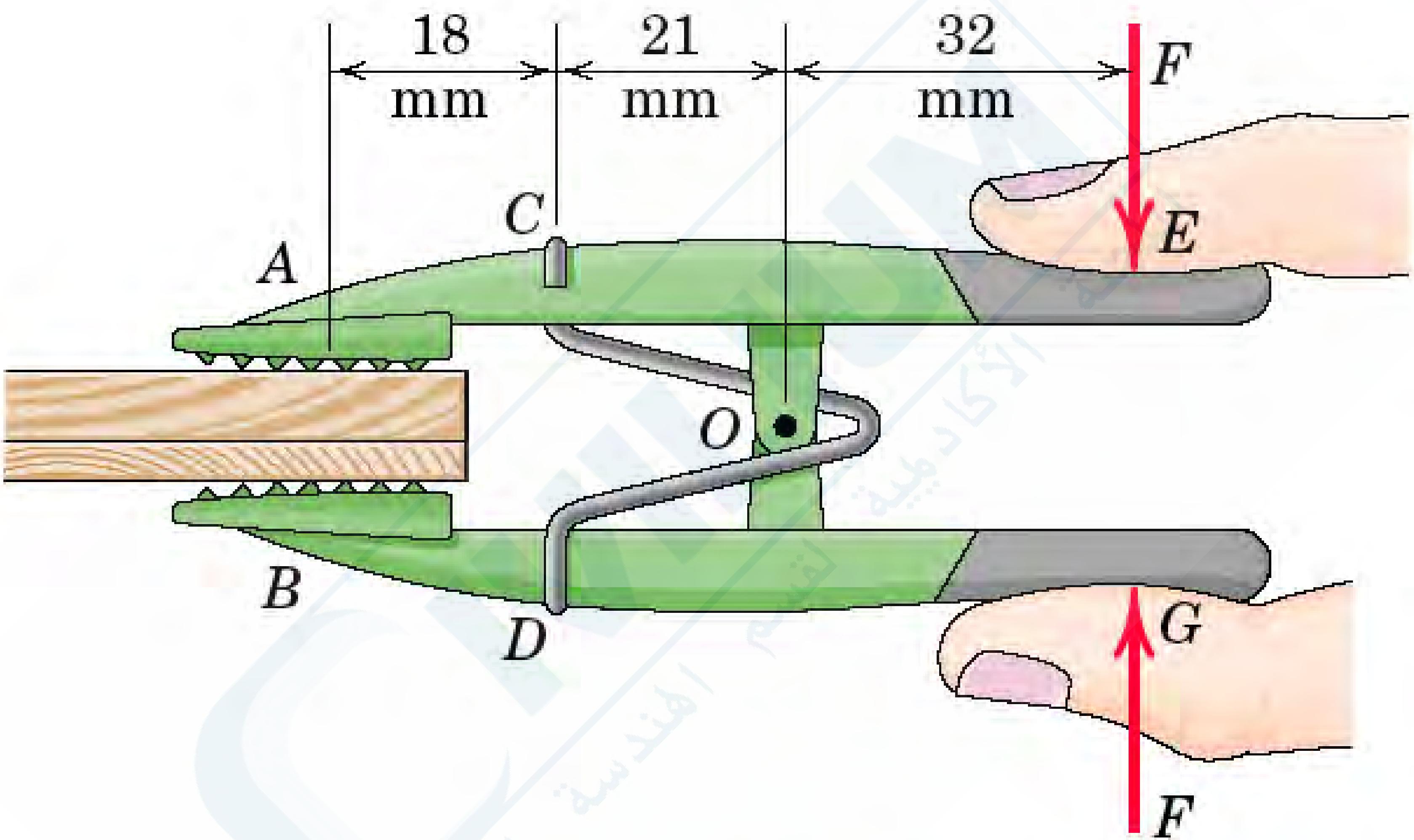


Problem 4/82

P 04 82



Problem 4/83



Problem 4/84

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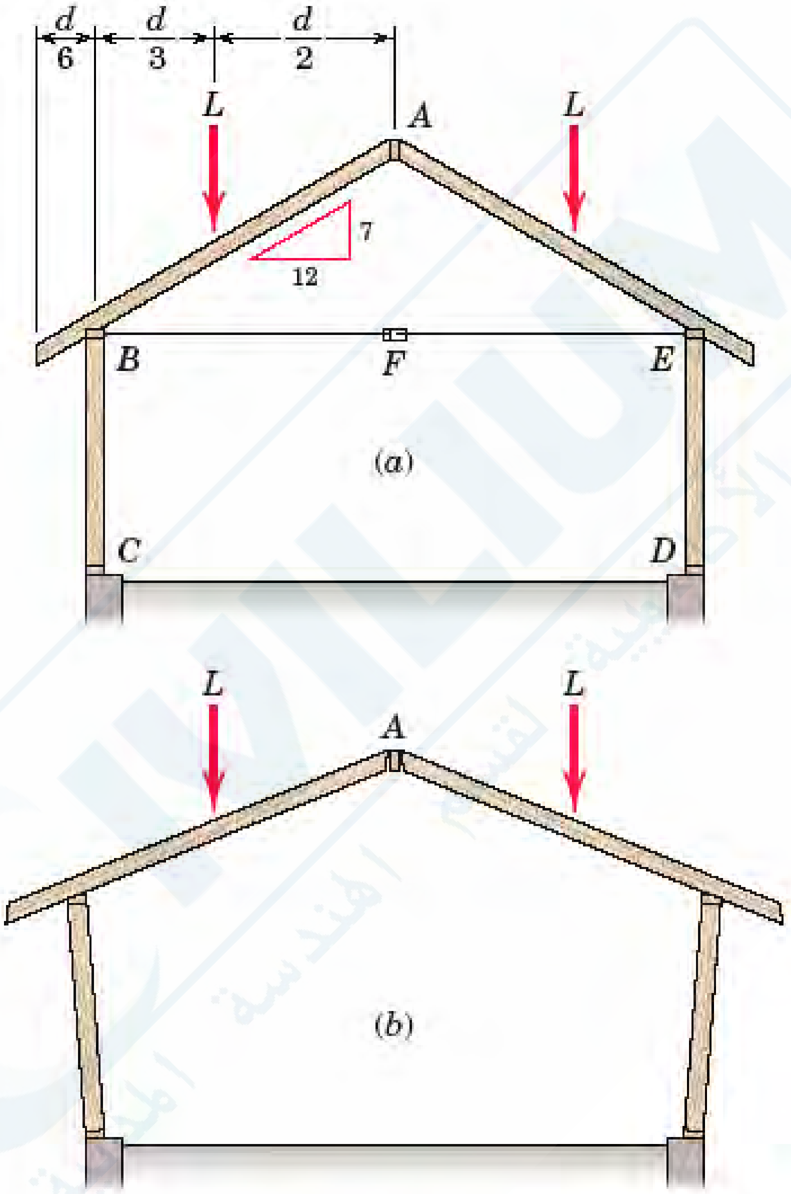

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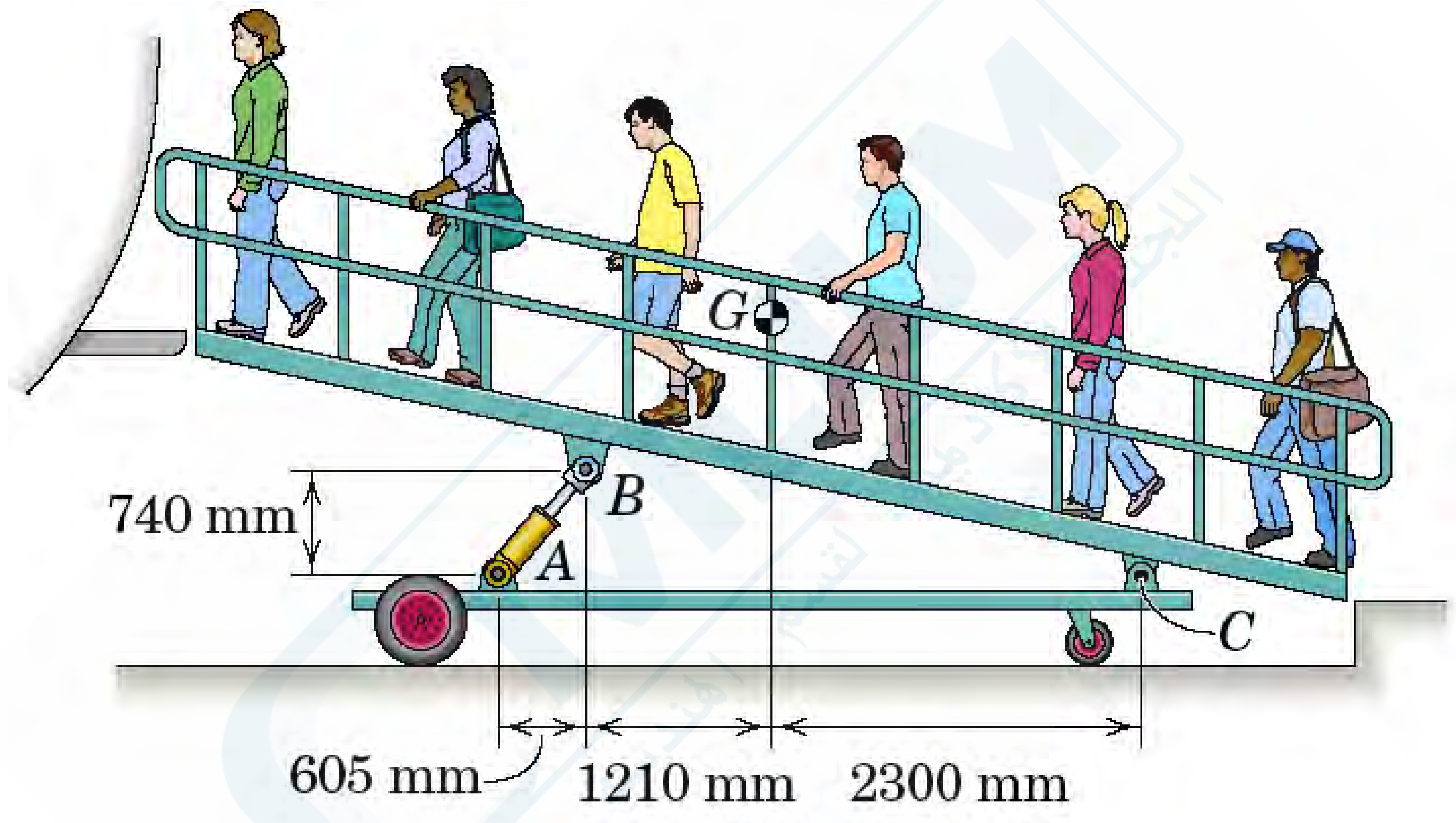

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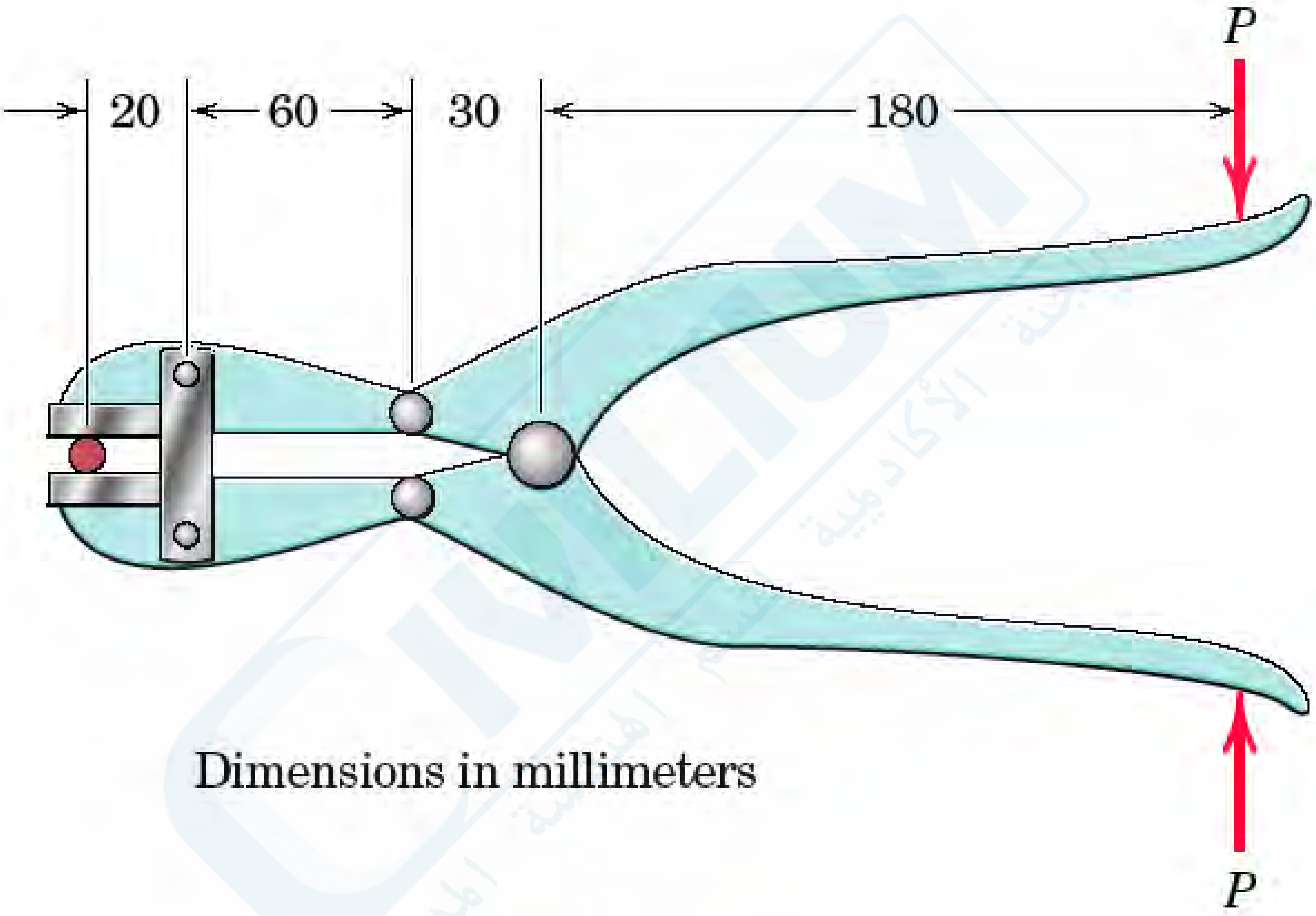

Problem 4/87

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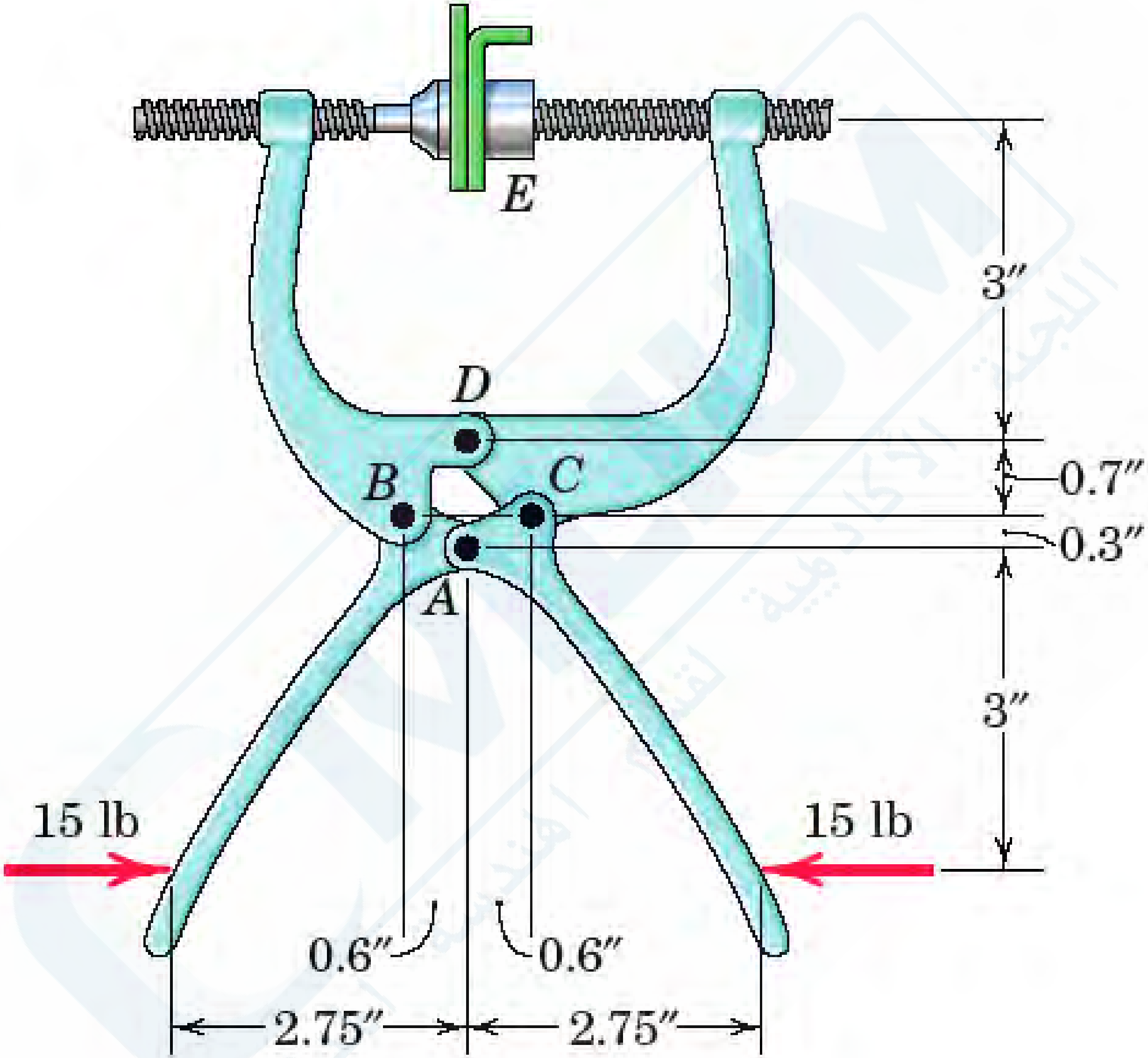
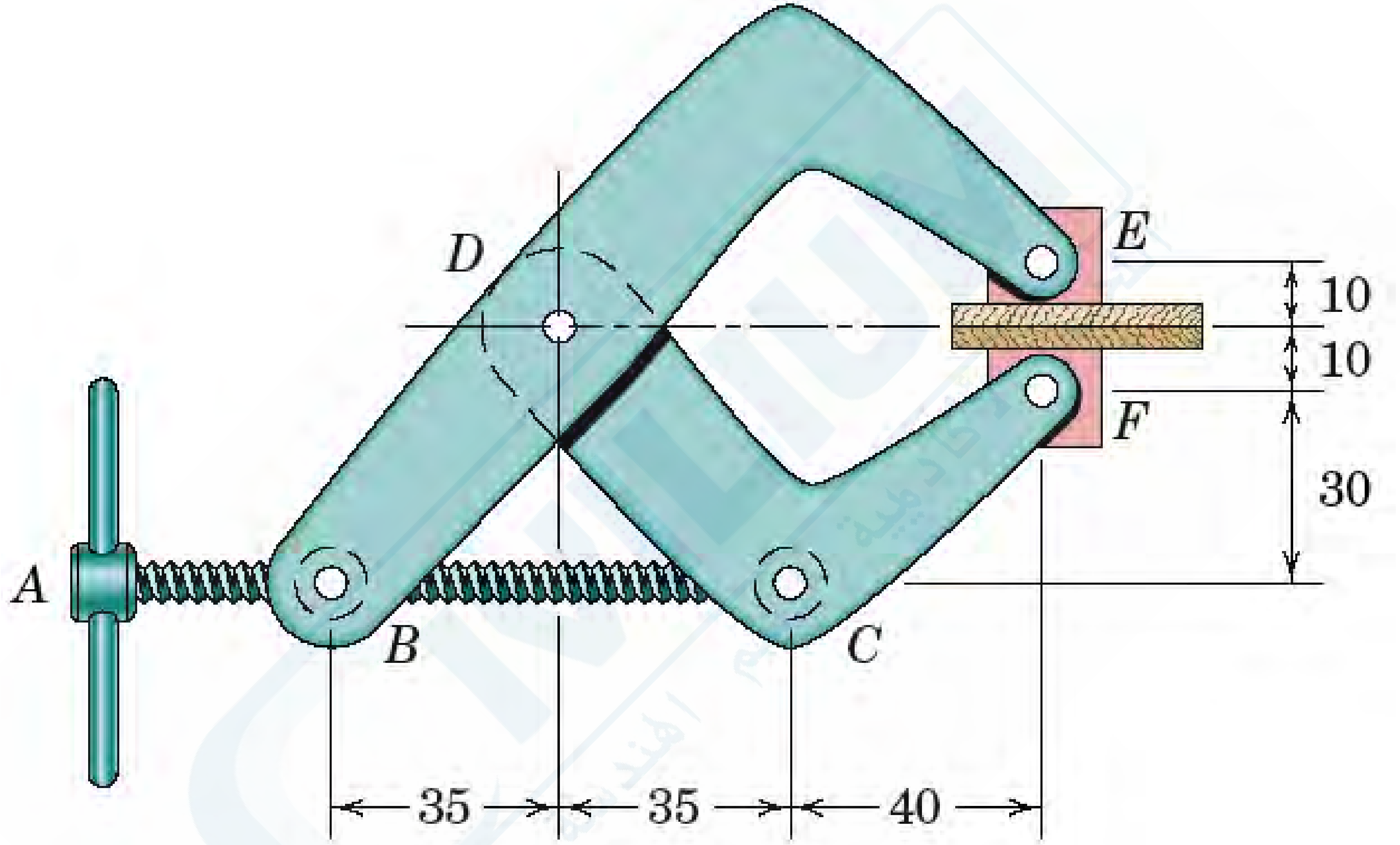


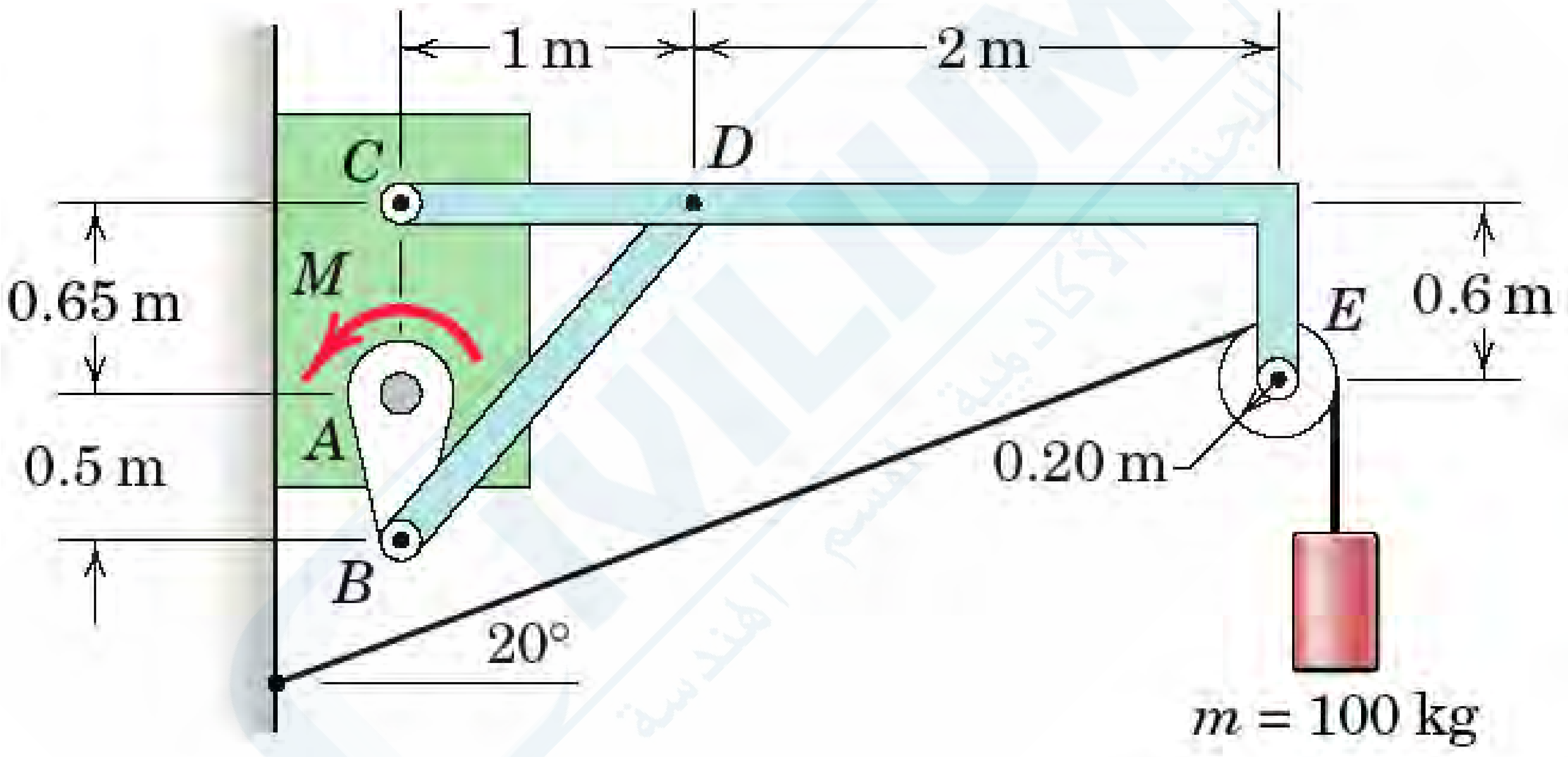
Problem 4/88

P 04 88



Dimensions in millimeters

Problem 4/89



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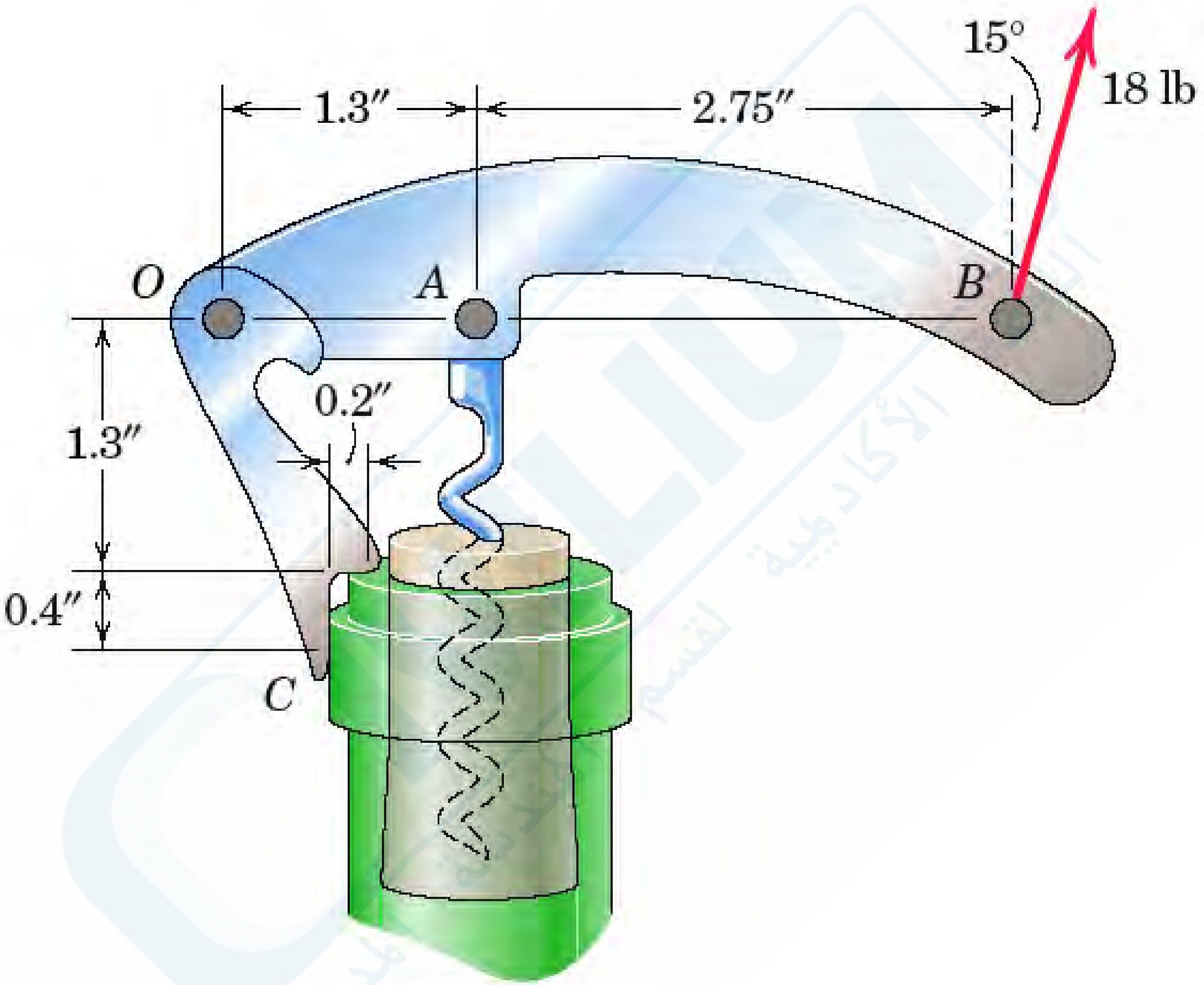
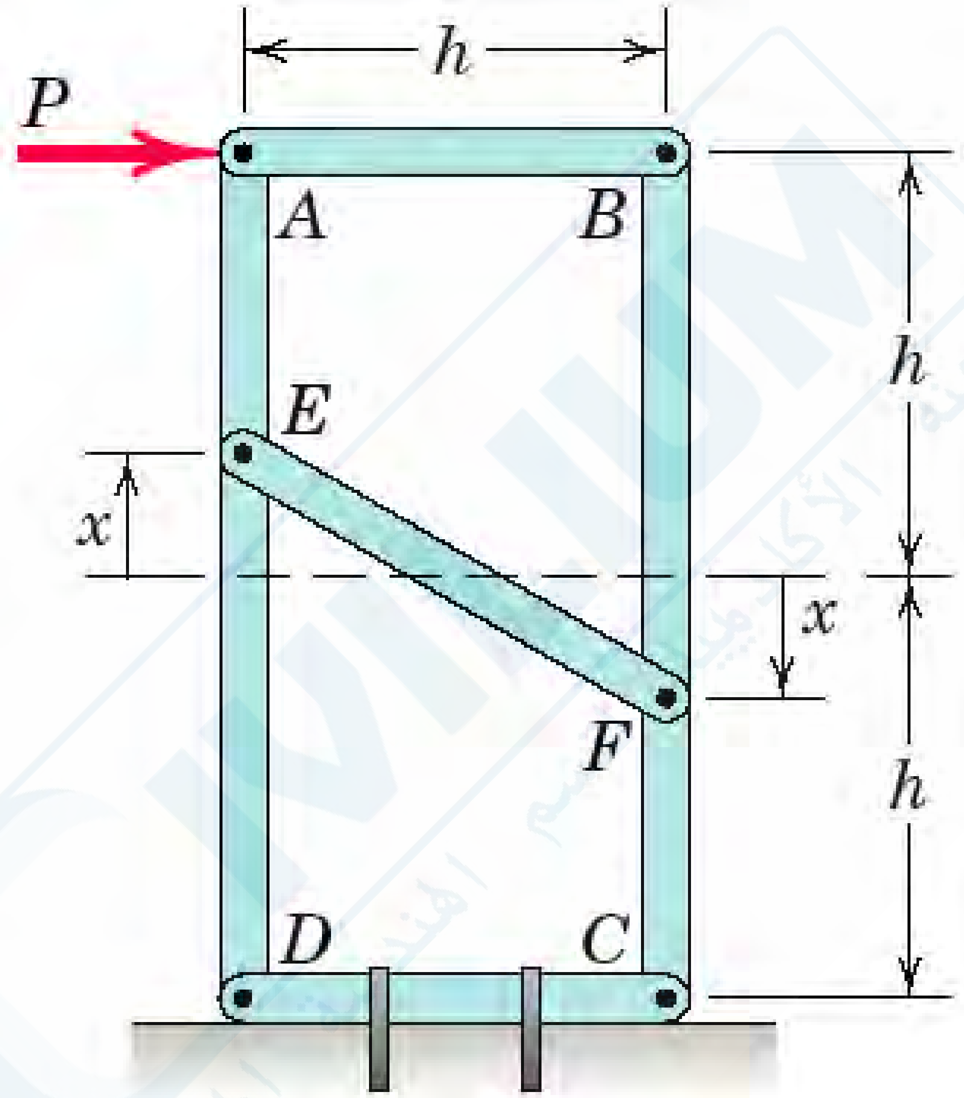


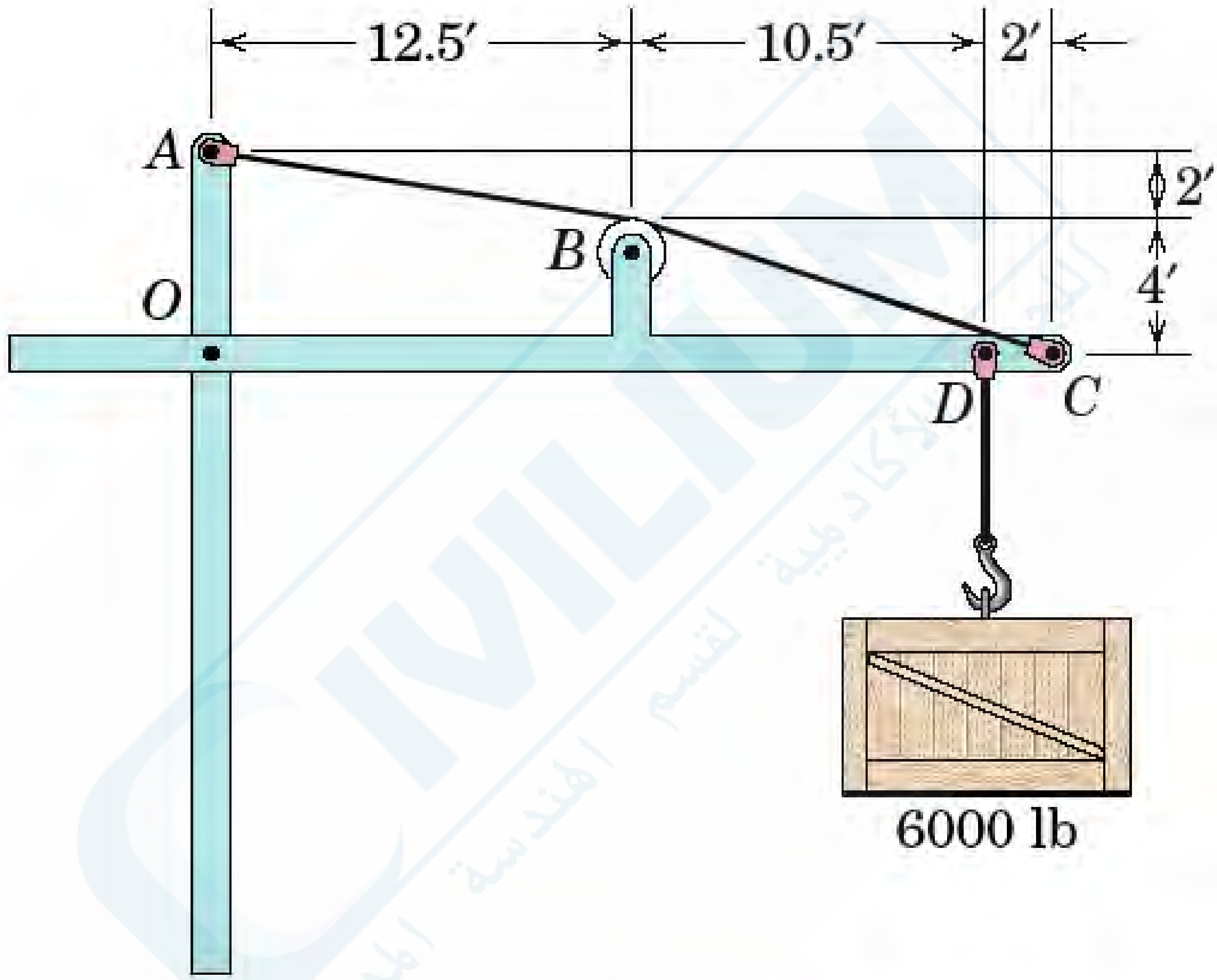
Problem 4/91

P_04_91



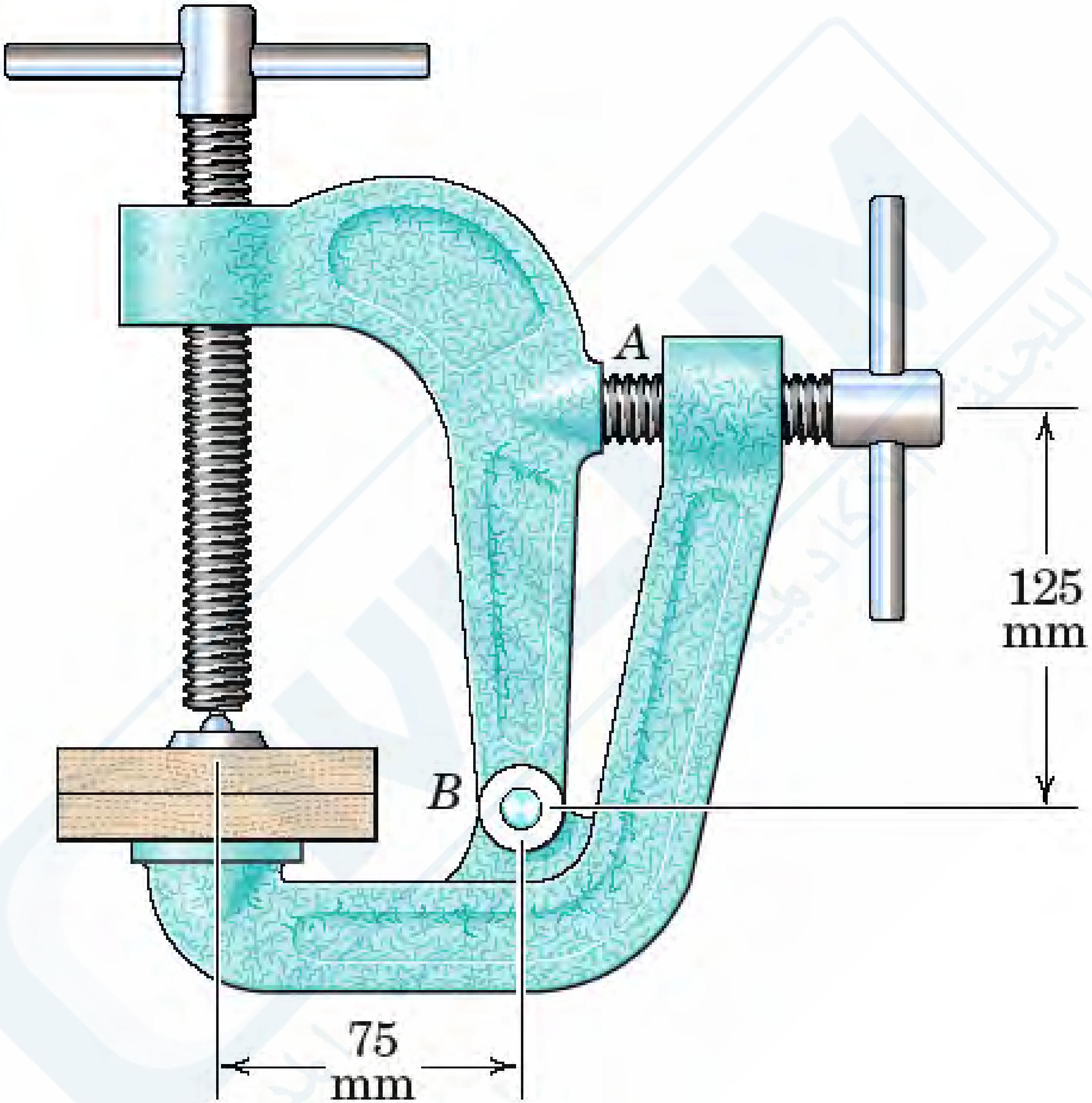
Problem 4/92

P_04_92



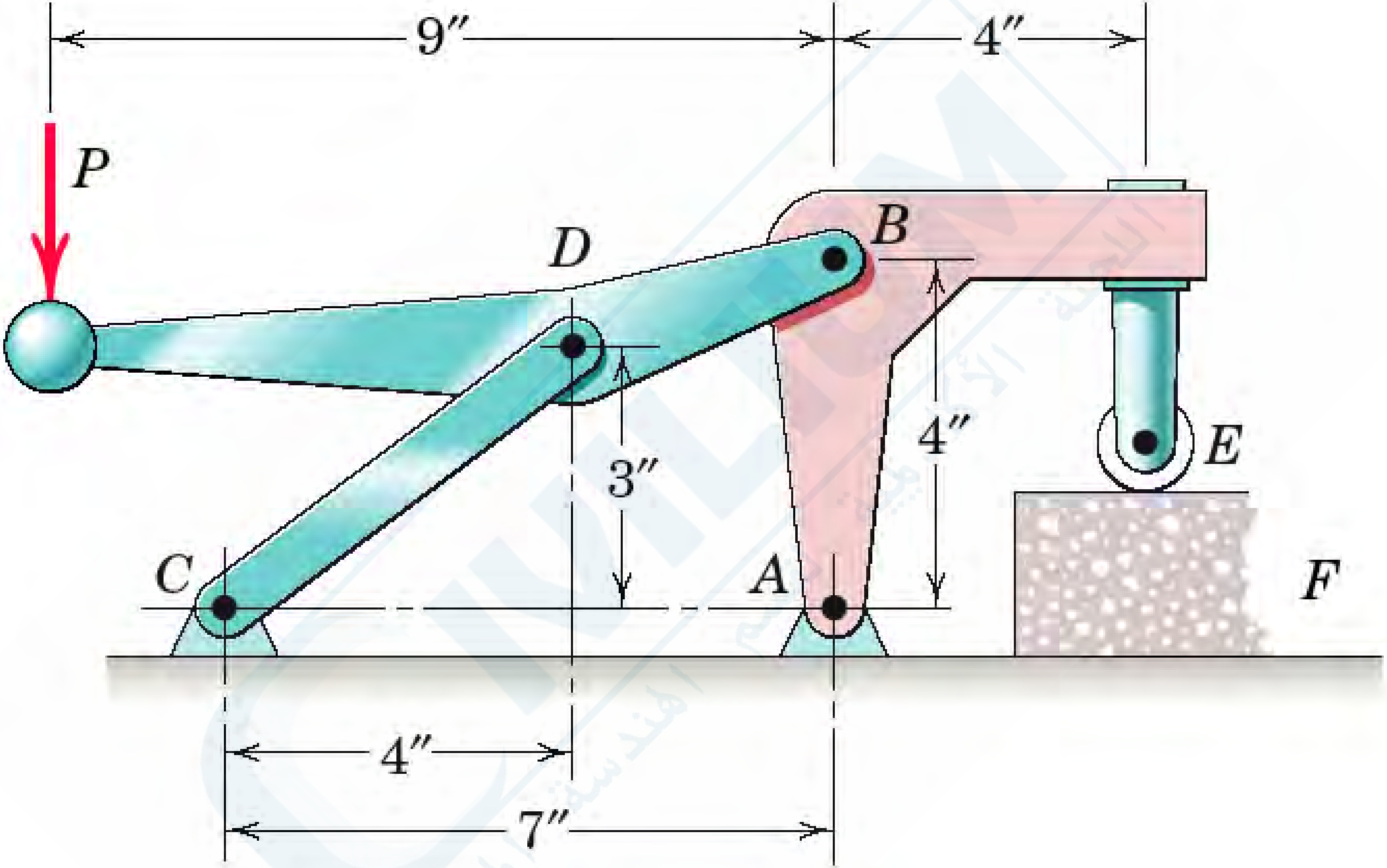
Problem 4/93

P 04 93



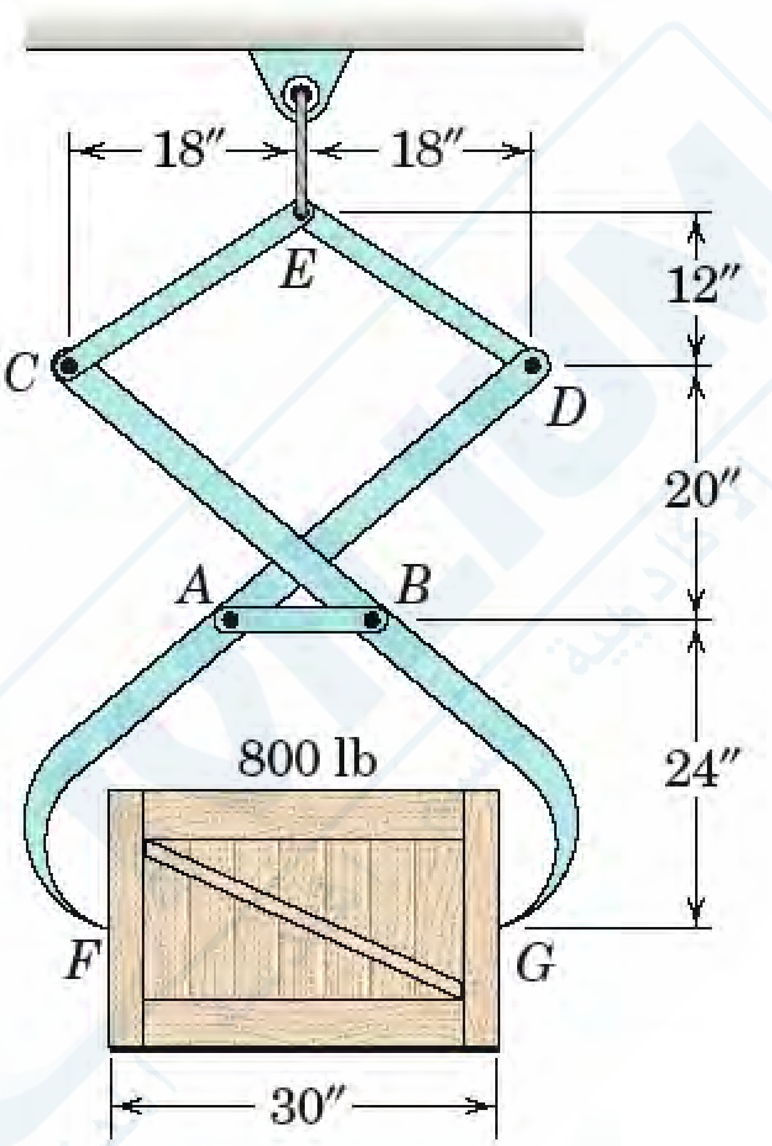
Problem 4/94

P_04_94



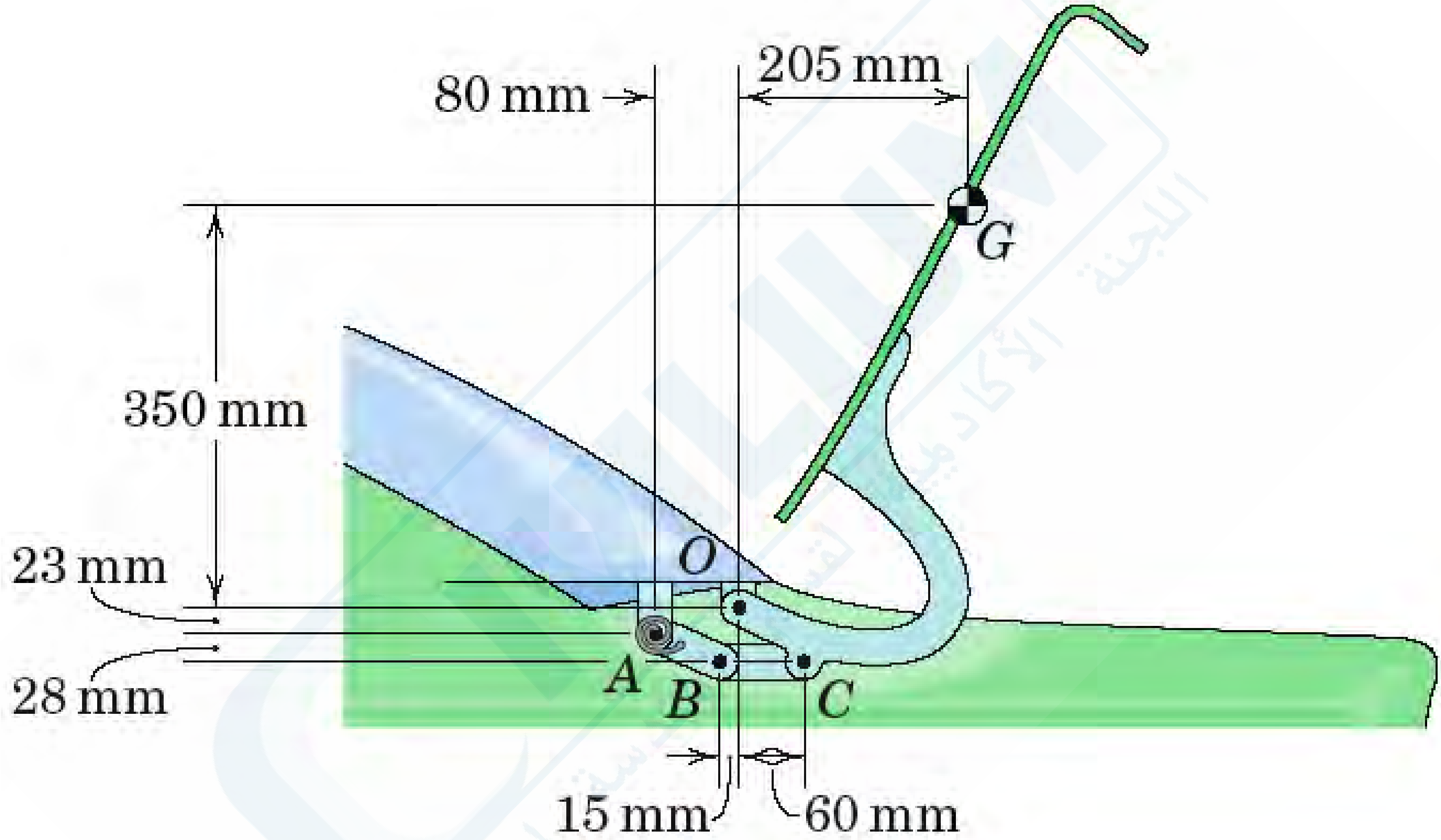
Problem 4/95

P_04_95



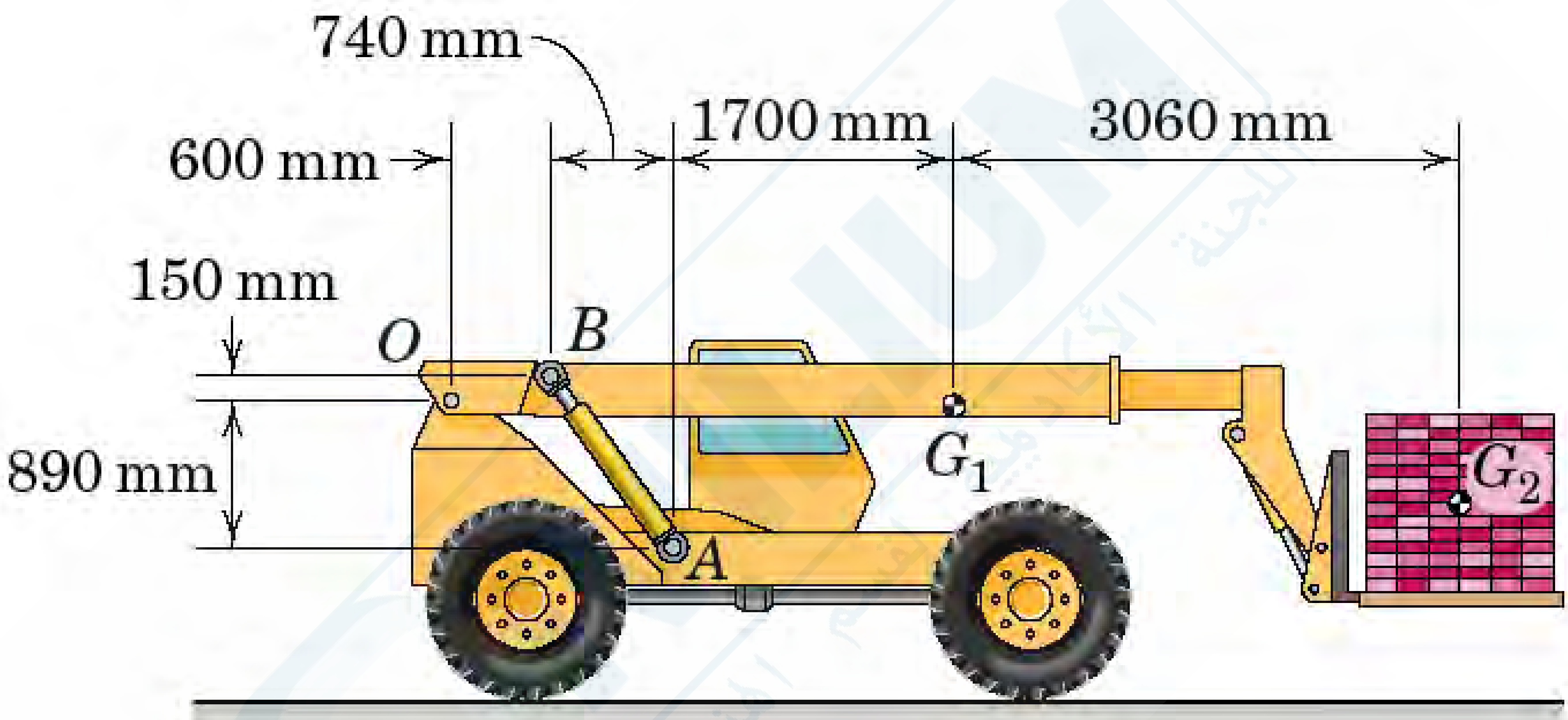
Problem 4/96

P 04 96



Problem 4/97

P_04_97



Problem 4/98

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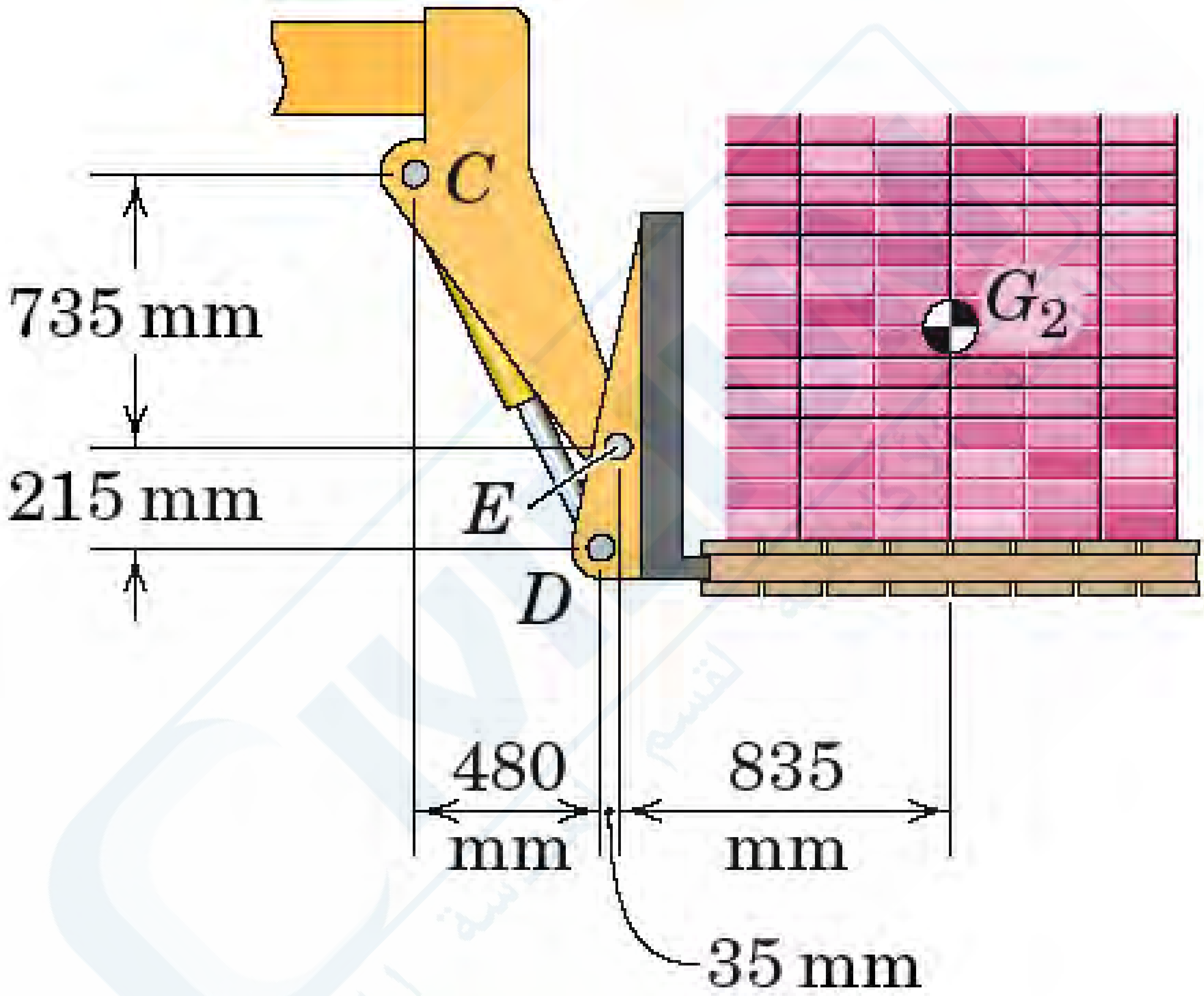


Problem 4/99

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 5 : Distributed Forces.

CIVILIUM
اللجنة الأكاديمية لقسم الهندسة المدنية

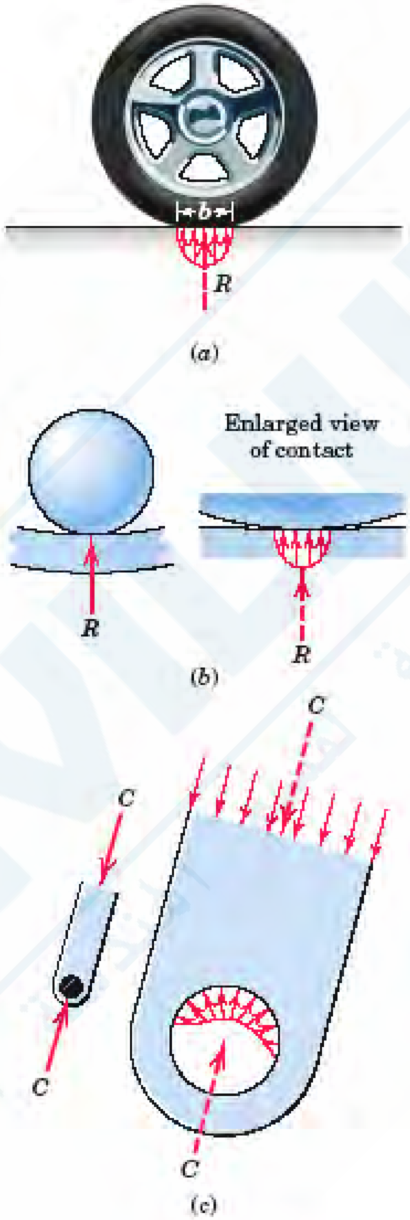
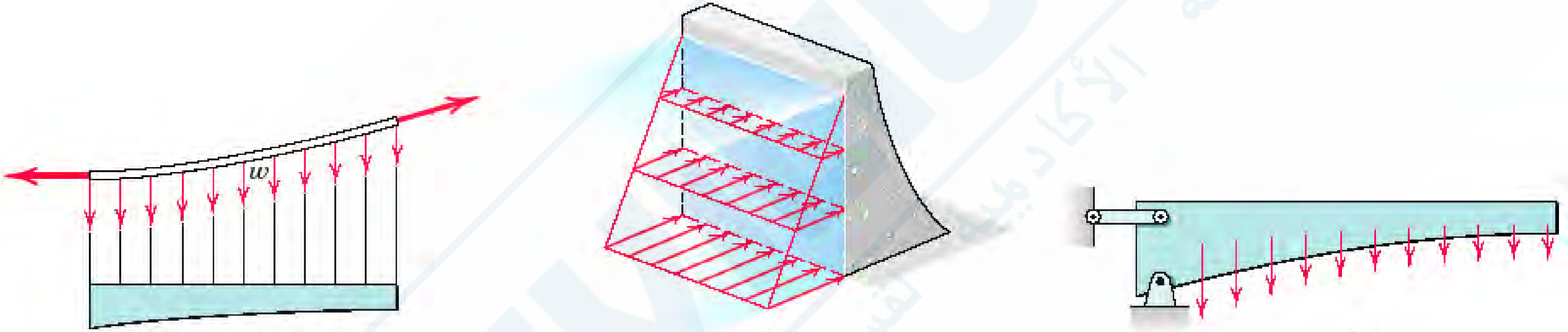


Figure 5/1



(a) (b) (c)

Figure 5/2

05_02

CIVILIUM
اللجنة الأكاديمية لقسم الهندسة المدنية

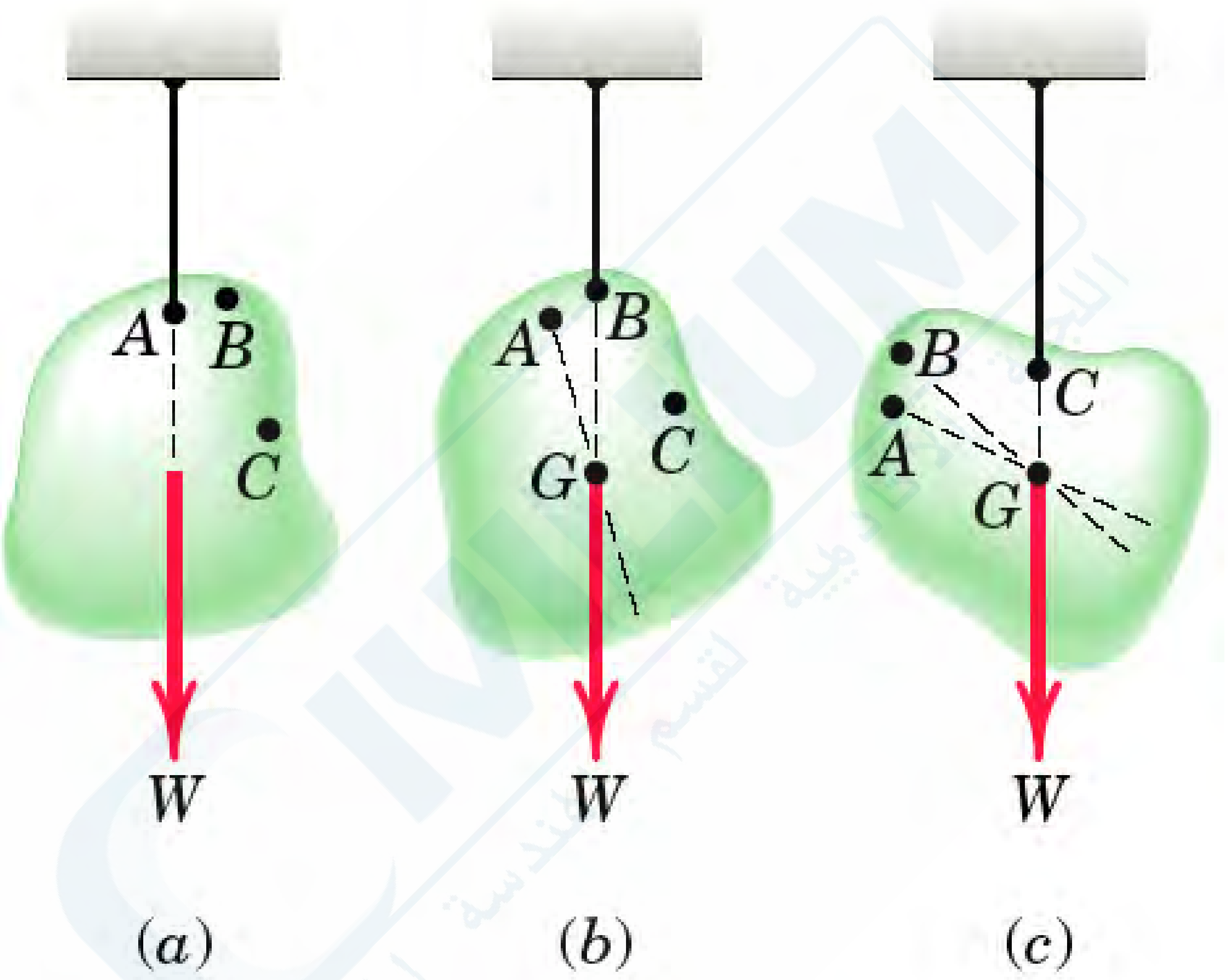


Figure 5/3

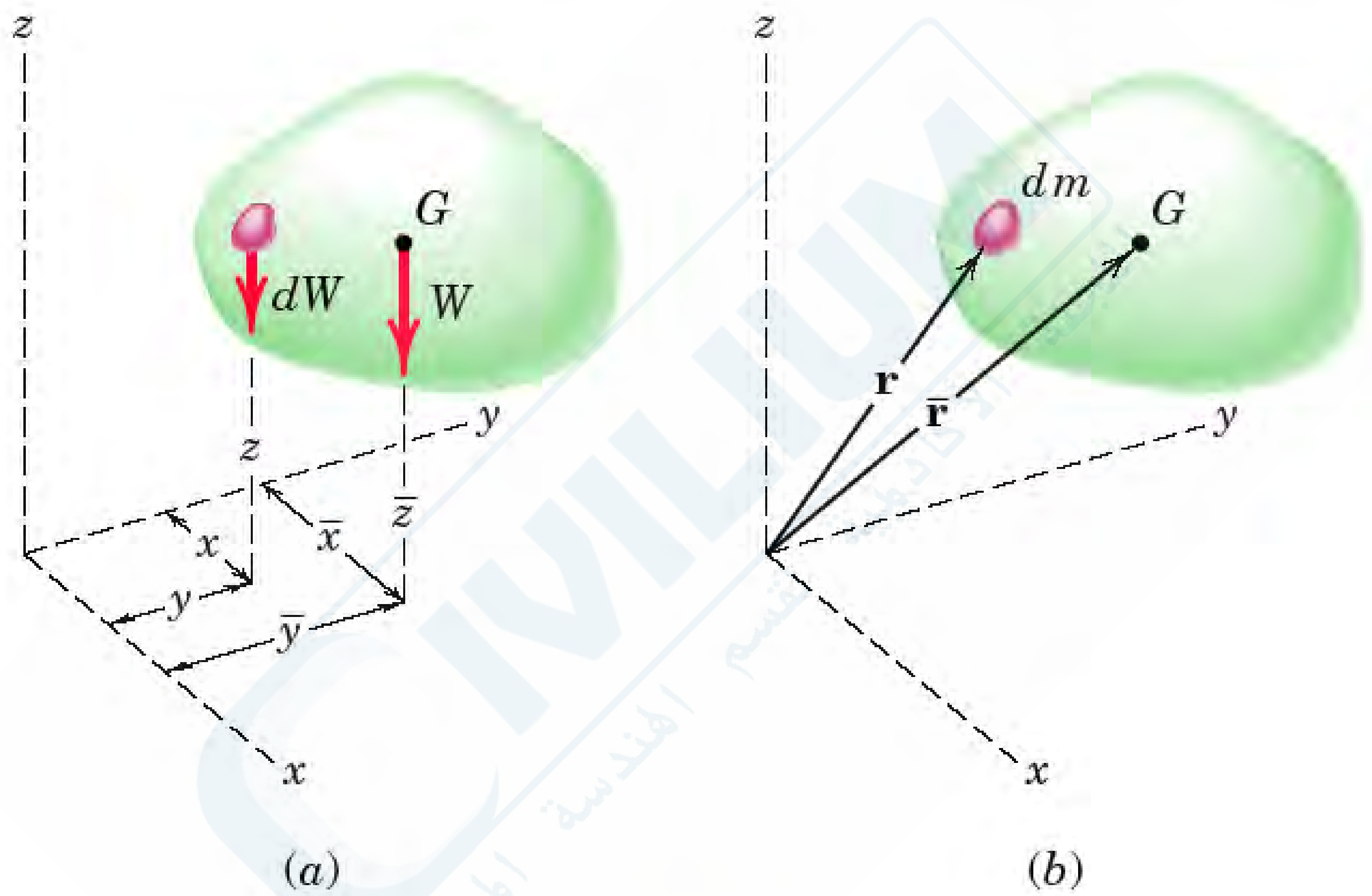


Figure 5/4

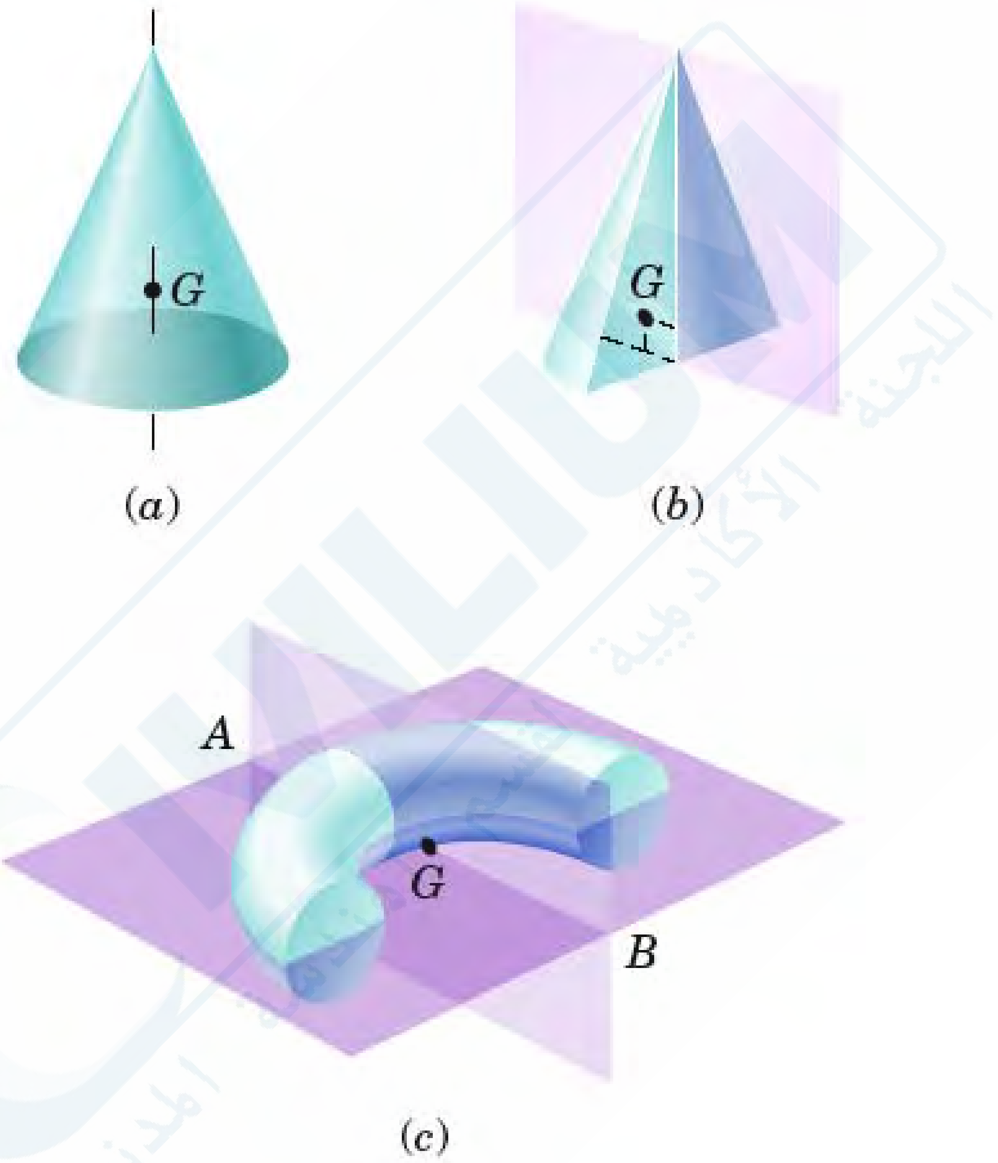


Figure 5/5

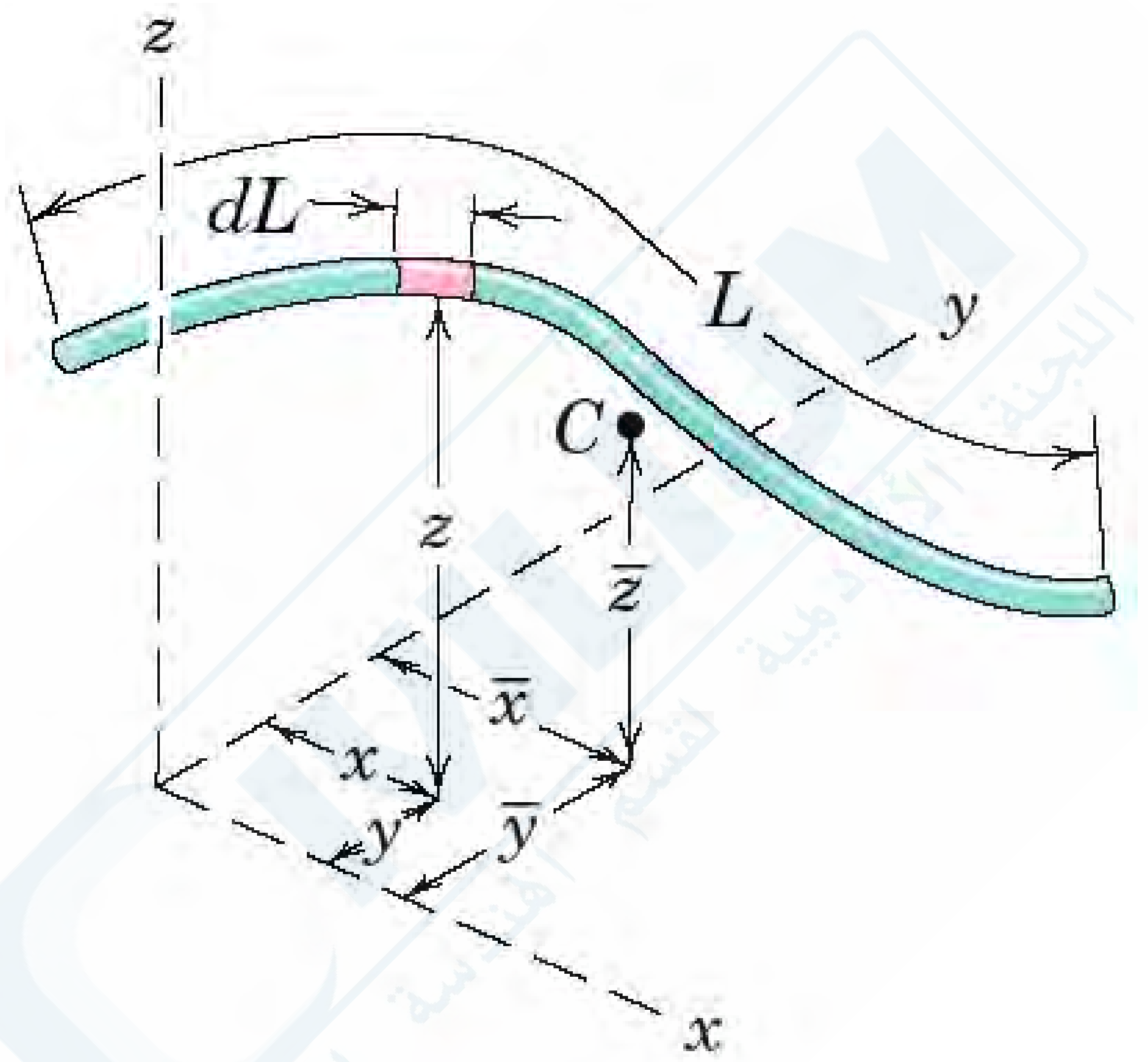


Figure 5/6

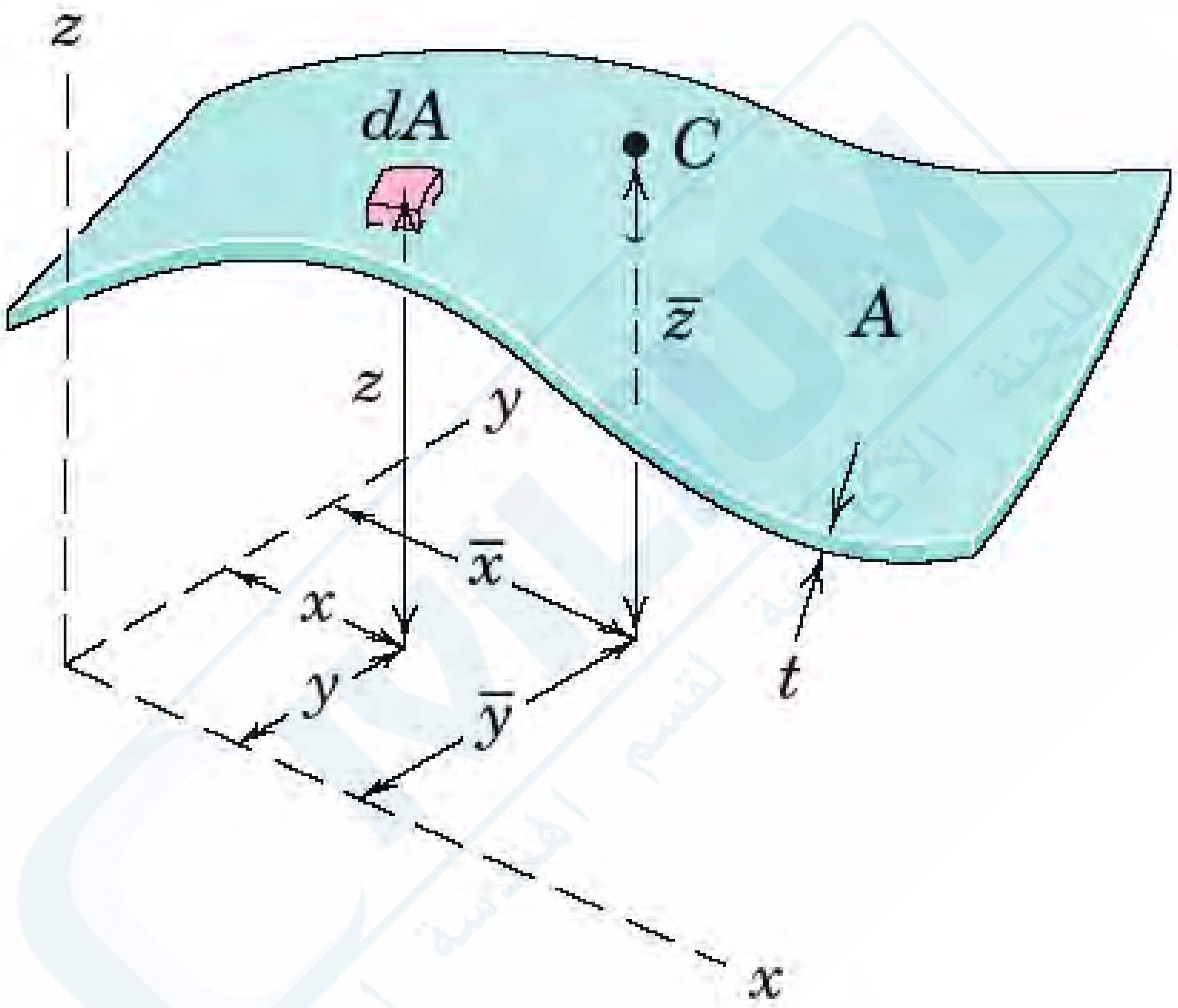
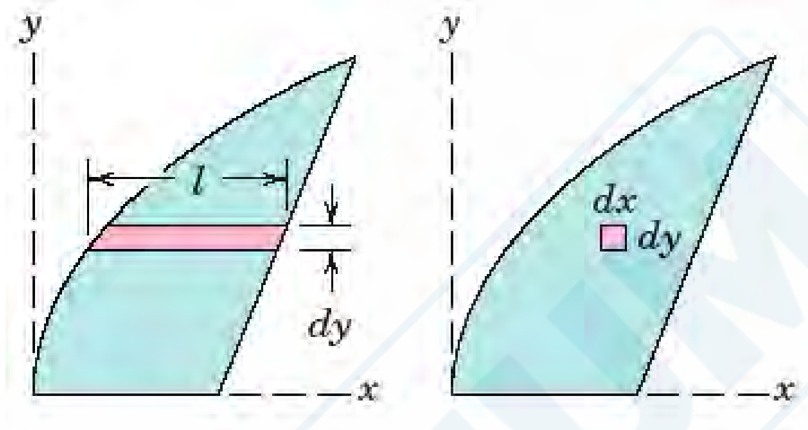
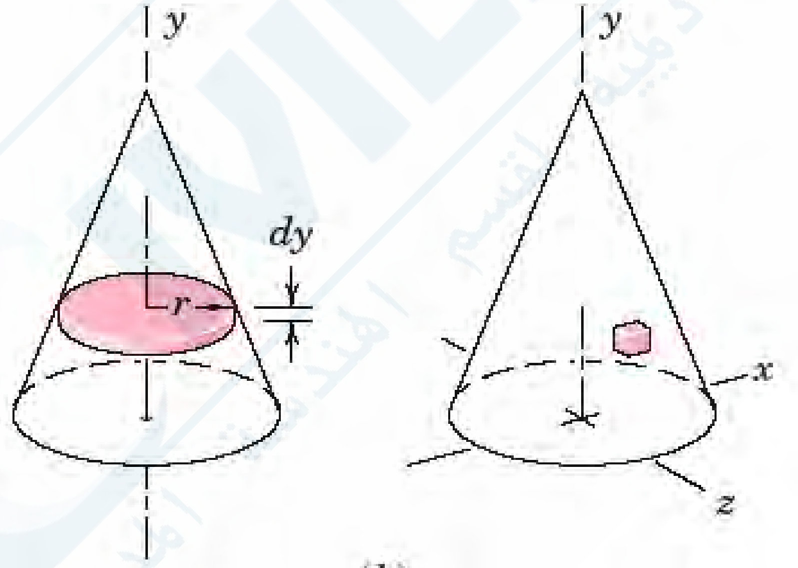


Figure 5/7



(a)



(b)

Figure 5/8

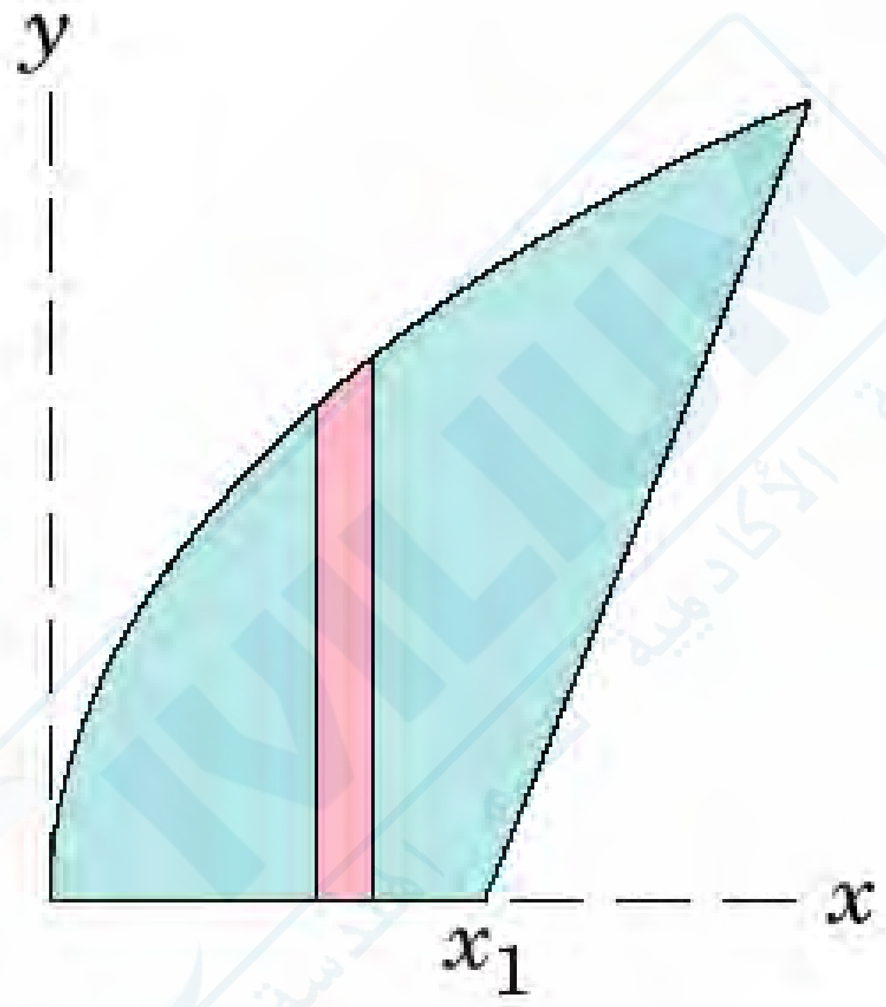


Figure 5/9

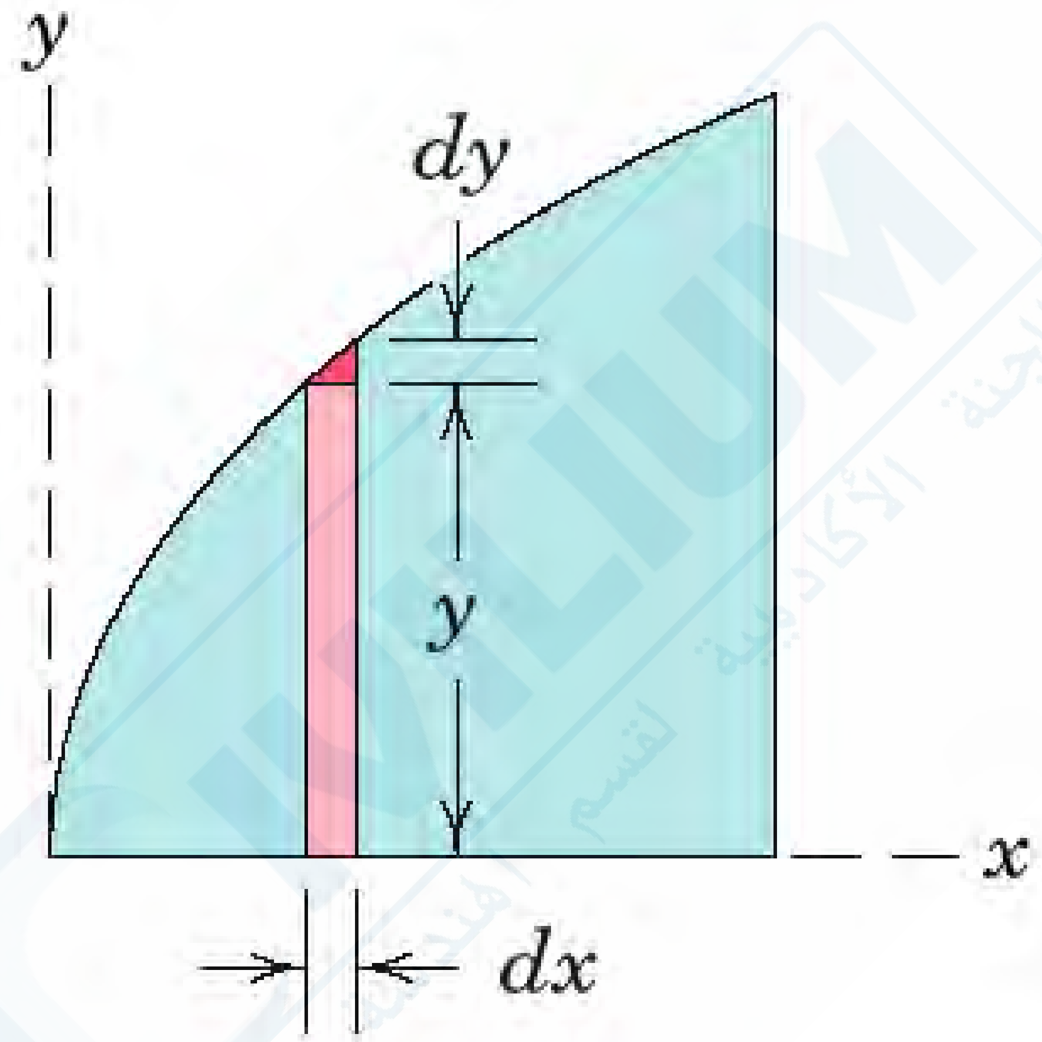


Figure 5/10

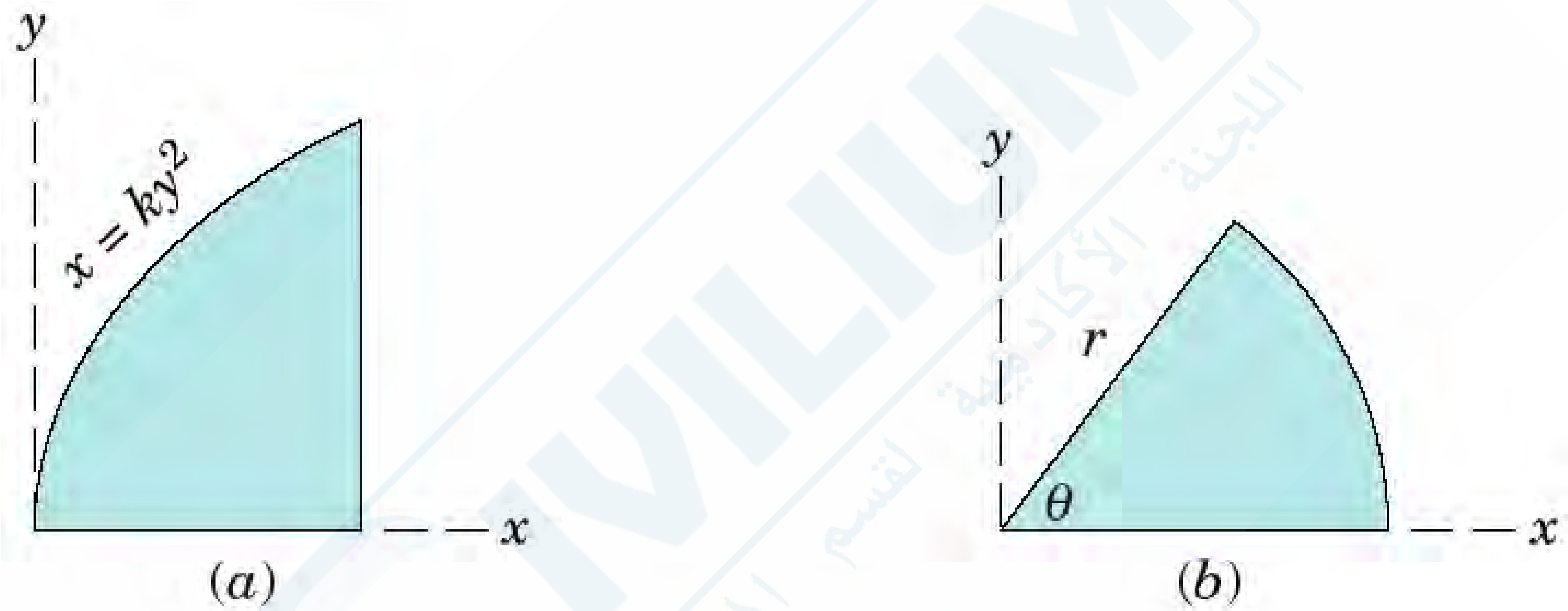


Figure 5/11

05_11

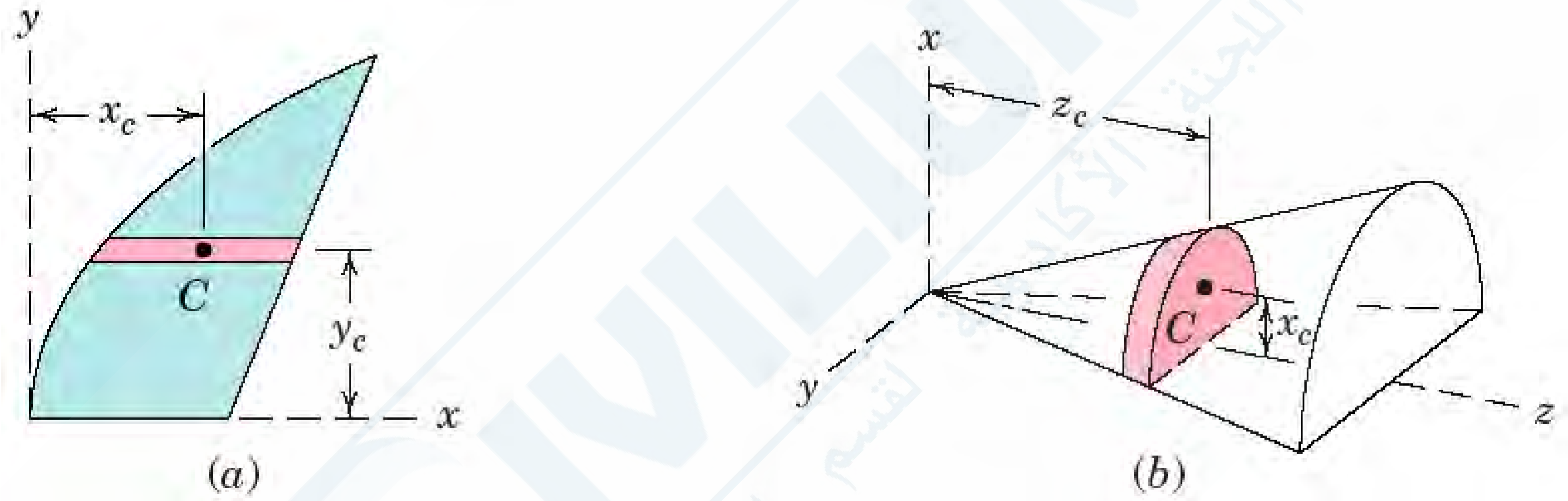


Figure 5/12

05_12

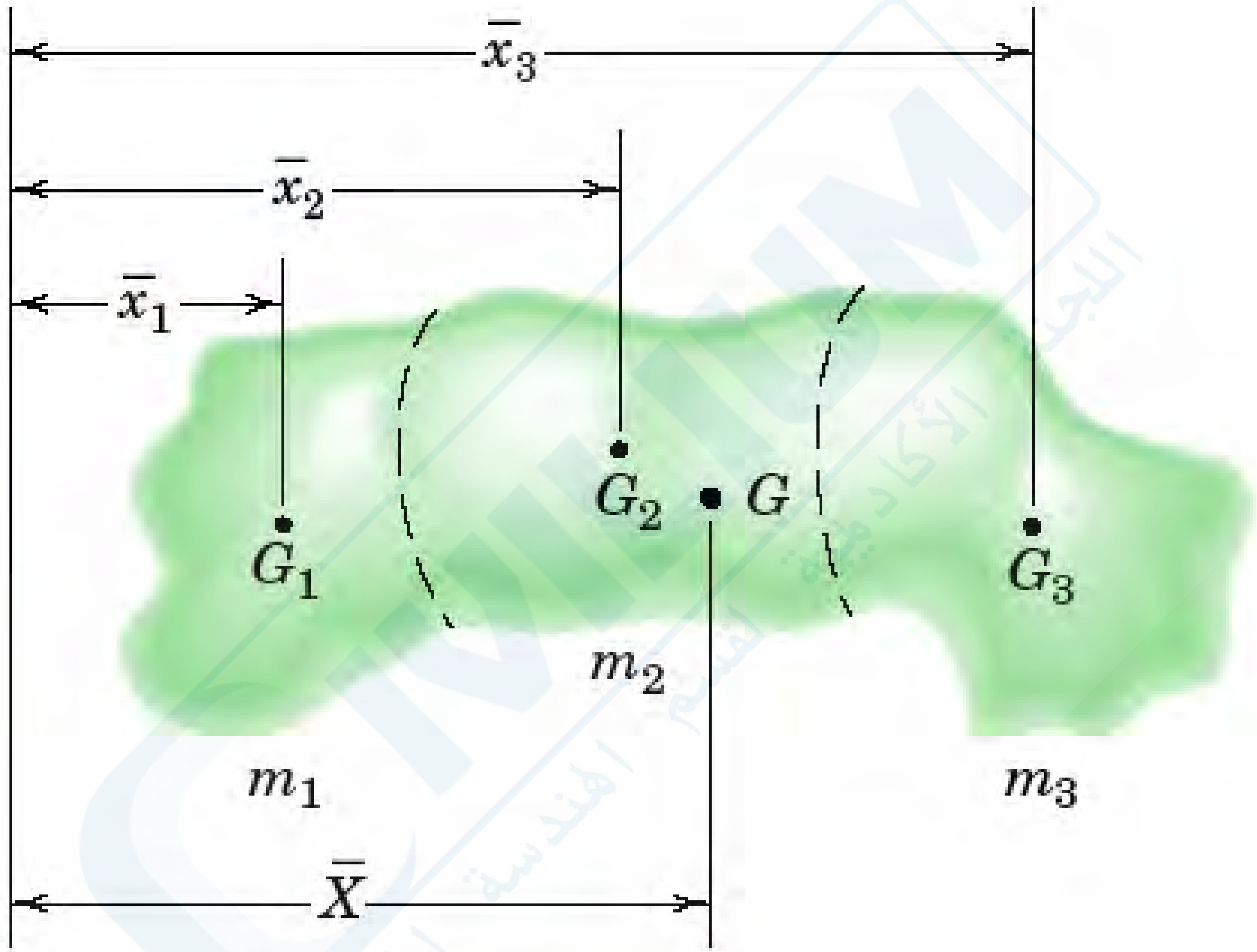


Figure 5/13

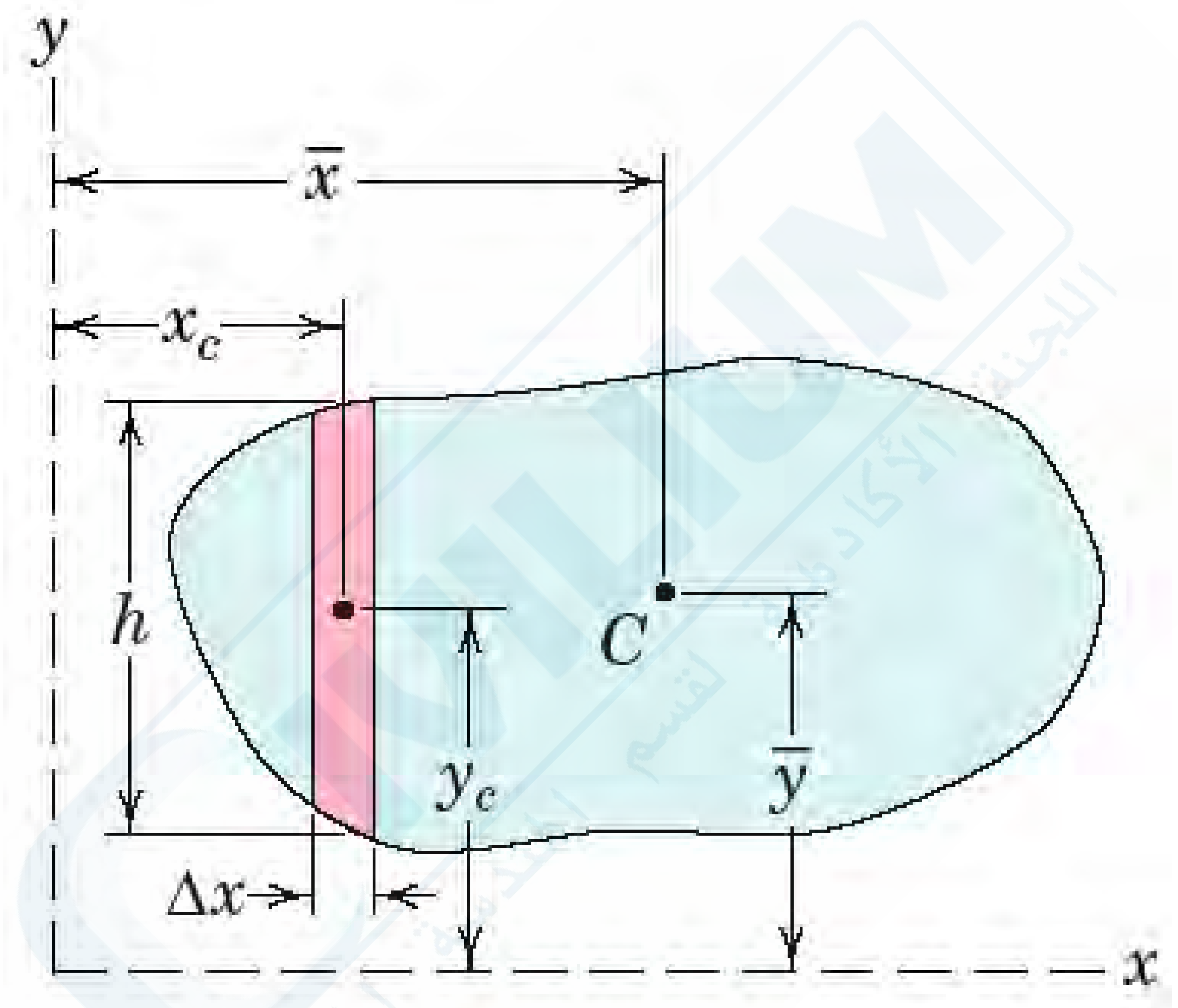


Figure 5/14

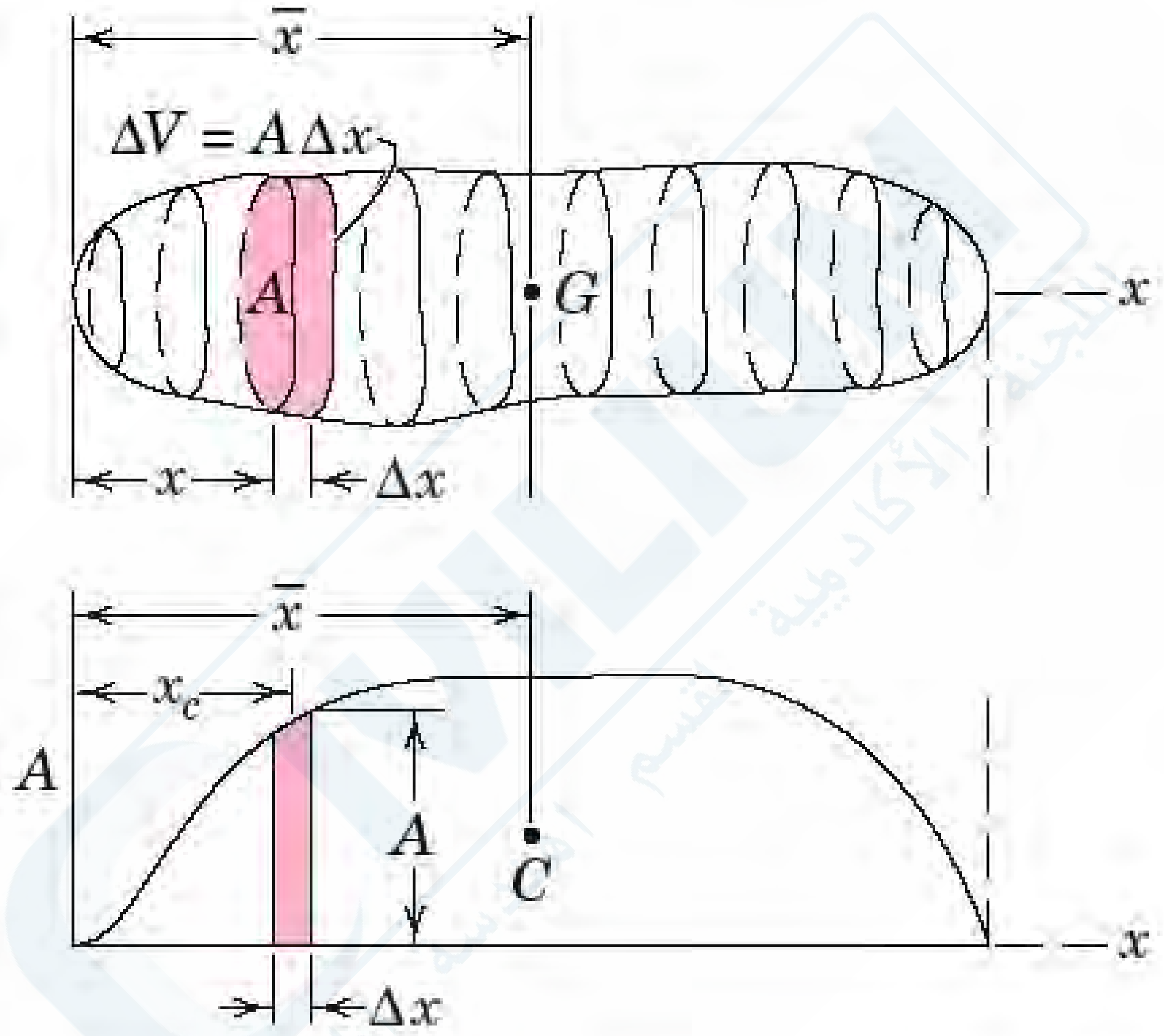


Figure 5/15

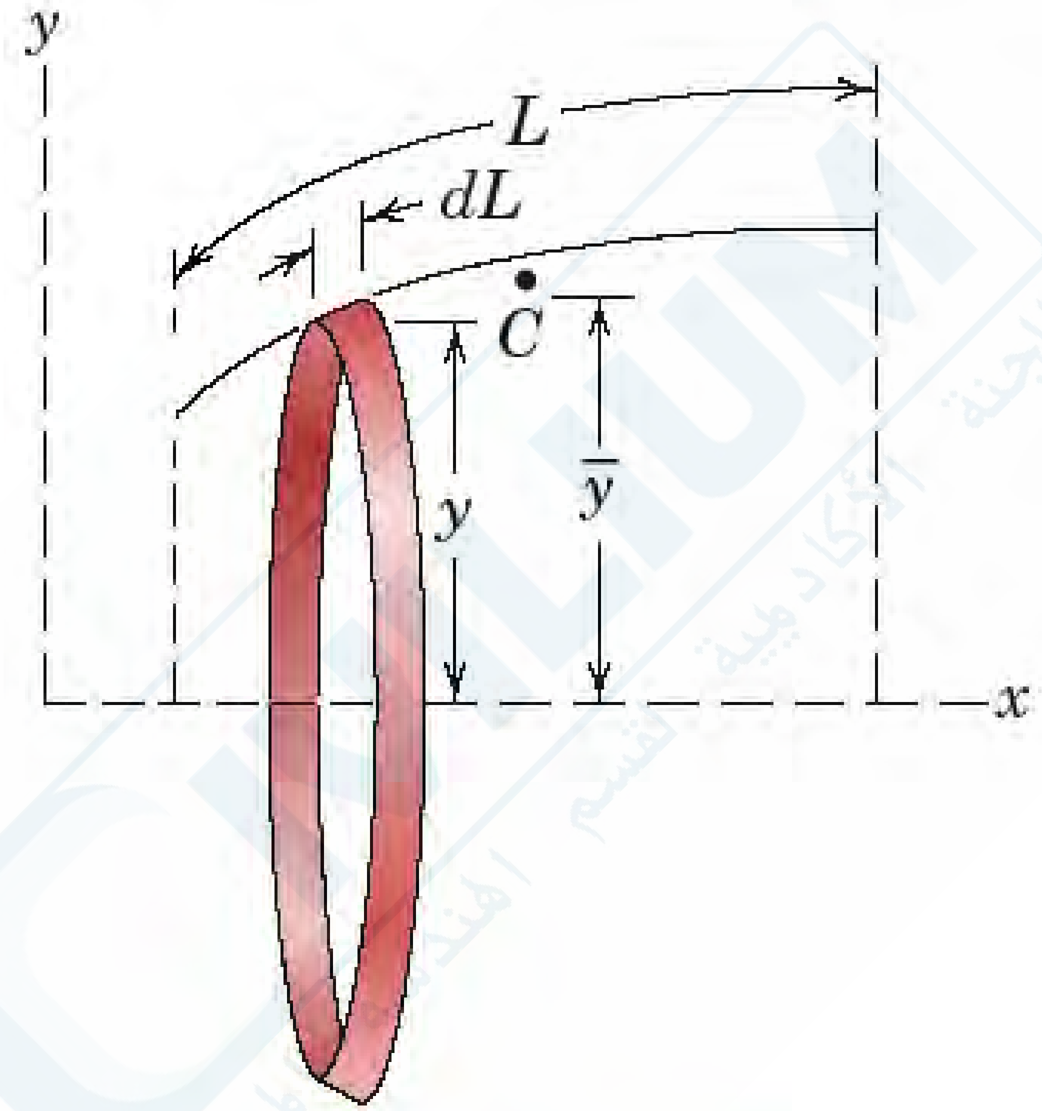


Figure 5/16

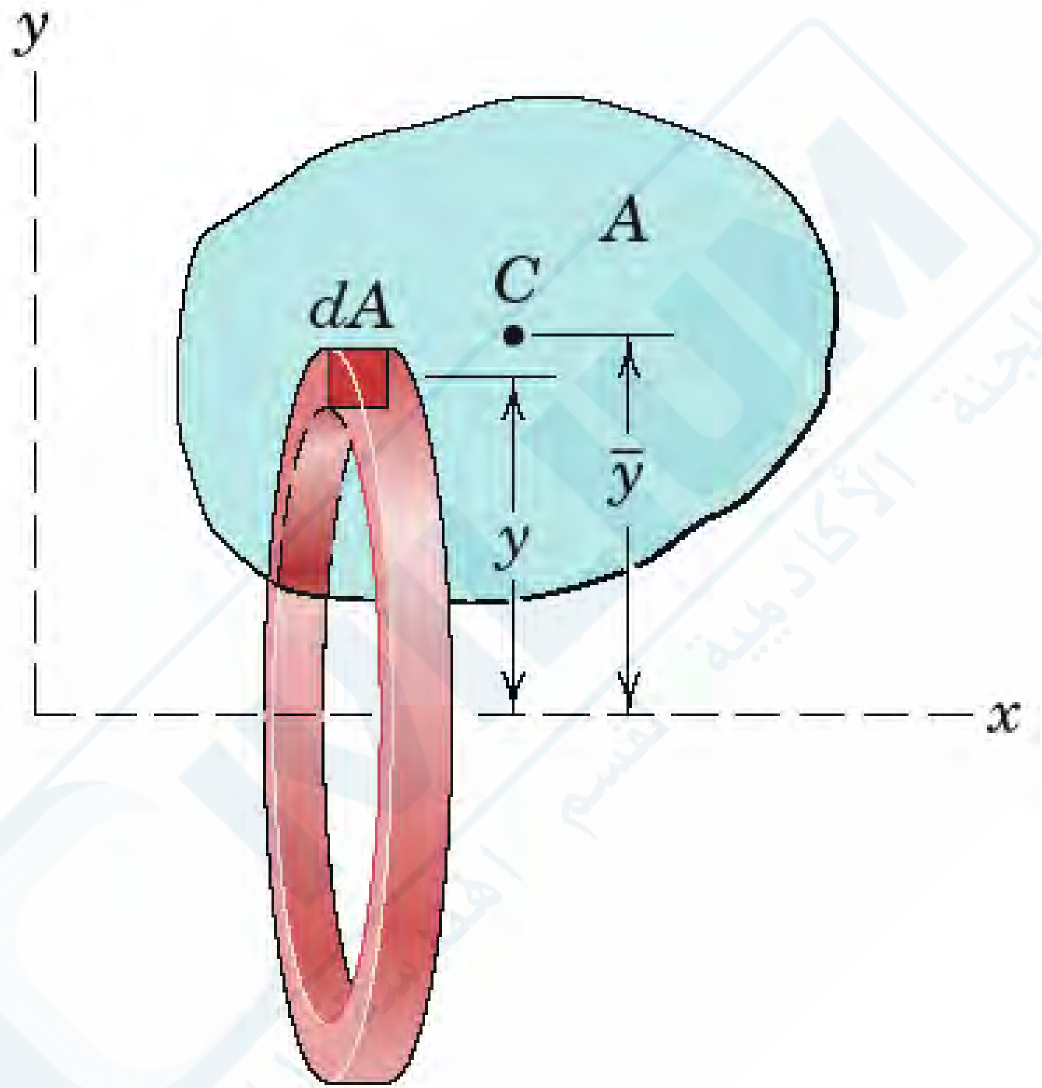


Figure 5/17

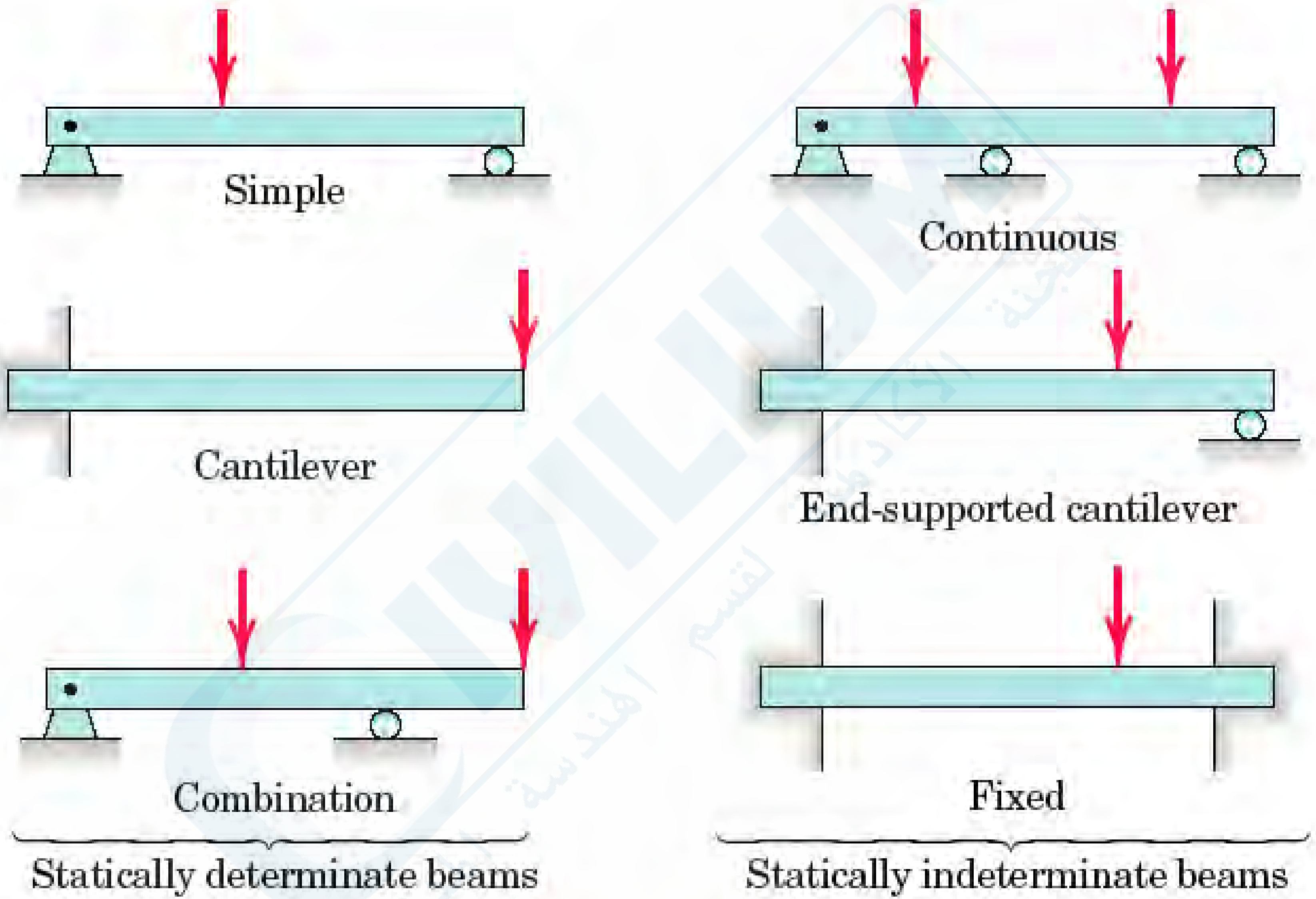

Simple
Continuous
Cantilever
End-supported cantilever
Combination
Fixed
Statically determinate beams
Statically indeterminate beams

Figure 5/18

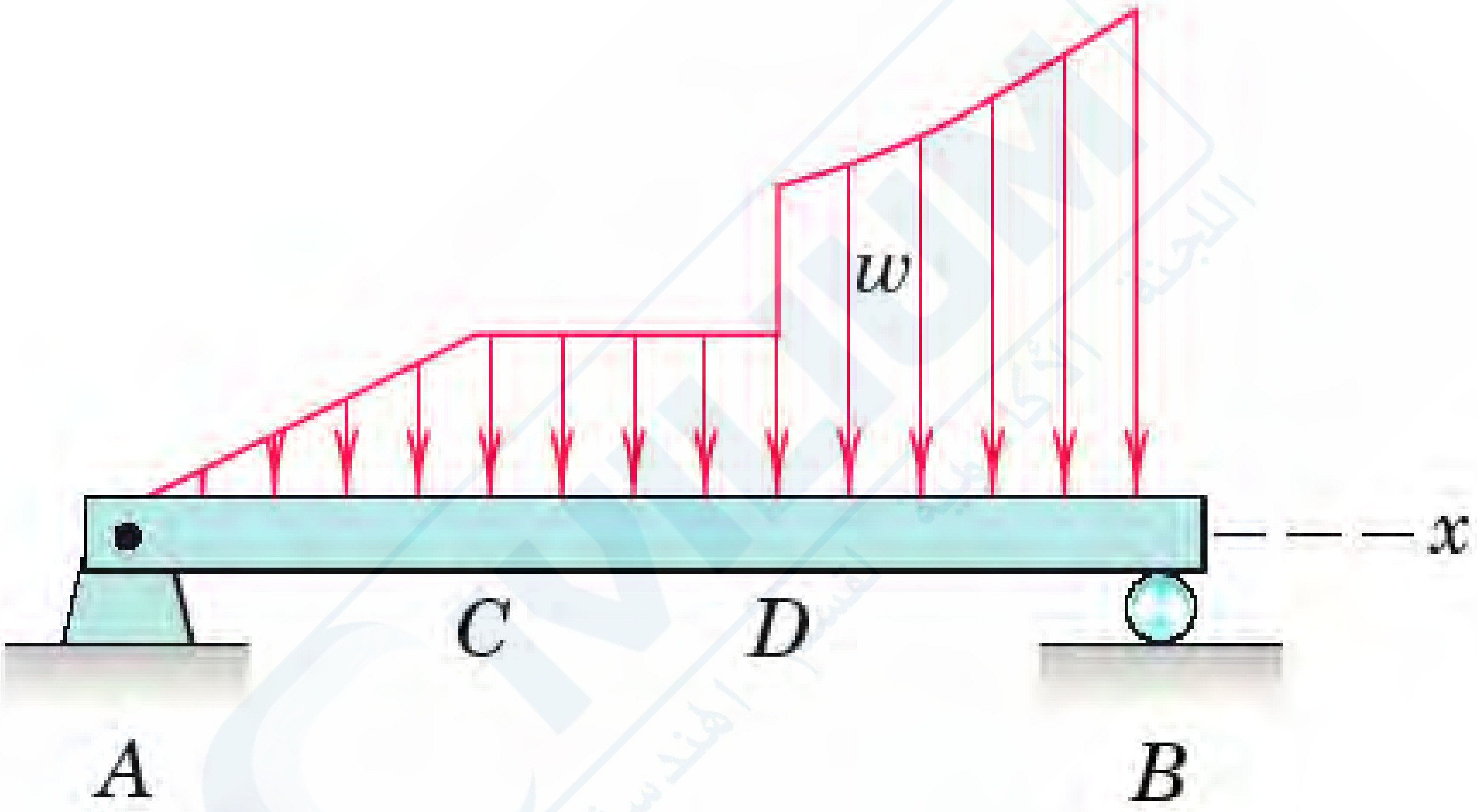


Figure 5/19

05_19

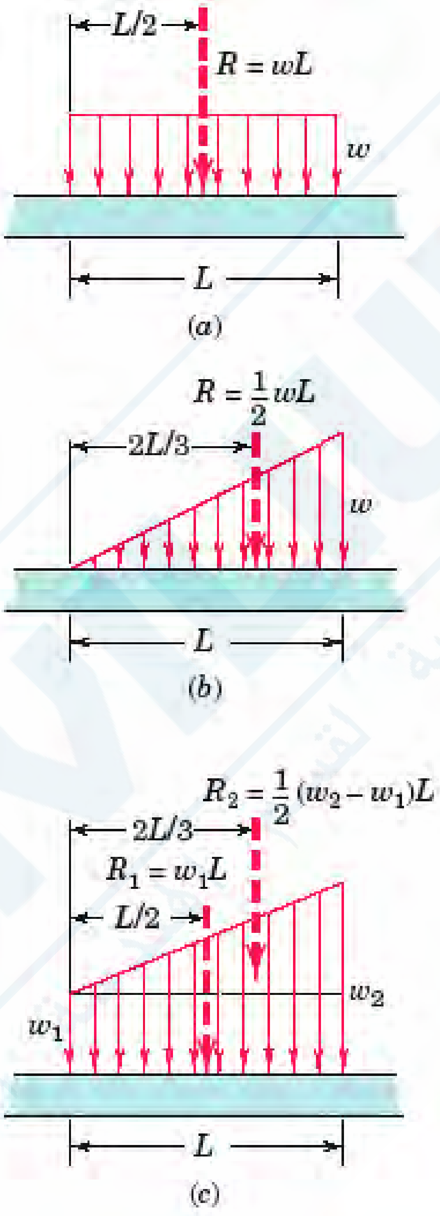


Figure 5/20

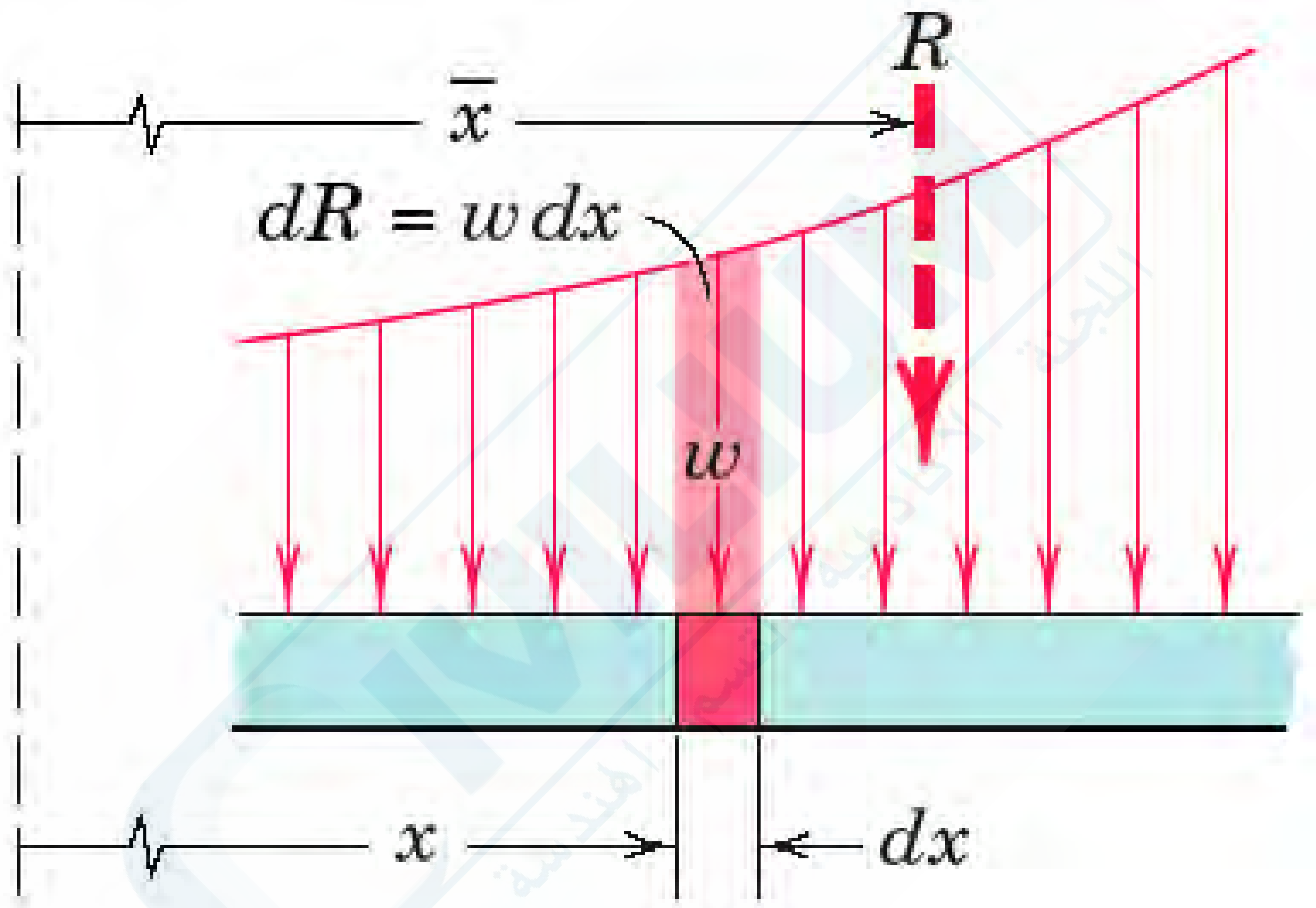


Figure 5/21

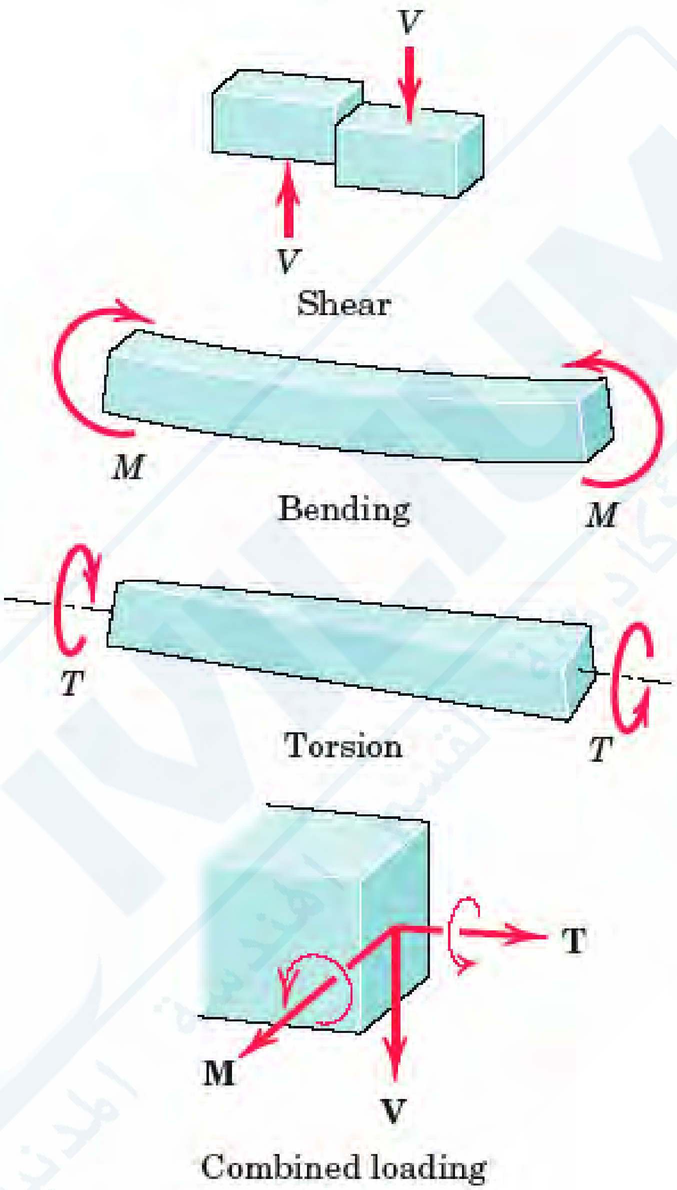


Figure 5/22

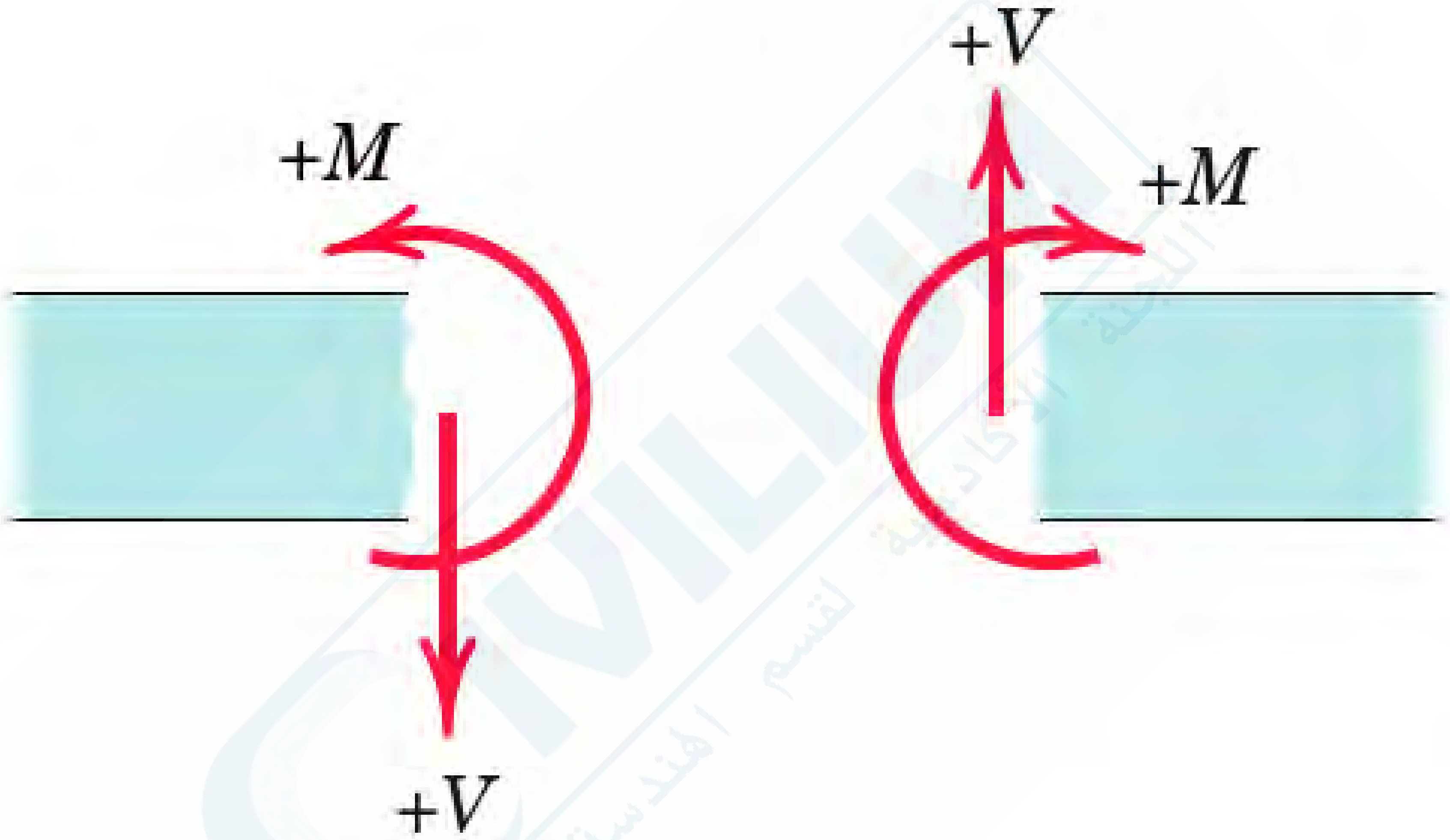


Figure 5/23

05_23

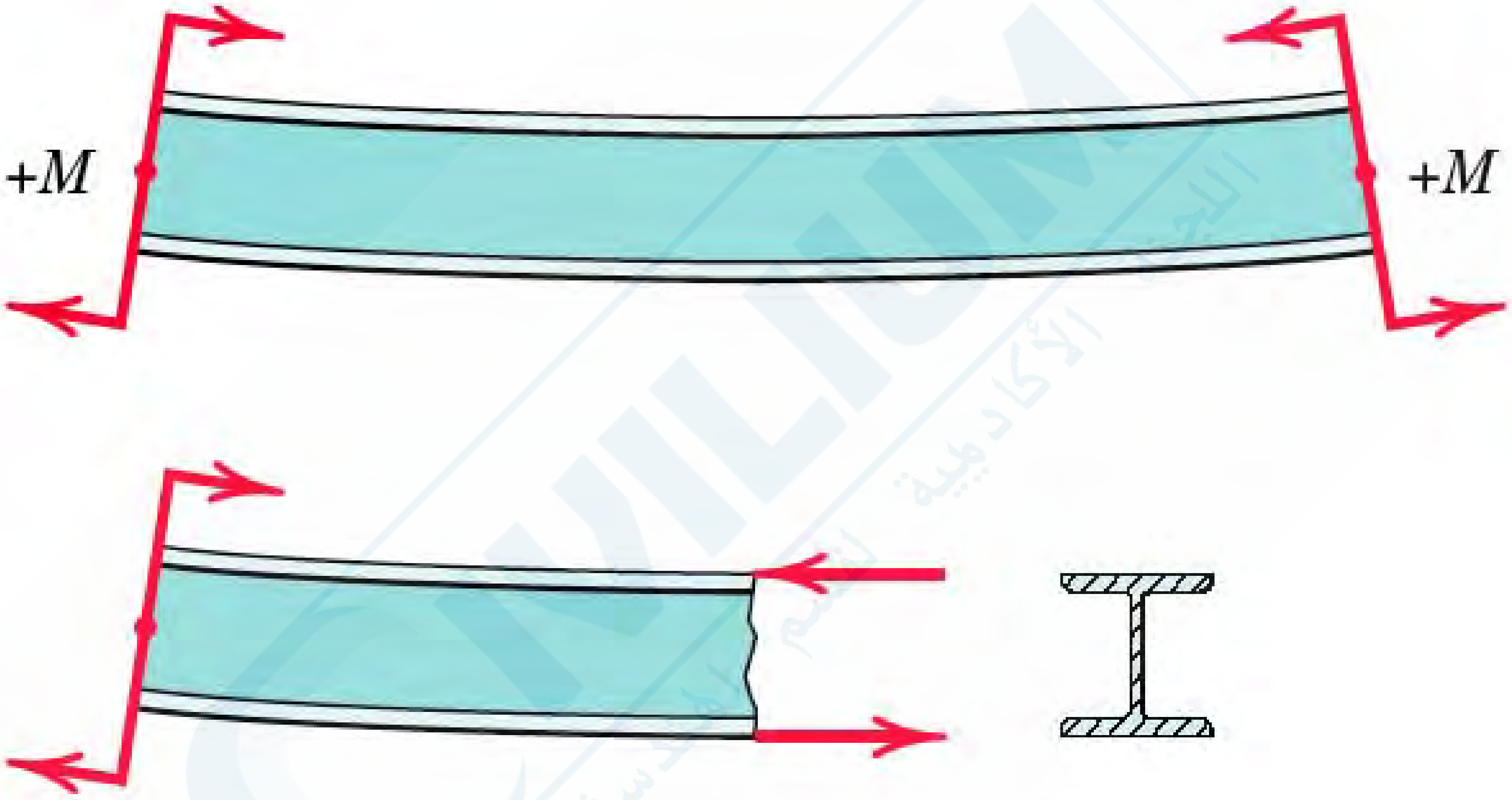


Figure 5/24

05_24

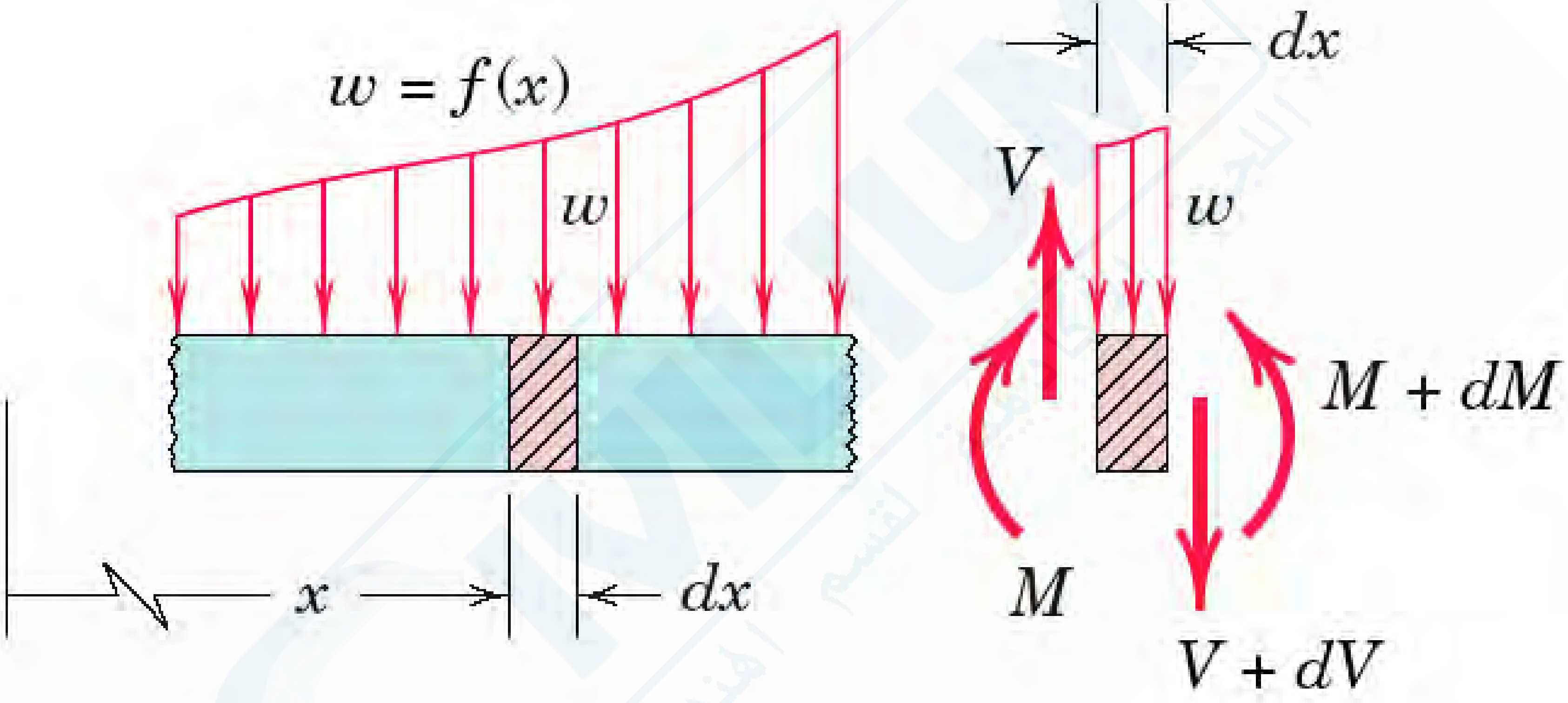


Figure 5/25

05_25

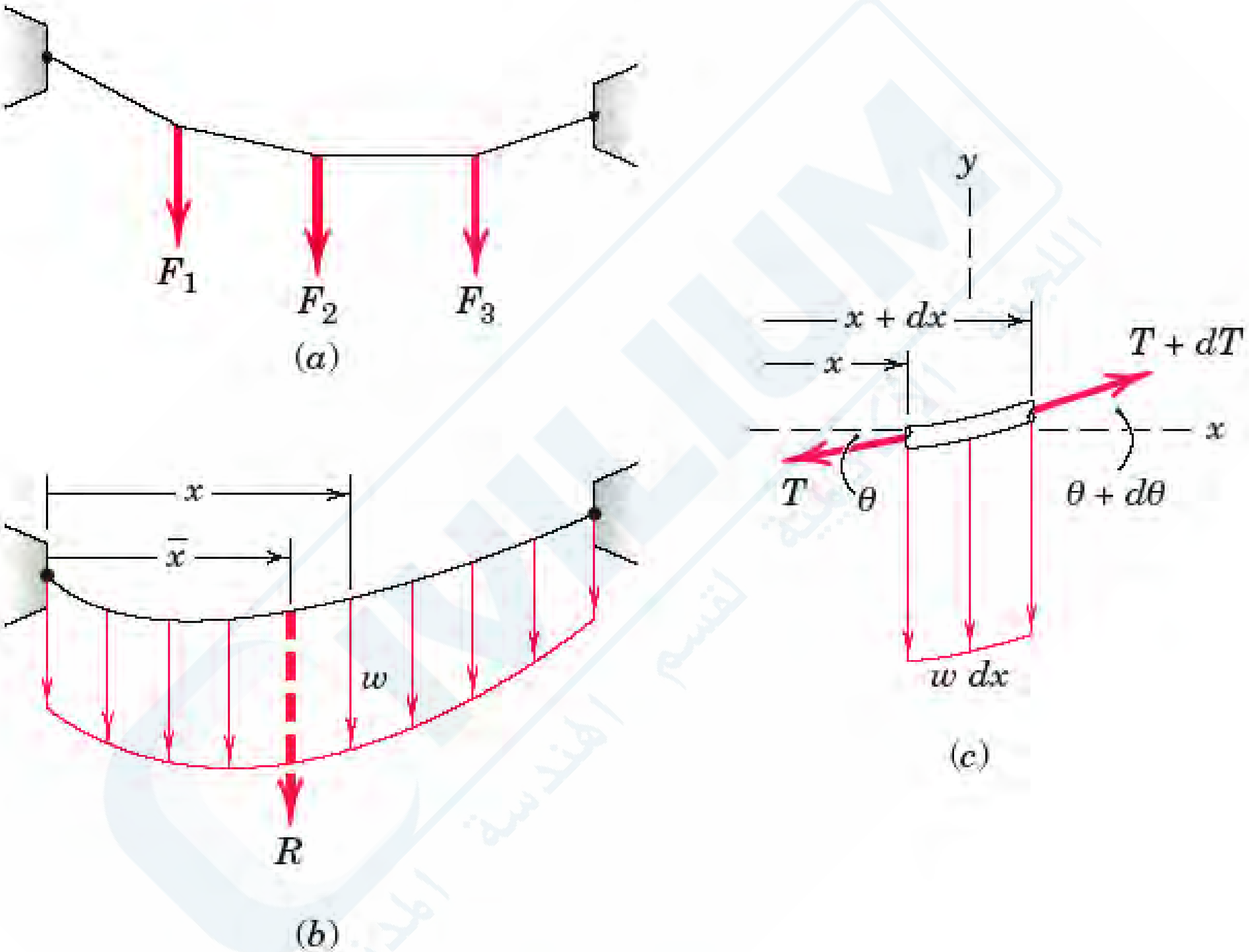


Figure 5/26

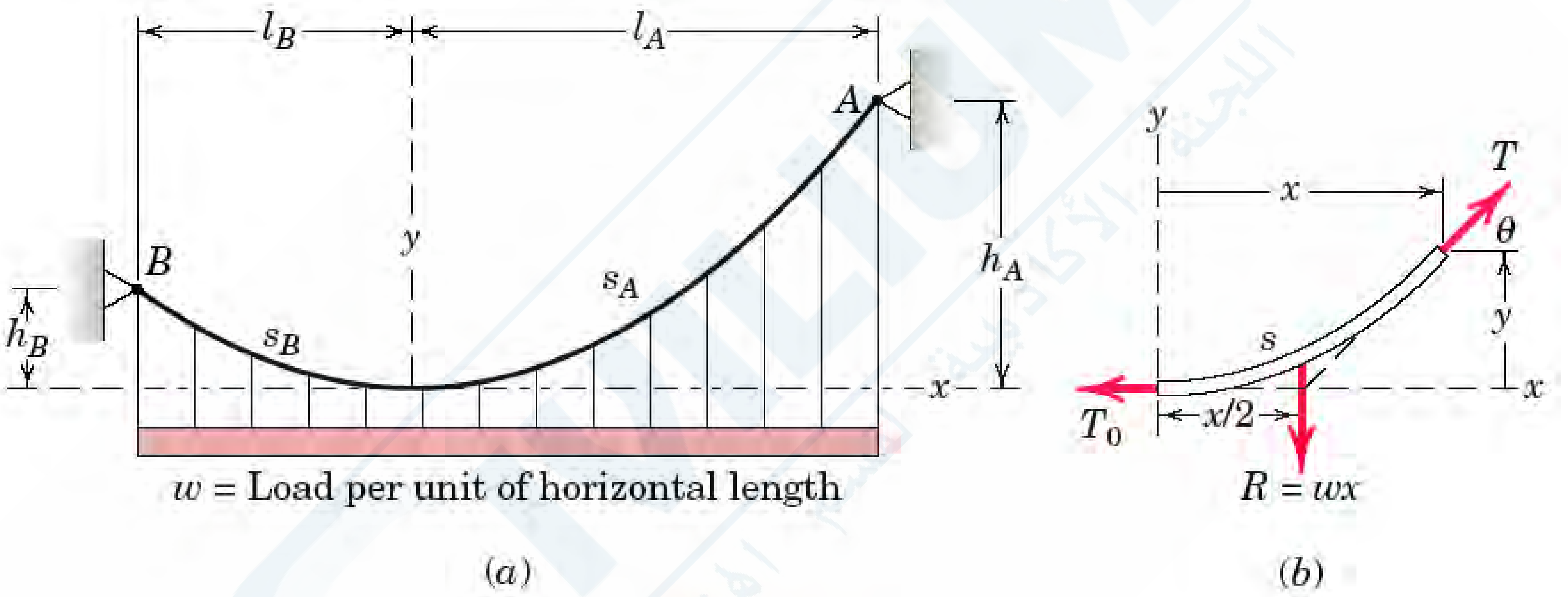


Figure 5/27

05_27

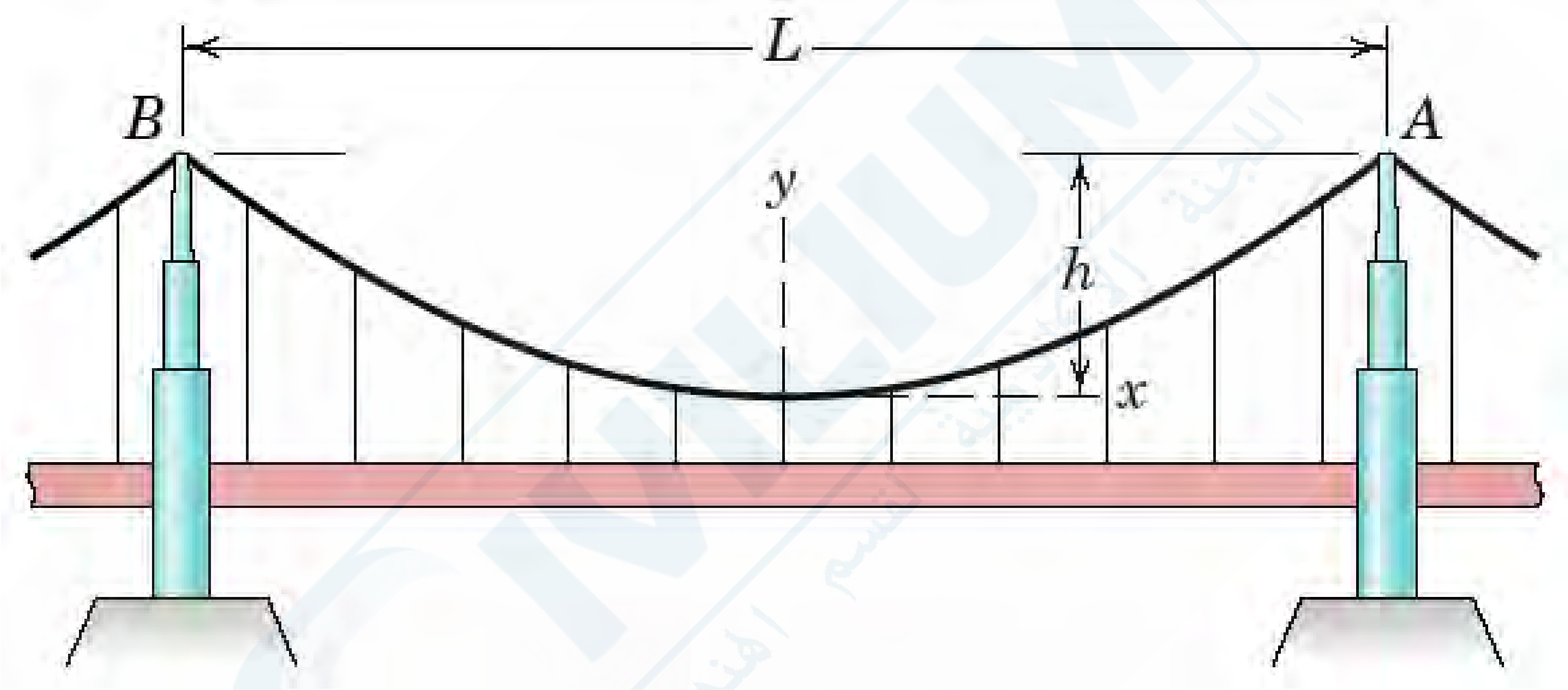


Figure 5/28

05_28

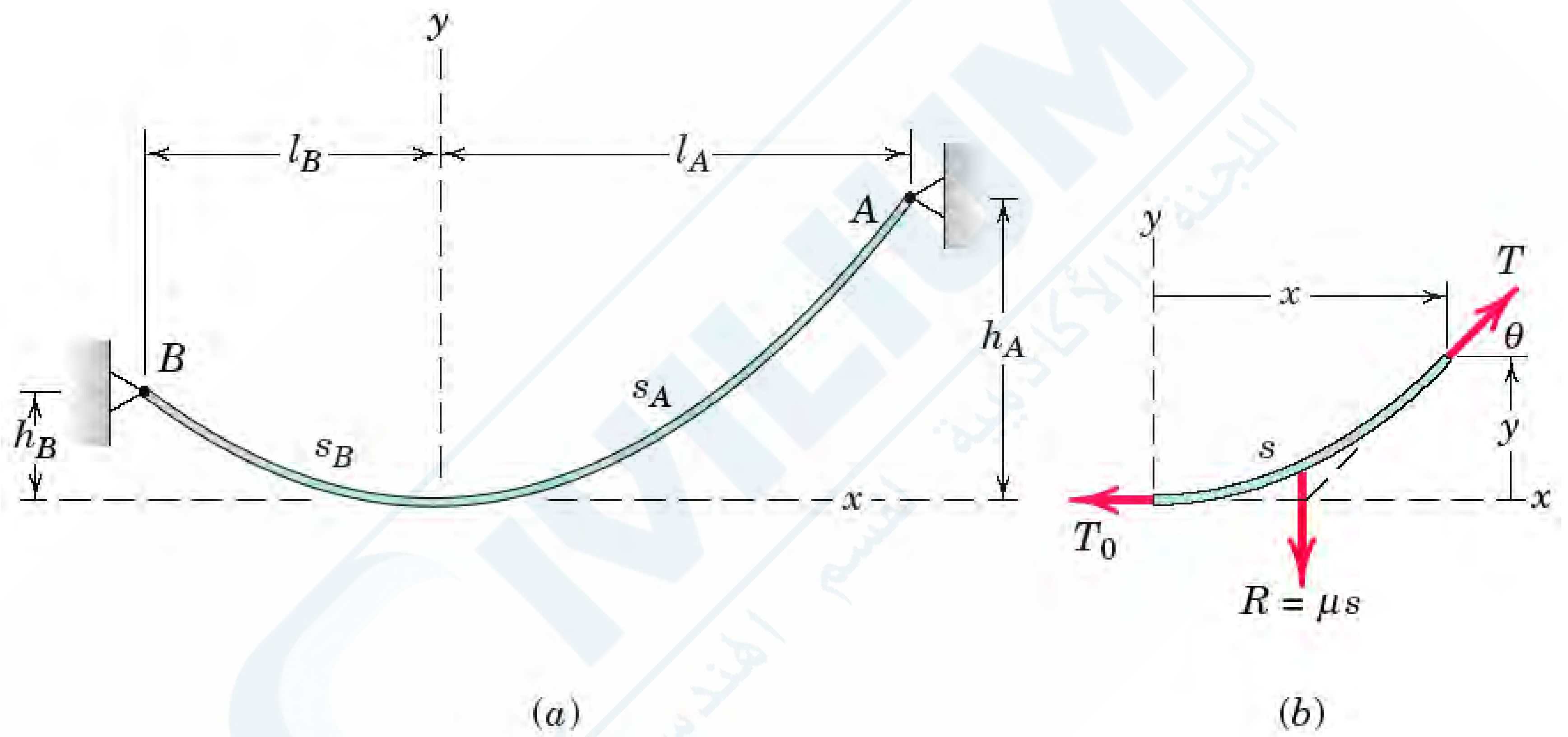


Figure 5/29

05_29

CIVILIUM

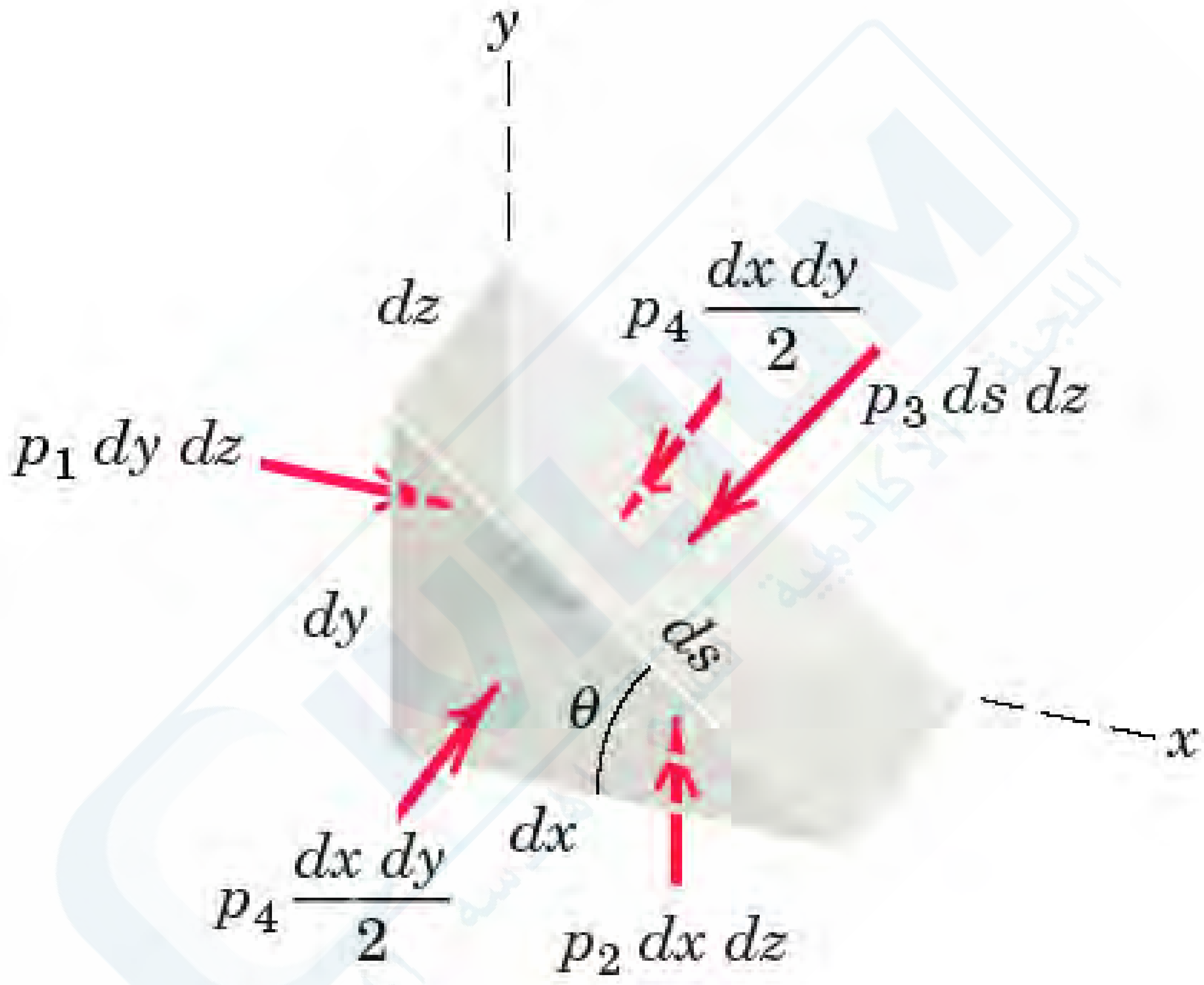

y
dz
$p_4 \frac{dx\,dy}{2}$
$p_3\,ds\,dz$
$p_1\,dy\,dz$
dy
ds
θ
x
dx
$p_4 \frac{dx\,dy}{2}$
$p_2\,dx\,dz$

Figure 5/30

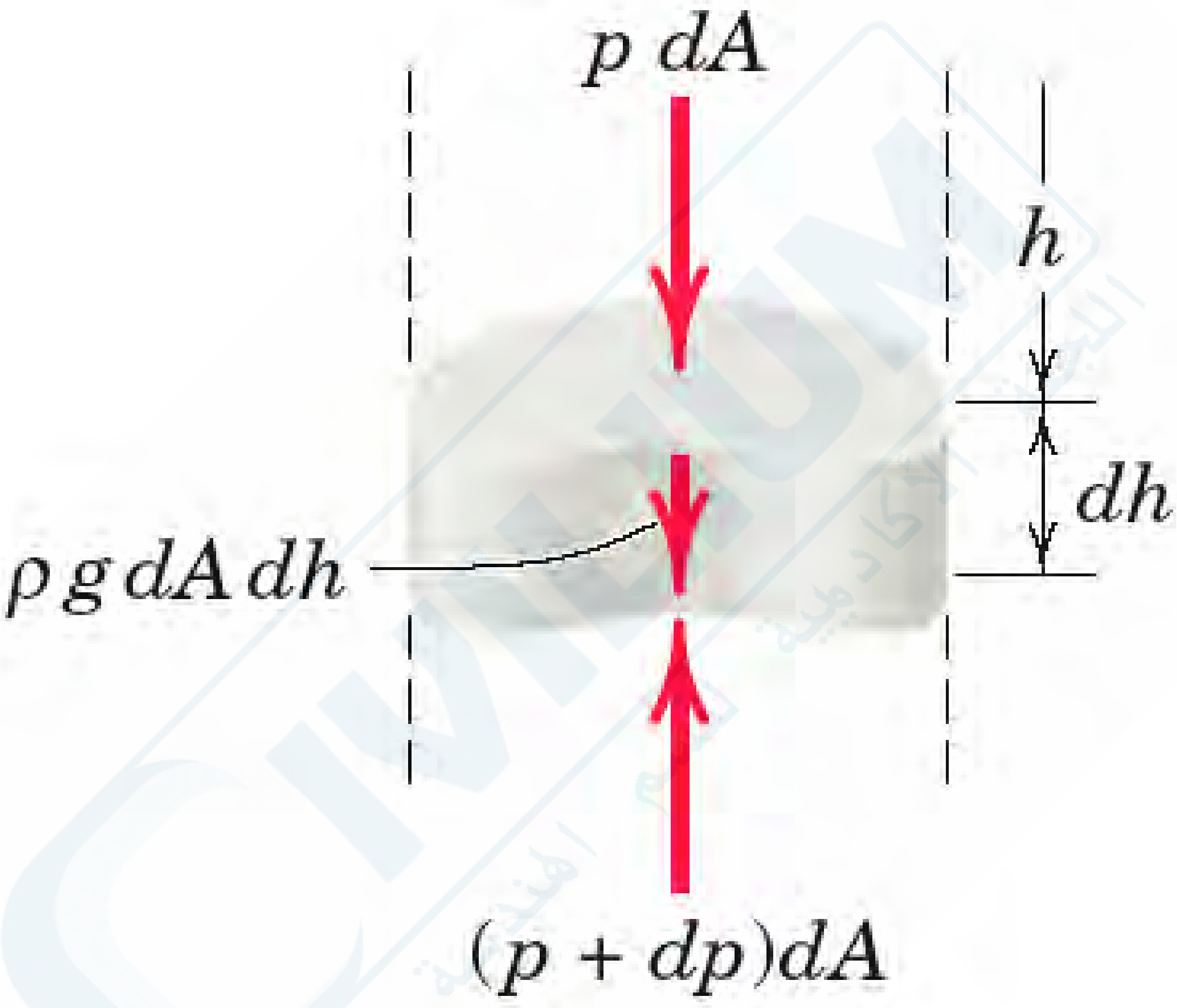


Figure 5/31

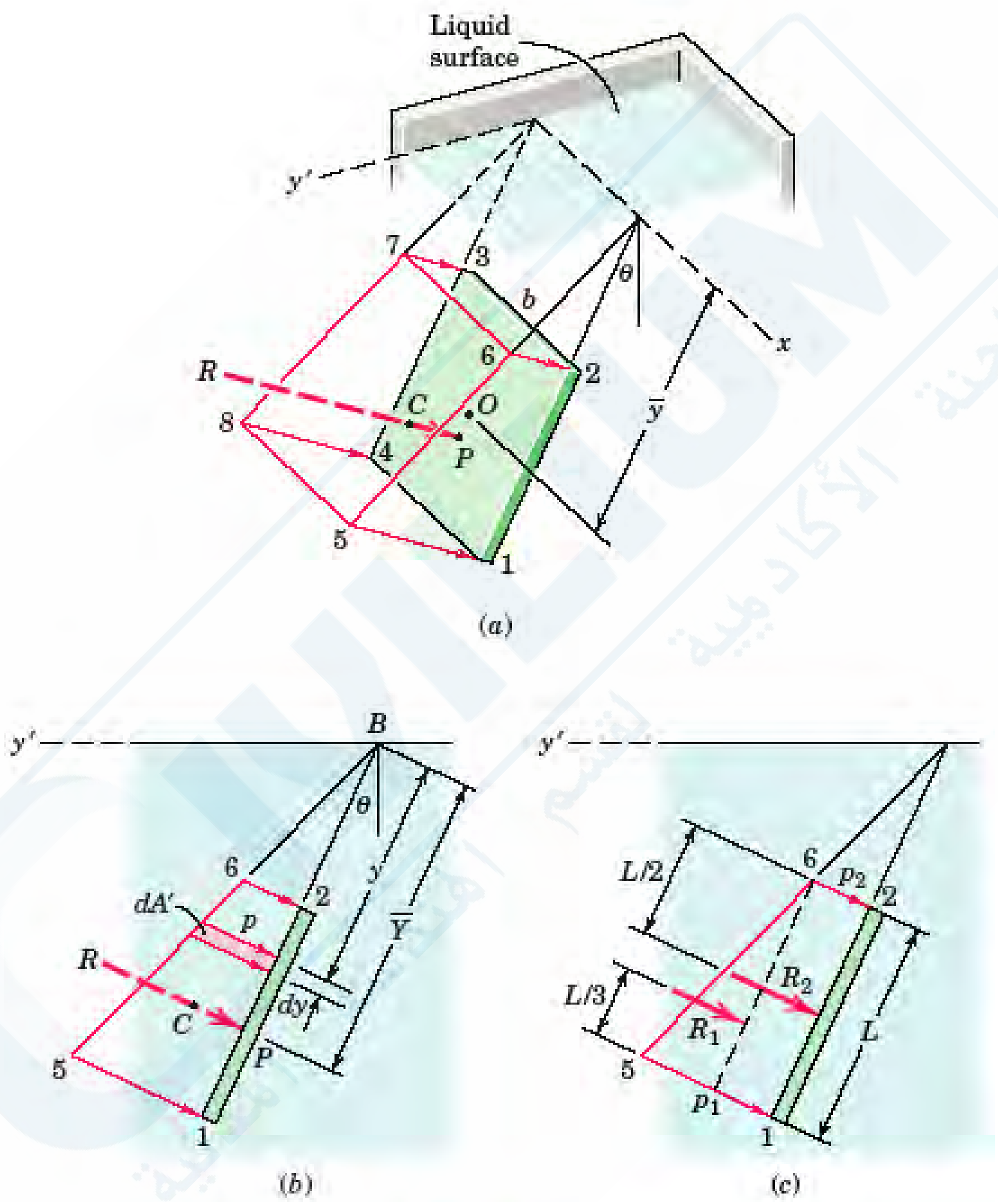


Figure 5/32

05_32

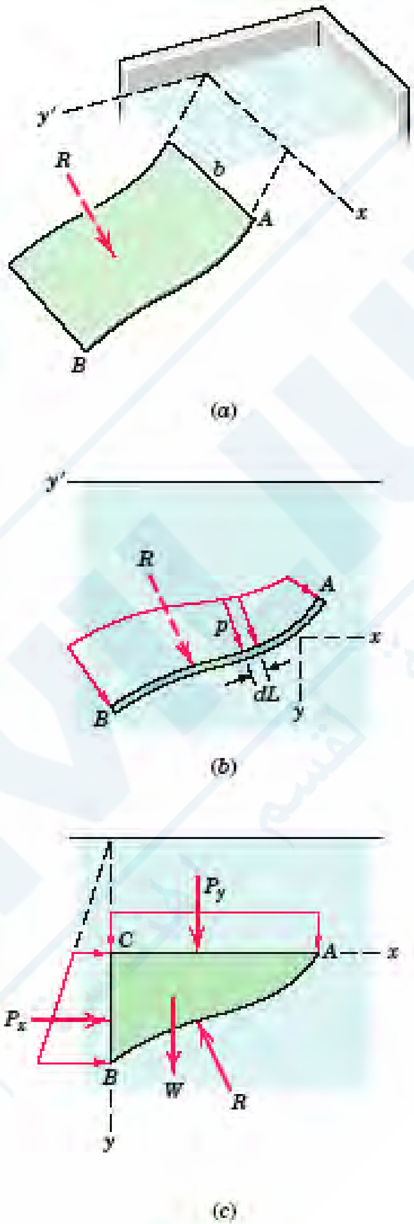


Figure 5/33

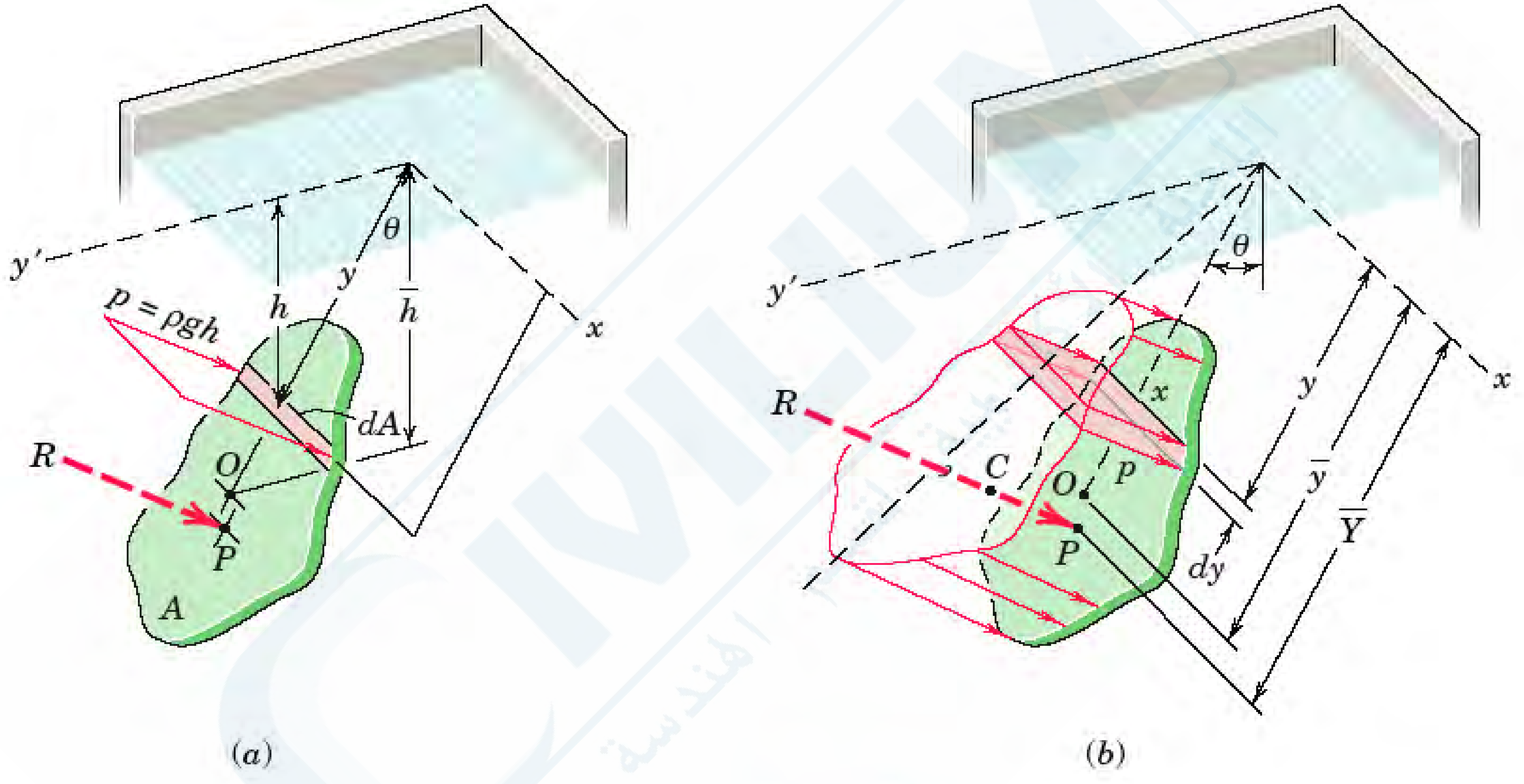


Figure 5/34

05_34

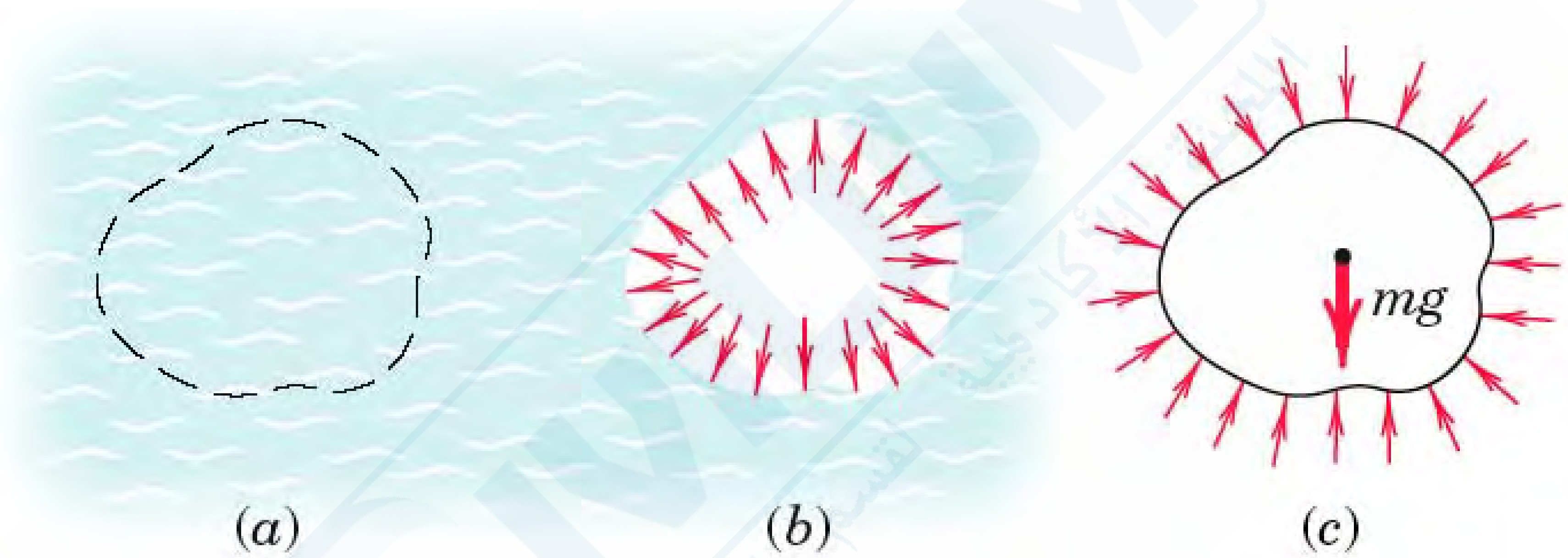


Figure 5/35

05_35

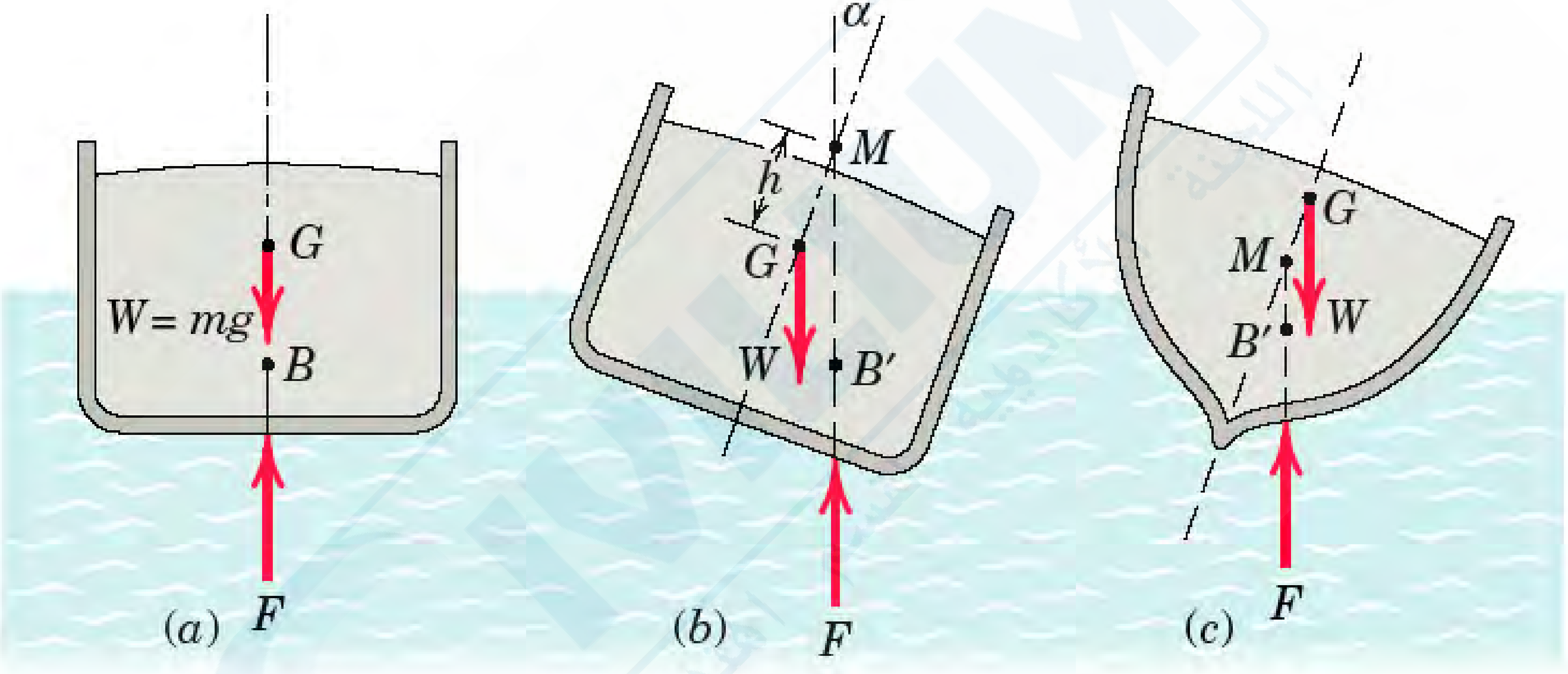
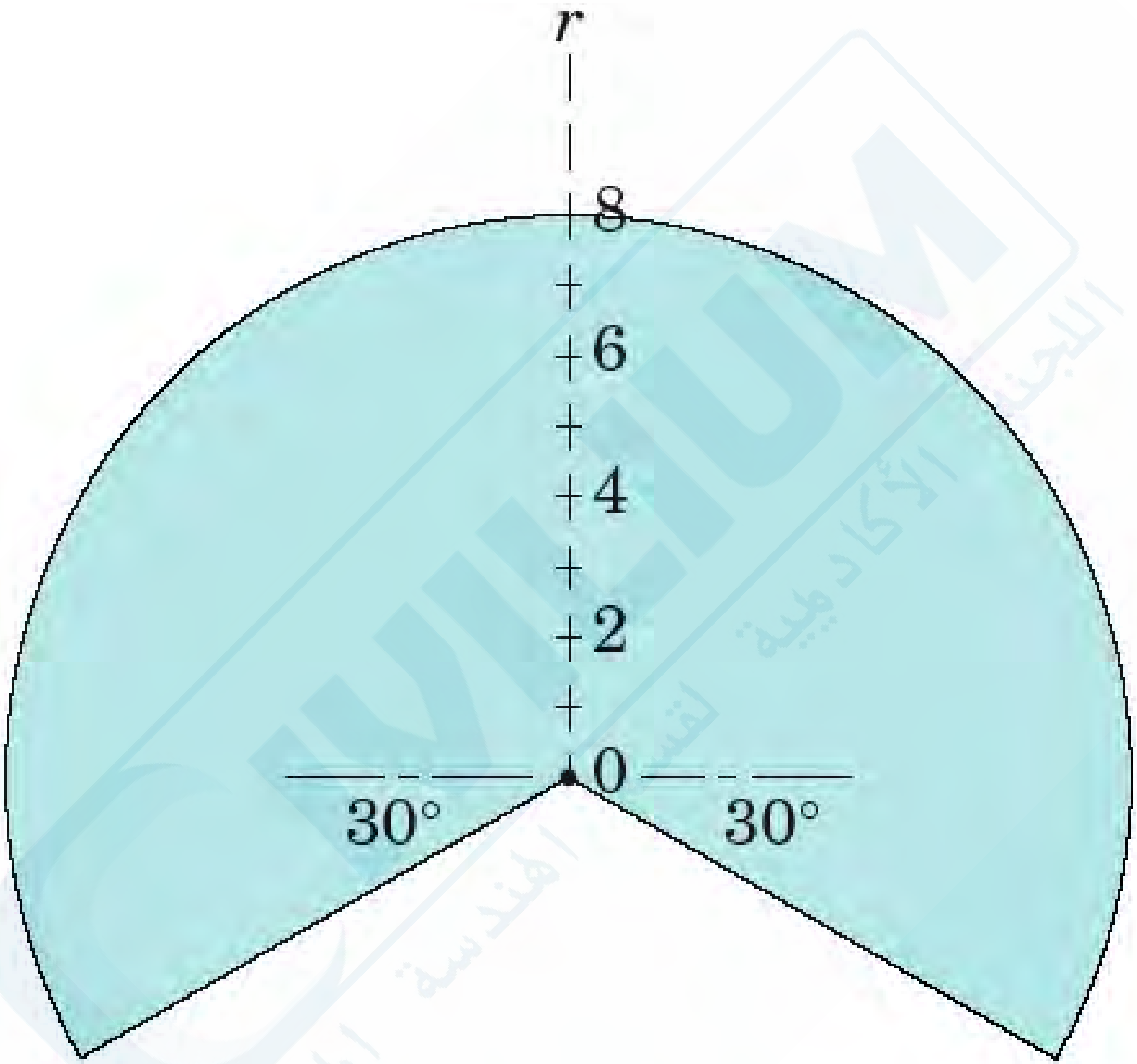


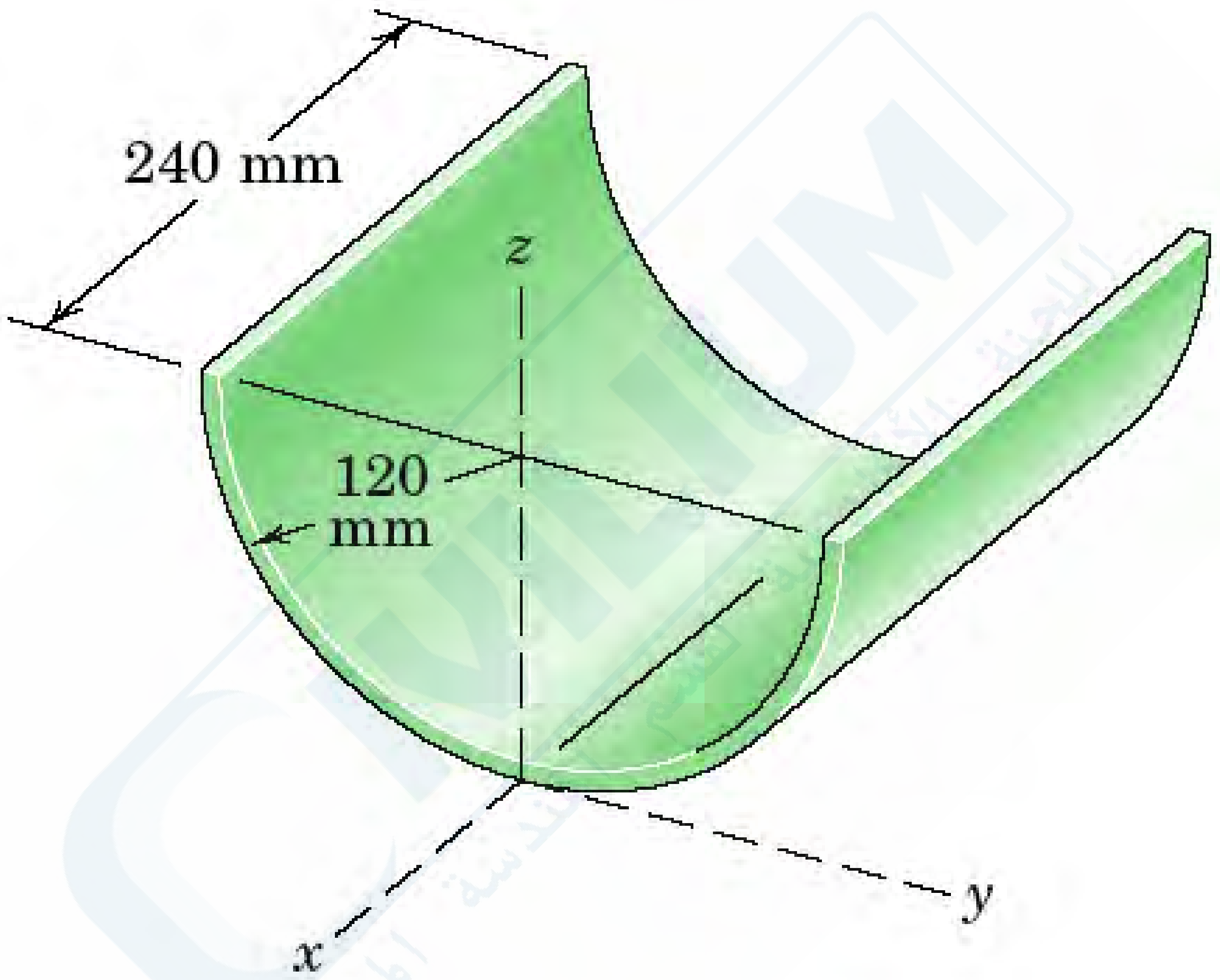
Figure 5/36

05_36

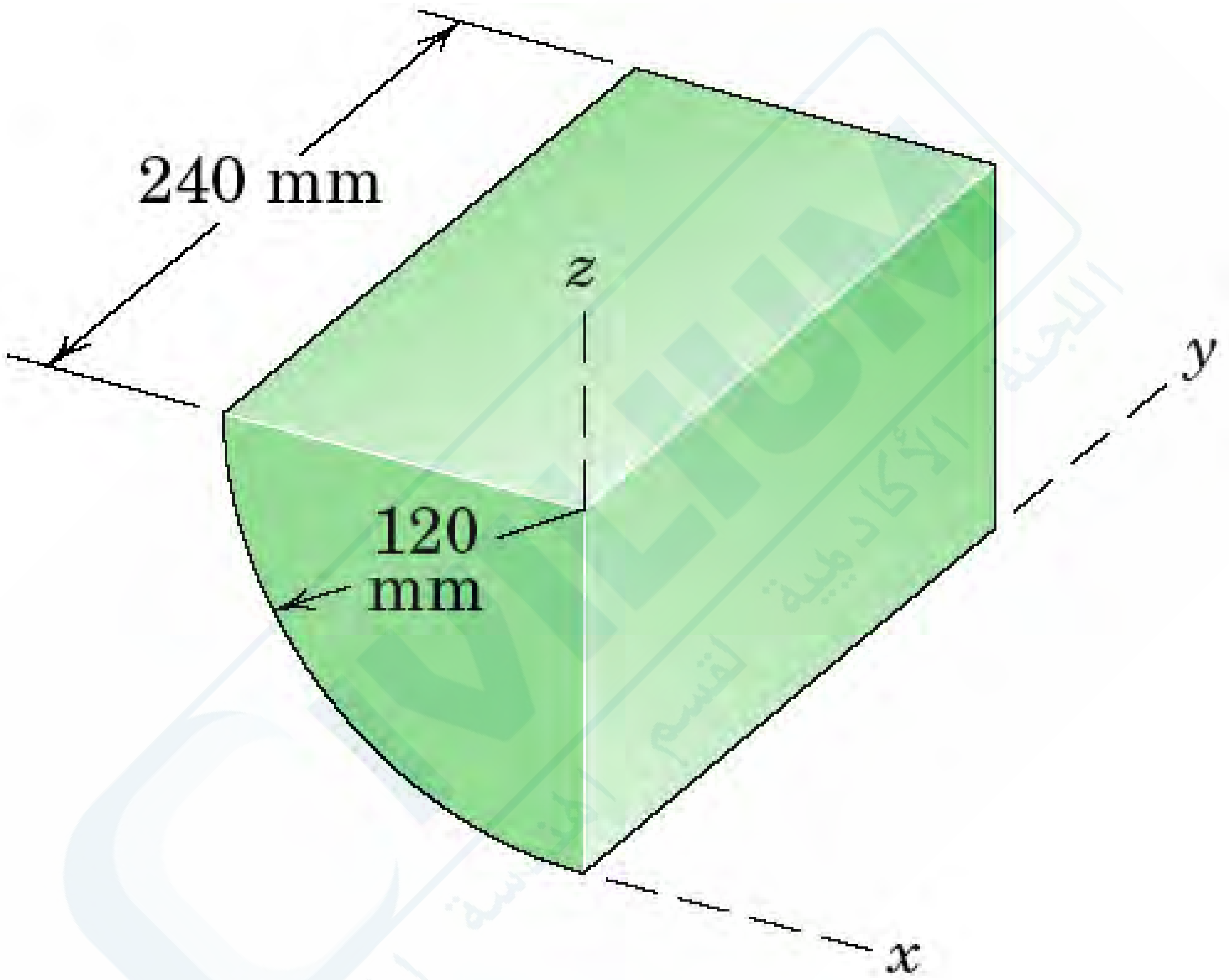


Problem 5/2

P_05_02

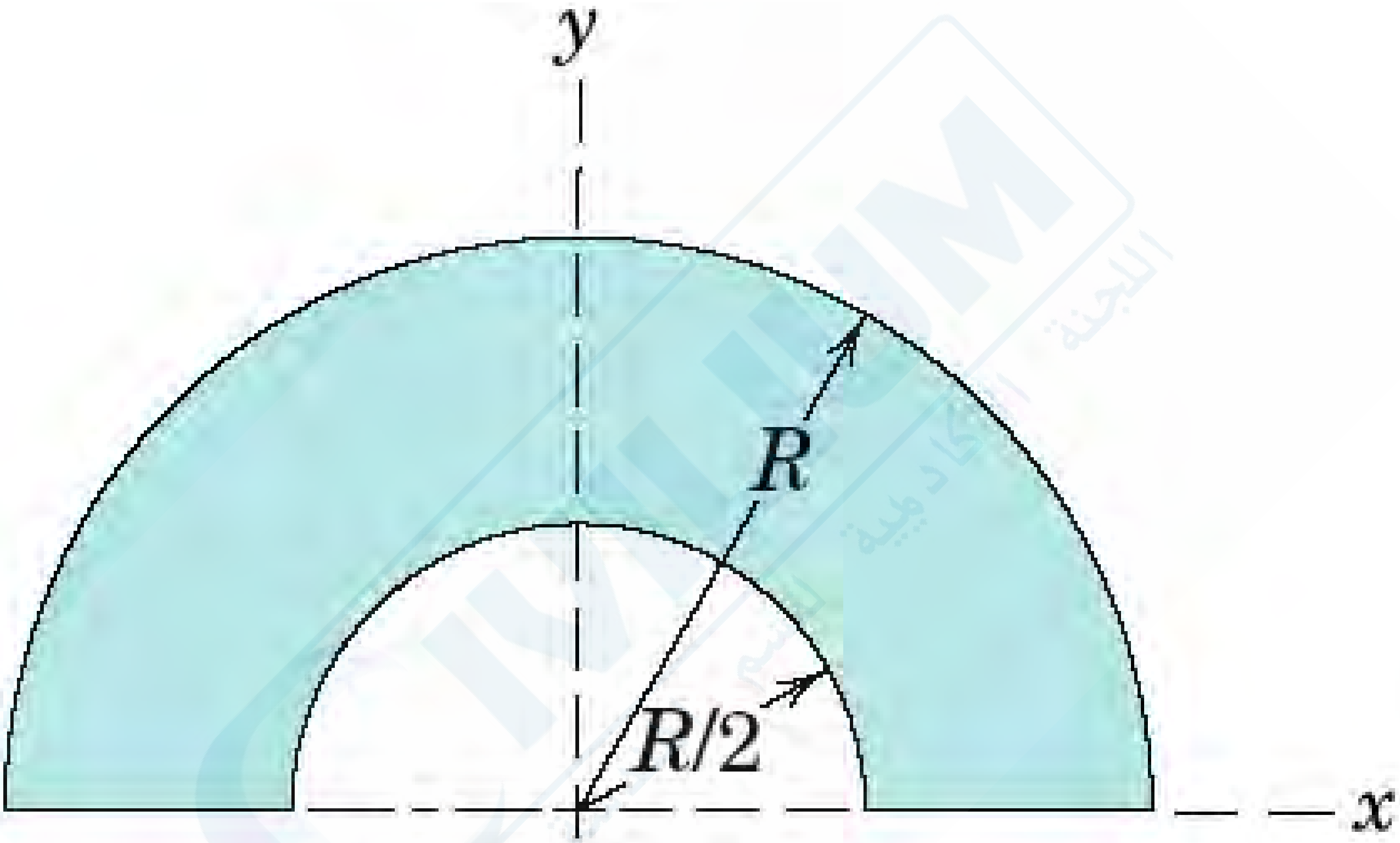


Problem 5/3



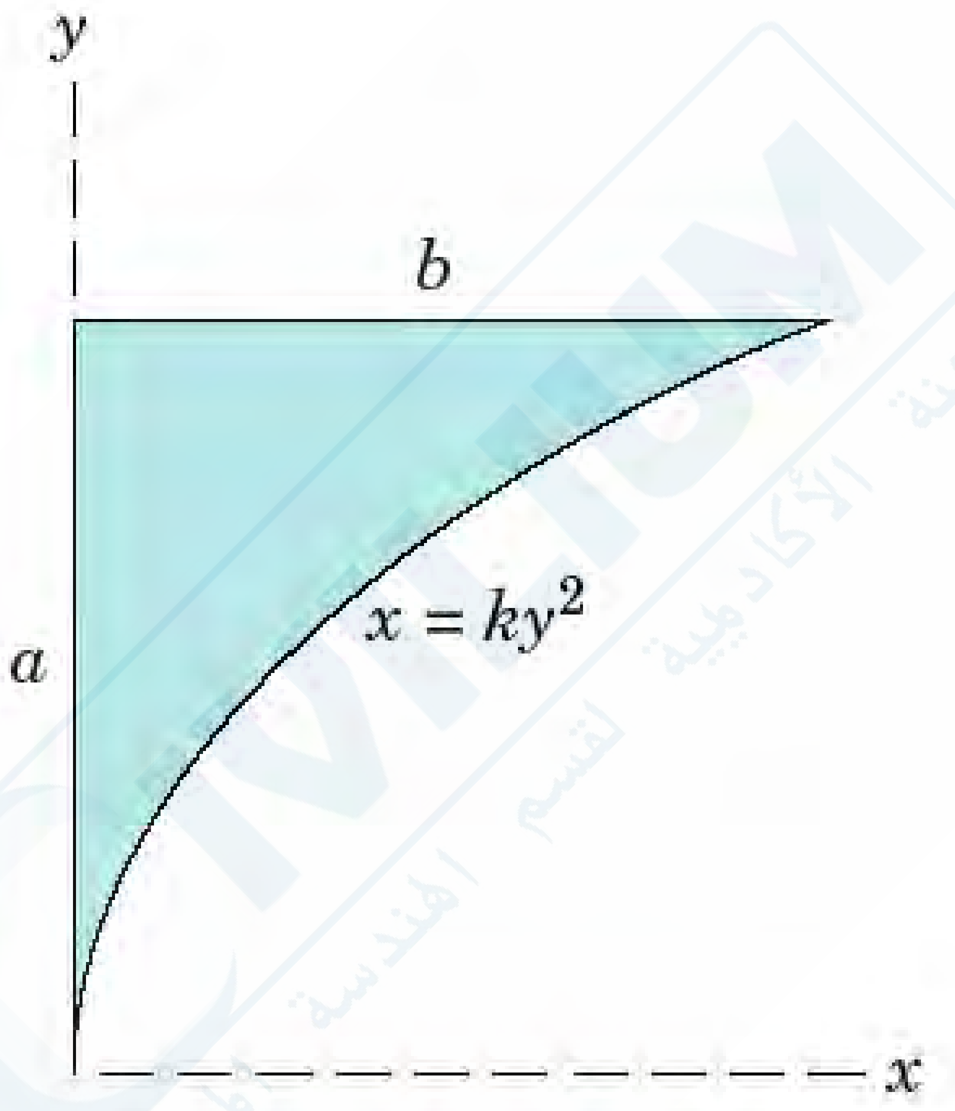
Problem 5/4

P_05_04



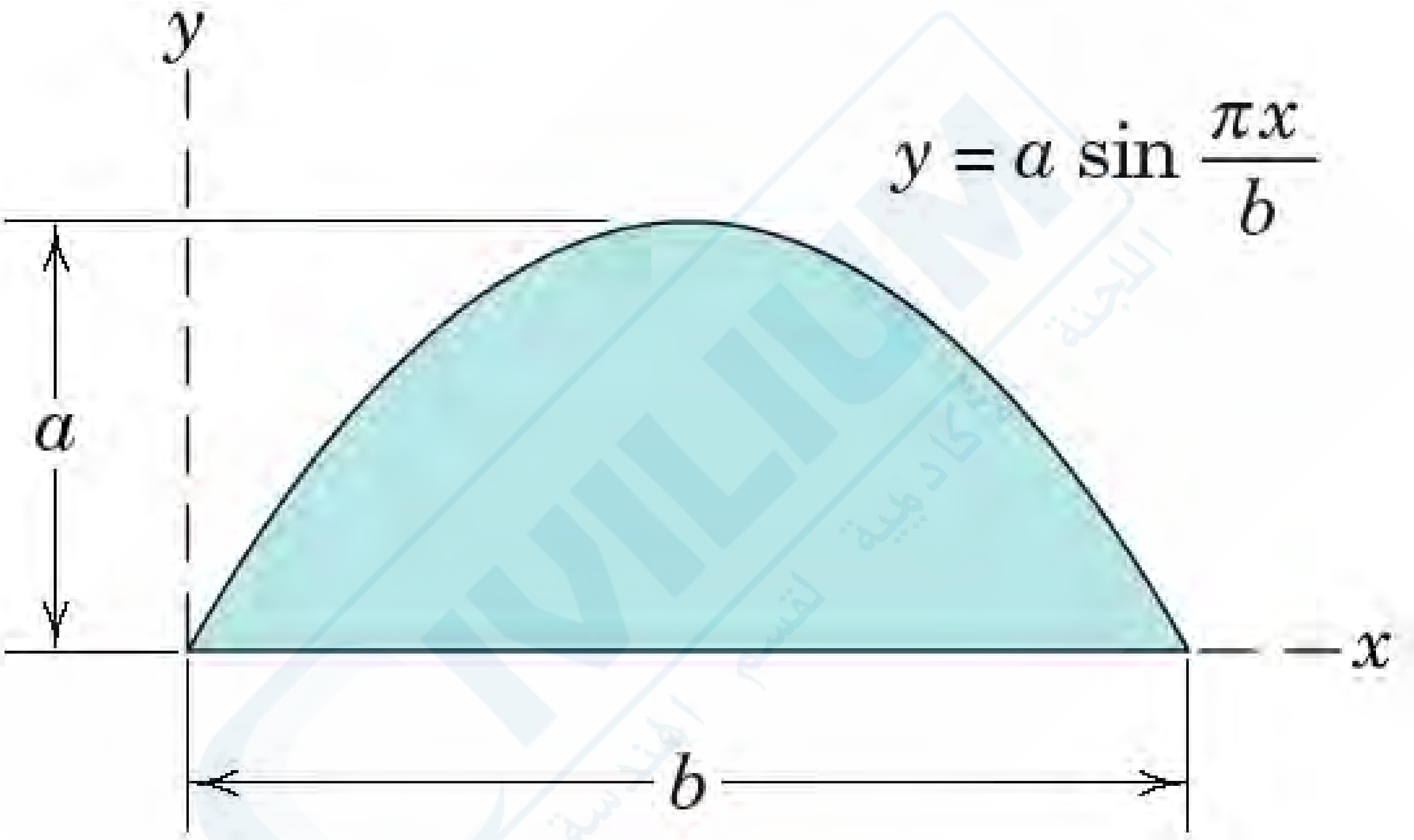
Problem 5/5

P_05_05



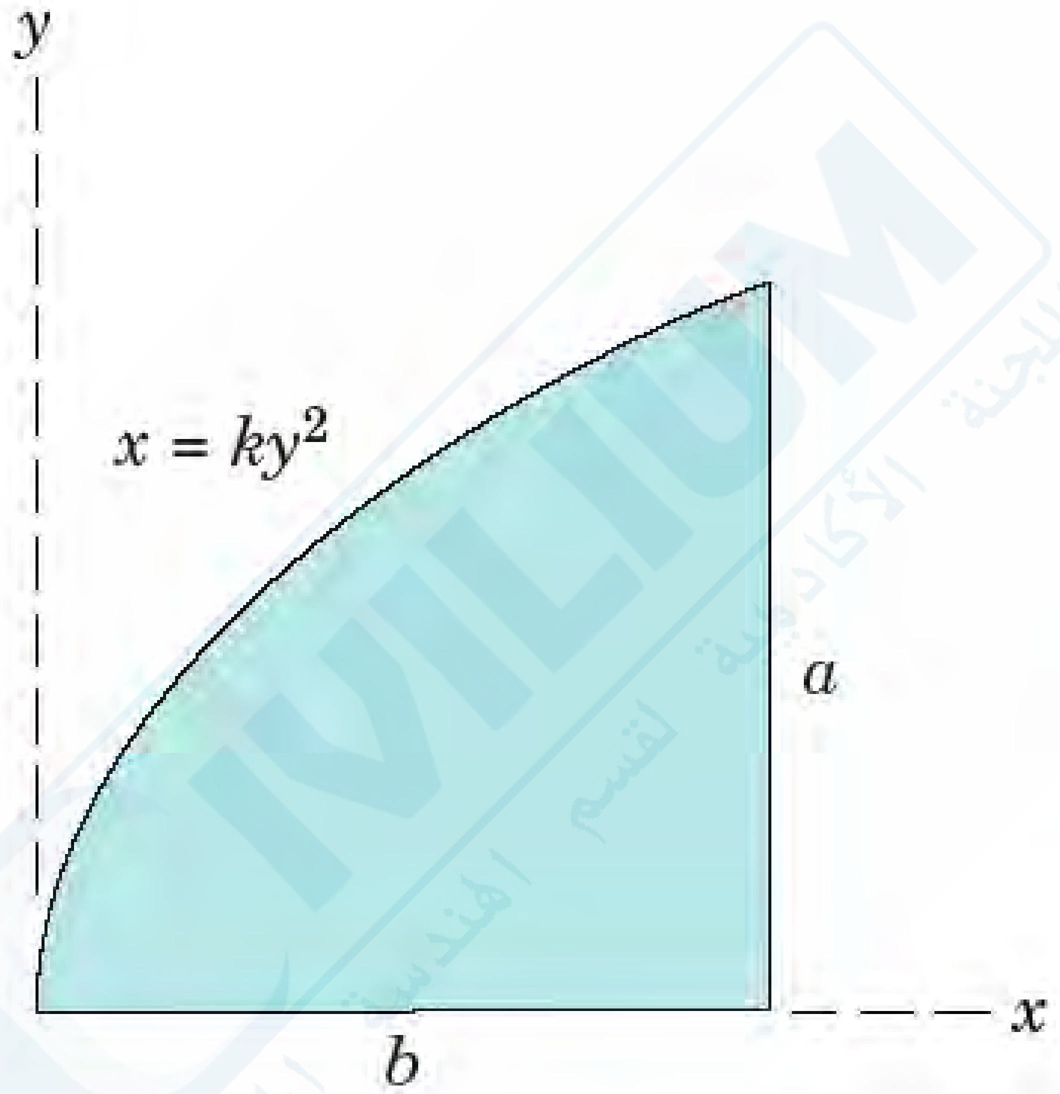
Problem 5/6

P_05_06



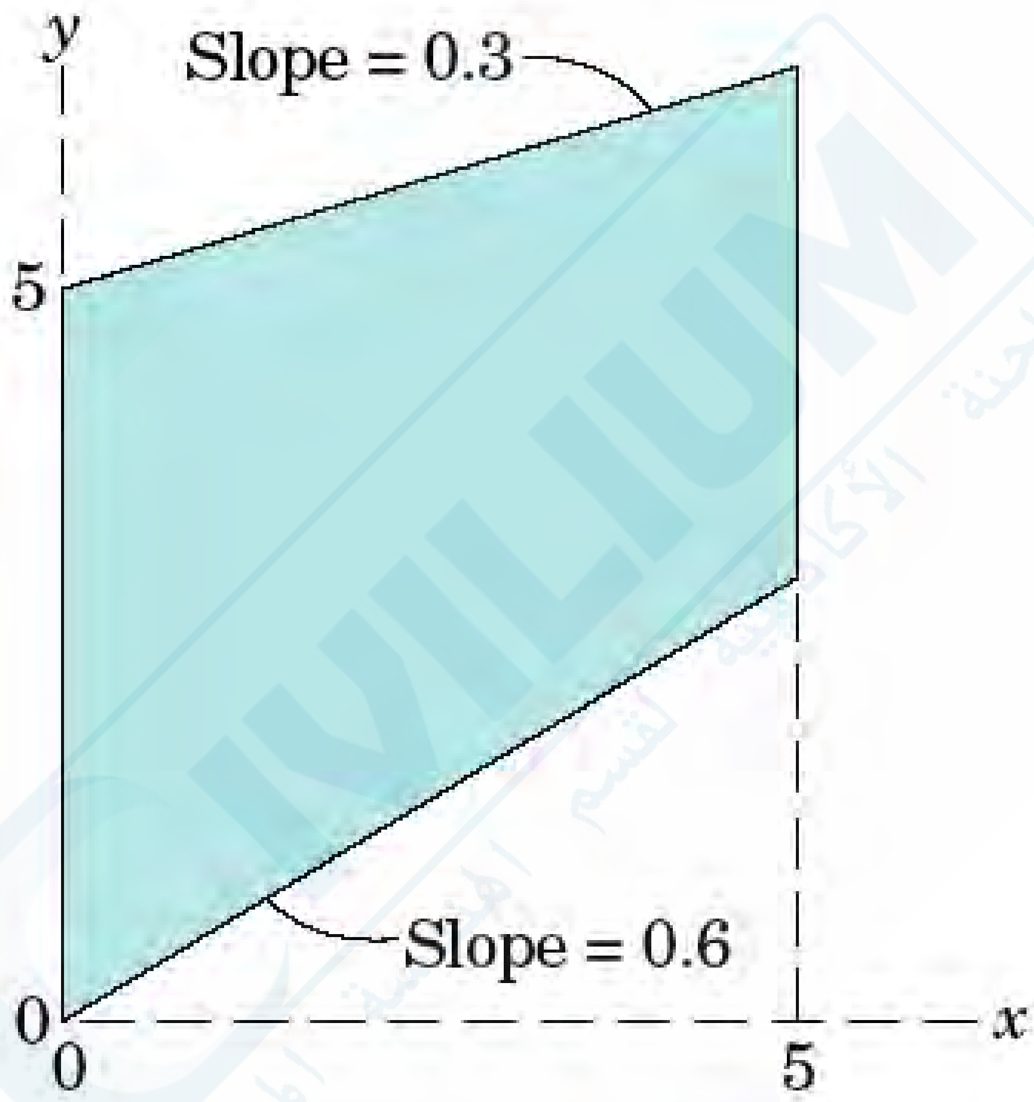
Problem 5/7

P_05_07



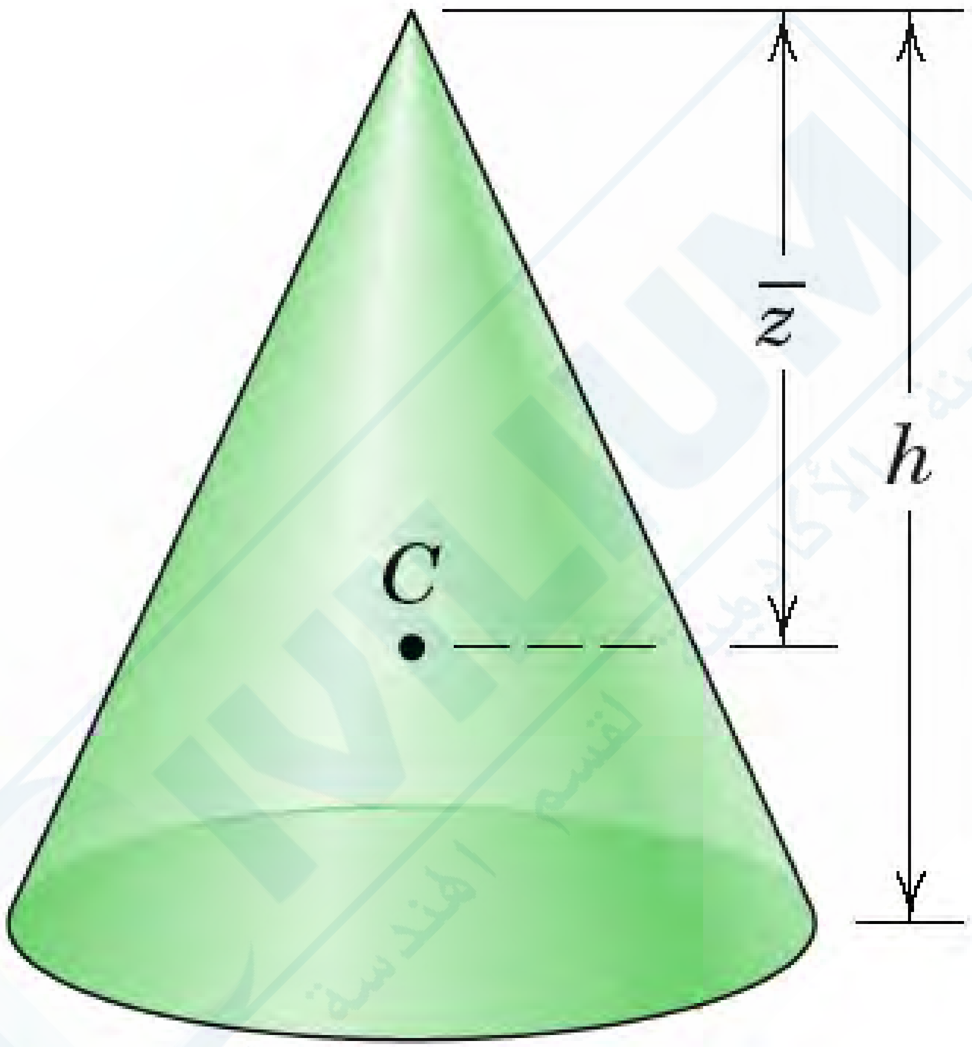
Problem 5/8

P_05_08

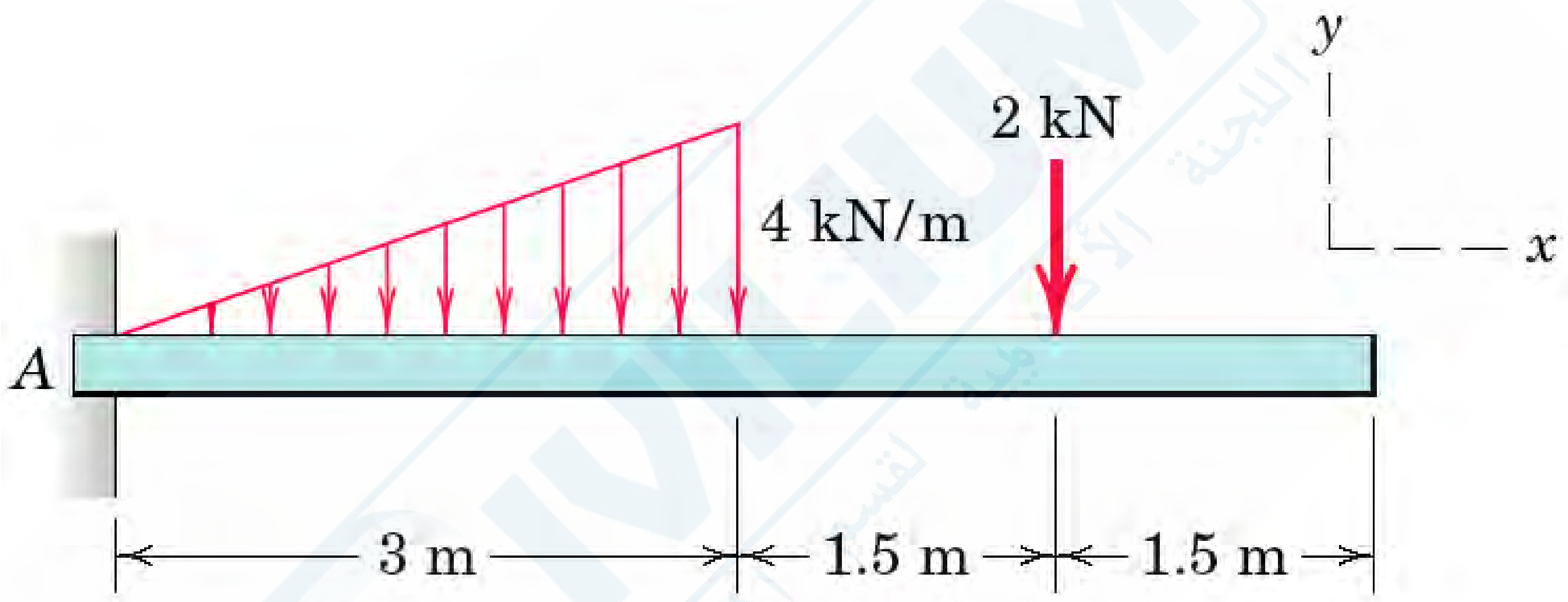


Problem 5/9

P_05_09

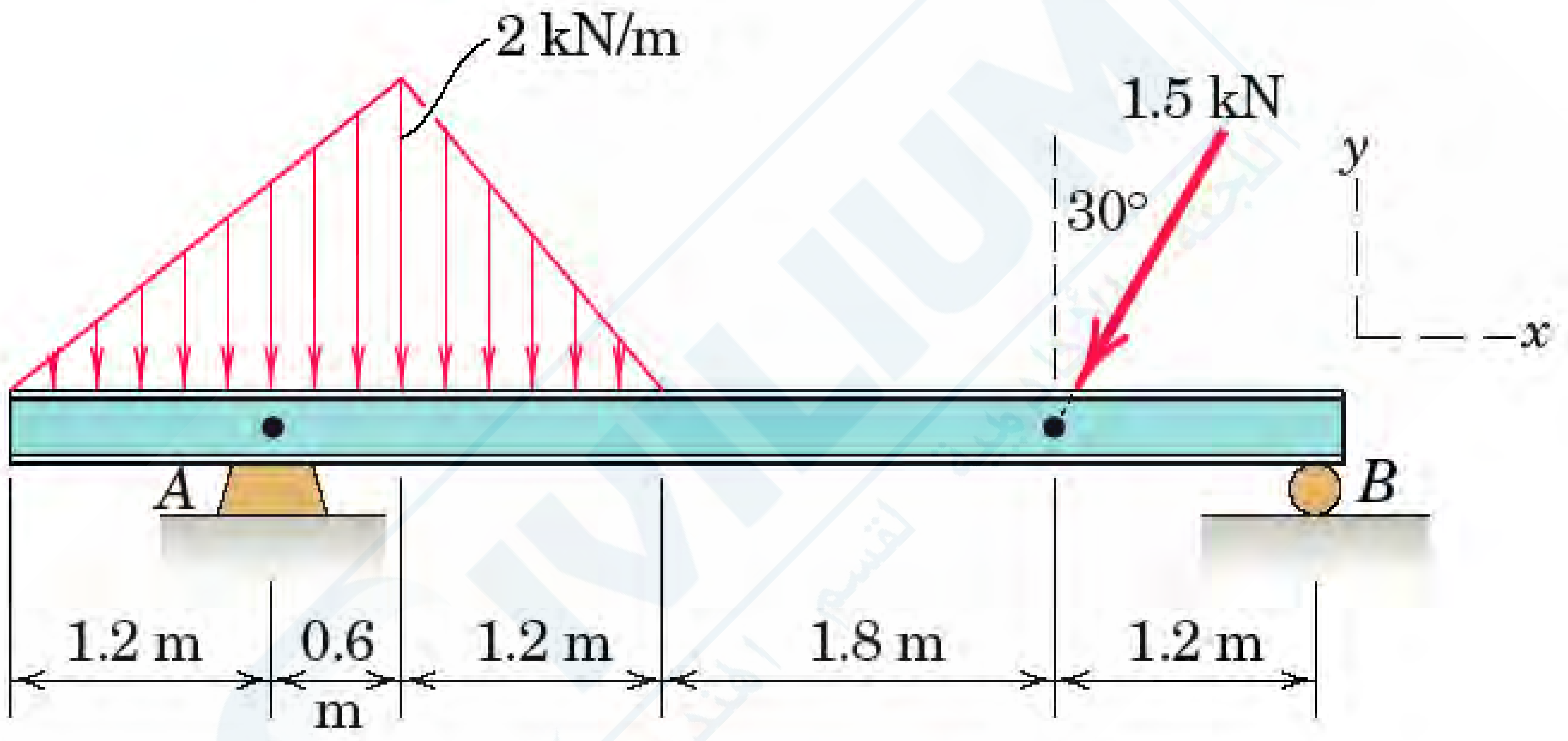


Problem 5/10



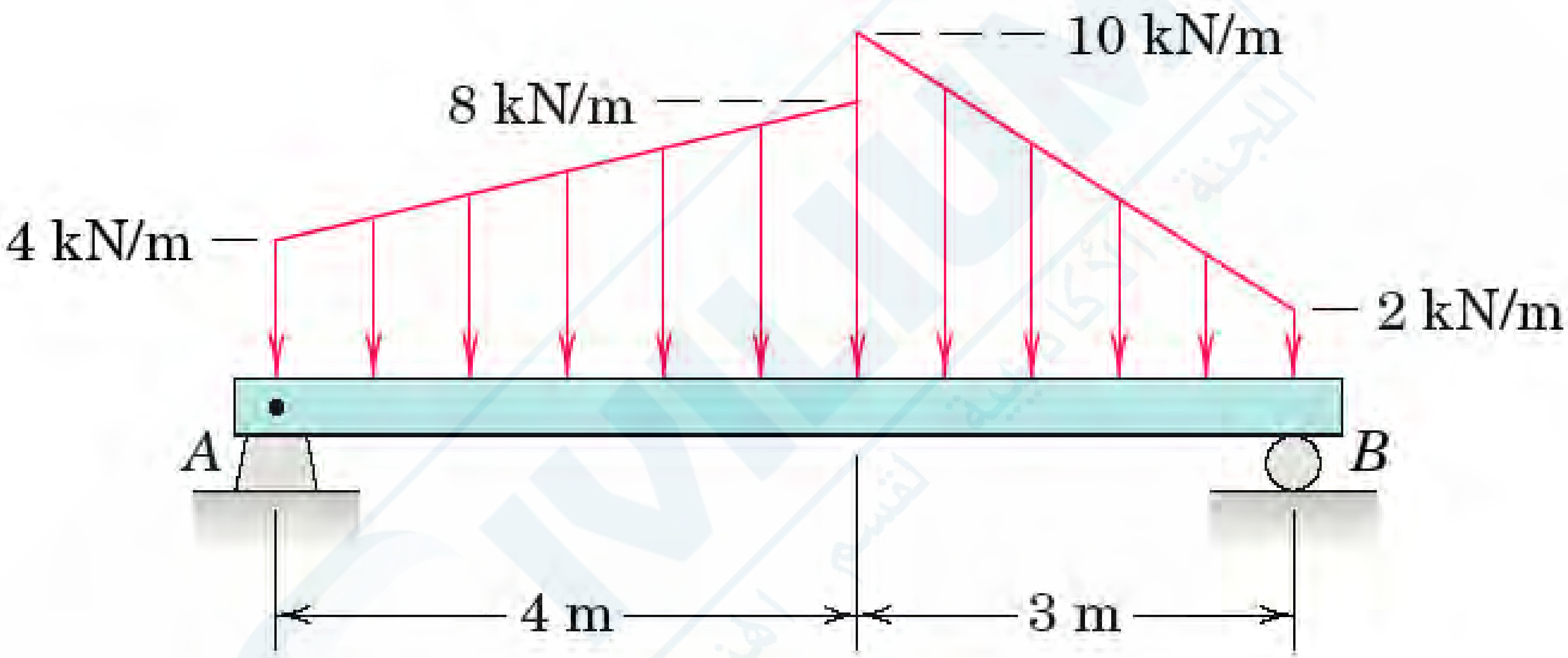
Problem 5/100

P_05_100



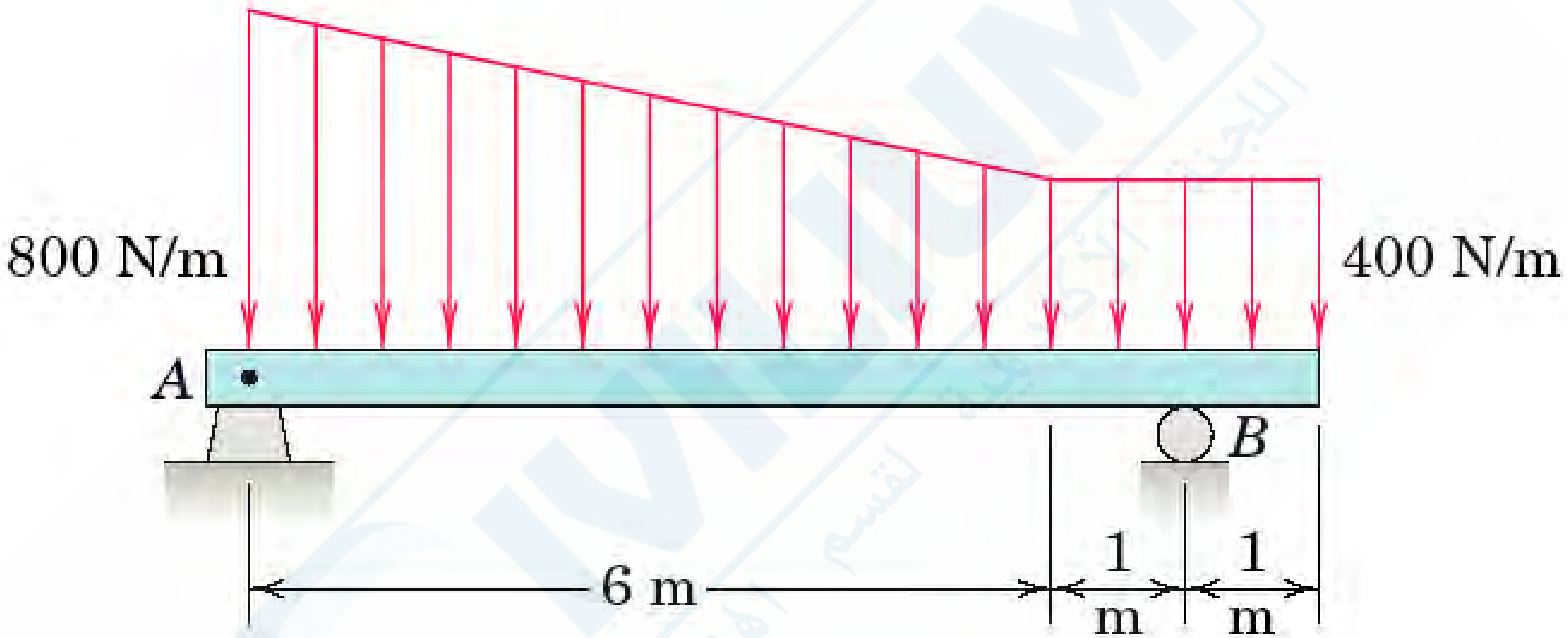
Problem 5/101

P_05_101



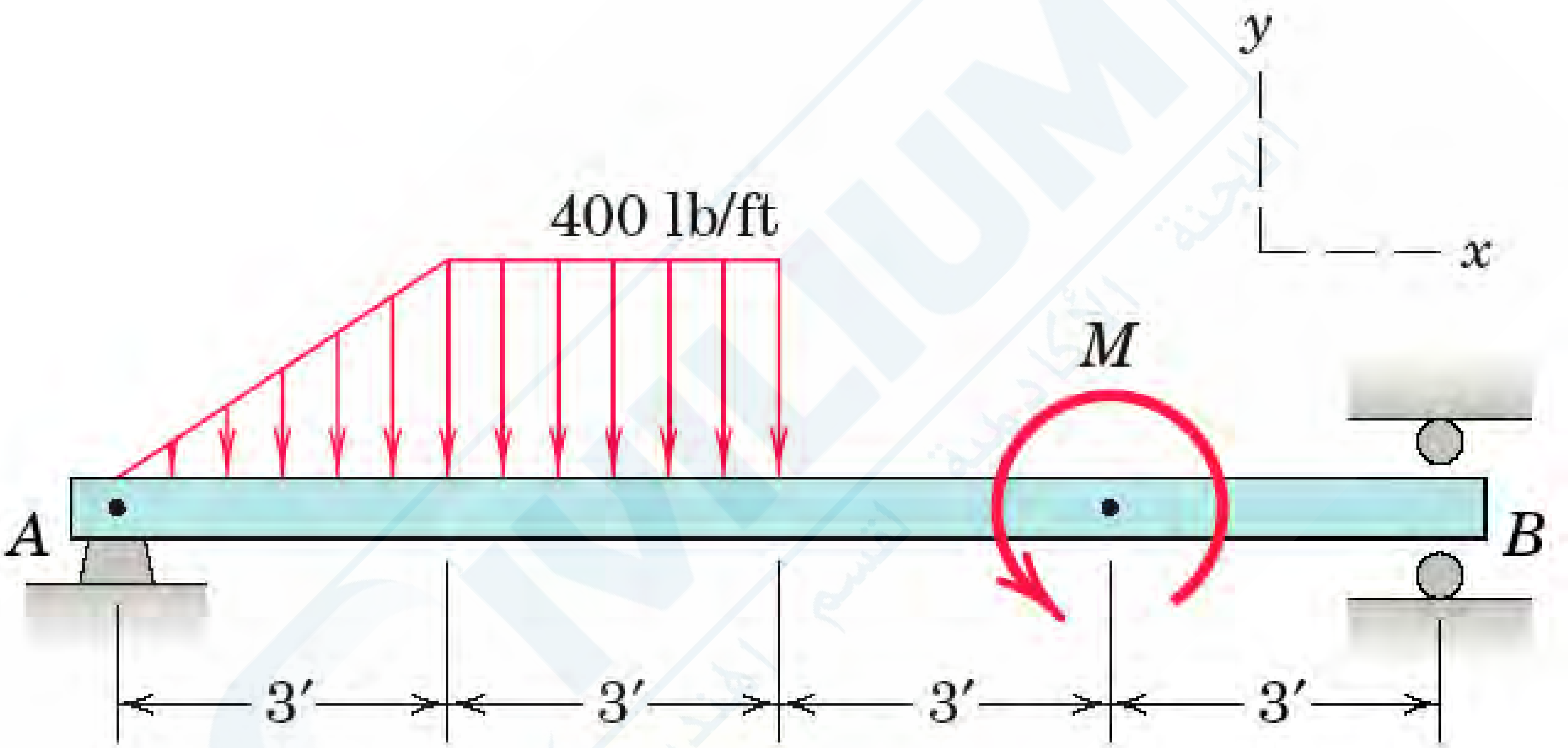
Problem 5/102

P_05_102



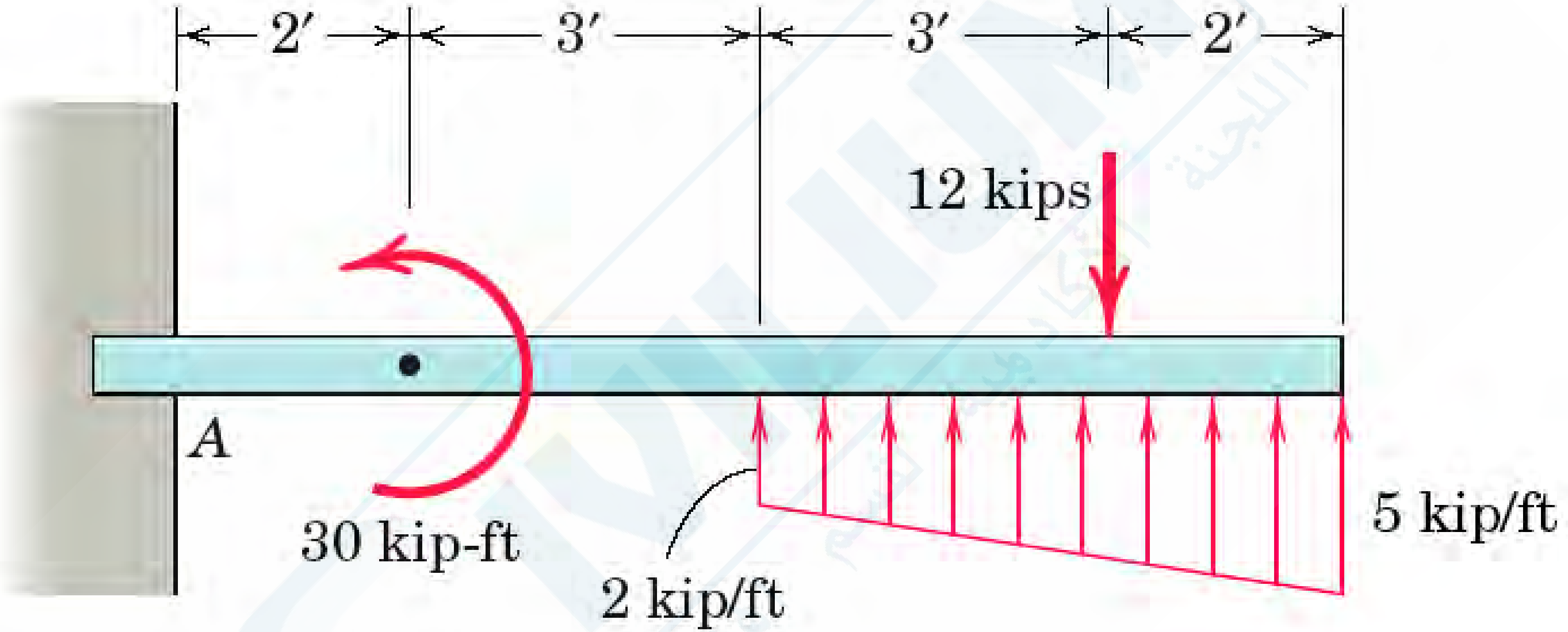
Problem 5/103

P_05_103



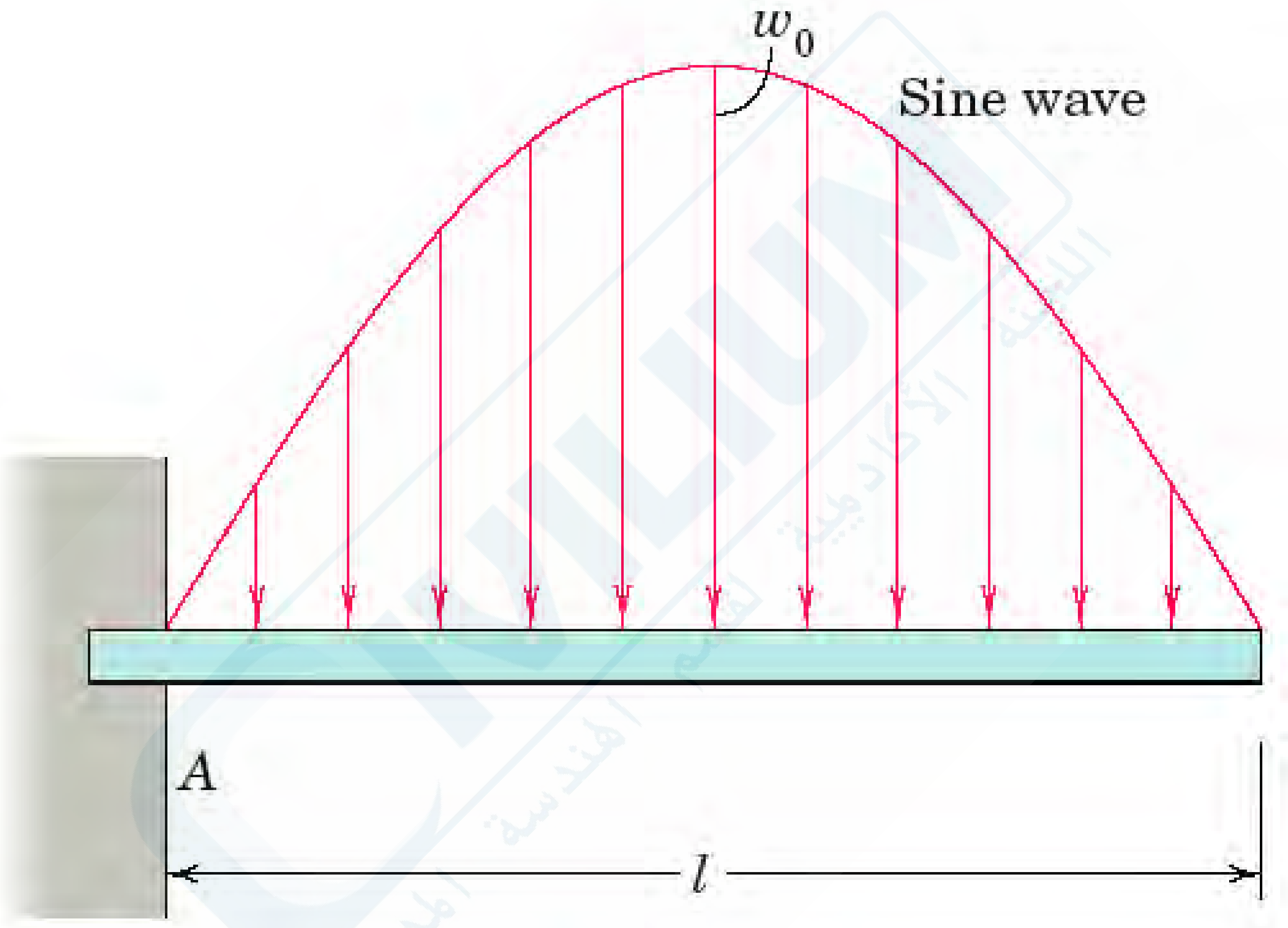
Problem 5/104

P_05_104



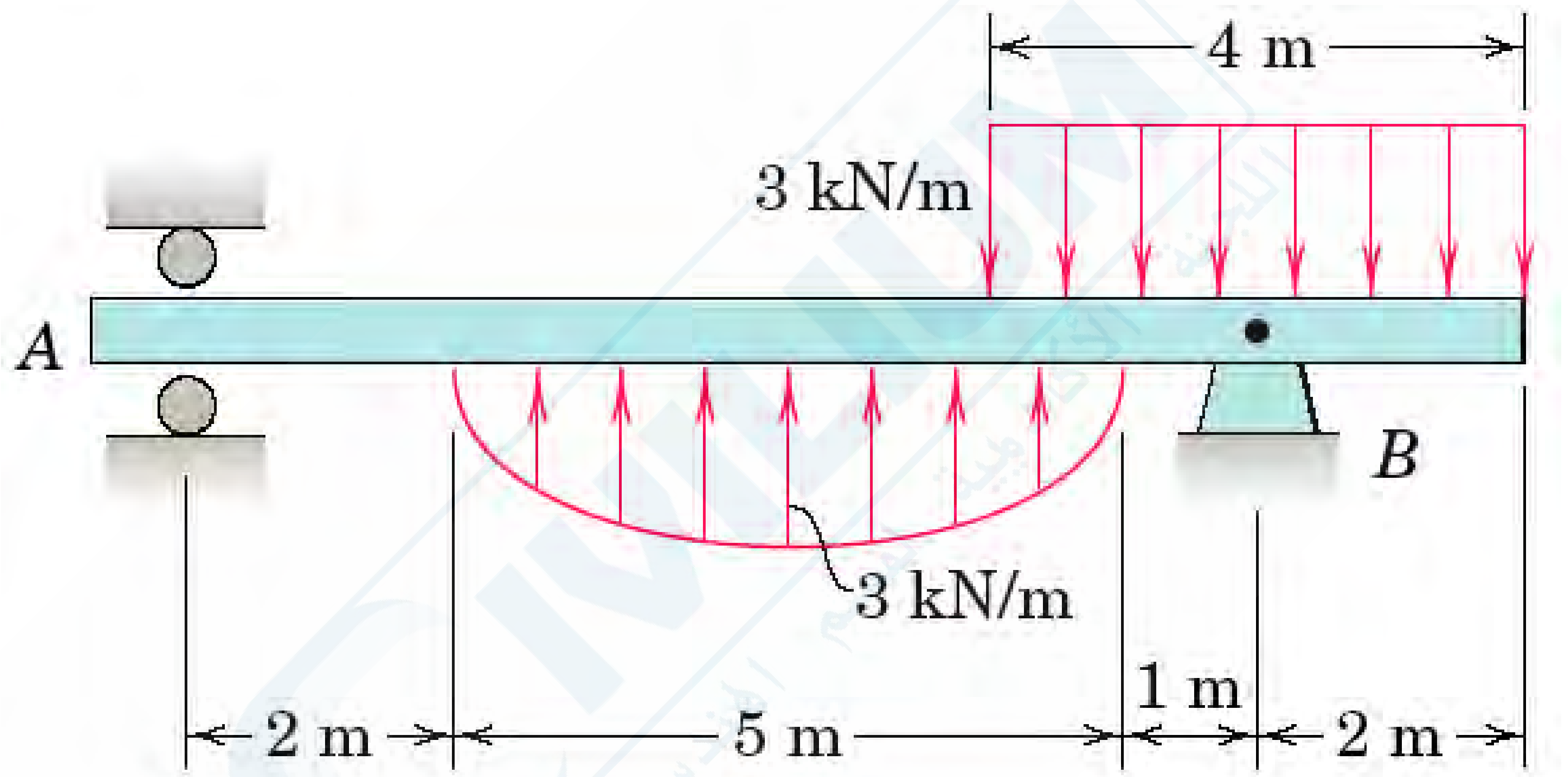
Problem 5/105

P_05_105



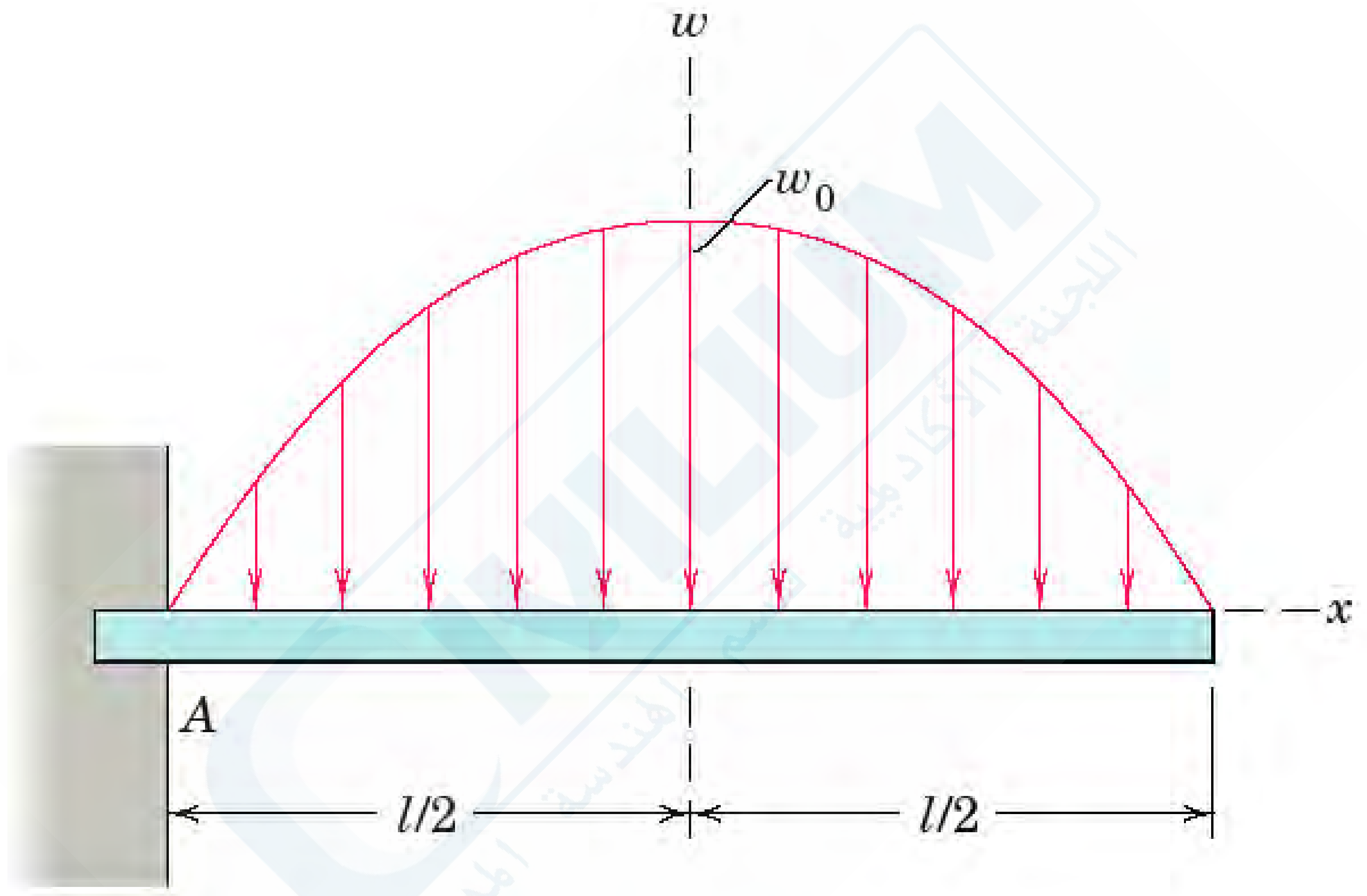
Problem 5/106

P_05_106

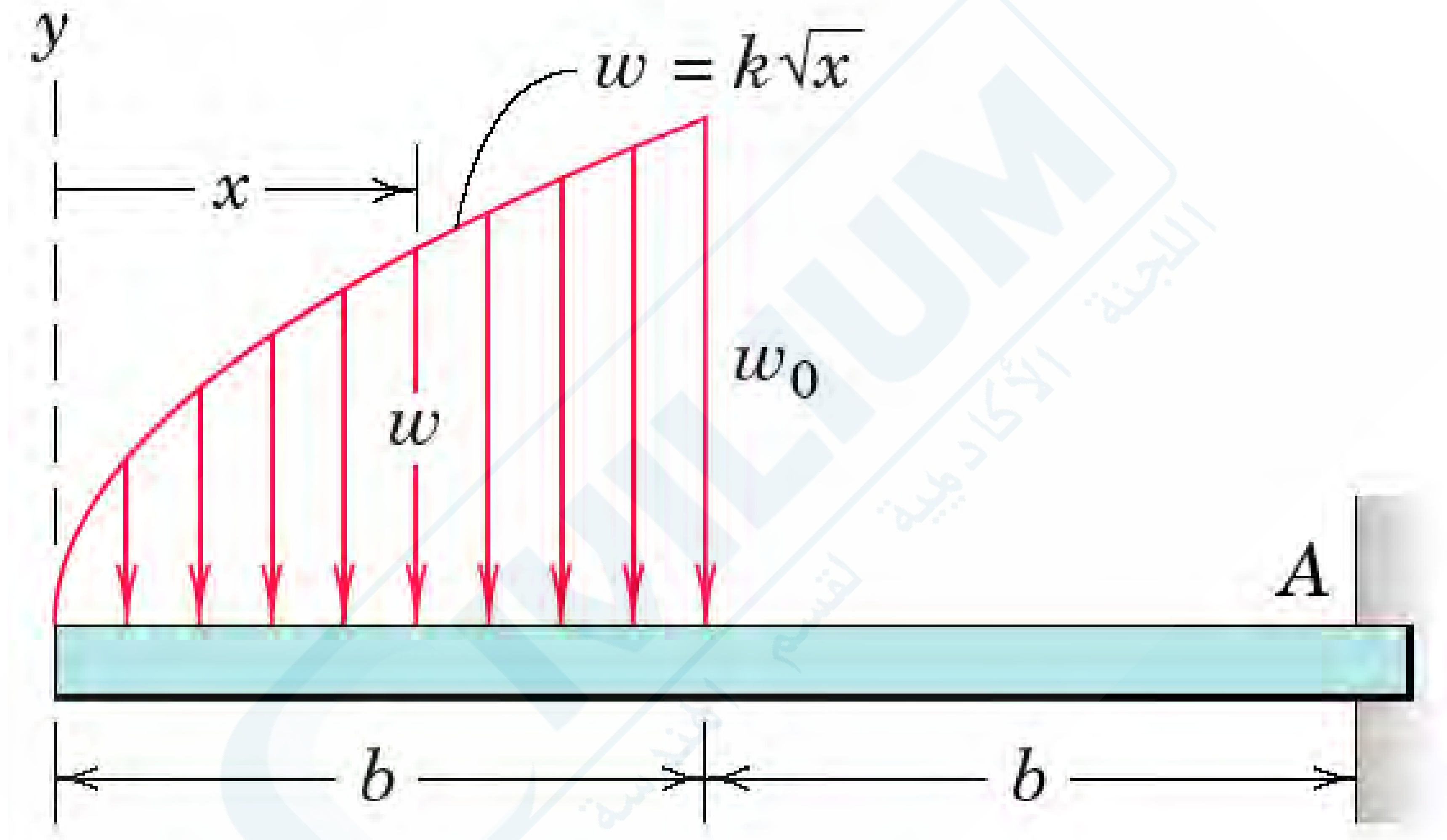


Problem 5/107

P_05_107

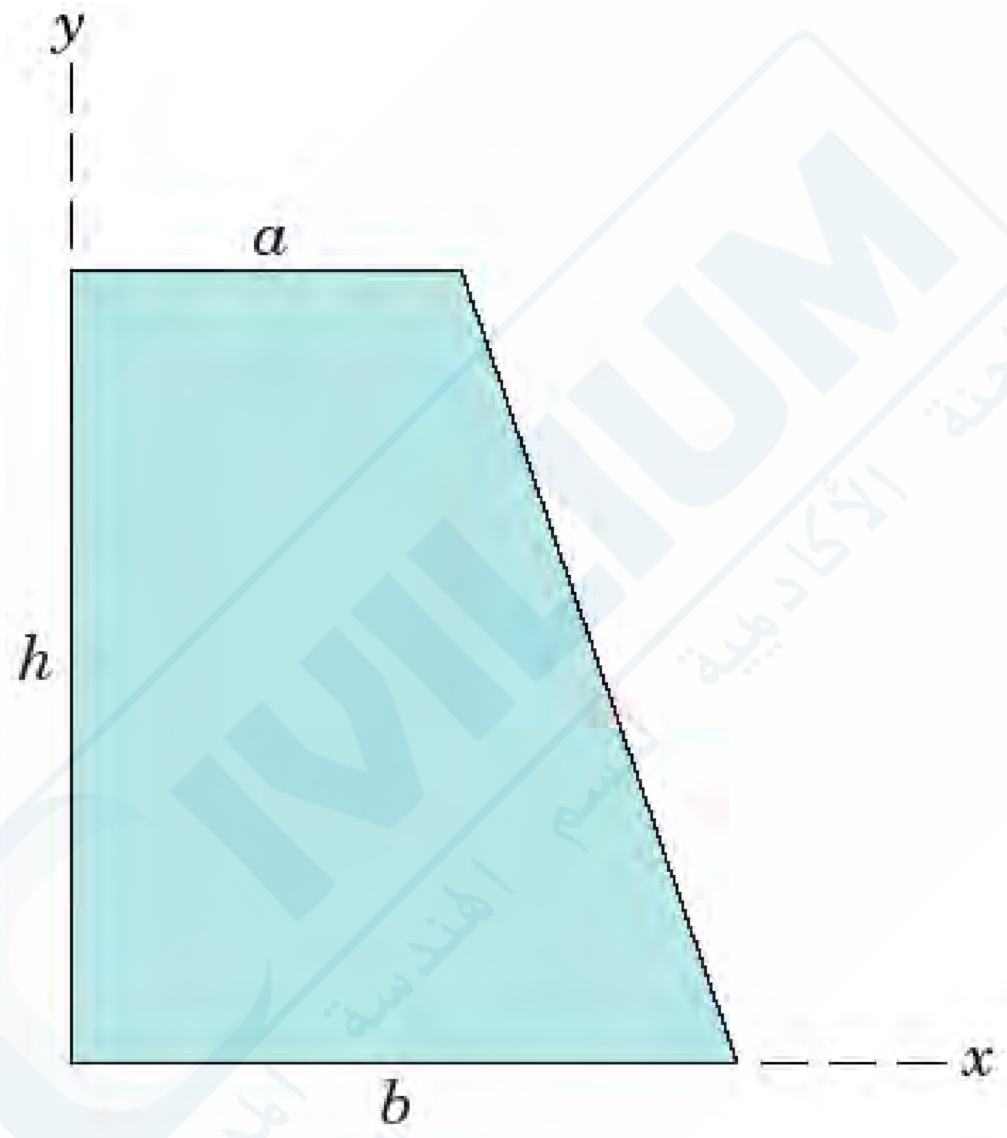


Problem 5/108



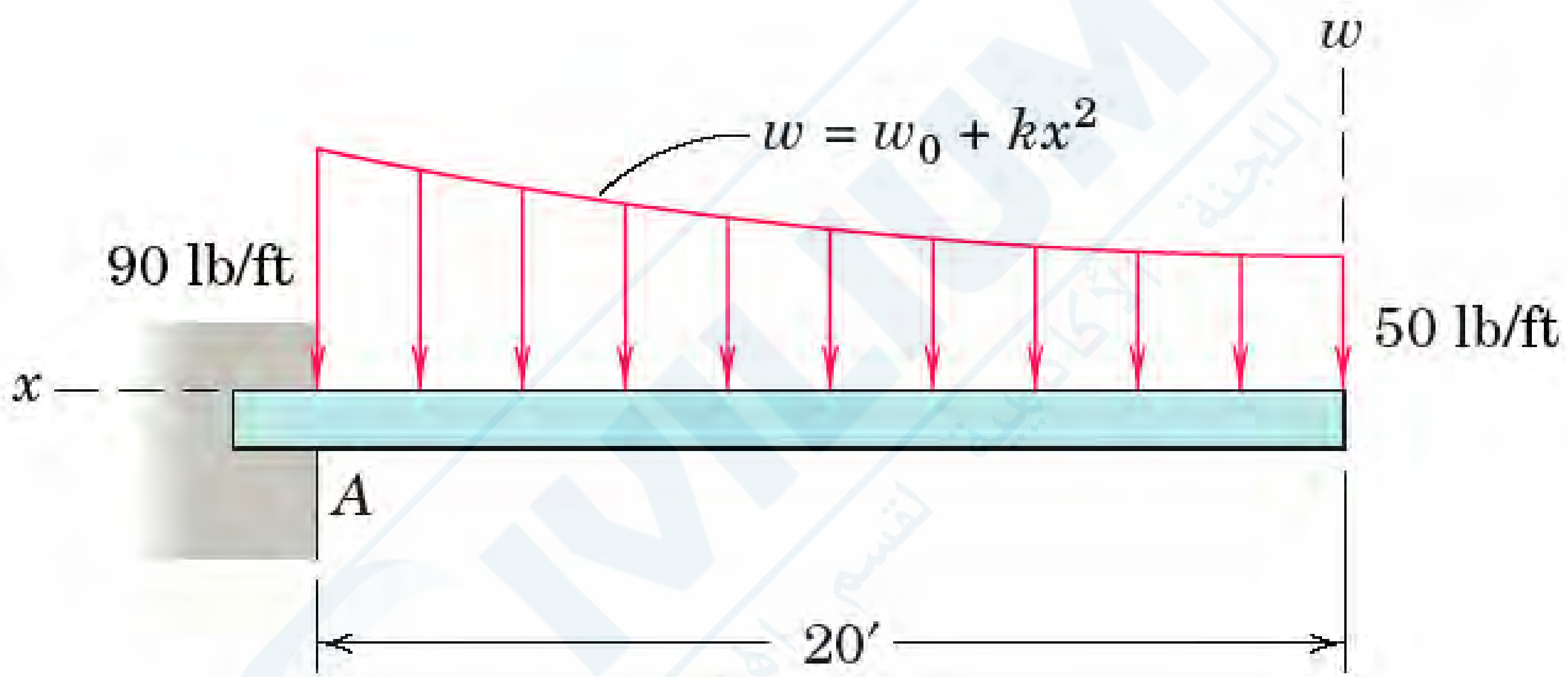
Problem 5/109

P_05_109



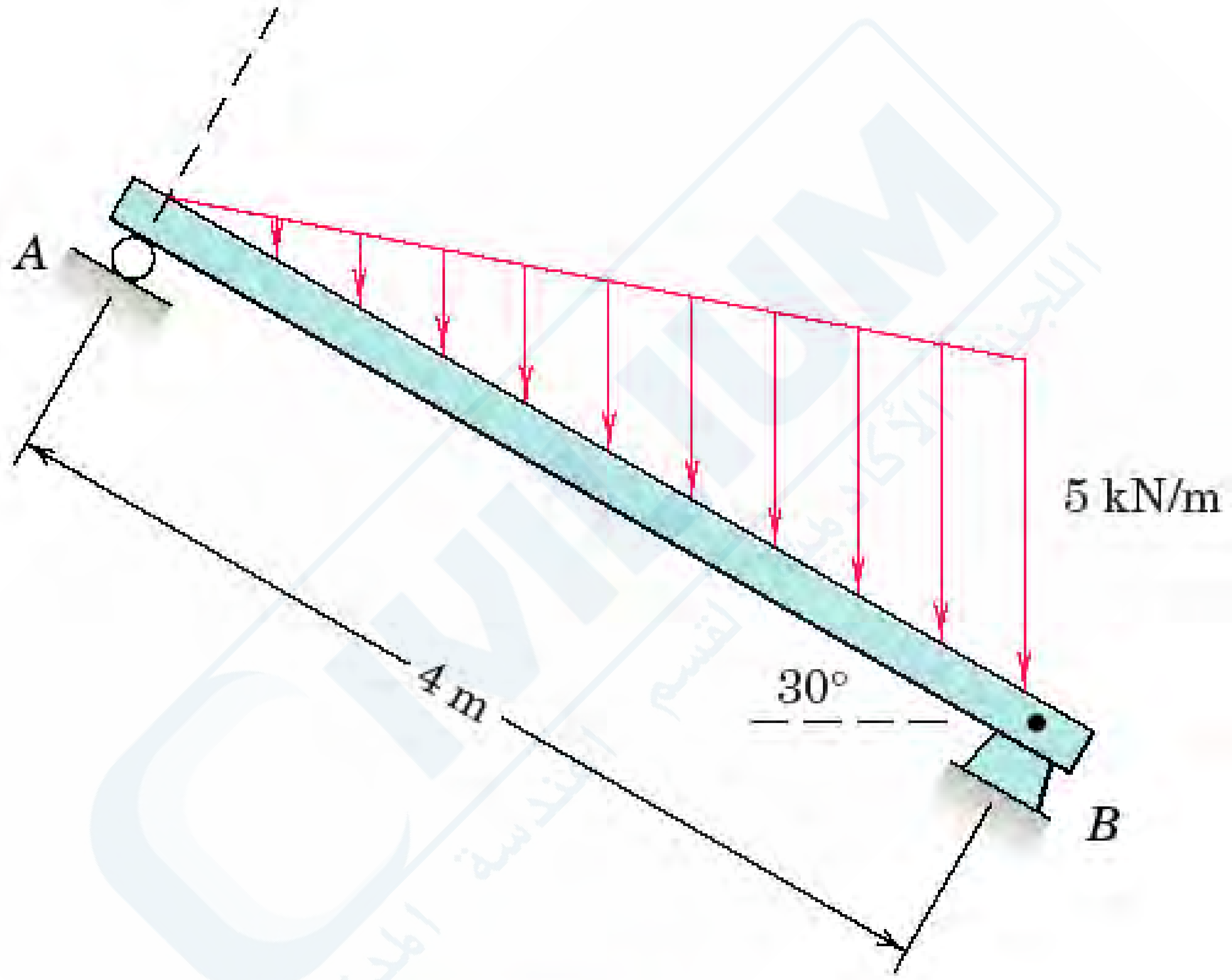
Problem 5/11

P_05_11

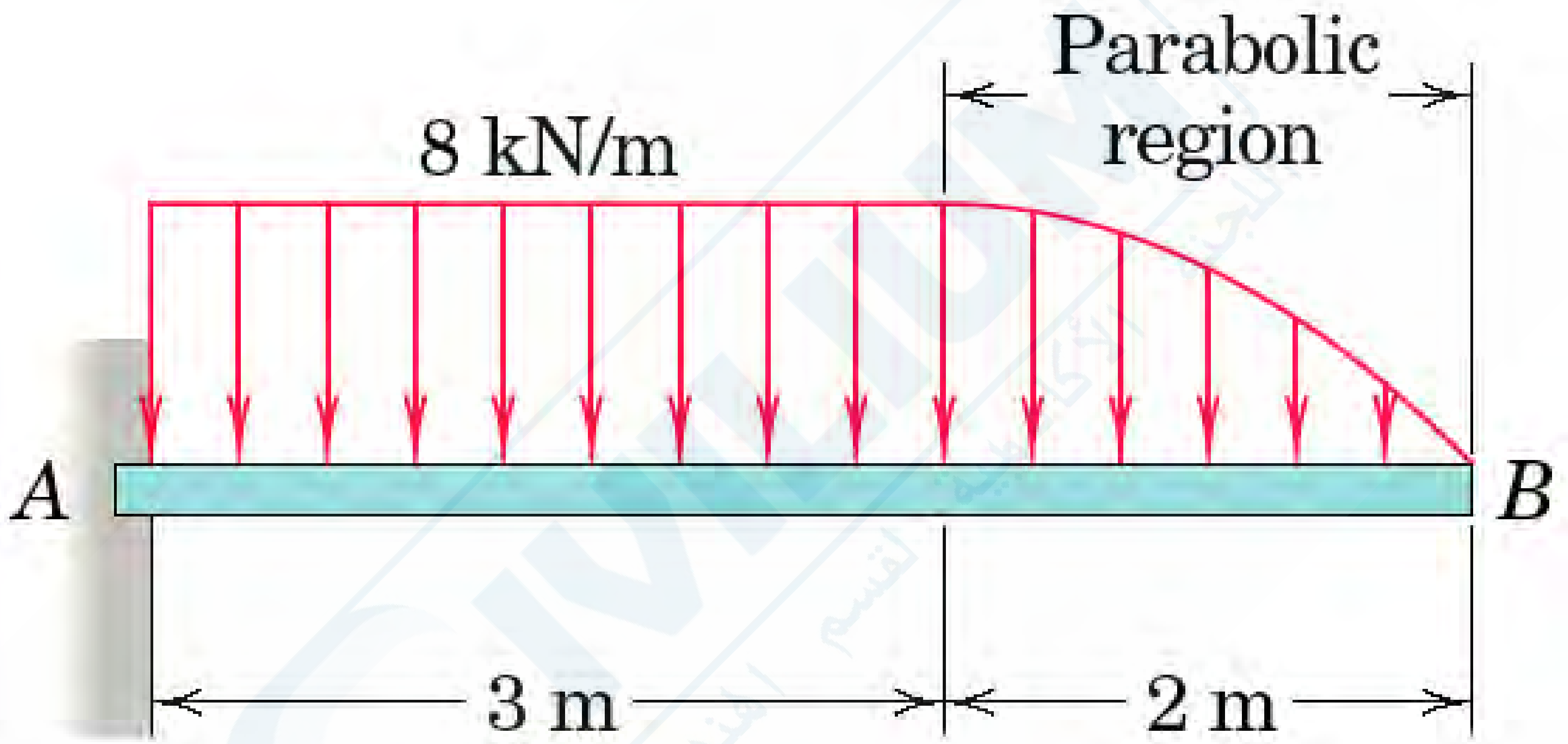


Problem 5/110

P_05_110

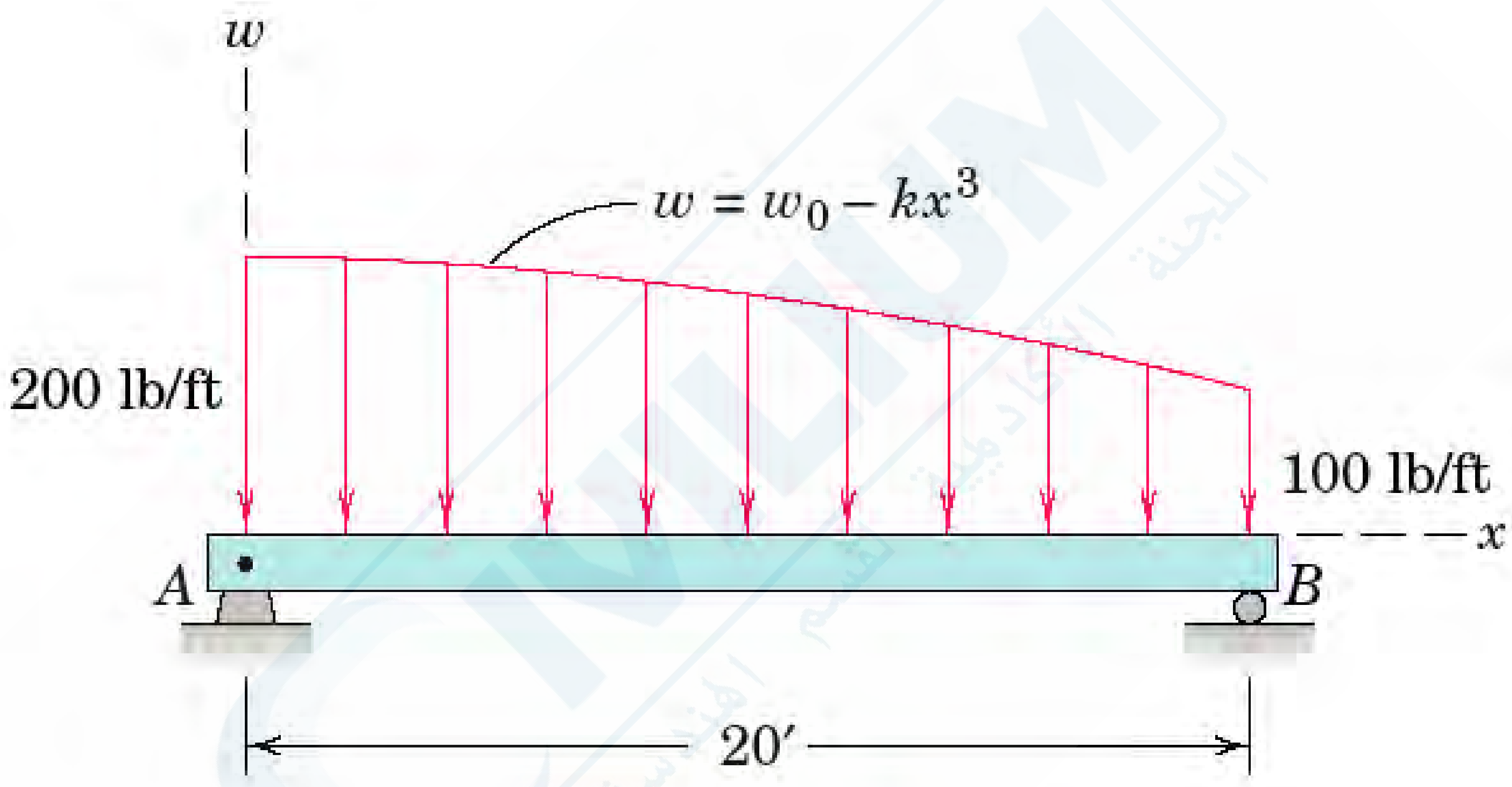


Problem 5/111



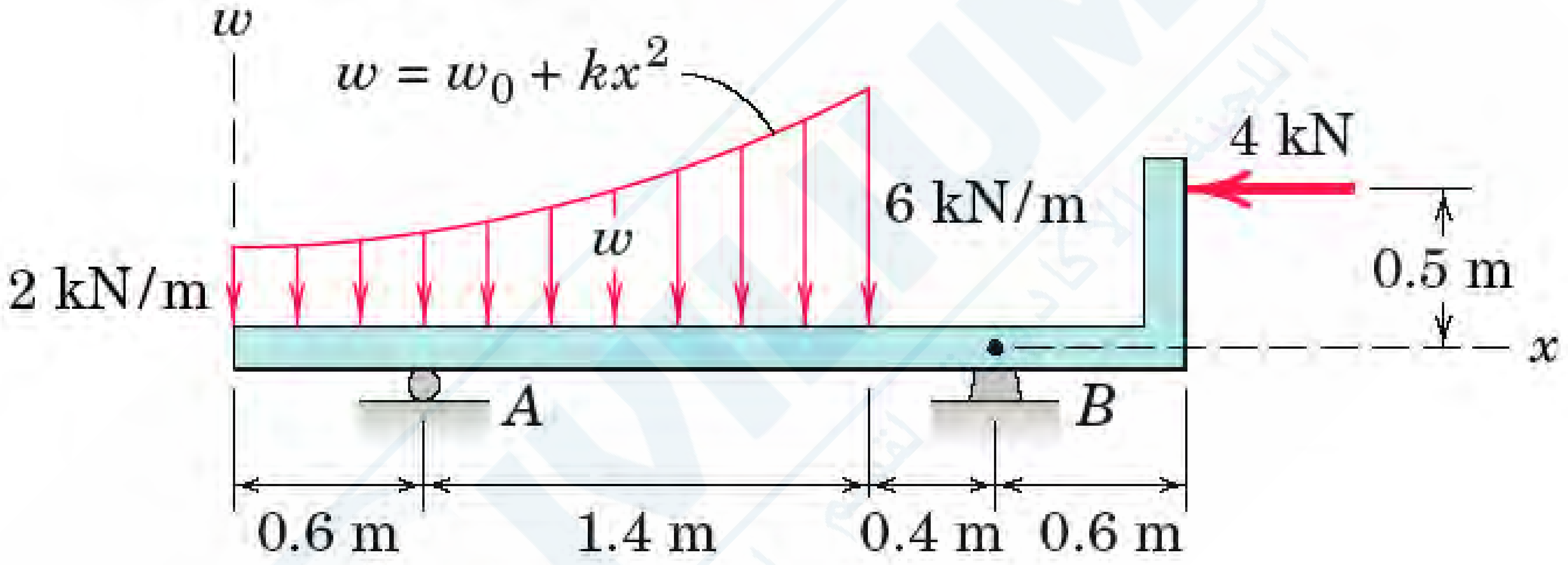
Problem 5/112

P_05_112



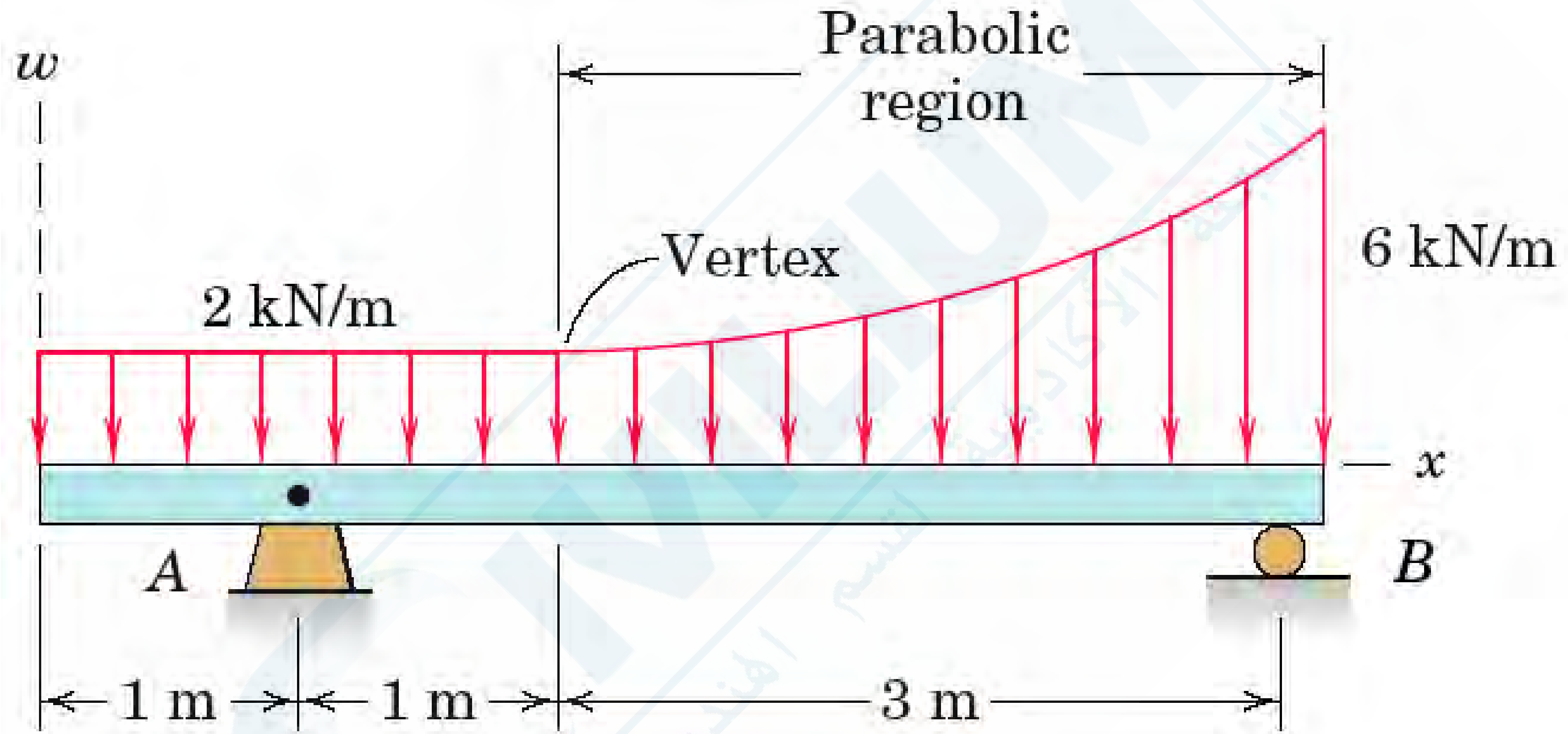
Problem 5/113

P_05_113



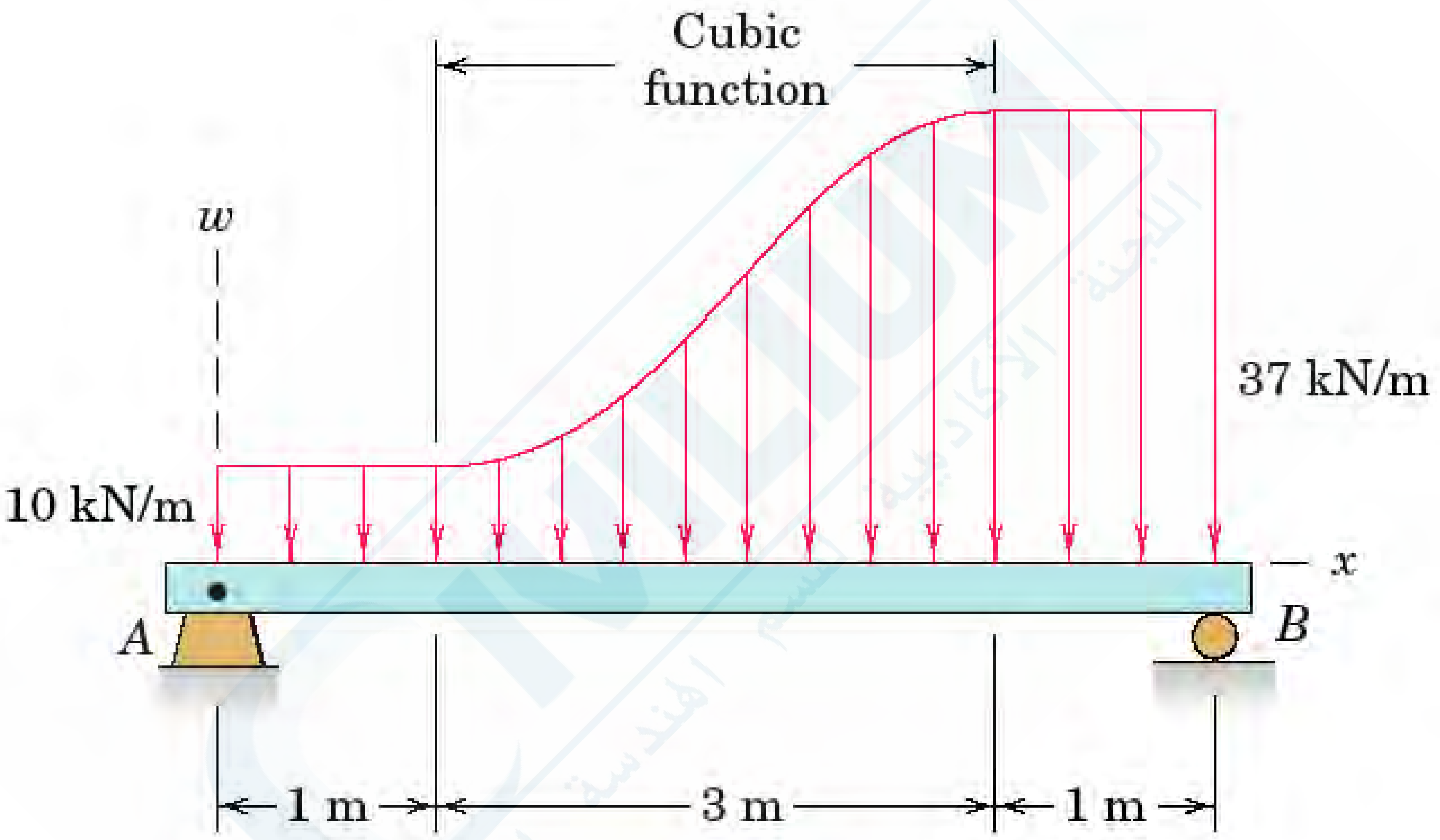
Problem 5/114

P_05_114



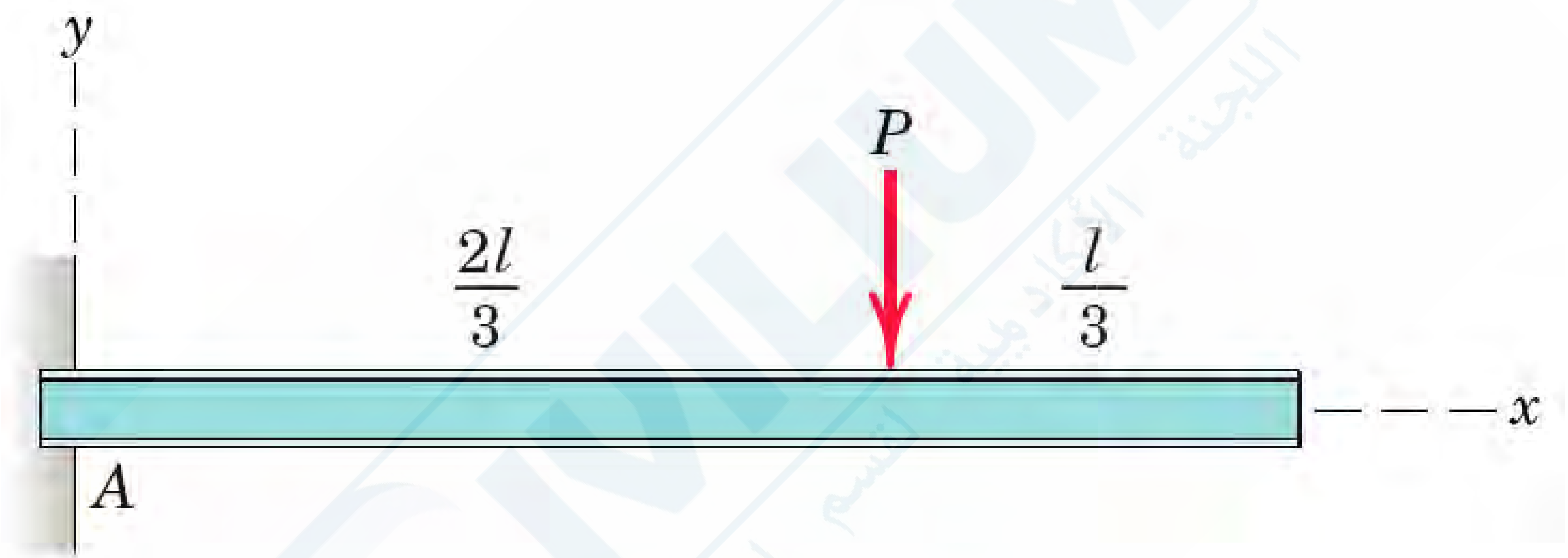
Problem 5/115

P_05_115



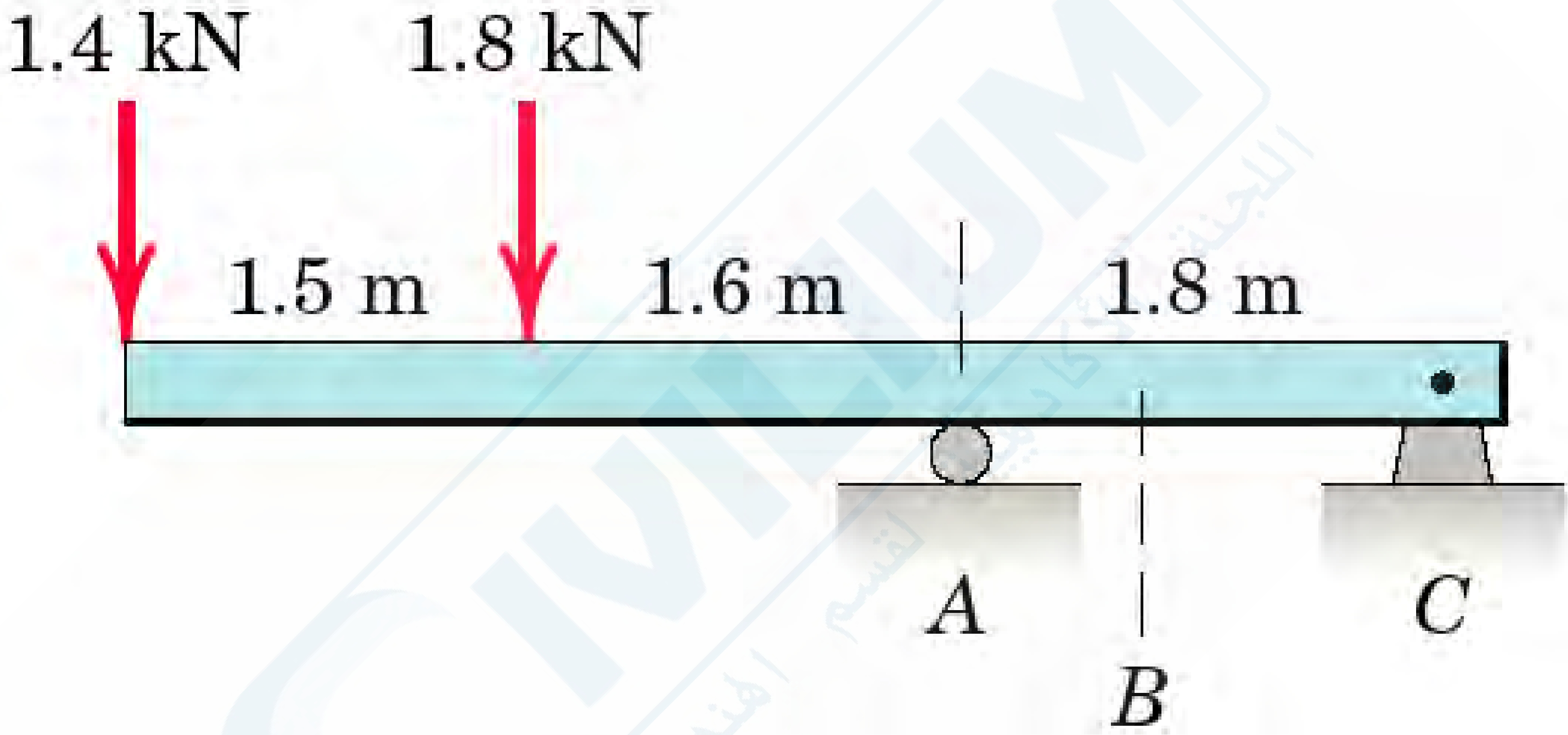
Problem 5/116

P_05_116



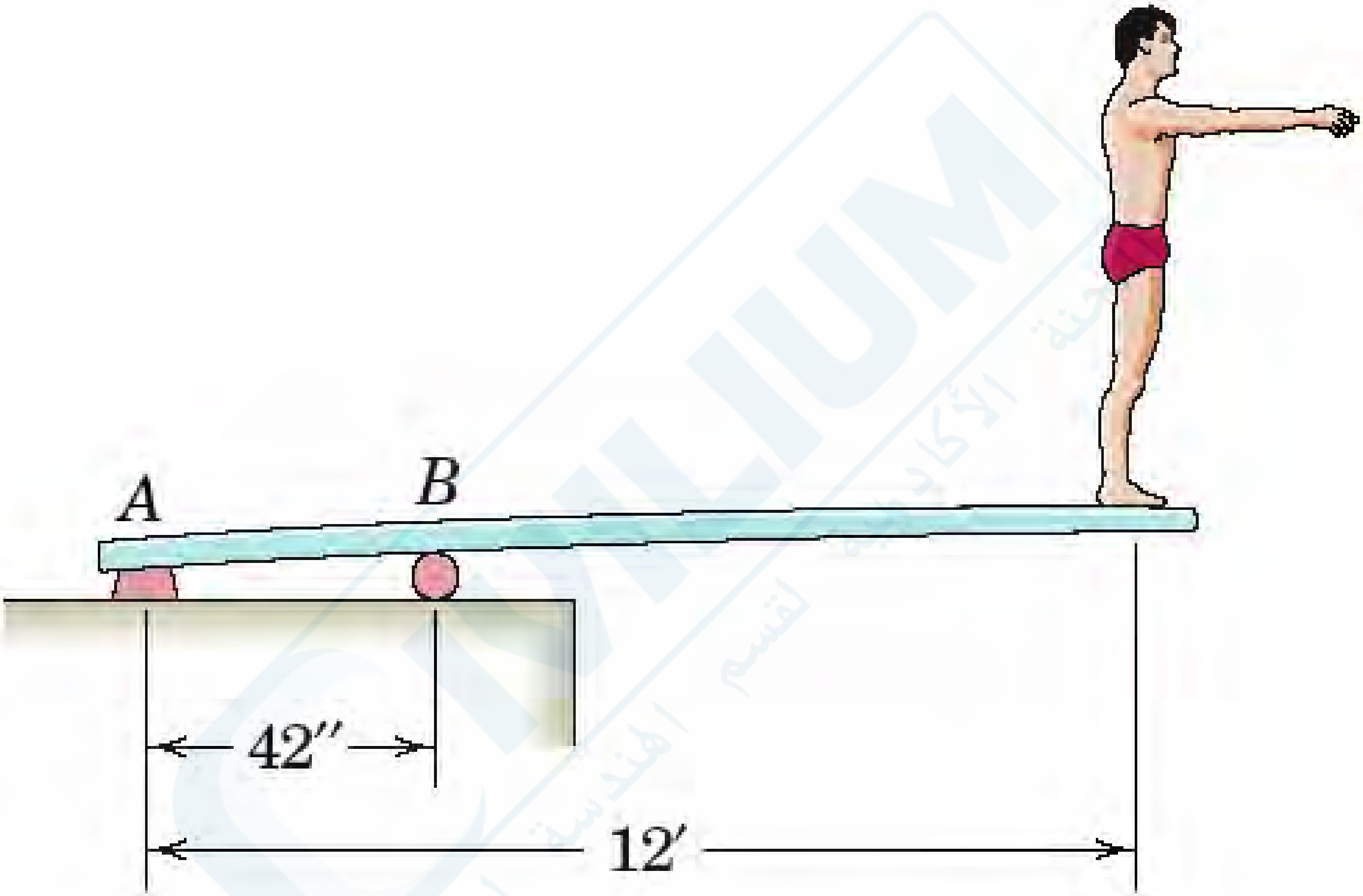
Problem 5/117

P_05_117



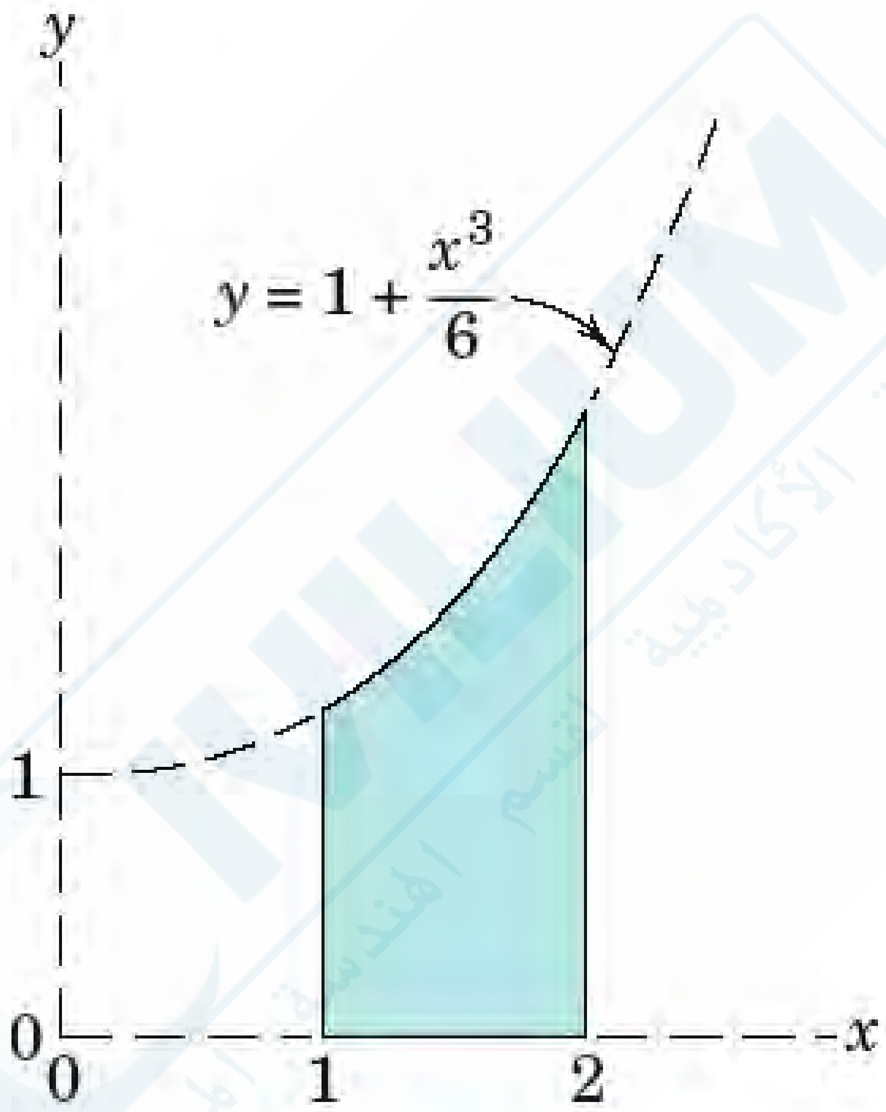
Problem 5/118

P_05_118



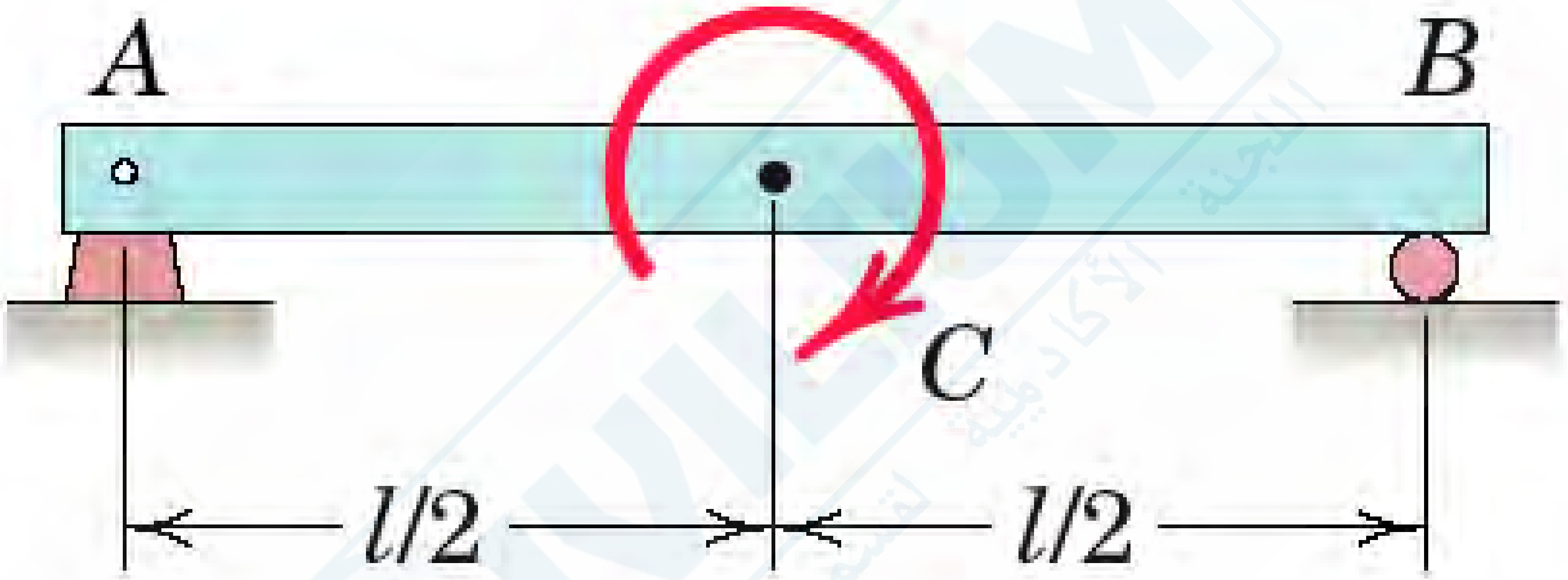
Problem 5/119

P_05_119



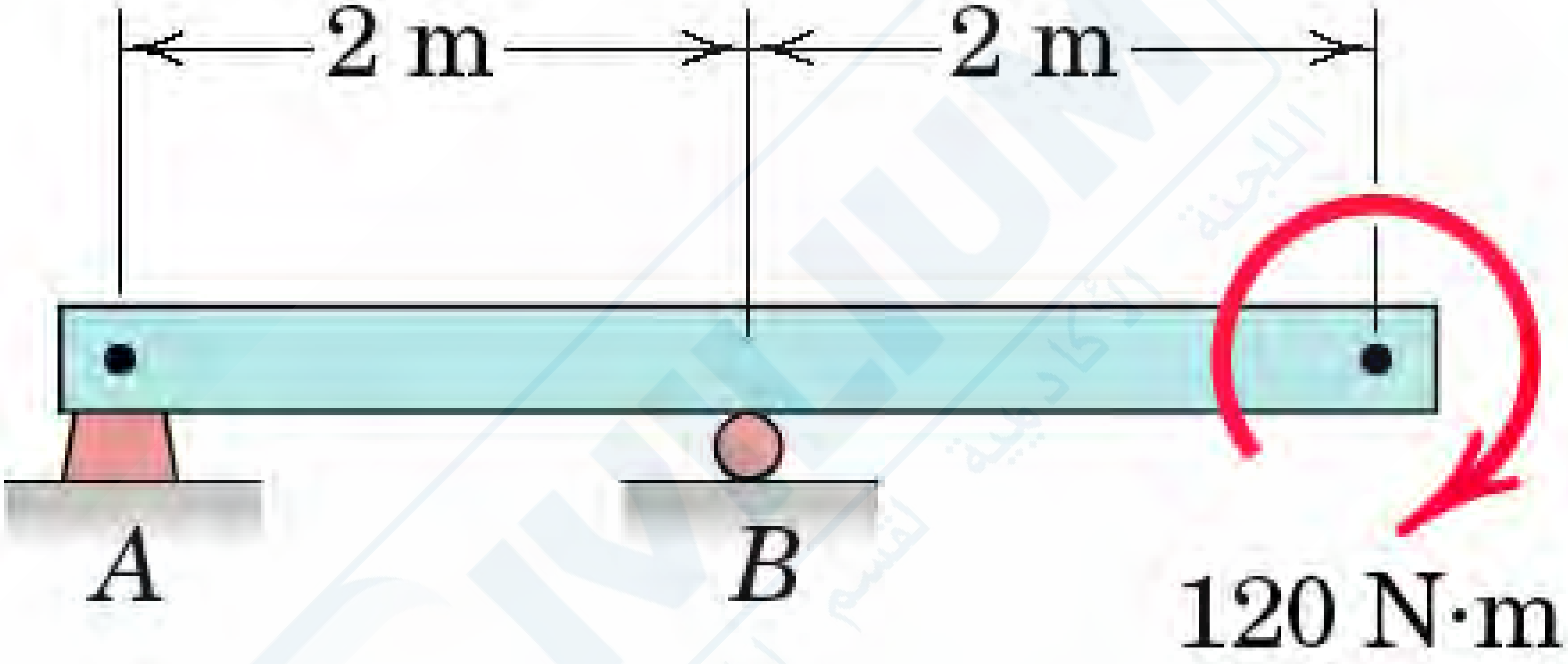
Problem 5/12

P_05_12



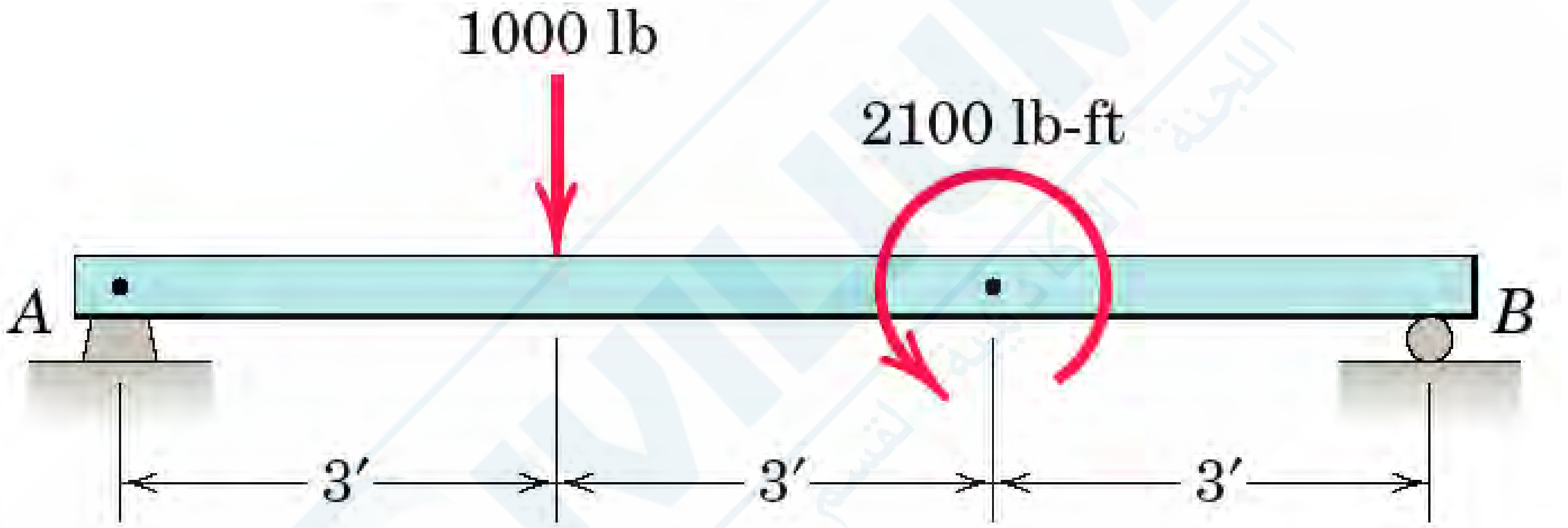
Problem 5/120

P_05_120



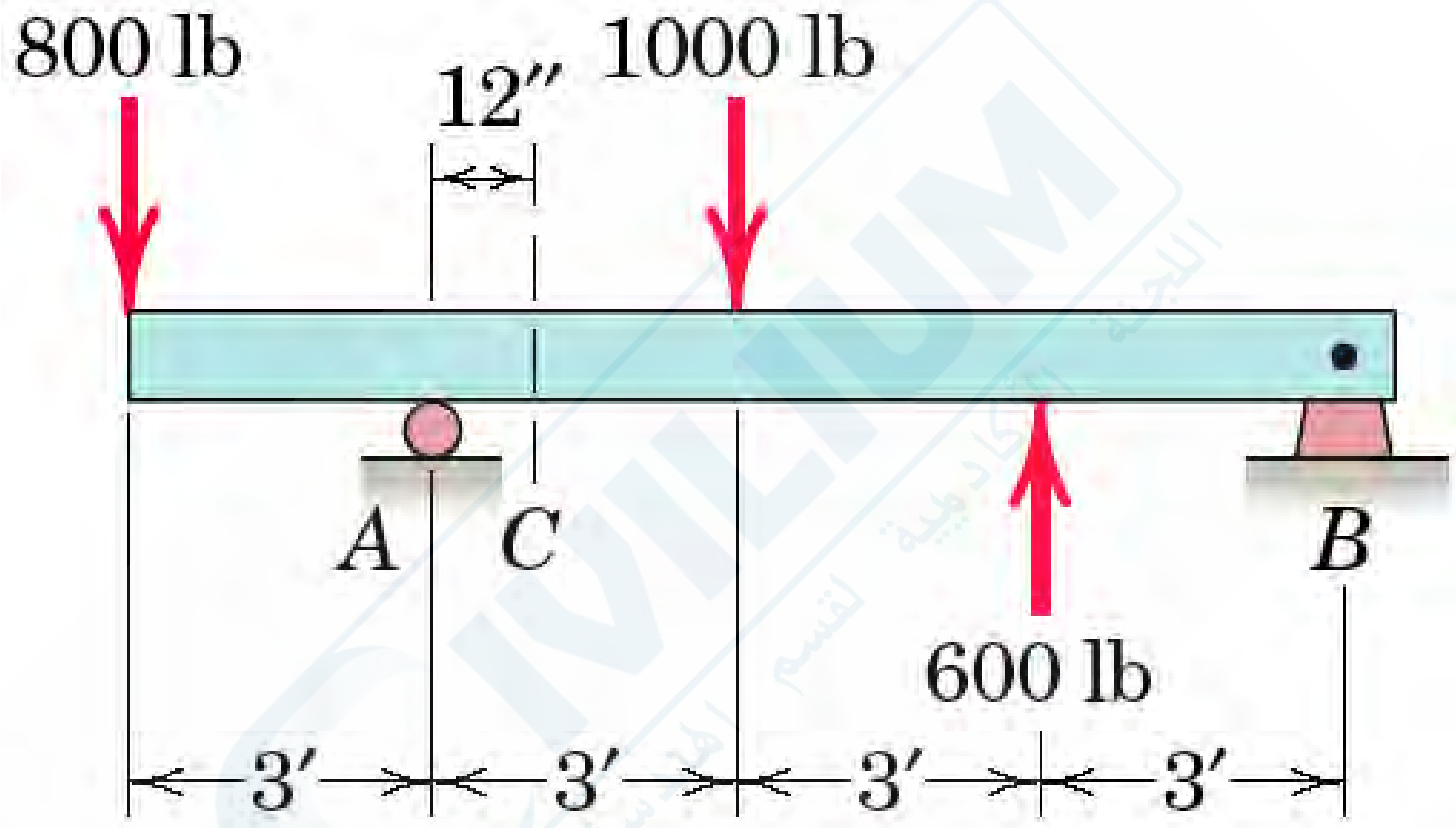
Problem 5/121

P_05_121



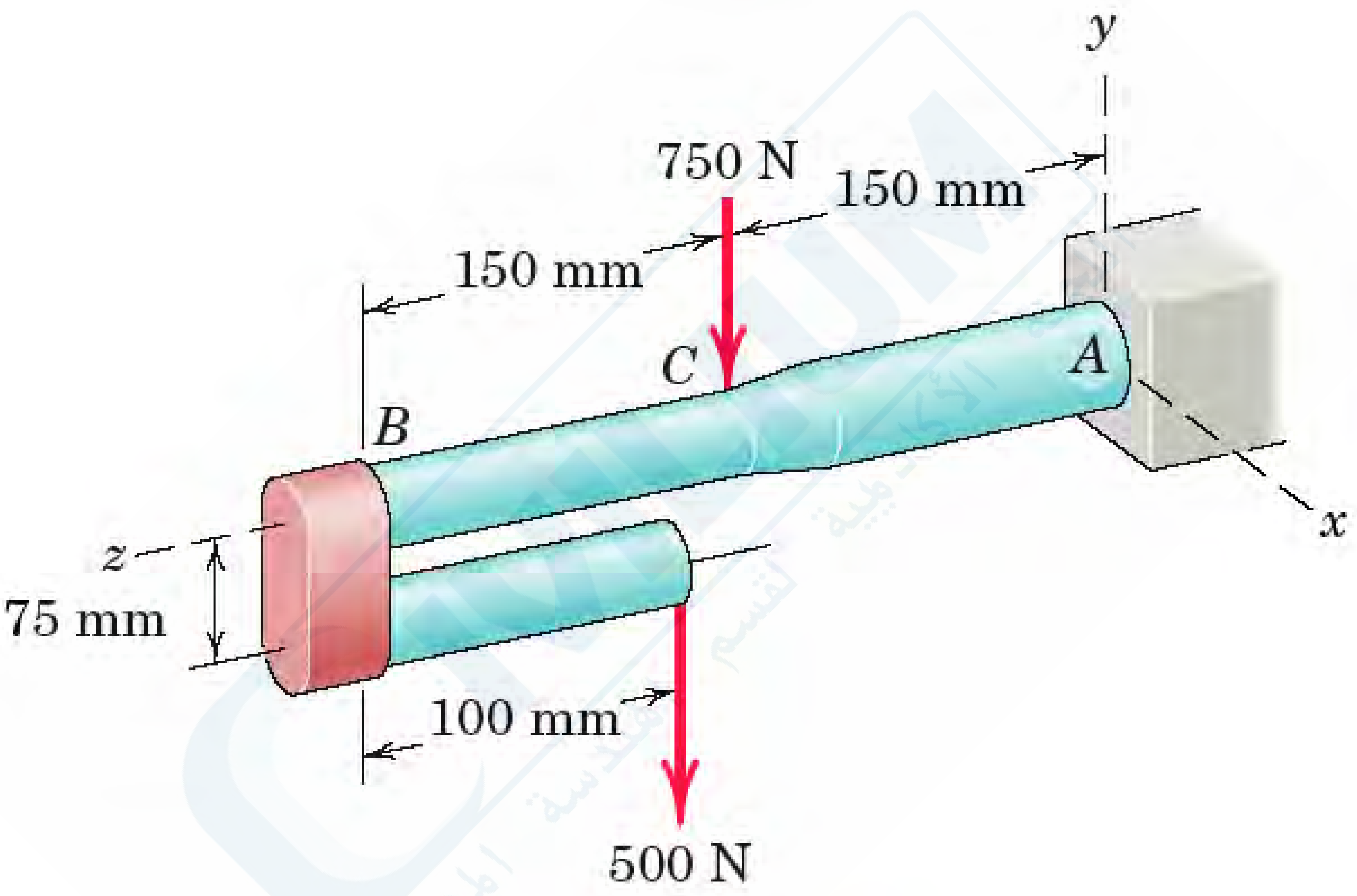
Problem 5/122

P_05_122



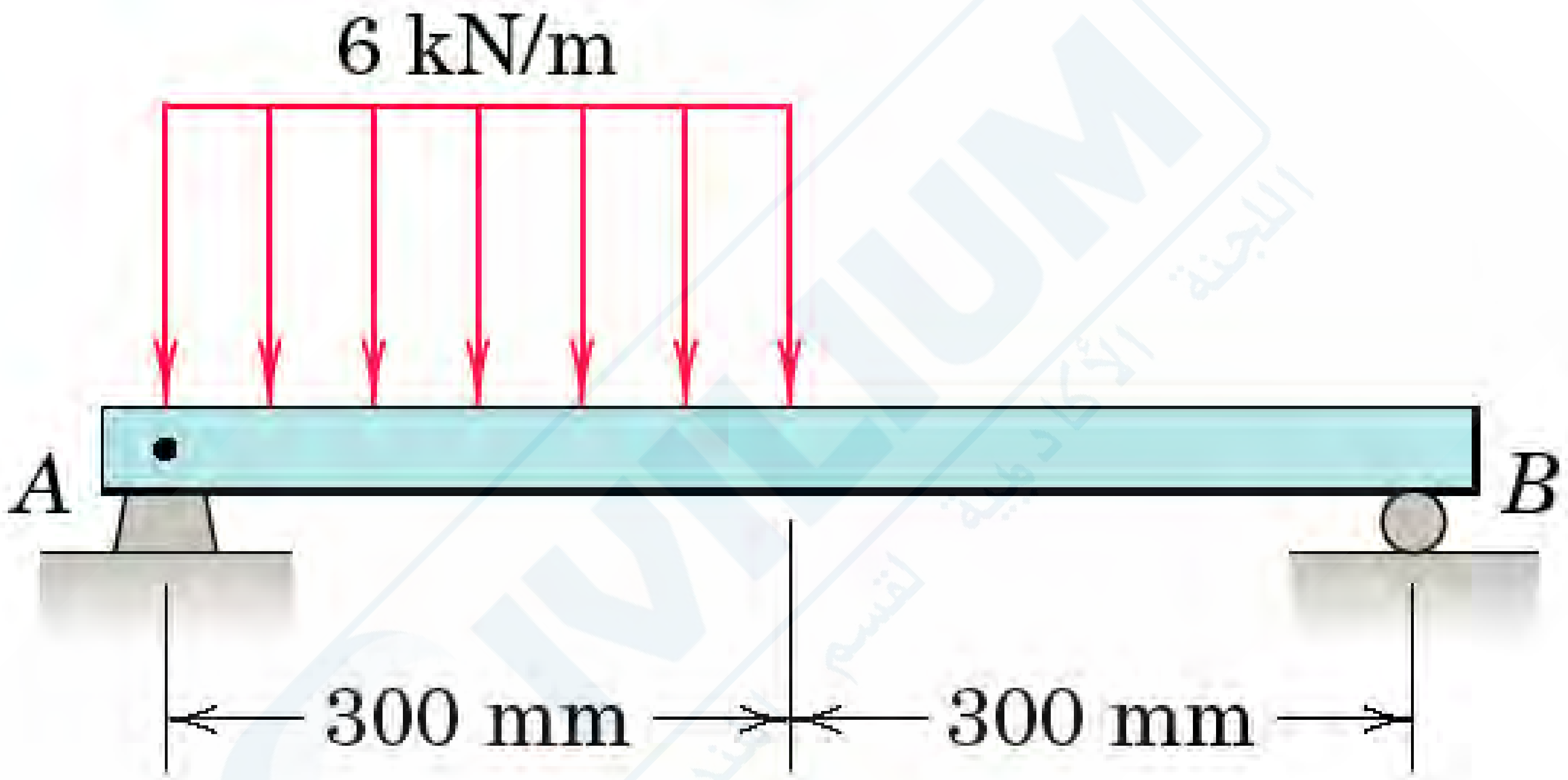
Problem 5/123

P_05_123



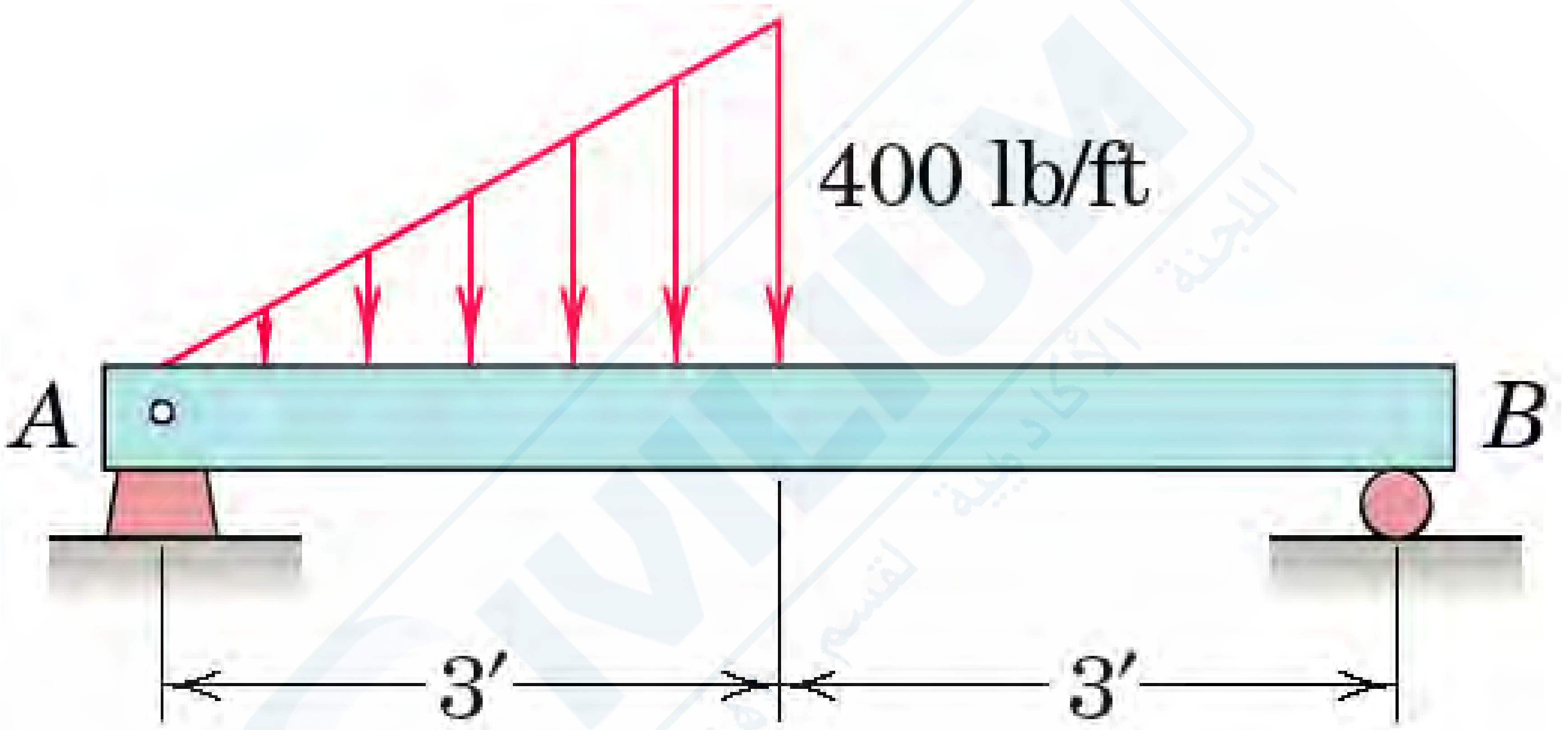
Problem 5/124

P_05_124



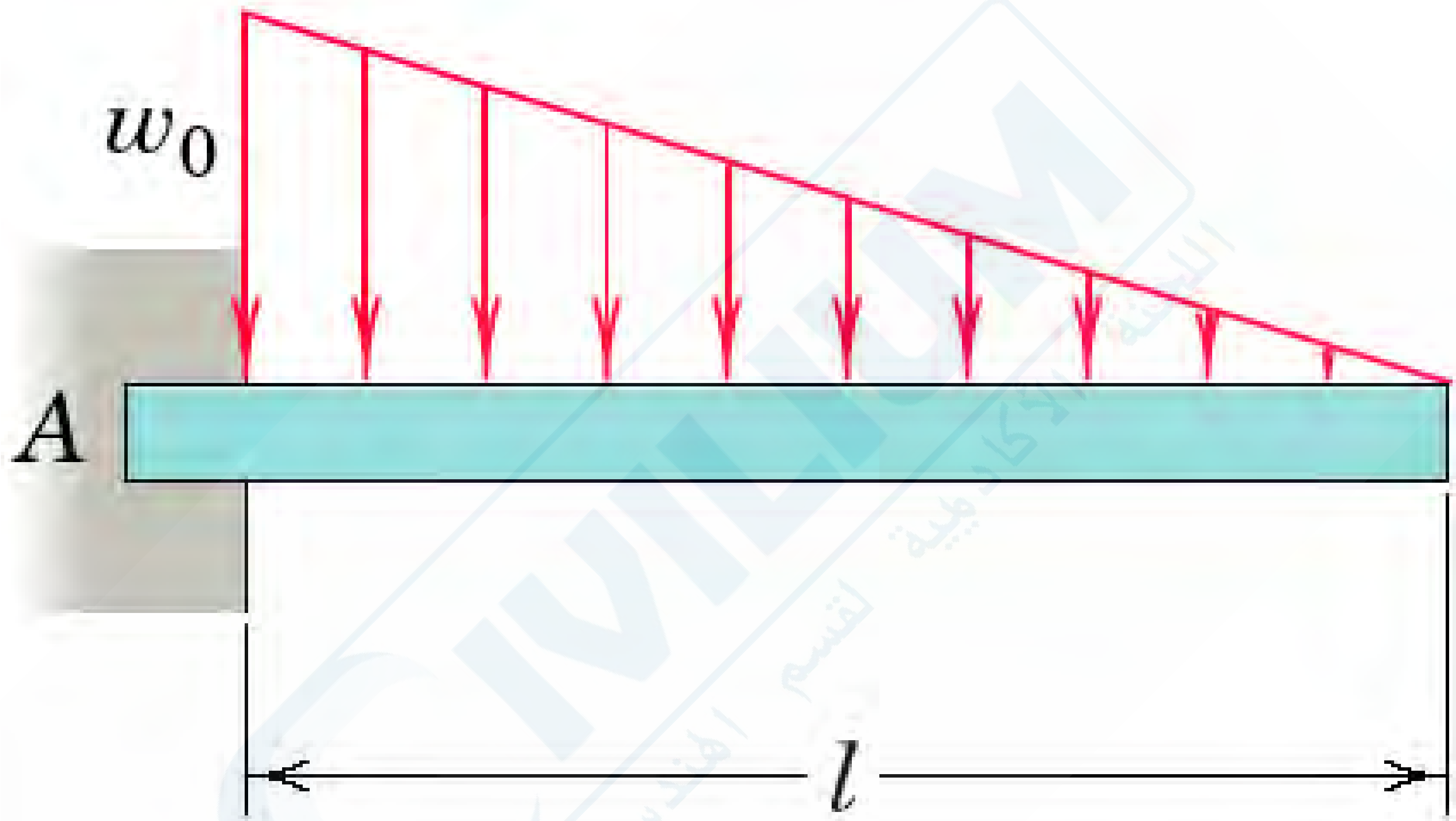
Problem 5/125

P_05_125



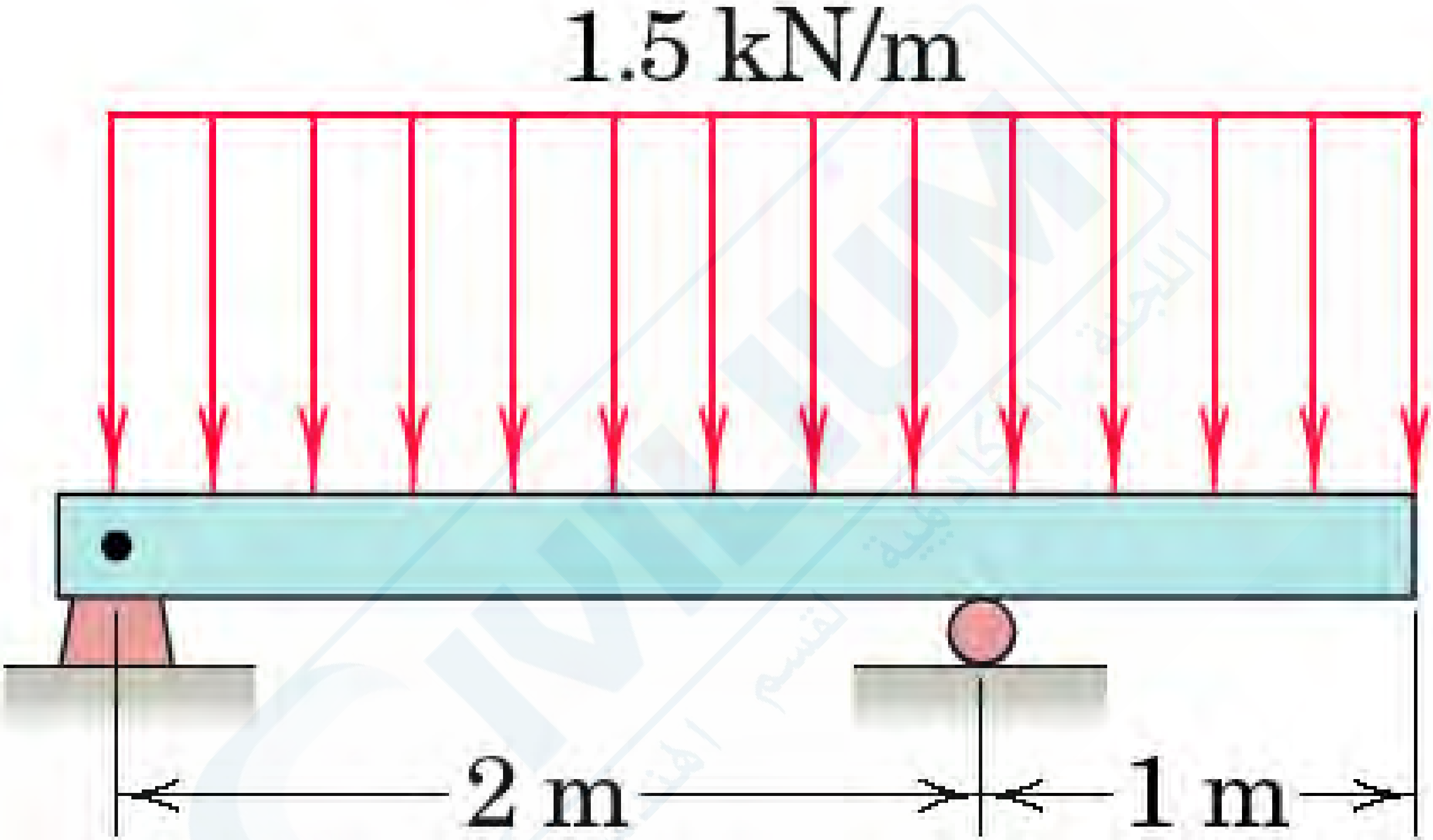
Problem 5/126

P_05_126



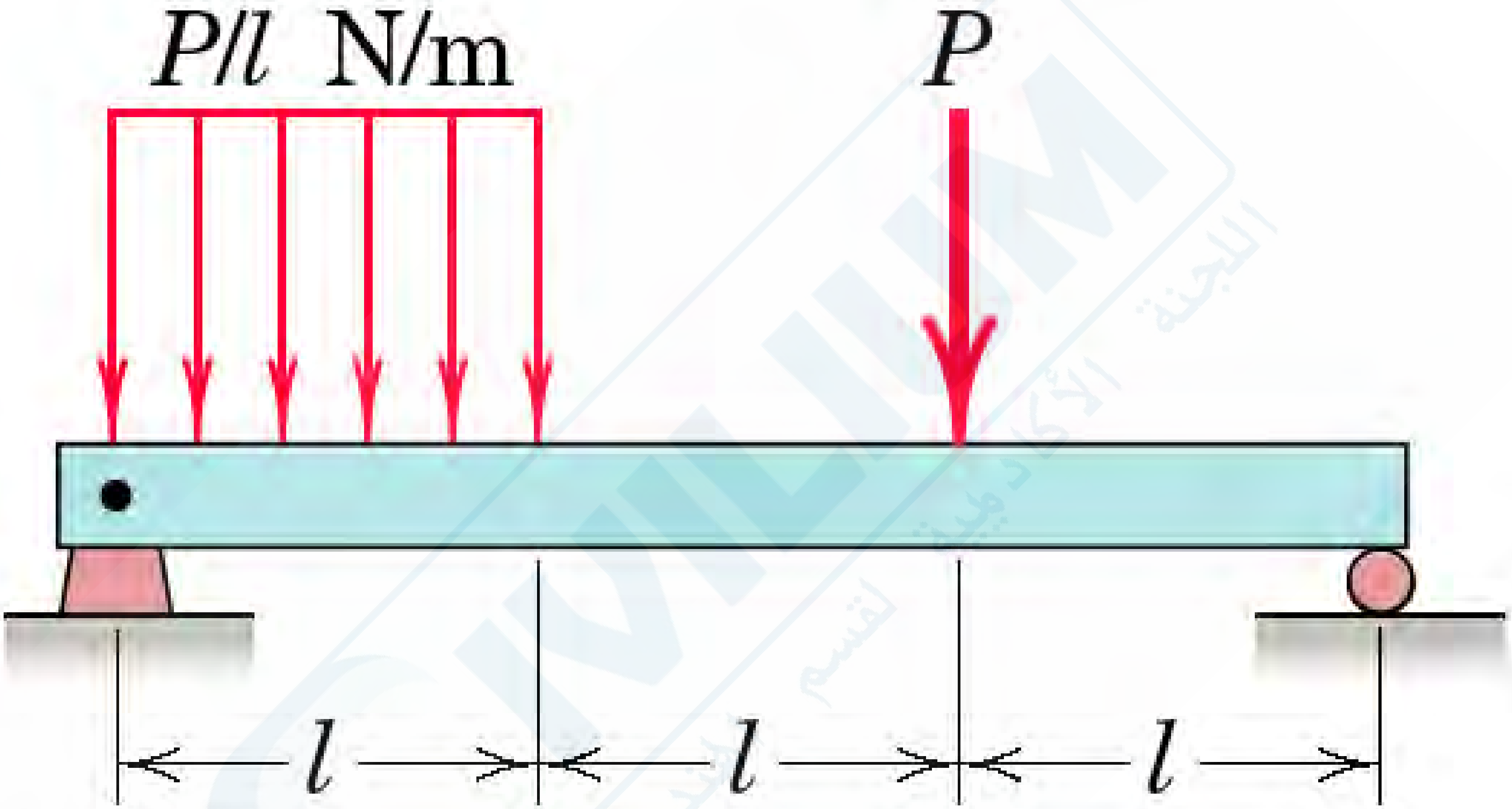
Problem 5/127

P 05 127



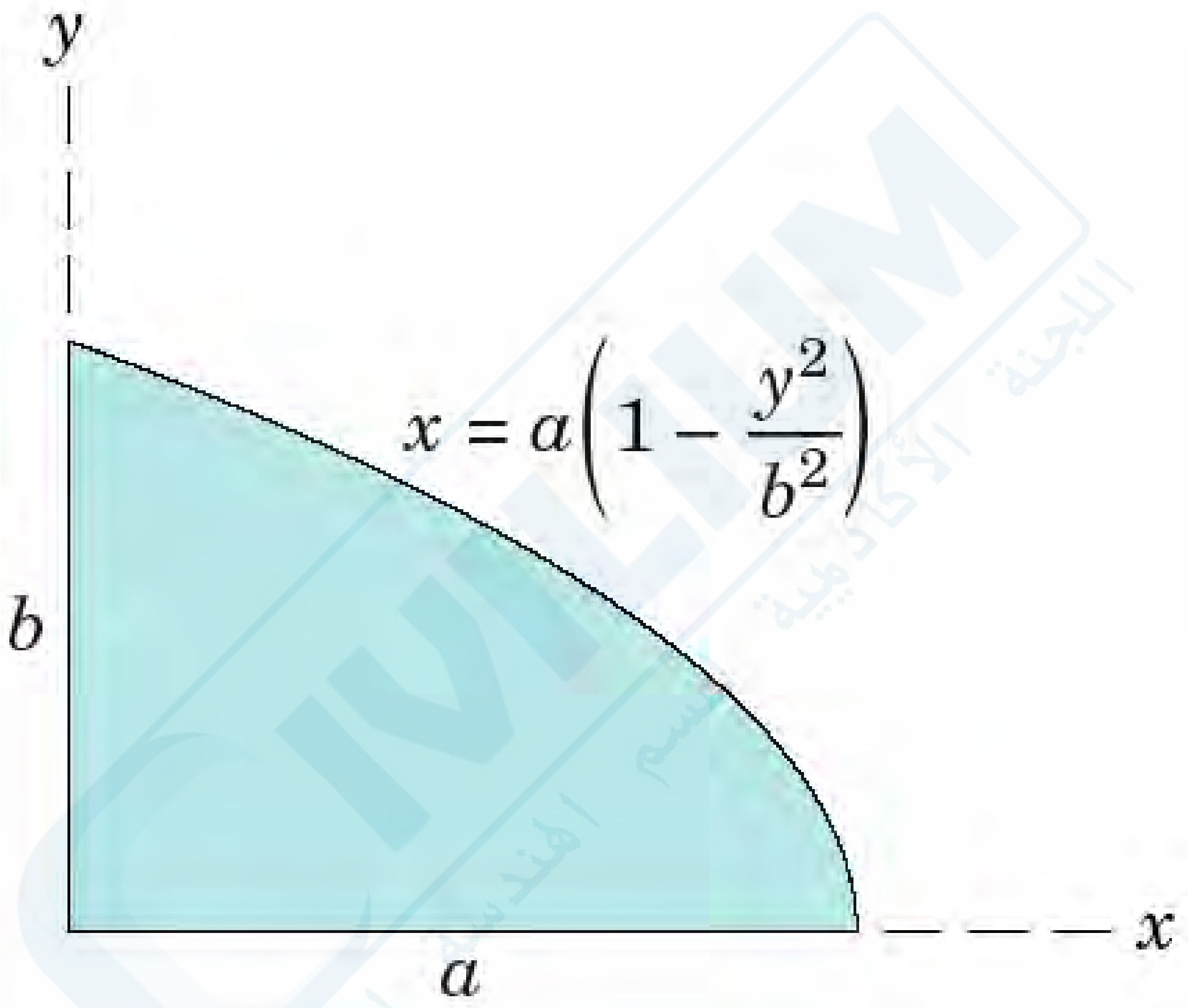
Problem 5/128

P 05 128

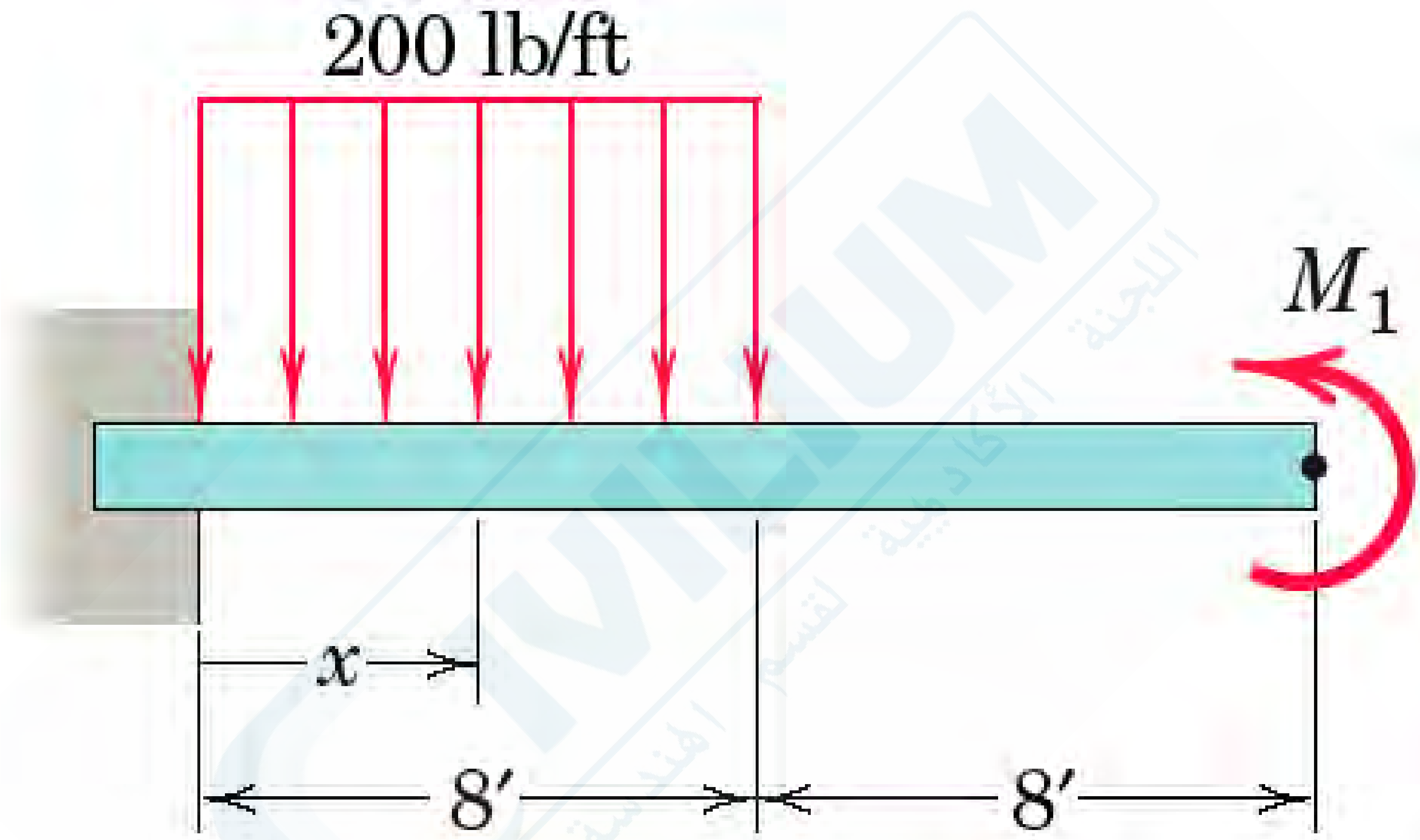


Problem 5/129

P_05_129

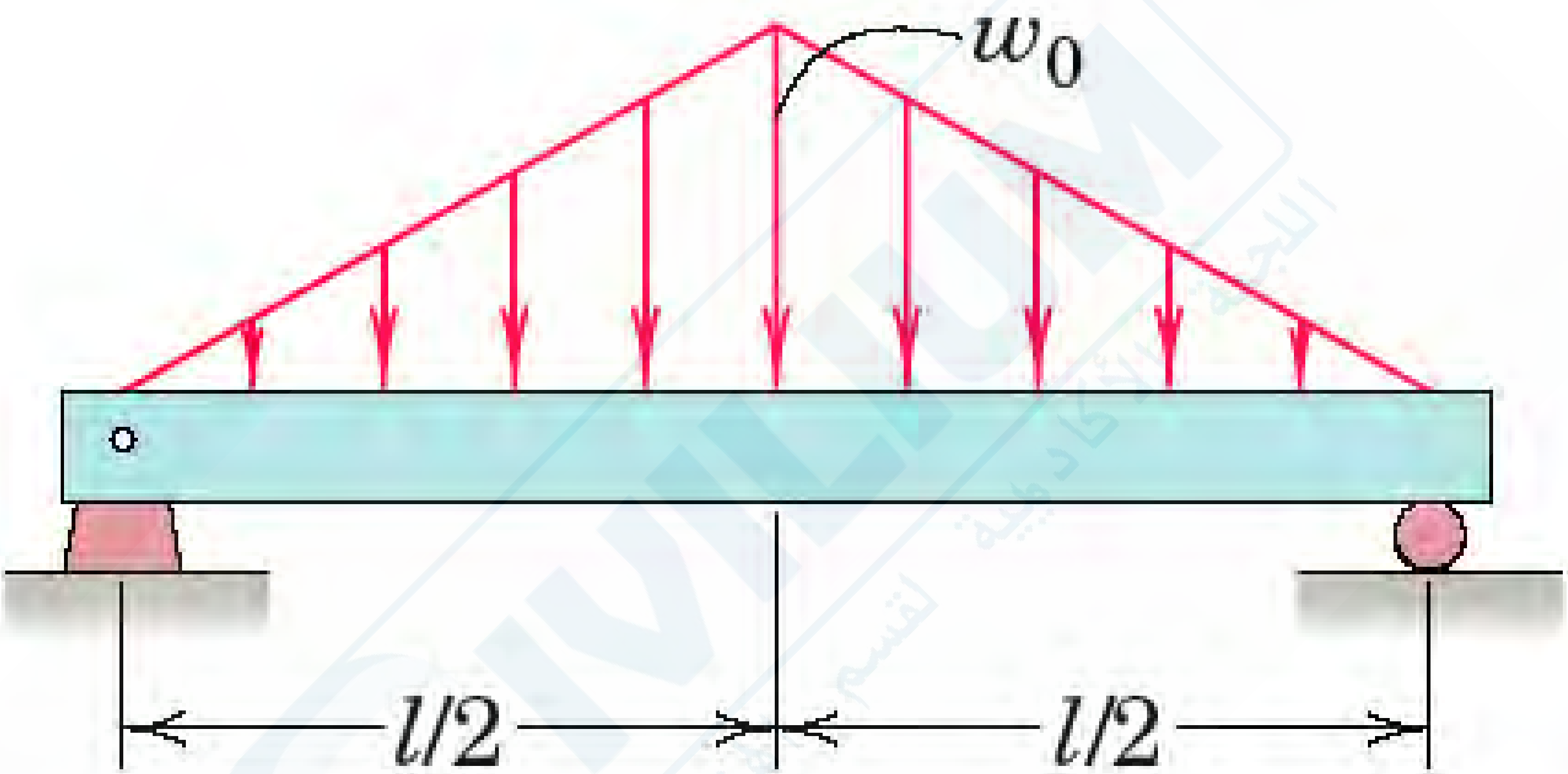


Problem 5/13



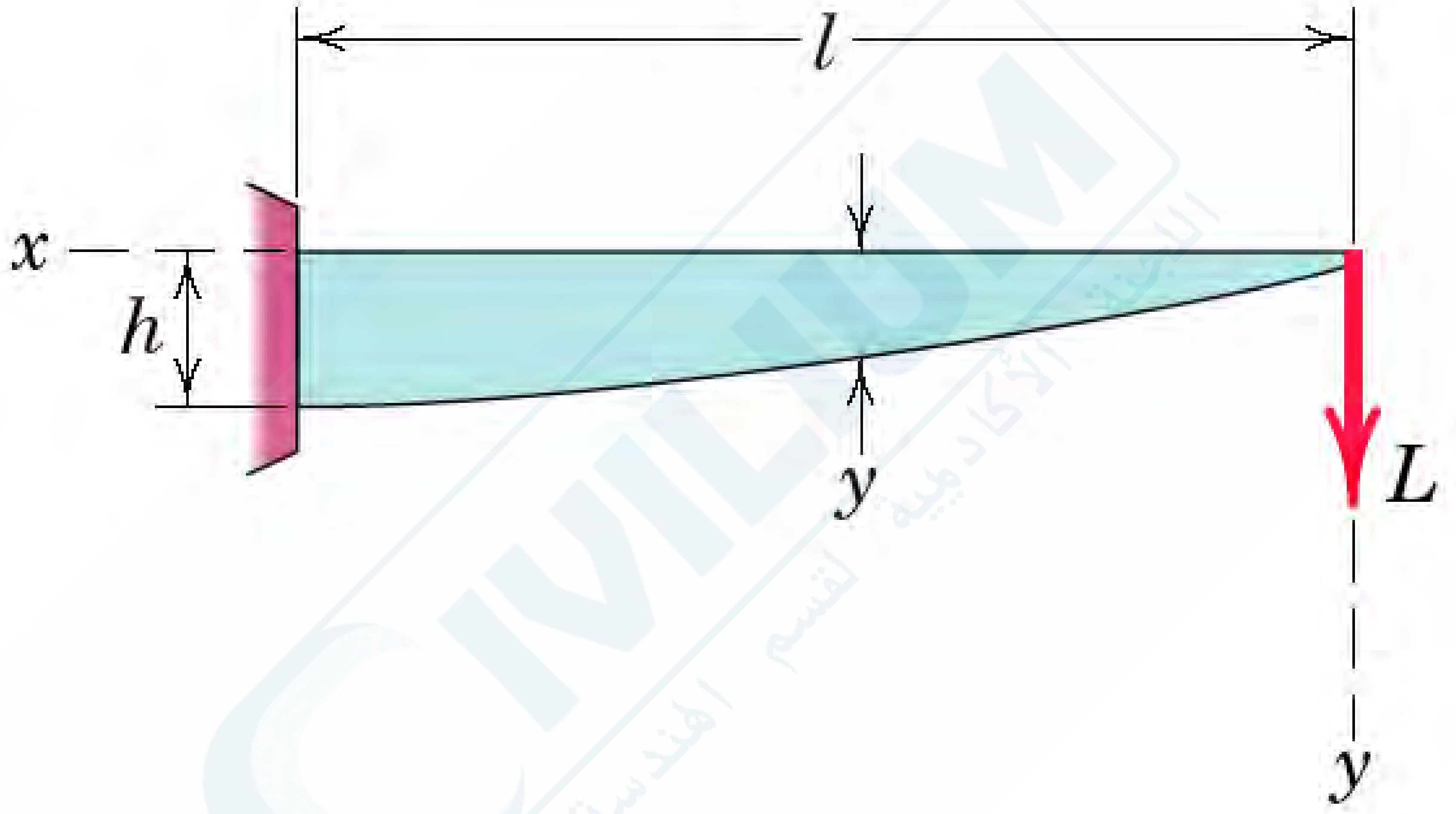
Problem 5/130

P_05_130



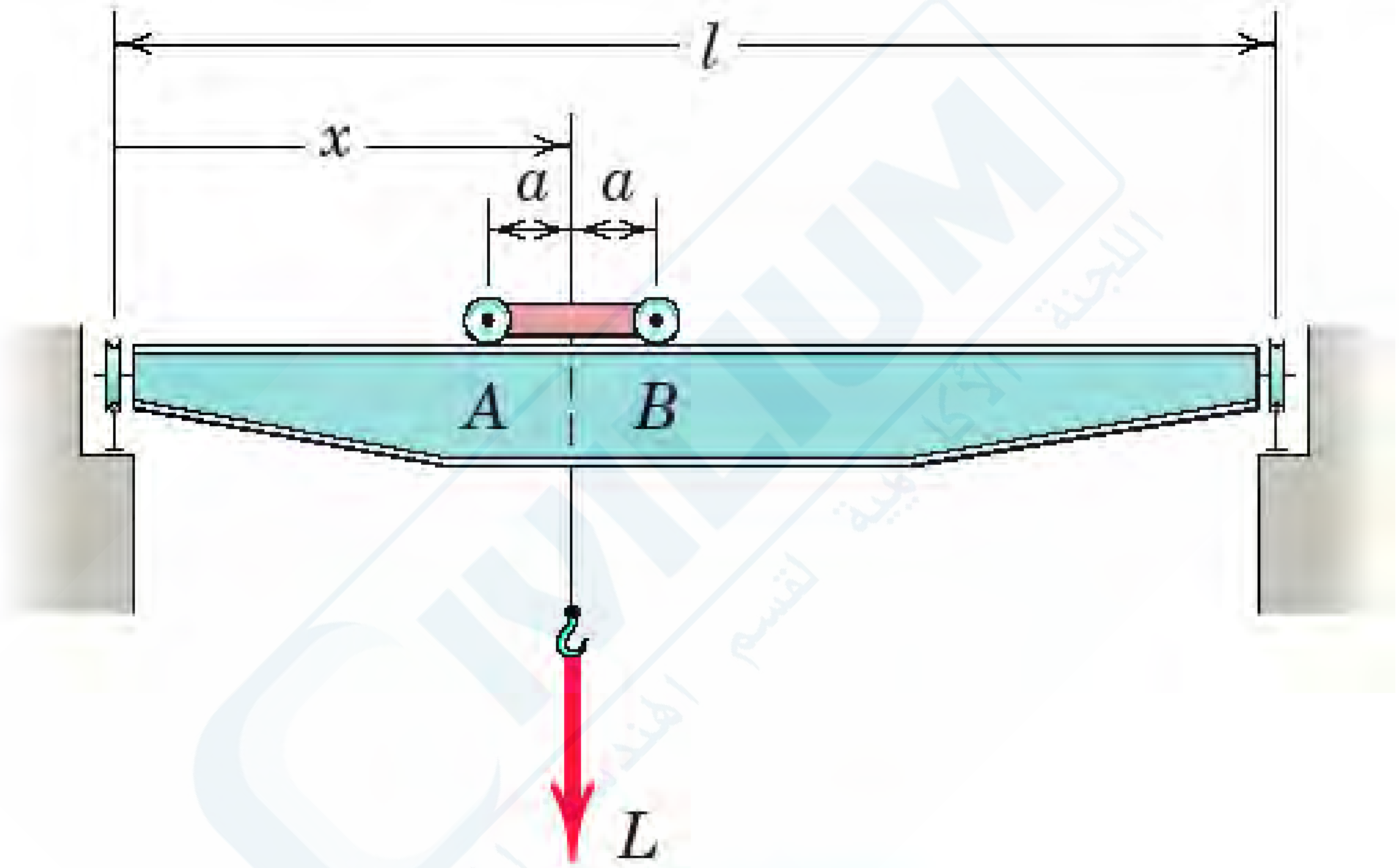
Problem 5/131

P_05_131



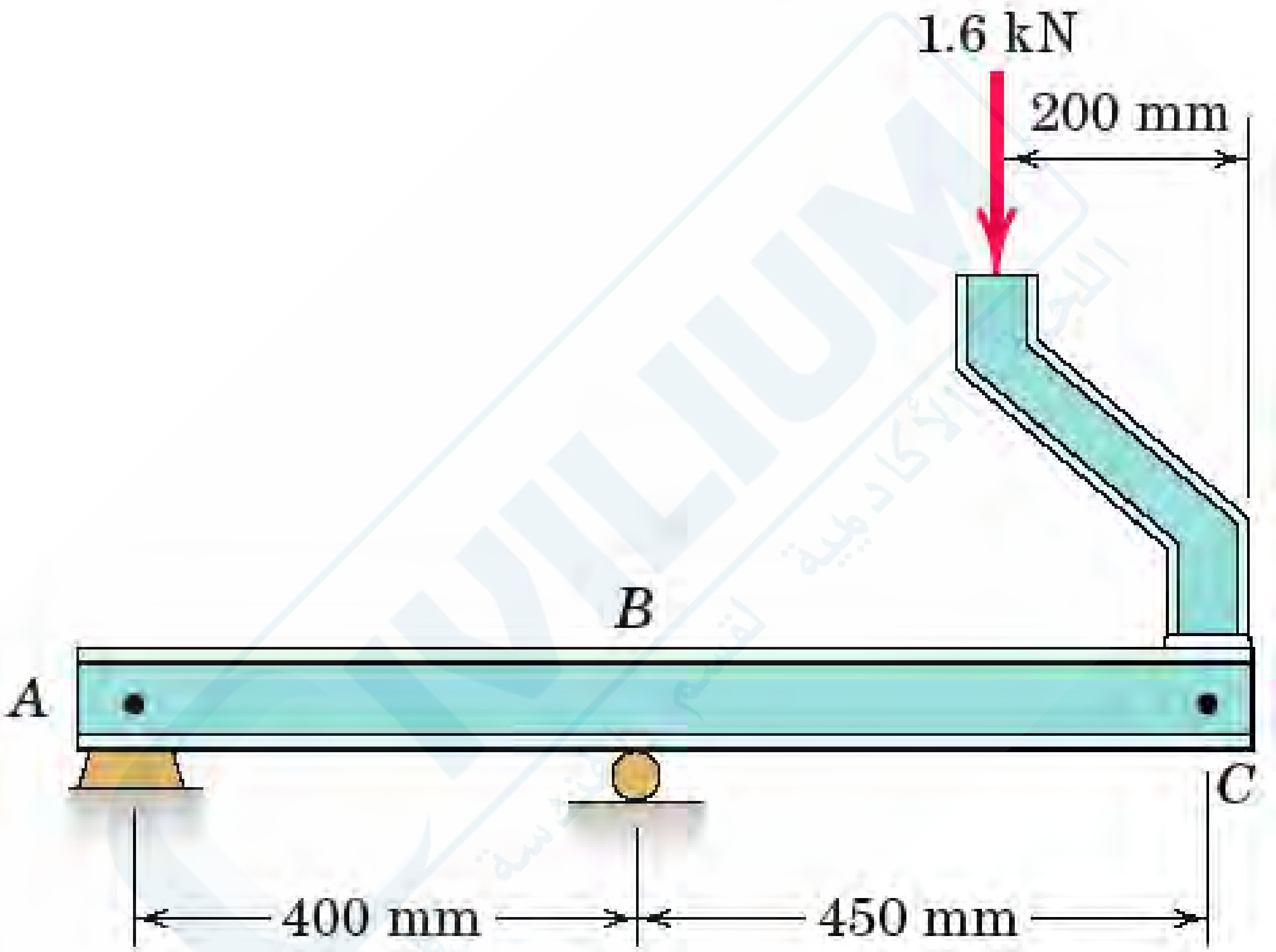
Problem 5/133

P_05_133



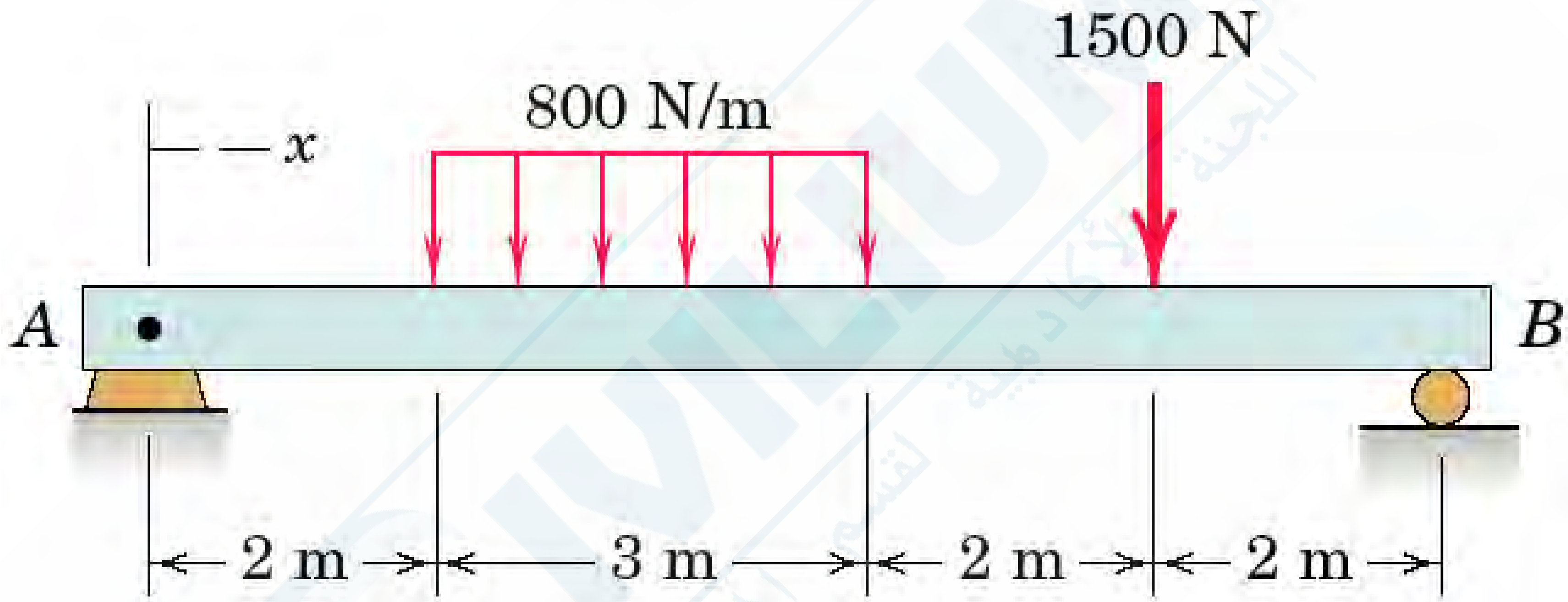
Problem 5/134

P_05_134



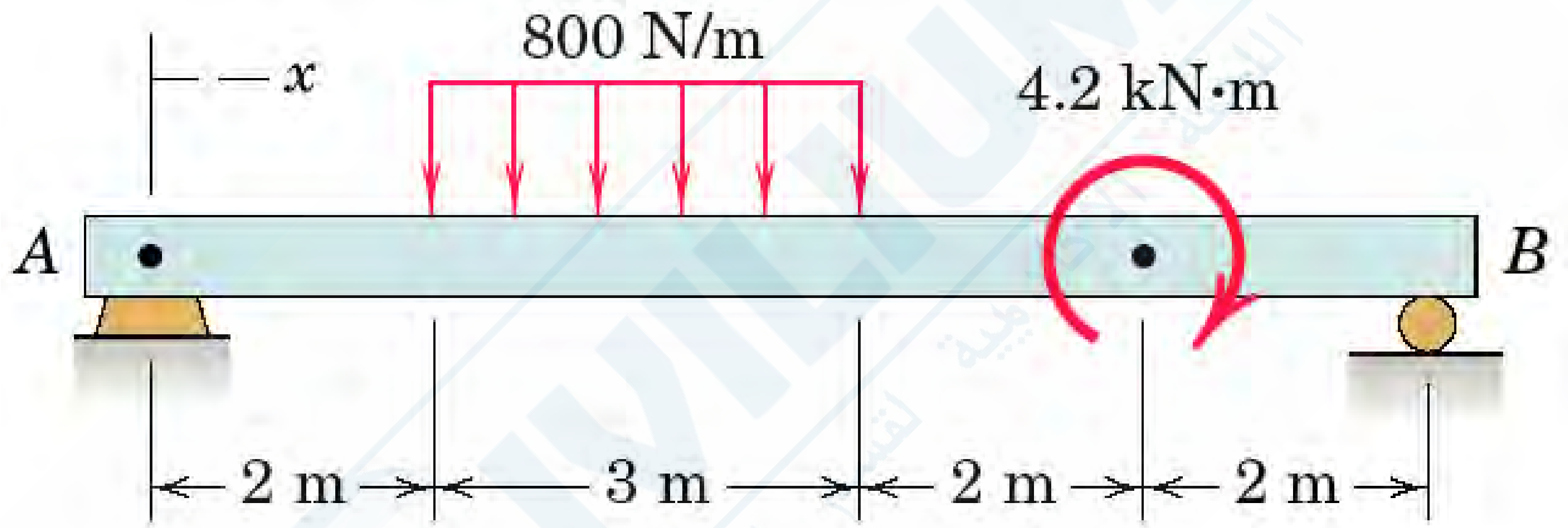
Problem 5/135

P_05_135



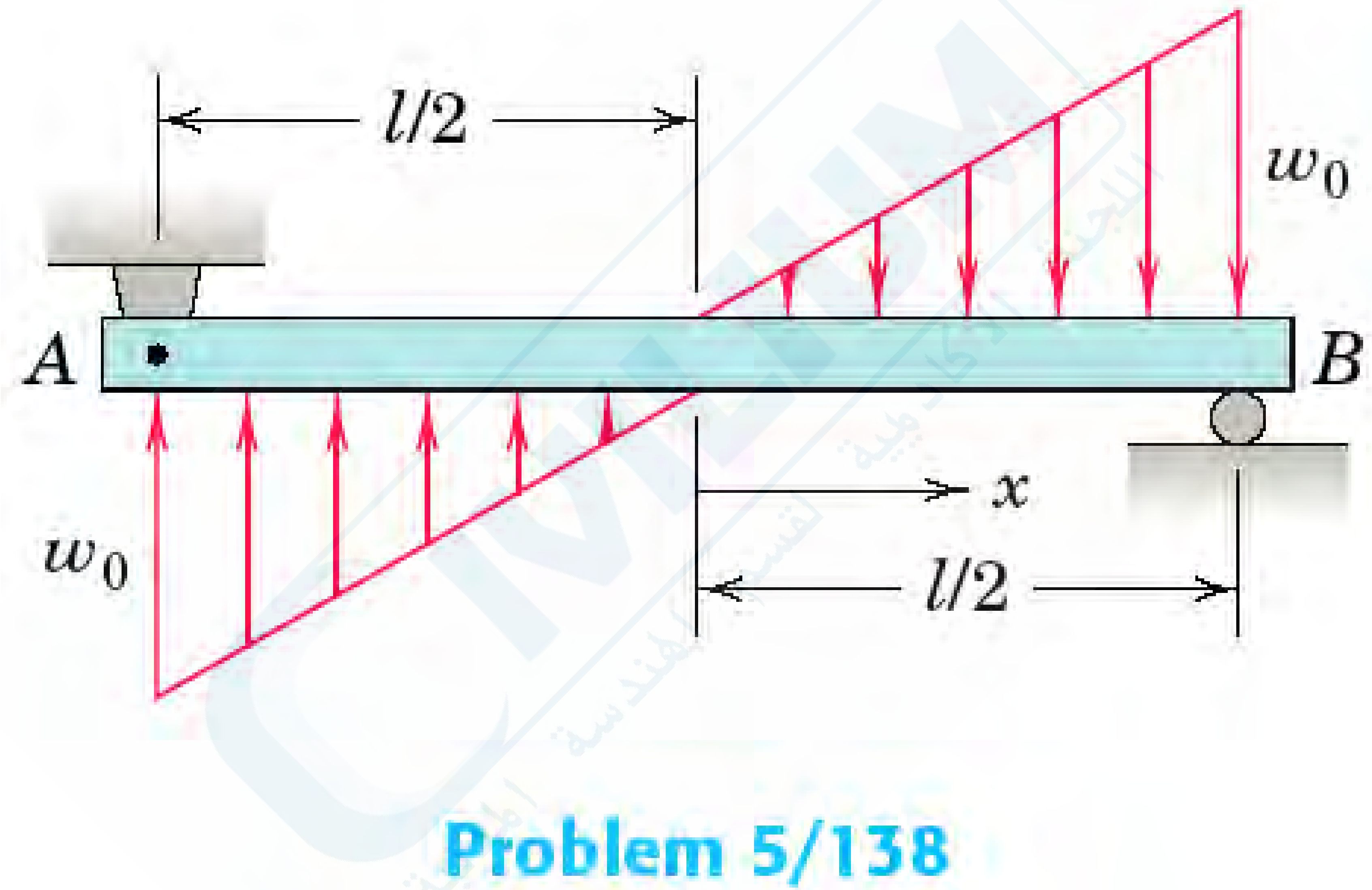
Problem 5/136

P_05_136

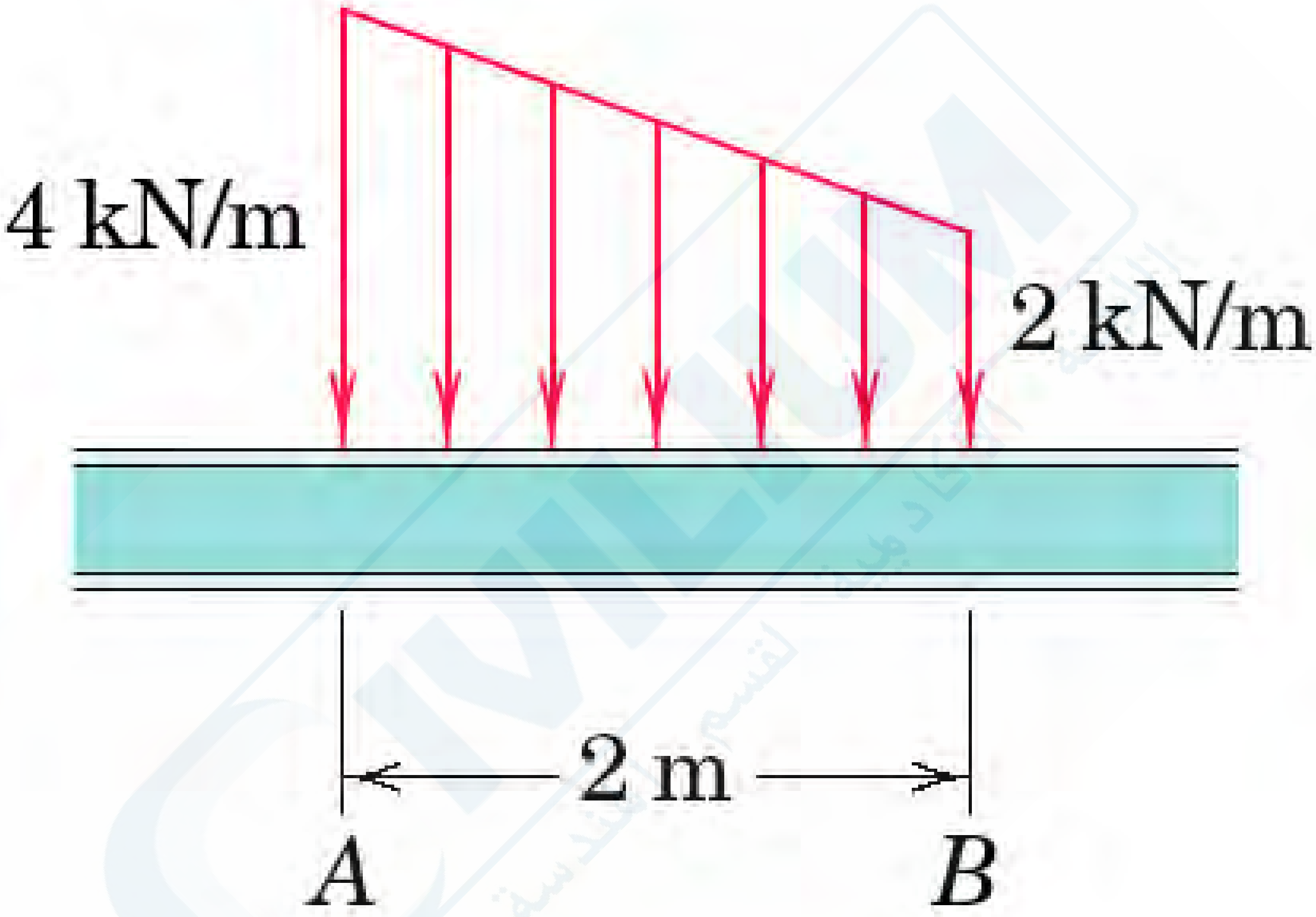


Problem 5/137

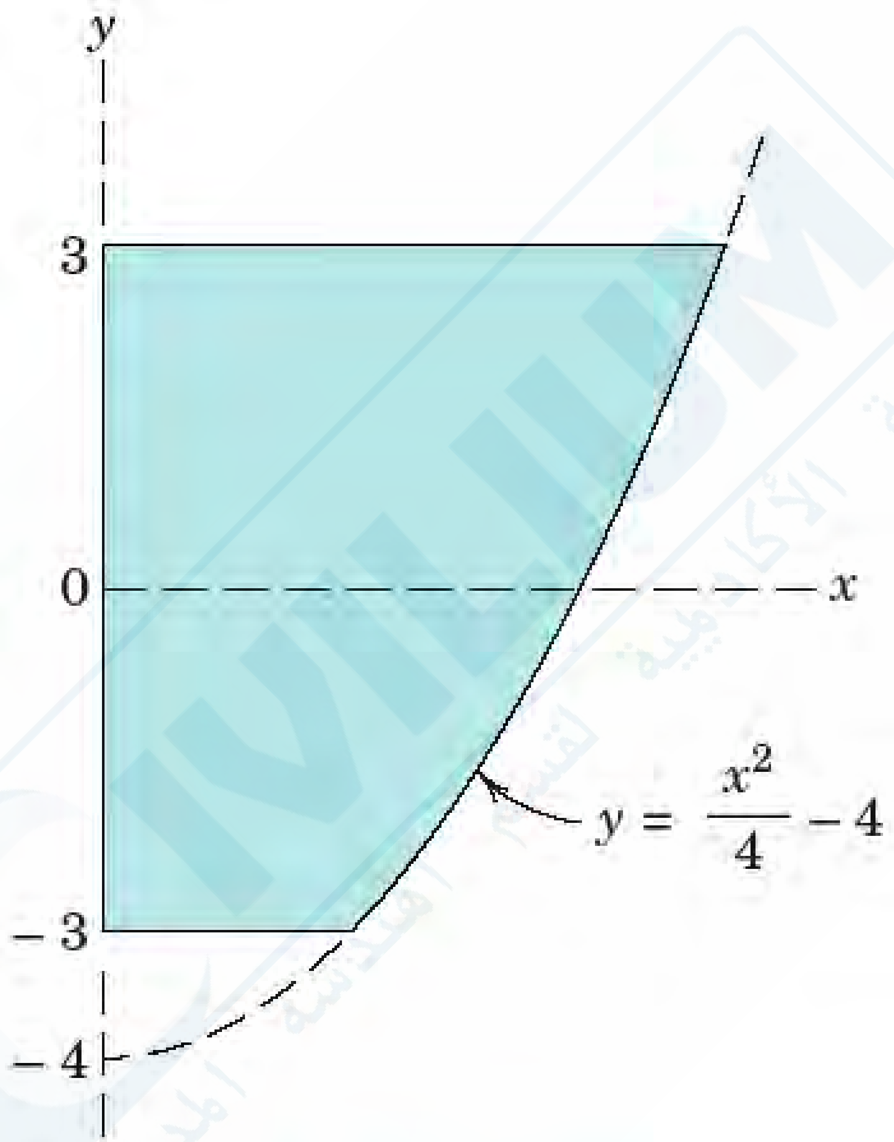
P_05_137



P_05_138

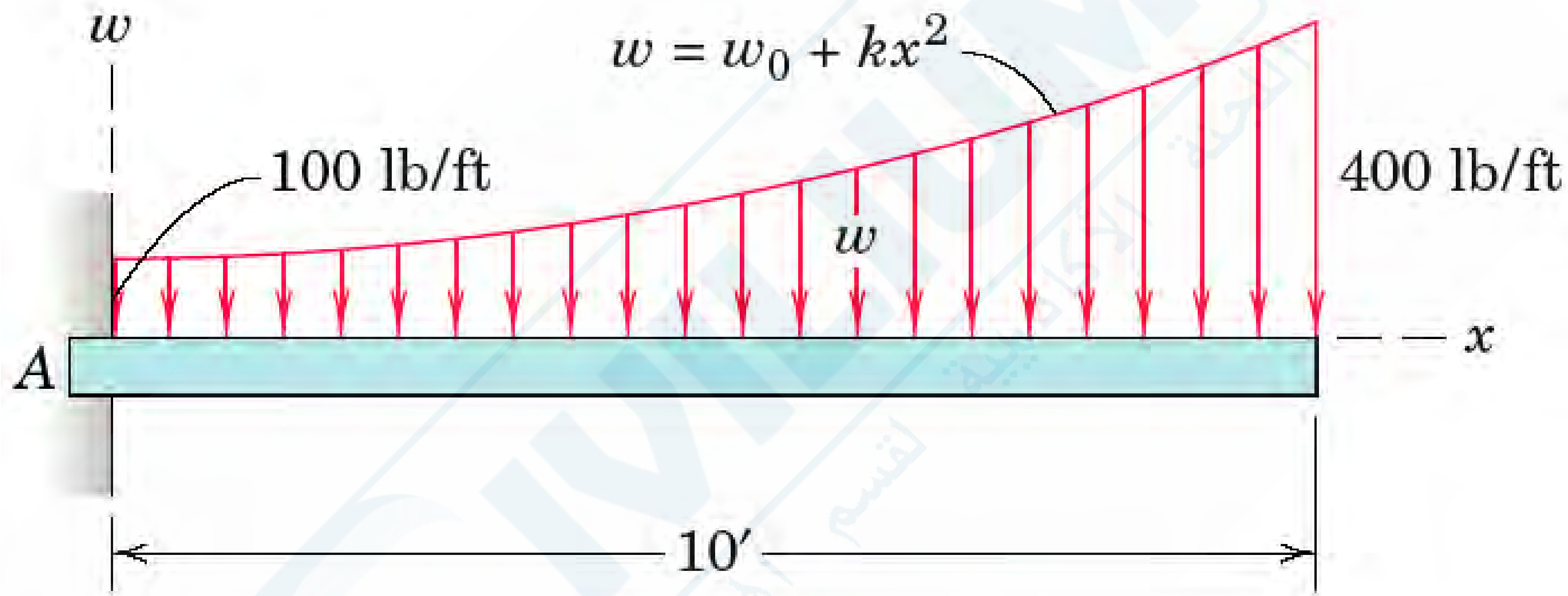


Problem 5/139



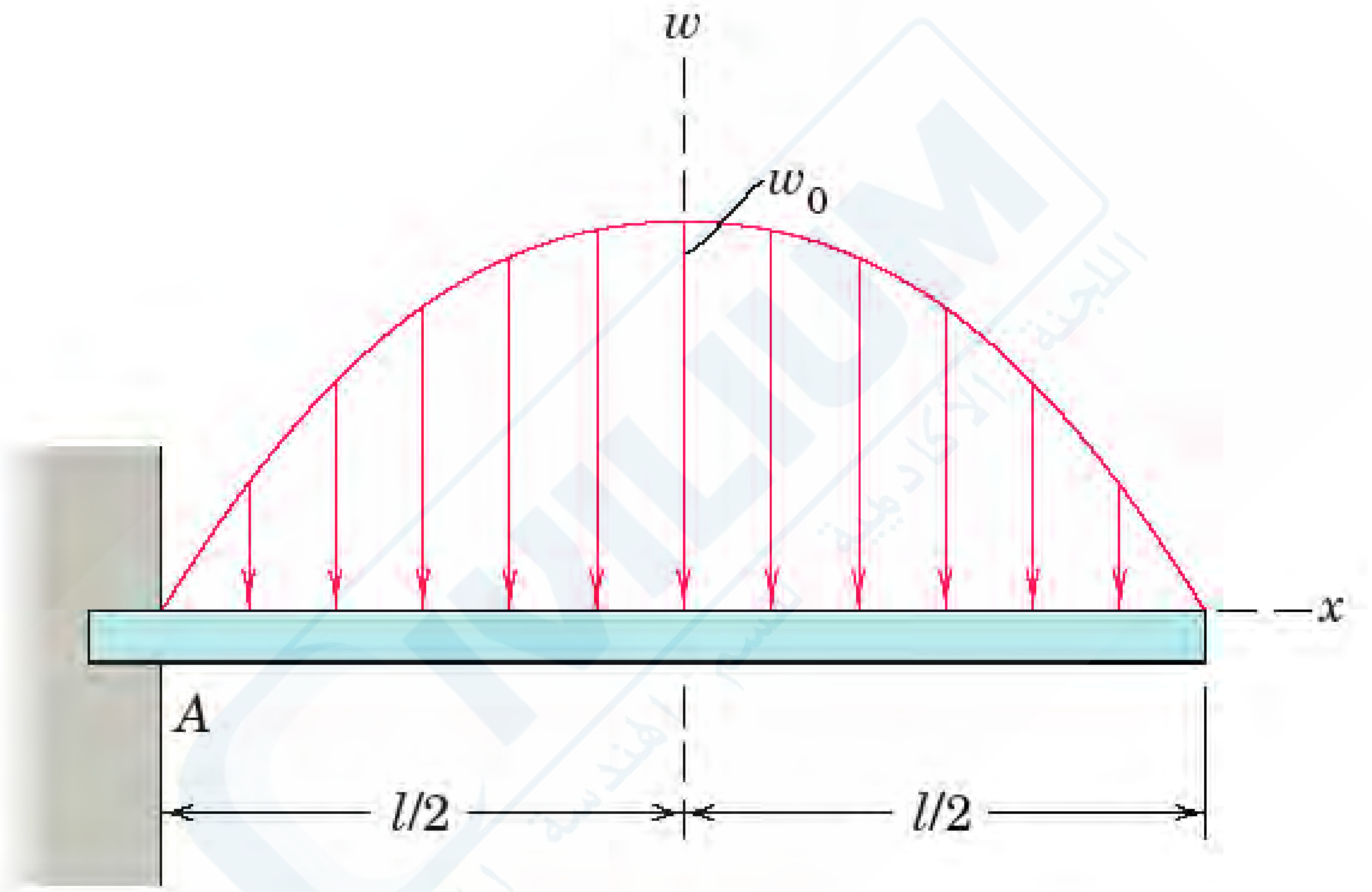
Problem 5/14

P_05_14



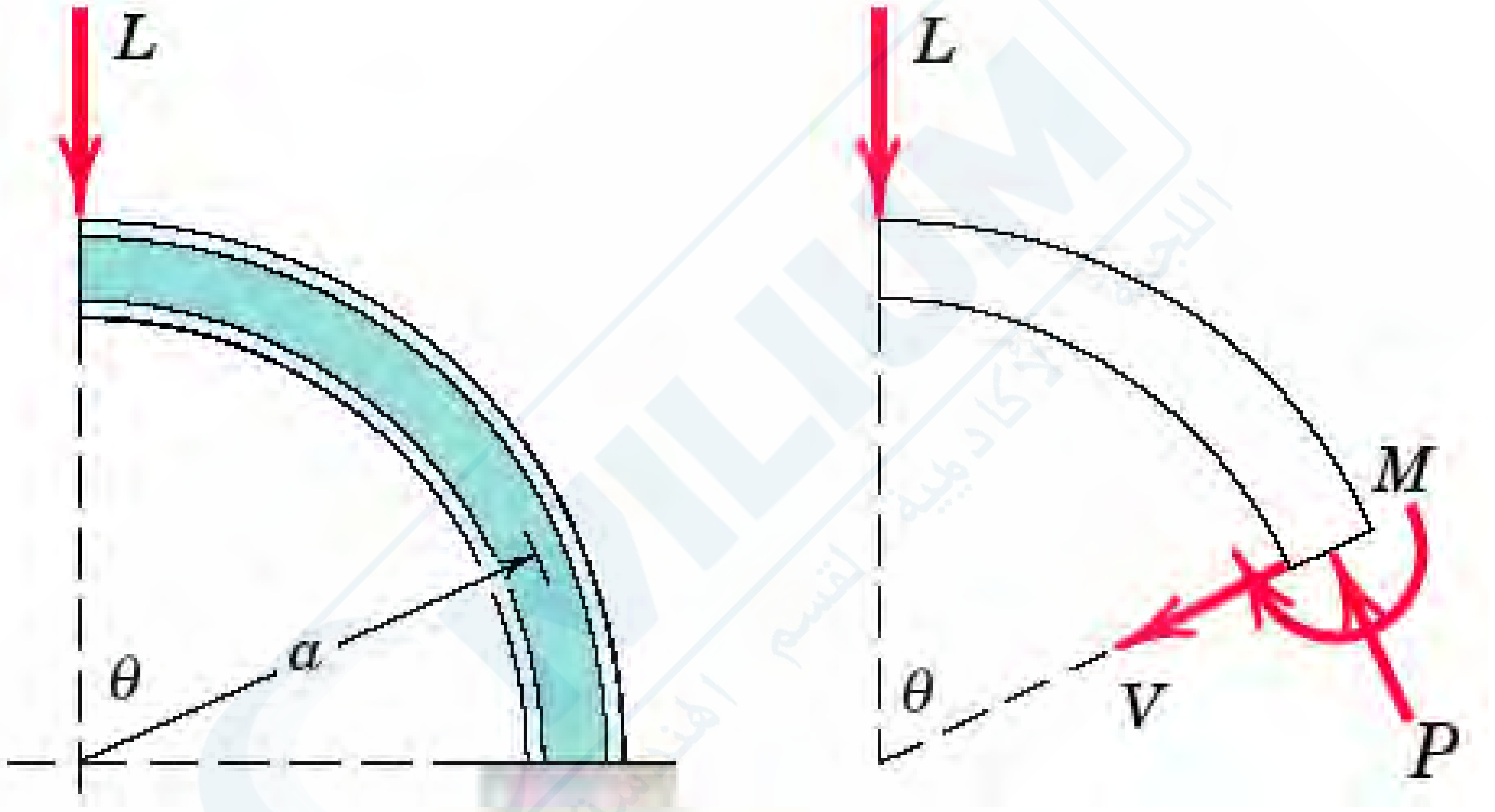
Problem 5/140

P_05_140



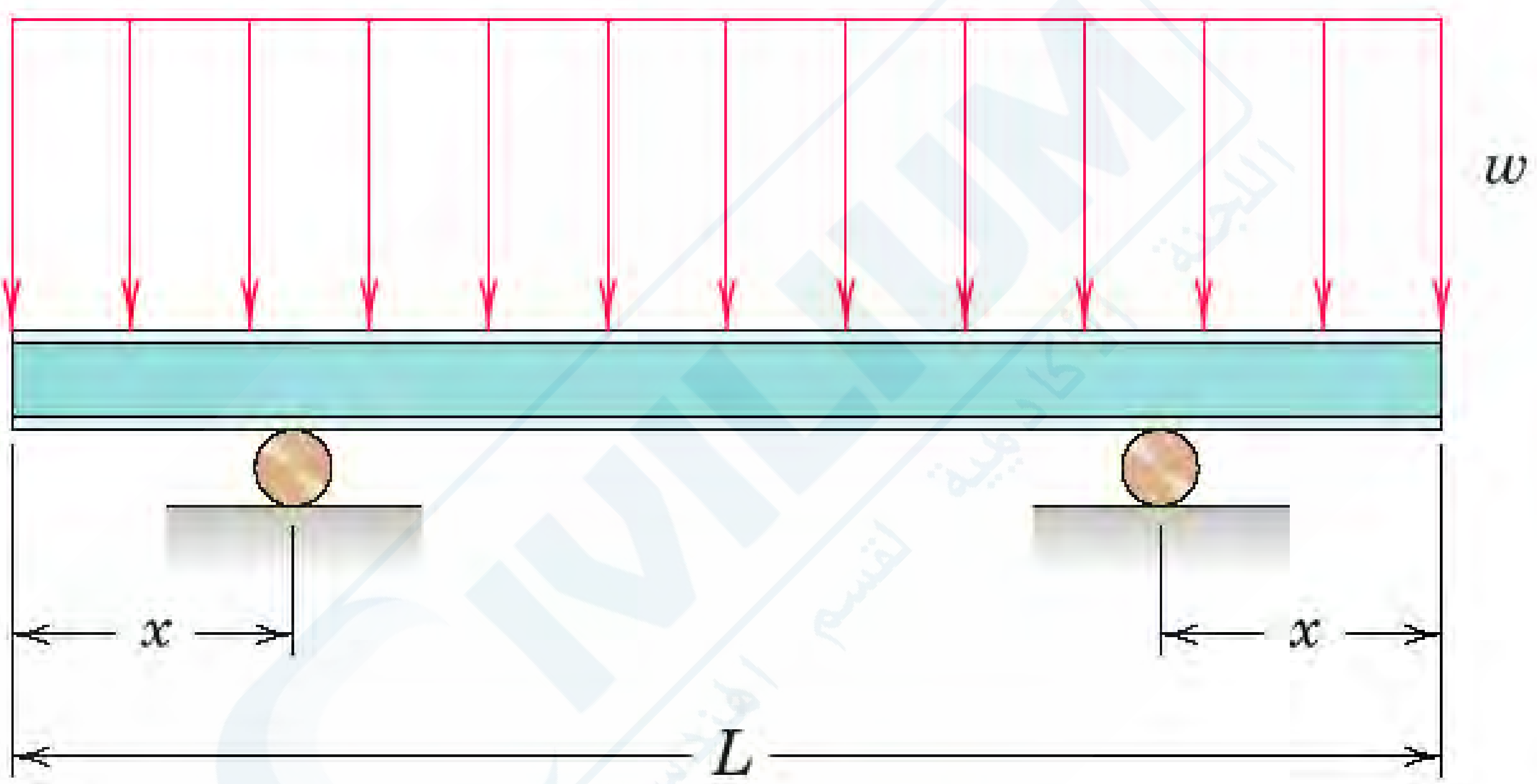
Problem 5/141

P 05 141



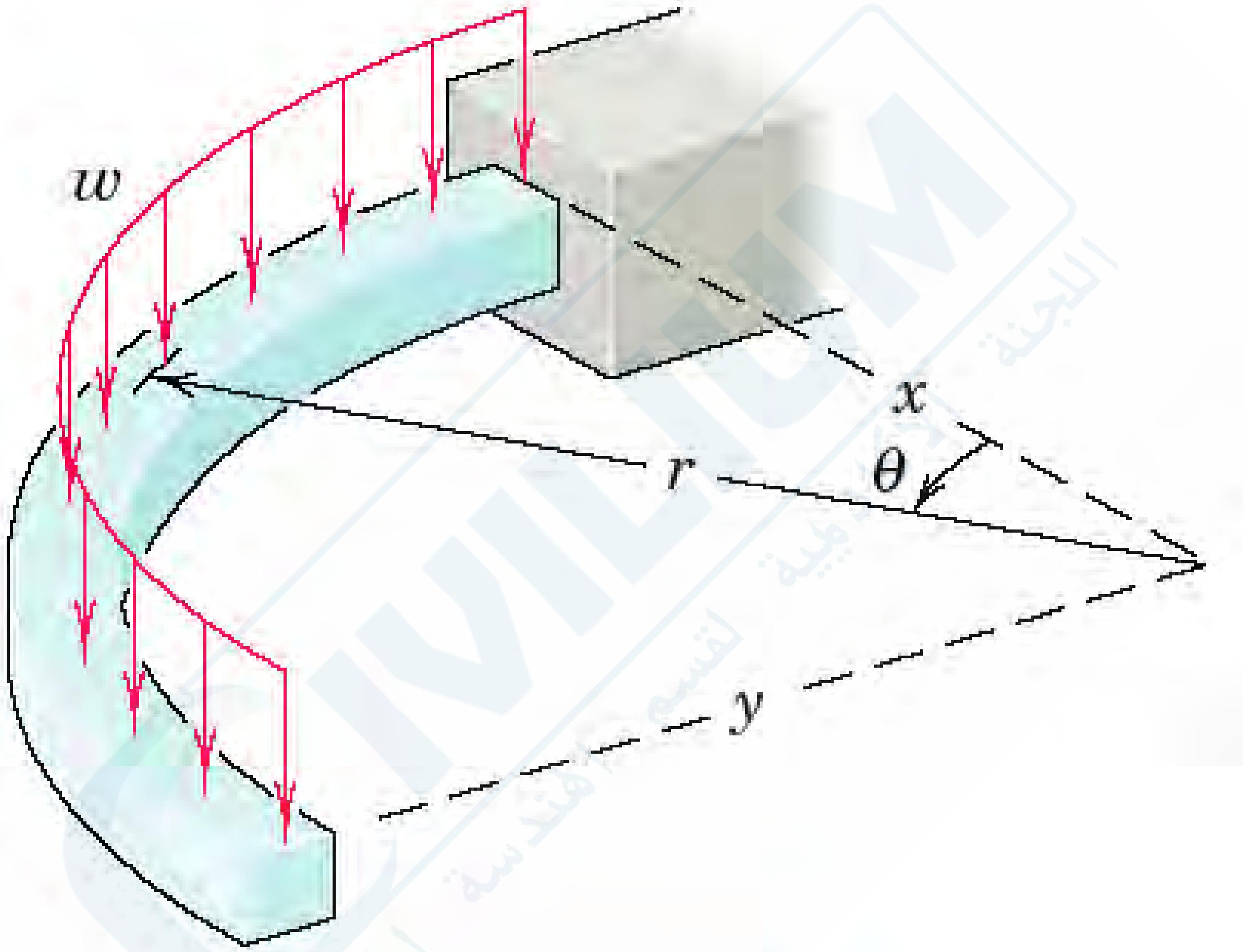
Problem 5/142

P_05_142



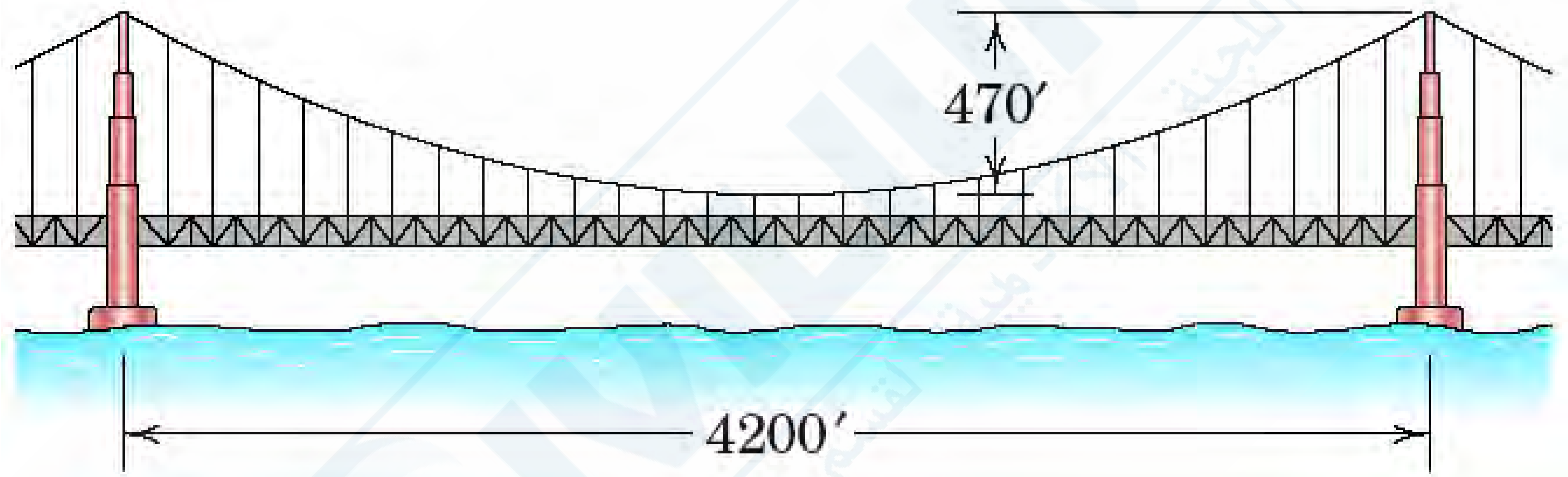
Problem 5/143

P_05_143



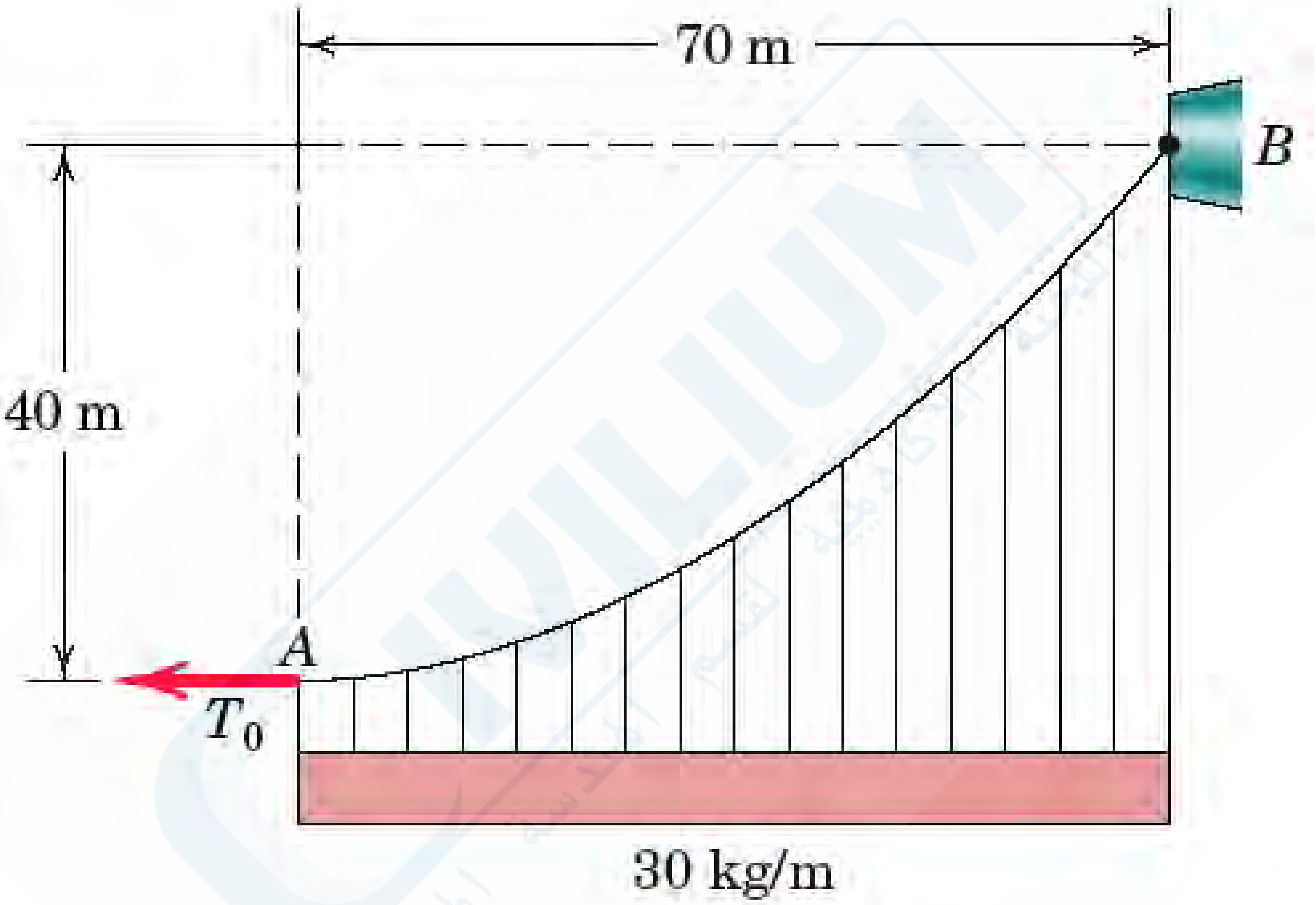
Problem 5/144

P 05 144



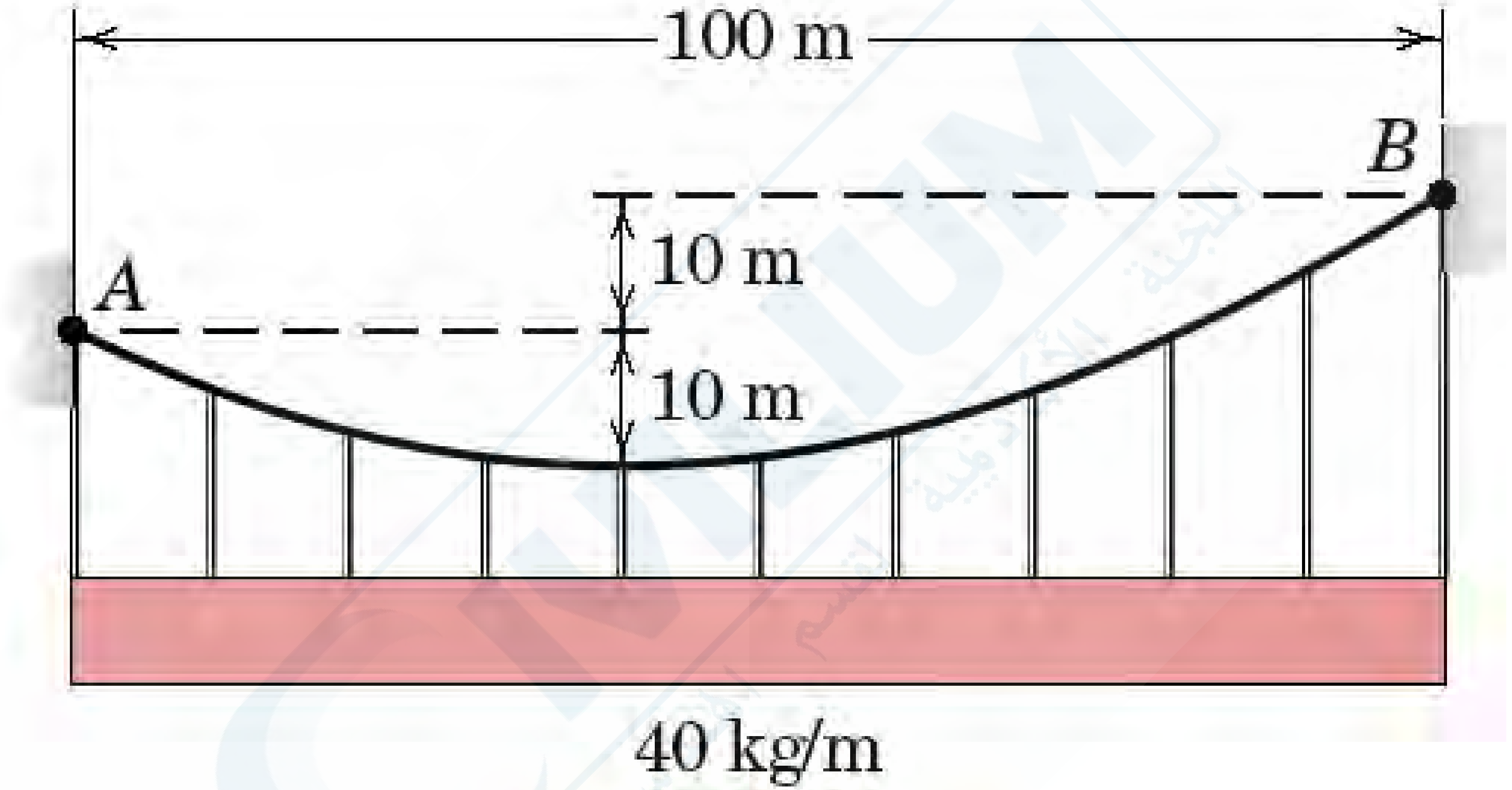
Problem 5/146

P_05_146



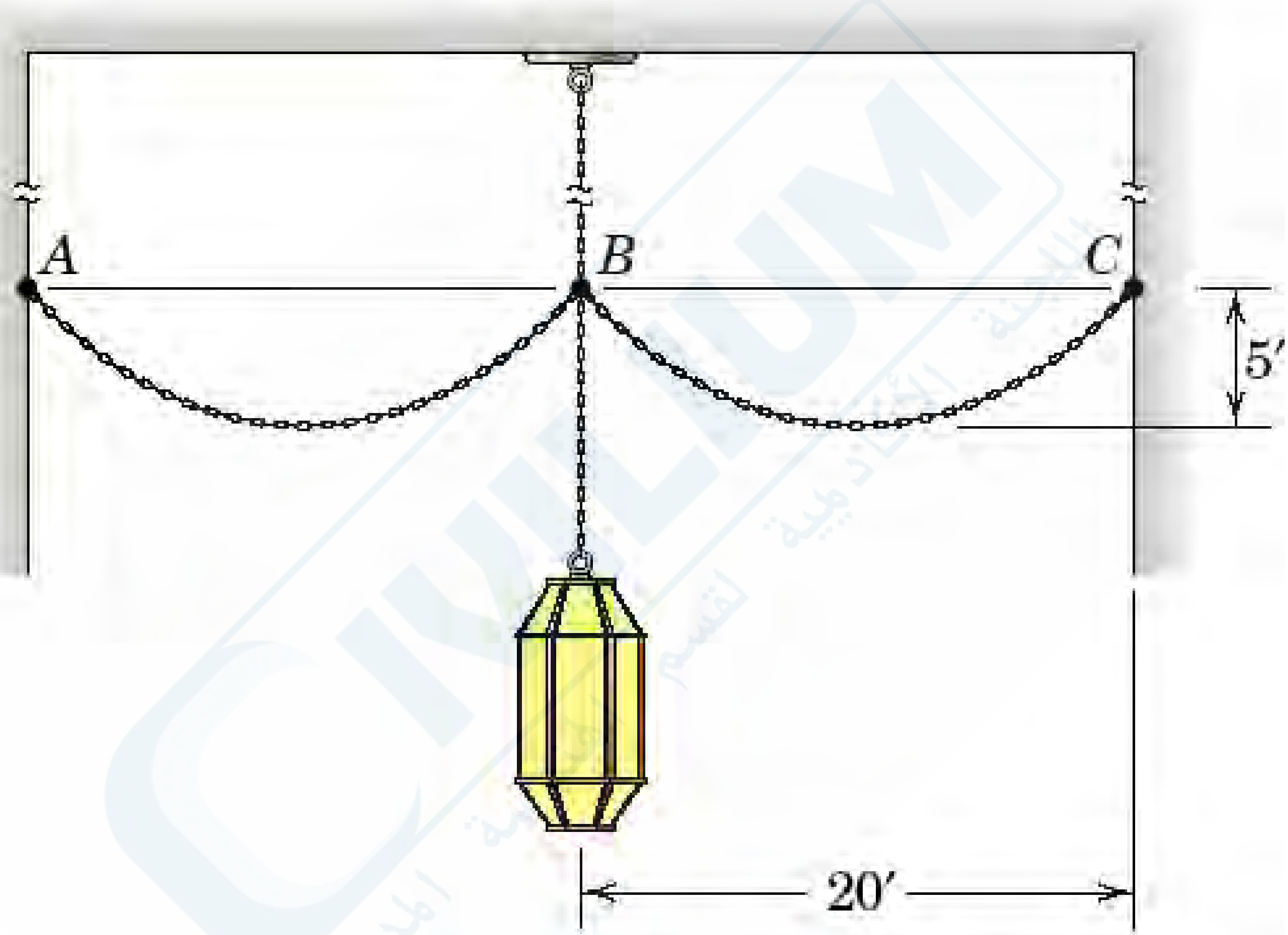
Problem 5/147

P_05_147



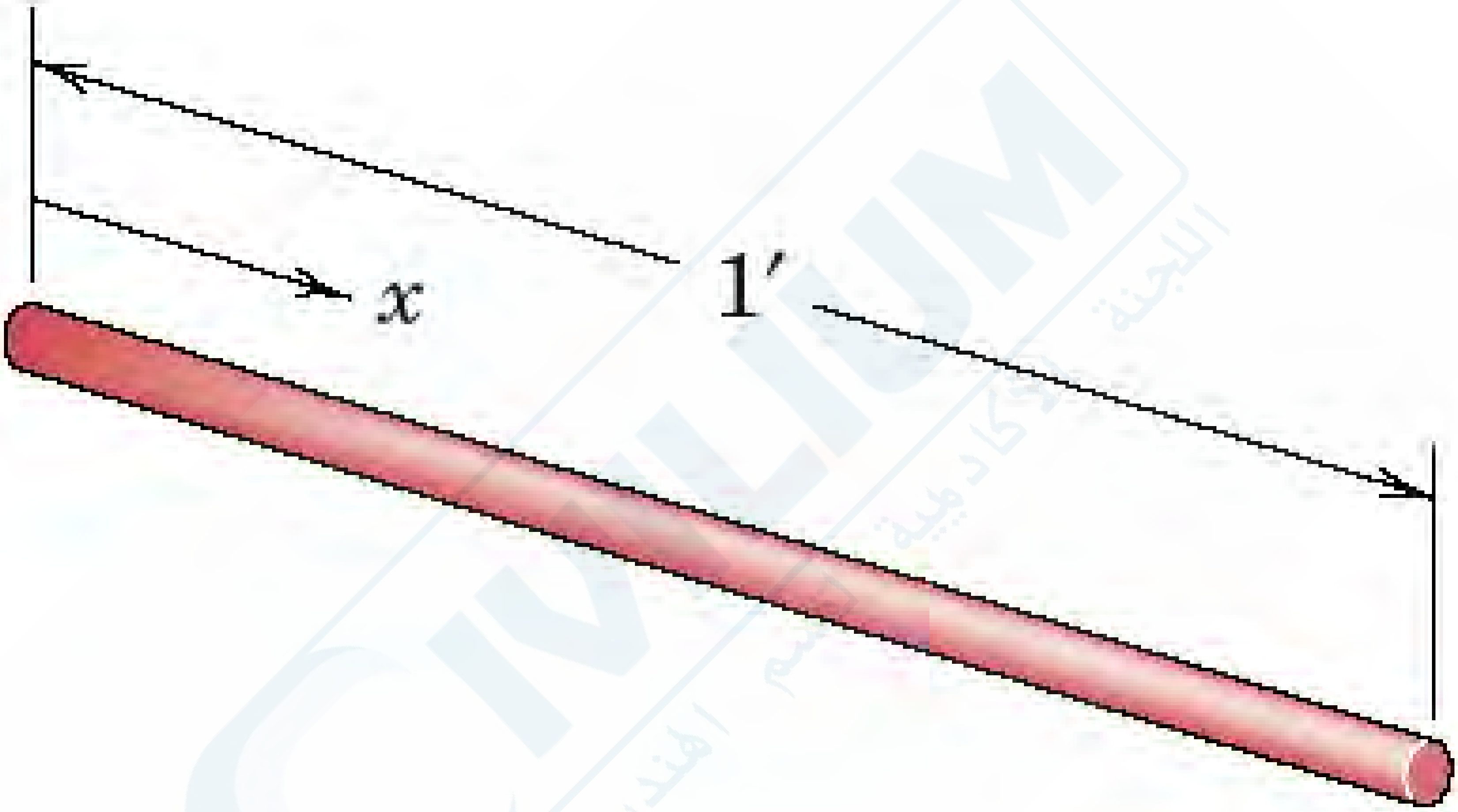
Problem 5/148

P_05_148



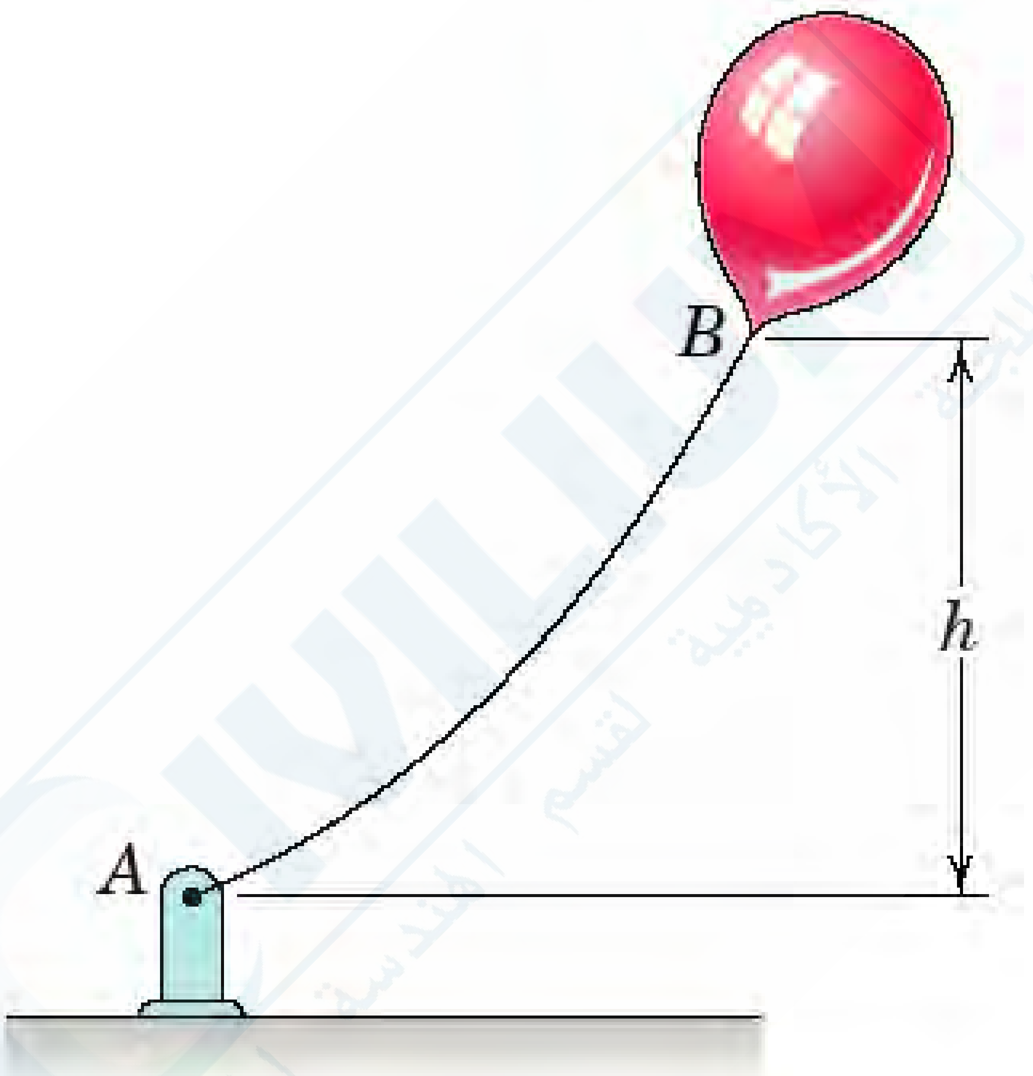
Problem 5/149

P_05_149



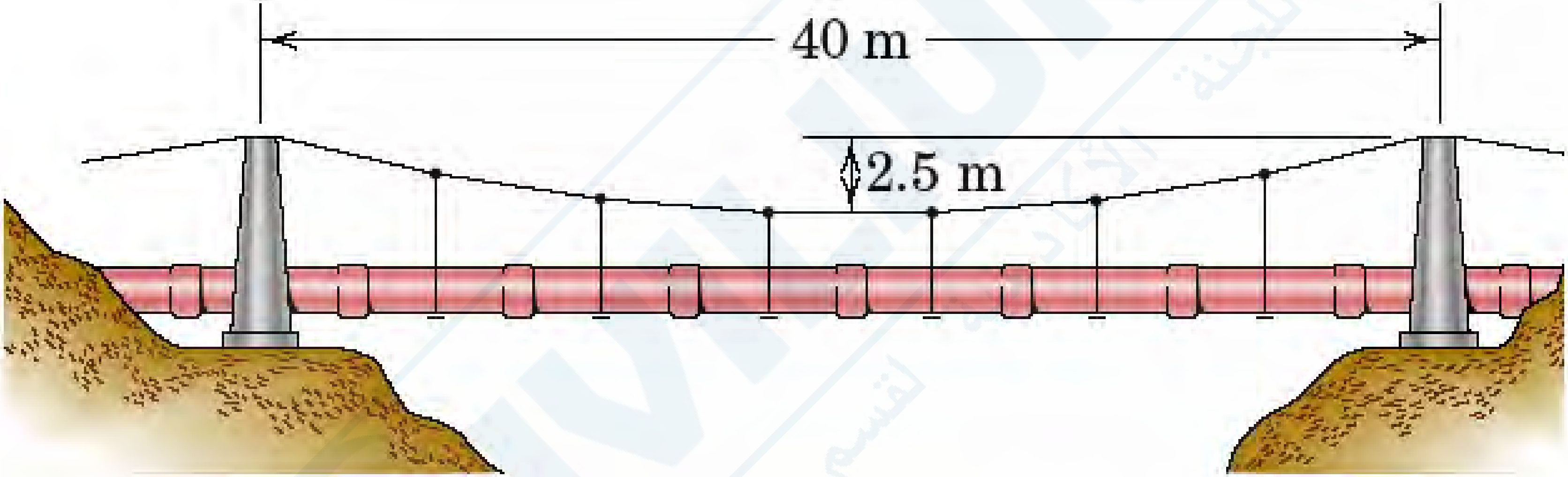
Problem 5/15

P_05_15



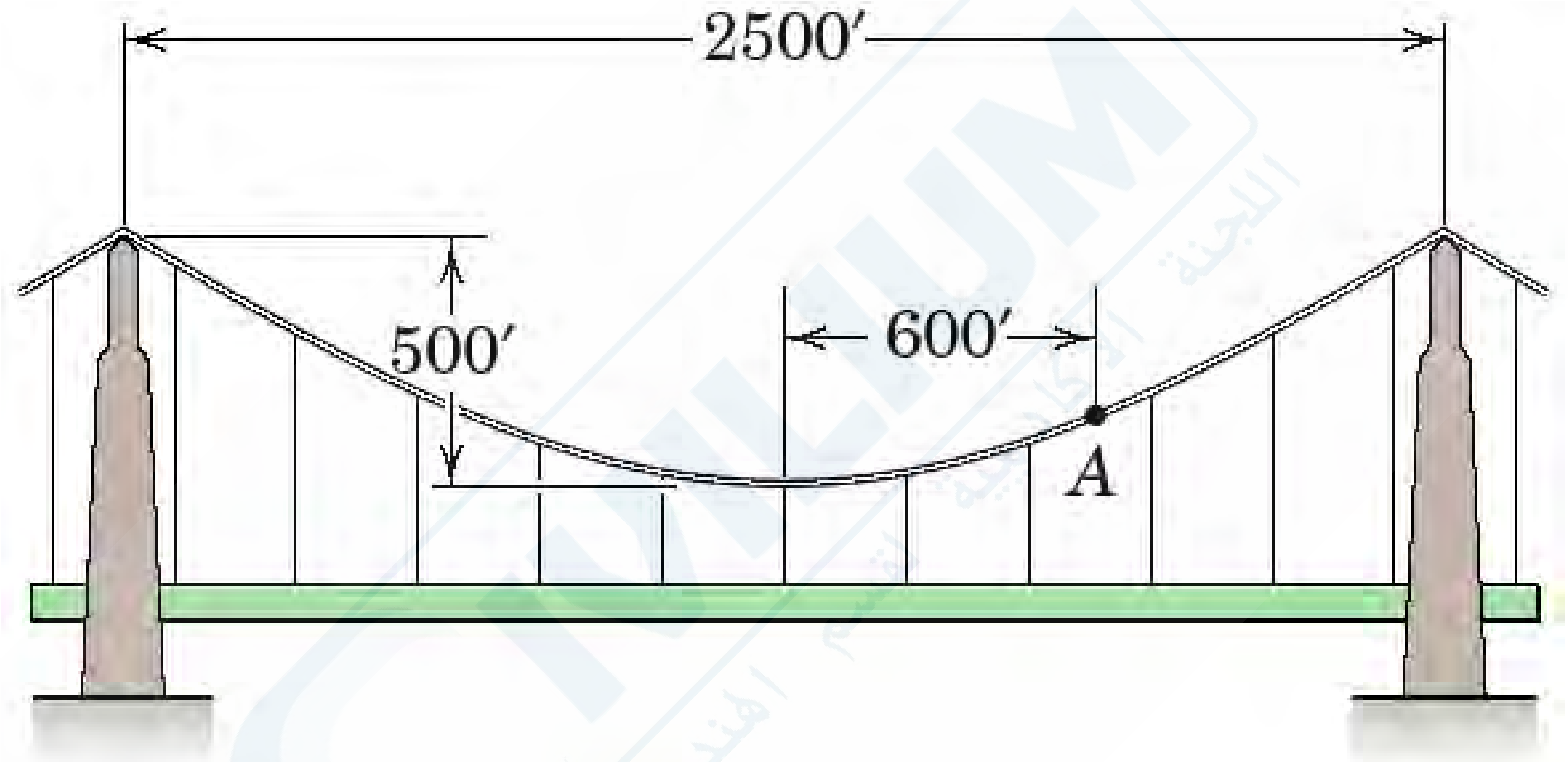
Problem 5/150

P_05_150



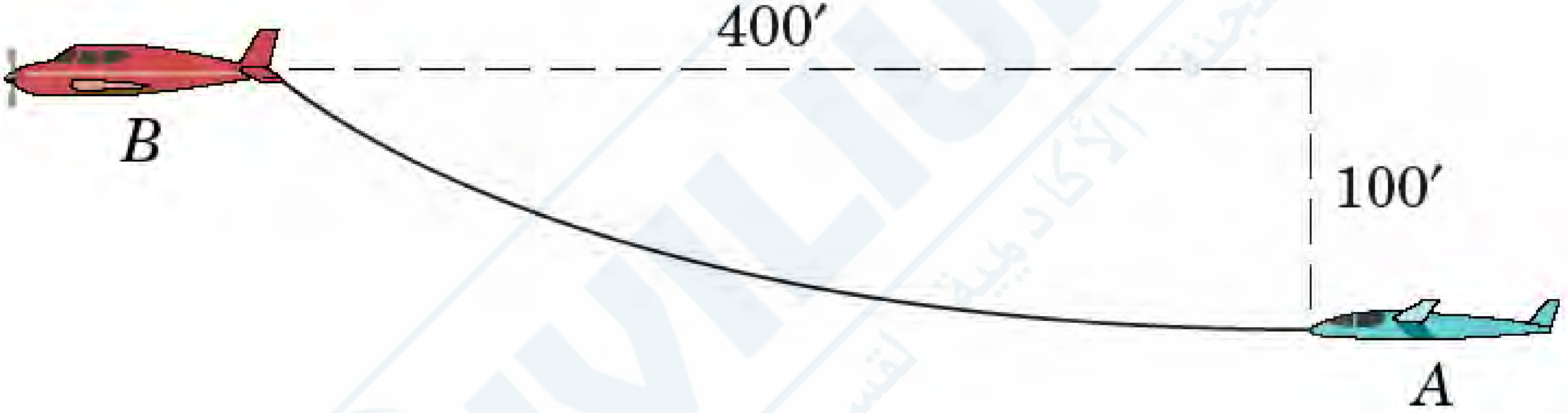
Problem 5/151

P_05_151



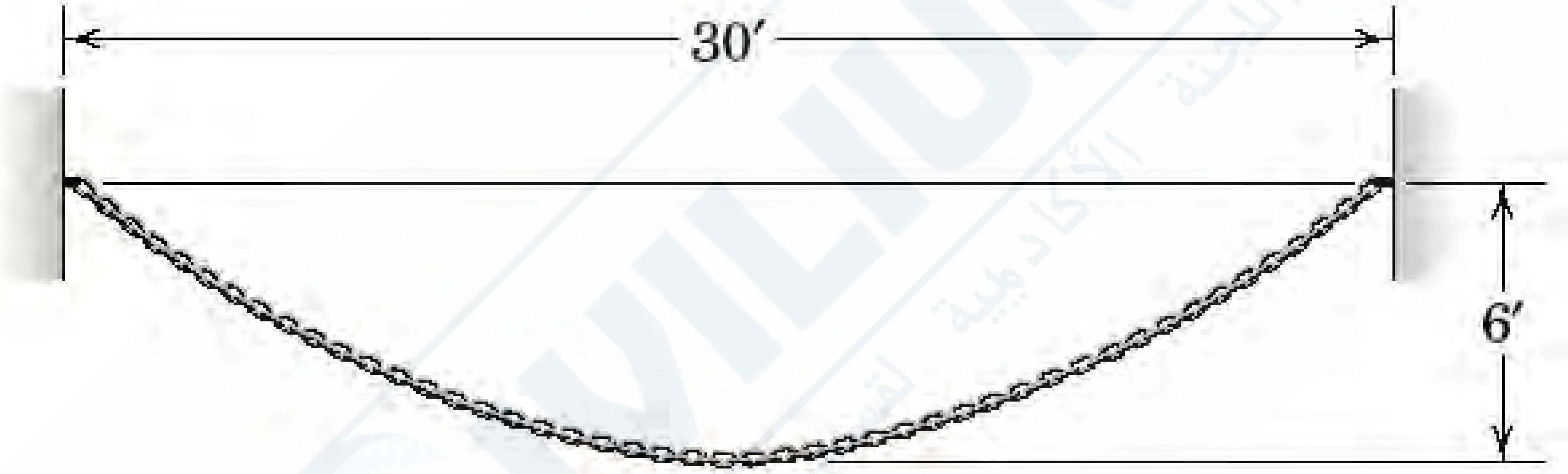
Problem 5/152

P_05_152



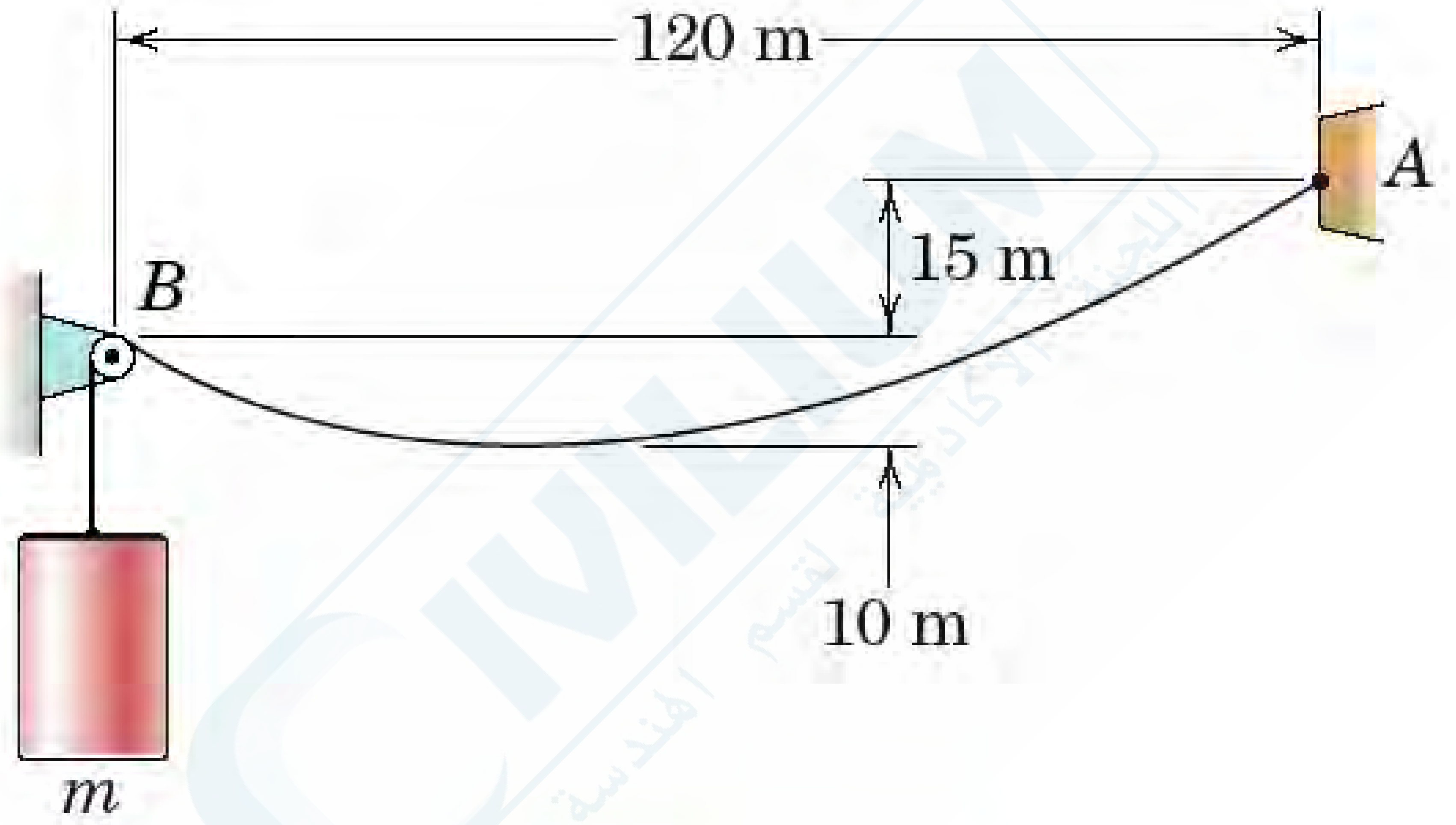
Problem 5/153

P_05_153



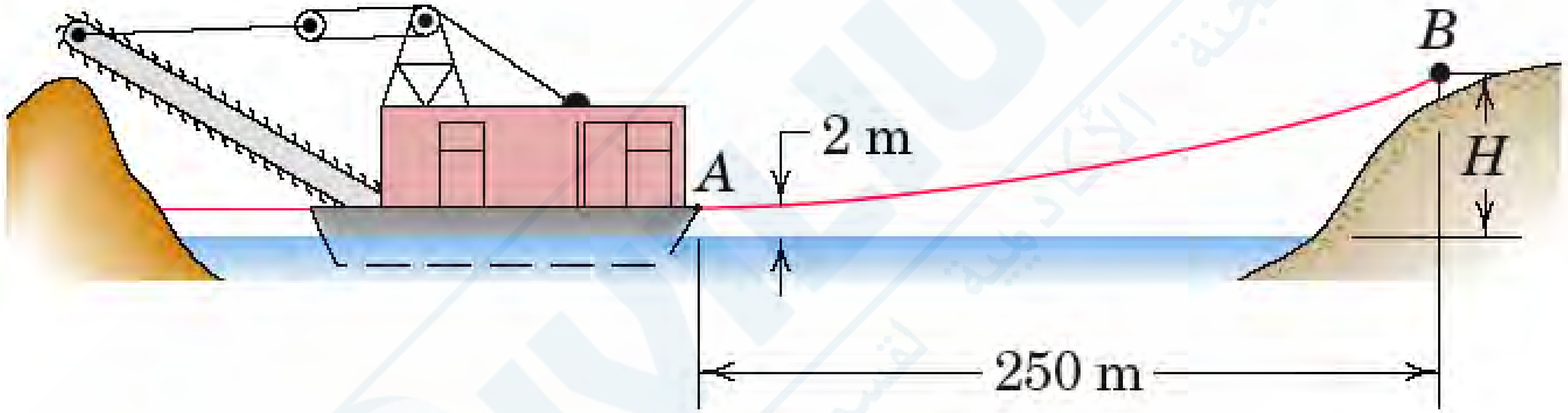
Problem 5/154

P_05_154



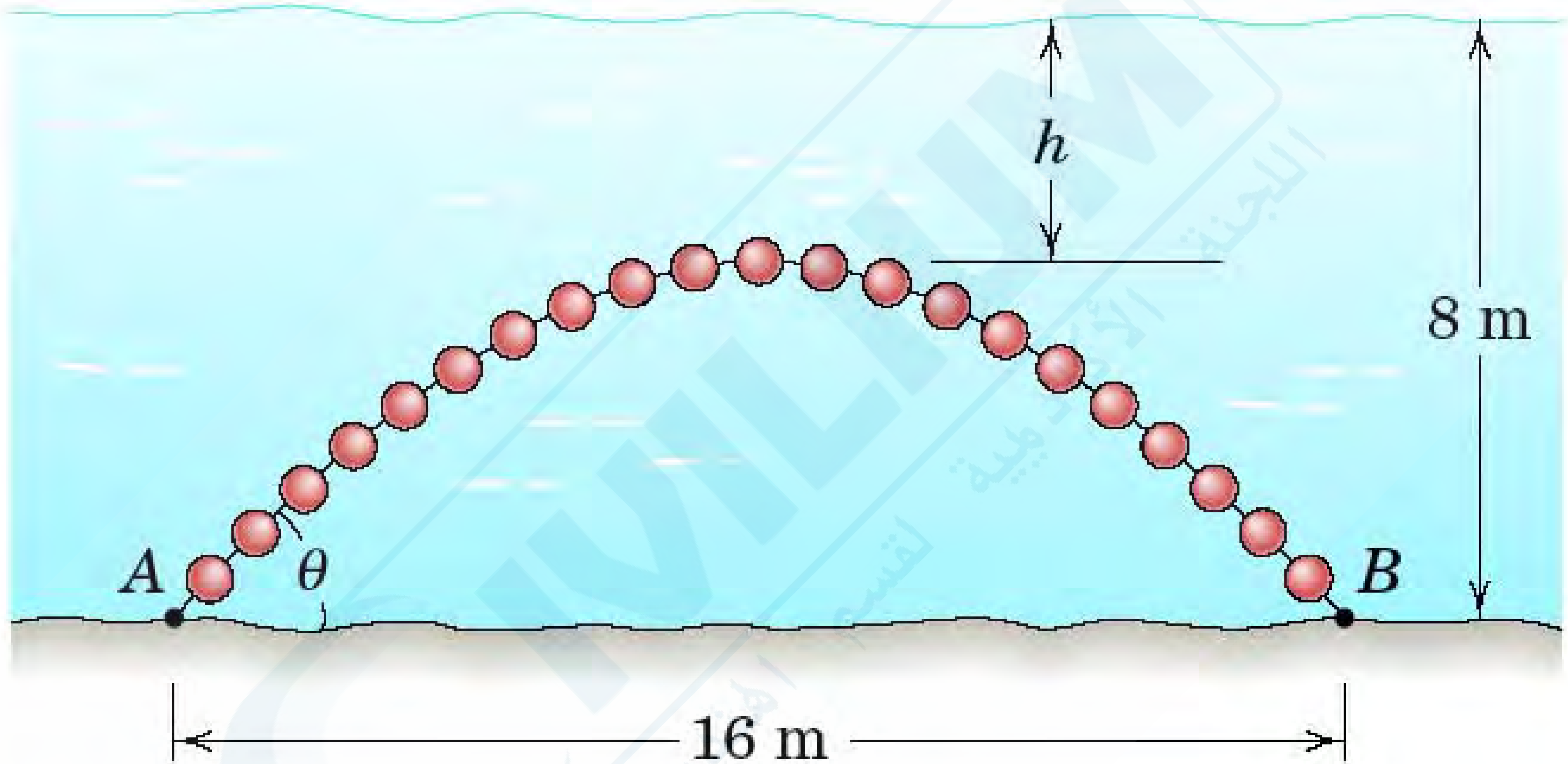
Problem 5/155

P_05_155



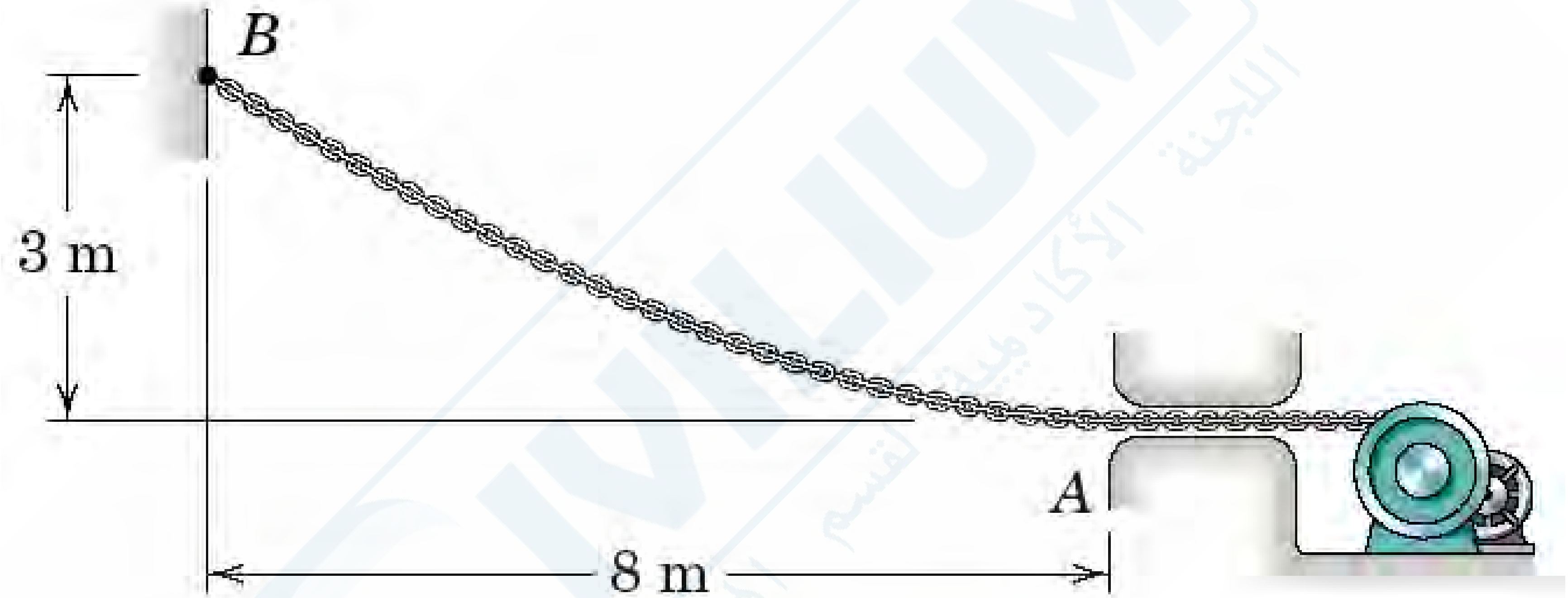
Problem 5/157

P_05_157



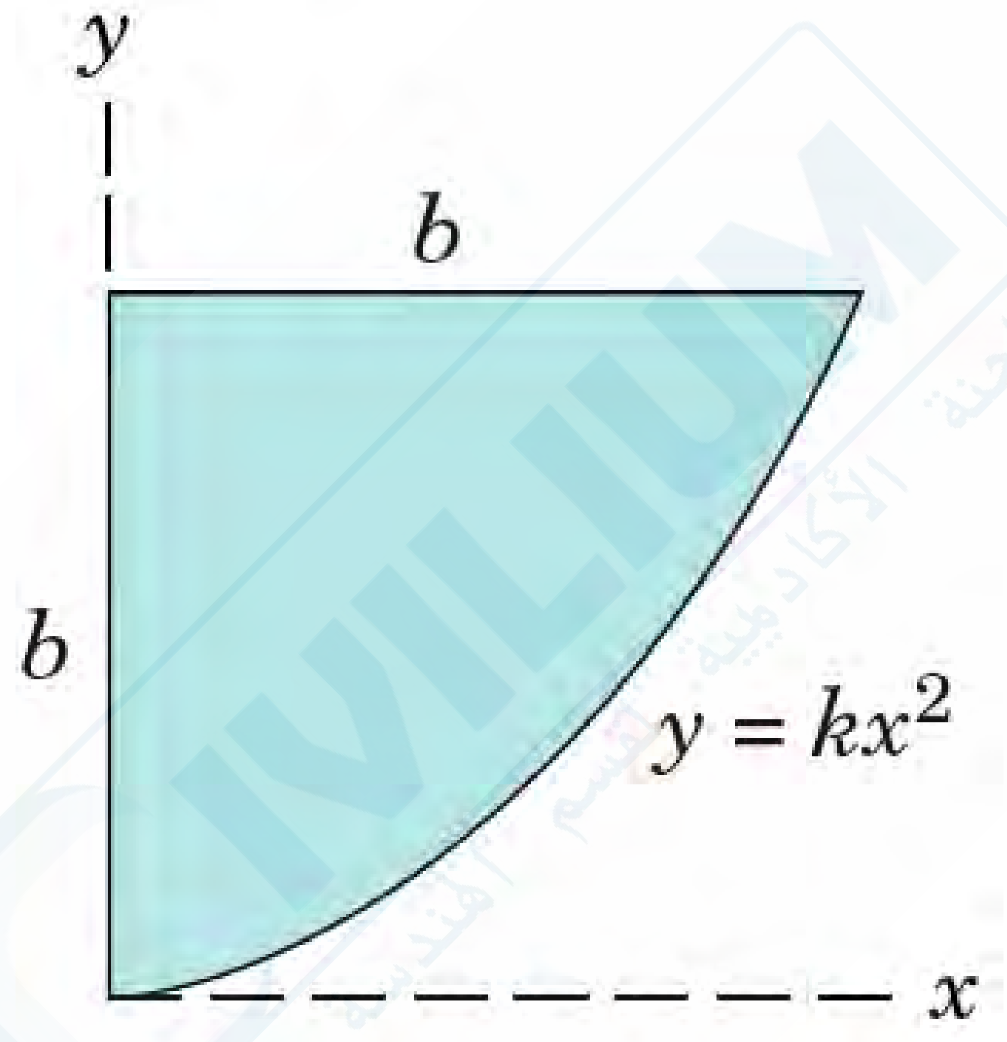
Problem 5/158

P_05_158



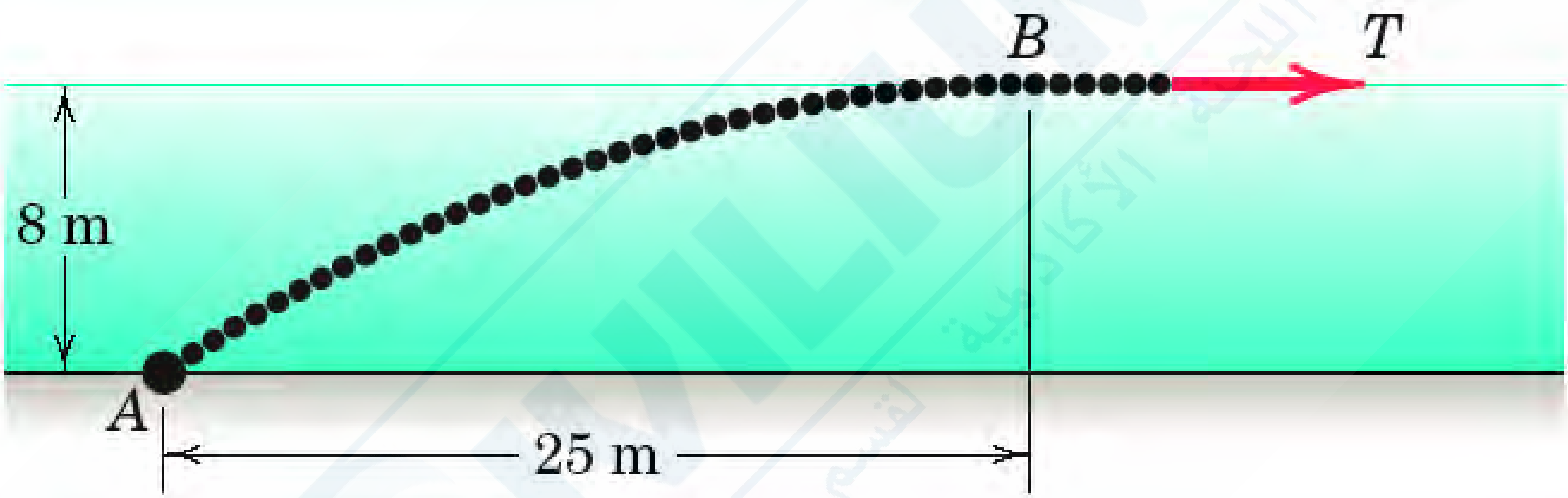
Problem 5/159

P_05_159



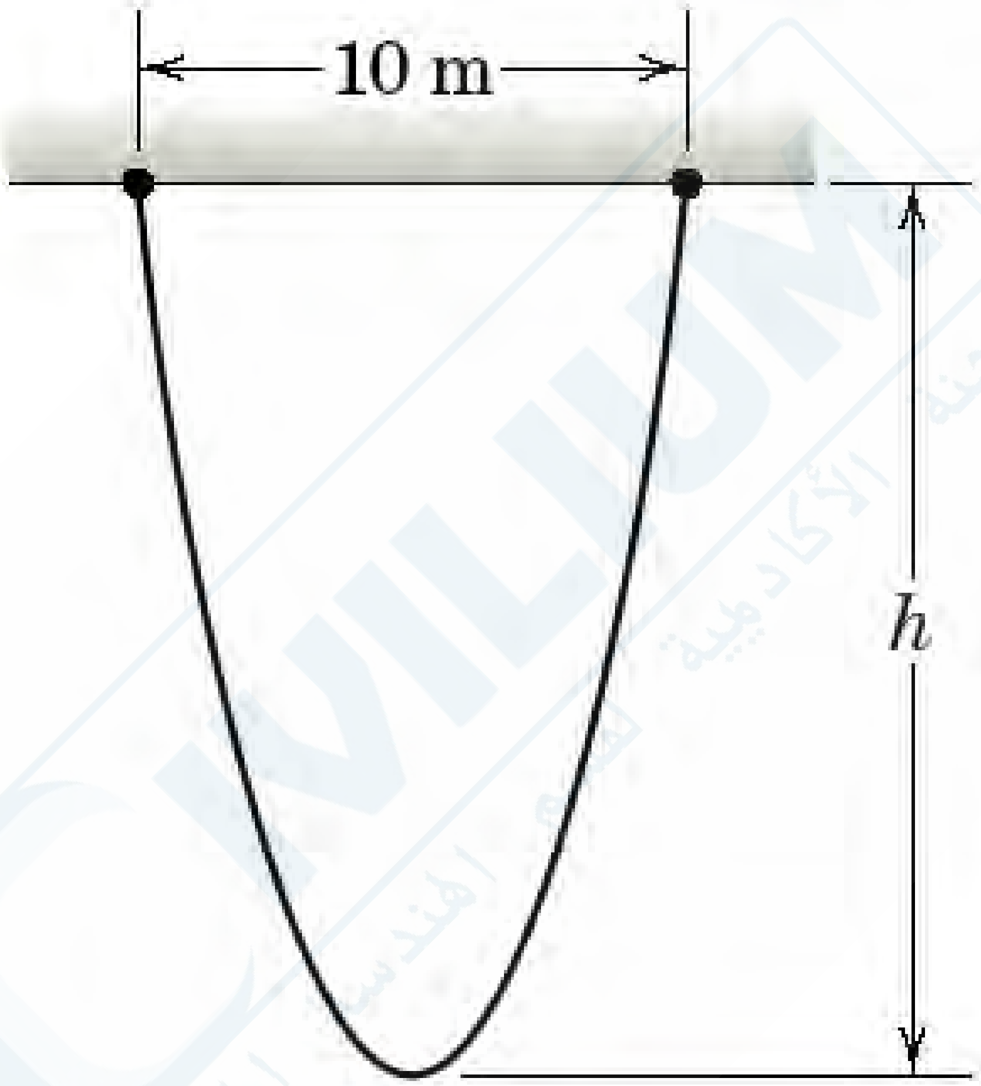
Problem 5/16

P_05_16



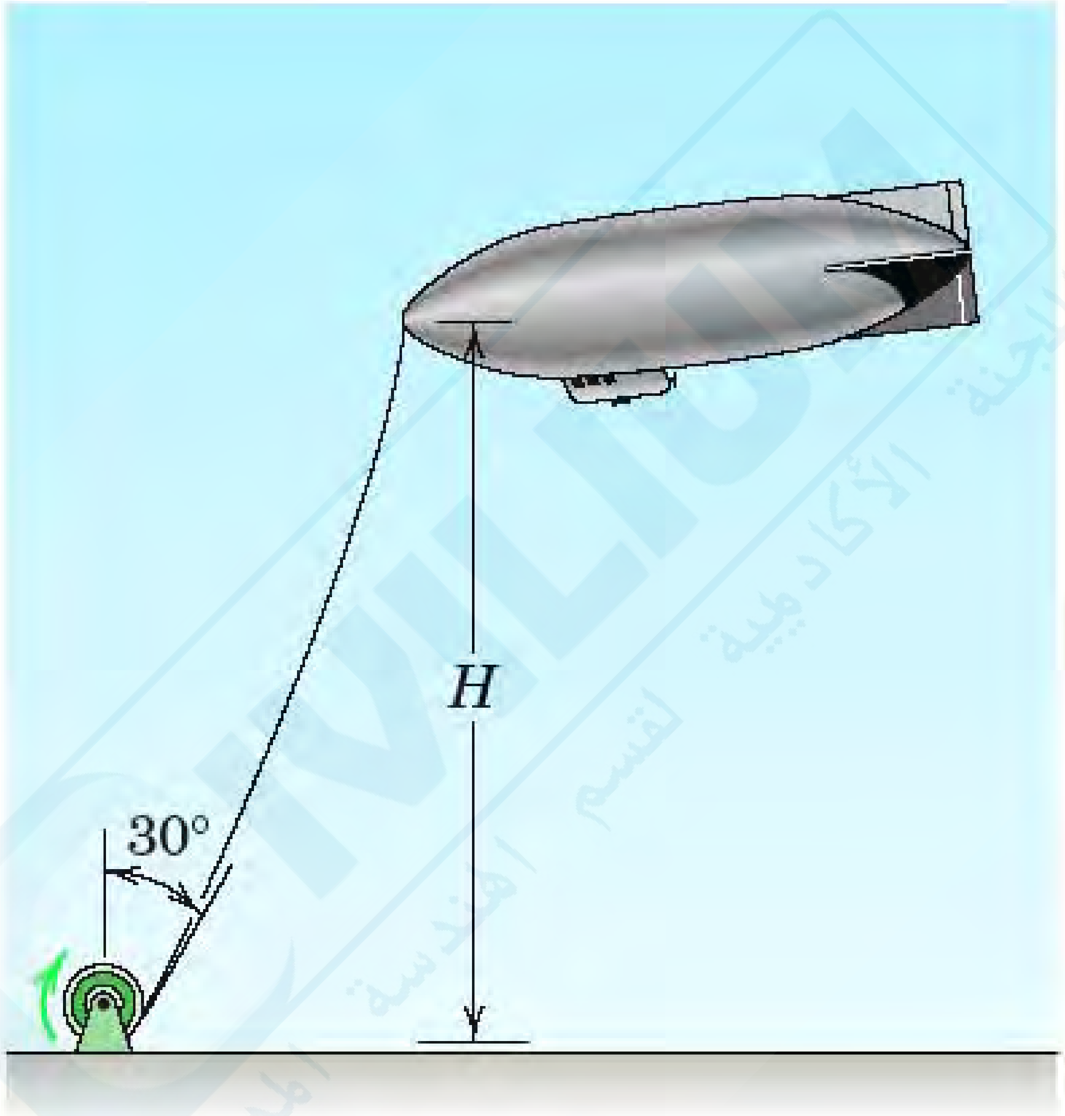
Problem 5/160

P_05_160



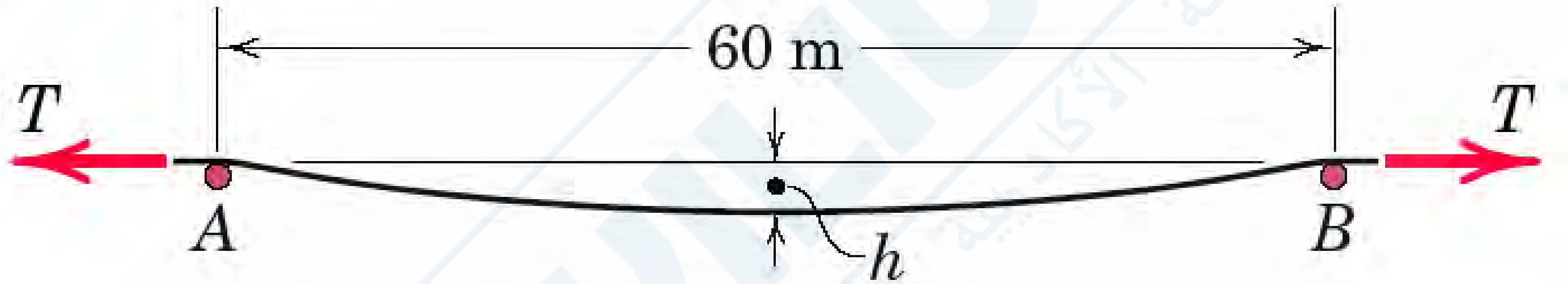
Problem 5/161

P 05 161



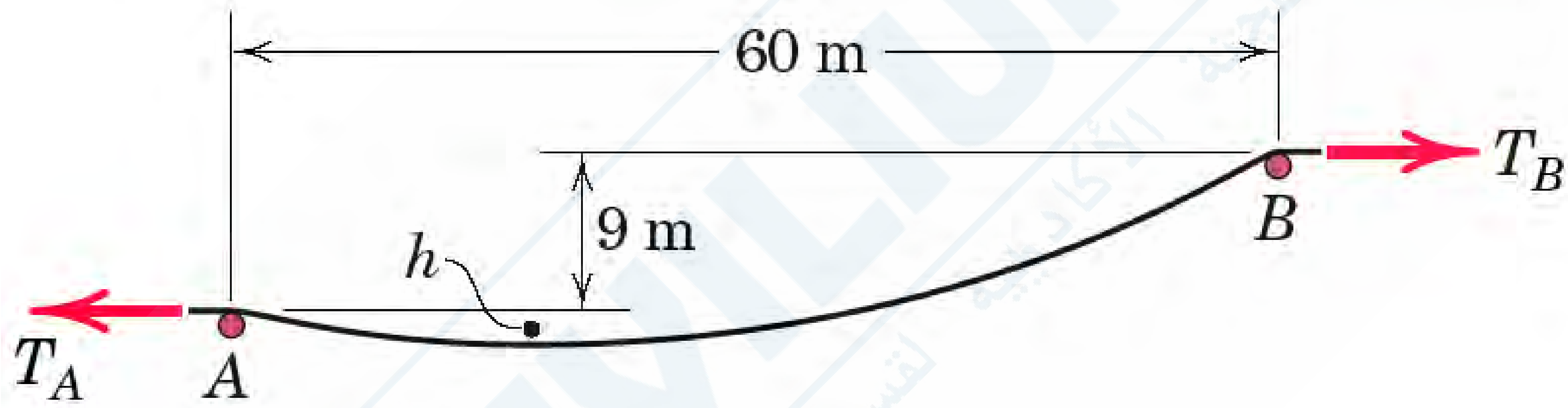
Problem 5/162

P 05 162



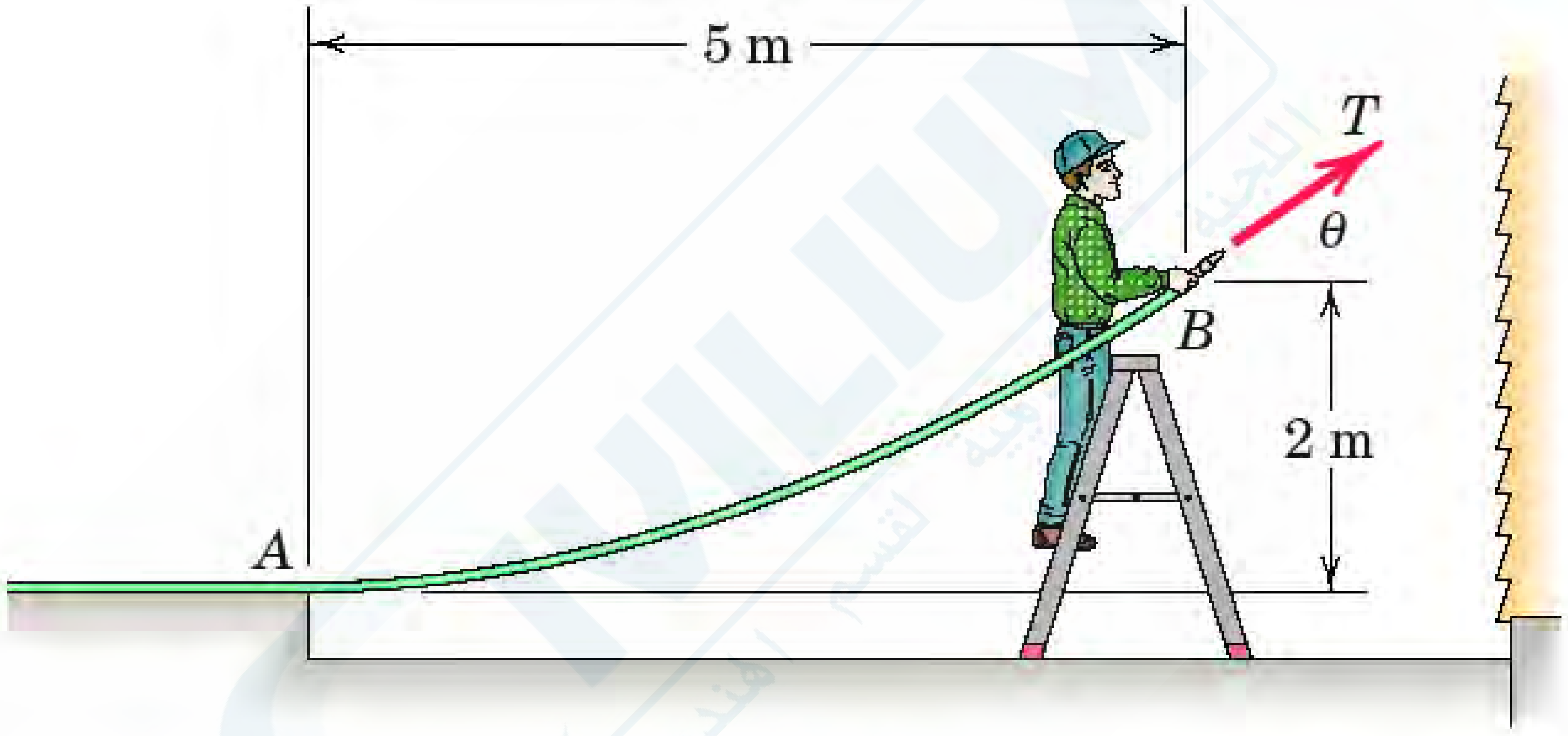
Problem 5/163

P_05_163



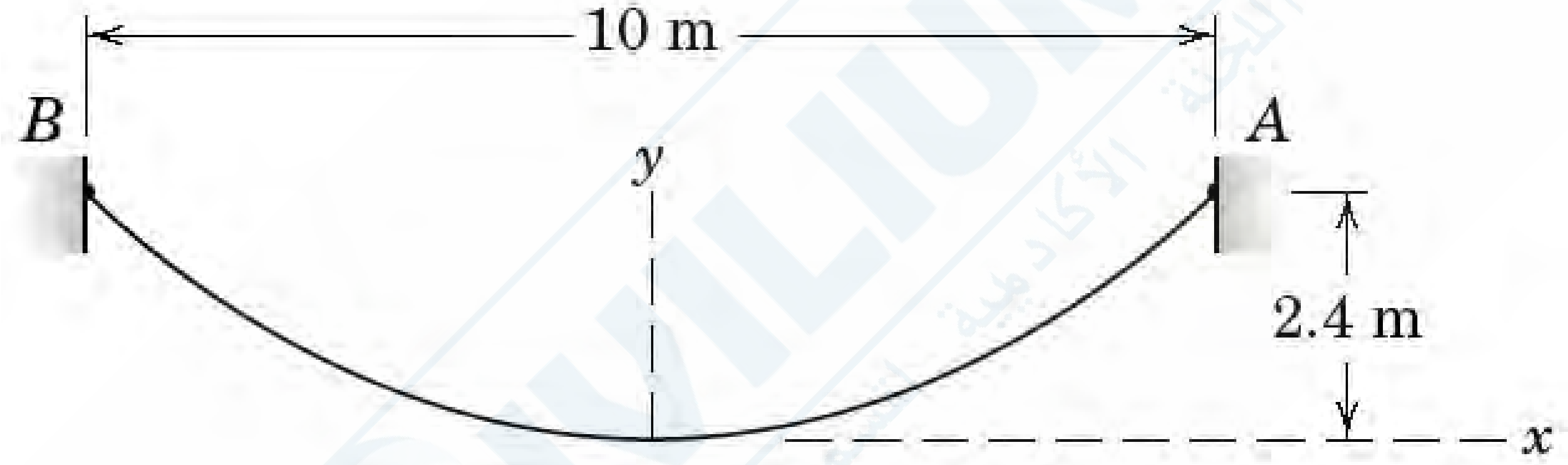
Problem 5/164

P_05_164



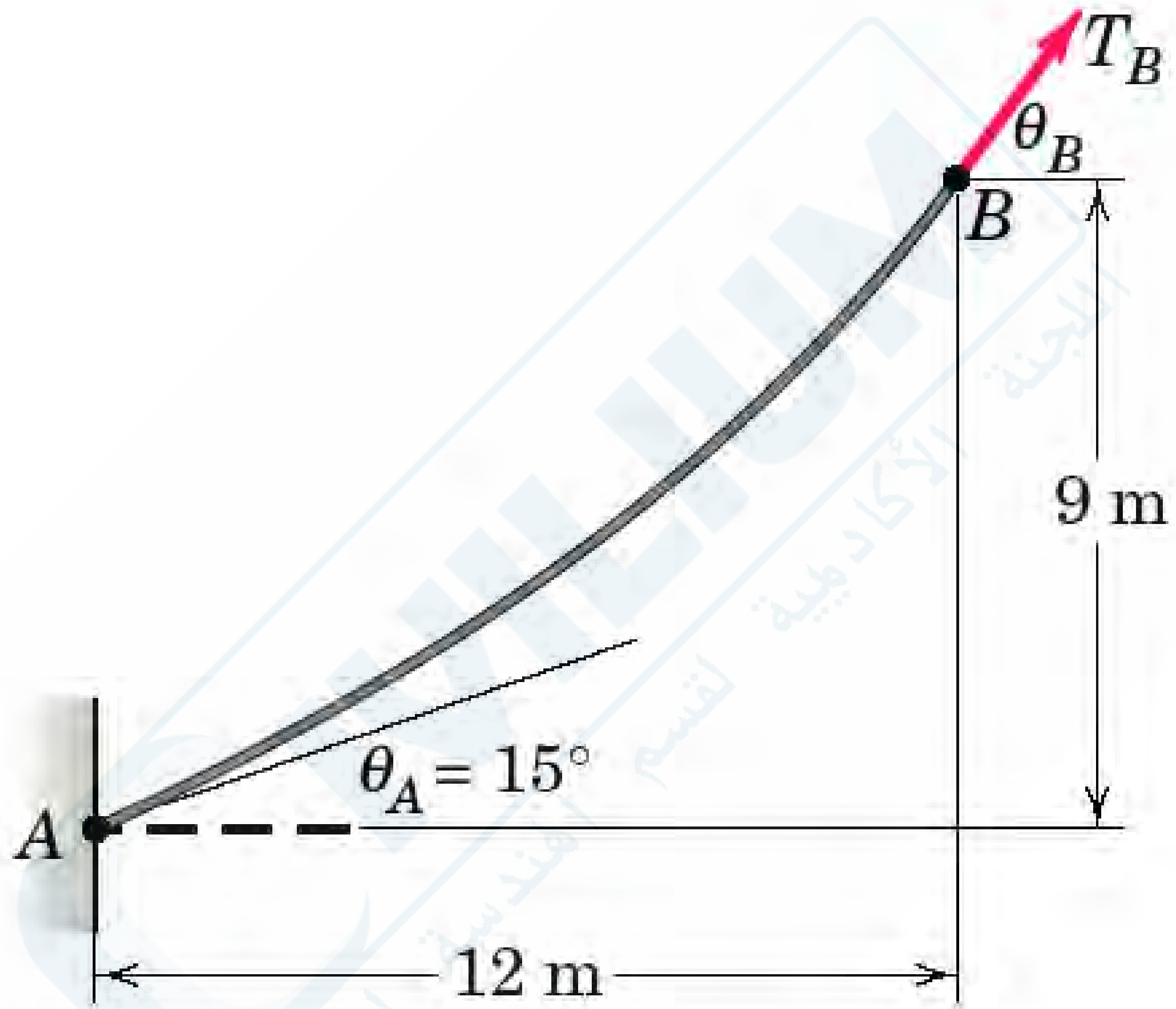
Problem 5/165

P_05_165



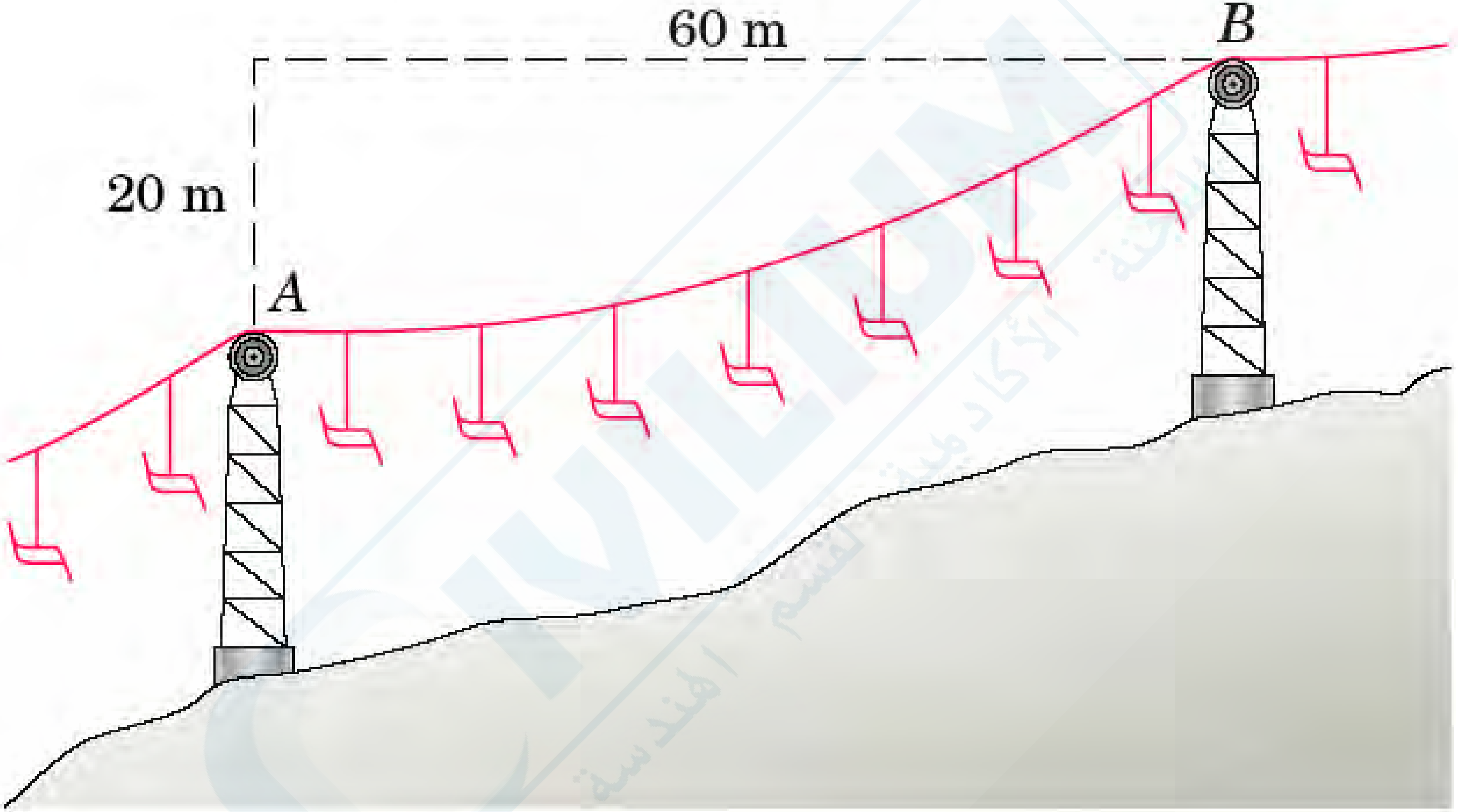
Problem 5/166

P_05_166



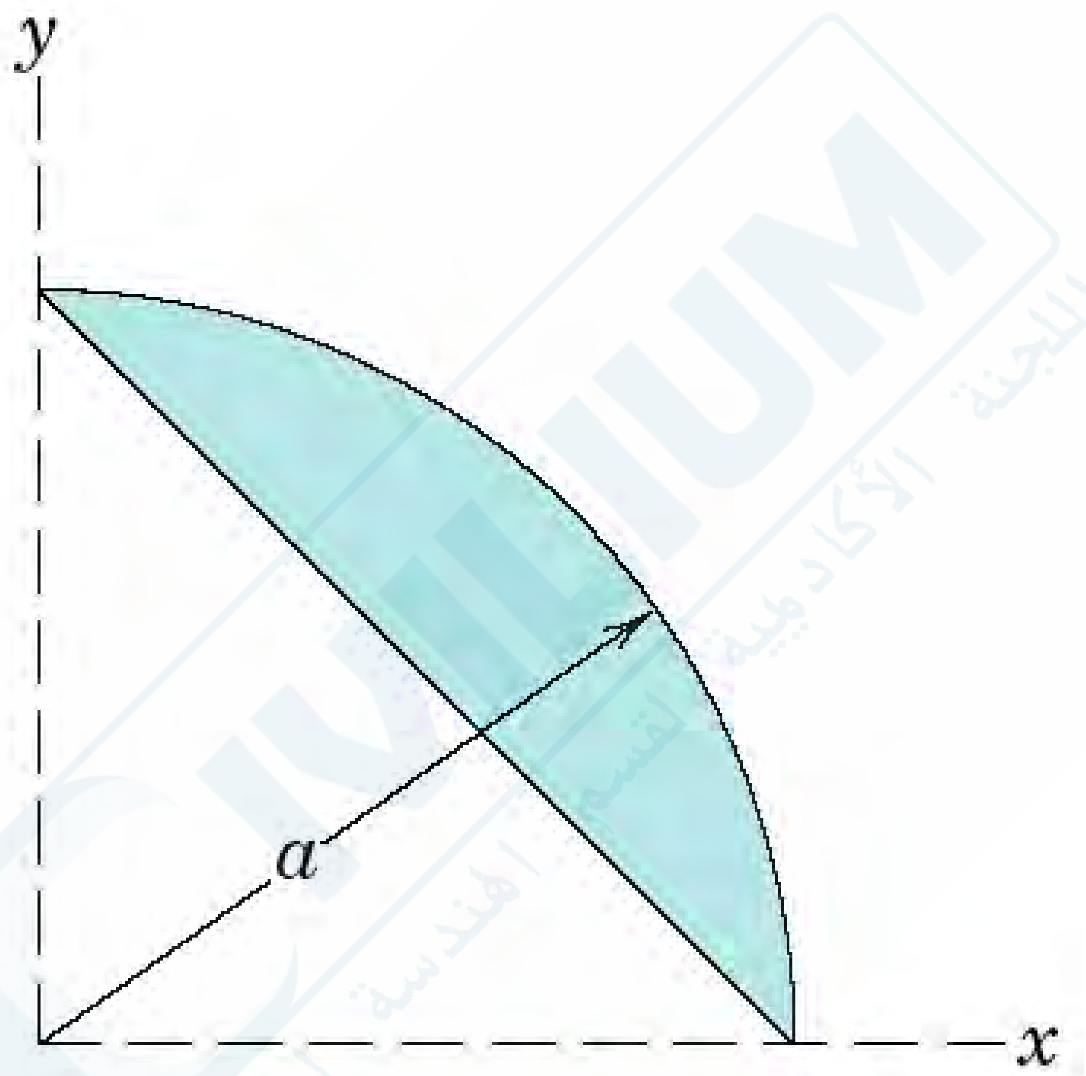
Problem 5/168

P_05_168

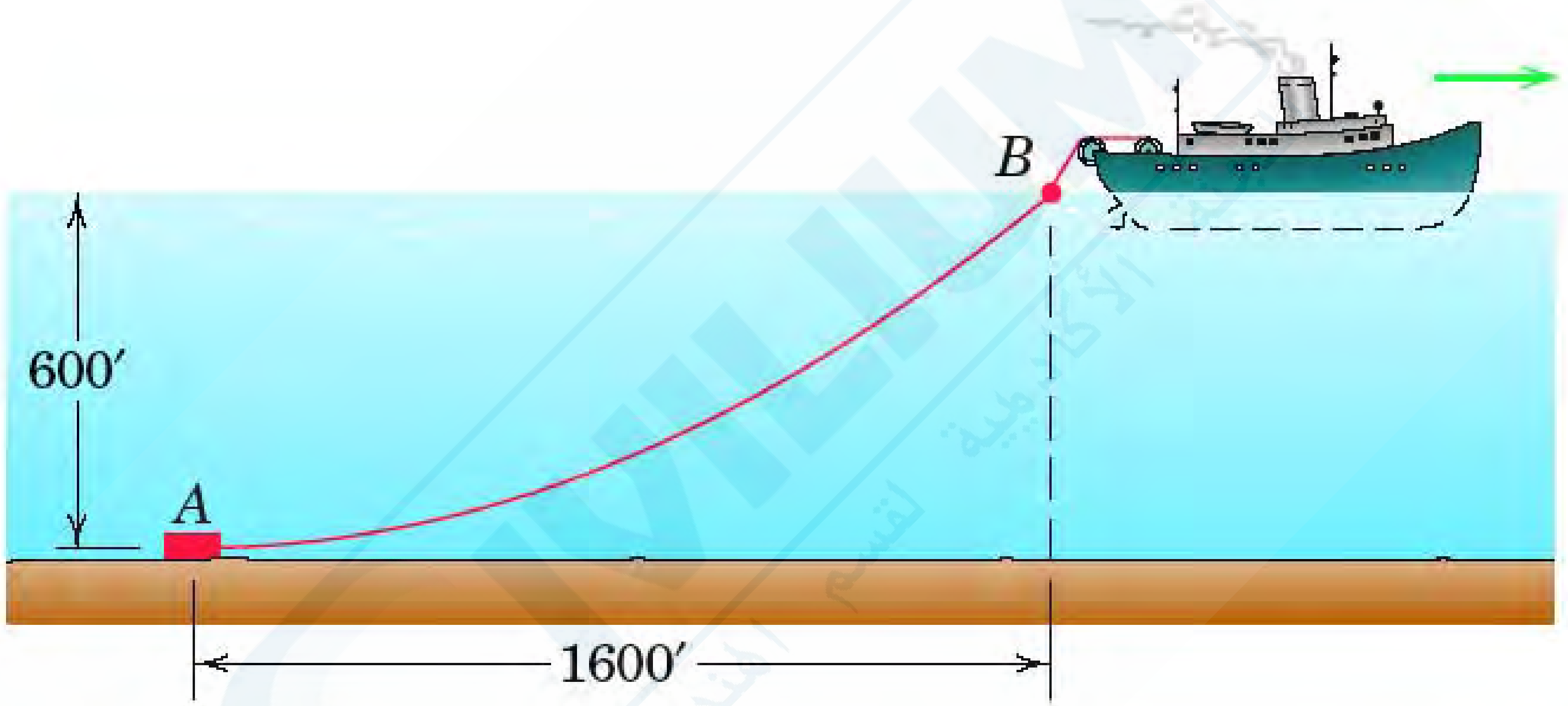


Problem 5/169

P_05_169

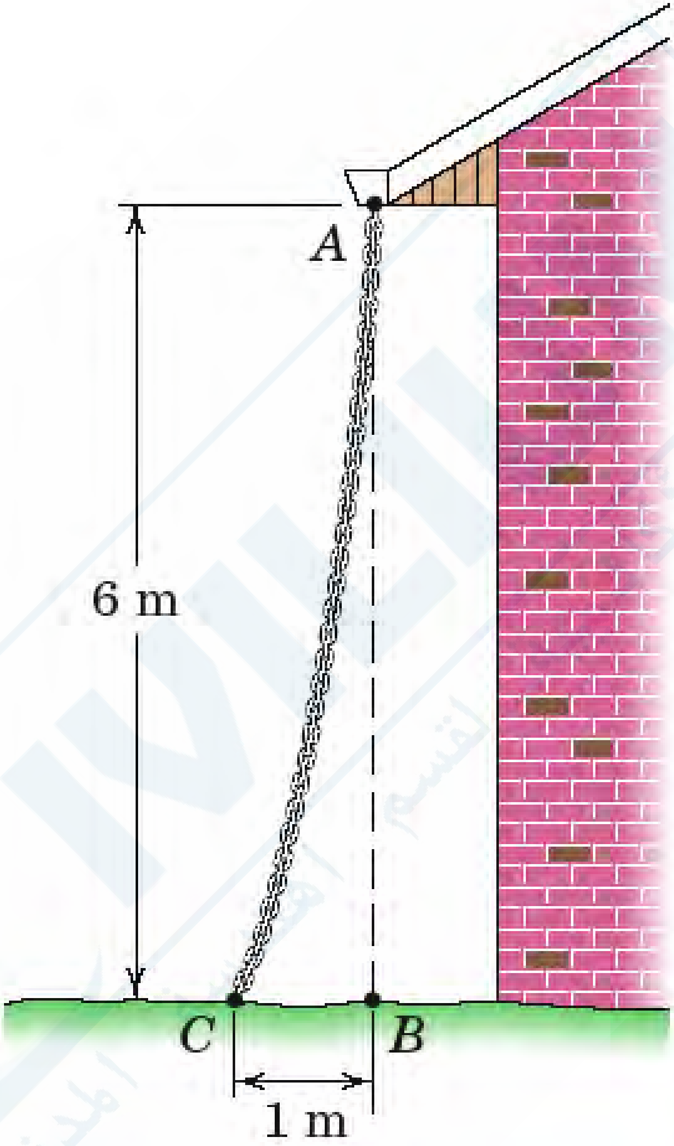


Problem 5/17



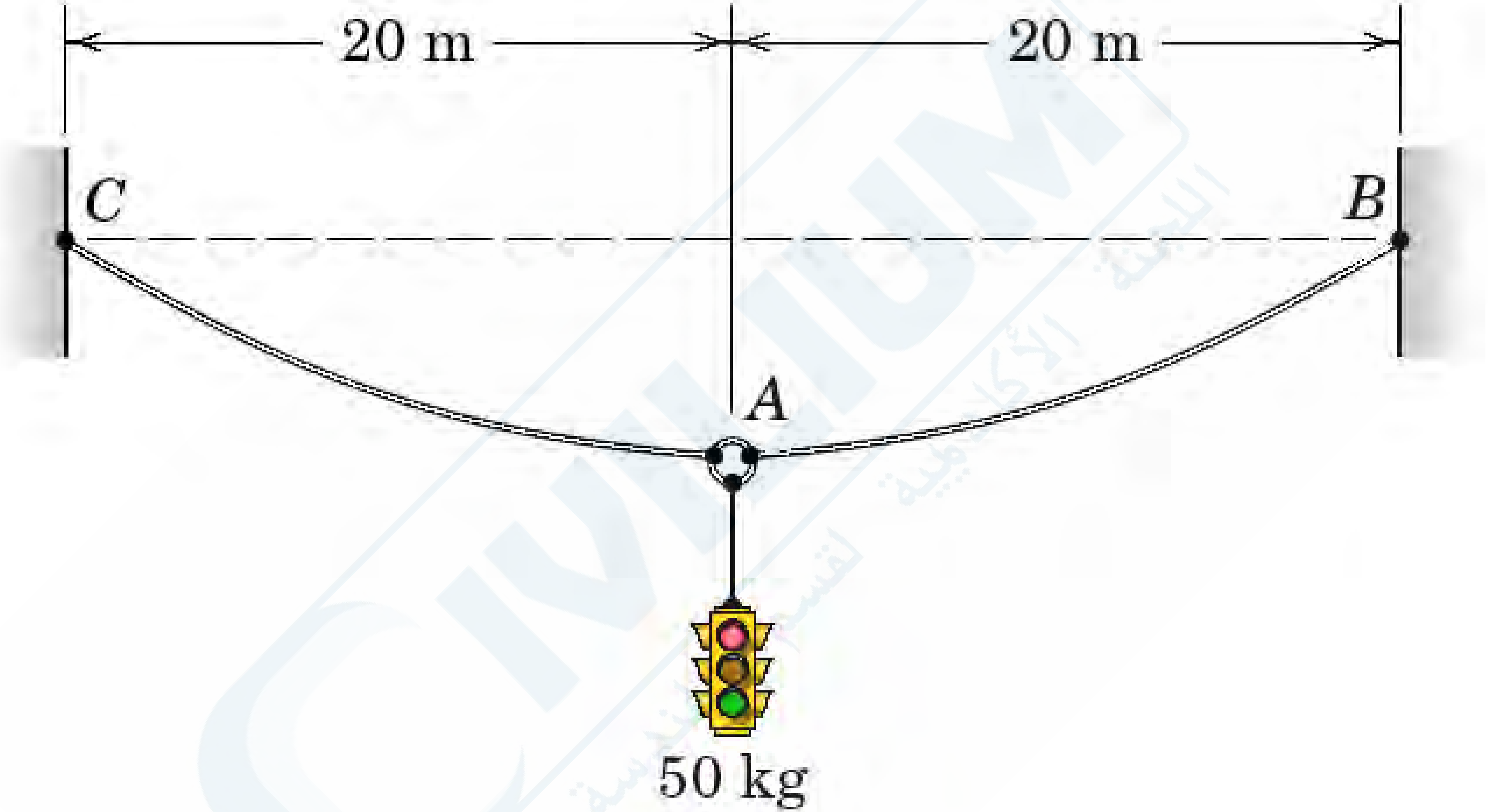
Problem 5/170

P_05_170



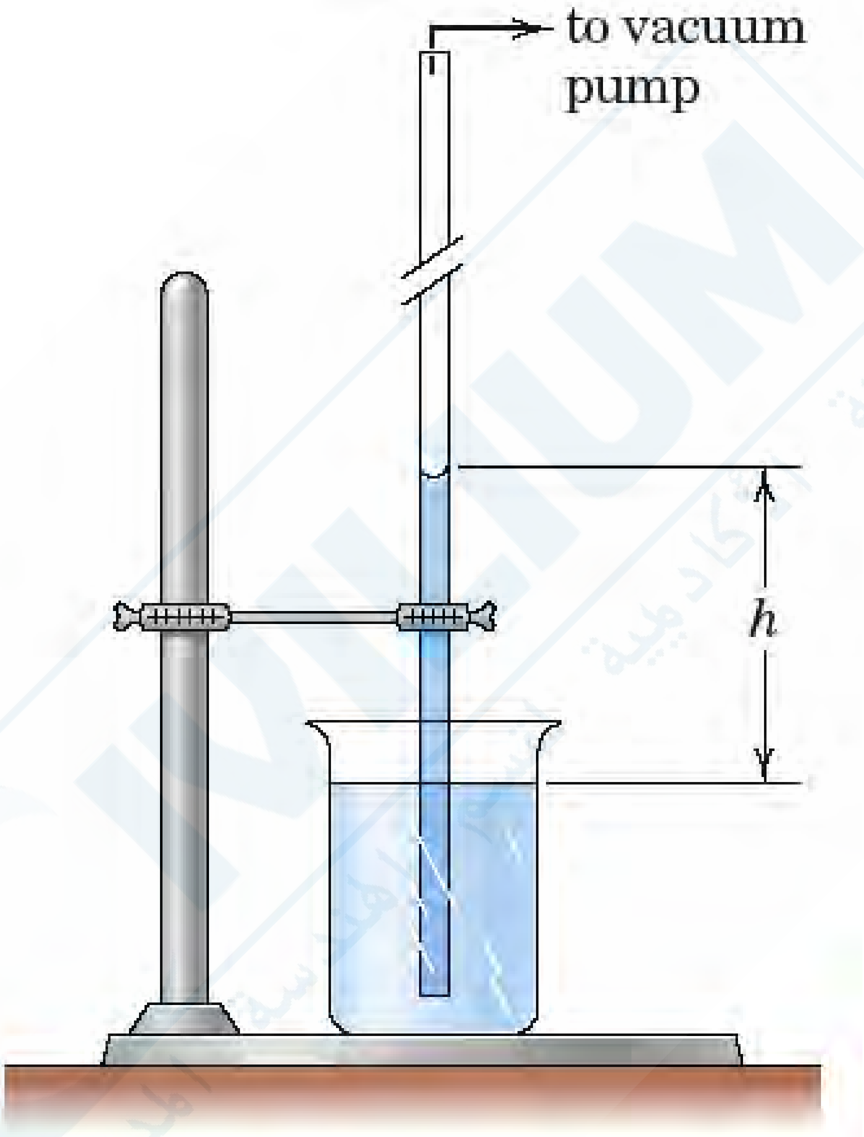
Problem 5/171

P 05 171



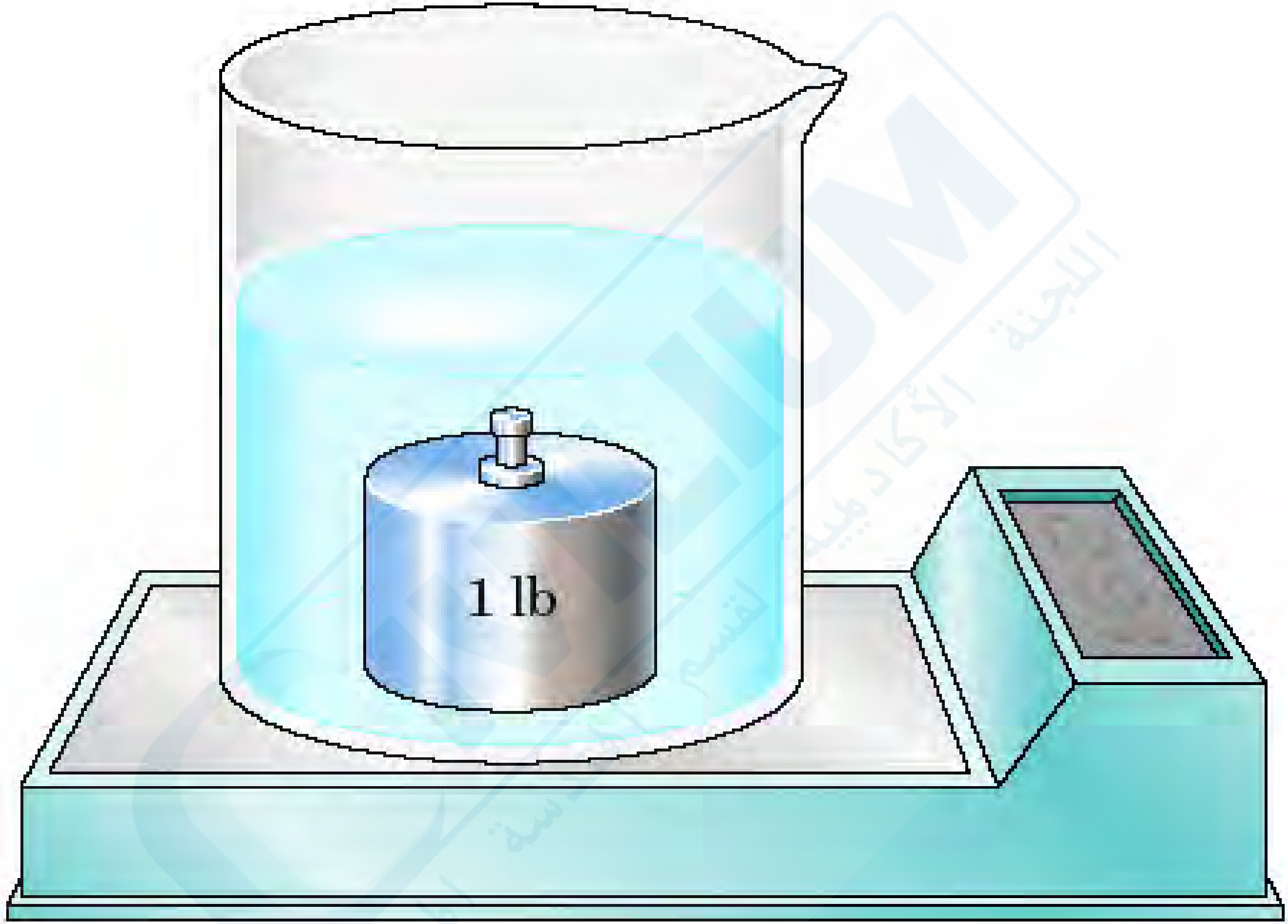
Problem 5/172

P_05_172



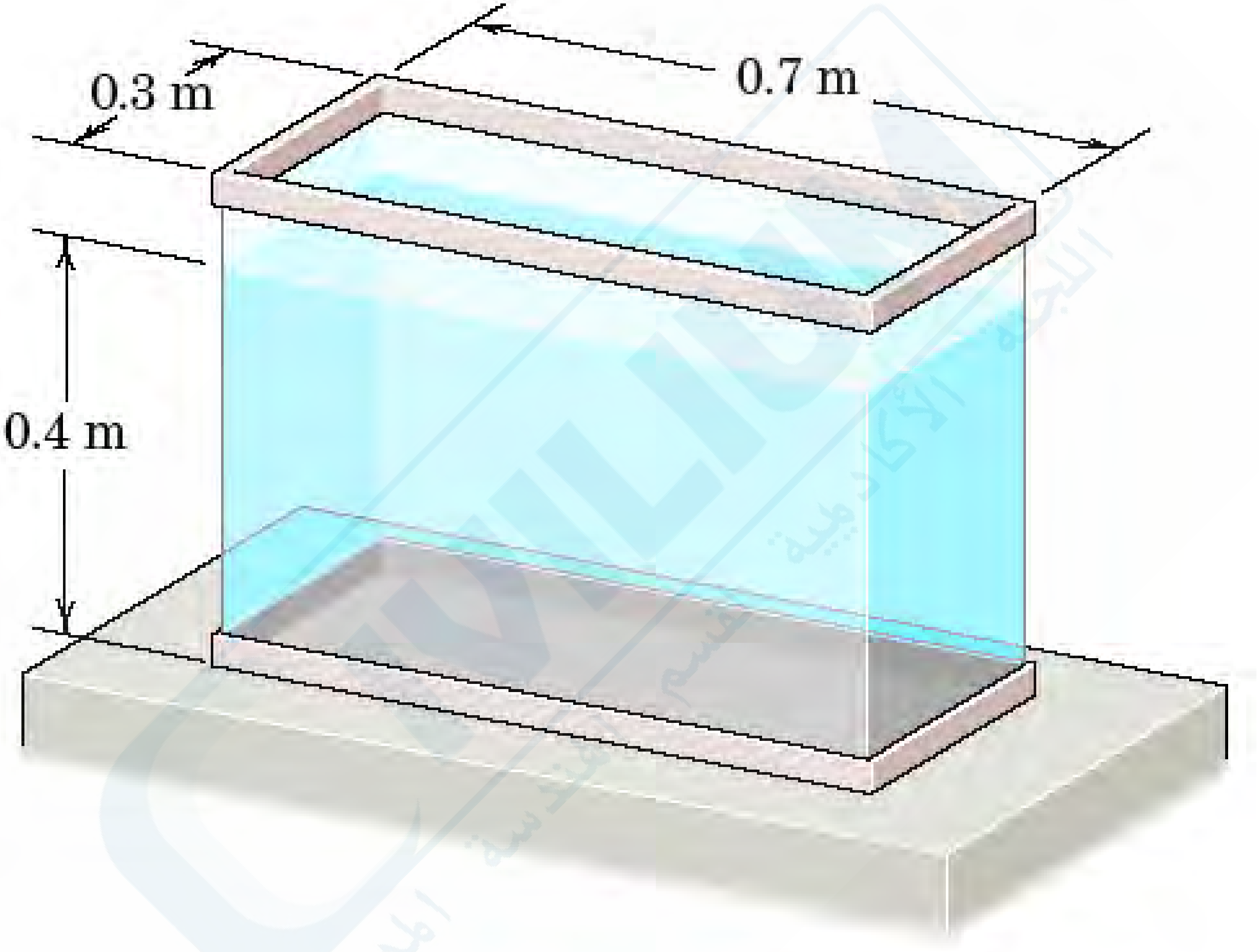
Problem 5/173

P_05_173



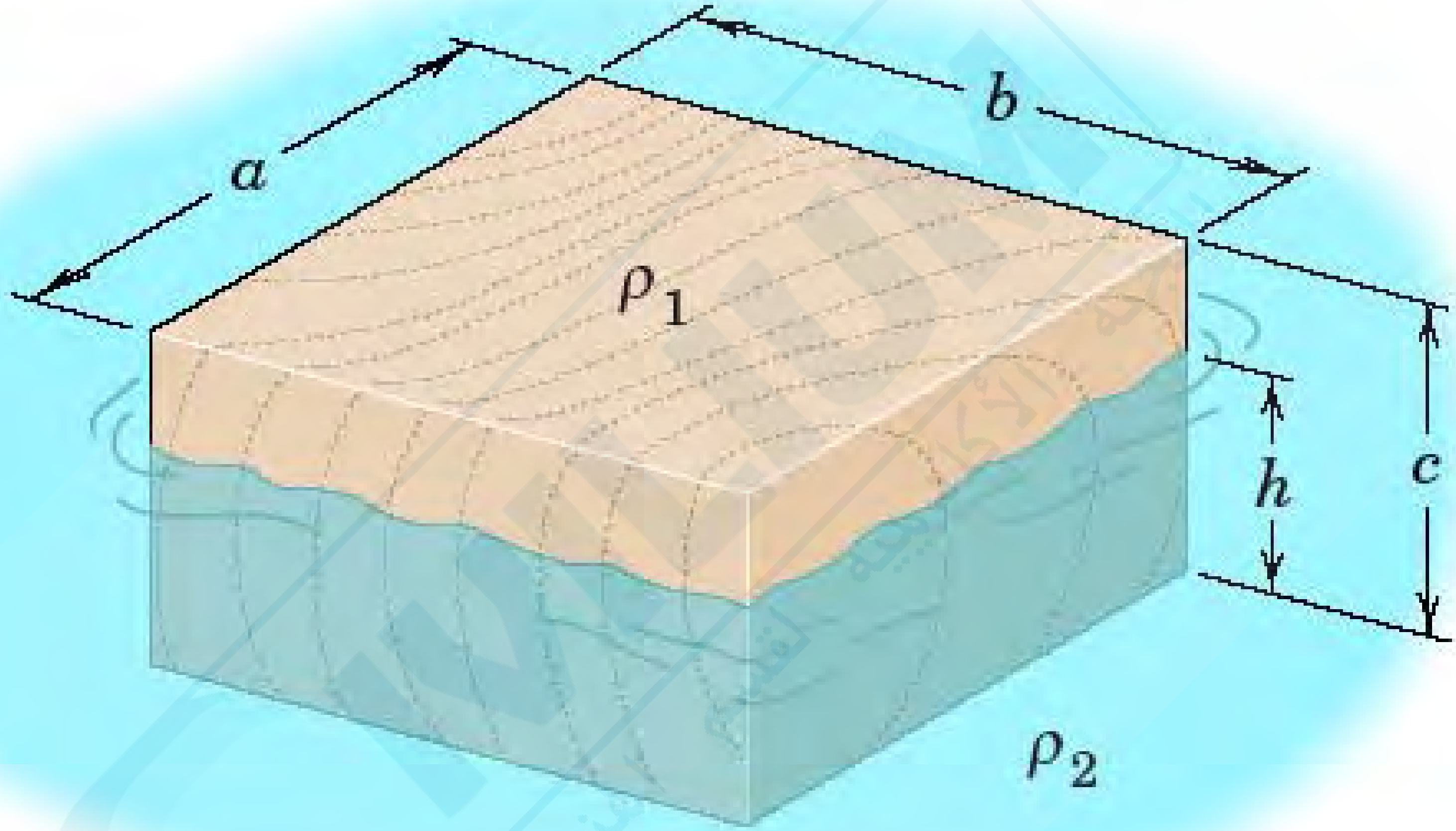
Problem 5/174

P 05 174



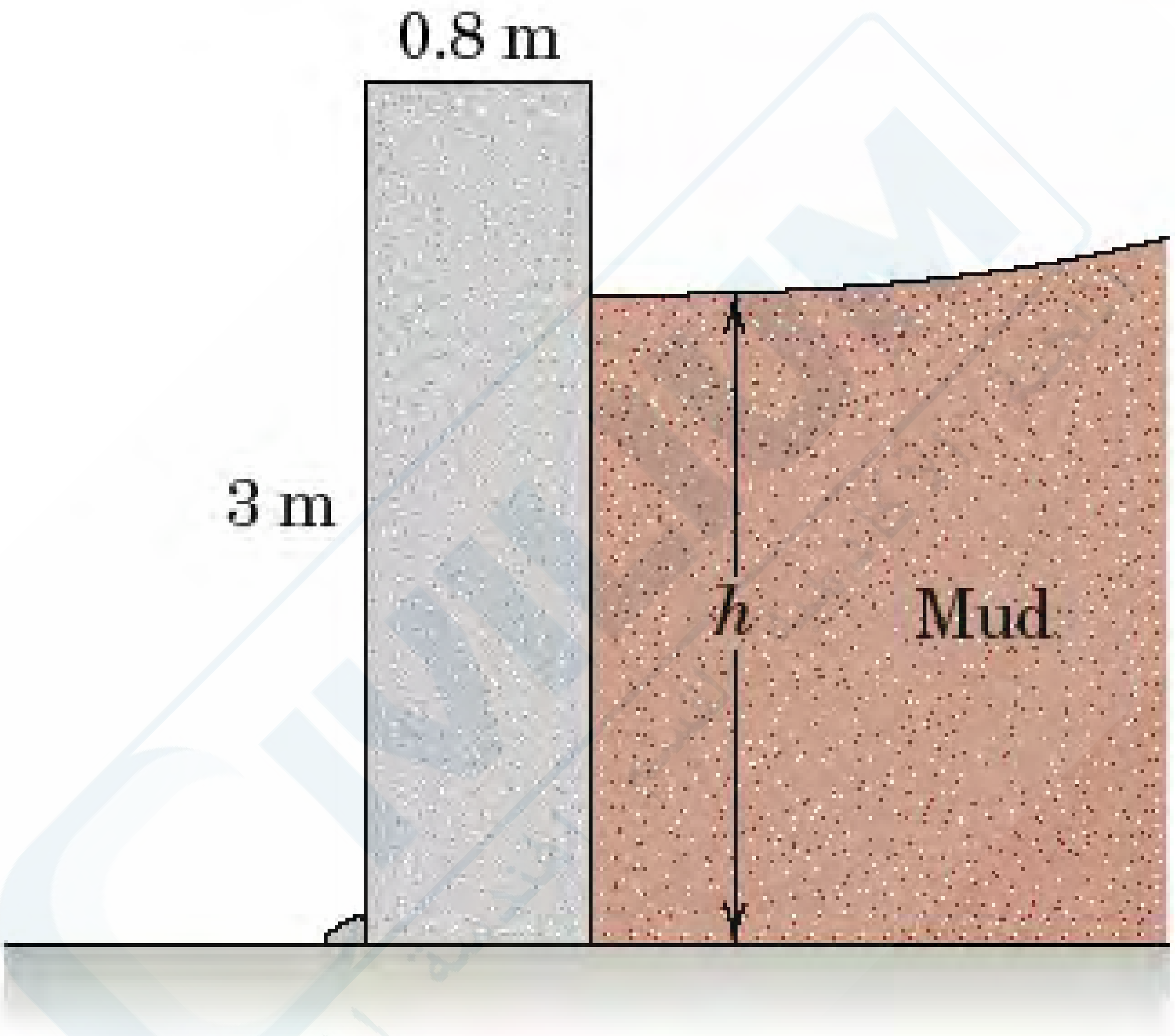
Problem 5/175

P_05_175



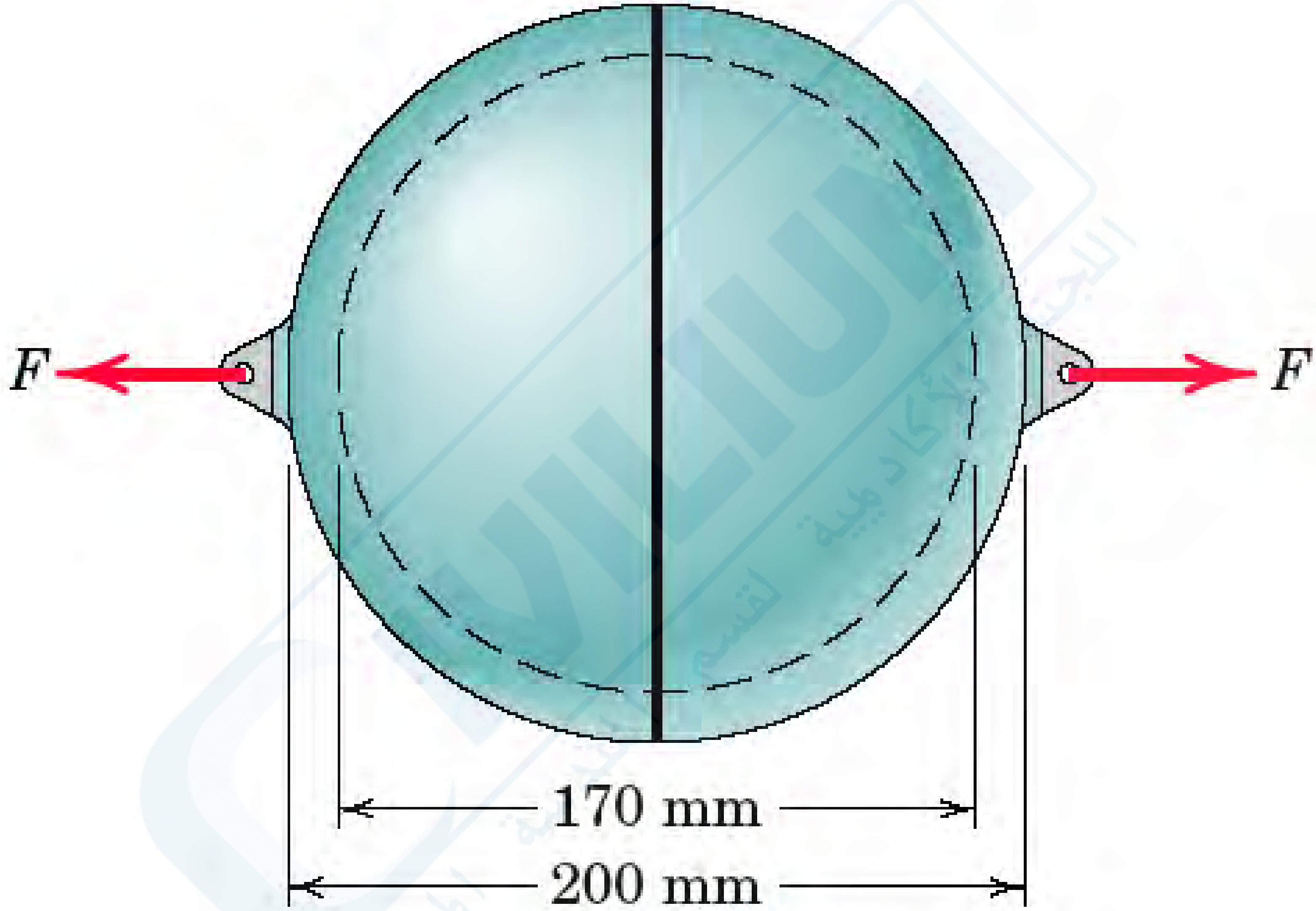
Problem 5/176

P_05_176



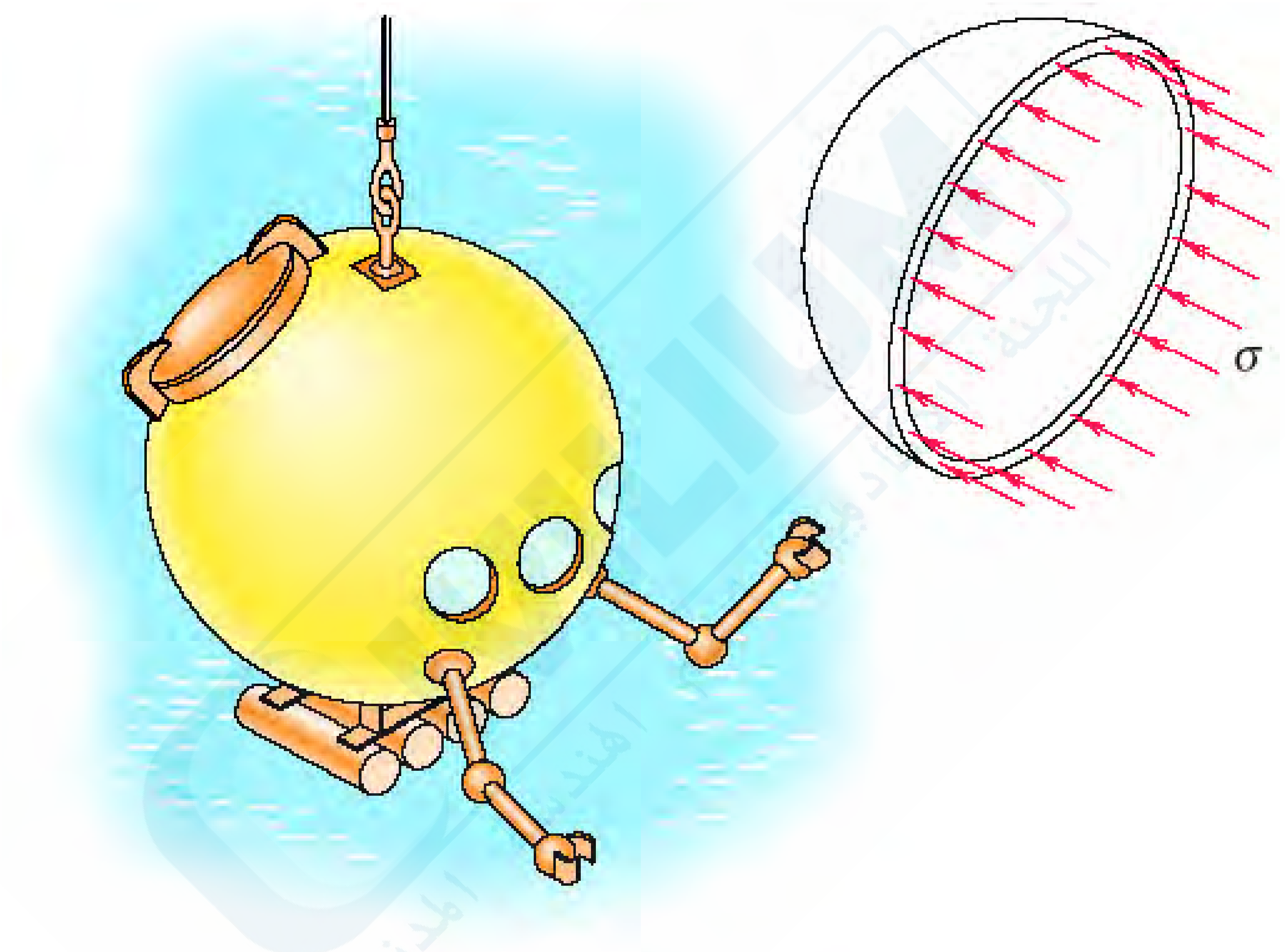
Problem 5/177

P_05_177



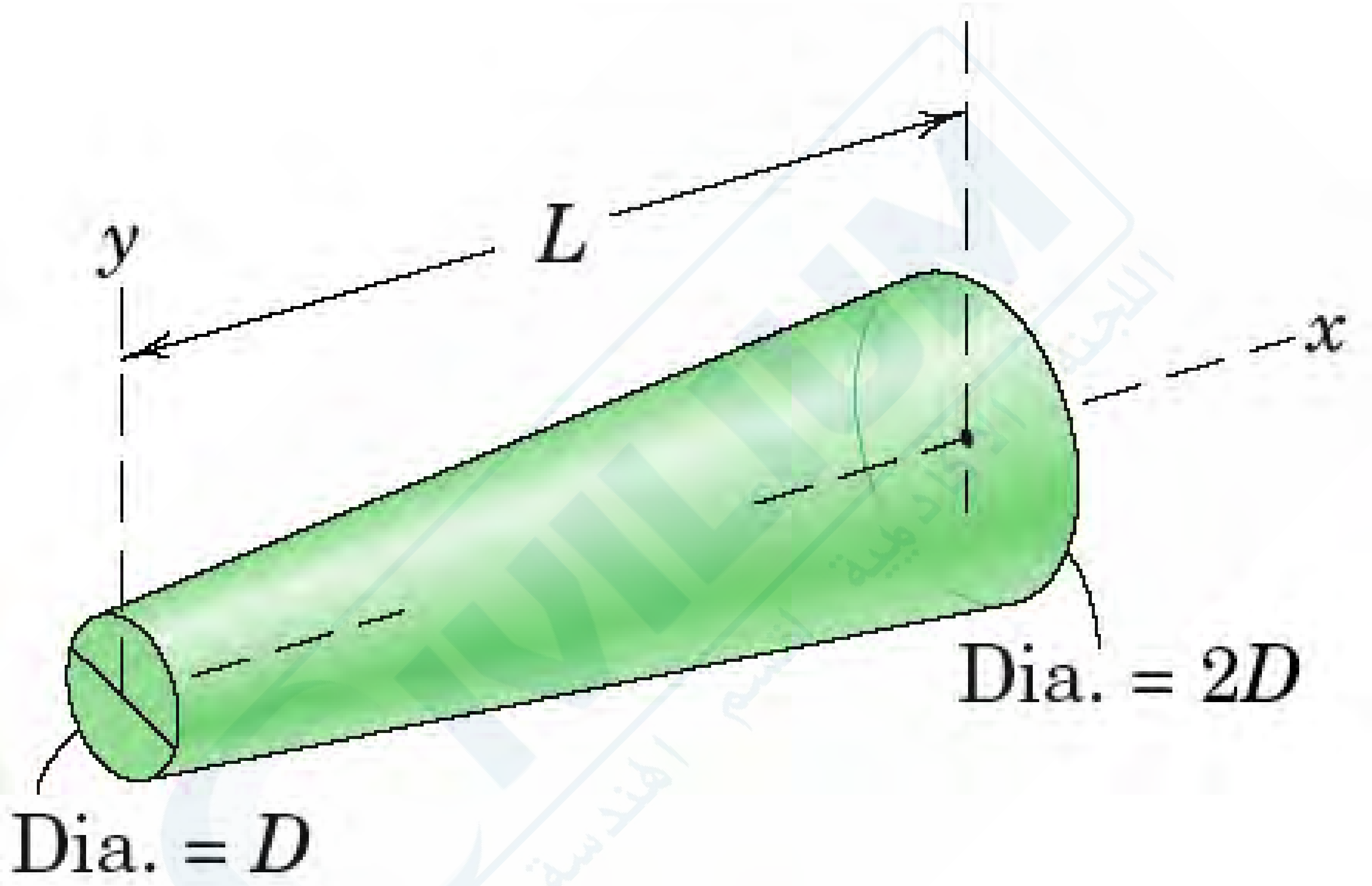
Problem 5/178

P 05 178



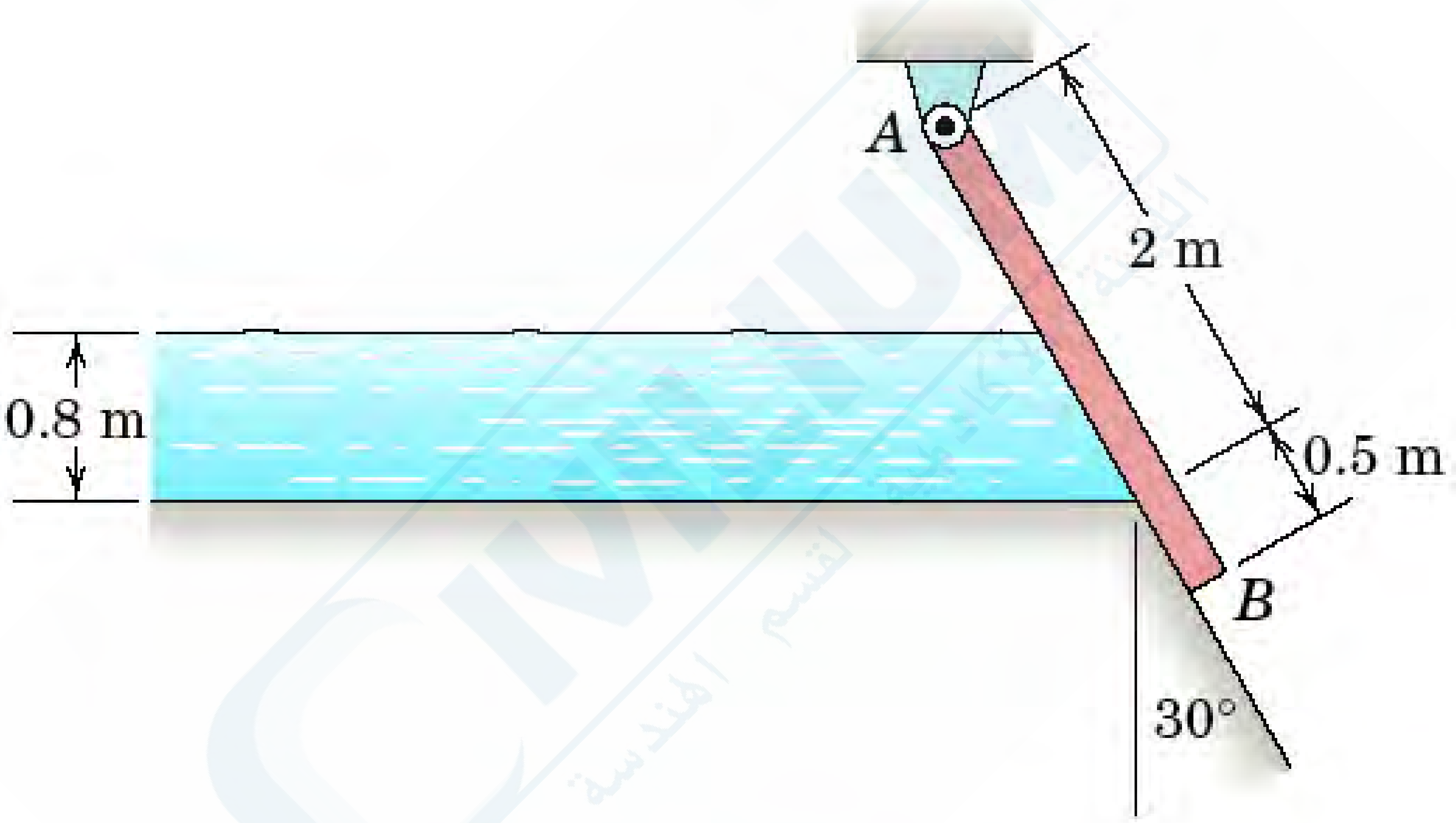
Problem 5/179

P_05_179



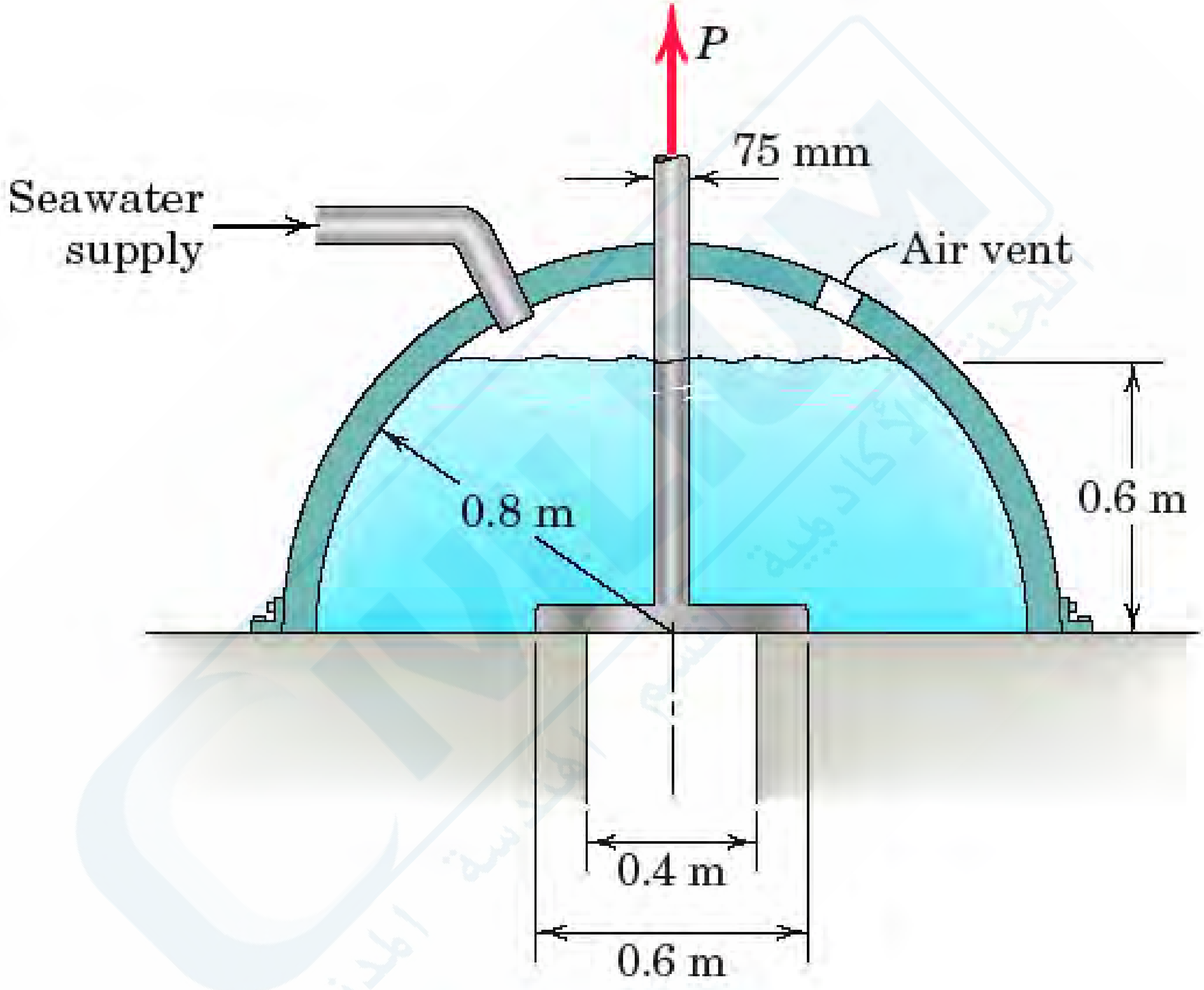
Problem 5/18

P_05_18



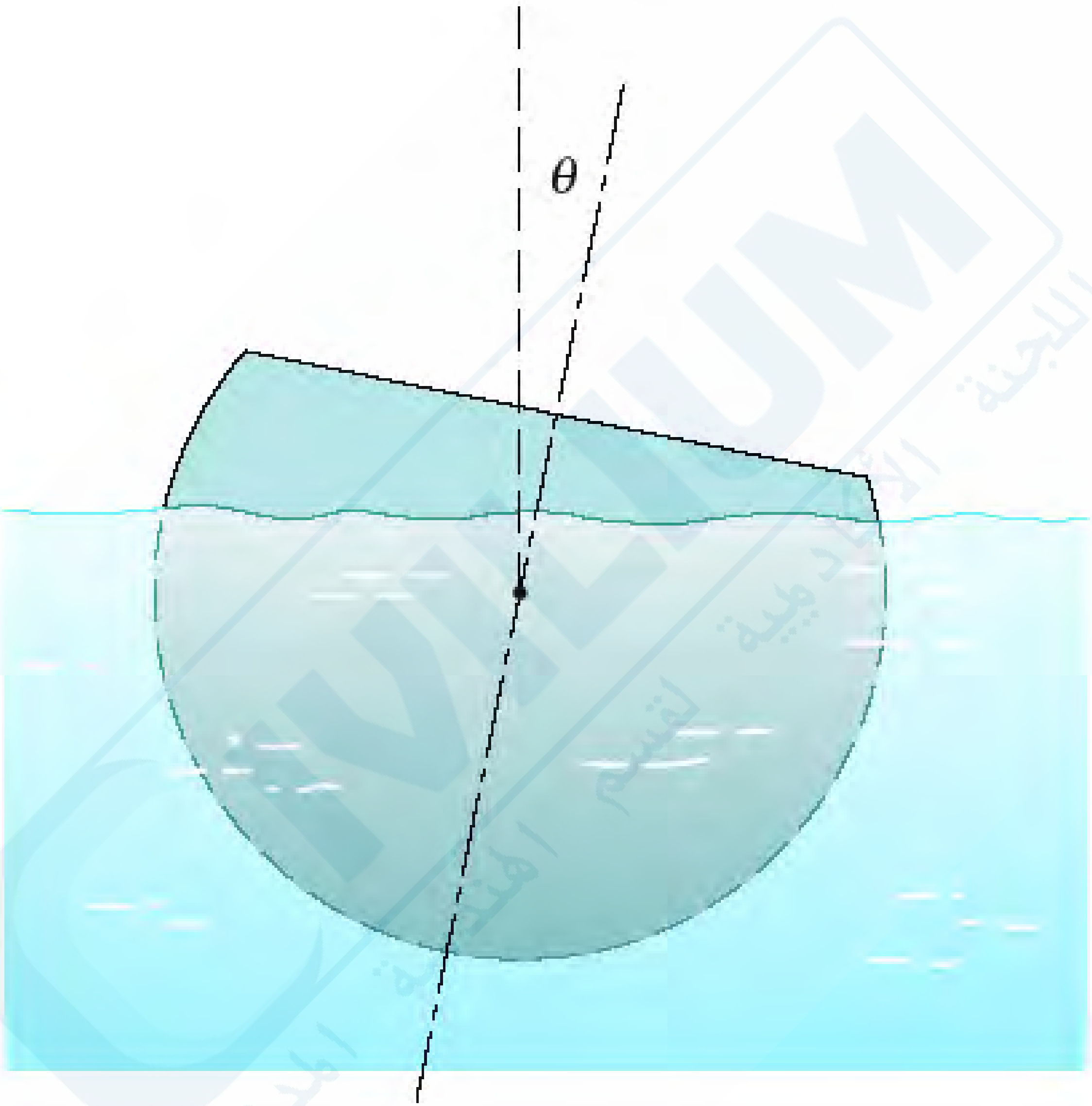
Problem 5/180

P_05_180



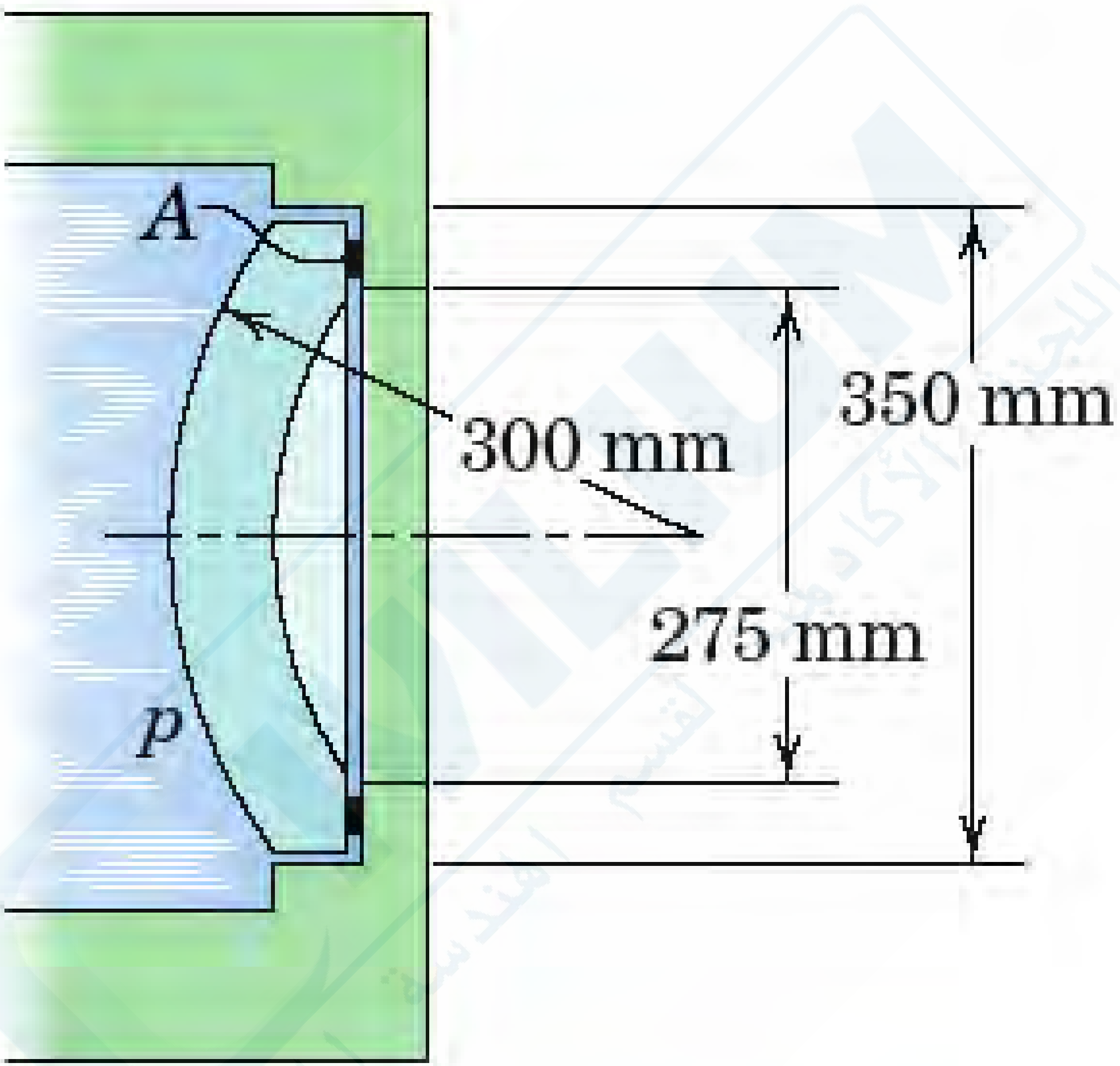
Problem 5/181

P_05_181

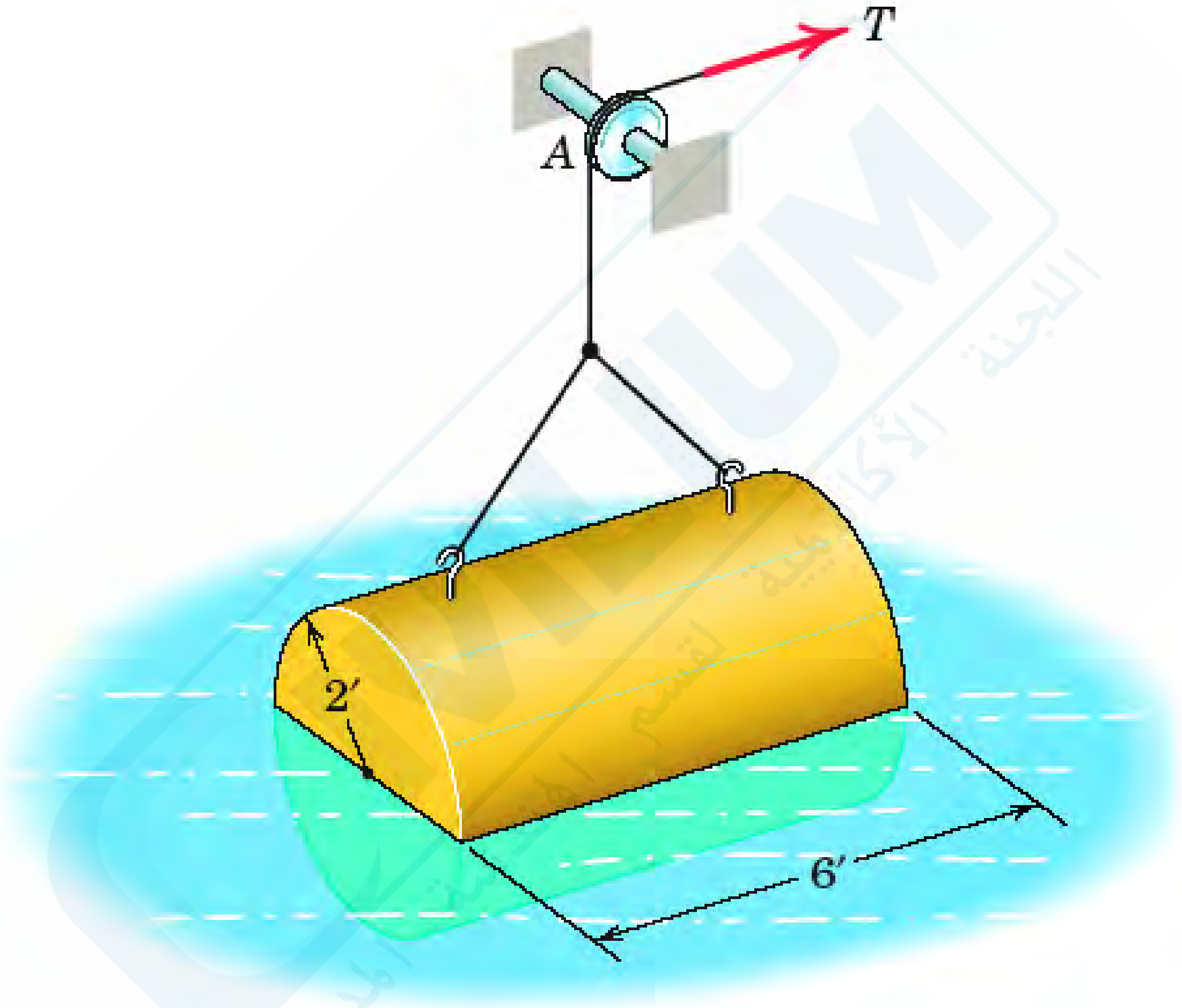


Problem 5/182

P_05_182

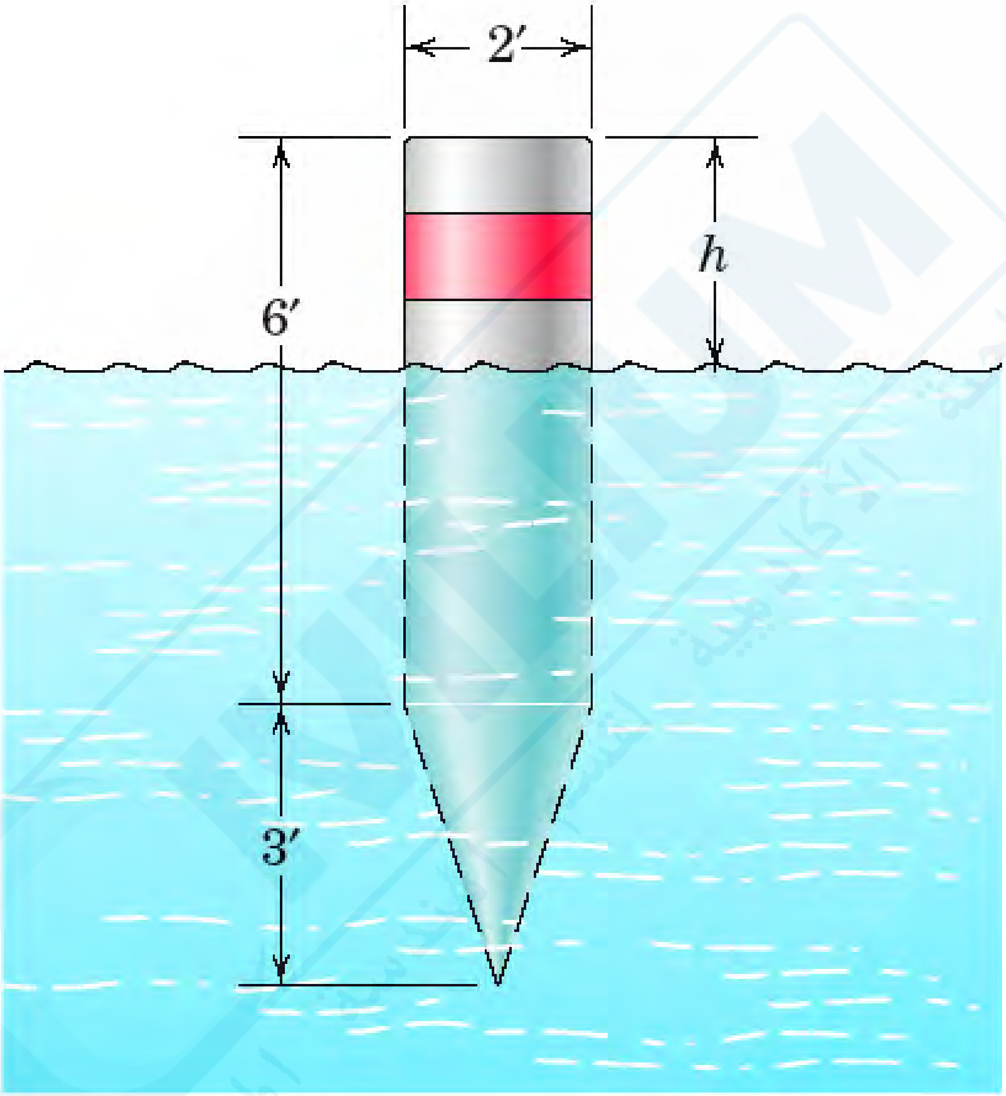


Problem 5/183



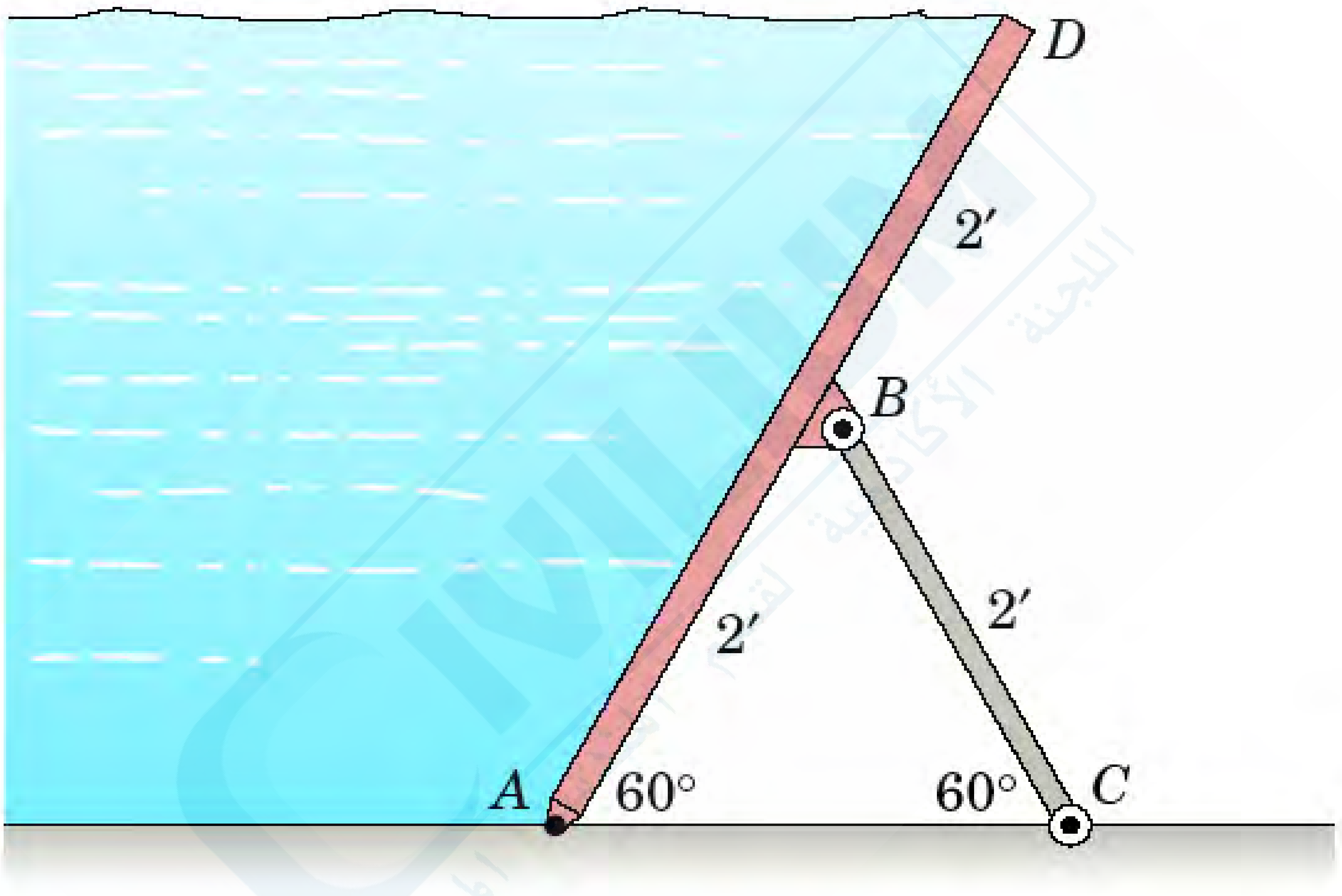
Problem 5/184

P 05 184



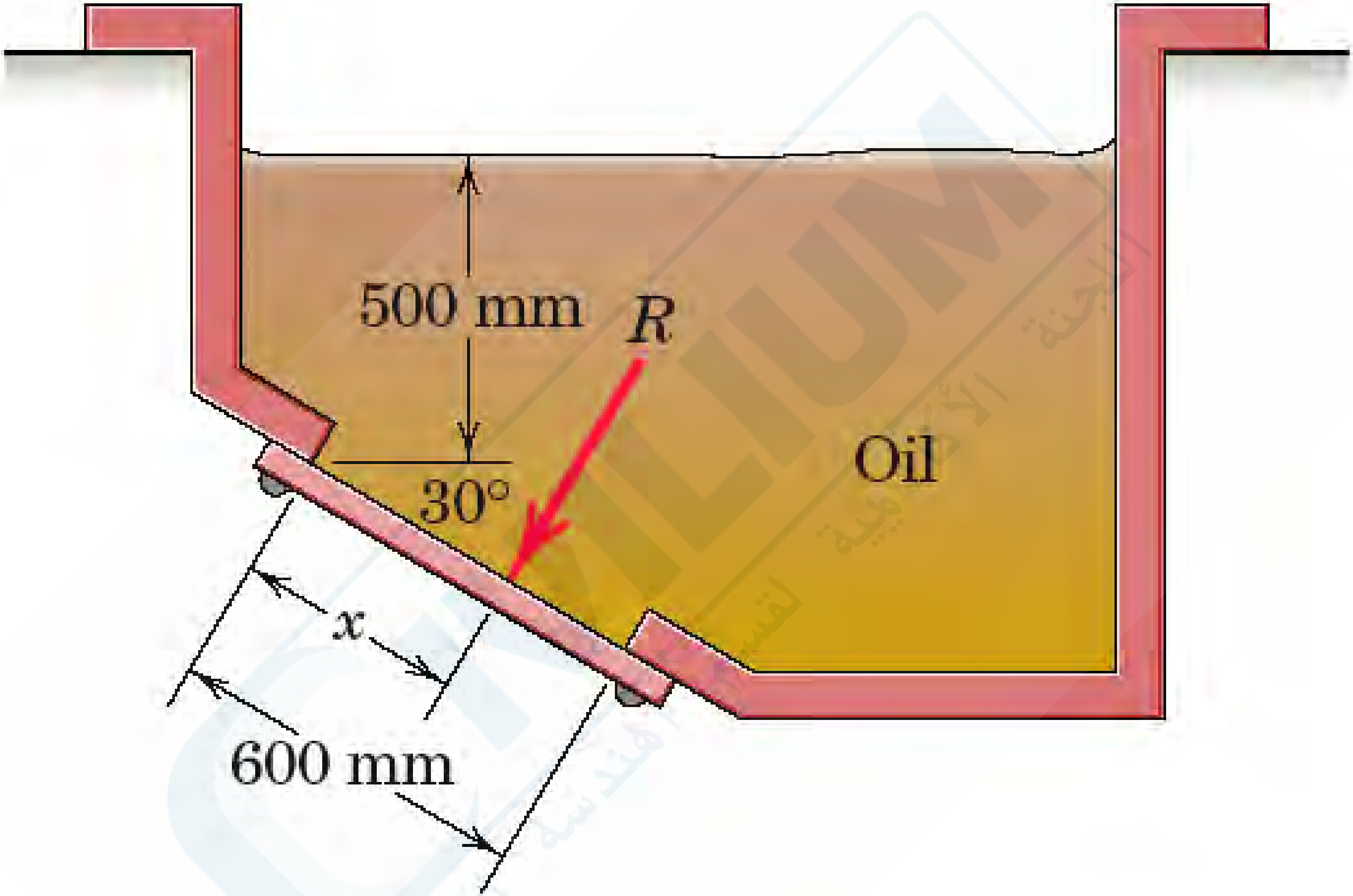
Problem 5/185

P 05 185



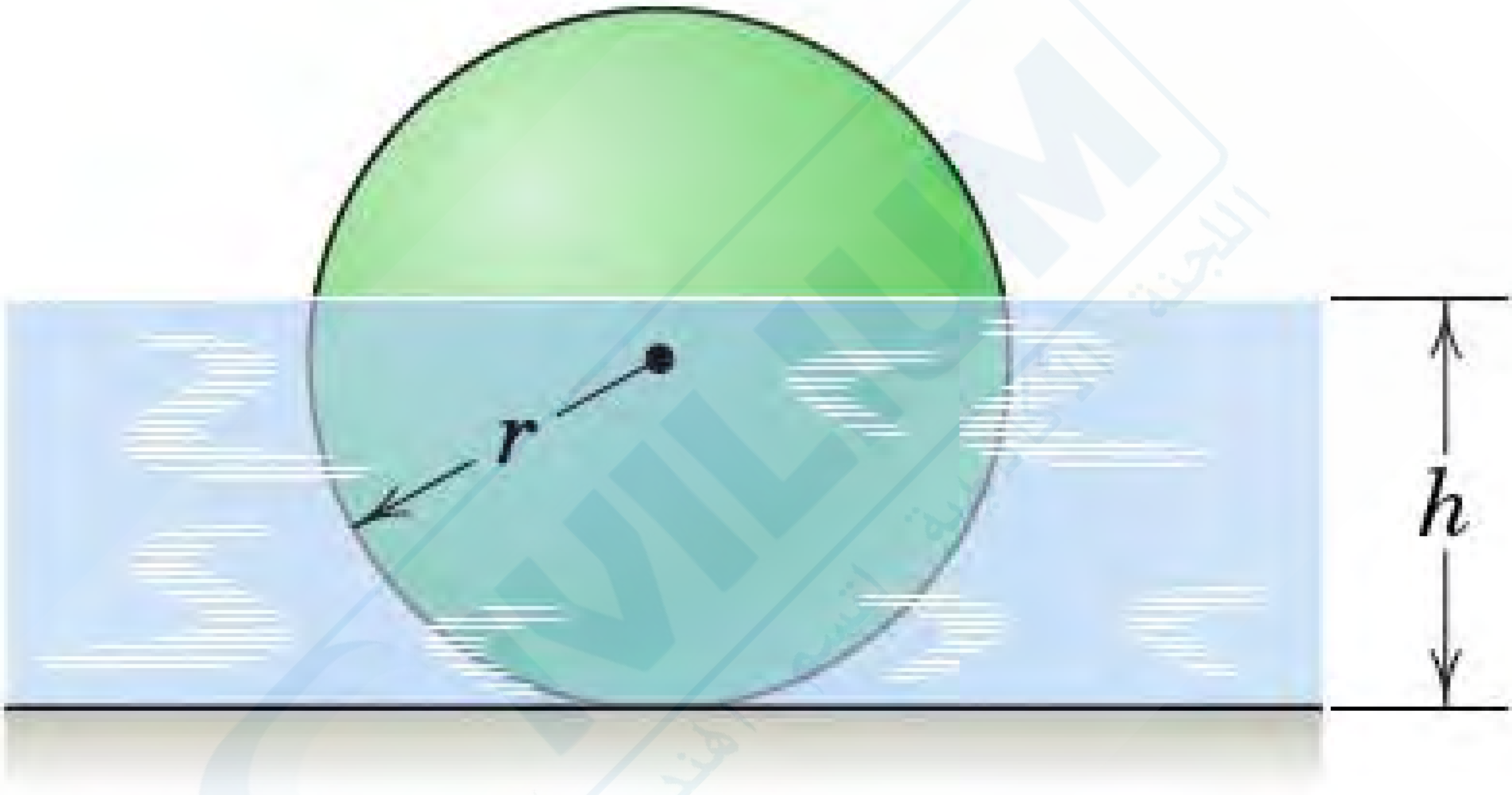
Problem 5/186

P_05_186



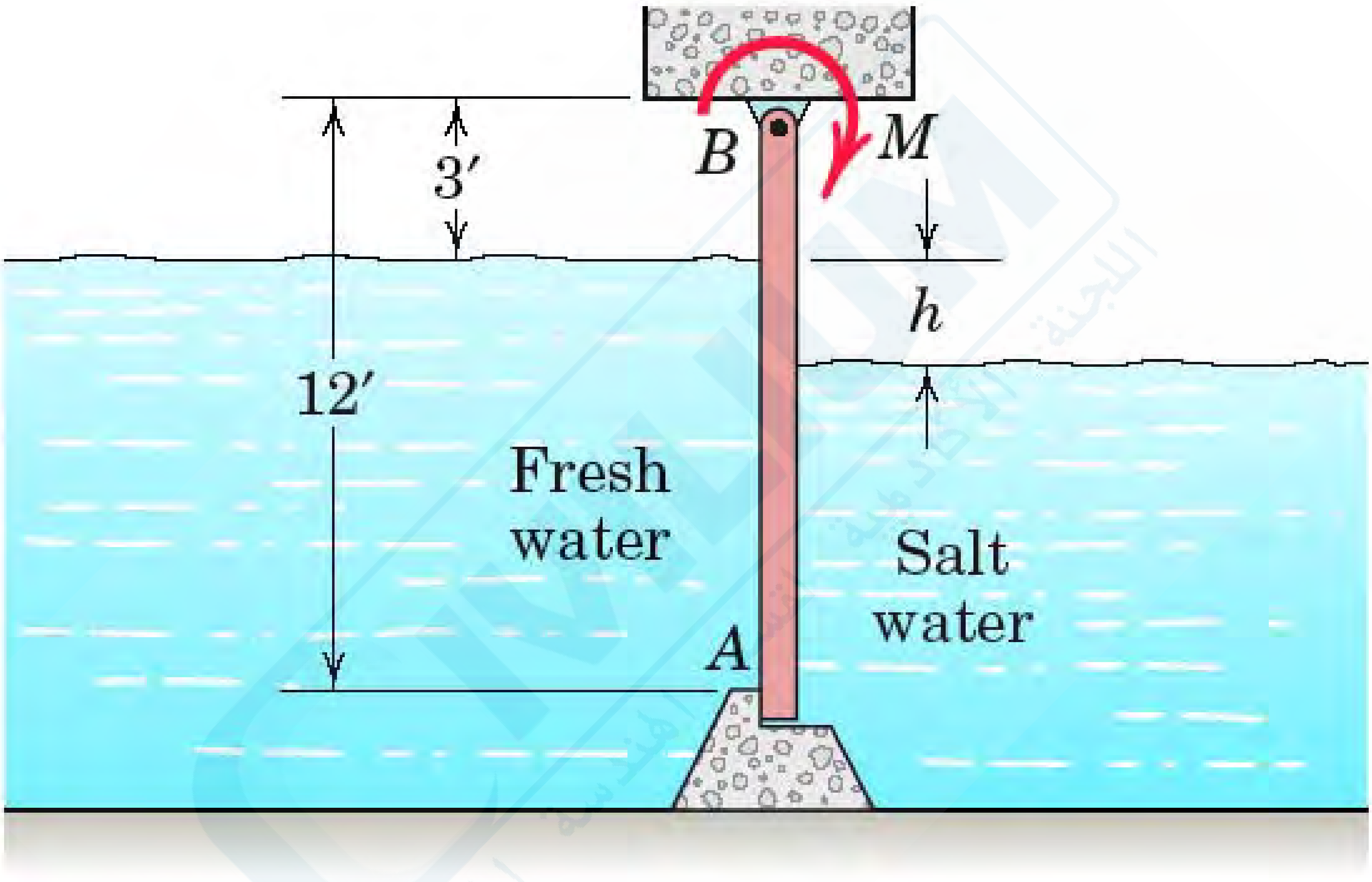
Problem 5/187

P_05_187

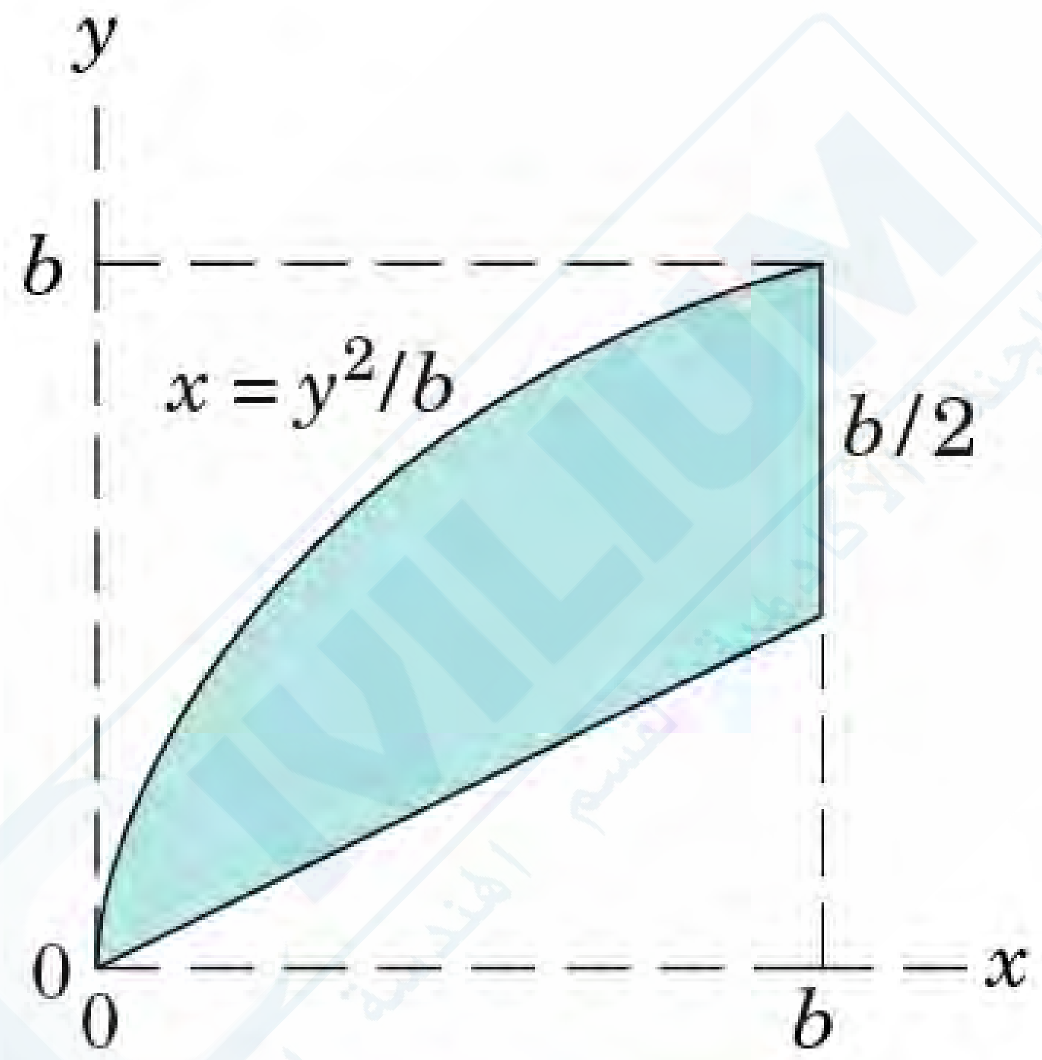


Problem 5/188

P_05_188

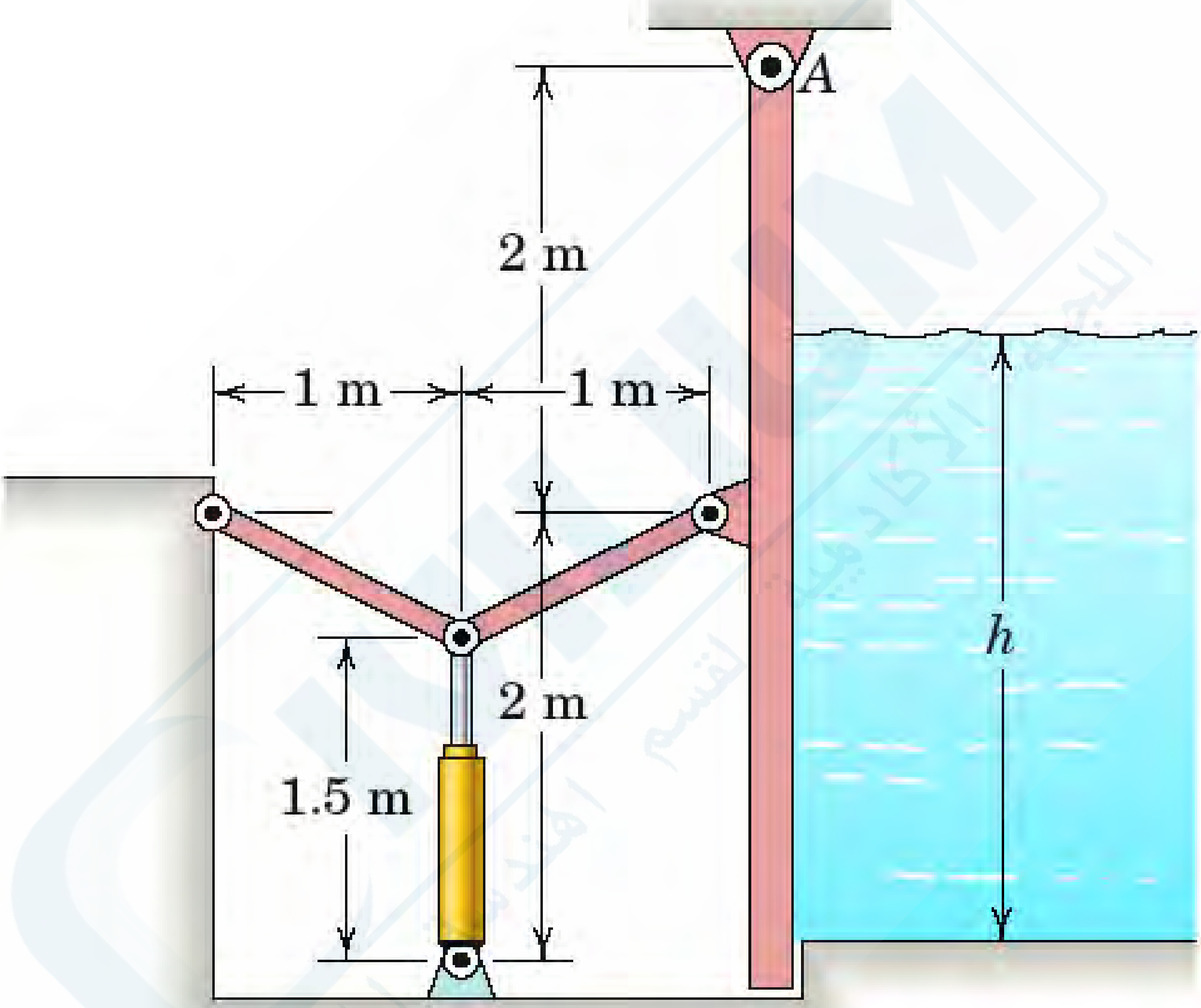


Problem 5/189



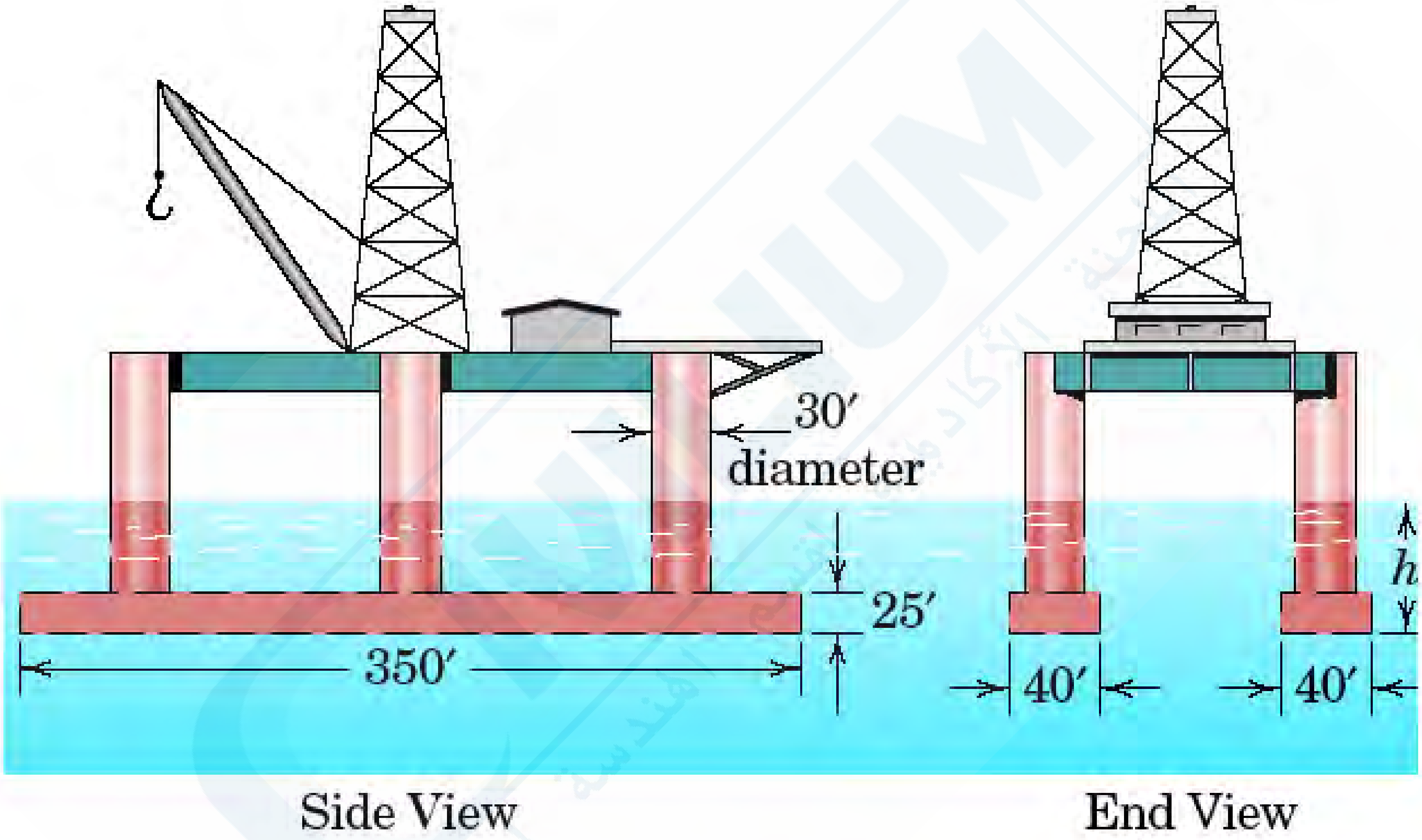
Problem 5/19

P_05_19



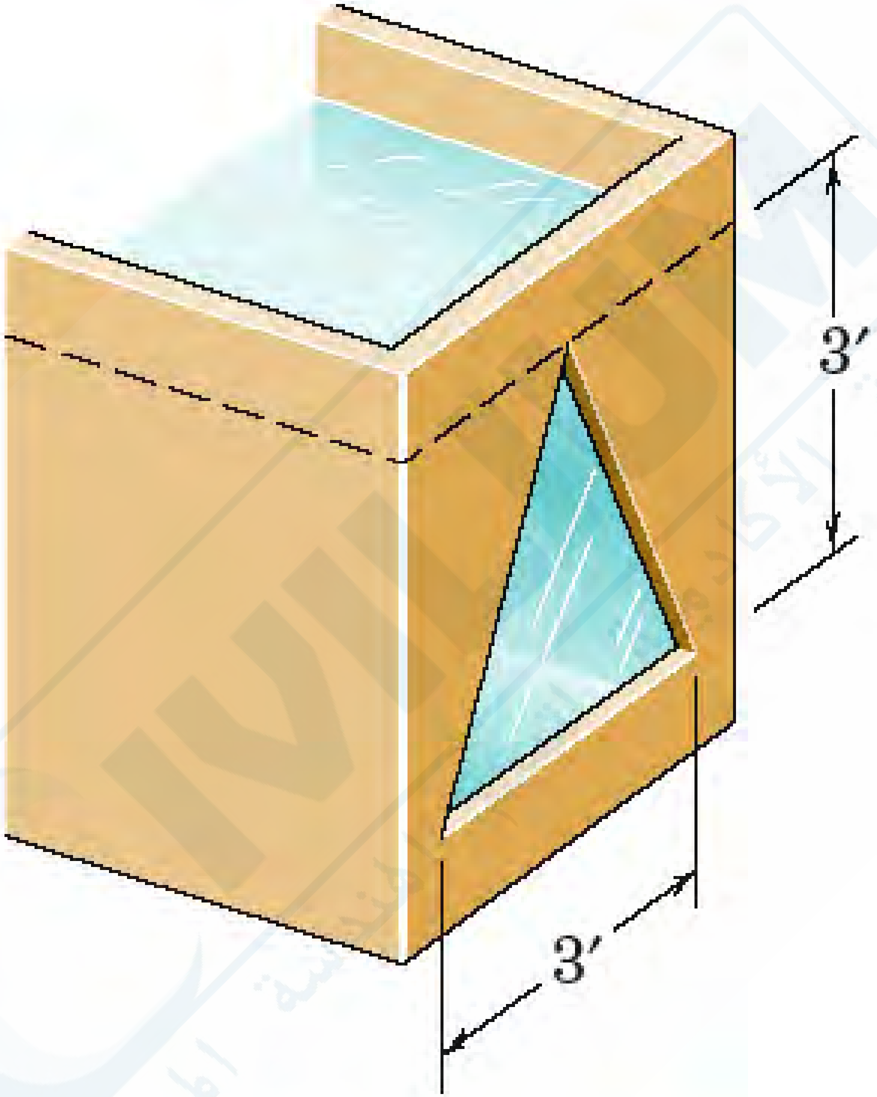
Problem 5/190

P 05 190



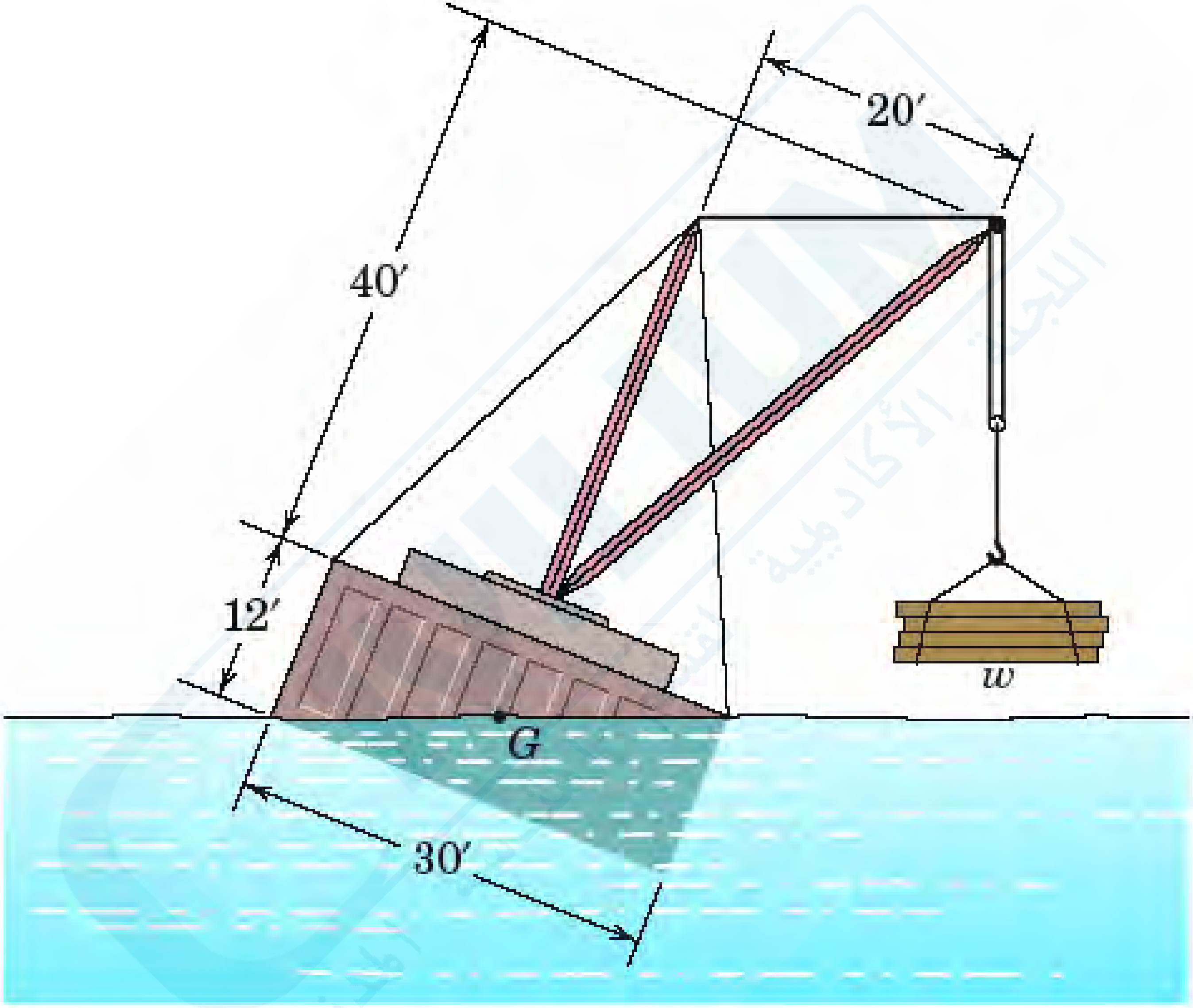
Problem 5/191

P_05_191



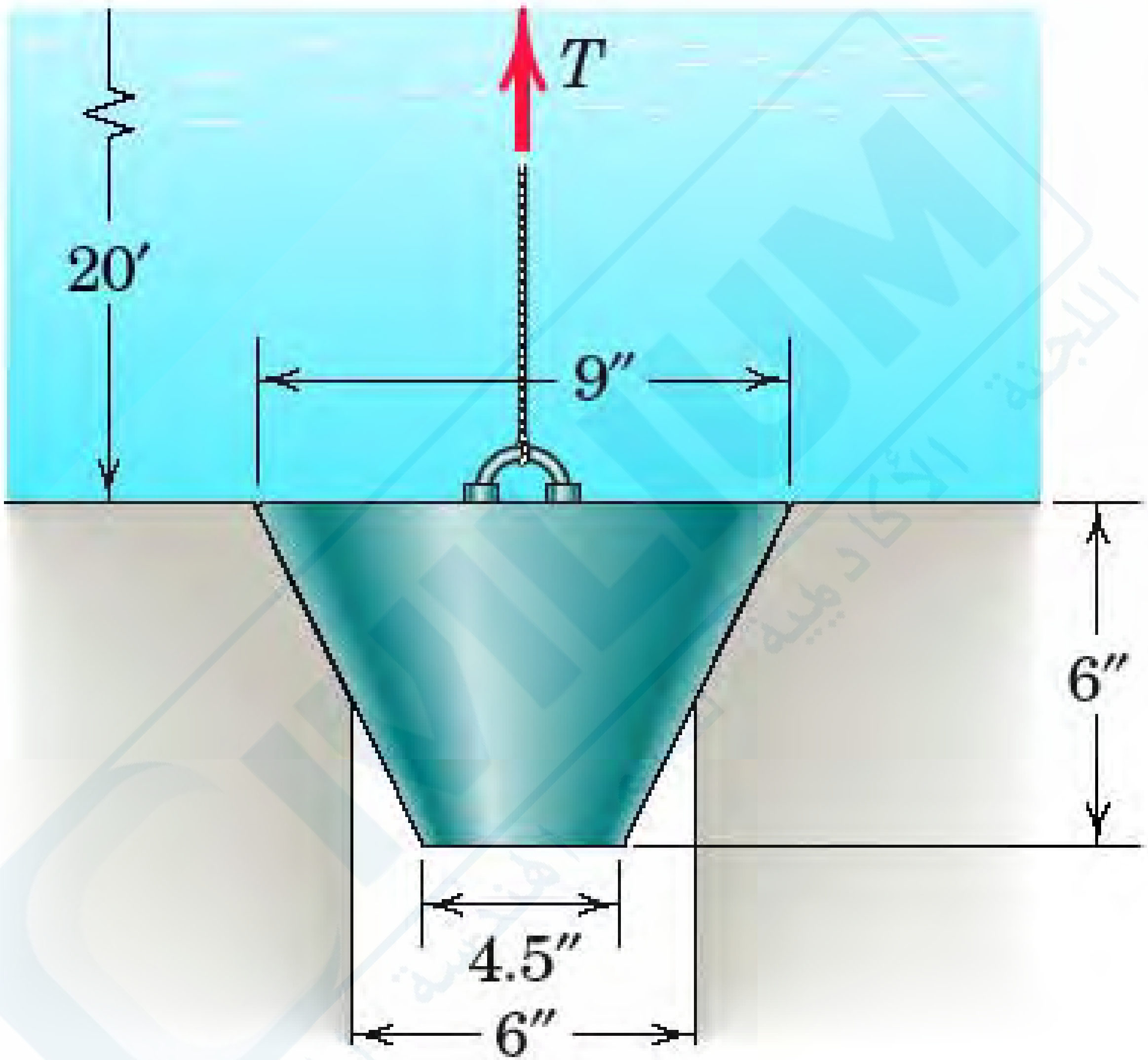
Problem 5/192

P_05_192



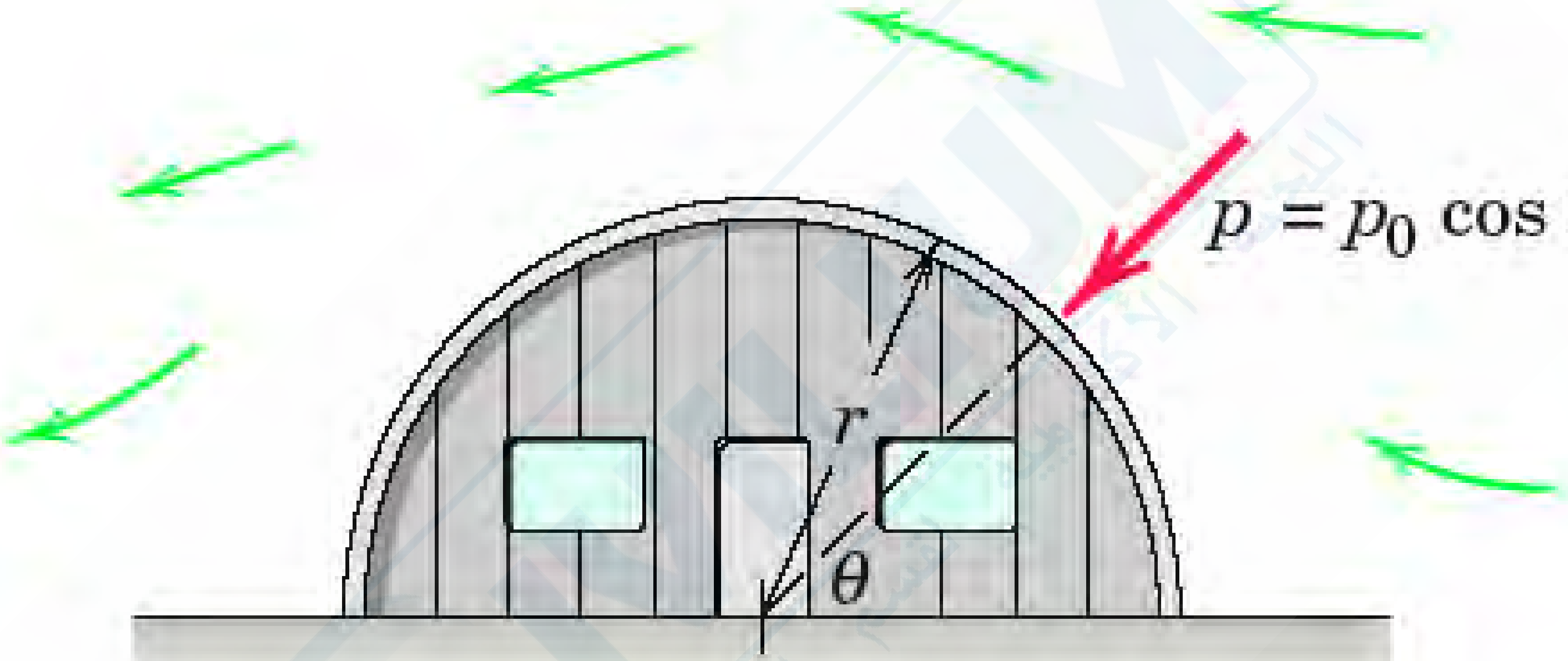
Problem 5/193

P_05_193



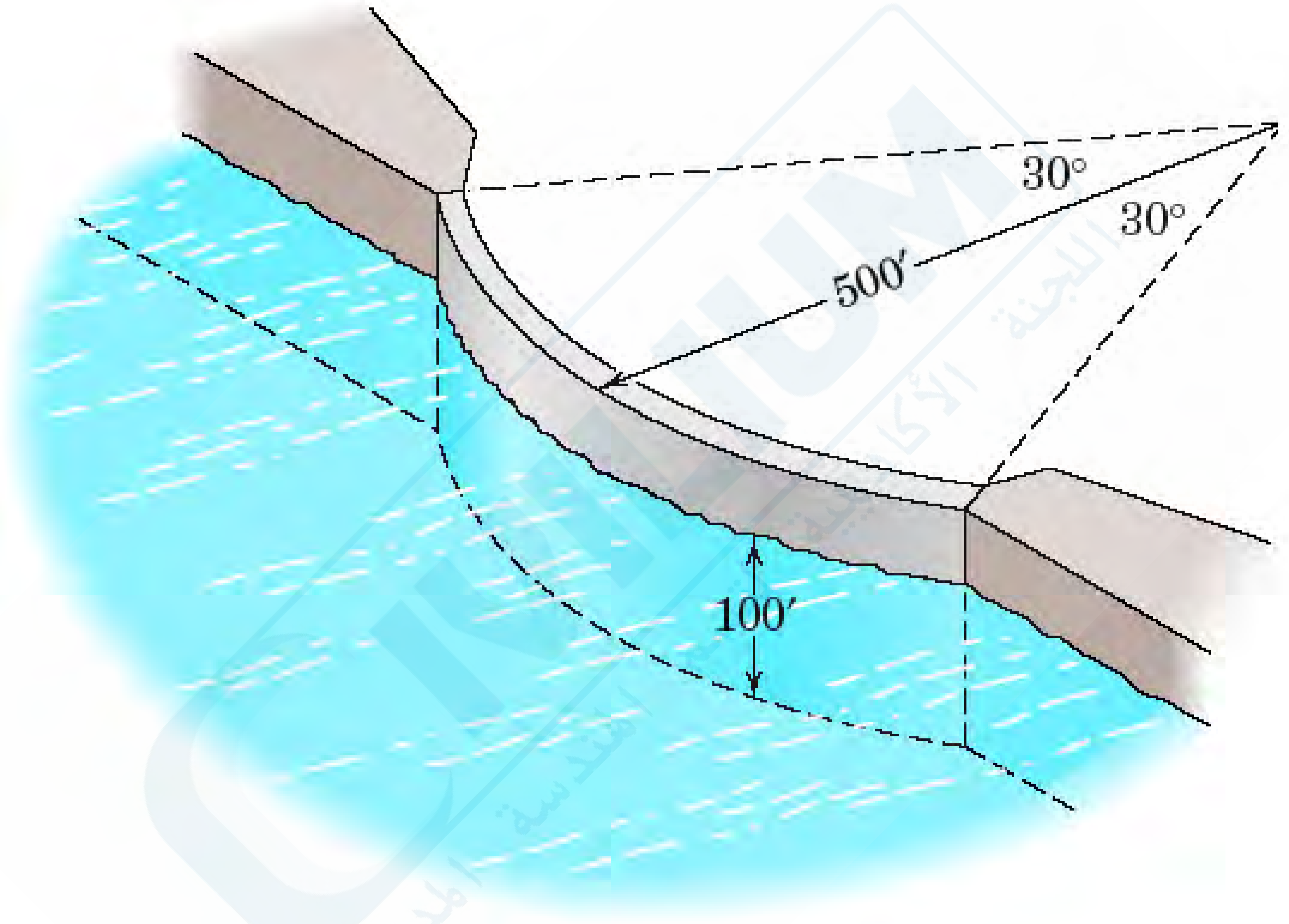
Problem 5/194

P_05_194



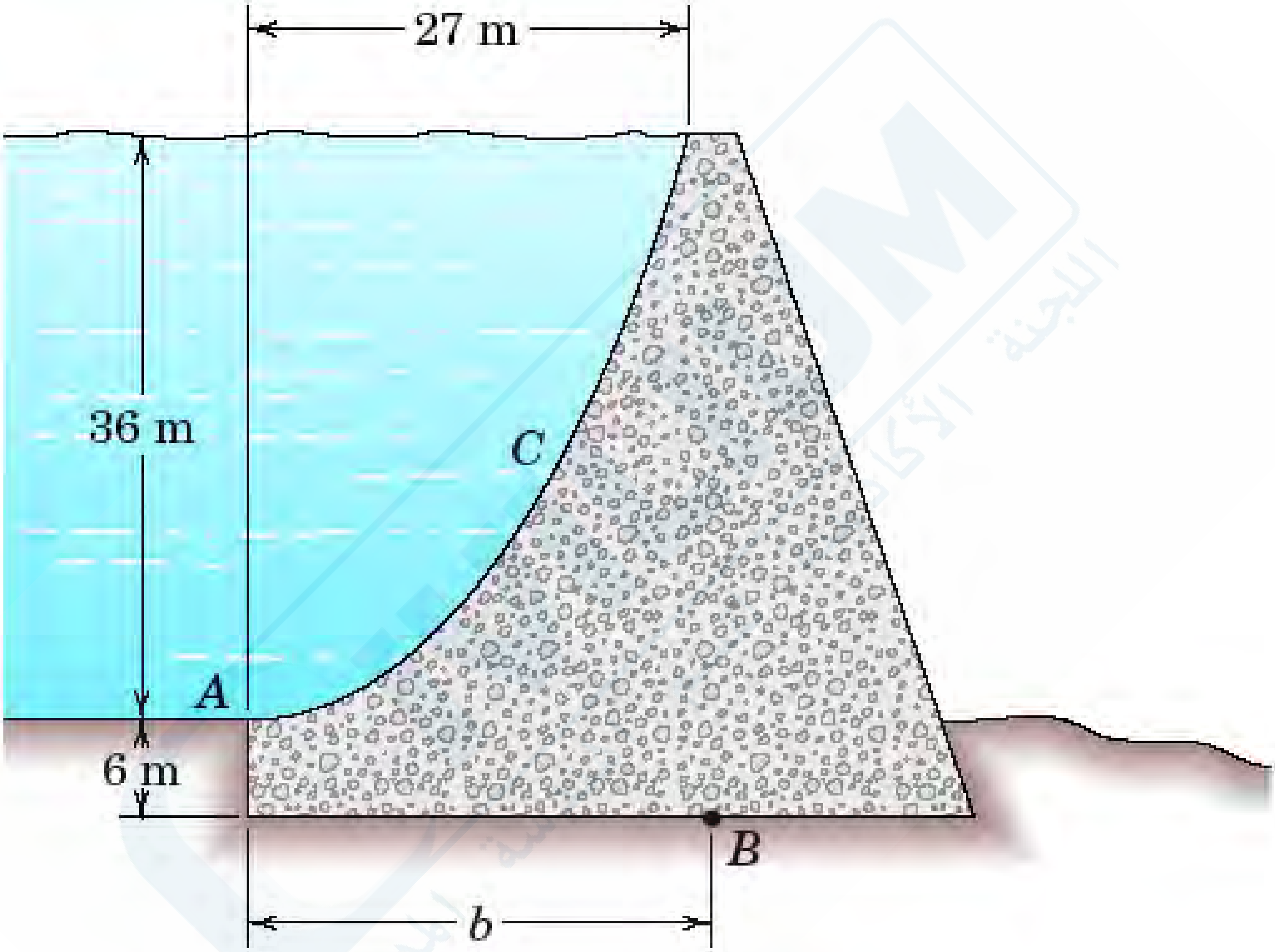
Problem 5/195

P_05_195

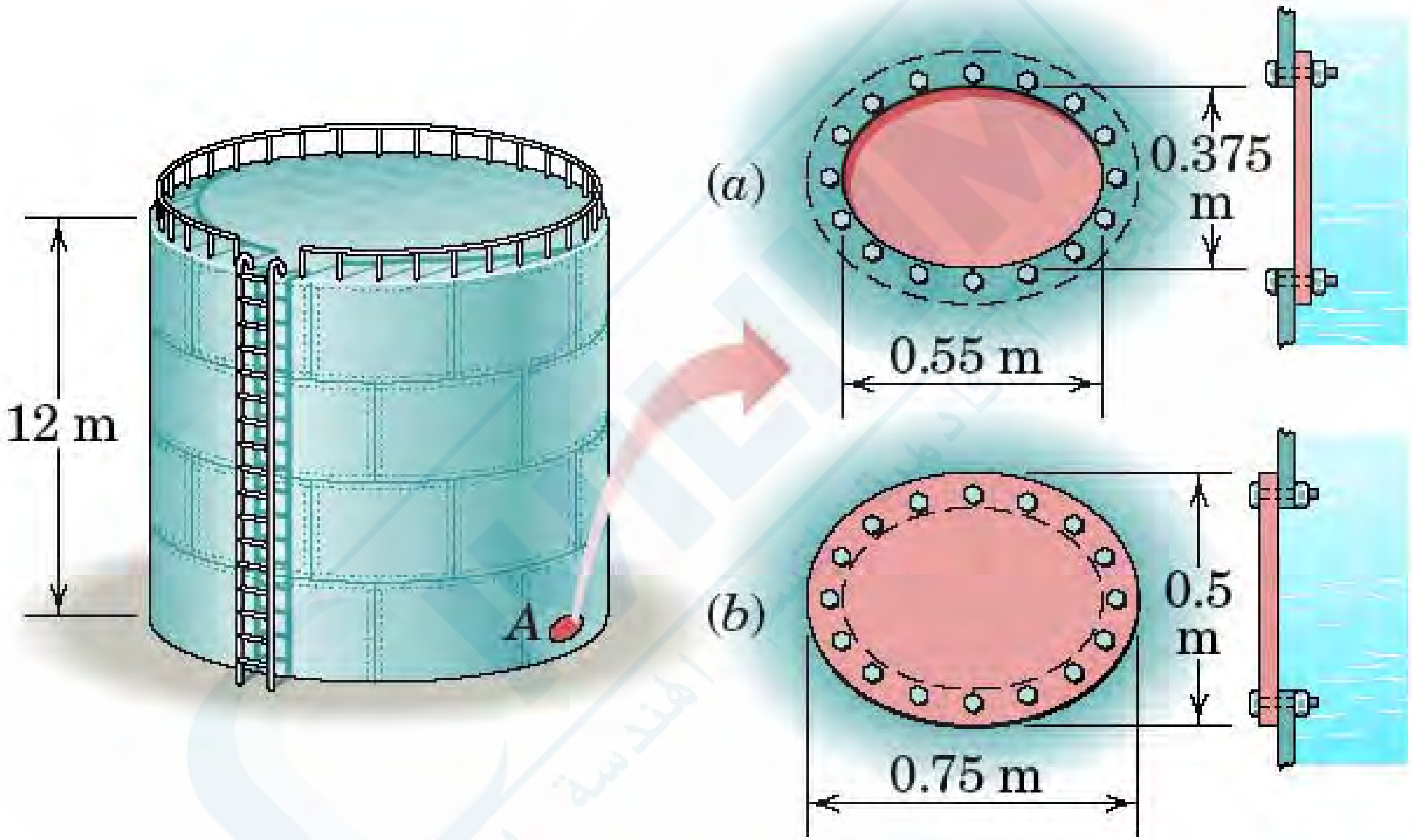


Problem 5/196

P 05 196

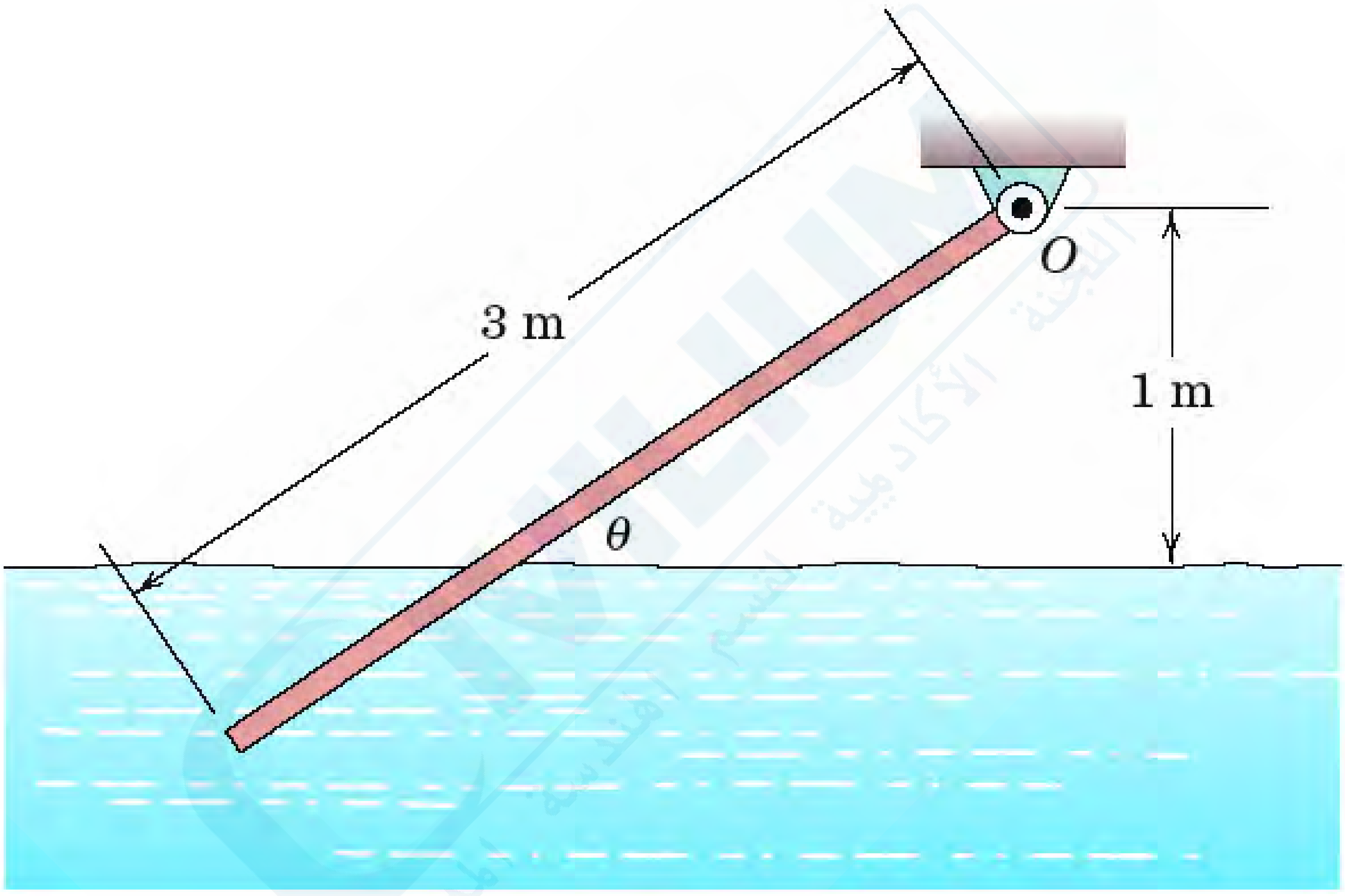


Problem 5/197



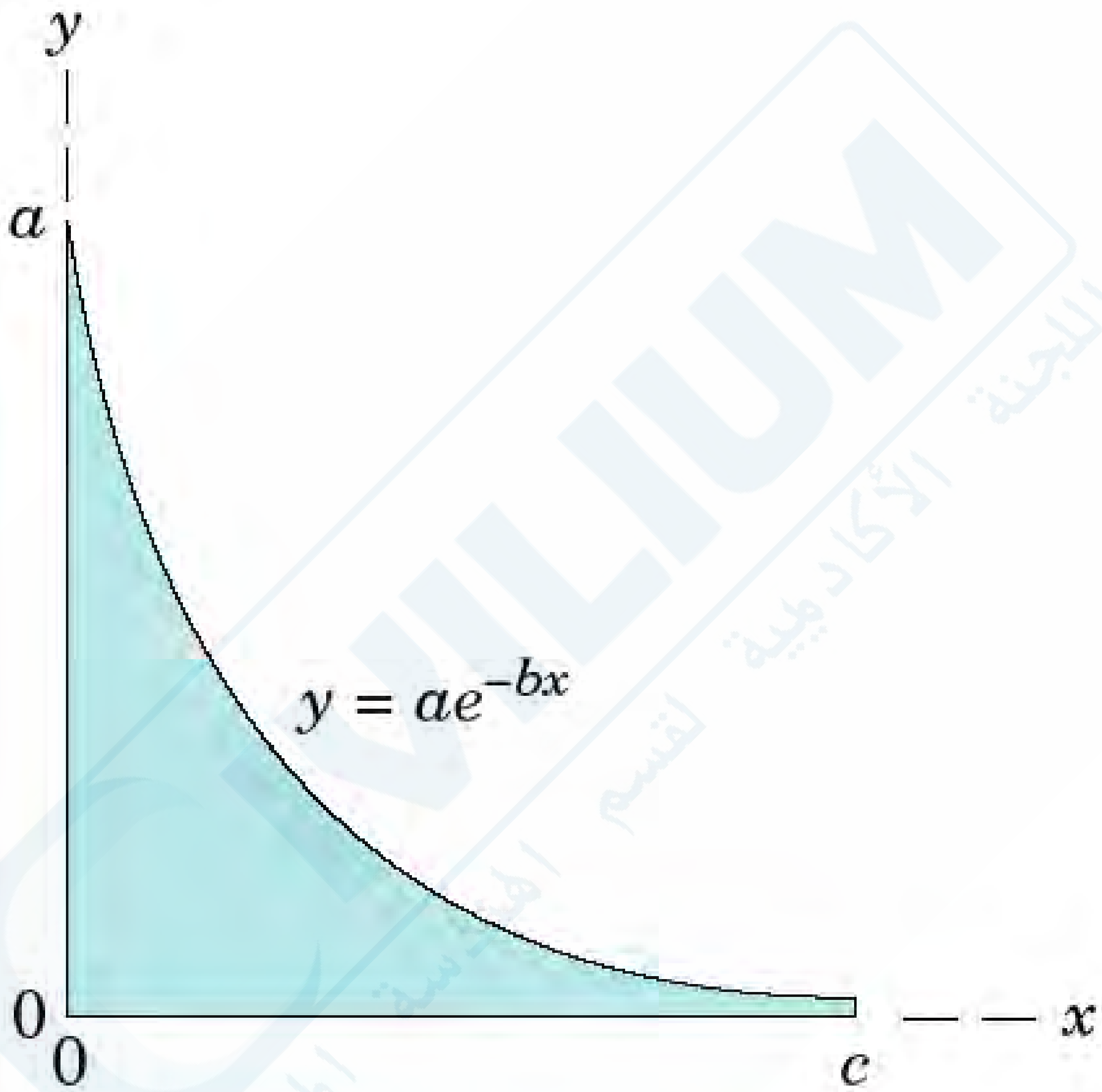
Problem 5/198

P_05_198

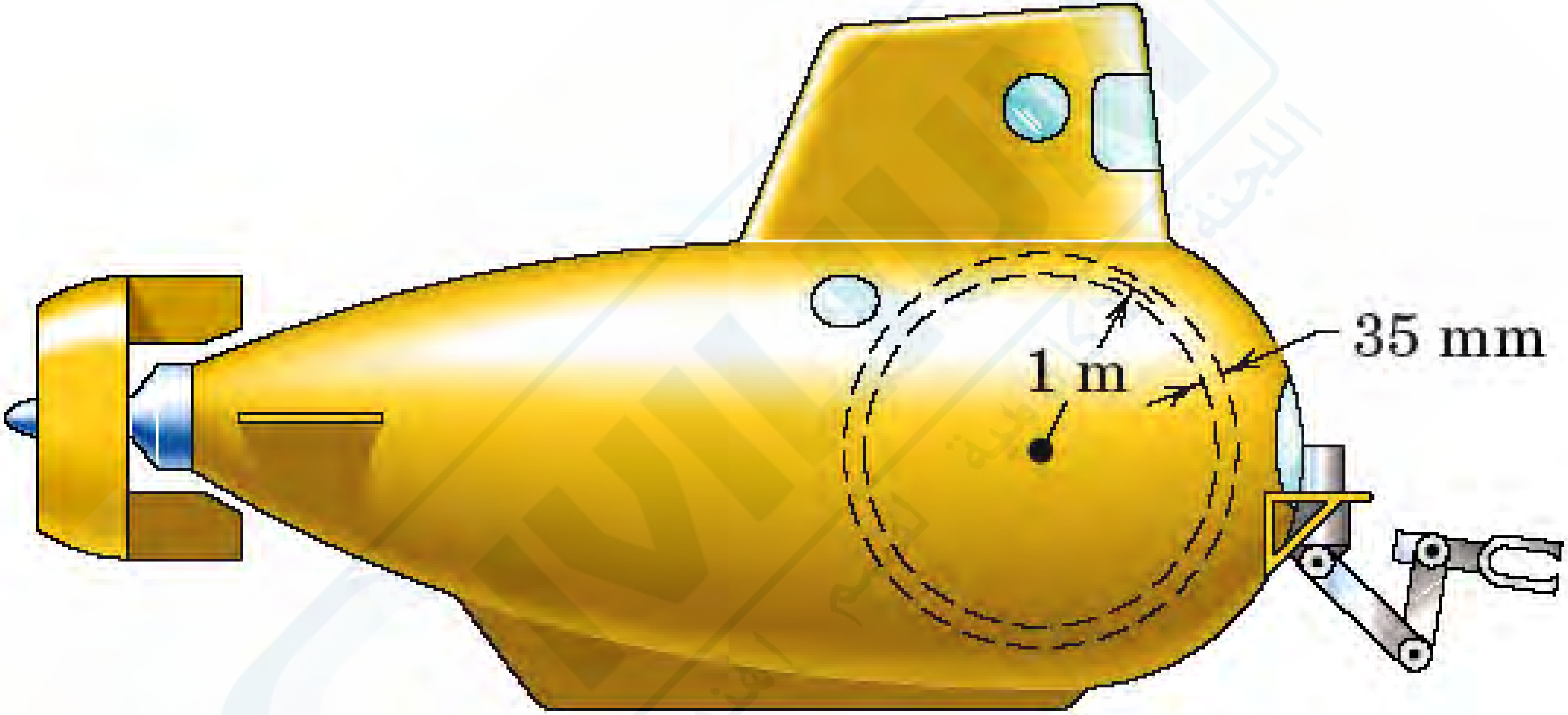


Problem 5/199

P_05_199

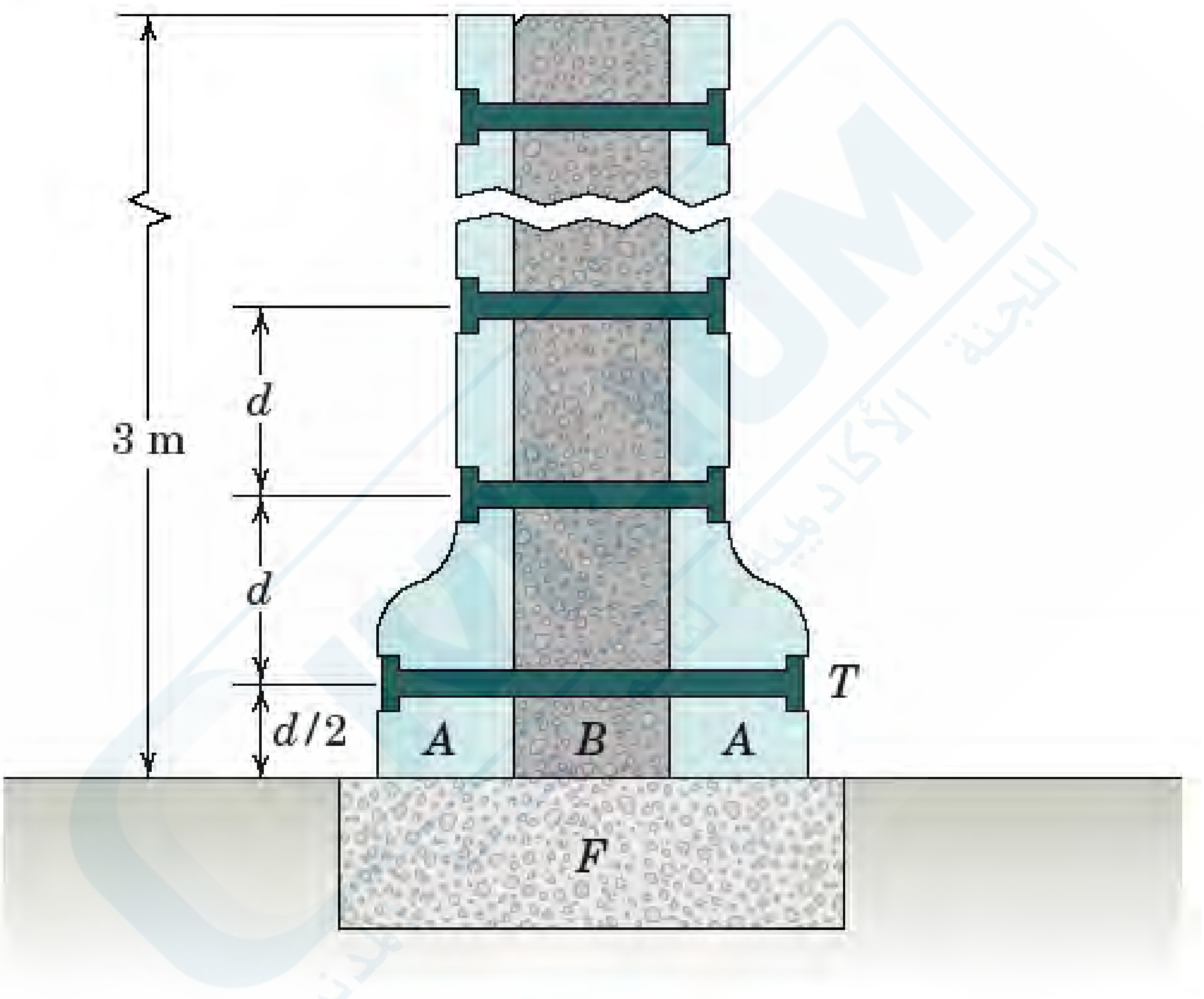


Problem 5/20

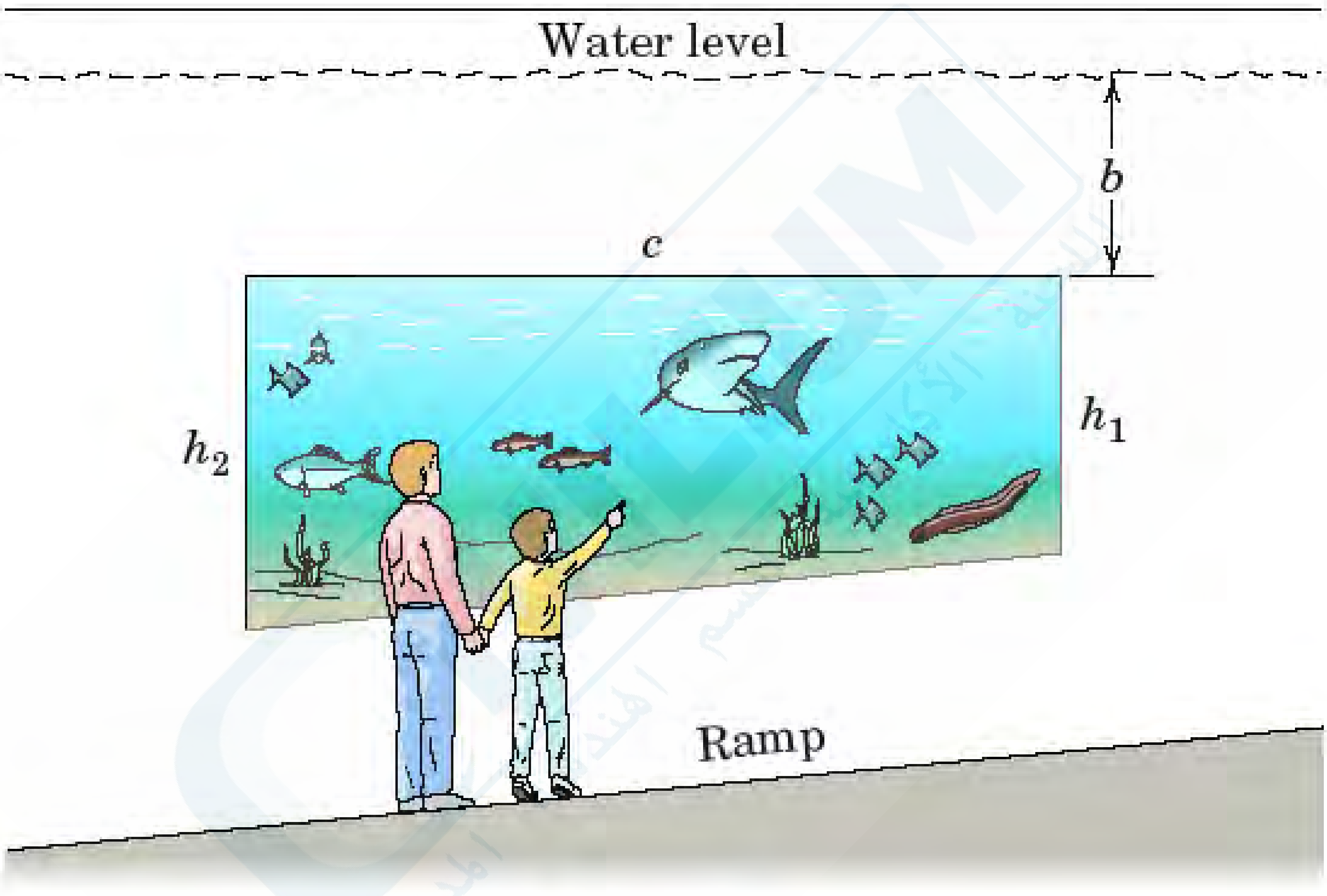


Problem 5/200

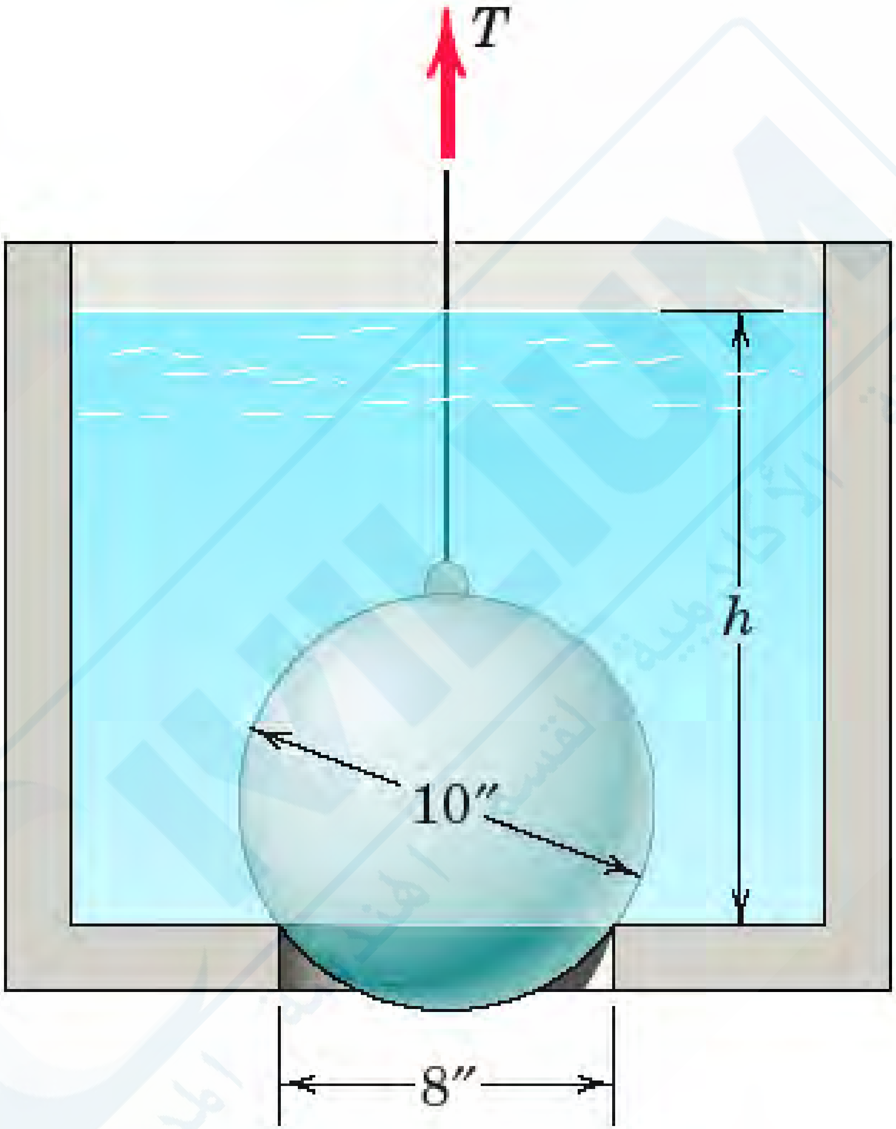
P_05_200



Problem 5/201

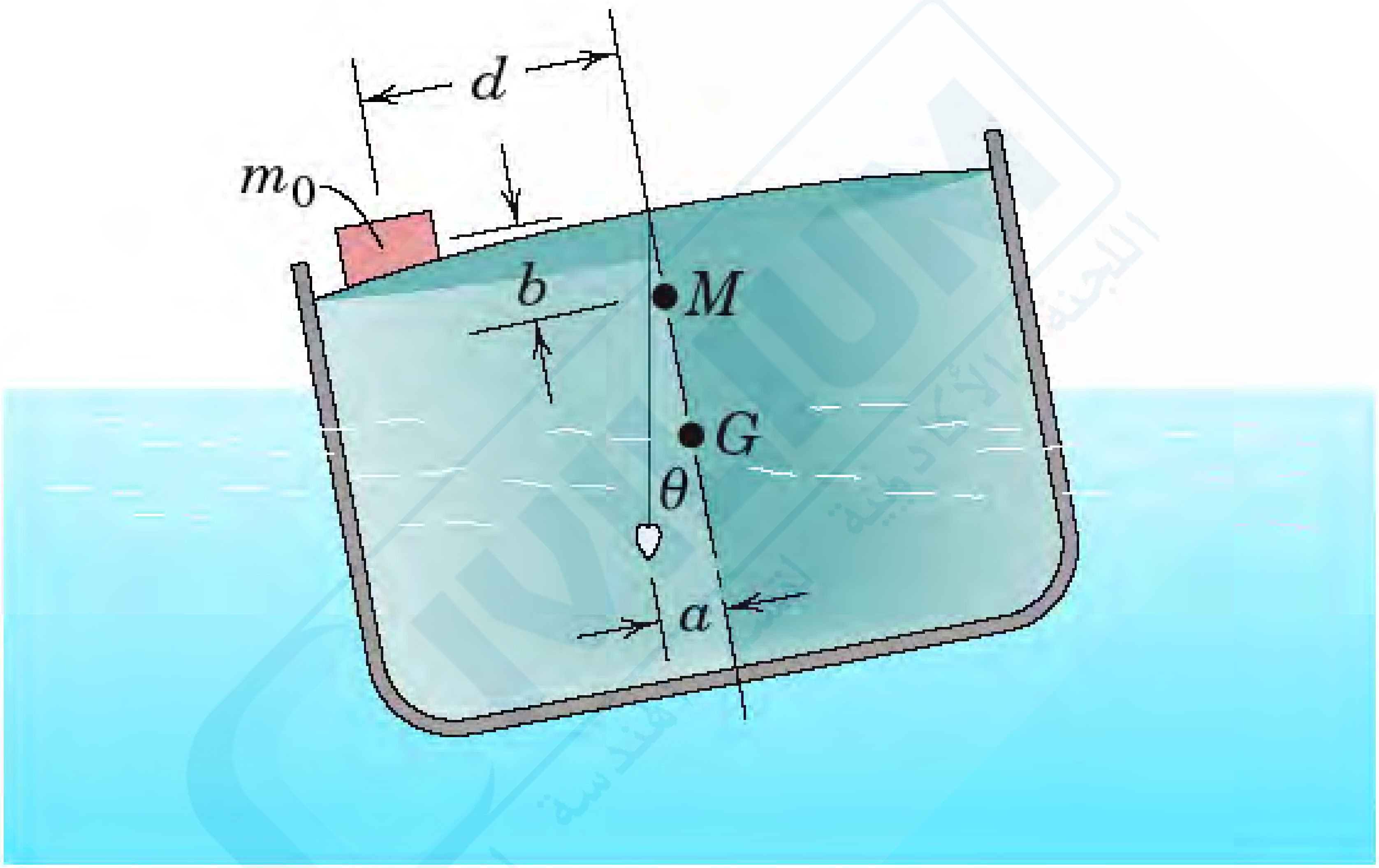


Problem 5/202



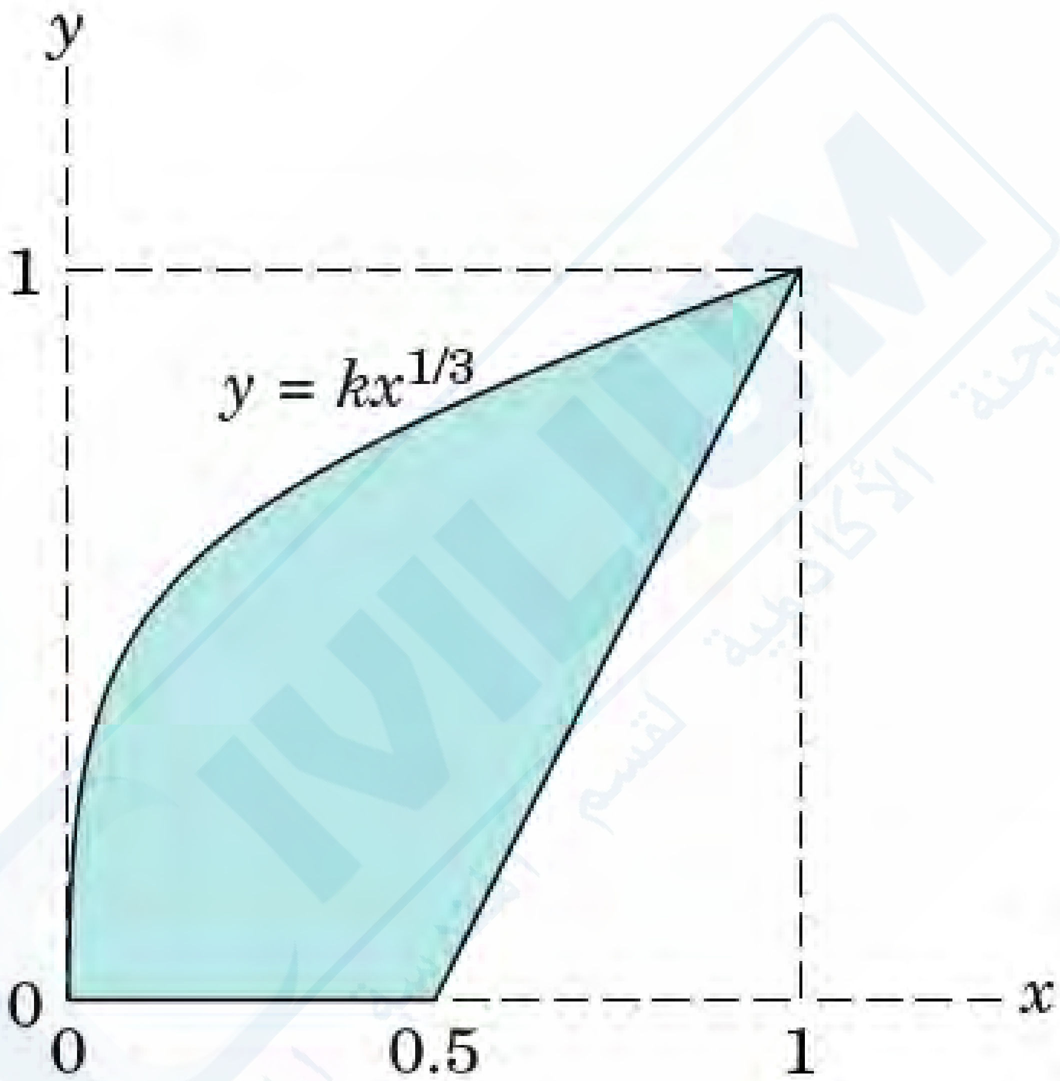
Problem 5/203

P 05 203



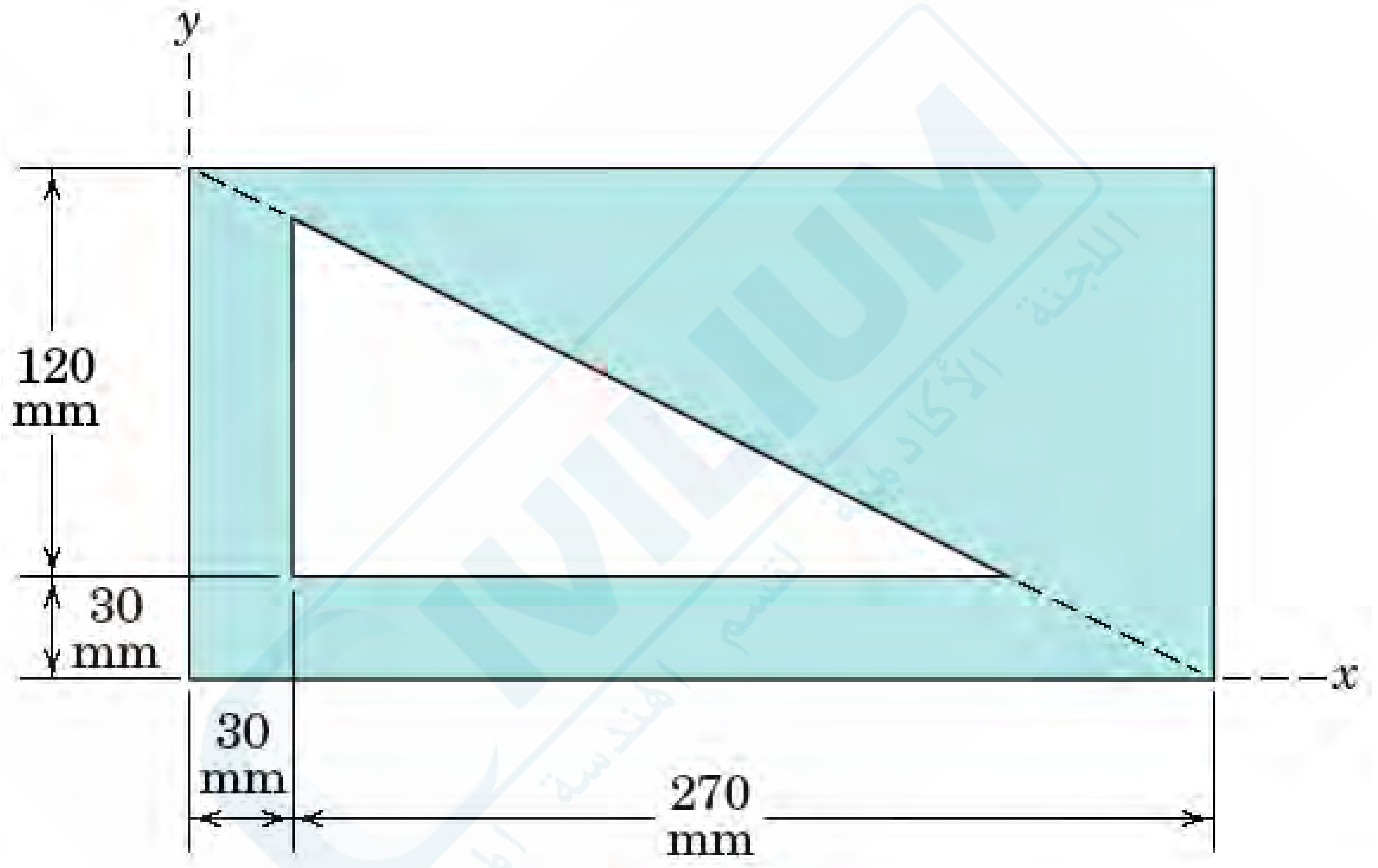
Problem 5/204

P_05_204



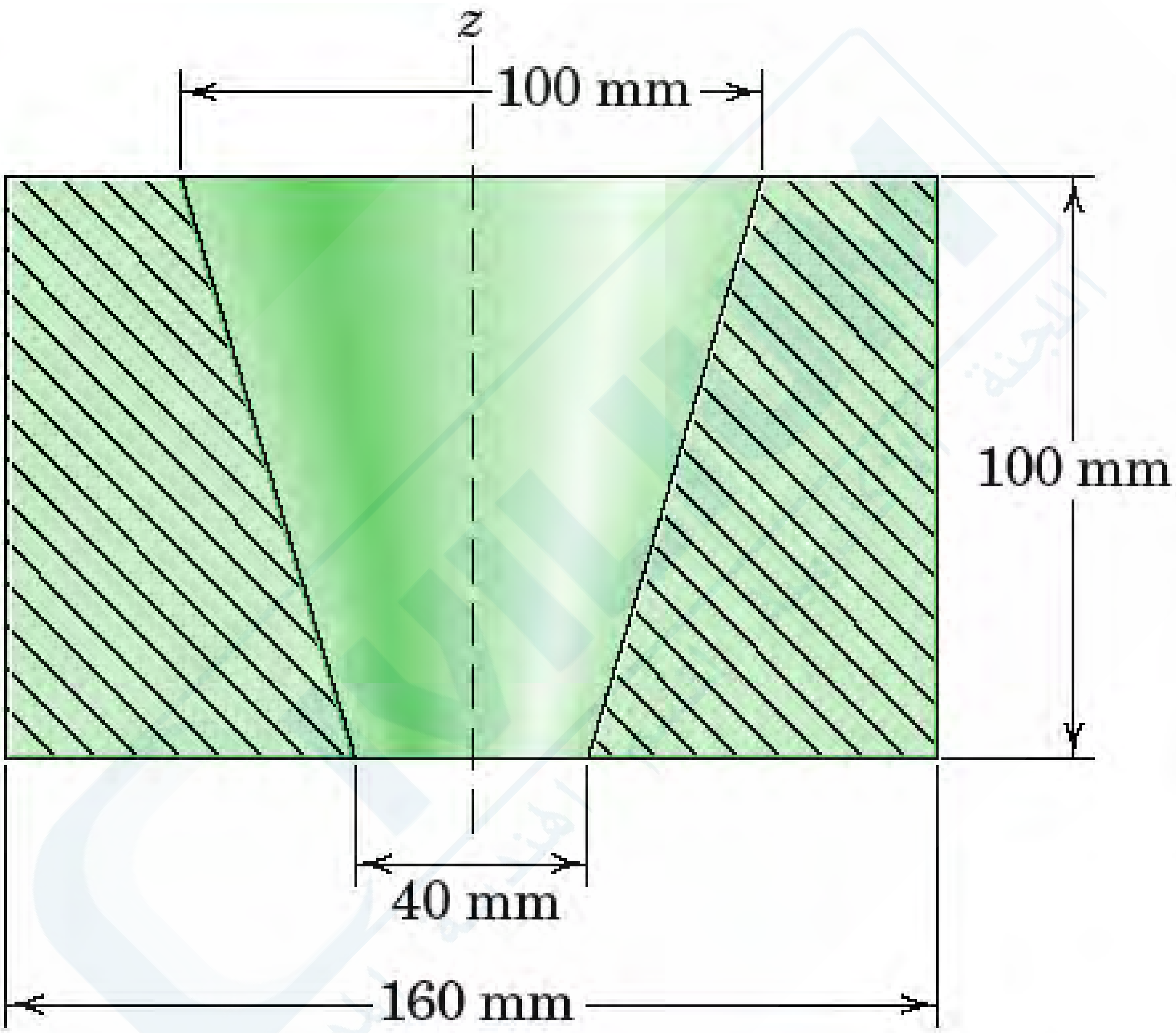
Problem 5/205

P_05_205



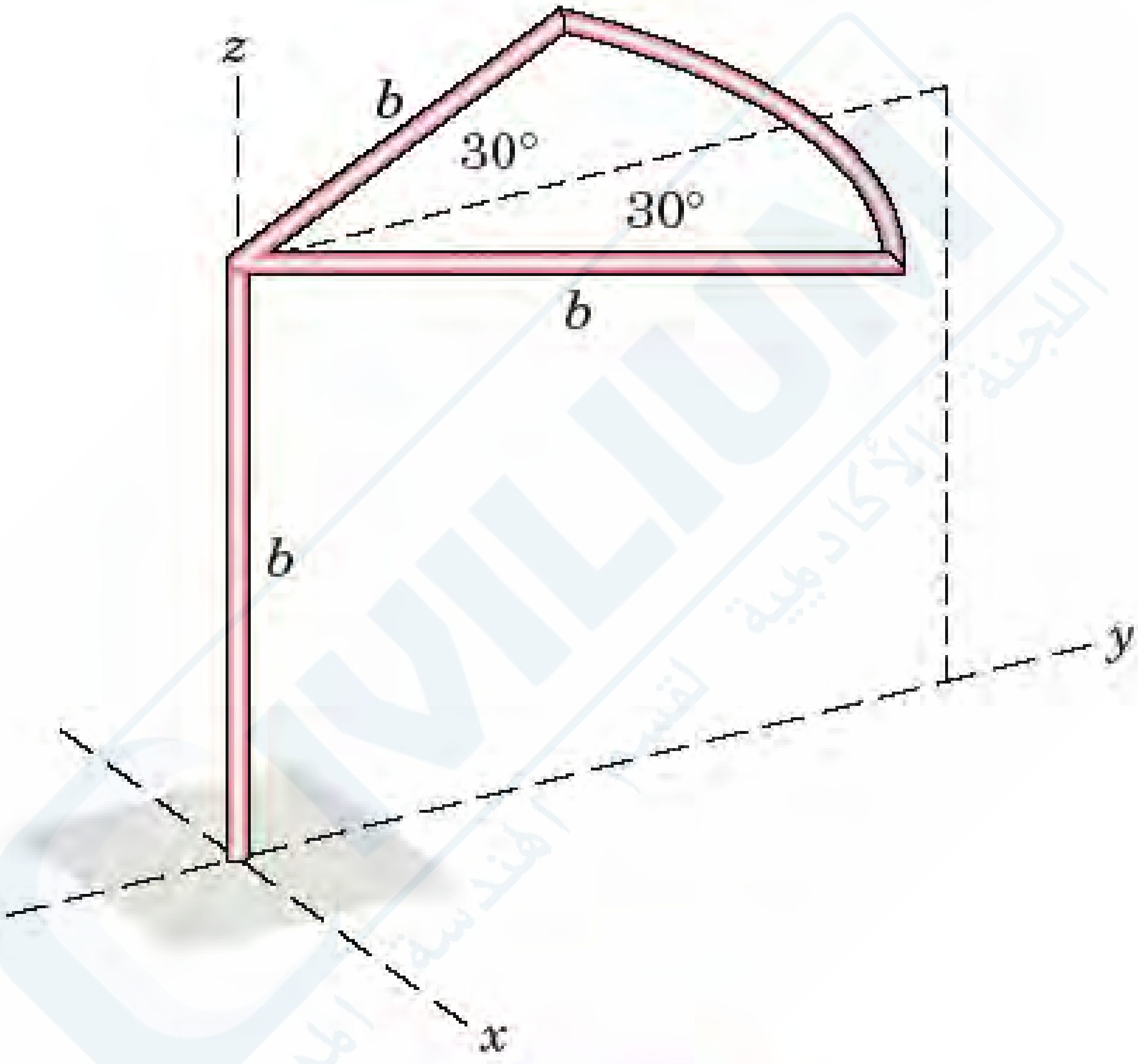
Problem 5/206

P_05_206



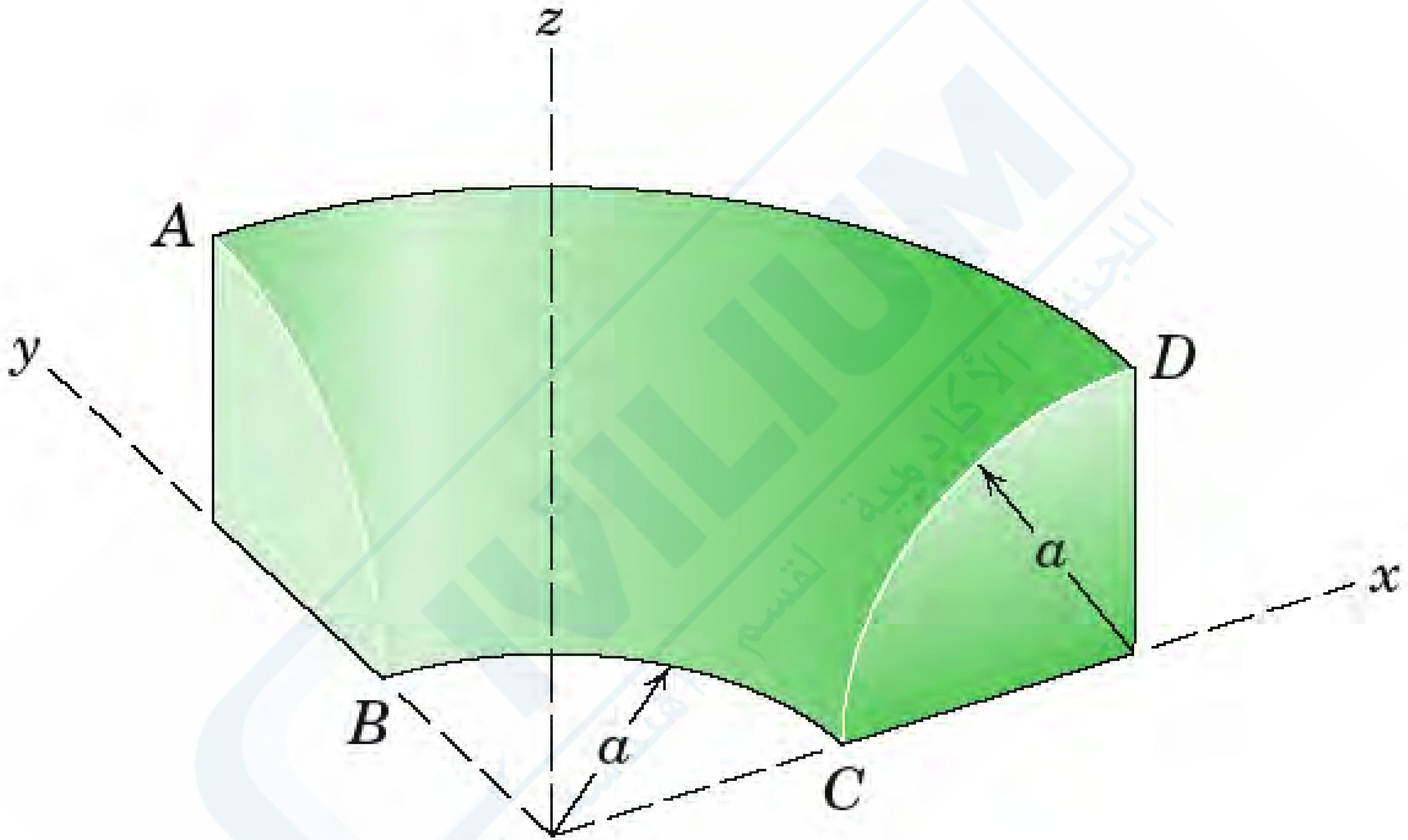
Problem 5/207

P_05_207



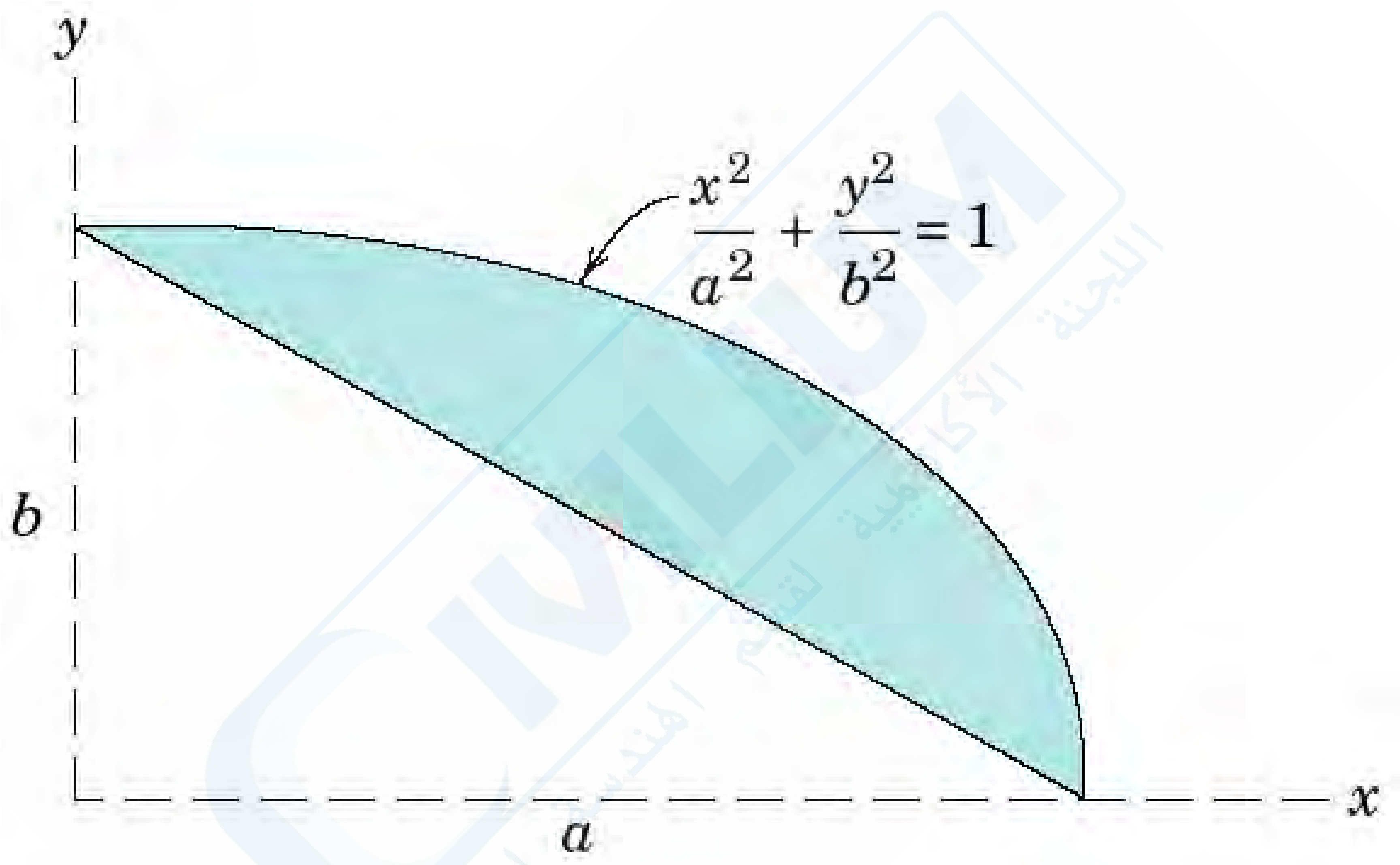
Problem 5/208

P_05_208



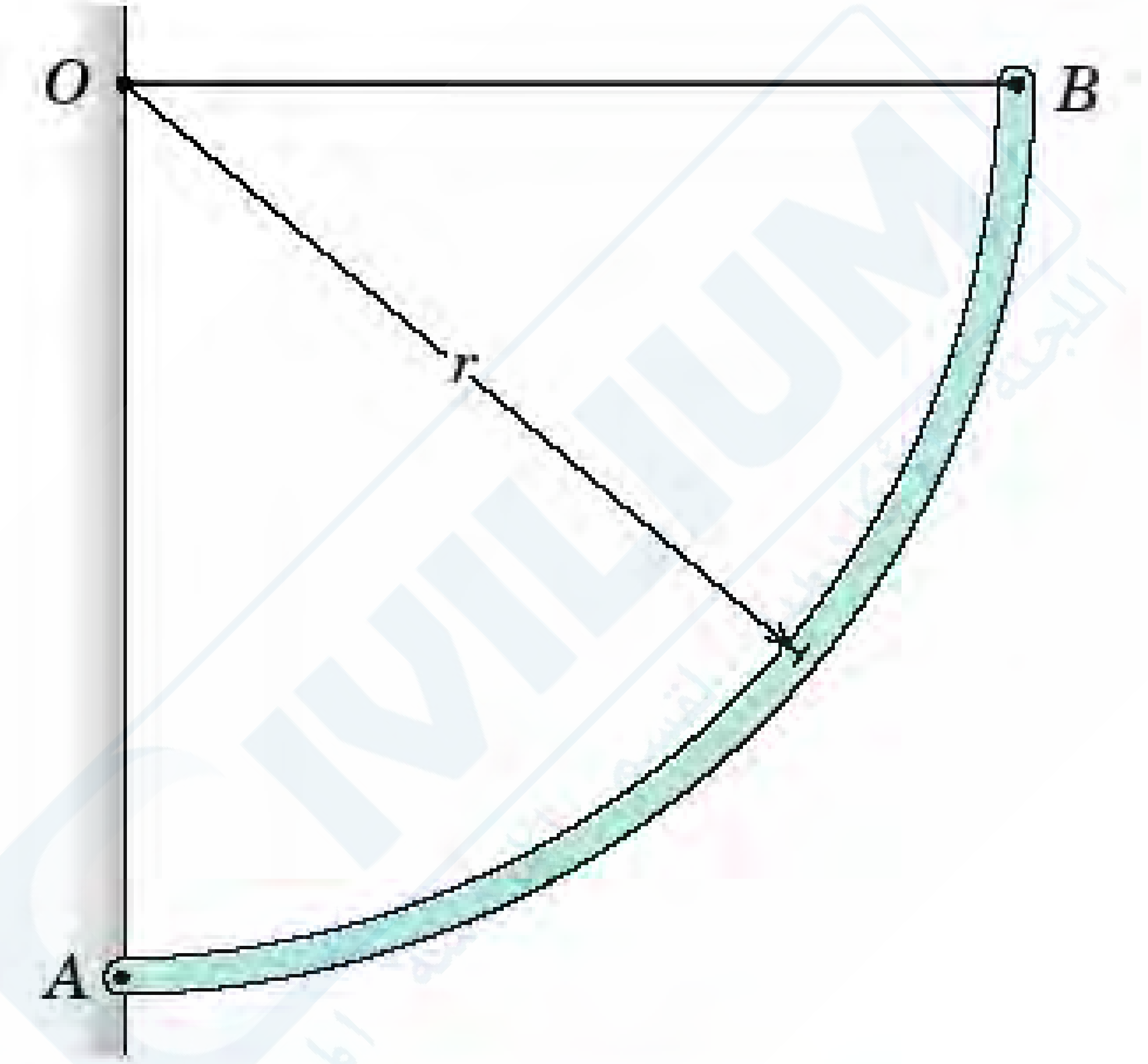
Problem 5/209

P_05_209



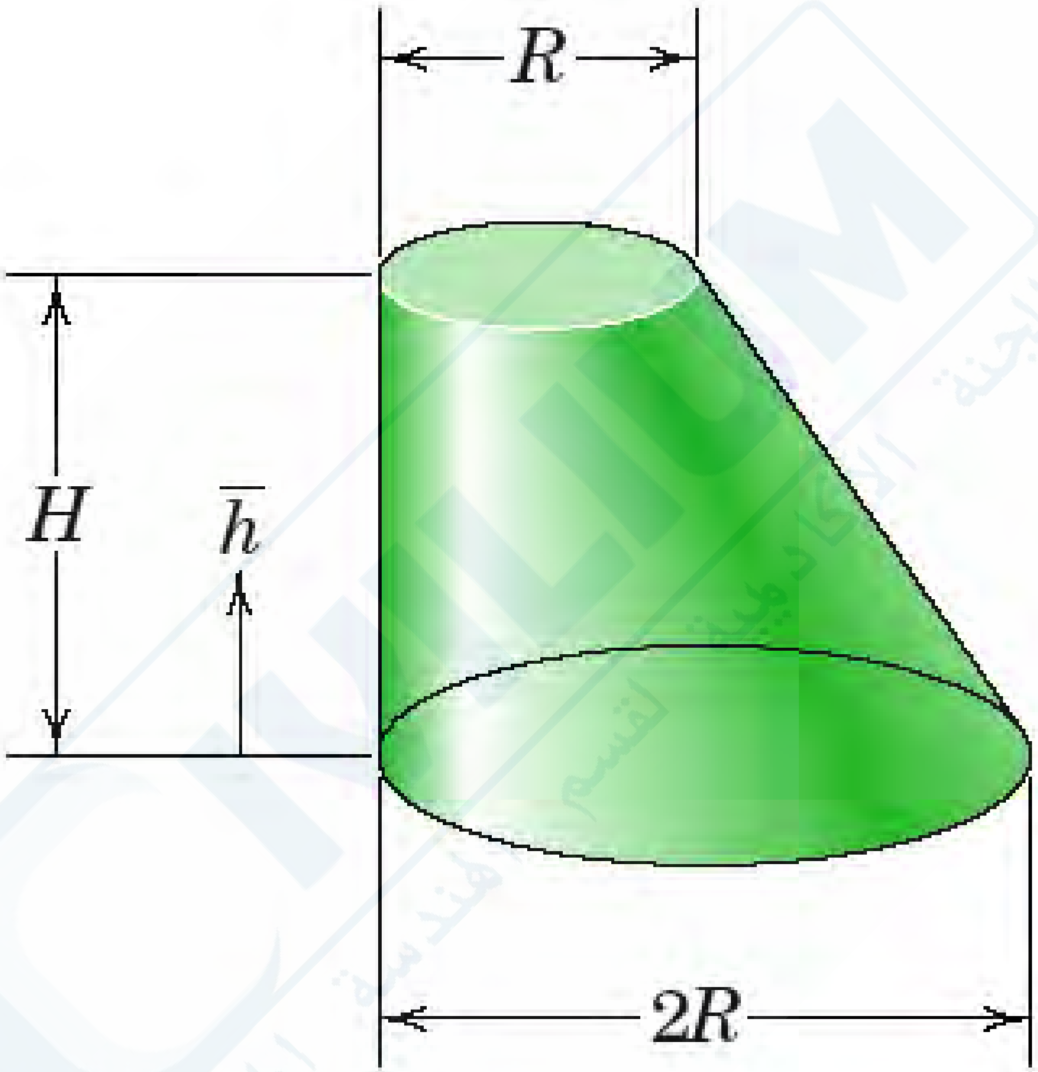
Problem 5/21

P_05_21



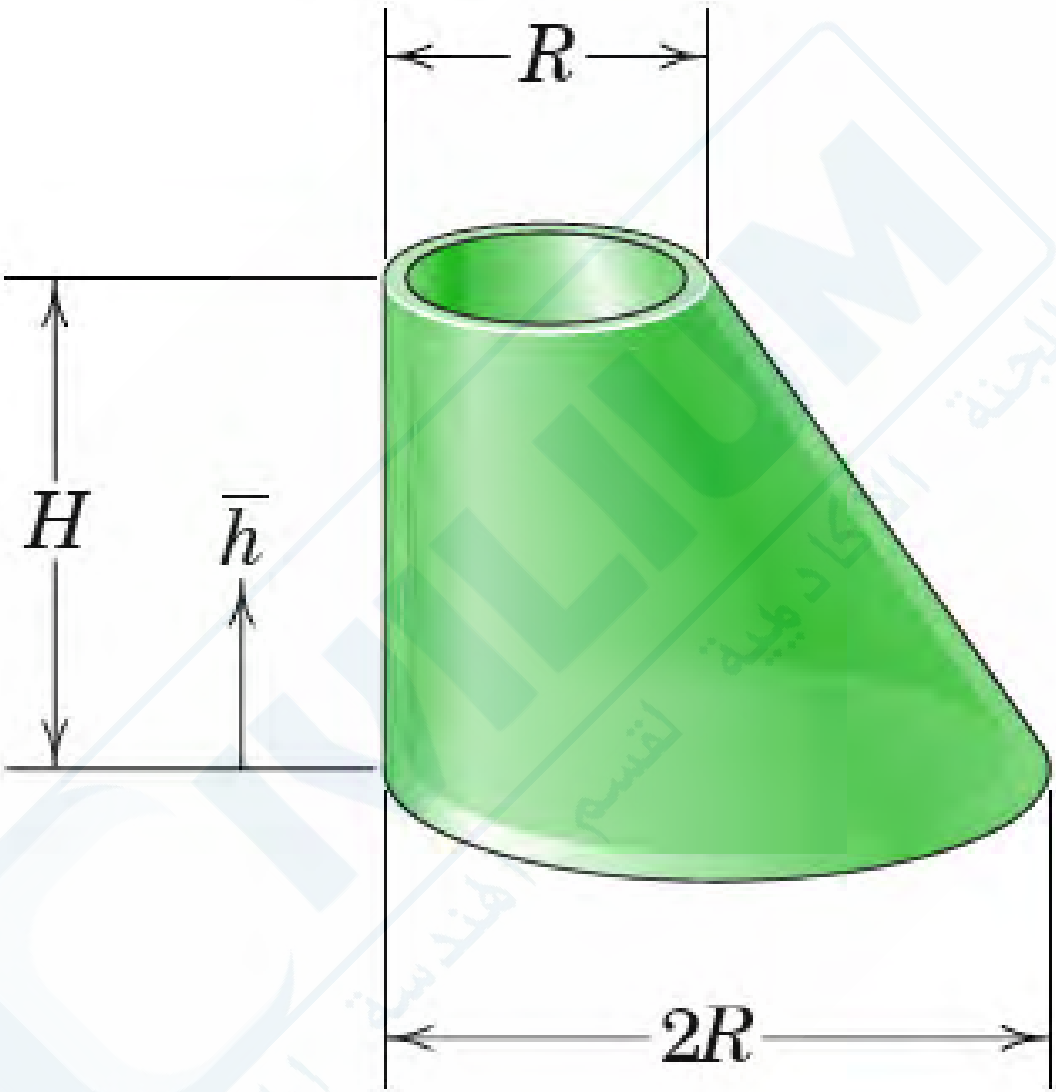
Problem 5/210

P_05_210



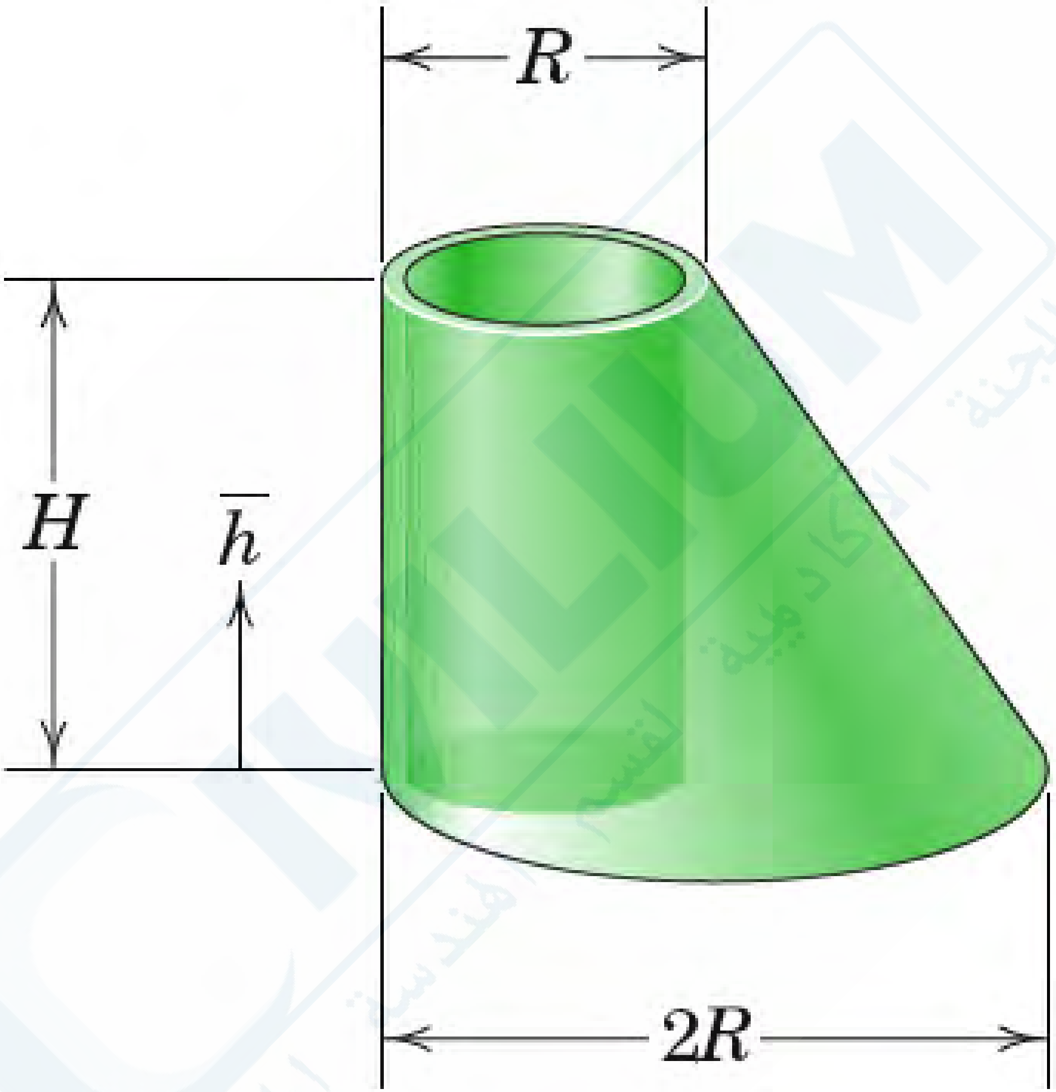
Problem 5/211

P_05_211



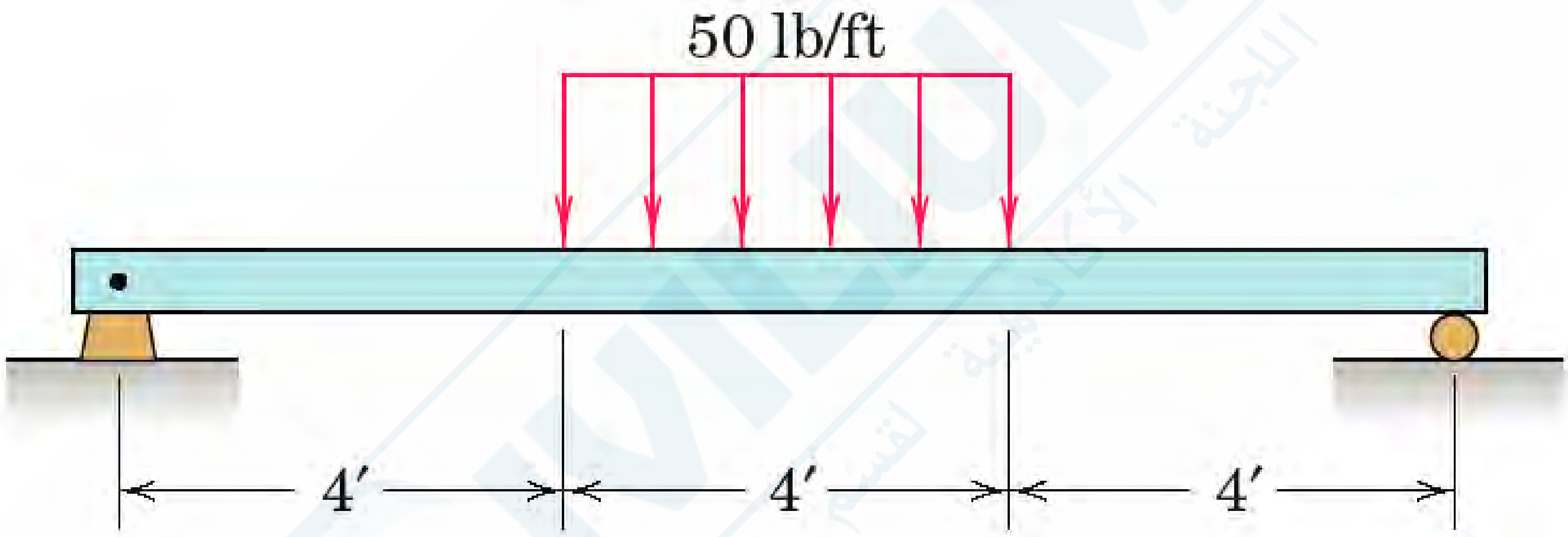
Problem 5/212

P 05 212



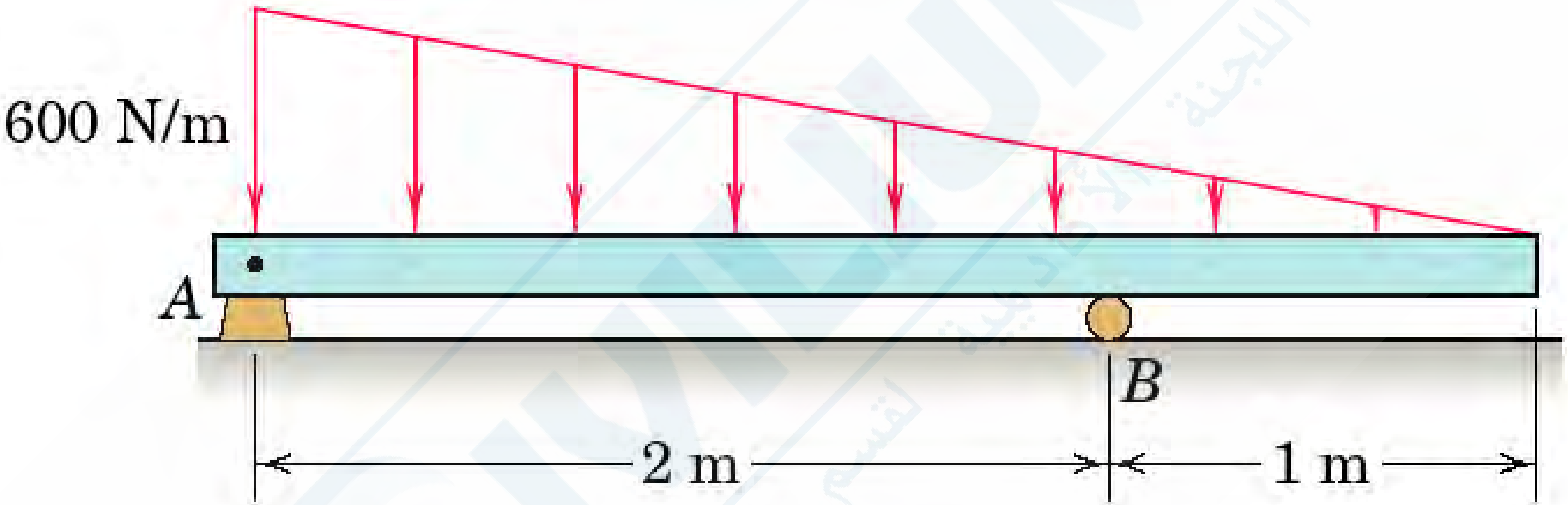
Problem 5/213

P_05_213



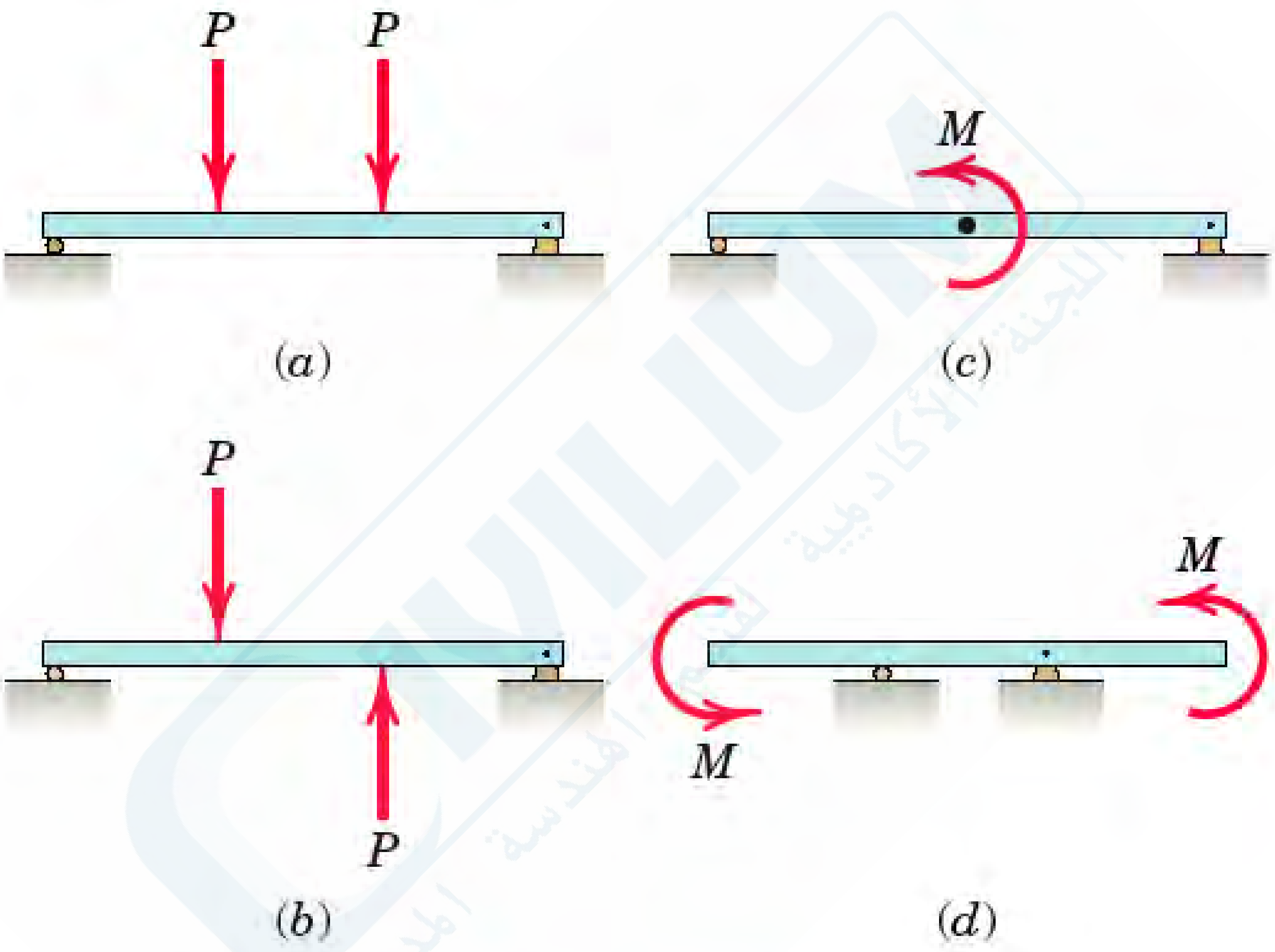
Problem 5/214

P_05_214



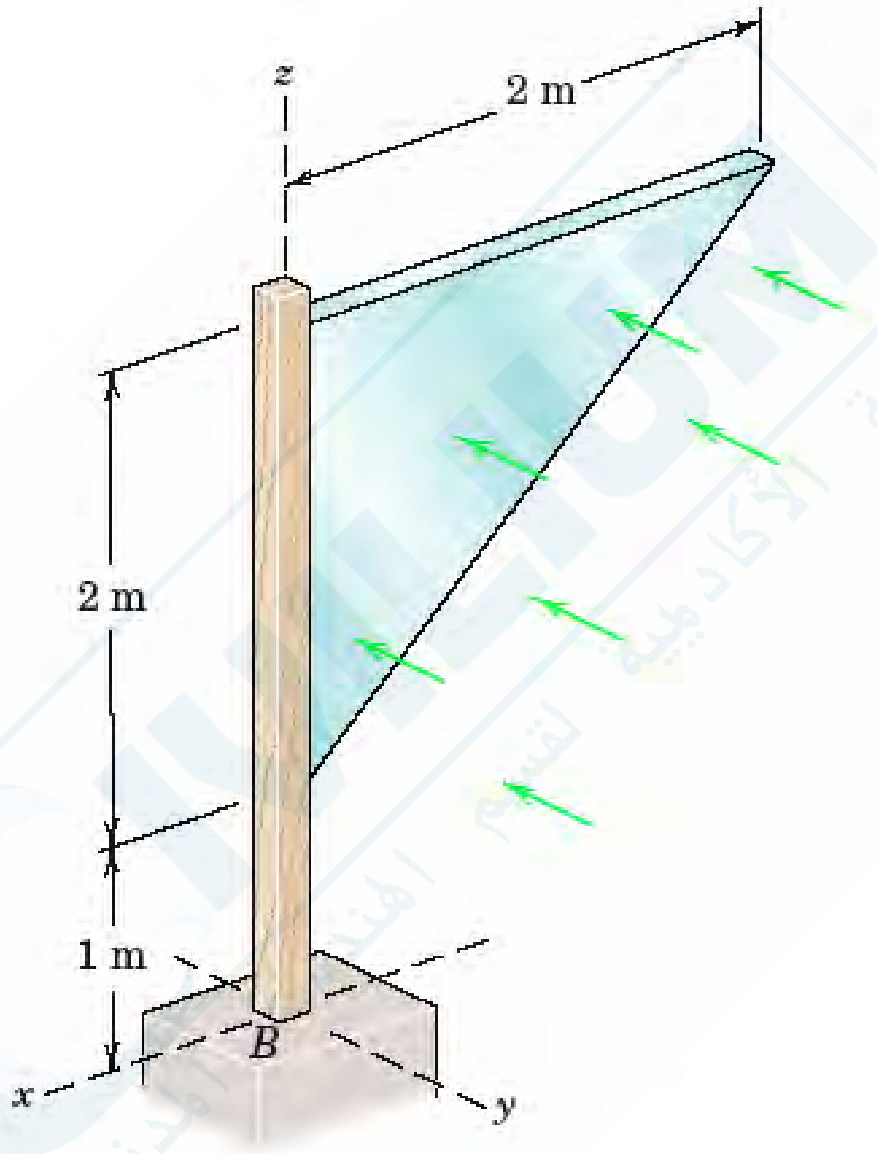
Problem 5/215

P_05_215



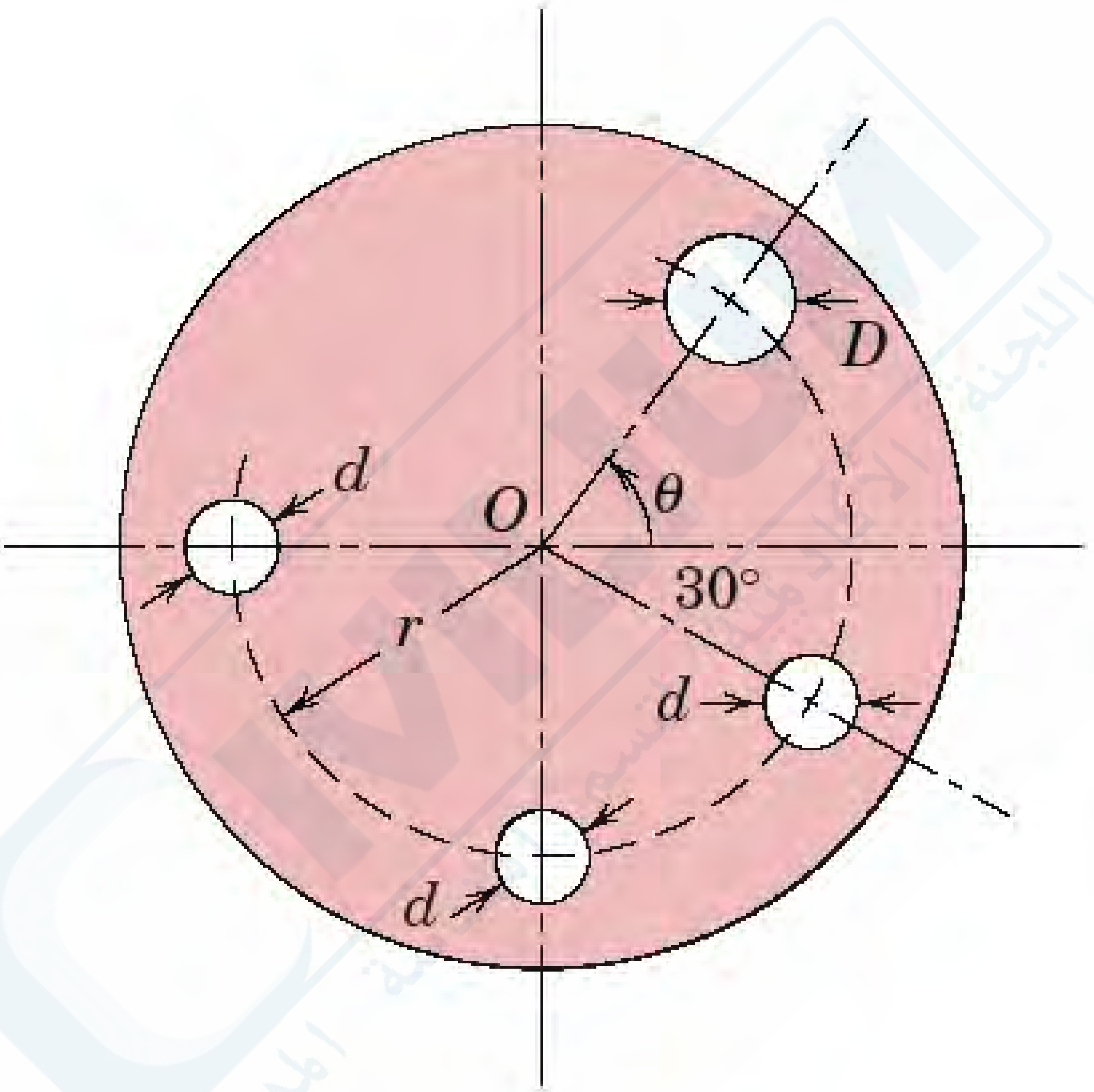
Problem 5/216

P_05_216



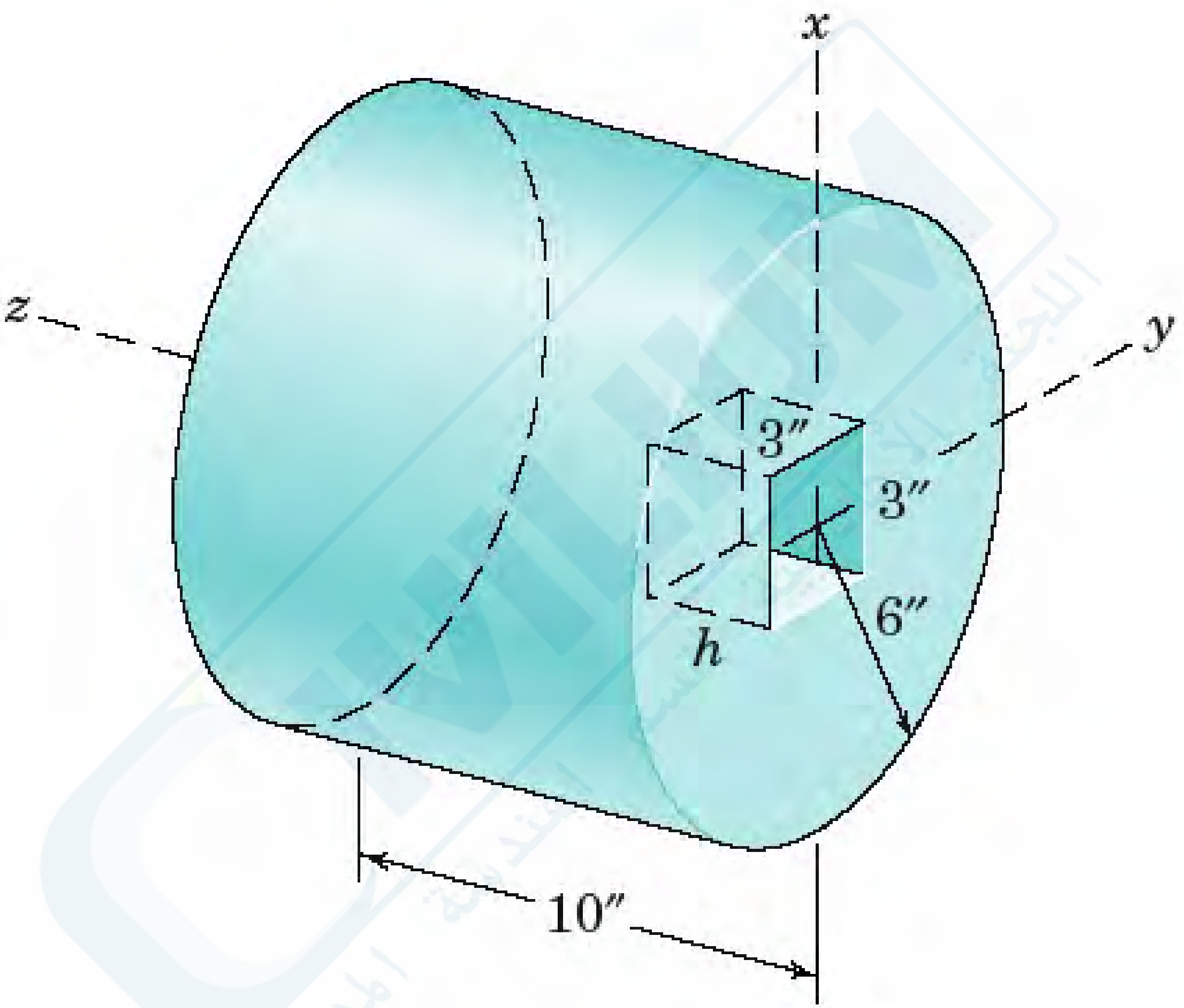
Problem 5/217

P_05_217



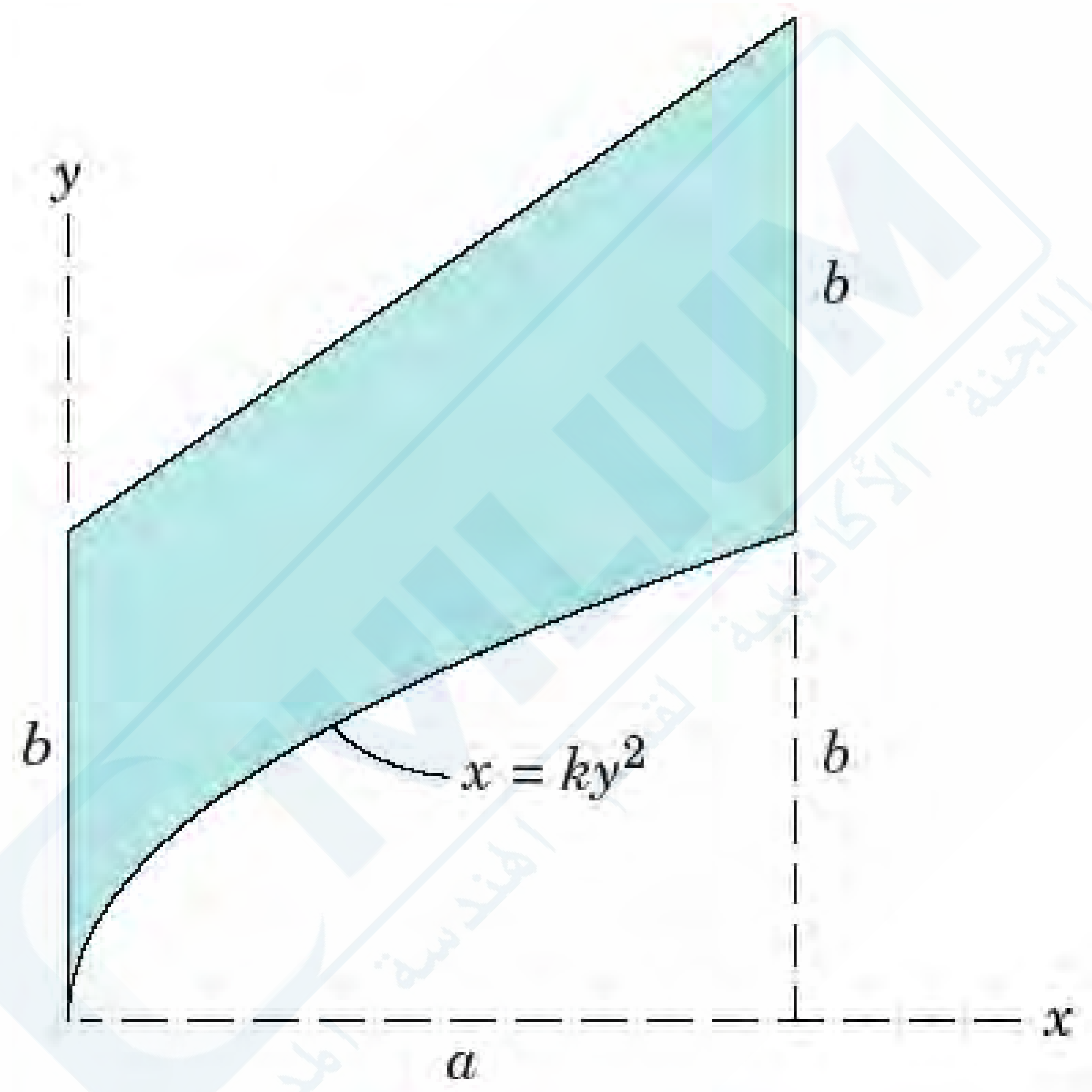
Problem 5/218

P 05 218



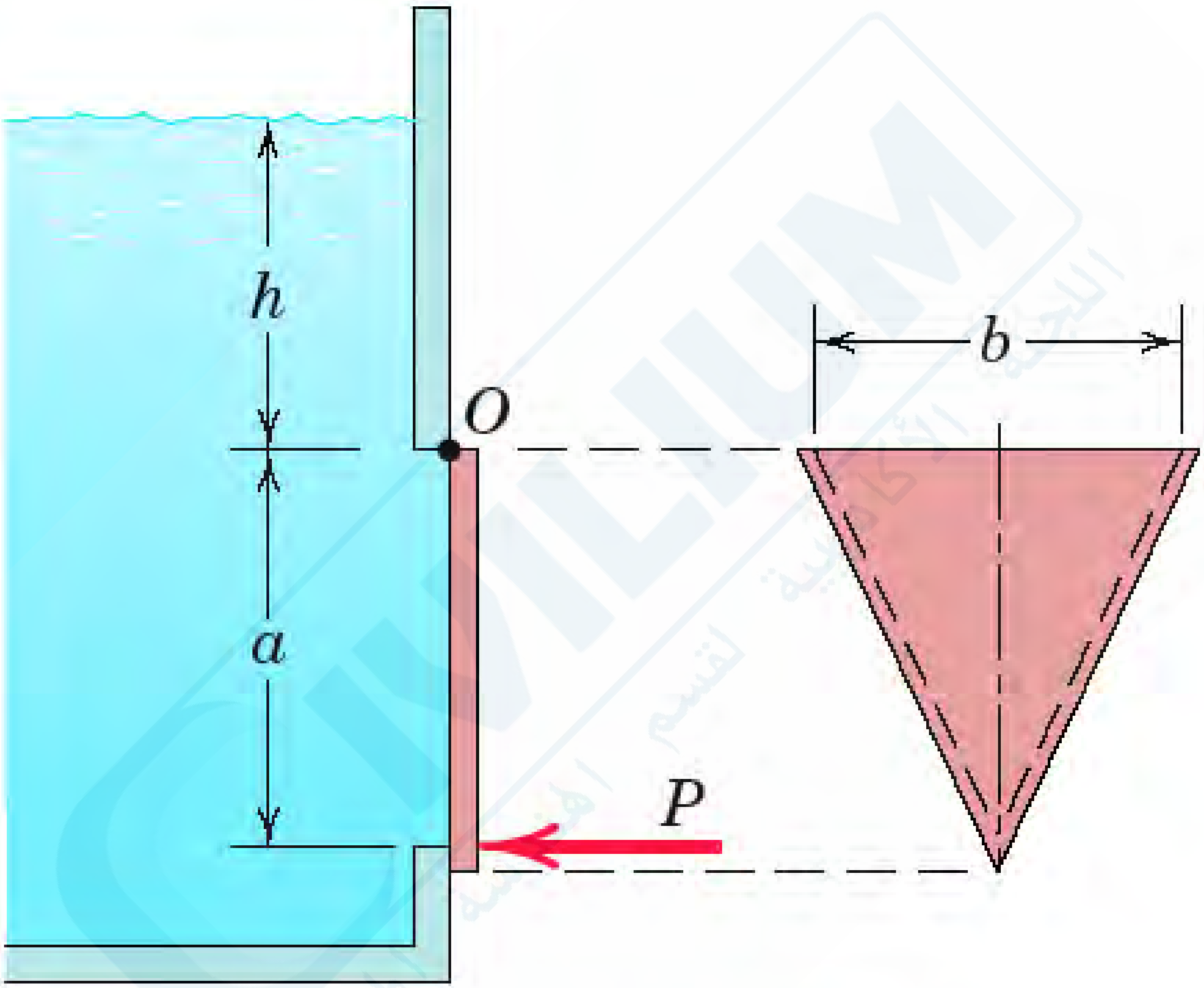
Problem 5/219

P_05_219



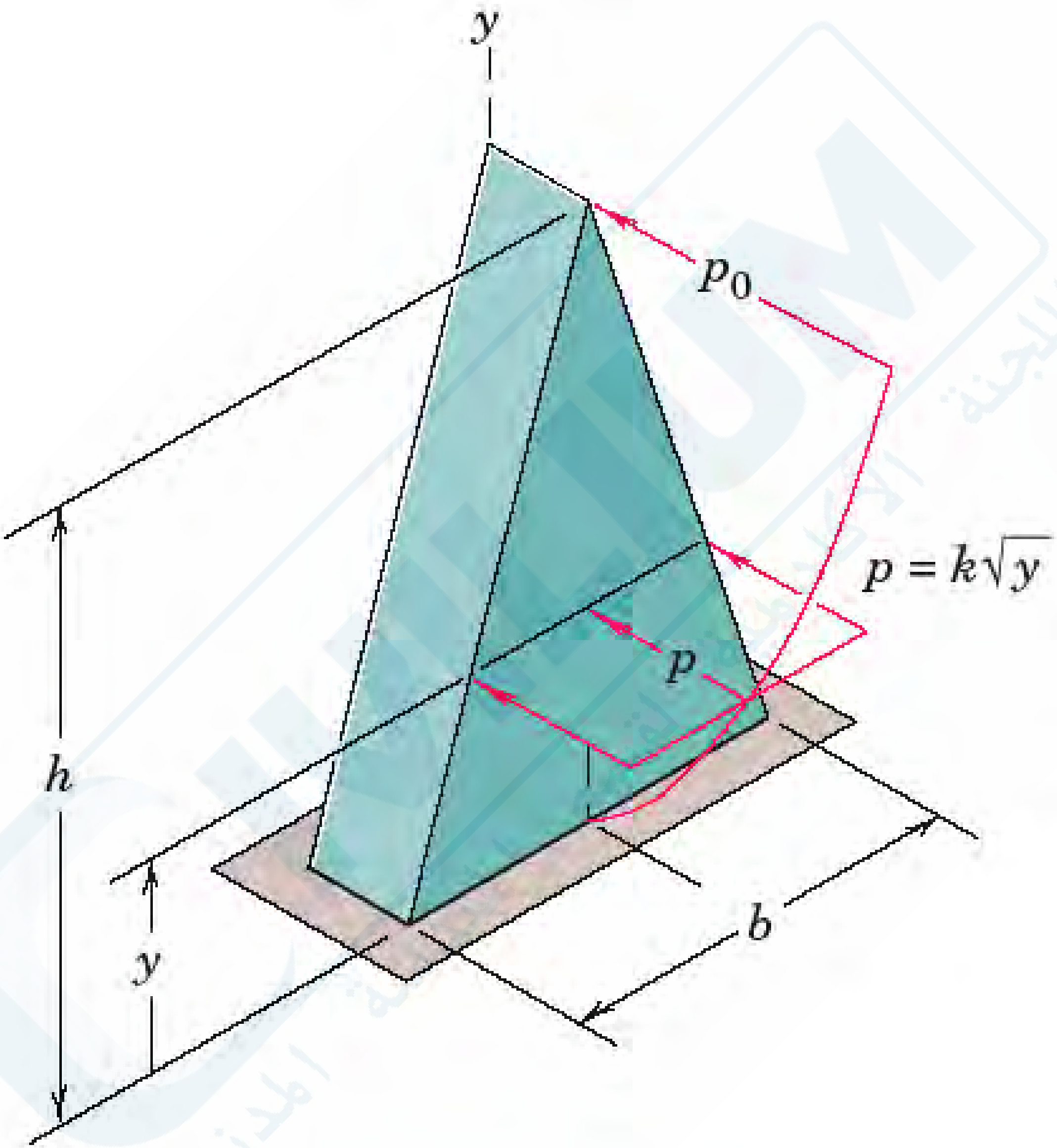
Problem 5/22

P_05_22

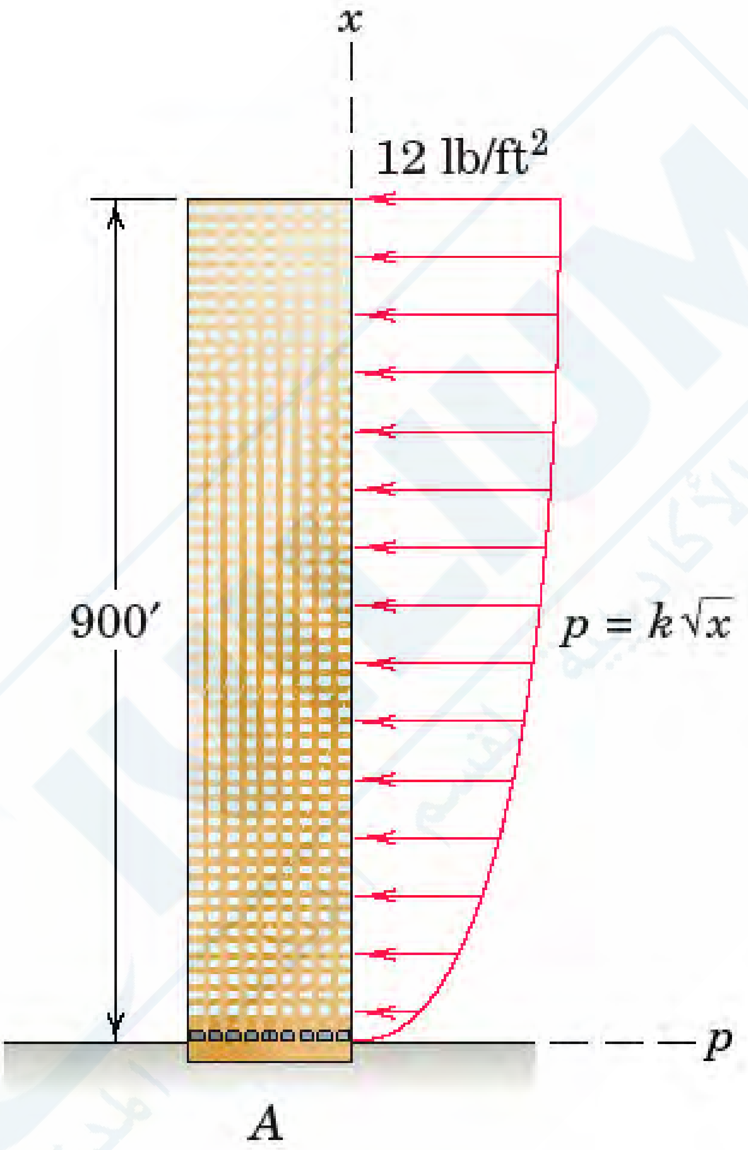


Problem 5/220

P_05_220

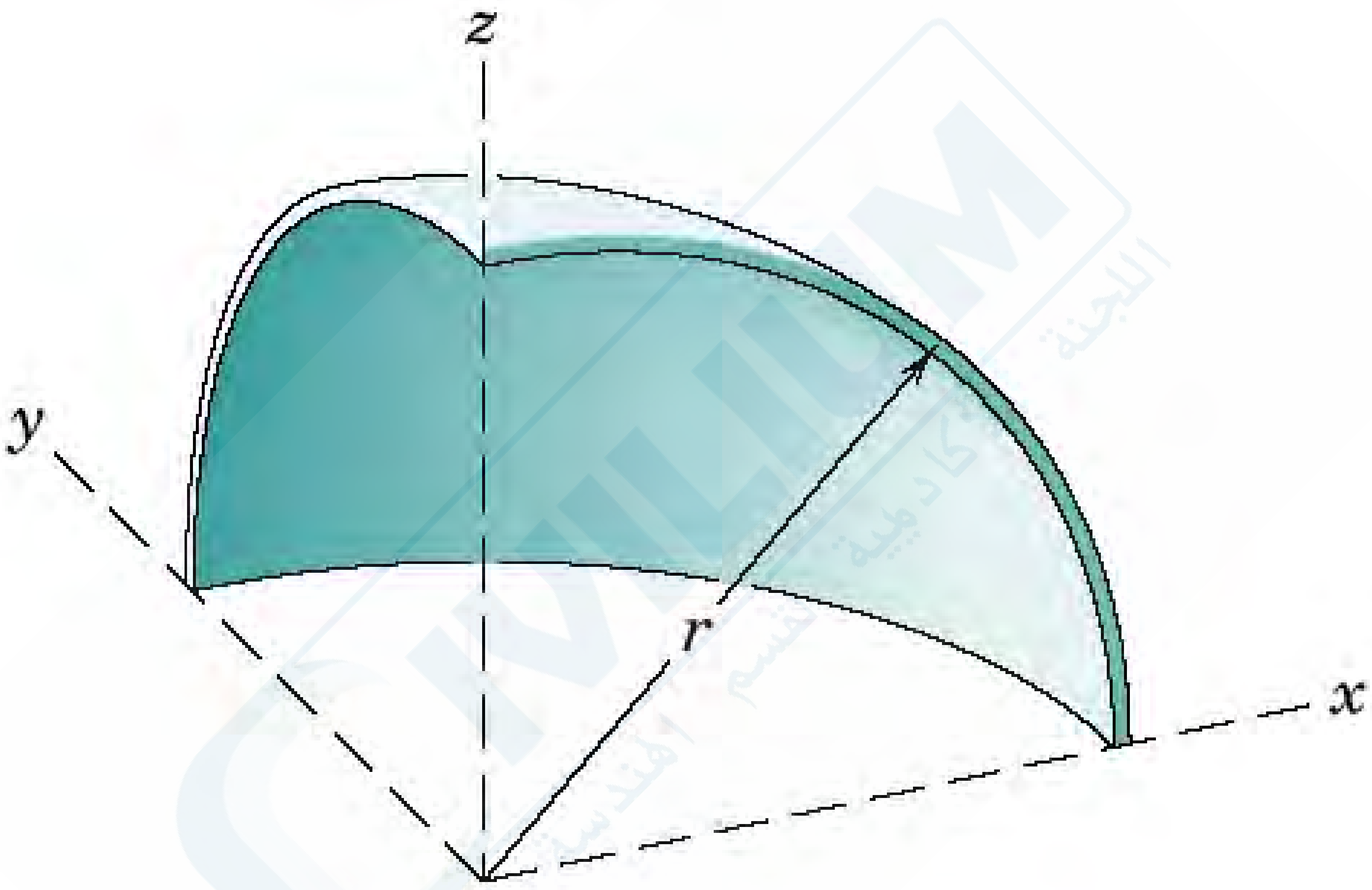


Problem 5/221



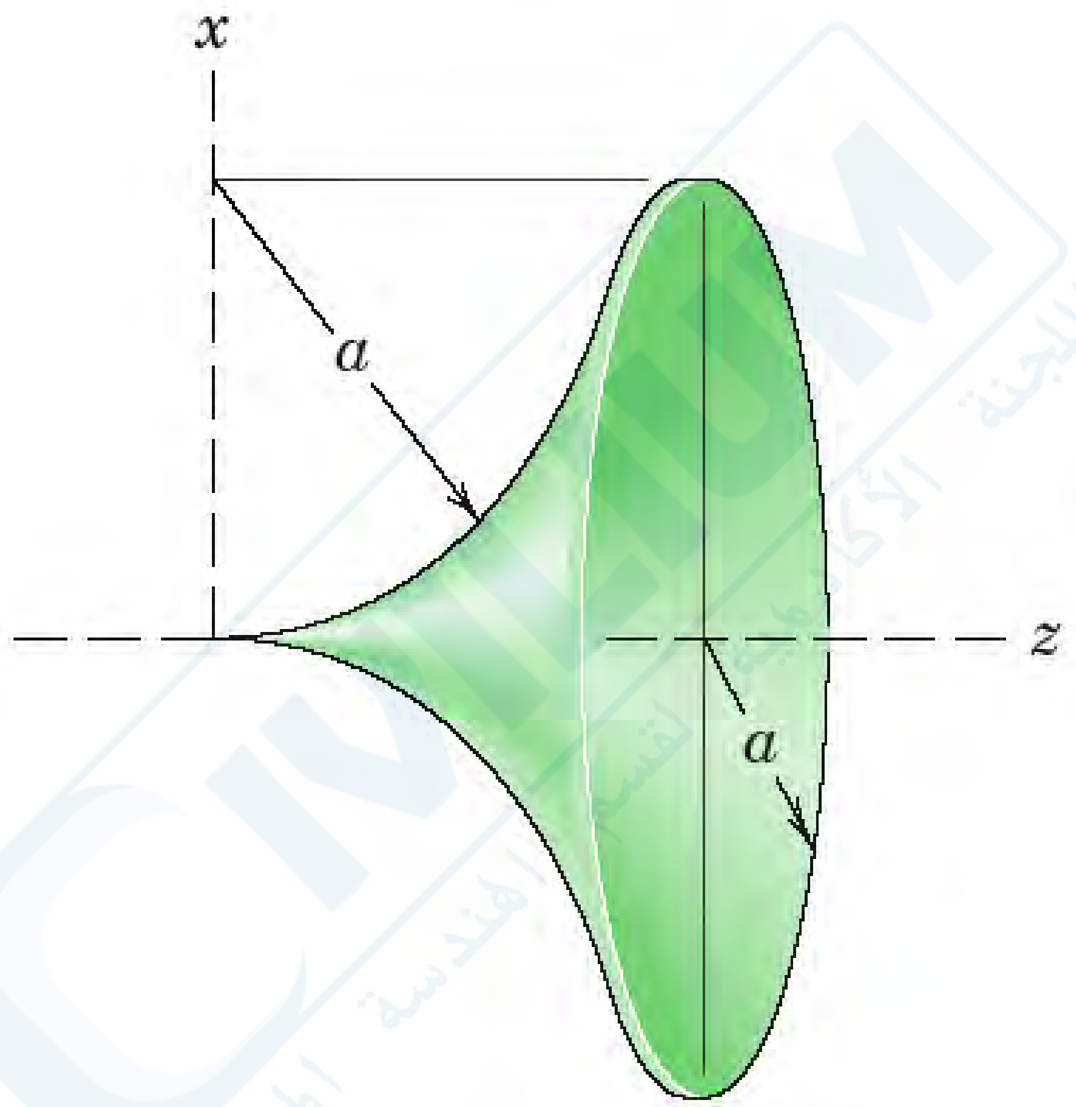
Problem 5/222

P_05_222



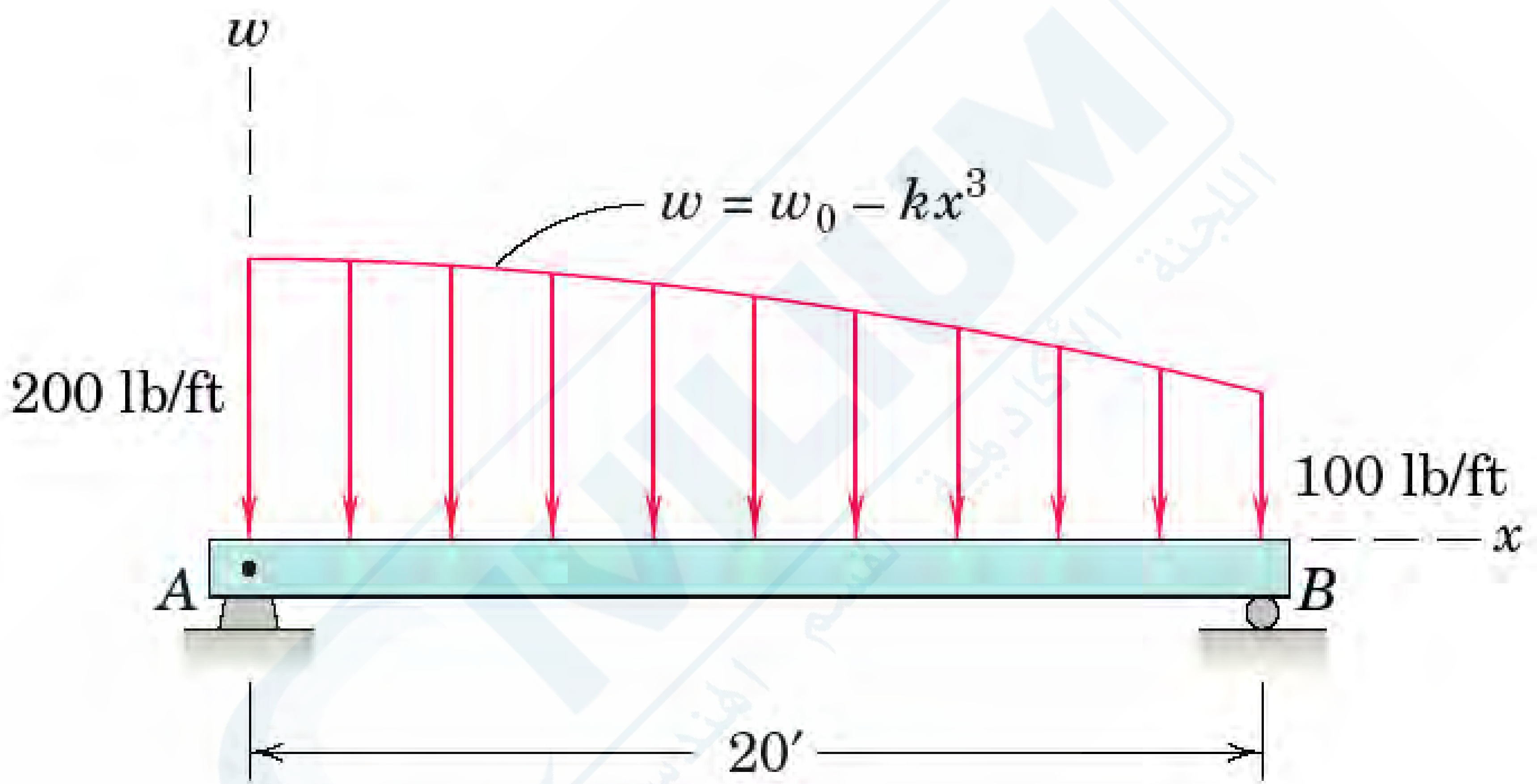
Problem 5/224

P 05 224



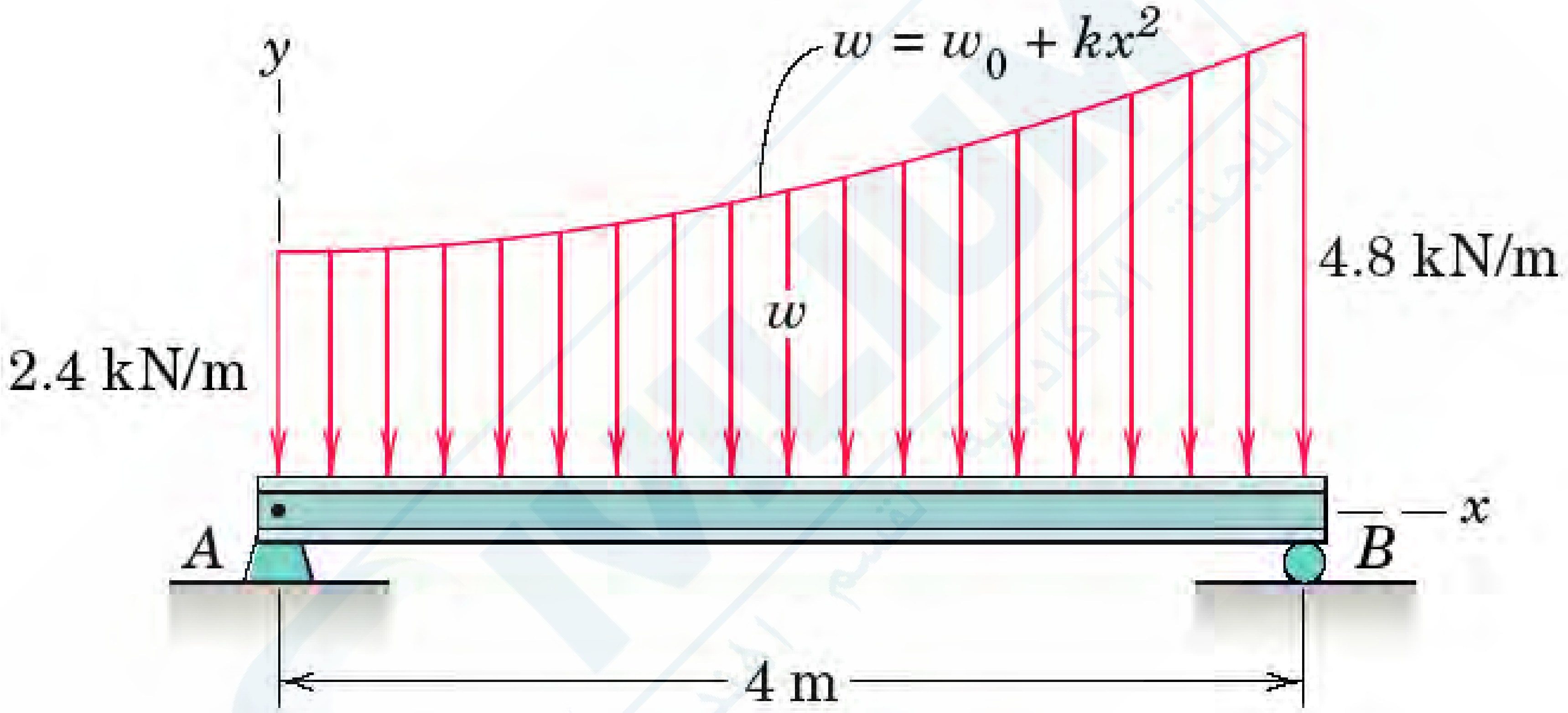
Problem 5/225

P 05 225



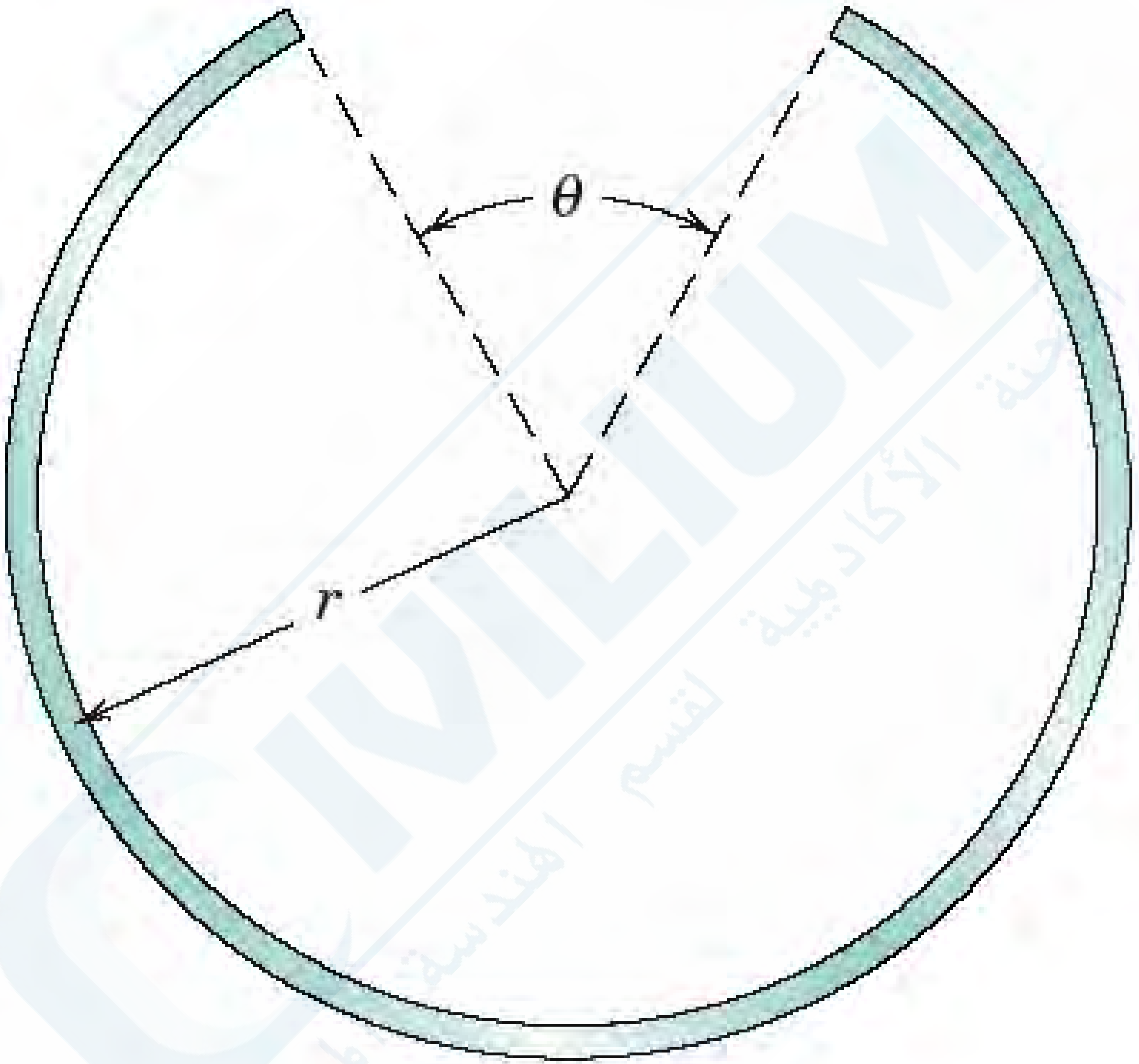
Problem 5/226

P_05_226



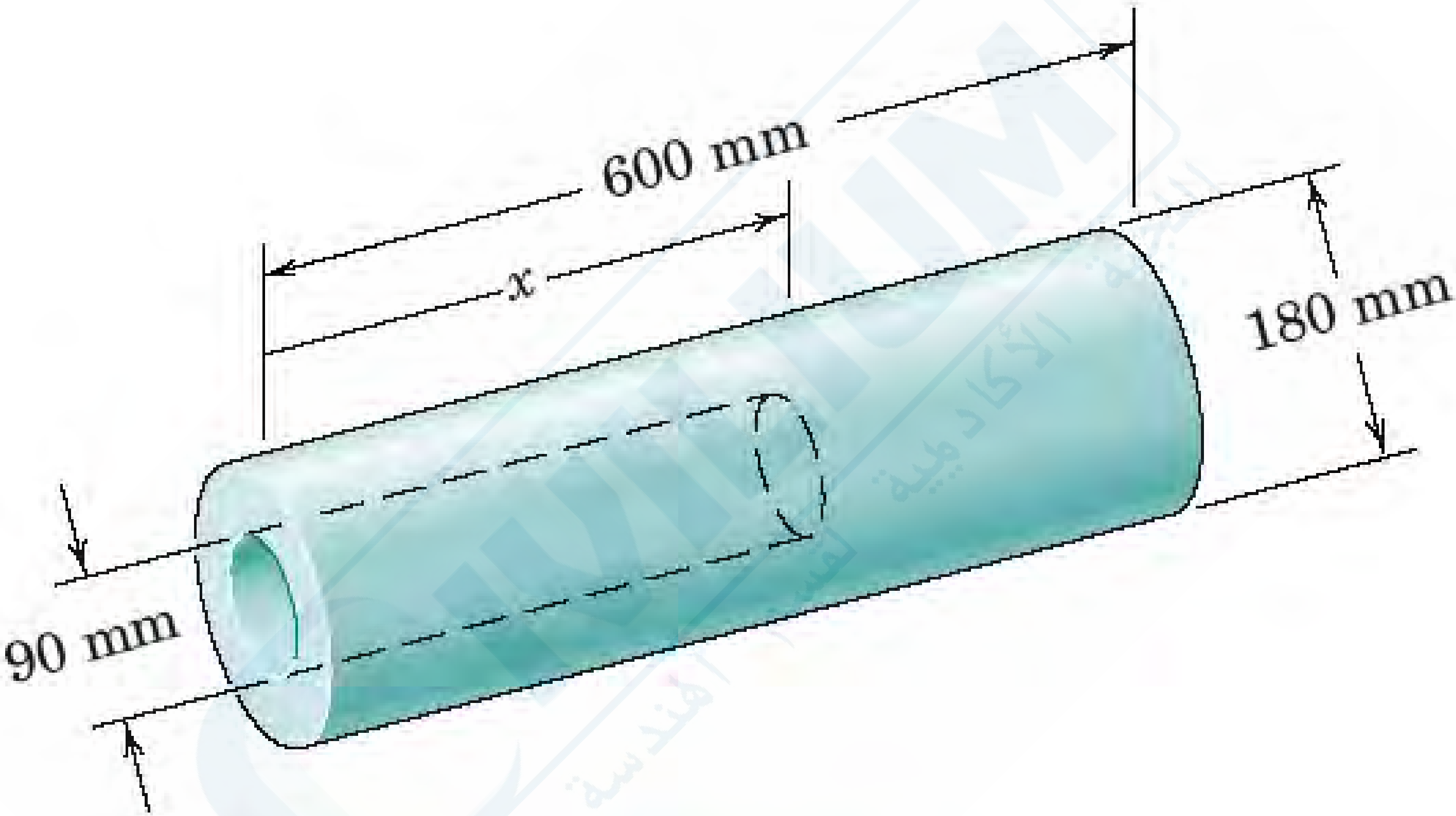
Problem 5/227

P_05_227



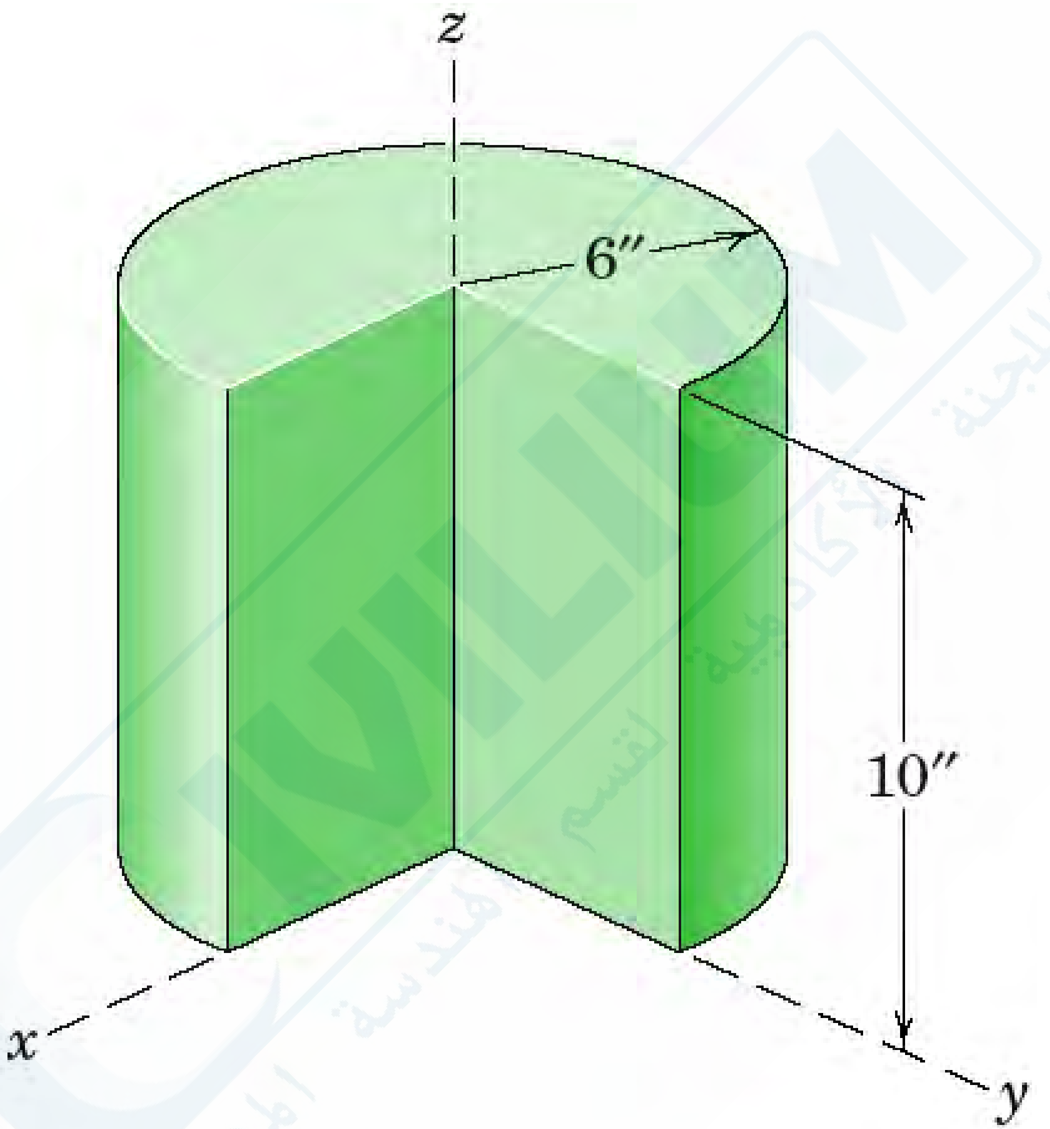
Problem 5/228

P_05_228



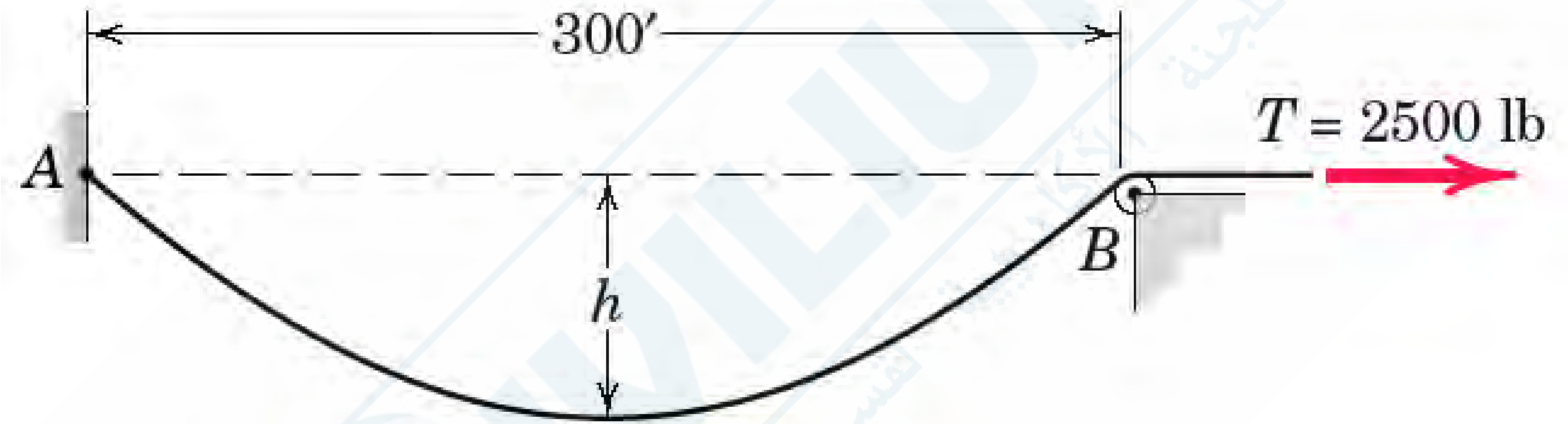
Problem 5/229

P_05_229



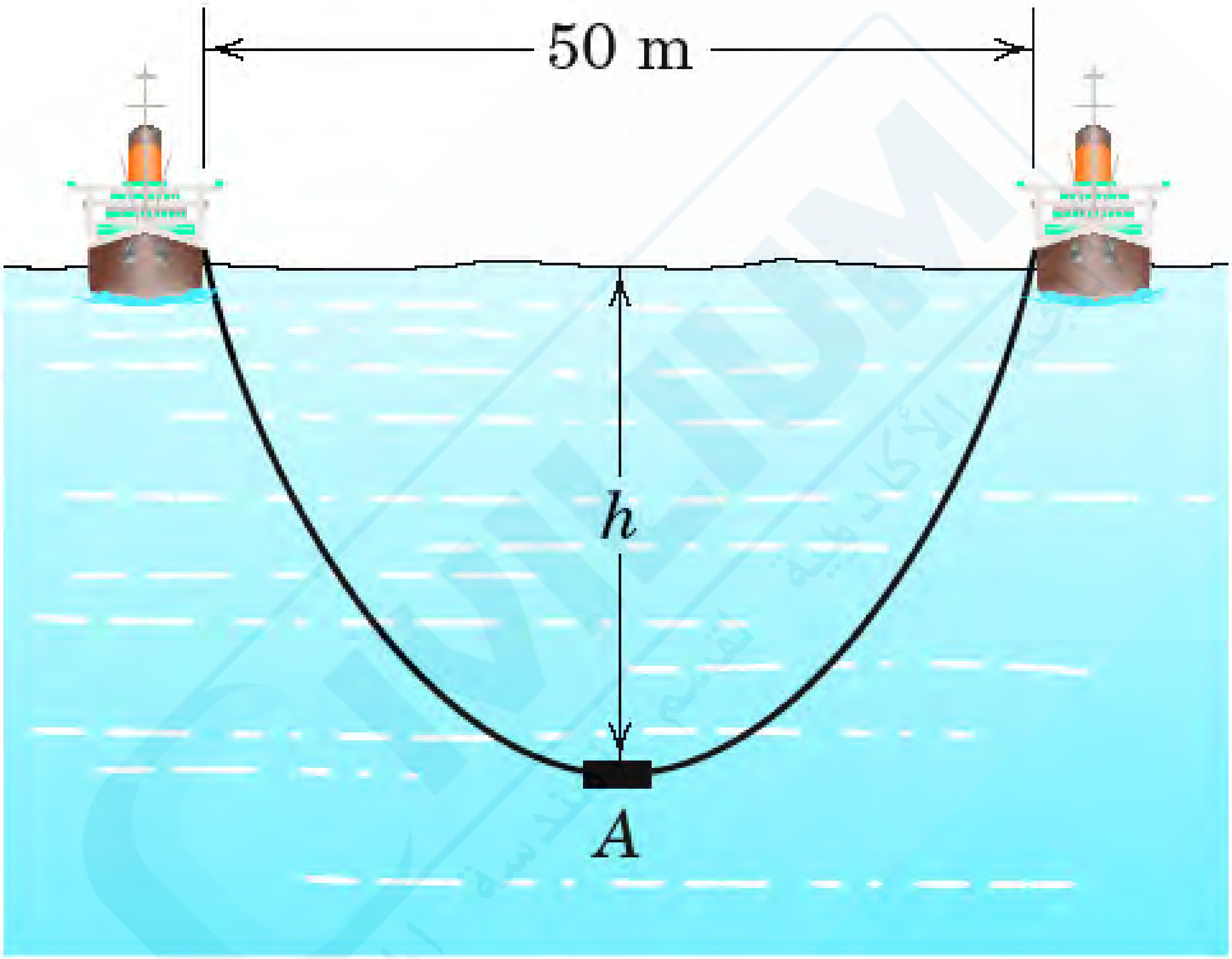
Problem 5/23

P_05_23



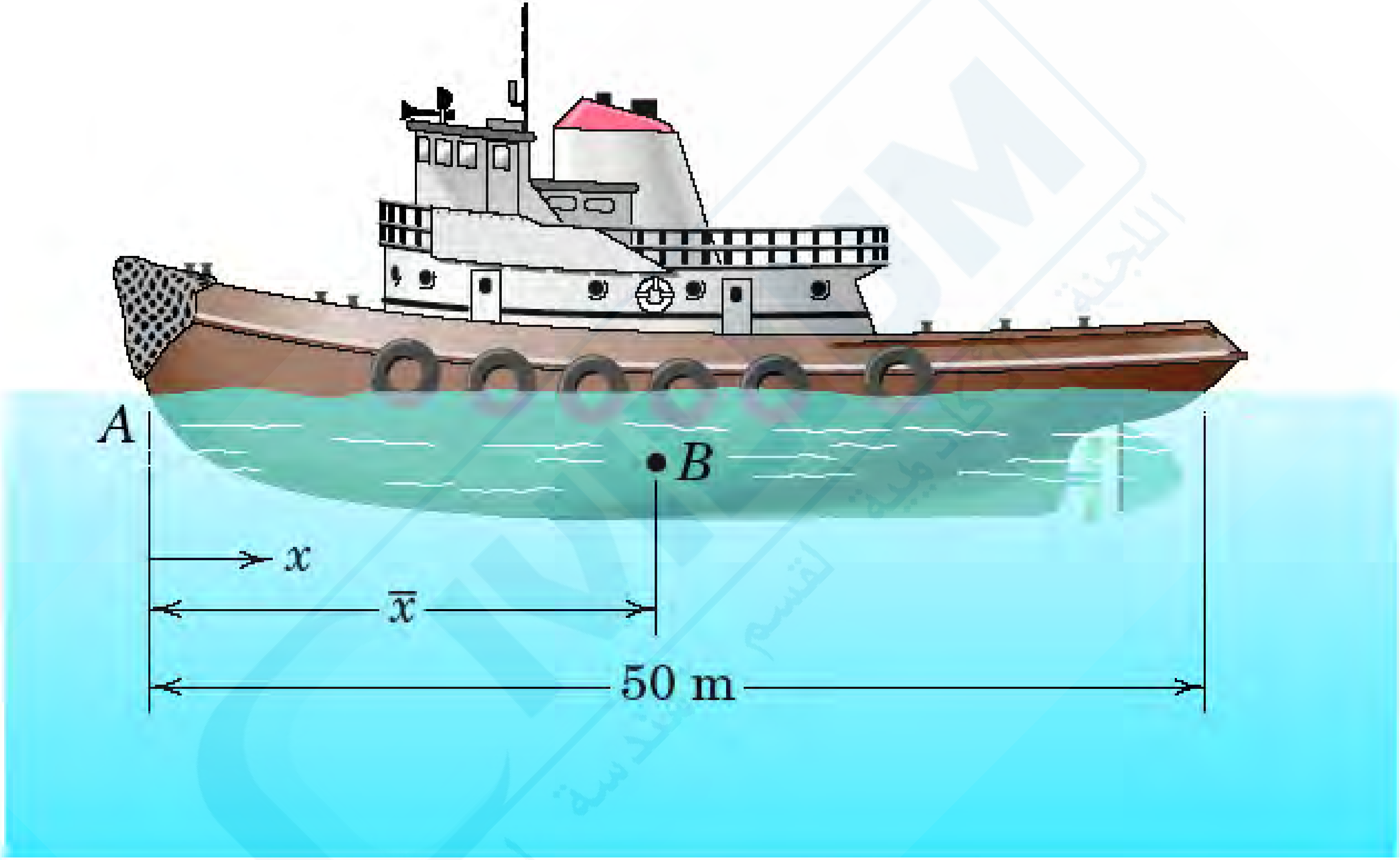
Problem 5/230

P_05_230



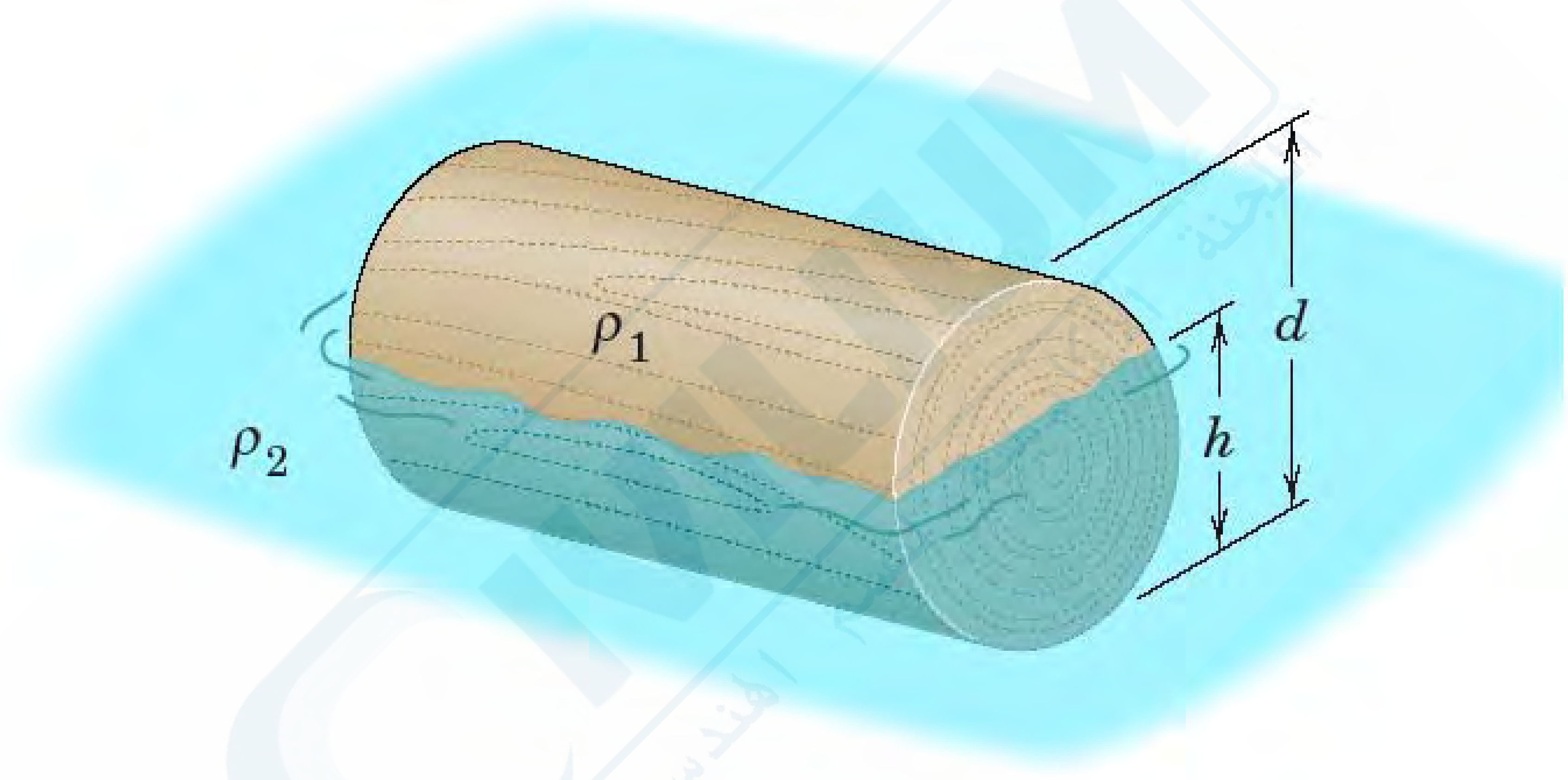
Problem 5/231

P_05_231



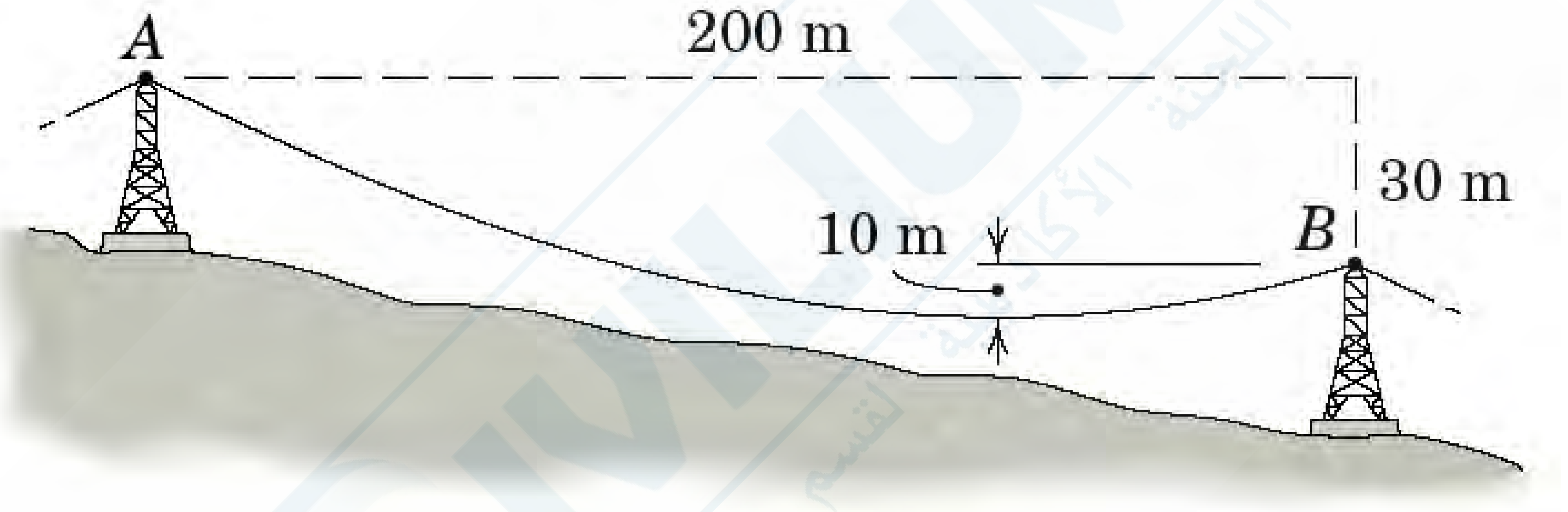
Problem 5/232

P_05_232



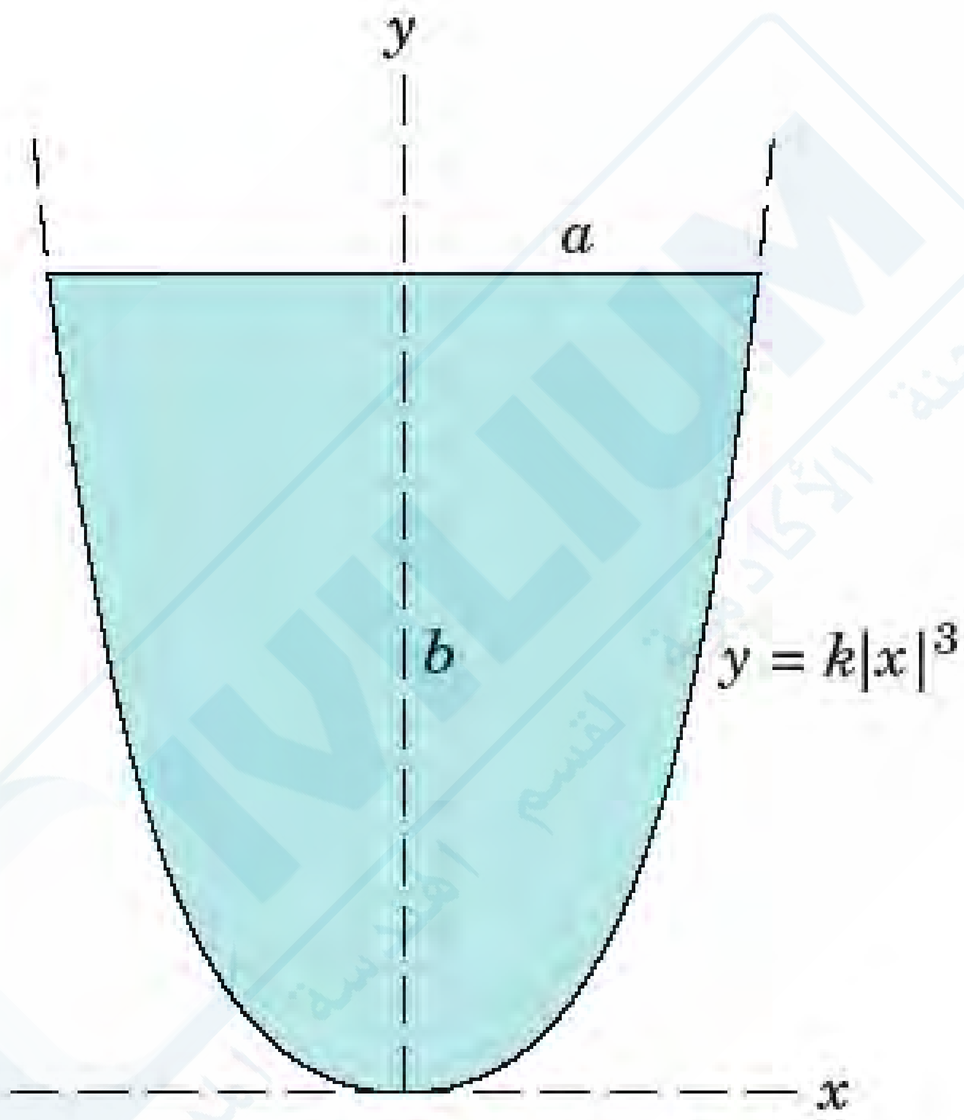
Problem 5/233

P_05_233



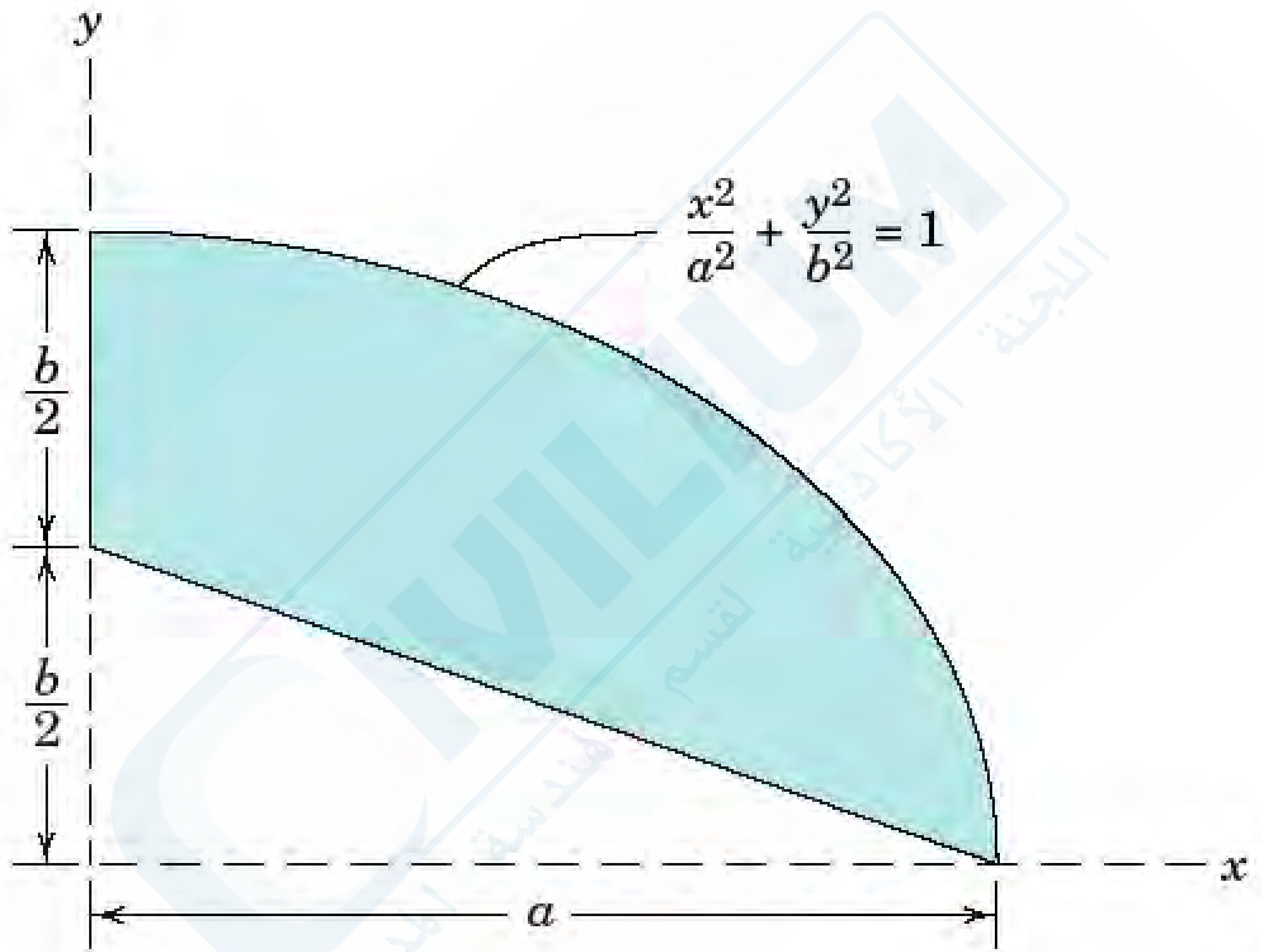
Problem 5/234

P_05_234



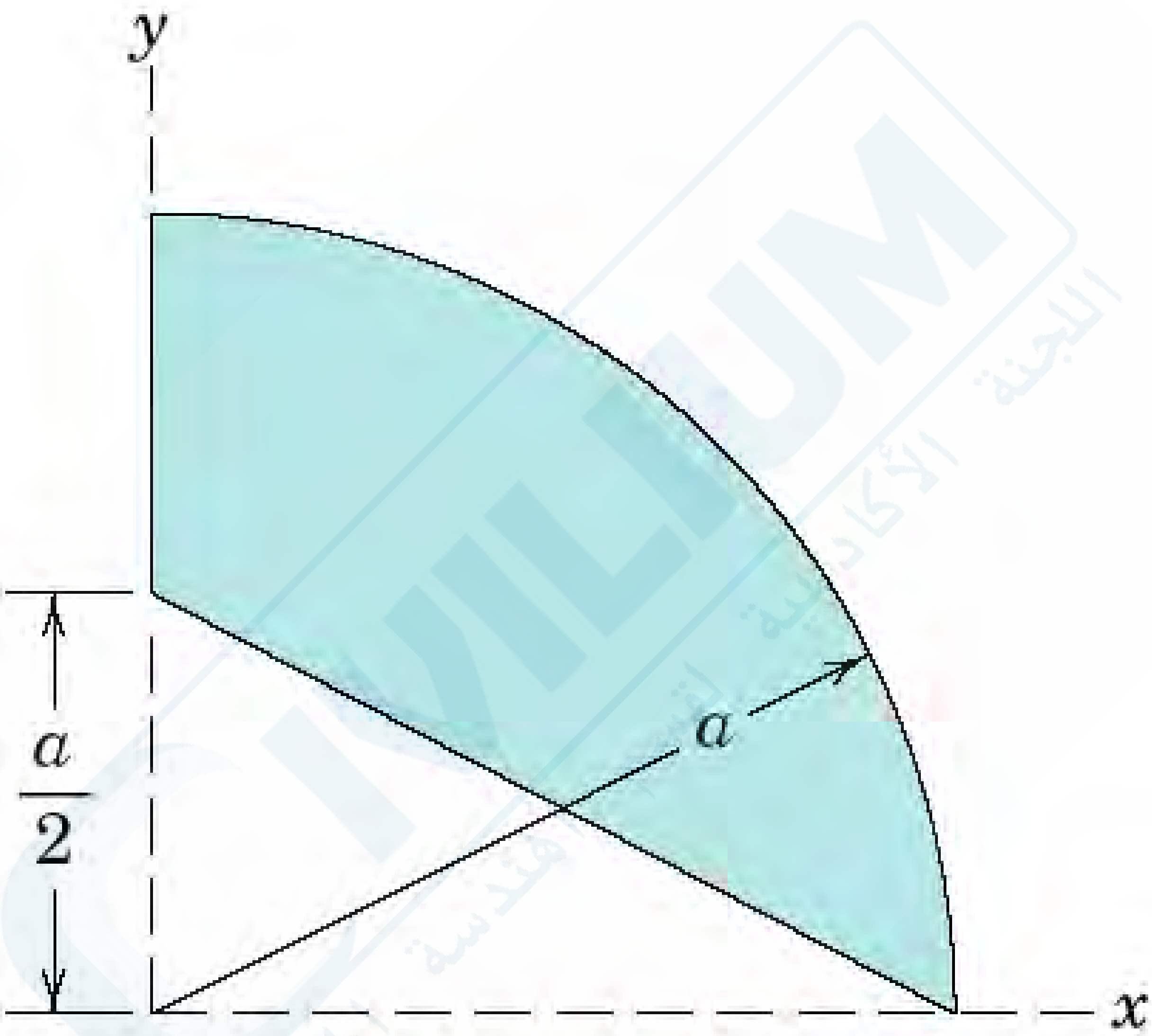
Problem 5/24

P_05_24



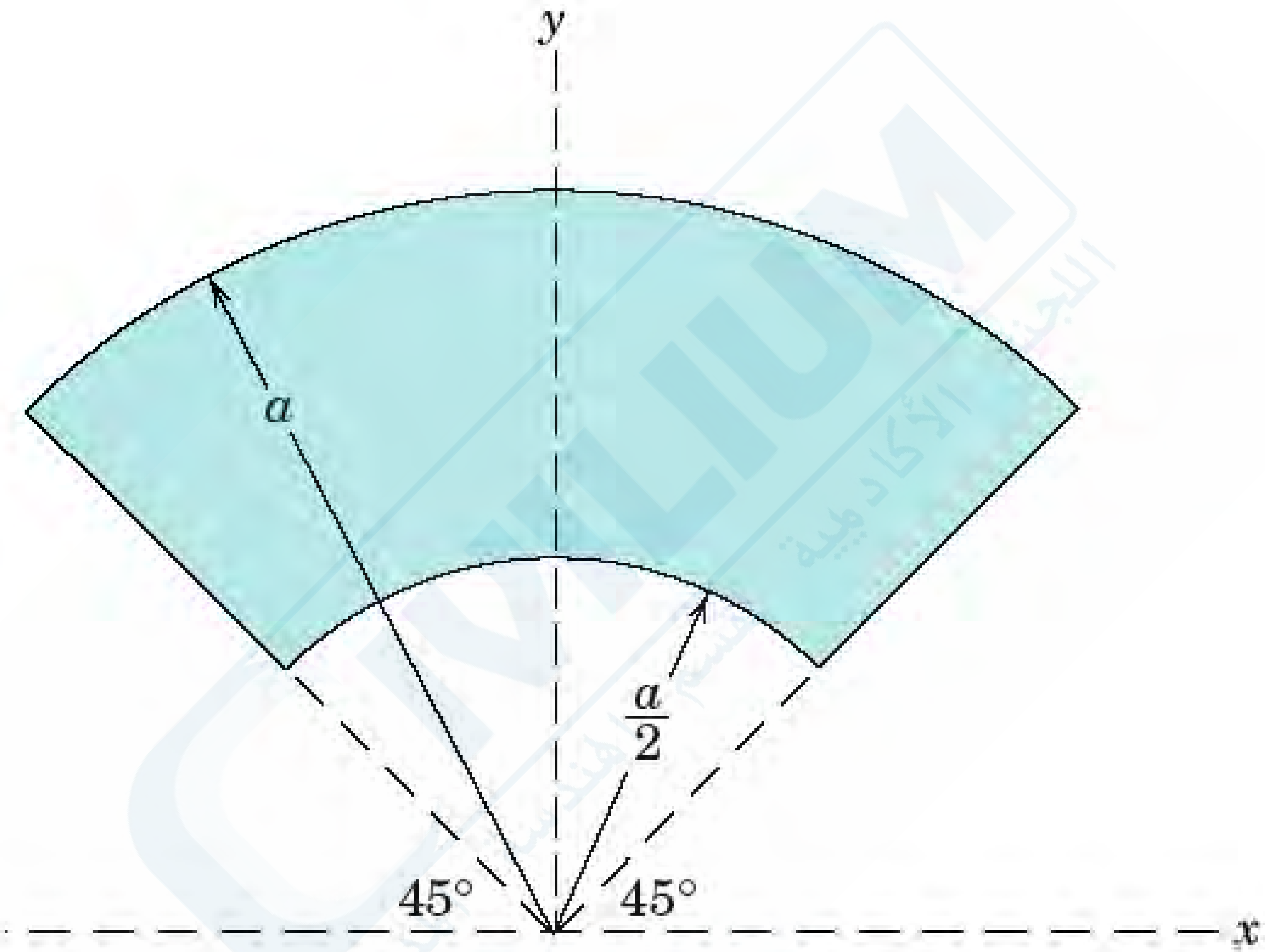
Problem 5/25

P_05_25



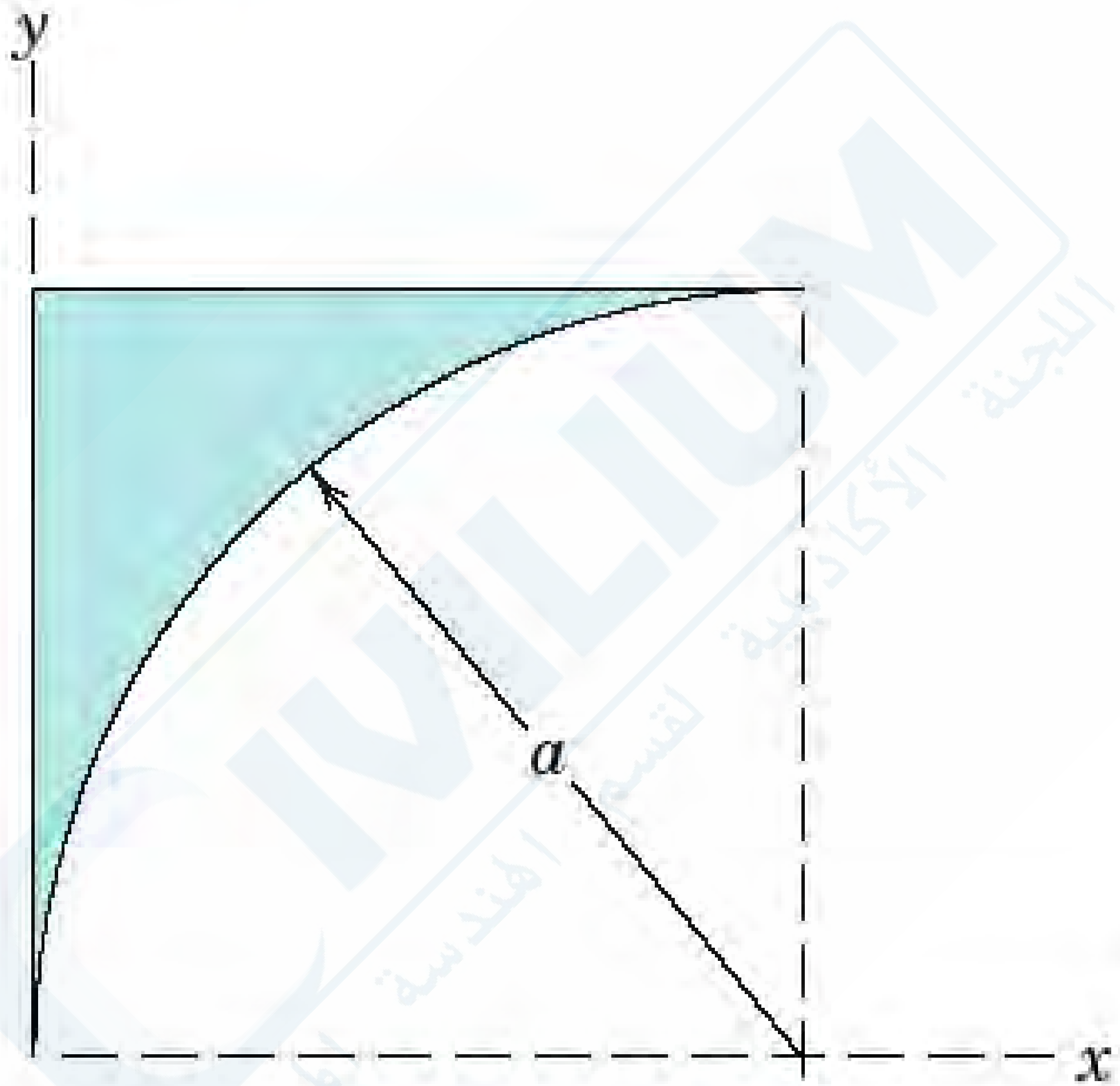
Problem 5/26

P_05_26



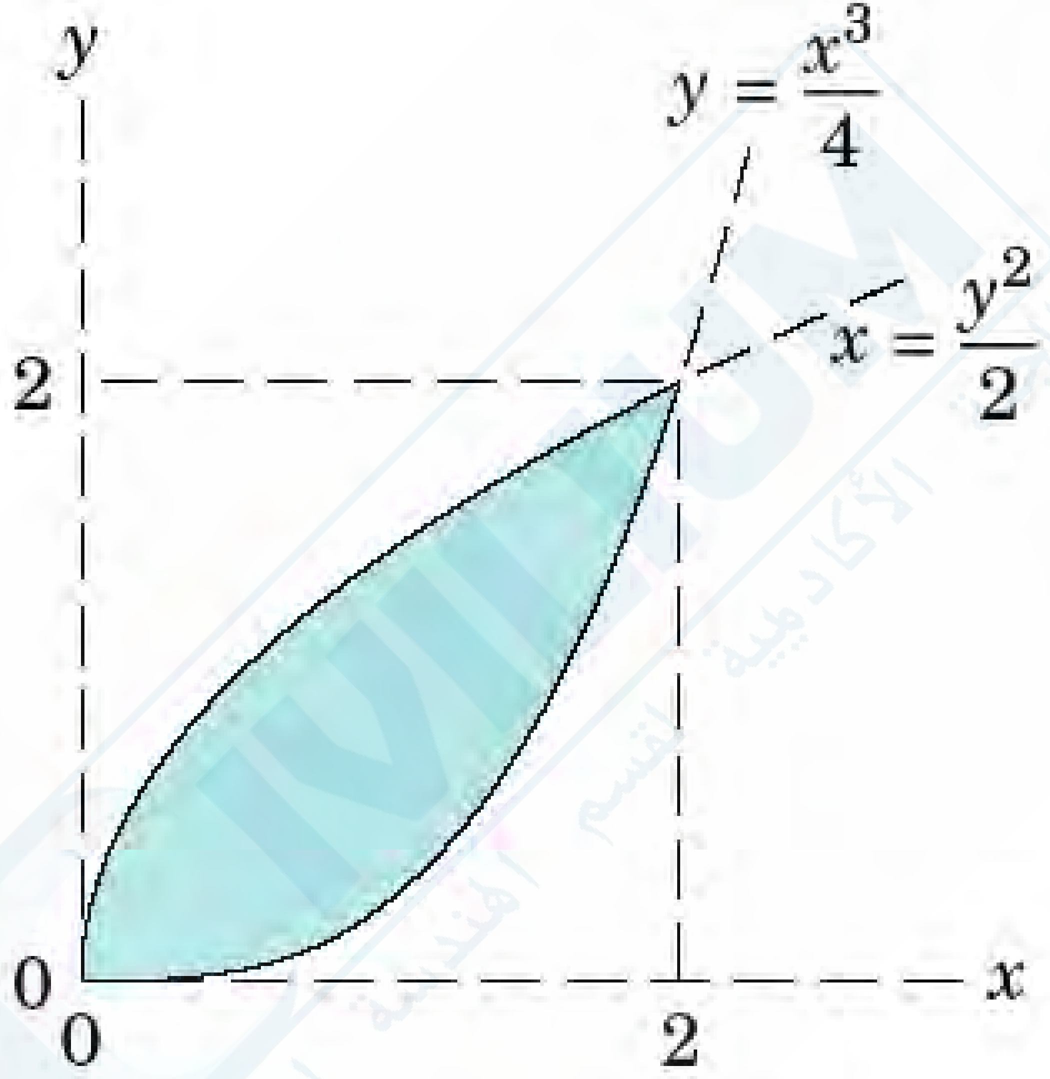
Problem 5/27

P_05_27

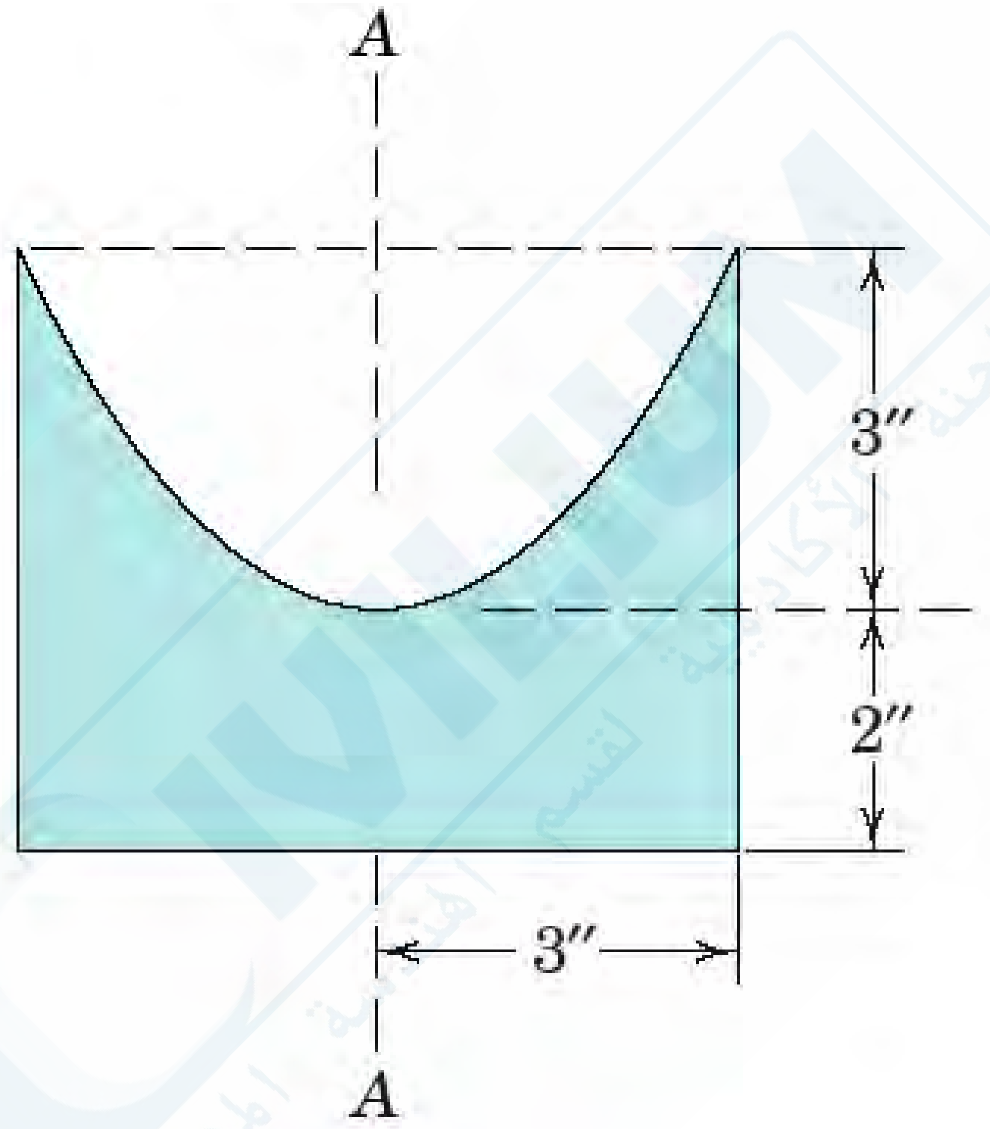


Problem 5/28

P_05_28

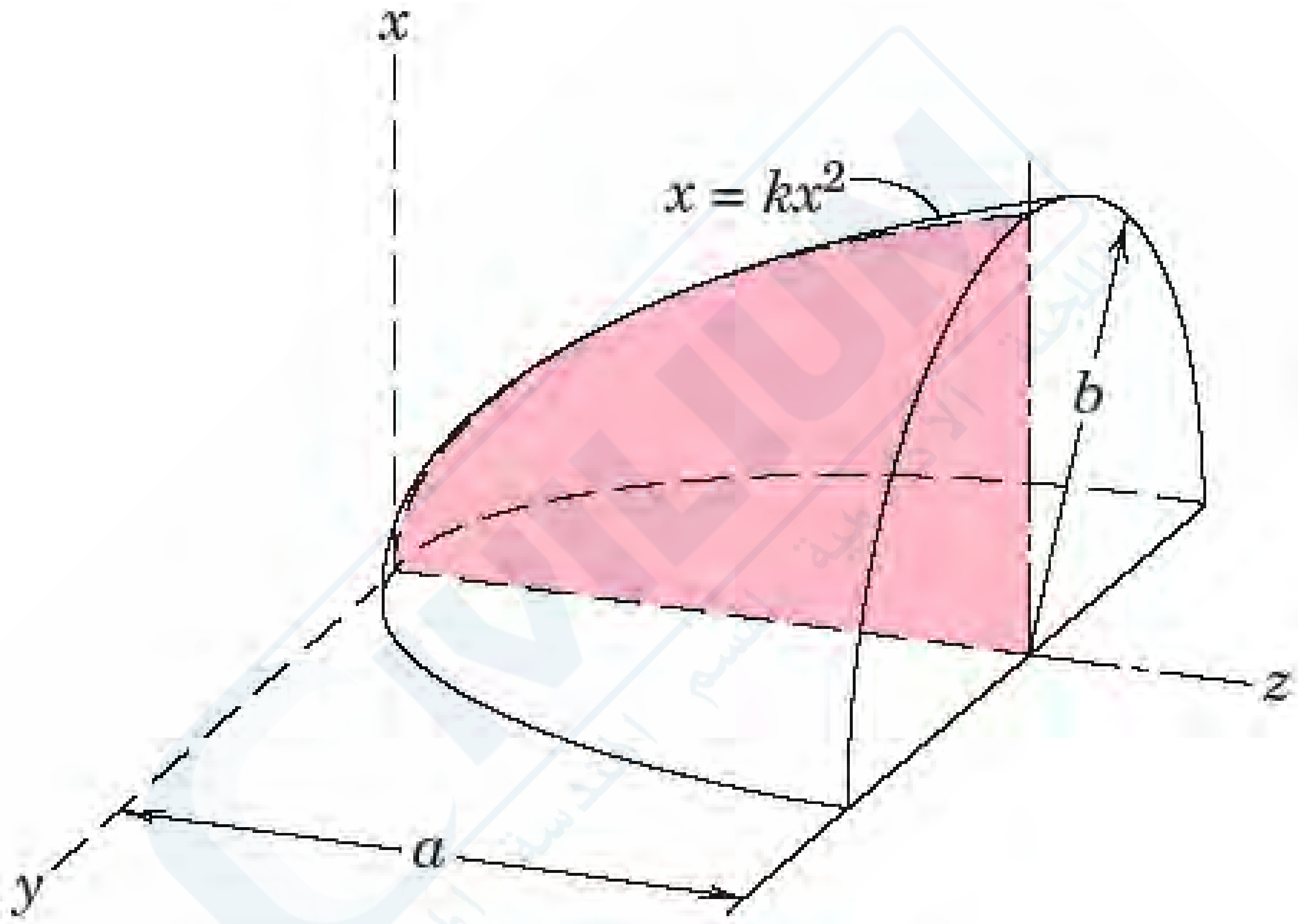


Problem 5/29



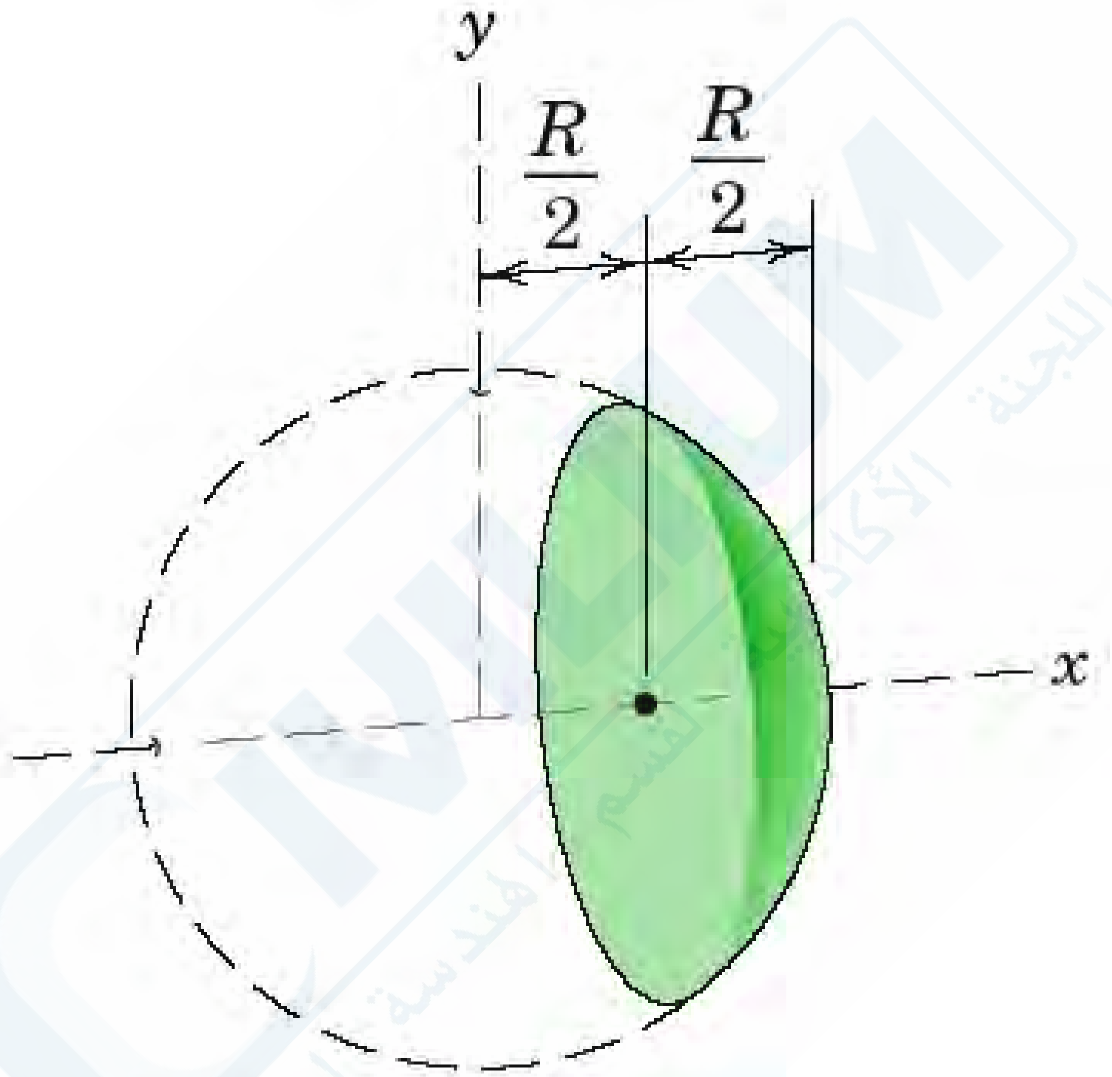
Problem 5/30

P_05_30



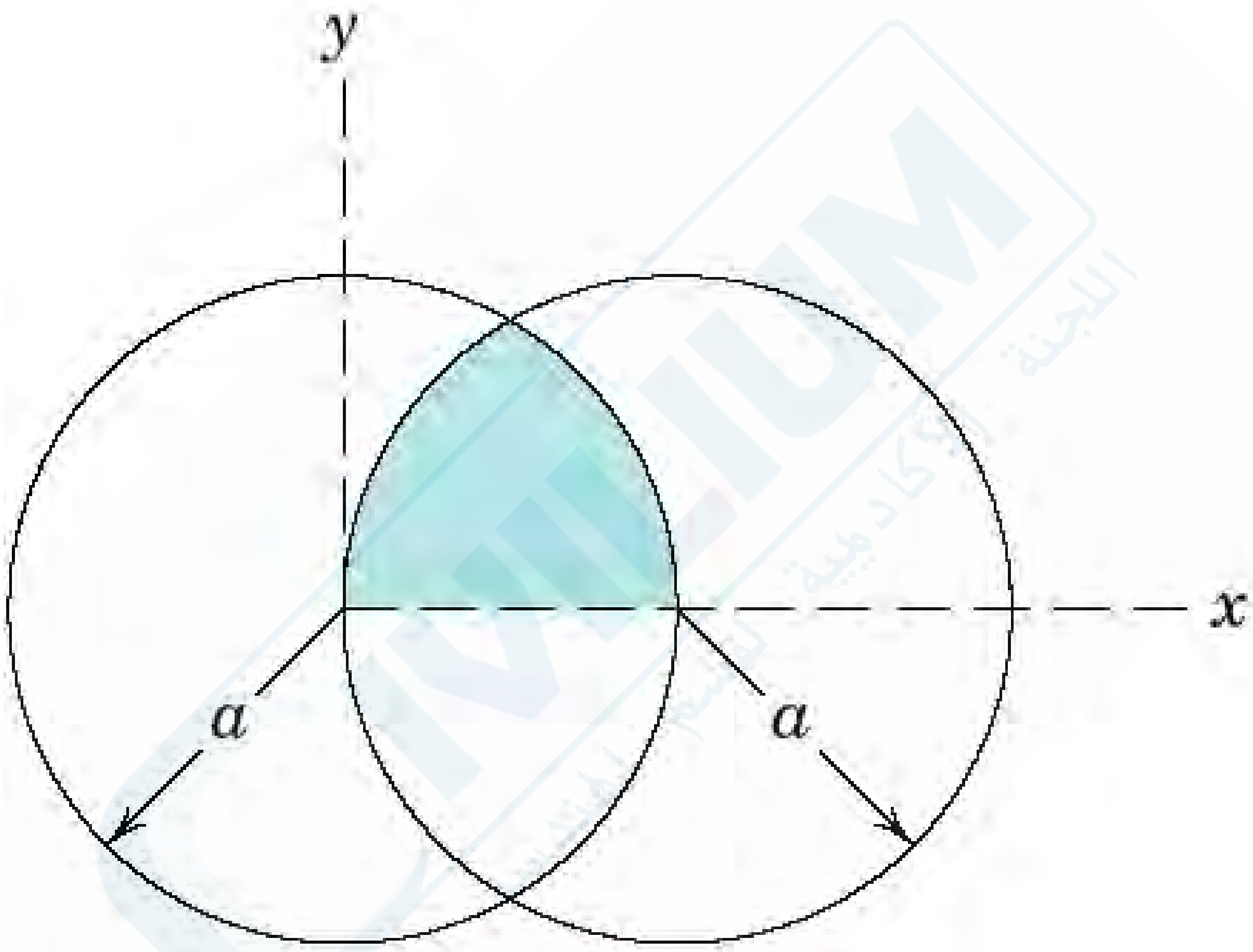
Problem 5/31

P_05_31



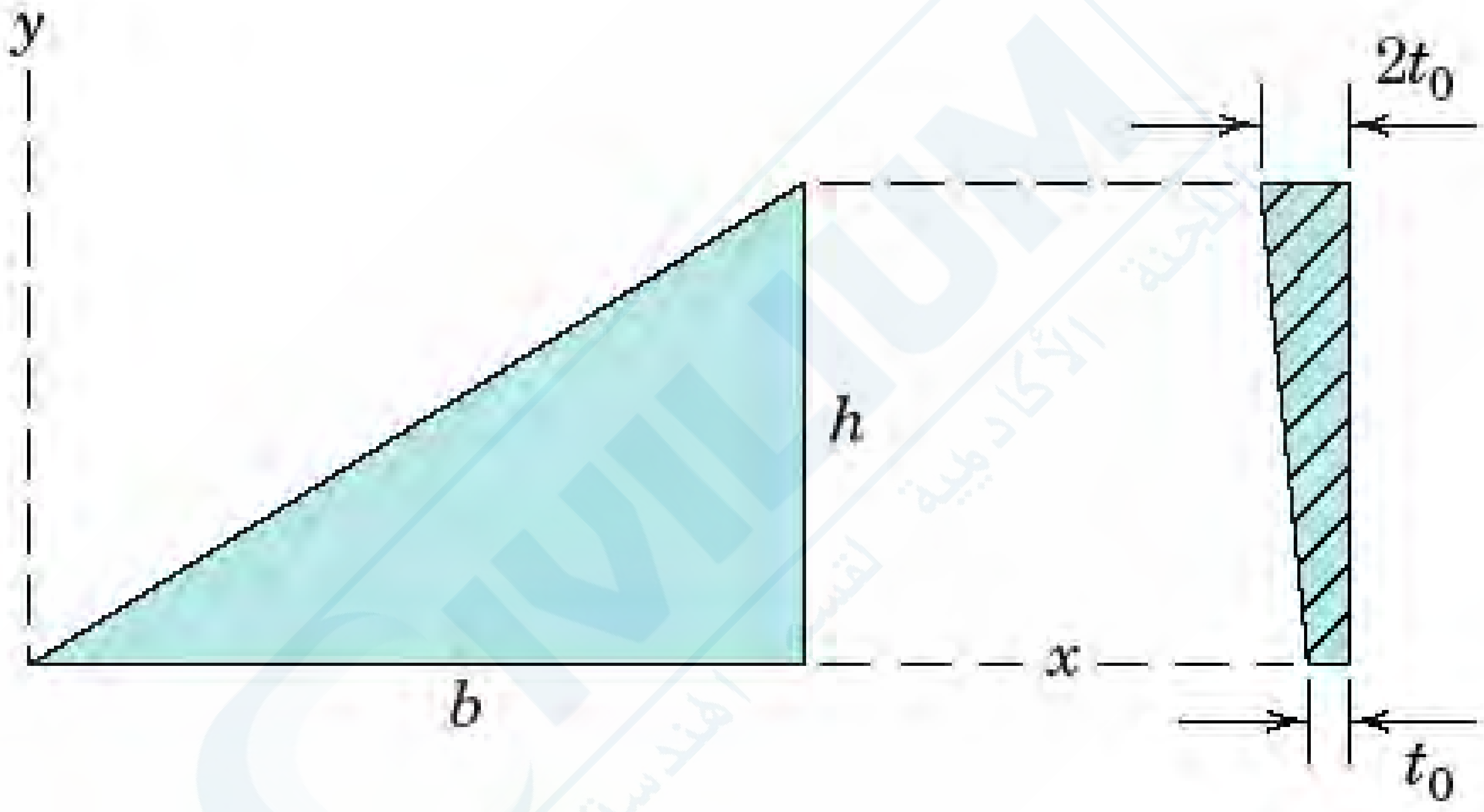
Problem 5/32

P_05_32



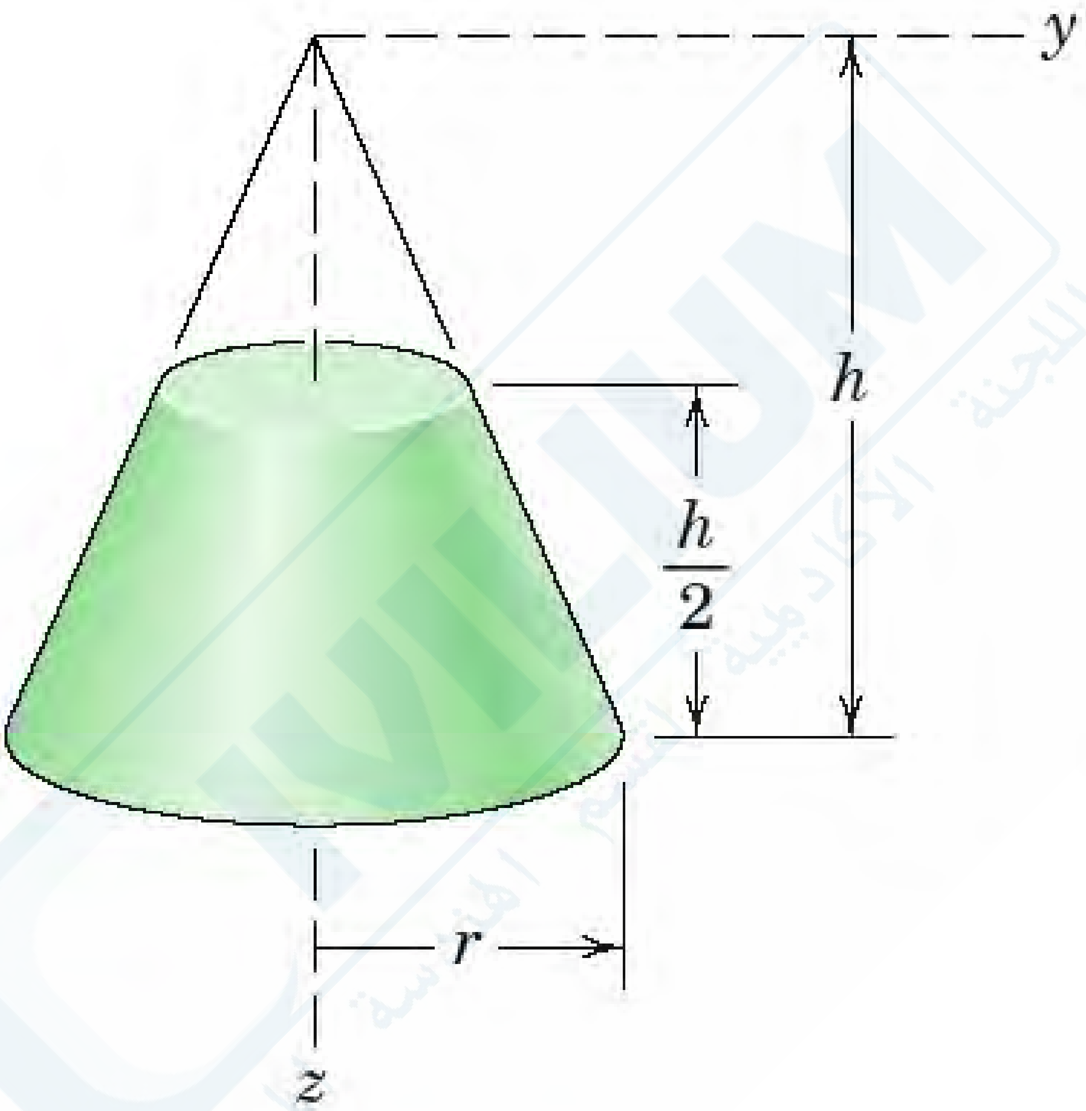
Problem 5/33

P 05 33



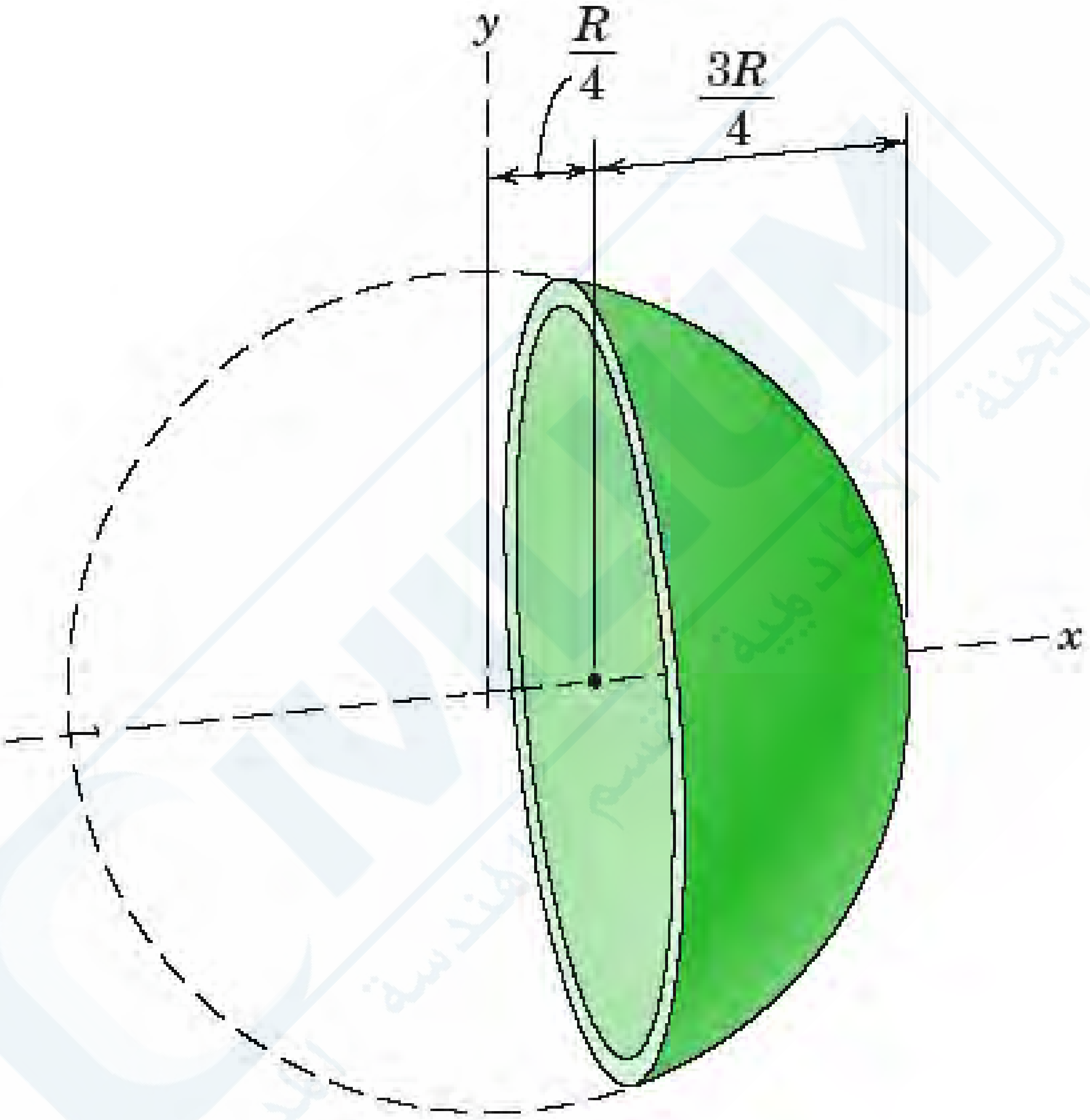
Problem 5/34

P_05_34



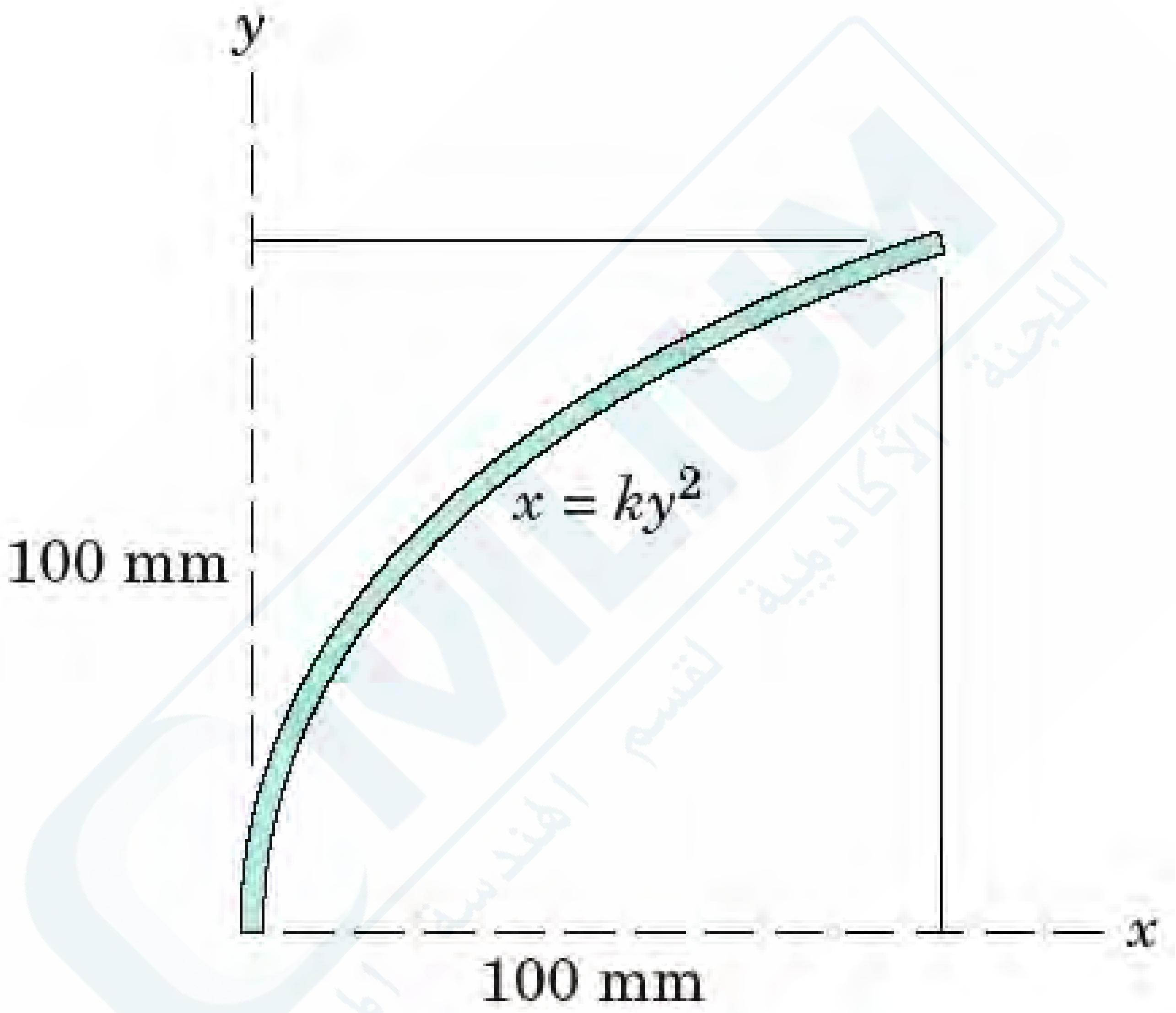
Problem 5/35

P_05_35



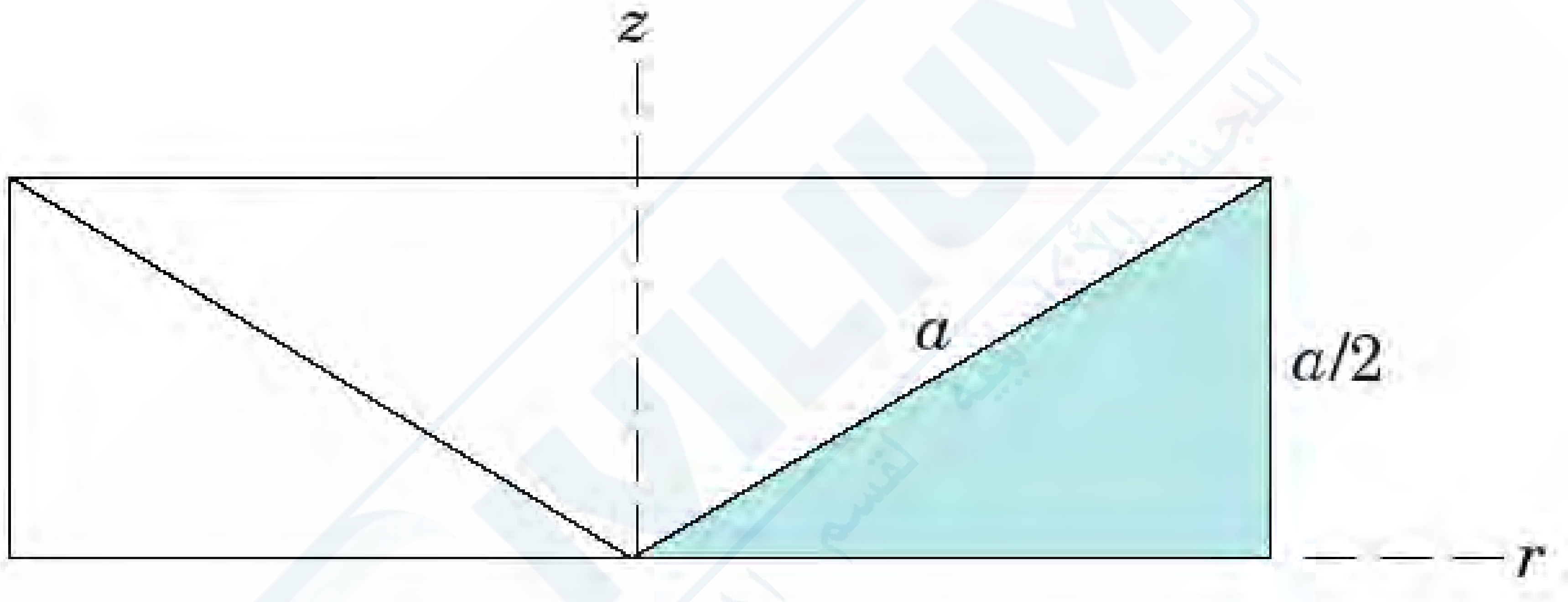
Problem 5/36

P_05_36



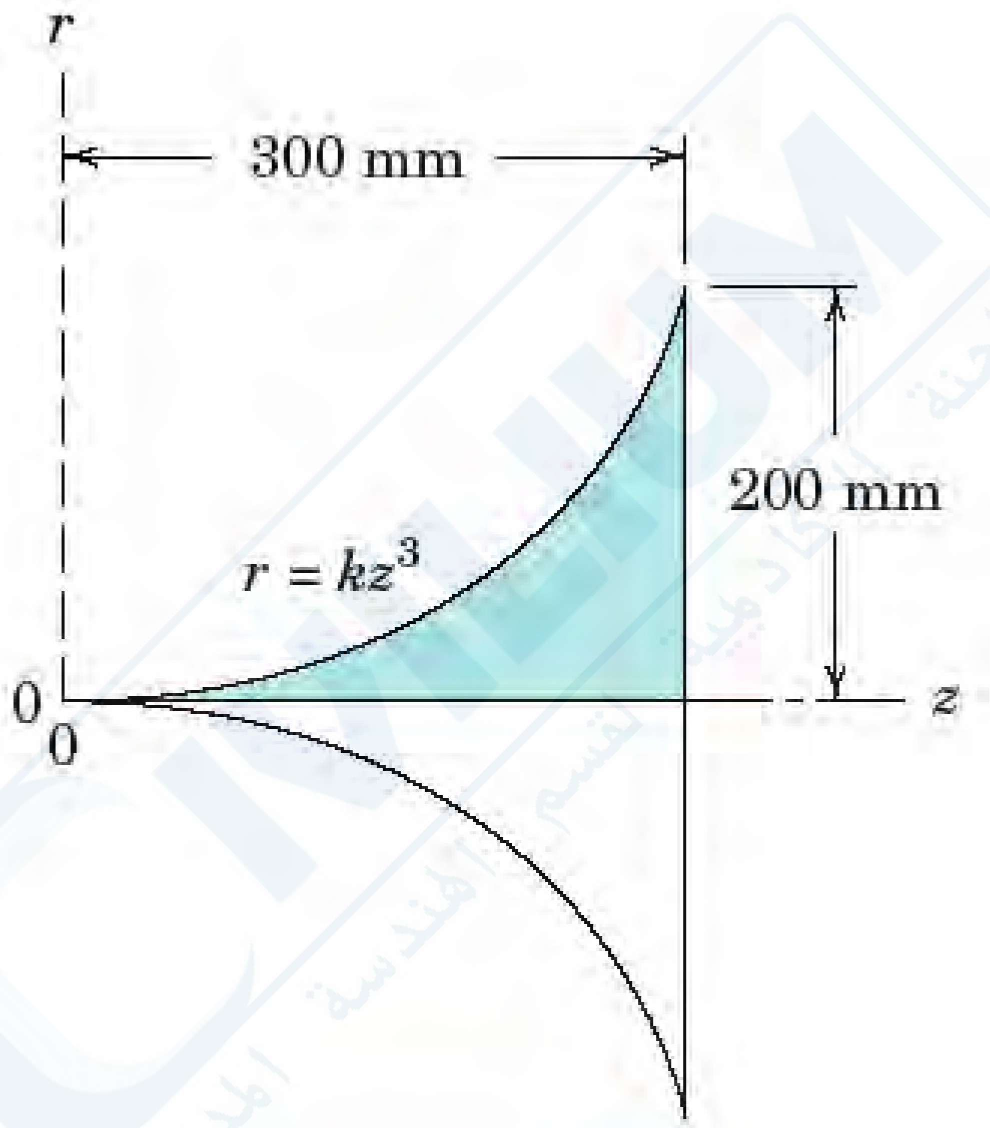
Problem 5/37

P_05_37



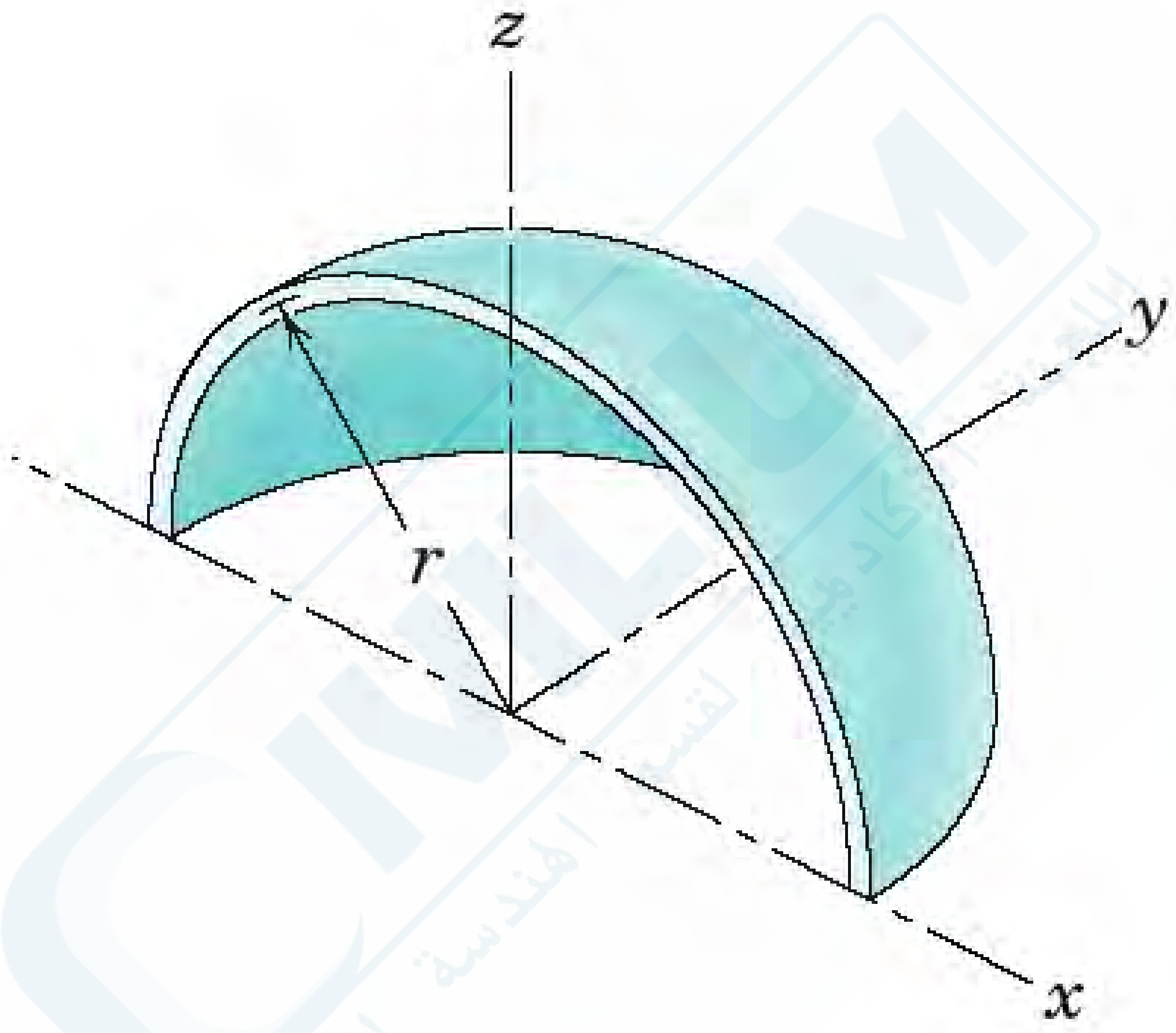
Problem 5/38

P_05_38



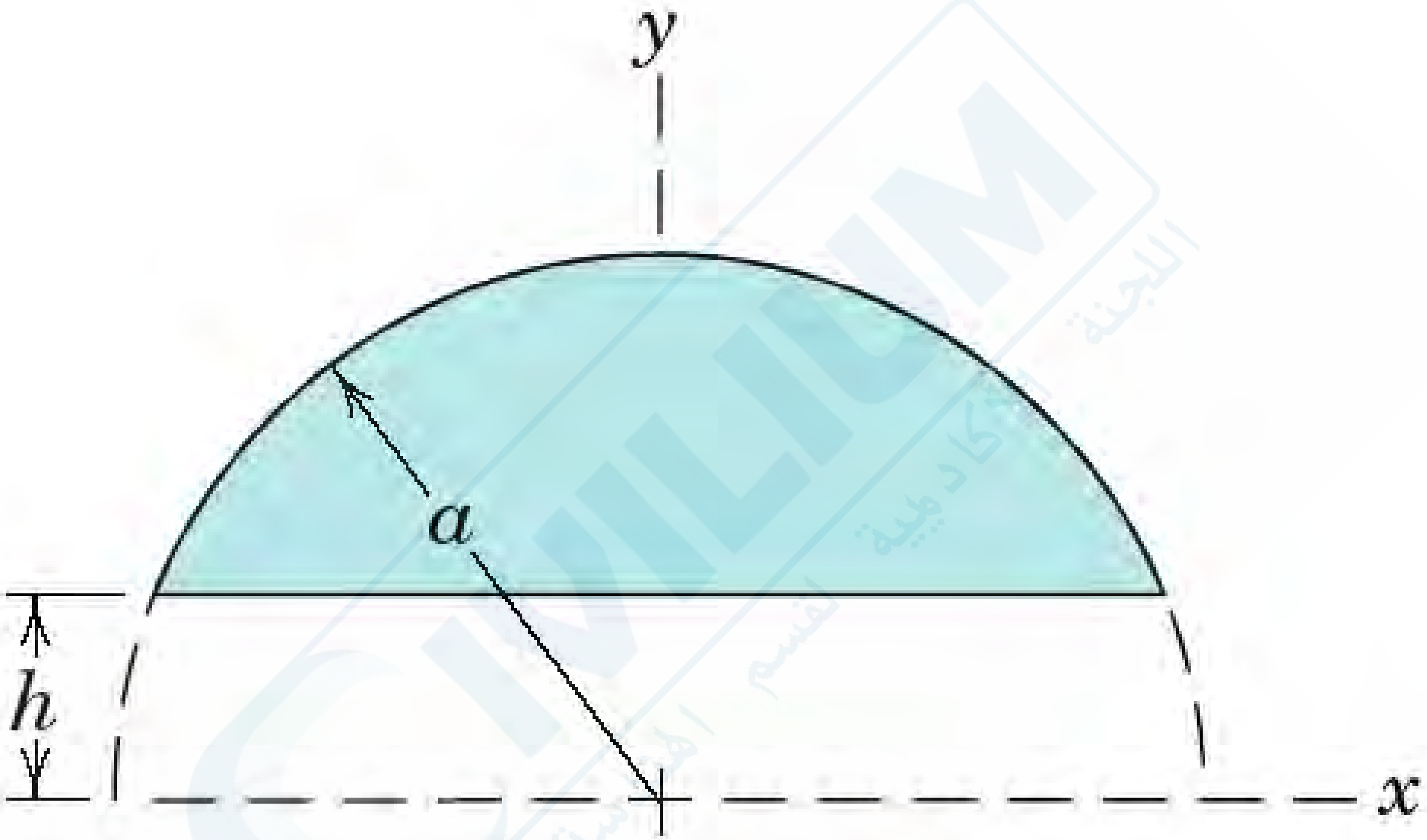
Problem 5/39

P_05_39



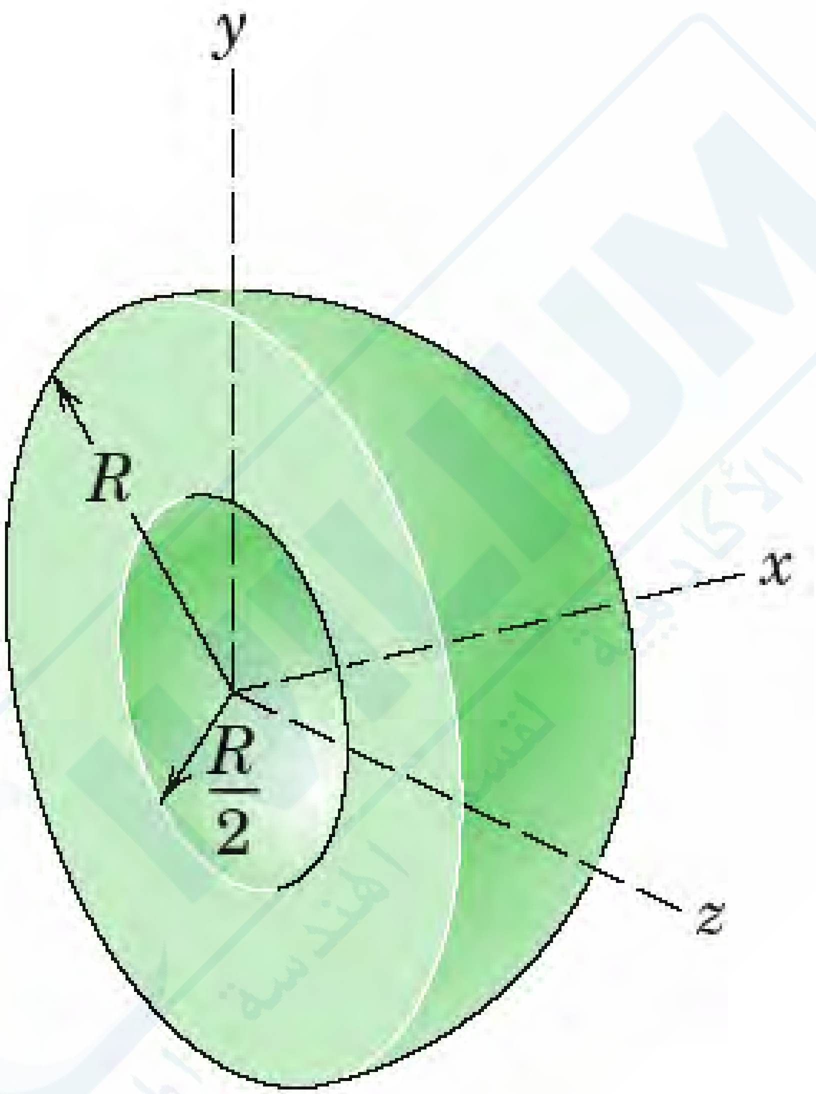
Problem 5/40

P_05_40



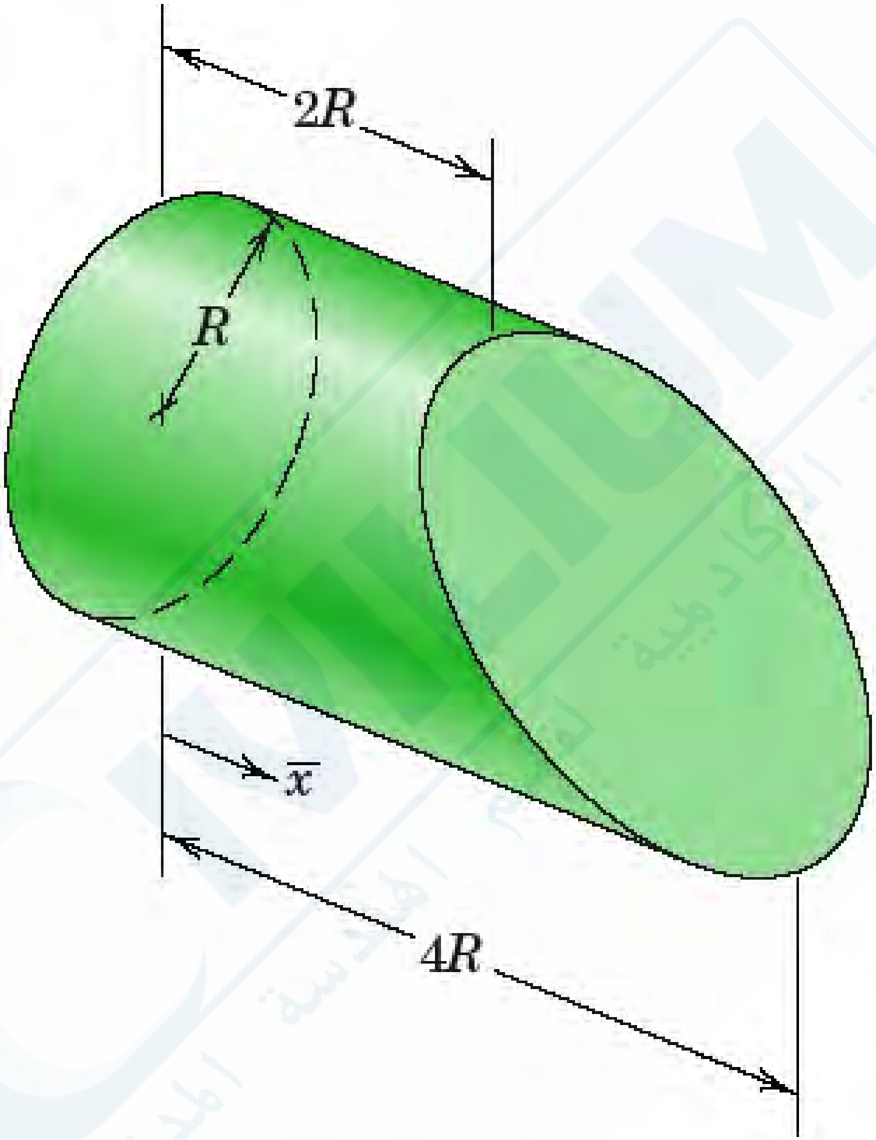
Problem 5/41

P_05_41



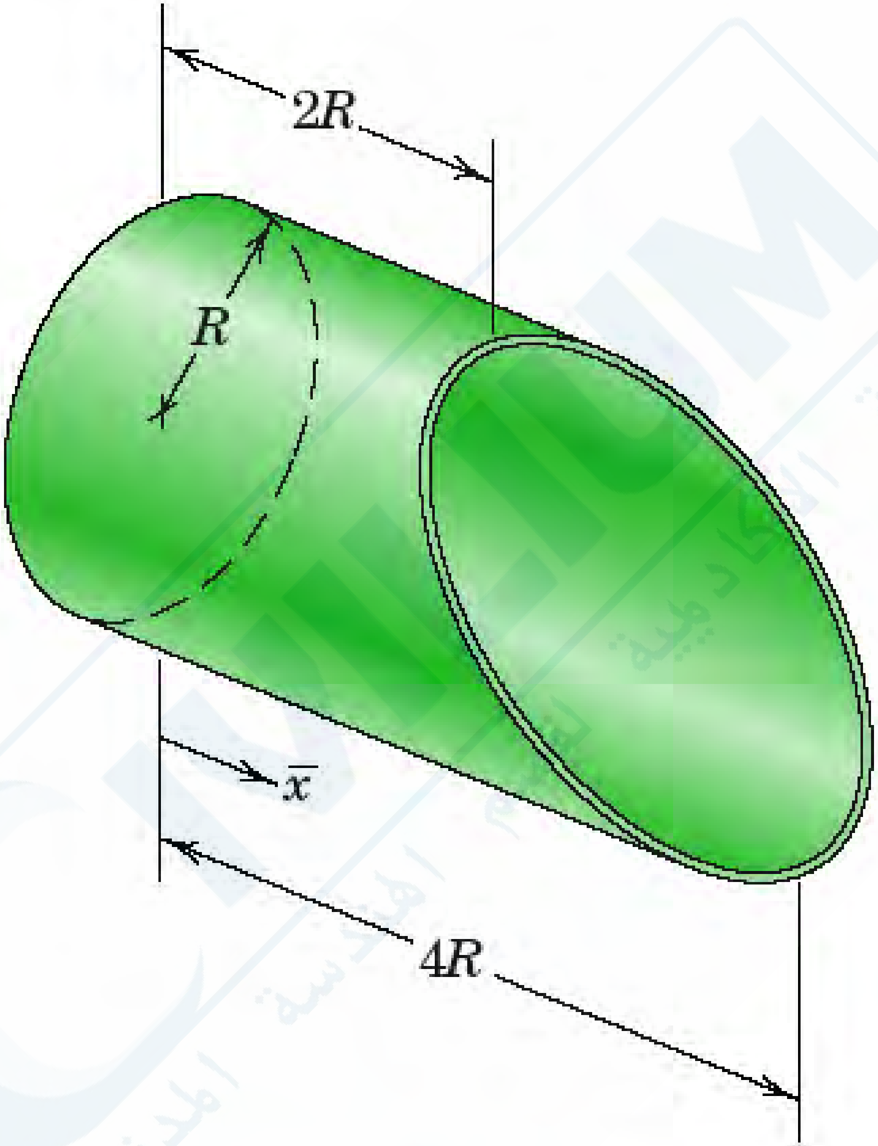
Problem 5/42

P_05_42



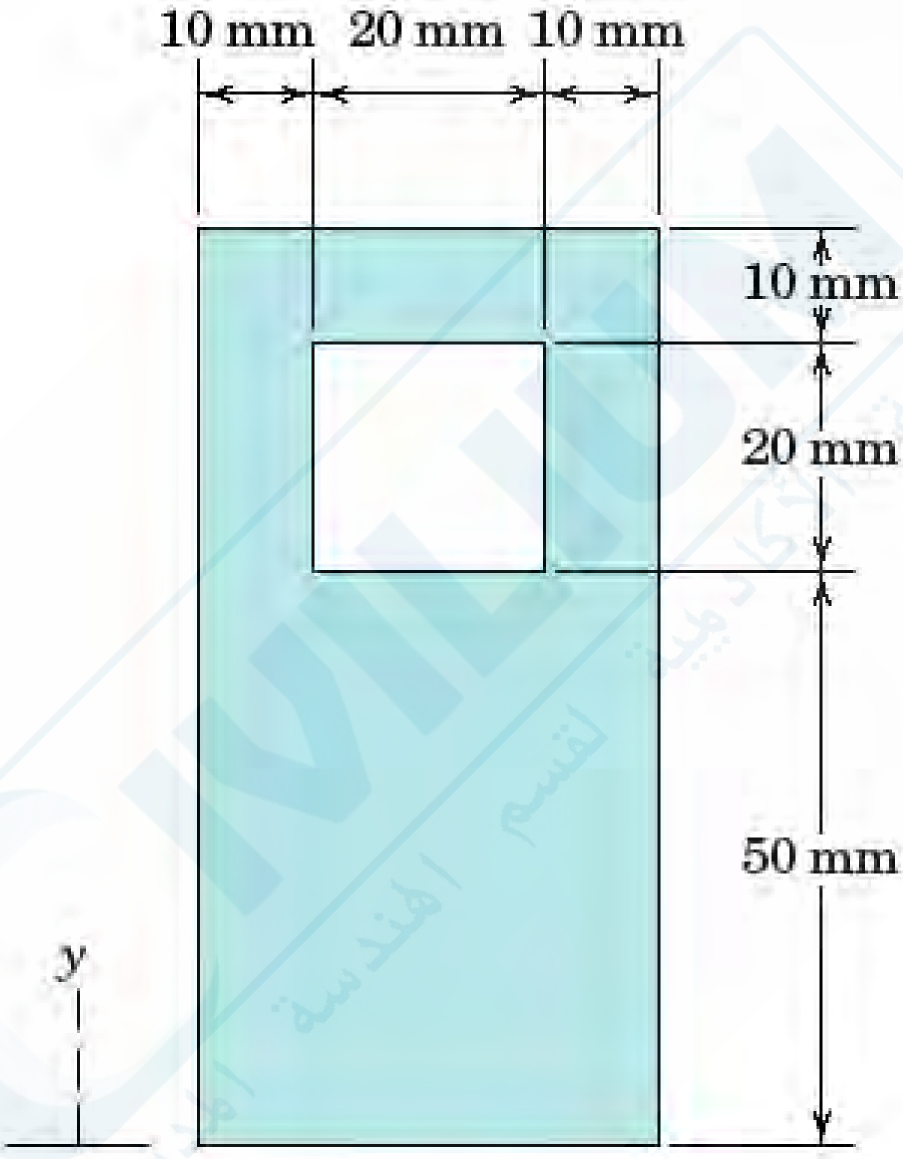
Problem 5/43

P 05 43



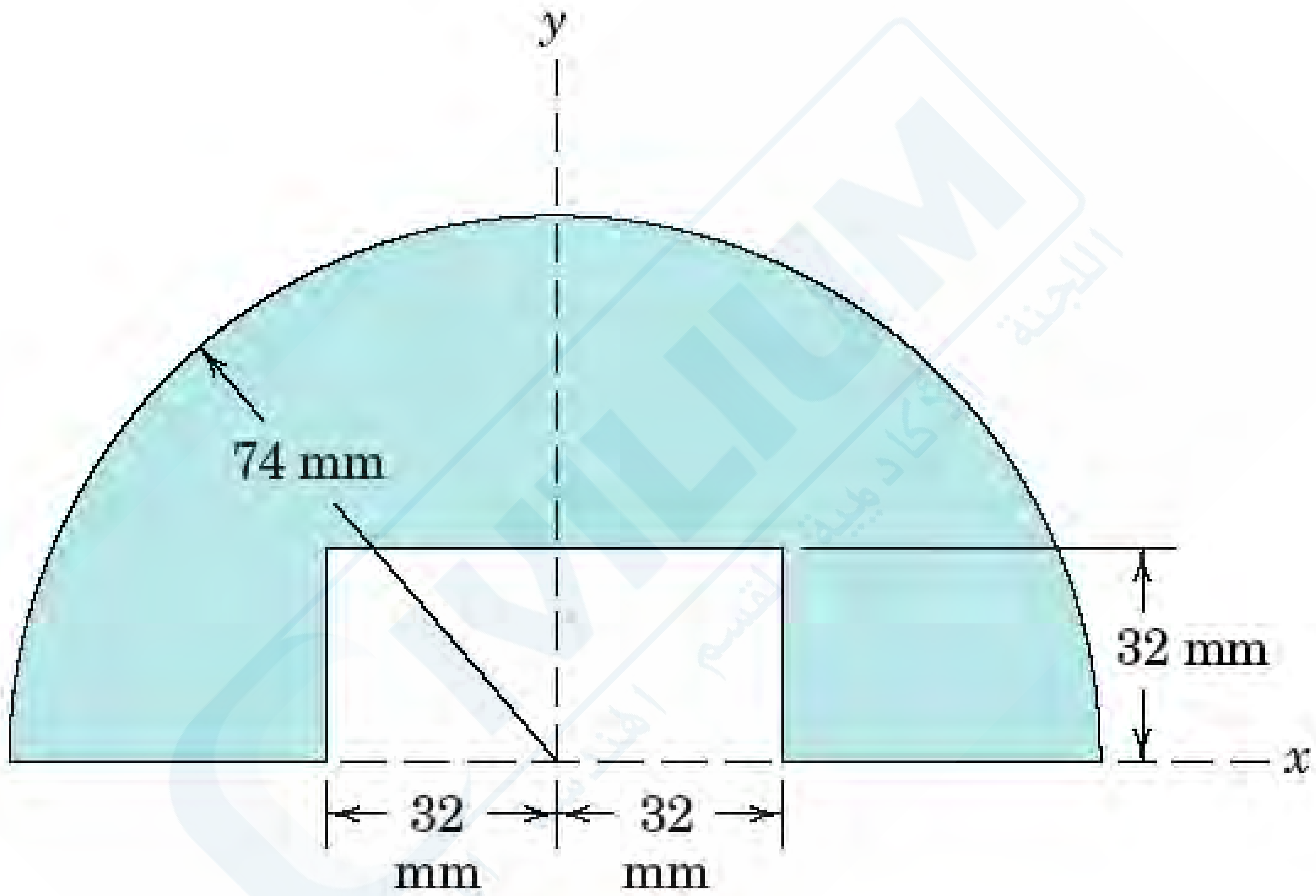
Problem 5/44

P_05_44



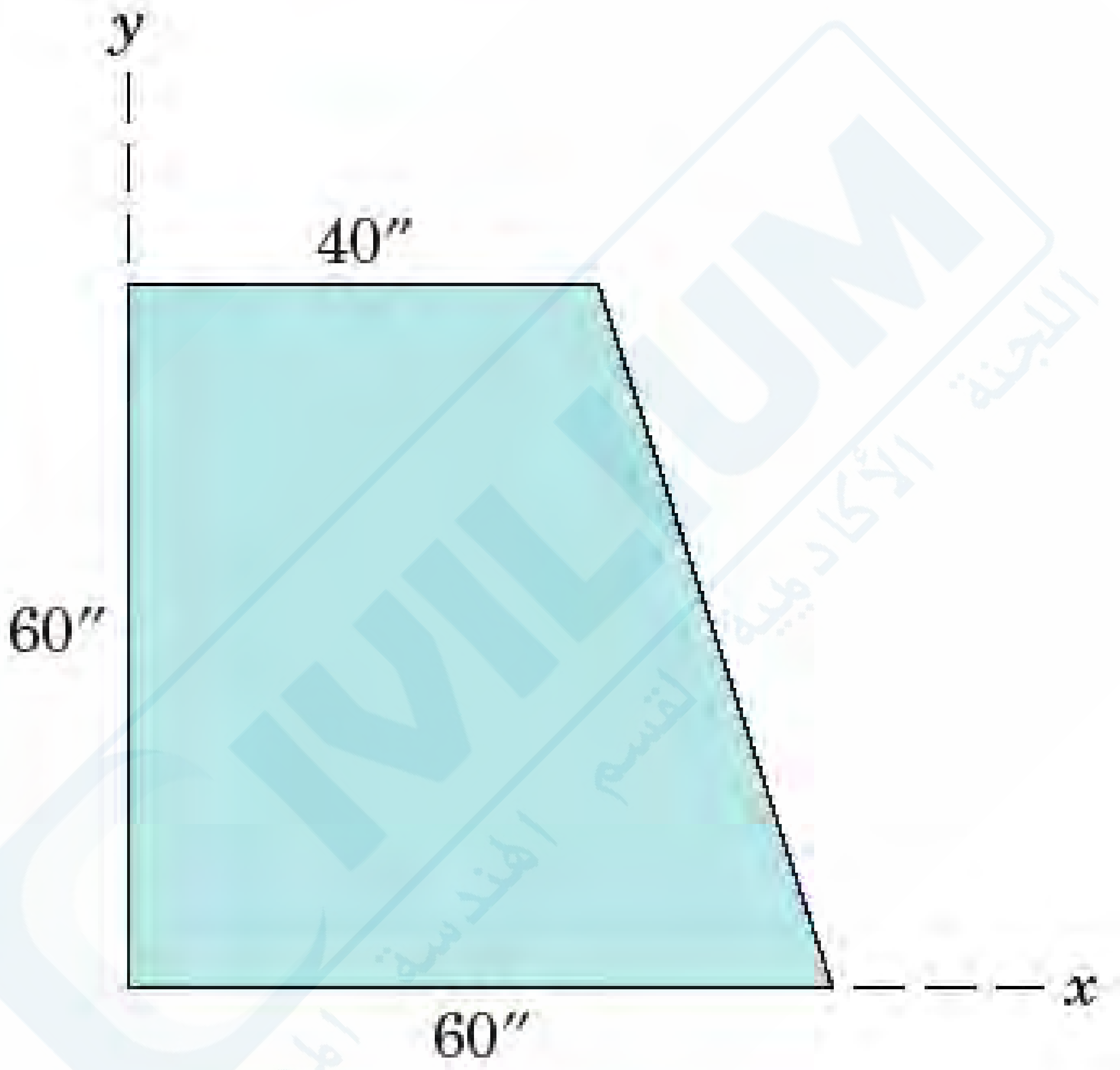
Problem 5/45

P_05_45



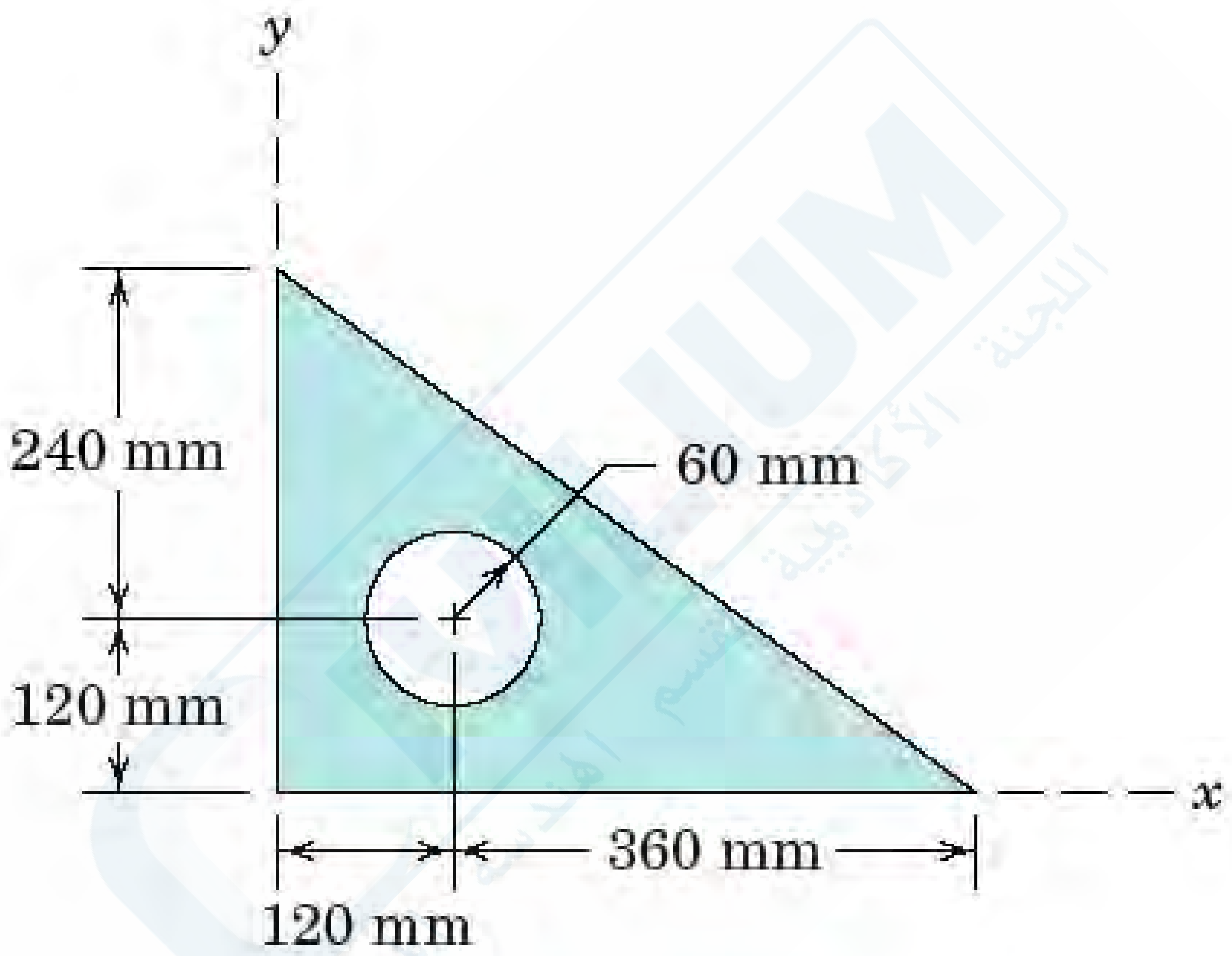
Problem 5/46

P_05_46



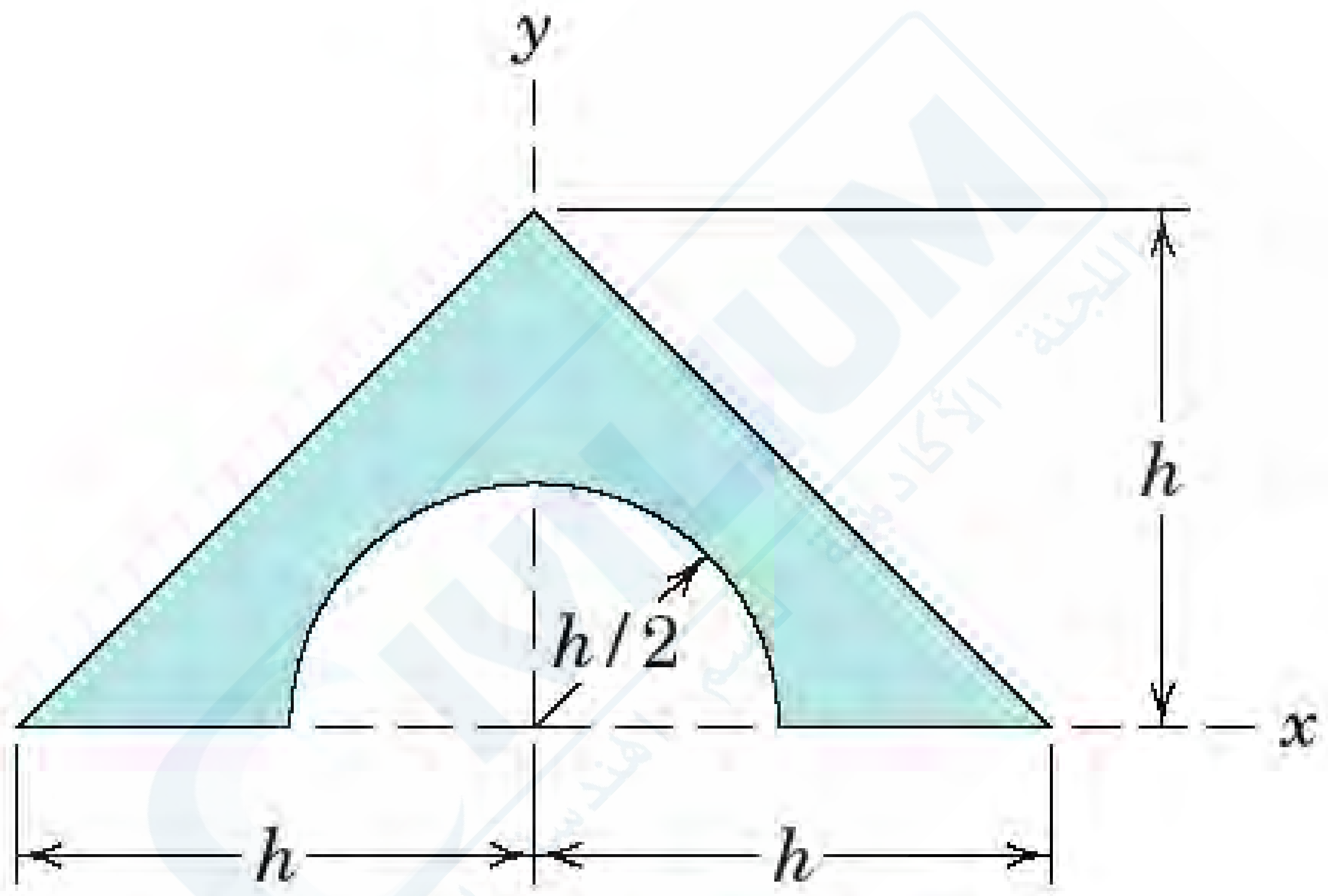
Problem 5/47

P_05_47



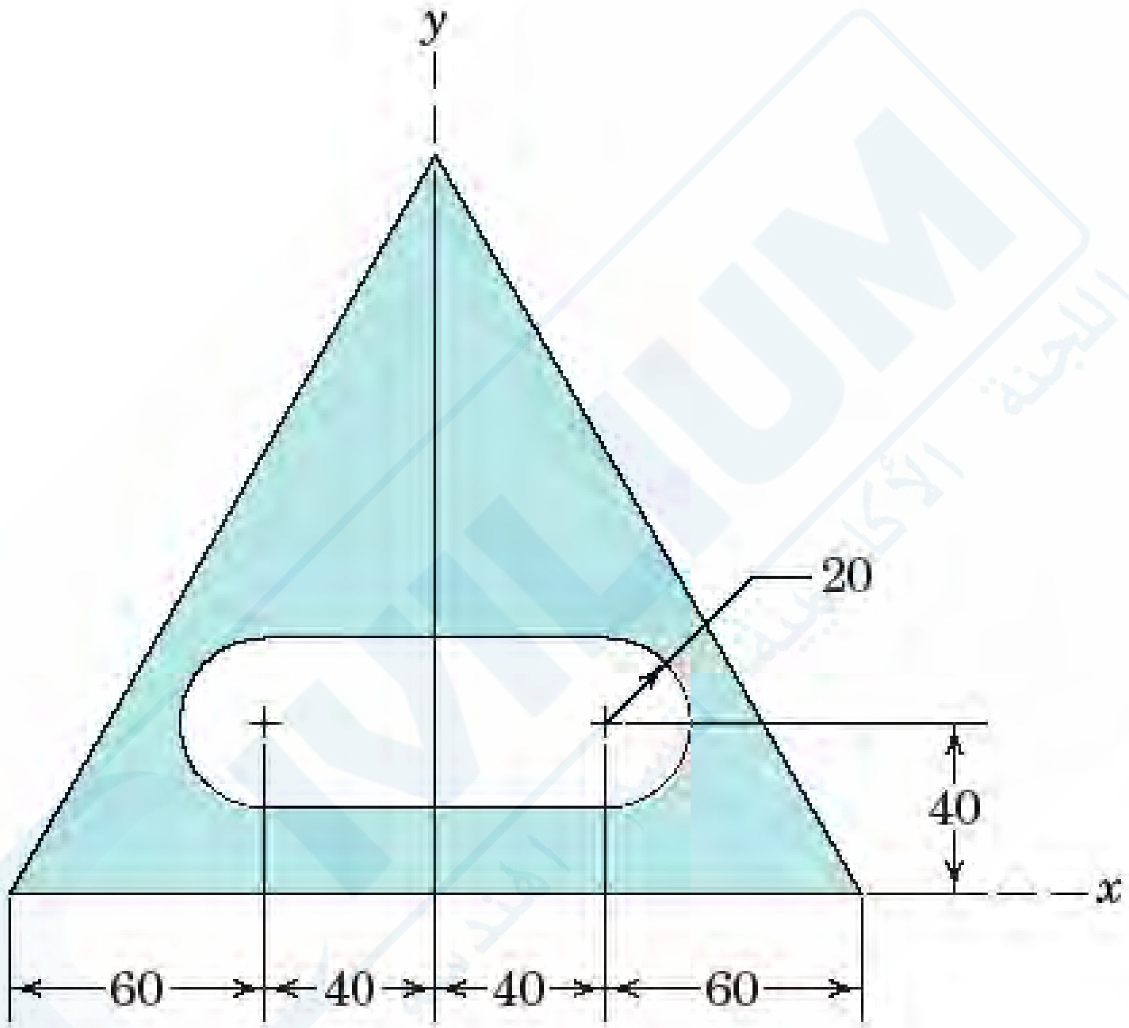
Problem 5/48

P_05_48



Problem 5/49

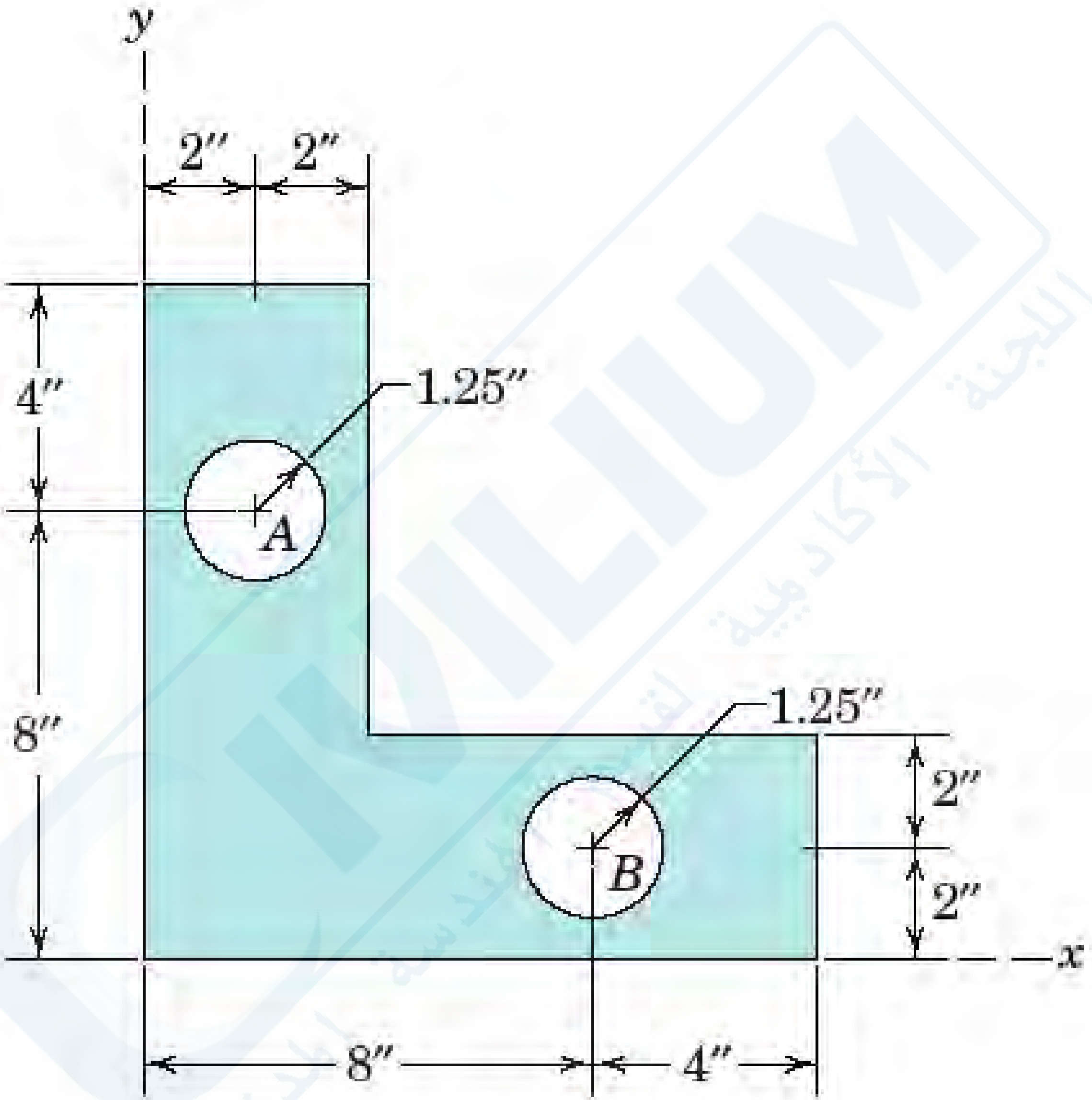
P_05_49



Dimensions in millimeters

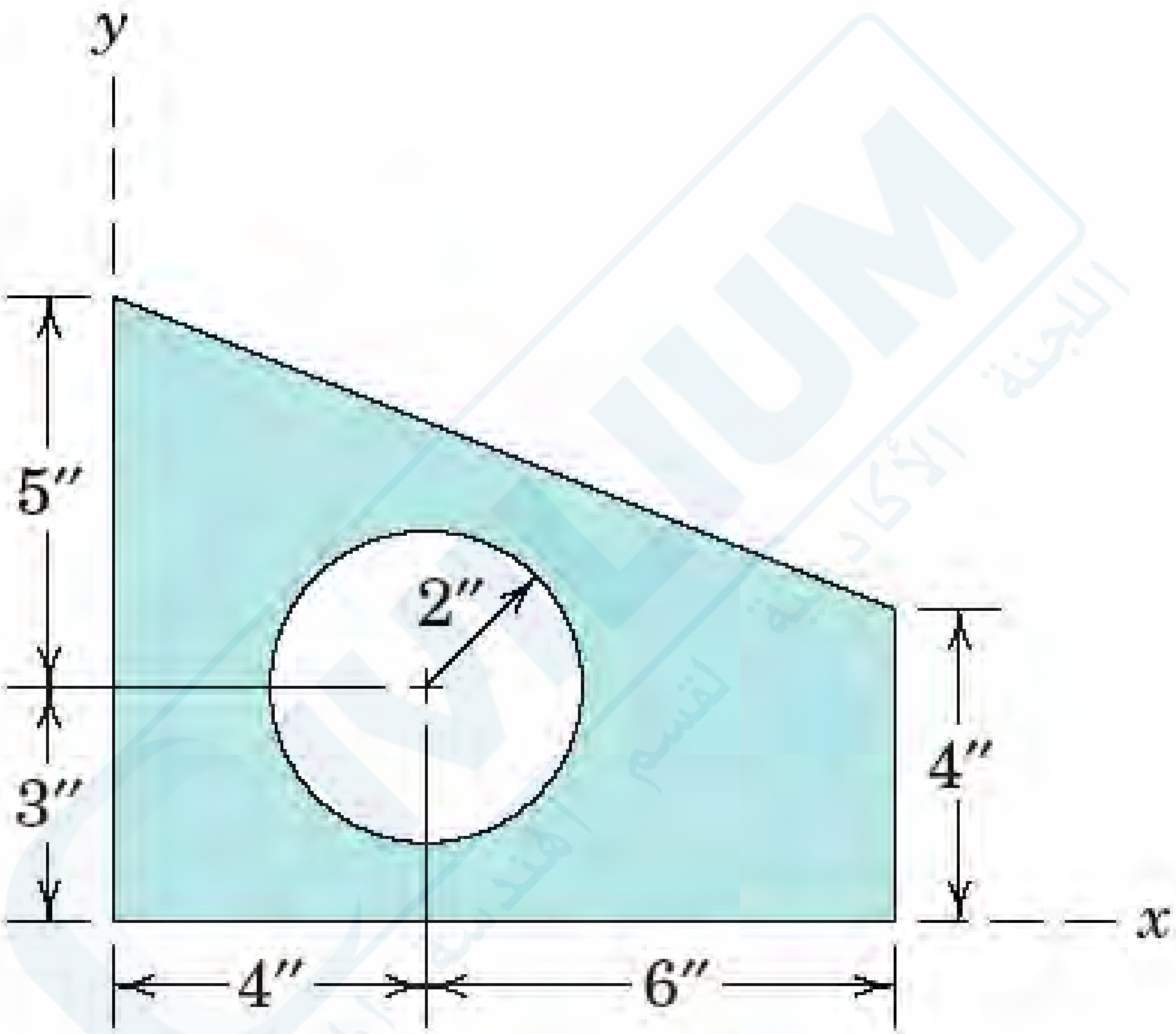
Problem 5/50

P_05_50



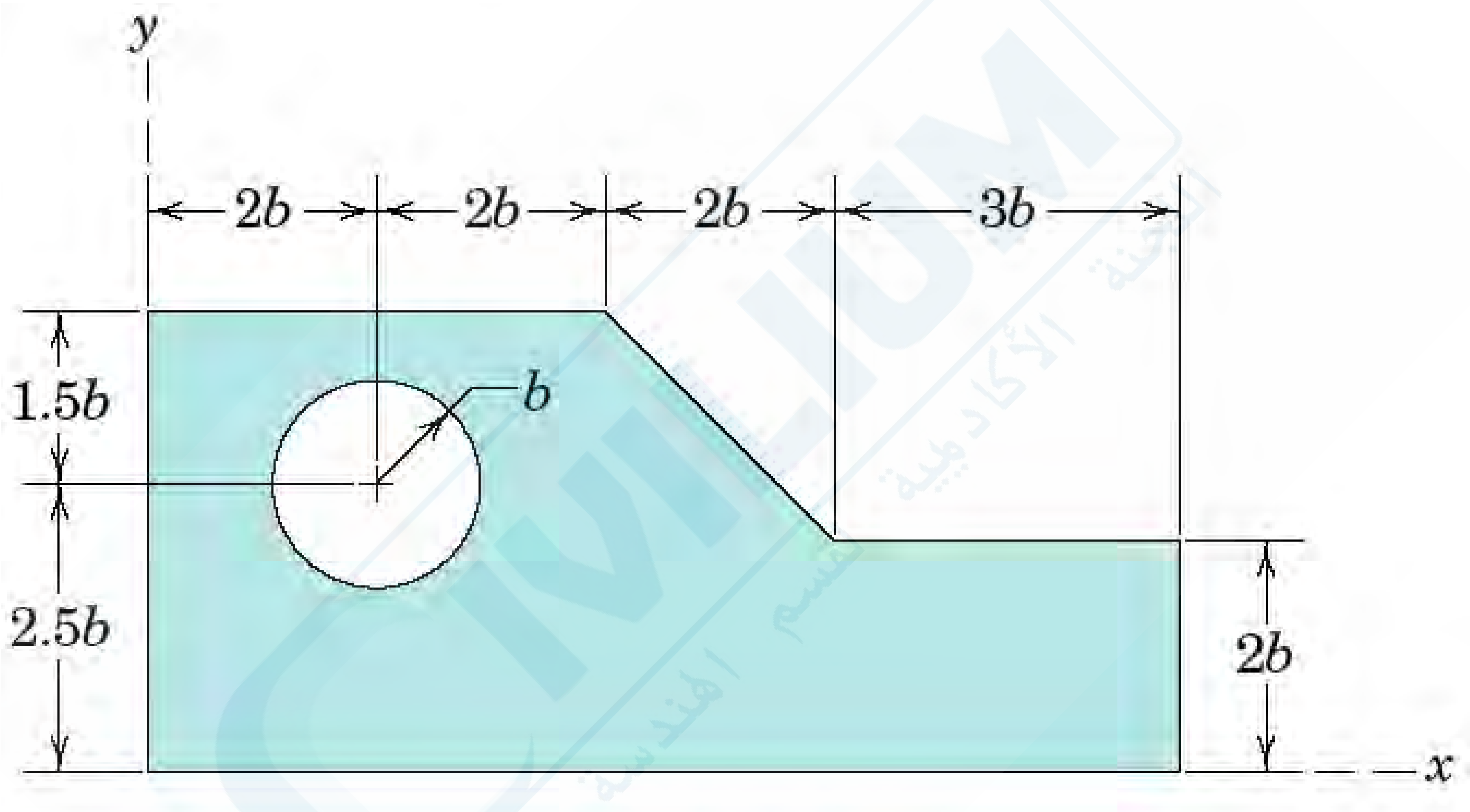
Problem 5/51

P_05_51



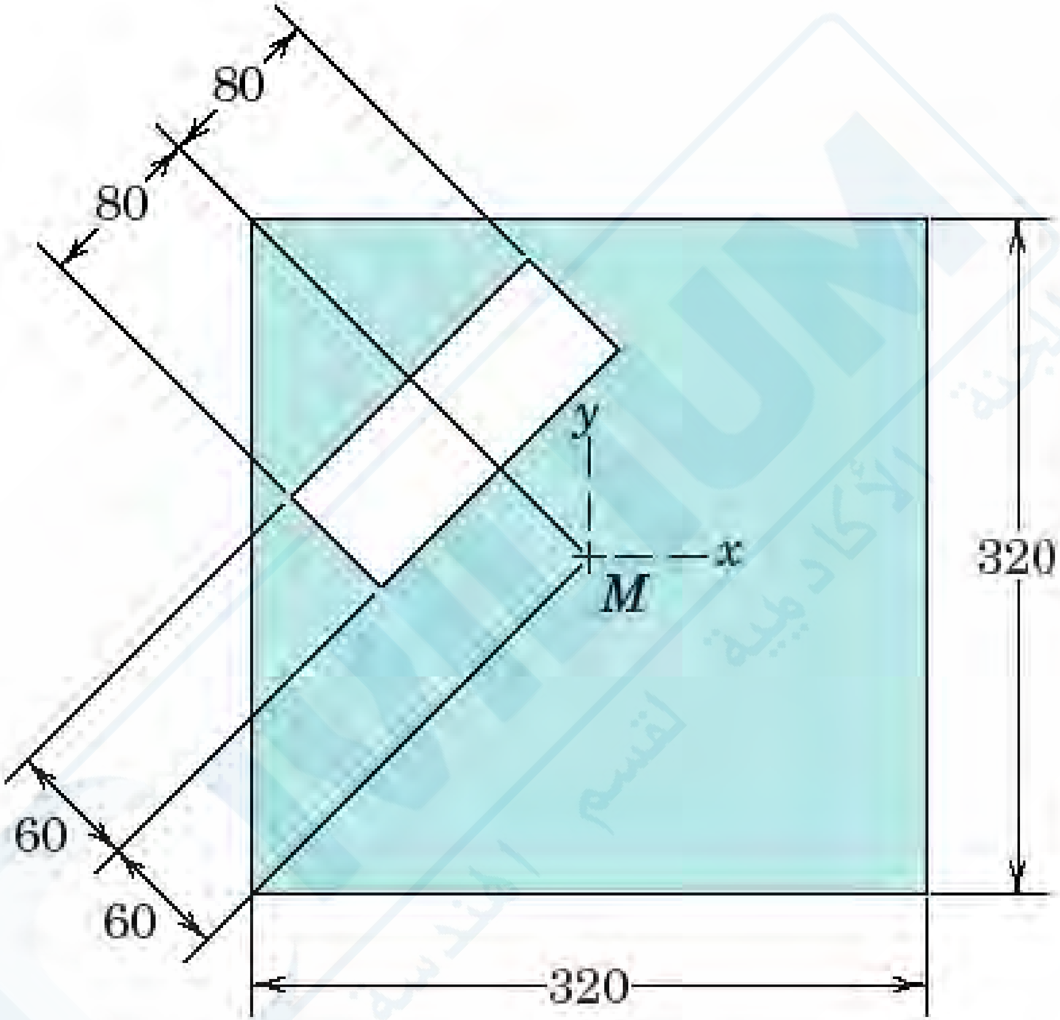
Problem 5/52

P_05_52



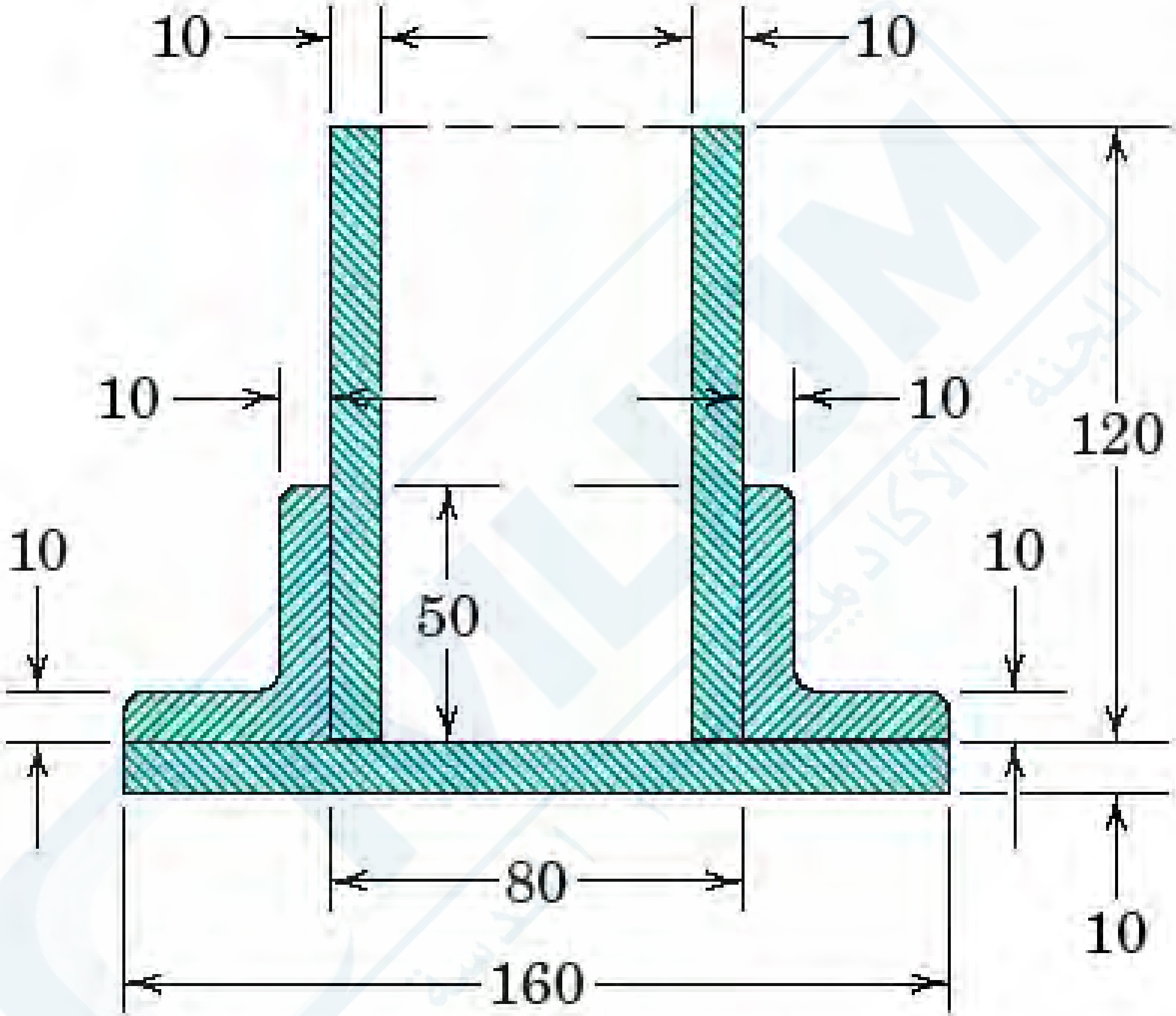
Problem 5/53

P_05_53



Problem 5/54

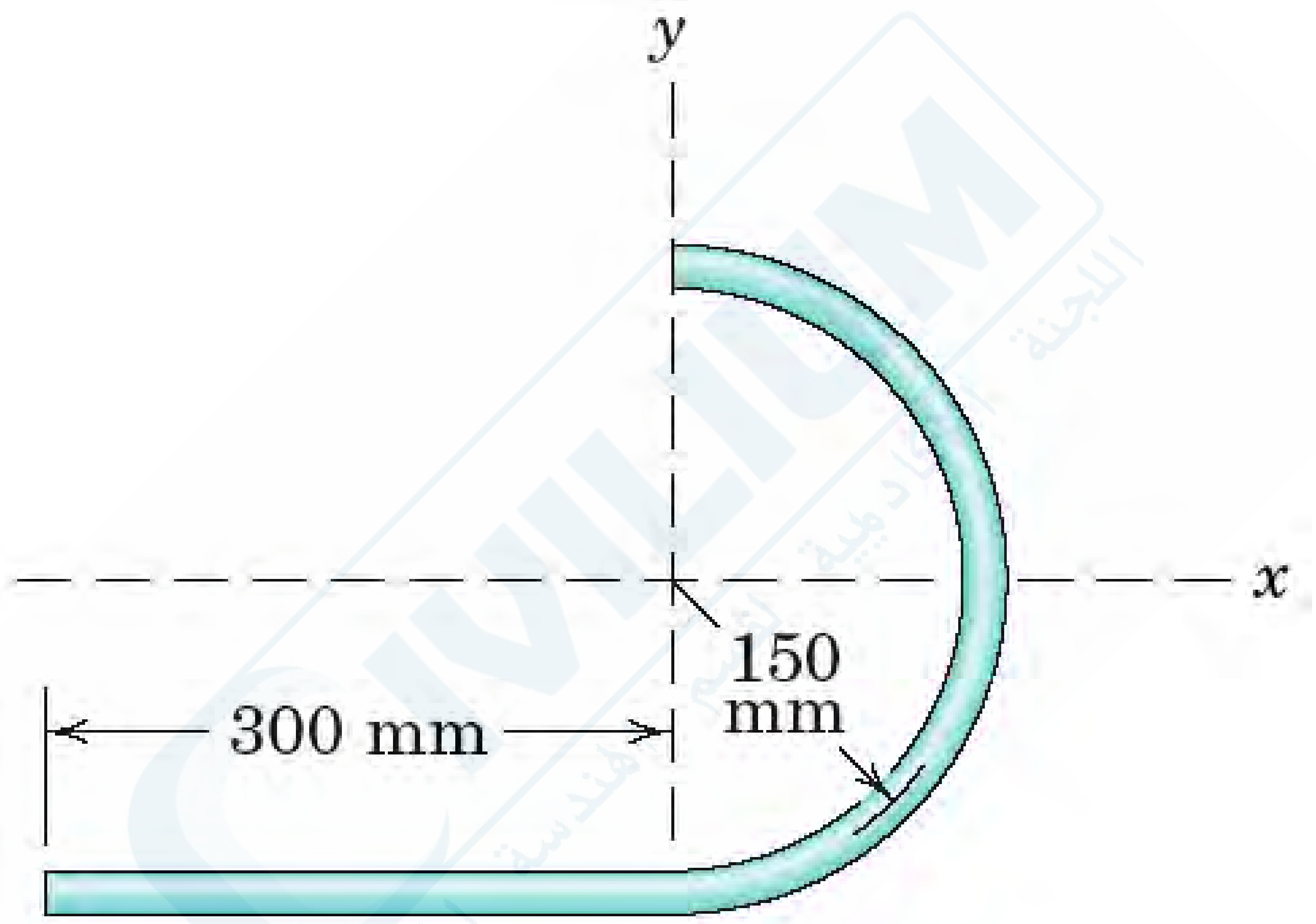
P_05_54



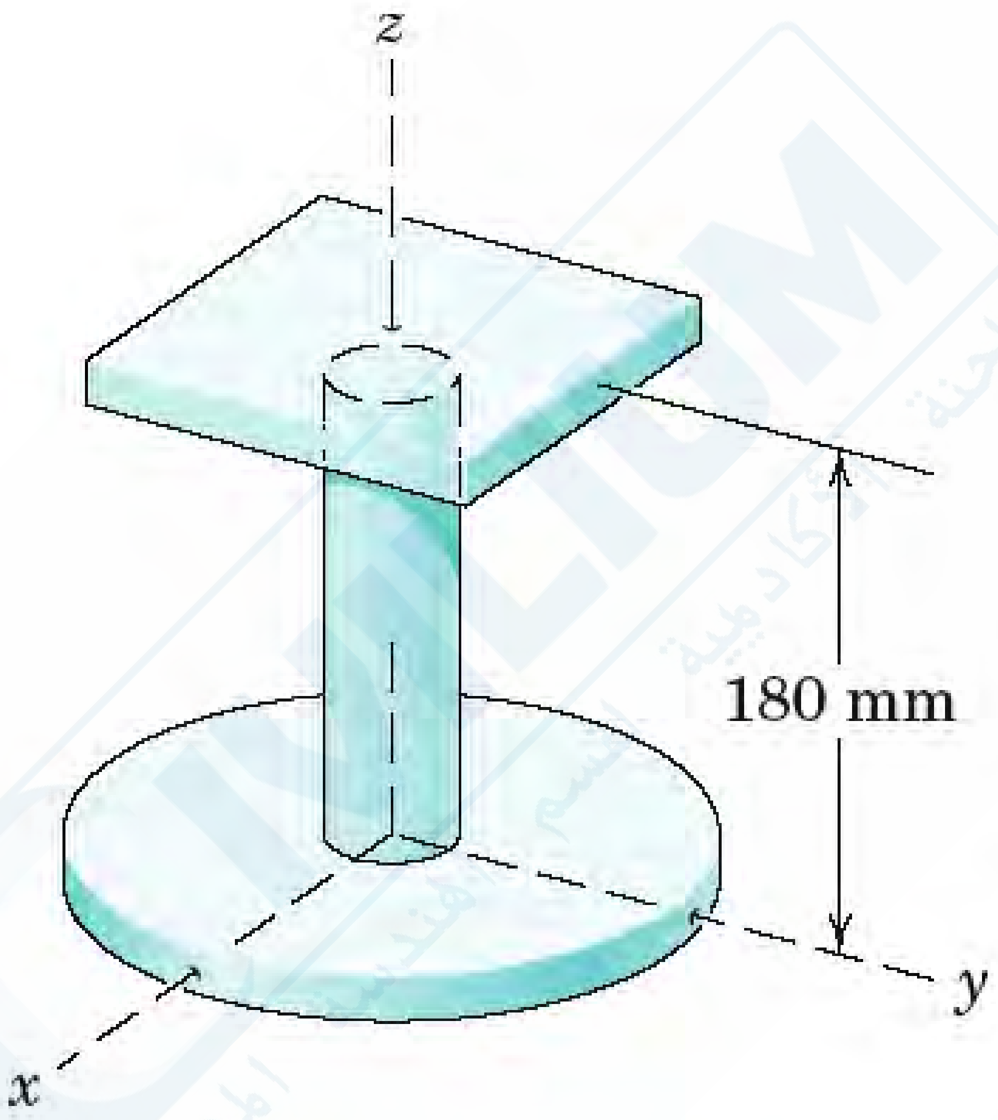
Dimensions in millimeters

Problem 5/55

P_05_55

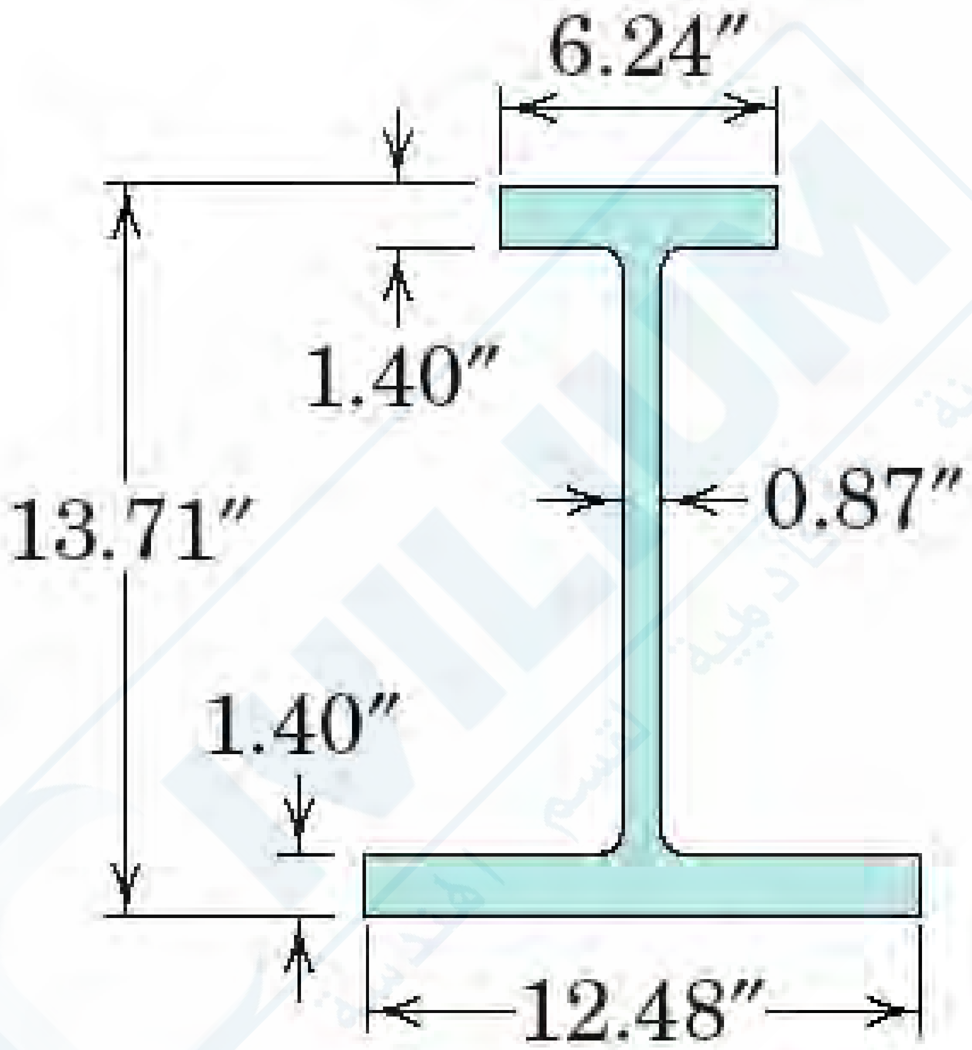


Problem 5/56



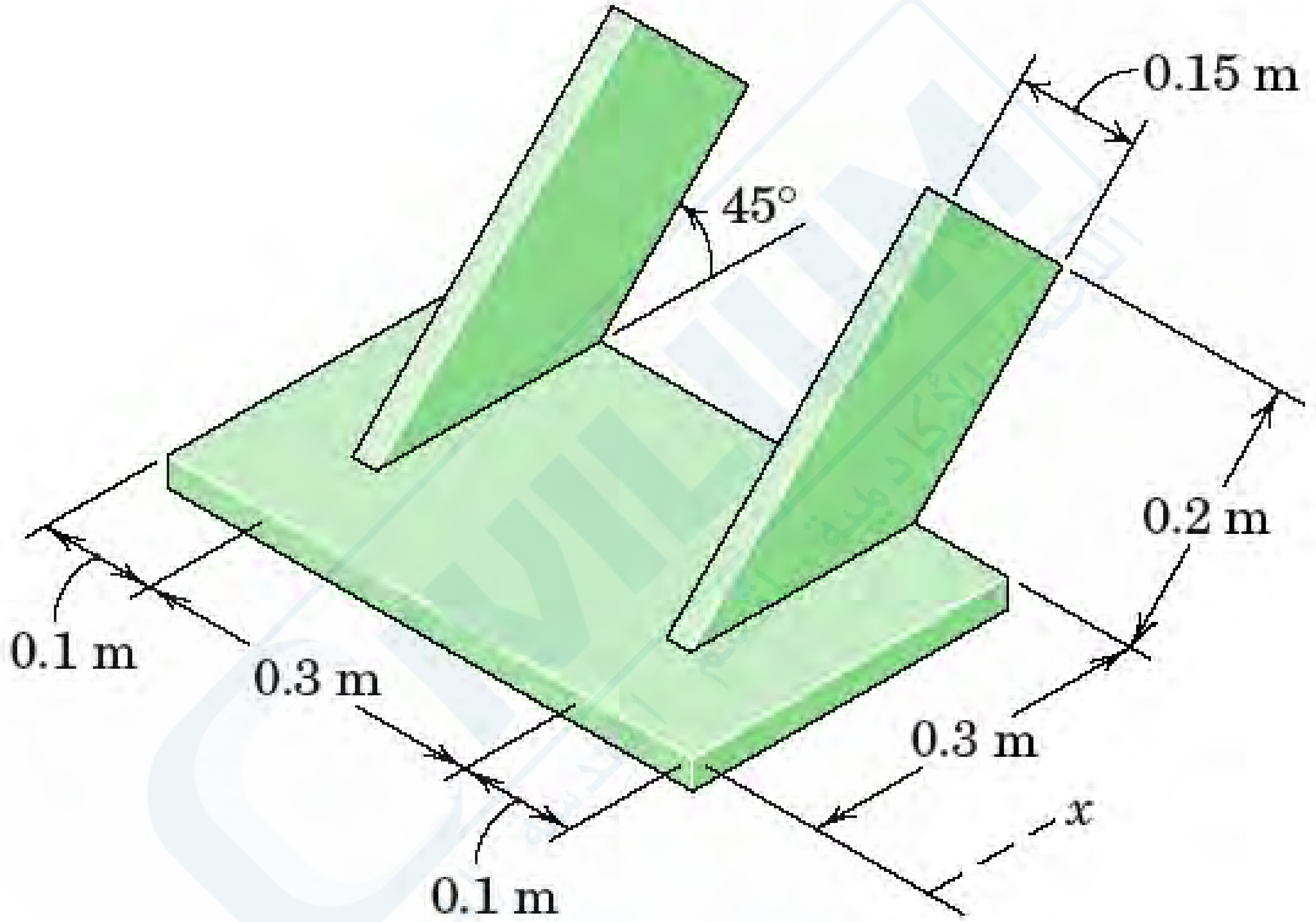
Problem 5/57

P_05_57



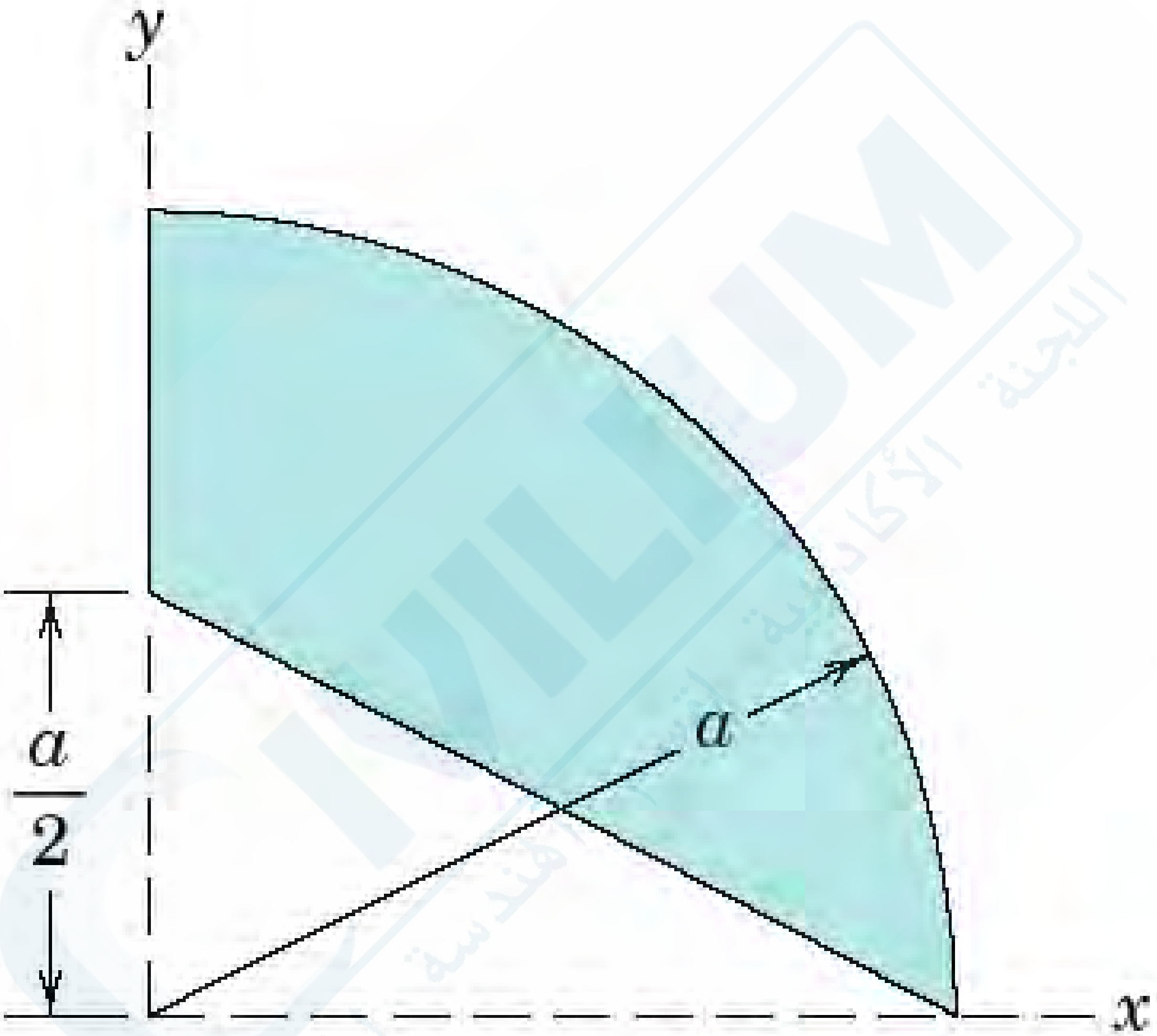
Problem 5/58

P_05_58



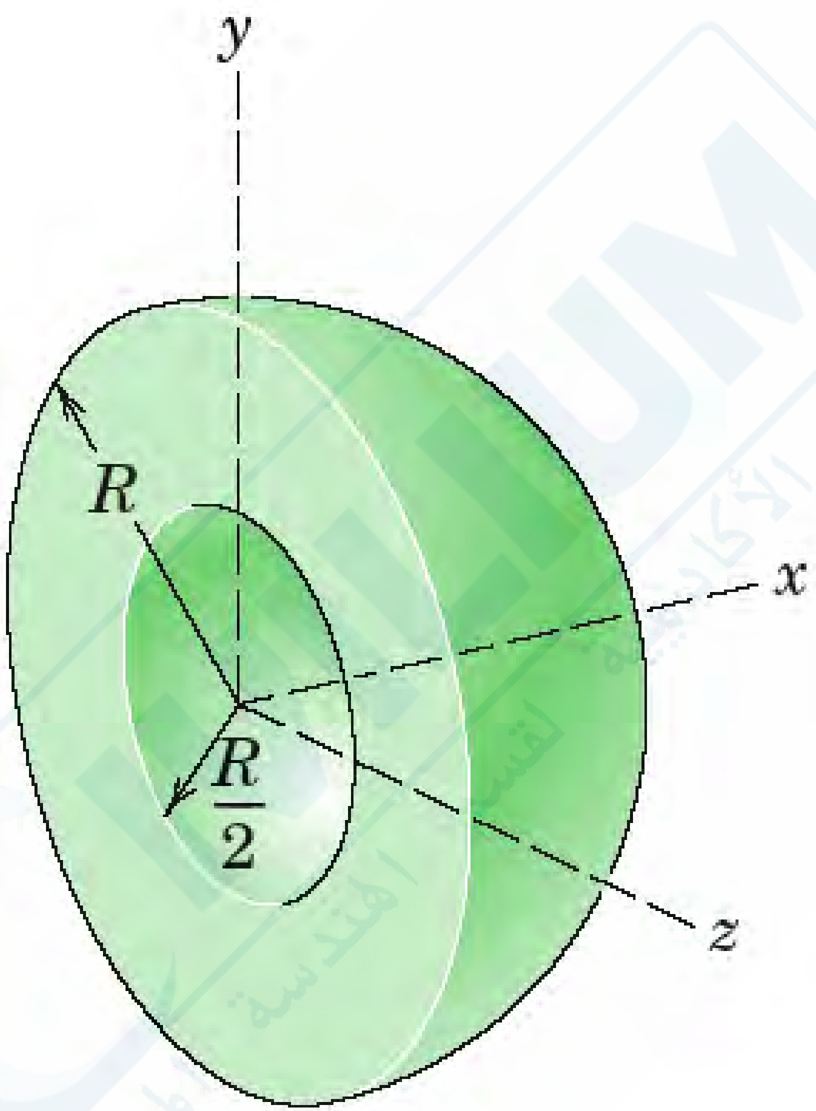
Problem 5/59

P_05_59



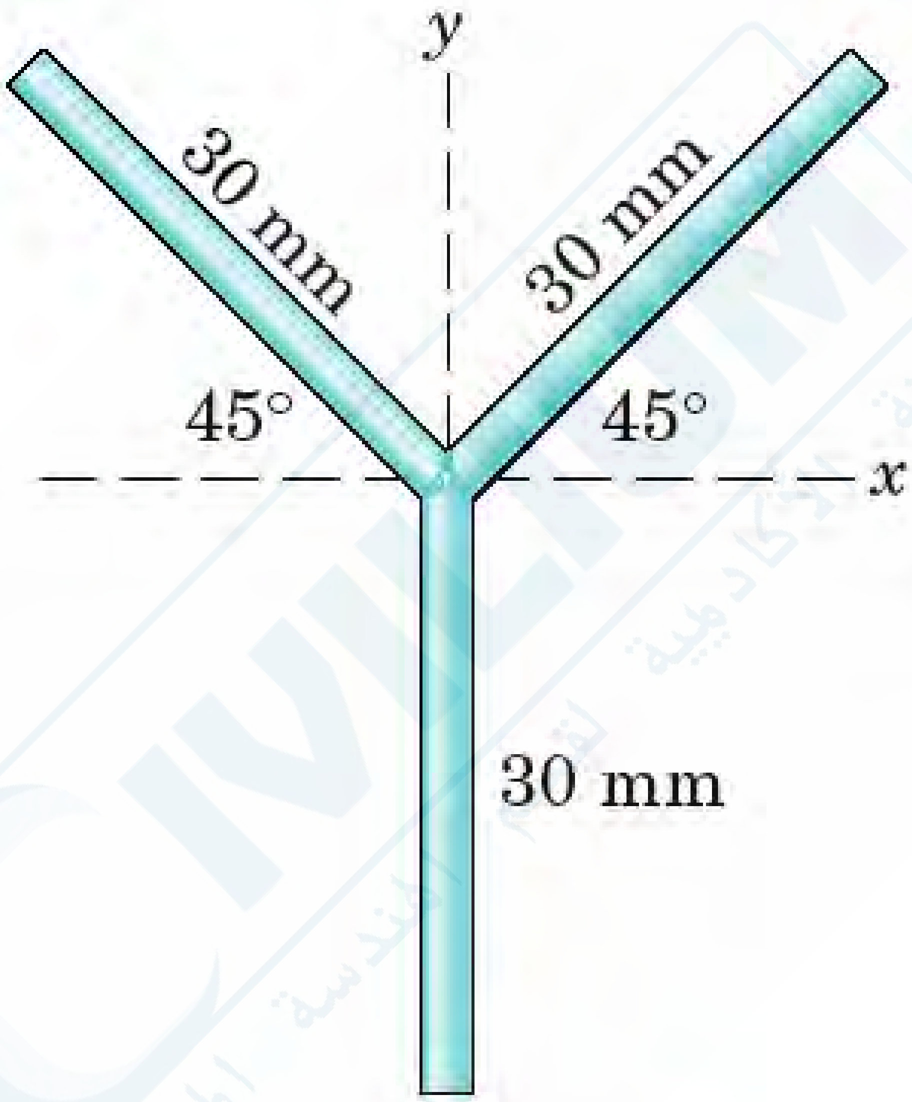
Problem 5/60

P_05_60



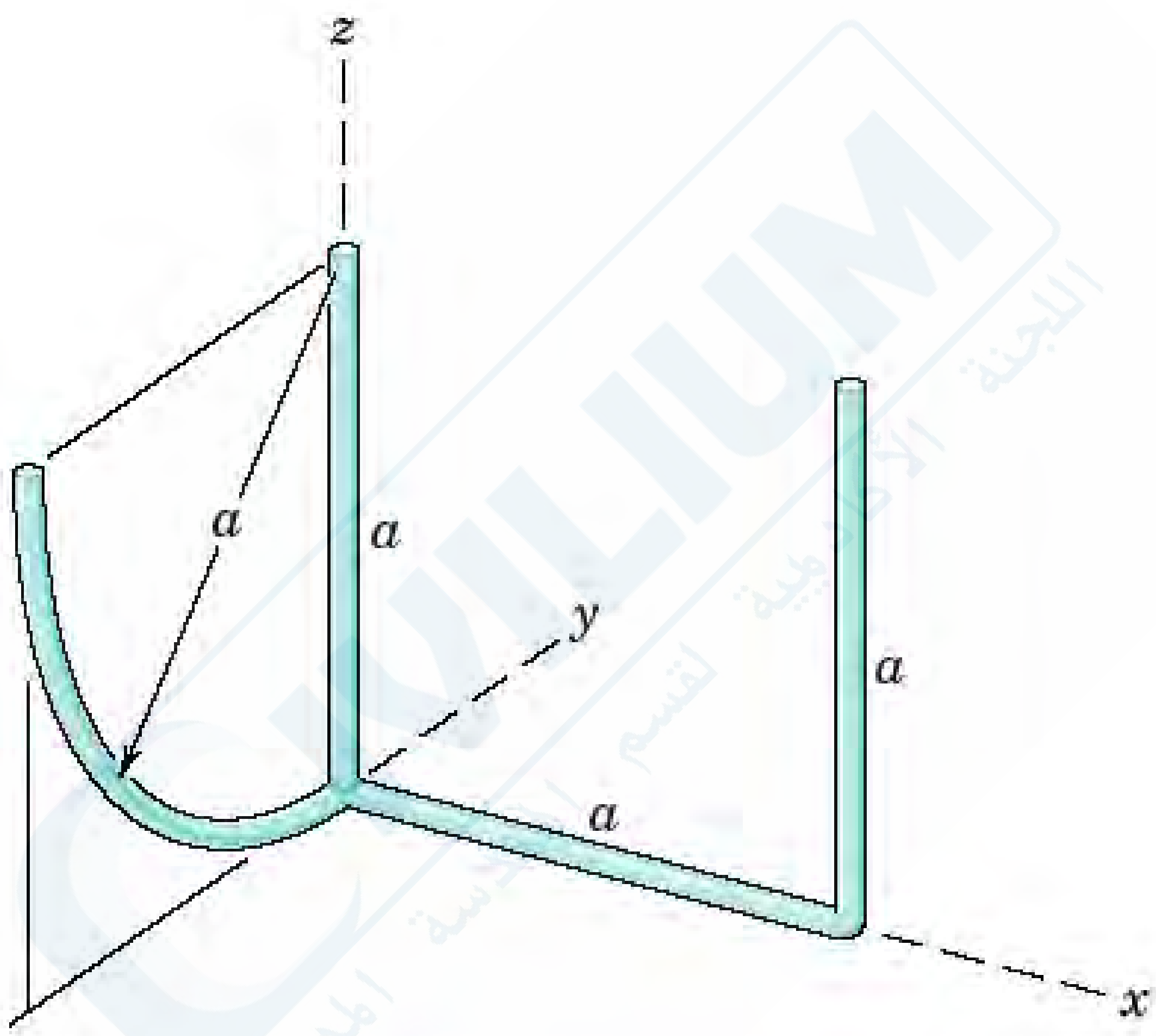
Problem 5/61

P_05_61



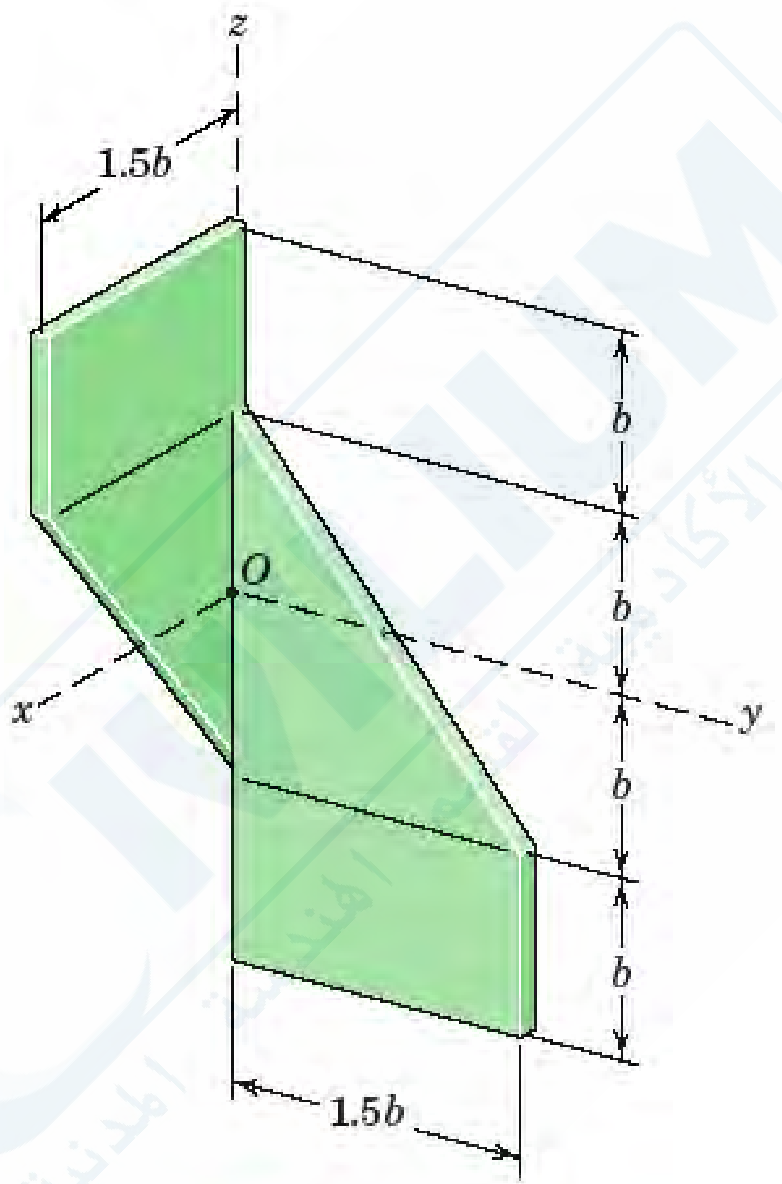
Problem 5/62

P_05_62



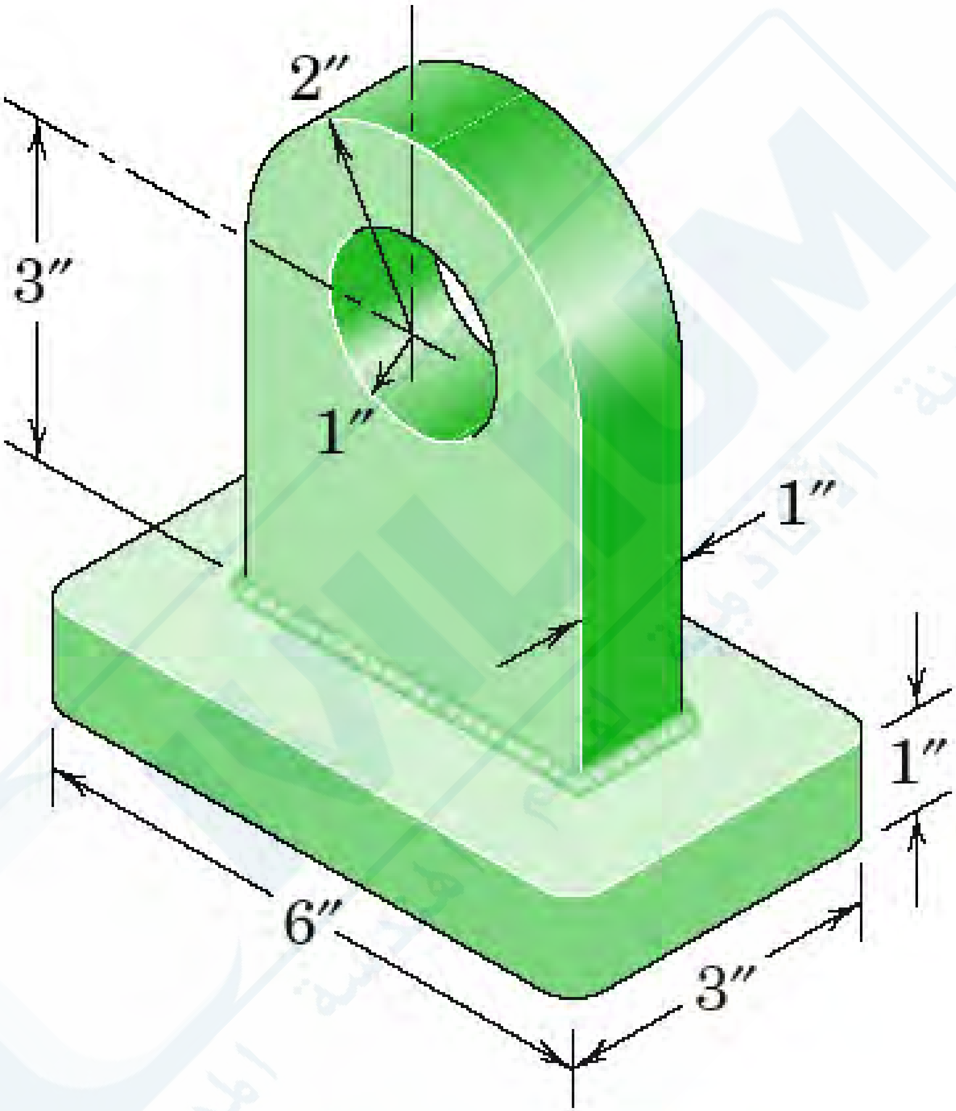
Problem 5/63

P 05 63



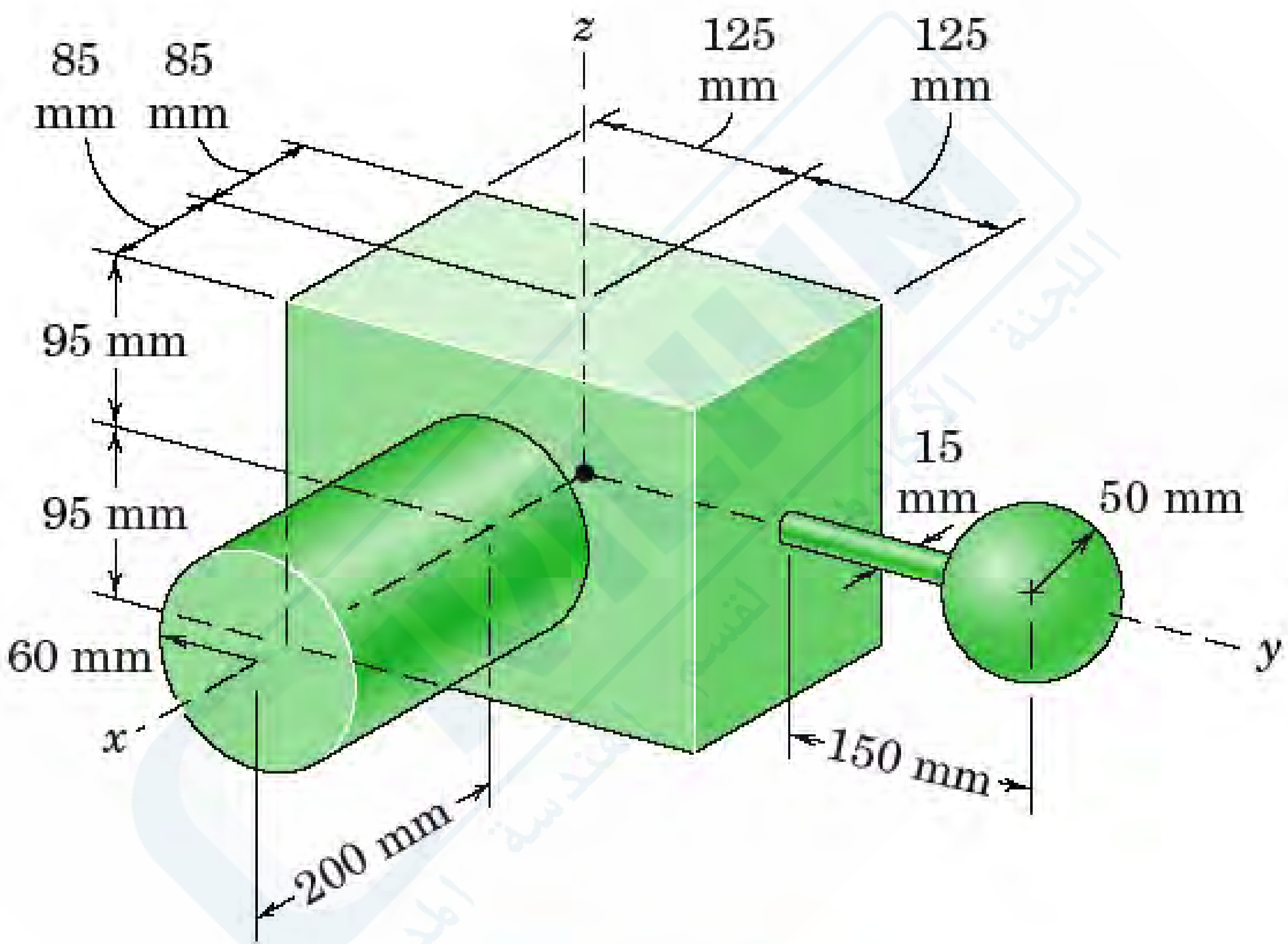
Problem 5/64

P_05_64



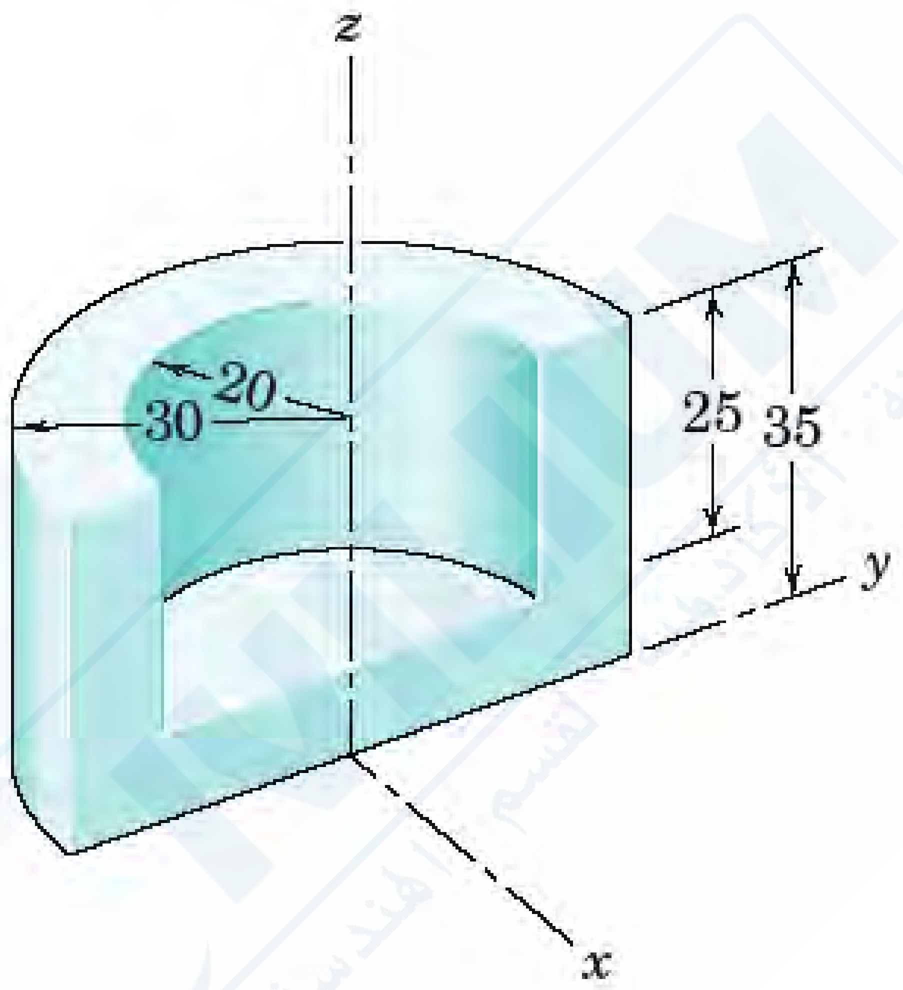
Problem 5/66

P_05_66



Problem 5/67

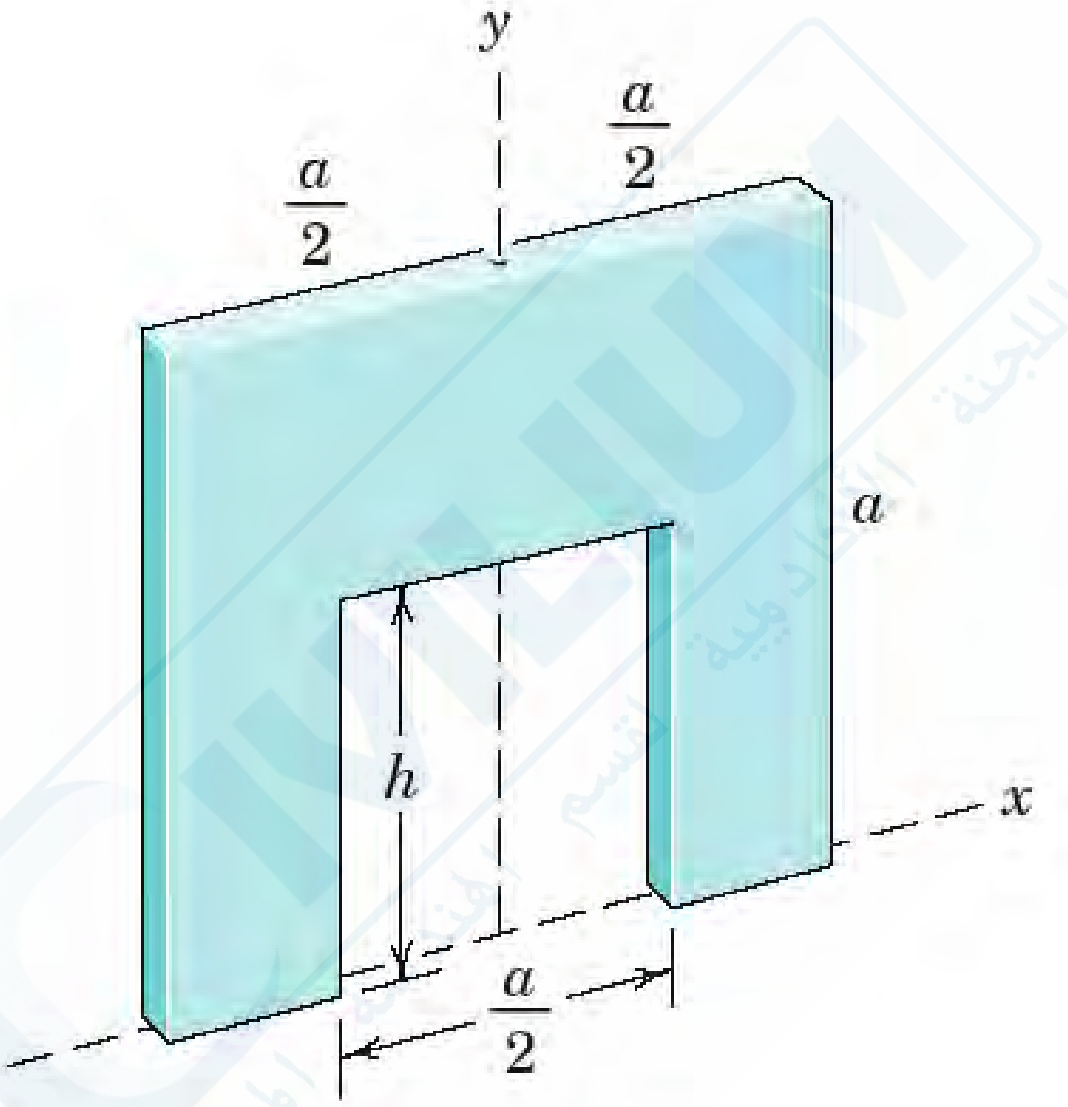
P 05 67



Dimensions in millimeters

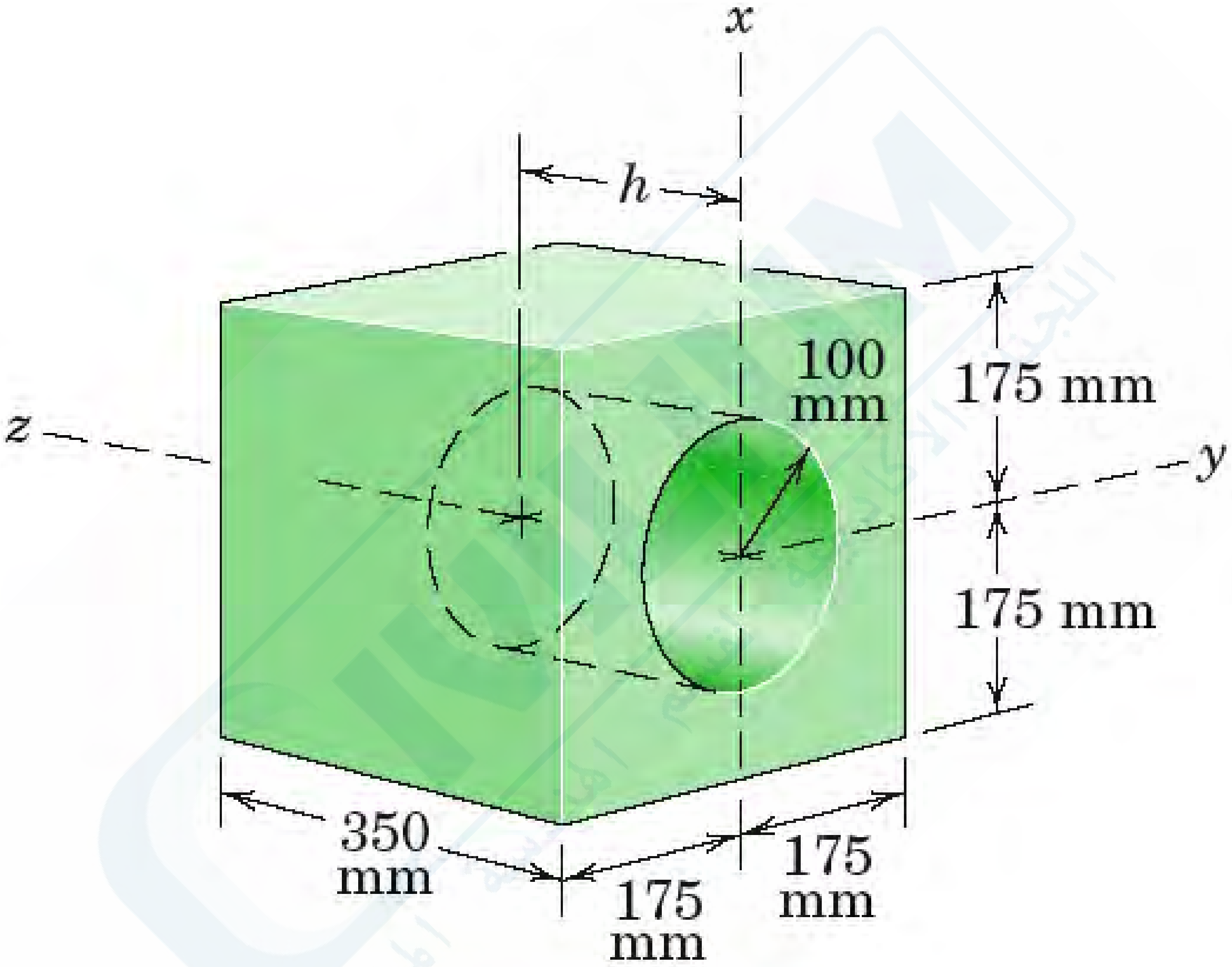
Problem 5/68

P_05_68



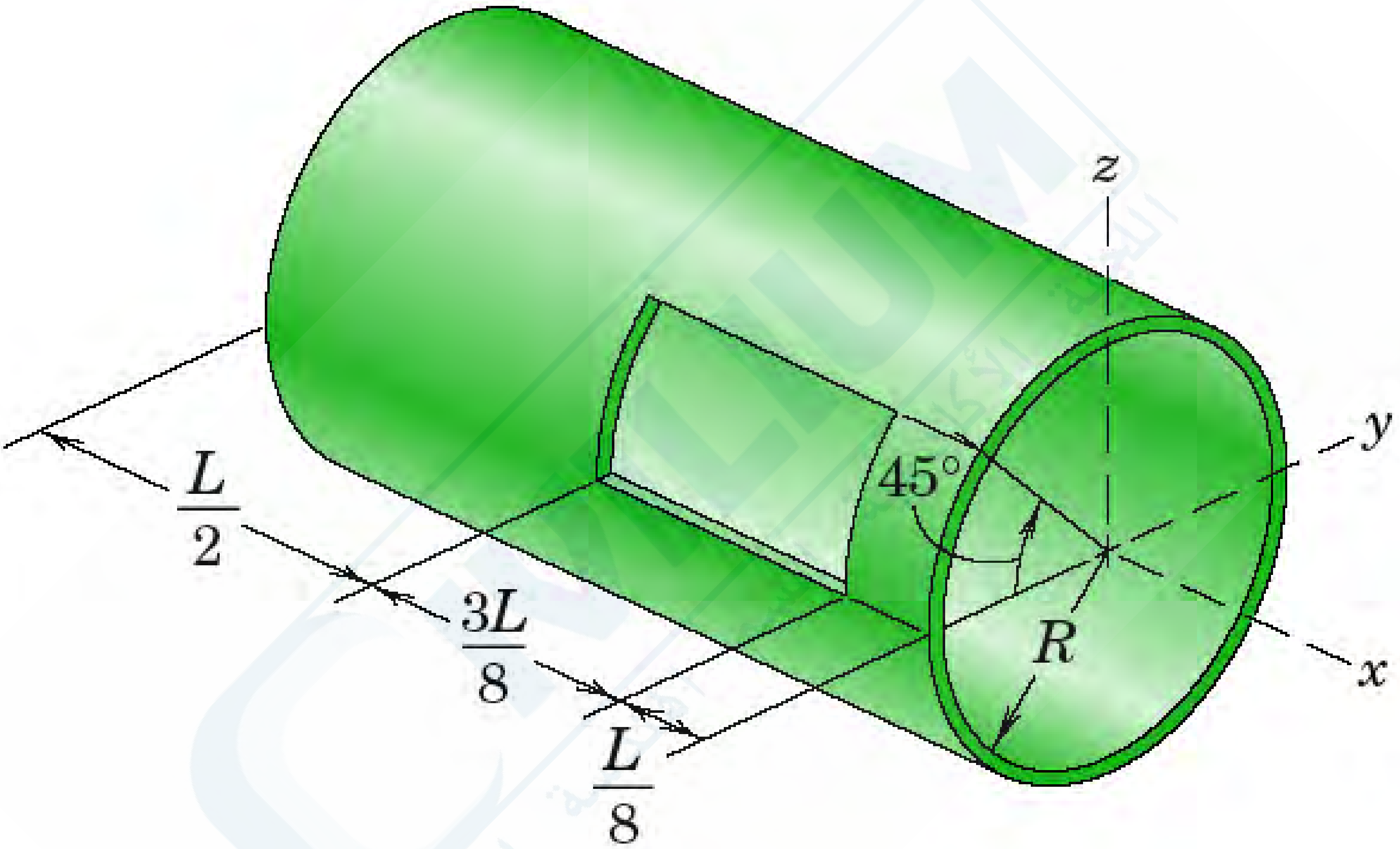
Problem 5/69

P_05_69



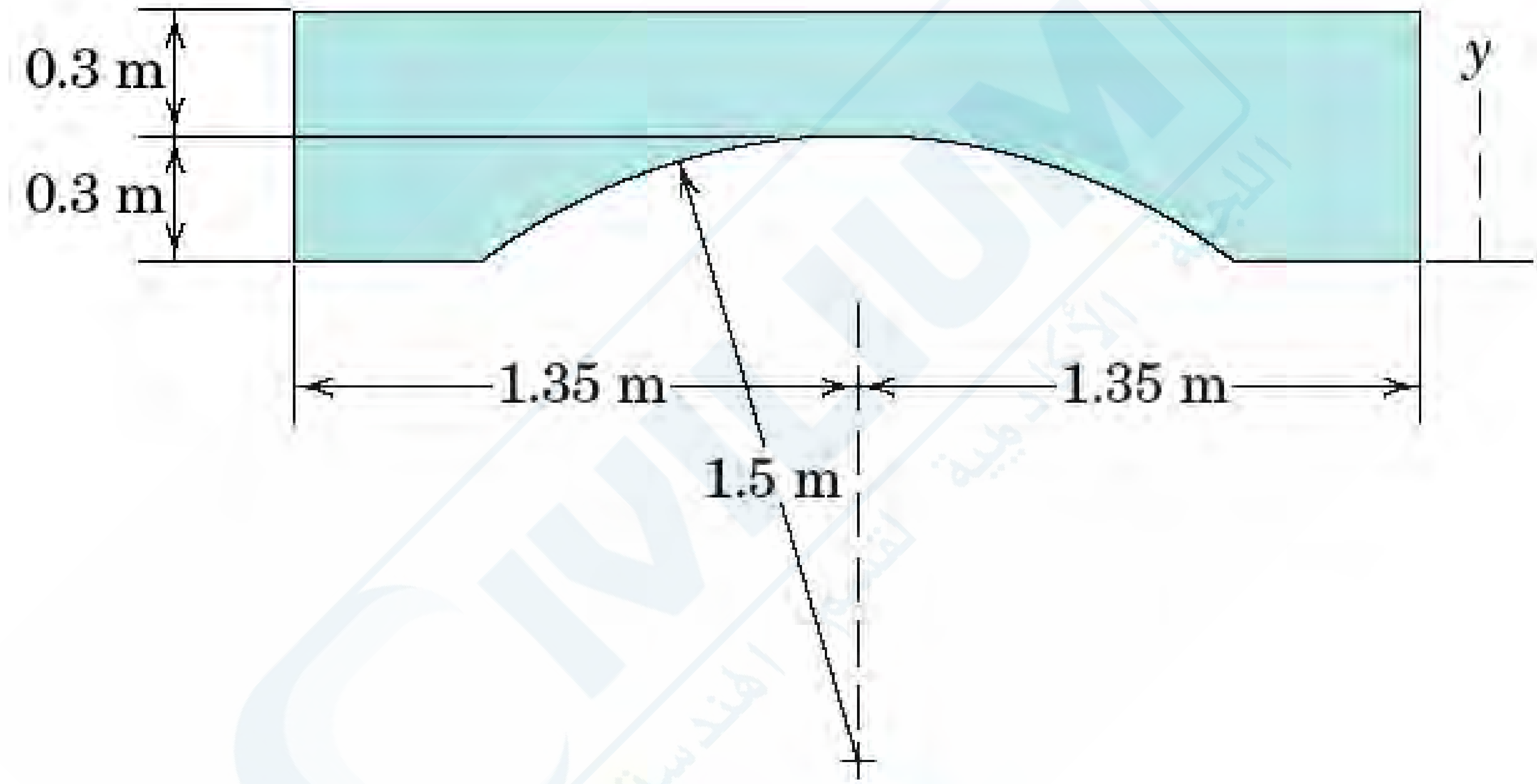
Problem 5/70

P_05_70



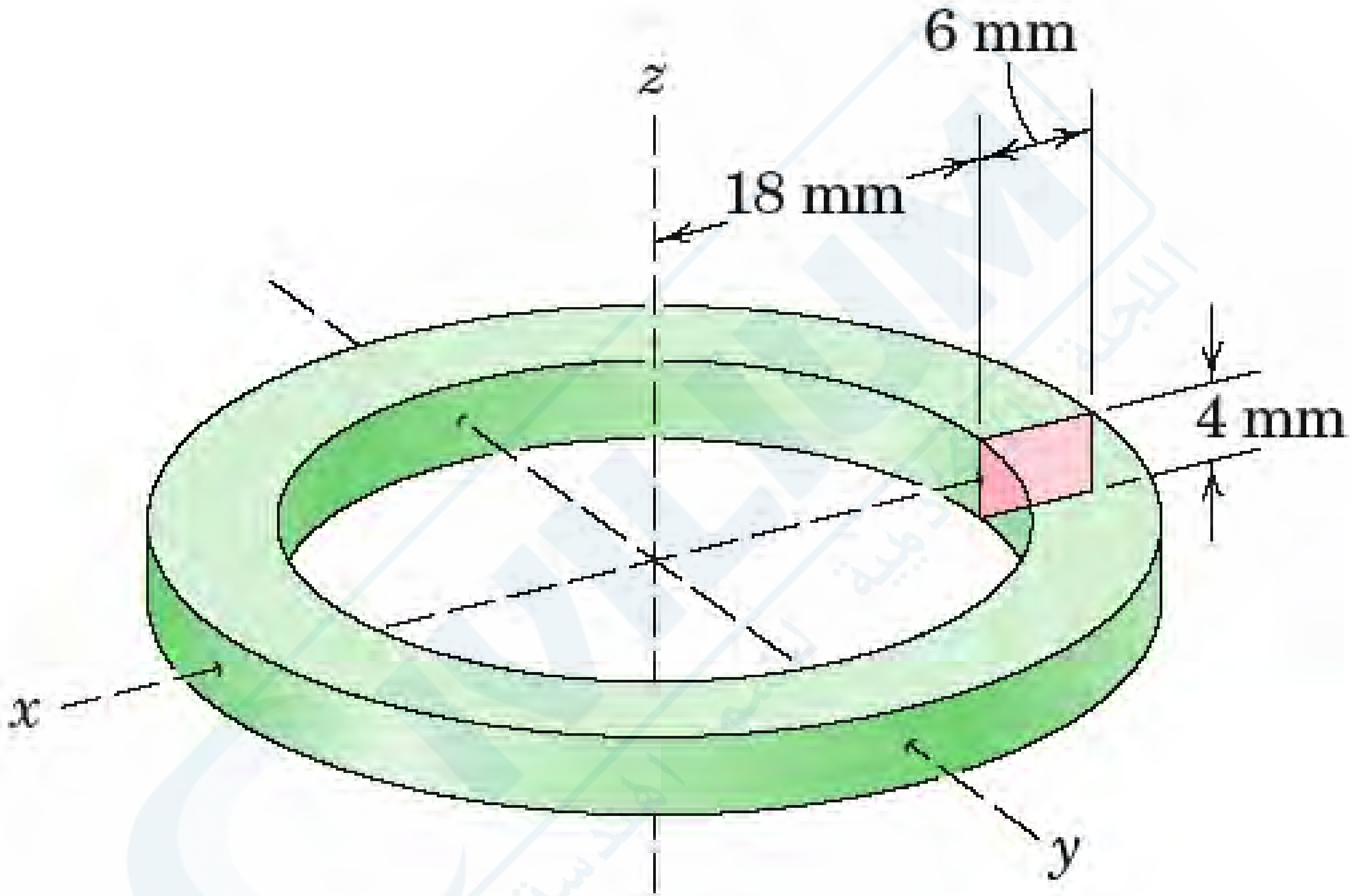
Problem 5/71

P_05_71

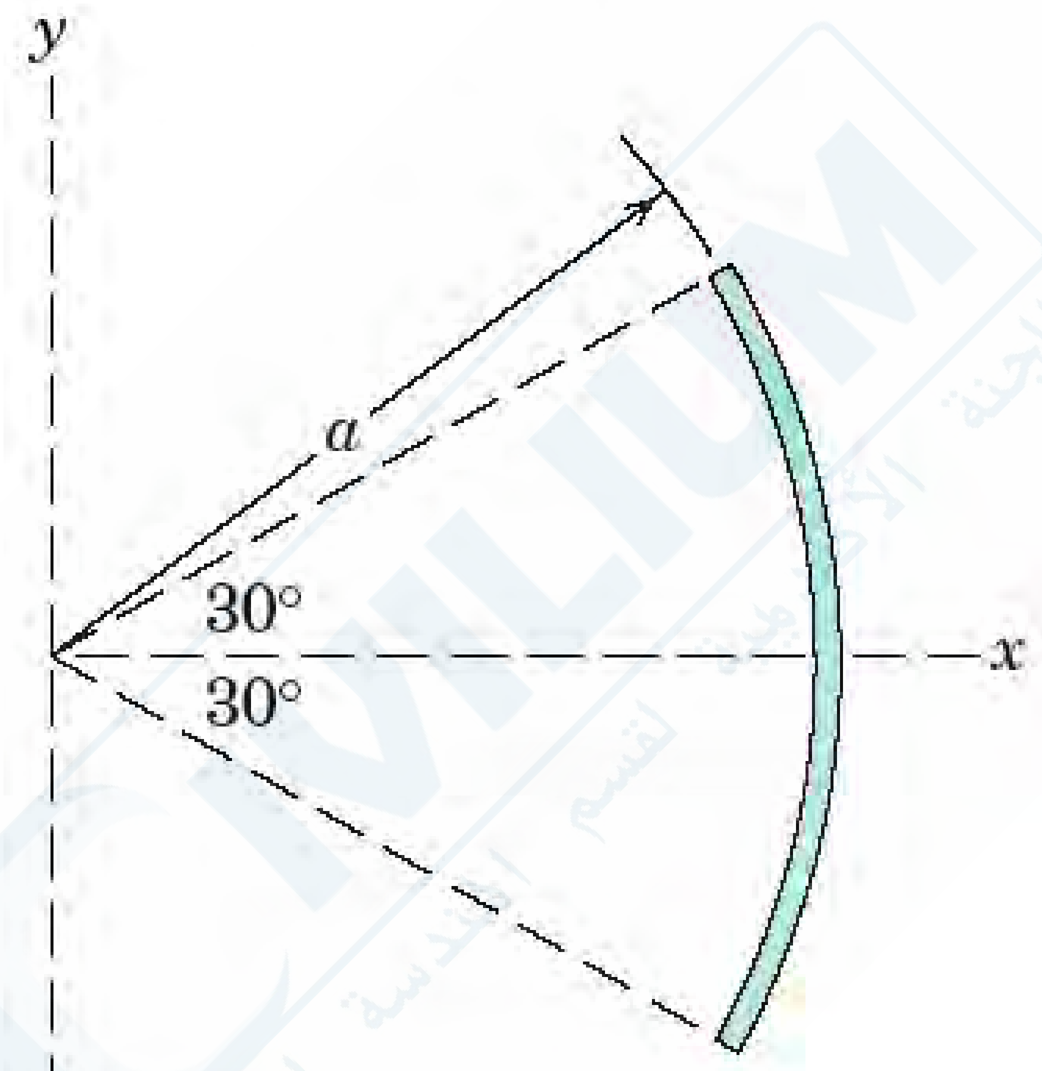


Problem 5/72

P_05_72

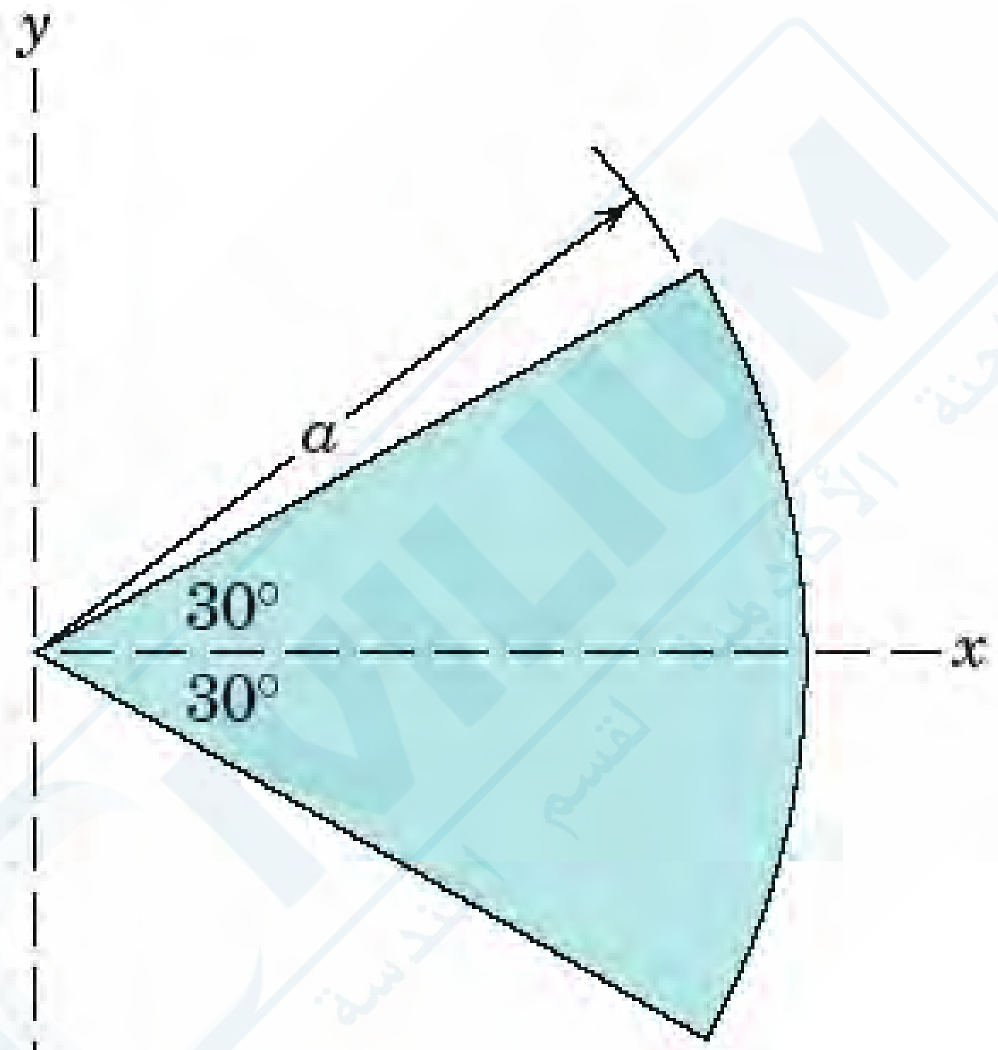


Problem 5/73



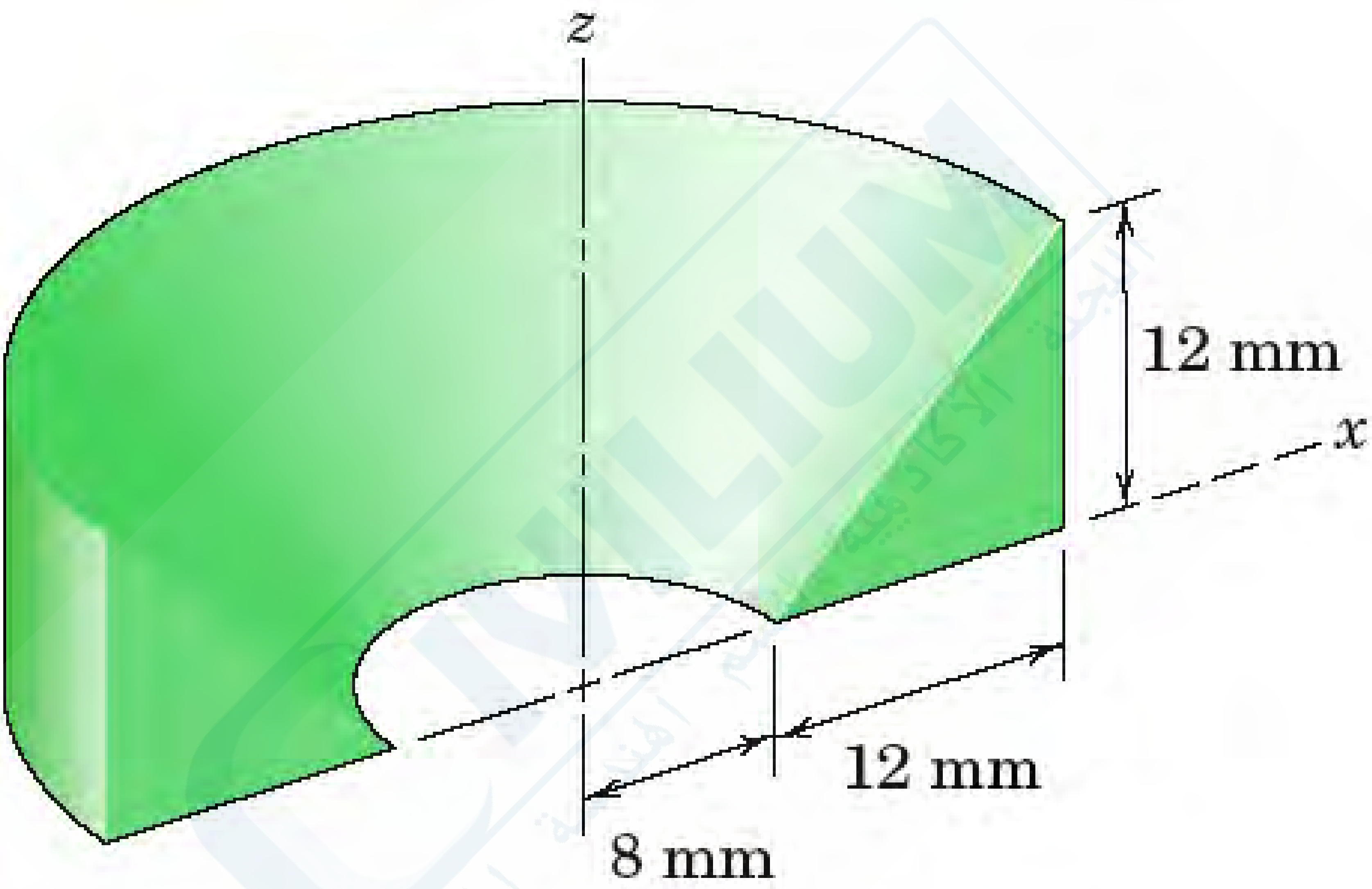
Problem 5/74

P_05_74



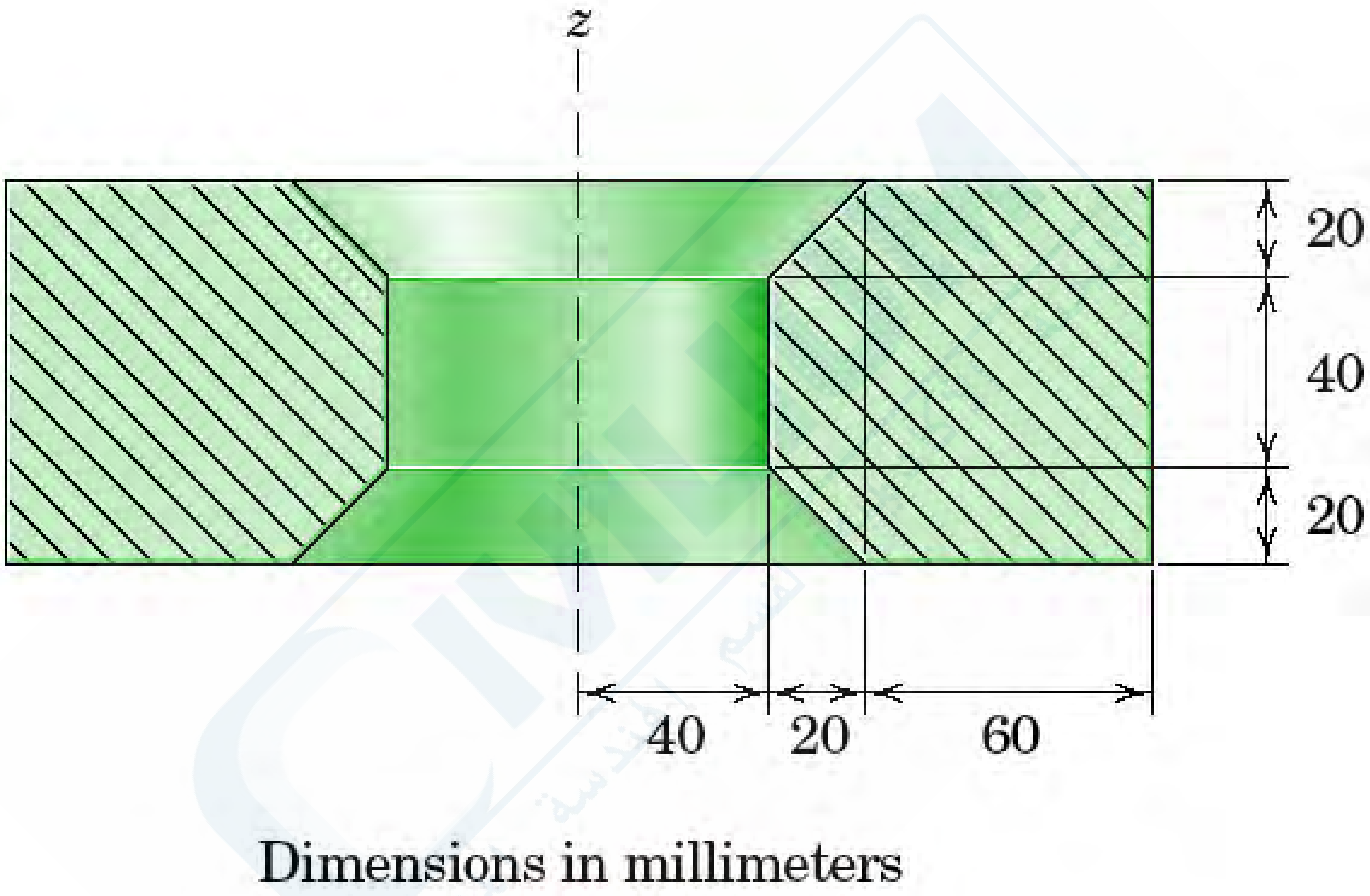
Problem 5/75

P_05_75



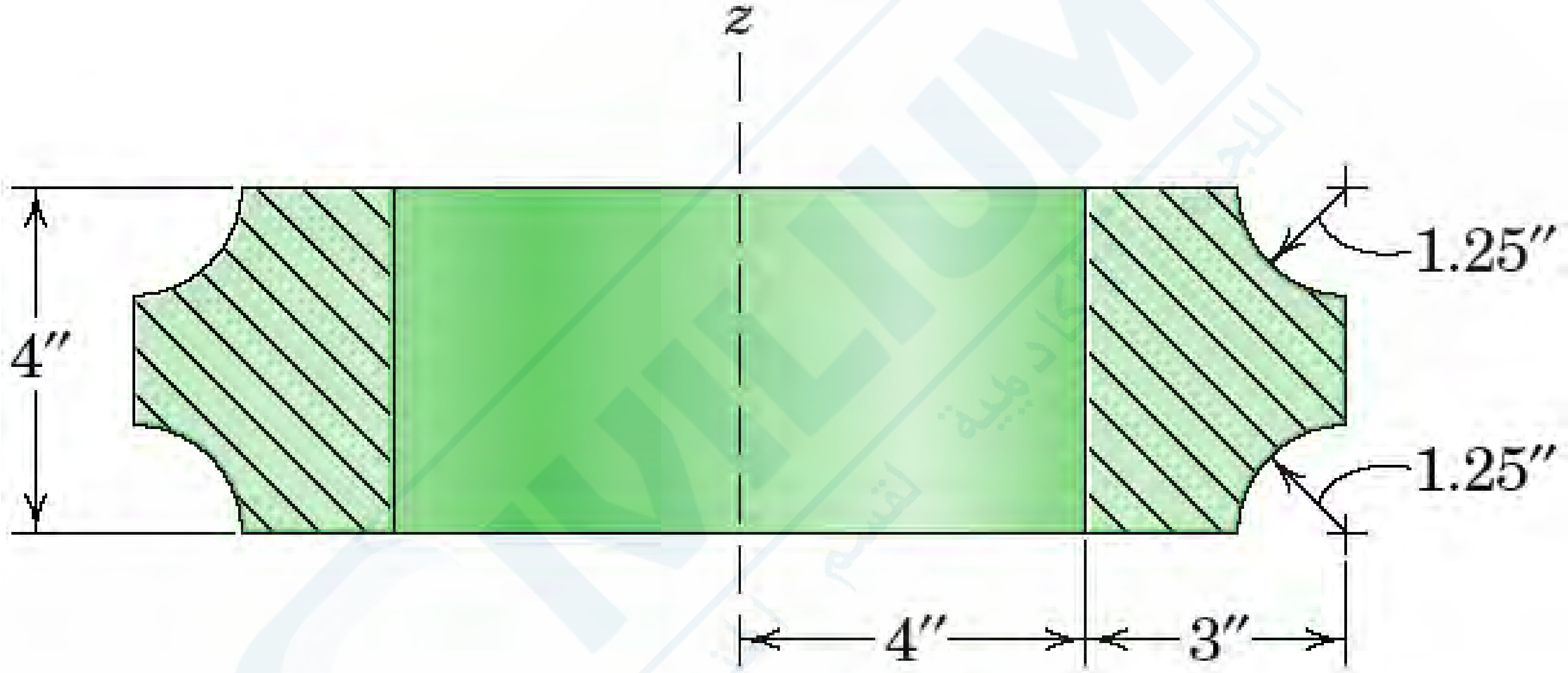
Problem 5/76

P_05_76



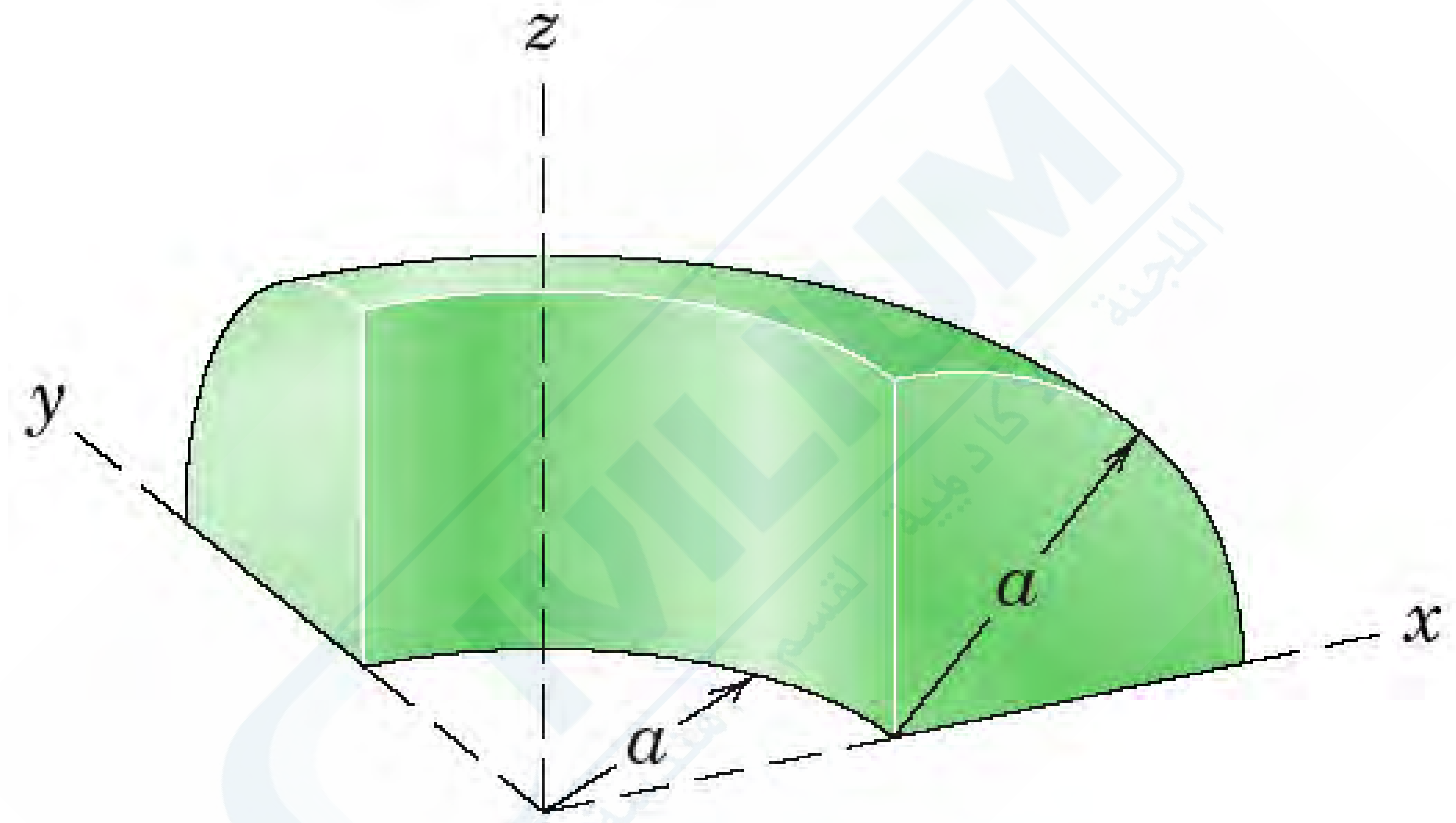
Problem 5/77

P_05_77



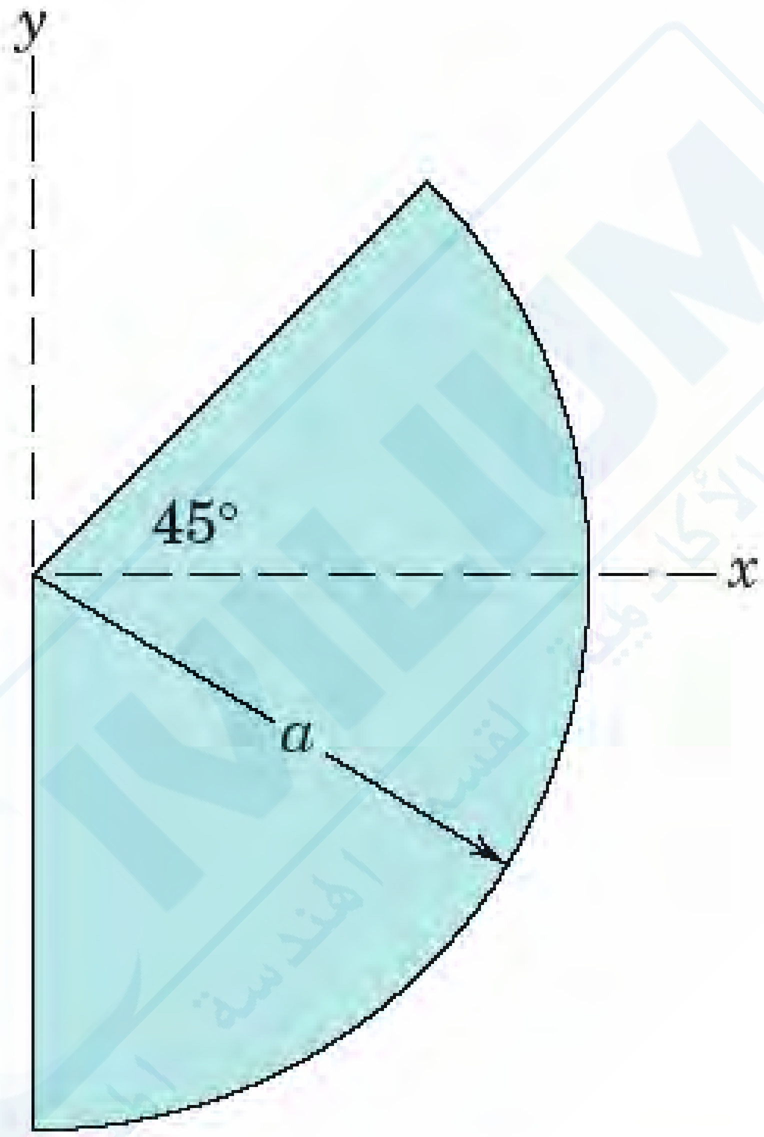
Problem 5/78

P_05_78



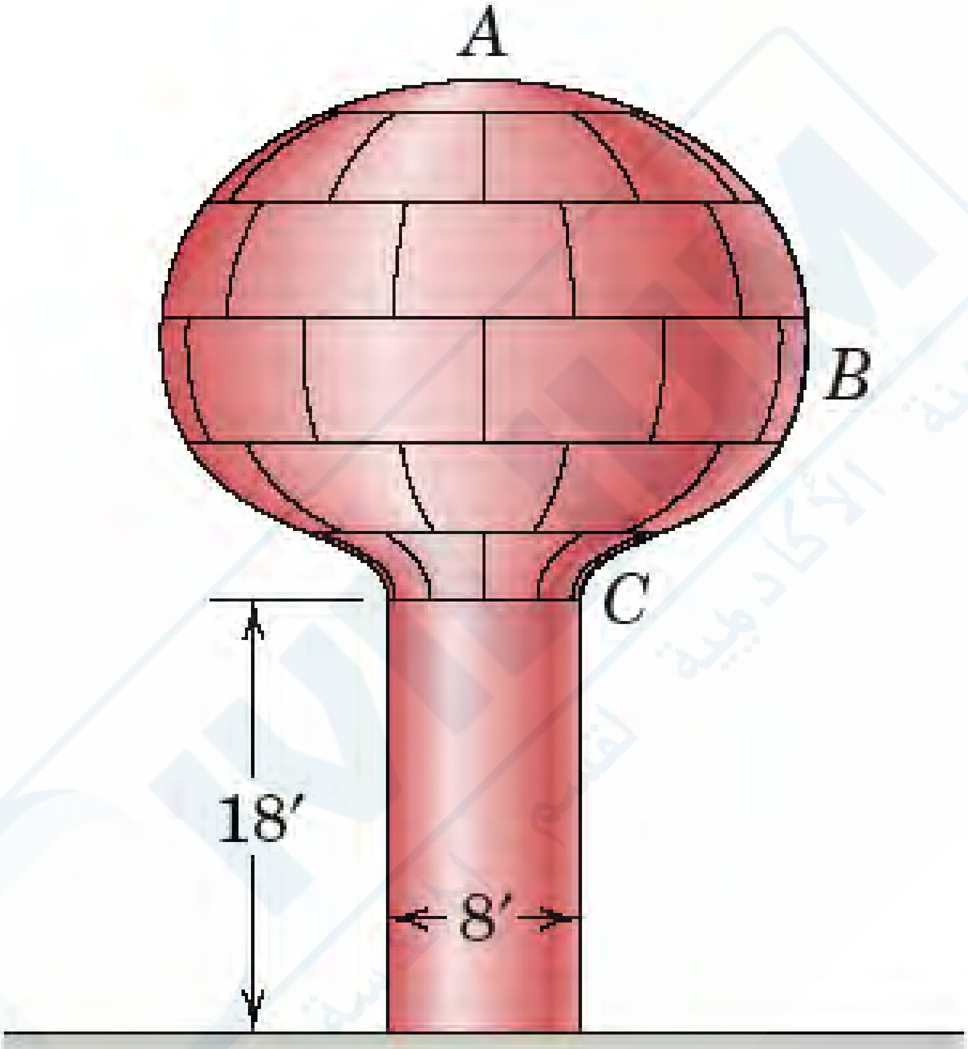
Problem 5/79

P_05_79



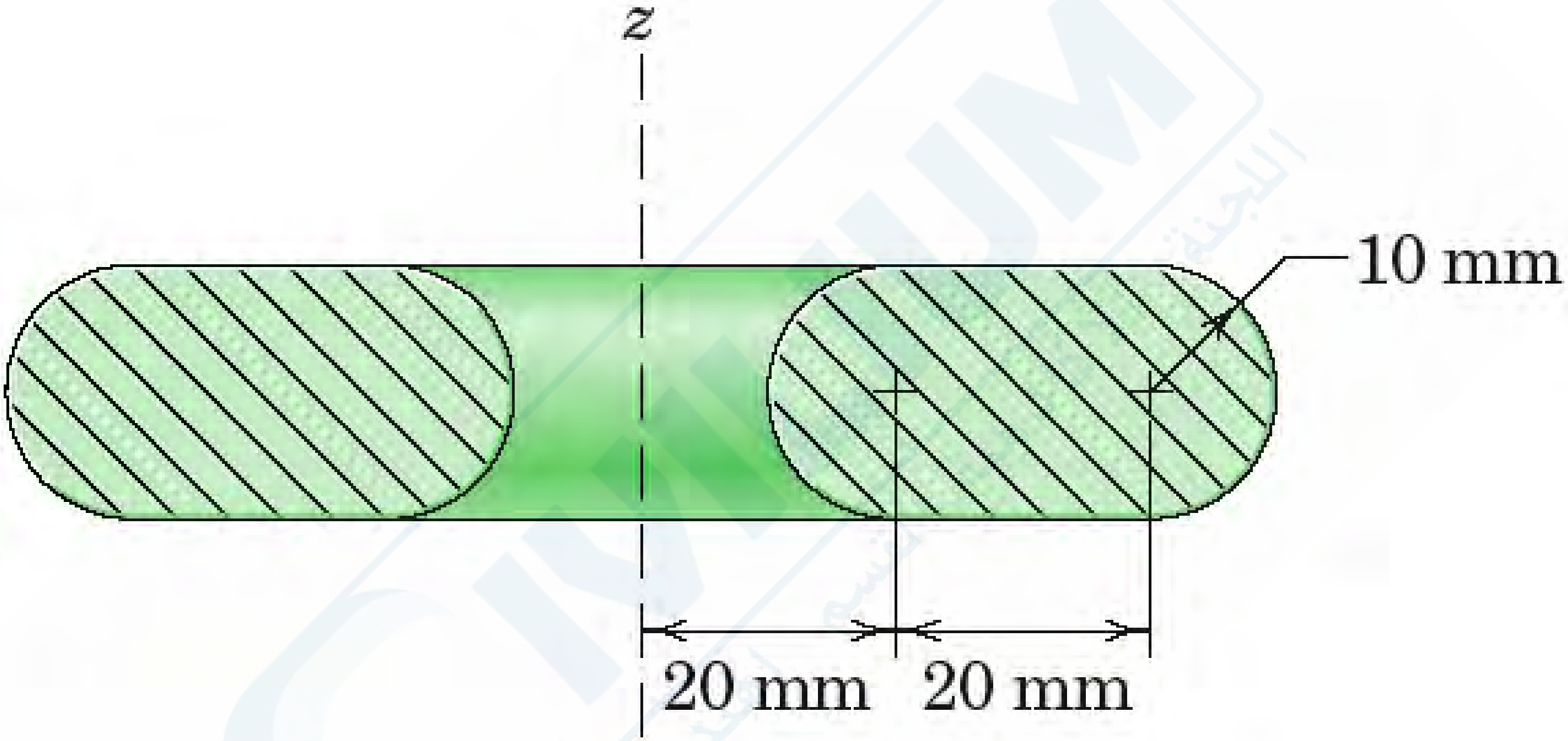
Problem 5/80

P_05_80



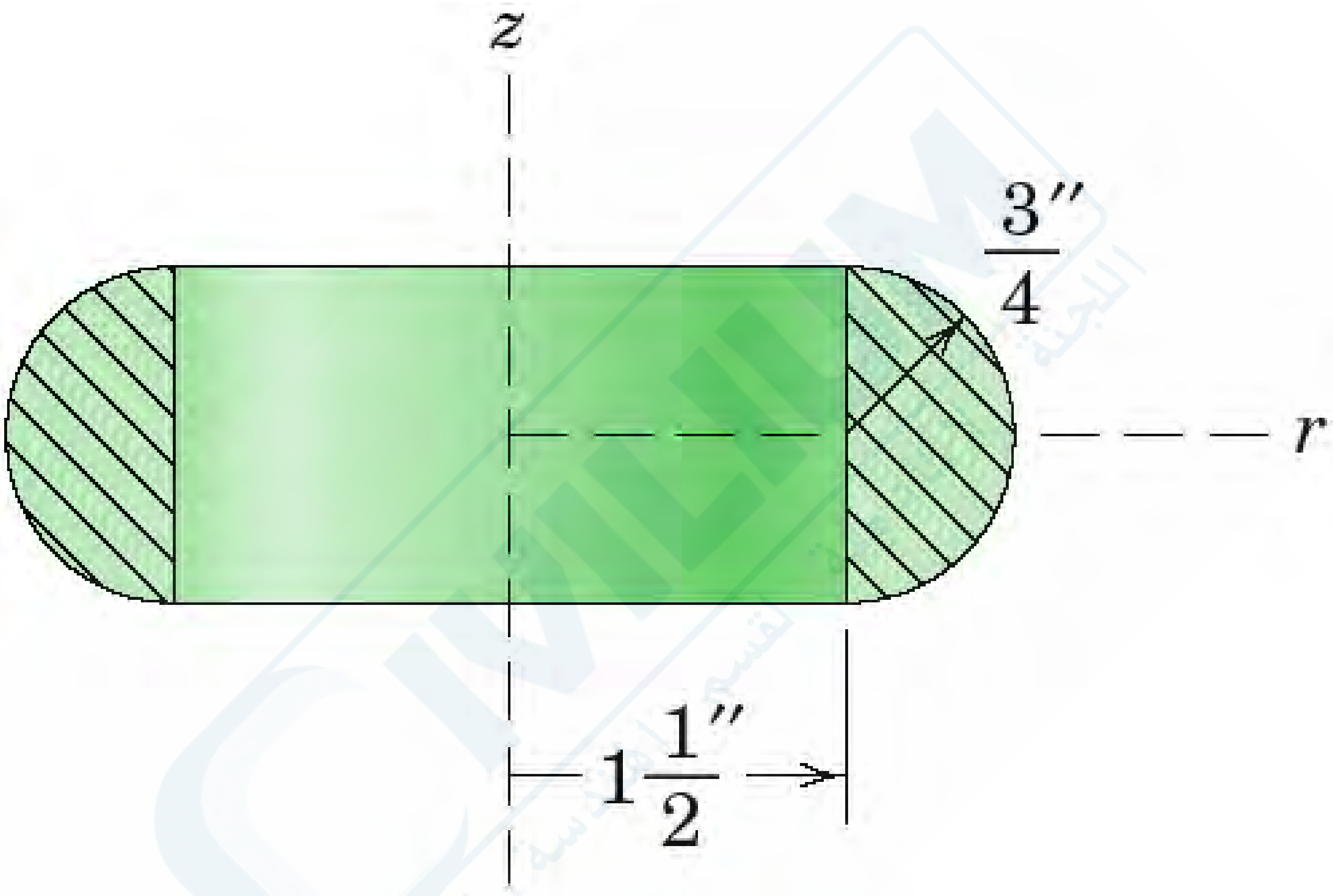
Problem 5/81

P_05_81



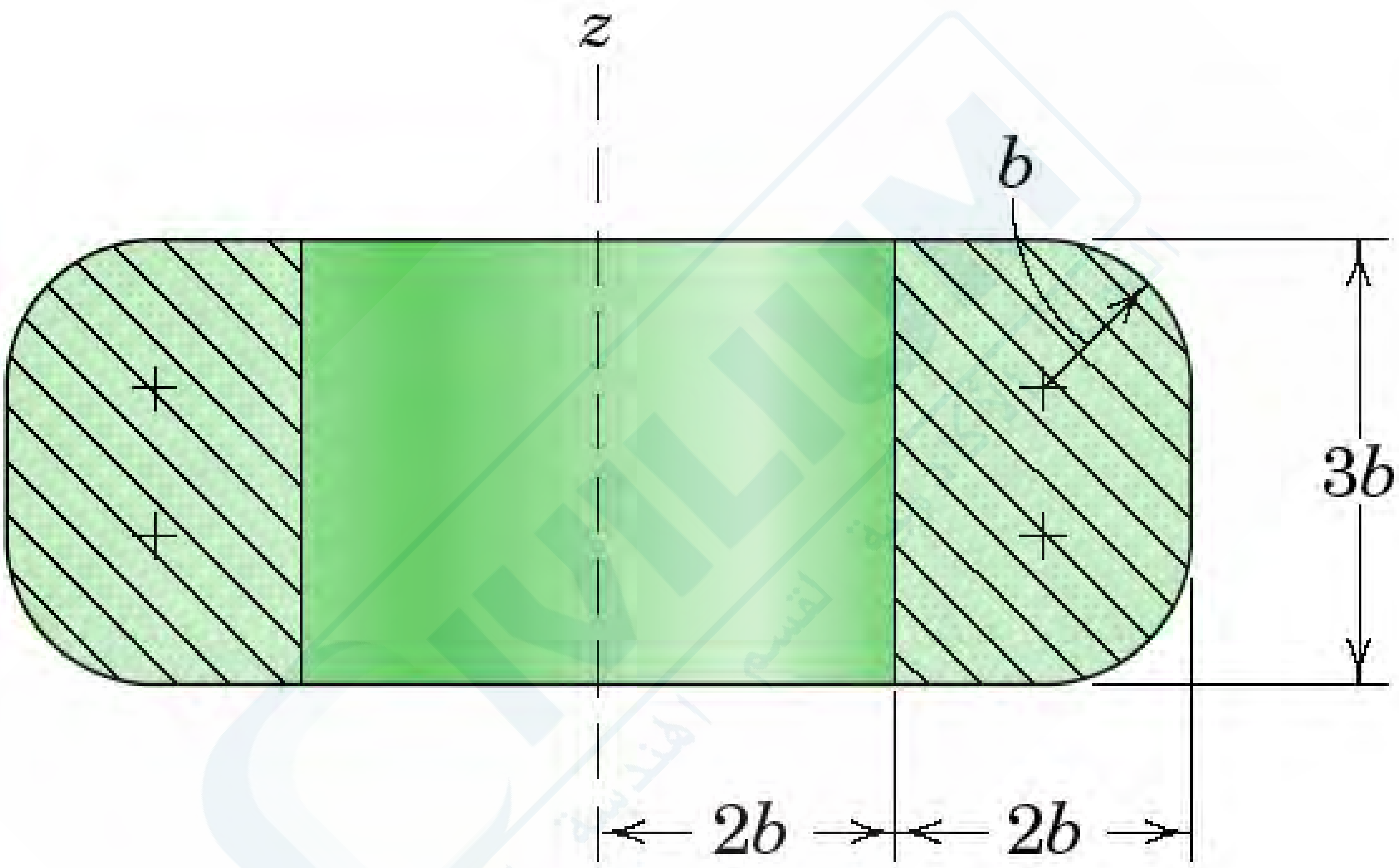
Problem 5/82

P_05_82



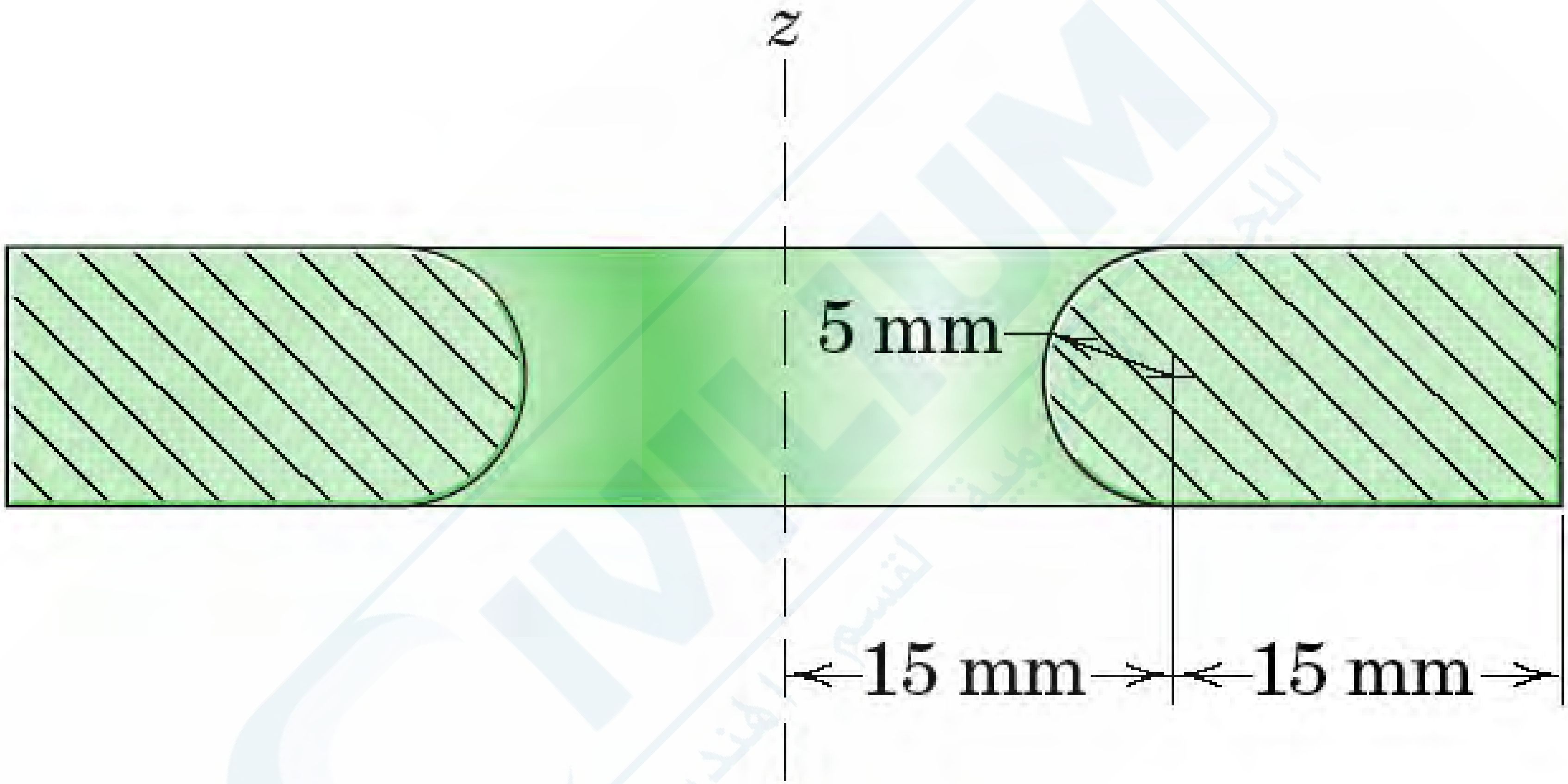
Problem 5/83

P_05_83



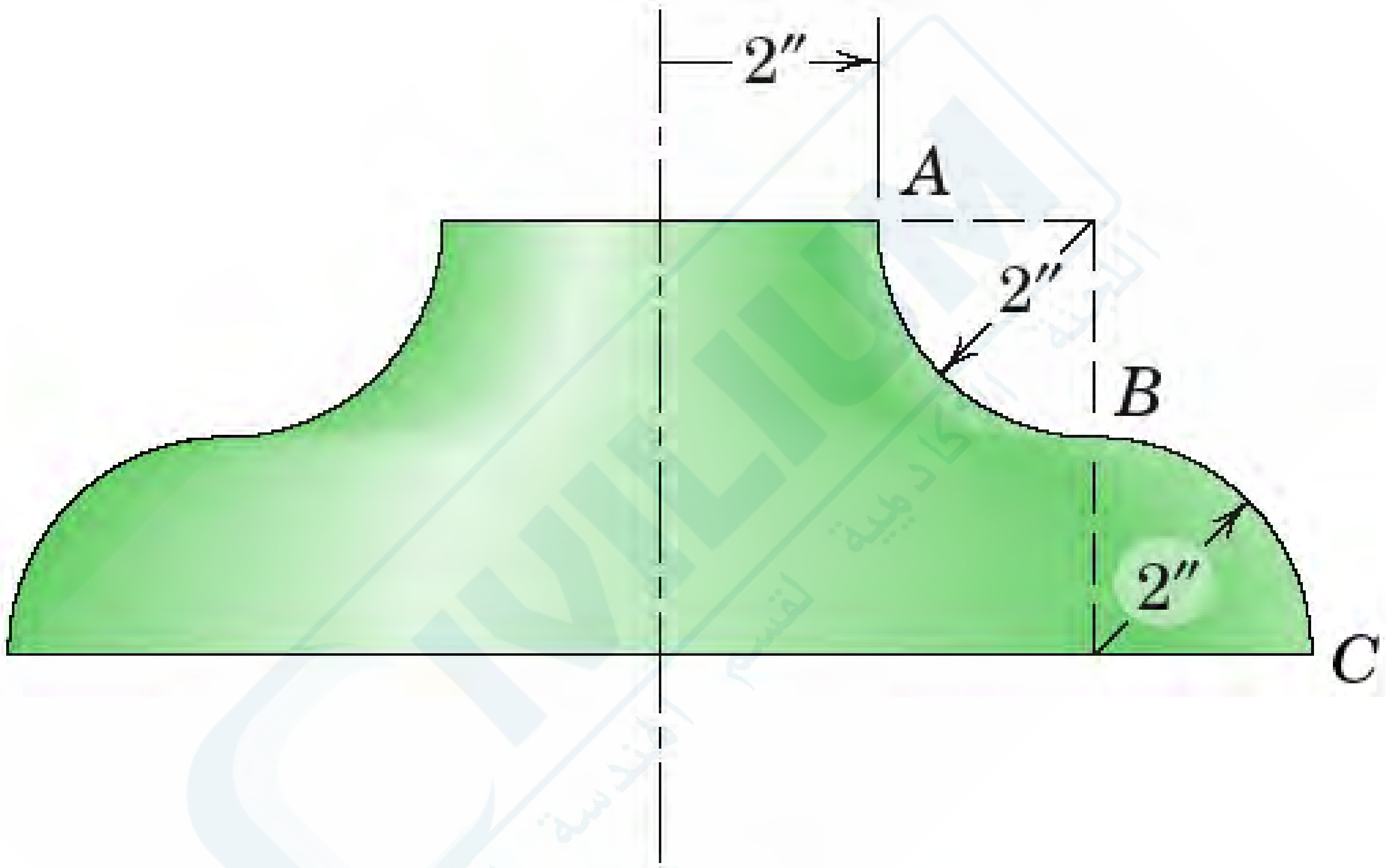
Problem 5/84

P_05_84



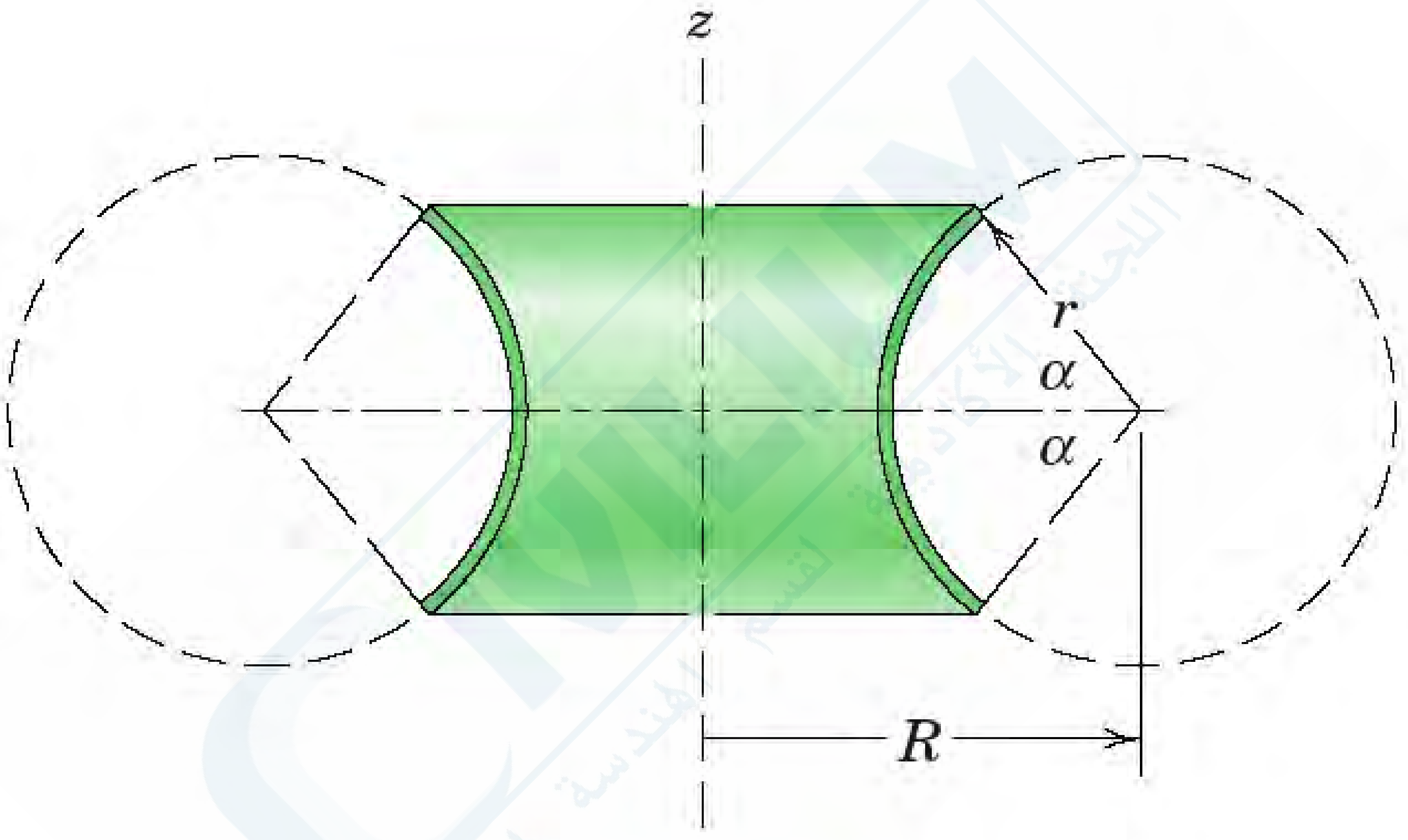
Problem 5/85

P_05_85



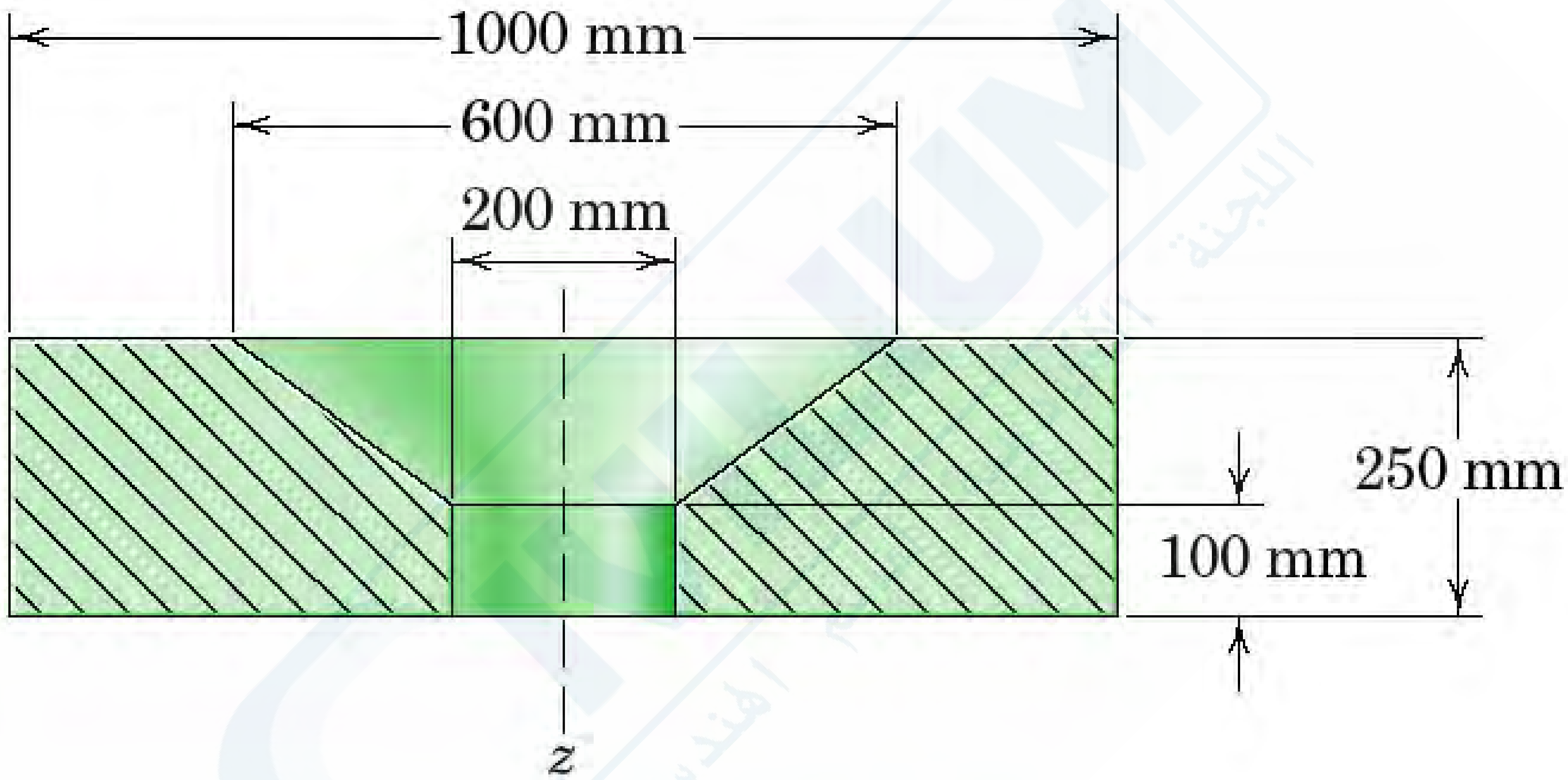
Problem 5/86

P_05_86



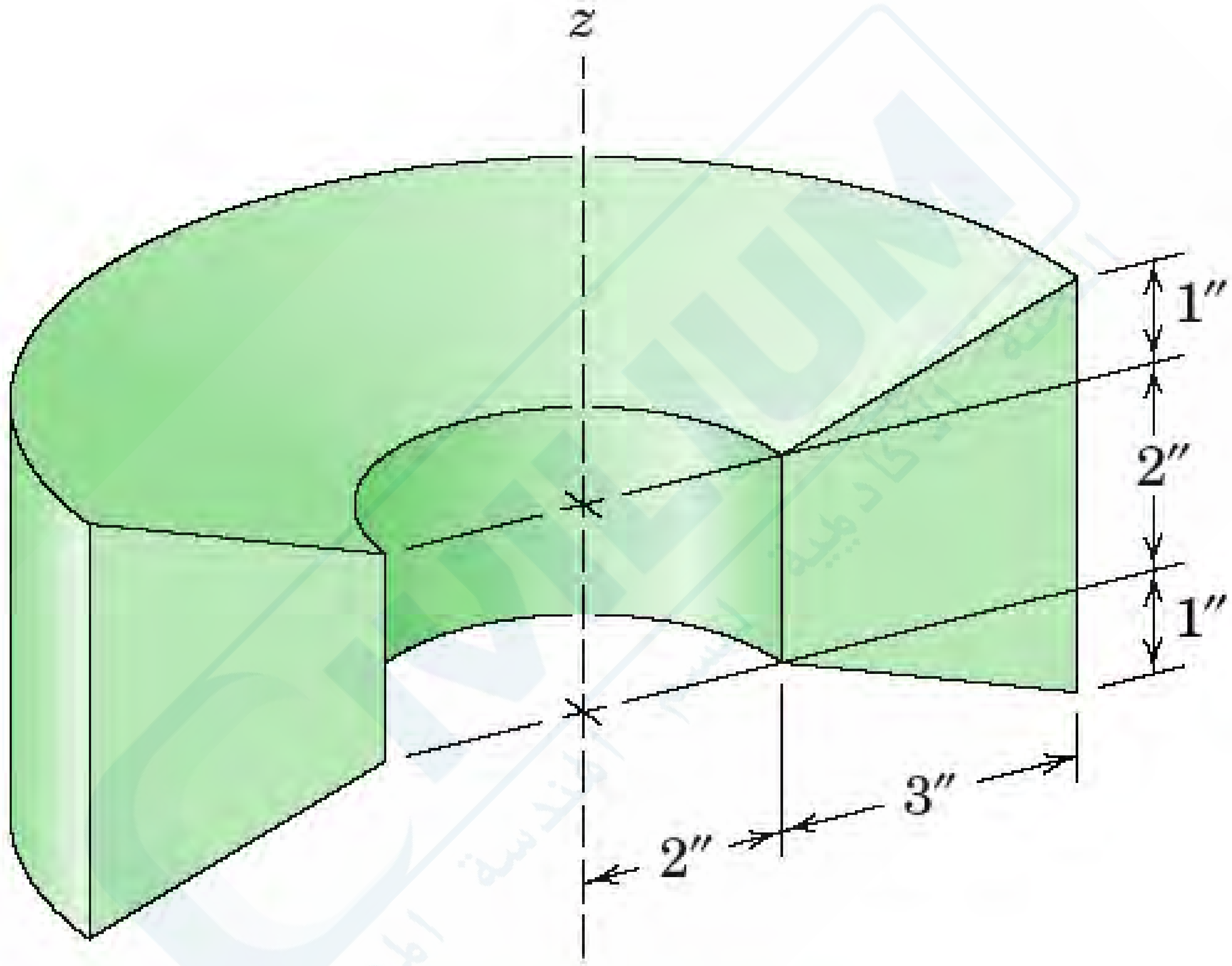
Problem 5/87

P_05_87



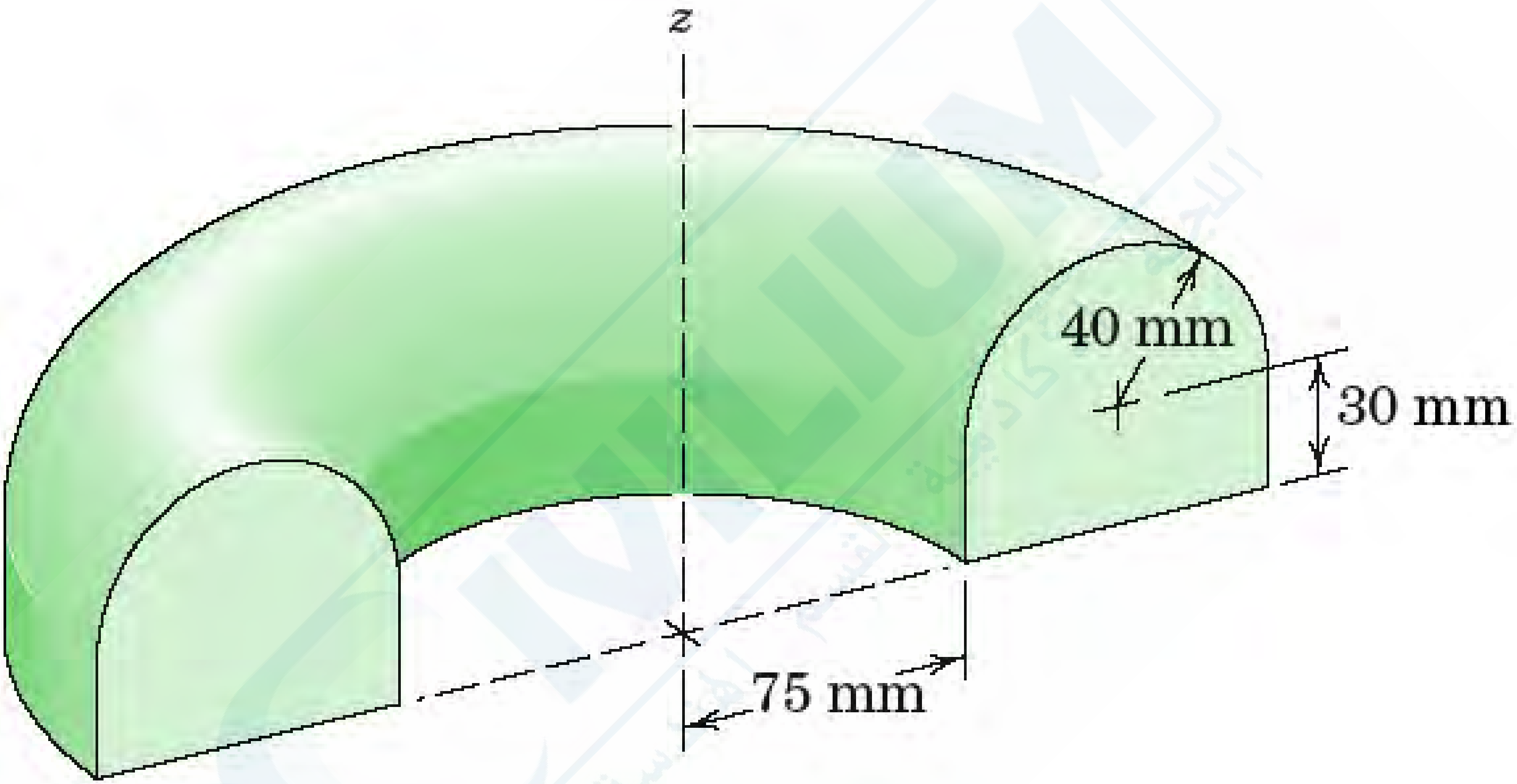
Problem 5/88

P_05_88



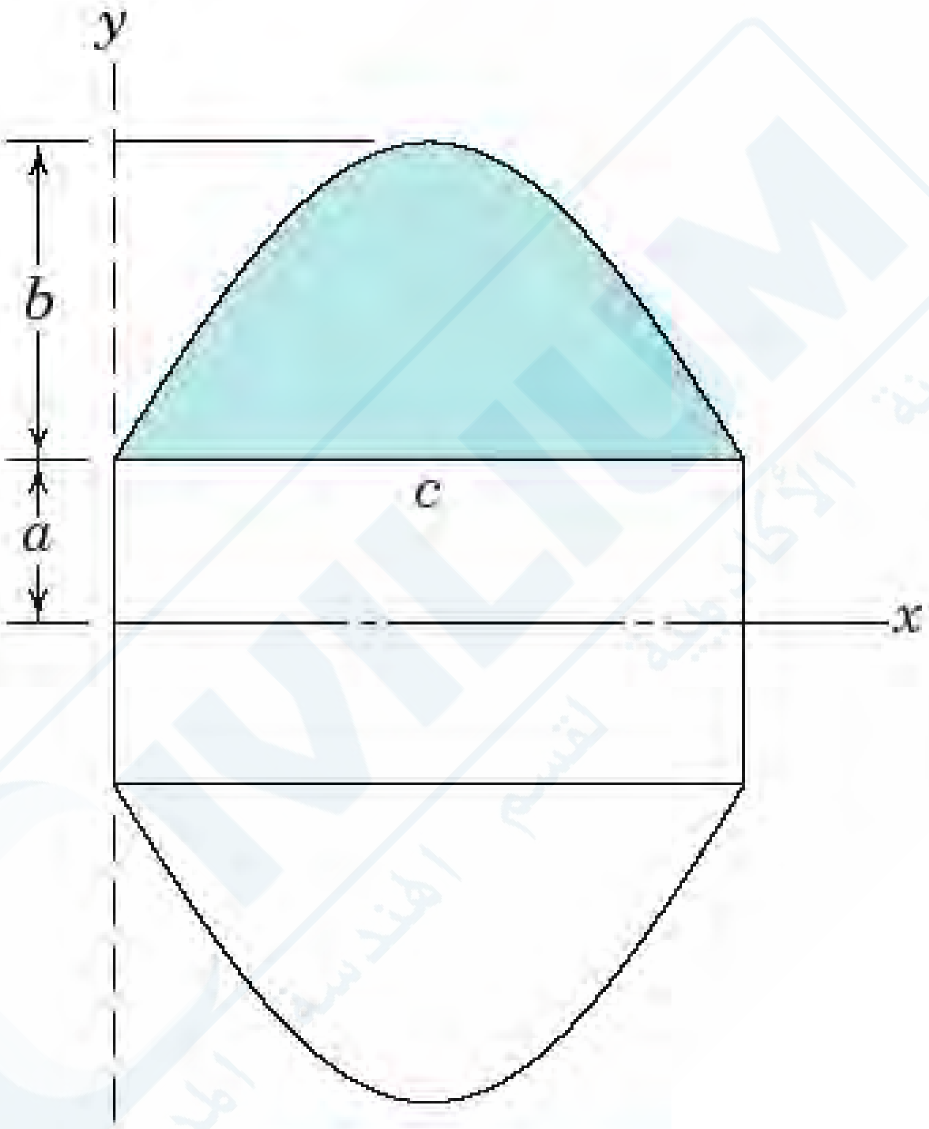
Problem 5/89

P_05_89



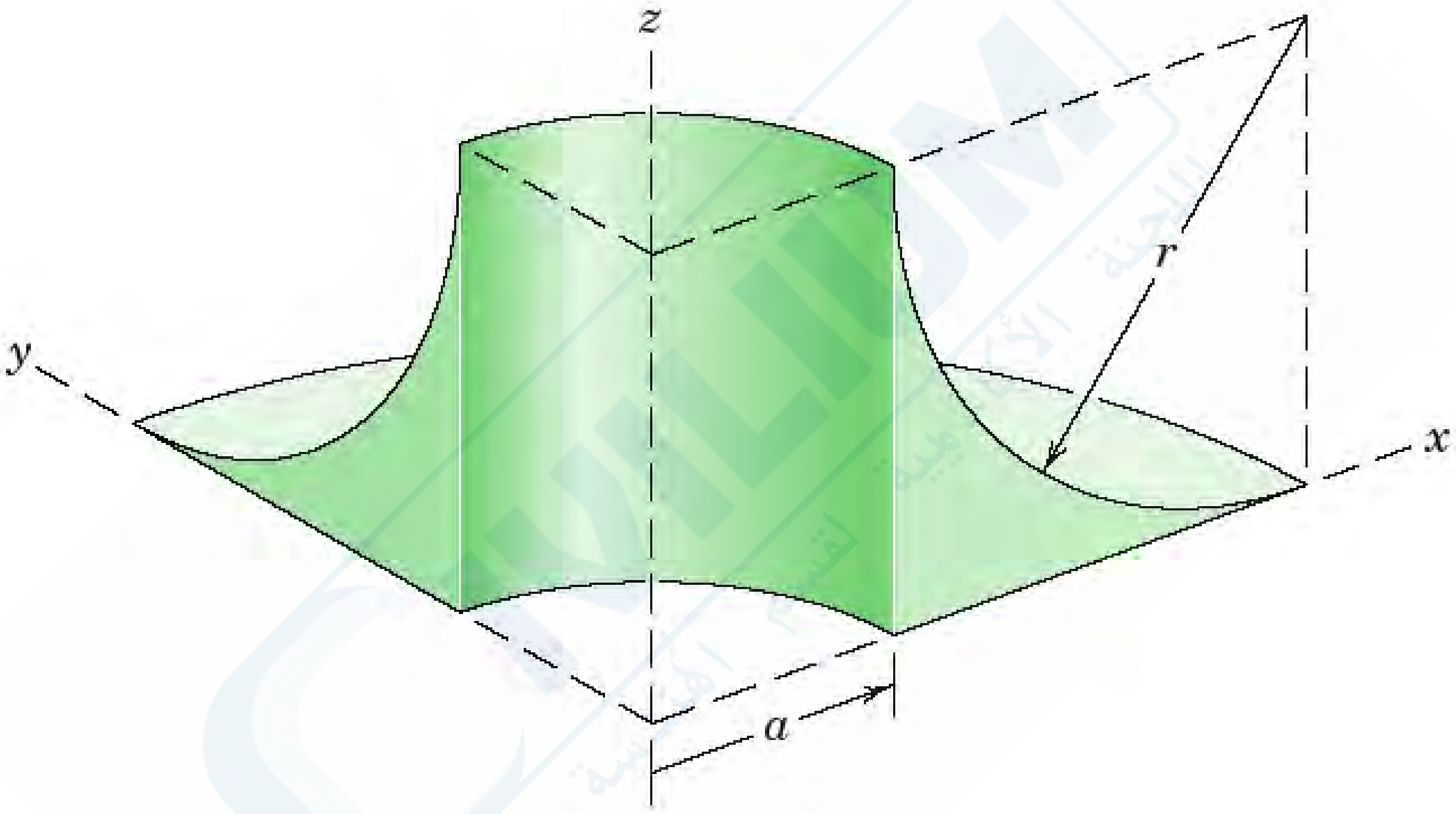
Problem 5/90

P_05_90



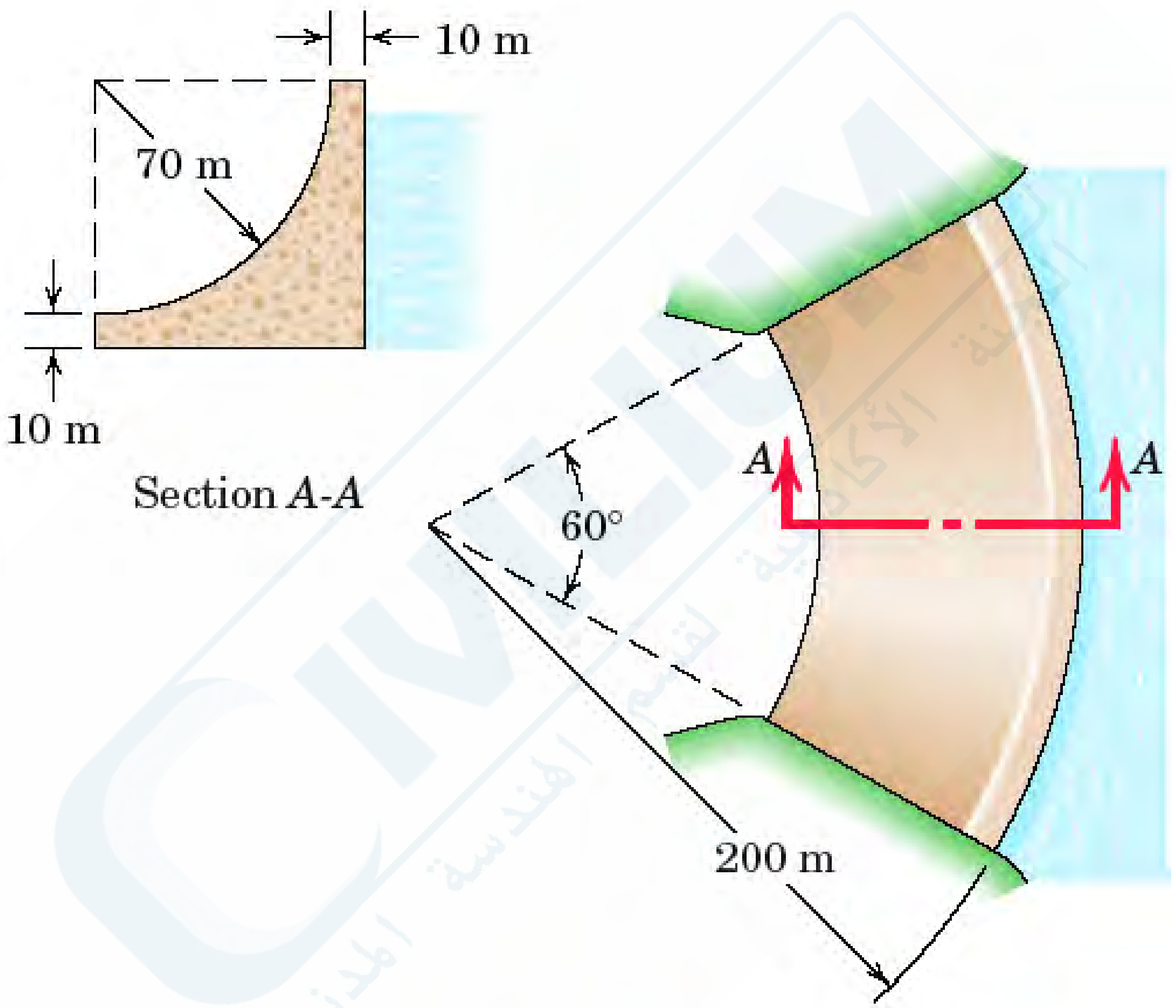
Problem 5/91

P_05_91



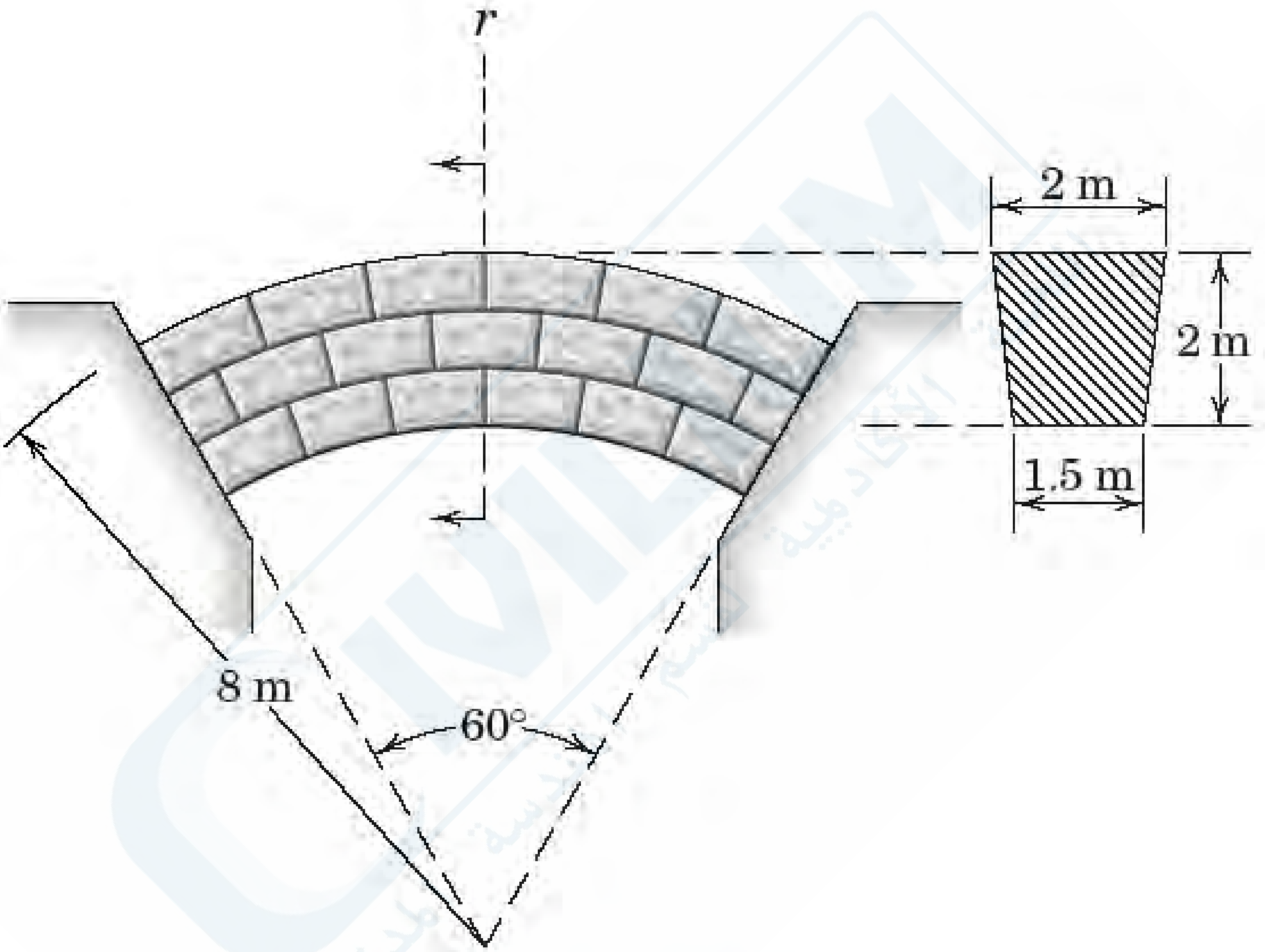
Problem 5/92

P_05_92



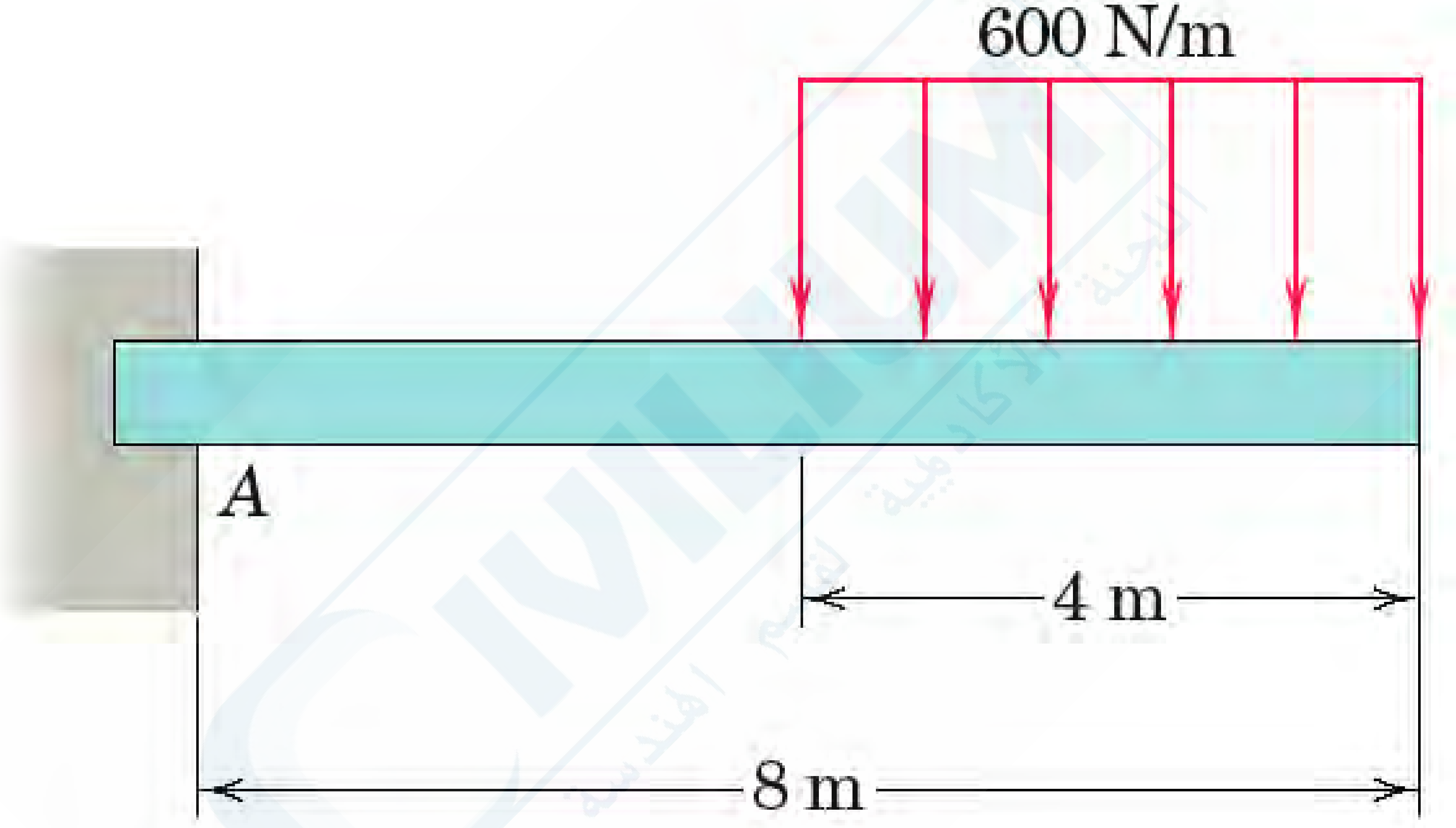
Problem 5/93

P_05_93



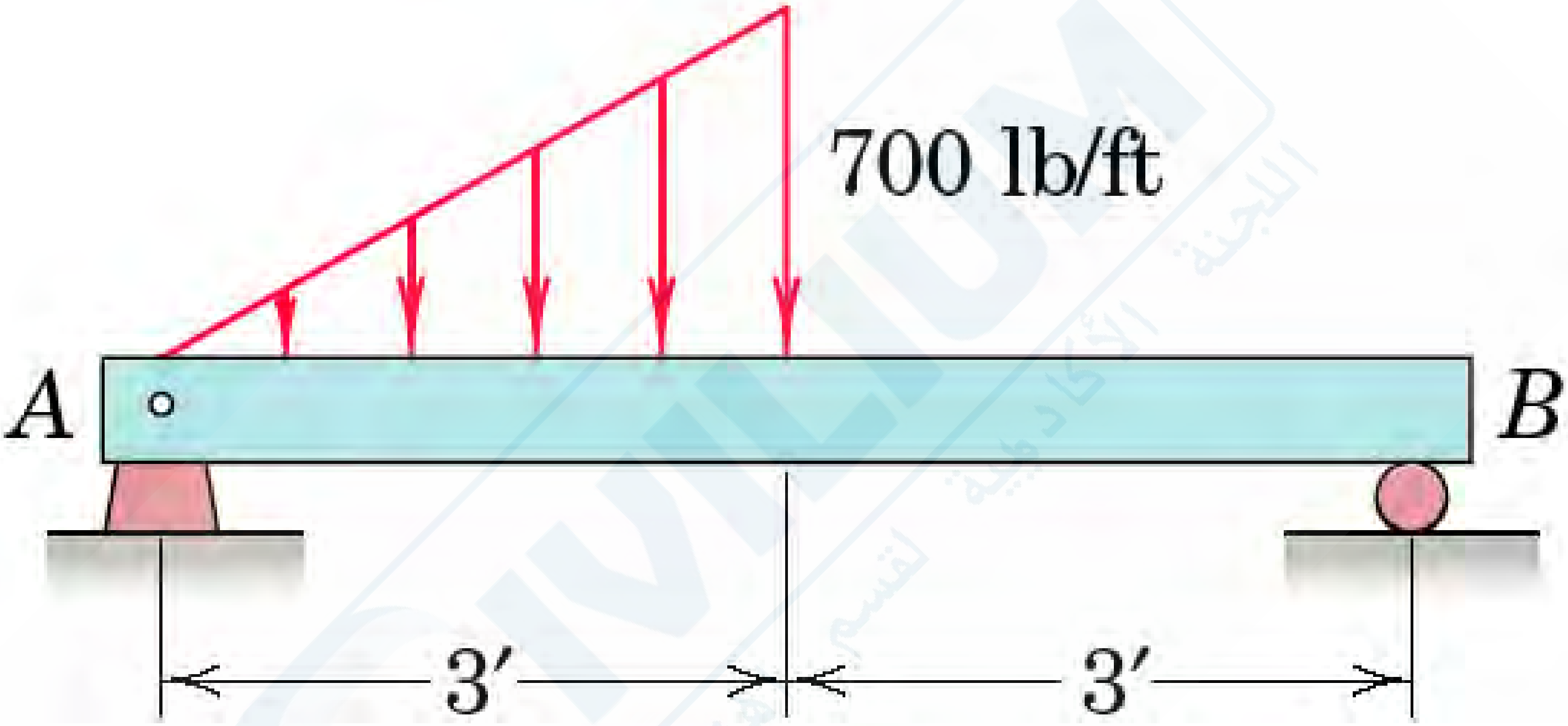
Problem 5/94

P_05_94



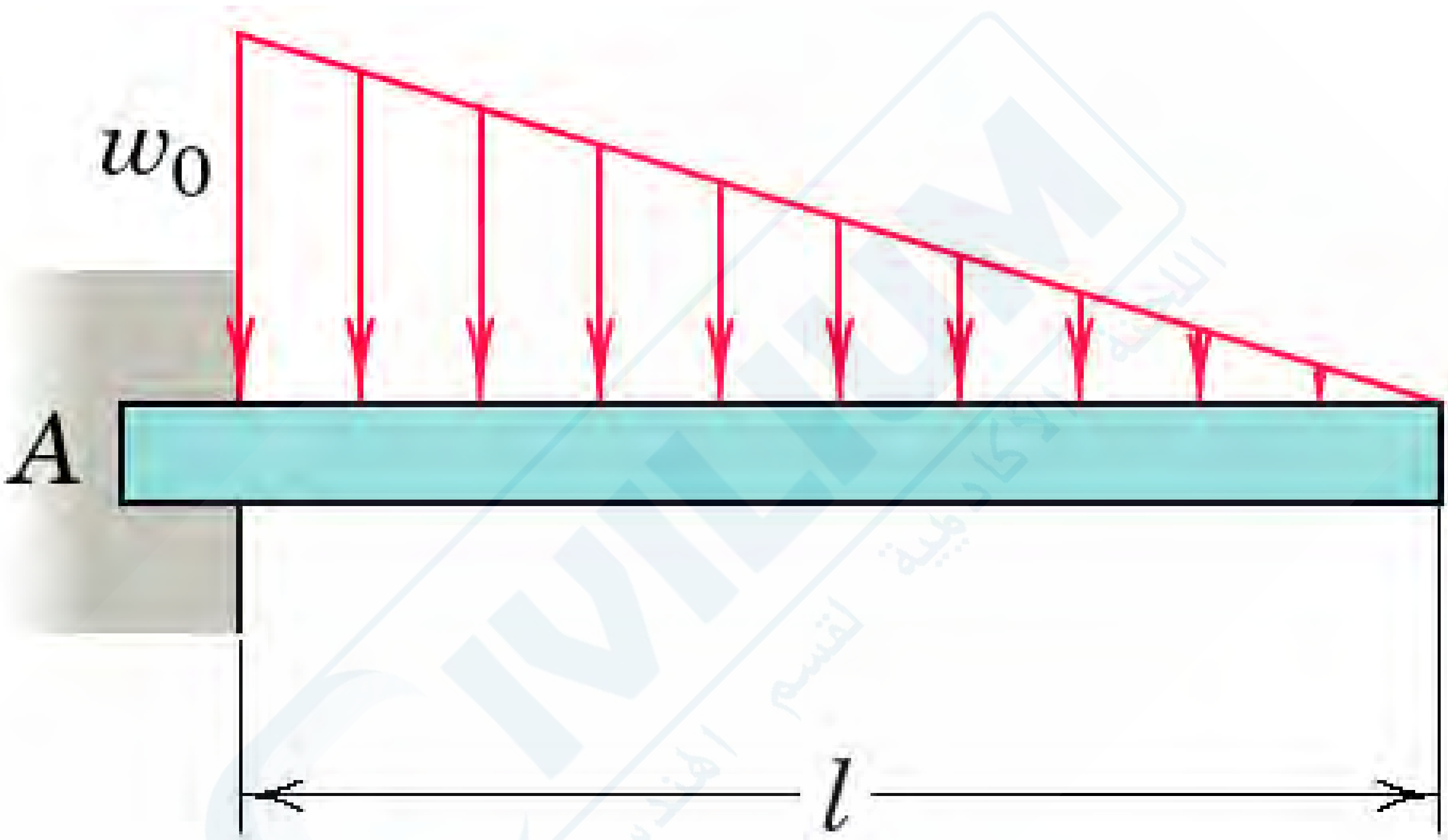
Problem 5/95

P_05_95



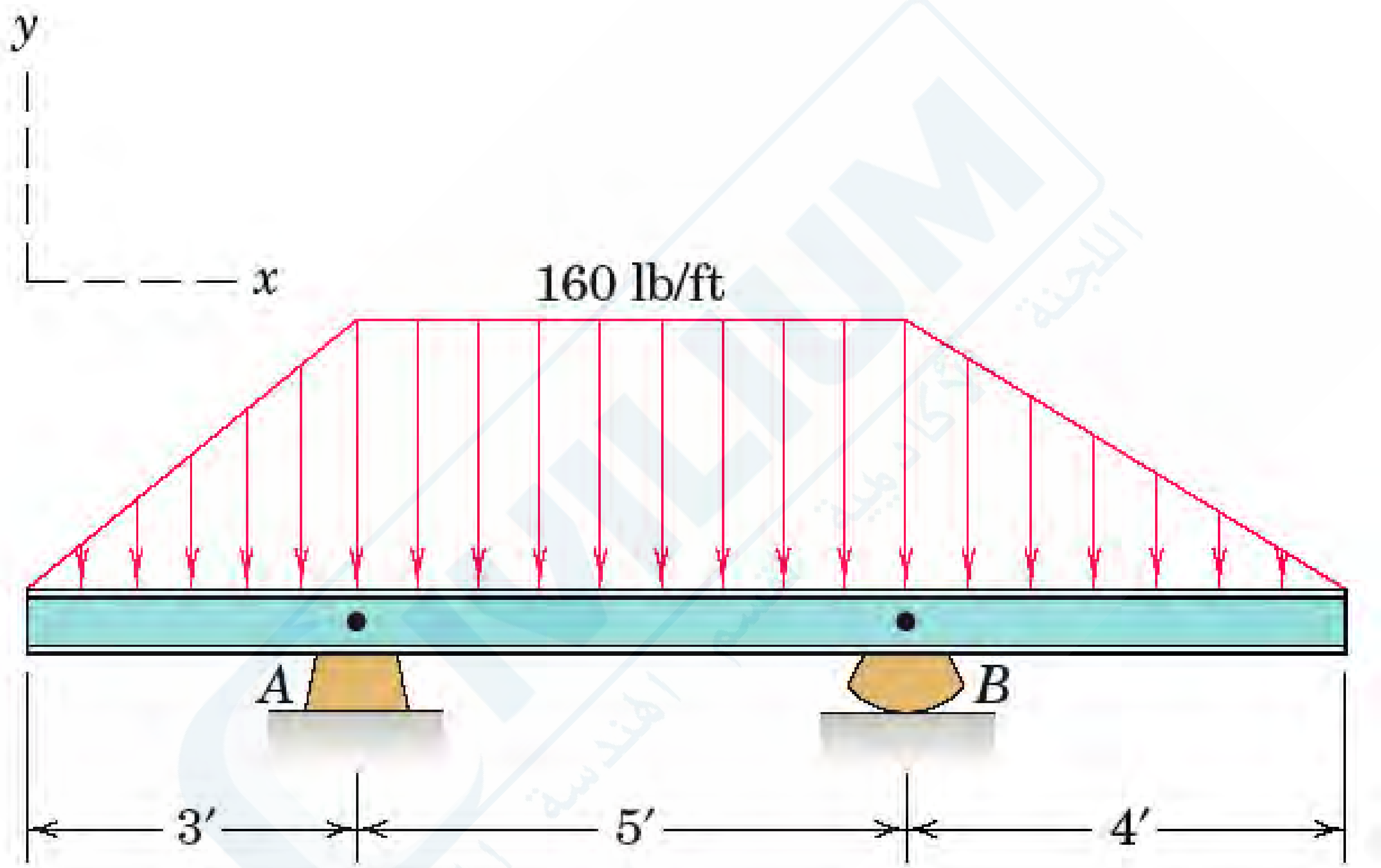
Problem 5/96

P_05_96



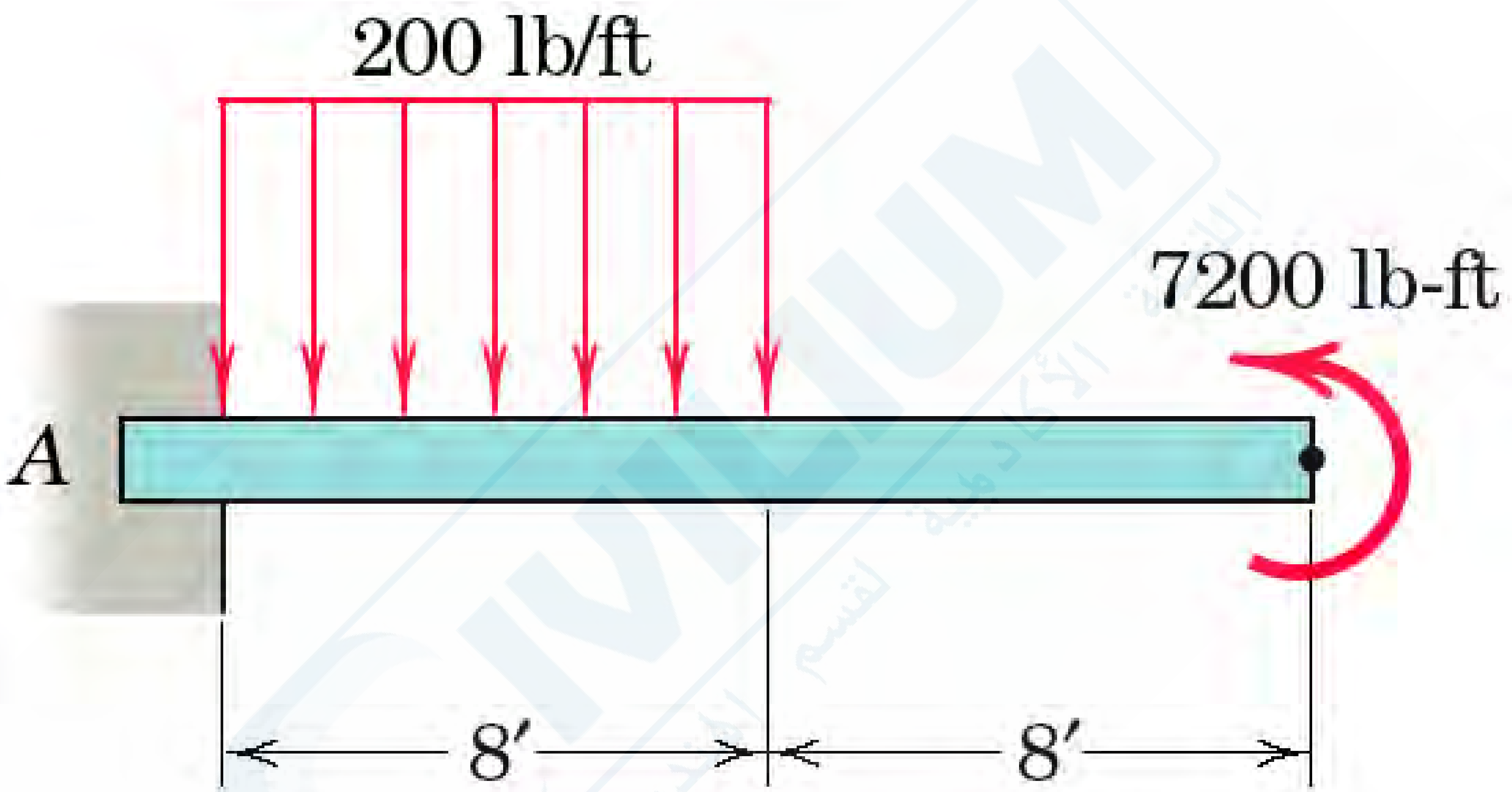
Problem 5/97

P_05_97



Problem 5/98

P_05_98



Problem 5/99

P_05_99

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 6 : Friction.

CIVILIUM اللجنة الأكاديمية لقسم الهندسة المدنية

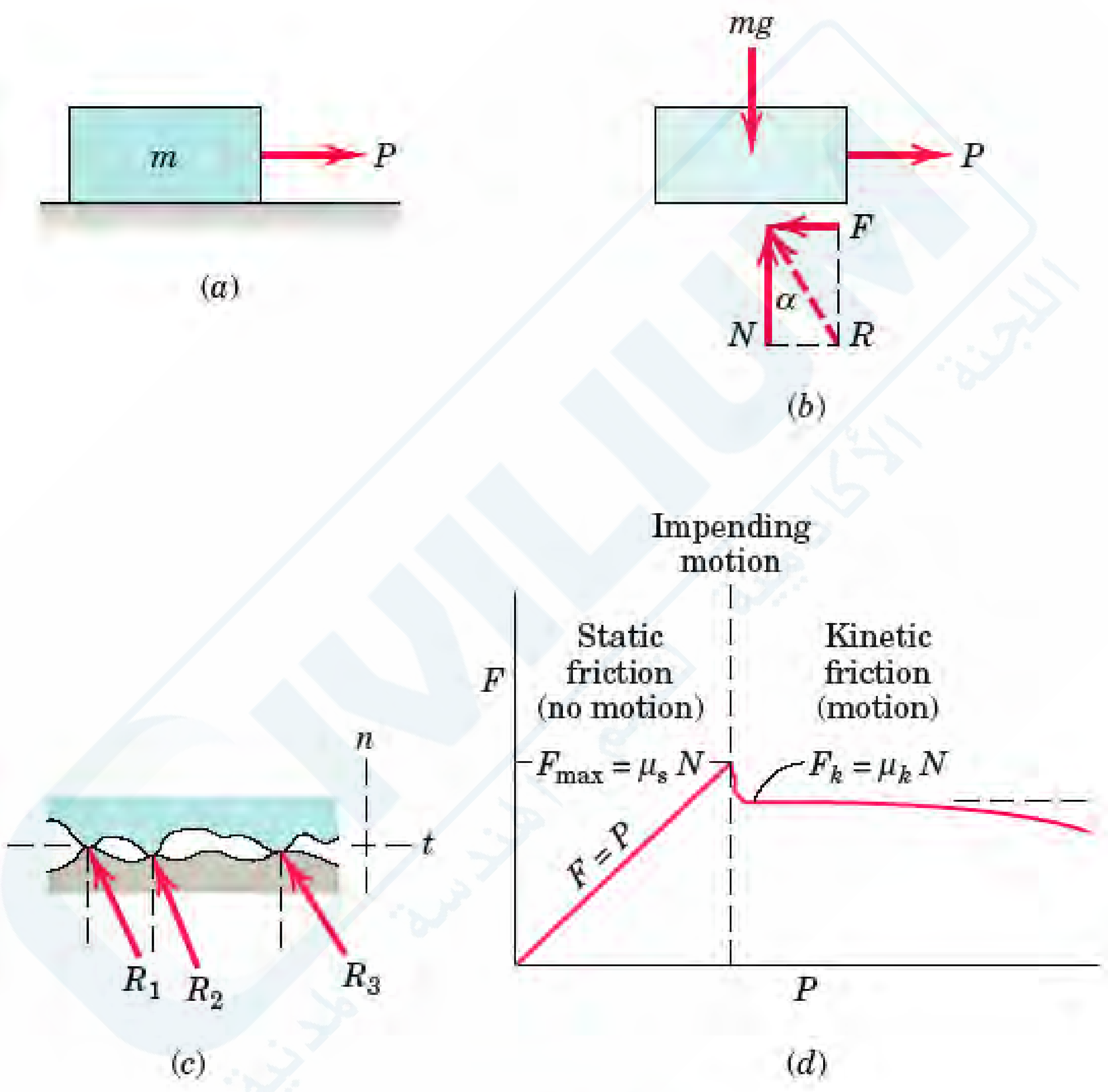


Figure 6/1

06_01

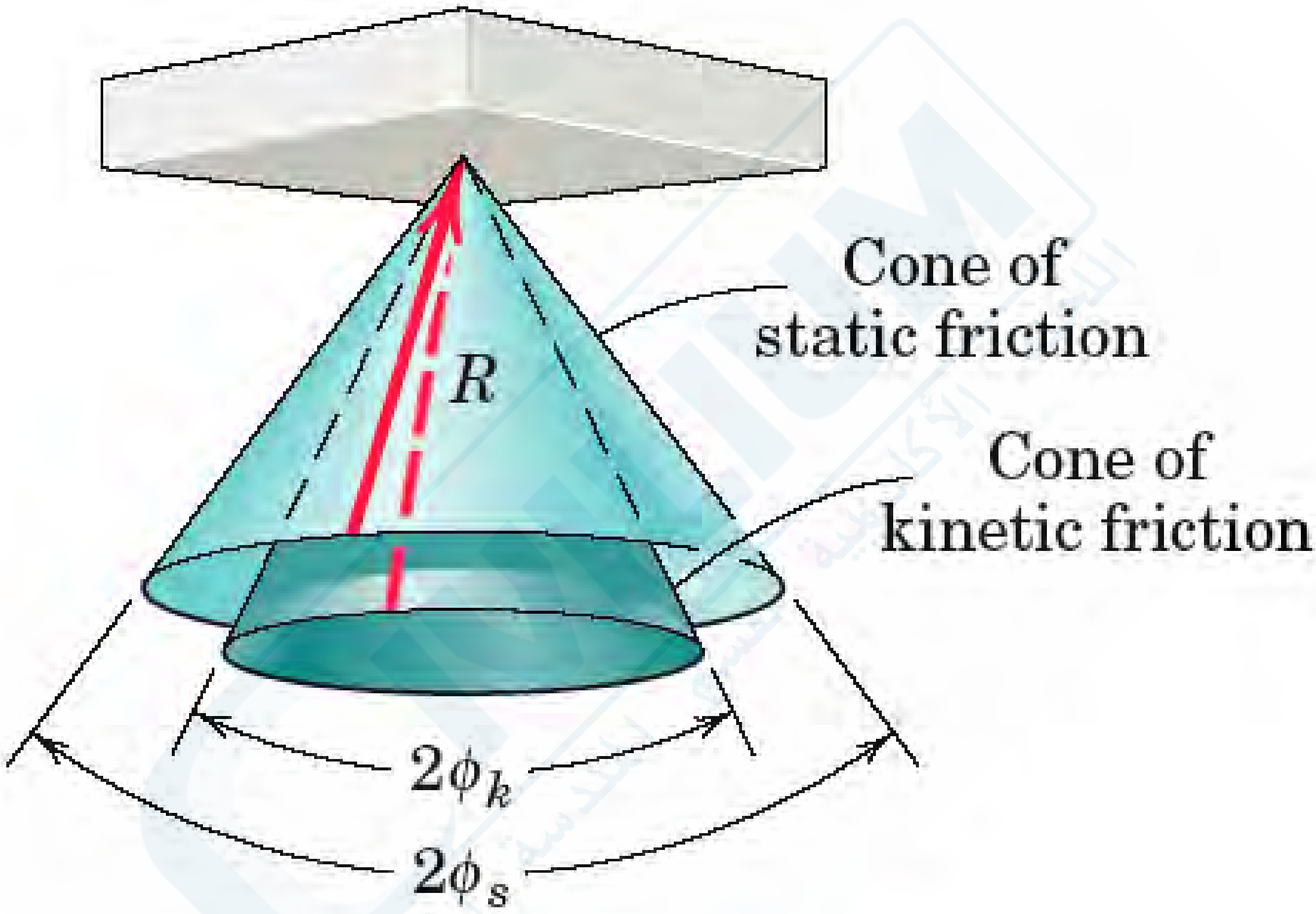
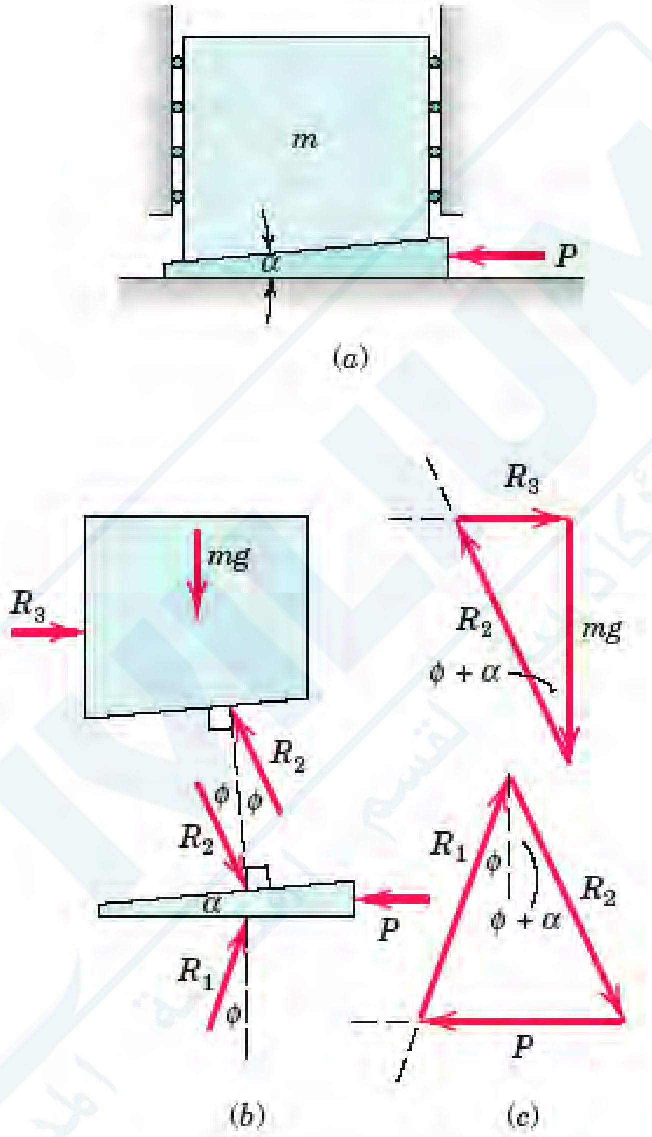


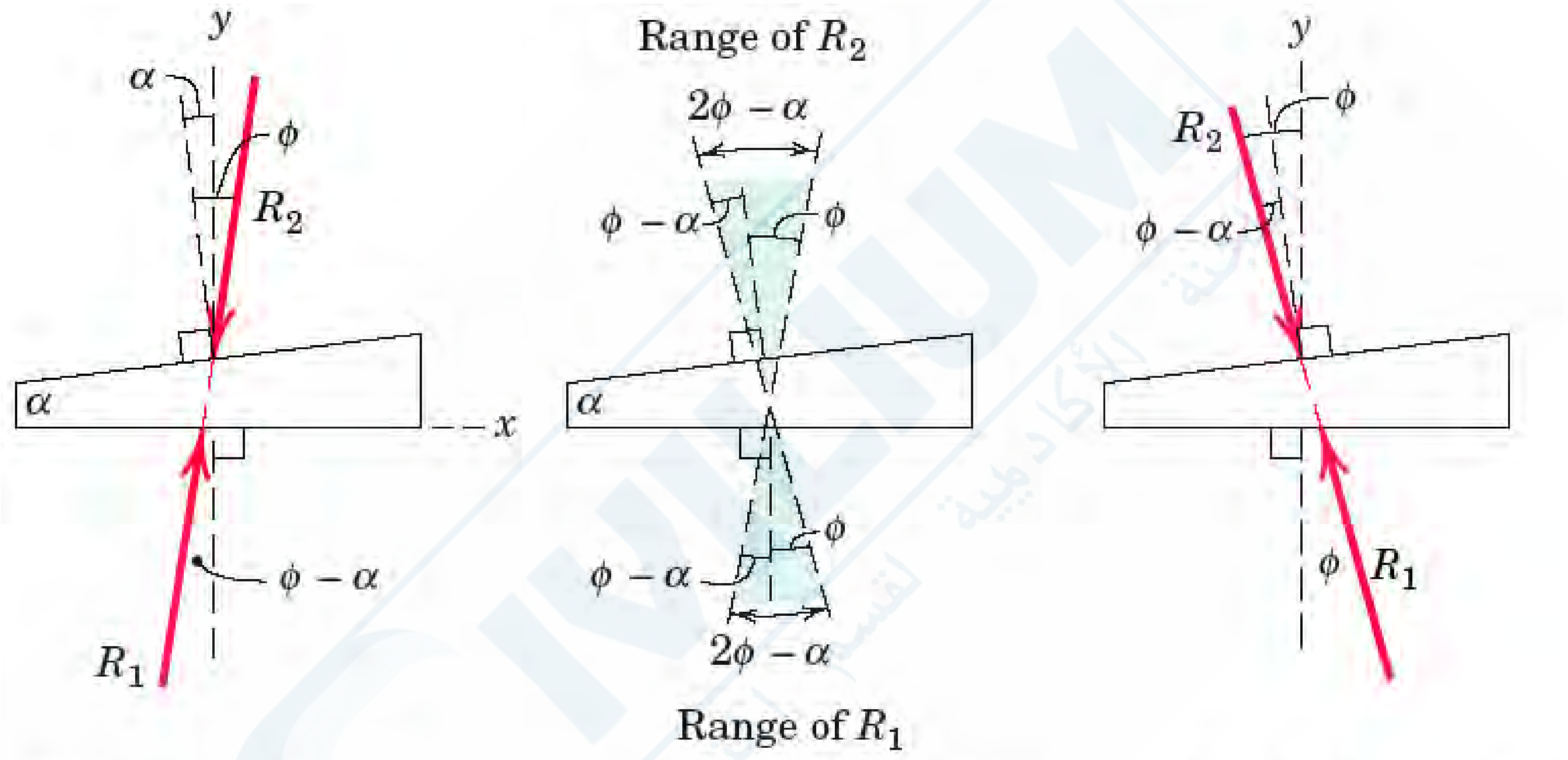
Figure 6/2

06_02



Forces to raise load

Figure 6/3



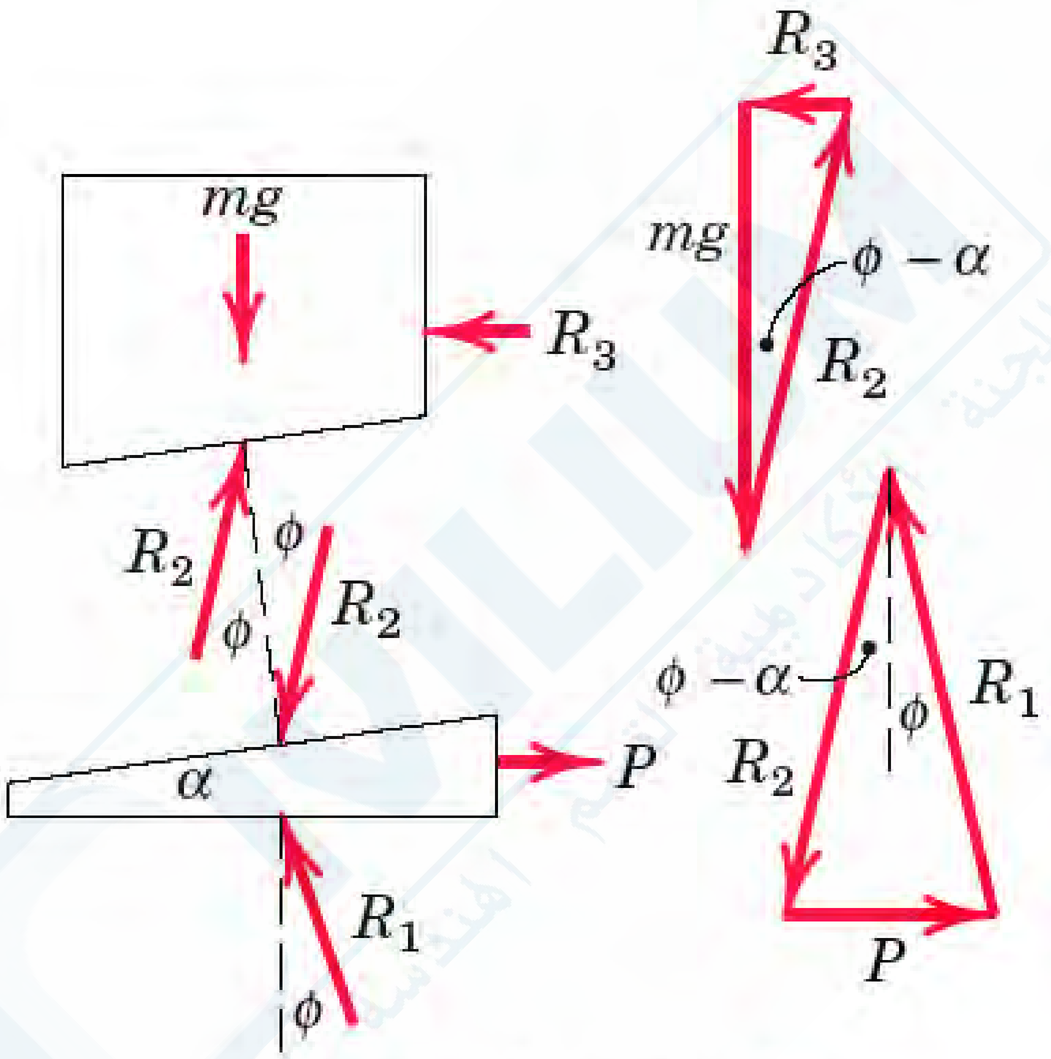
(*a*) Slipping impending at upper surface

(*b*) Range of $R_1 = R_2$ for no slip

(*c*) Slipping impending at lower surface

Figure 6/4

06_04



Forces to lower load

Figure 6/5

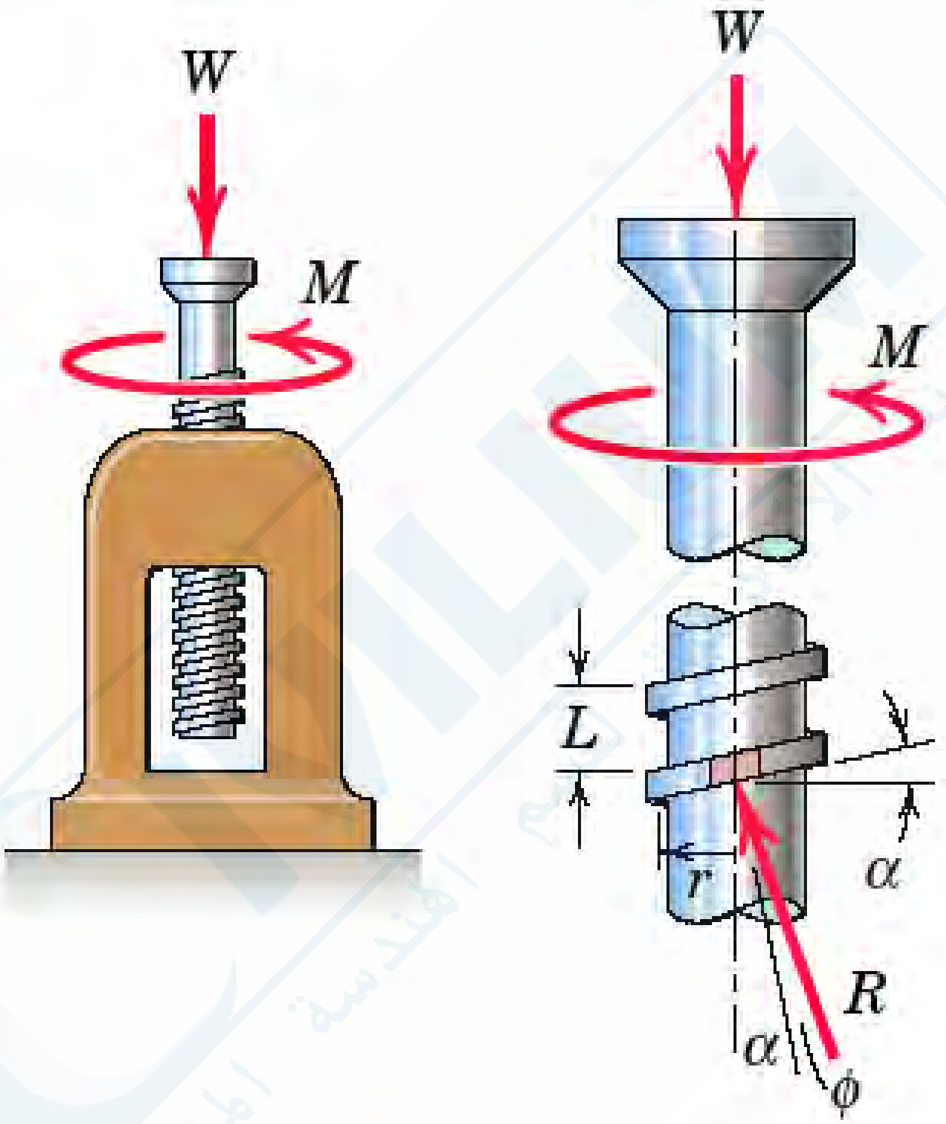


Figure 6/6

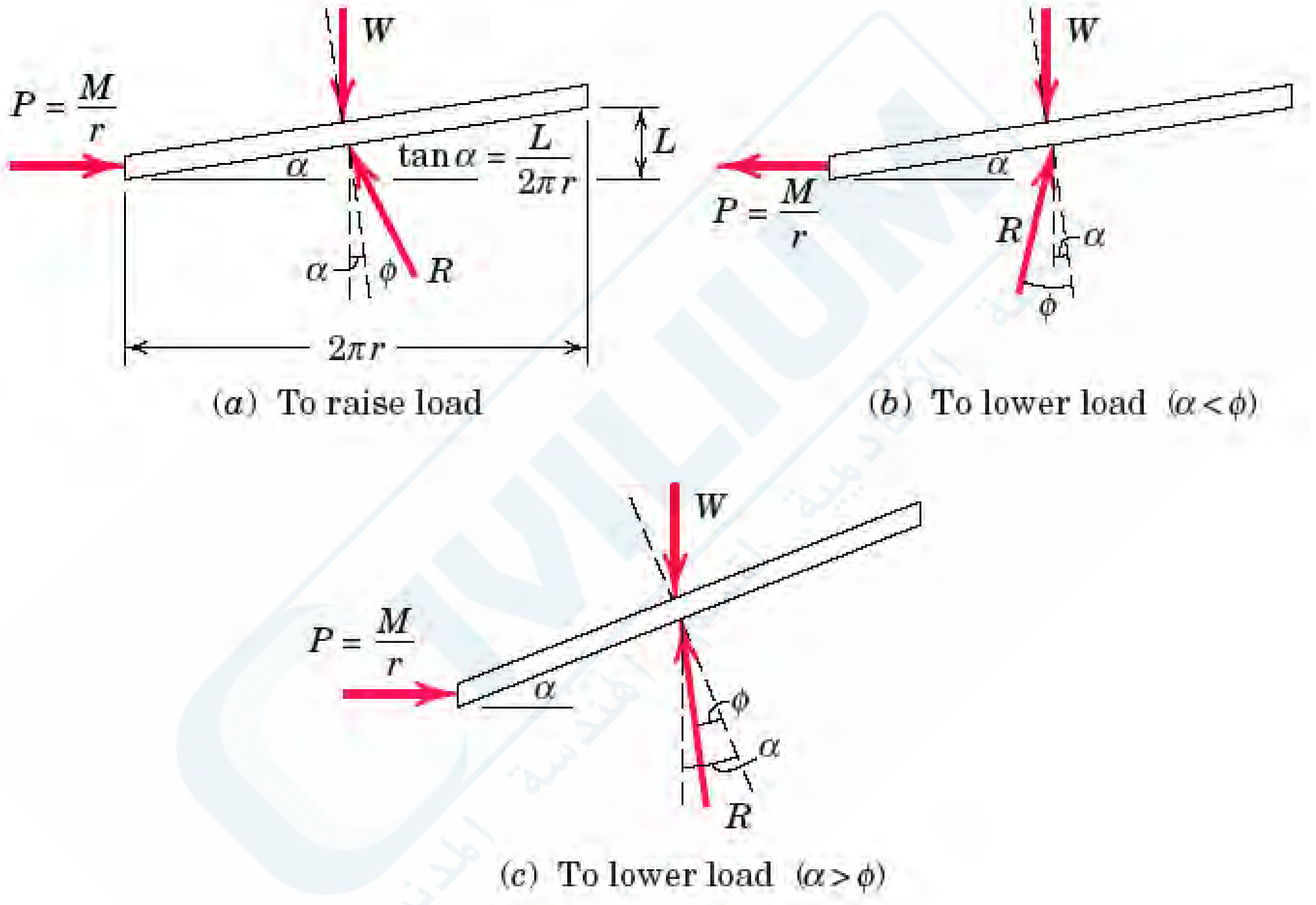


Figure 6/7

06_07

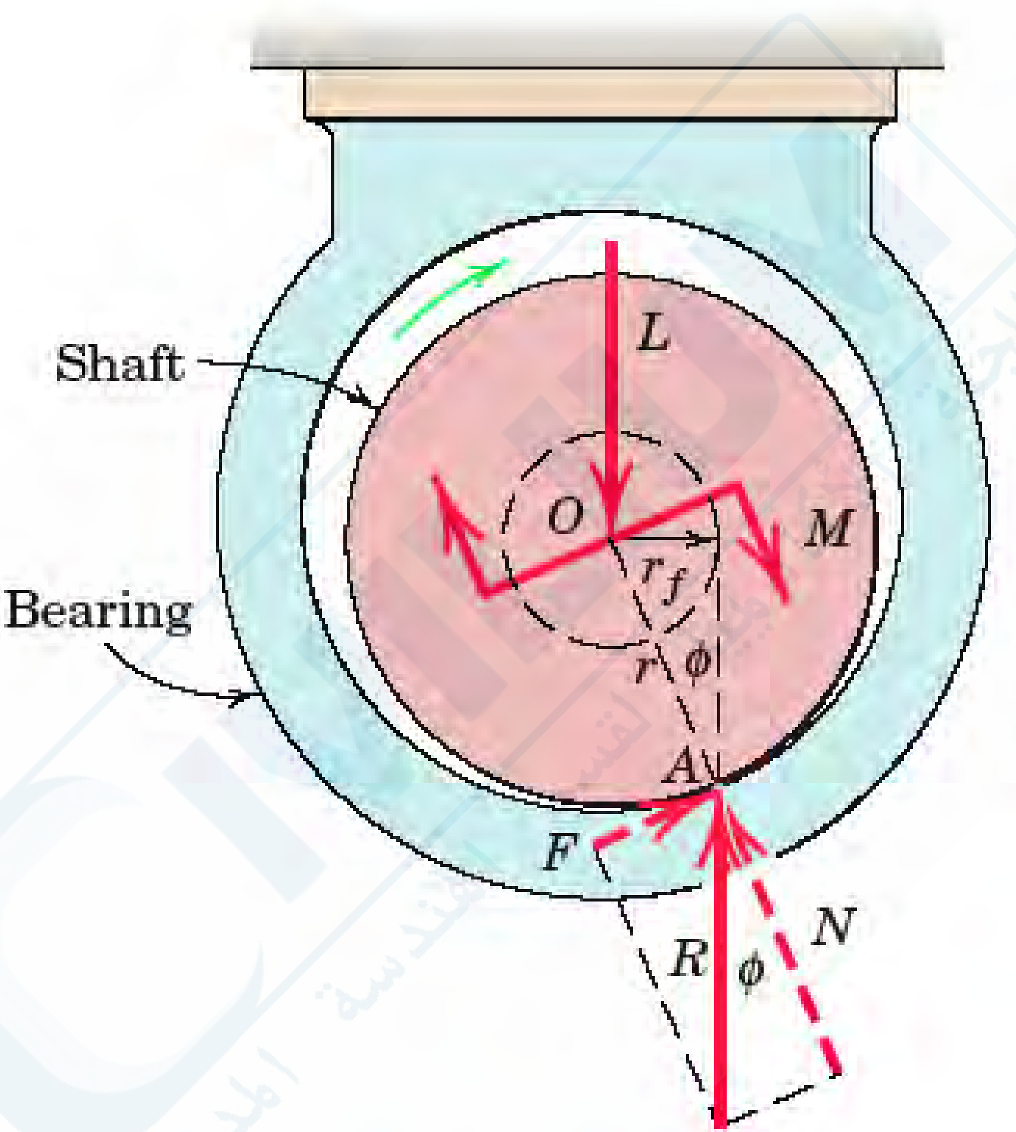


Figure 6/8

06_08

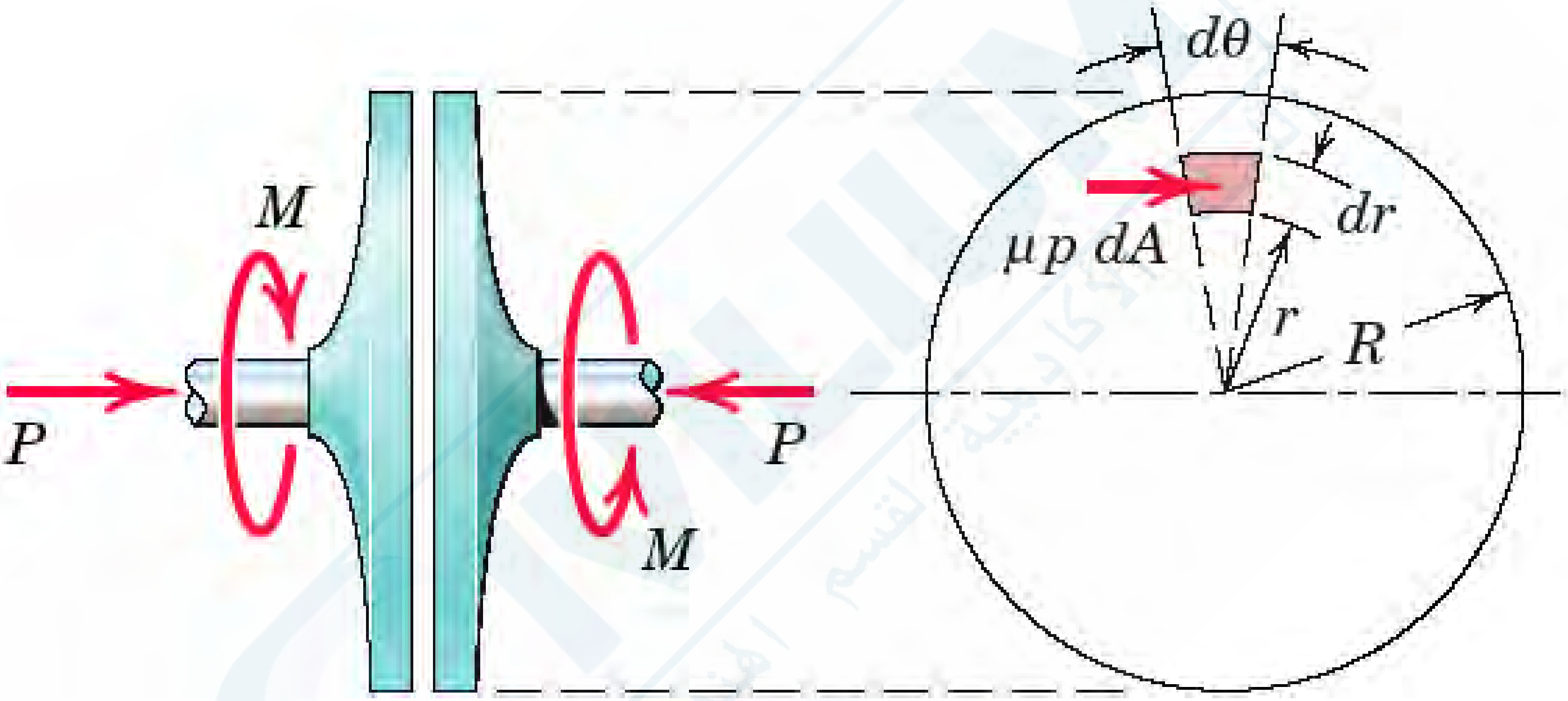


Figure 6/9

06_09

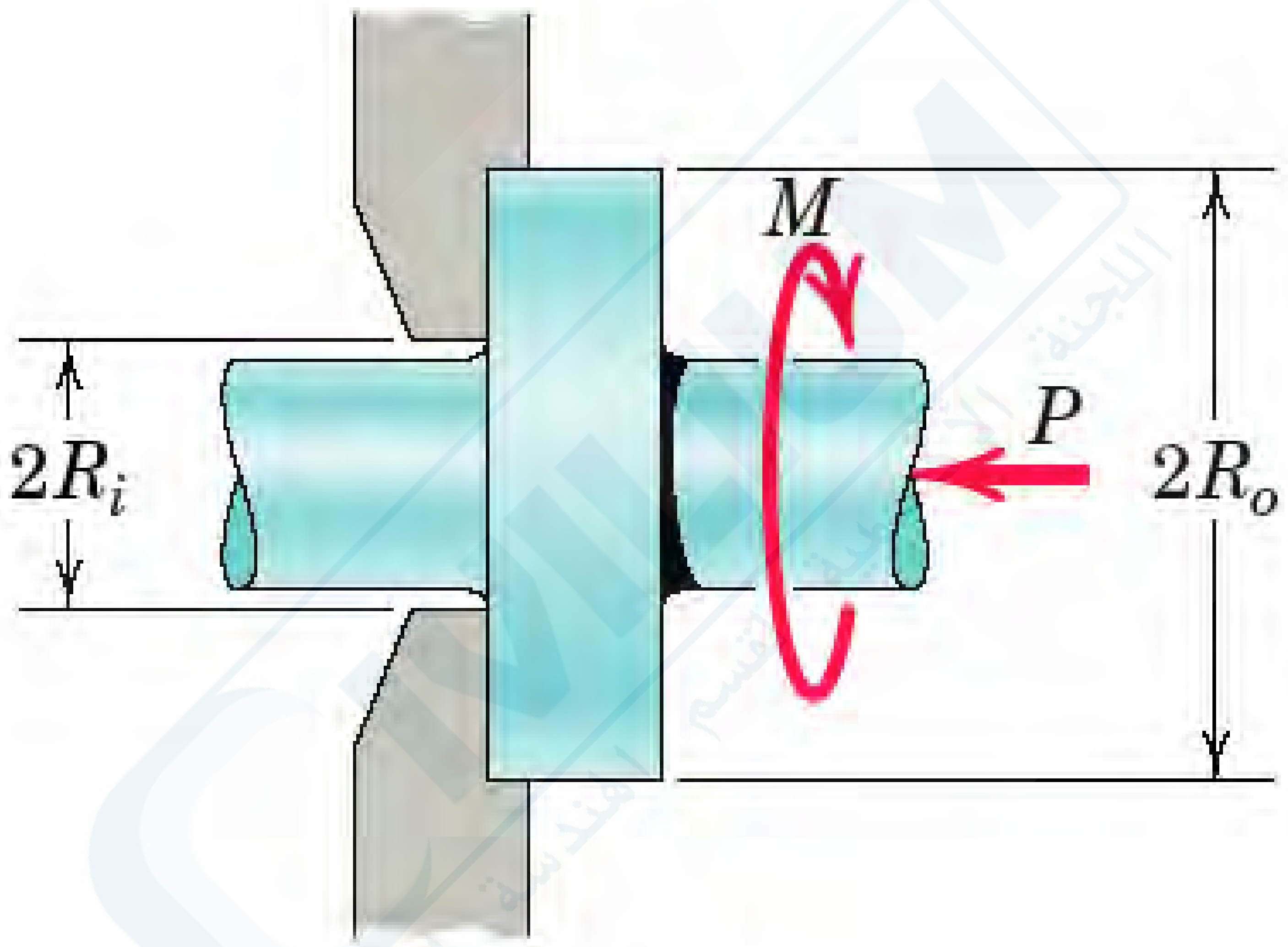


Figure 6/10

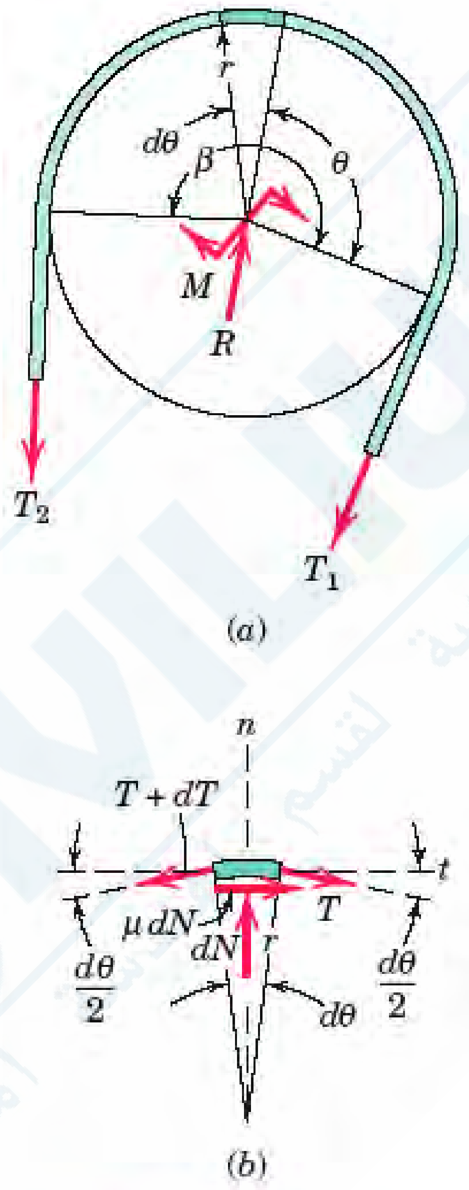


Figure 6/11

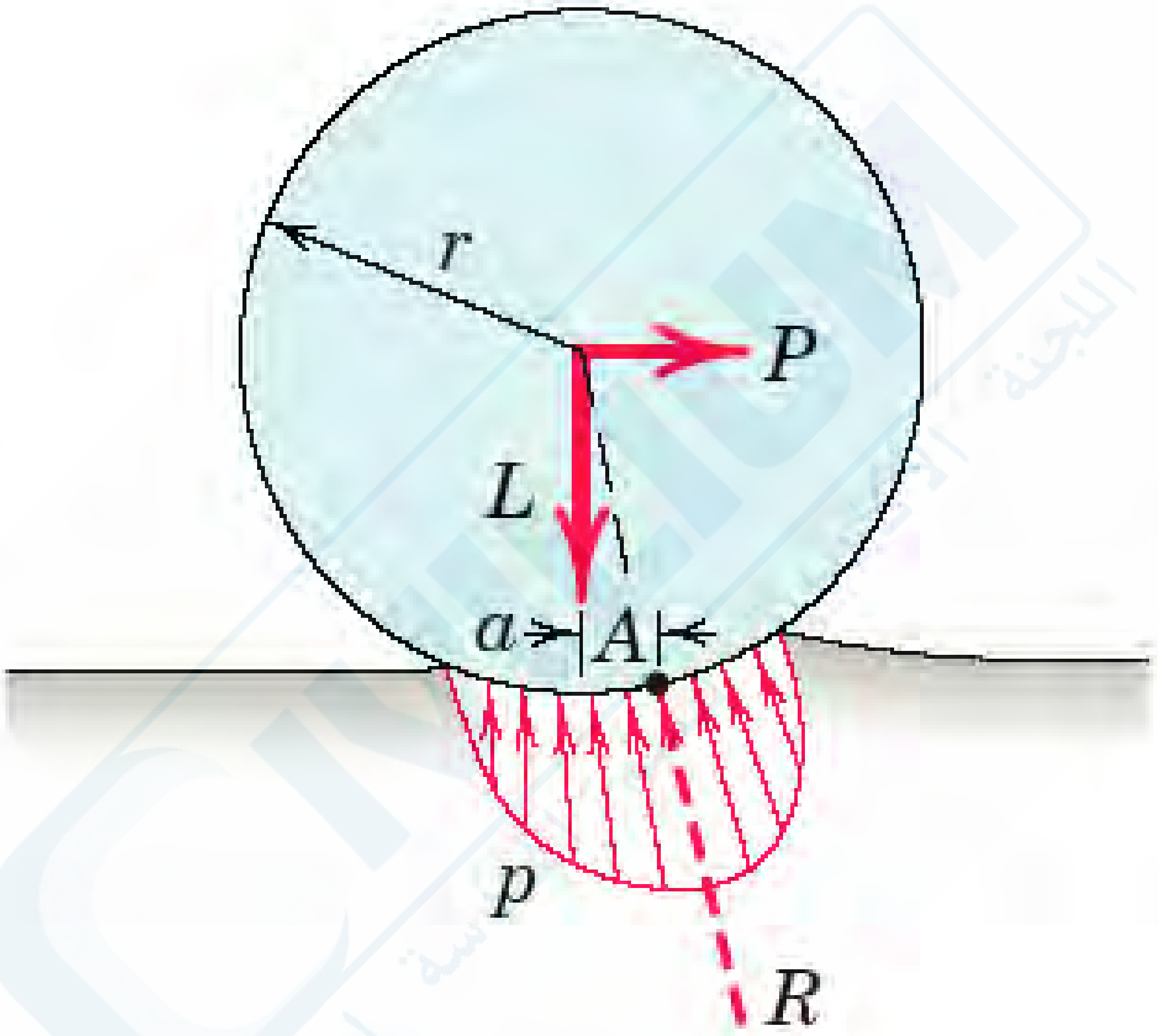
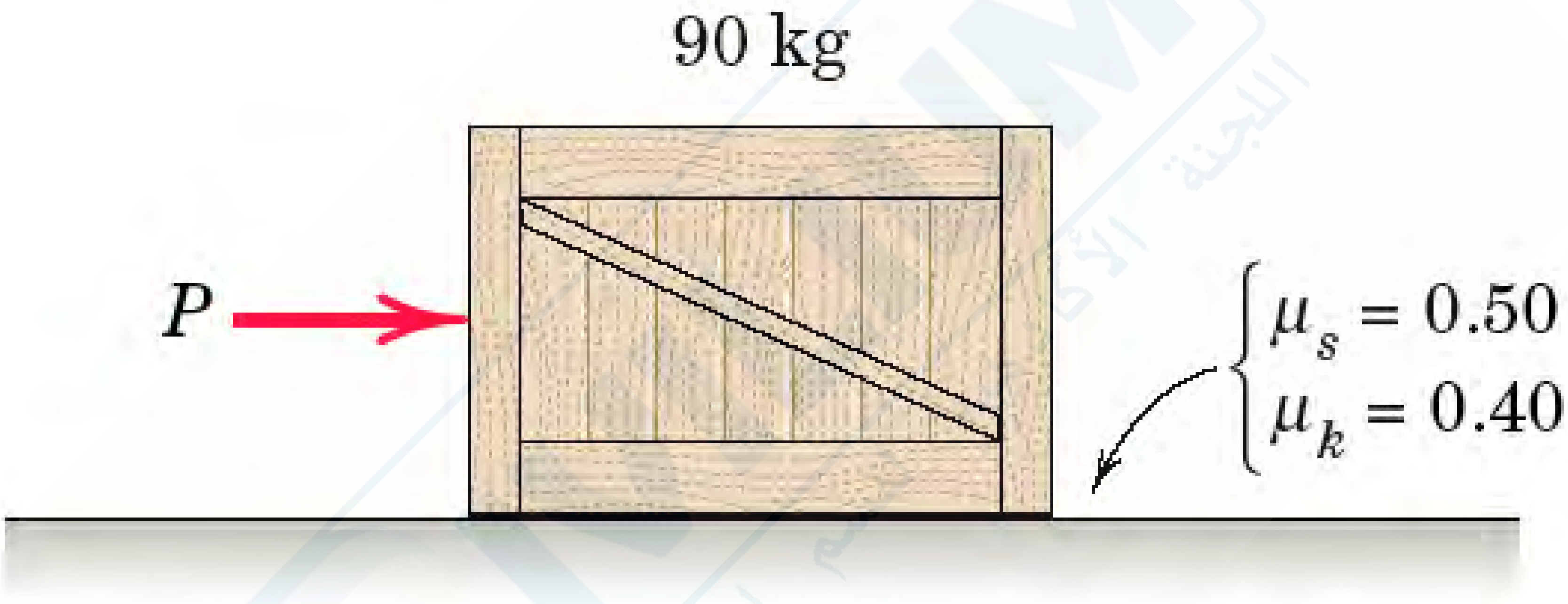


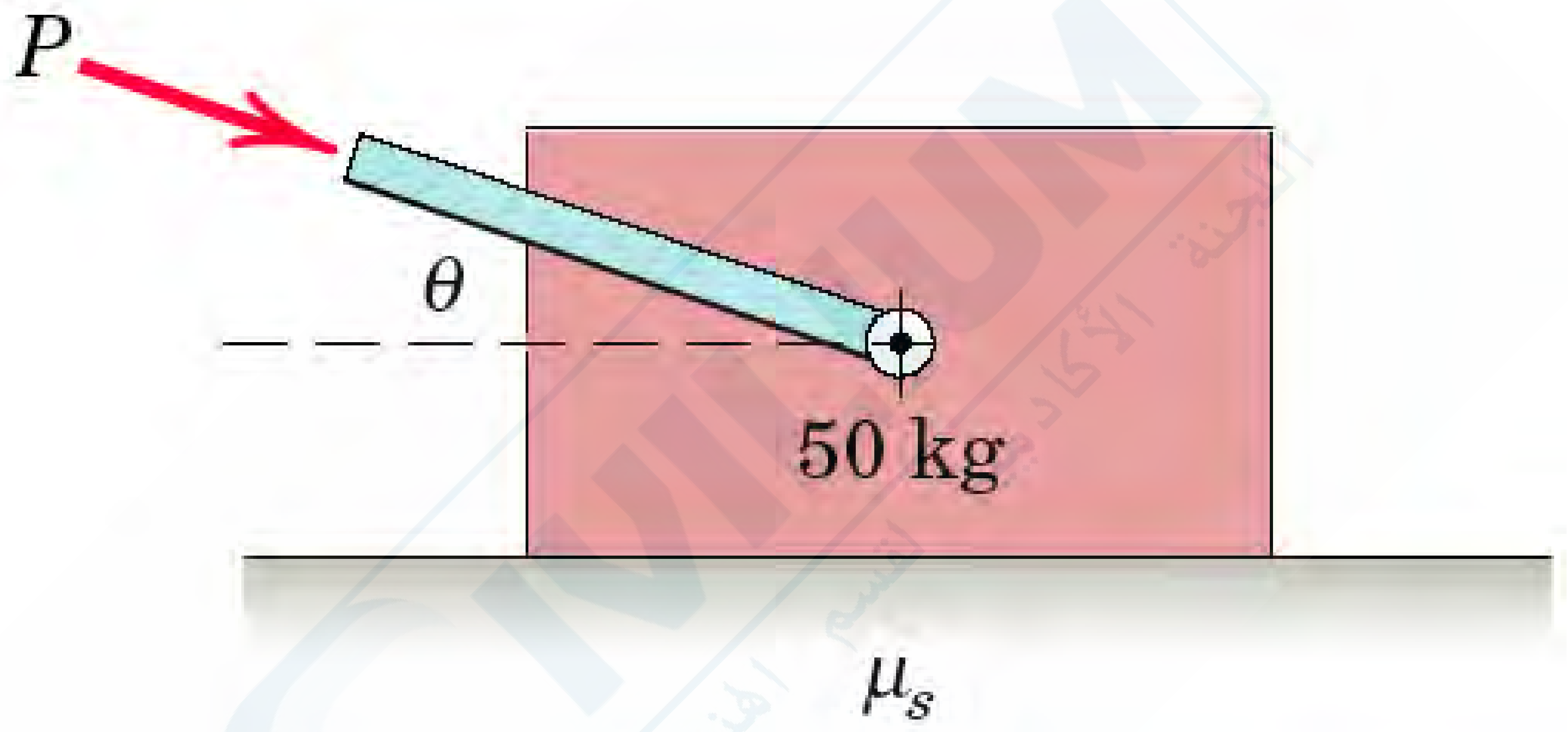
Figure 6/12

06_12



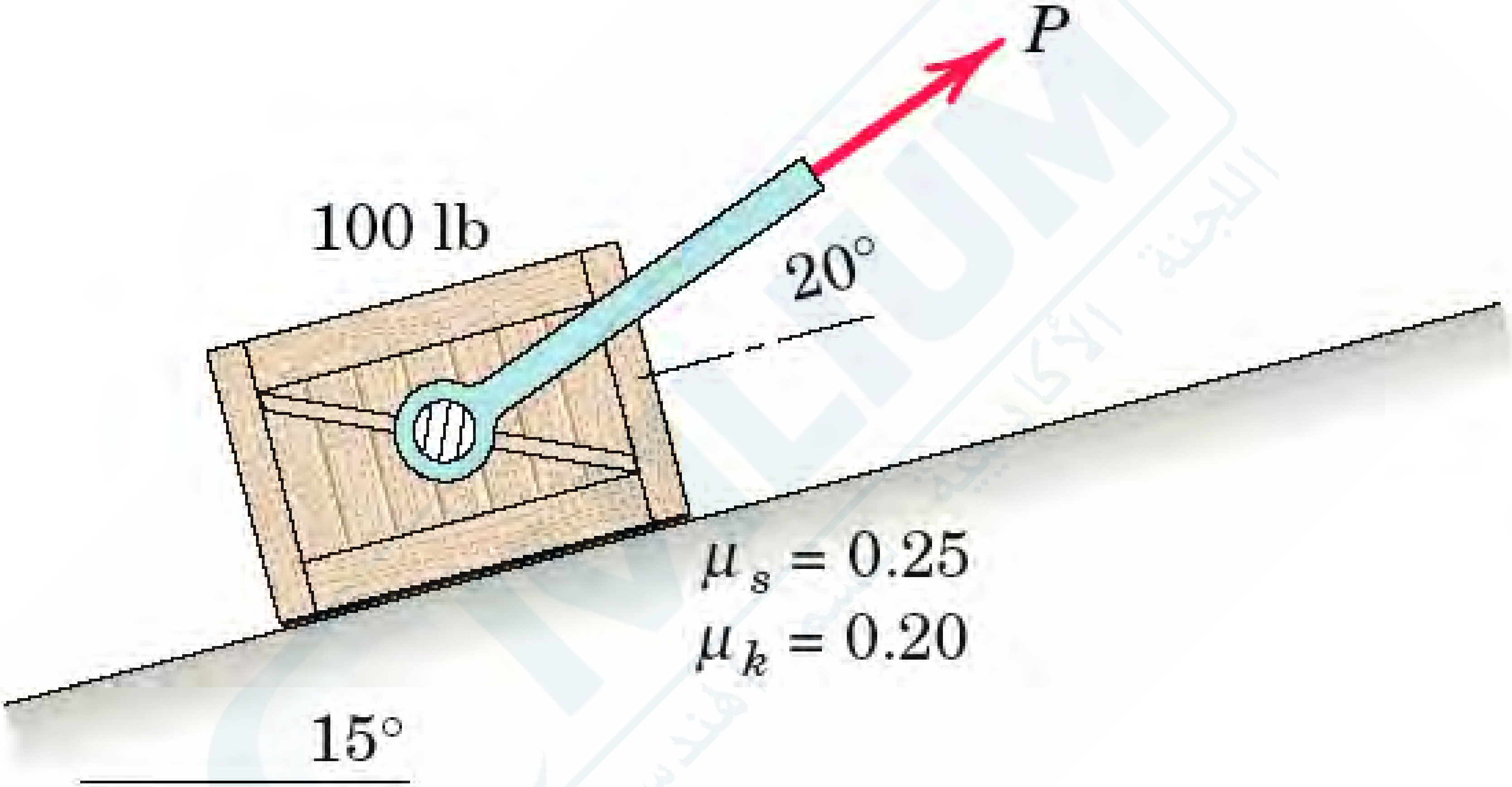
Problem 6/1

P_06_01



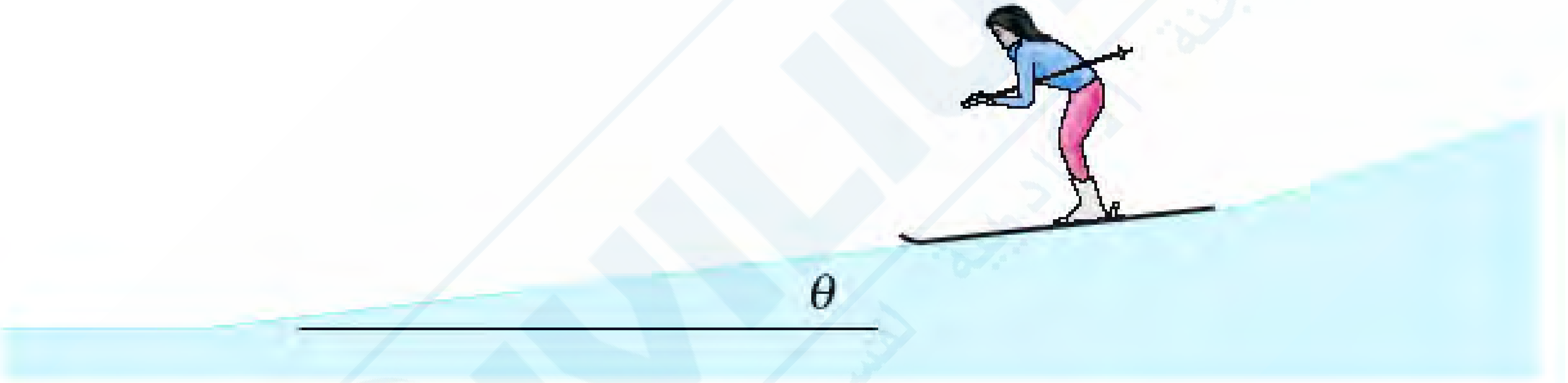
Problem 6/2

P_06_02



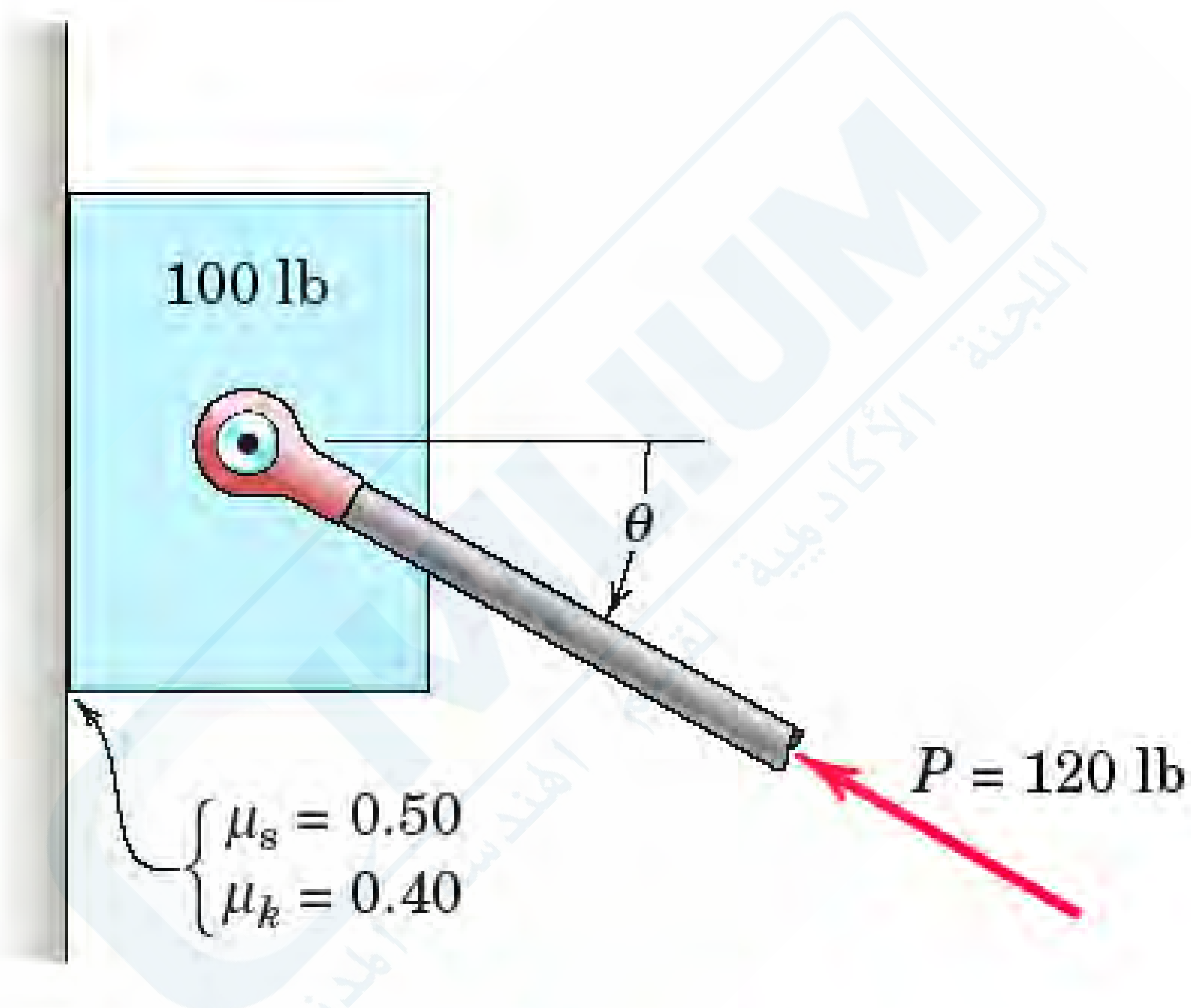
Problem 6/3

P_06_03



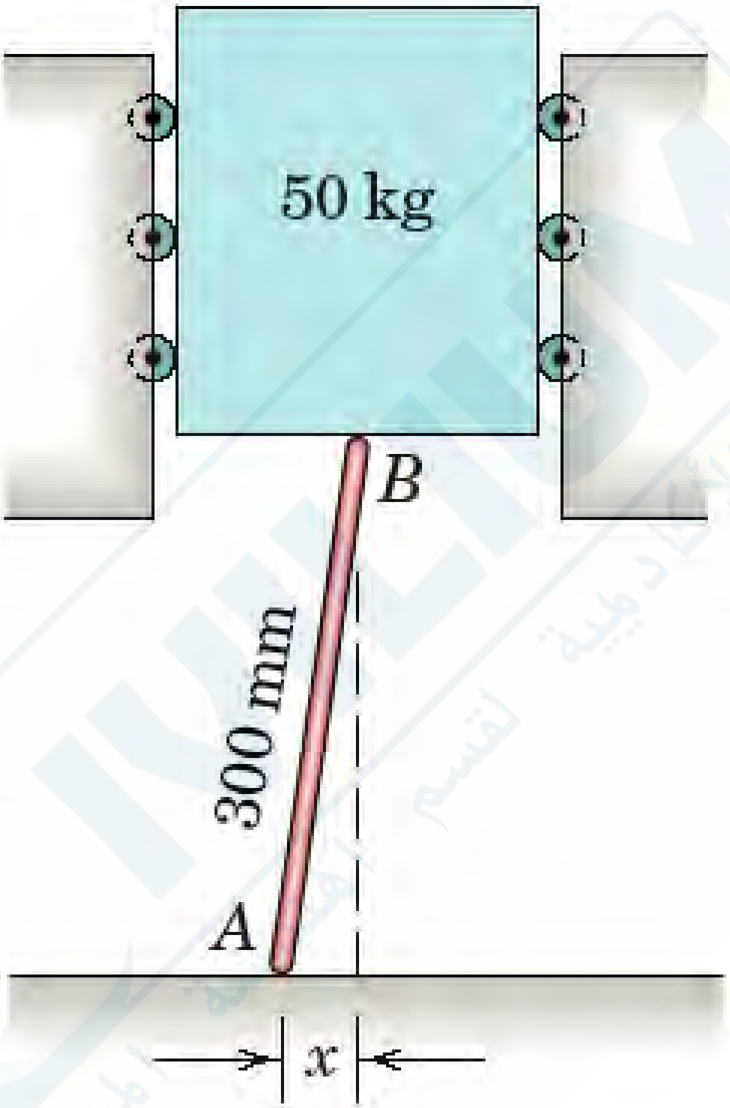
Problem 6/4

P_06_04



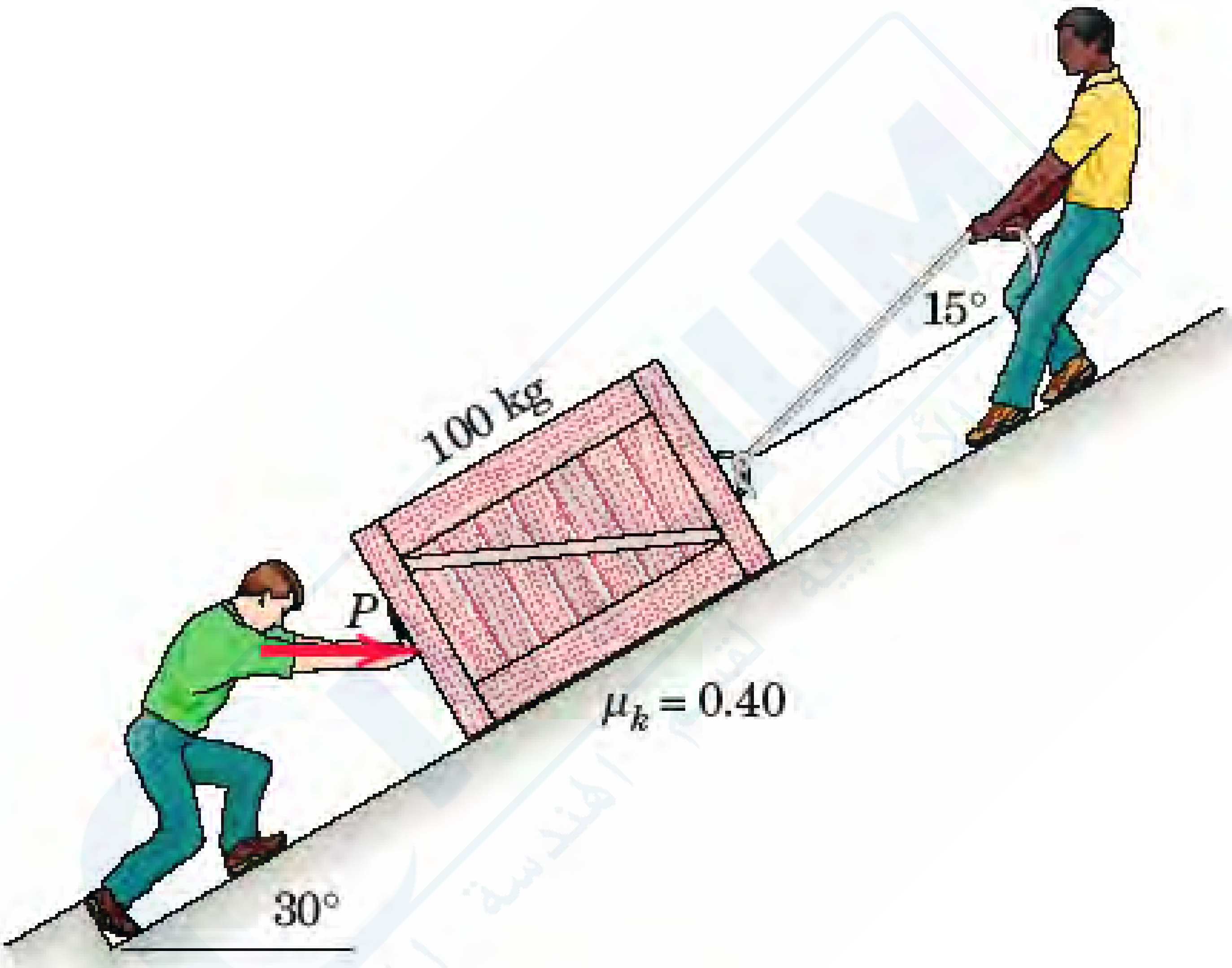
Problem 6/5

P_06_05



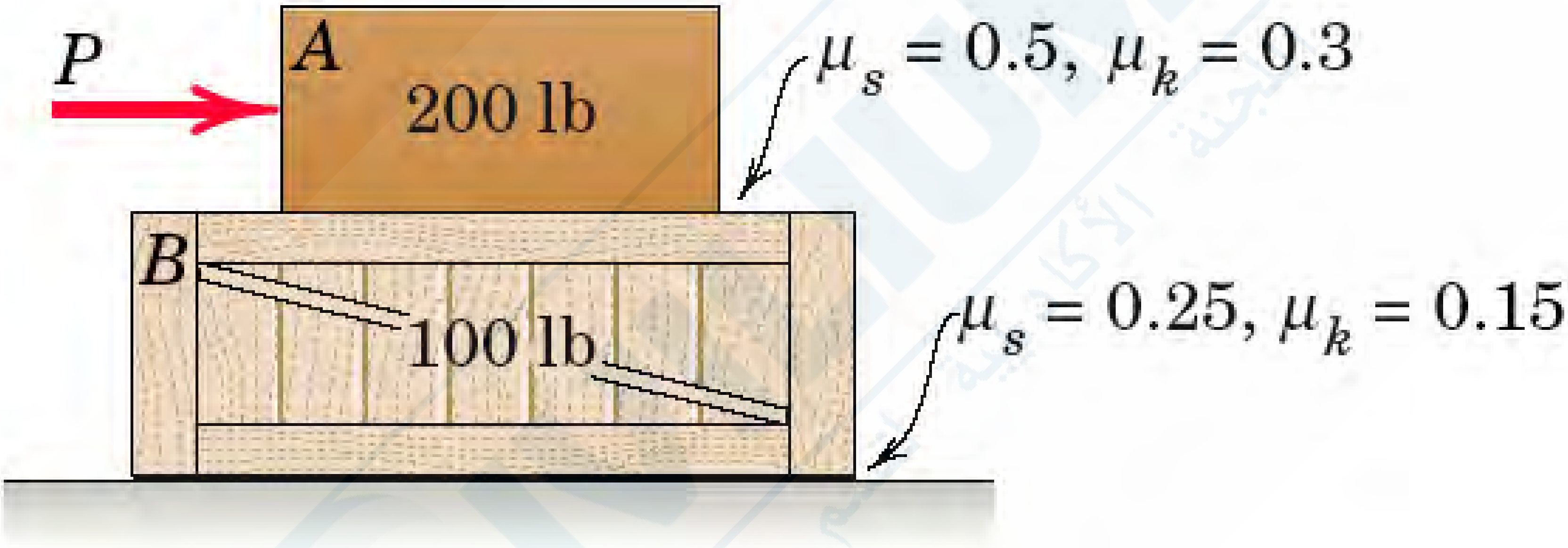
Problem 6/6

P_06_06



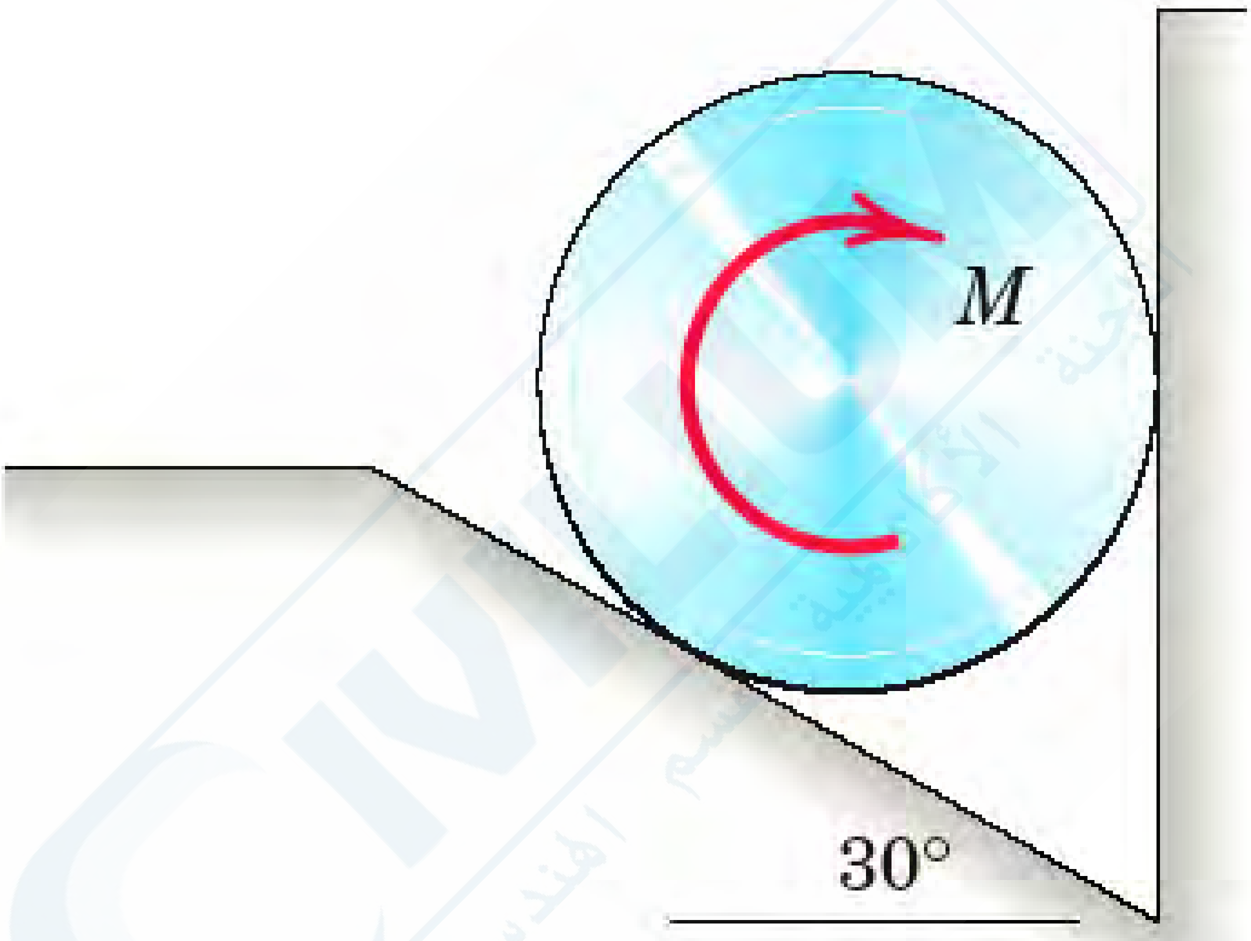
Problem 6/7

P_06_07



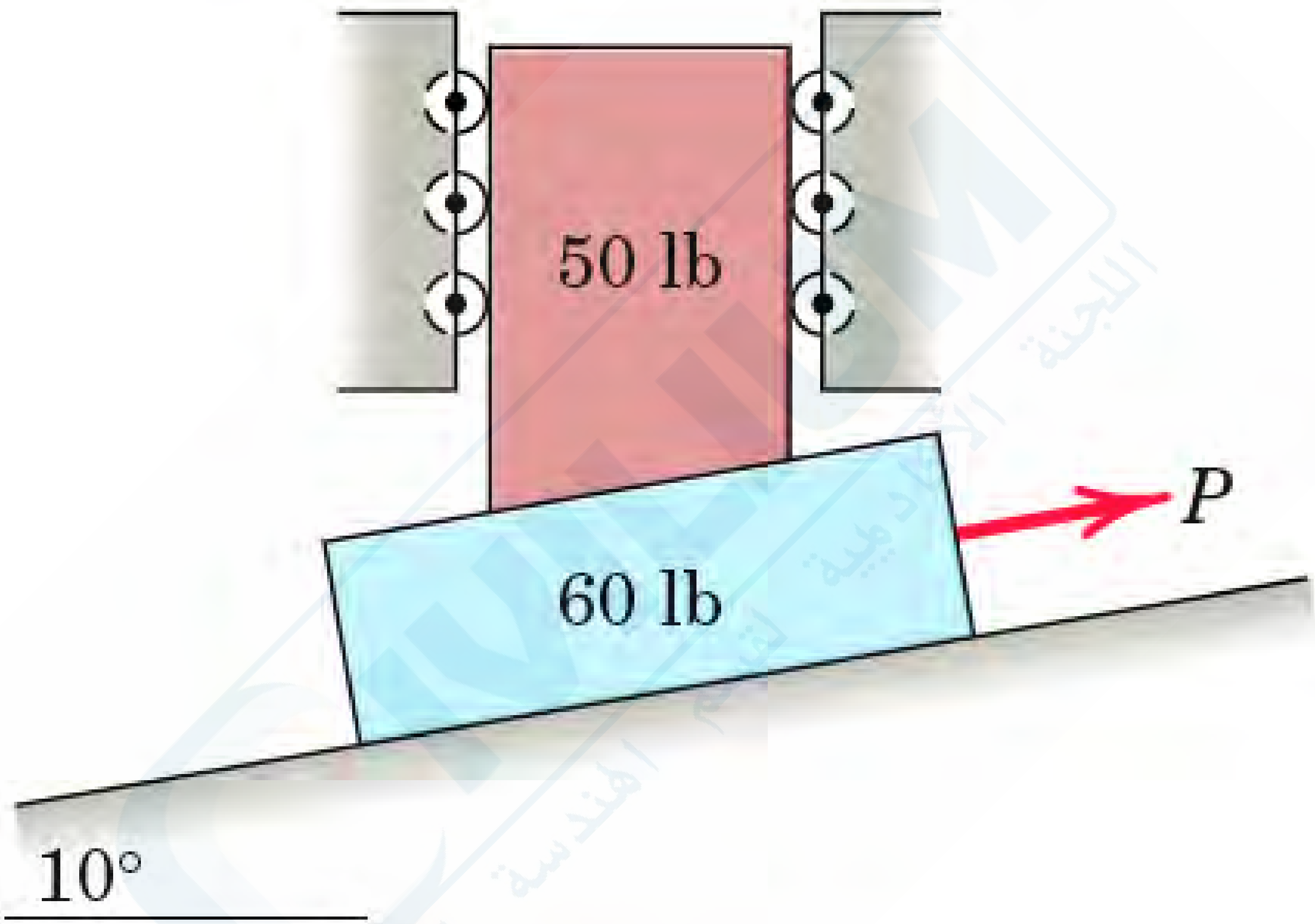
Problem 6/8

P_06_08



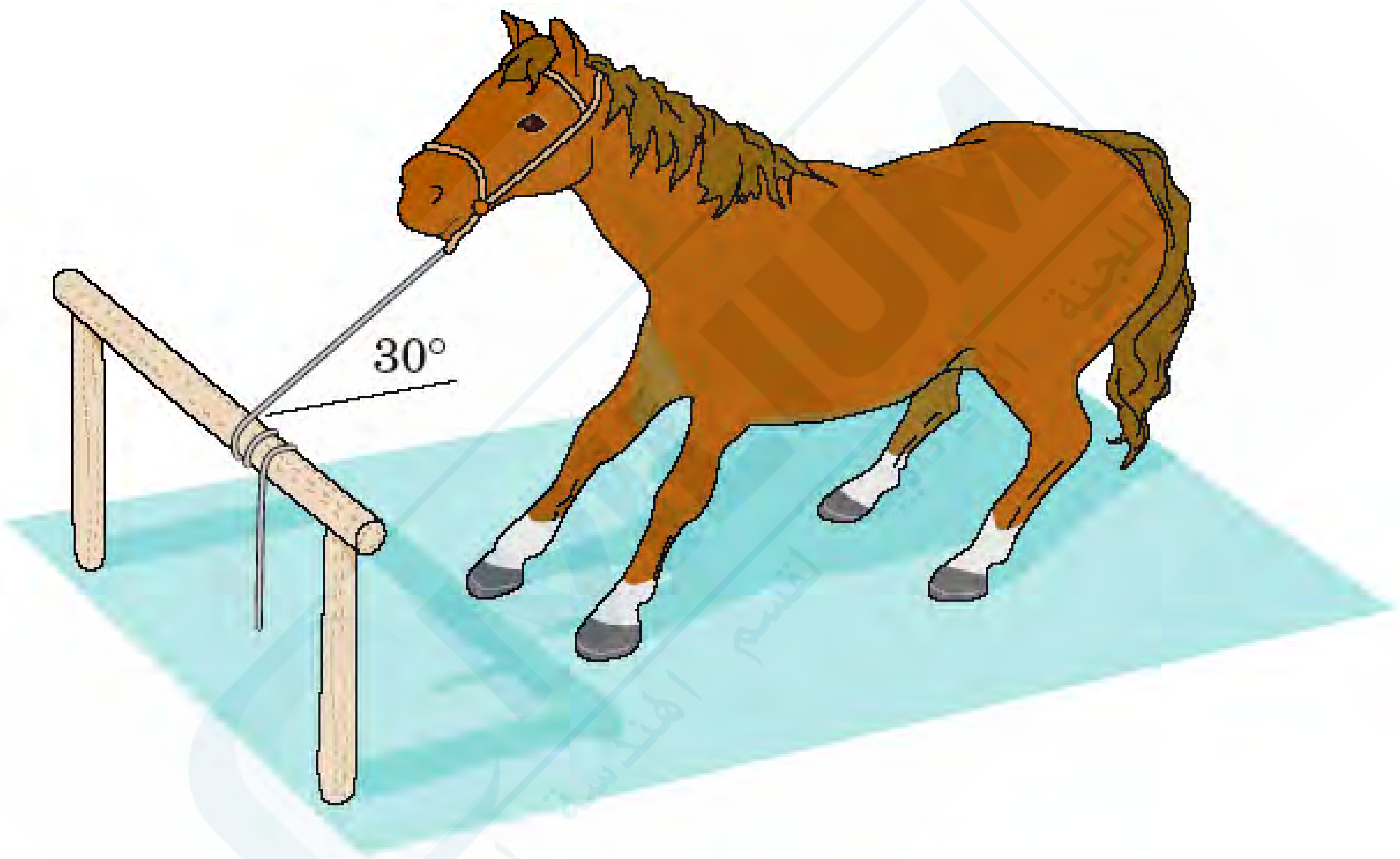
Problem 6/9

P_06_09



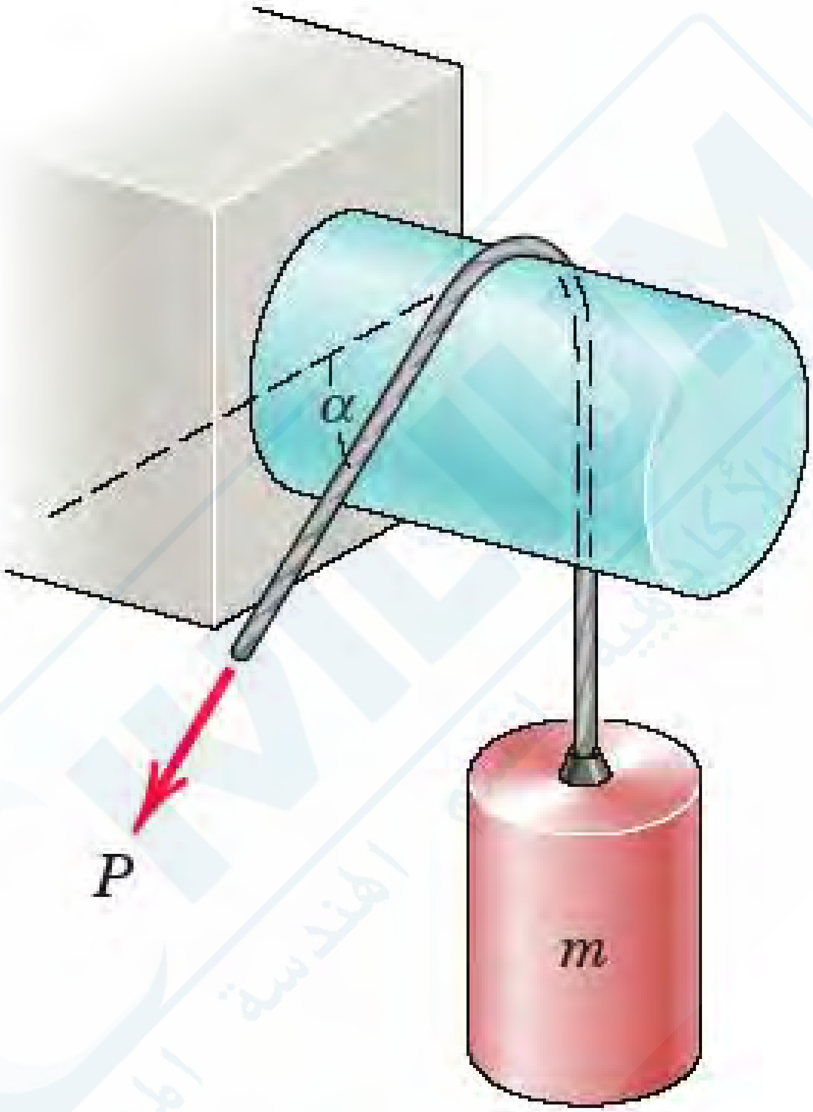
Problem 6/10

P_06_10



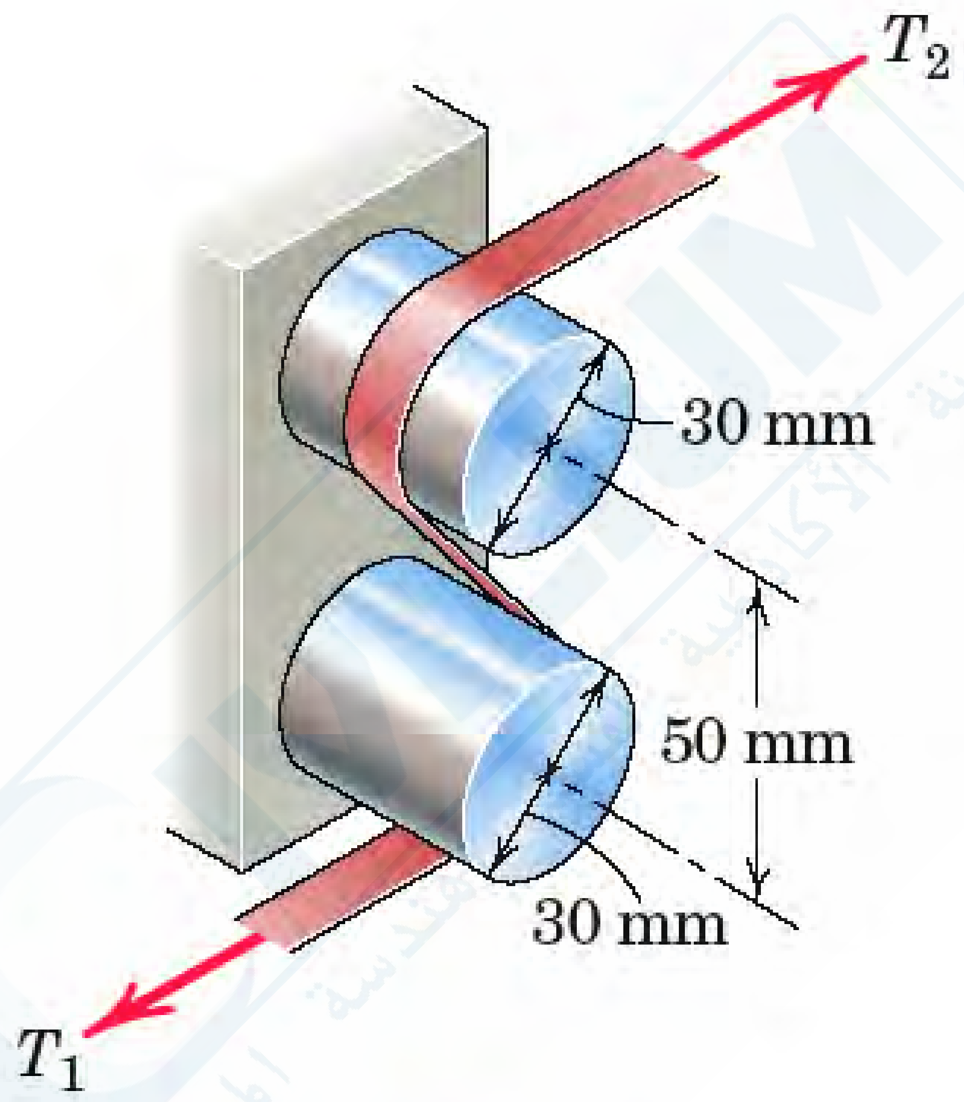
Problem 6/100

P_06_100



Problem 6/101

P_06_101

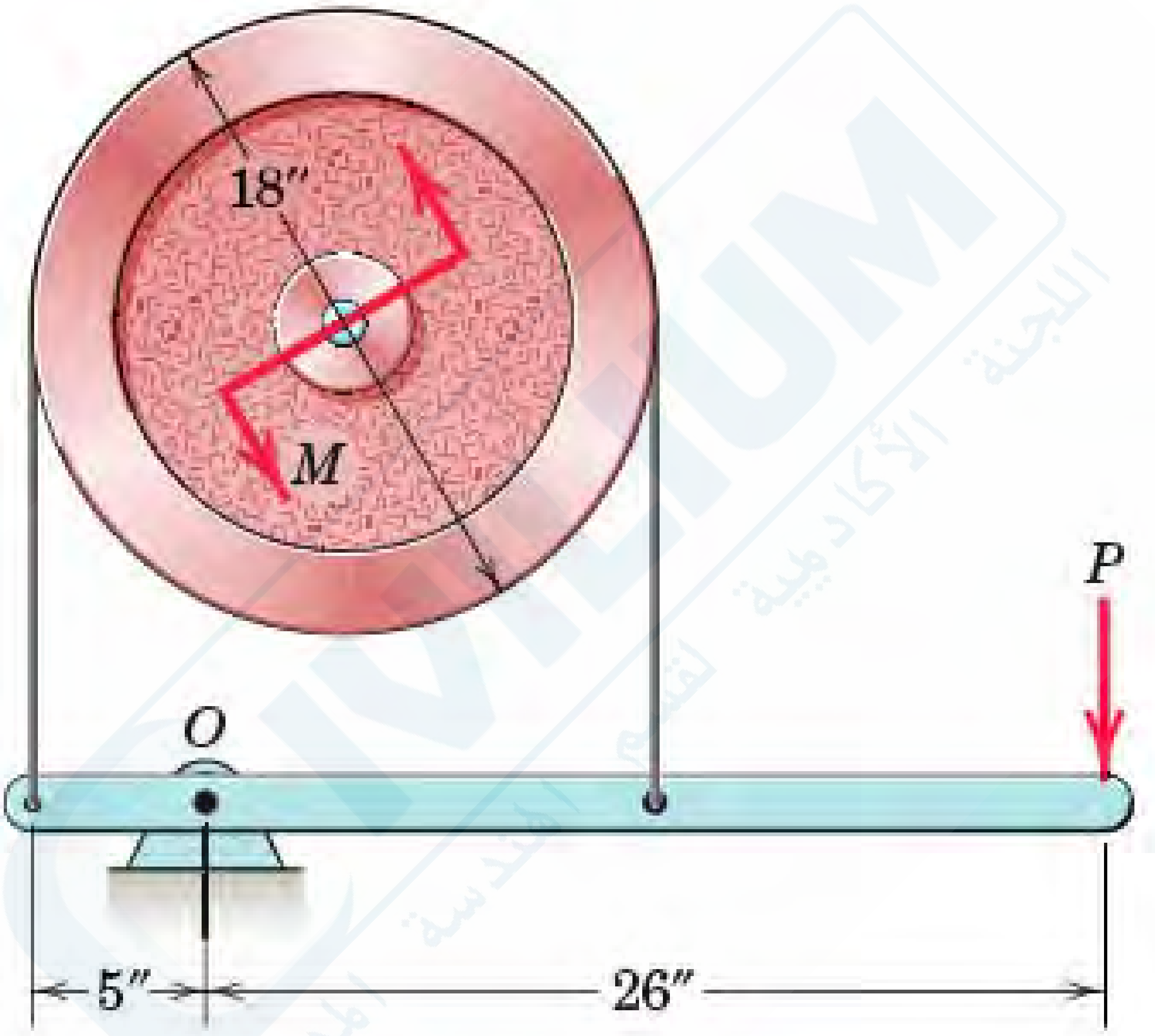


Problem 6/102

P_06_102

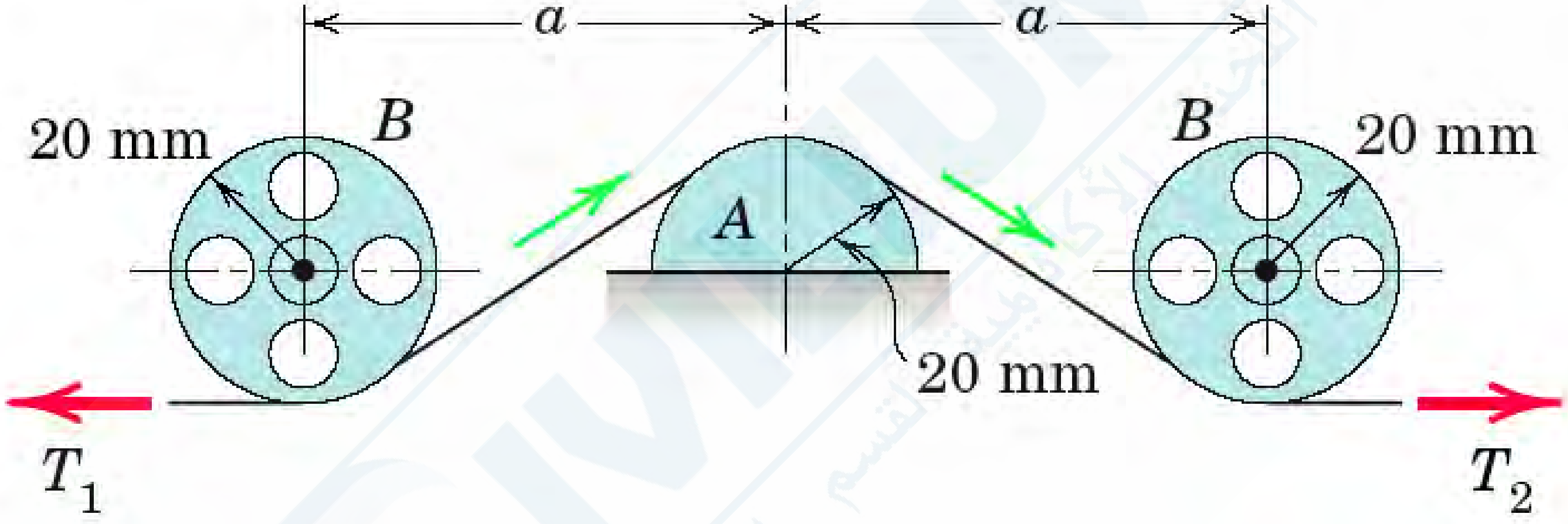
Problem 6/103

P_06_103



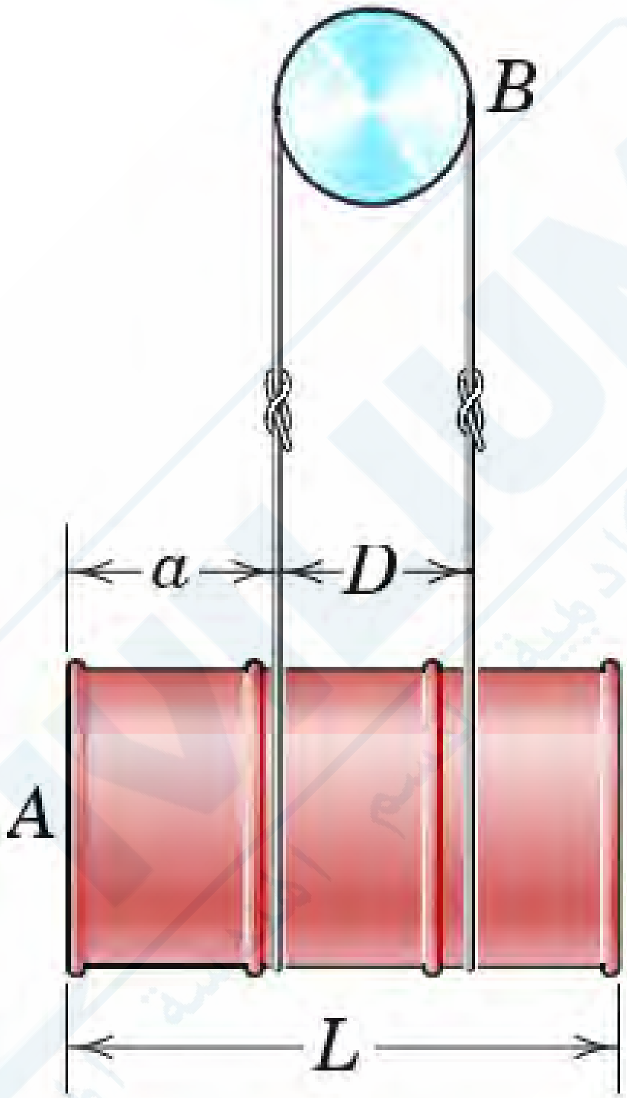
Problem 6/104

P_06_104



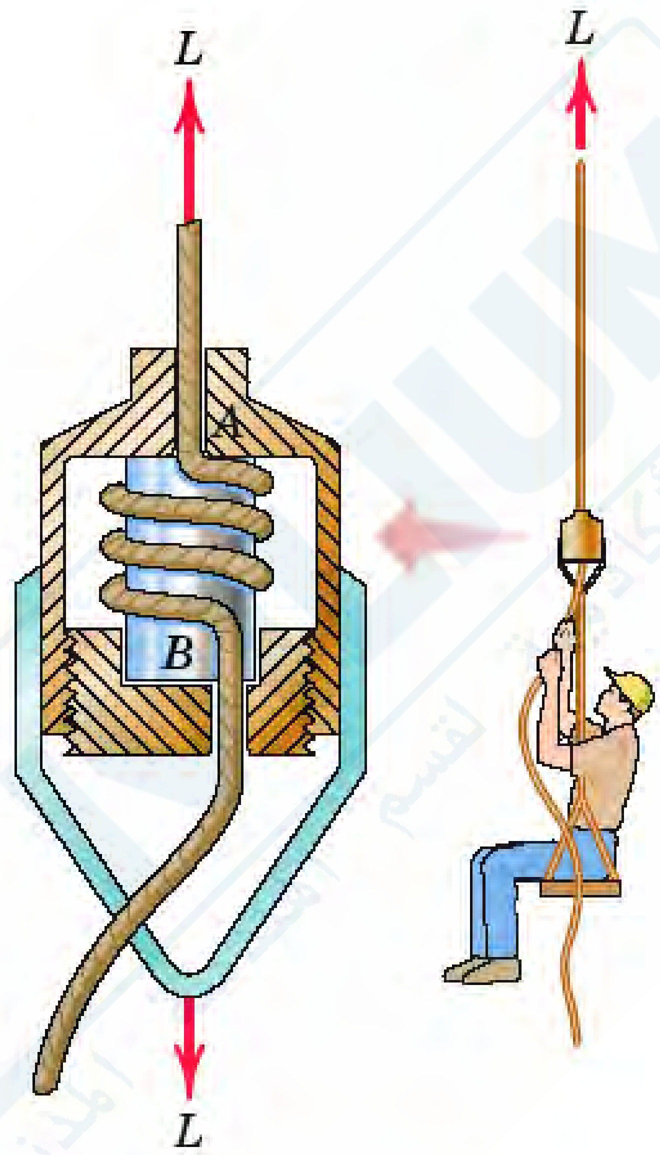
Problem 6/105

P_06_105



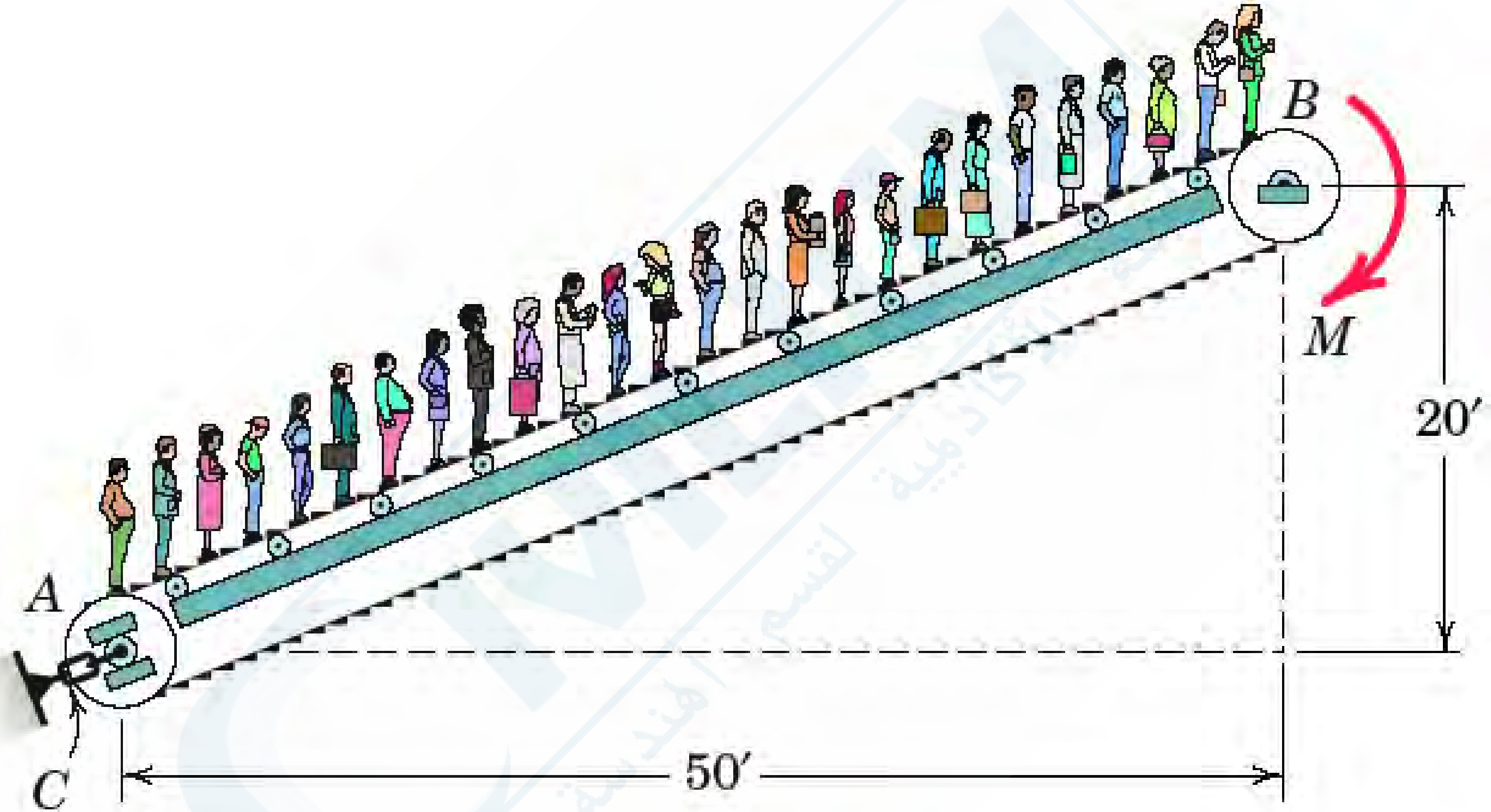
Problem 6/106

P_06_106



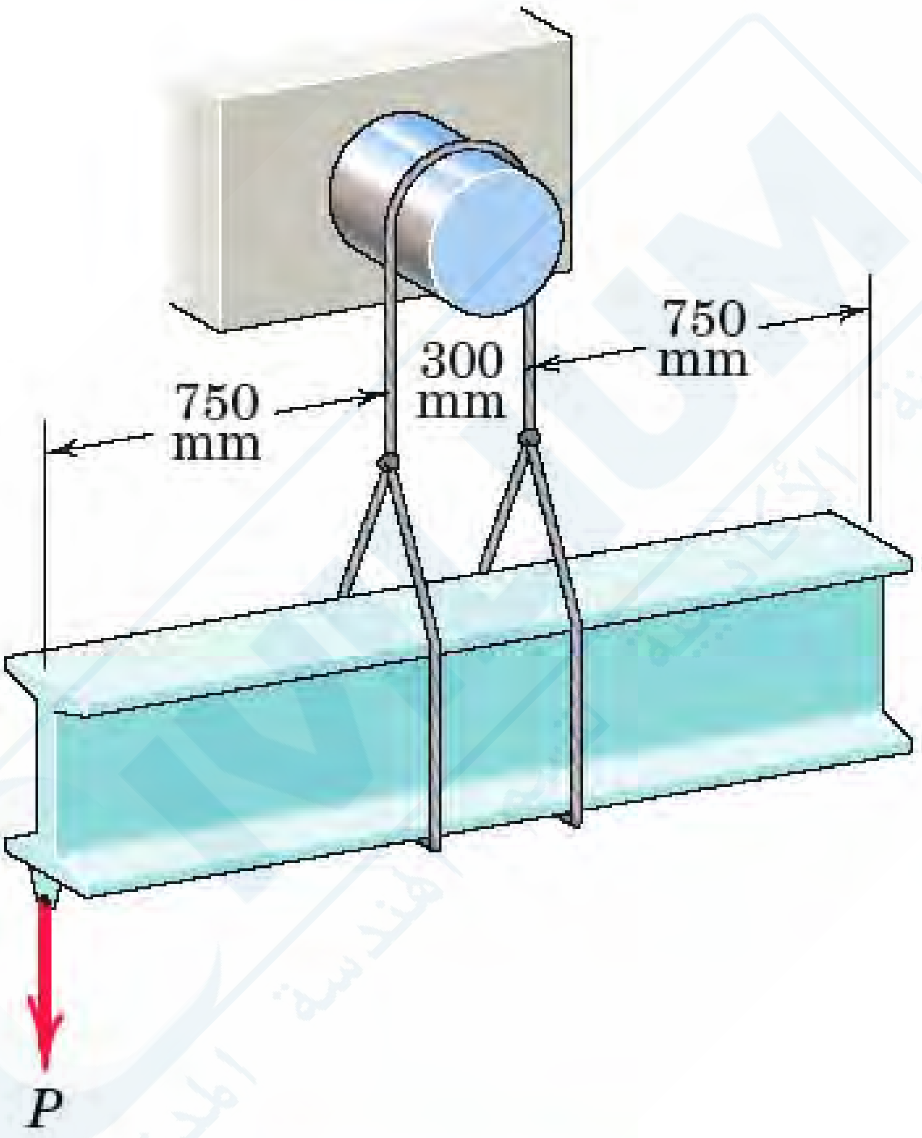
Problem 6/107

P_06_107



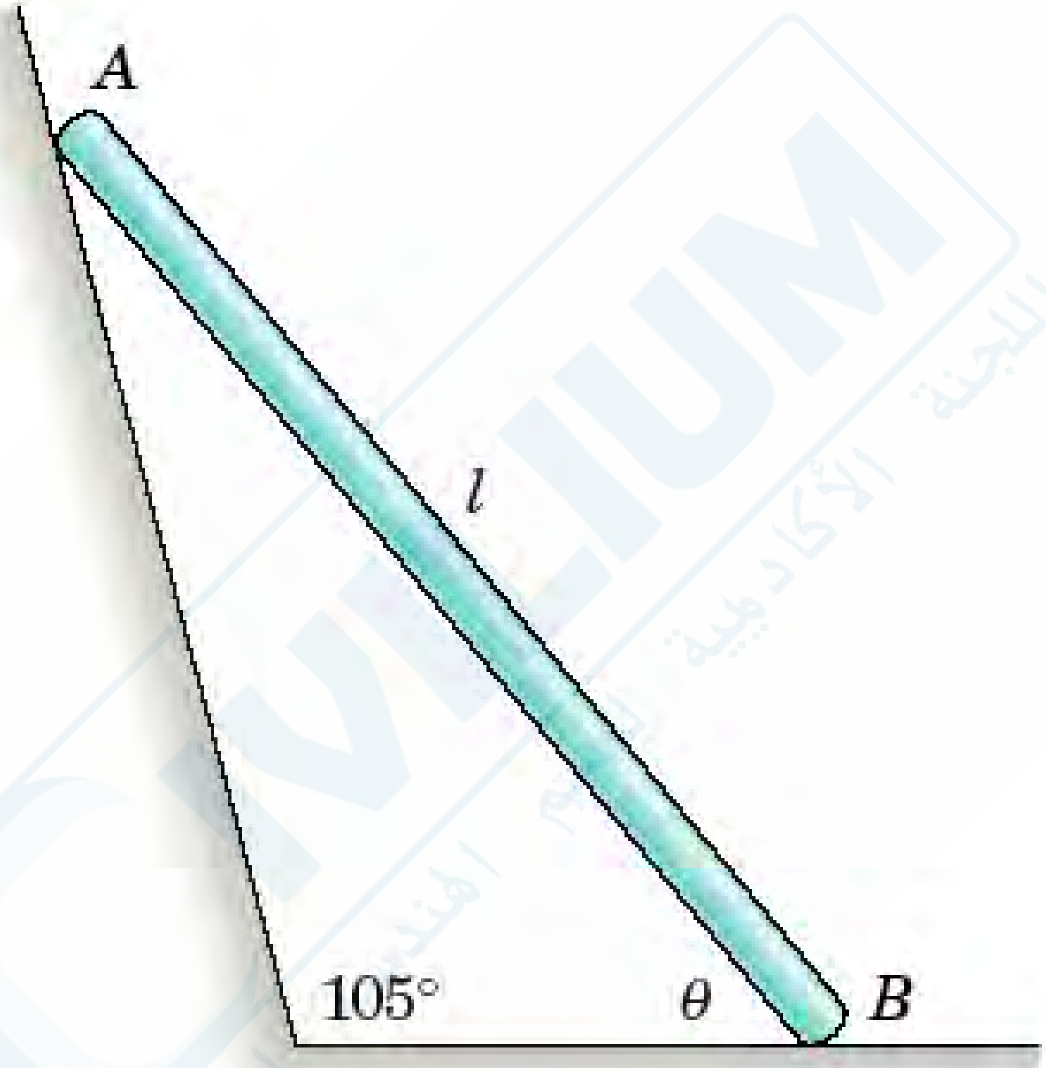
Problem 6/108

P_06_108



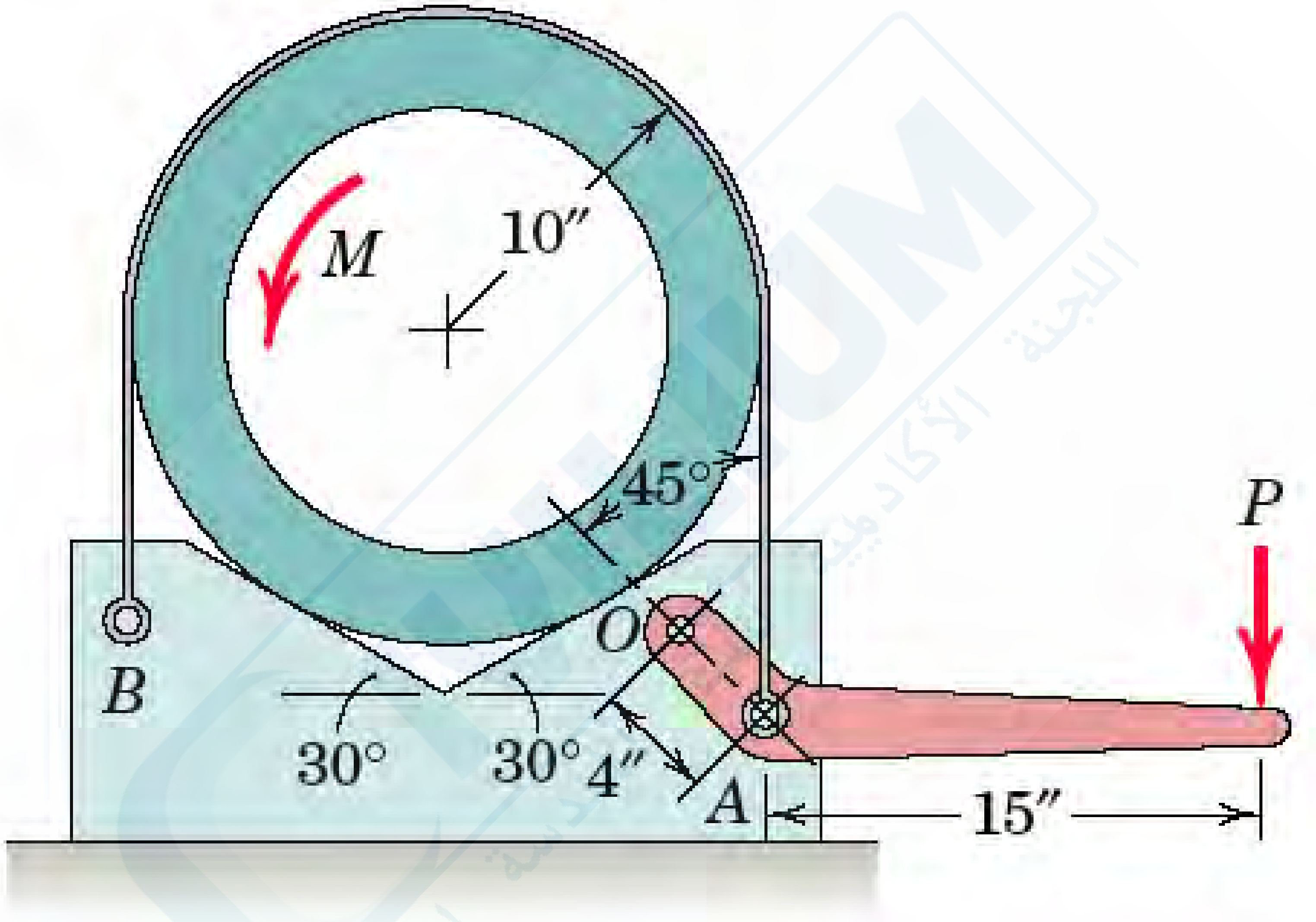
Problem 6/109

P_06_109



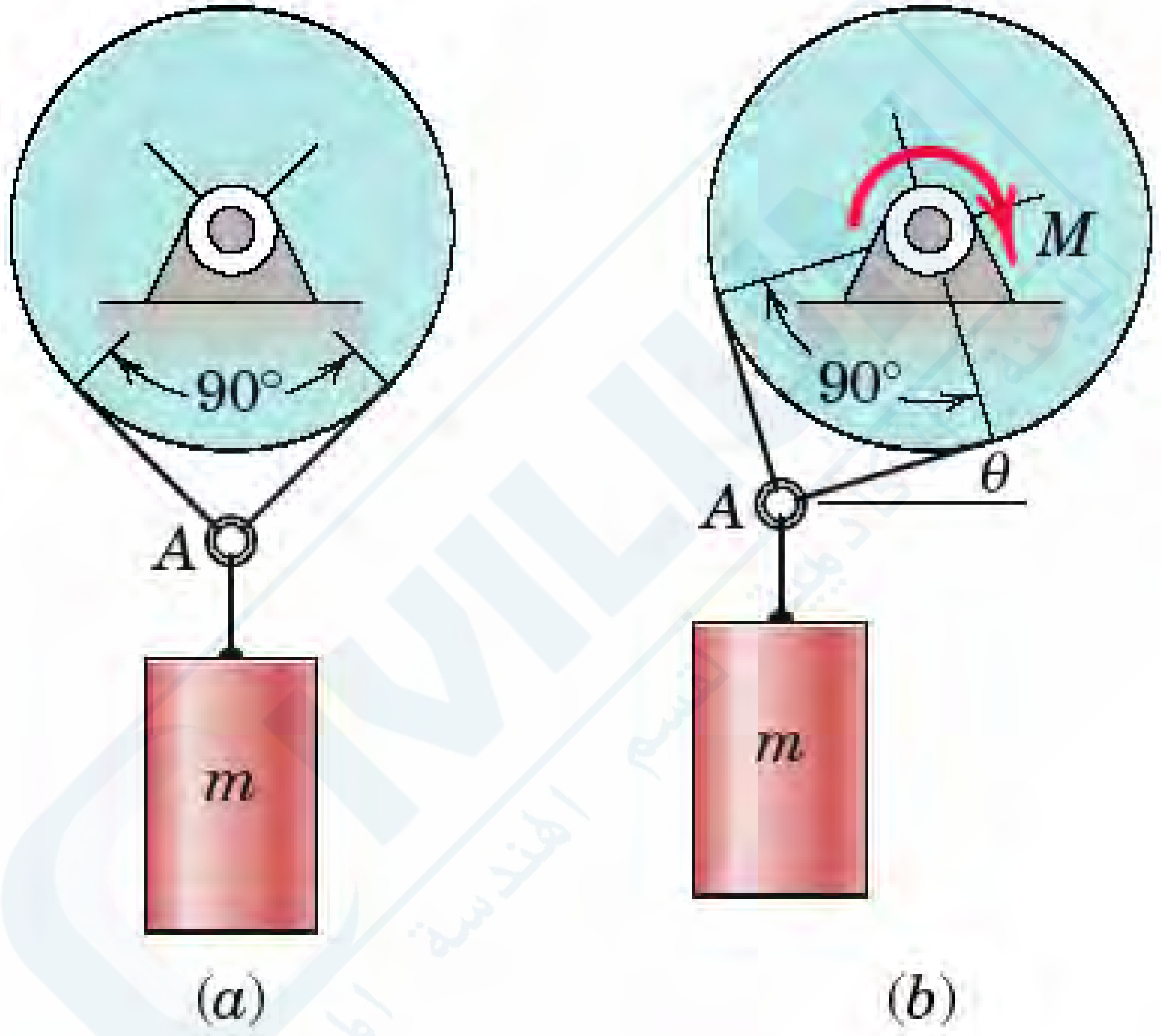
Problem 6/11

P_06_11



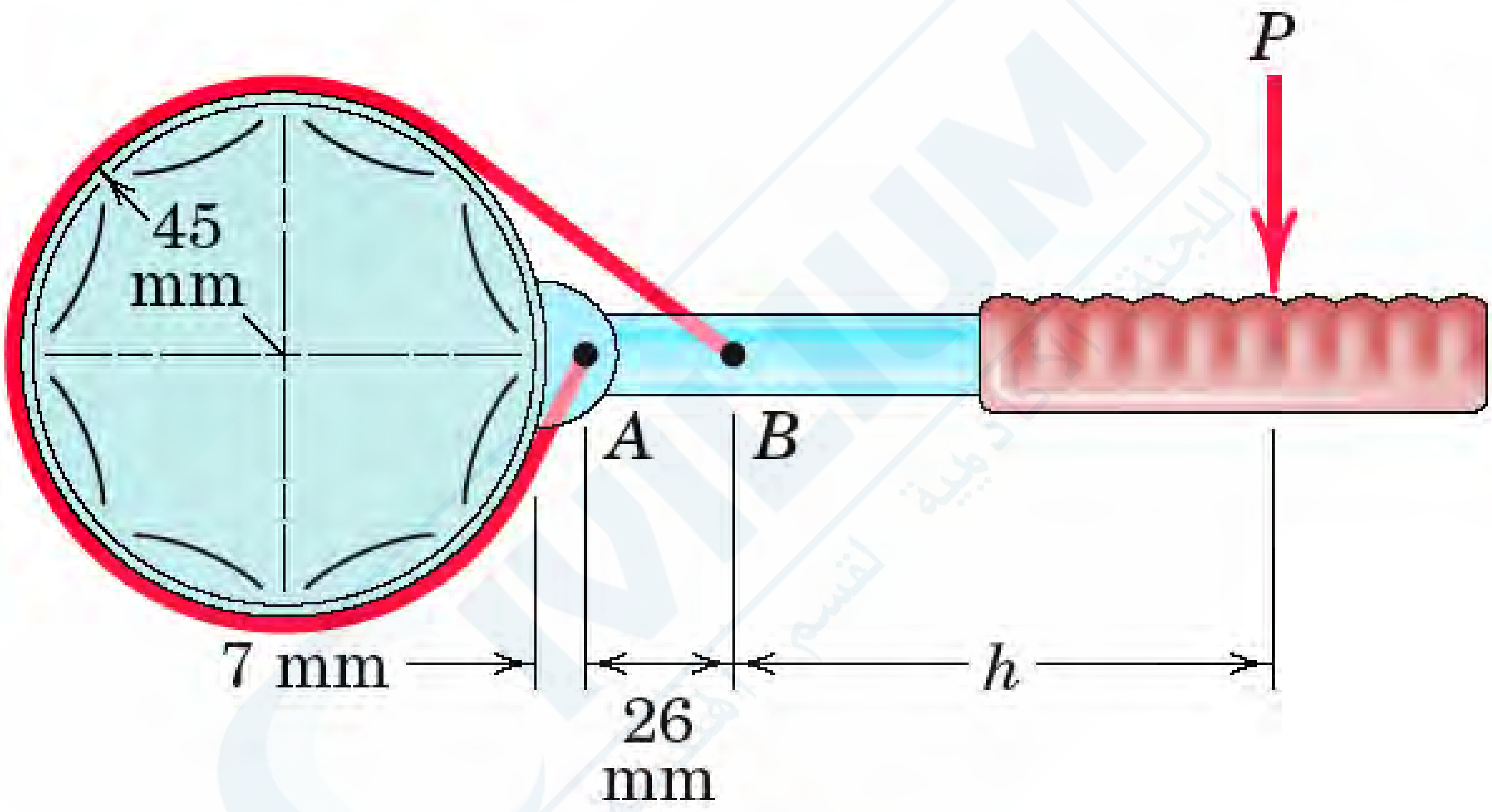
Problem 6/110

P_06_110



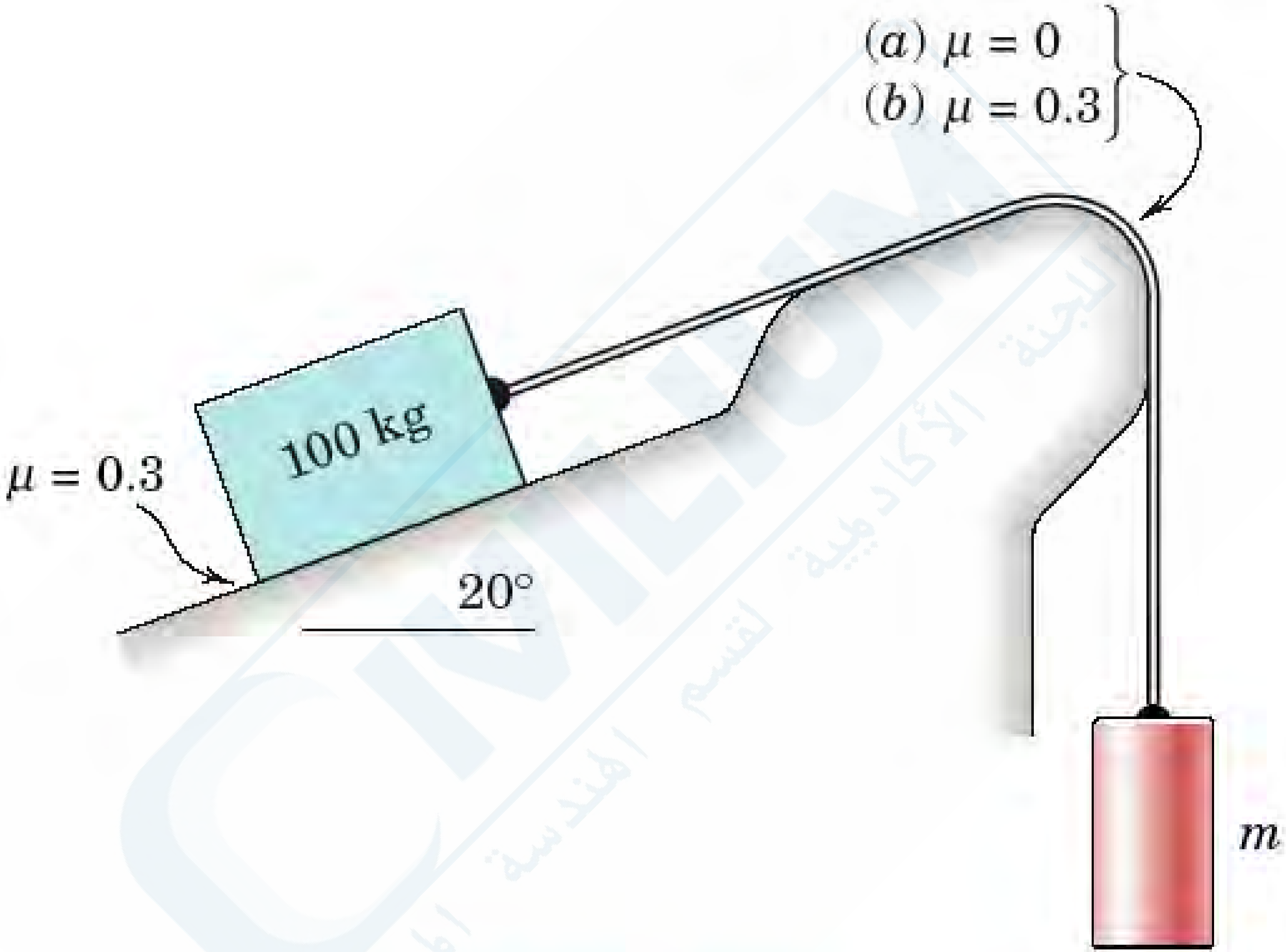
Problem 6/111

P_06_111



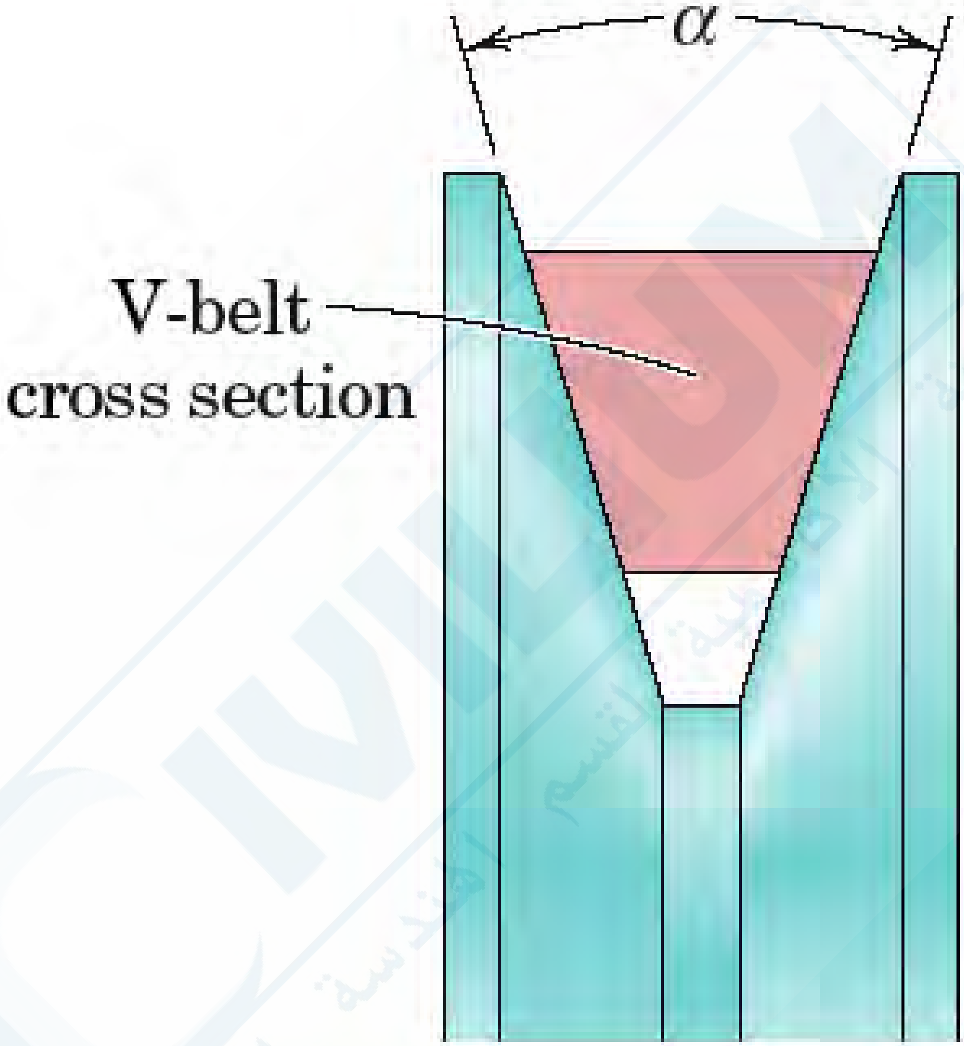
Problem 6/112

P_06_112



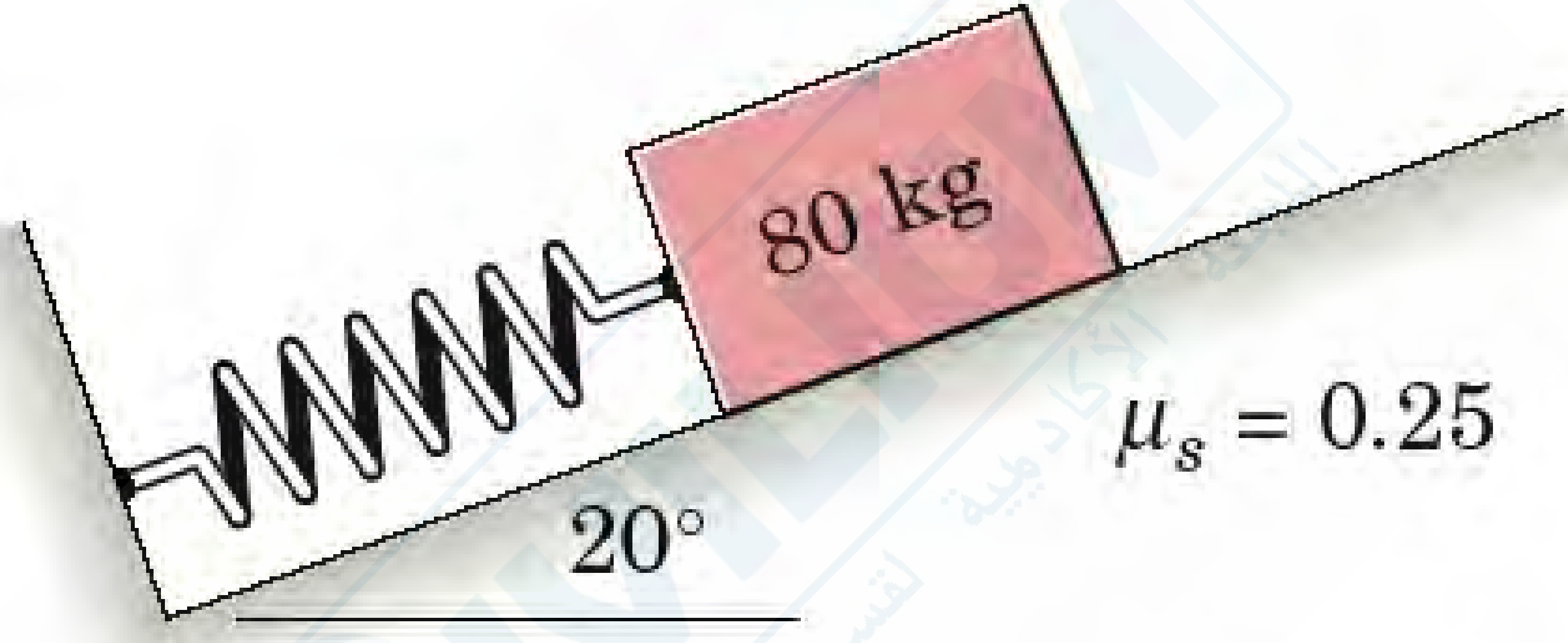
Problem 6/113

P_06_113



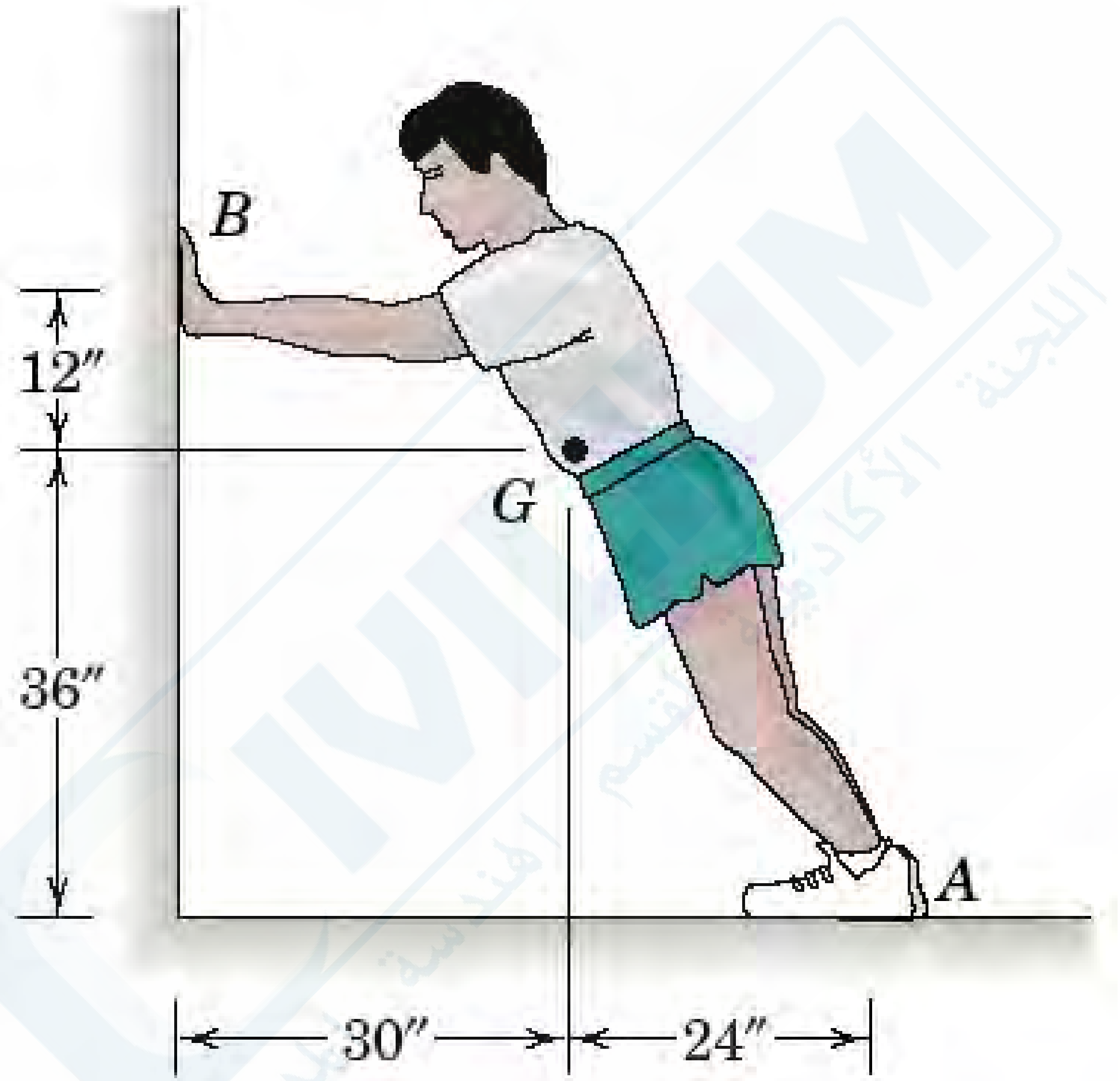
Problem 6/114

P_06_114



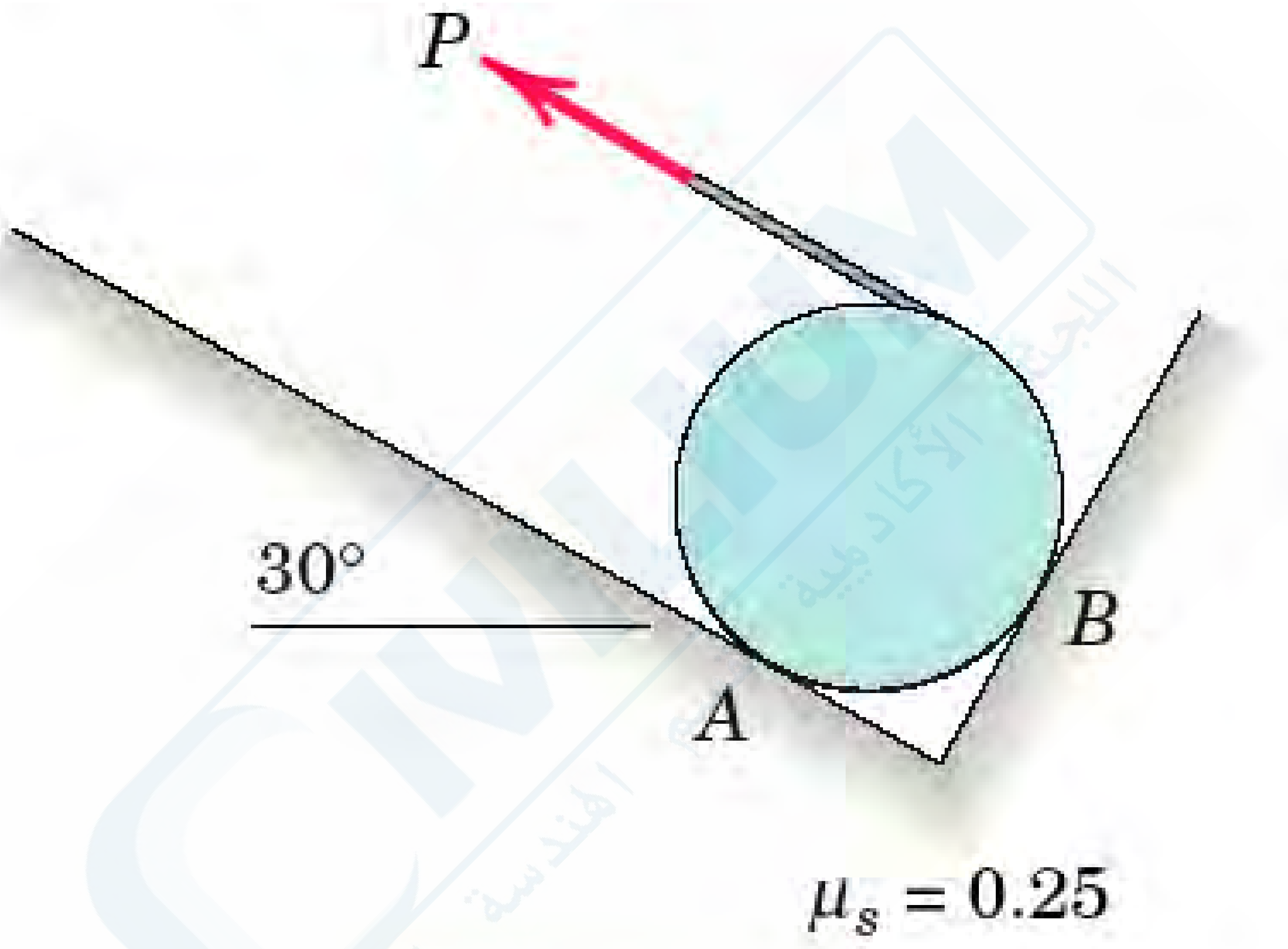
Problem 6/115

P_06_115



Problem 6/116

P_06_116



Problem 6/117

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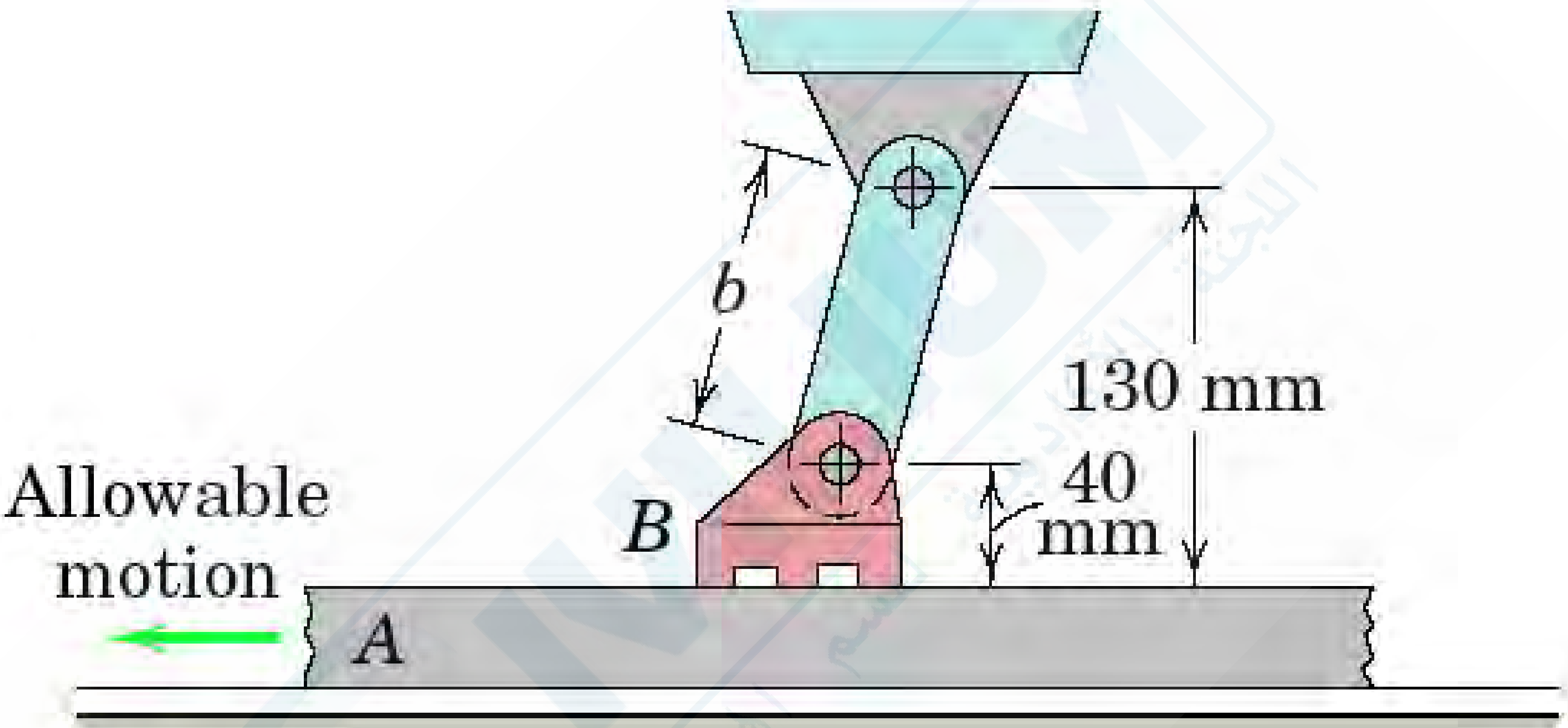
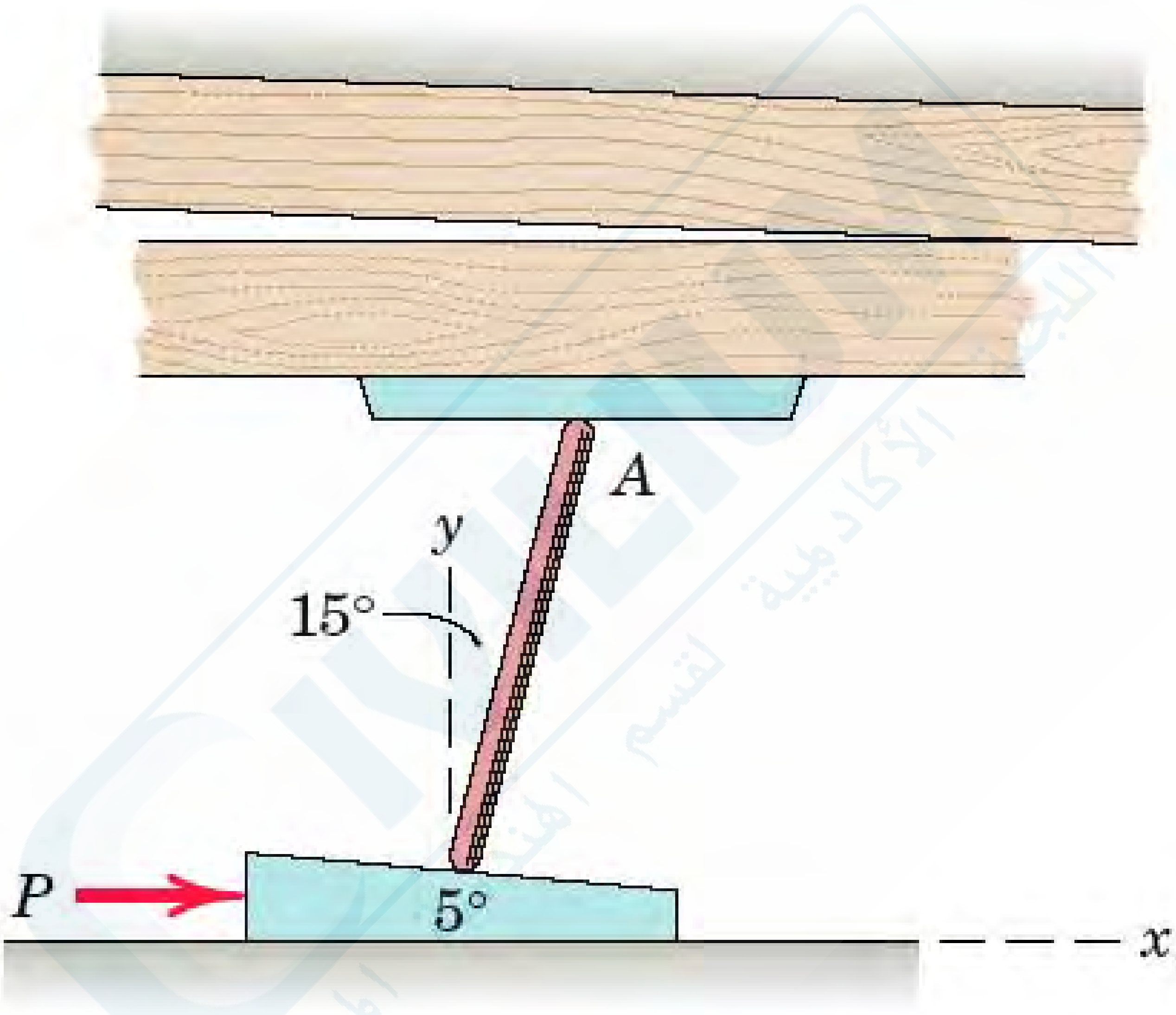
P_06_117



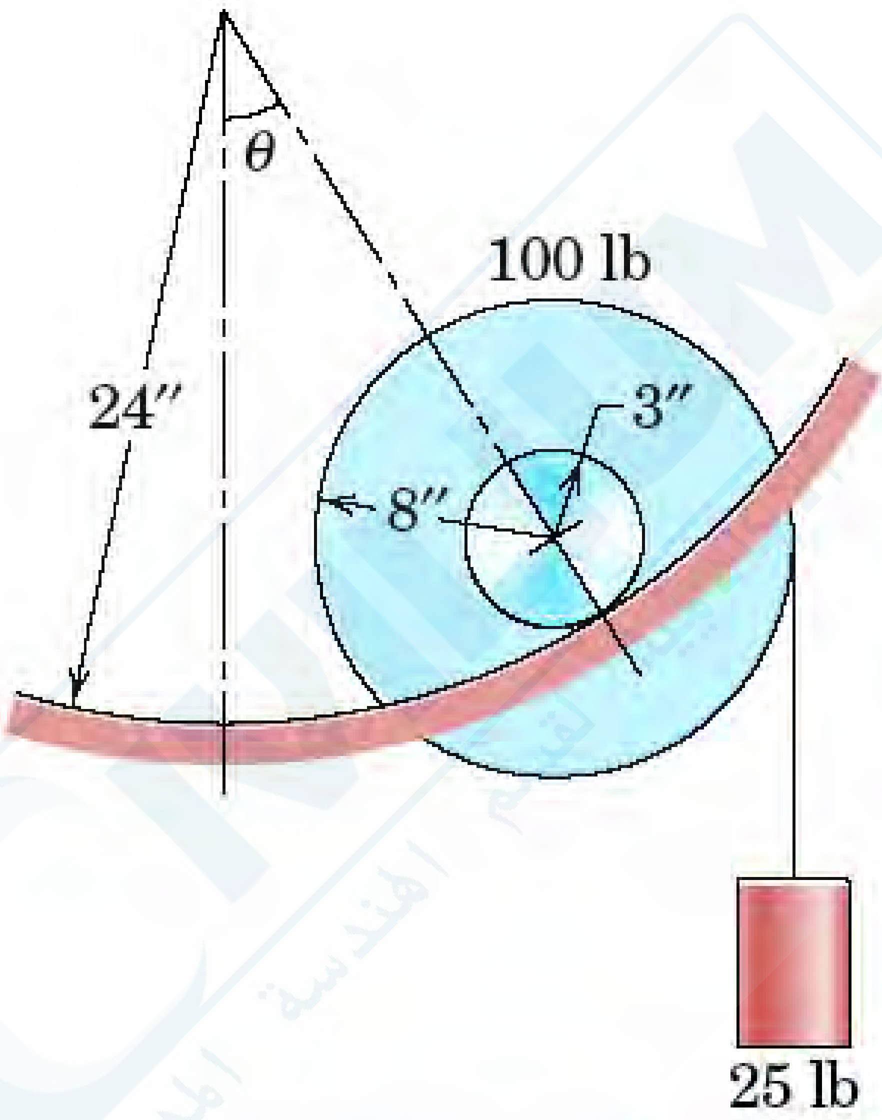
Problem 6/118

P_06_118



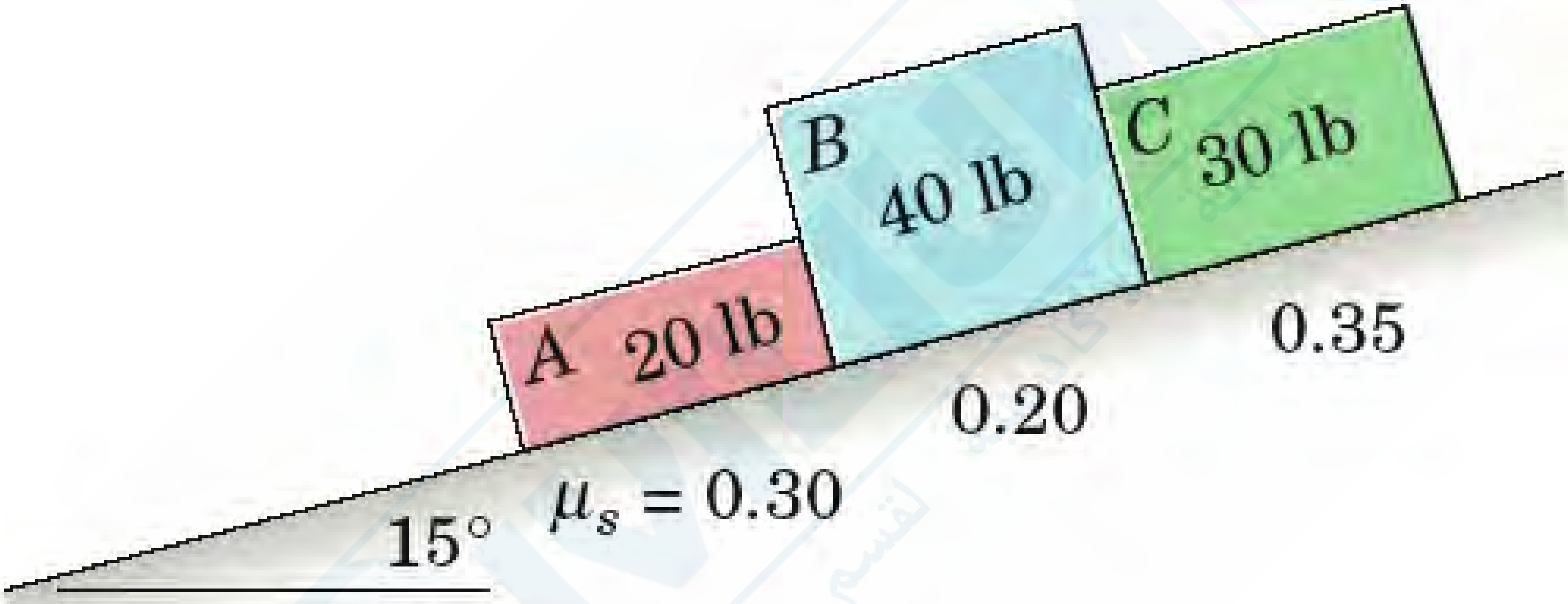
Problem 6/119

P_06_119



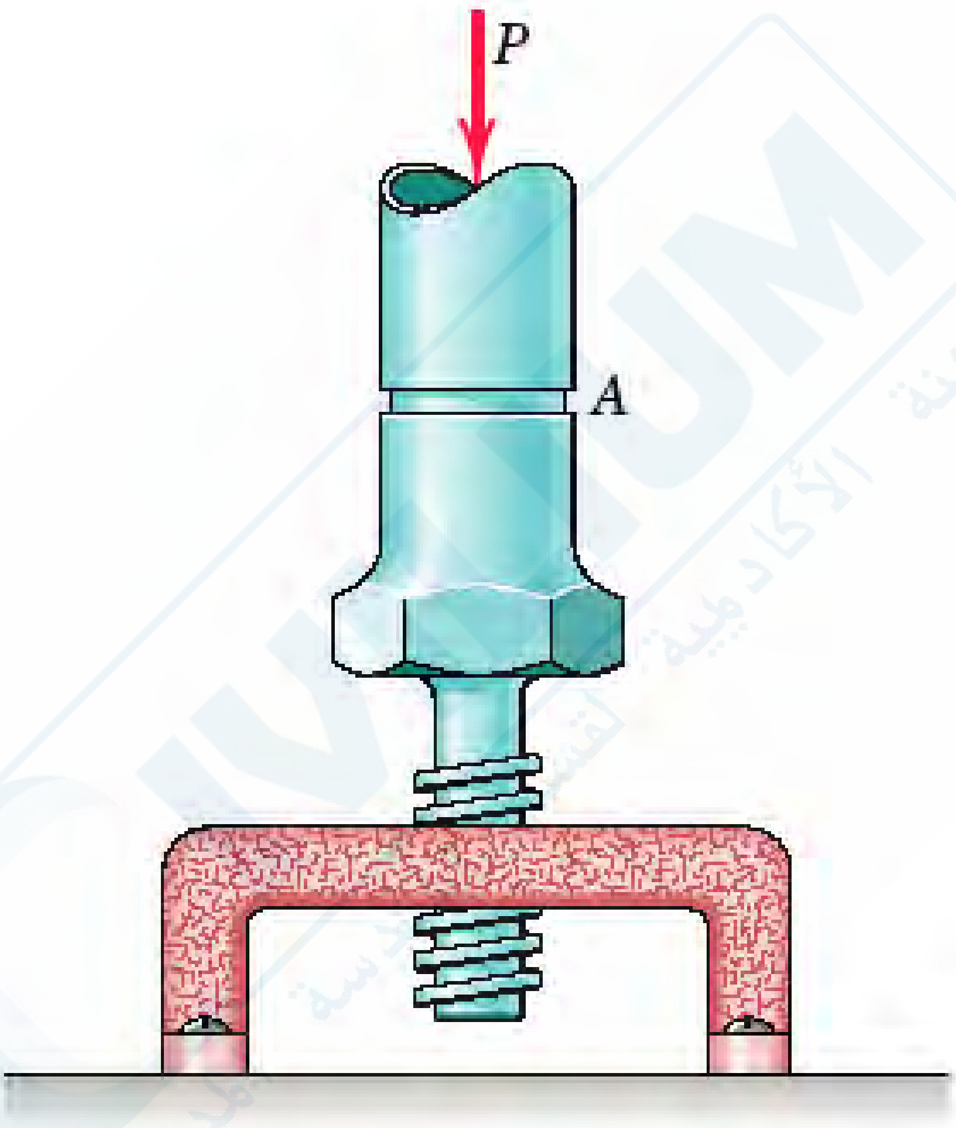
Problem 6/12

P_06_12



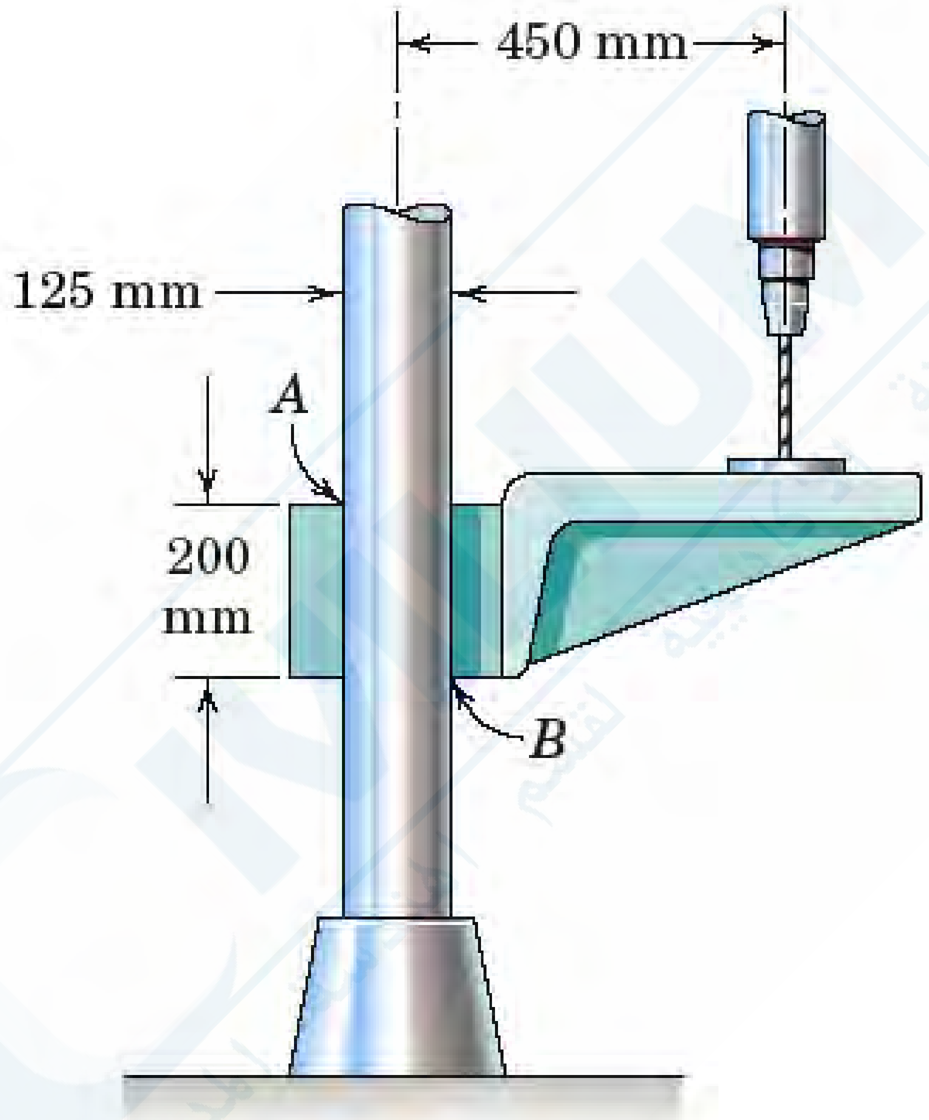
Problem 6/120

P_06_120



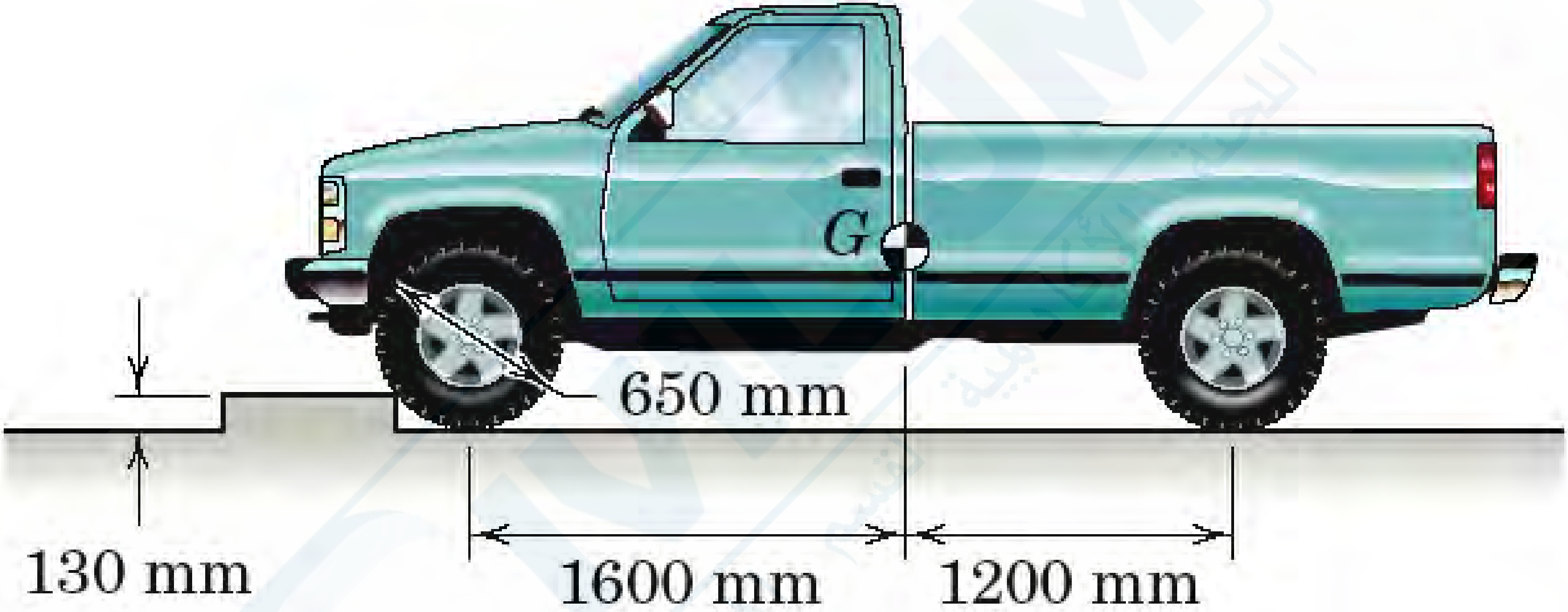
Problem 6/121

P_06_121



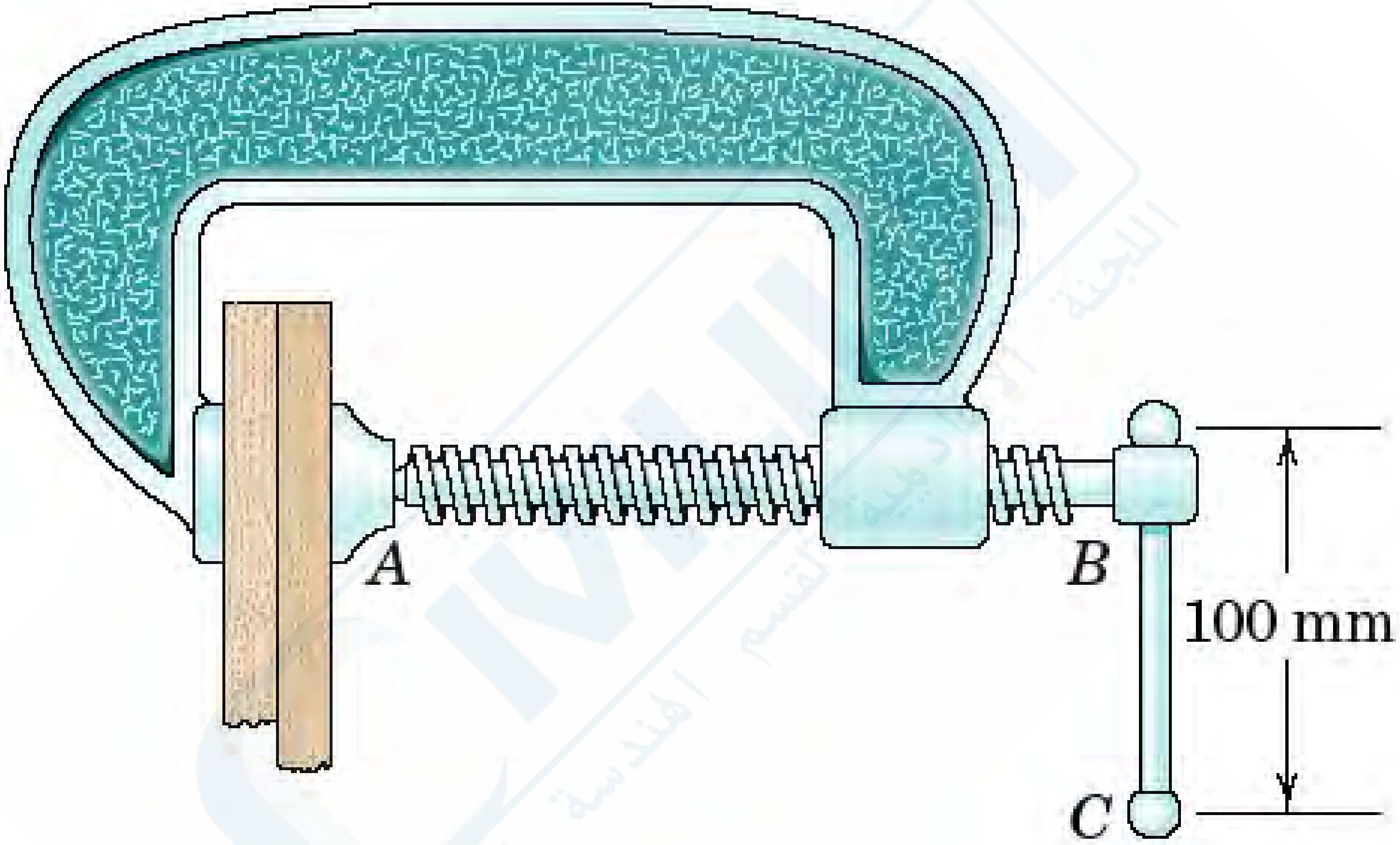
Problem 6/122

P_06_122



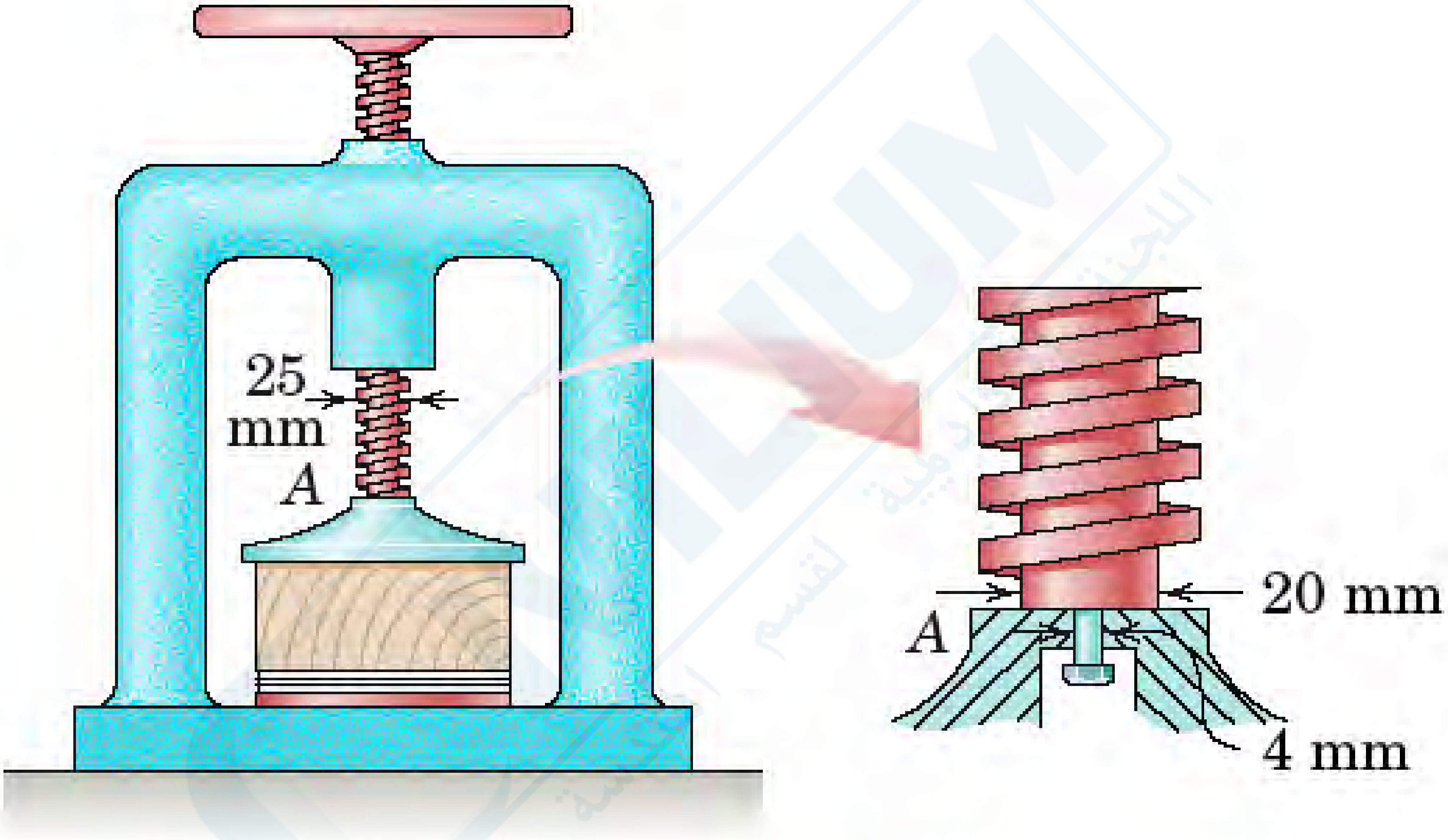
Problem 6/123

P_06_123



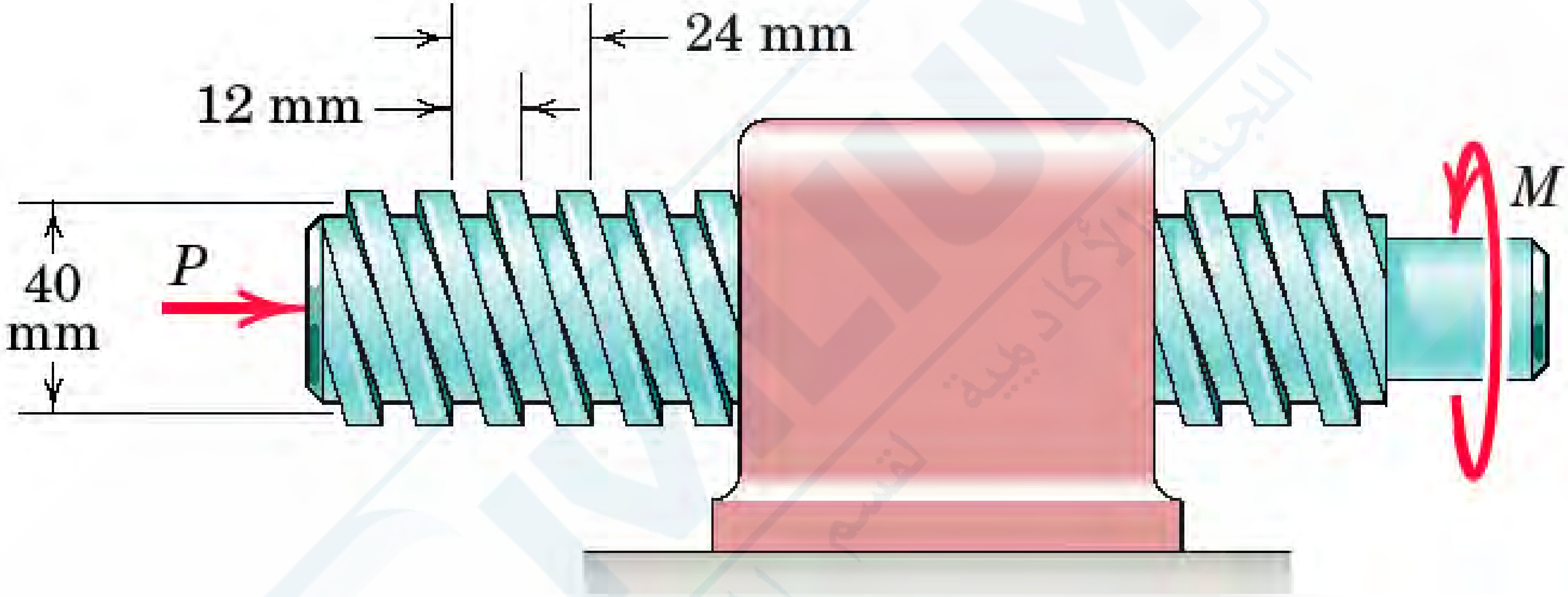
Problem 6/124

P_06_124



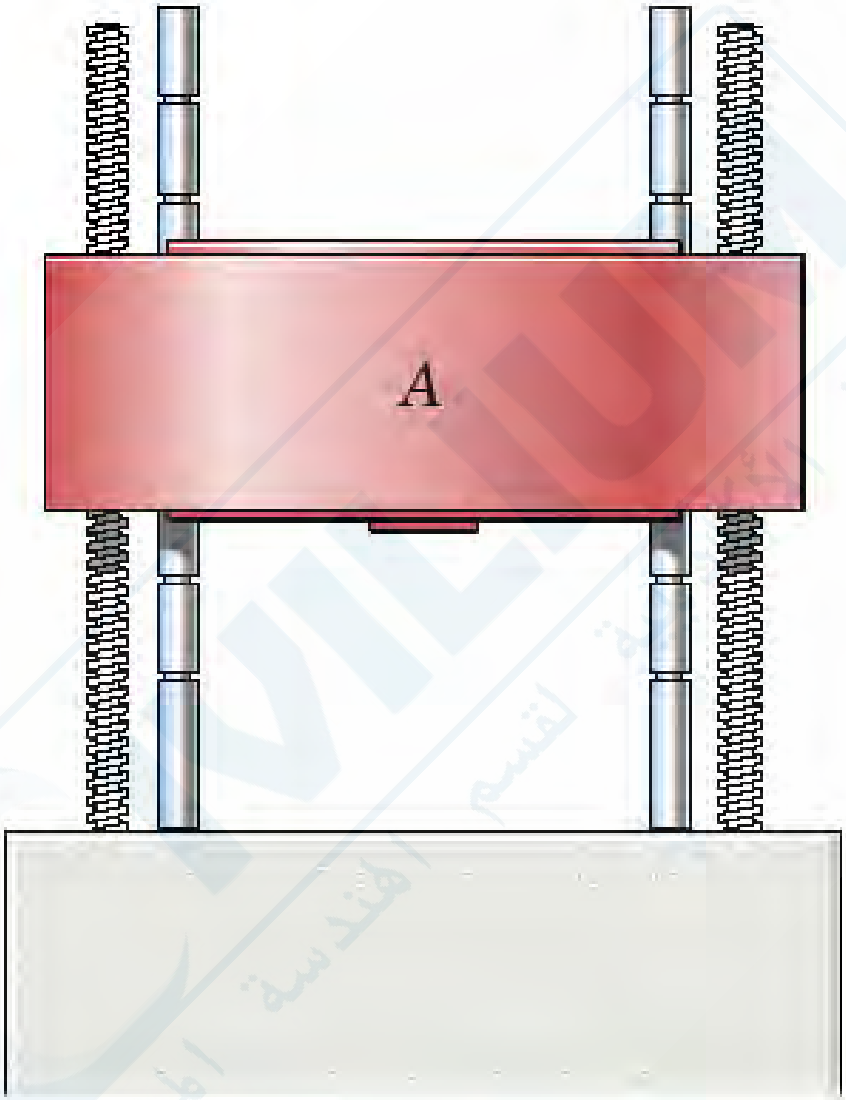
Problem 6/125

P_06_125



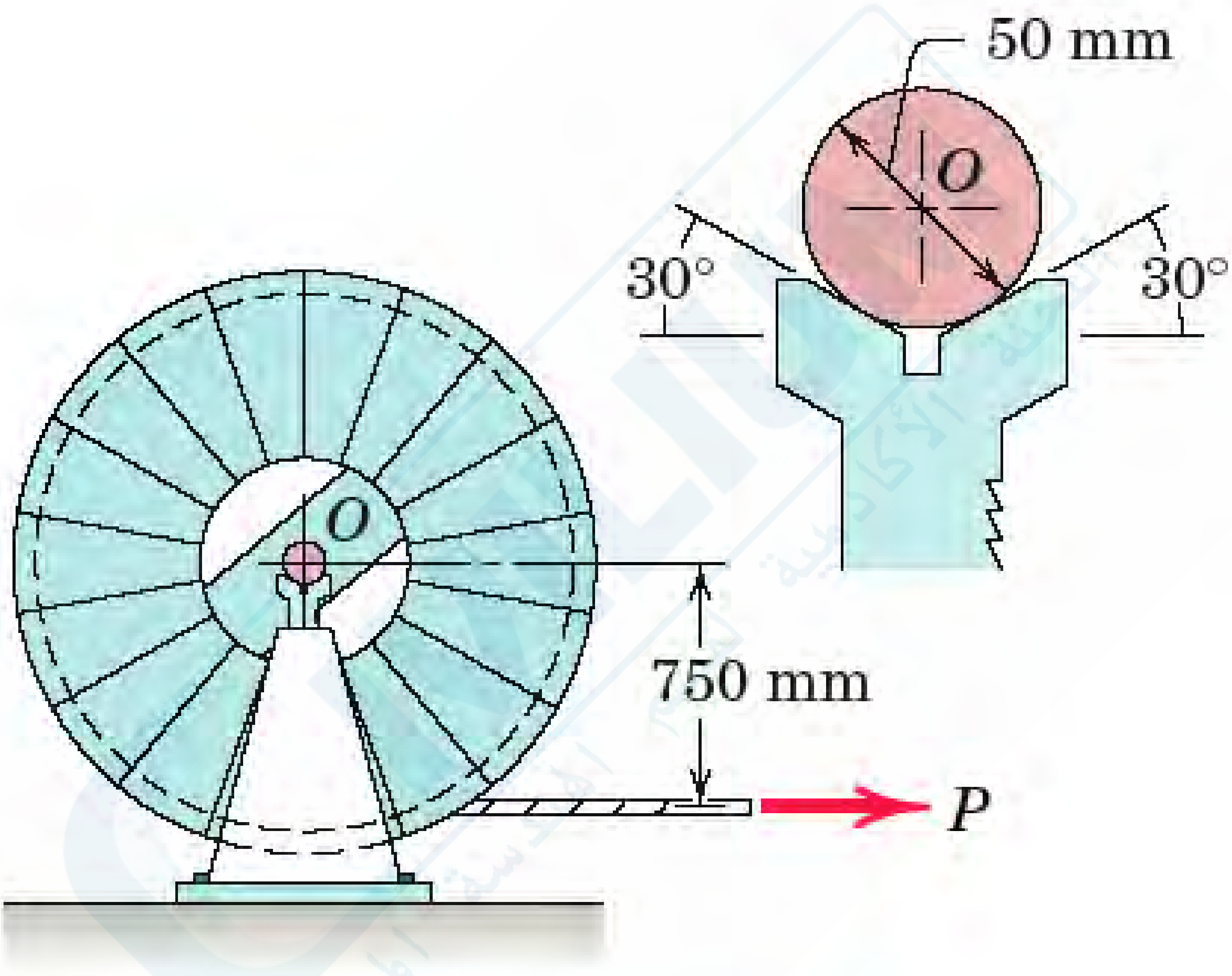
Problem 6/126

P_06_126



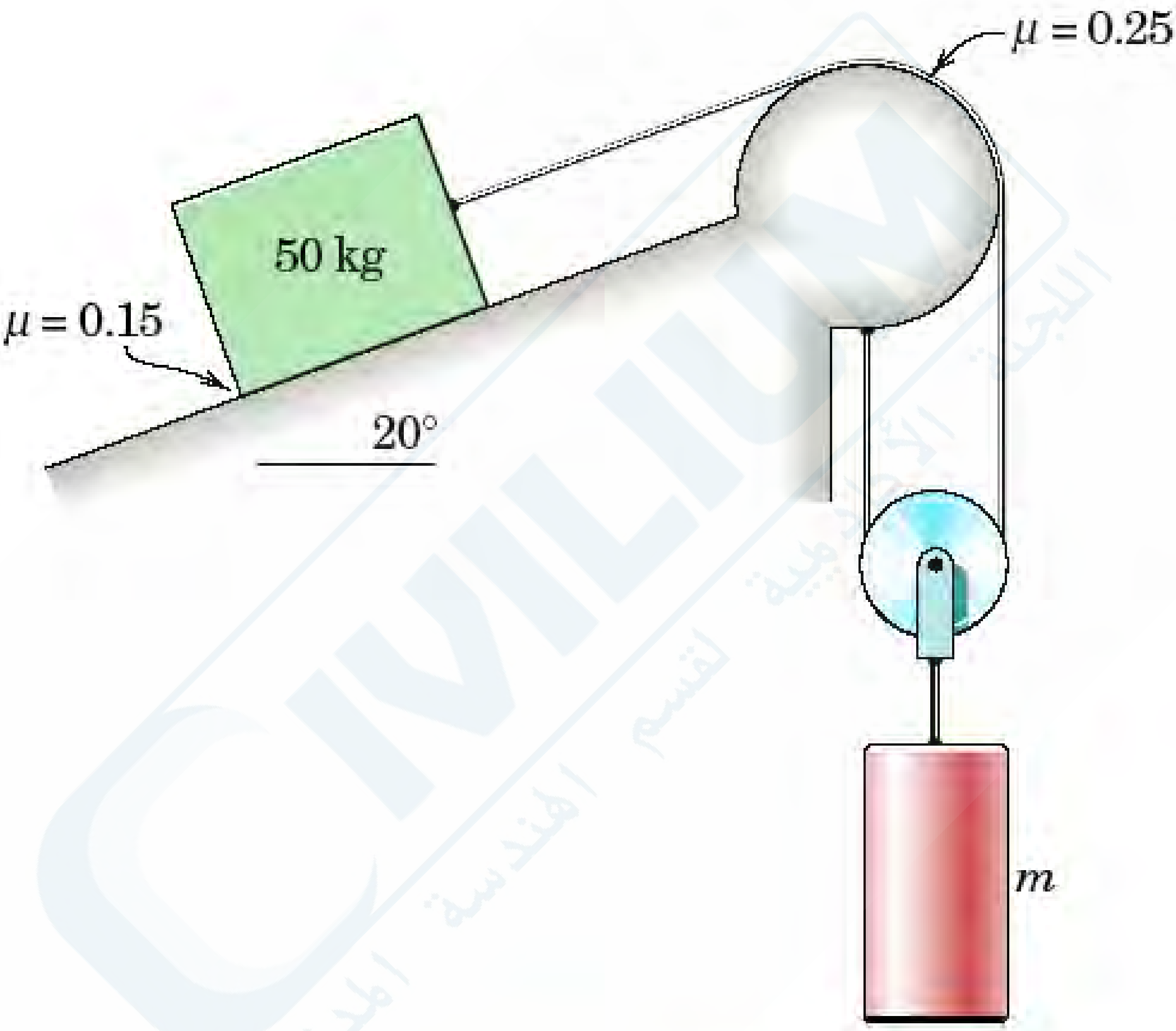
Problem 6/127

P_06_127



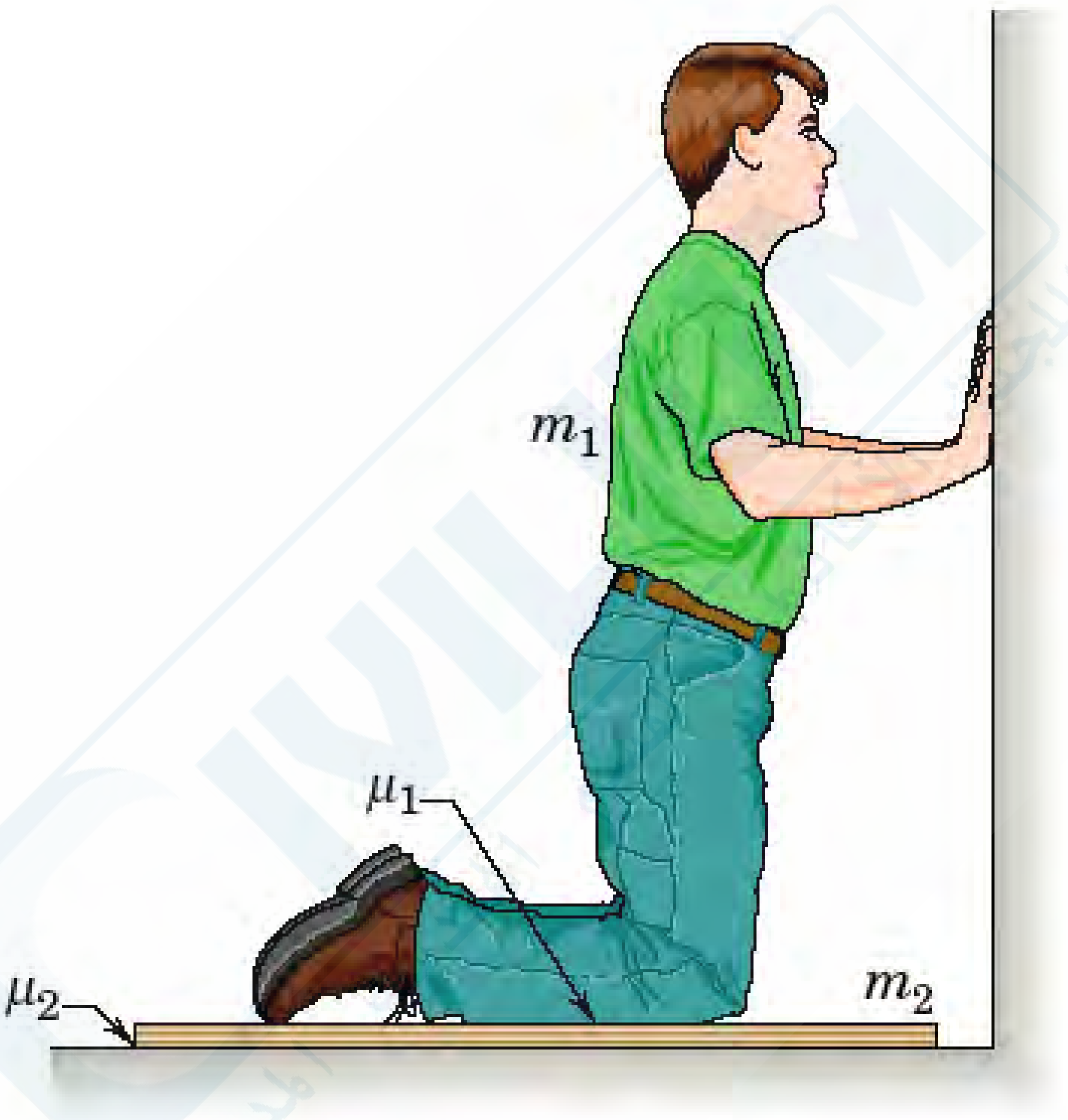
Problem 6/128

P_06_128



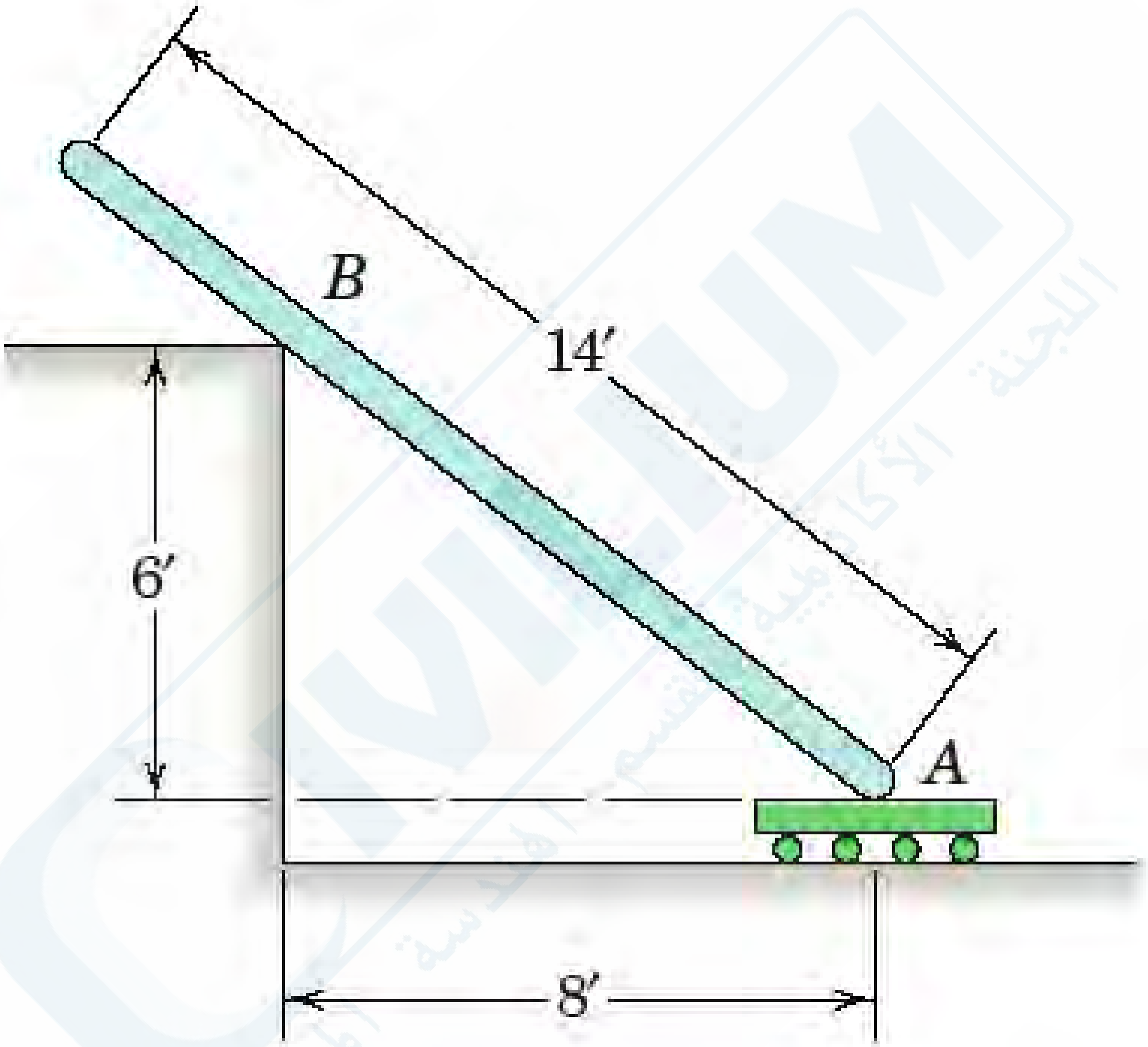
Problem 6/129

P_06_129



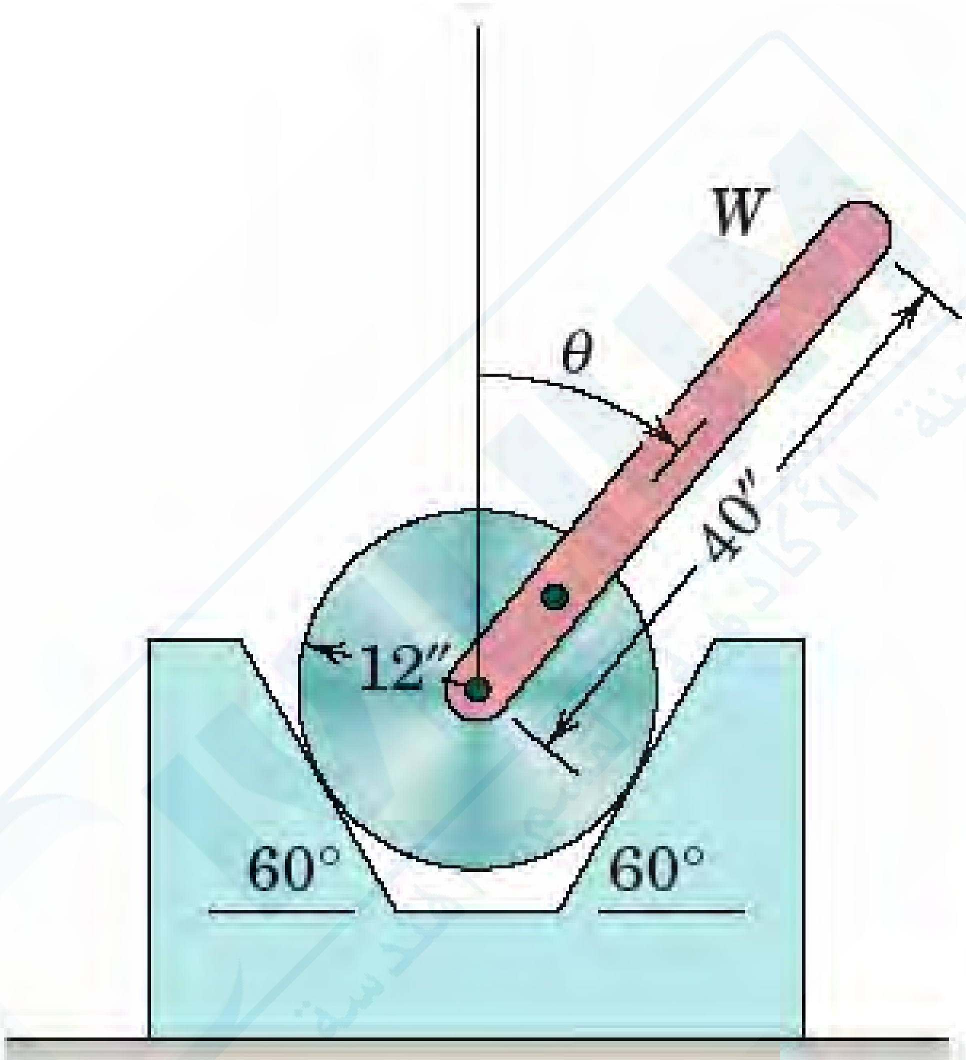
Problem 6/13

P_06_13



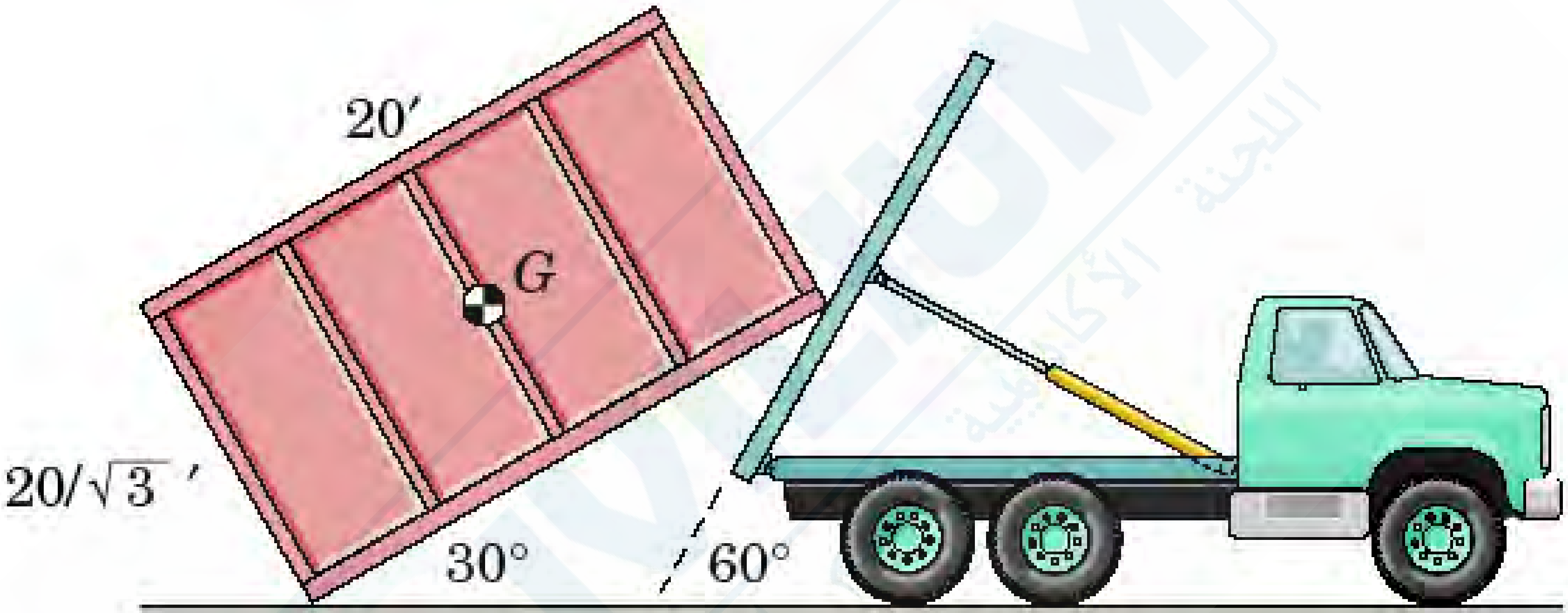
Problem 6/130

P_06_130



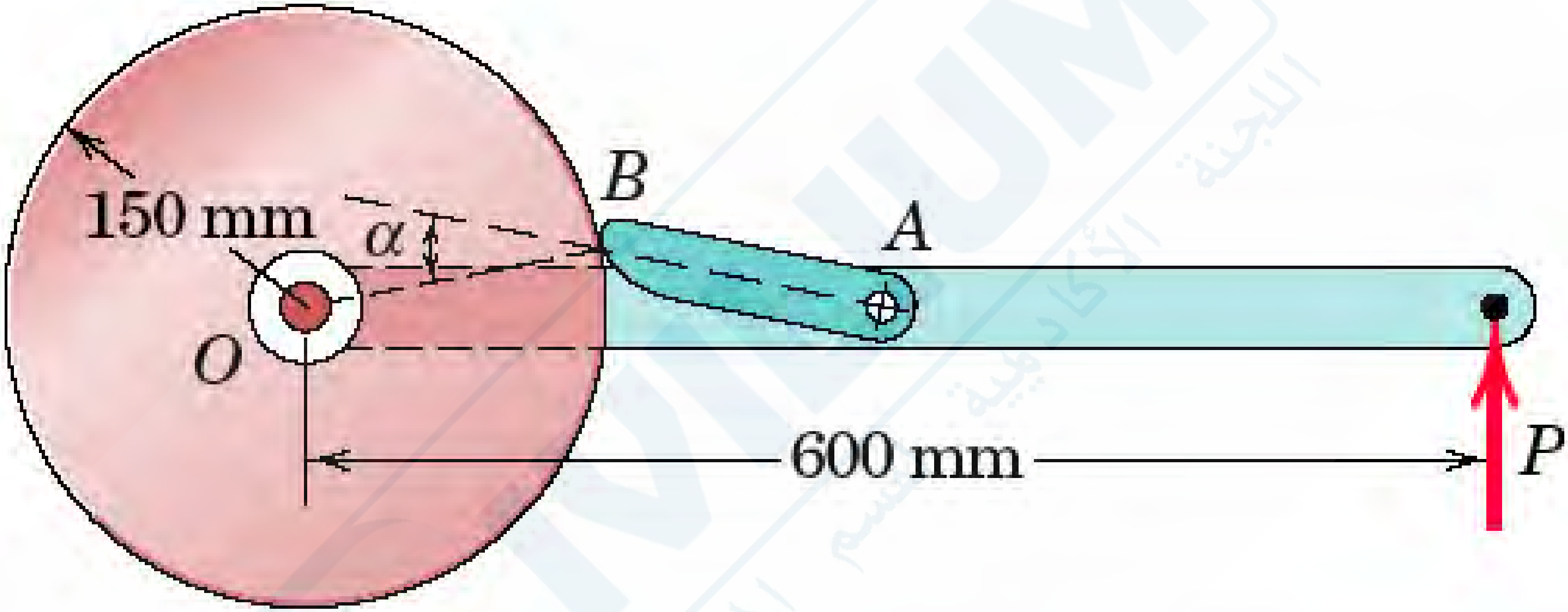
Problem 6/131

P_06_131



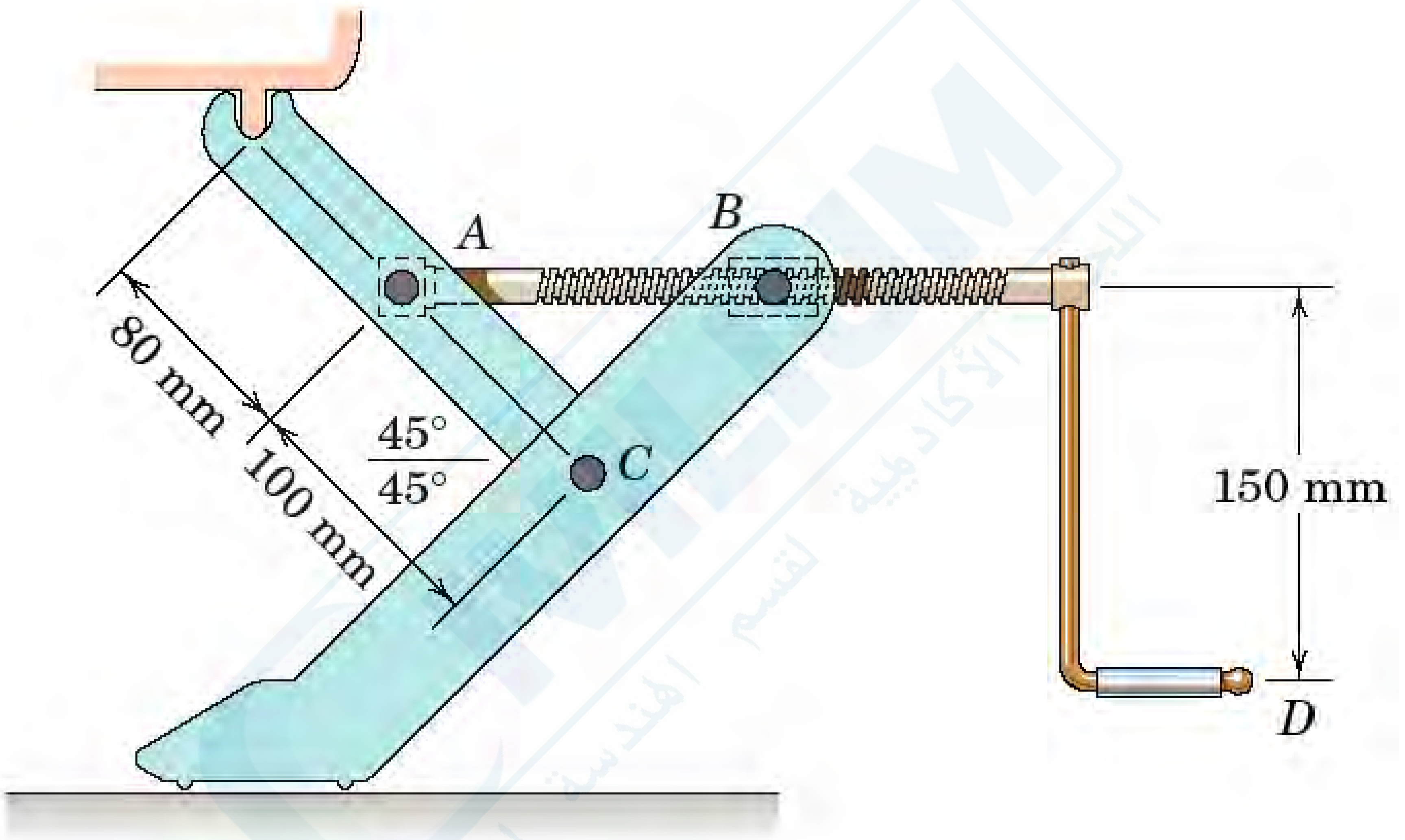
Problem 6/132

P_06_132

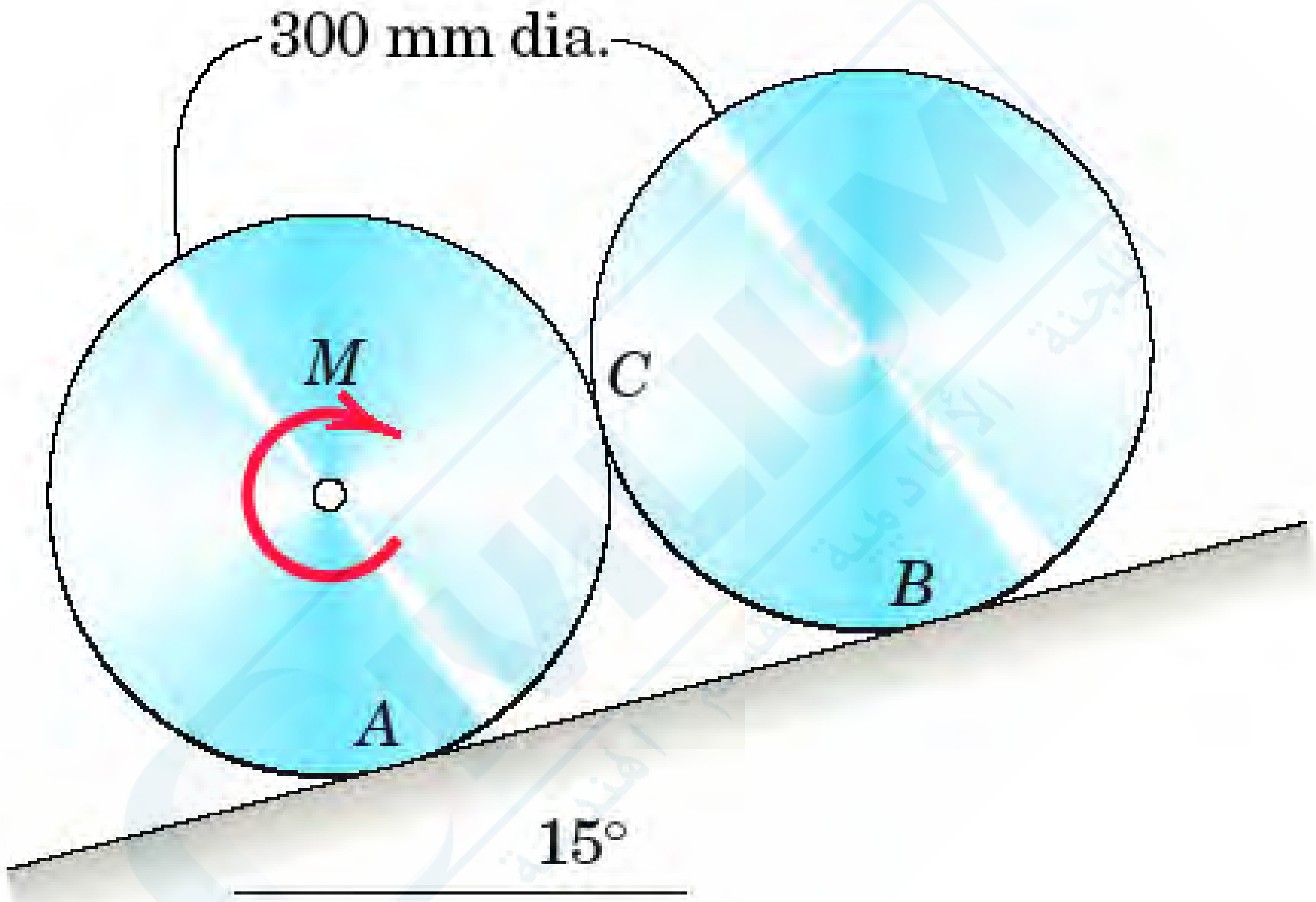


Problem 6/133

P_06_133

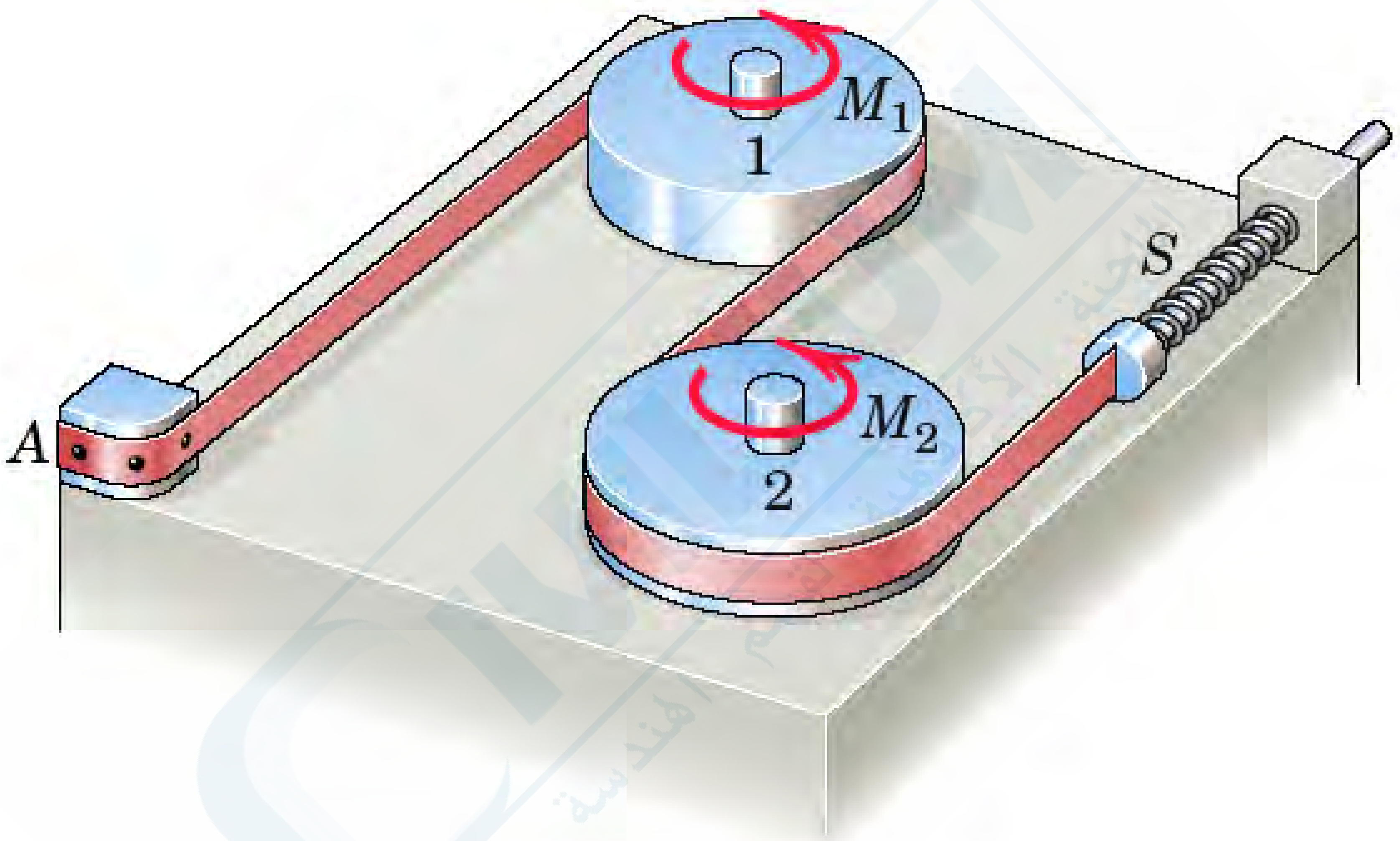


Problem 6/134



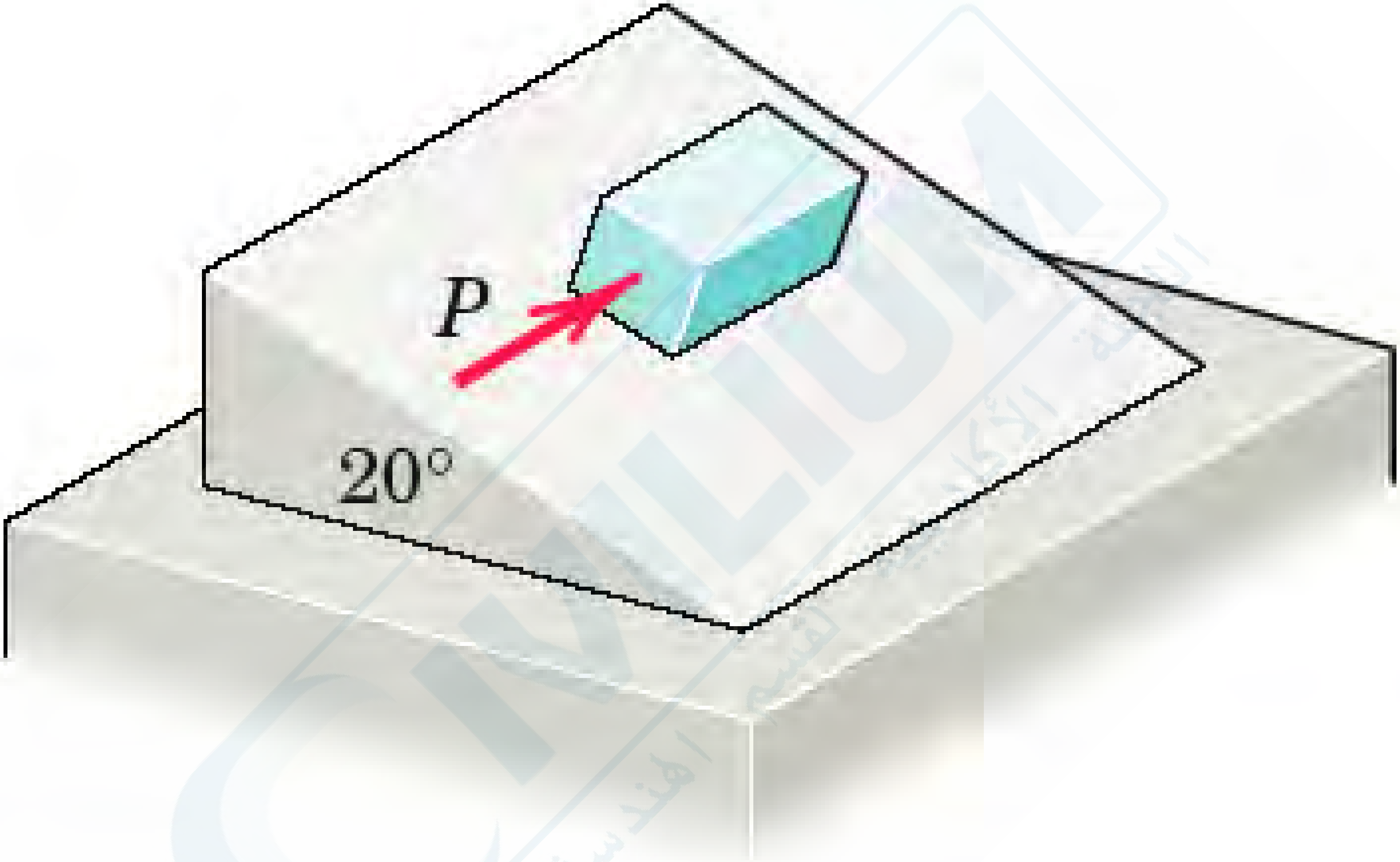
Problem 6/135

P_06_135



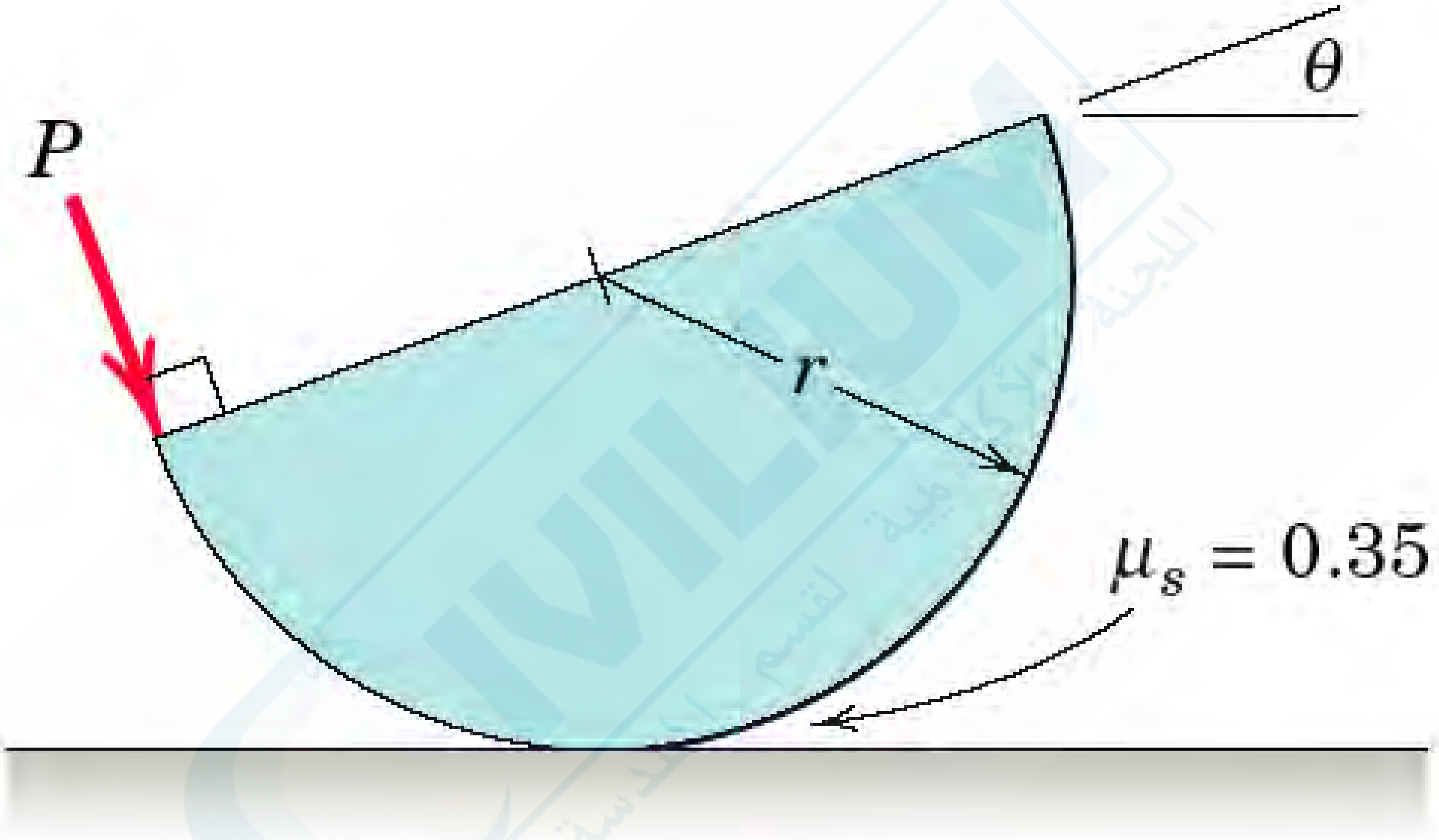
Problem 6/136

P_06_136



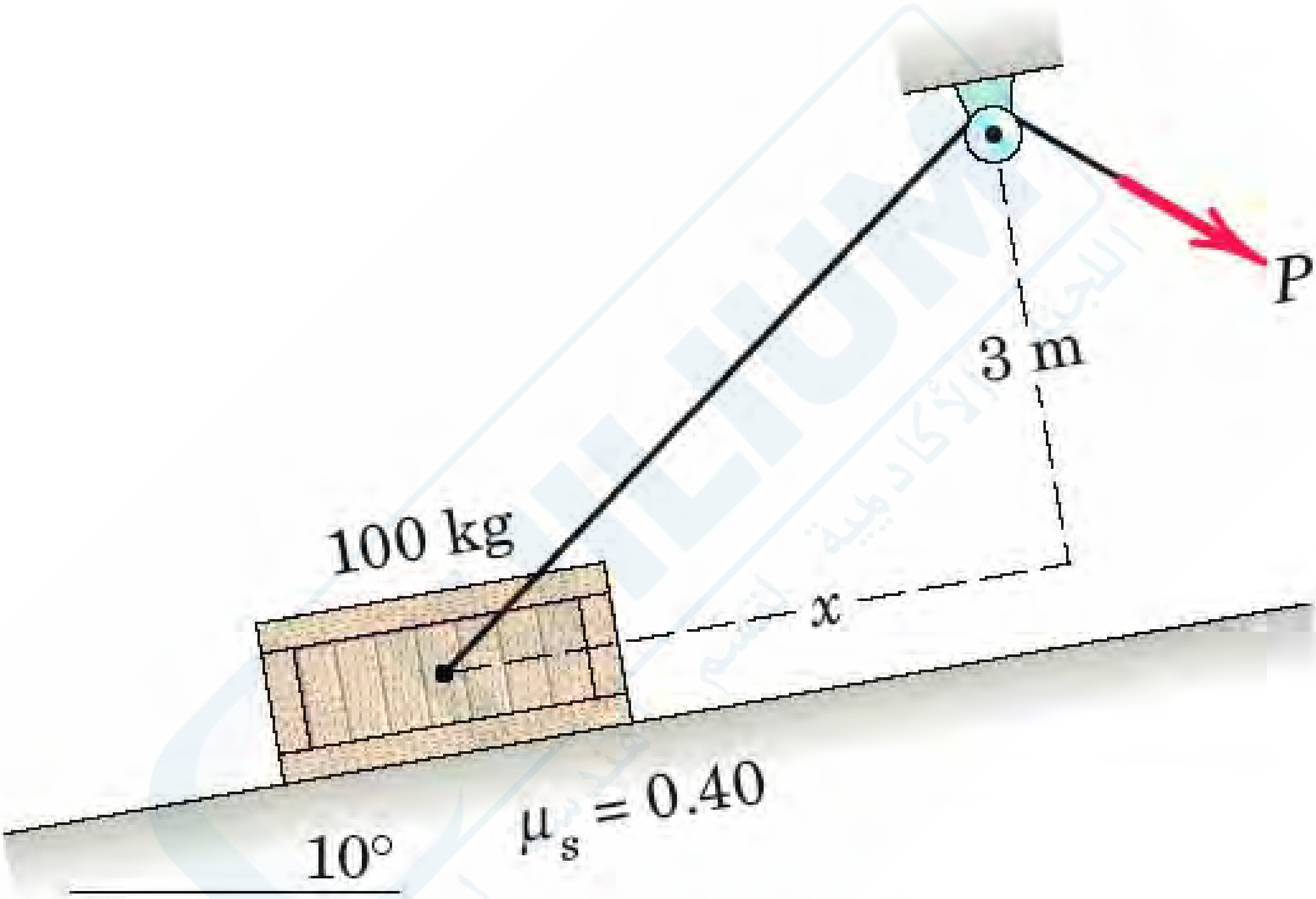
Problem 6/137

P_06_137

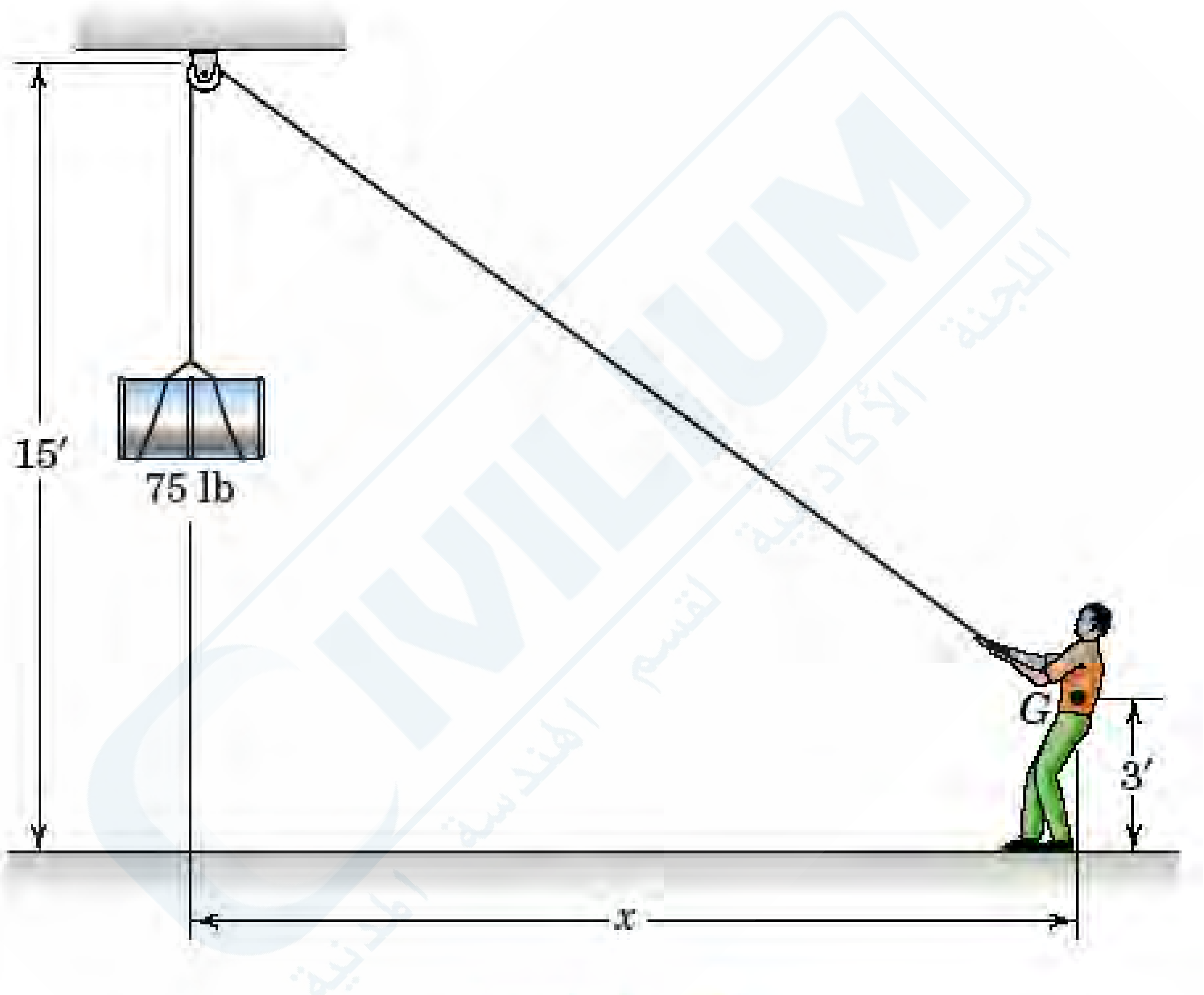


Problem 6/138

P_06_138

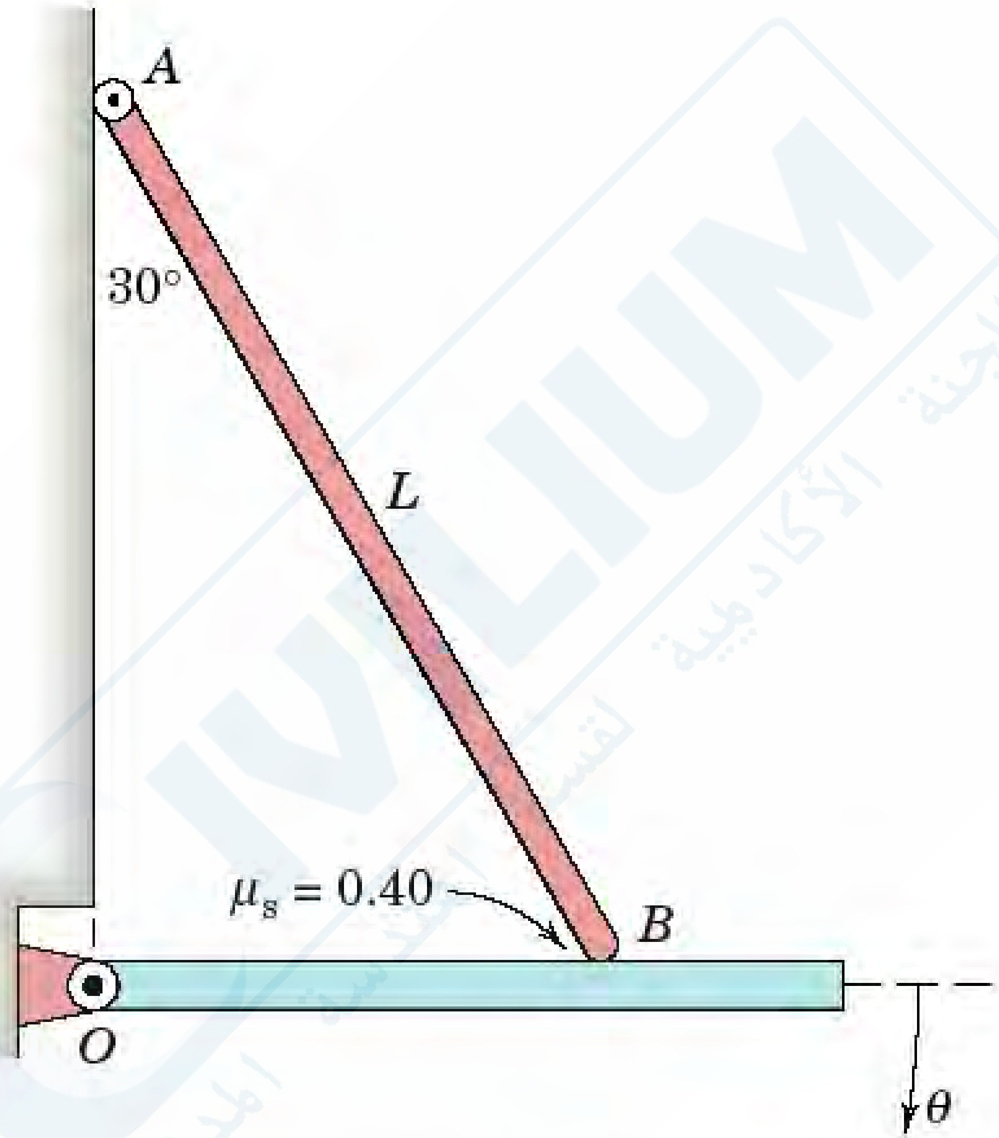


Problem 6/139



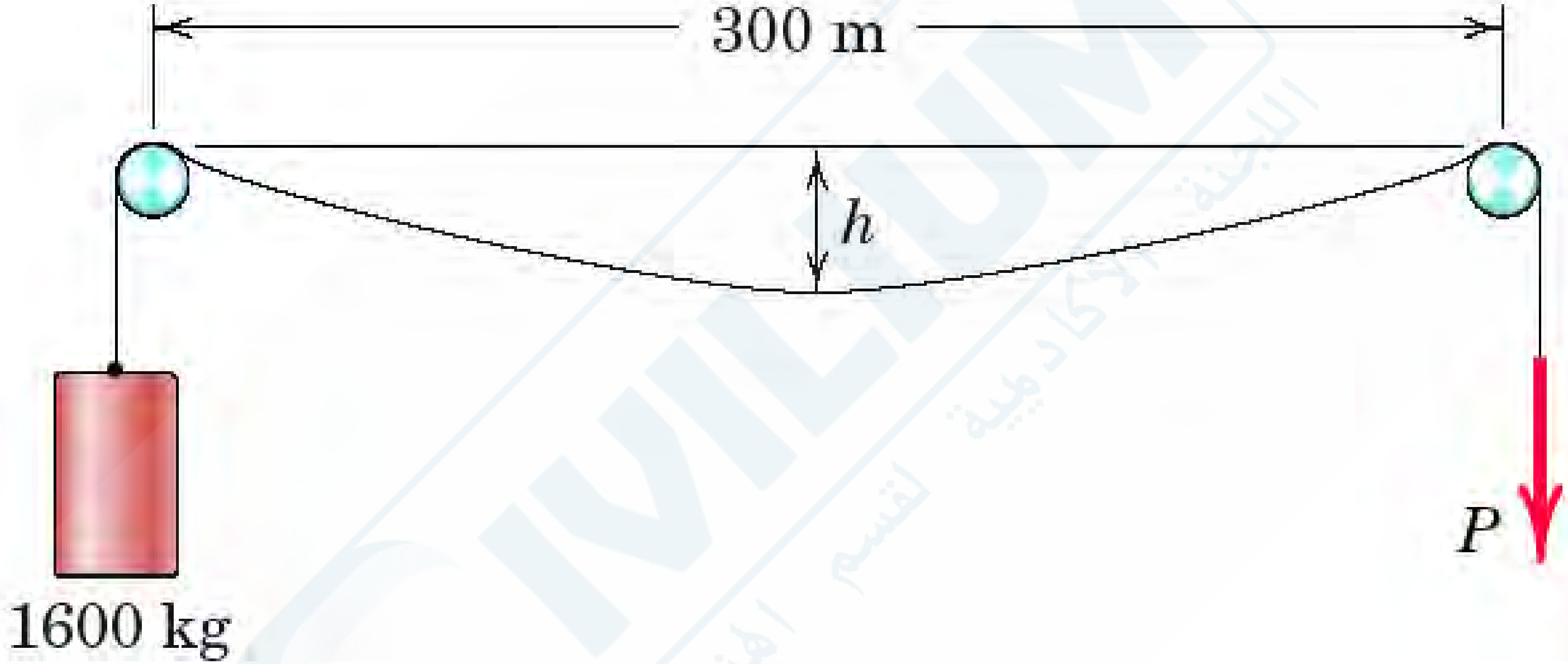
Problem 6/14

P_06_14



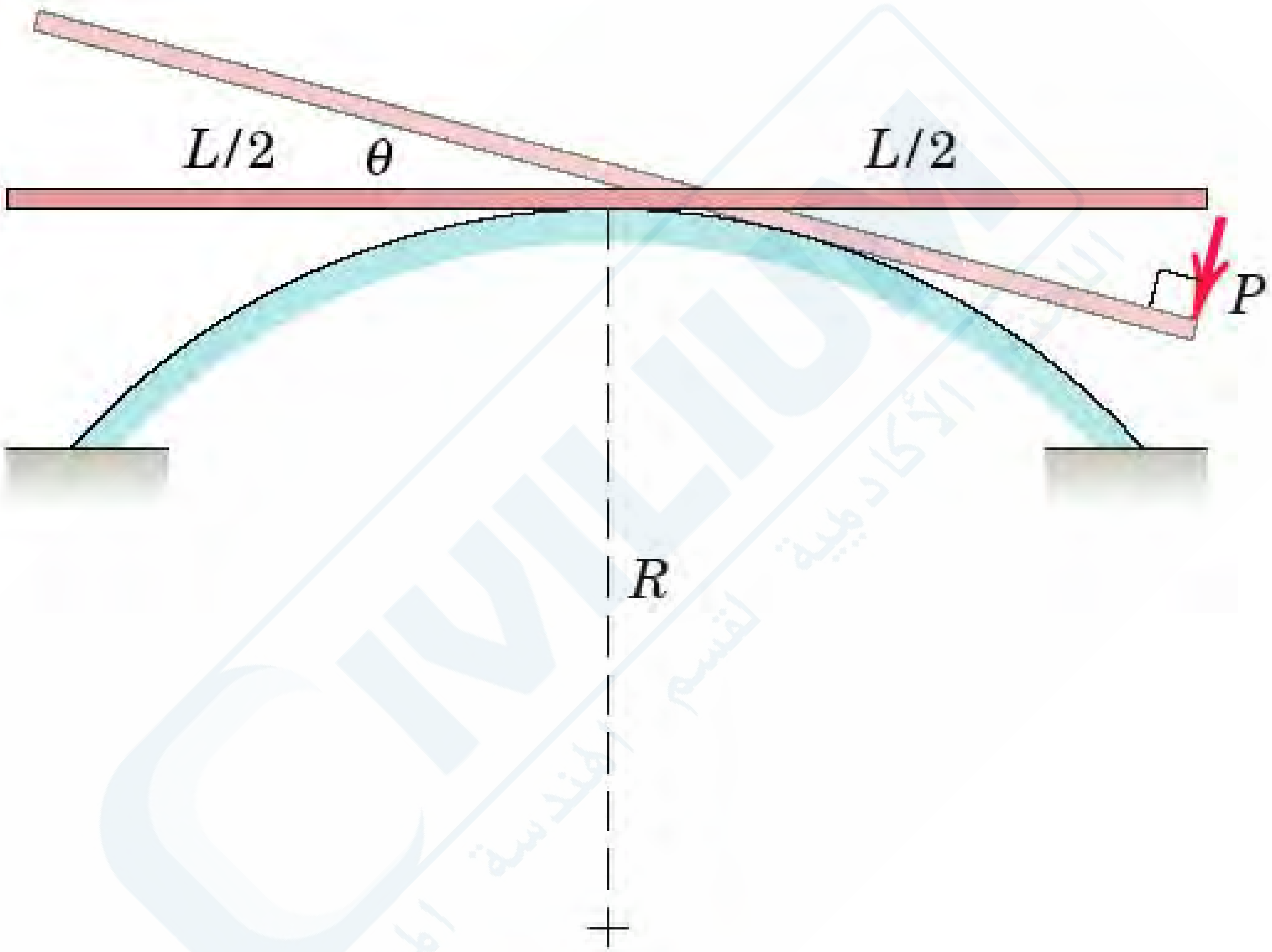
Problem 6/140

P_06_140



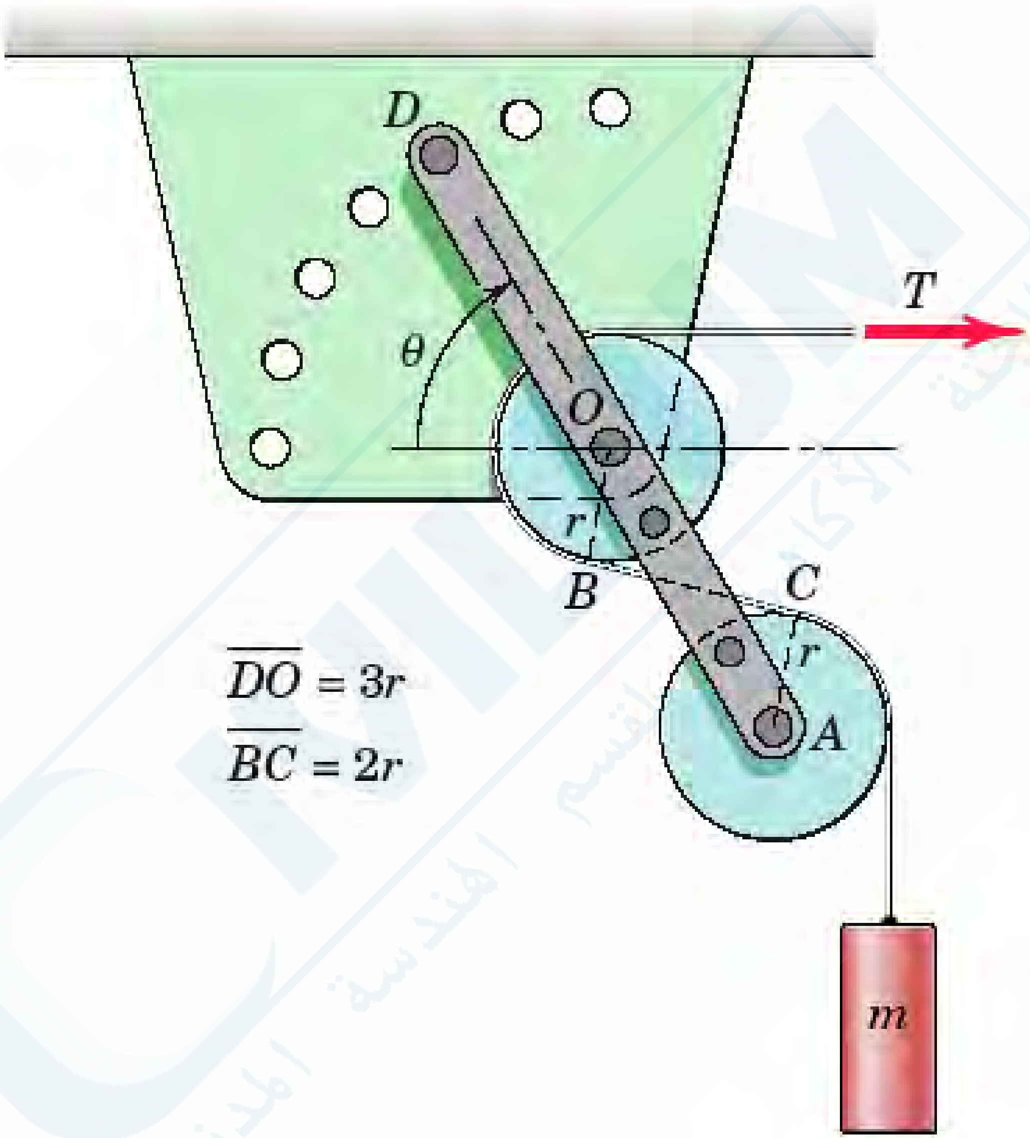
Problem 6/141

P_06_141



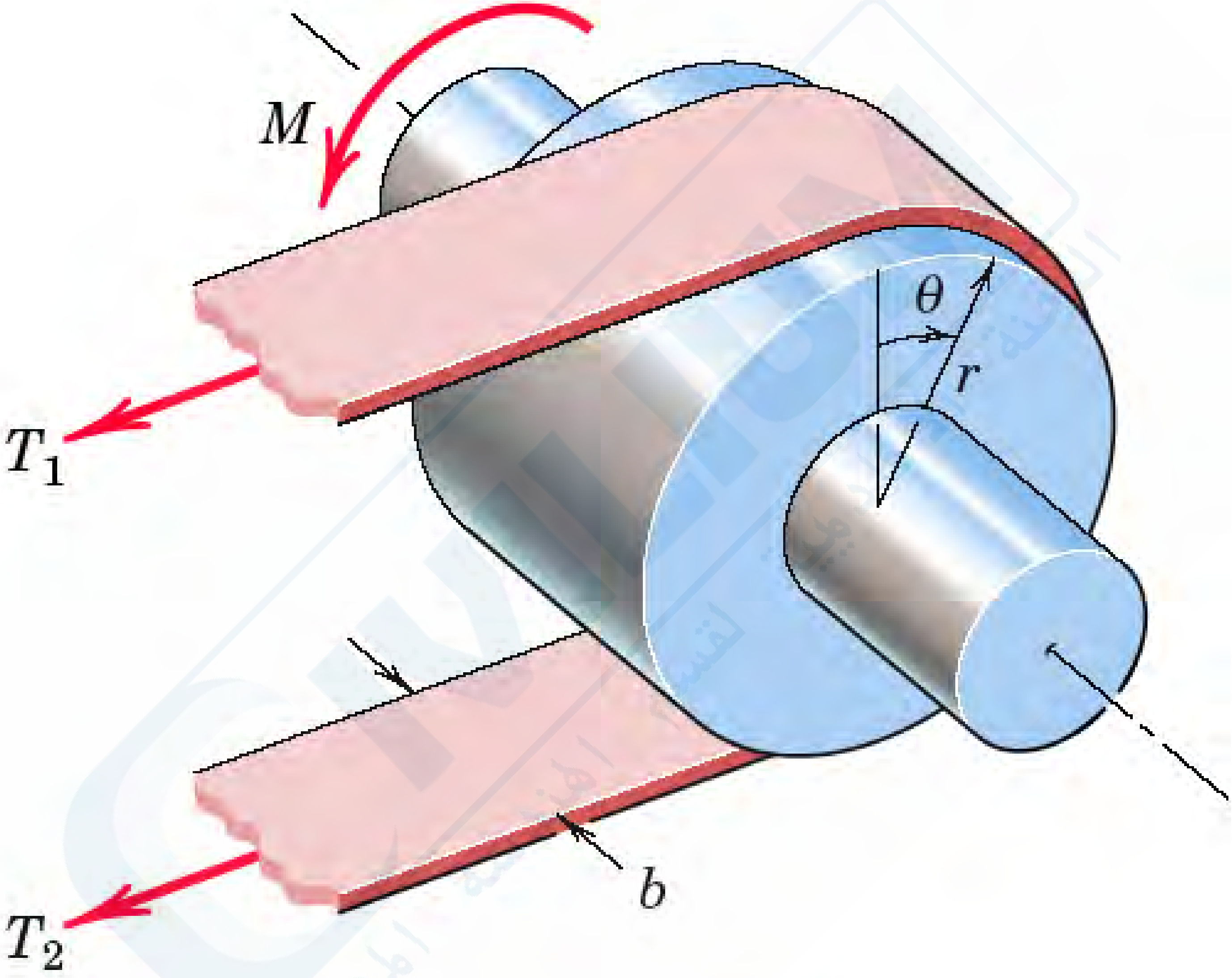
Problem 6/142

P_06_142



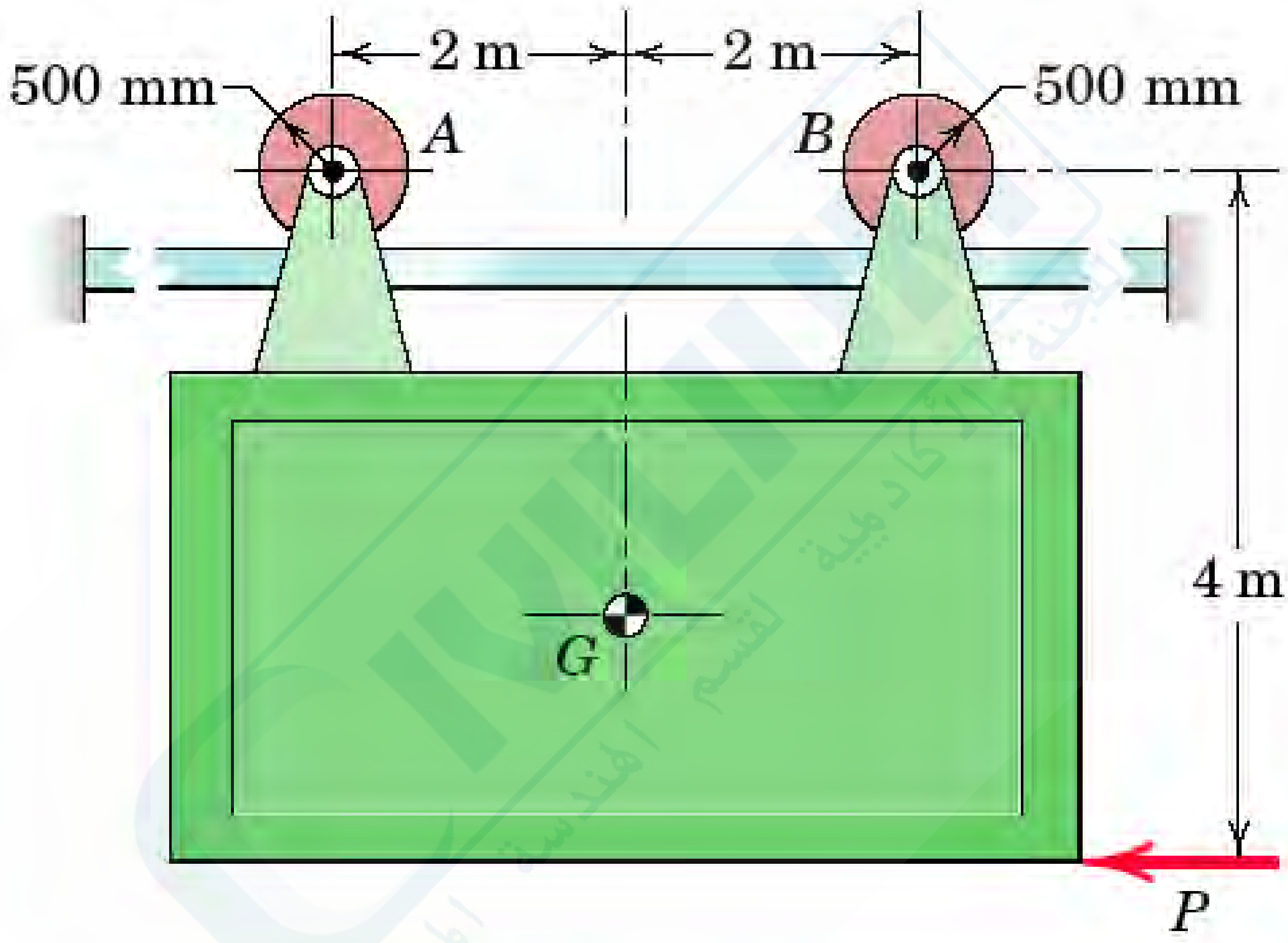
Problem 6/144

P_06_144



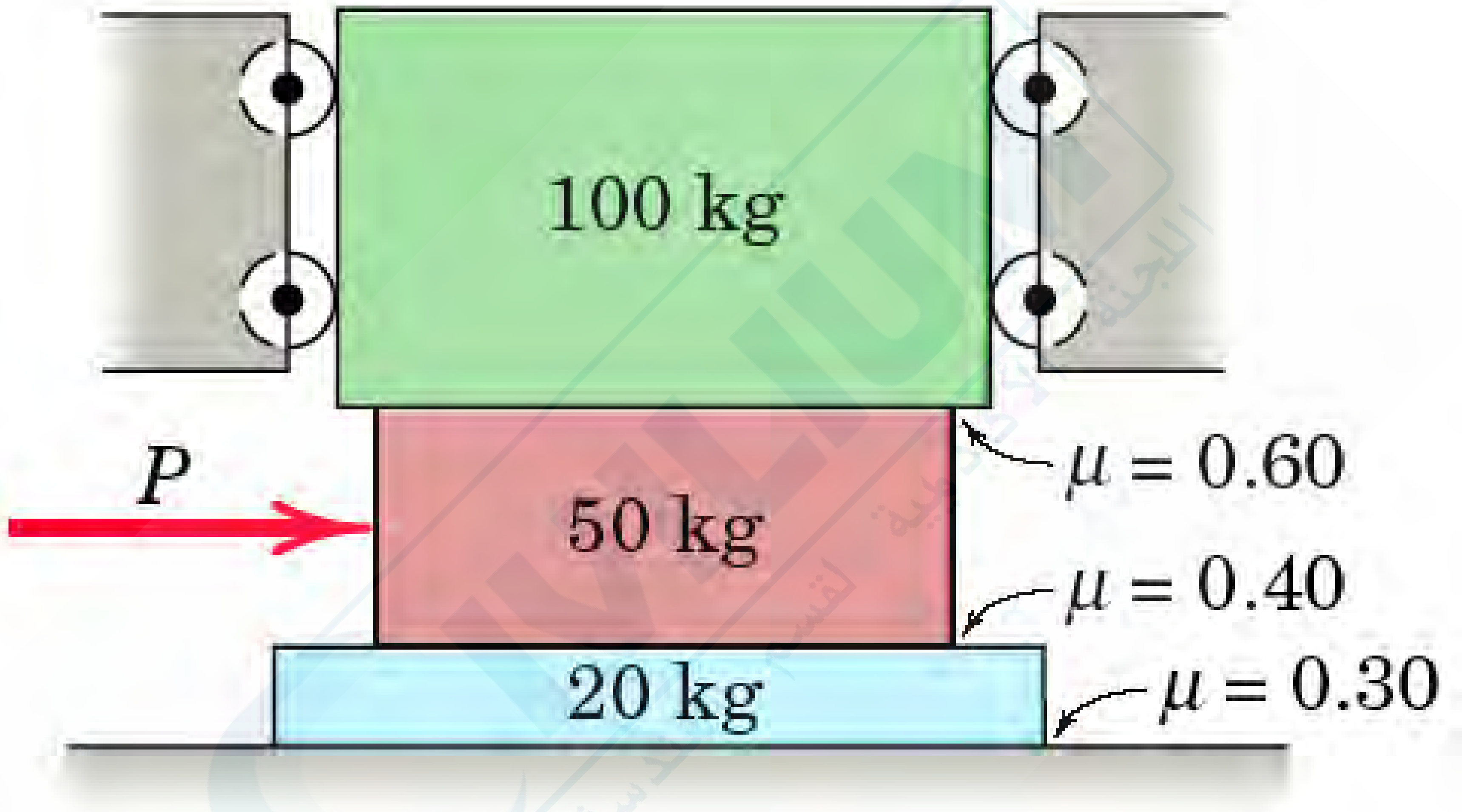
Problem 6/145

P_06_145



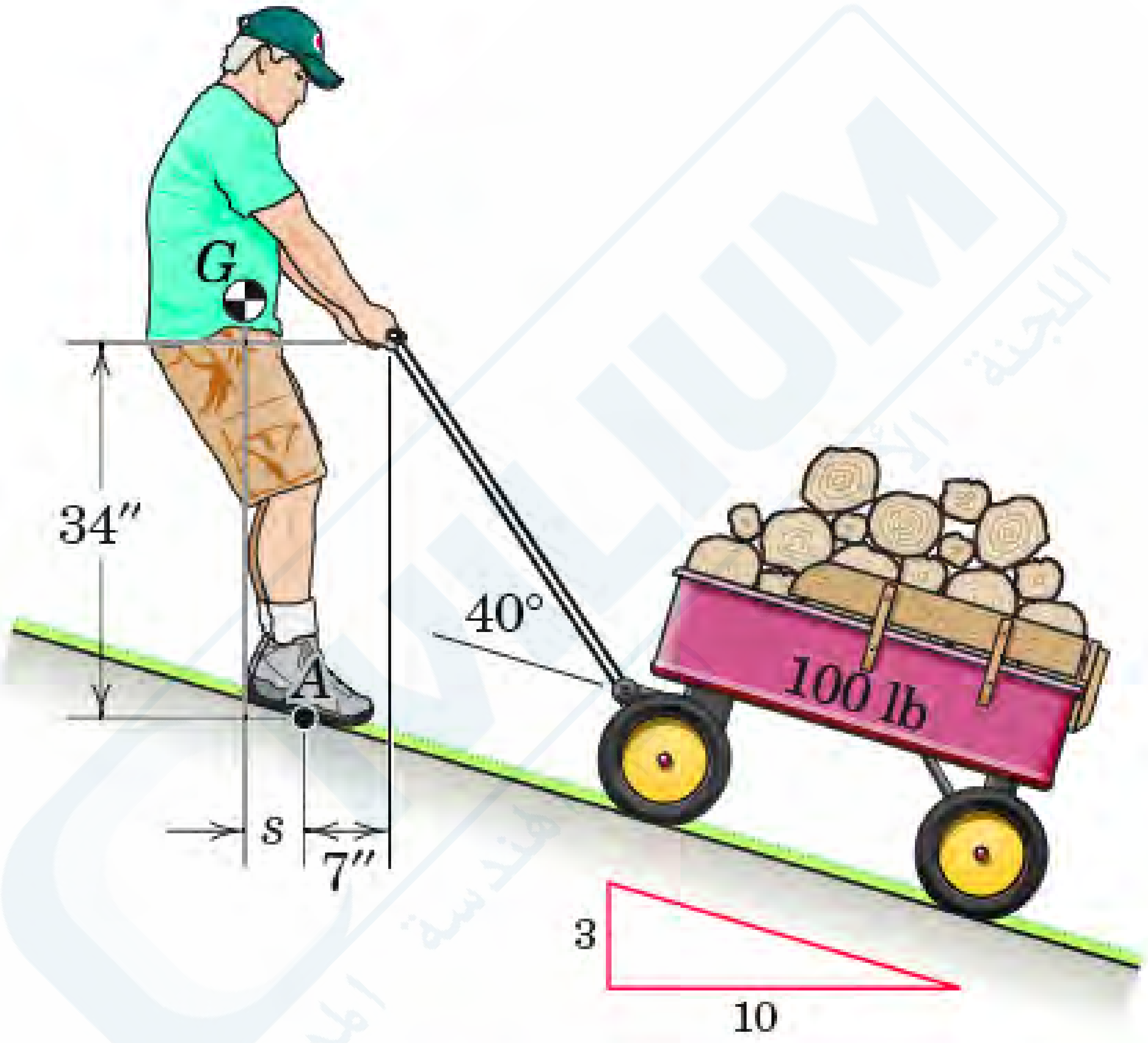
Problem 6/15

P_06_15



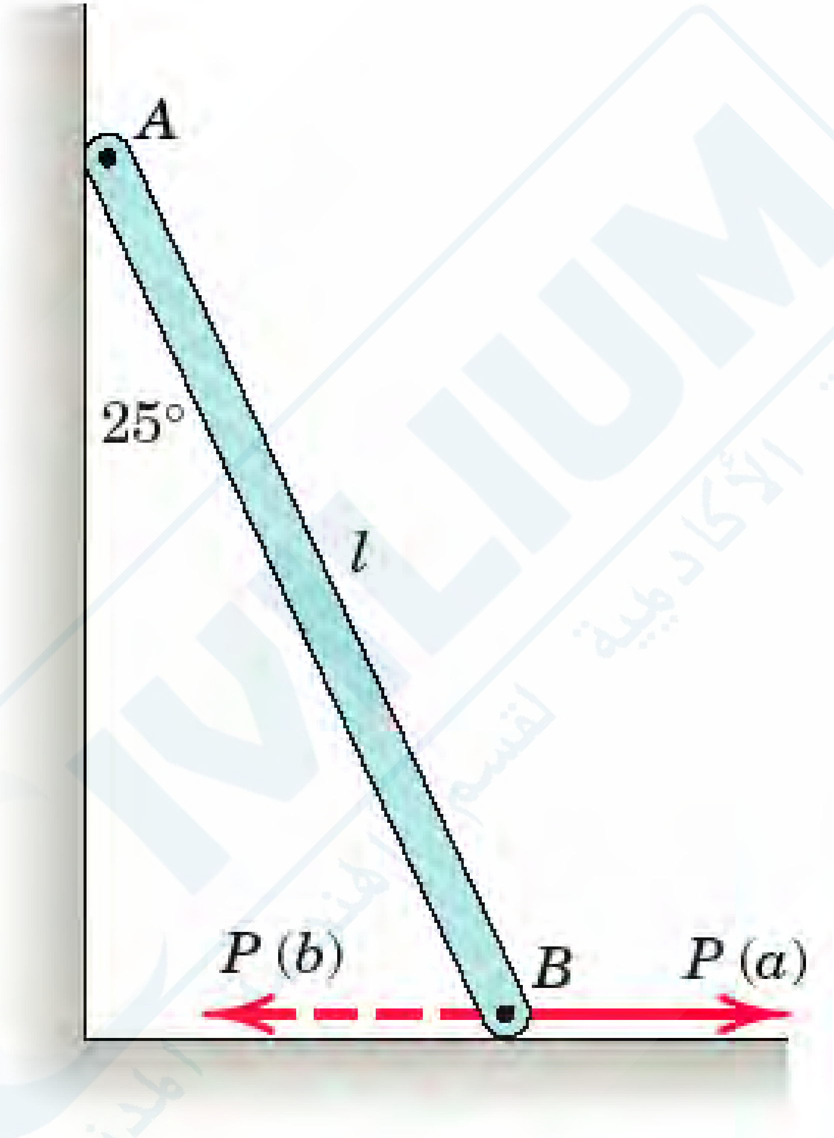
Problem 6/16

P_06_16



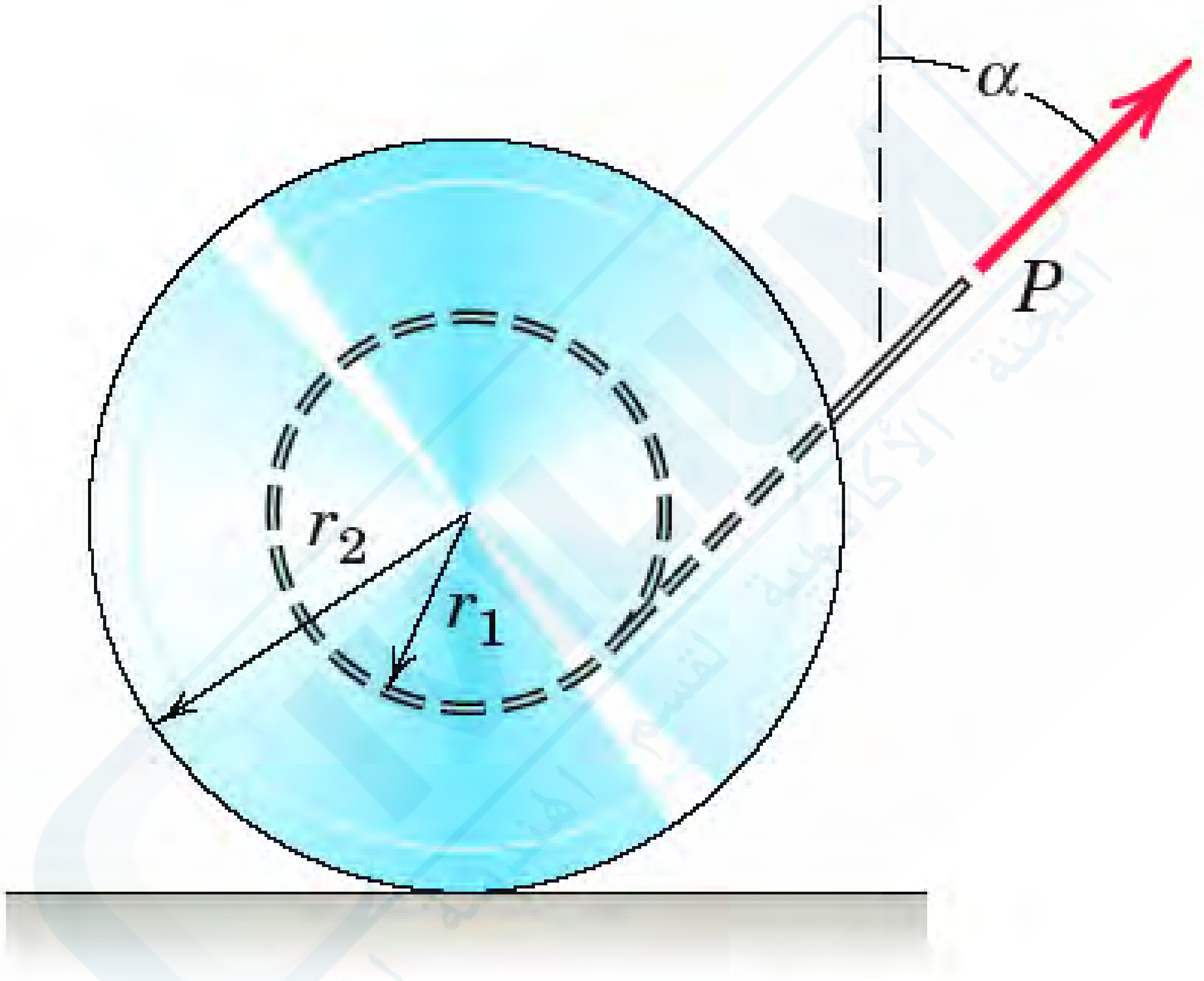
Problem 6/17

P_06_17



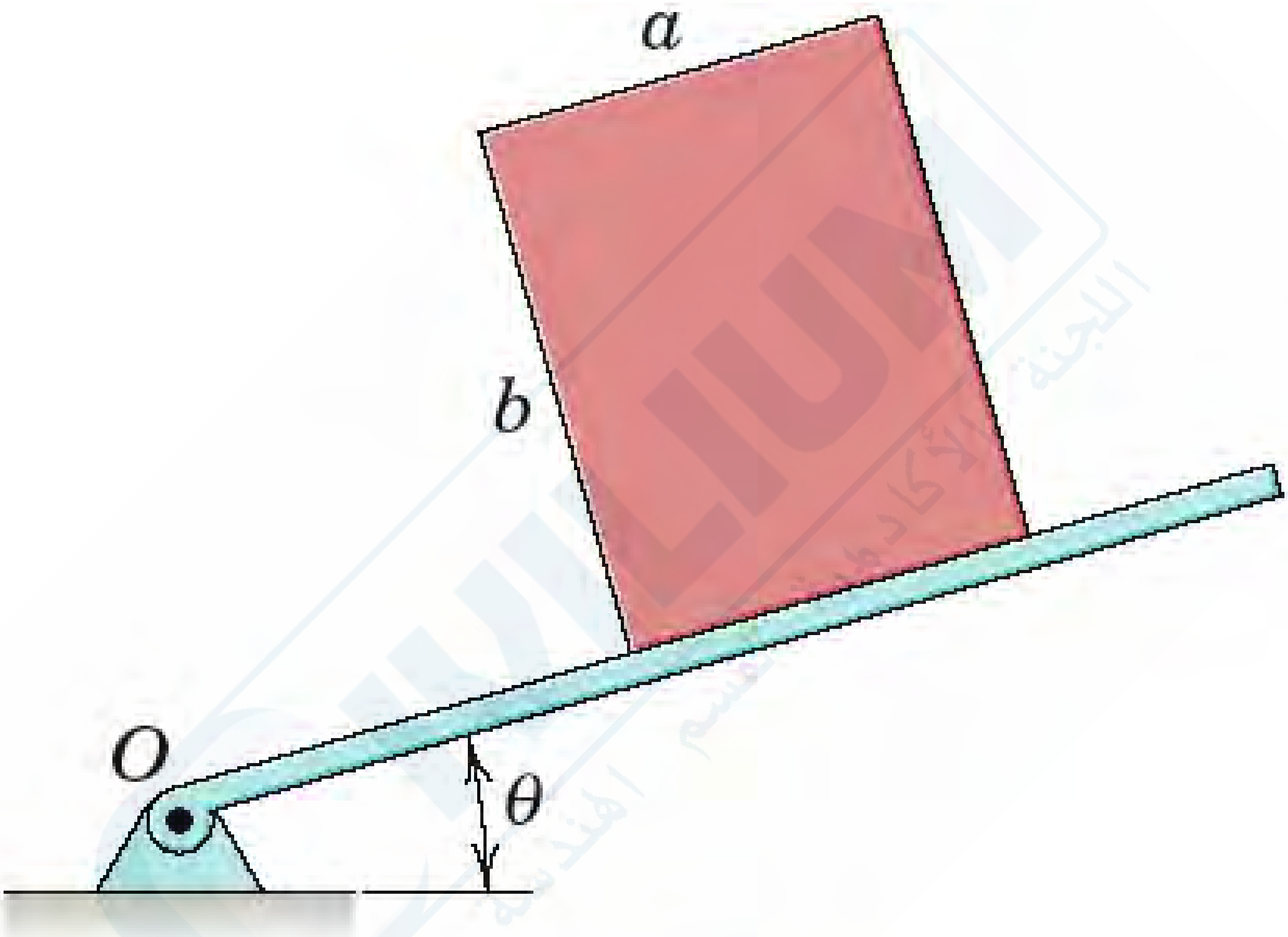
Problem 6/18

P_06_18



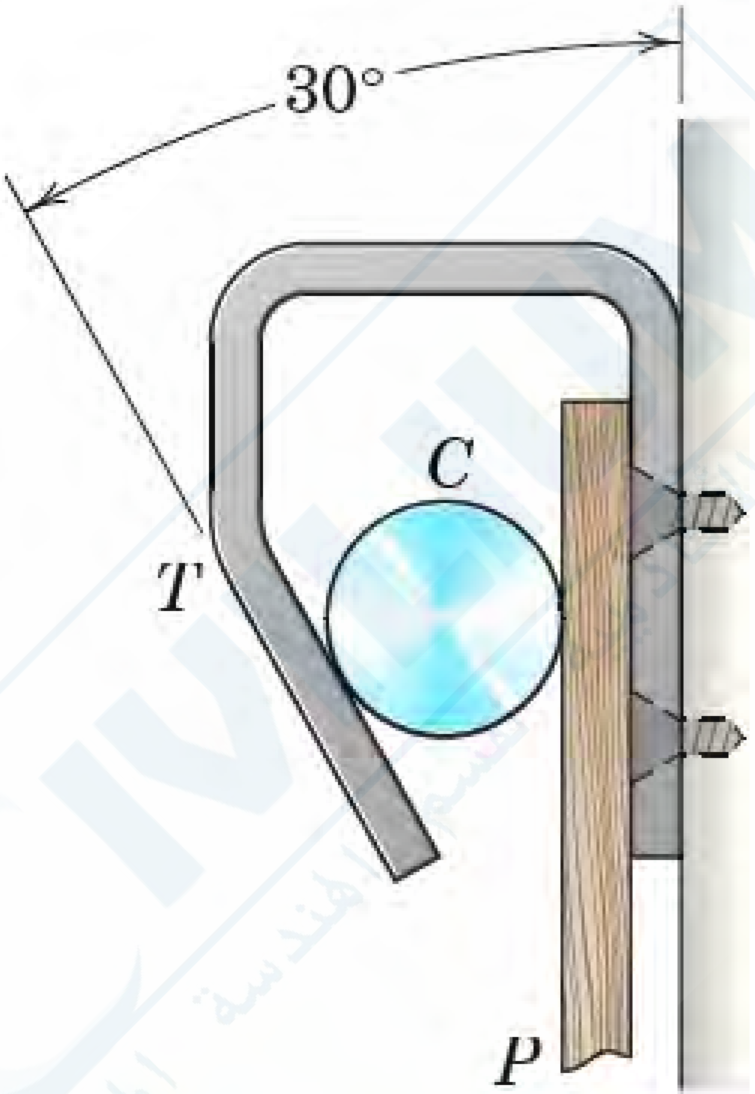
Problem 6/19

P_06_19



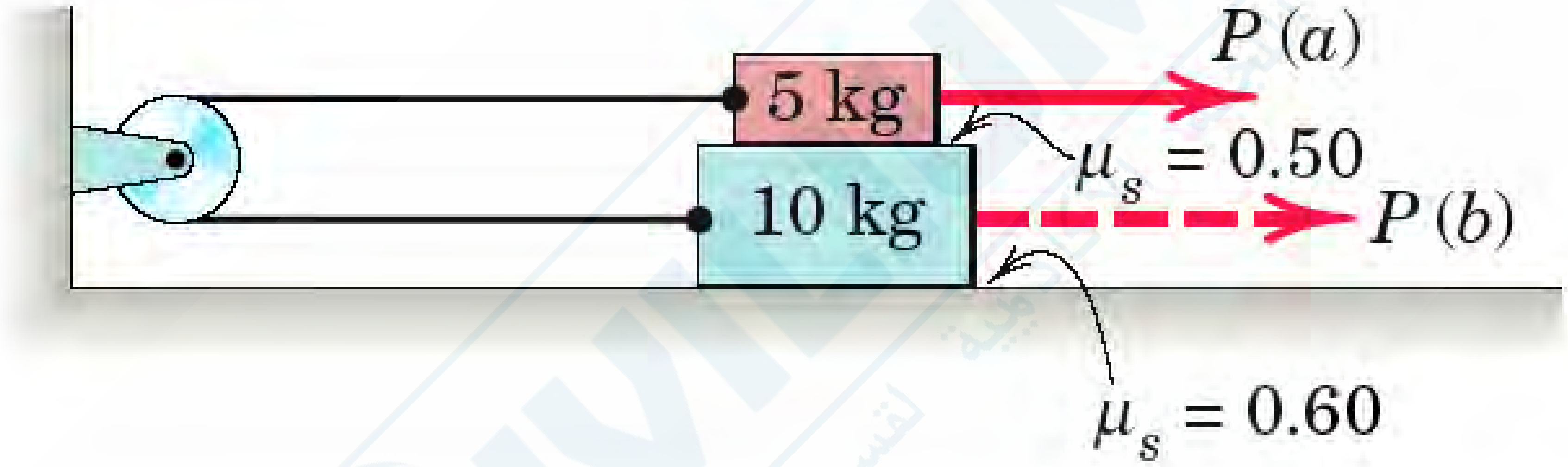
Problem 6/20

P_06_20



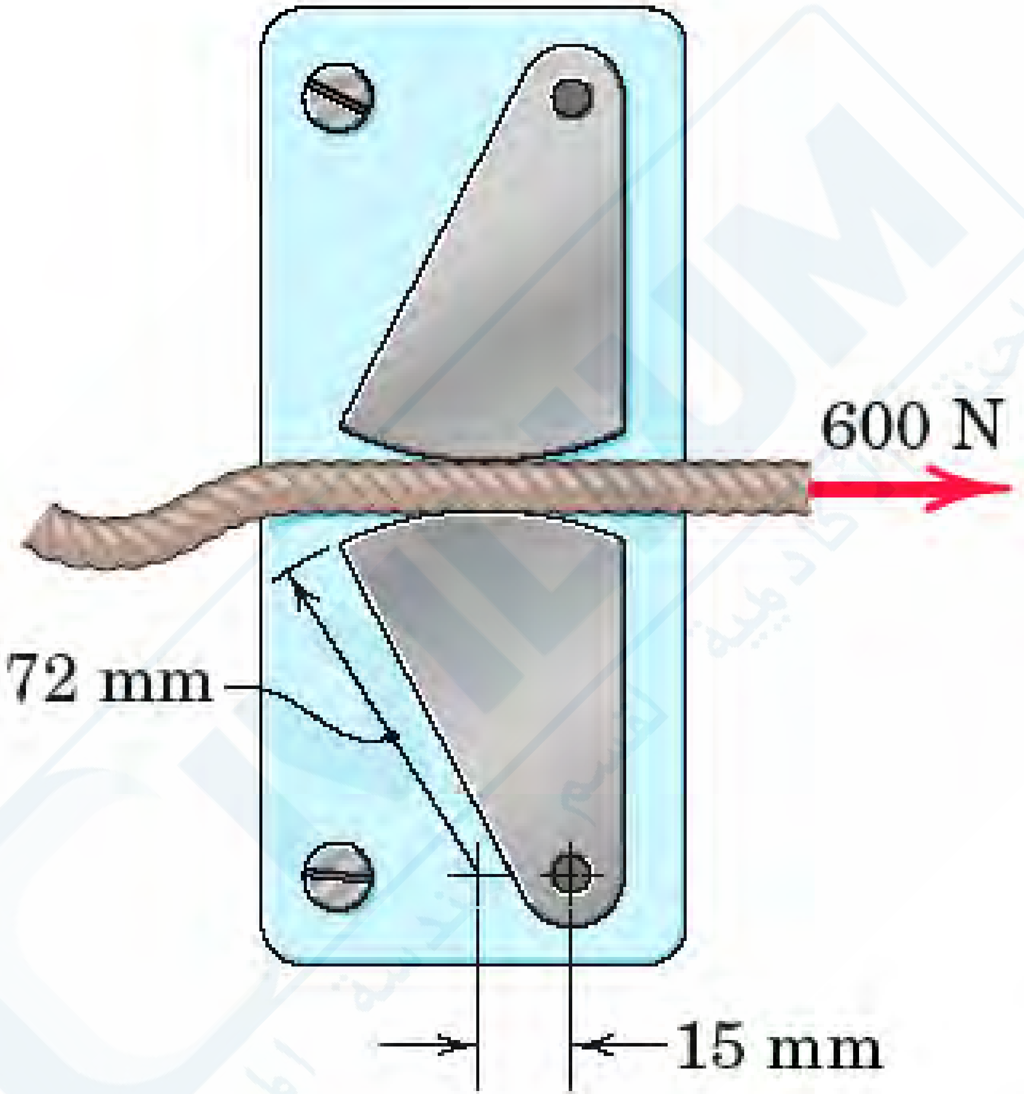
Problem 6/21

P_06_21



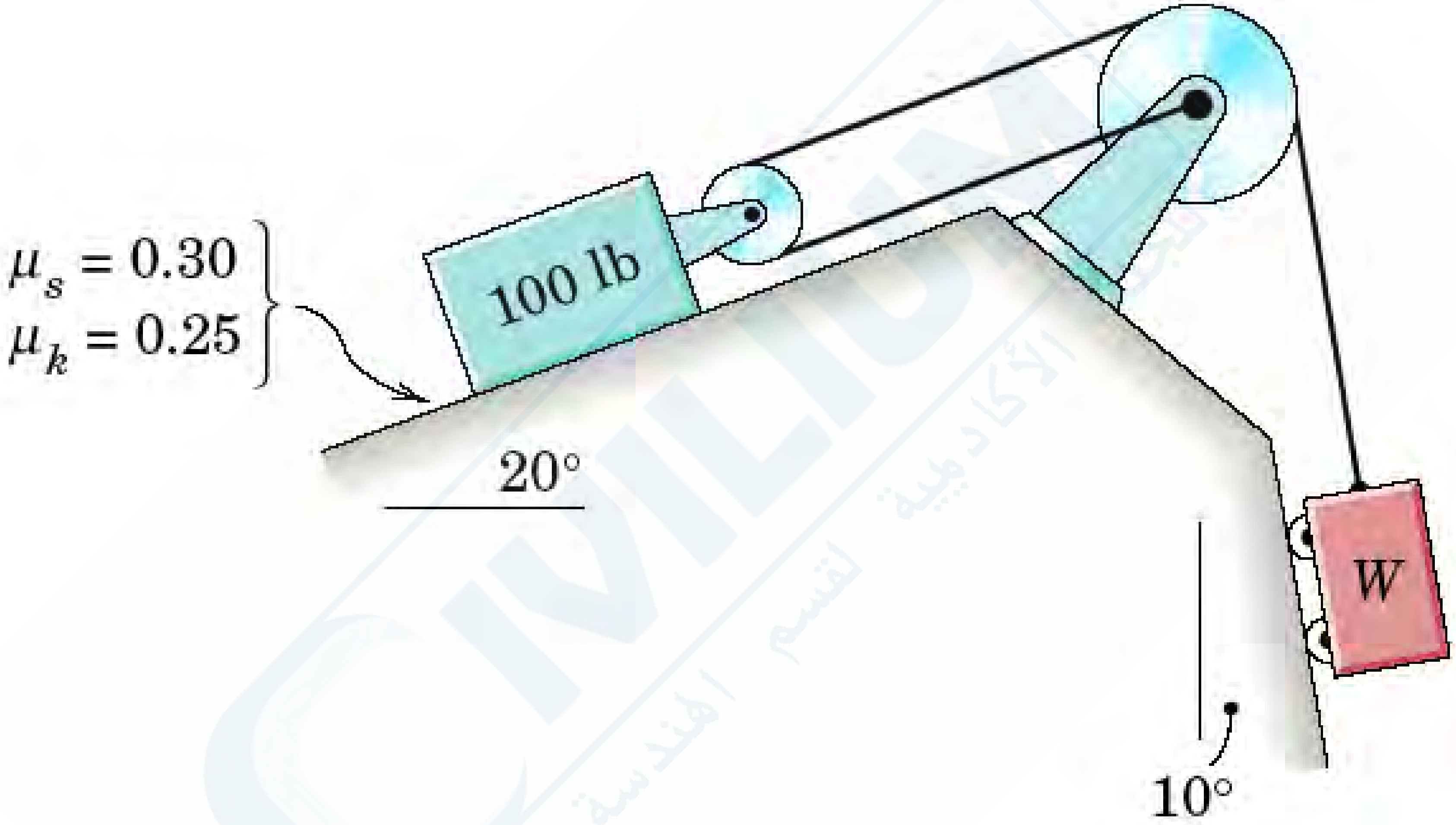
Problem 6/22

P_06_22

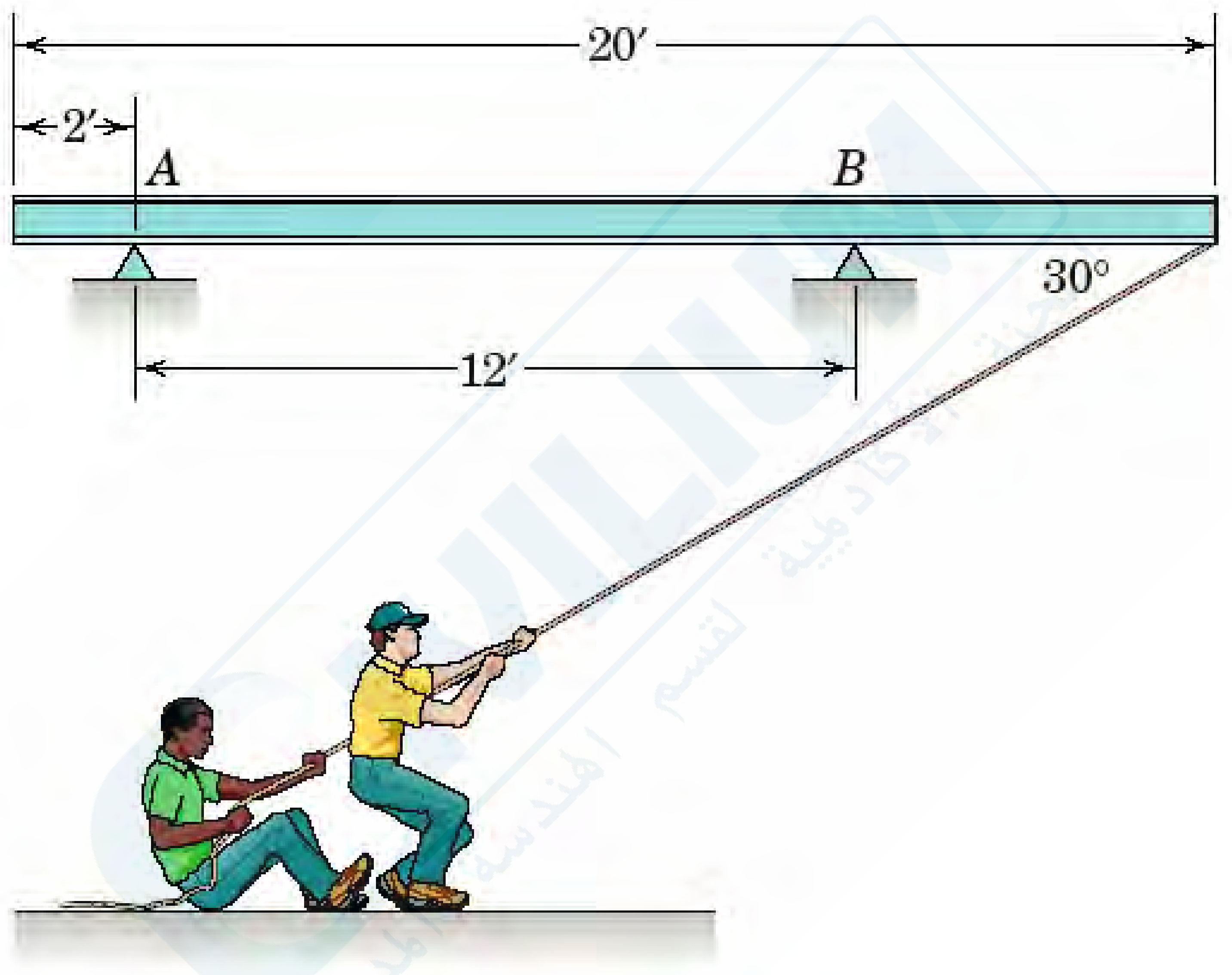


Problem 6/23

P_06_23

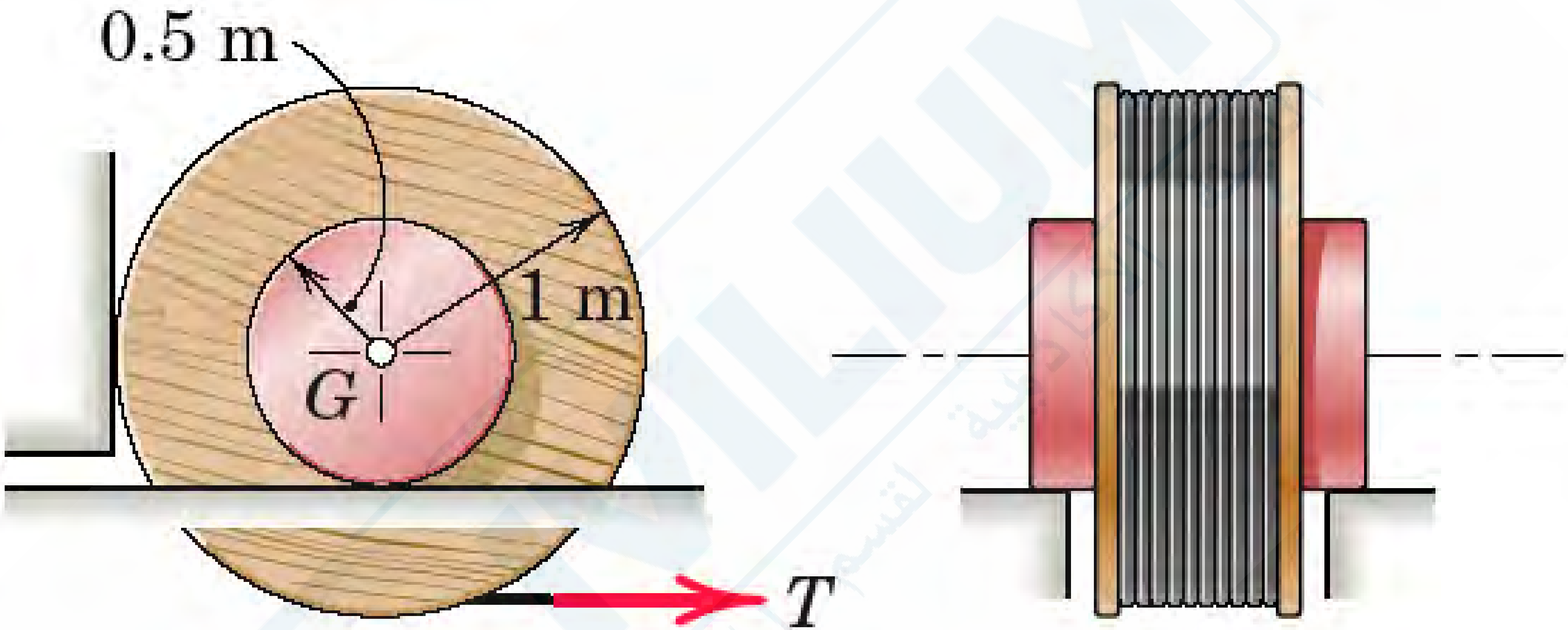


Problem 6/24



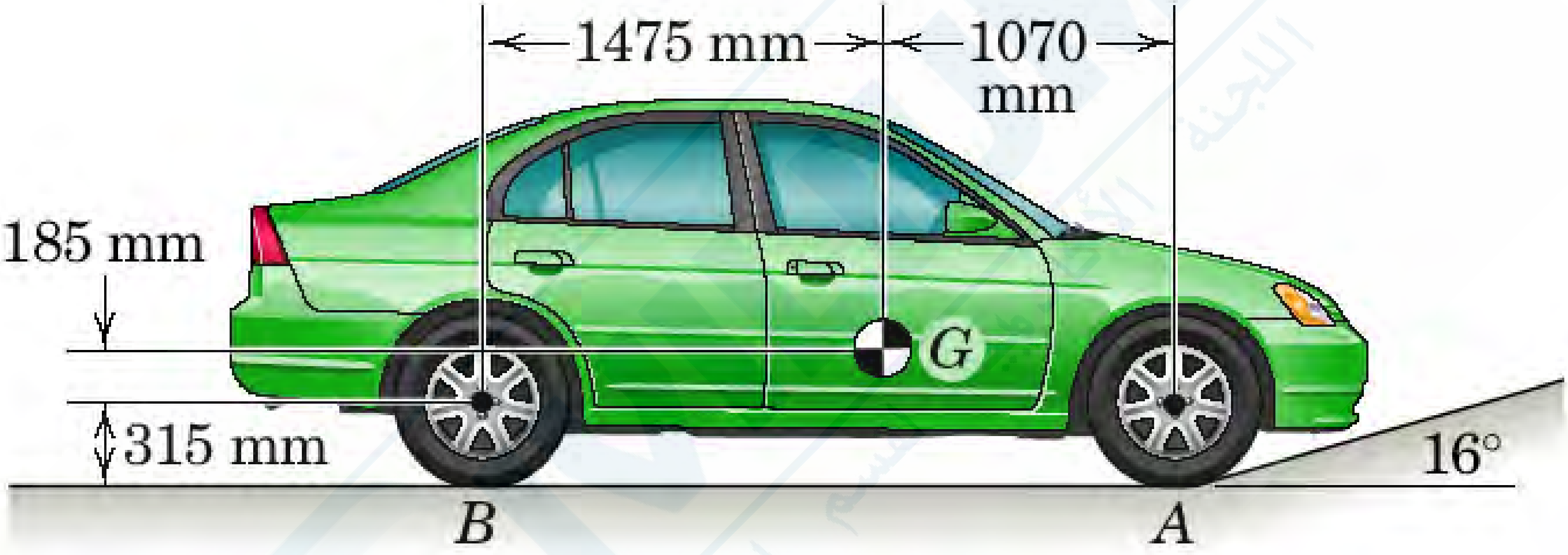
Problem 6/25

P_06_25



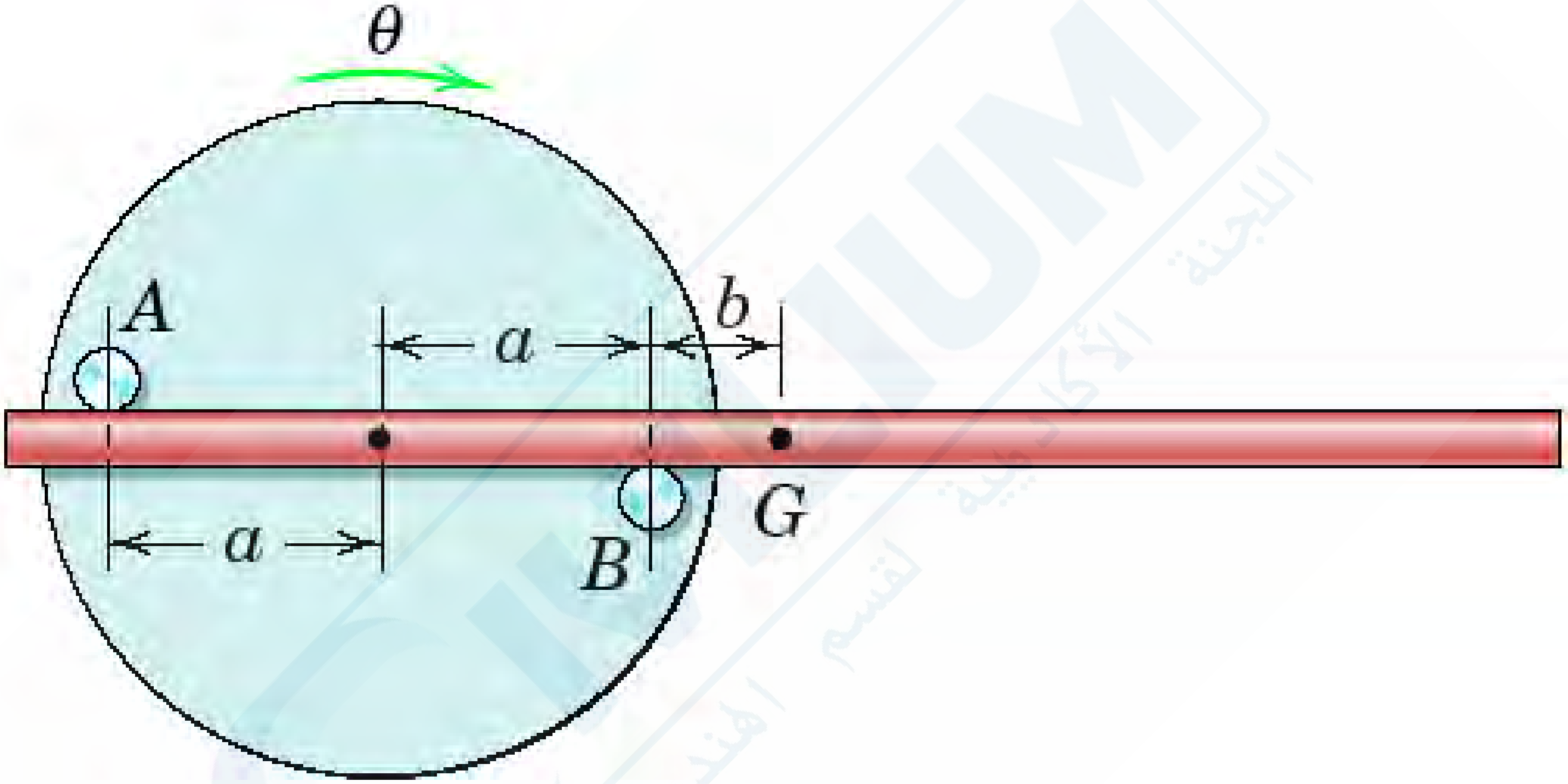
Problem 6/26

P_06_26



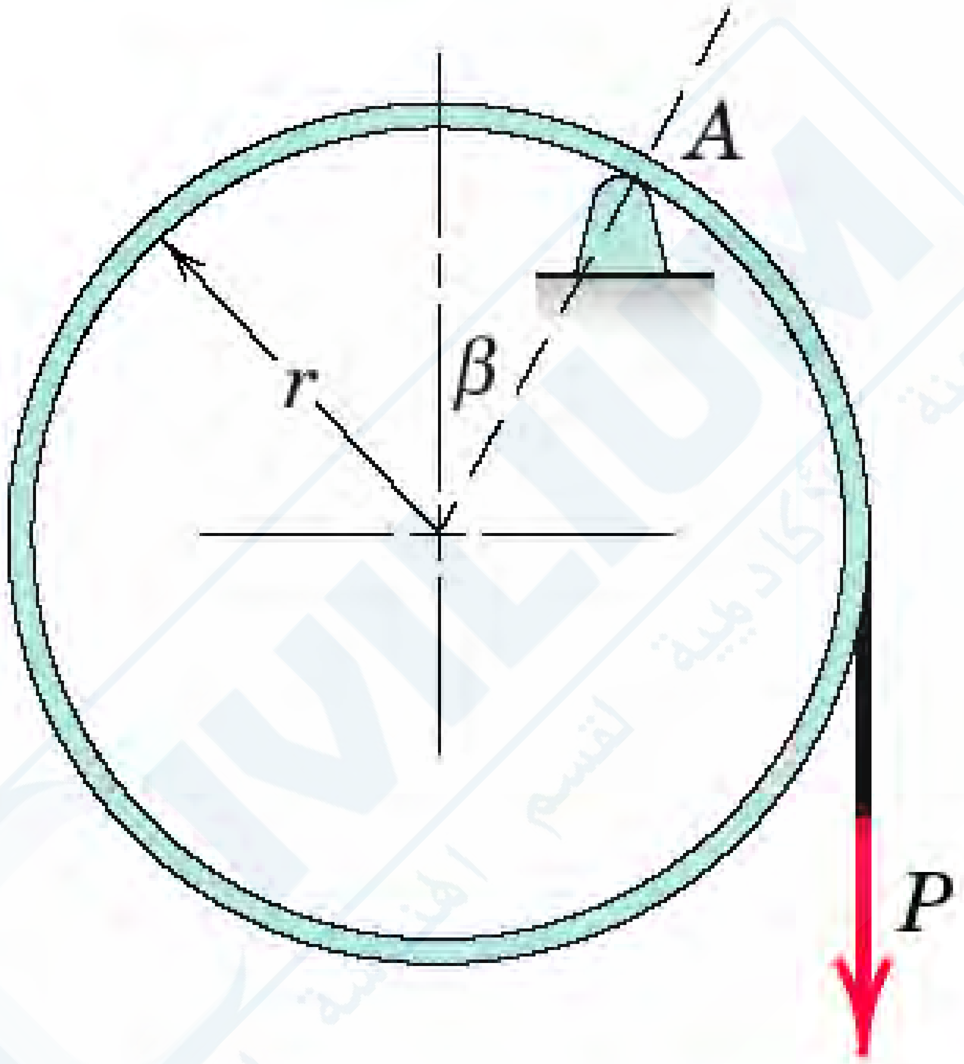
Problem 6/27

P_06_27



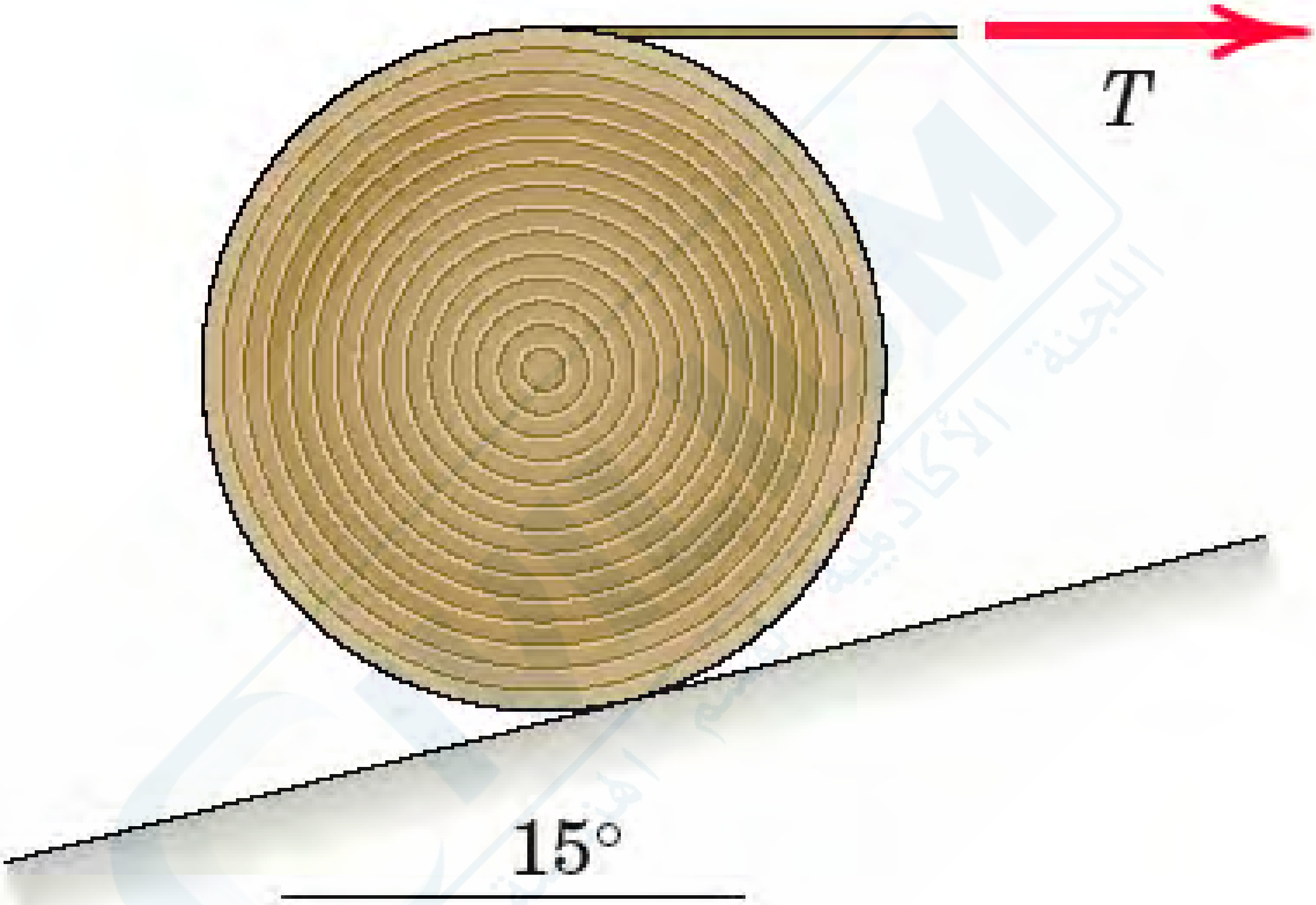
Problem 6/30

P_06_30



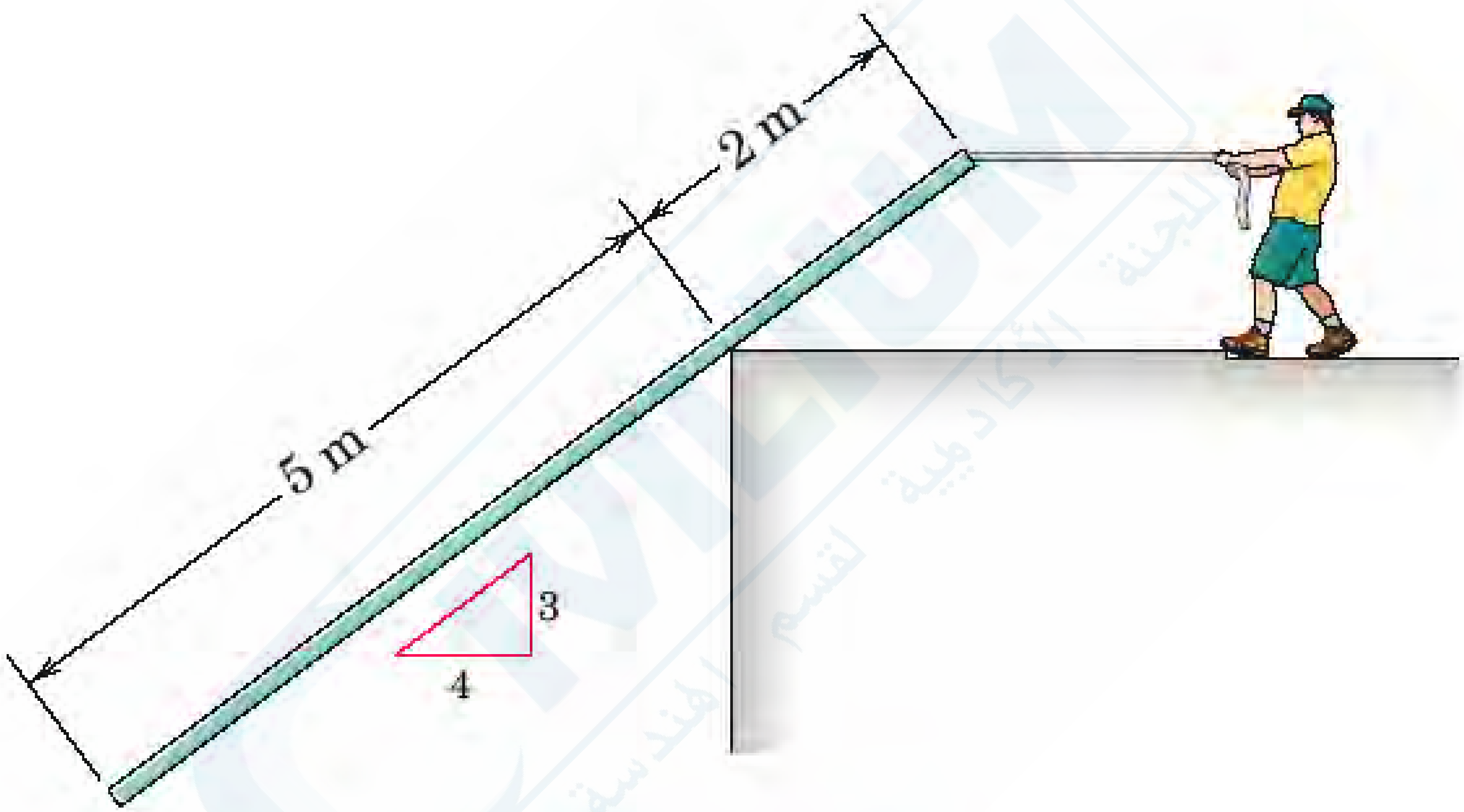
Problem 6/31

P_06_31



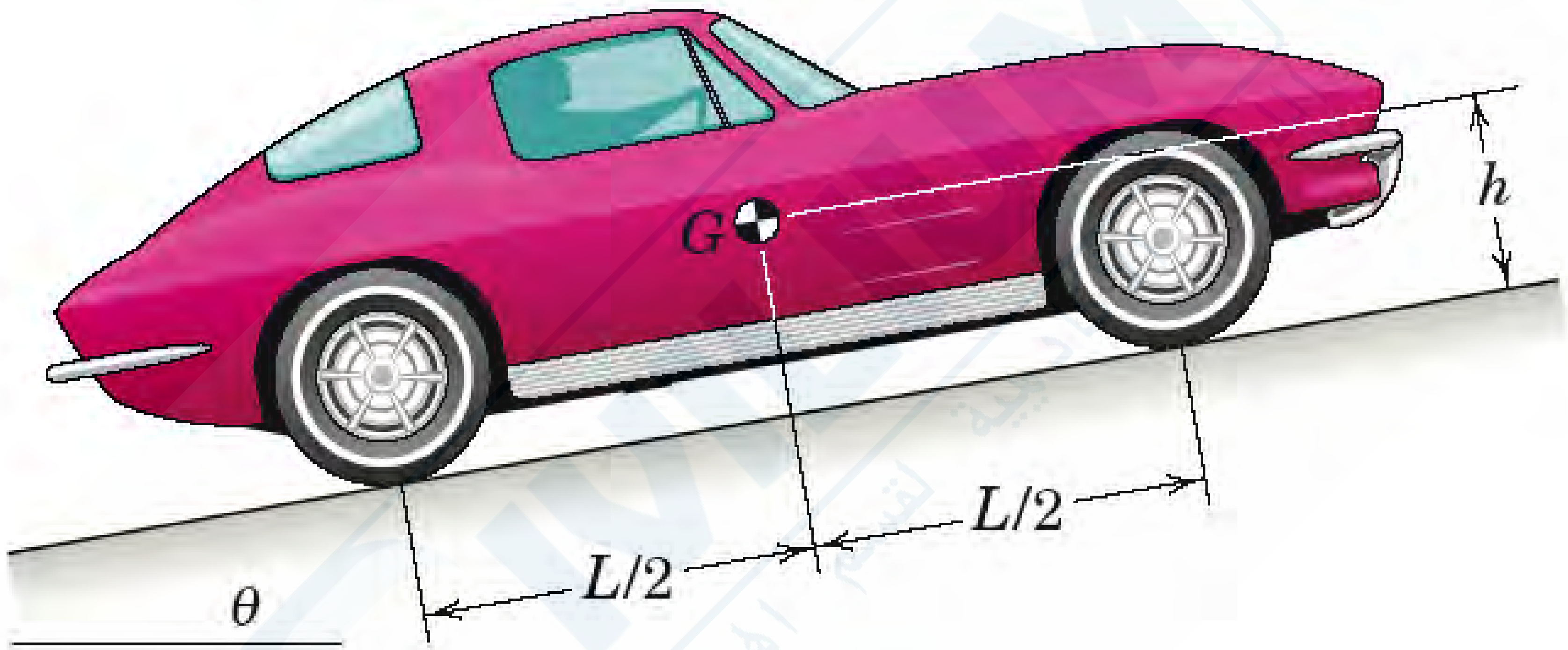
Problem 6/32

P_06_32



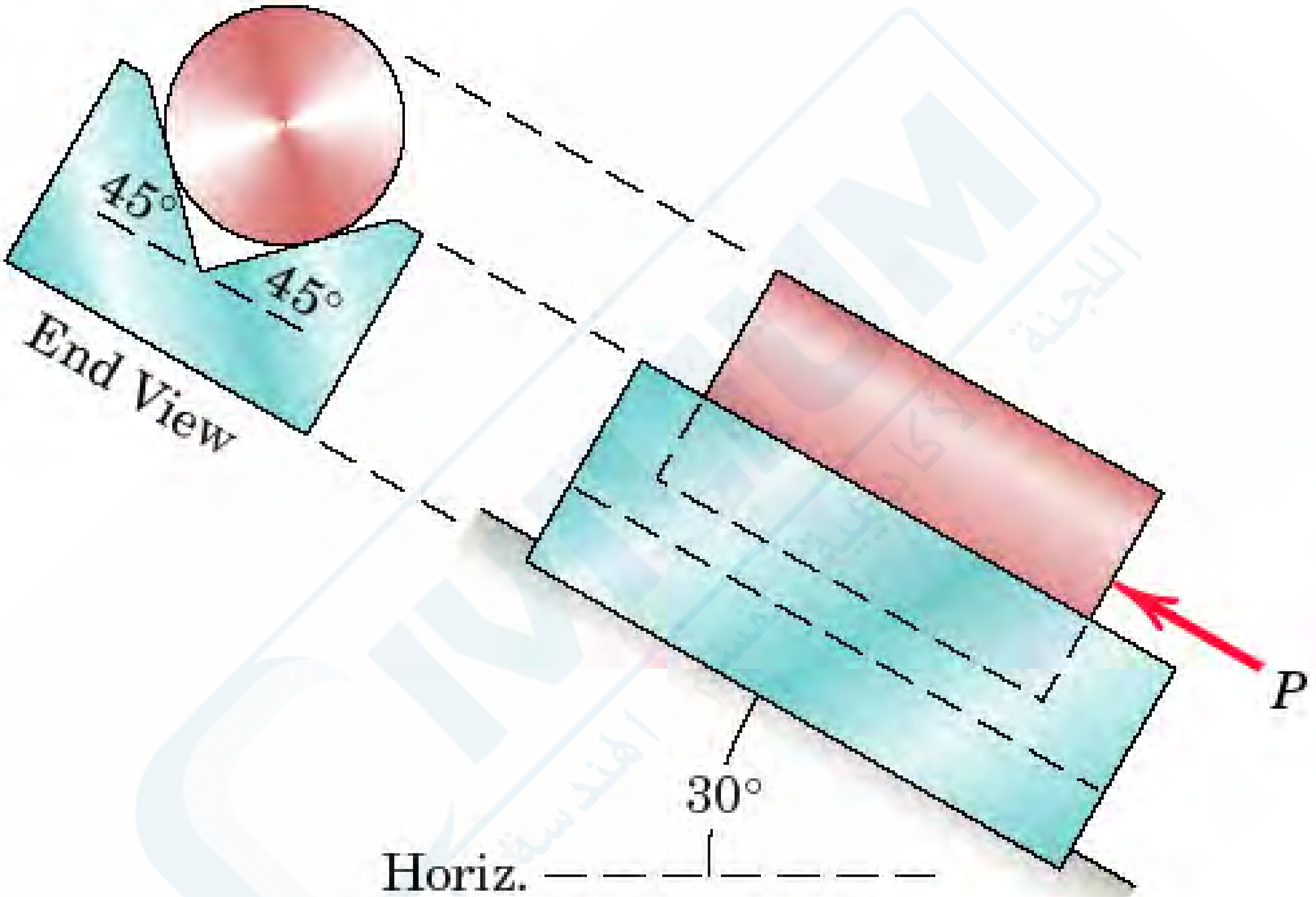
Problem 6/33

P_06_33

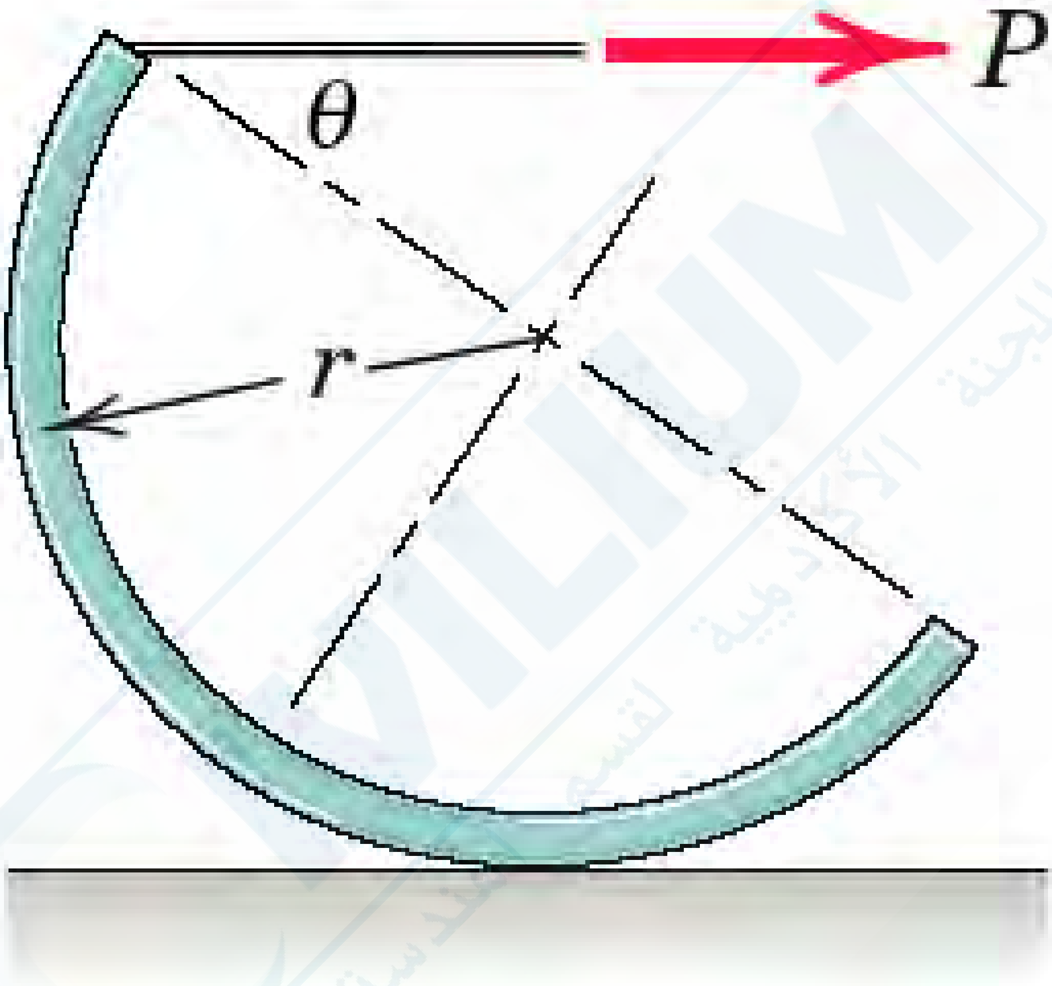


Problem 6/34

P_06_34



Problem 6/35



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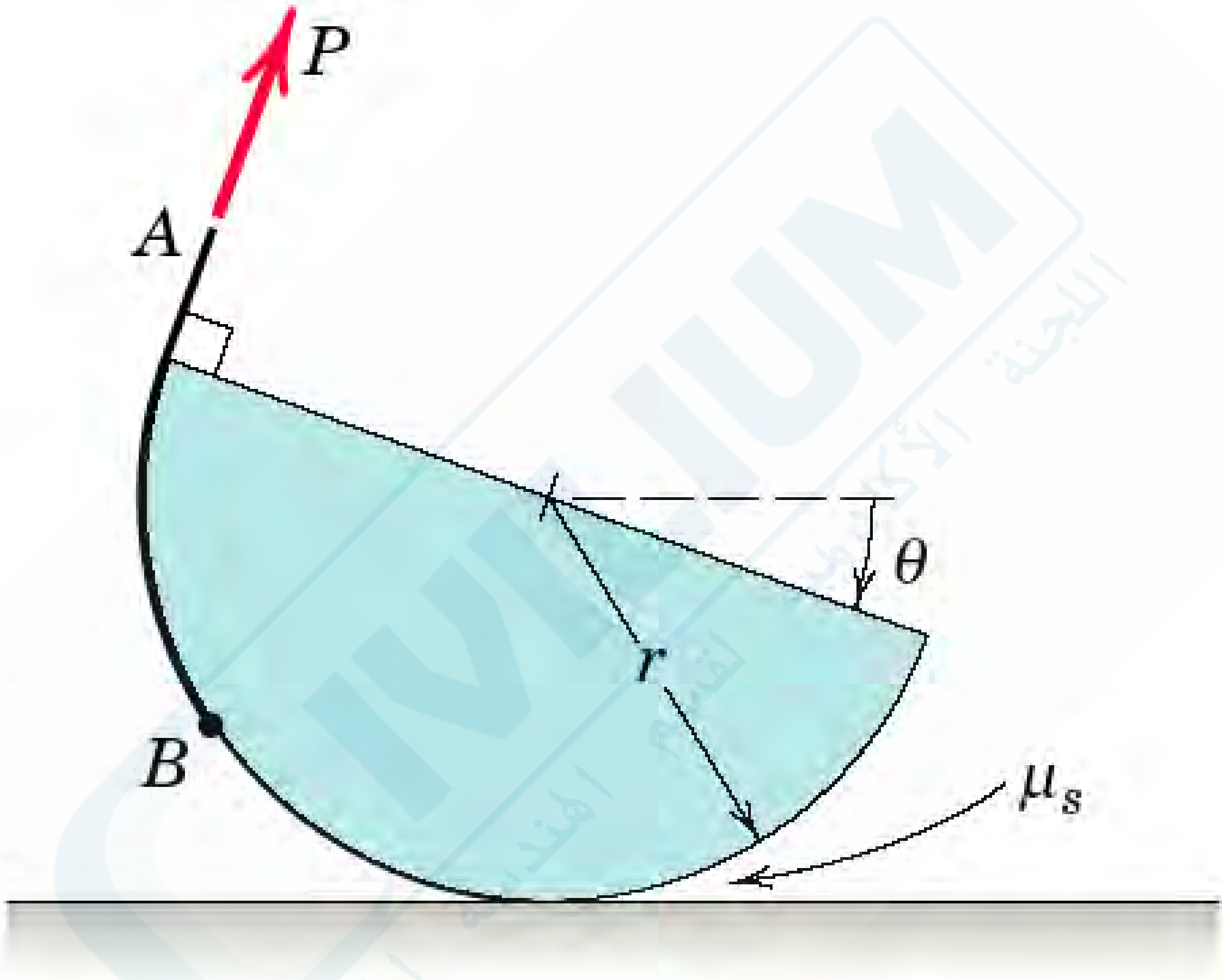
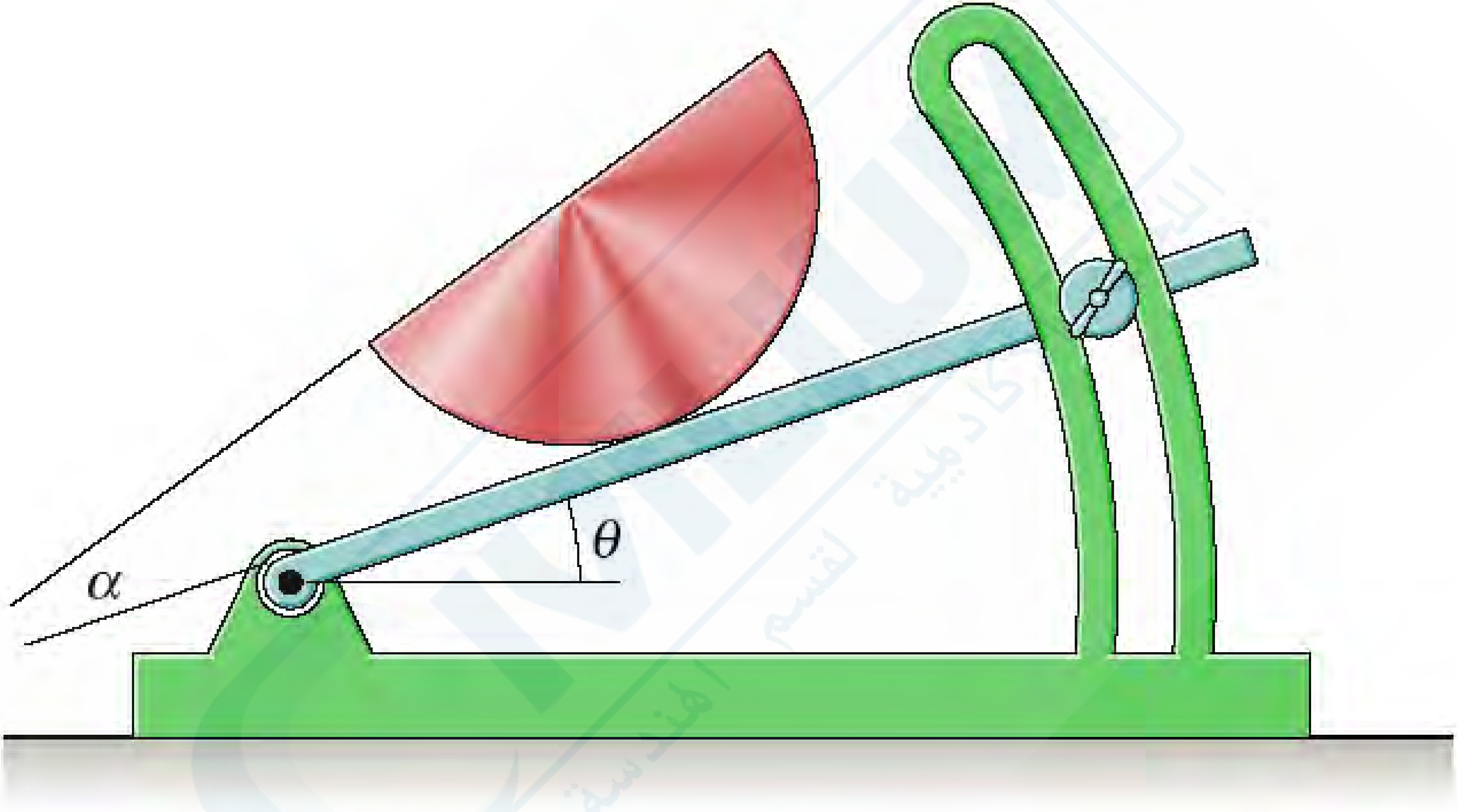
Problem 6/36



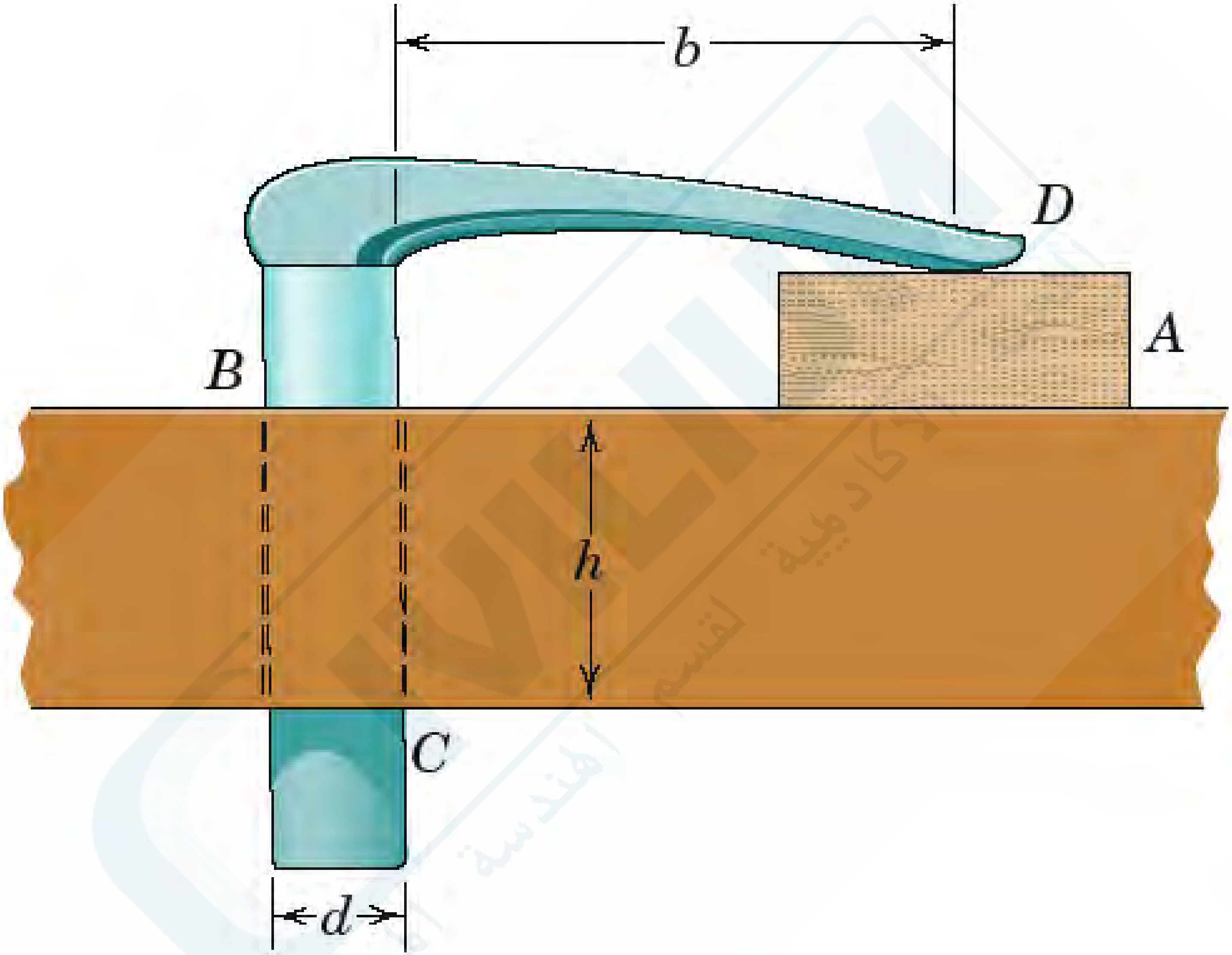
Problem 6/37

P_06_37



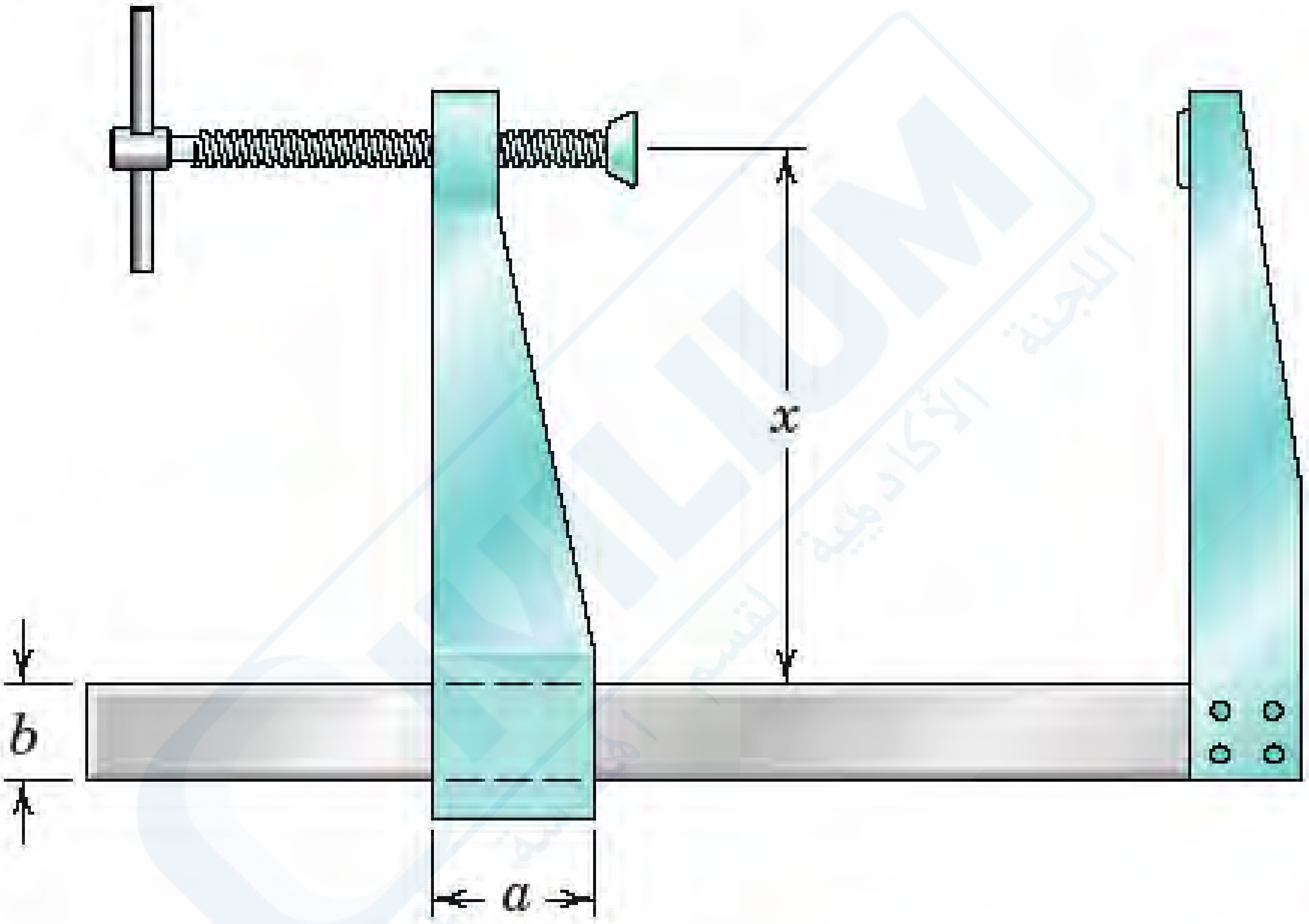
Problem 6/38

P_06_38



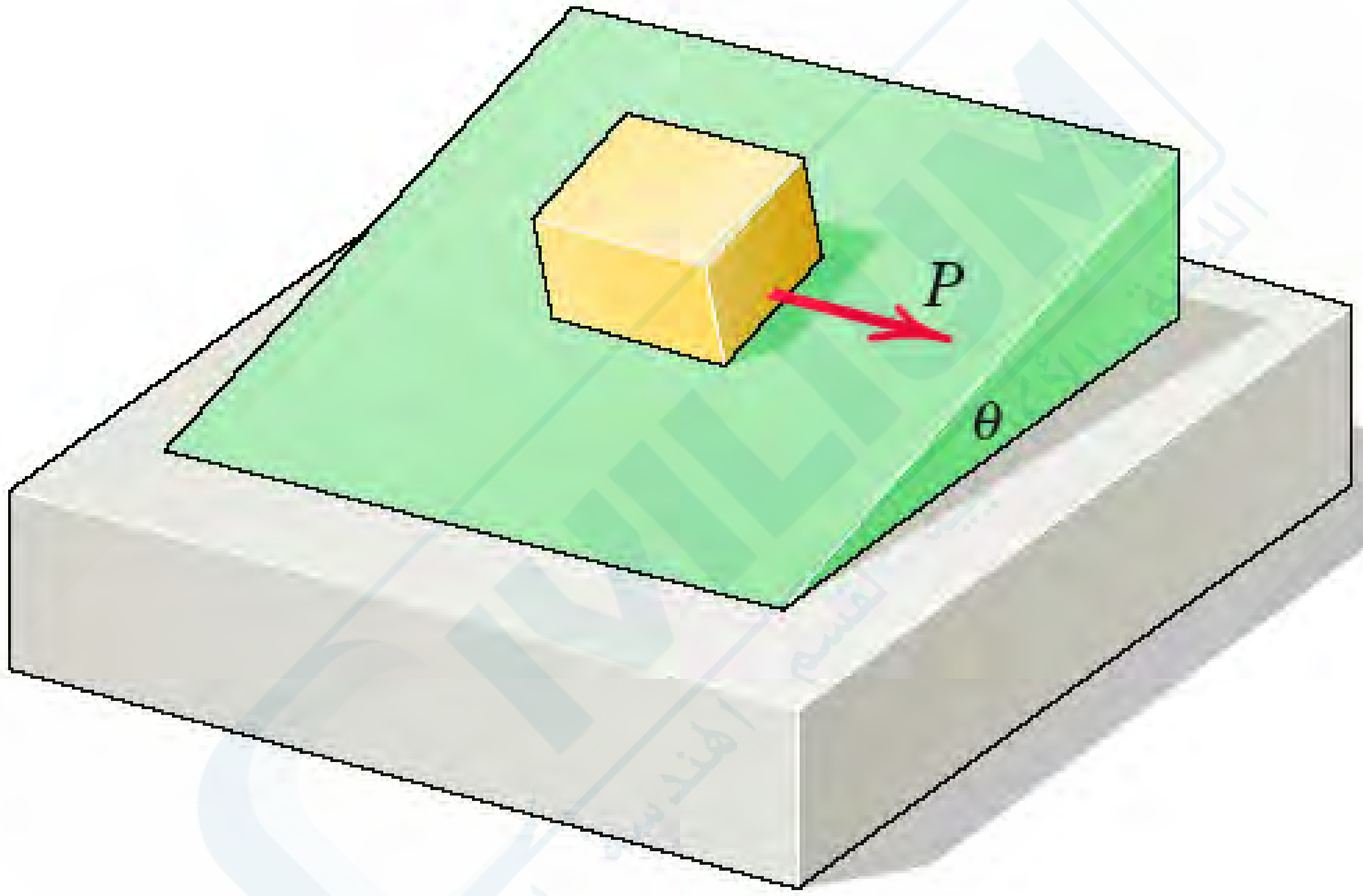
Problem 6/39

P_06_39



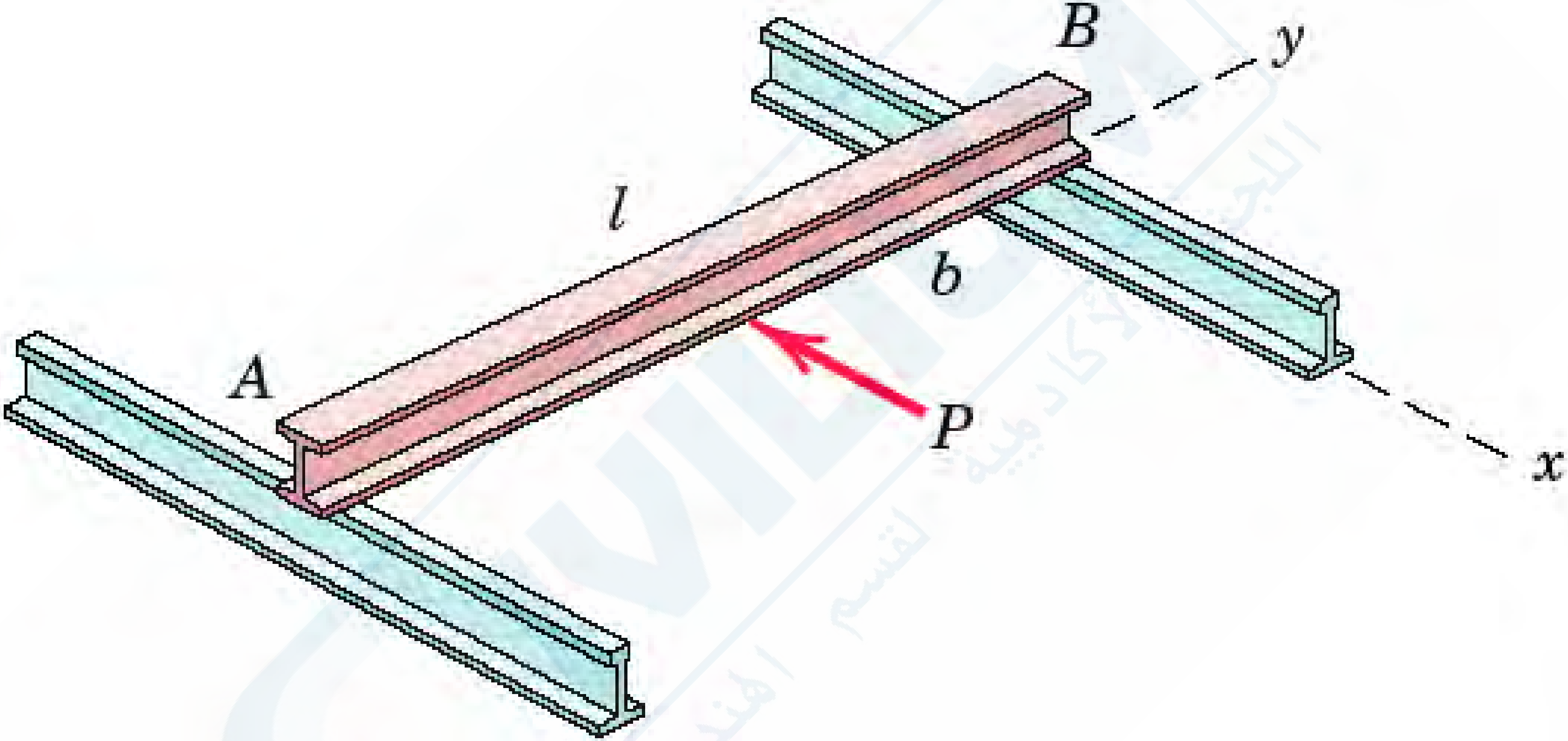
Problem 6/40

P_06_40



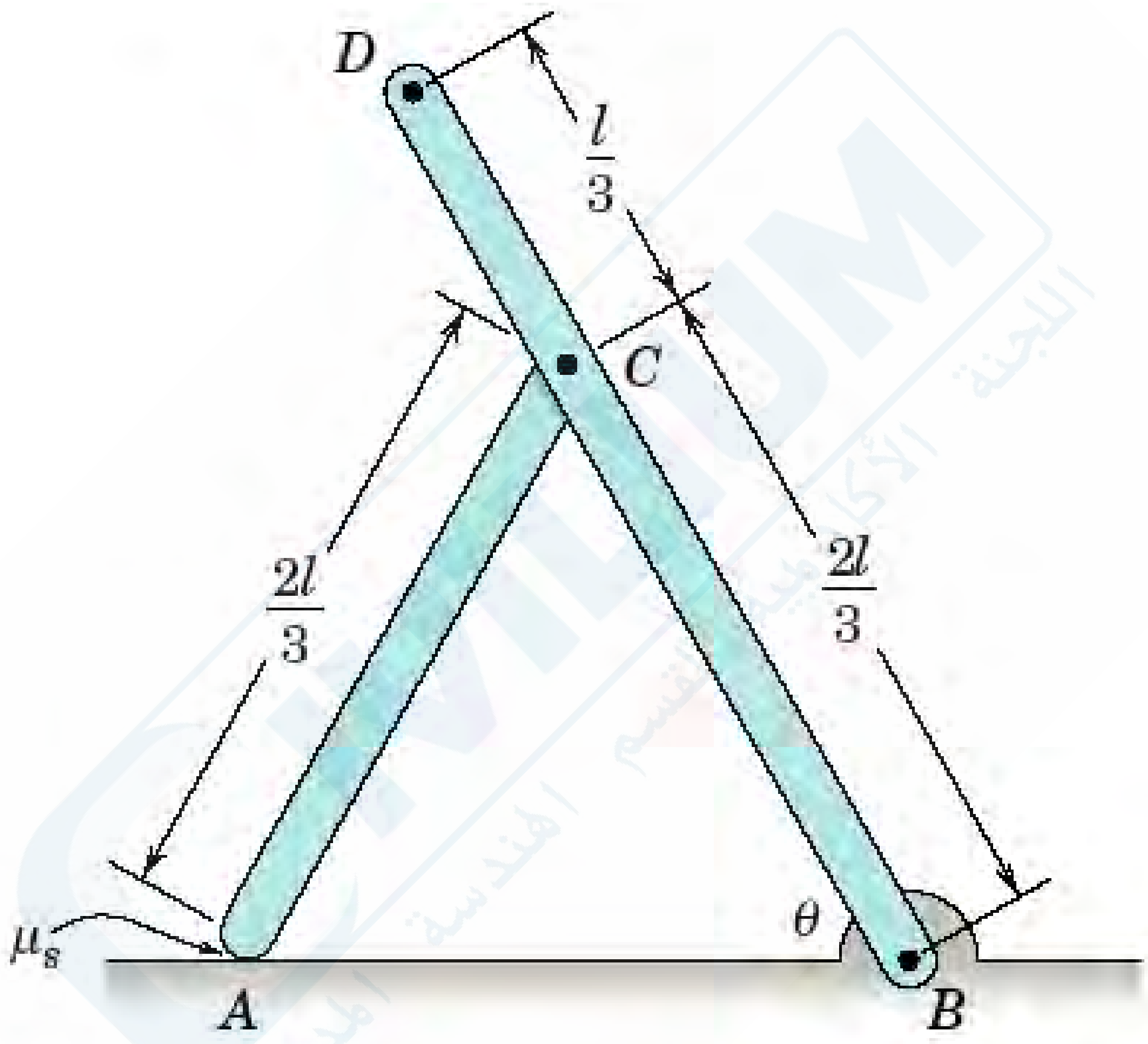
Problem 6/41

P_06_41



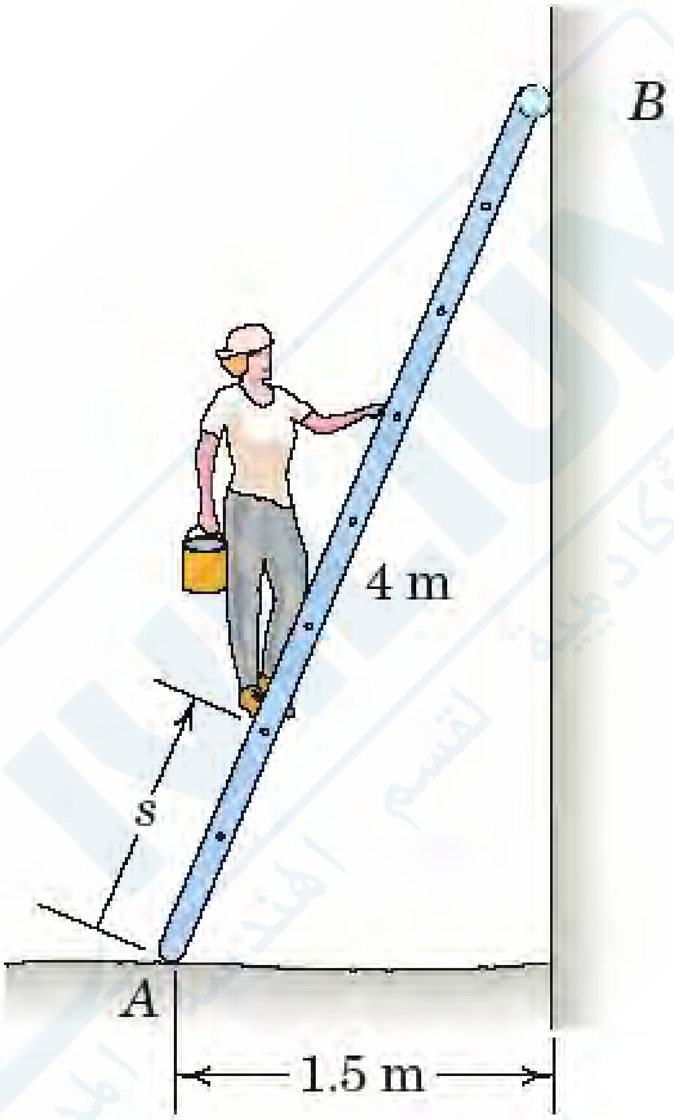
Problem 6/42

P_06_42



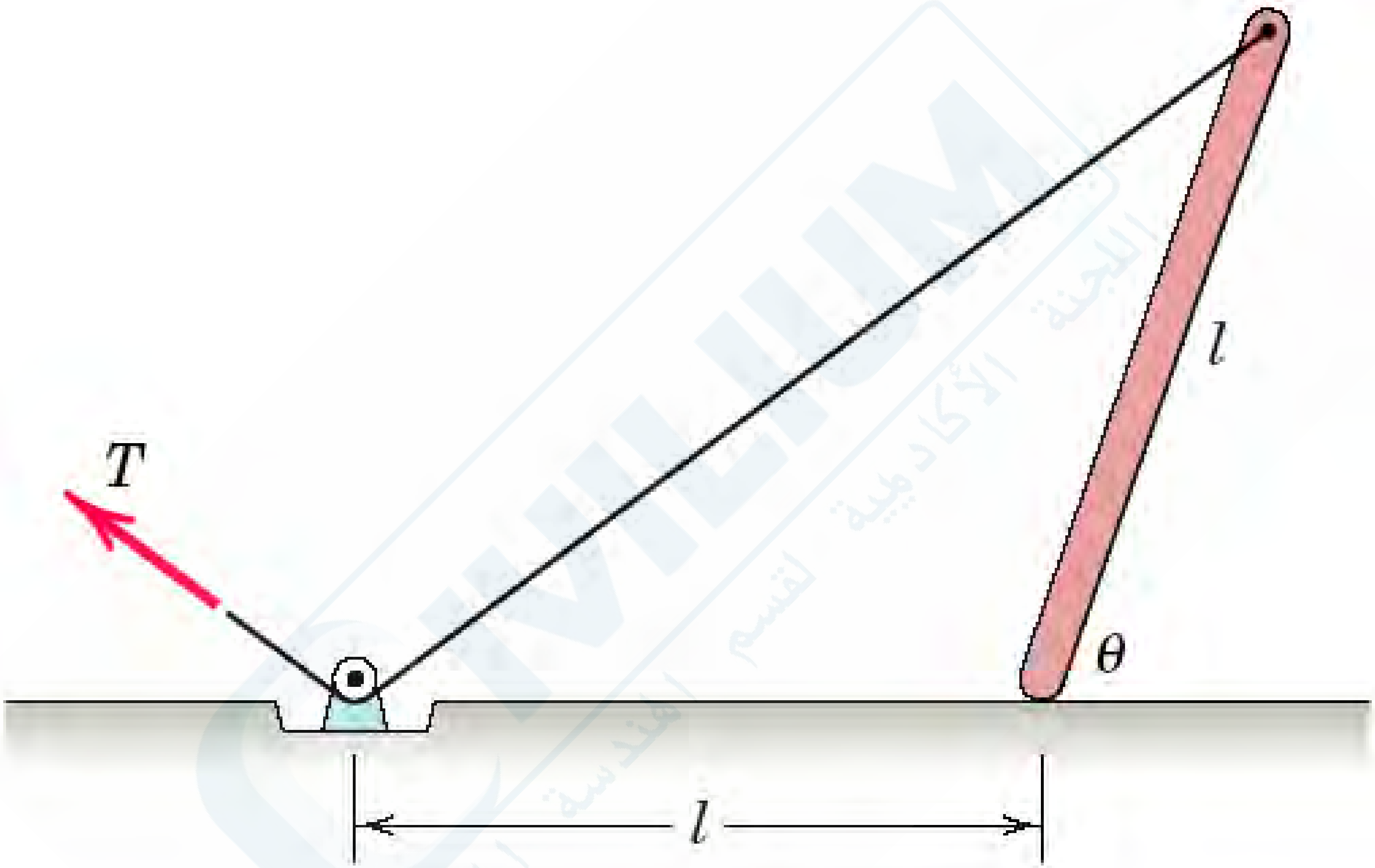
Problem 6/43

P_06_43



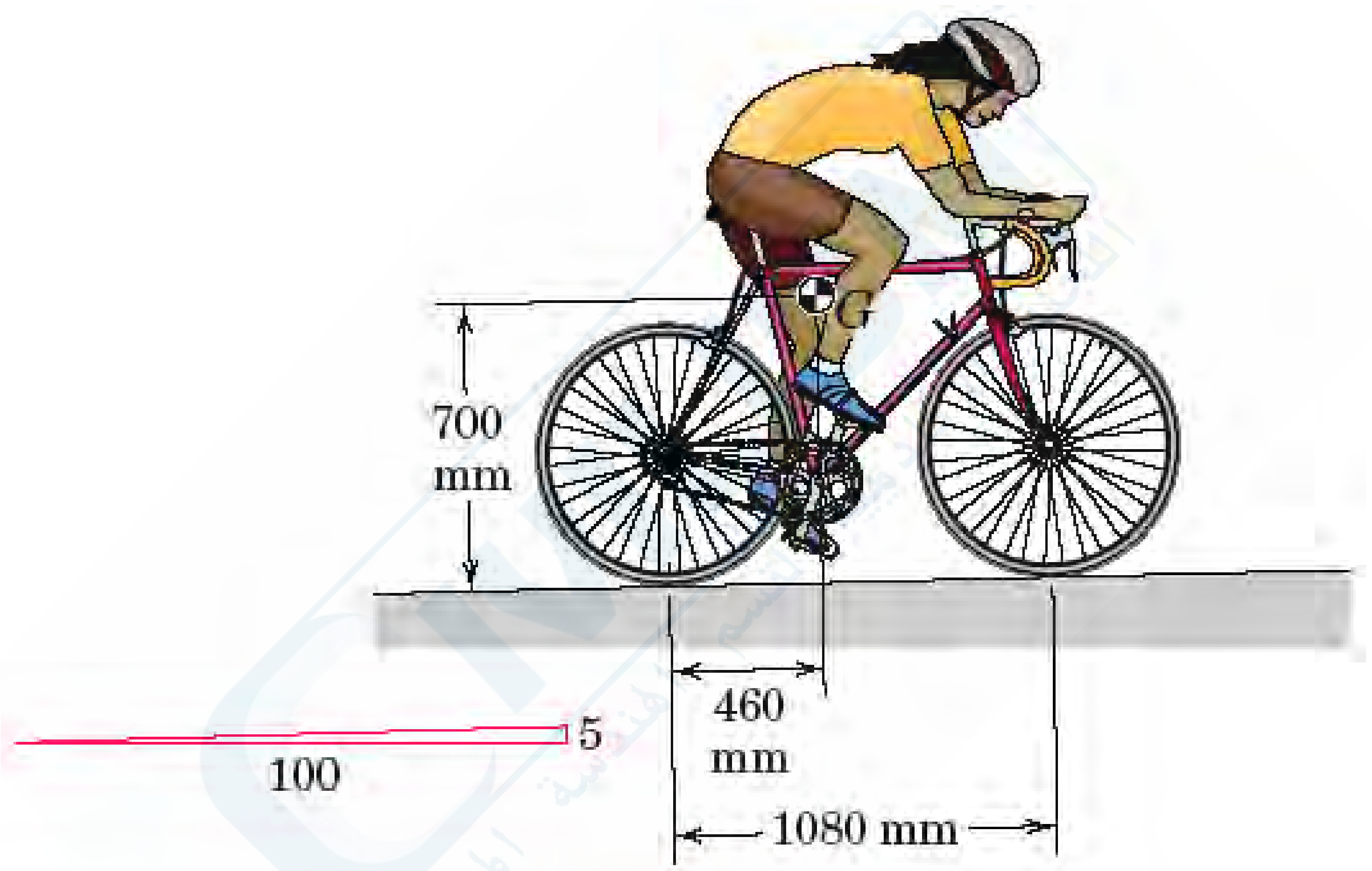
Problem 6/44

P_06_44



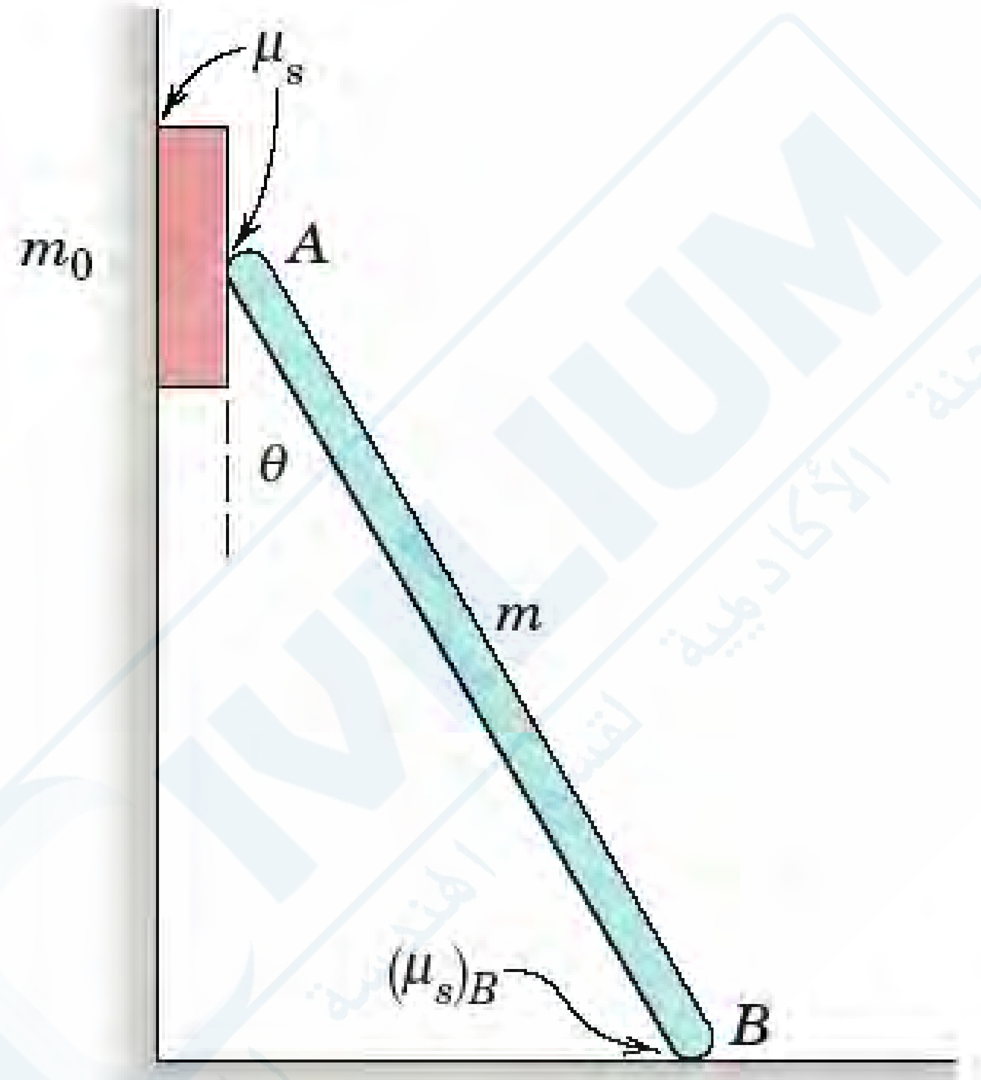
Problem 6/45

P_06_45



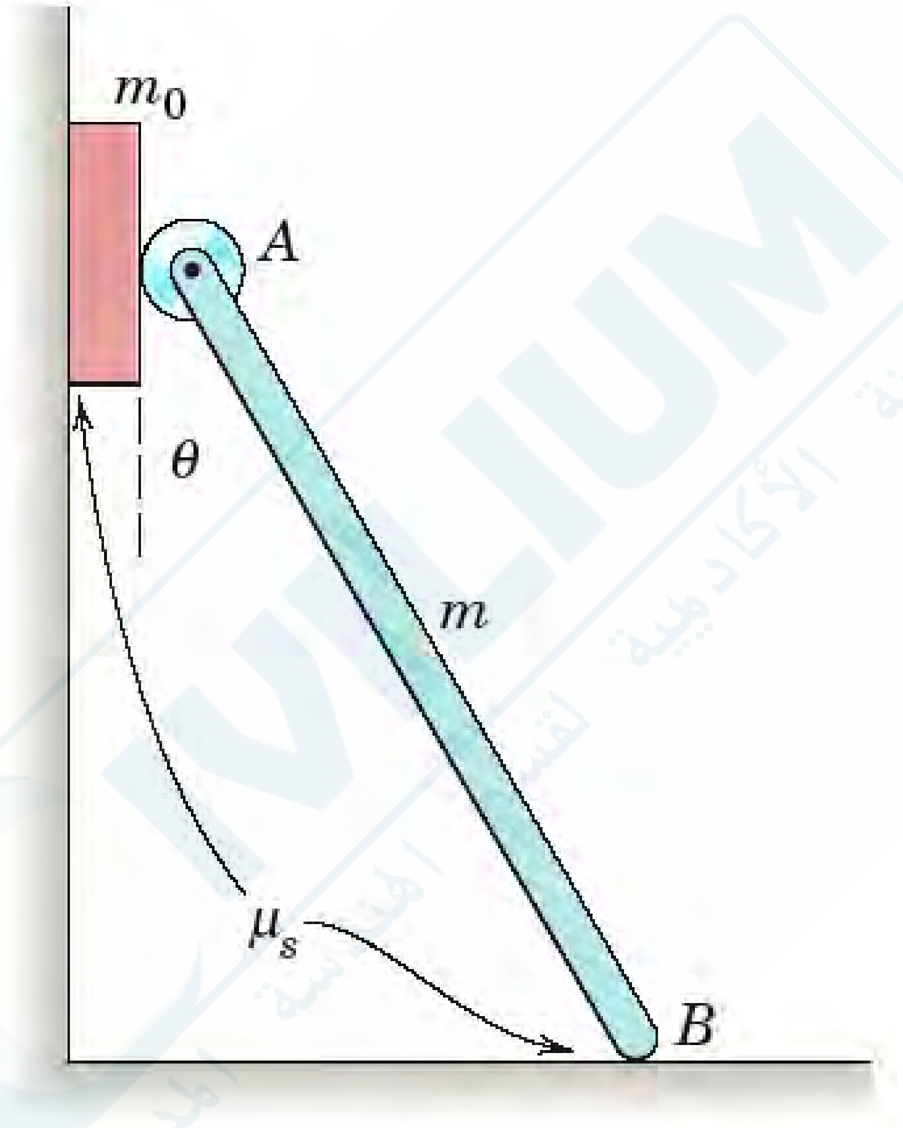
Problem 6/46

P_06_46



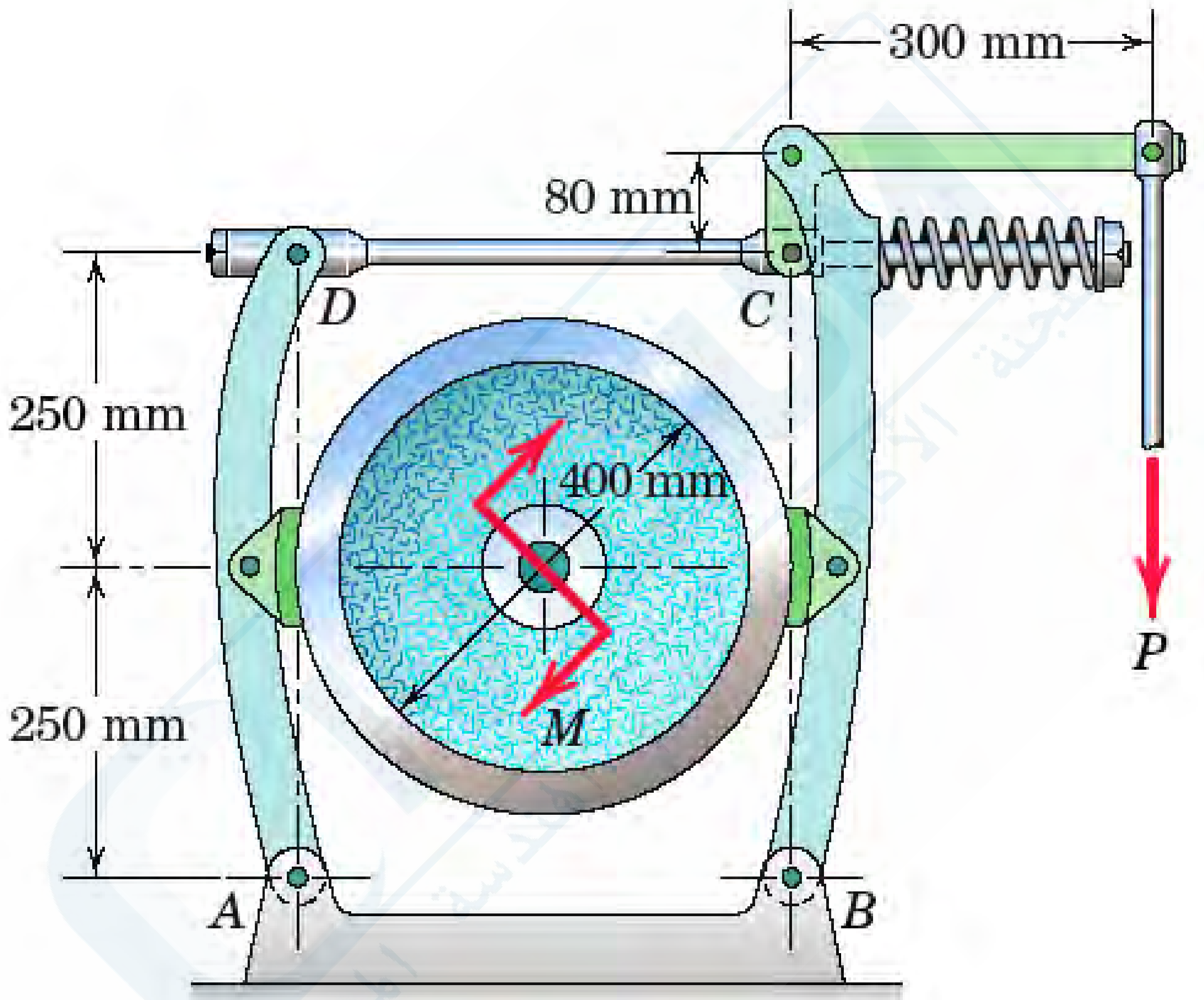
Problem 6/47

P_06_47



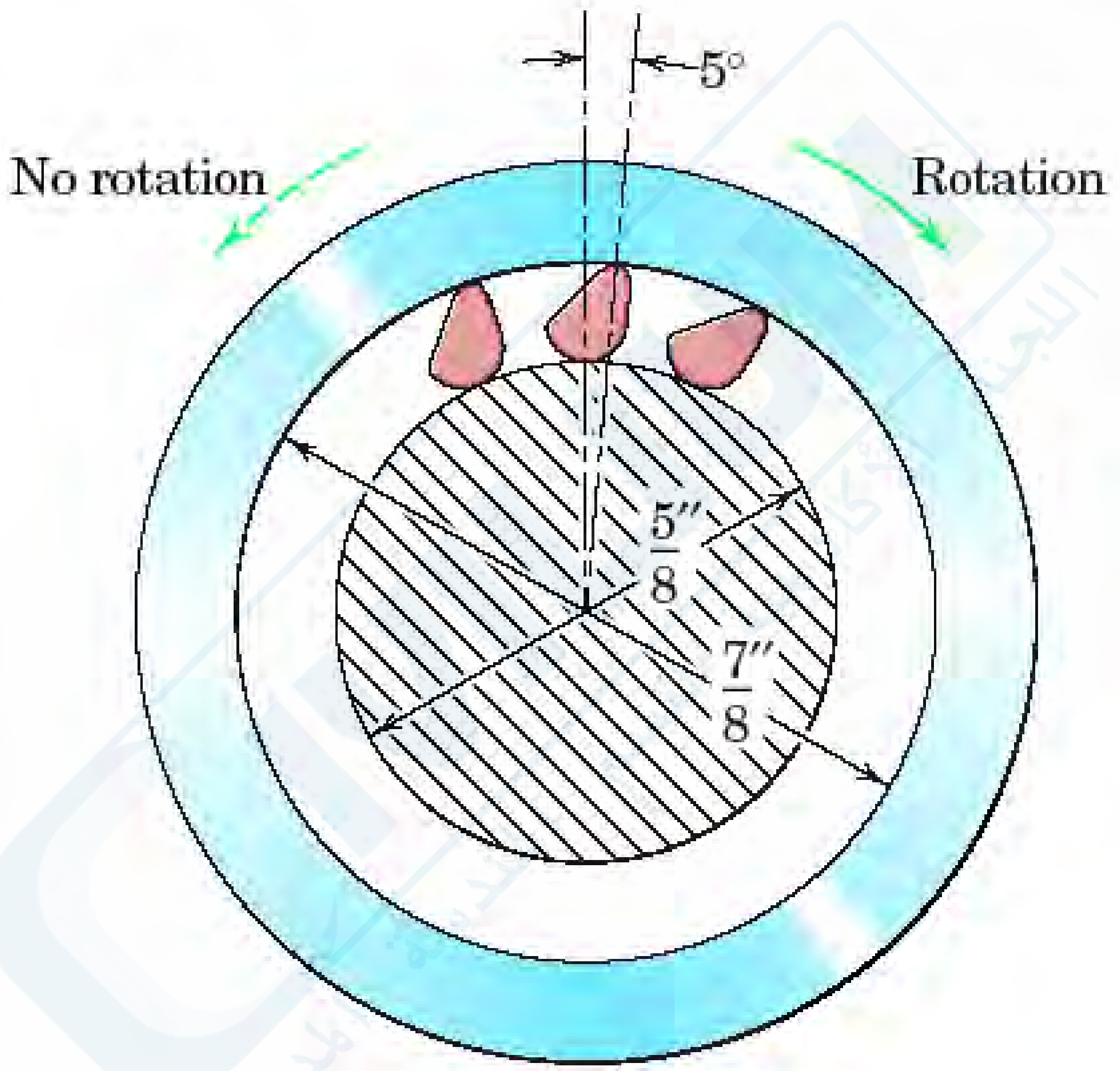
Problem 6/48

P_06_48



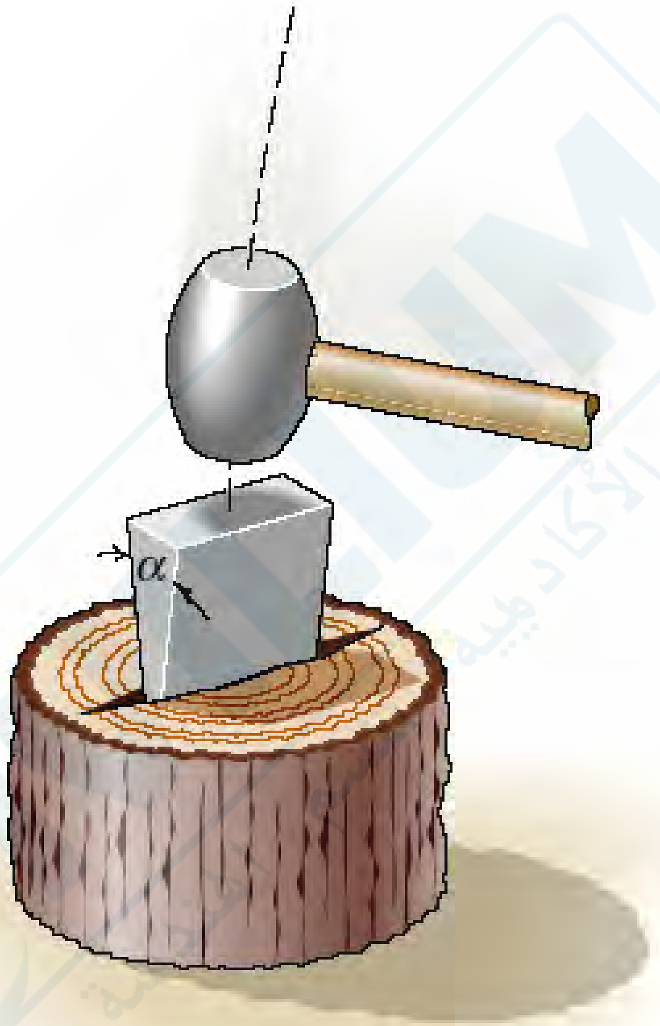
Problem 6/49

P_06_49



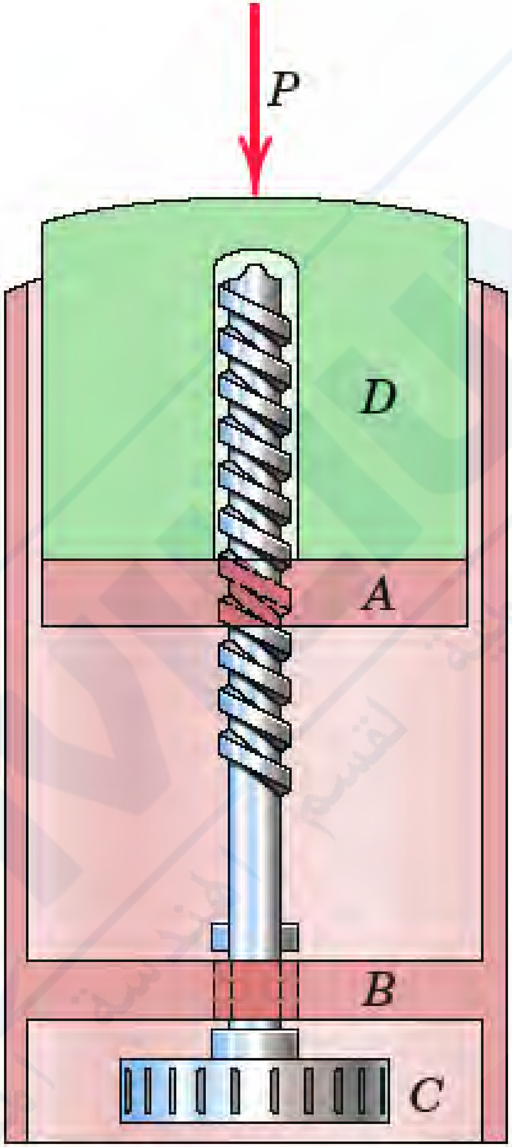
Problem 6/50

P_06_50



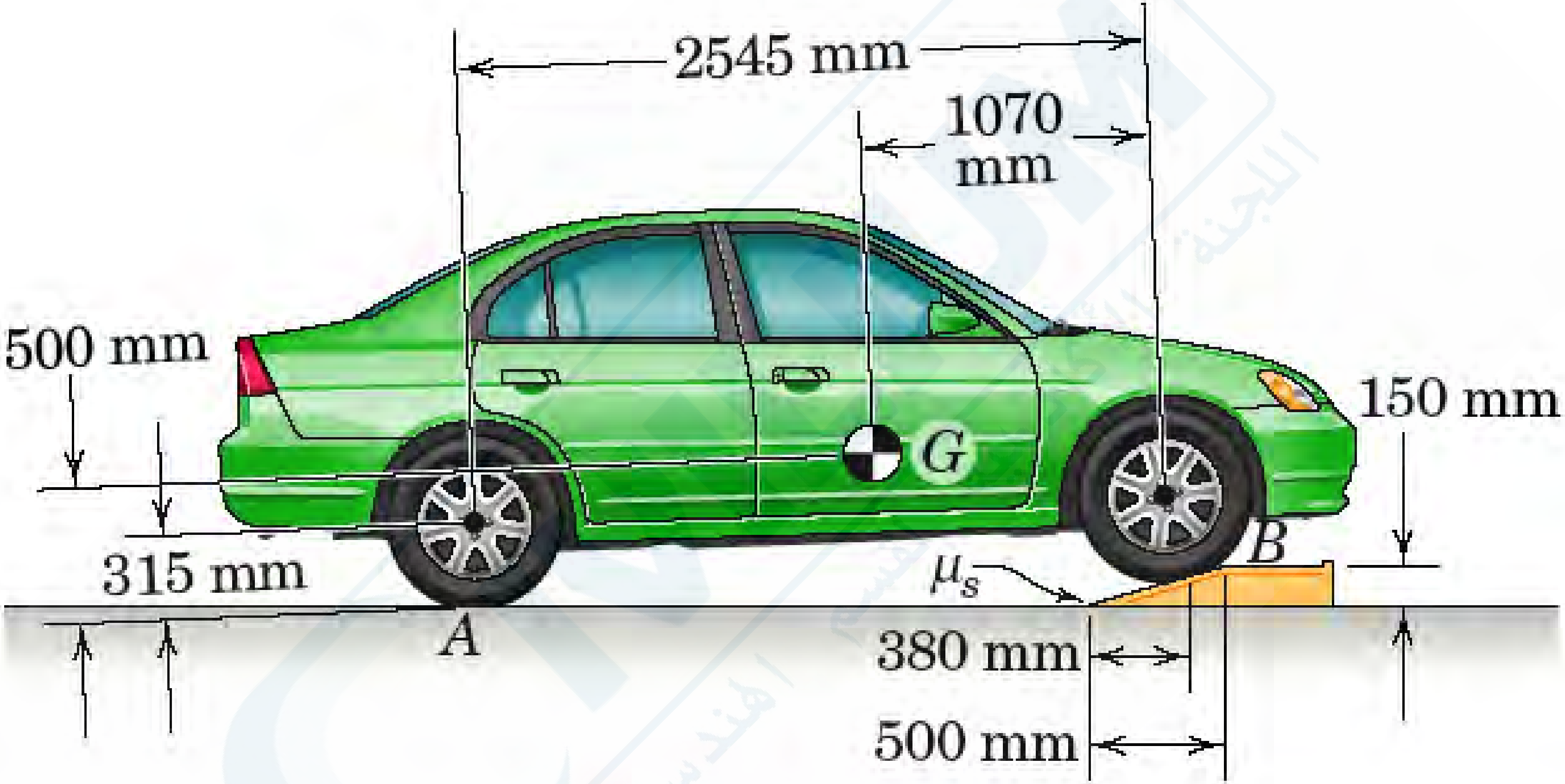
Problem 6/51

P_06_51



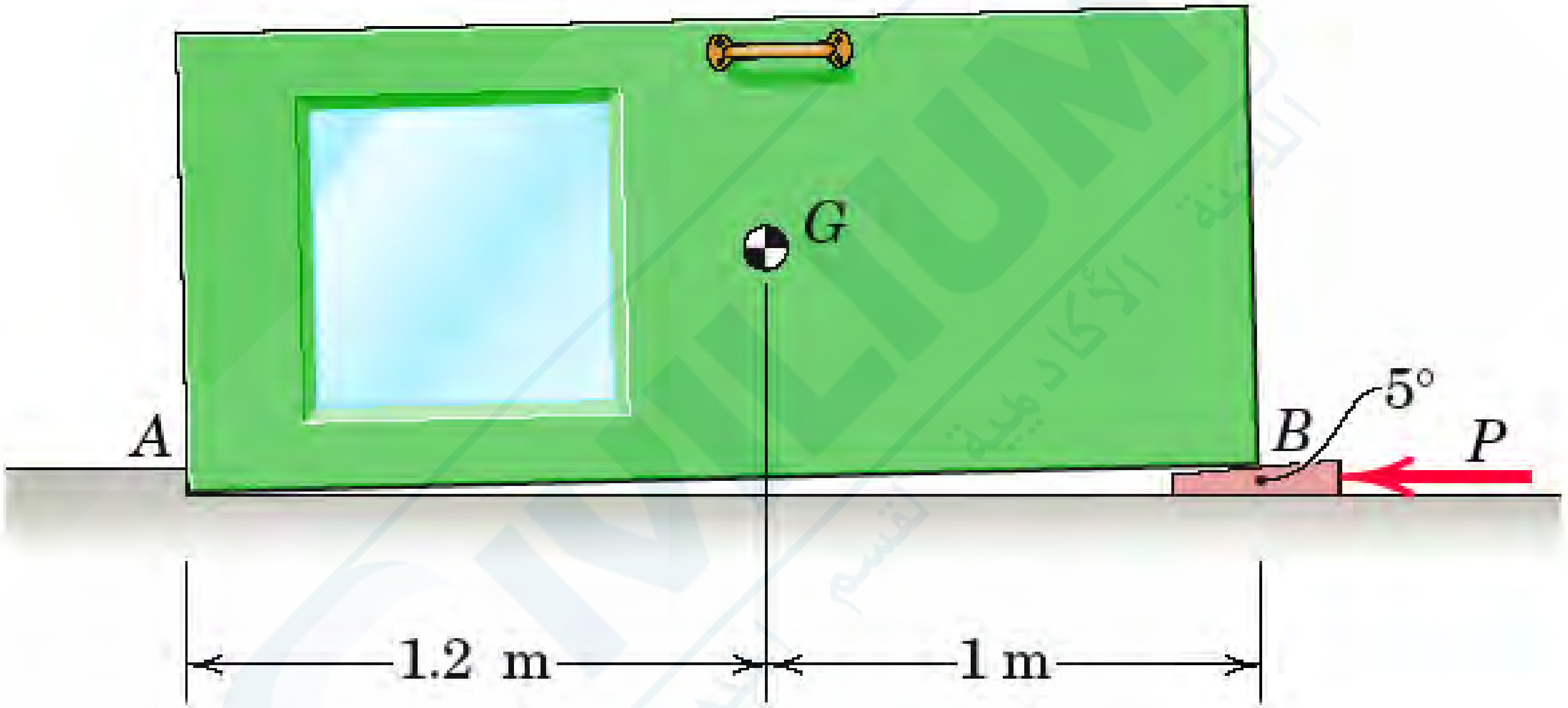
Problem 6/52

P_06_52



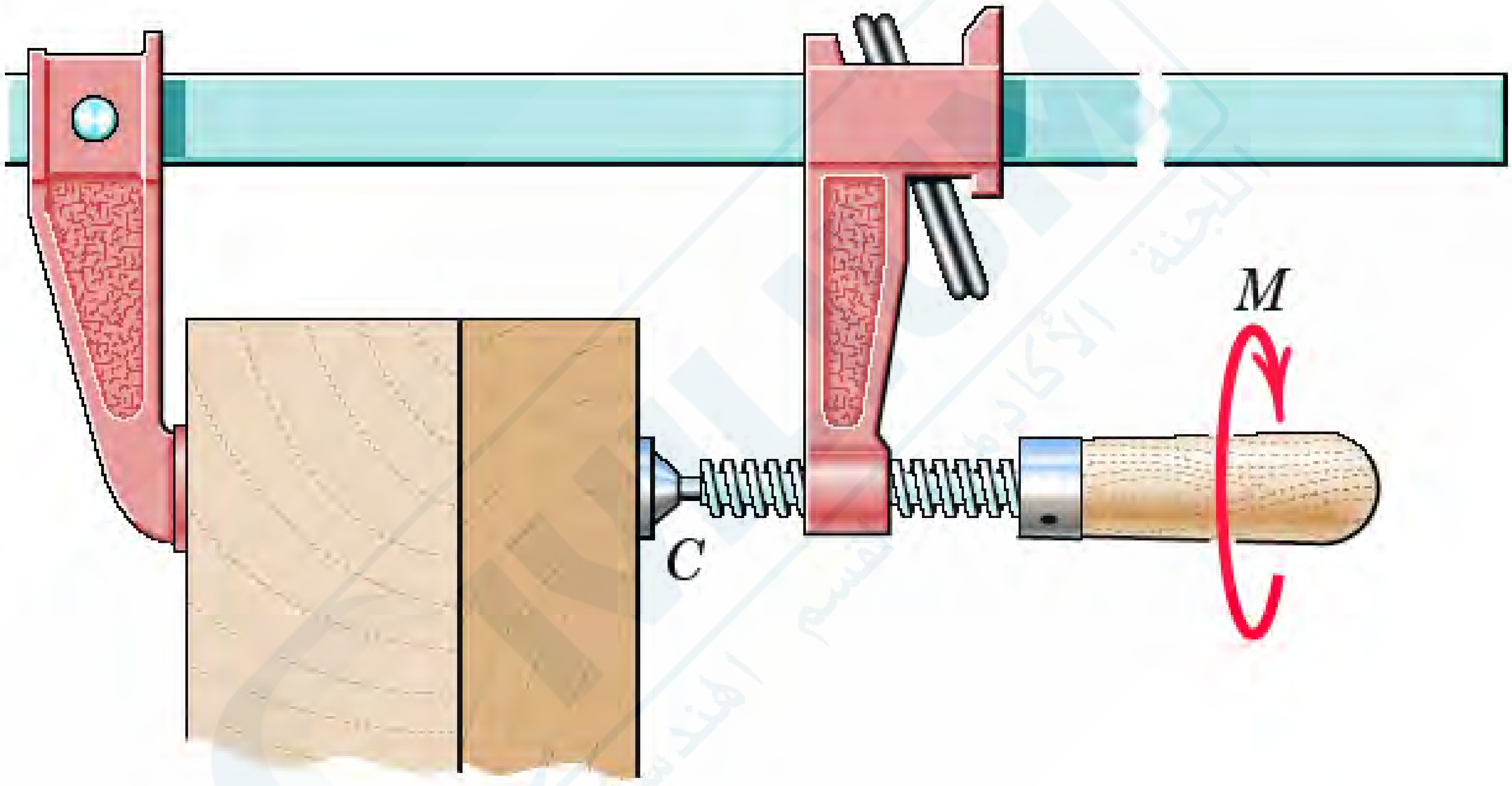
Problem 6/53

P_06_53



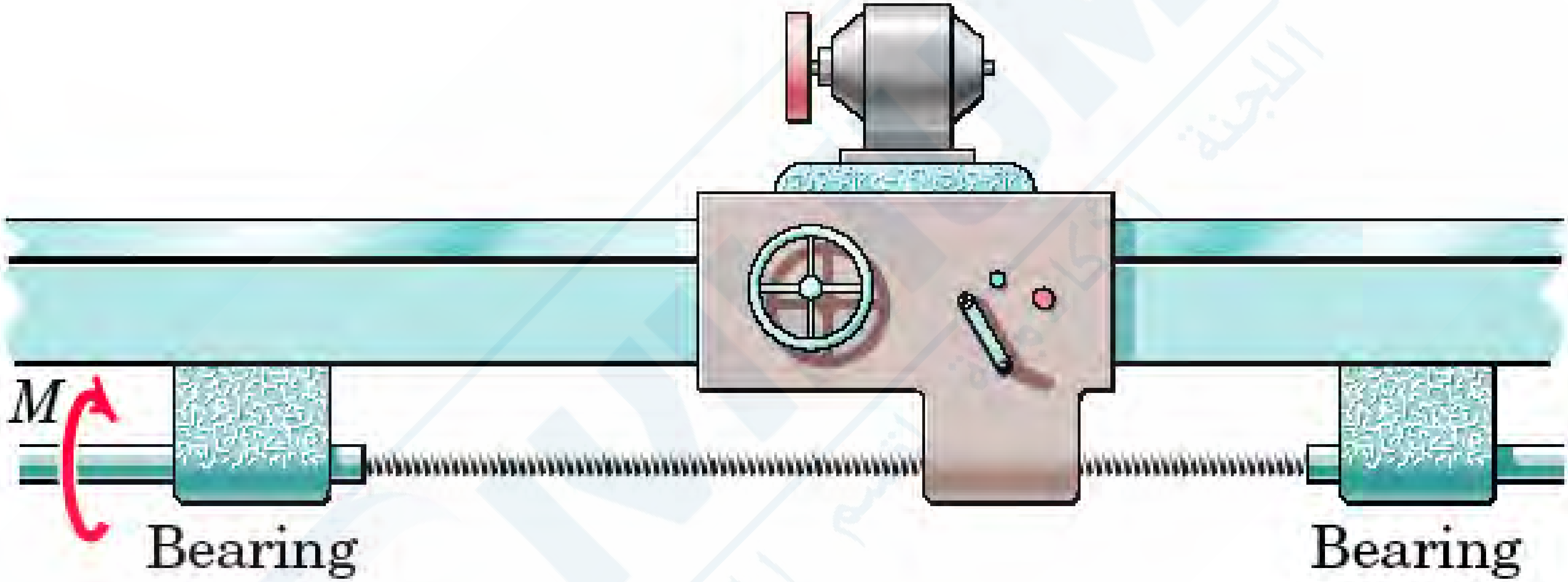
Problem 6/54

P_06_54



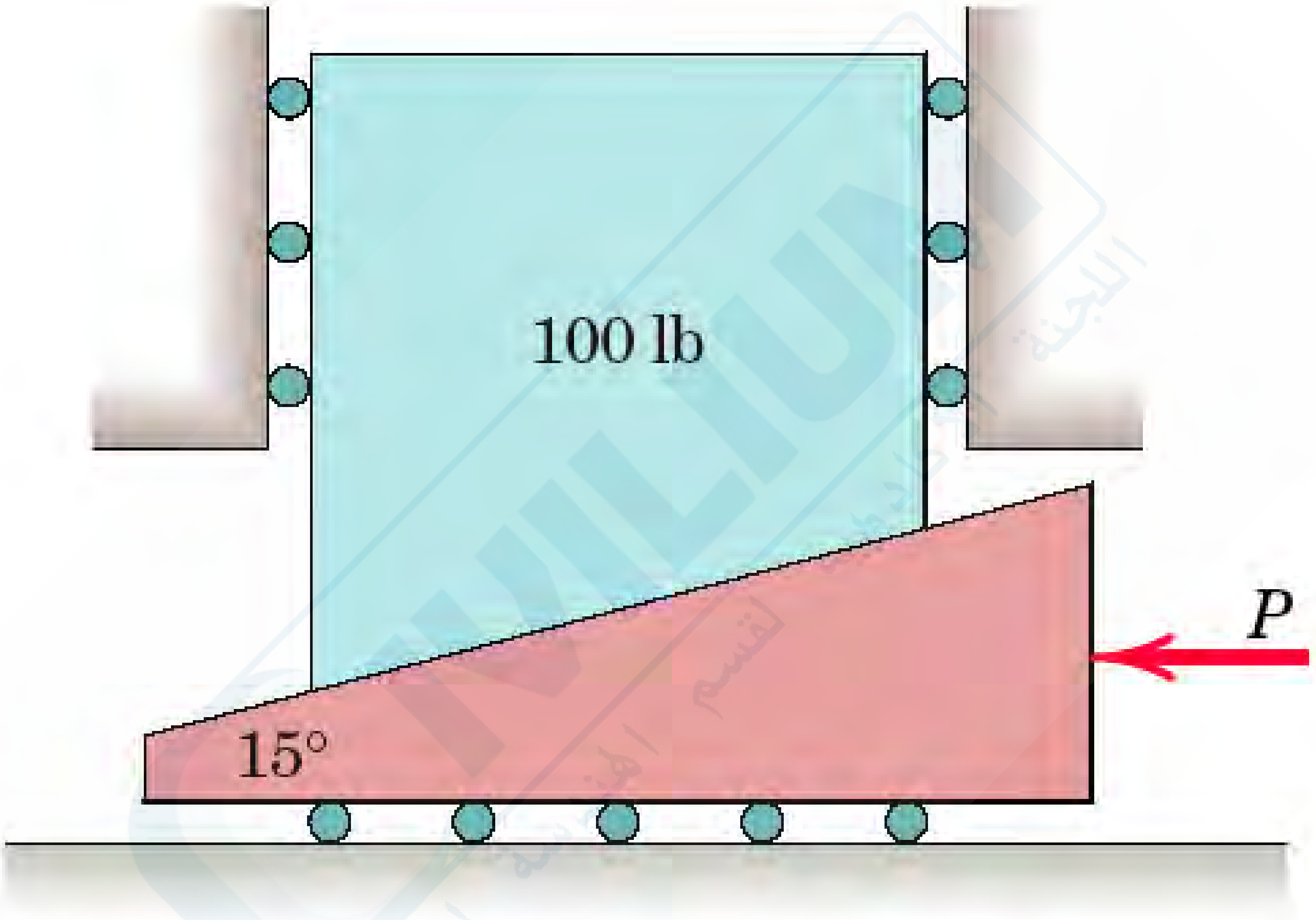
Problem 6/56

P_06_56



Problem 6/57

P_06_57

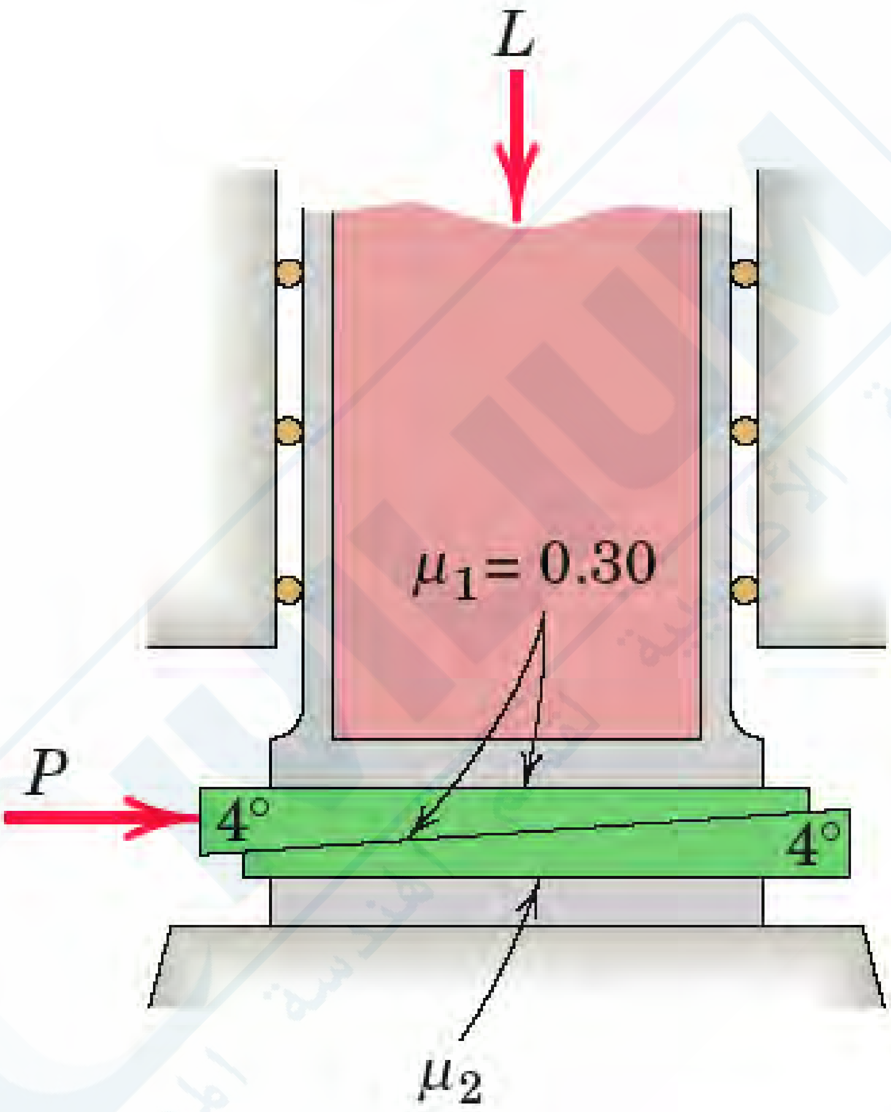


Problem 6/58

P_06_58

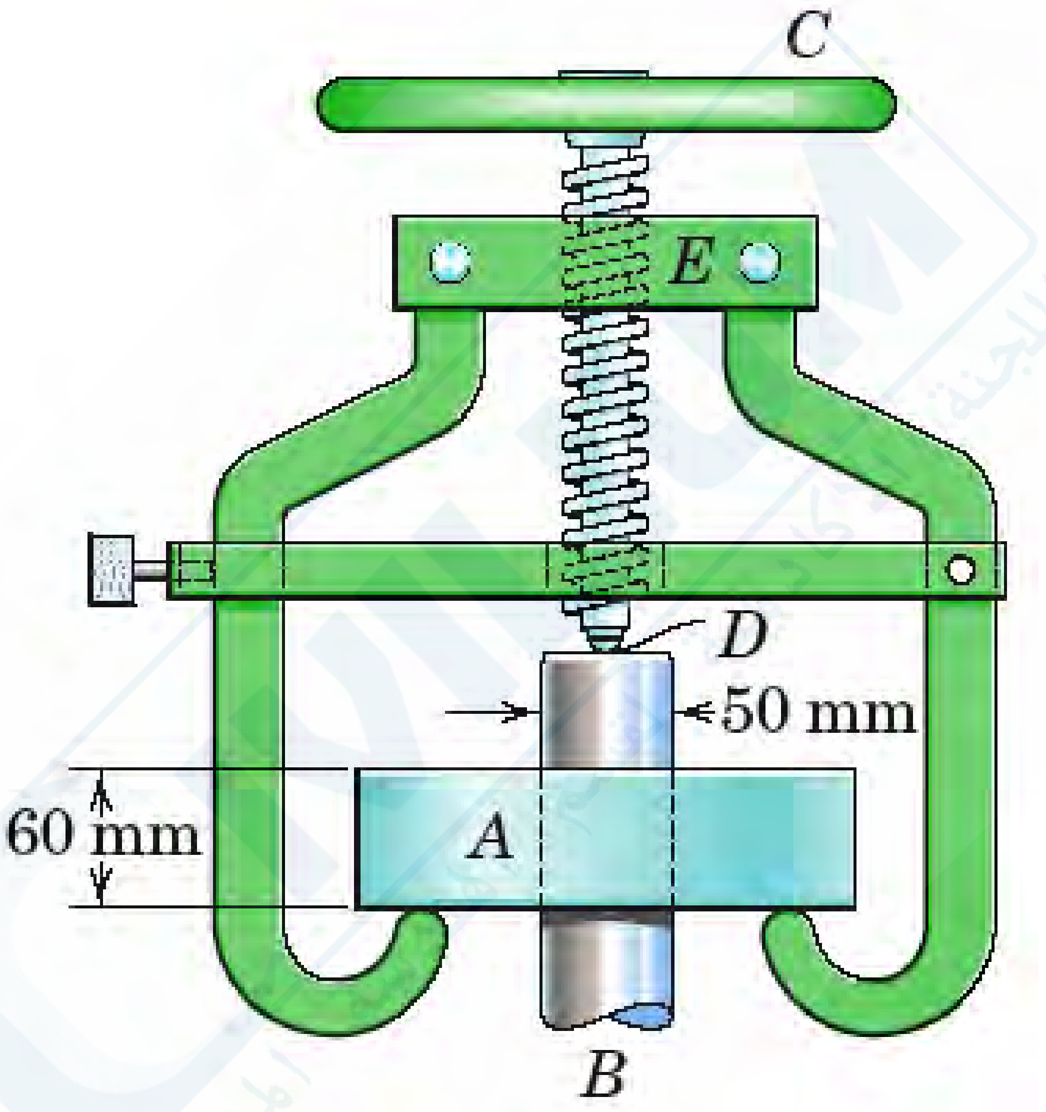
Problem 6/60

P_06_60



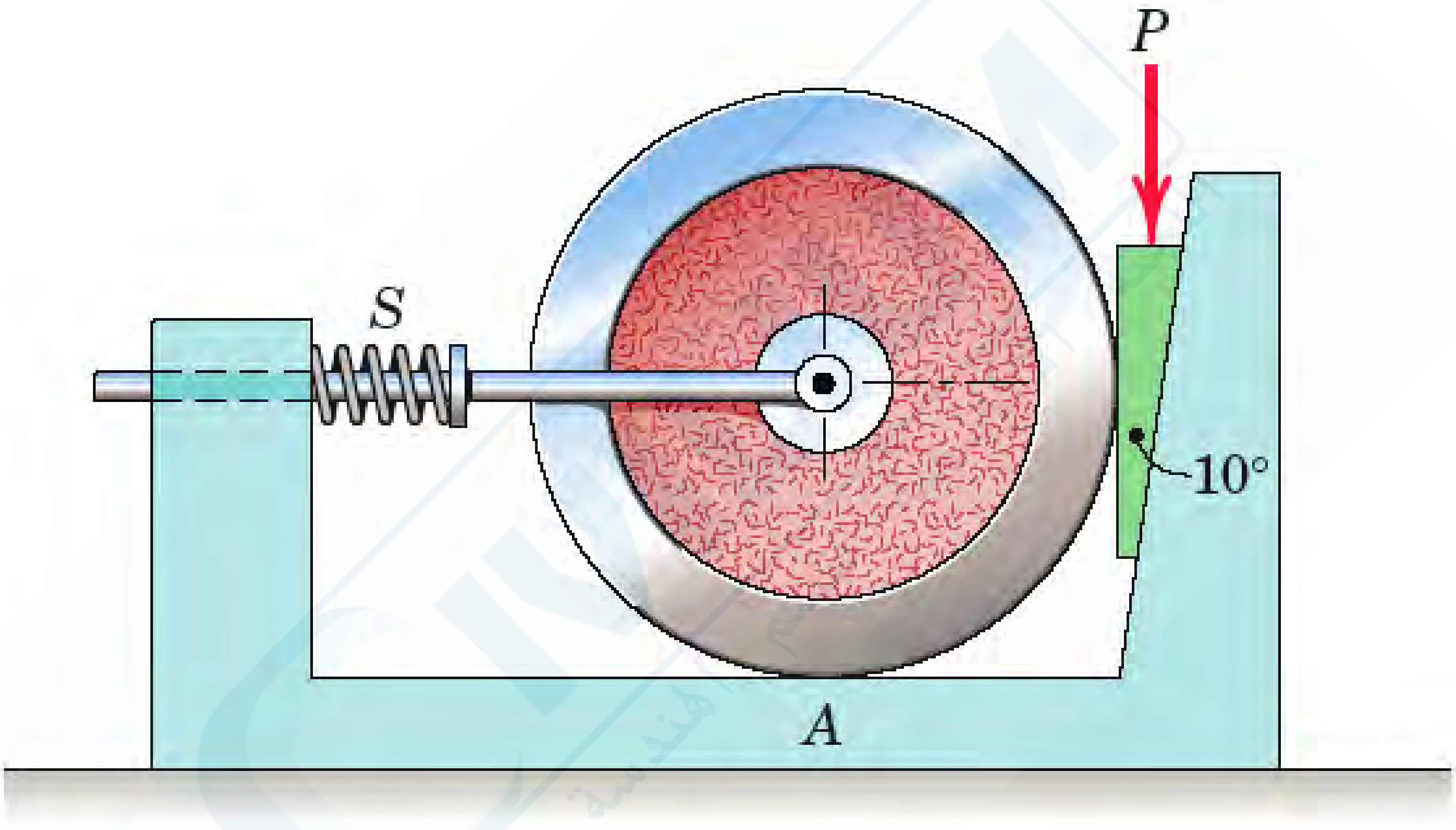
Problem 6/61

P_06_61



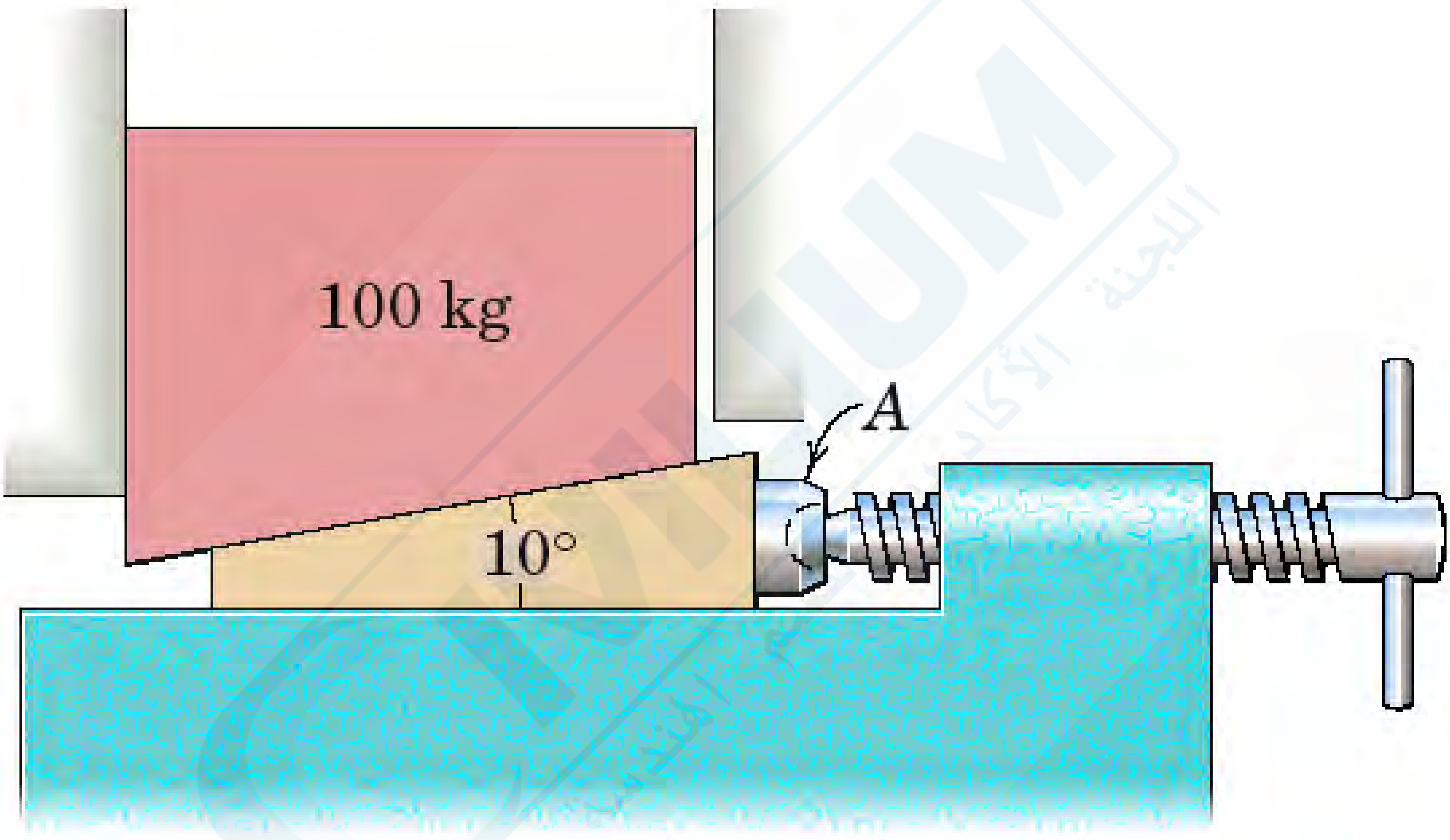
Problem 6/62

P_06_62



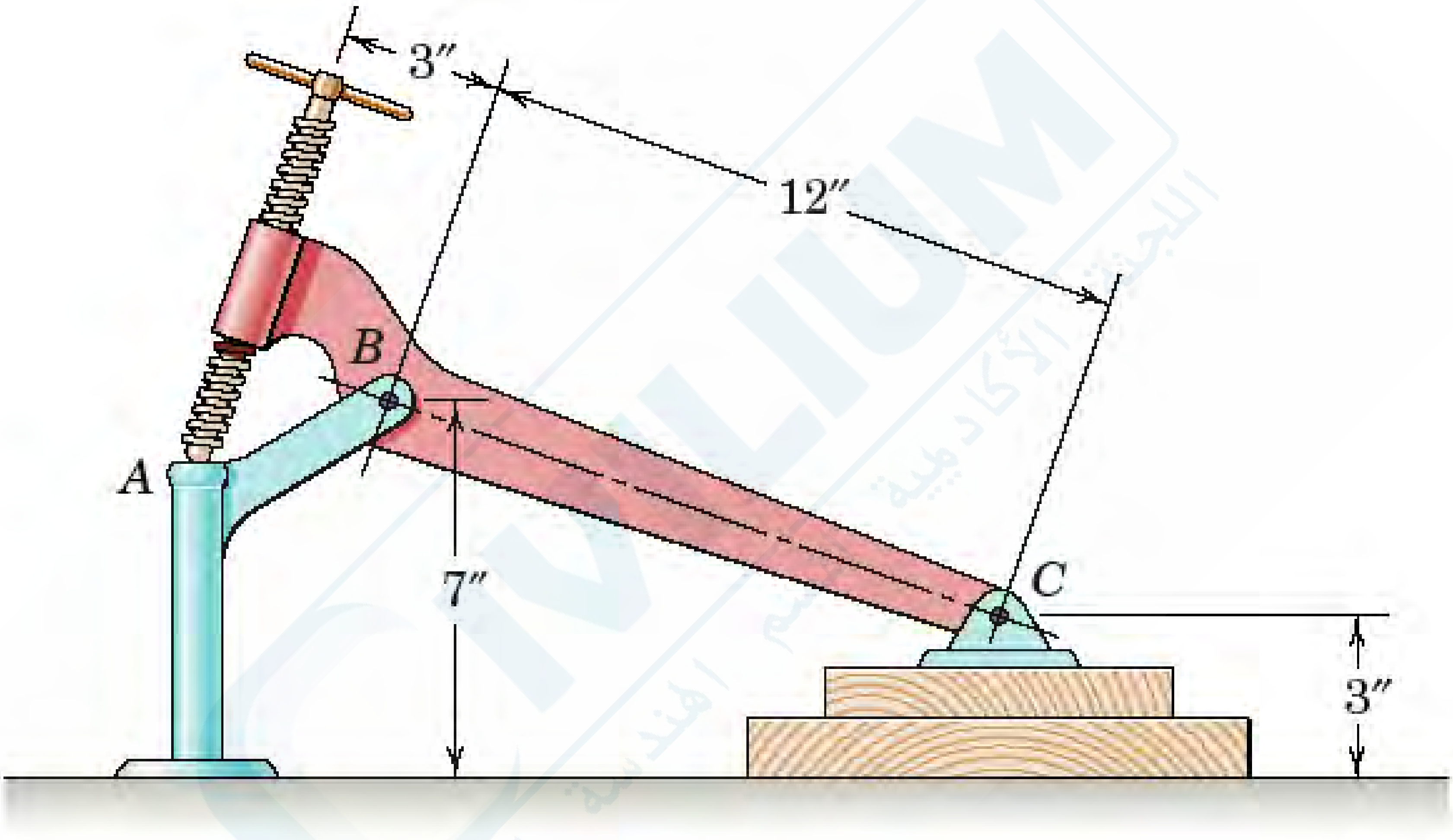
Problem 6/63

P_06_63



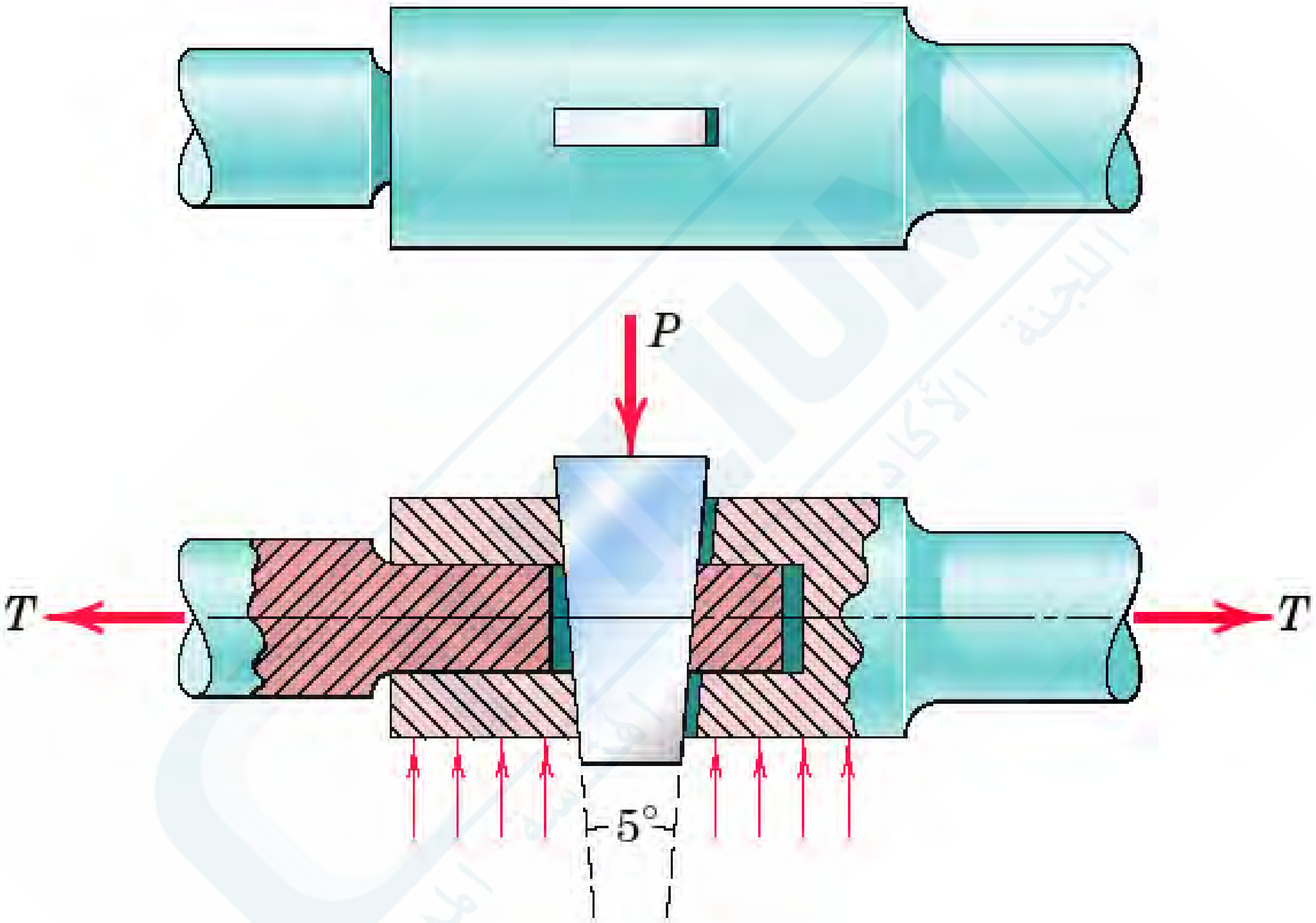
Problem 6/65

P_06_65



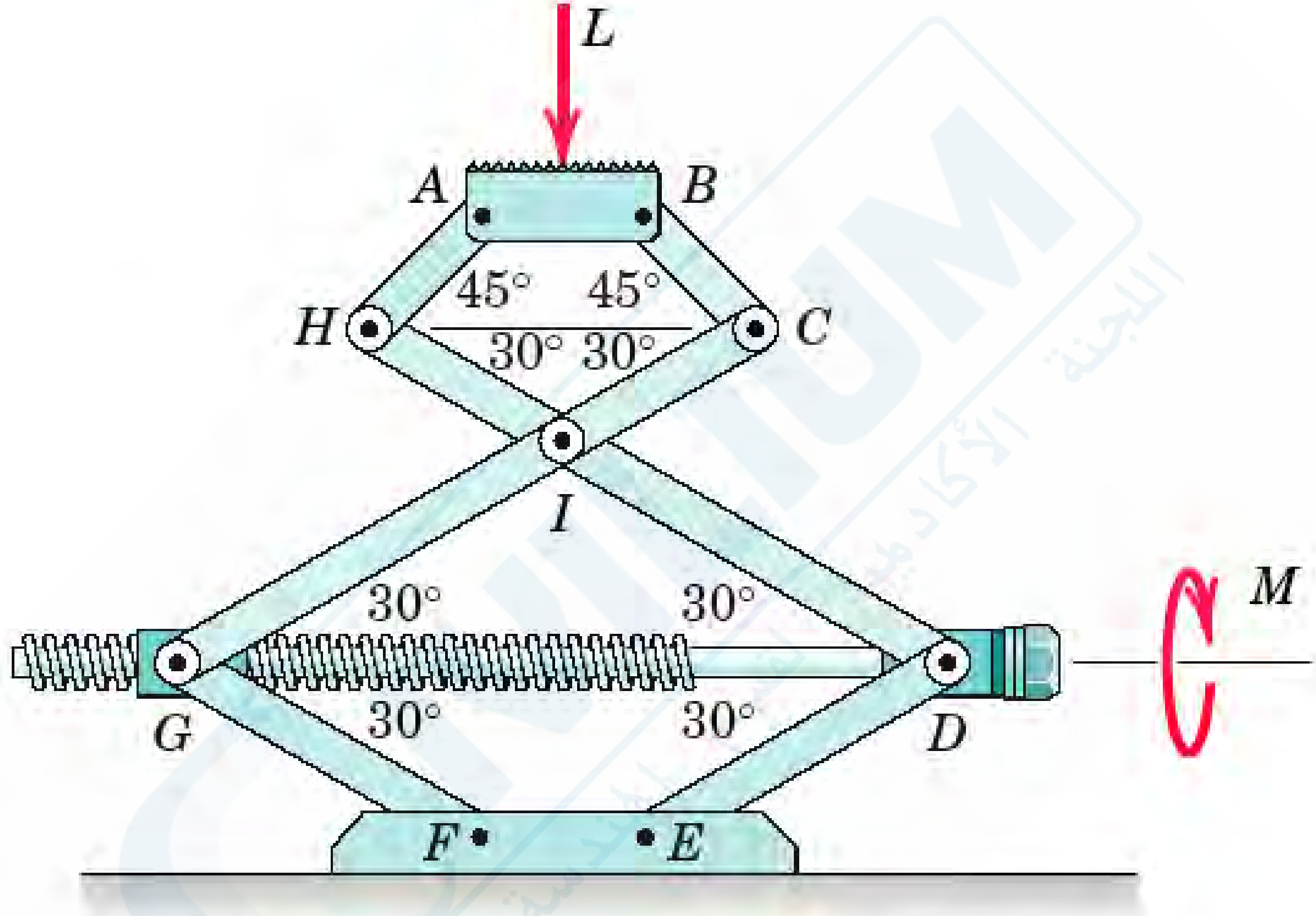
Problem 6/67

P_06_67



Problem 6/68

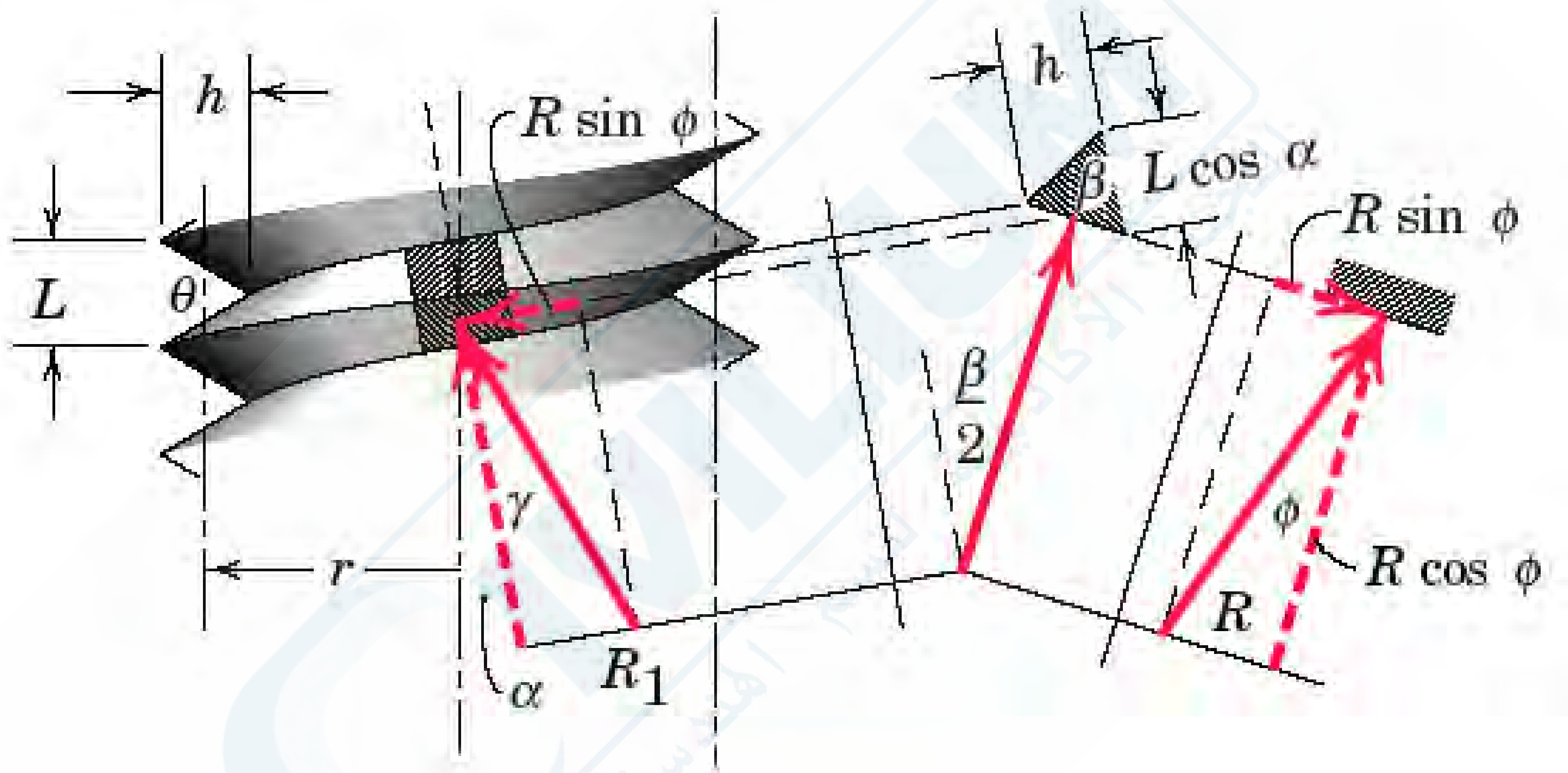
P_06_68



$\overline{GI} = \overline{DI} = 160$ mm, $\overline{HI} = \overline{CI} = 80$ mm

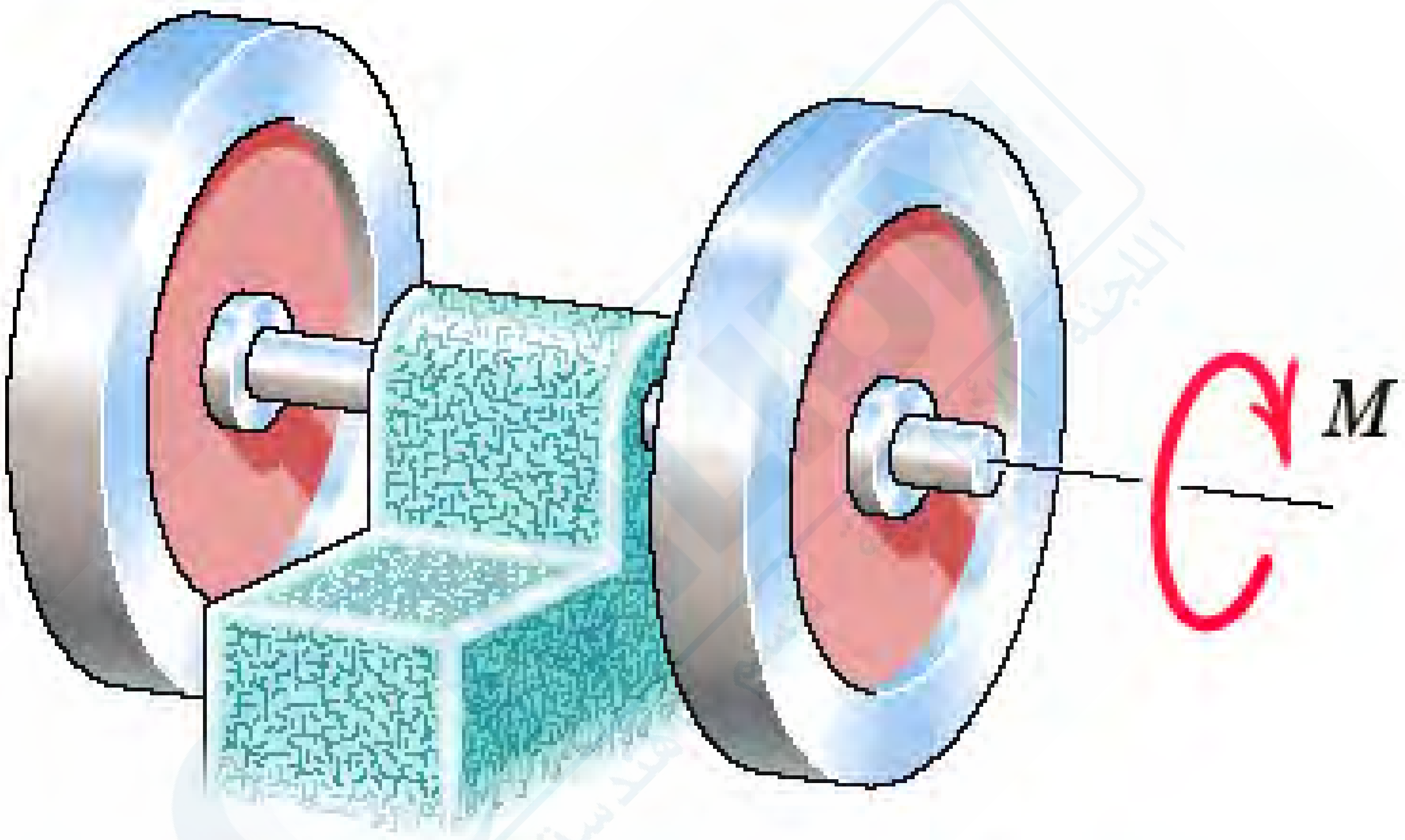
Problem 6/69

P_06_69



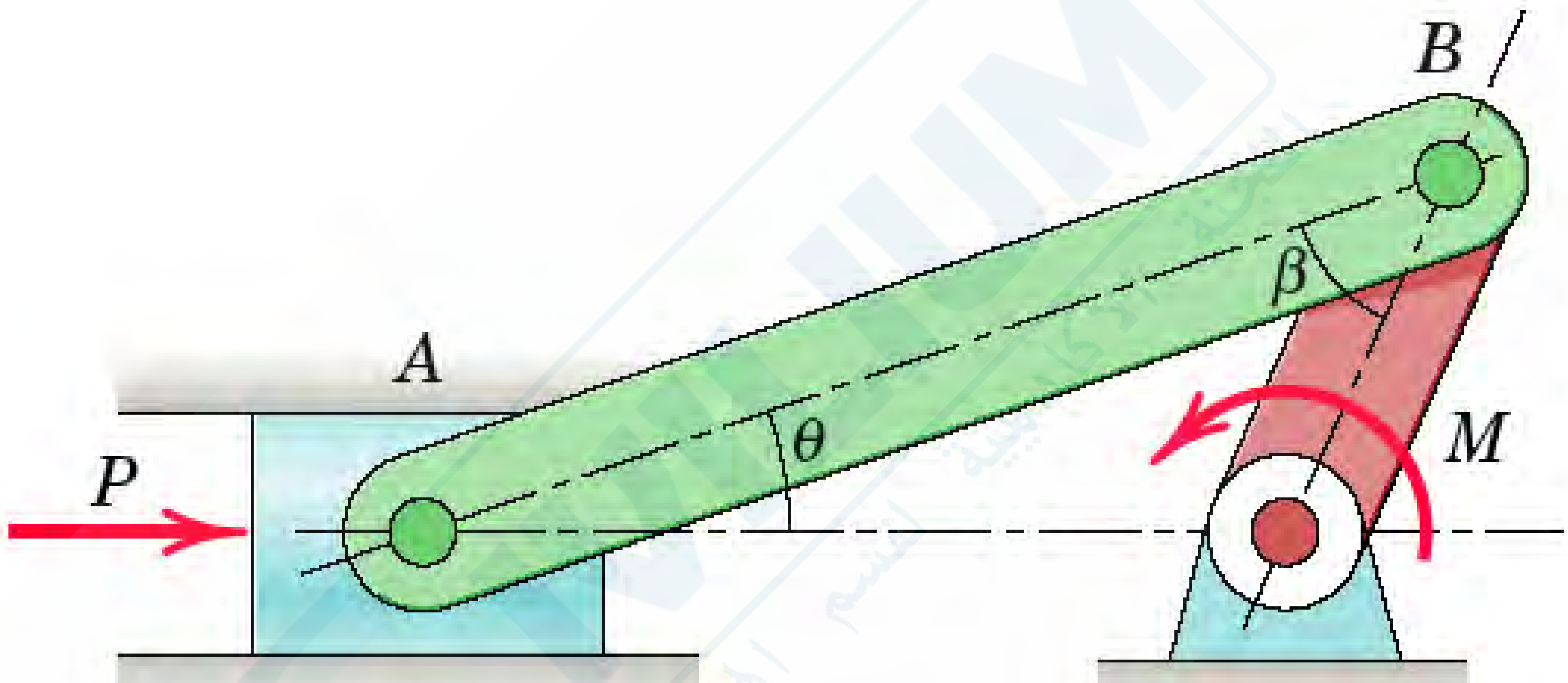
Problem 6/70

P_06_70



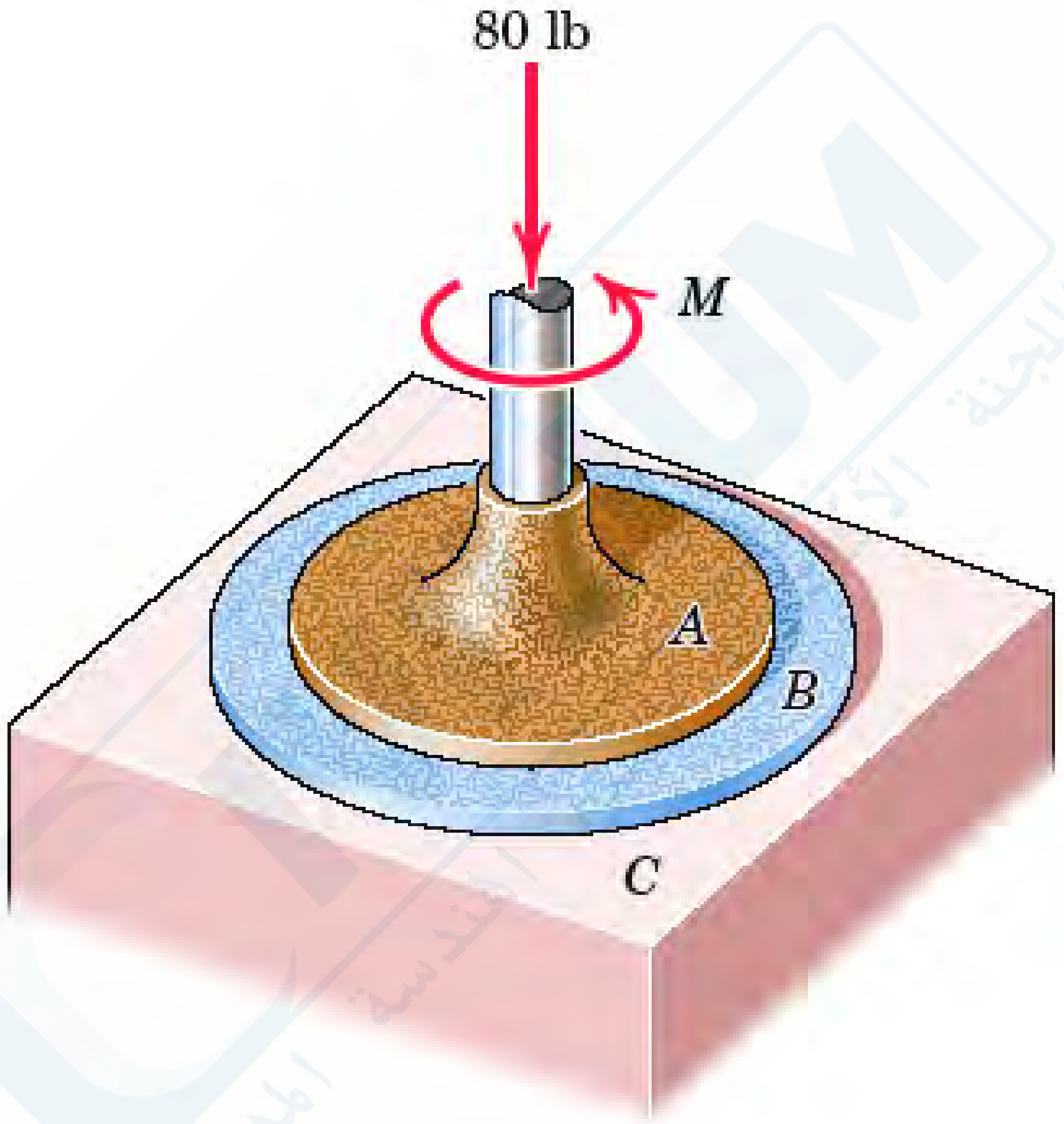
Problem 6/71

P_06_71



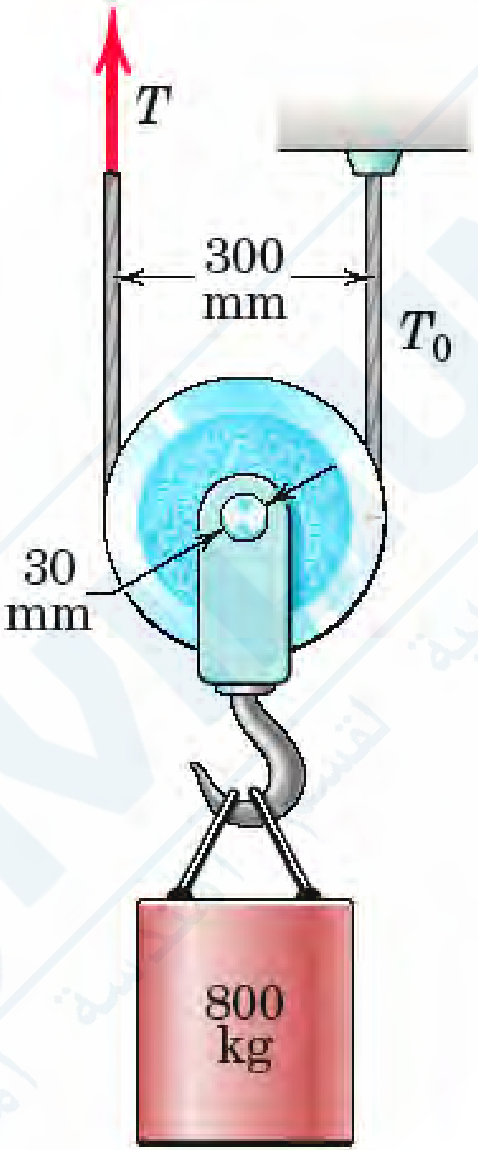
Problem 6/72

P_06_72



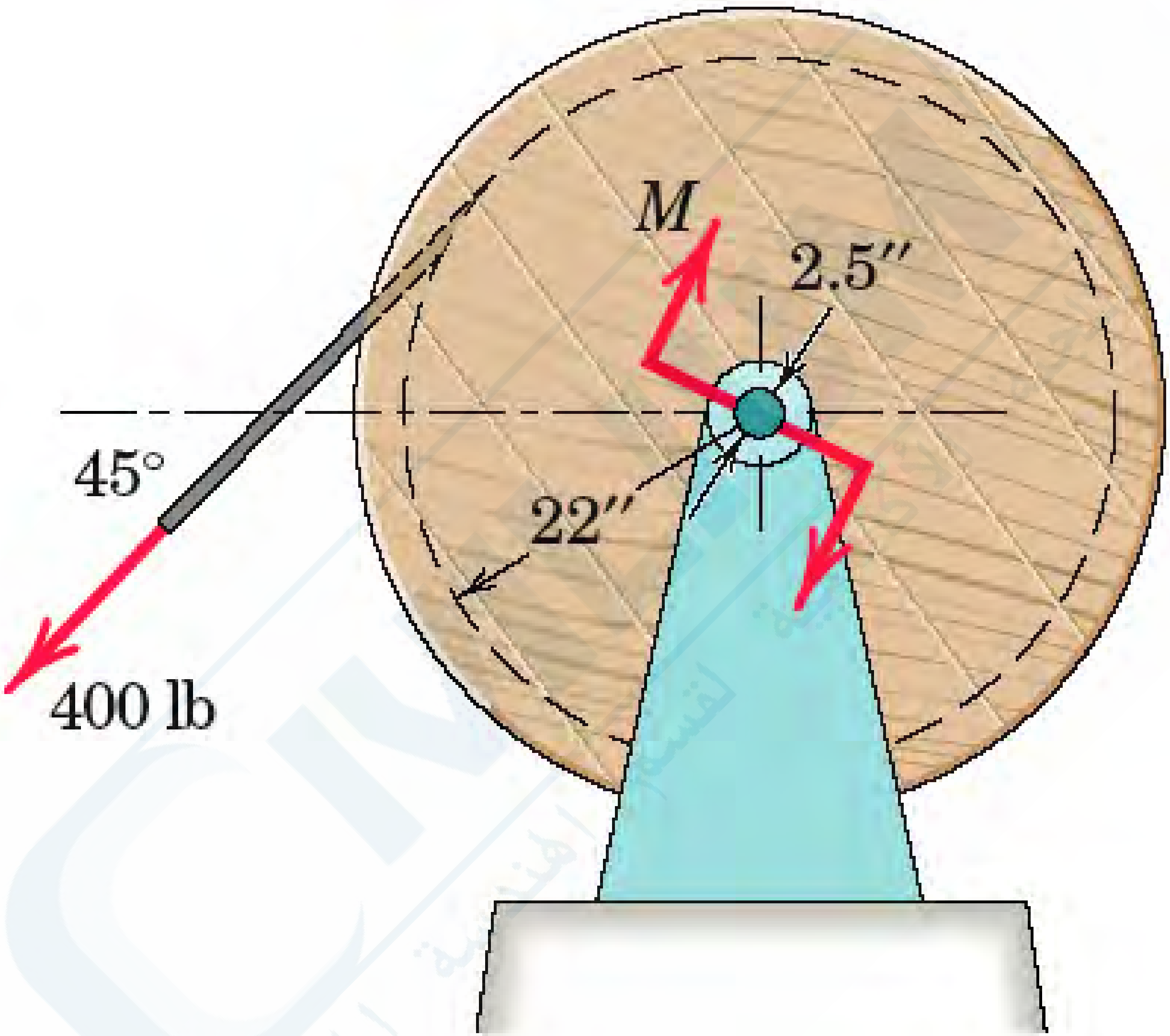
Problem 6/73

P_06_73



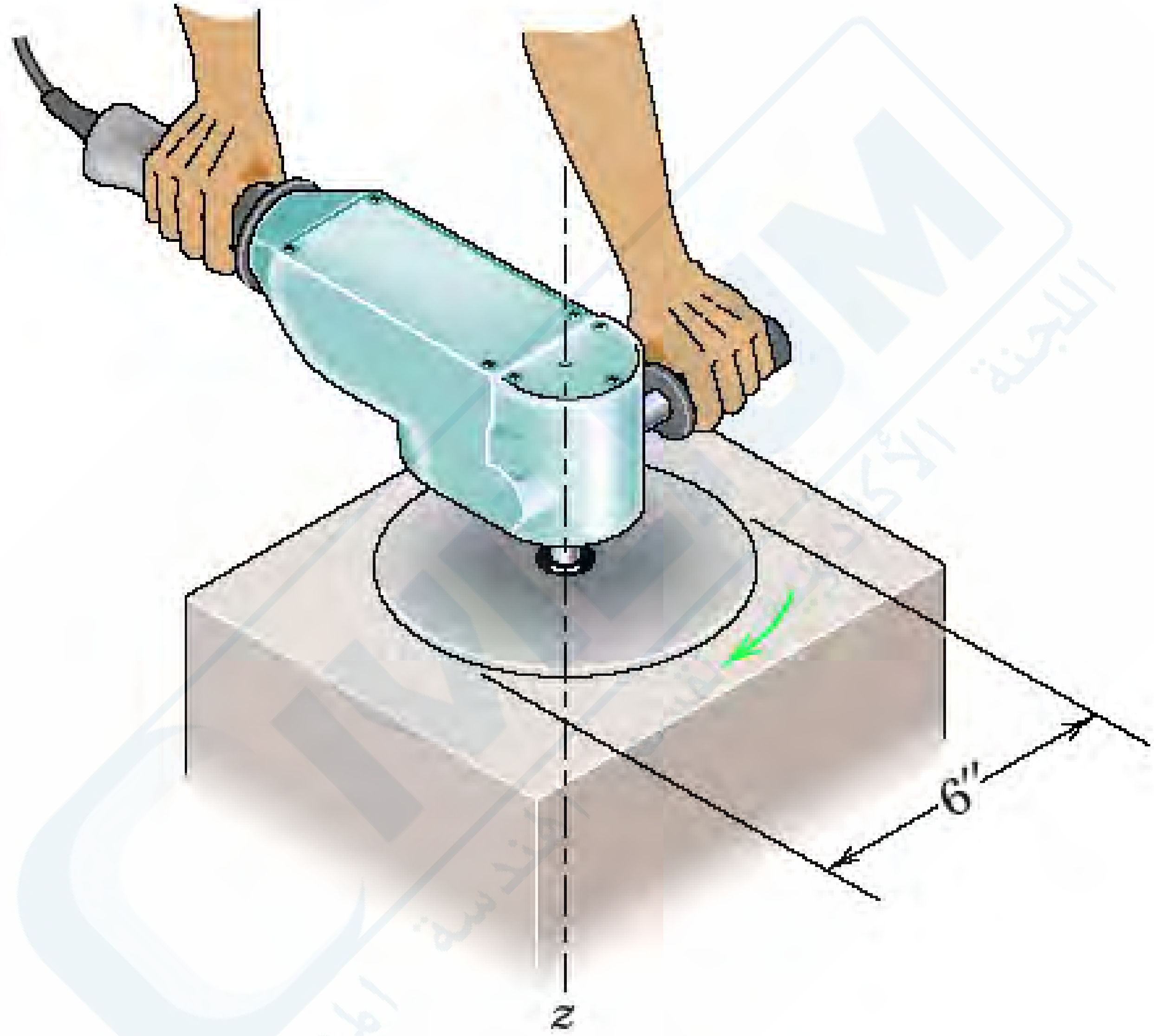
Problem 6/74

P_06_74



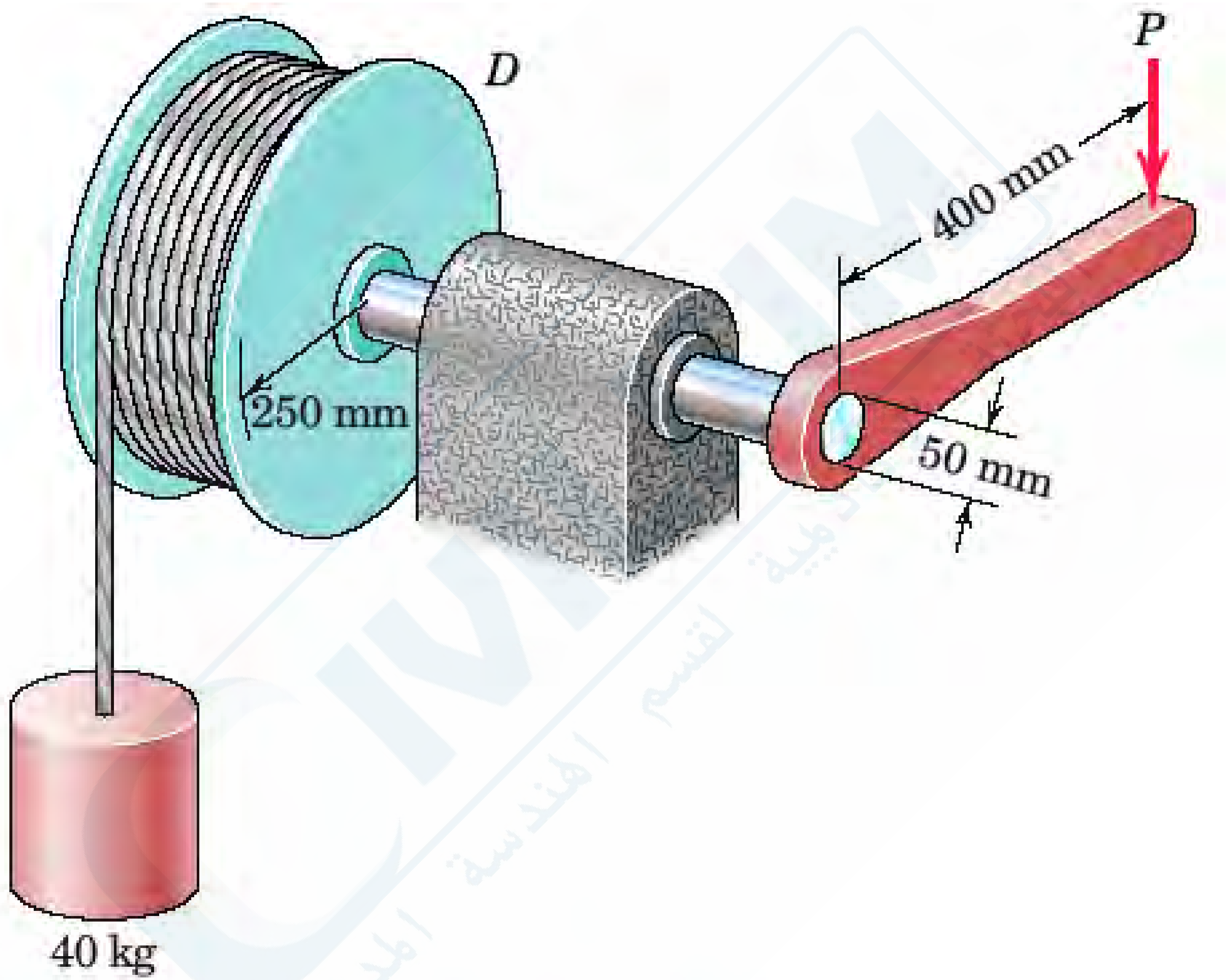
Problem 6/76

P_06_76



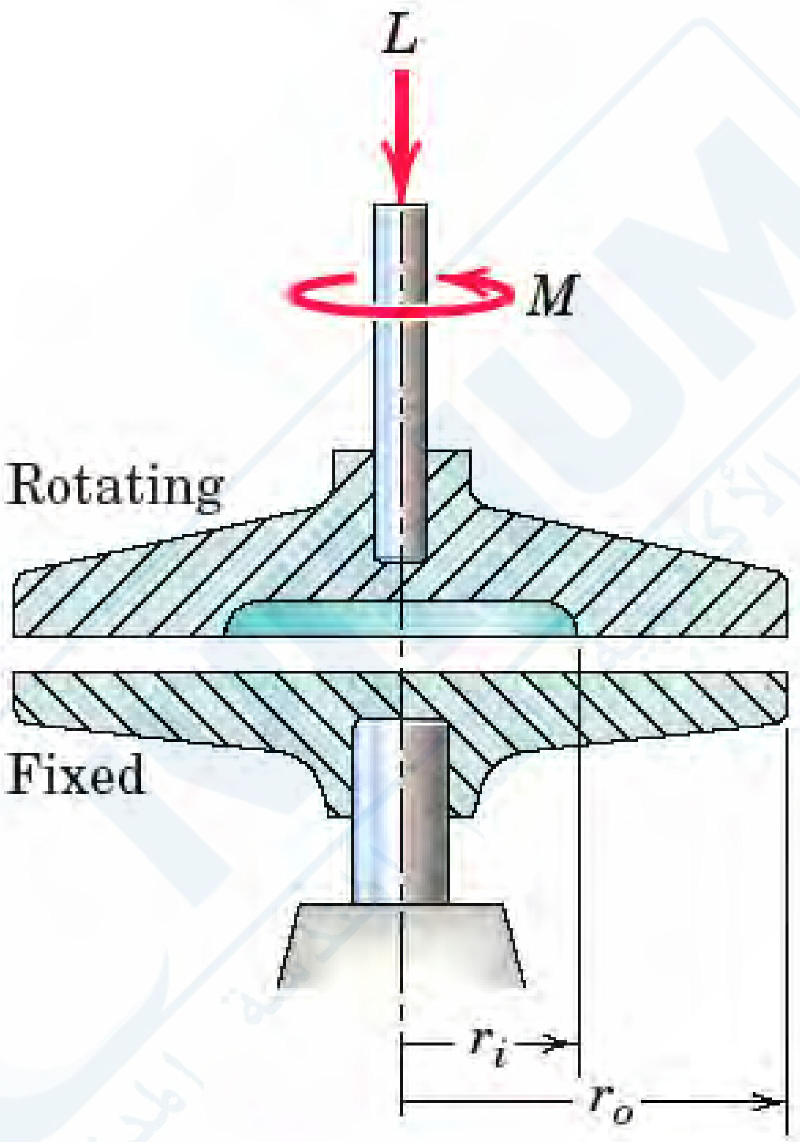
Problem 6/78

P_06_78

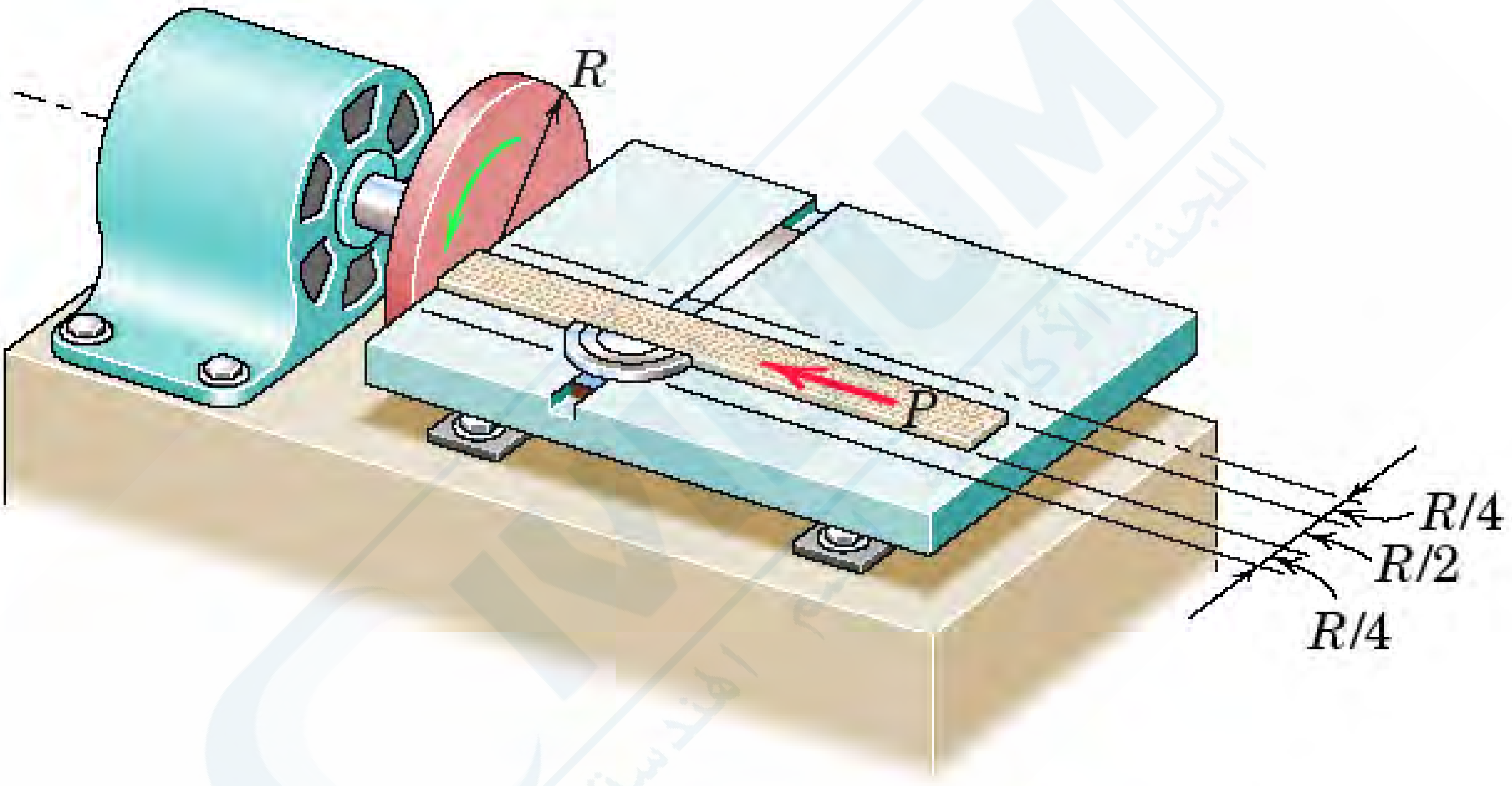


Problem 6/79

P_06_79

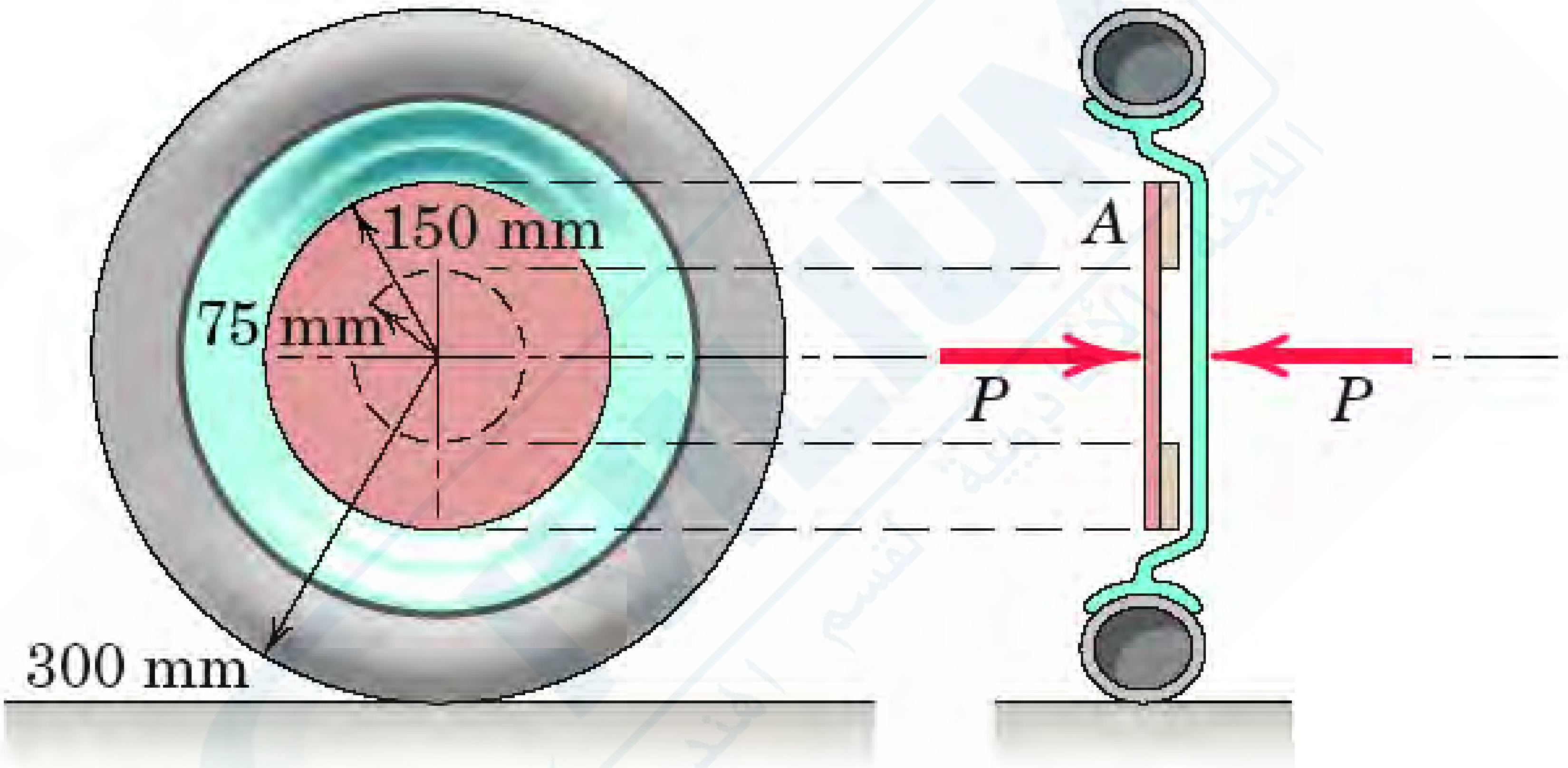


Problem 6/81



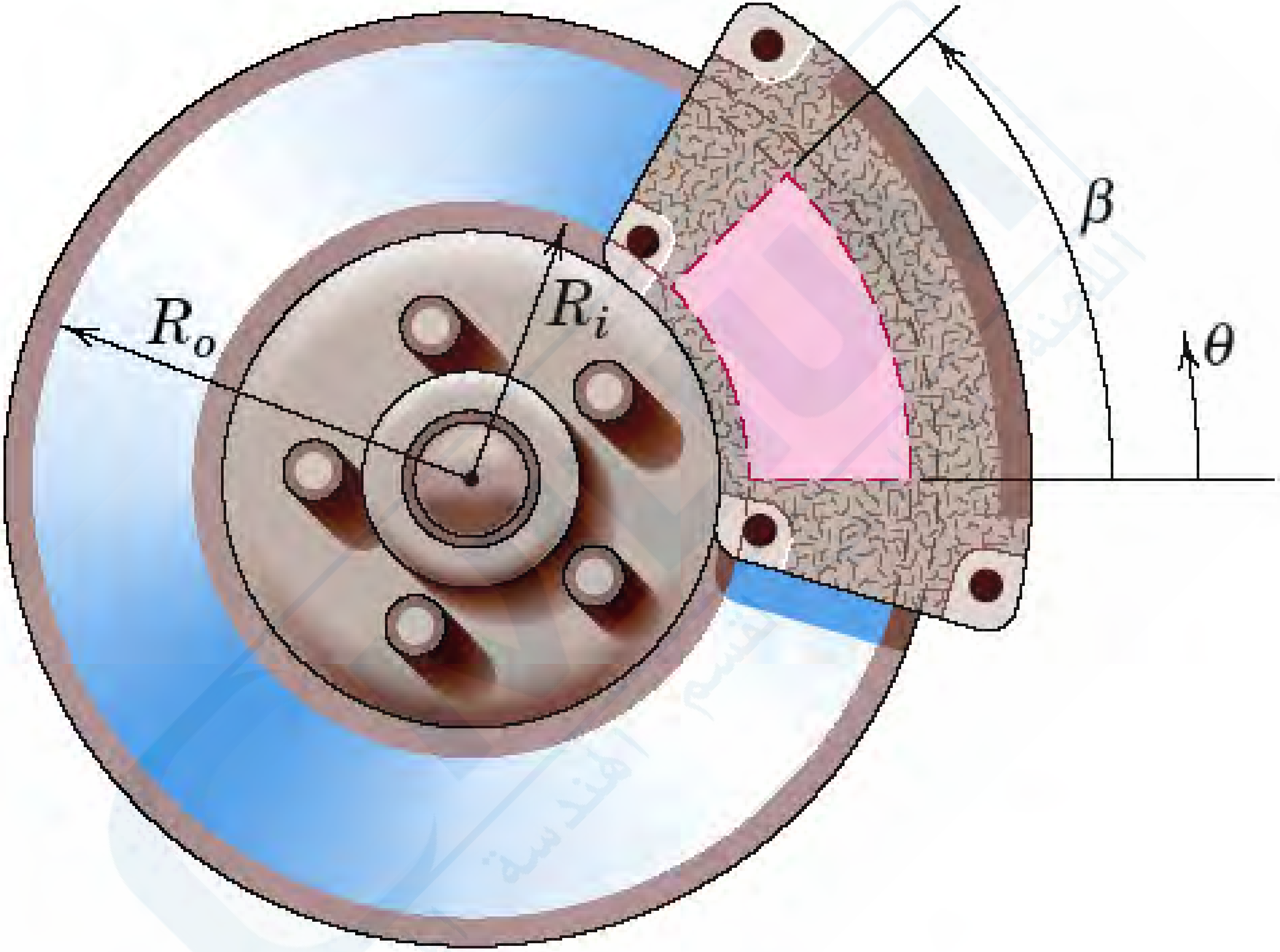
Problem 6/82

P_06_82



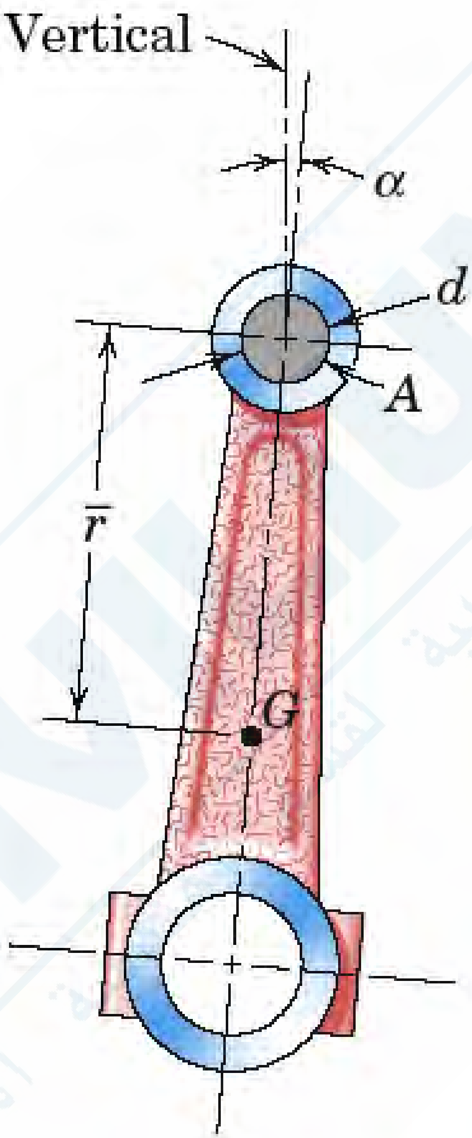
Problem 6/83

P_06_83



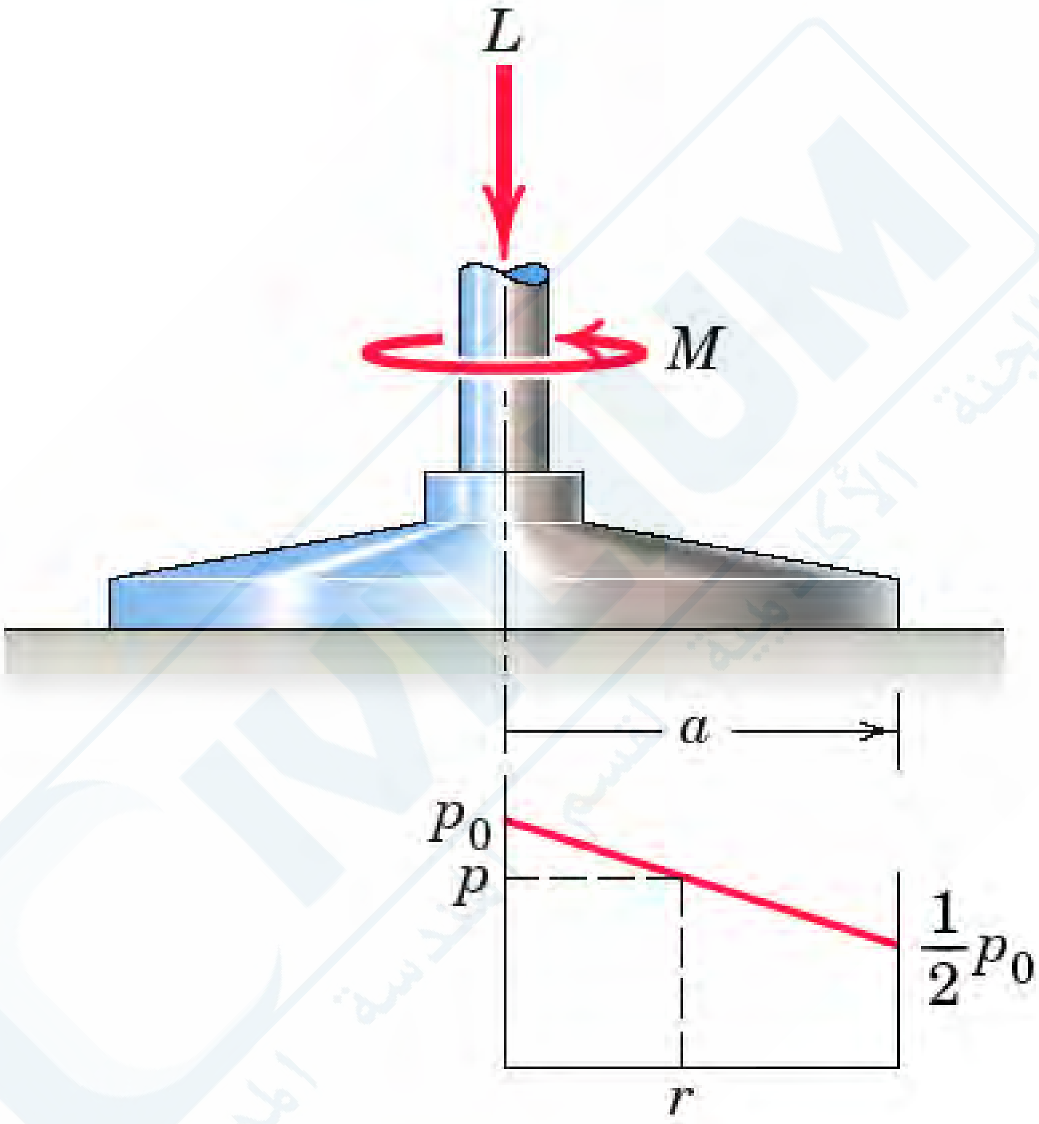
Problem 6/84

P_06_84



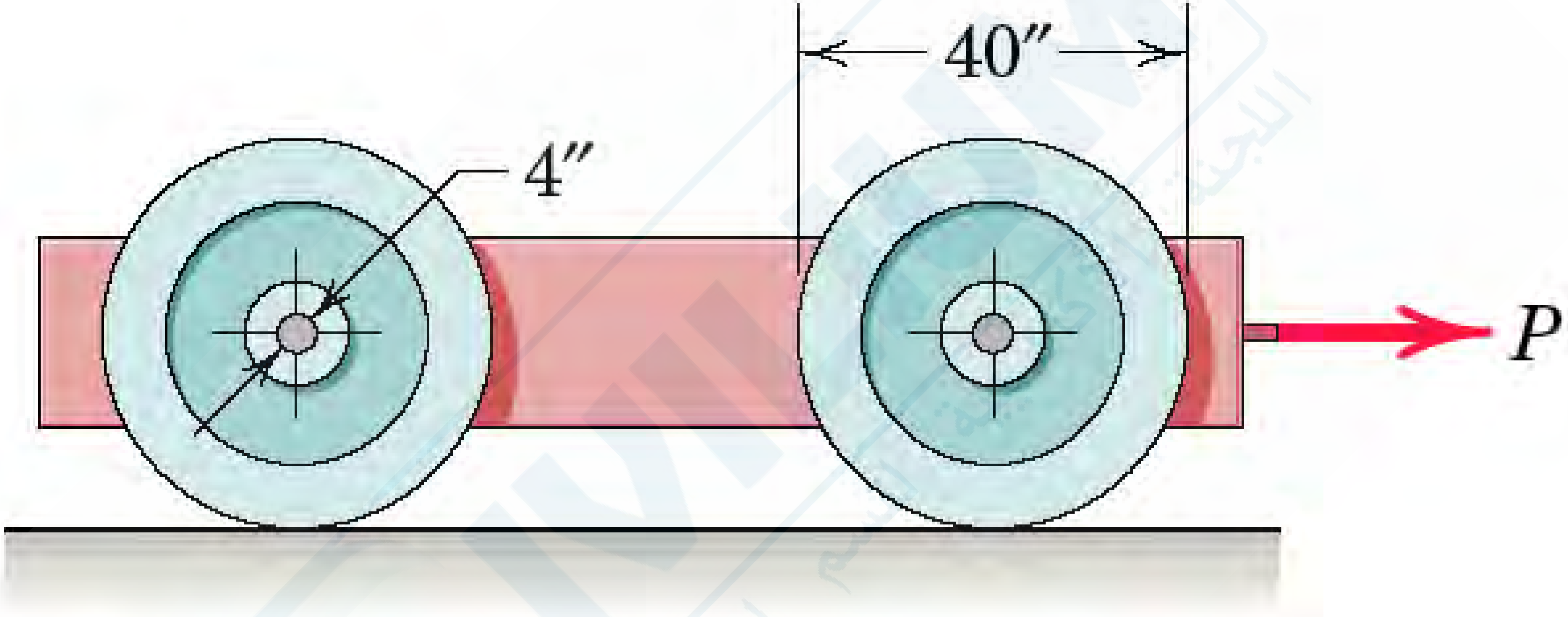
Problem 6/85

P_06_85



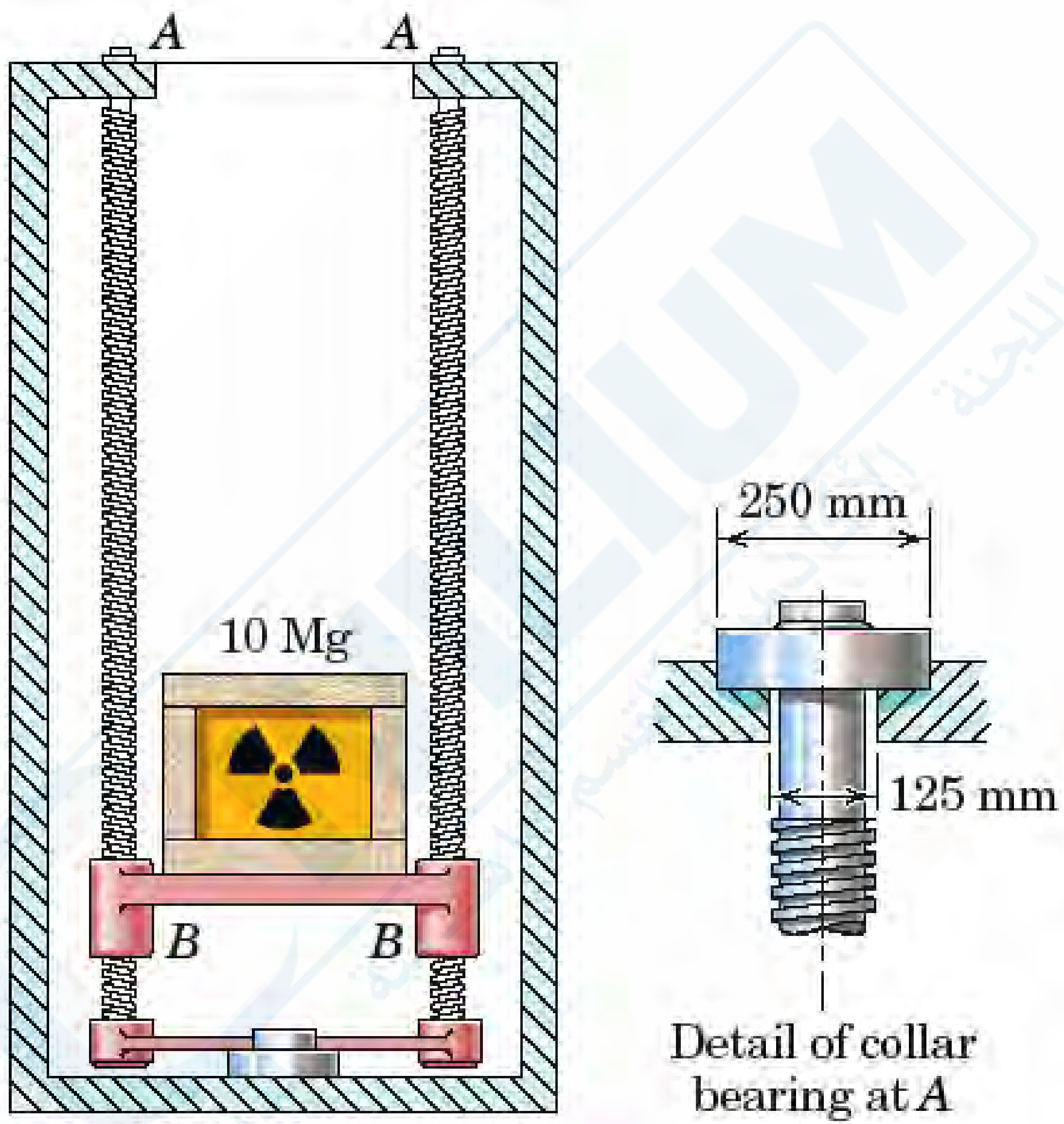
Problem 6/86

P_06_86

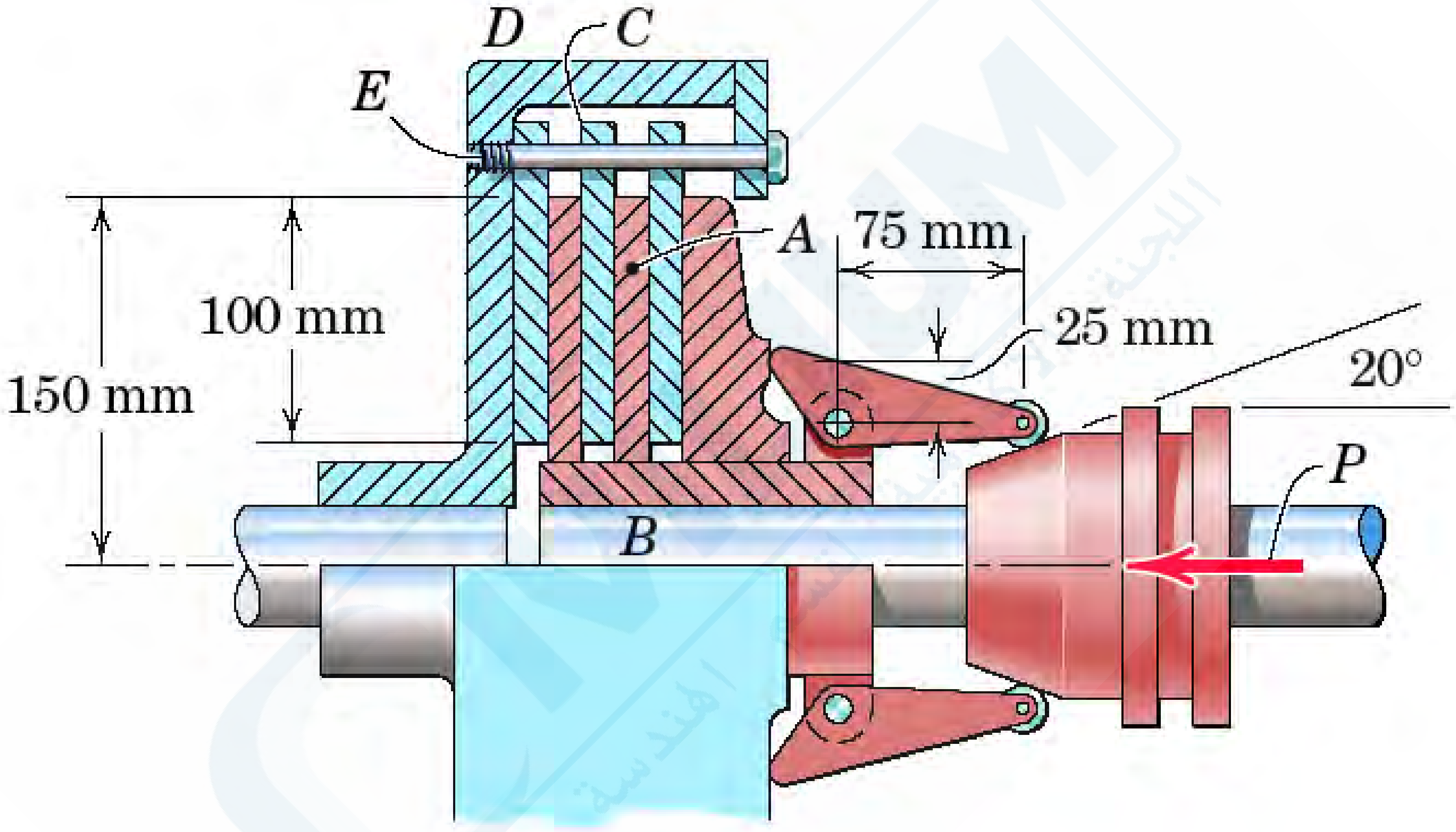


Problem 6/87

P_06_87

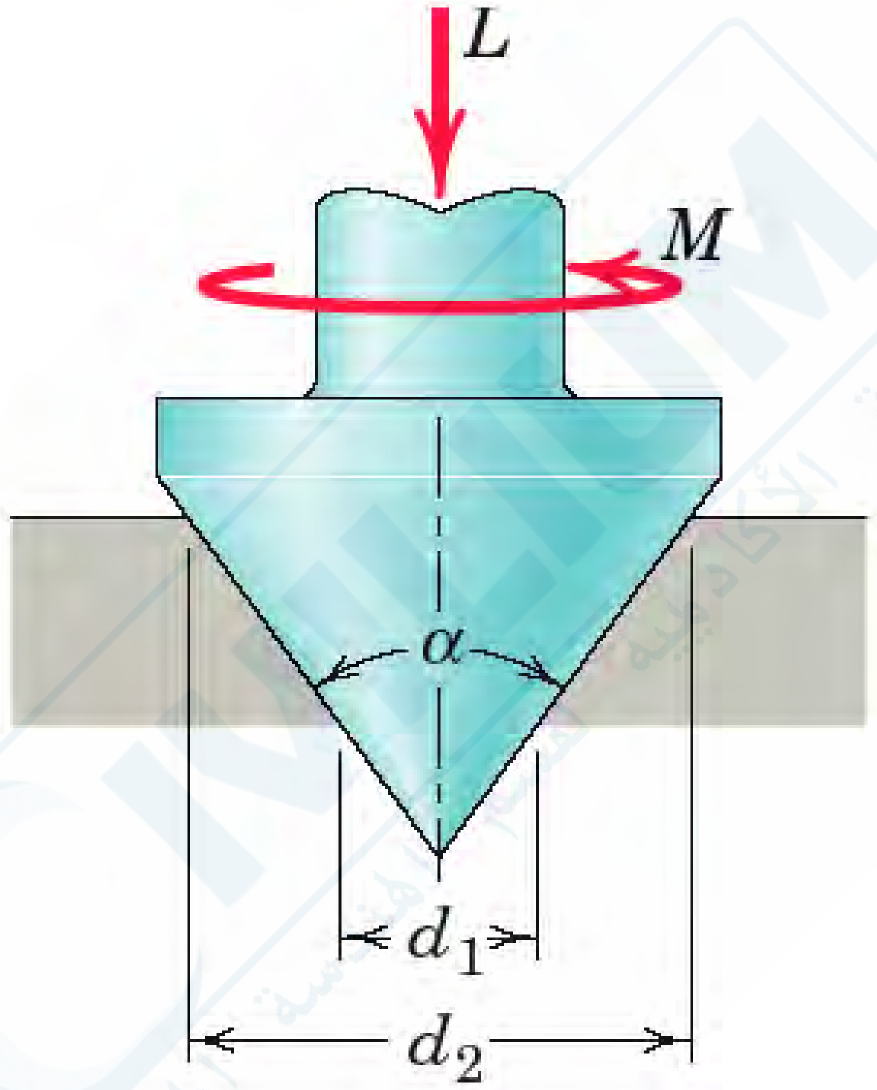


Problem 6/89



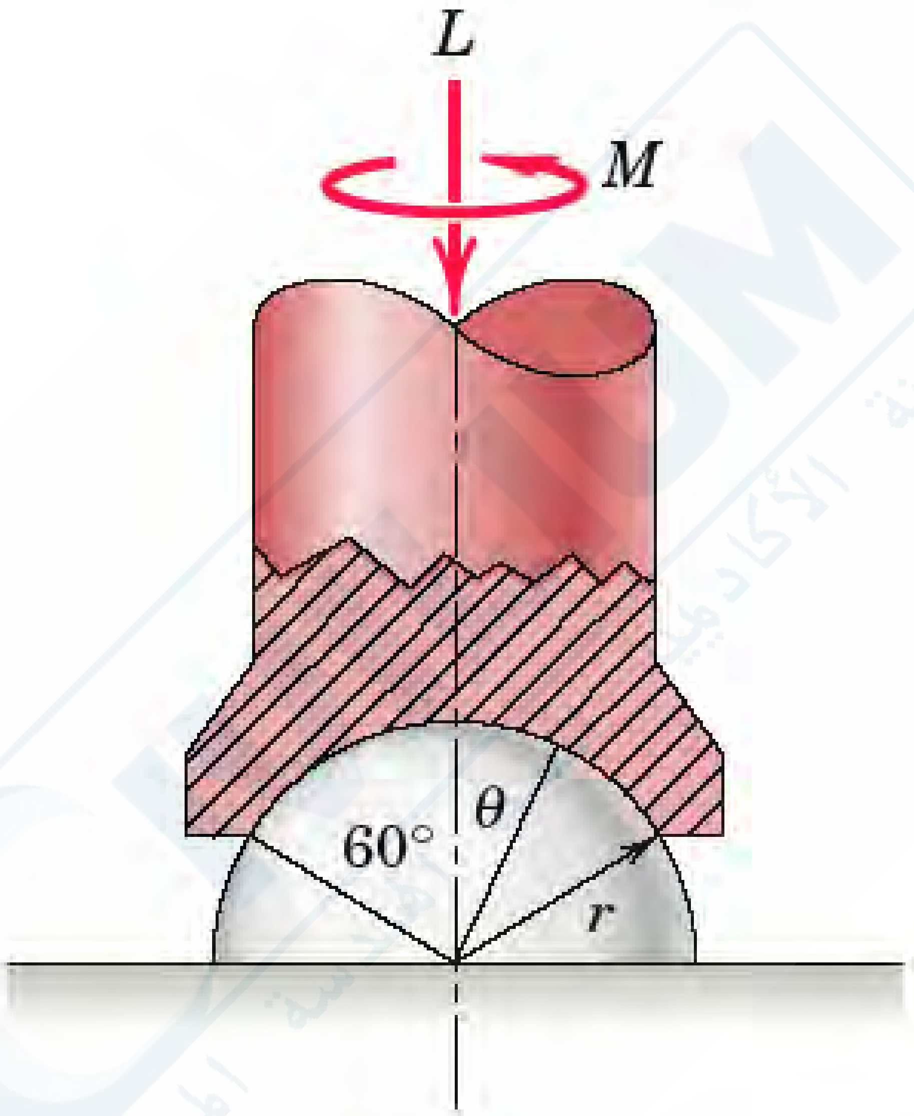
Problem 6/90

P_06_90



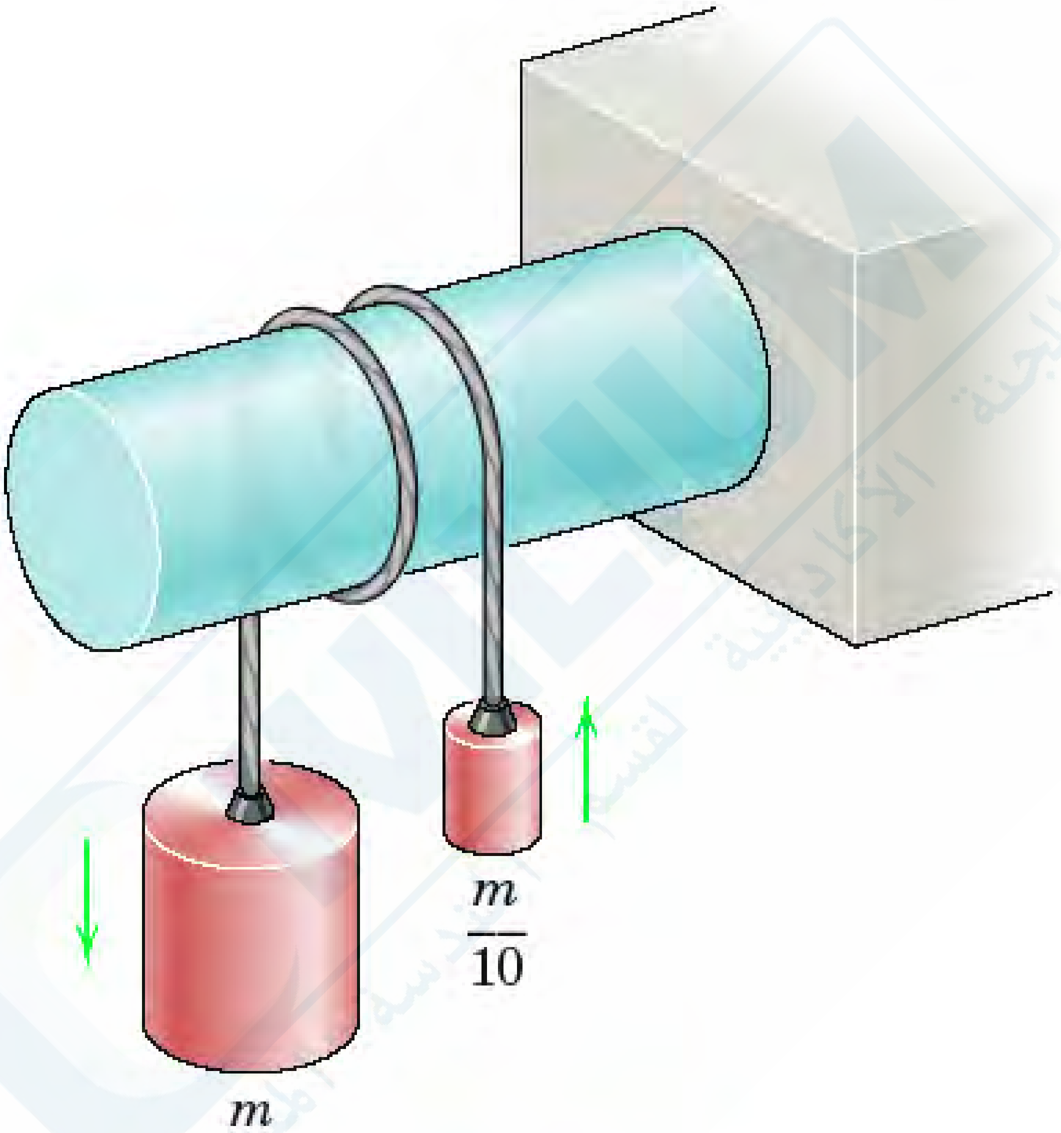
Problem 6/91

P_06_91



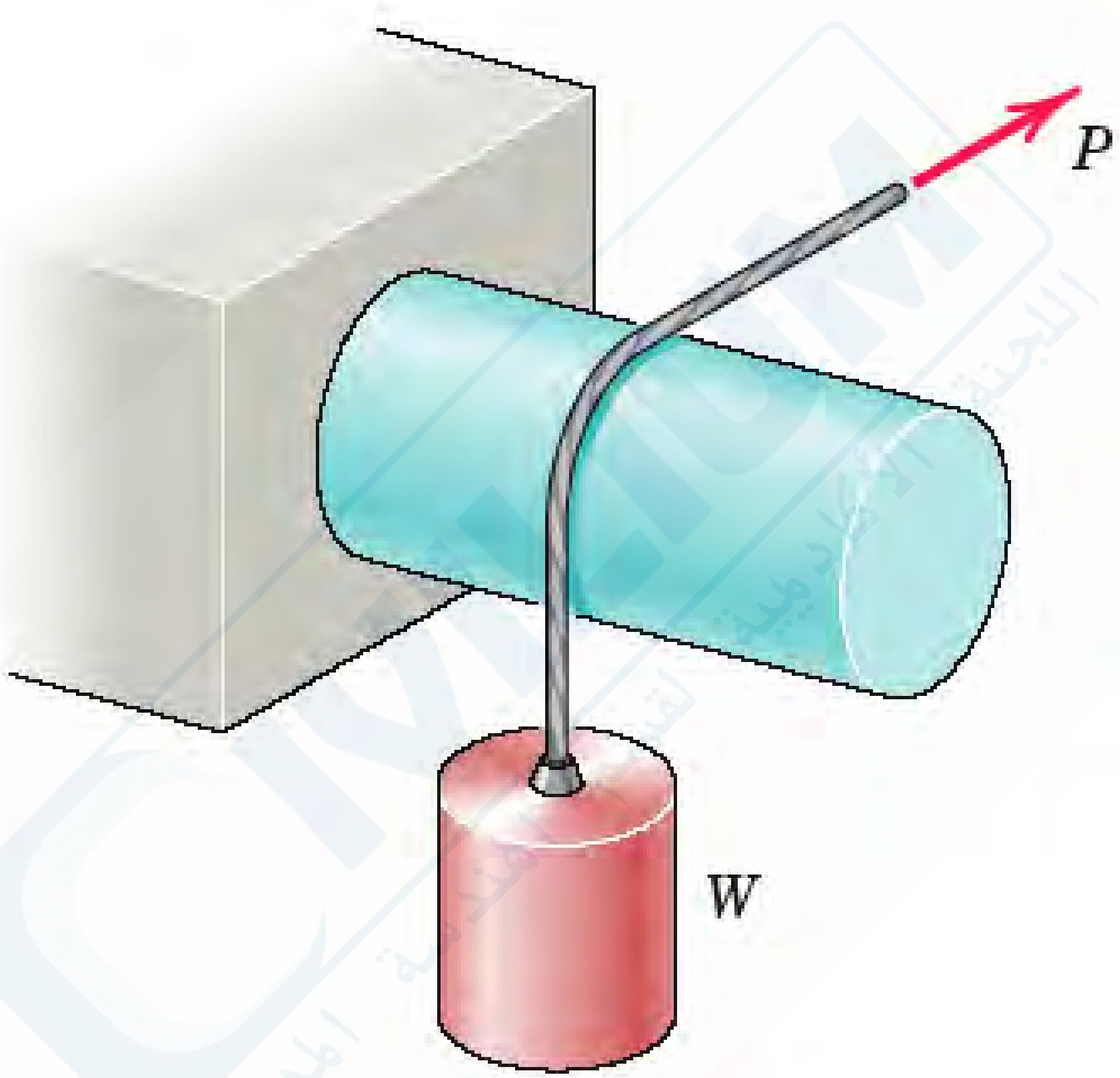
Problem 6/92

P_06_92



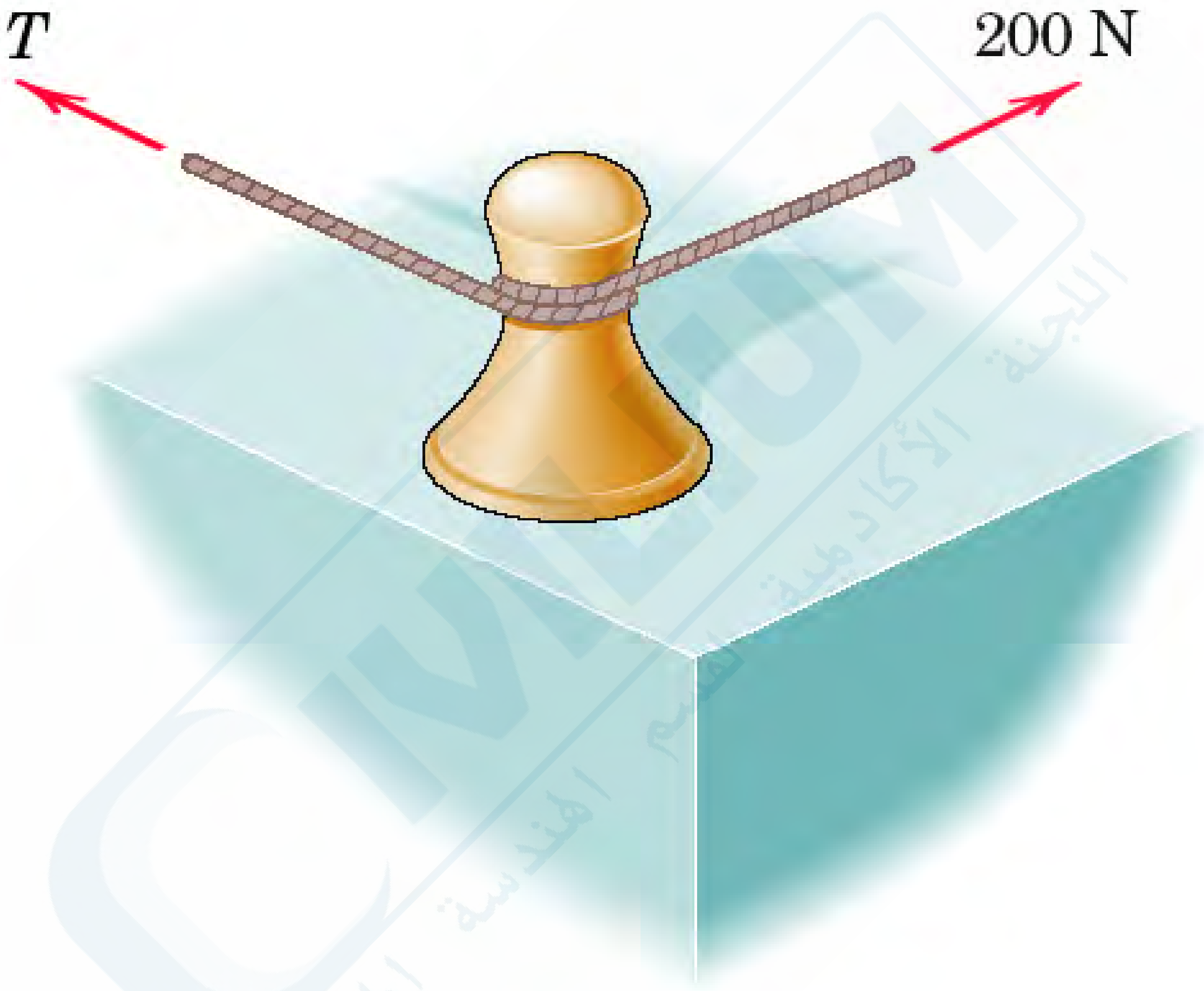
Problem 6/93

P_06_93



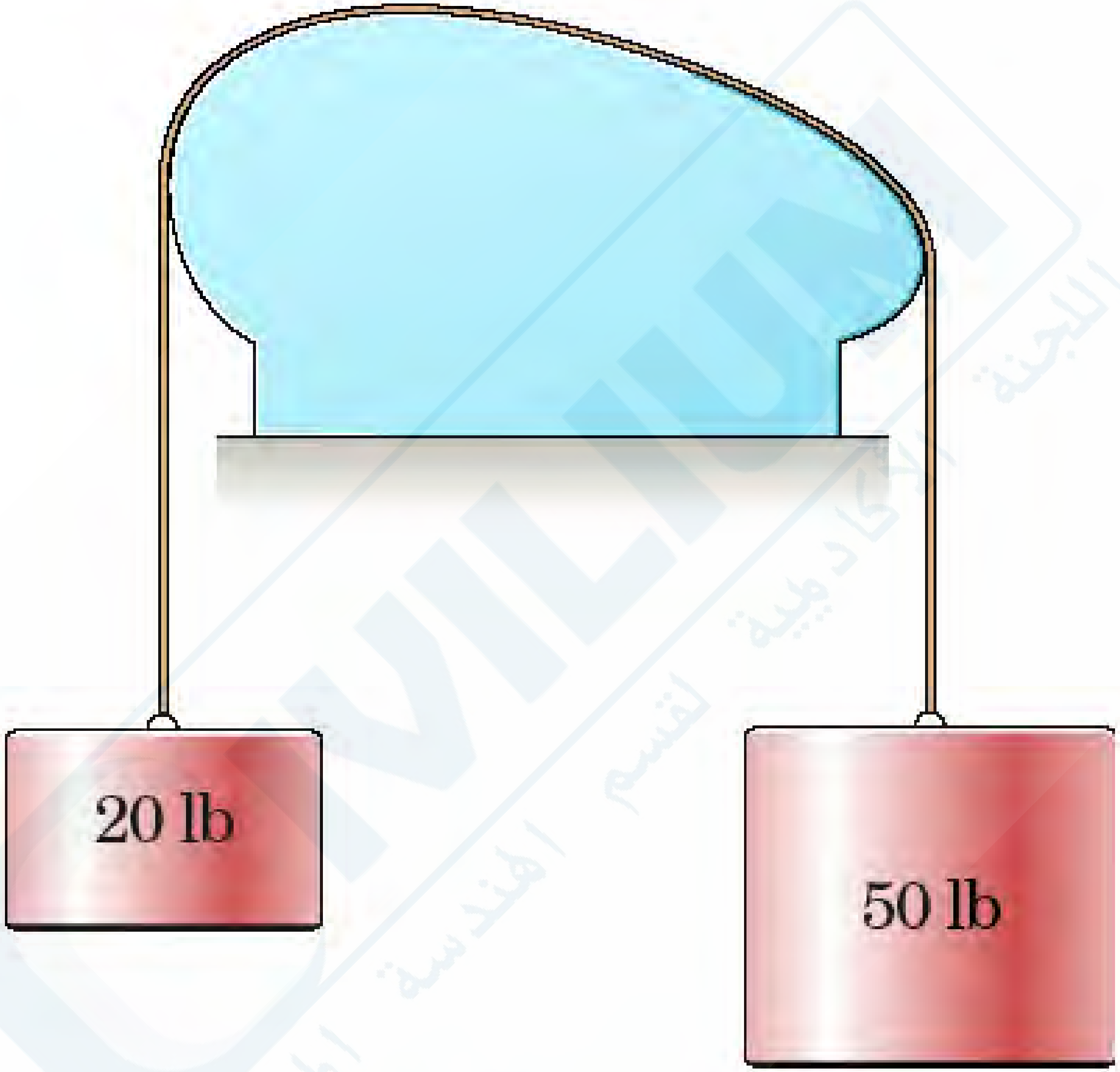
Problem 6/94

P_06_94



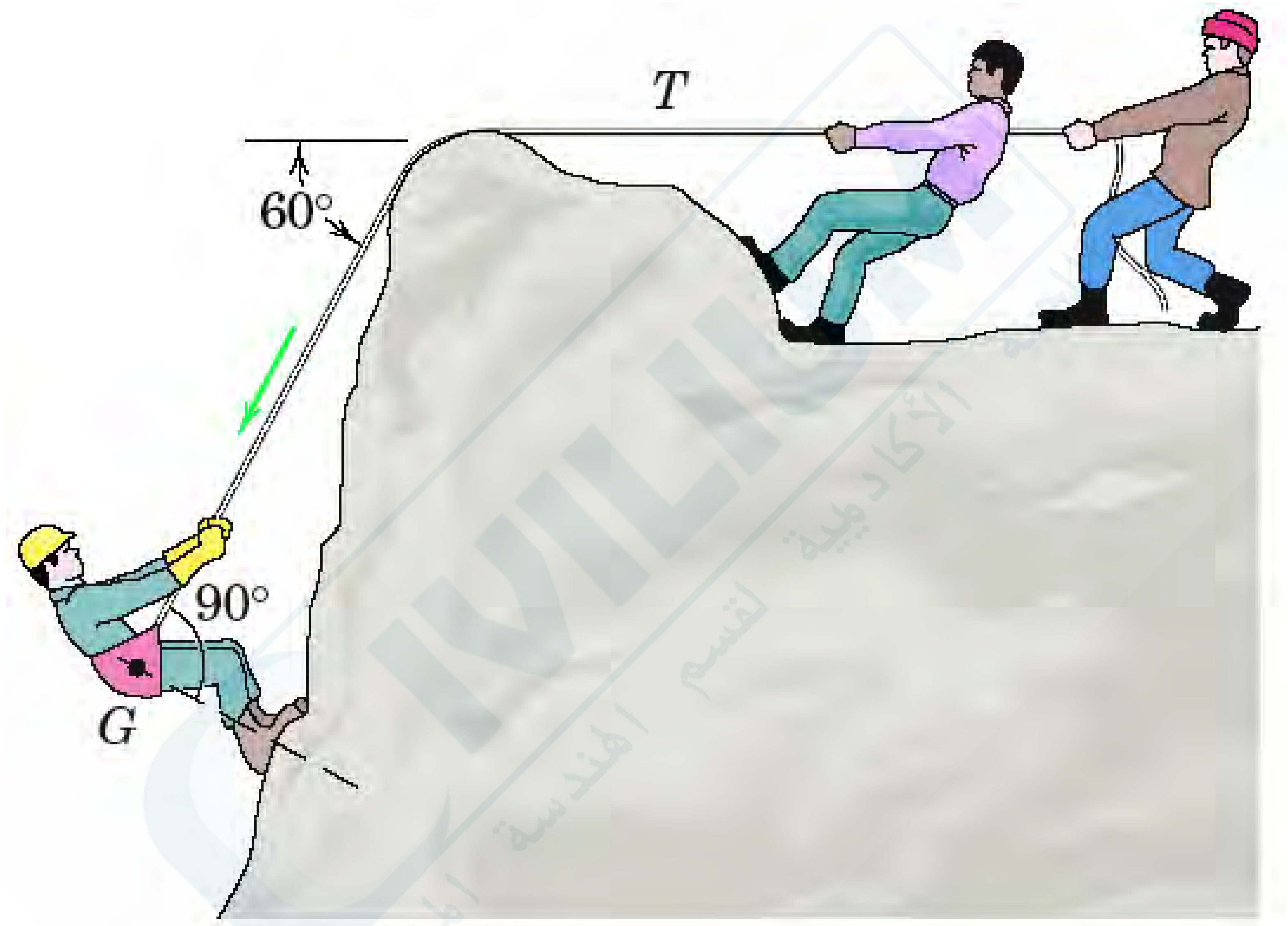
Problem 6/95

P_06_95



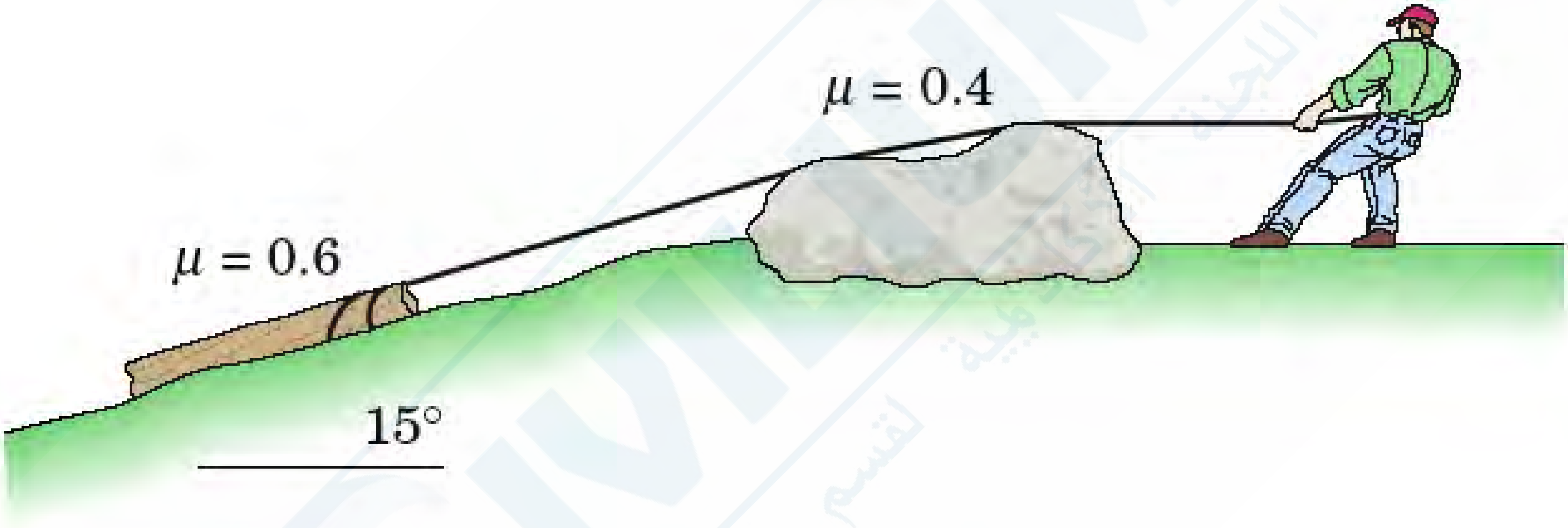
Problem 6/97

P_06_97



Problem 6/98

P_06_98



Problem 6/99

P_06_99

Text Illustrations

To Accompany

Meriam/Kraige – Statics 6e

Chapter 7 : Virtual Work.

CIVILIUM
اللجنة الأكاديمية لقسم الهندسة المدنية

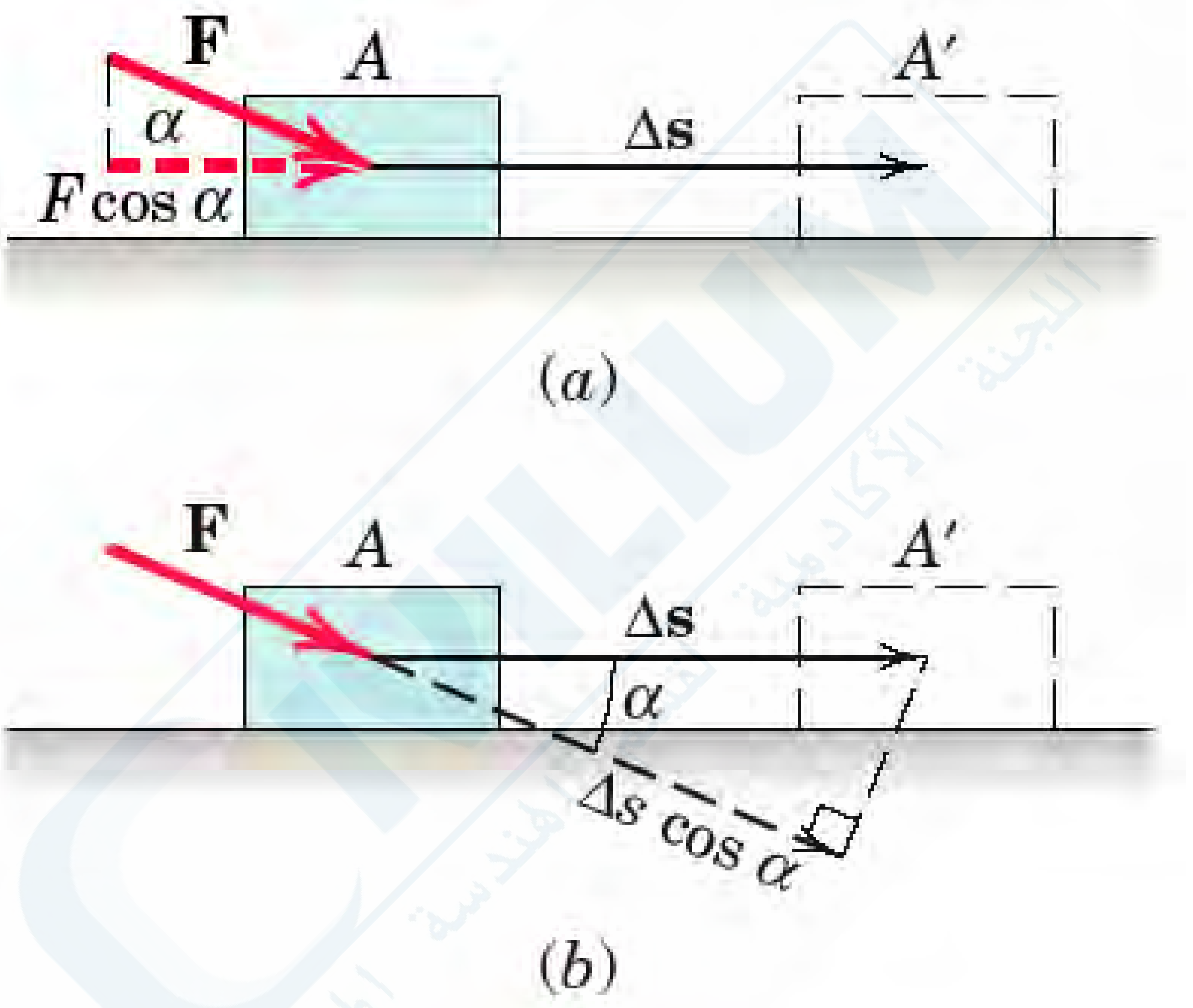


Figure 7/1

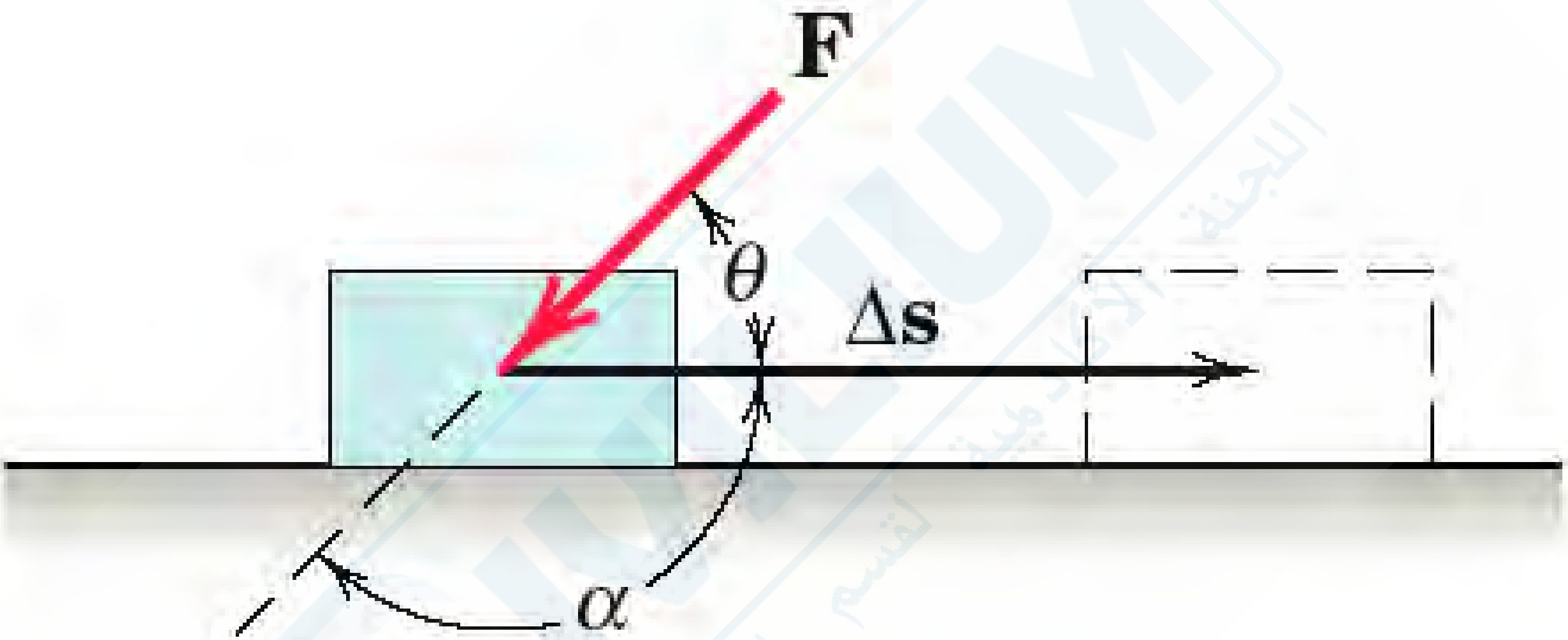


Figure 7/2

07_02

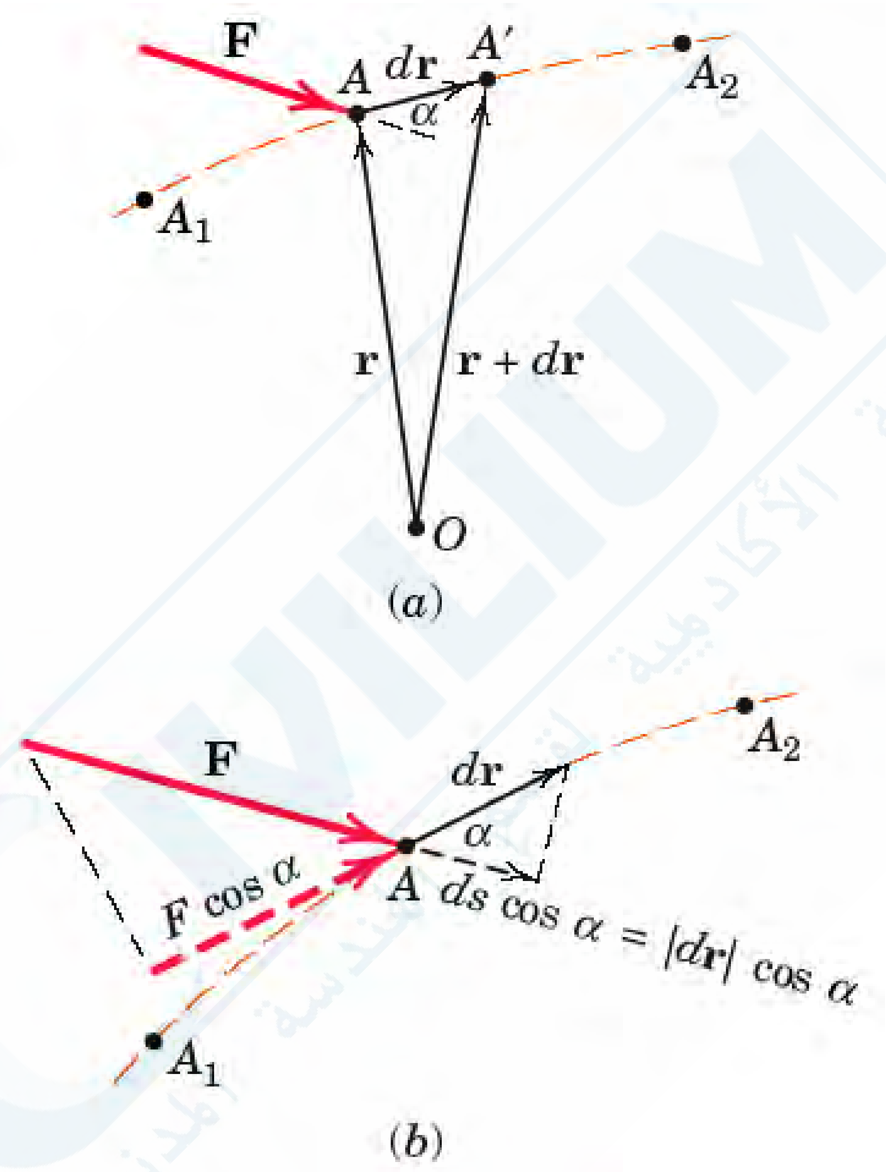


Figure 7/3

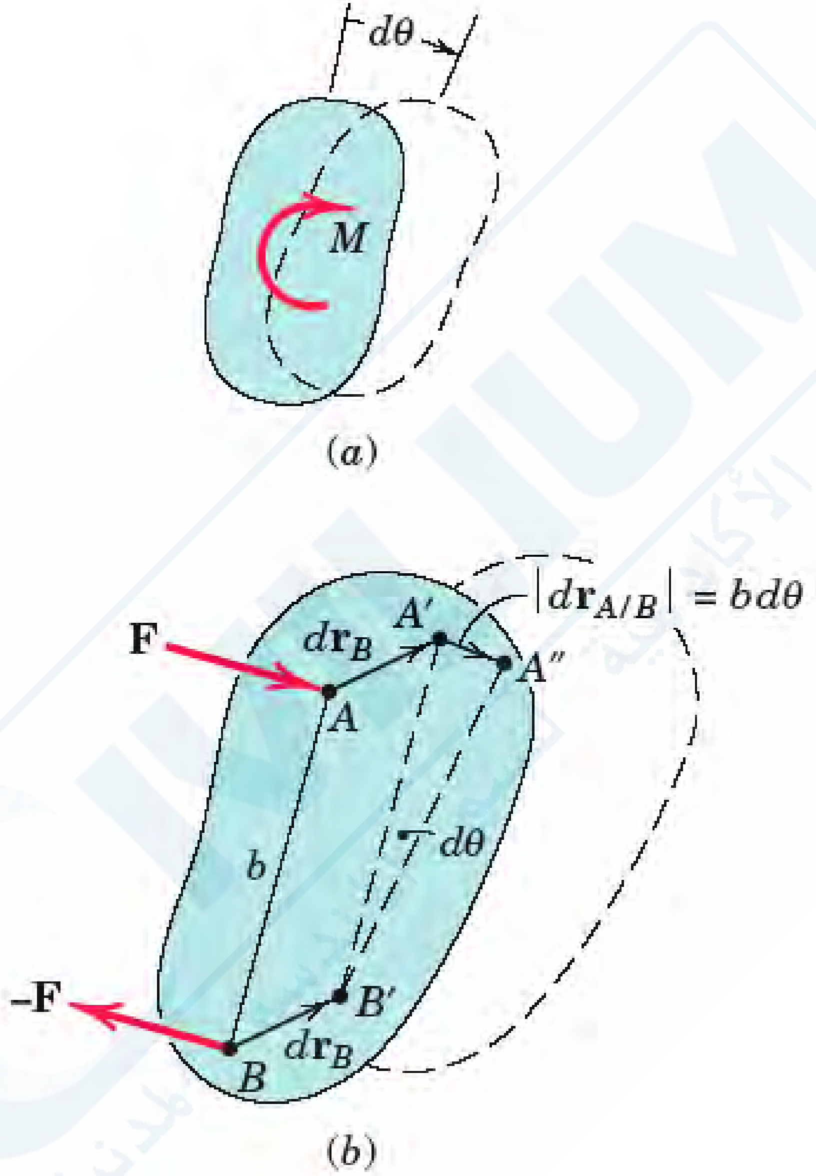


Figure 7/4

07_04

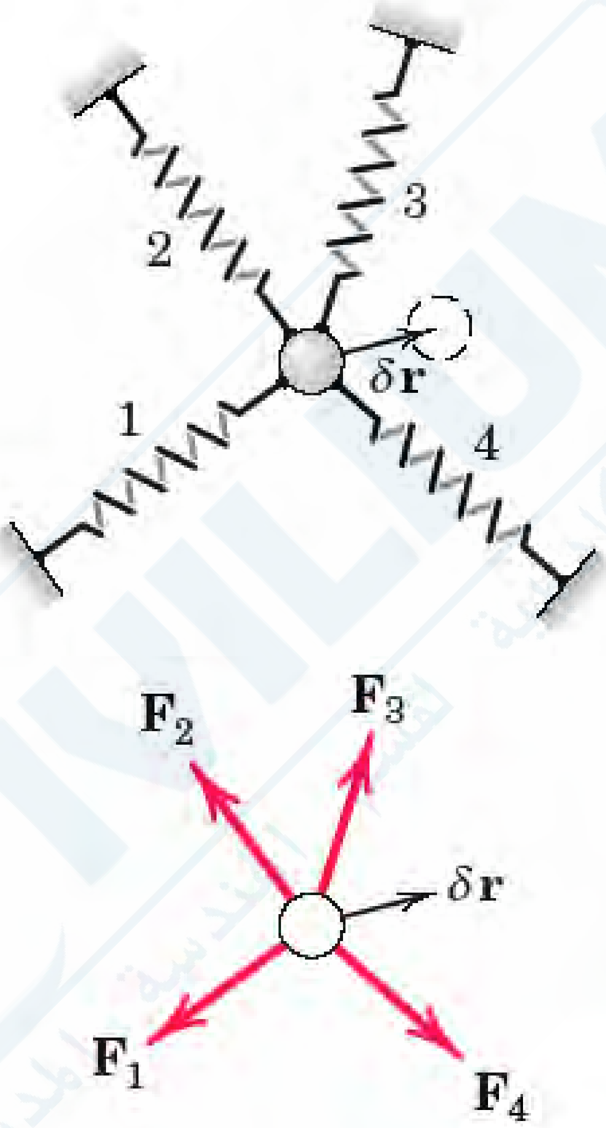


Figure 7/5

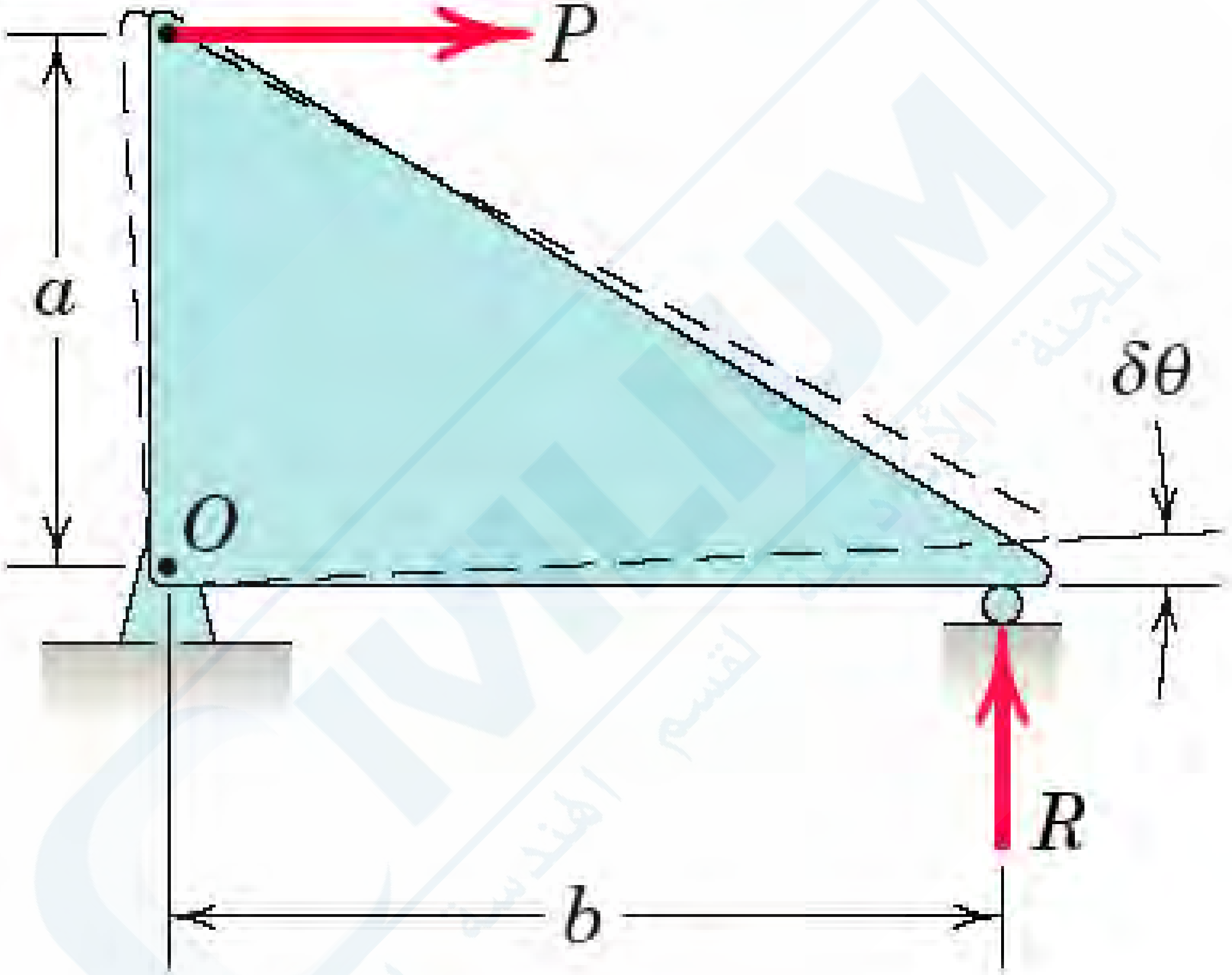
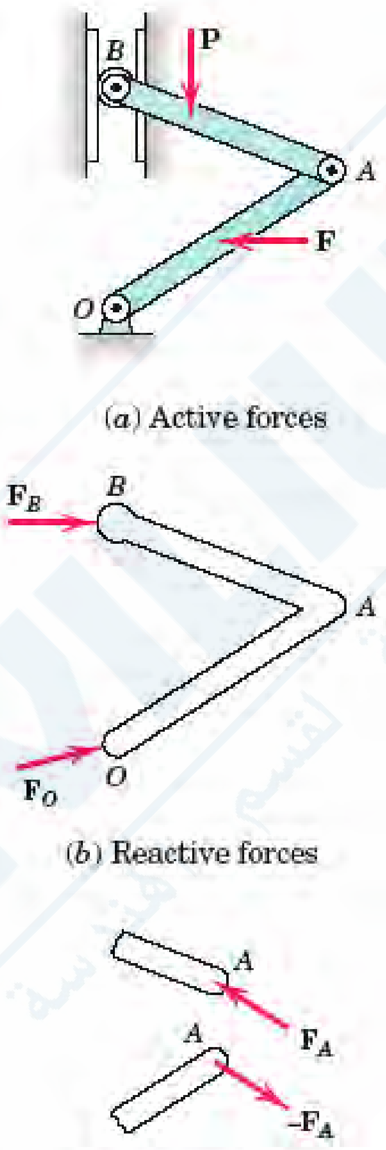


Figure 7/6



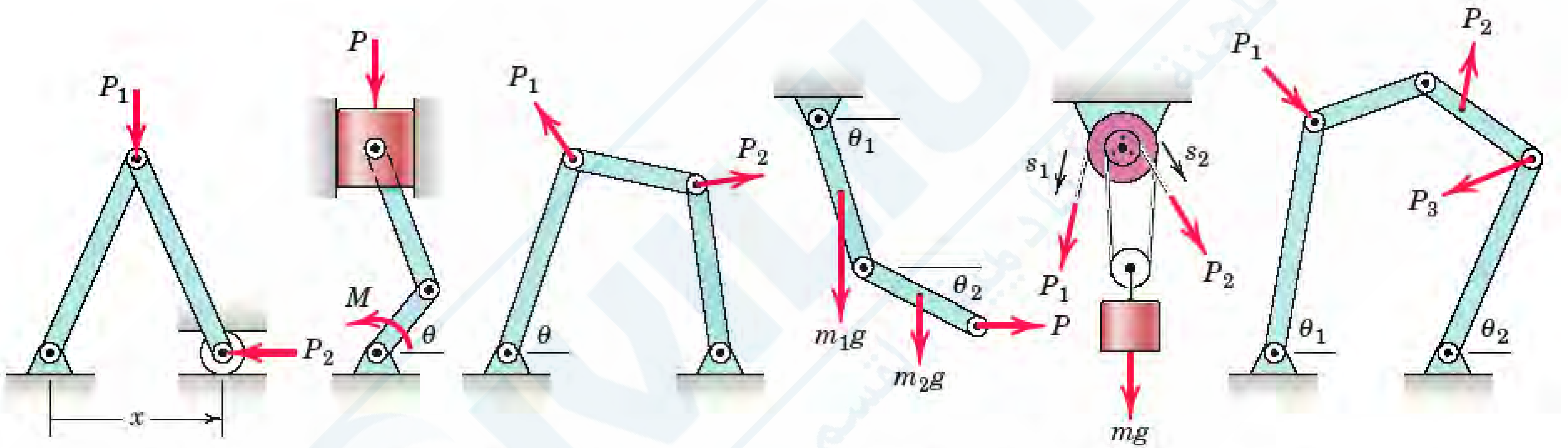
(*a*) Active forces

(*b*) Reactive forces

(*c*) Internal forces

Figure 7/7

CIVILIUM
اللجنة الأكاديمية لقسم الهندسة المدنية



(*a*) Examples of one-degree-of-freedom systems (*b*) Examples of two-degree-of-freedom systems

Figure 7/8

07_08

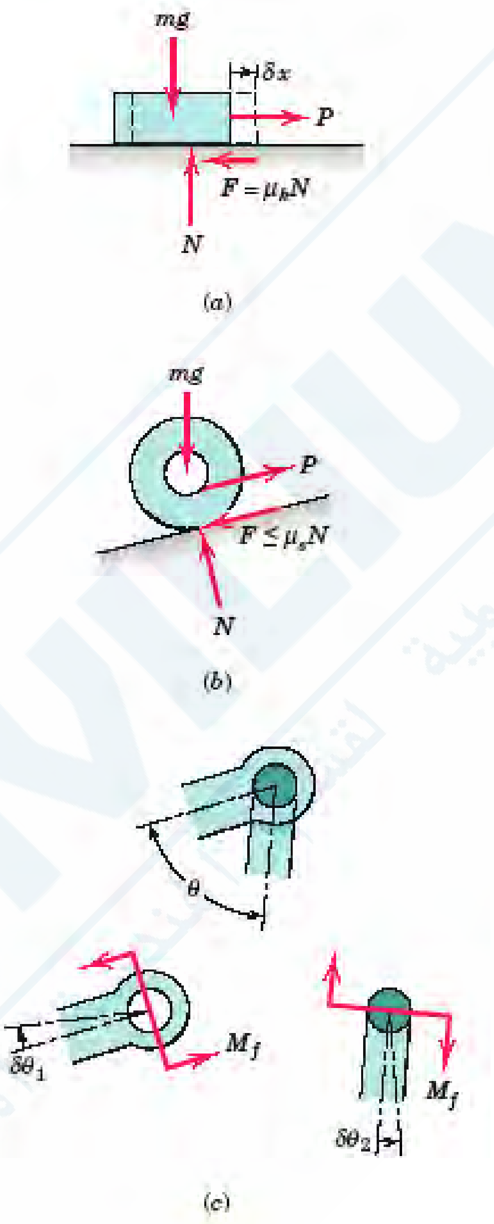


Figure 7/9

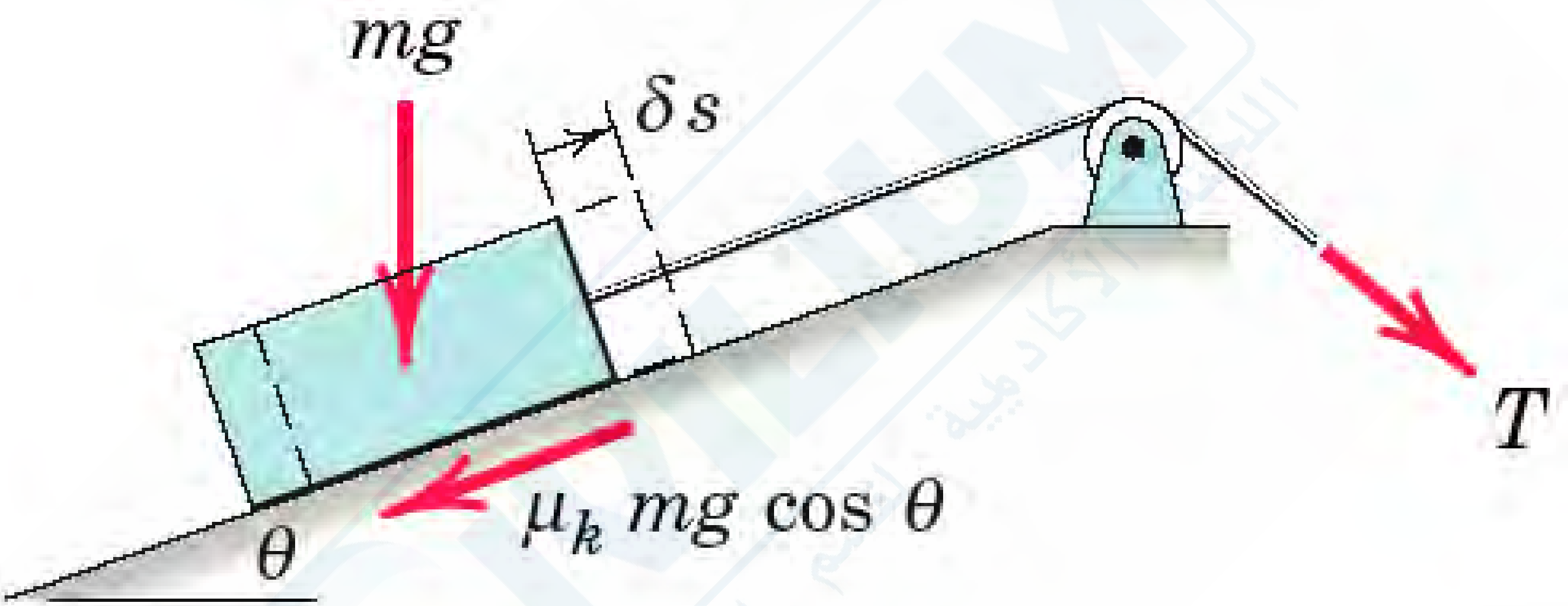


Figure 7/10

07_10

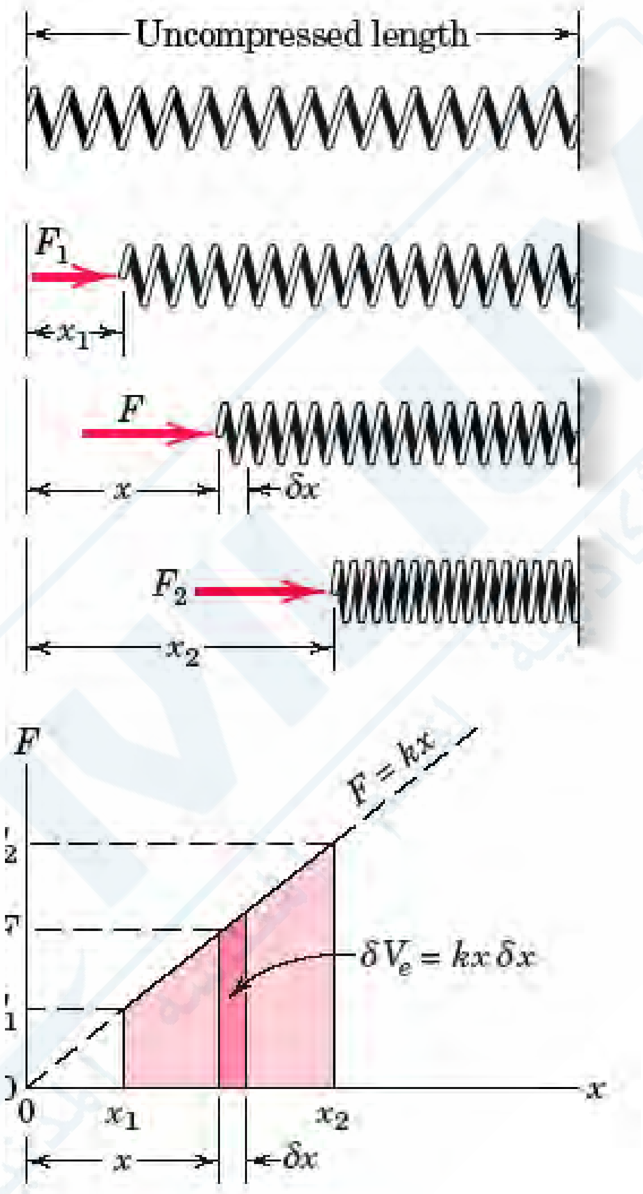
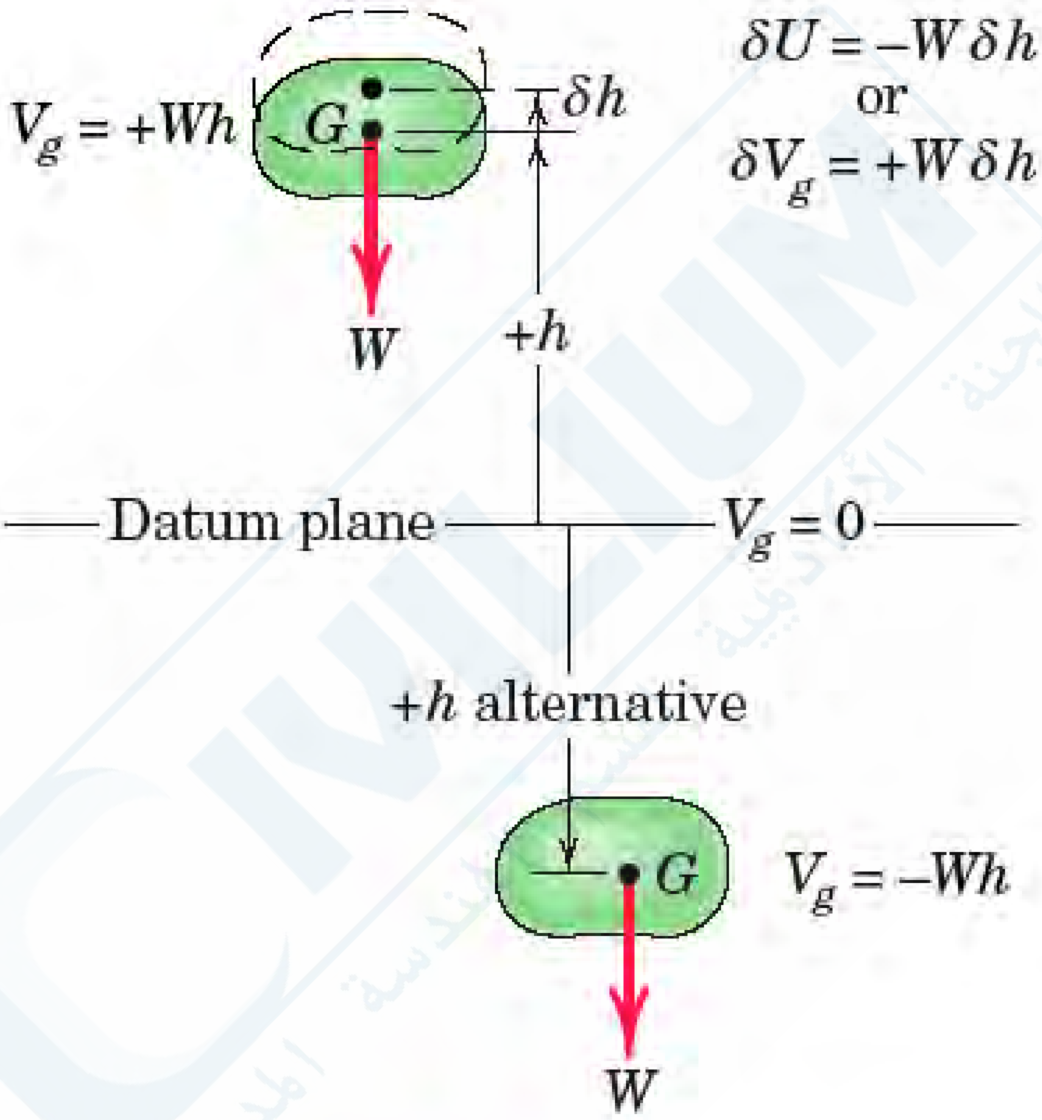

Uncompressed length
F1
x1
F
x
δx
F2
x2
F
F = kx
δVe = kx δx
0
x1
x2
x
x
δx

Figure 7/11



$\delta U = -W\,\delta h$
or
$\delta V_g = +W\,\delta h$
$V_g = +Wh$
G
δh
W
$+h$
Datum plane
$V_g = 0$
$+h$ alternative
G
$V_g = -Wh$
W

Figure 7/12

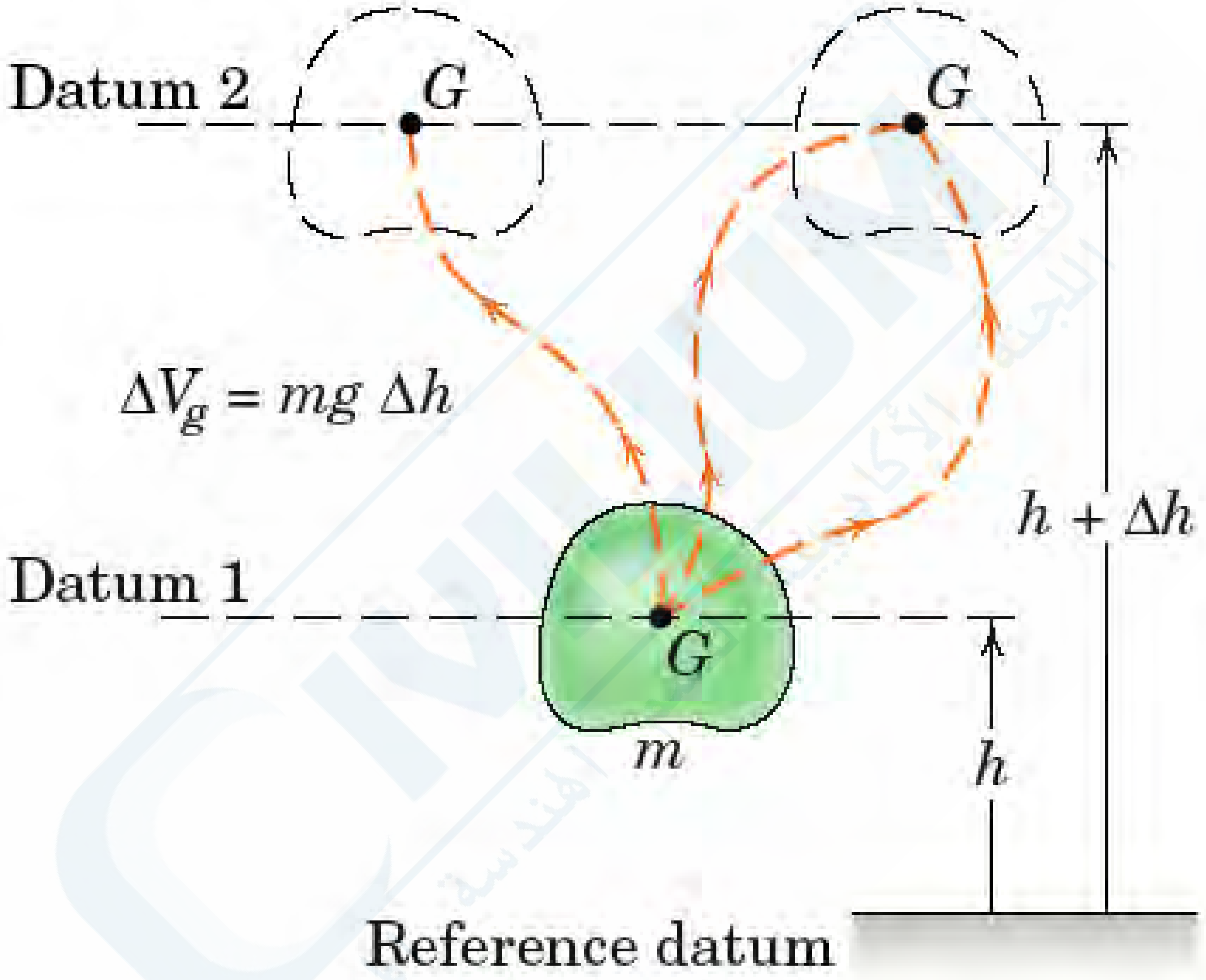
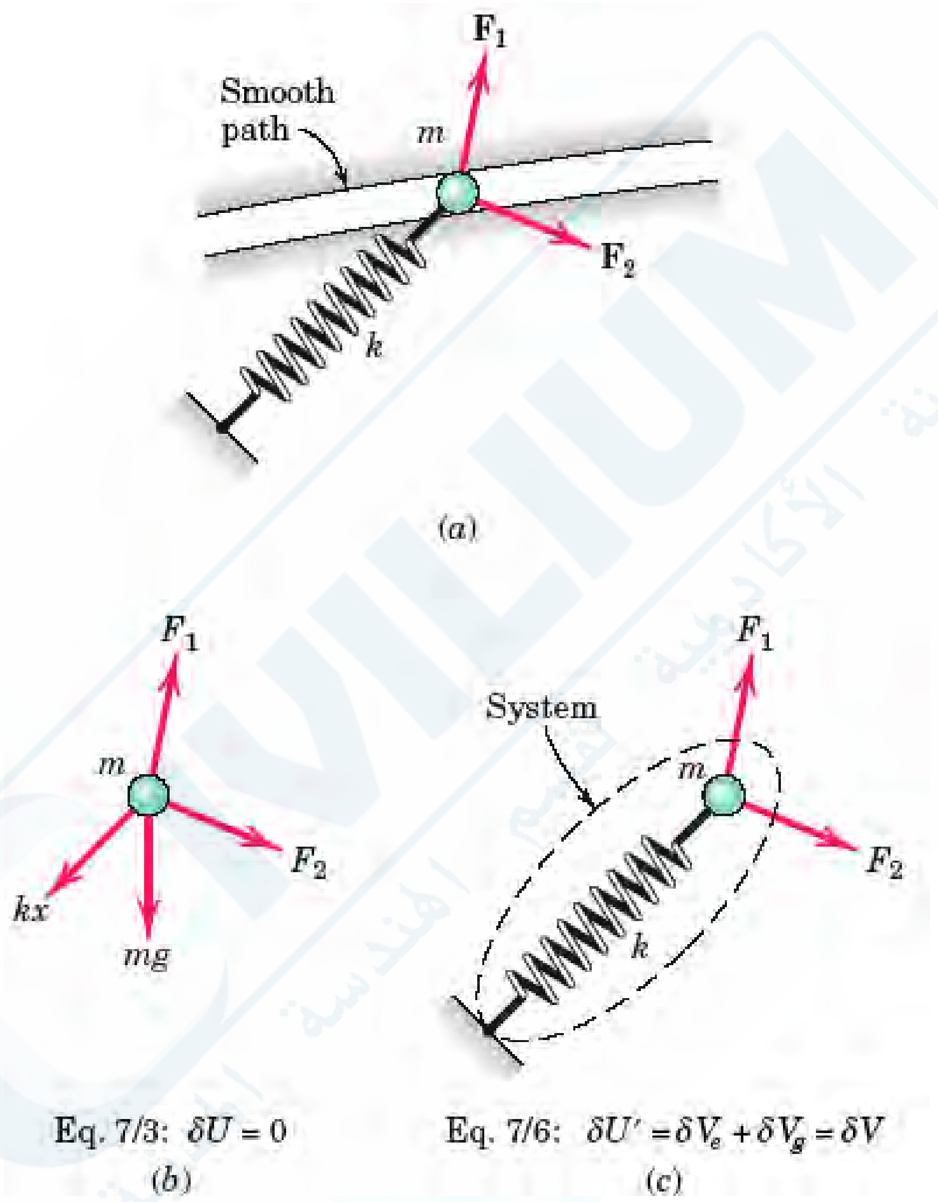

Datum 2
G
G
$\Delta V_g = mg\,\Delta h$
$h + \Delta h$
Datum 1
G
m
h
Reference datum

Figure 7/13

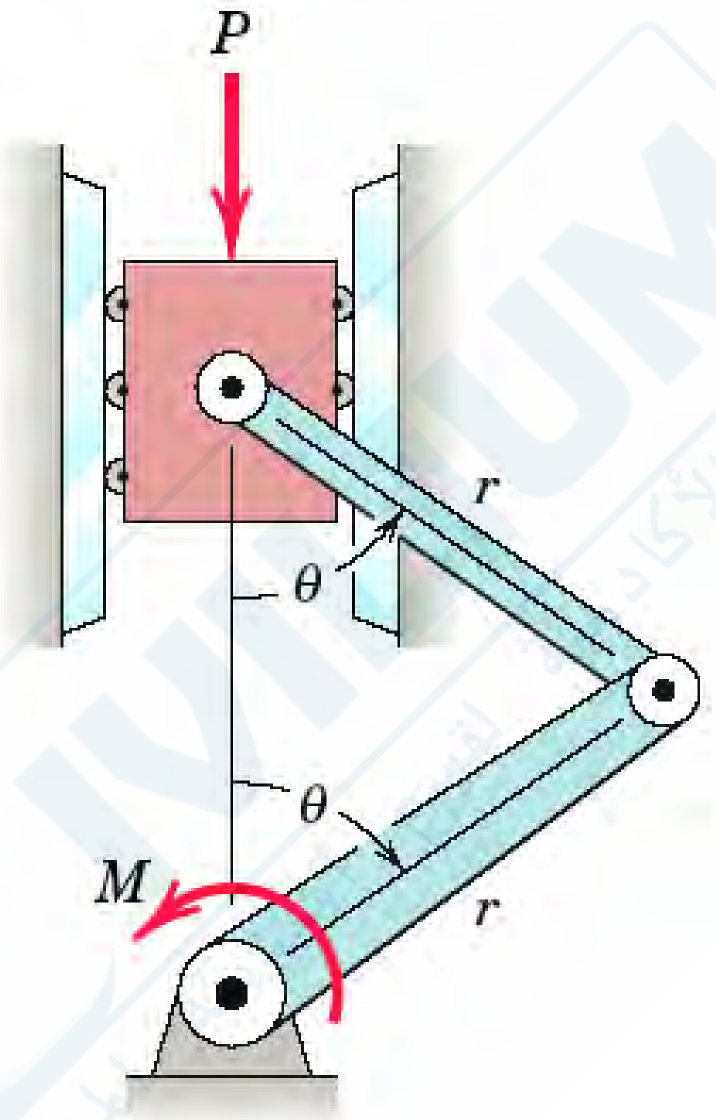


F1
Smooth path
m
F2
k
(a)
F1
m
F2
kx
mg
Eq. 7/3: $\delta U = 0$
(b)
F1
System
m
k
F2
Eq. 7/6: $\delta U' = \delta V_e + \delta V_g = \delta V$
(c)

Figure 7/14

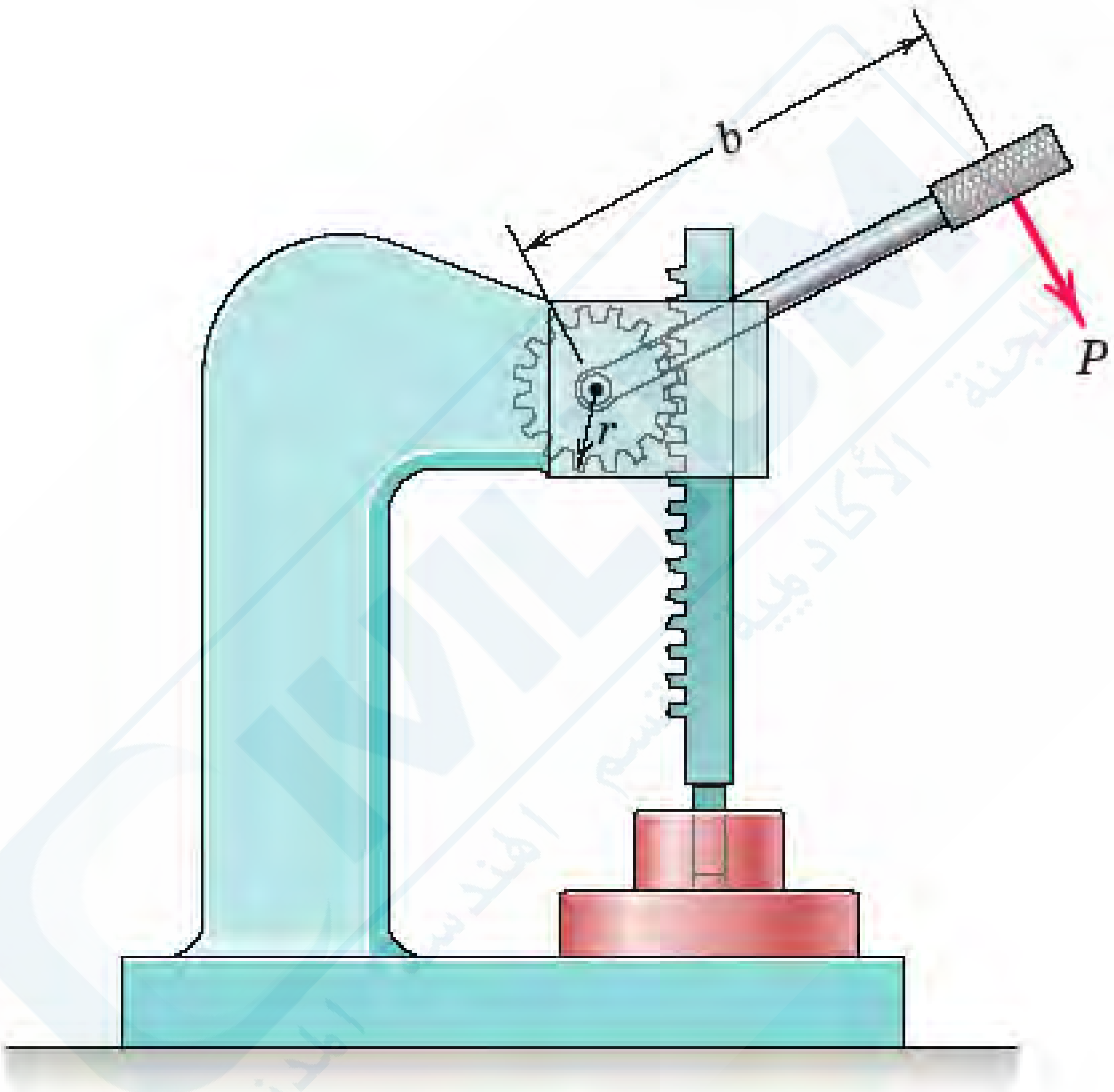
Figure 7/15

07_15



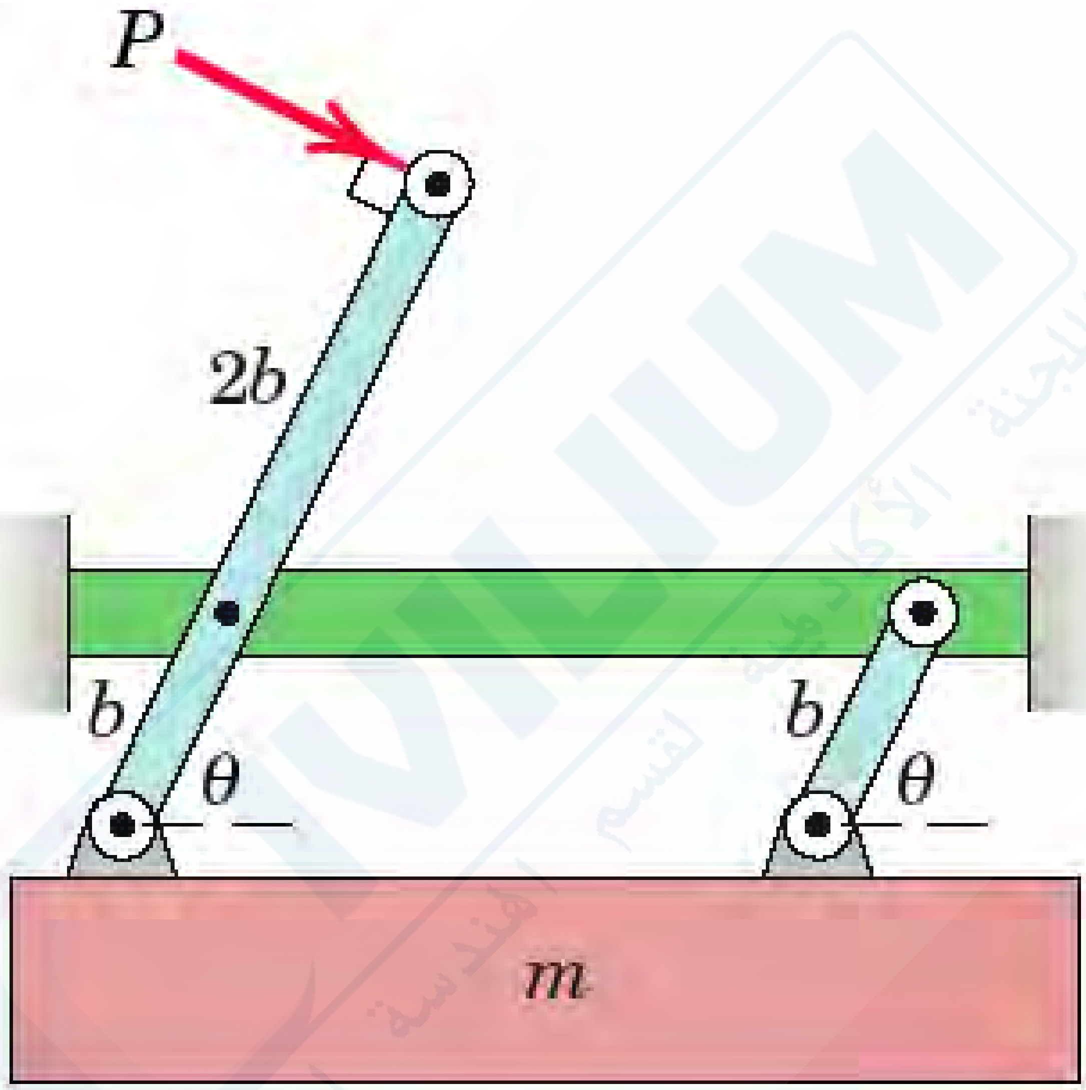
Problem 7/1

P_07_01



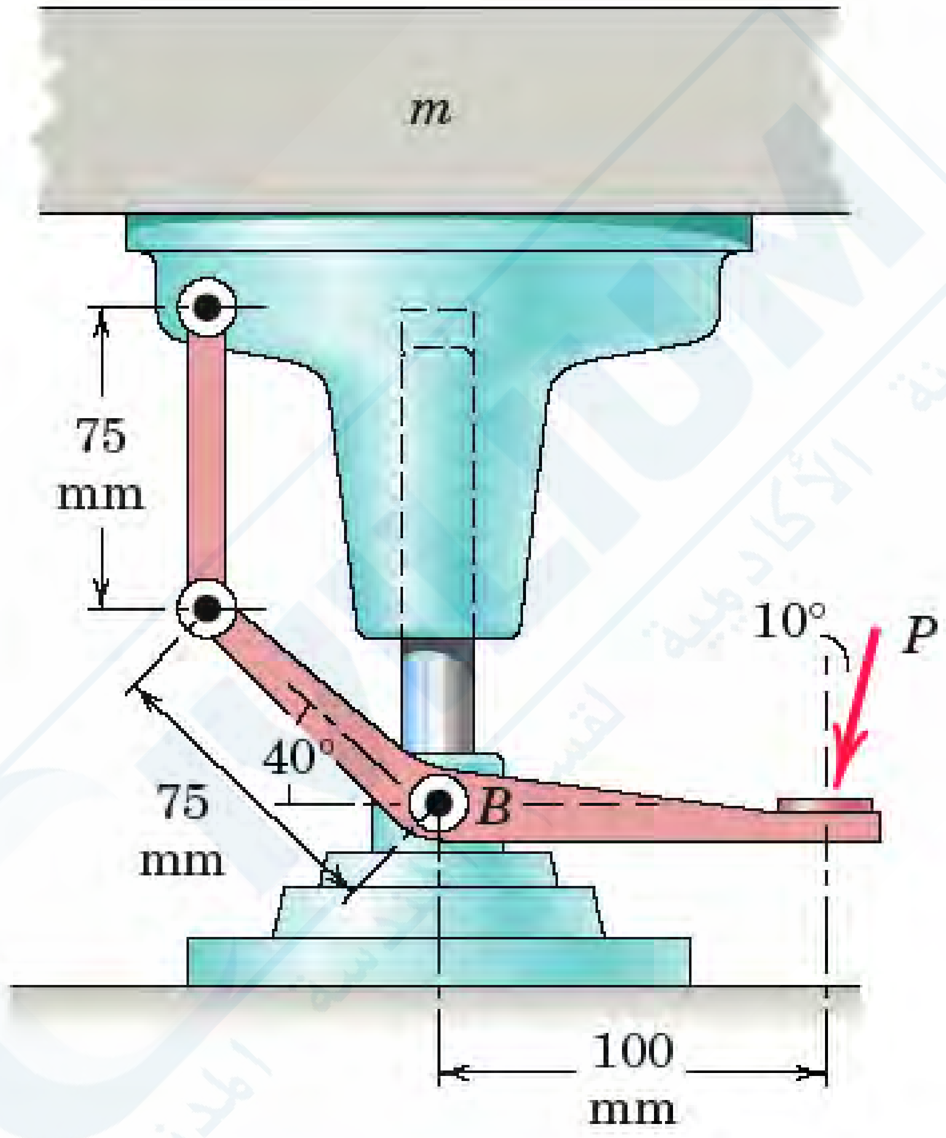
Problem 7/2

P_07_02



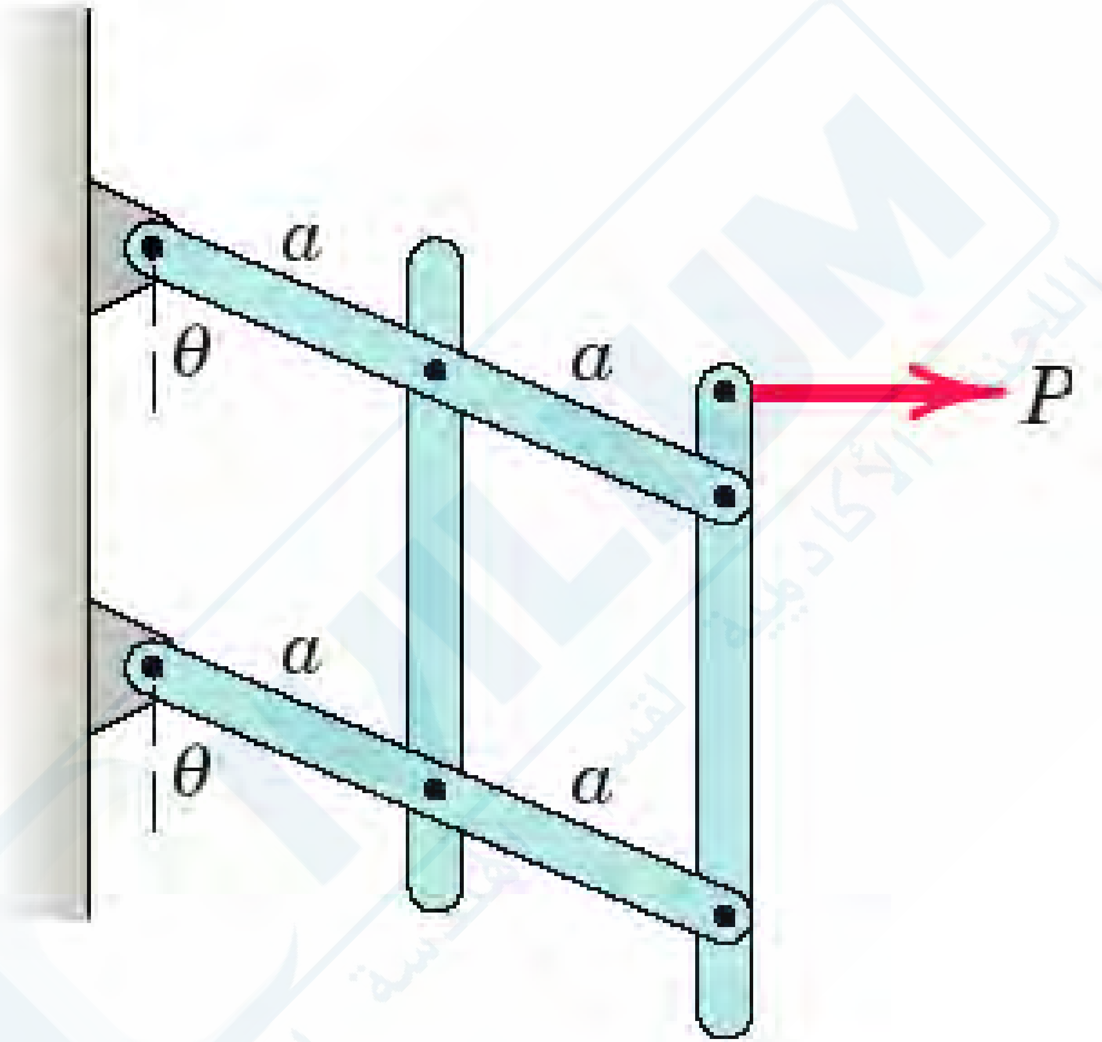
Problem 7/3

P_07_03



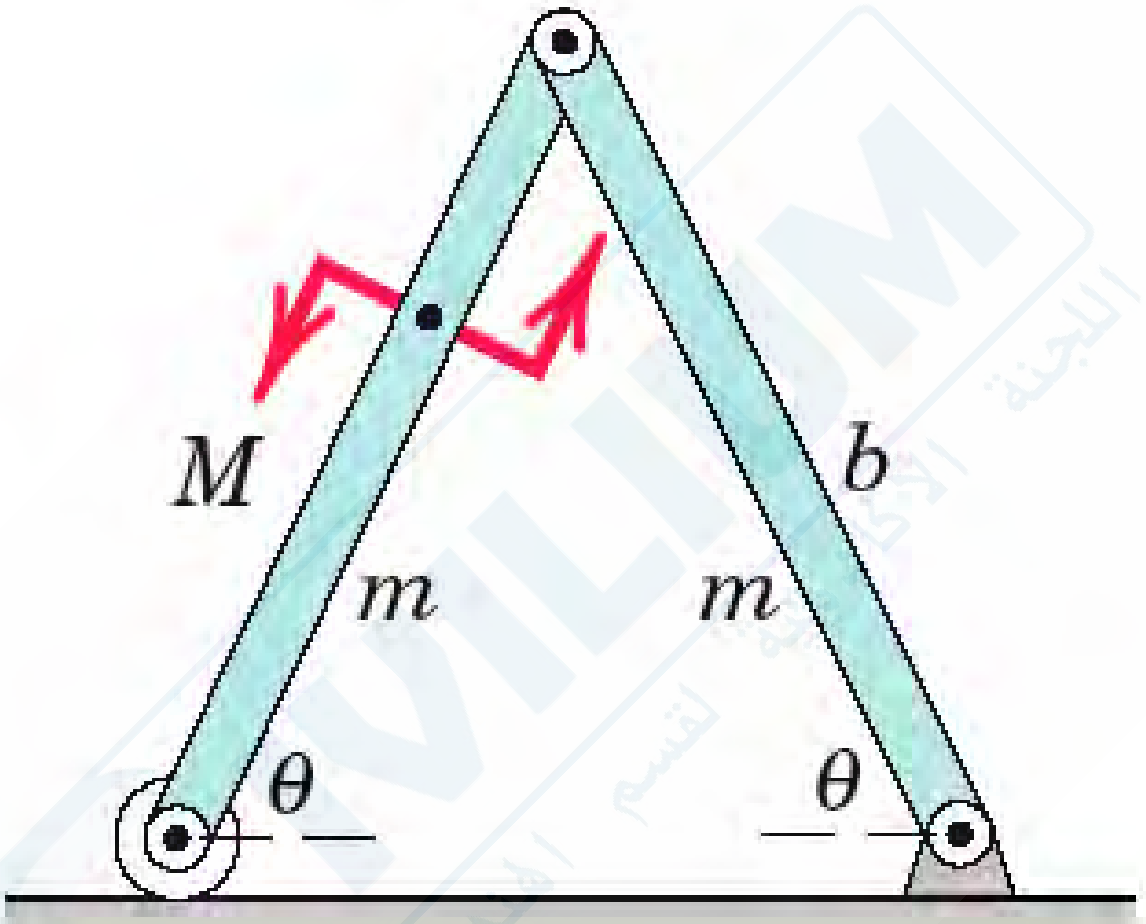
Problem 7/4

P_07_04



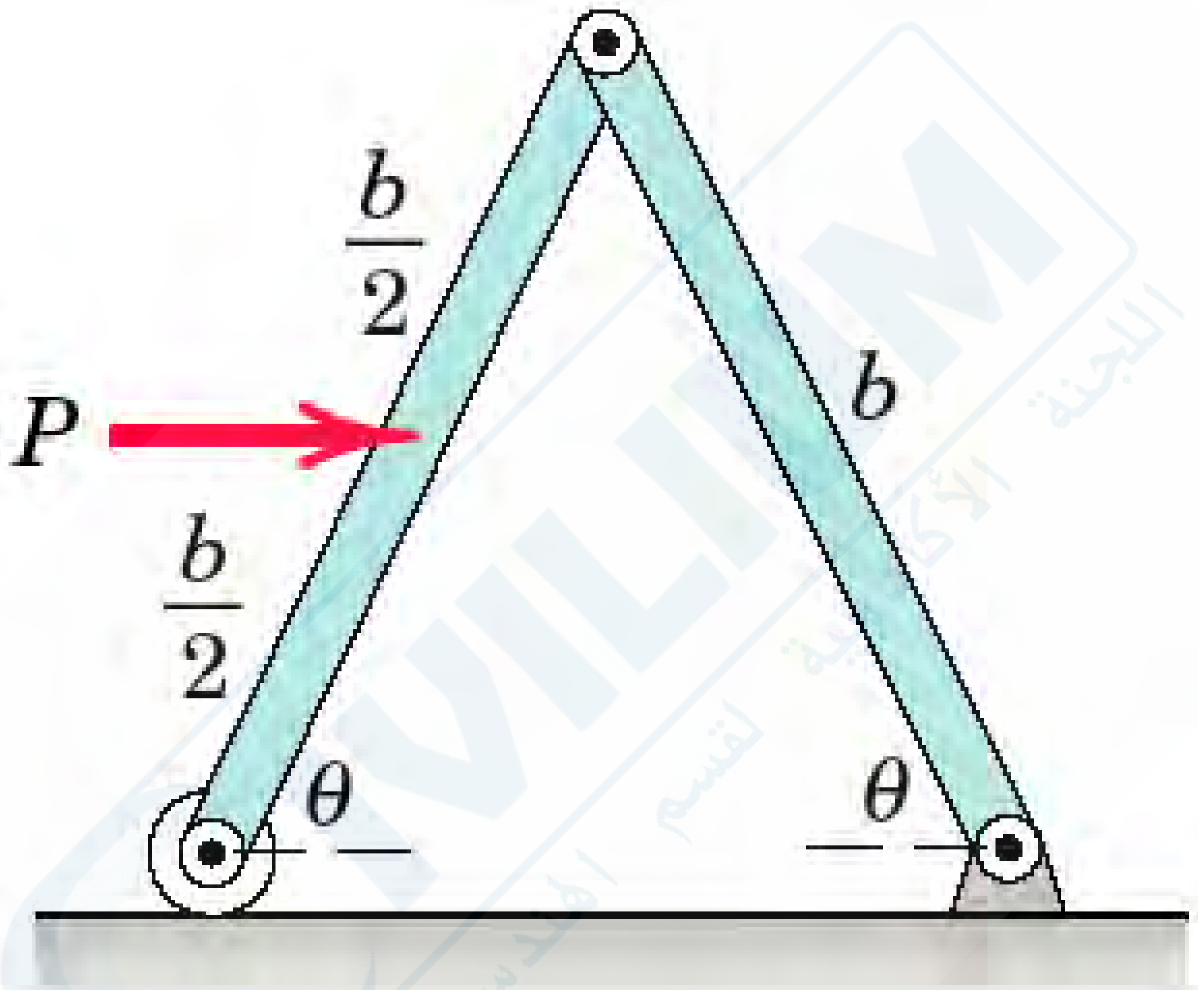
Problem 7/5

P_07_05



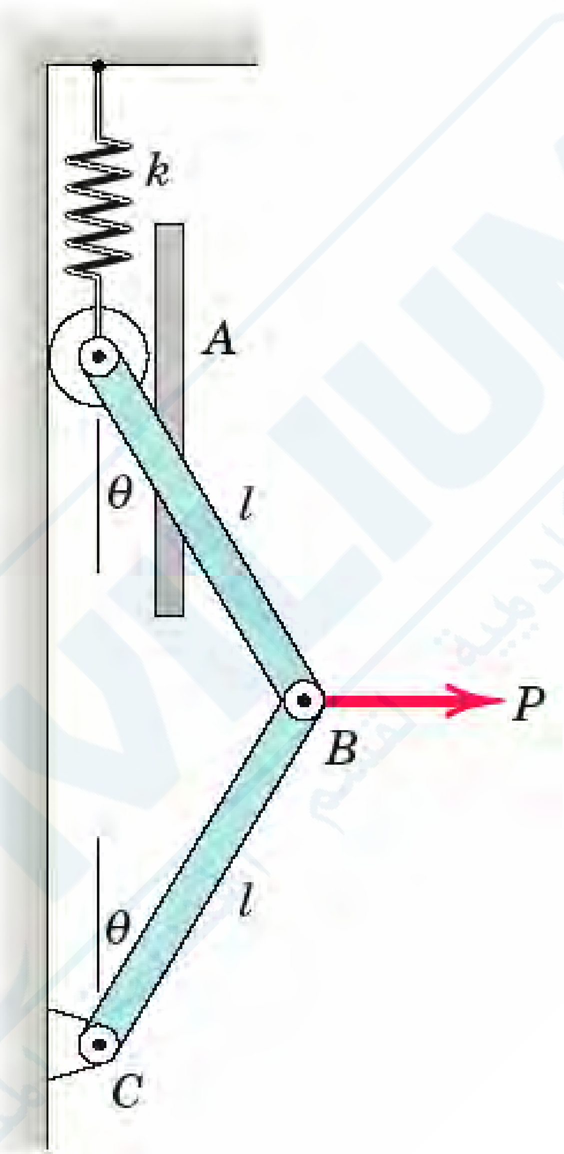
Problem 7/6

P_07_06

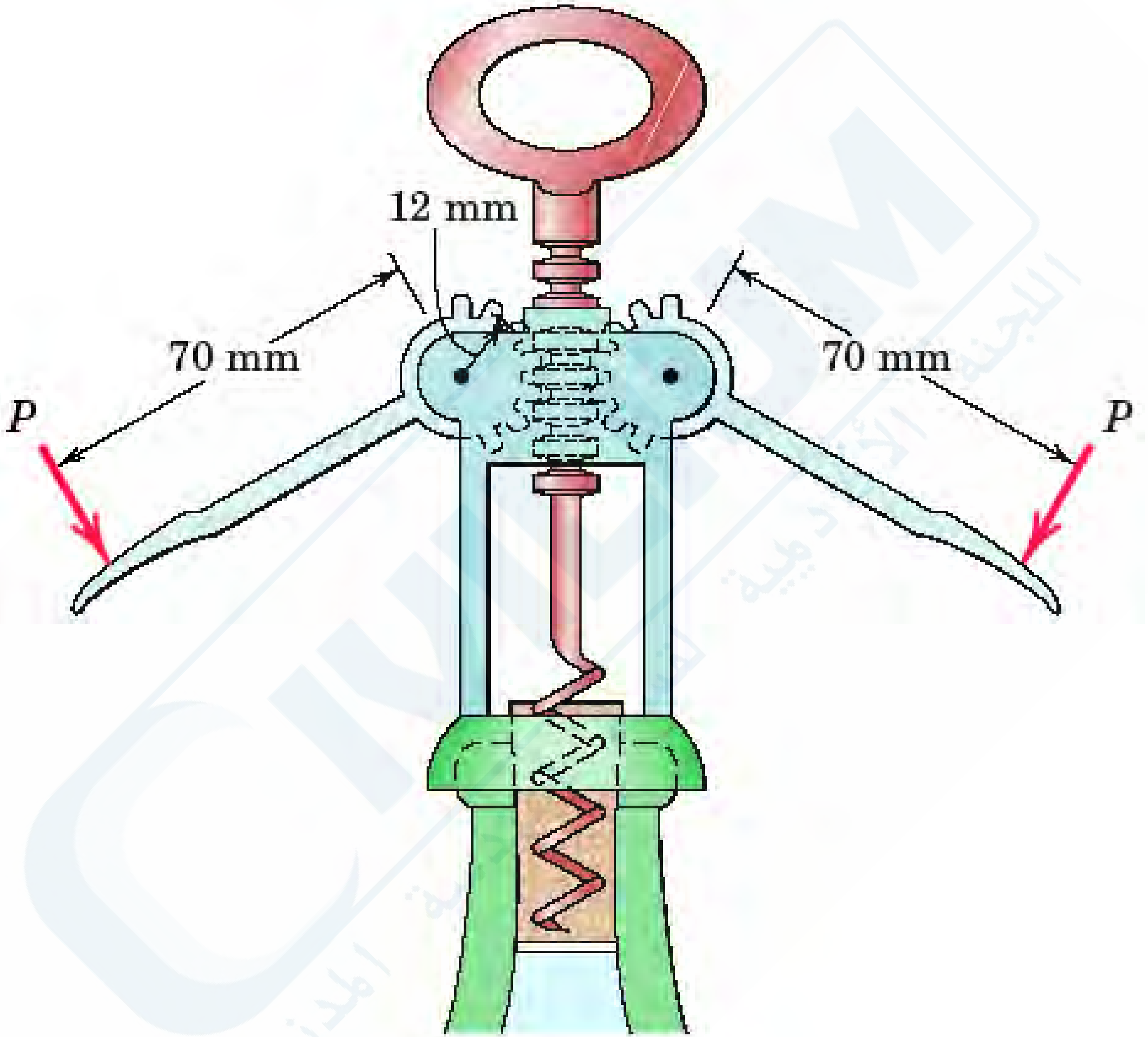


Problem 7/7

P_07_07

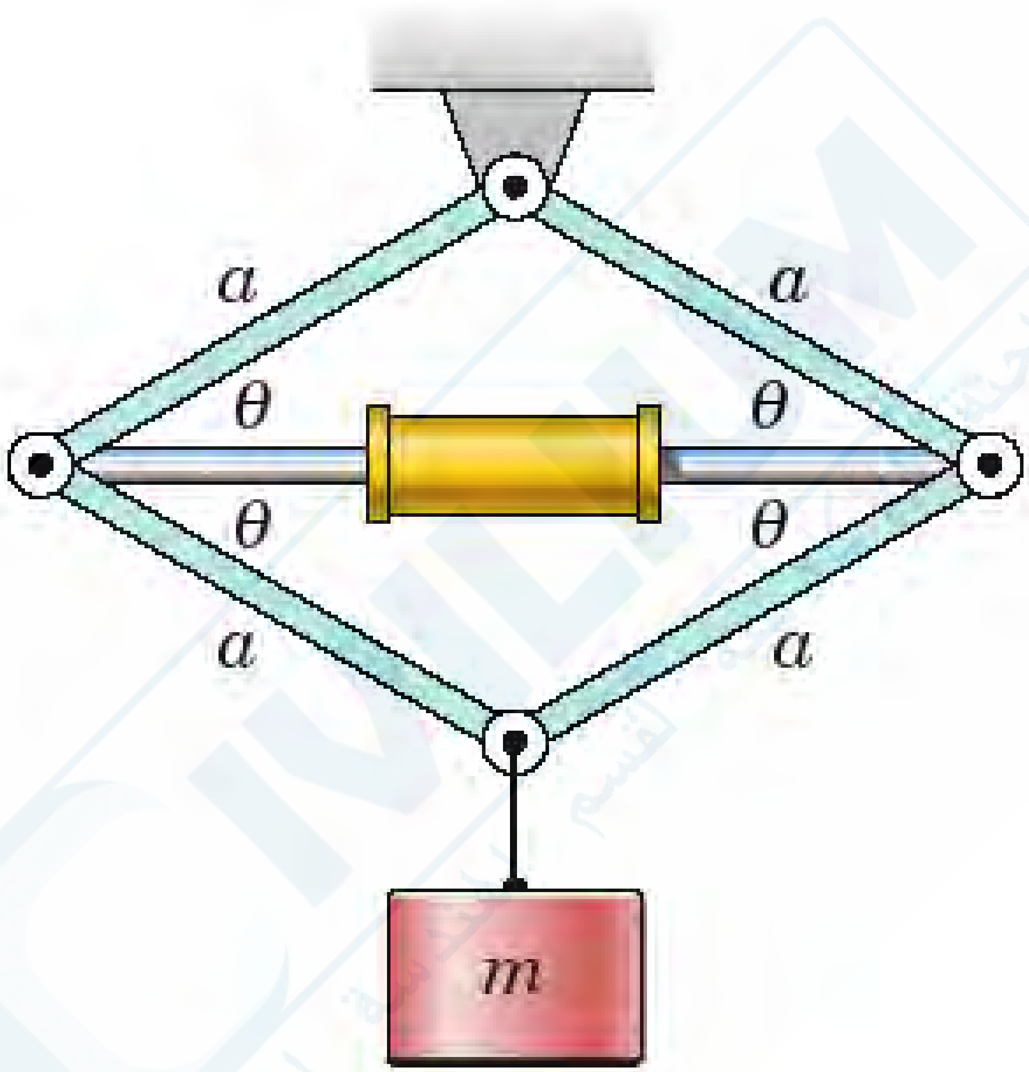


Problem 7/8



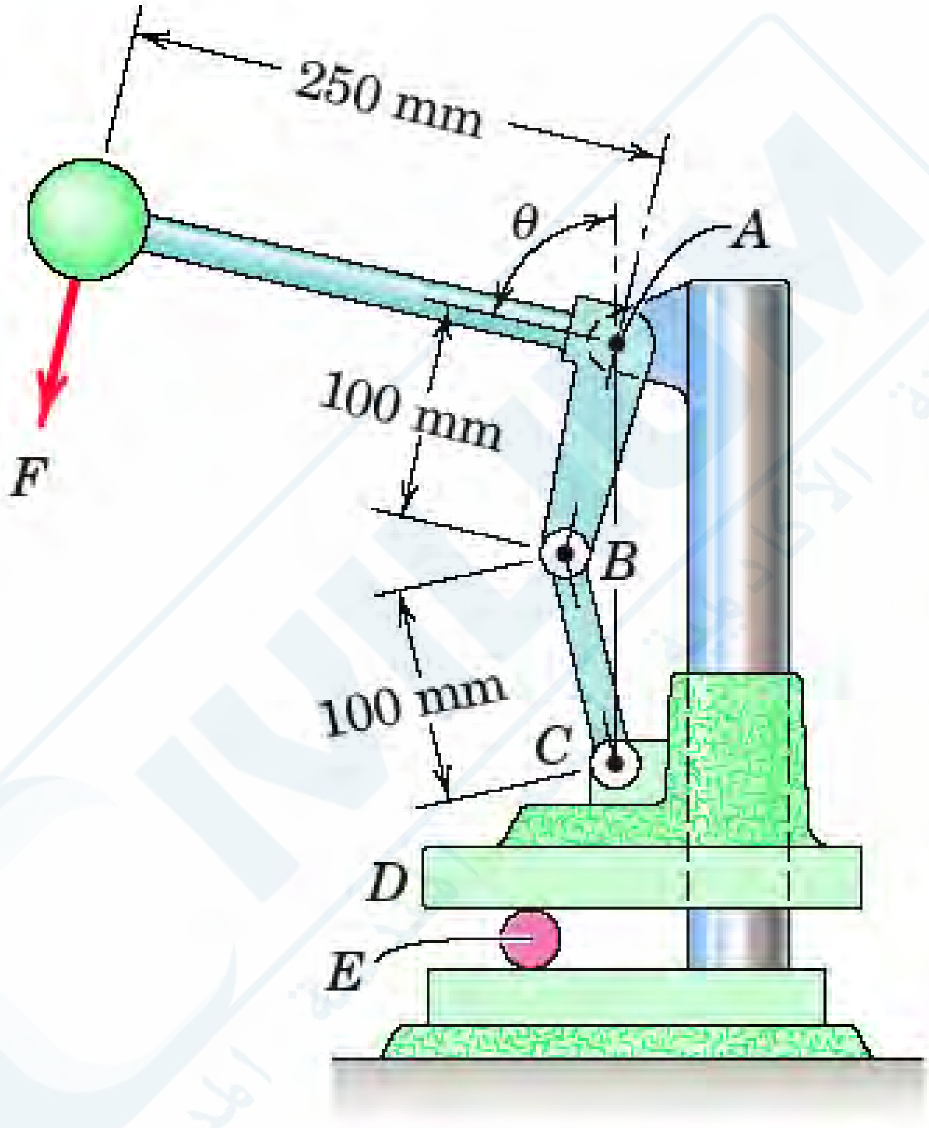
Problem 7/9

P_07_09



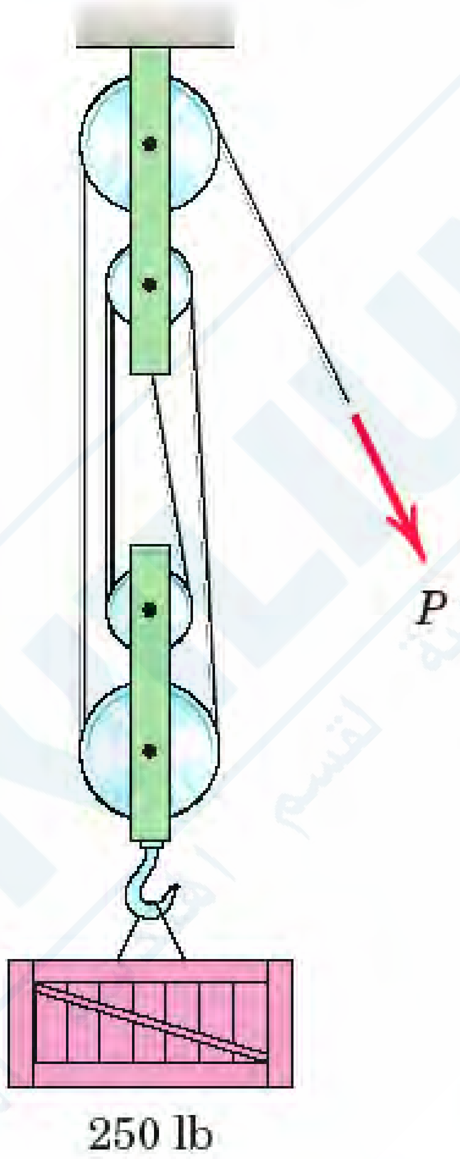
Problem 7/10

P_07_10



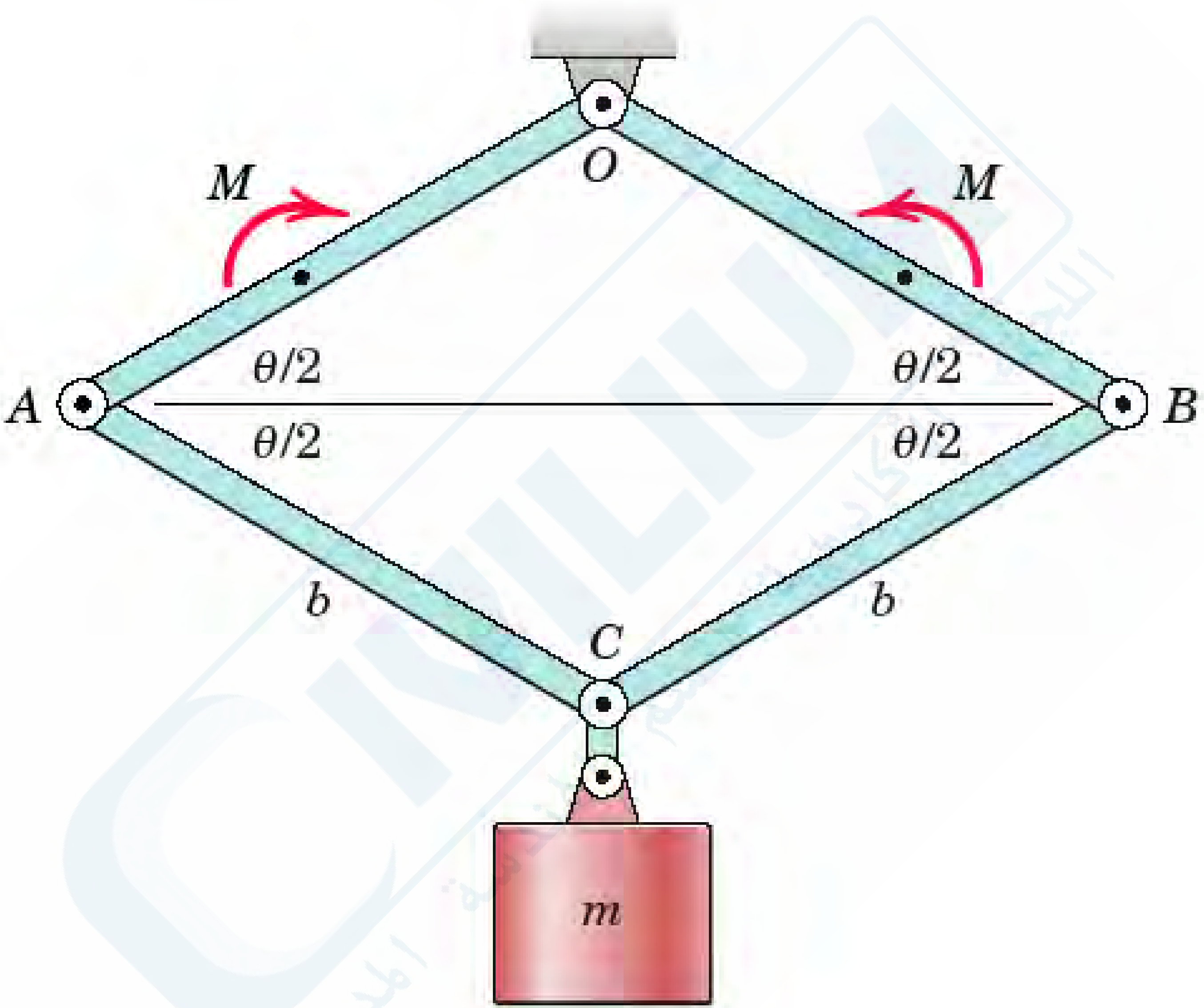
Problem 7/11

P_07_11



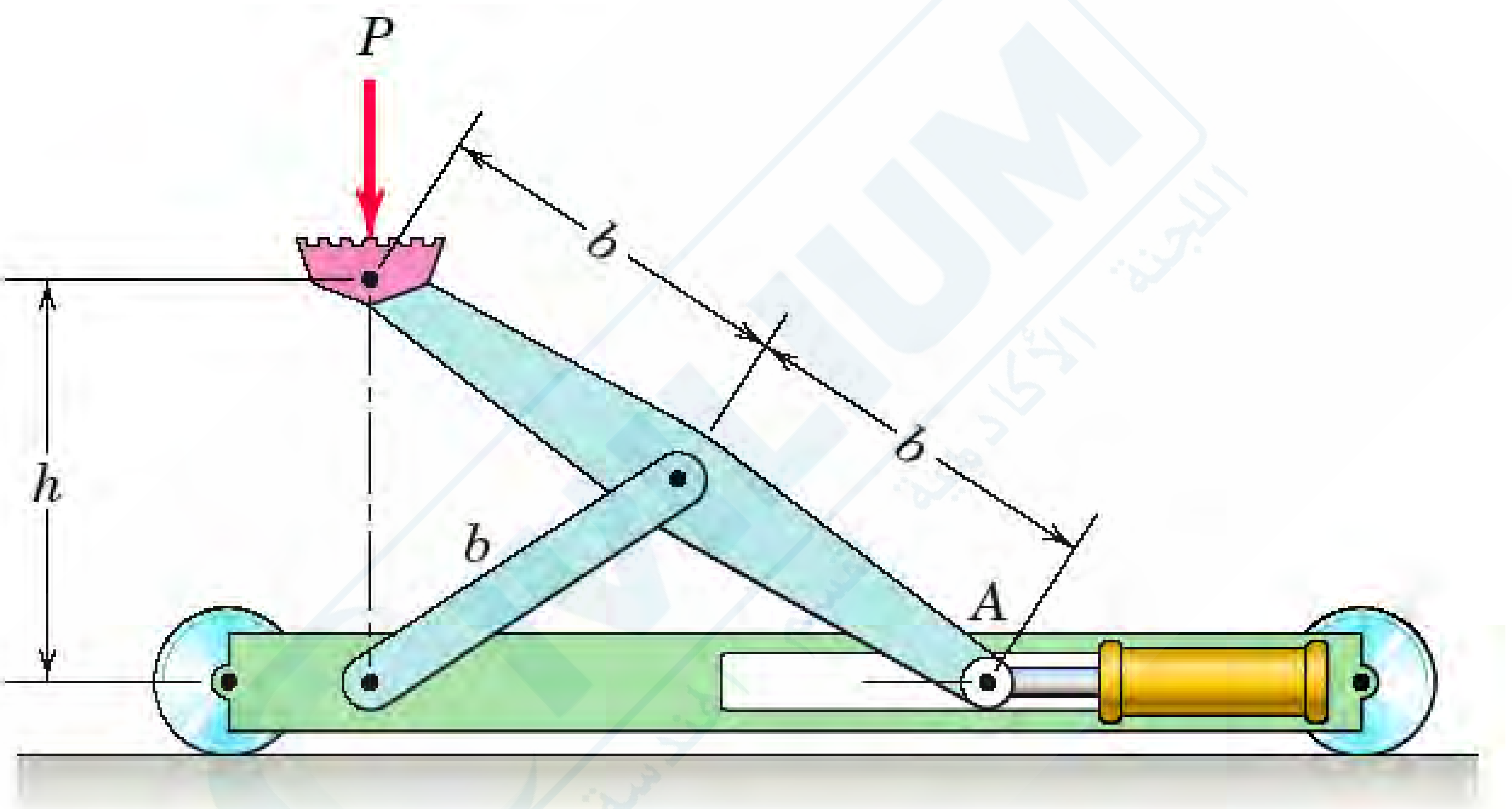
Problem 7/12

P_07_12



Problem 7/13

P_07_13



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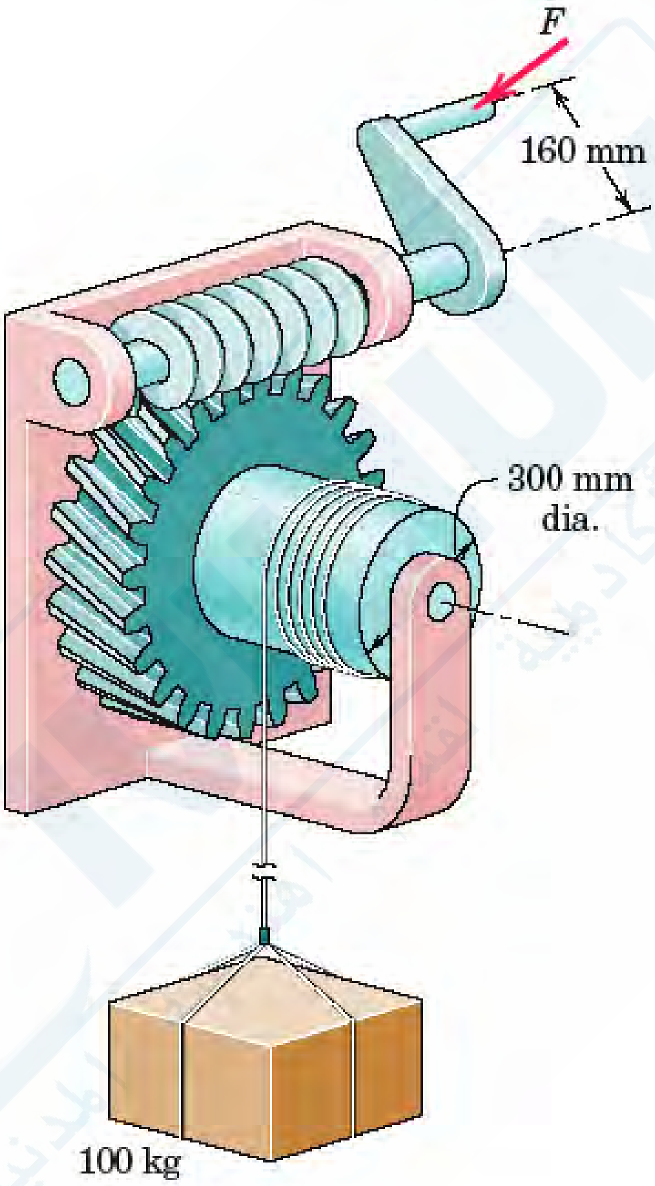
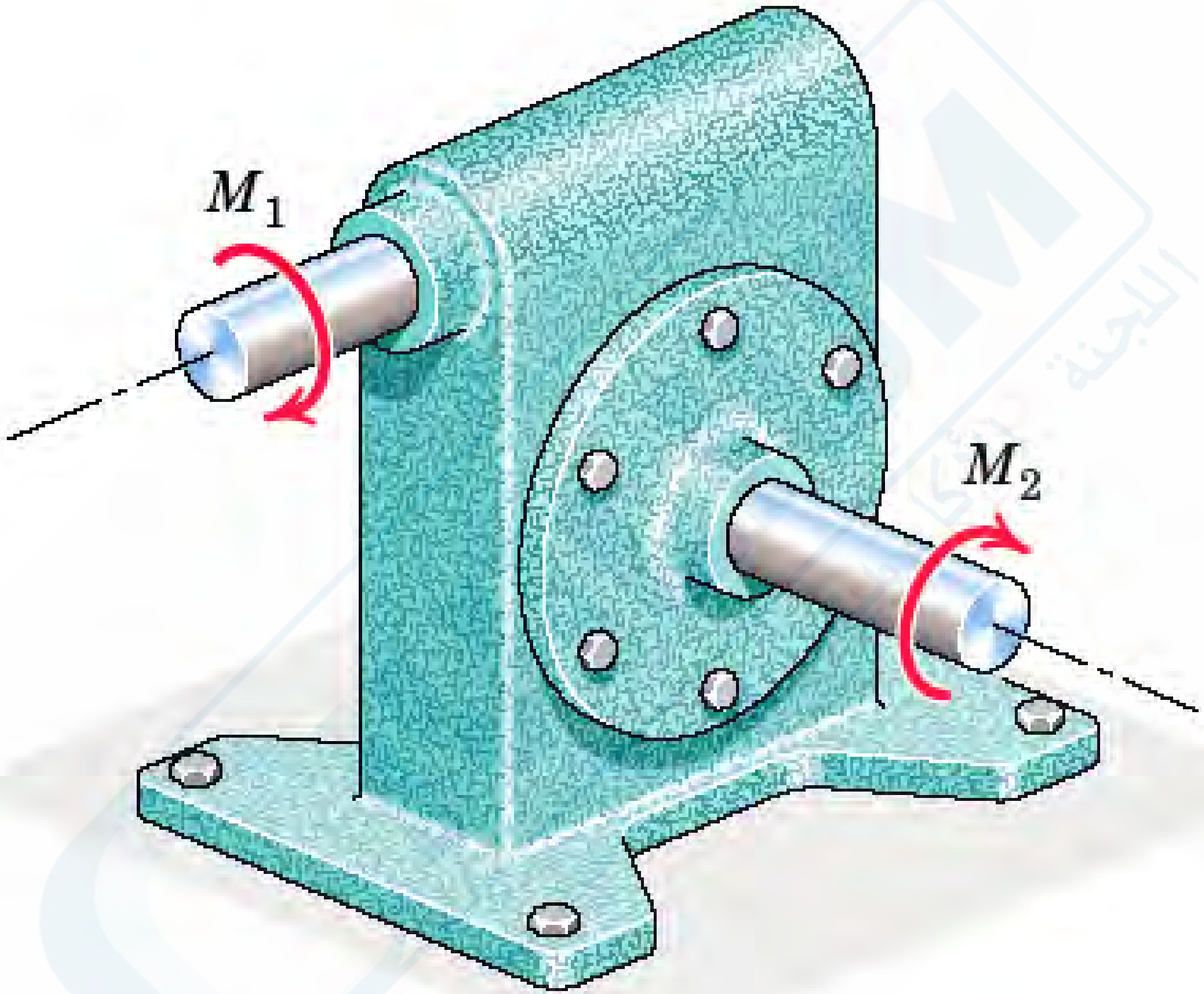


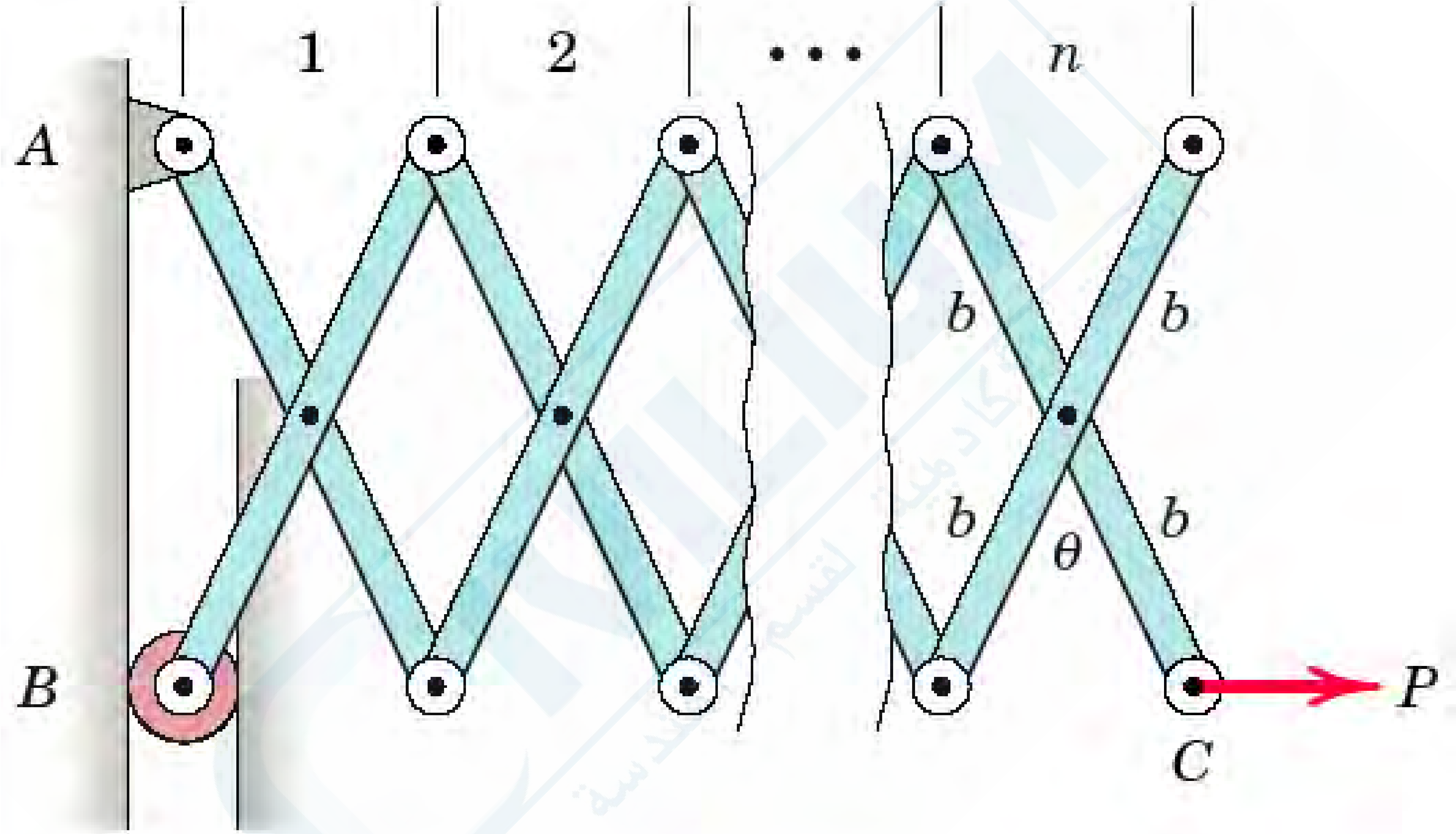
Problem 7/15

P_07_15



Problem 7/16

P_07_16



Problem 7/17

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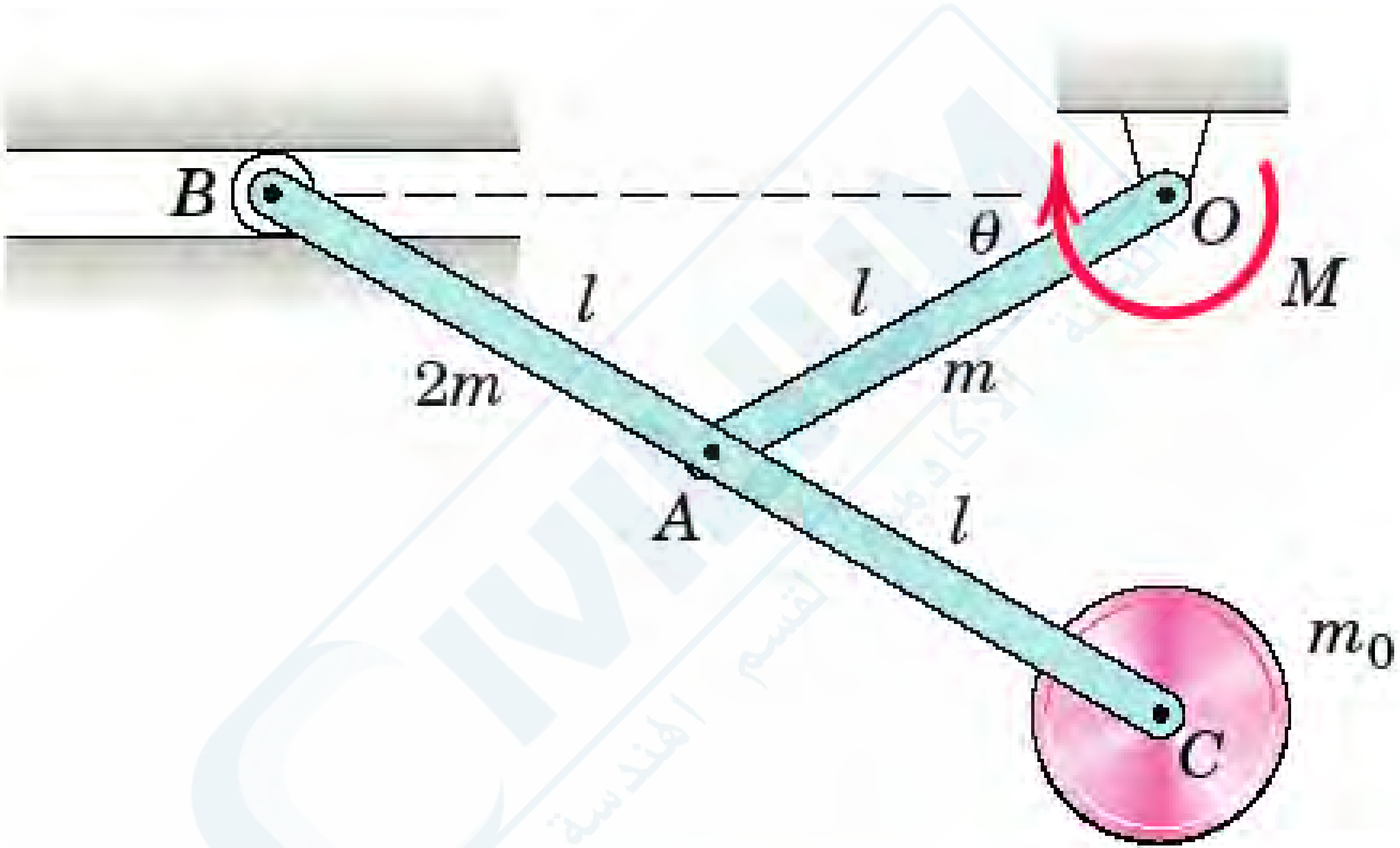
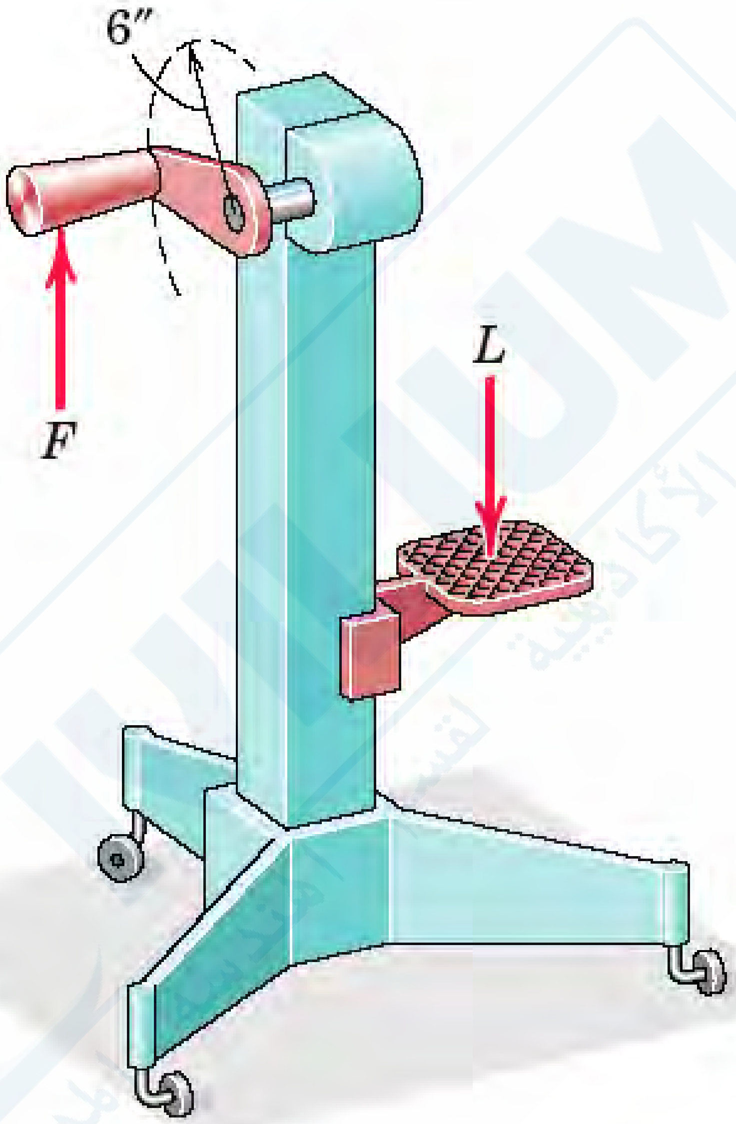


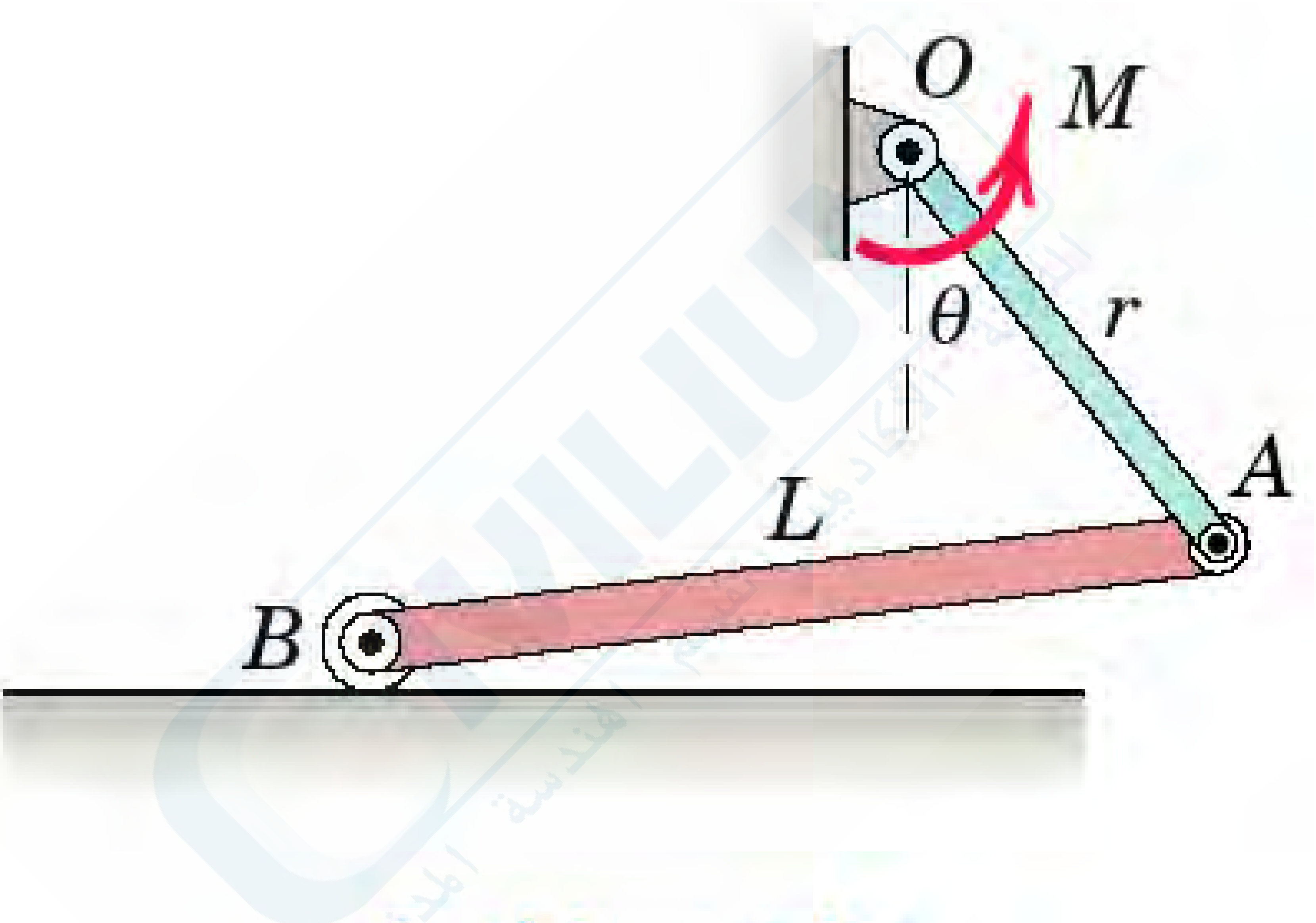
Problem 7/19

P_07_19



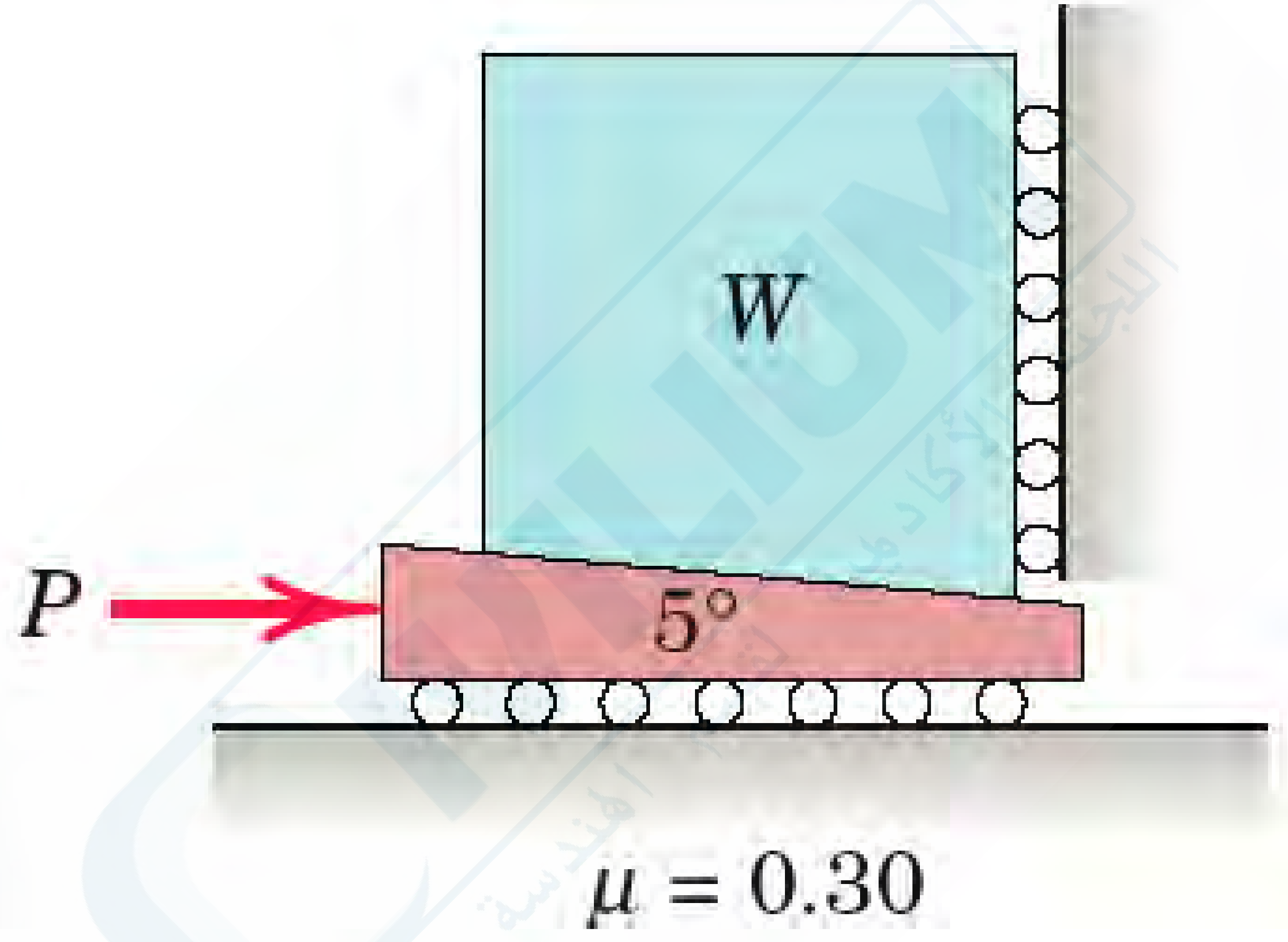
Problem 7/20

P_07_20



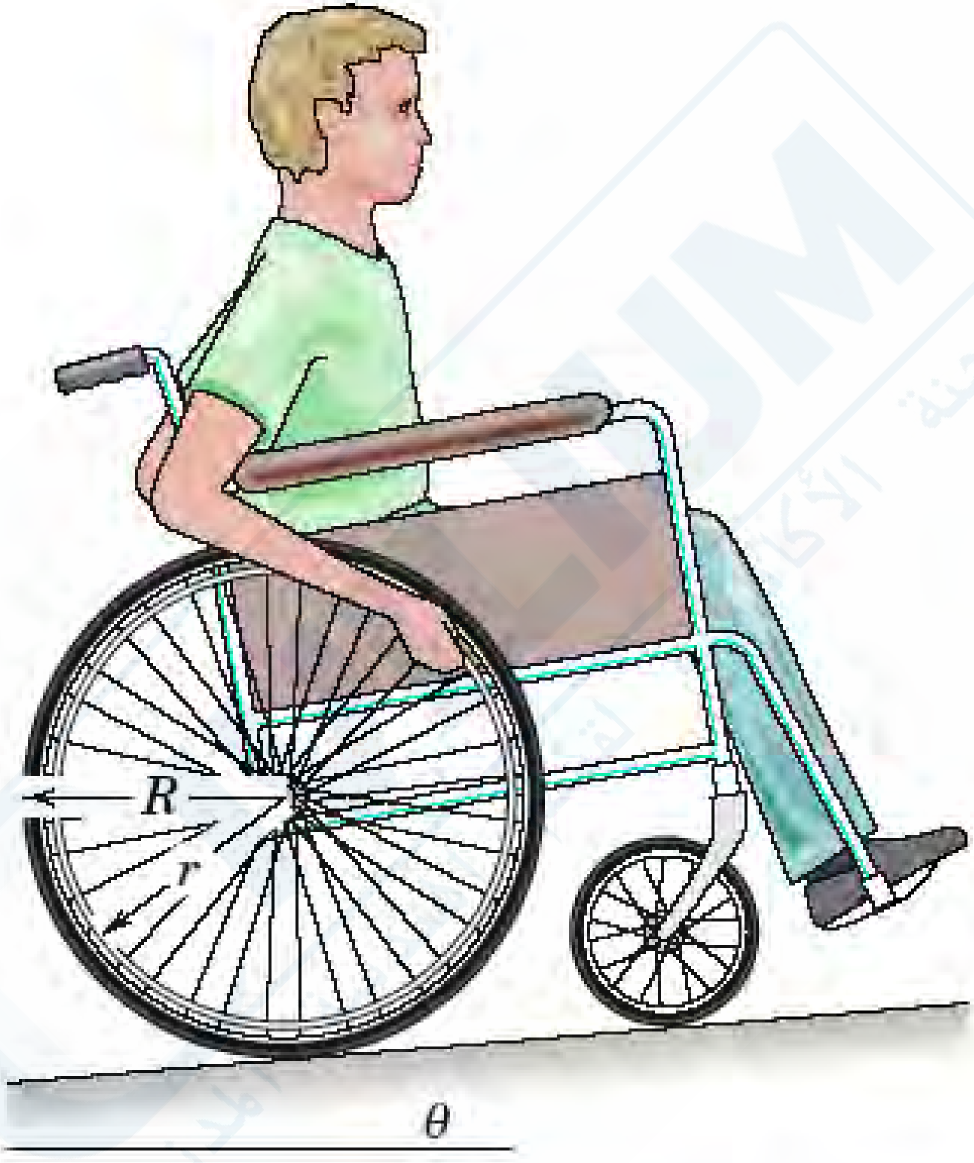
Problem 7/21

P_07_21



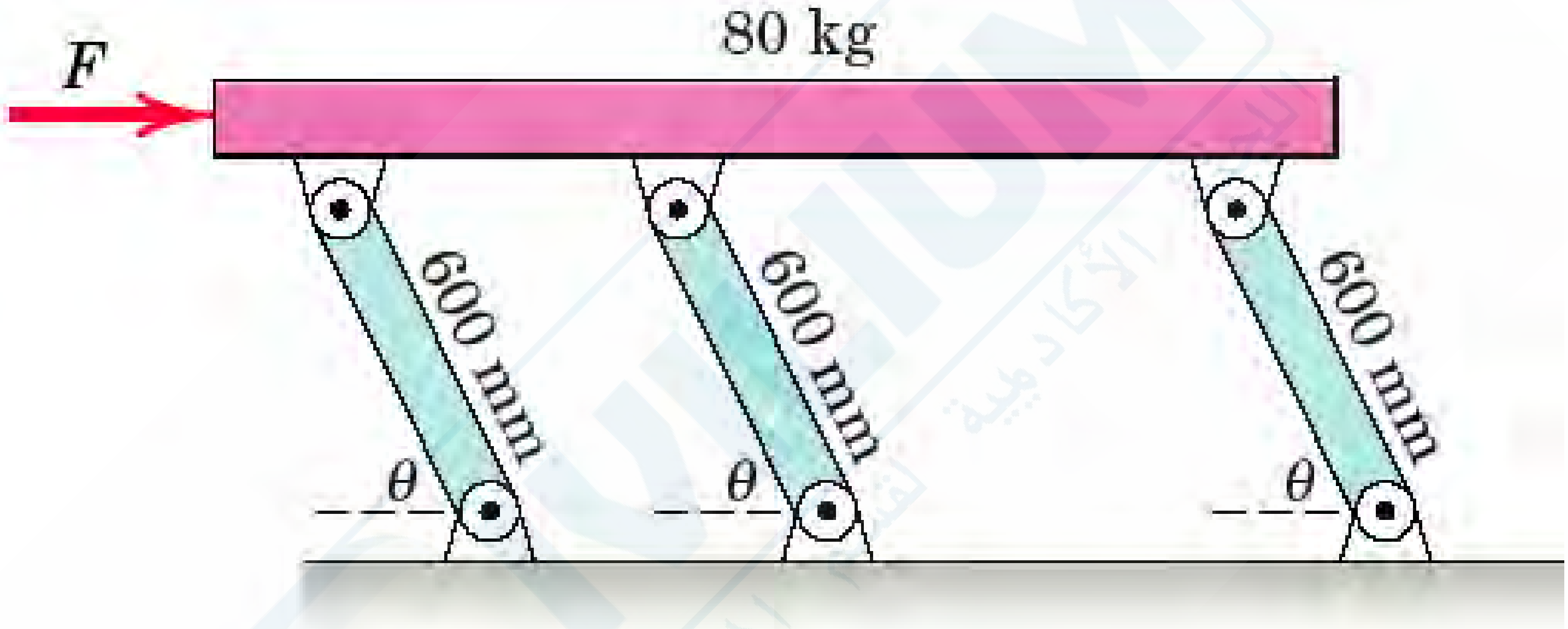
Problem 7/22

P_07_22



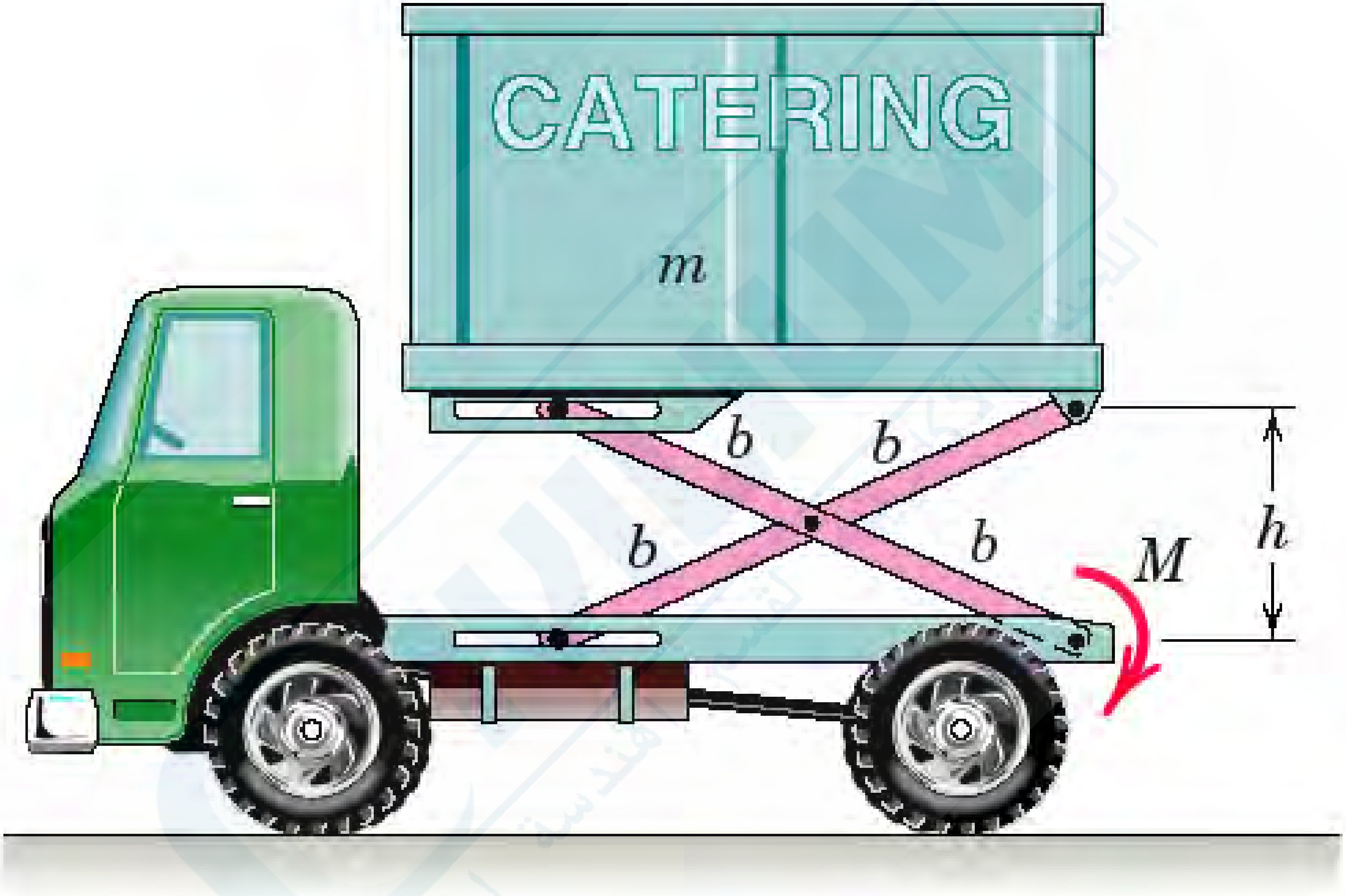
Problem 7/23

P_07_23



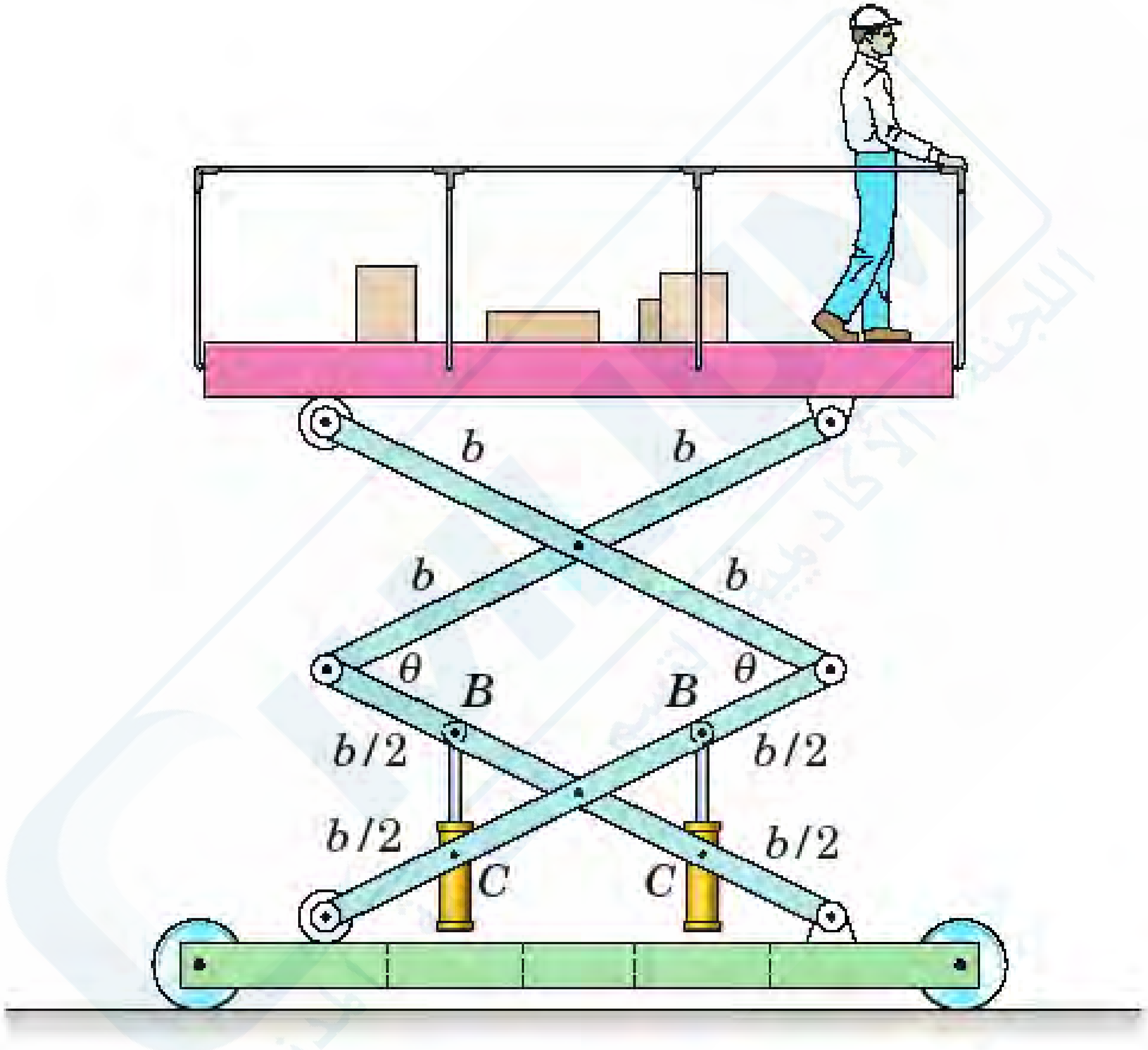
Problem 7/24

P_07_24



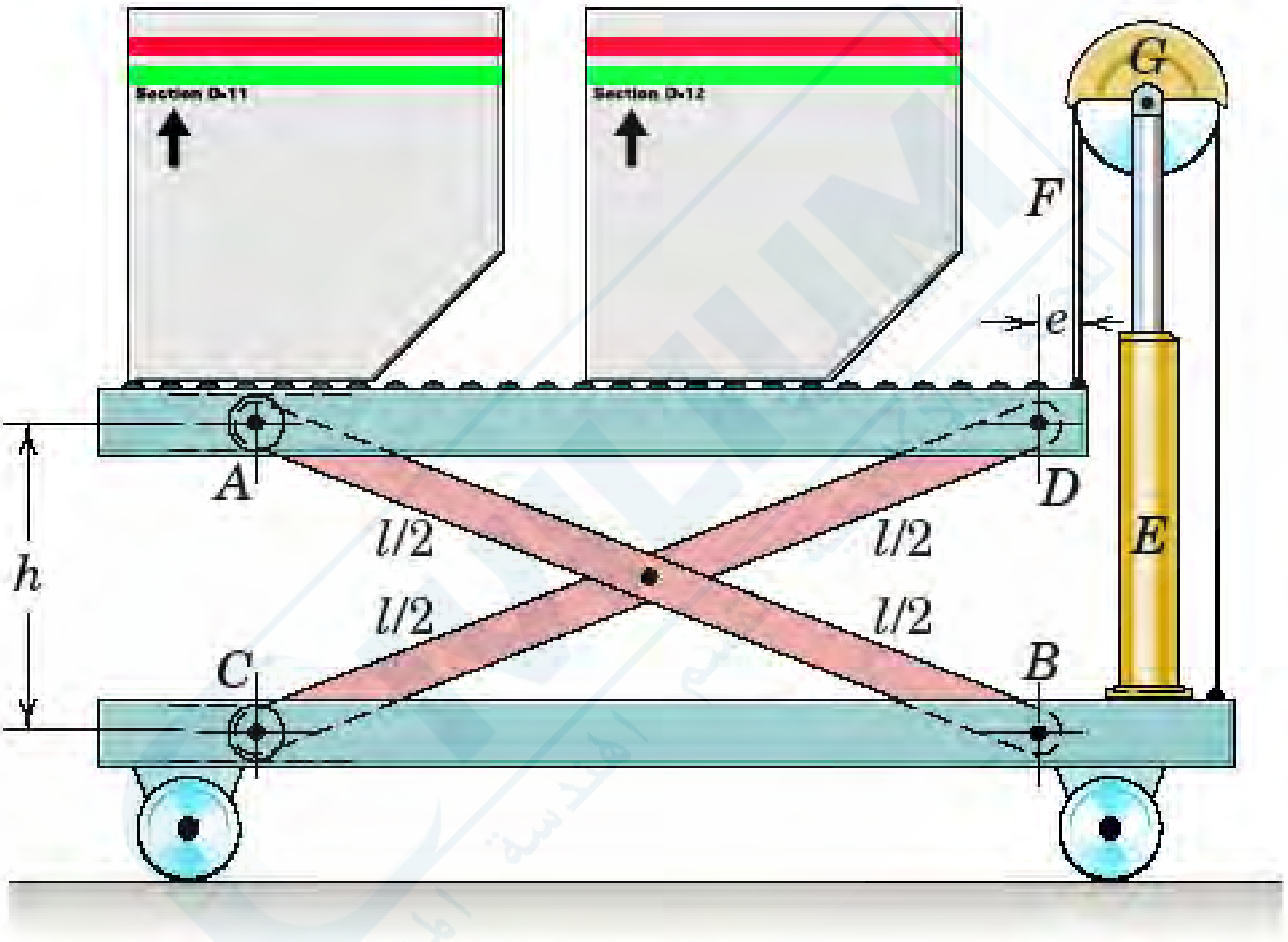
Problem 7/25

P_07_25



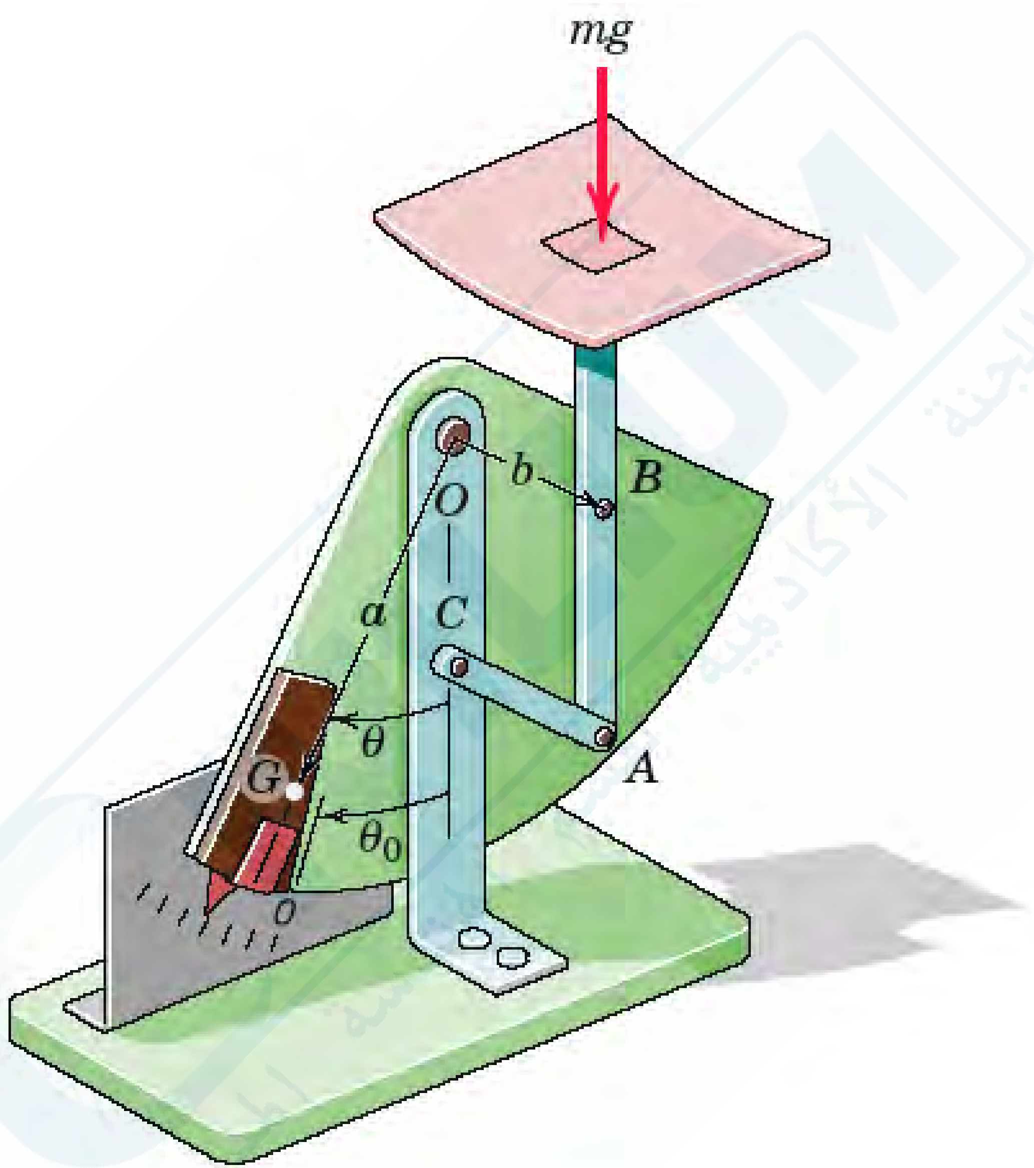
Problem 7/26

P_07_26



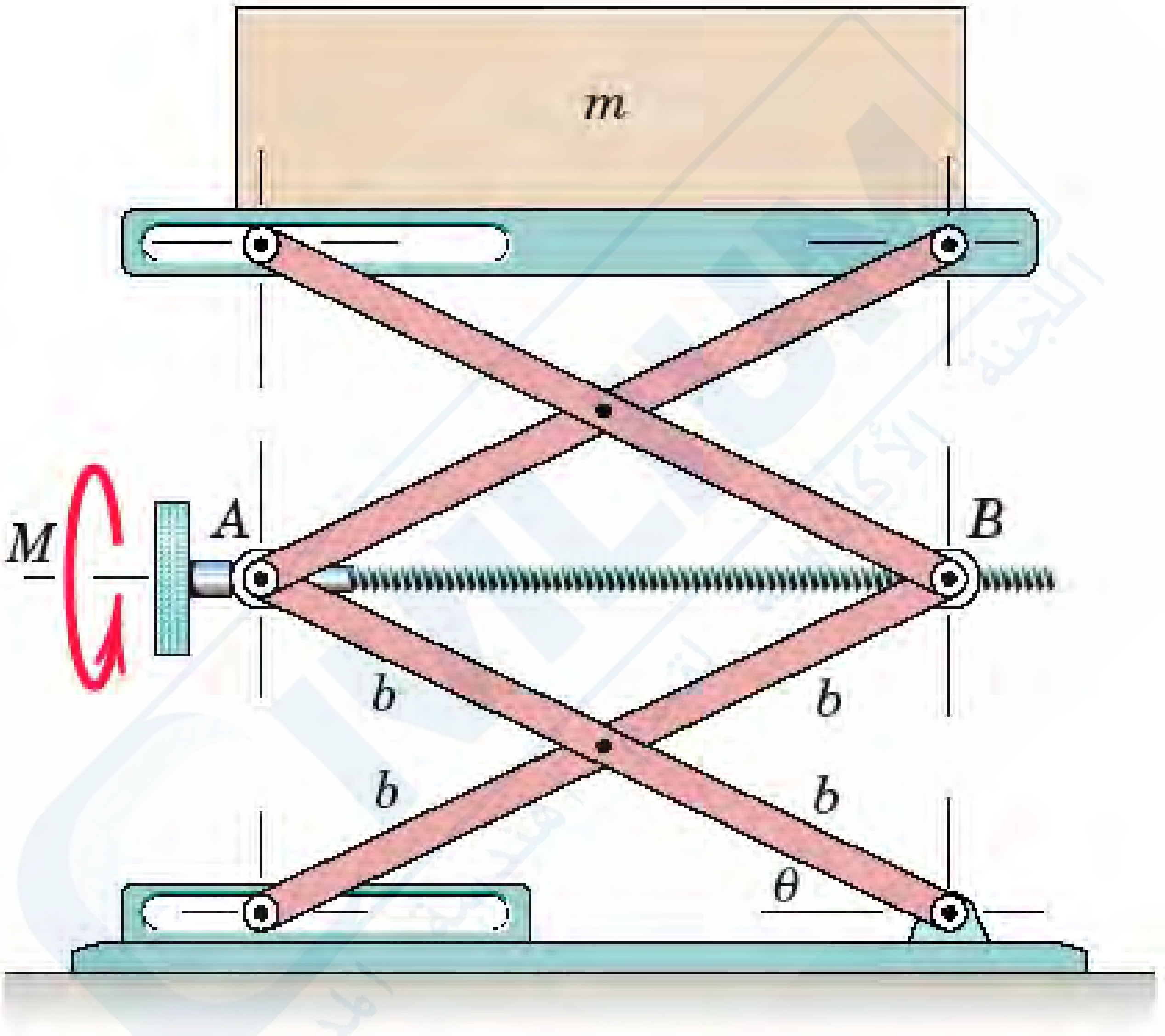
Problem 7/27

P_07_27



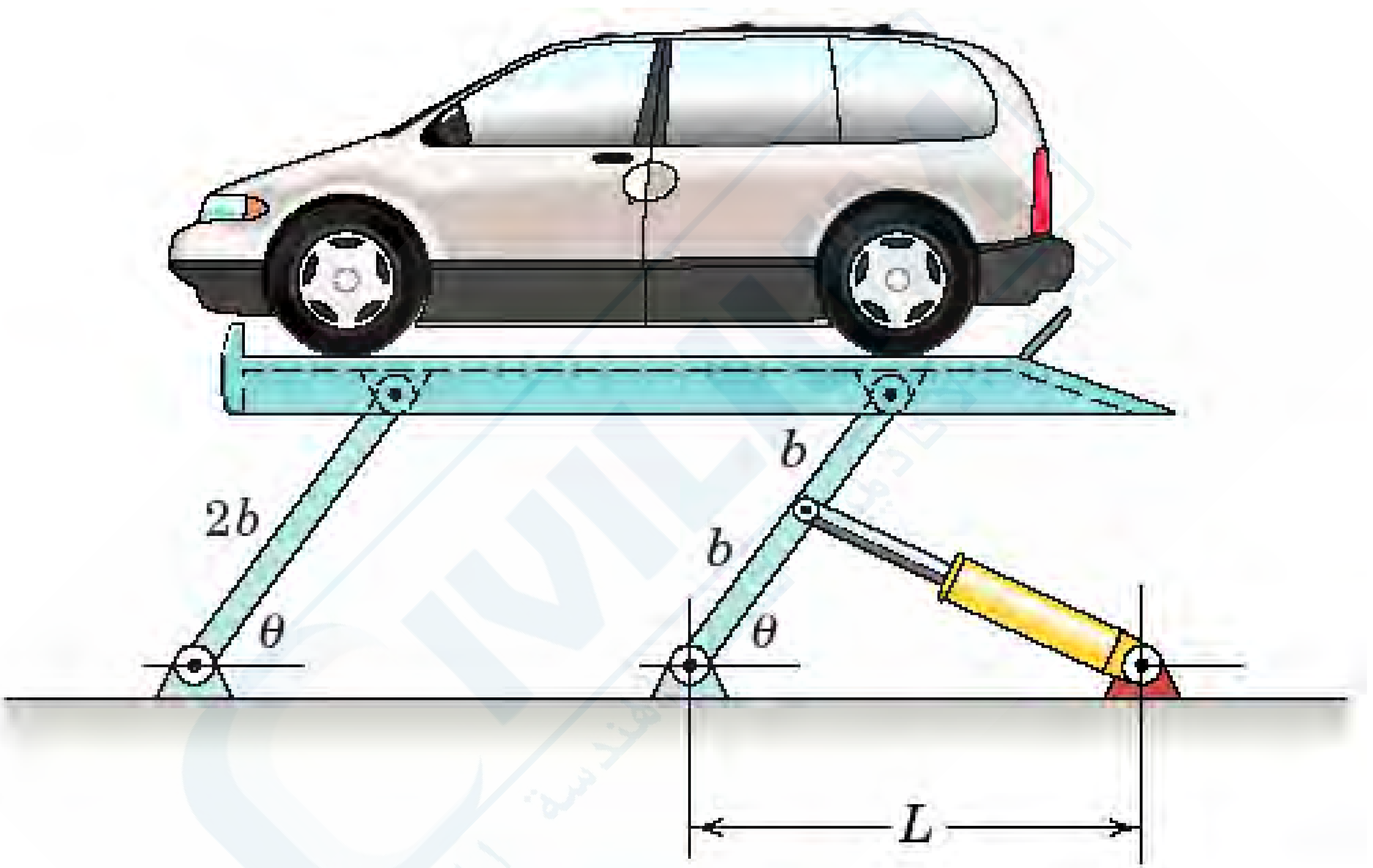
Problem 7/28

P_07_28



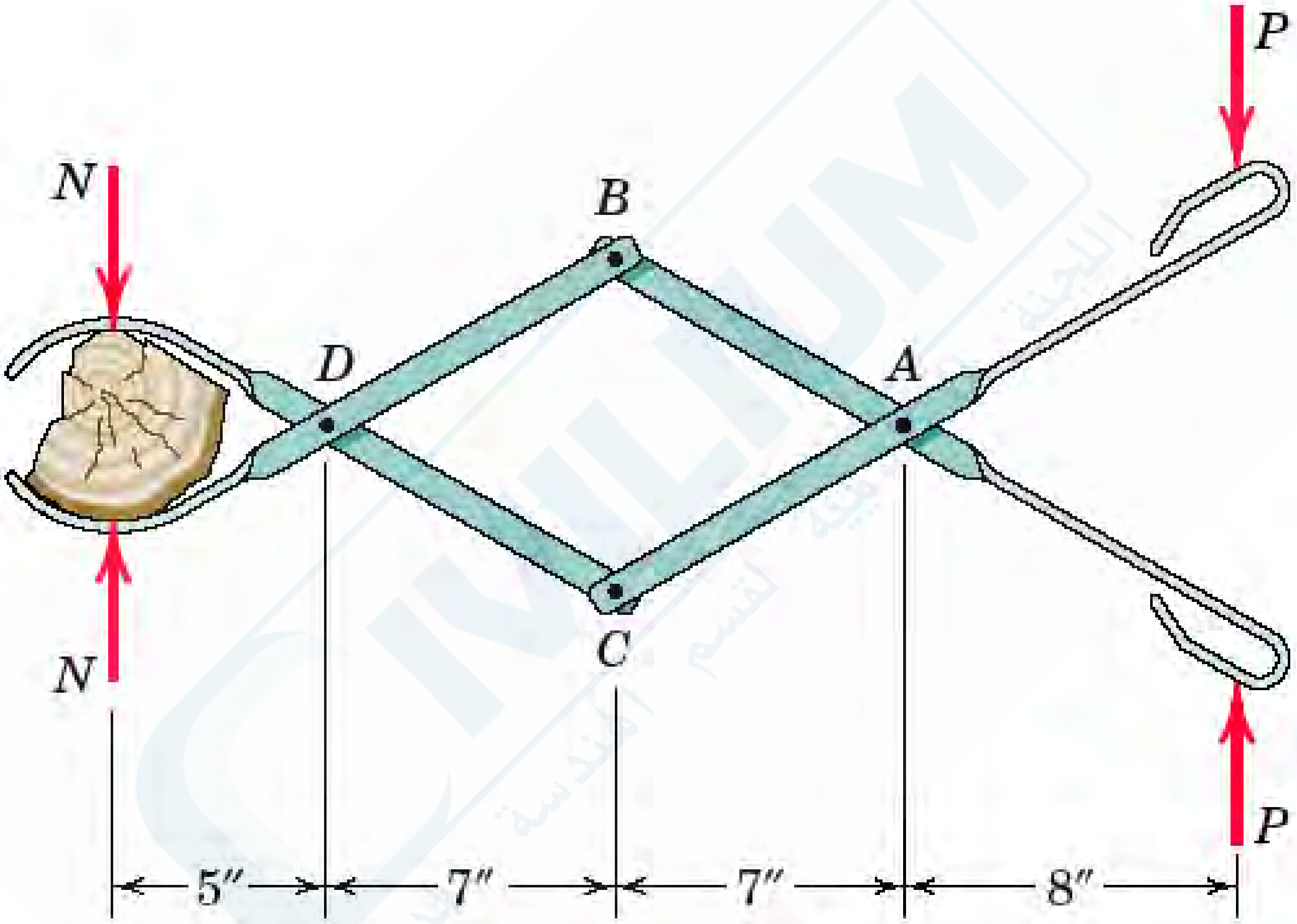
Problem 7/29

P_07_29



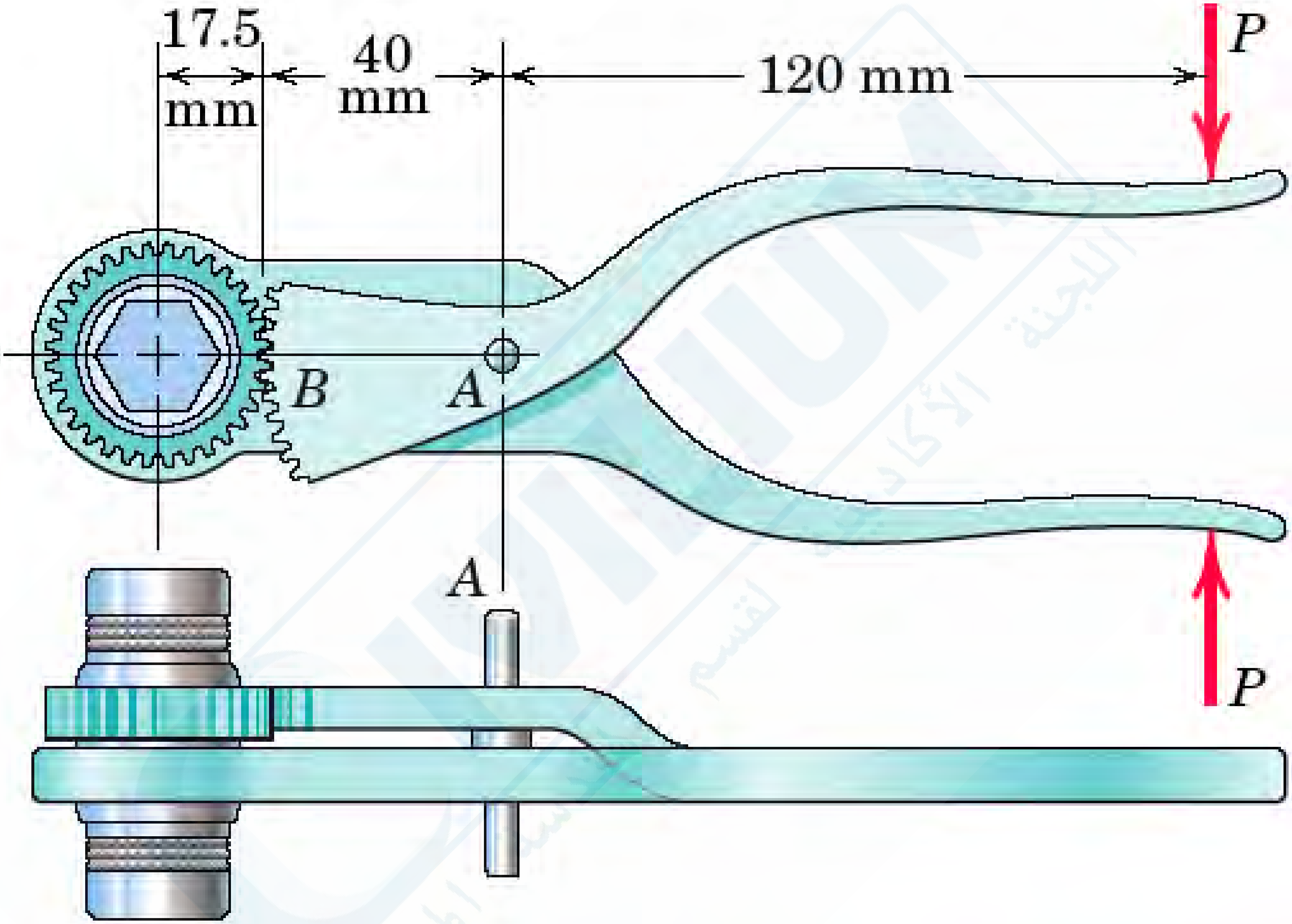
Problem 7/30

P_07_30



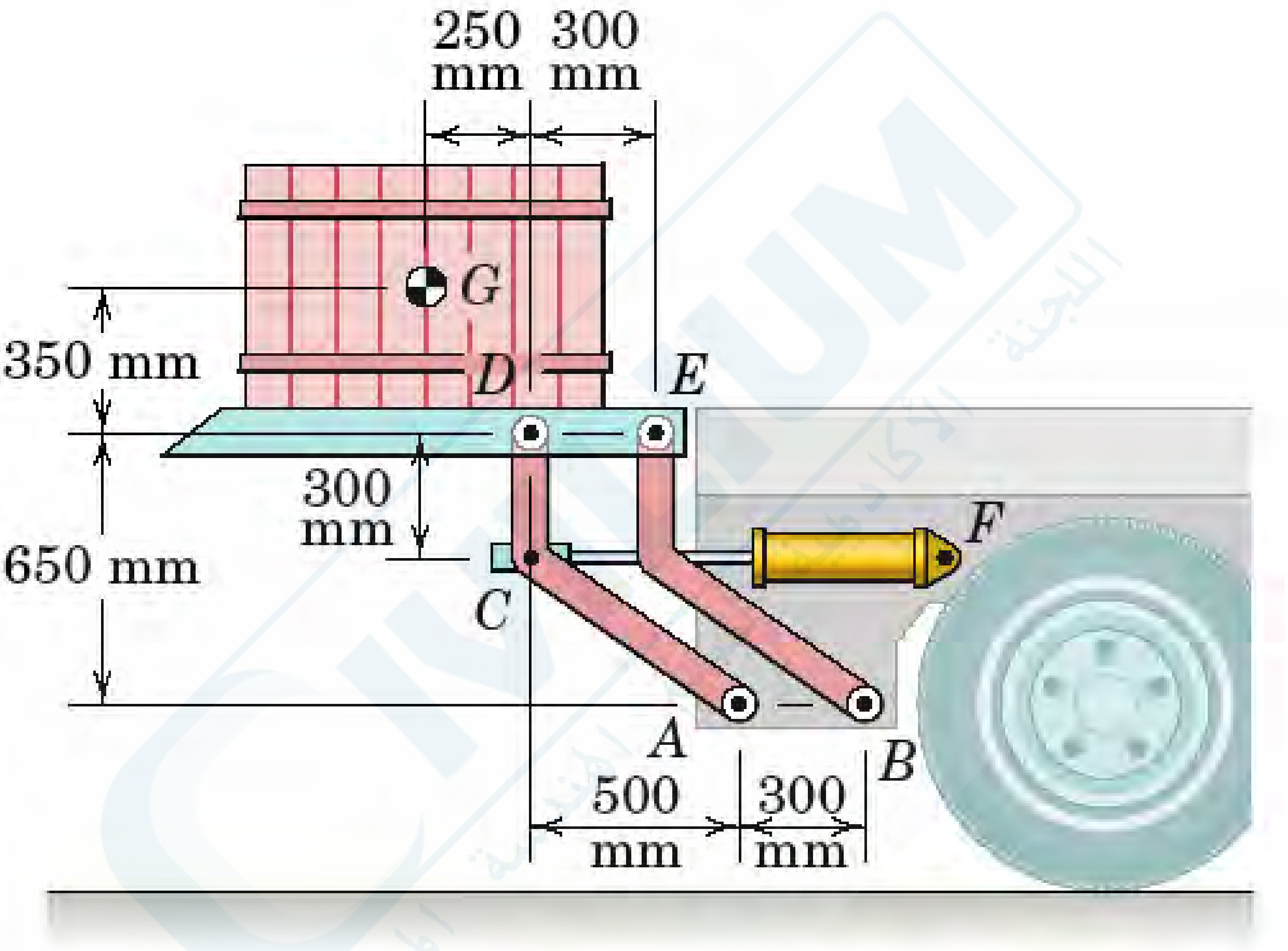
Problem 7/31

P_07_31



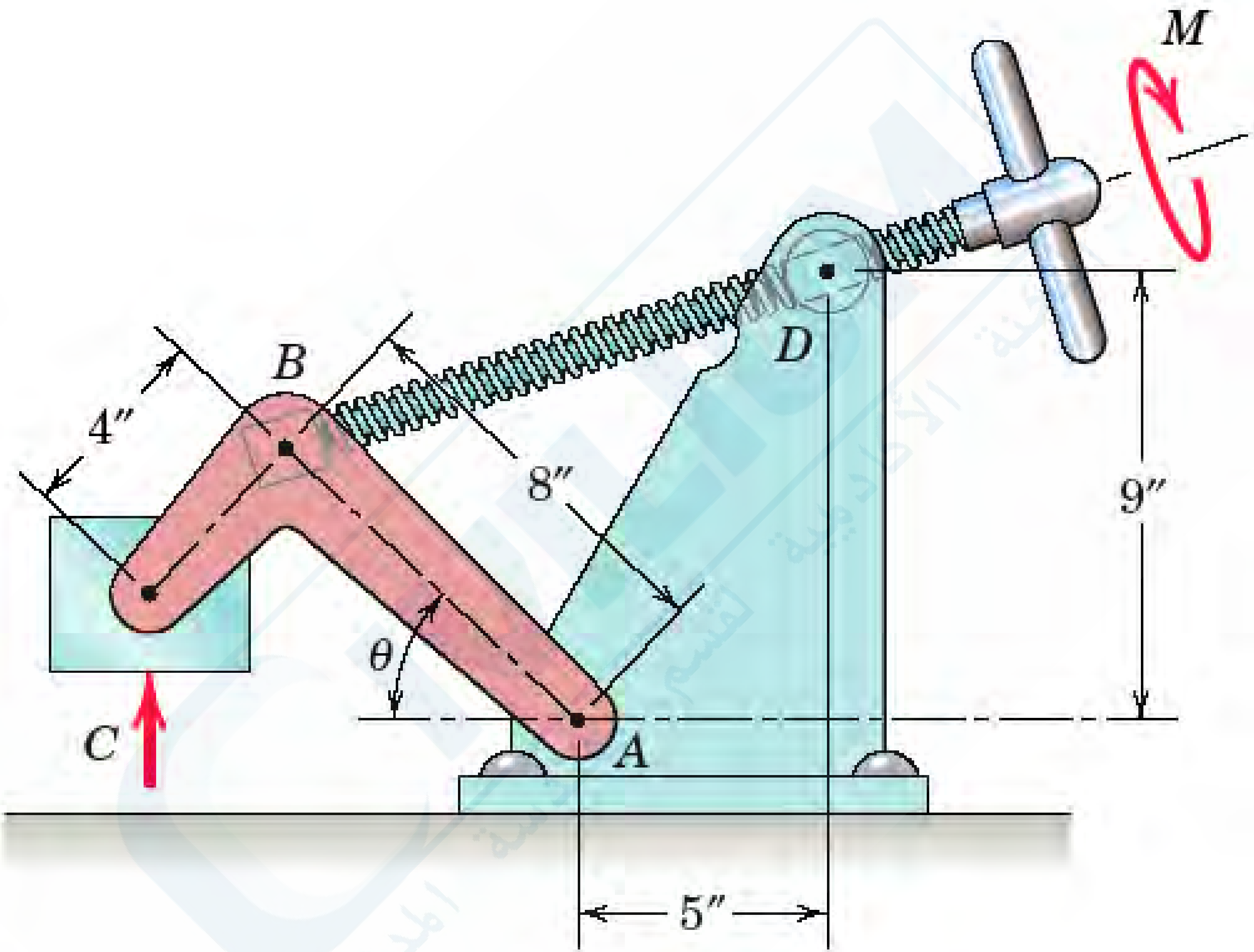
Problem 7/32

P_07_32

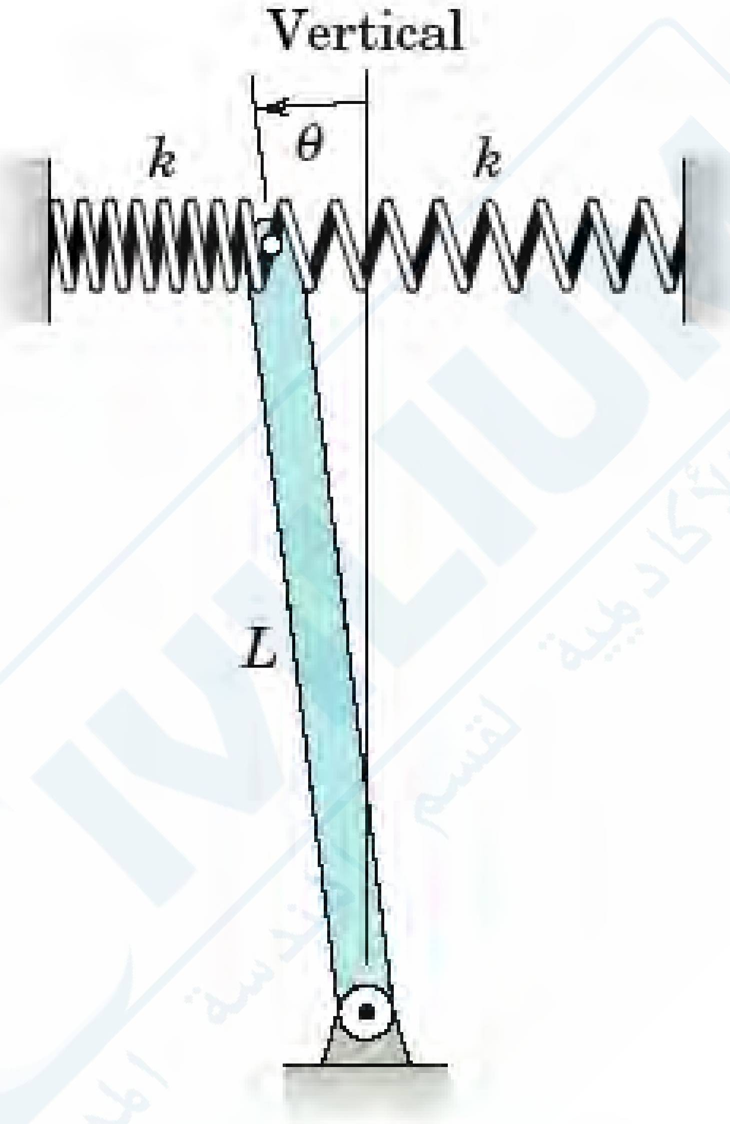


Problem 7/33

P_07_33

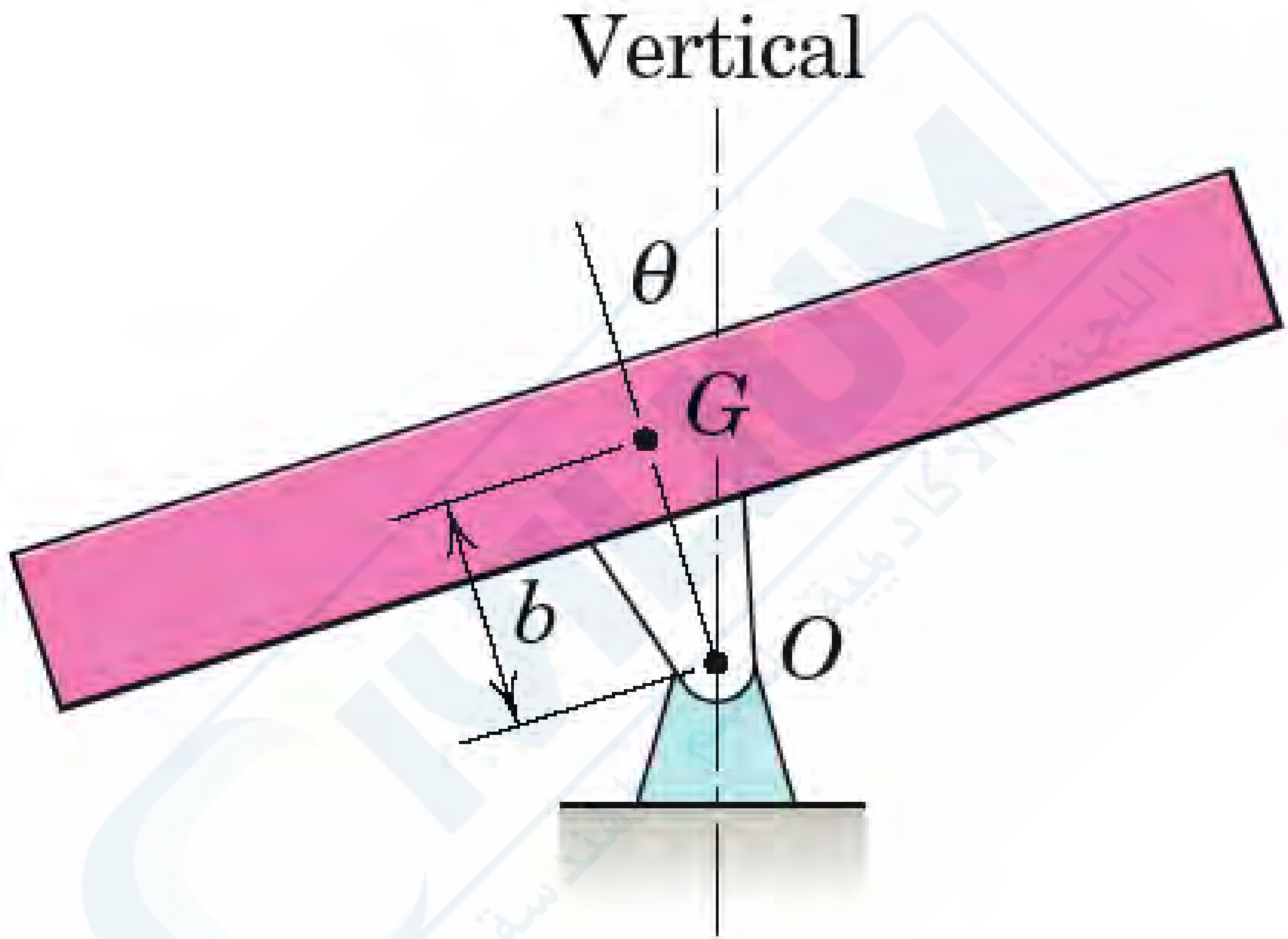


Problem 7/34

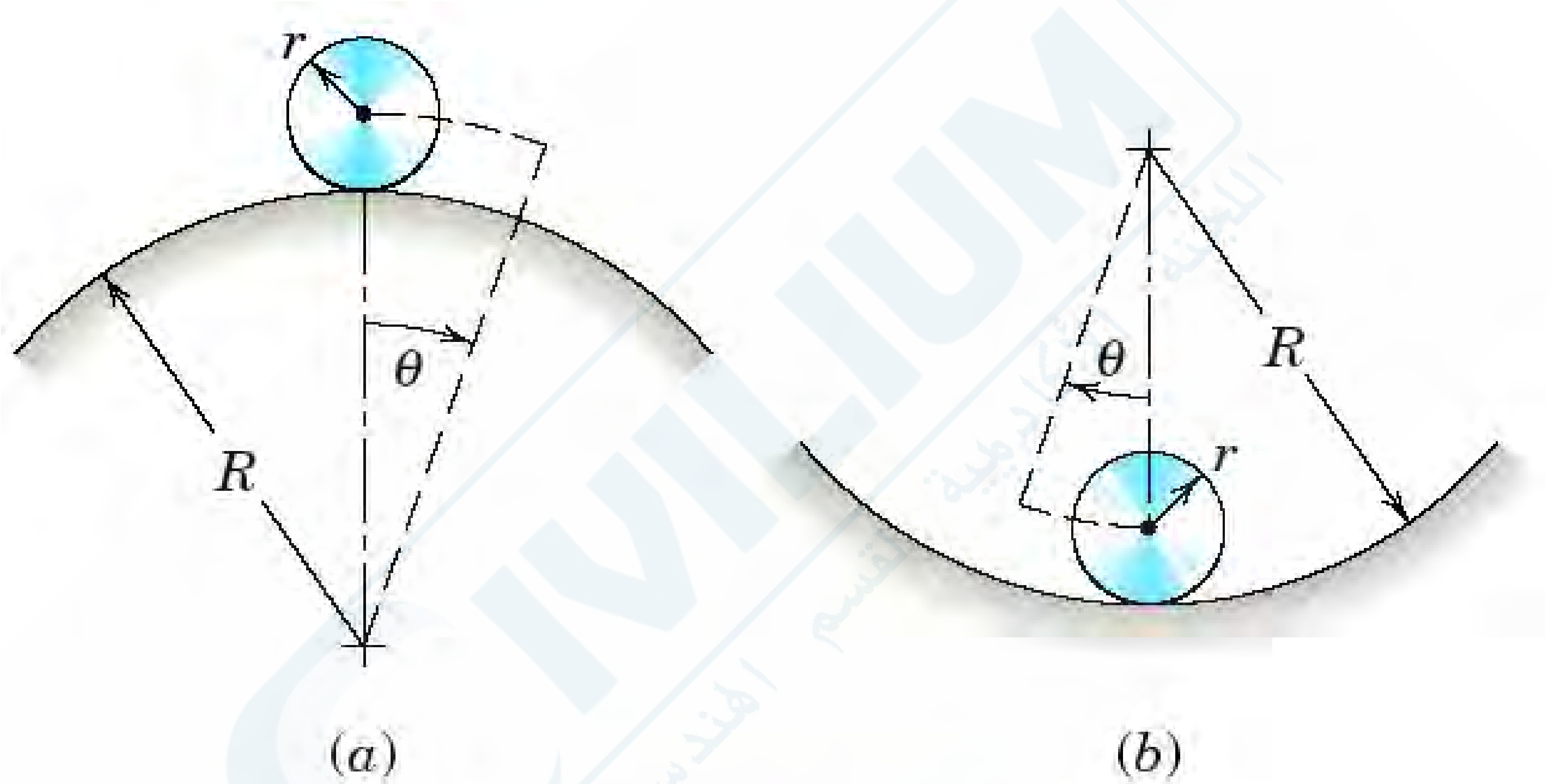


Problem 7/36

P_07_36

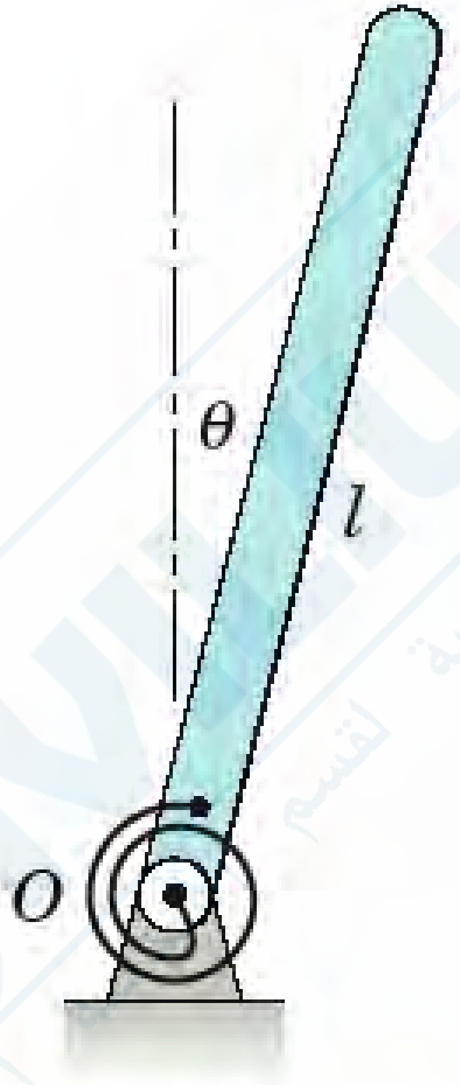


Problem 7/37



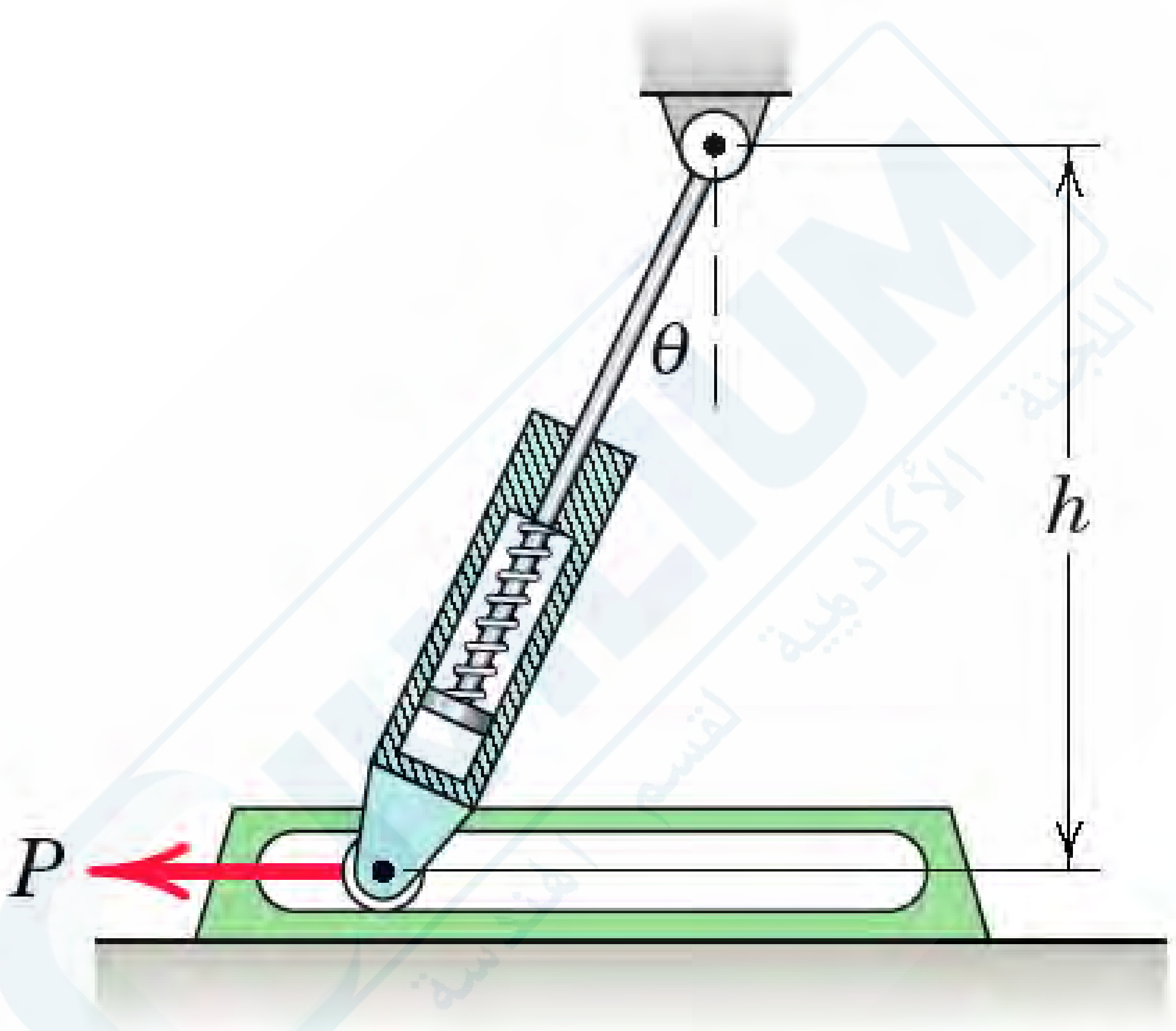
Problem 7/38

P_07_38



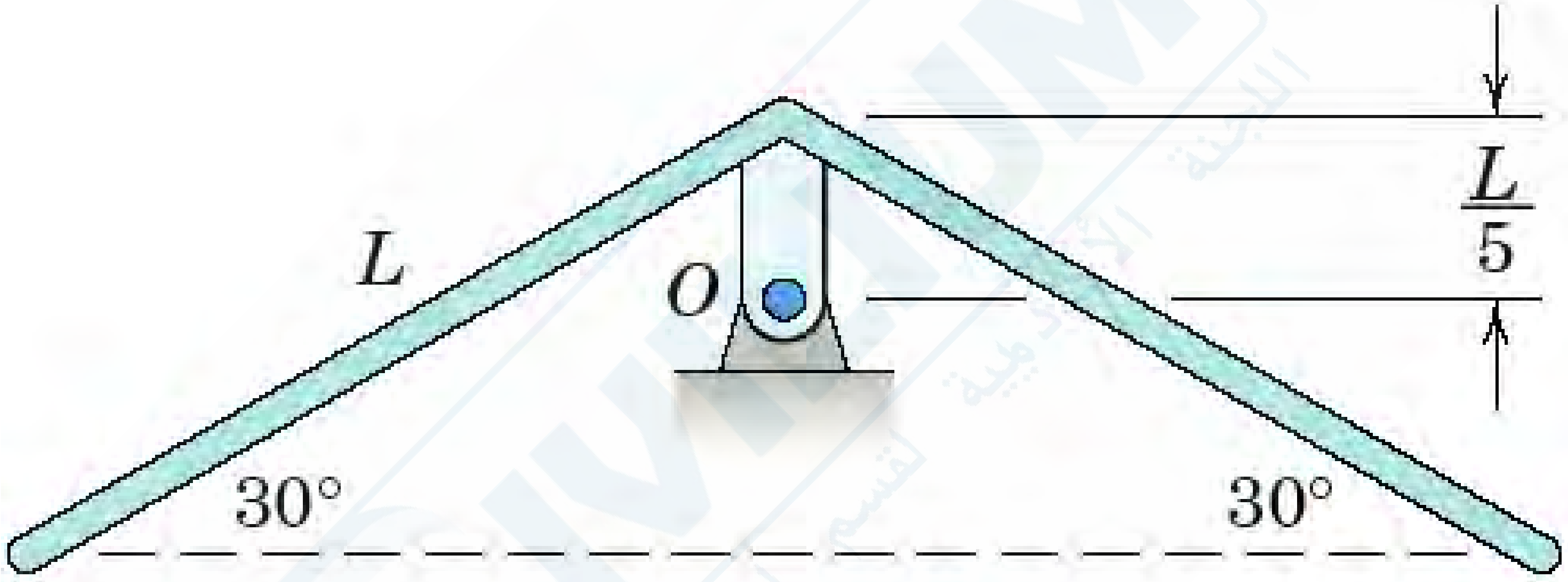
Problem 7/39

P_07_39



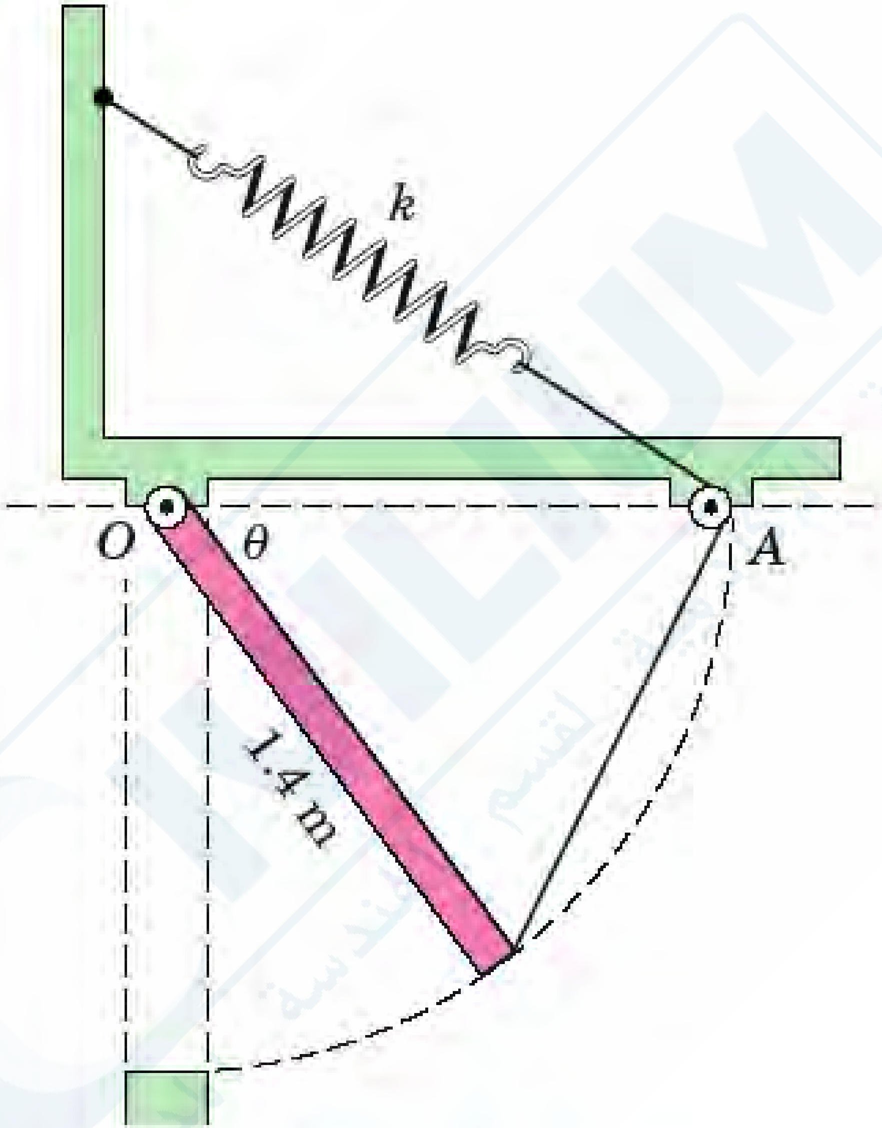
Problem 7/40

P_07_40



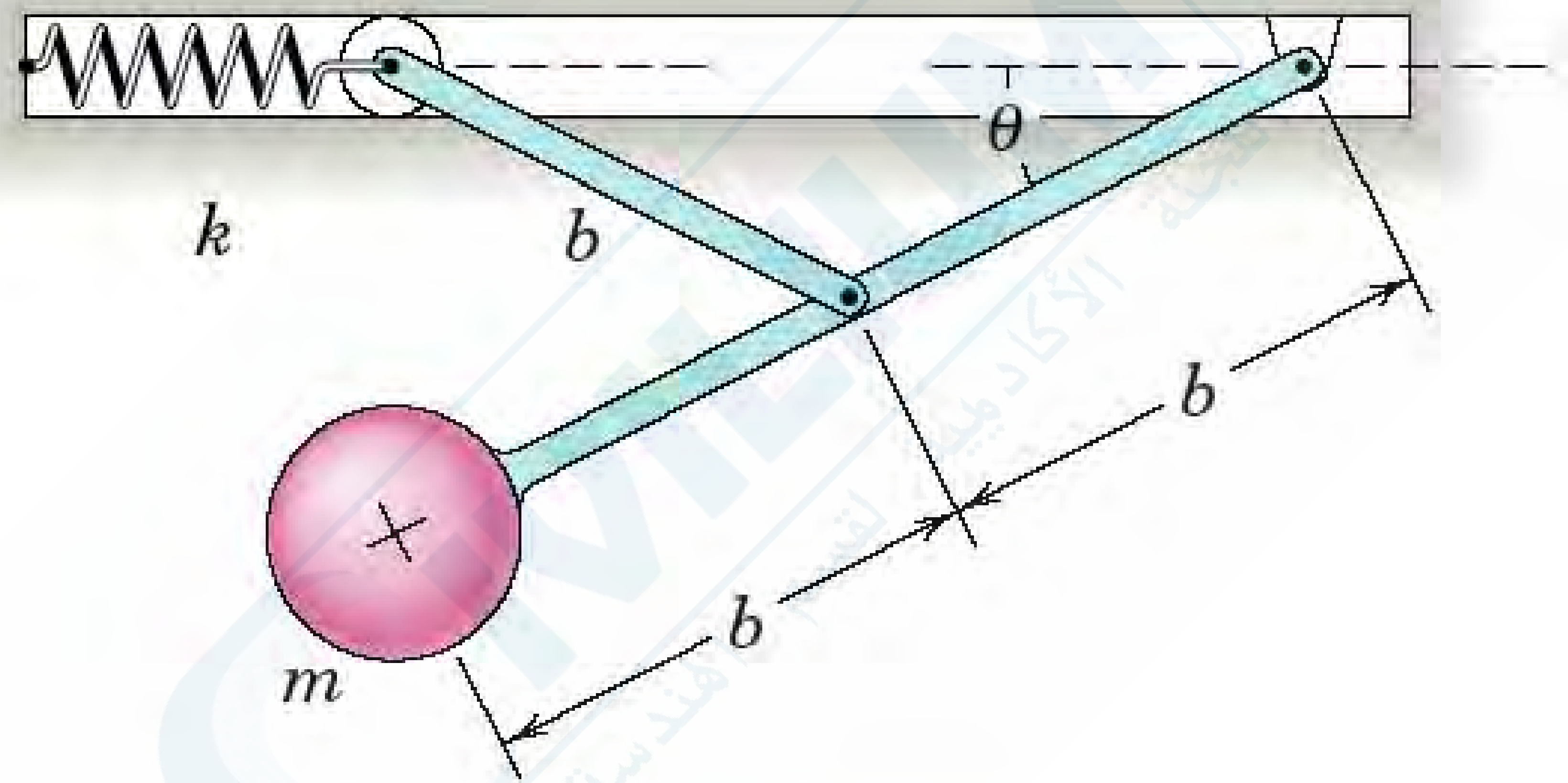
Problem 7/41

P_07_41



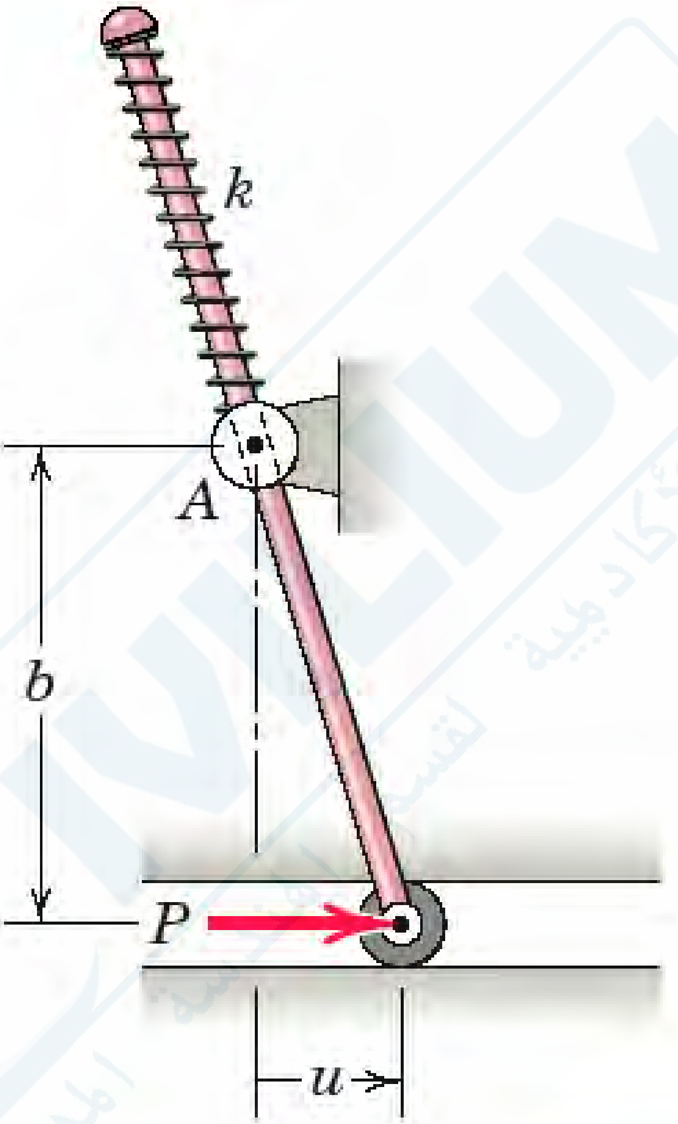
Problem 7/42

P_07_42



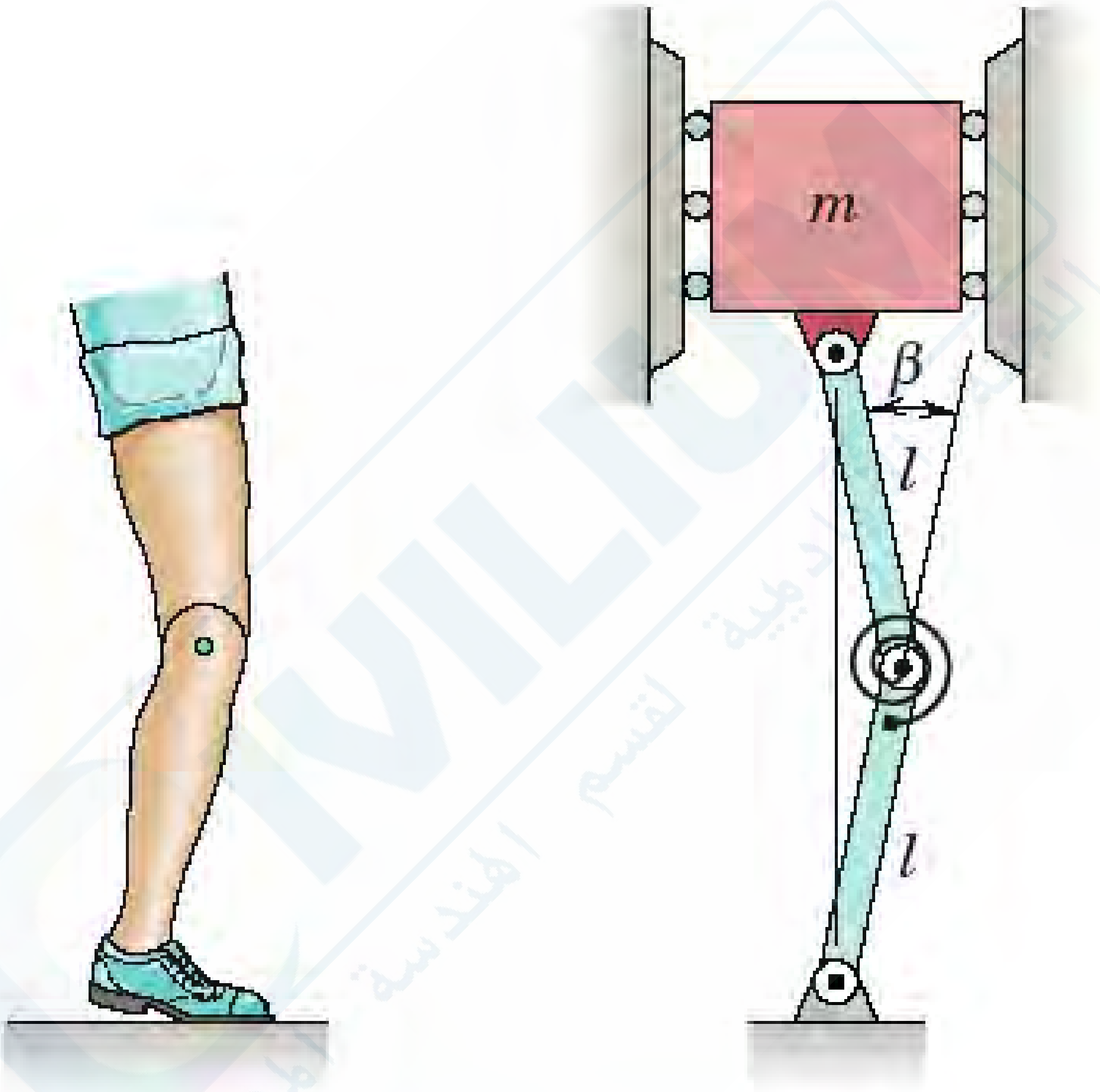
Problem 7/43

P_07_43



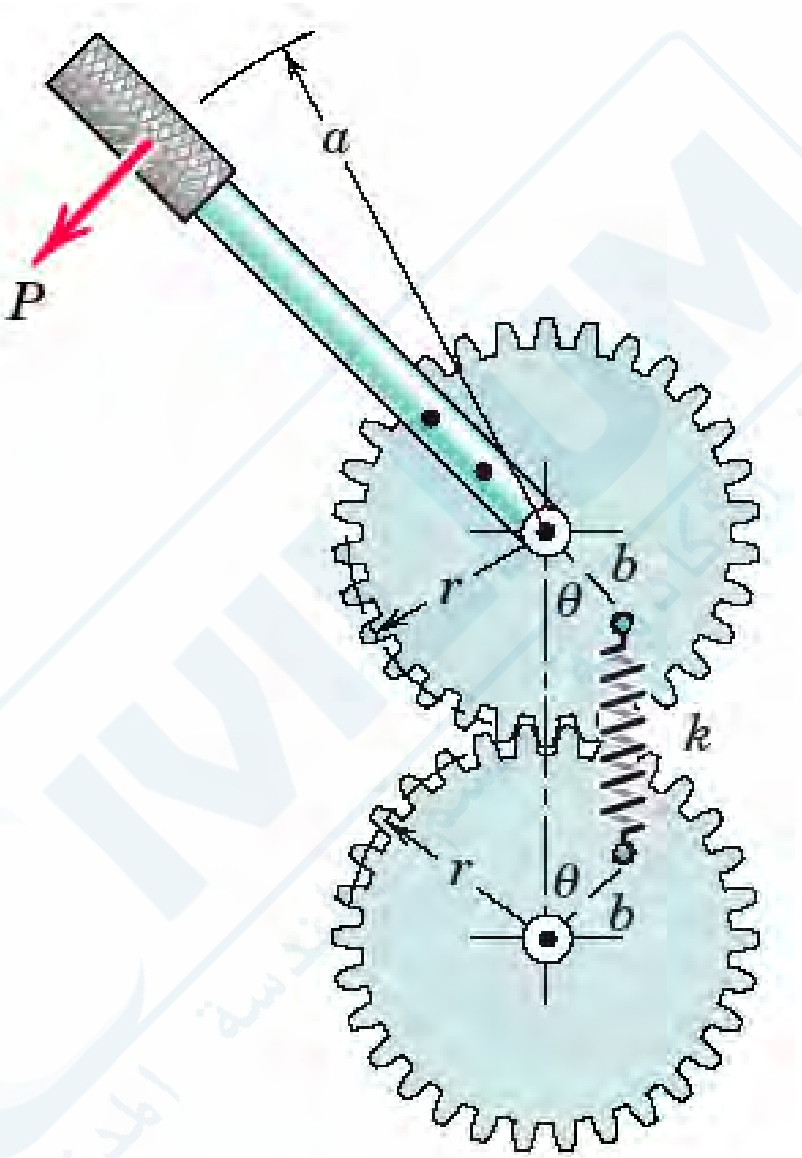
Problem 7/44

P_07_44



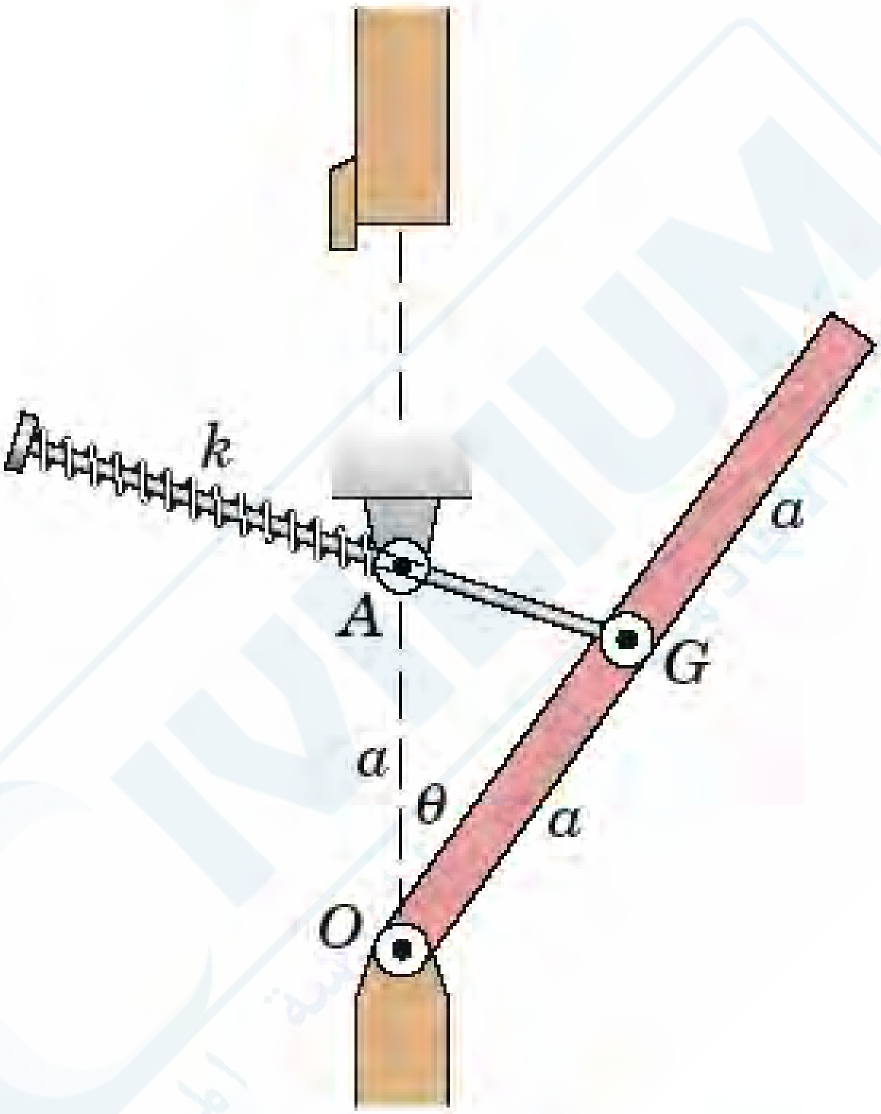
Problem 7/45

P_07_45



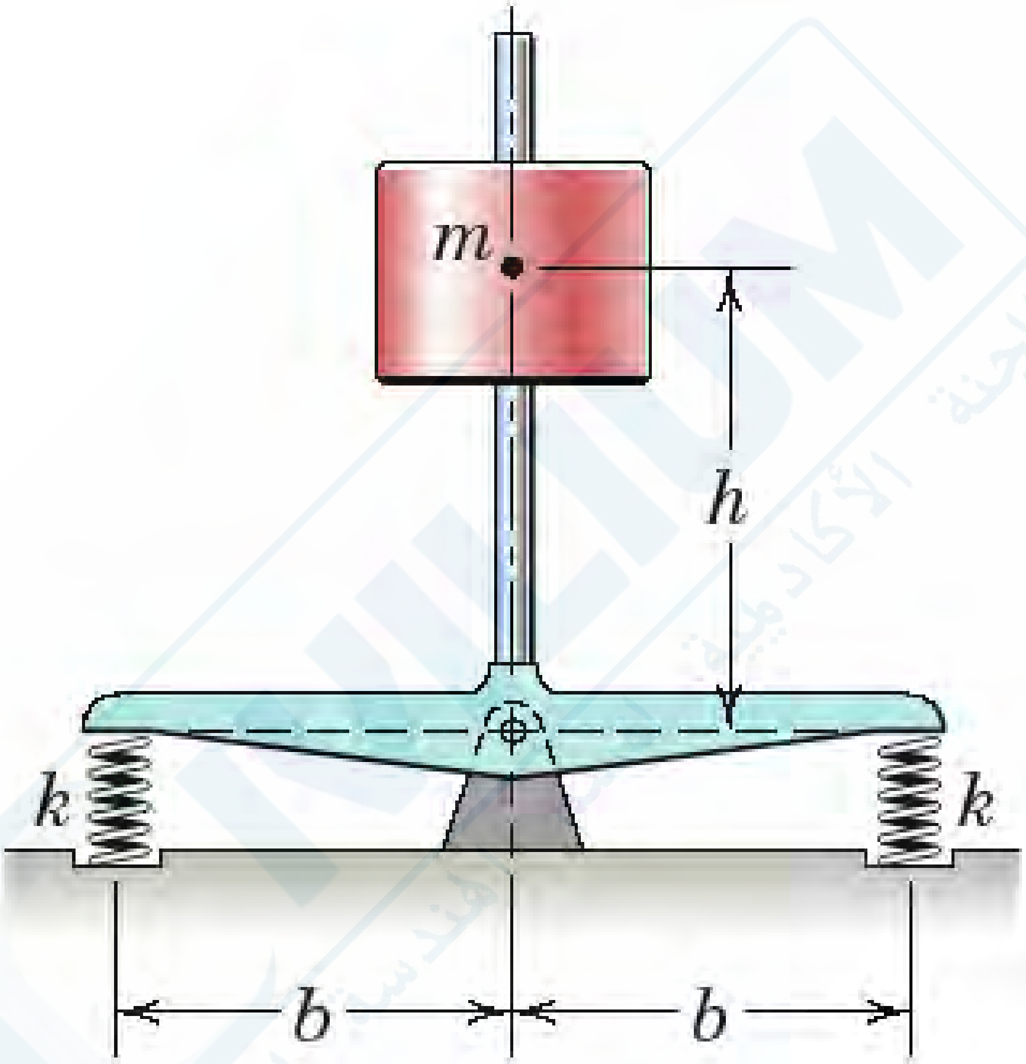
Problem 7/46

P_07_46



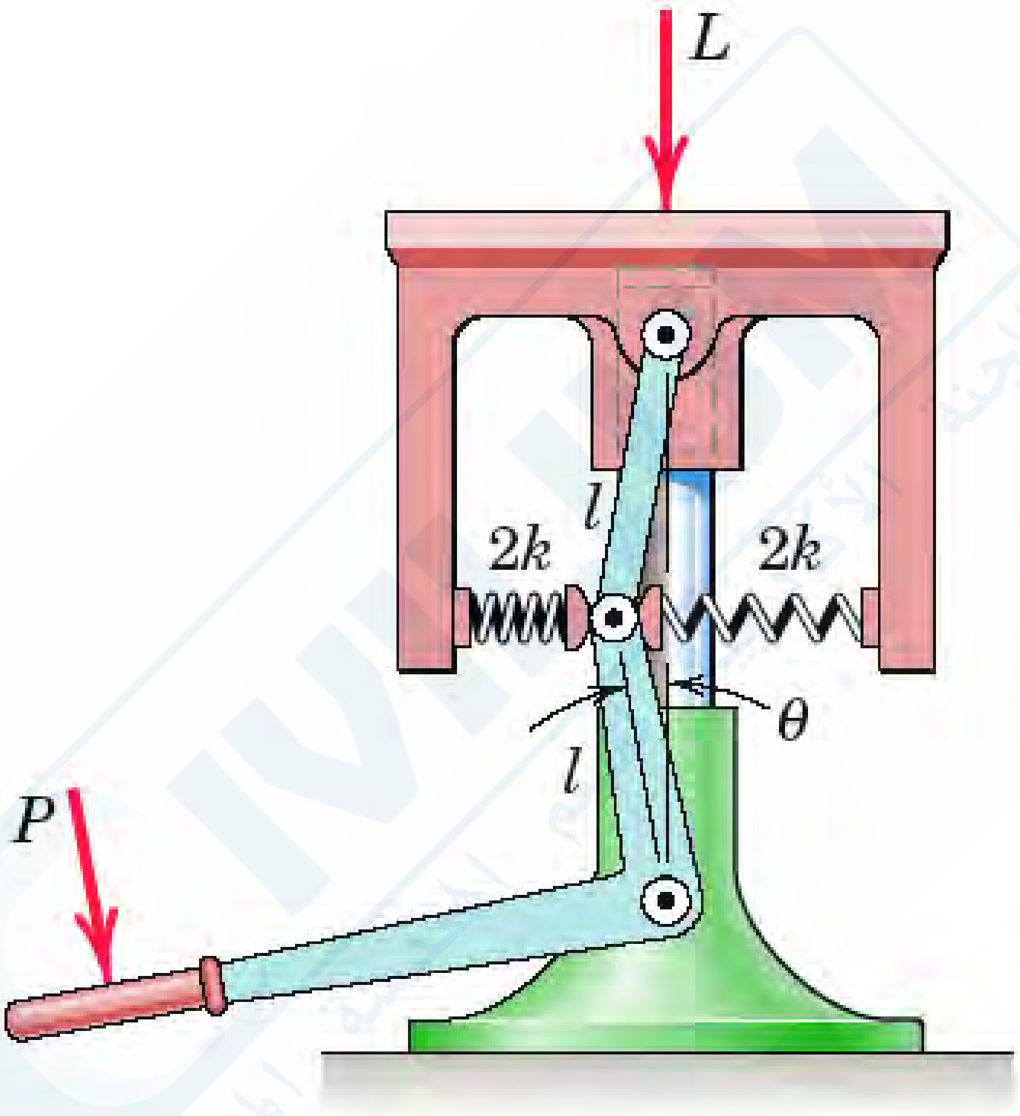
Problem 7/47

P_07_47



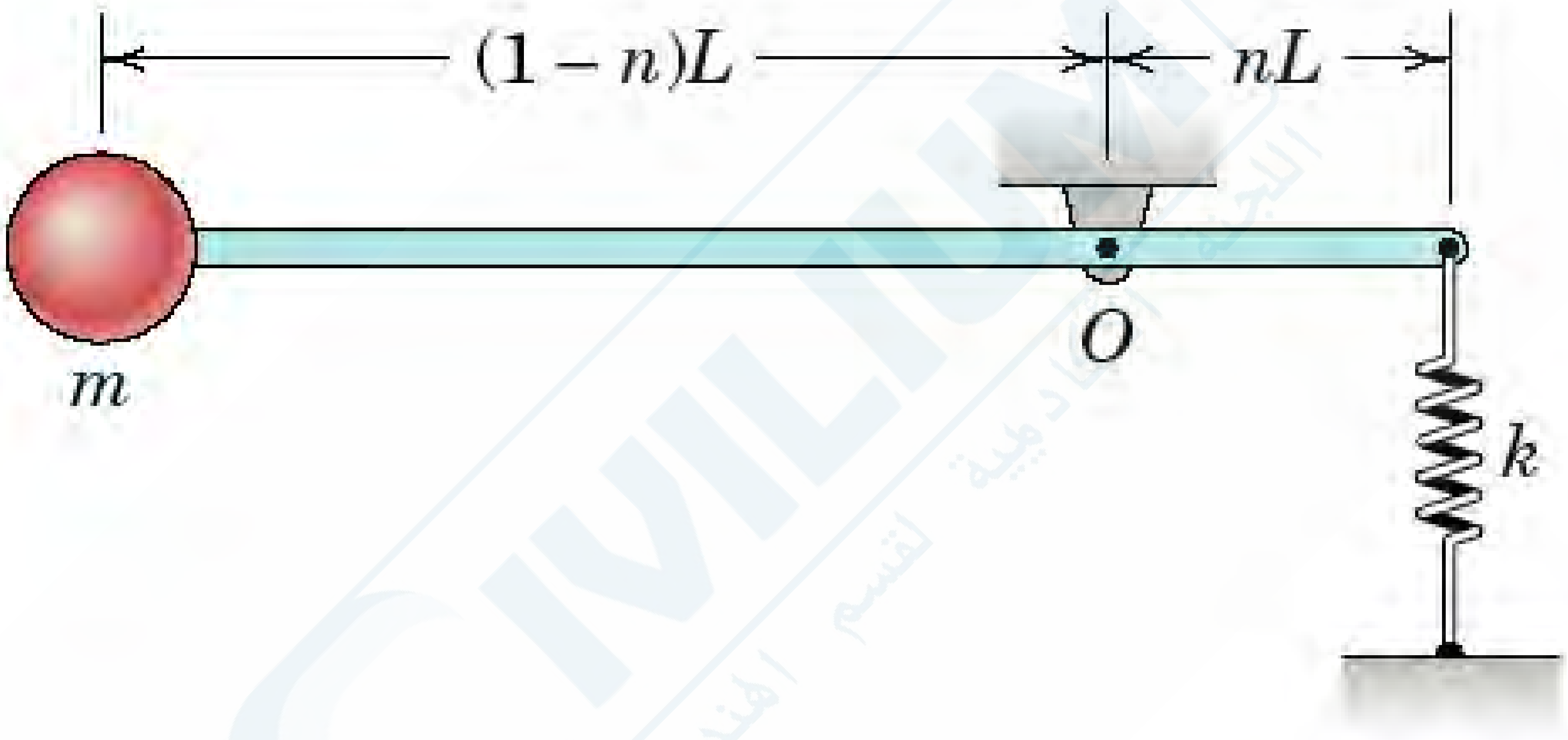
Problem 7/48

P_07_48



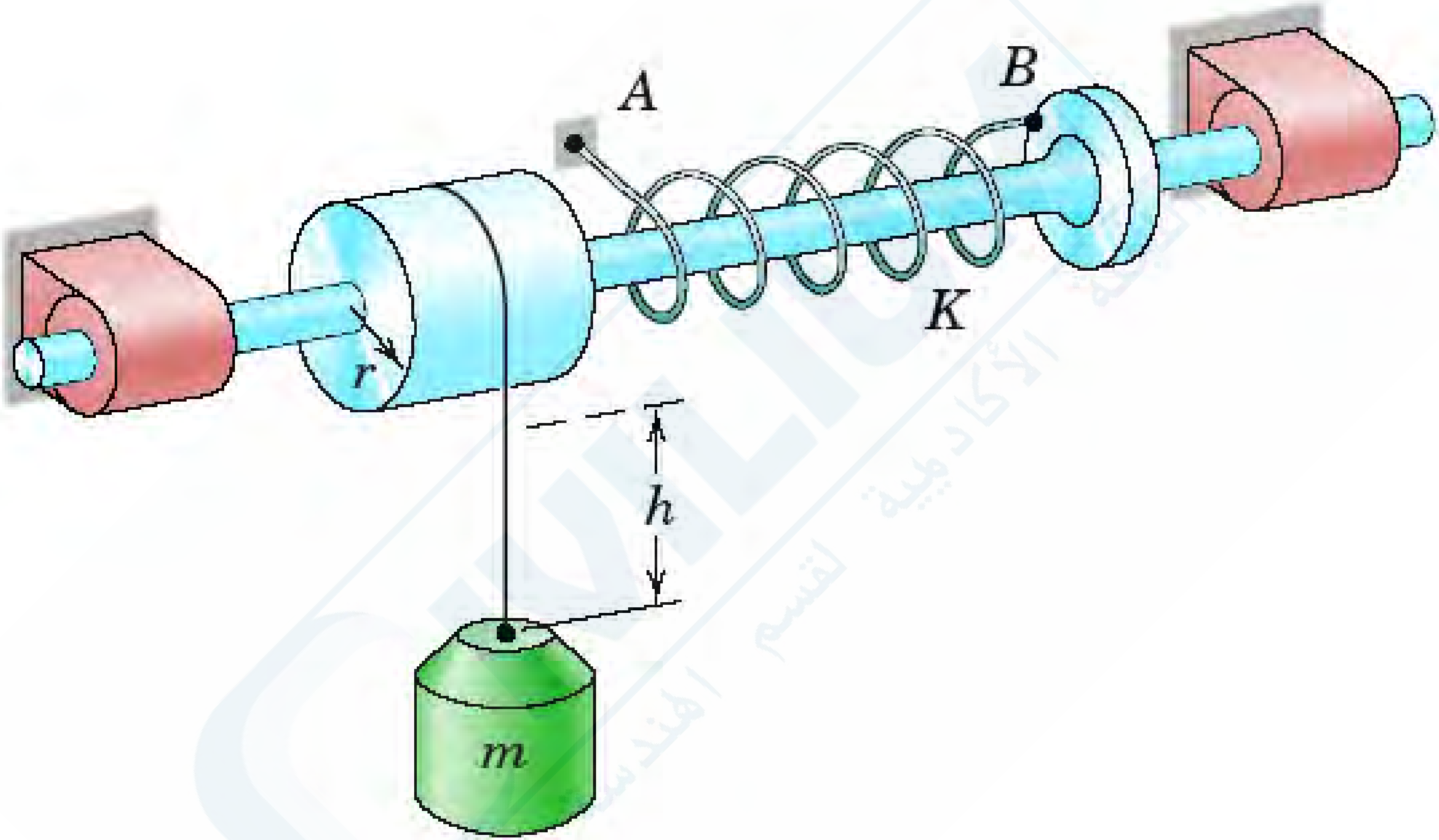
Problem 7/49

P_07_49



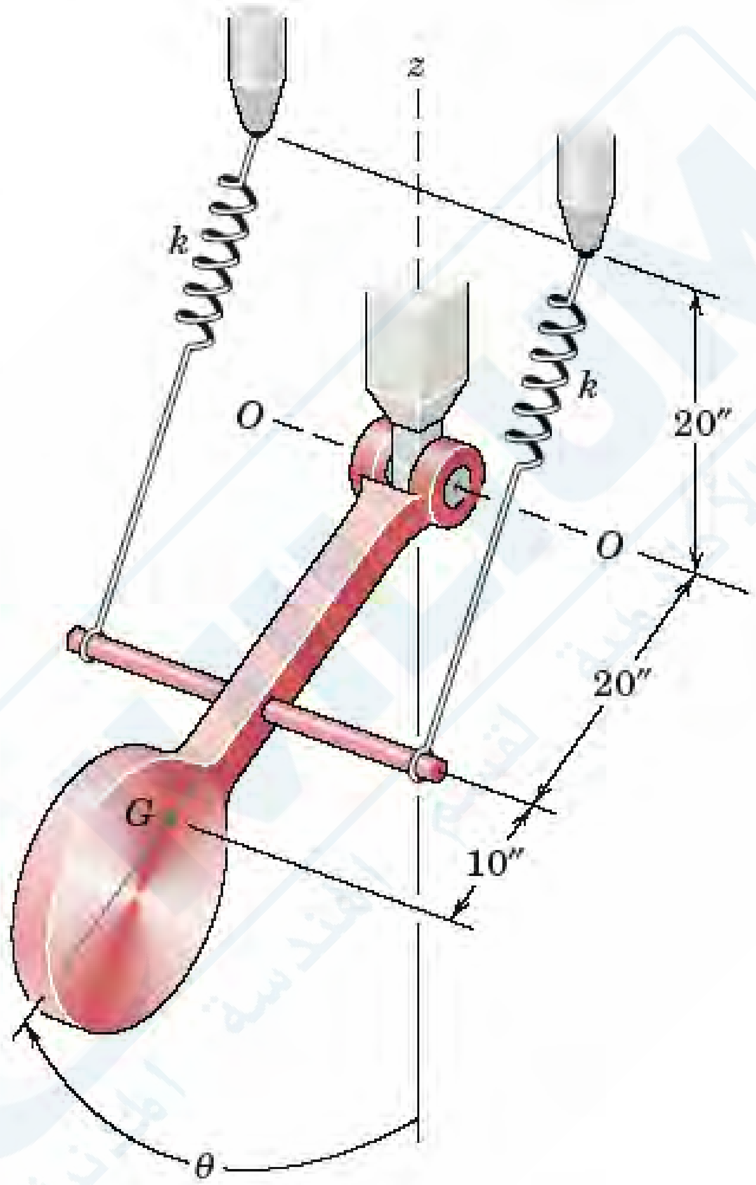
Problem 7/50

P_07_50



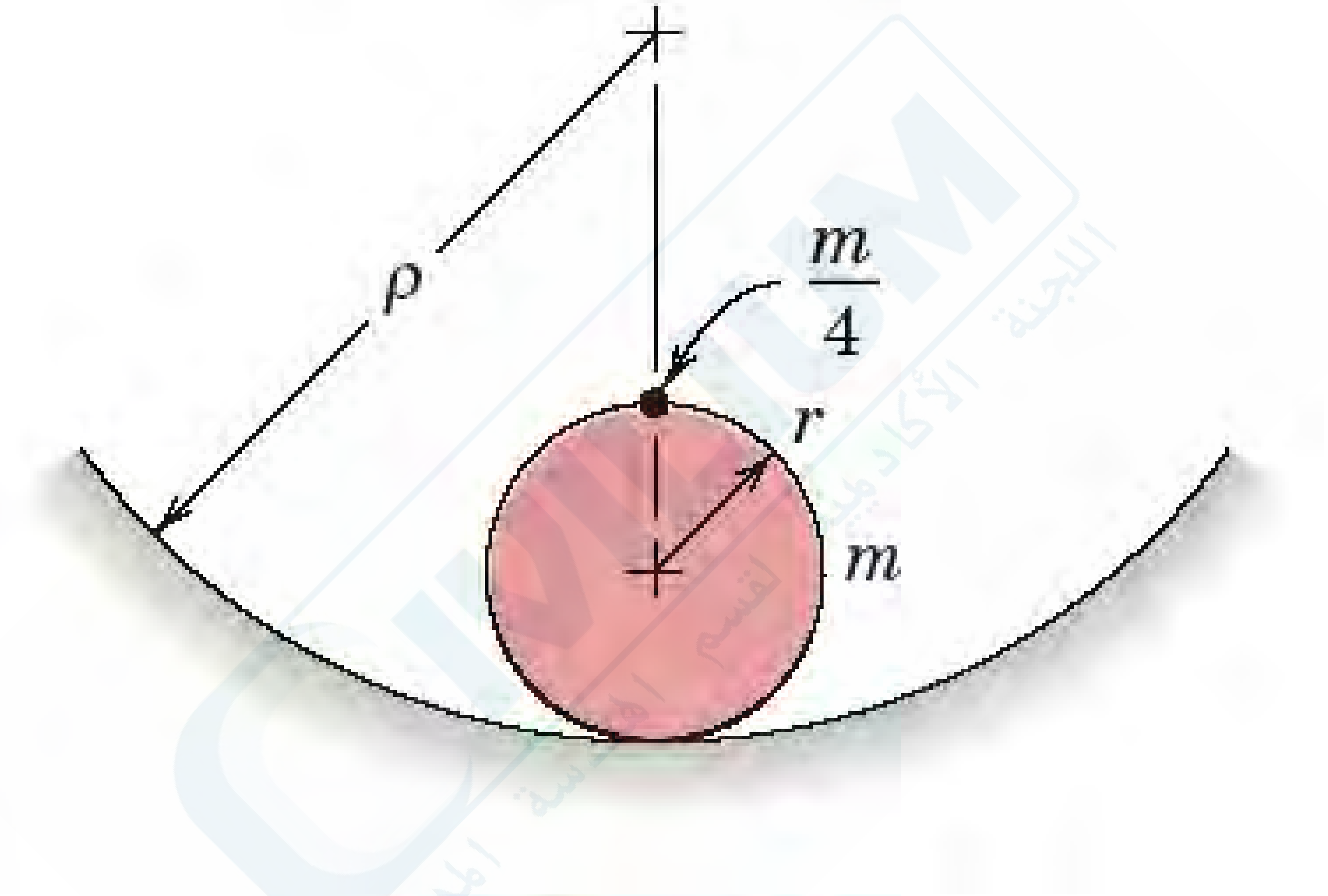
Problem 7/51

P_07_51



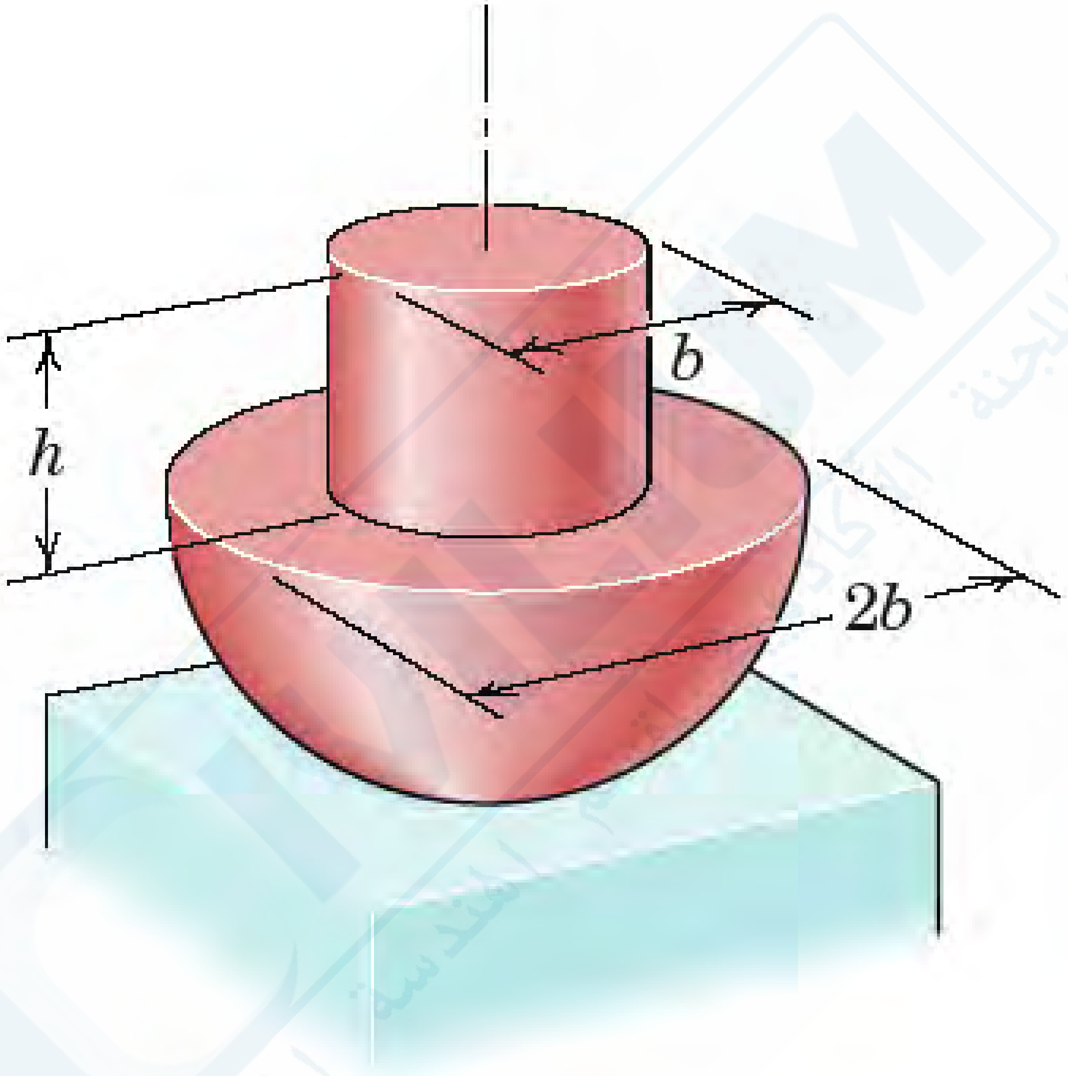
Problem 7/52

P_07_52



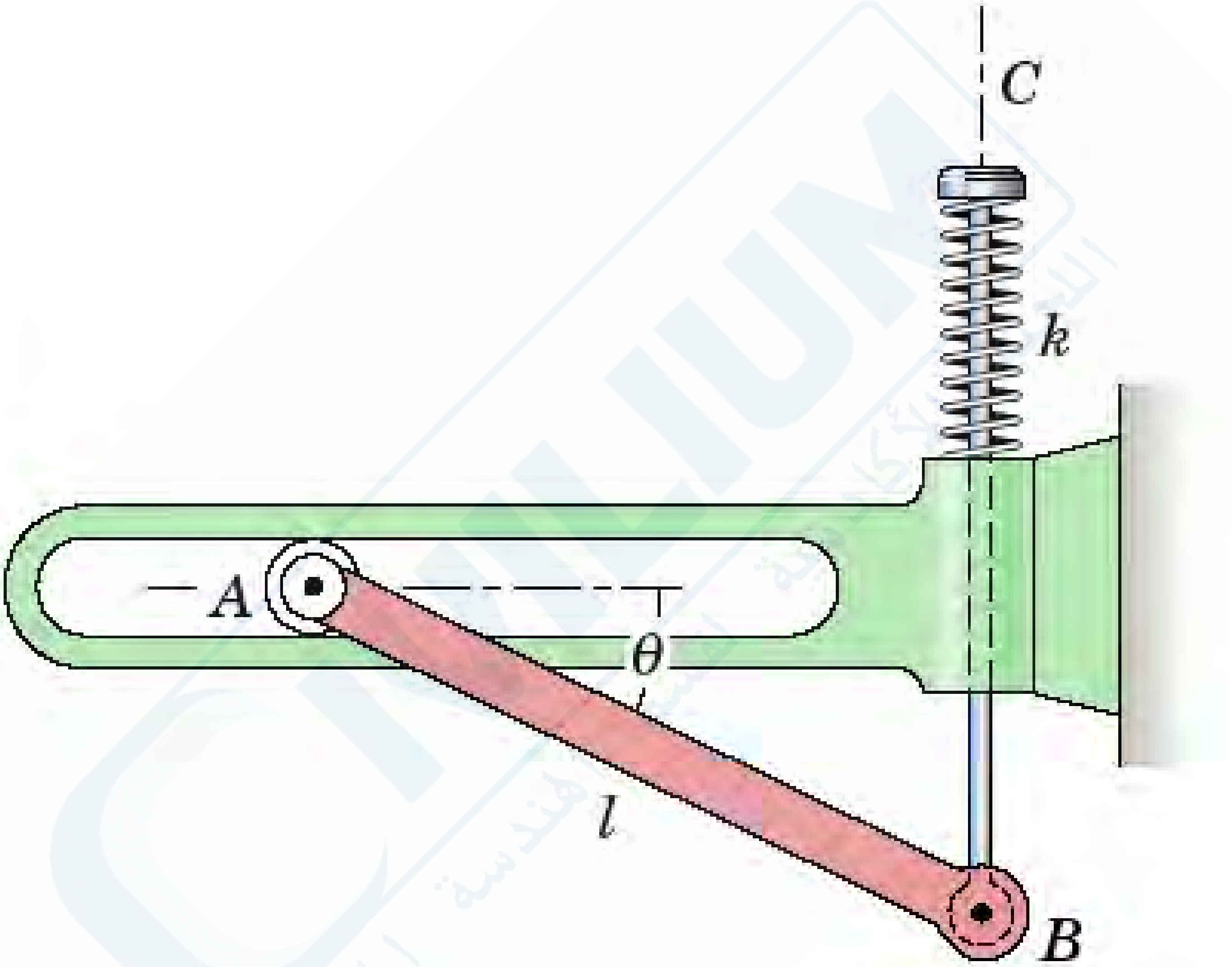
Problem 7/53

P_07_53



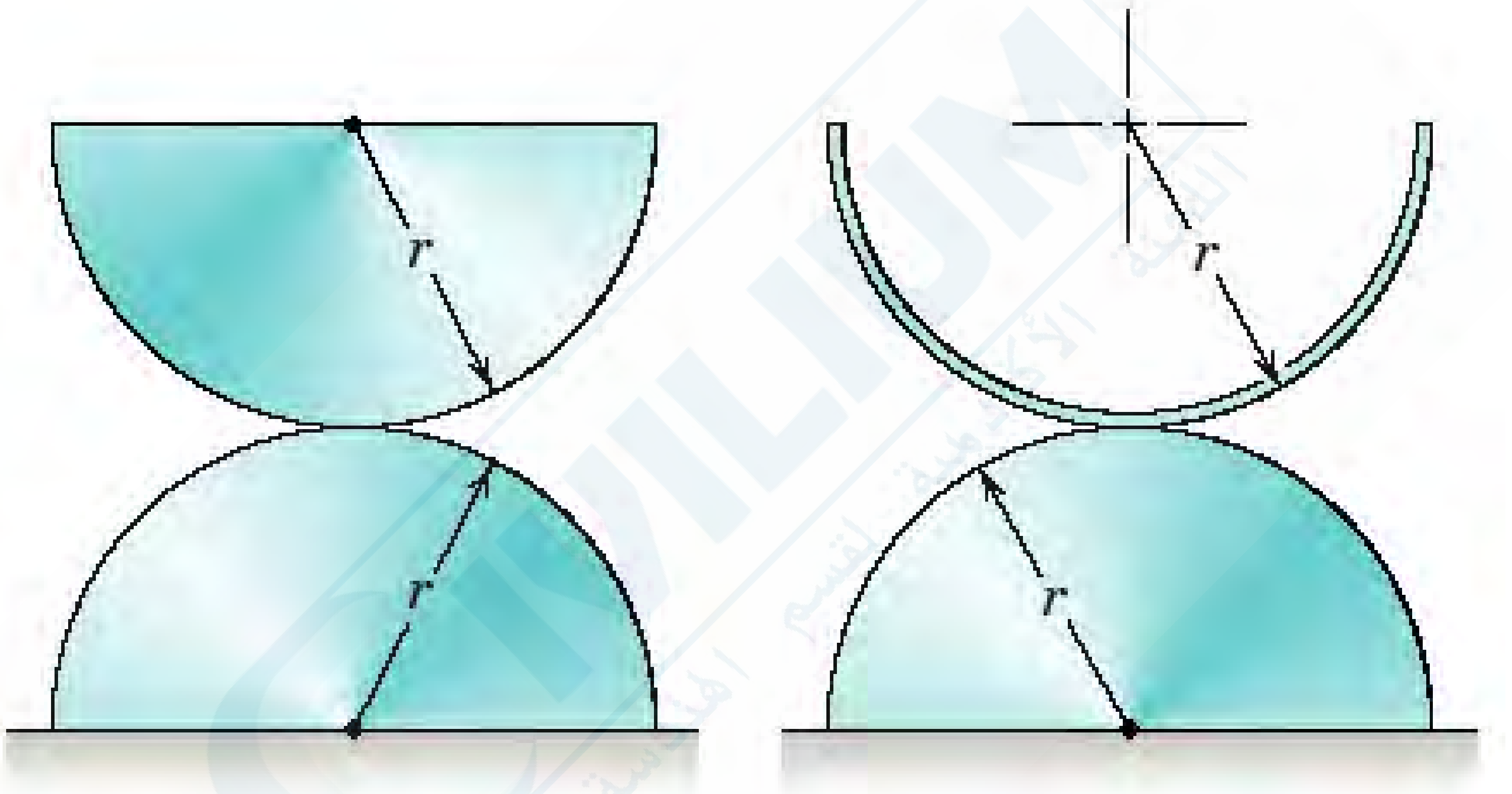
Problem 7/54

P_07_54



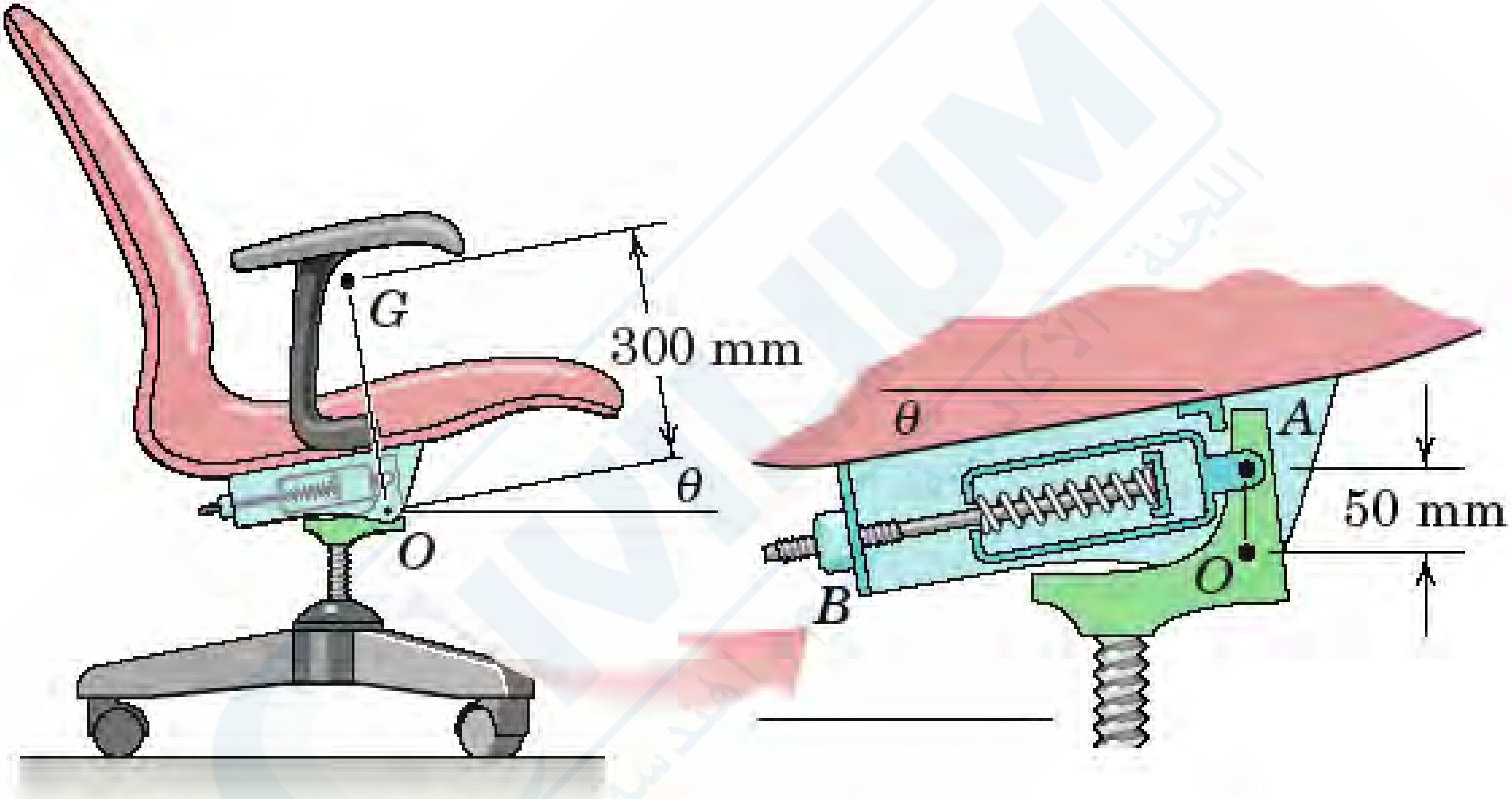
Problem 7/55

P_07_55



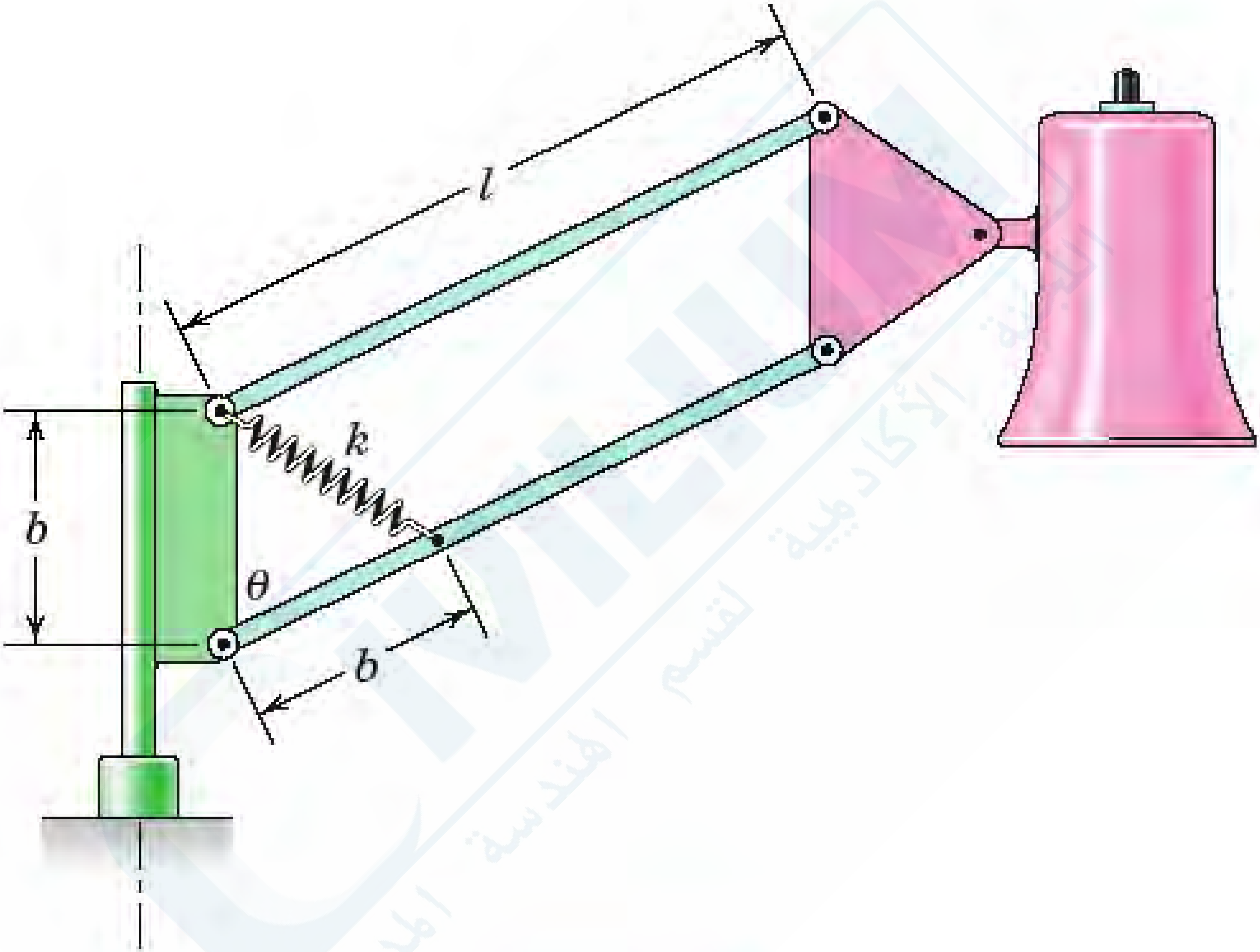
Problem 7/56

P_07_56



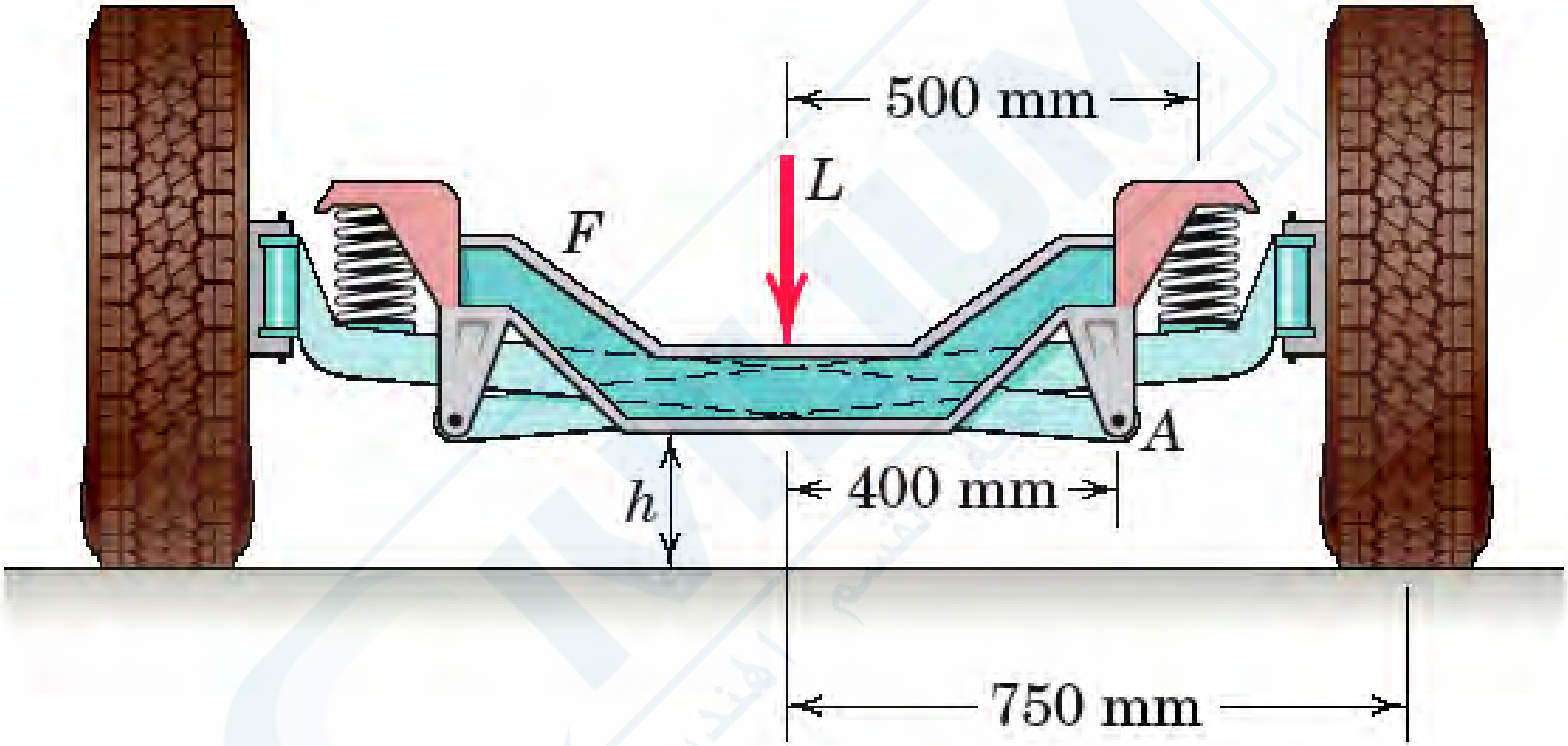
Problem 7/57

P_07_57



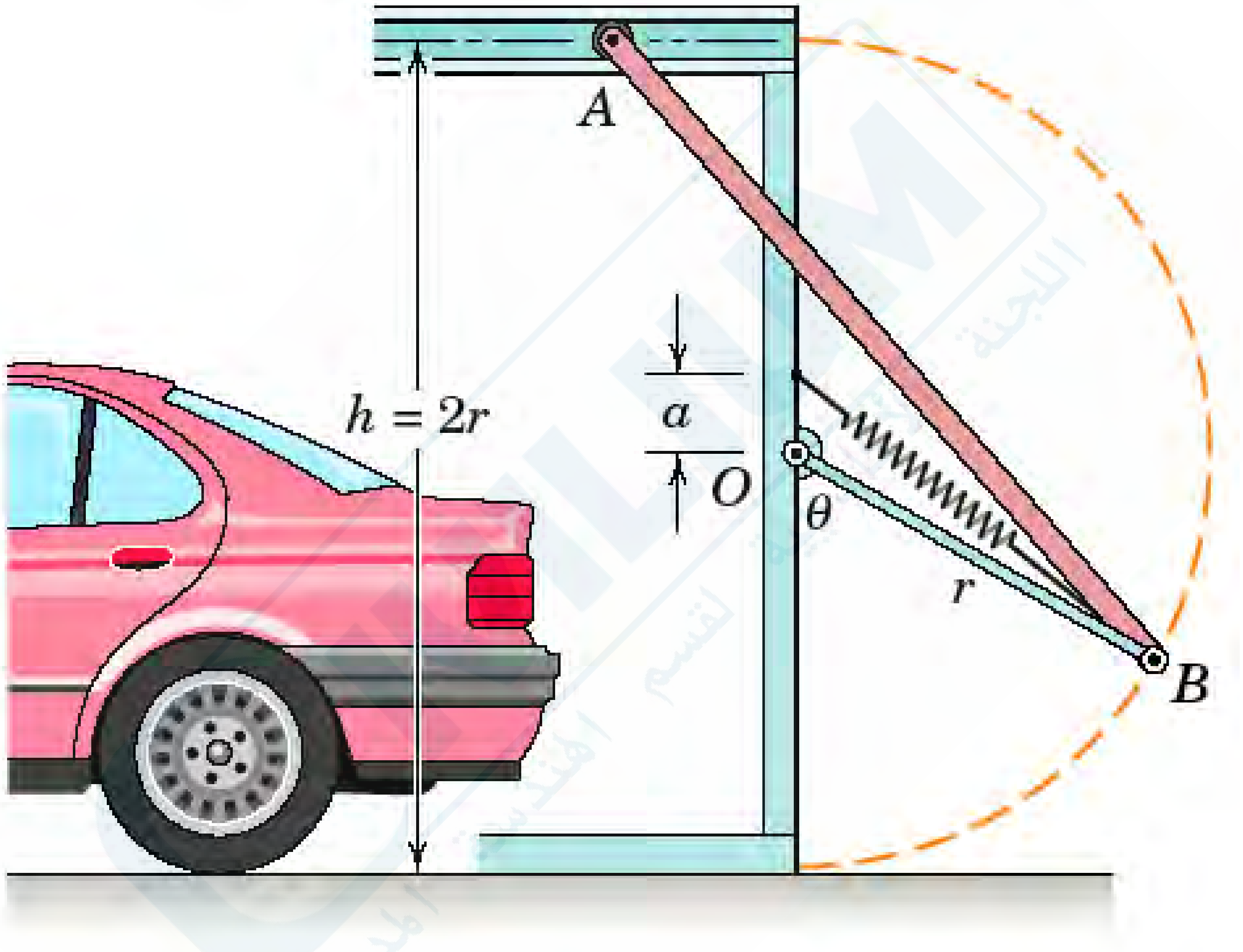
Problem 7/58

P_07_58



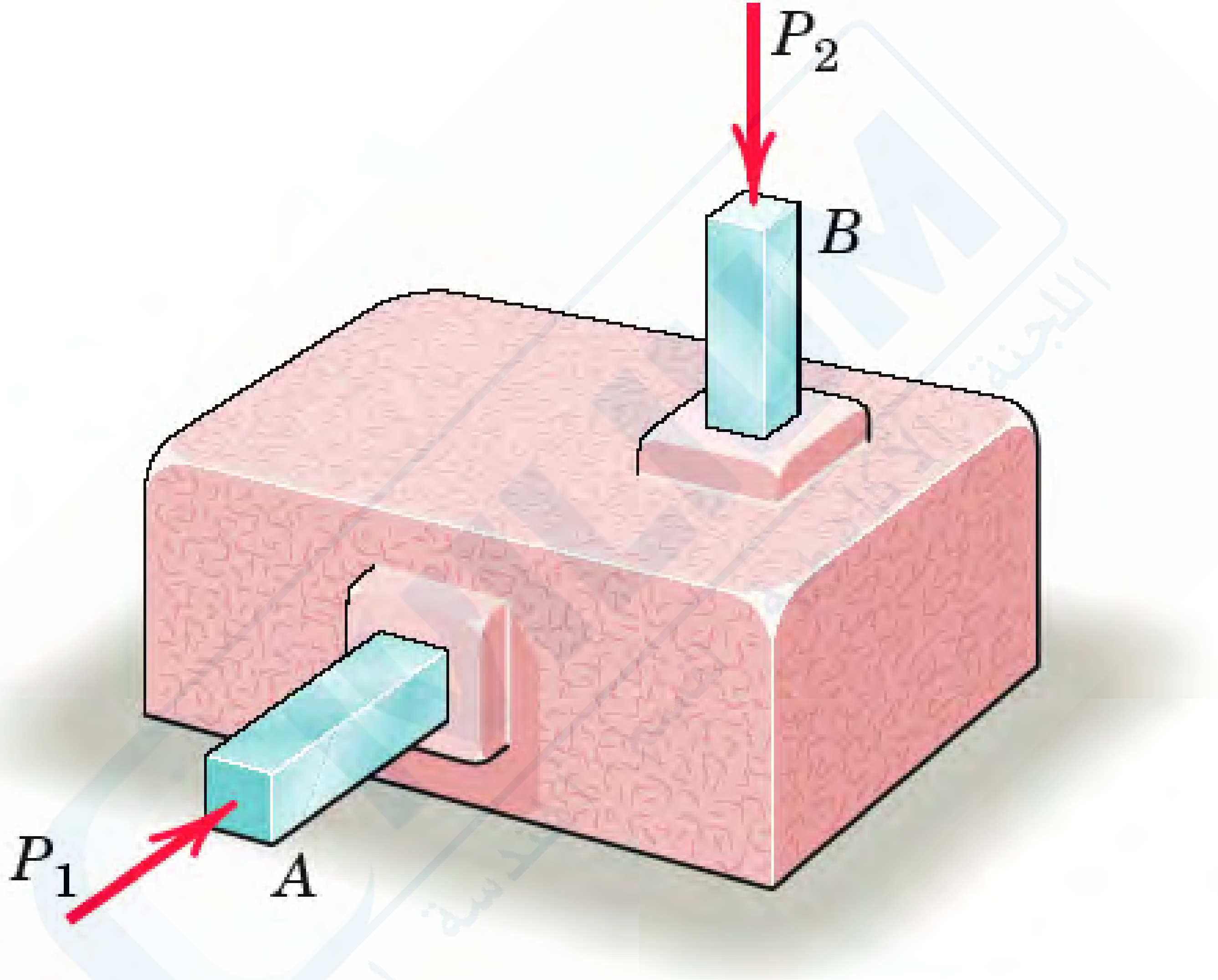
Problem 7/59

P_07_59



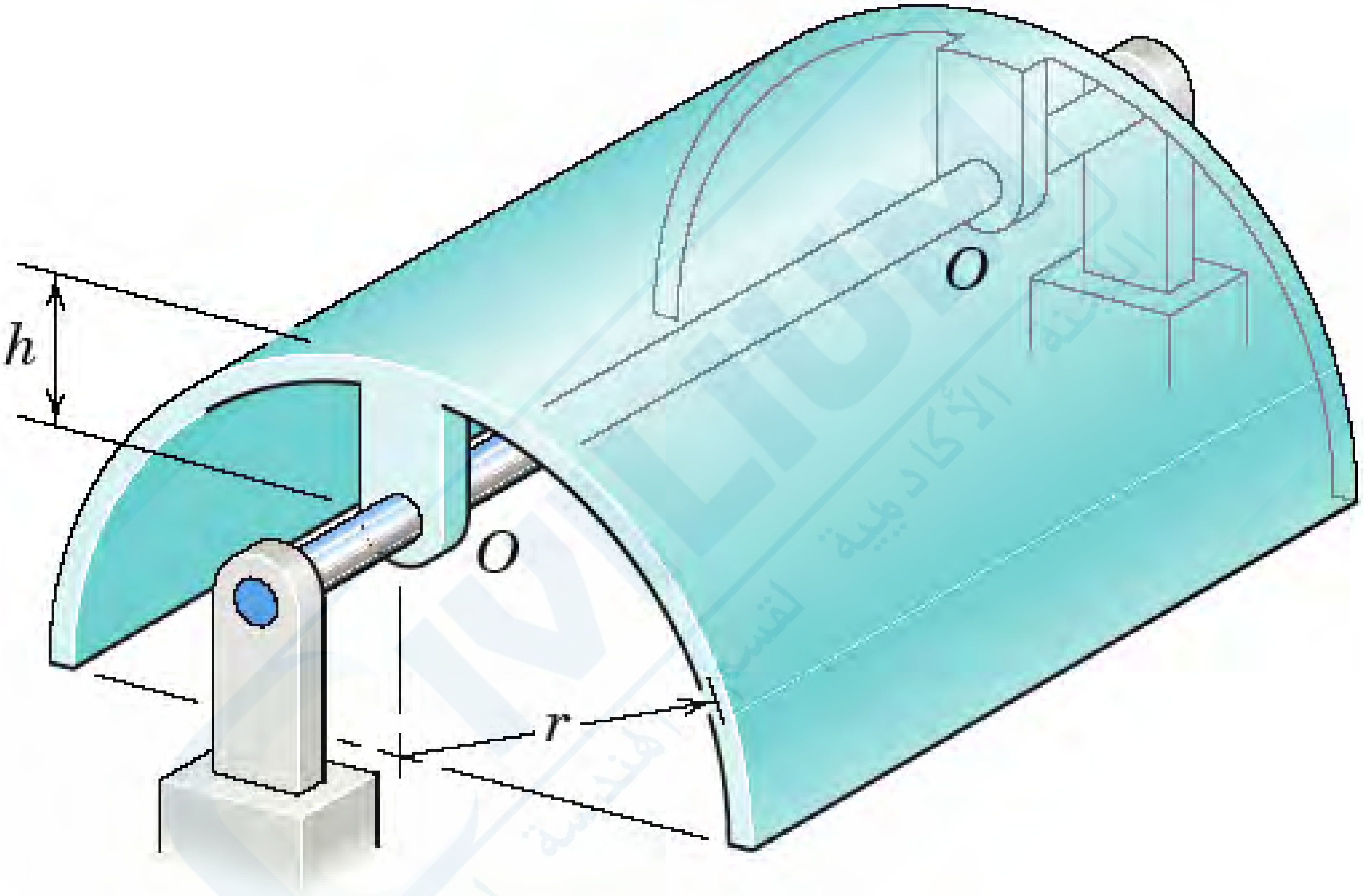
Problem 7/60

P_07_60



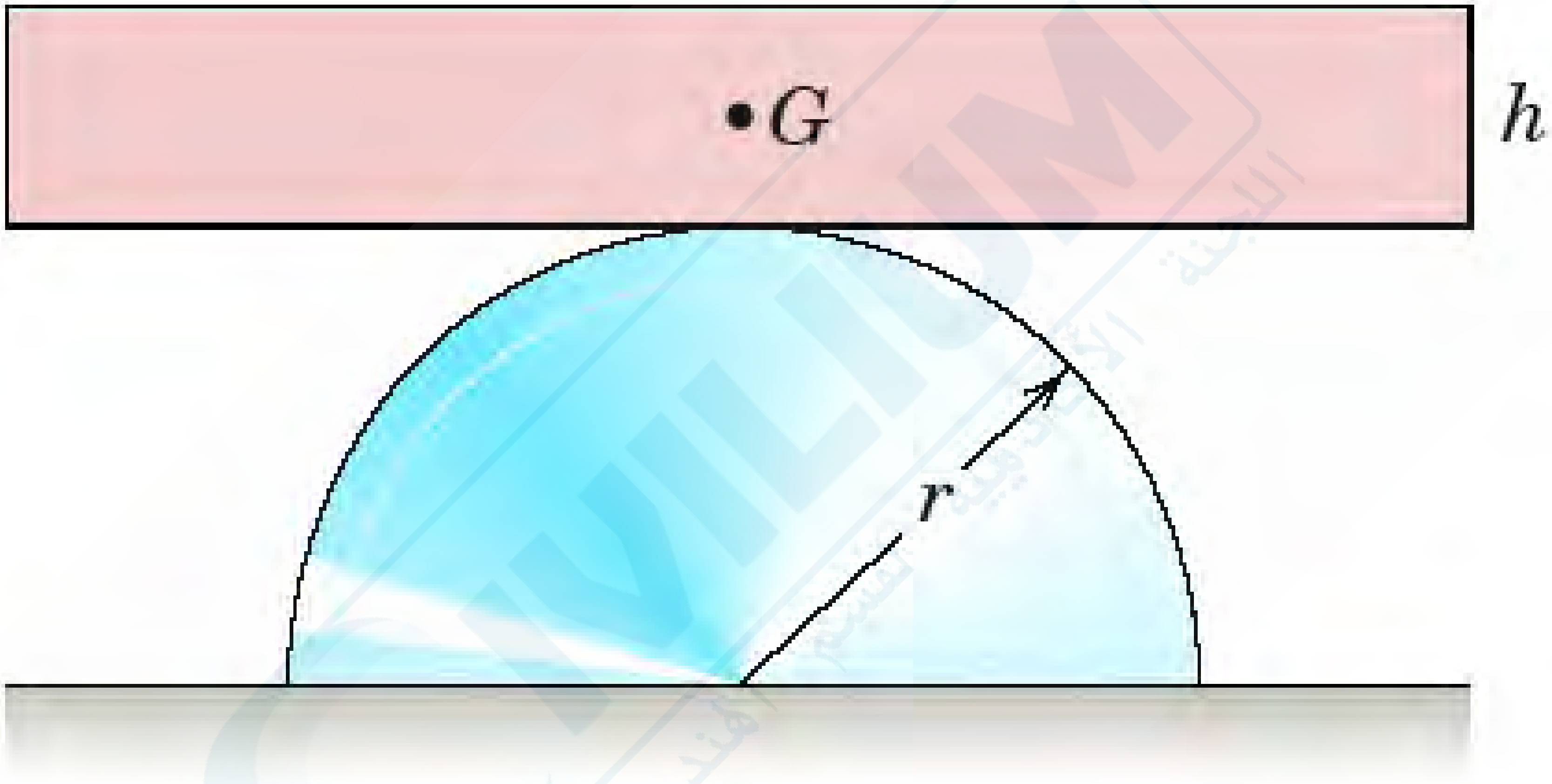
Problem 7/61

P_07_61



Problem 7/62

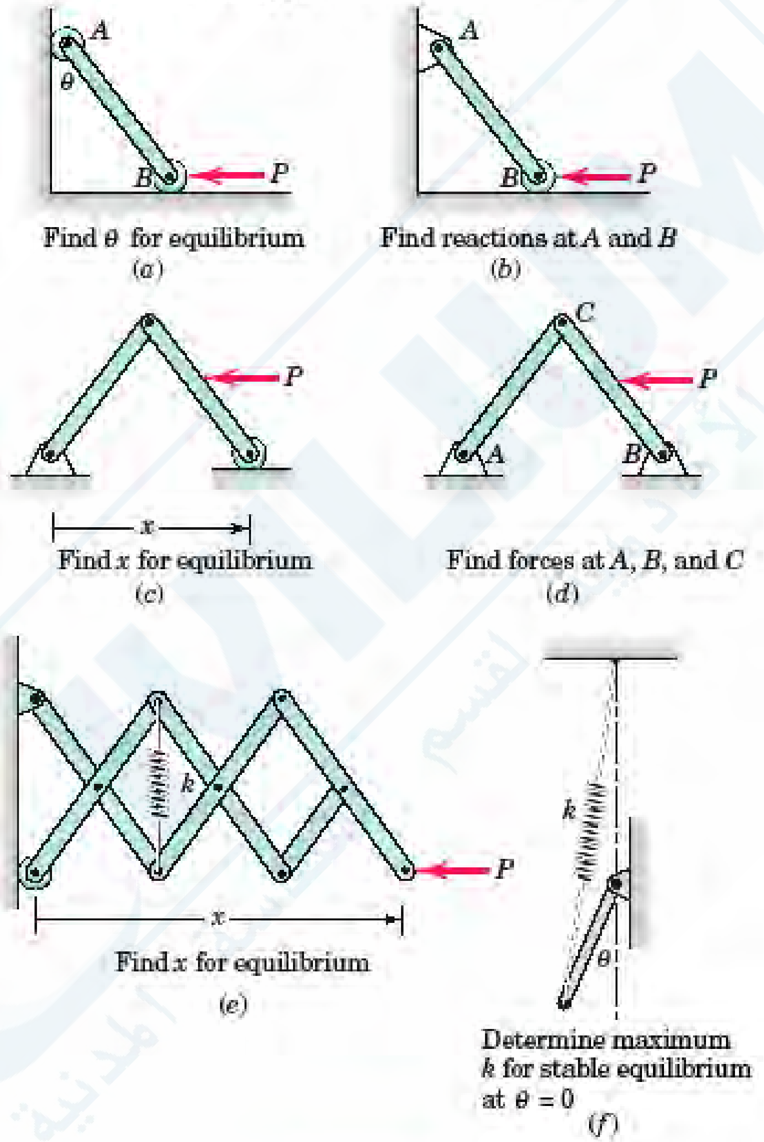
P_07_62



Problem 7/63

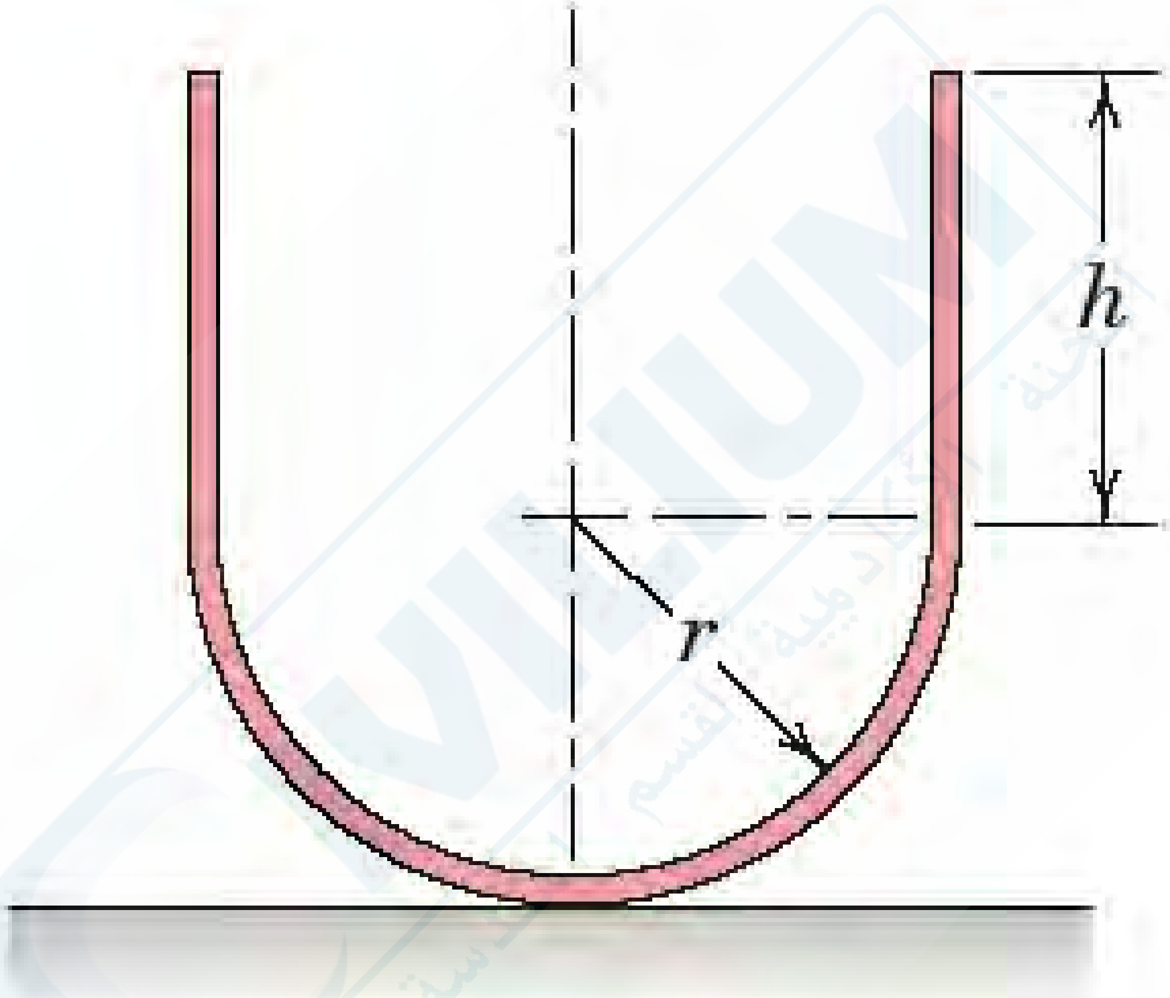
P_07_63

Plane of each figure is vertical. Size and mass of each member and applied force are known.

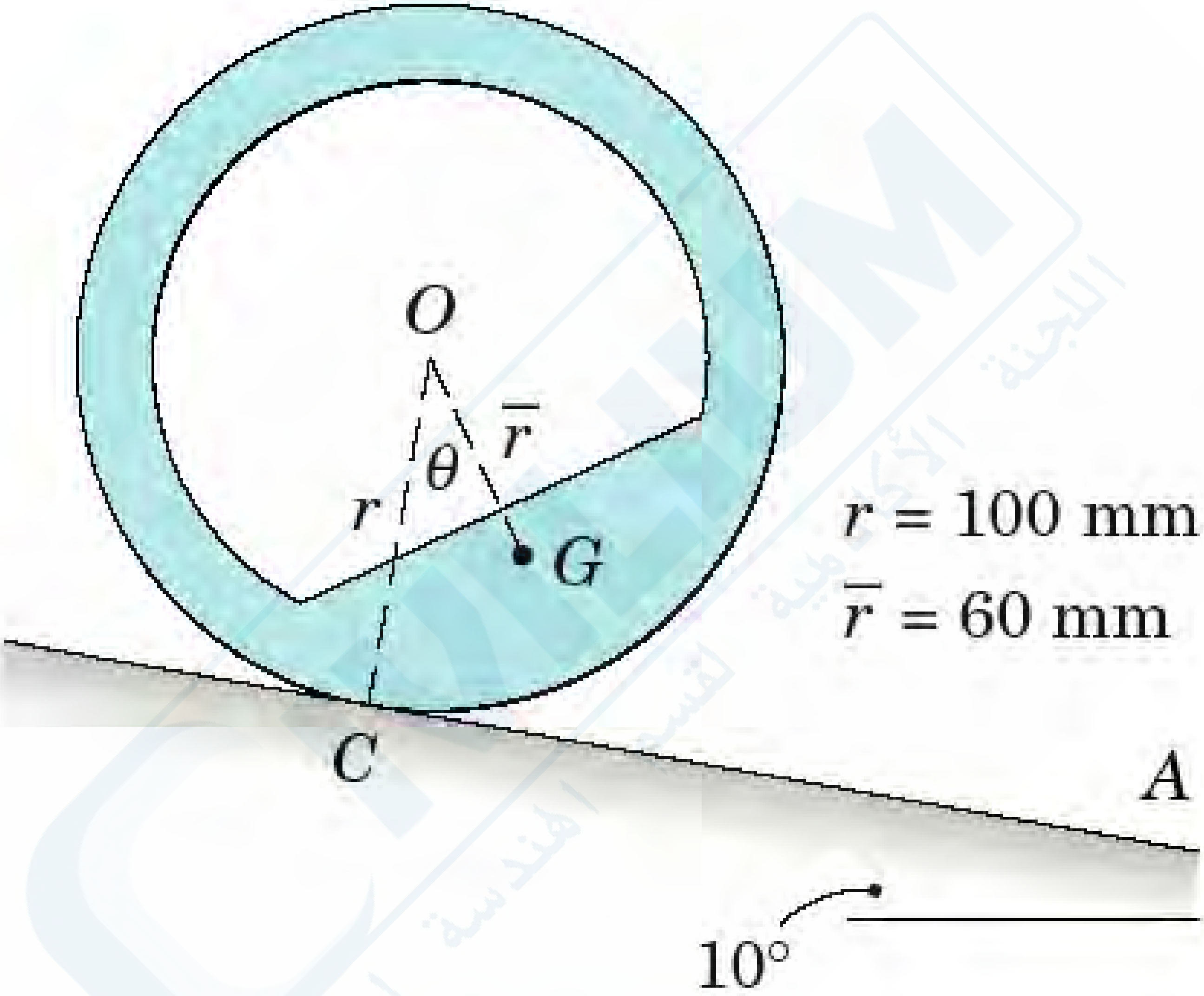


Problem 7/64

P_07_64

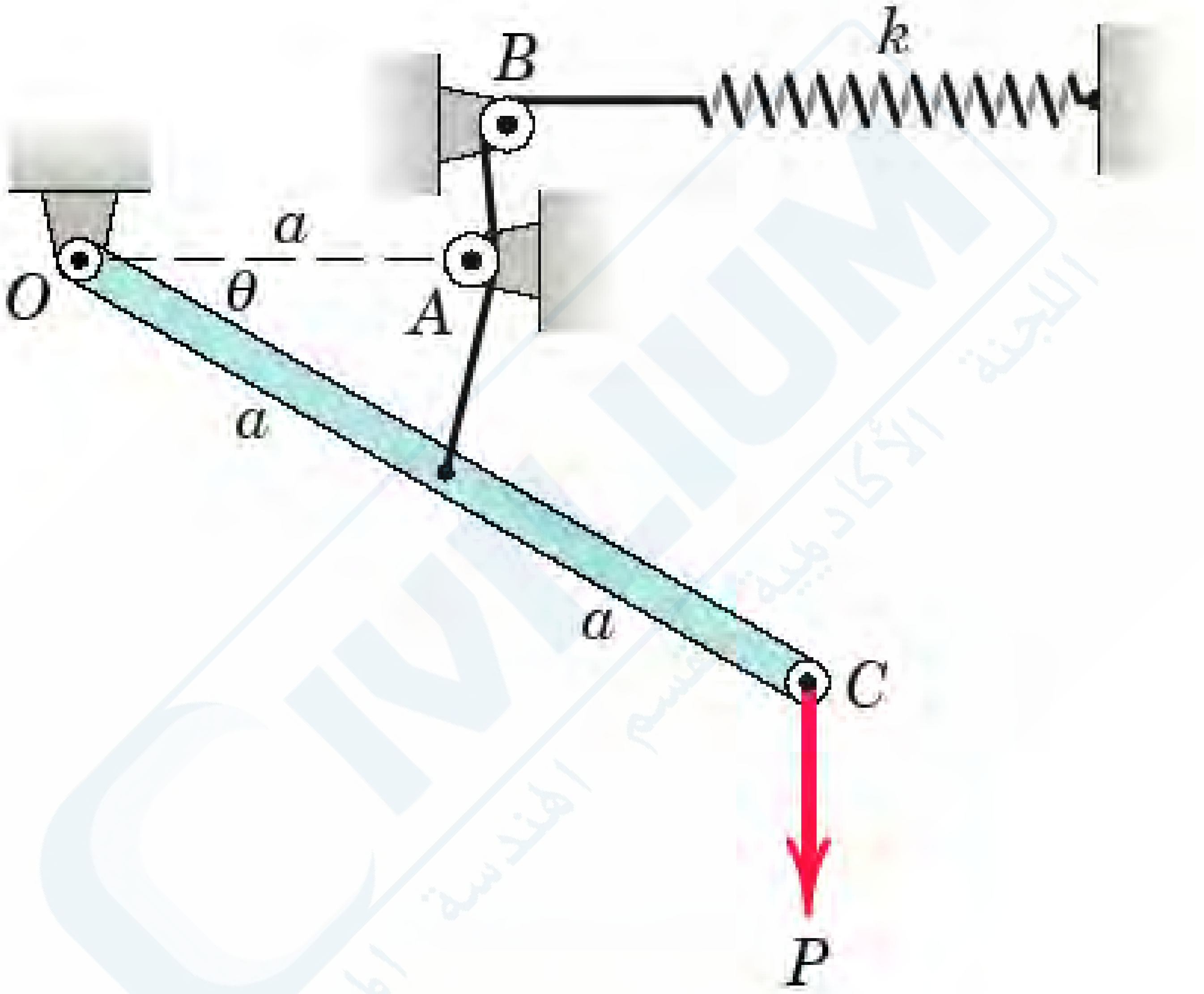


Problem 7/65



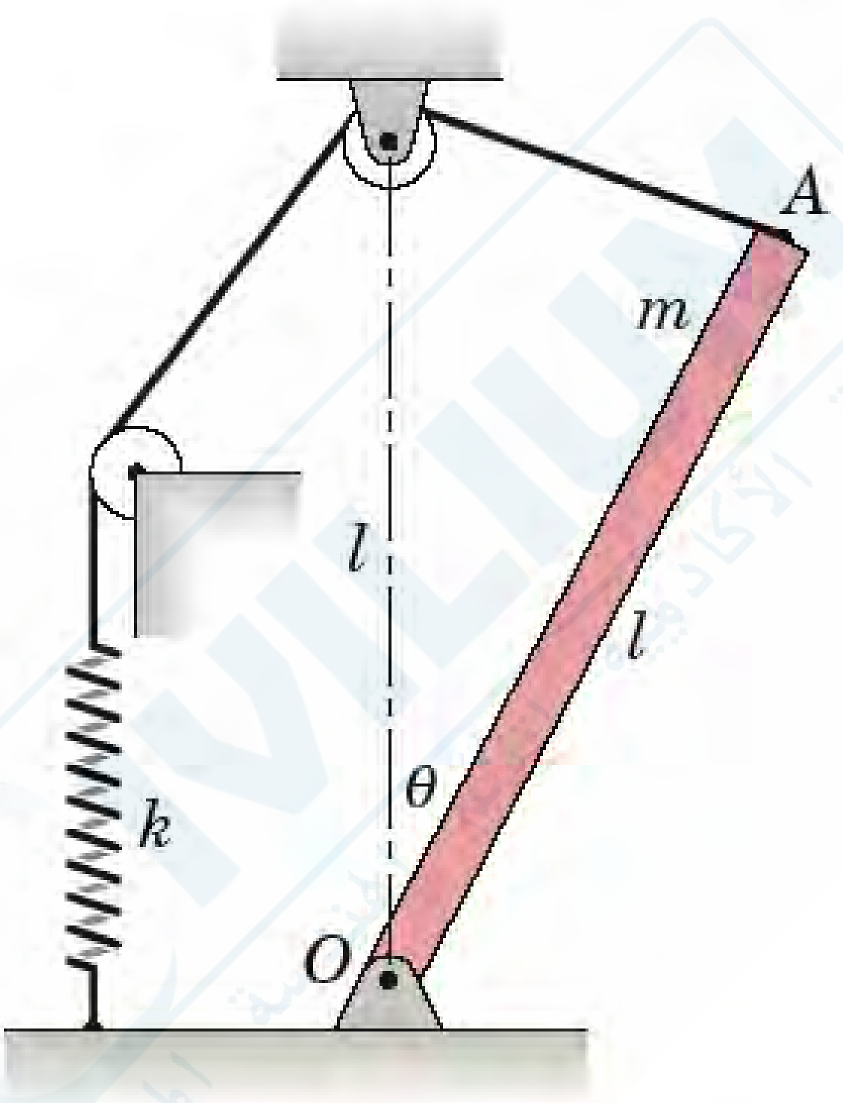
Problem 7/66

P_07_66



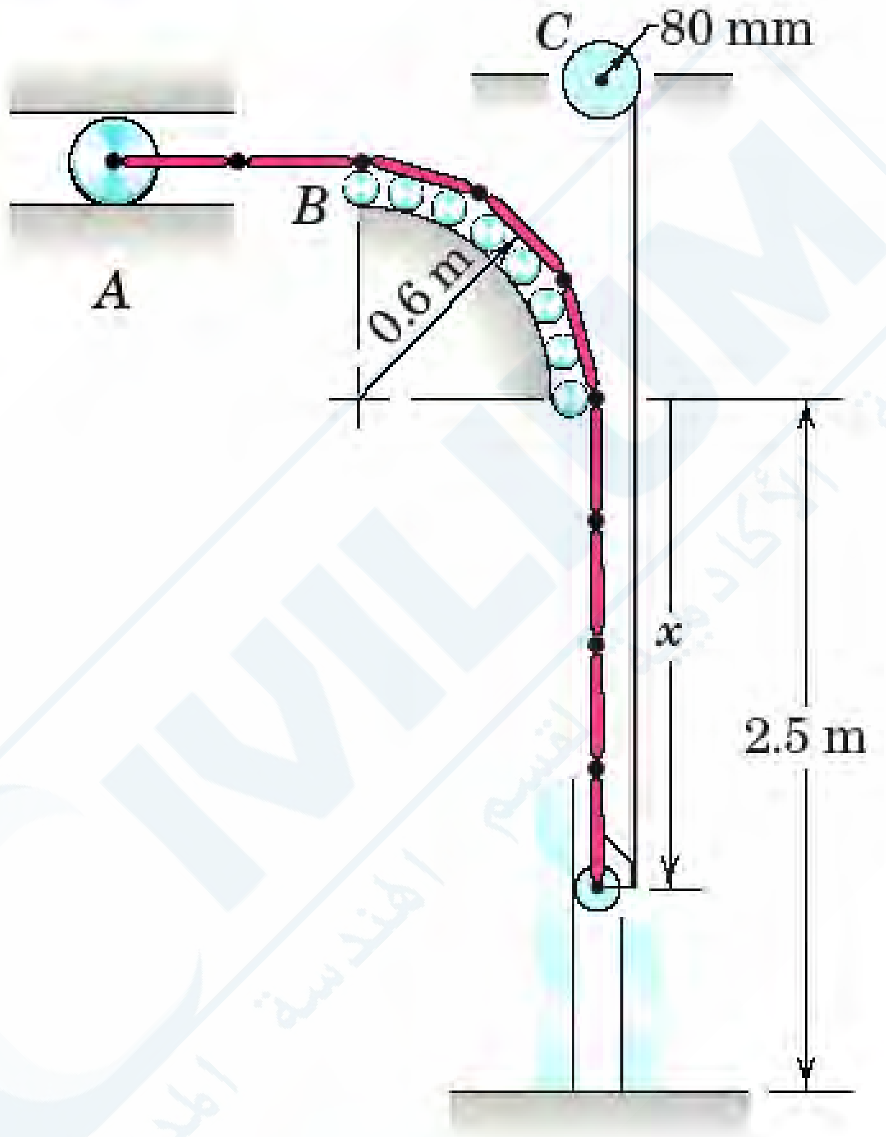
Problem 7/67

P_07_67



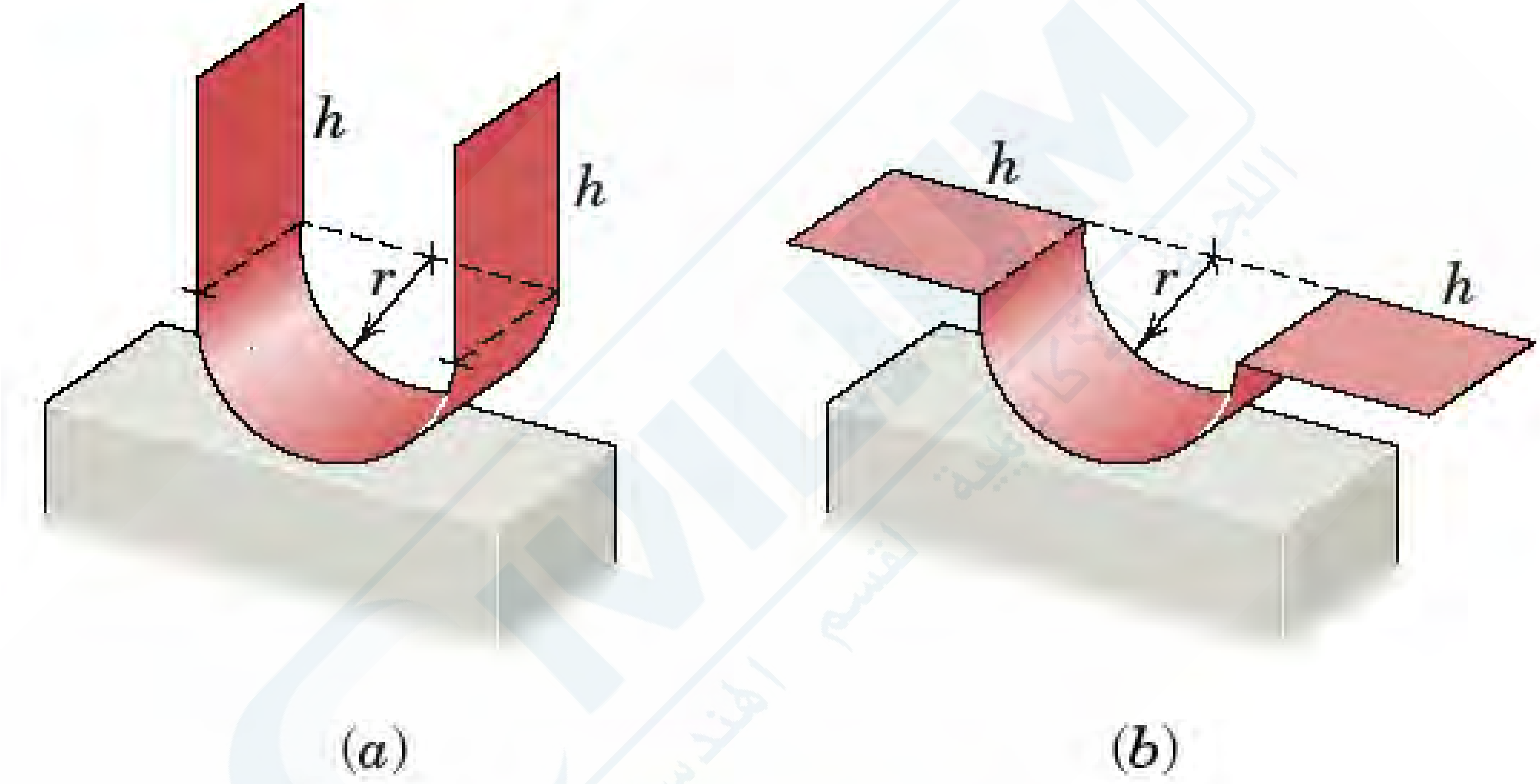
Problem 7/68

P_07_68



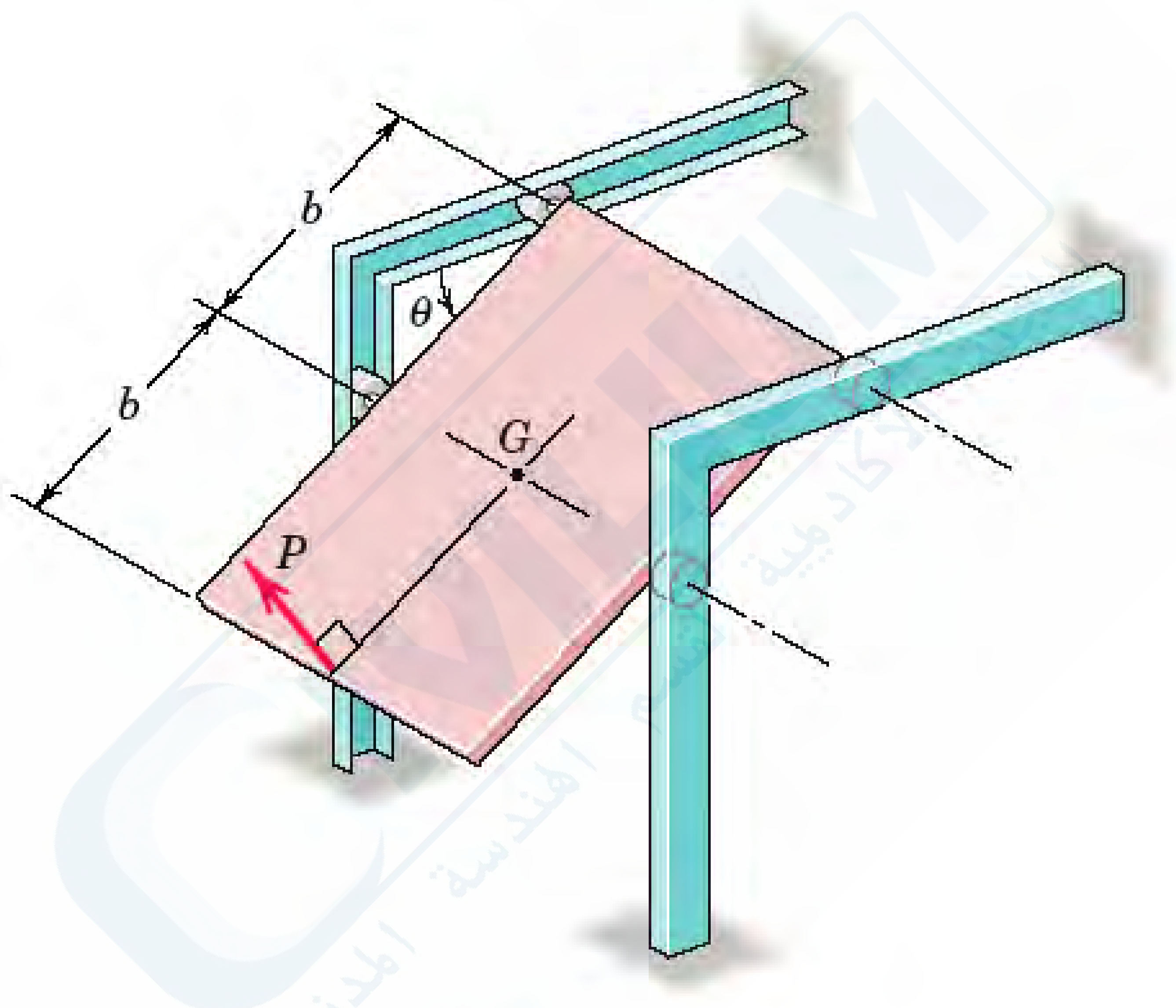
Problem 7/69

P_07_69



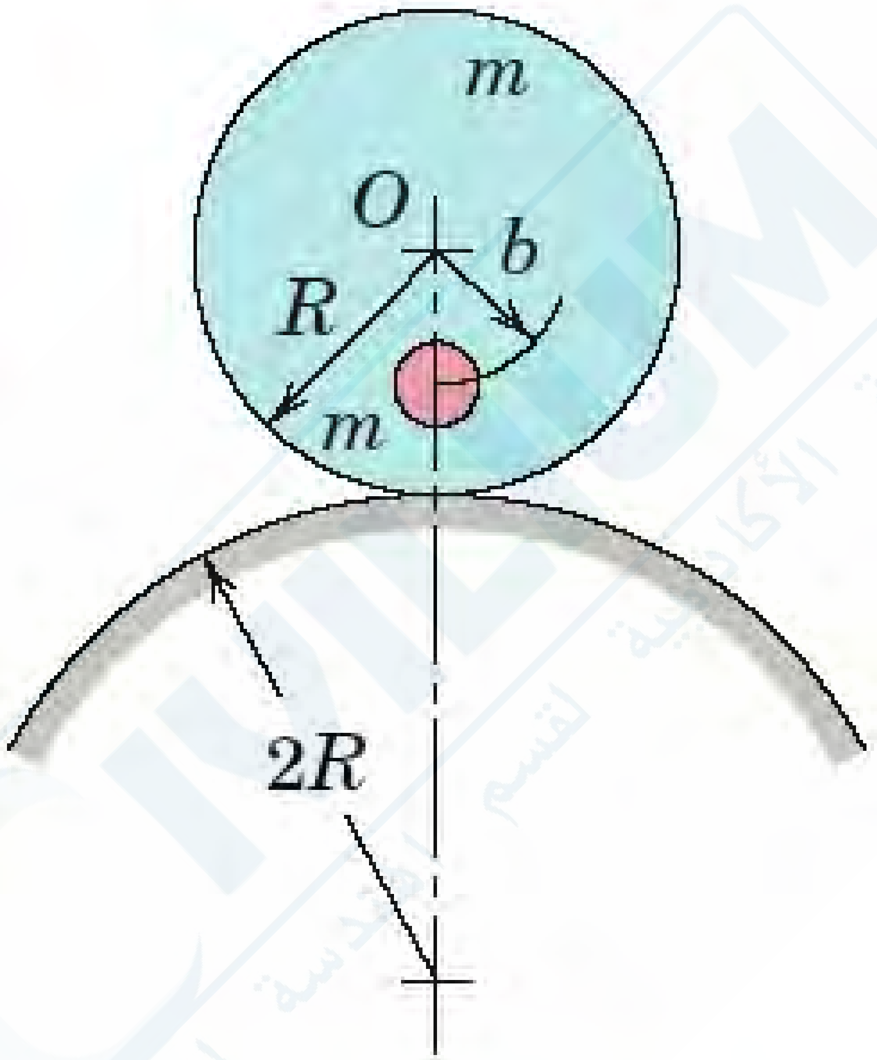
Problem 7/70

P_07_70



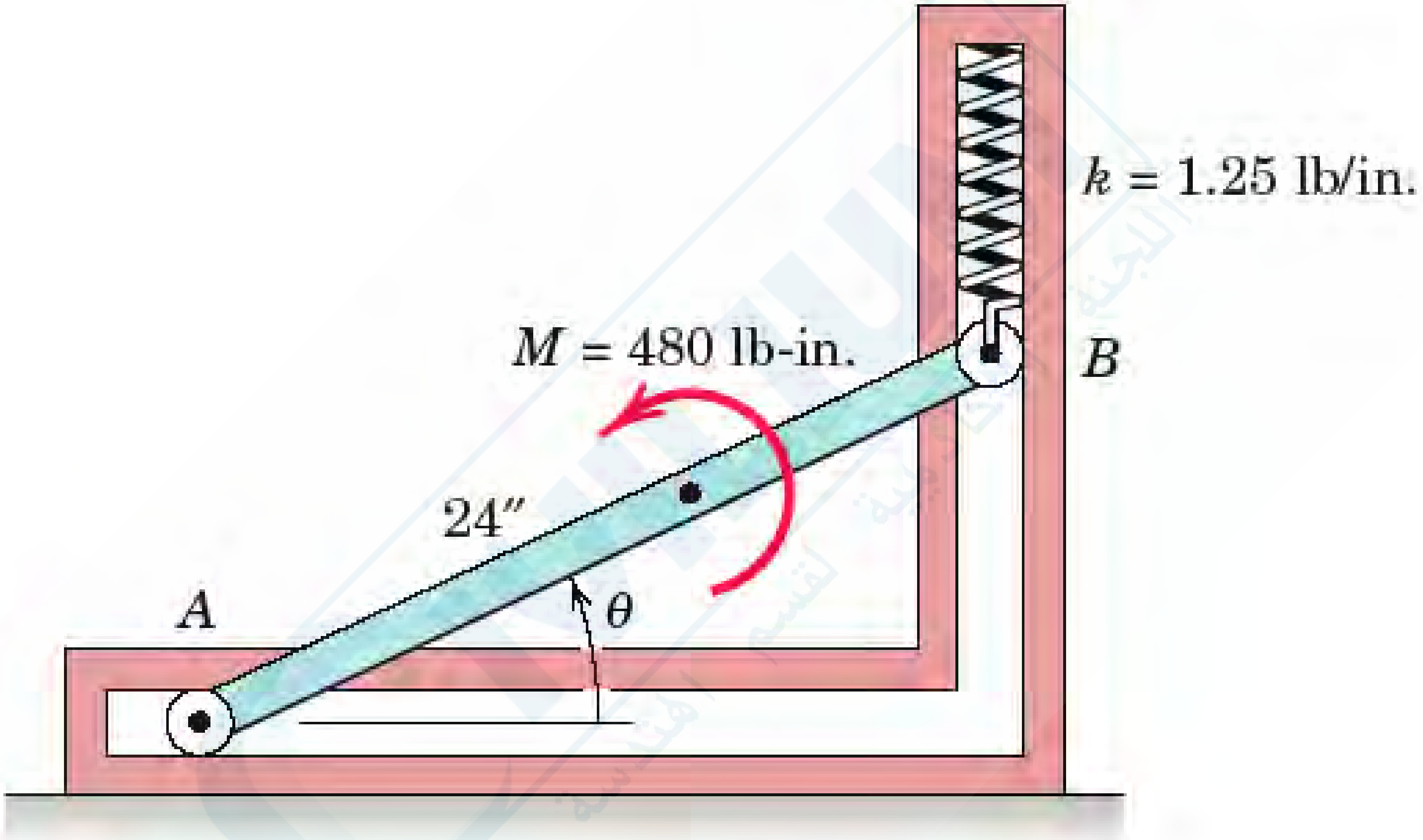
Problem 7/71

P_07_71



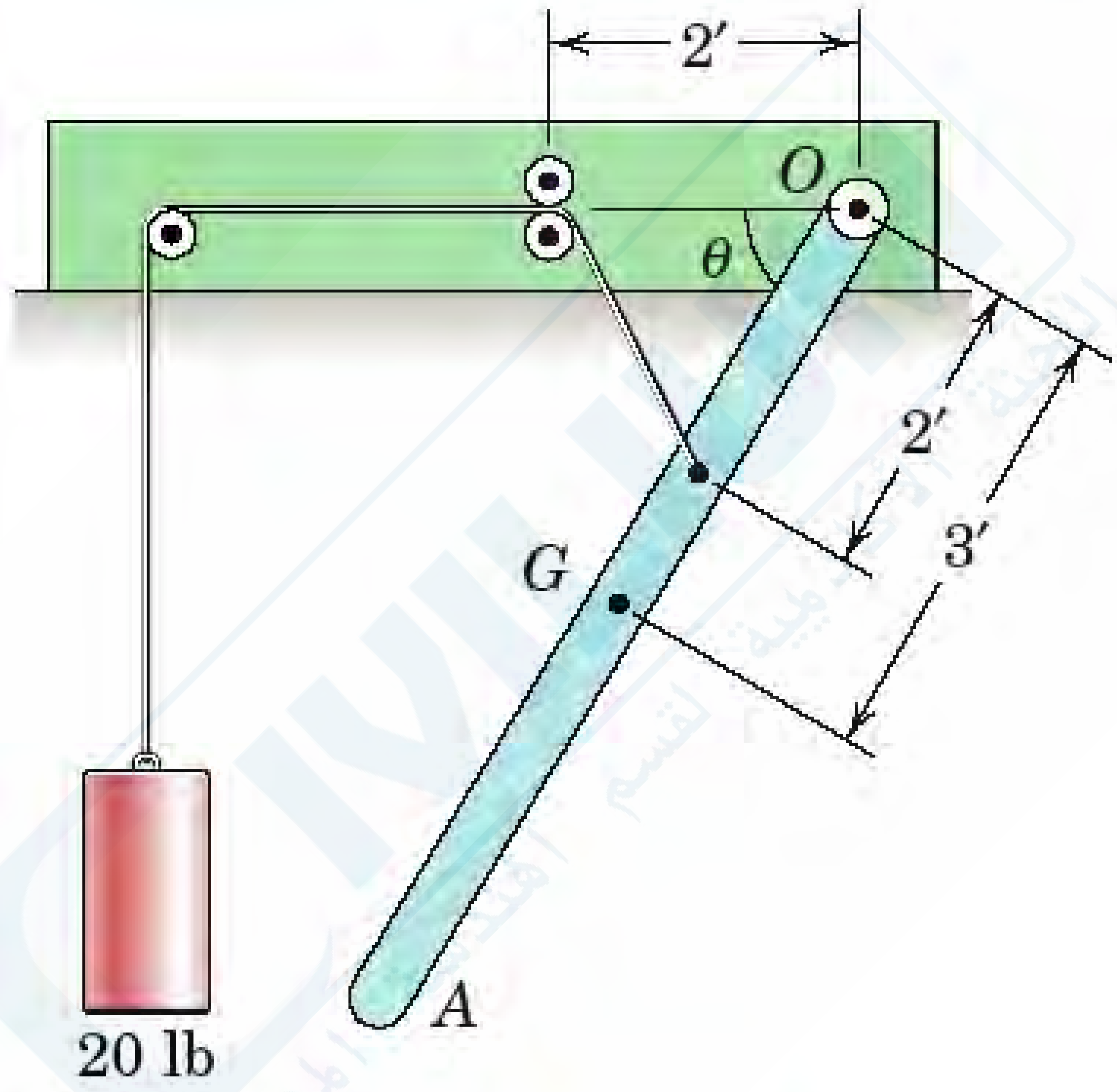
Problem 7/72

P_07_72



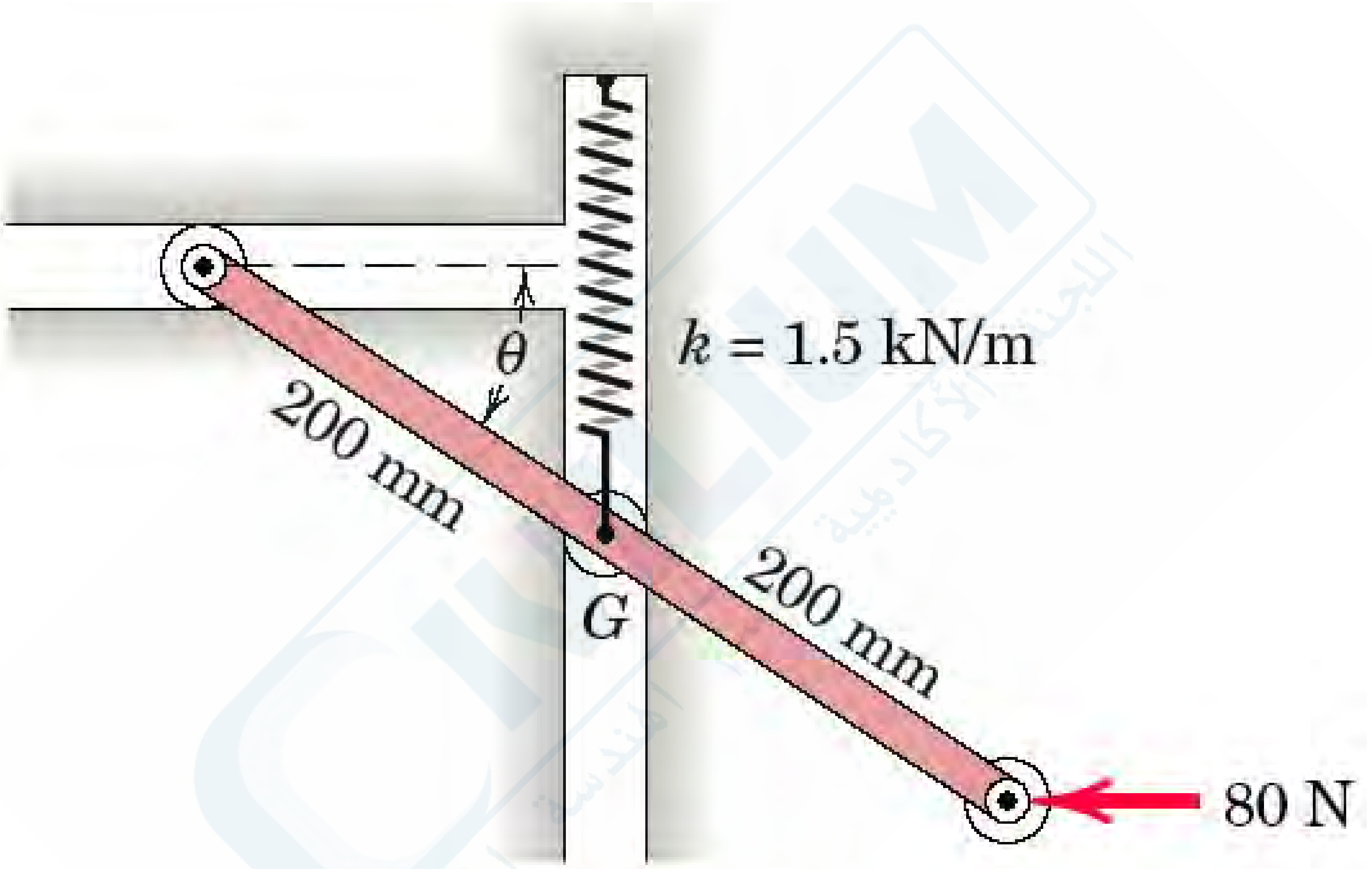
Problem 7/73

P_07_73



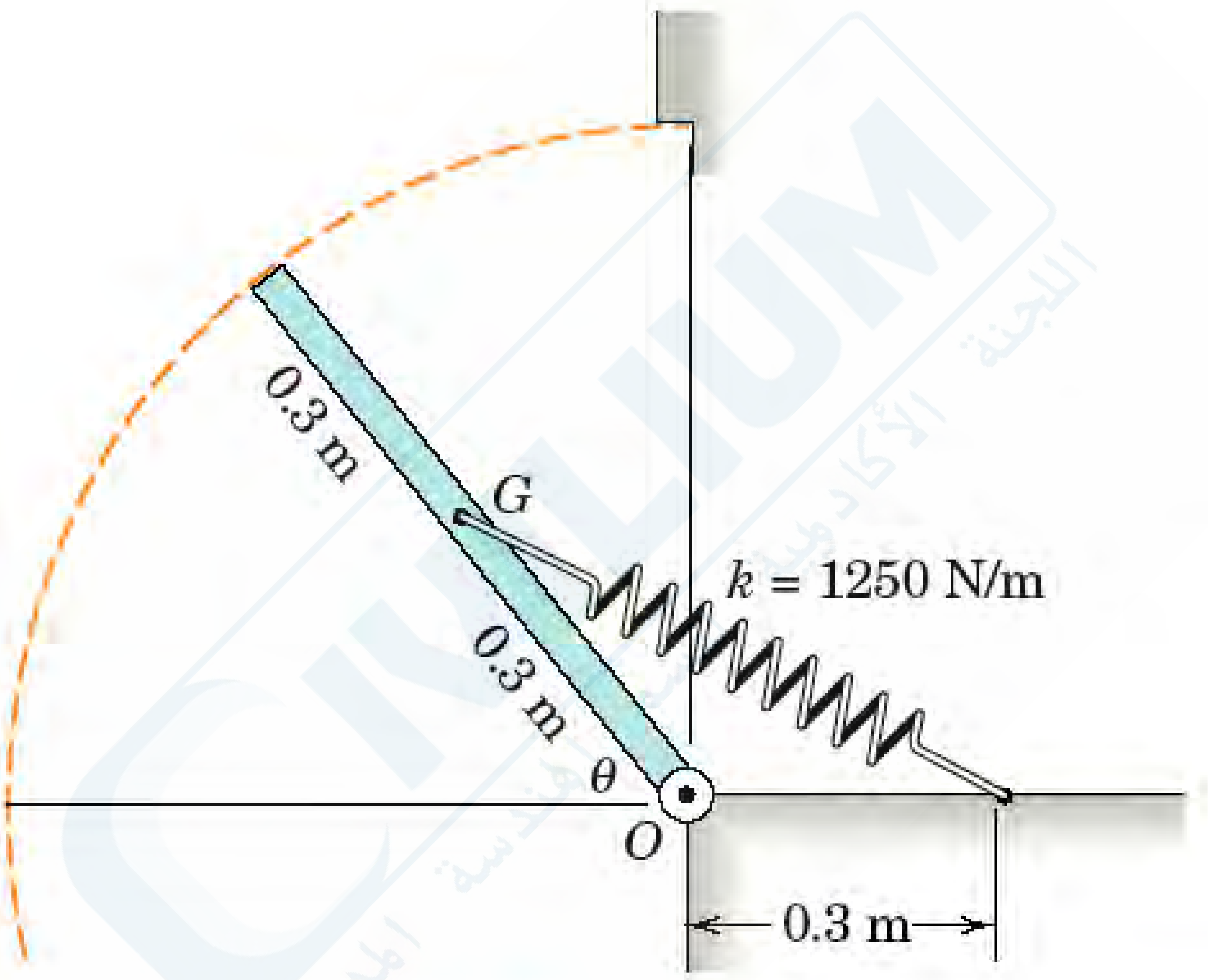
Problem 7/75

P_07_75



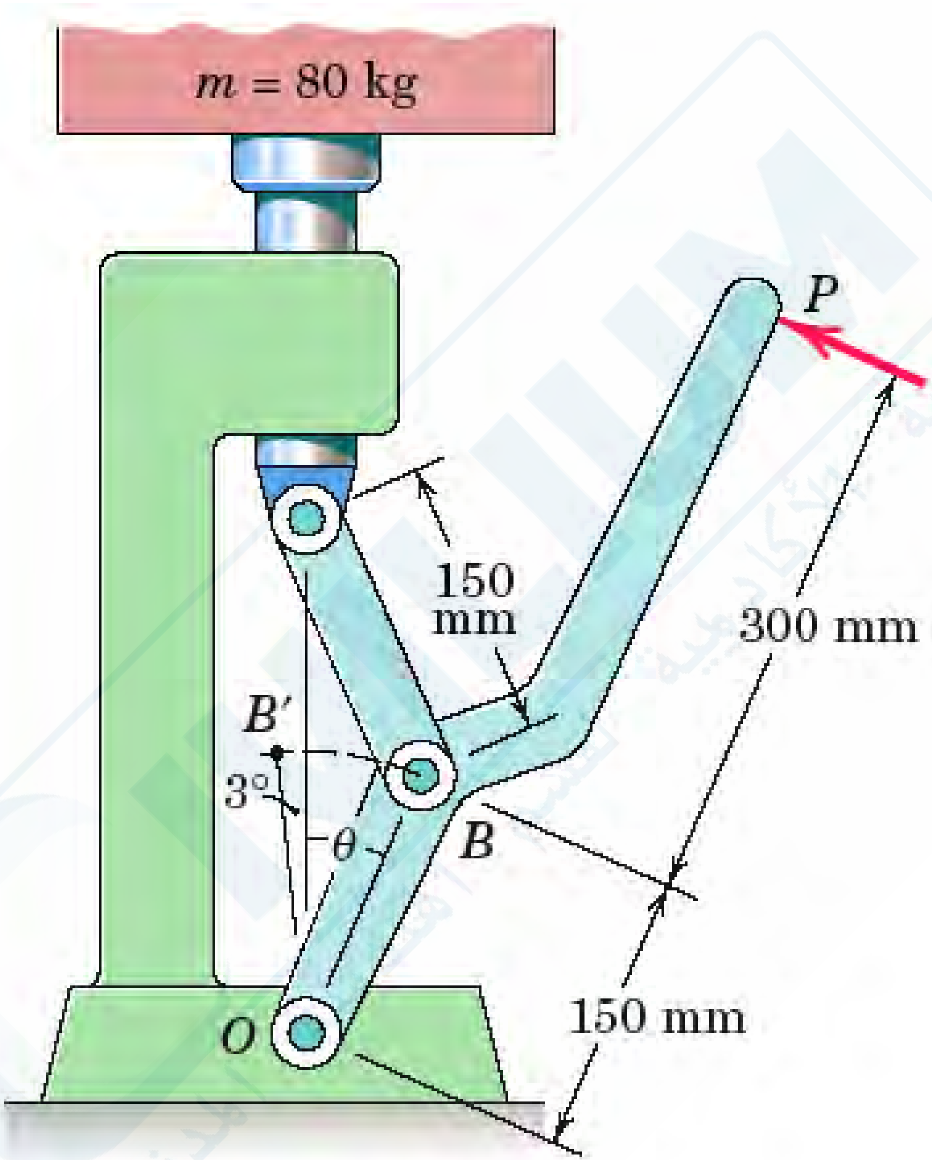
Problem 7/76

P_07_76



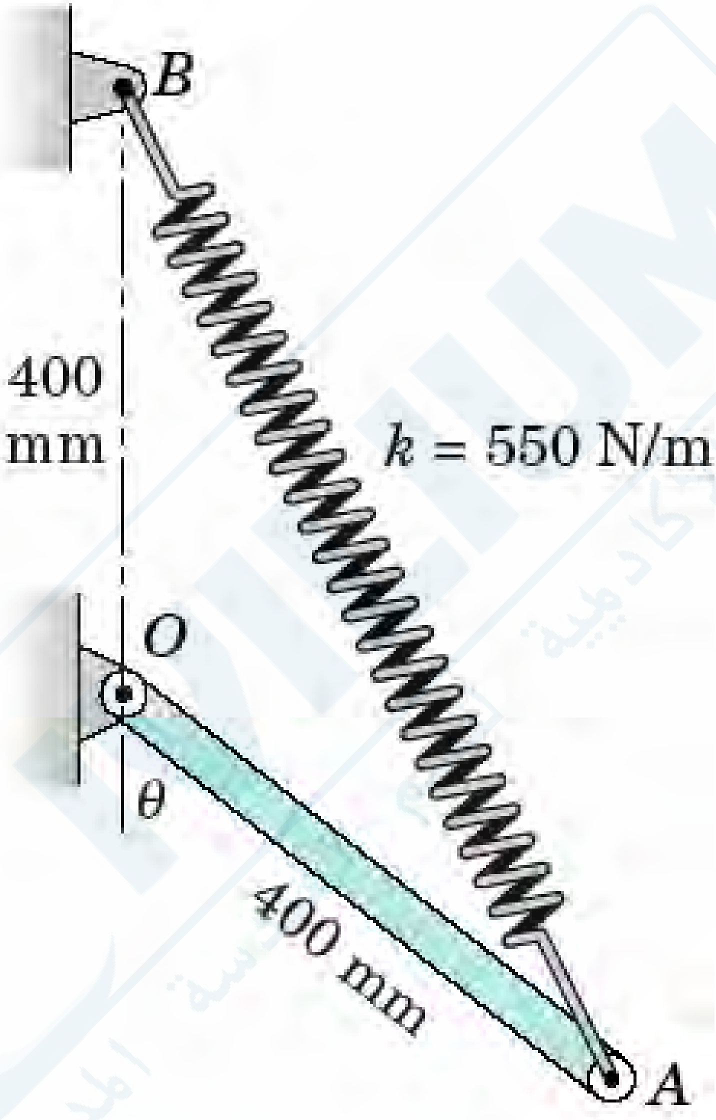
Problem 7/77

P_07_77



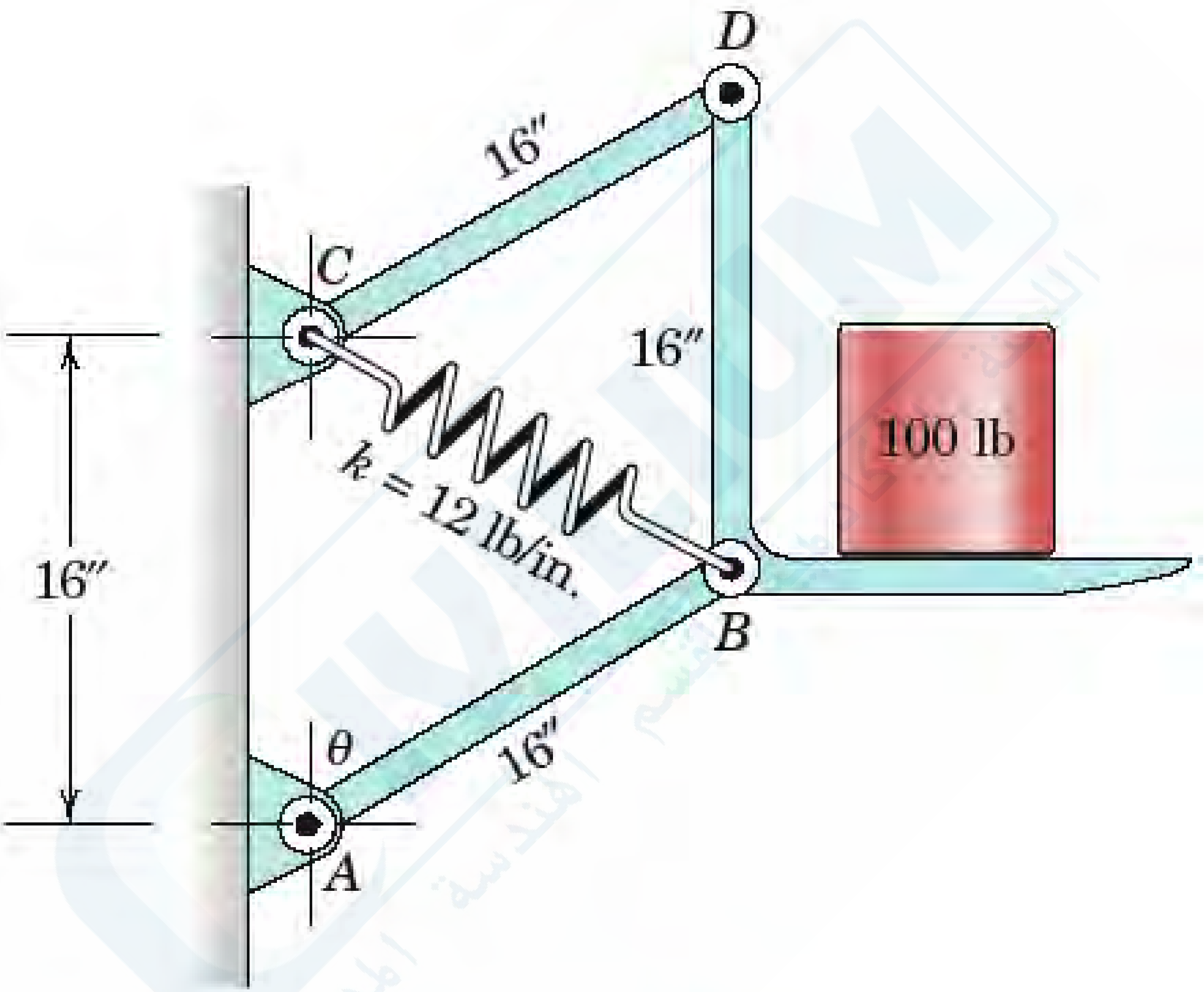
Problem 7/78

P_07_78



Problem 7/79

P_07_79



Problem 7/80

P_07_80